6.6 Symmetries of Regular Polygons

A Solidify Understanding Task

A line that reflects a figure onto itself is called a line of symmetry. A figure that can be carried onto itself by a rotation is said to have rotational symmetry. A diagonal of a polygon is any line segment that connects non-consecutive vertices of the polygon.

For each of the following regular polygons, describe the rotations and reflections that carry it onto itself: (be as specific as possible in your descriptions, such as specifying the angle of rotation)

1. An equilateral triangle

2. A square

3. A regular pentagon

4. A regular hexagon
5. A regular octagon

6. A regular nonagon

What patterns do you notice in terms of the number and characteristics of the lines of symmetry in a regular polygon?

What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon?
6.6 Symmetries of Regular Polygons – Teacher Notes

A Solidify Understanding Task

**Purpose:** In this task, students continue to focus on classes of geometric figures that can be carried onto themselves by a transformation—figures that possess a line of symmetry or rotational symmetry. Students solidify the idea of “symmetry” relative to finding lines that reflect a figure onto itself, or determining if a figure has rotational symmetry by finding a center of rotation about which a figure can be rotated onto itself. They also look for and describe the structure that determines if a figure possesses some type of symmetry. This work can be experimental (e.g., folding paper, using transparencies, using technology, measuring with ruler and protractor, etc.), or theoretical, with the definitions of reflection and rotation being called upon to support students’ claims that a figure possesses some type of symmetry.

The particular classes of geometric figures considered in this task are various types of regular polygons, and students will look for patterns in the types and numbers of lines of symmetry a regular polygon with an odd number of sides possesses, versus those with an even number of sides. They should also note a pattern between the smallest angle of rotation that carries a regular polygon onto itself and the number of sides of the polygon.

**Core Standards Focus:**

**G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

**G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure.

**Related Standards:** G.CO.4, G.CO.5
Standards for Mathematical Practice of focus in the task:

SMP 7 – Look for and make use of structure
SMP 6 – Attend to precision
SMP 8 – Look for and express regularity in repeated reasoning

Additional Resources for Teachers:
Images of each of the regular polygons have been provided on a master copy that can be reproduced and distributed to students. Students can cut out the individual polygons in order to manipulate them, or they can trace them on tracing paper in order to fold them along a diagonal or rotate them about a point.

The Teaching Cycle:
Launch (Whole Class):
Based on the level of student thinking that exists in your class, decide if you want the work of this task to be experimental (e.g., using tools, cut-outs and/or dynamic geometric software), or theoretical, making decisions based on the definitions of reflections and rotations. Introduce the task as being similar in nature to the previous task. If you have decided to provide experimental tools, outline what is available for students (note that a handout of the figures is provided at the end of the teacher notes, if needed.) You might also choose to just hand out the task and let students decide if they want to draw upon reasoning or tools to support their work. Or, you might want to press students to analyze the figures using reasoning based on the definitions of reflection and rotation.

Before having students start on the task, read the last two questions together, and point out that this is the goal of the task: to look for patterns in the number and characteristics of the lines of symmetry in a regular $n$-gon, and to look for patterns that describe the nature of the rotational symmetry in a regular $n$-gon. Encourage students to keep these goals in mind as they explore.

Explore (Small Group):
Listen for how students are determining the types of symmetry that exists for each regular polygon, and make sure they are identifying both types of symmetry—lines of symmetry and rotational
symmetry. For rotational symmetry, make sure they are identifying all possible angles of rotation. As you observe students work, you may want to suggest additional tools for exploration, or set tools aside if you feel students are capable of noting the symmetries by reasoning with the definitions of rotation and reflection and the properties of regular polygons.

It is important for students to examine the last two questions in small groups, before moving to a whole class discussion. Keep reminding students of these goals, even before they have examined all of the listed regular polygons. That way, students will be able to attend to the presentations during the discussion, even if they haven’t examined the complete set of polygons.

**Discuss (Whole Class):**
The discussion should focus on the last two questions, and students might draw upon specific examples of regular polygons to support their conjectures.

Ask students to state a conjecture as to the number of lines of symmetry in a regular $n$-gon, and to provide some justification of their conjecture. If students have not noticed any patterns in the number of lines of symmetry, make a table on the board consisting of “number of sides” as the input and “number of lines of symmetry” as the output. Ask what they may have noticed about the types of lines of symmetry in a regular polygon with an even number of sides versus an odd number of sides. What accounts for the differences in the ways they located the lines of symmetry?

Students should notice that in a regular polygon with an odd number of sides they can only draw lines of symmetry (or locate a crease line that folds the polygon onto itself) by using the lines that pass through a vertex point and the midpoint of the opposite side. Since such a line can be drawn through each vertex, a regular $n$-gon with an odd number of sides will possess $n$ lines of symmetry.

In a regular polygon with an even number of sides you can draw (or fold) a line of reflection through opposite vertices. Since only one line of symmetry exists for each pair of opposite vertices, there are $n/2$ such lines of symmetry. You can also draw lines of symmetry through the midpoints of opposite sides of the polygon—the sides that are parallel to each other. Since only one line of symmetry
exists for each pair of opposite sides, there are also \( n/2 \) such lines of symmetry. Consequently, a regular polygon with an even number of sides also has \( n/2 + n/2 = n \) lines of symmetry, but for different reasons than in the case of regular polygons with an odd number of sides. Make sure that the arguments for \( n \) lines of symmetry in any regular polygon are based on the structure of the geometric figures themselves, and not just on the pattern observed in the table.

Turn the focus of the discussion to the second question: What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon? Again it may be helpful to create a table with the input representing “number of sides” and output representing “the smallest angle of rotation”. Point out that every regular polygon can be rotated onto itself by rotating \( 360^\circ \) about the point of intersection of the diagonals of the polygon. How might the smallest angle of rotation be related to this \( 360^\circ \) rotation? You might draw the line segments between a pair of consecutive vertices and the center of rotation and ask, “What is the measure of this angle, and how do you know?” Students should notice that the smallest angle of rotation in a regular \( n \)-gon is \( 360^\circ/n \) and they should be able to justify why this is so. They should also note that any whole-number multiple of this smallest angle of rotation is also an angle of rotation for the polygon.

**Aligned Ready, Set, Go: Transformations and Symmetry 6.6**
Topic: Rotational symmetry, connected to fractions of a turn and degrees.

1. What fraction of a turn does the wagon wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?

2. What fraction of a turn does the propeller below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?

3. What fraction of a turn does the model of a Ferris wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?
SET
Topic: Finding angles of rotational symmetry for regular polygons, lines of symmetry and diagonals

4. Draw the lines of symmetry for each regular polygon, fill in the table including an expression for the number of lines of symmetry in a \( n \)-sided polygon.

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<tr>
<th>Number of Sides</th>
<th>Number of lines of symmetry</th>
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5. Draw all of the diagonals in each regular polygon. Fill in the table and find a pattern, is it linear, exponential or neither? How do you know? Attempt to find an expression for the number of diagonals in a \( n \)-sided polygon.

<table>
<thead>
<tr>
<th>Number of Sides</th>
<th>Number of diagonals</th>
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<tbody>
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6. Find the angle(s) of rotation that will carry the 12 sided polygon below onto itself.

7. What are the angles of rotation for a 20-gon? How many lines of symmetry (lines of reflection) will it have?

8. What are the angles of rotation for a 15-gon? How many line of symmetry (lines of reflection) will it have?

9. How many sides does a regular polygon have that has an angle of rotation equal to $180^\circ$? Explain.

10. How many sides does a regular polygon have that has an angle of rotation equal to $20^\circ$? How many lines of symmetry will it have?
GO

Topic: Reflecting and rotating points on the coordinate plane.
(The coordinate grid, compass, ruler and other tools may be helpful in doing this work.)

9. Reflect point $A$ over the line of reflection and label the image $A'$.

10. Reflect point $A$ over the line of reflection and label the image $A'$.

11. Reflect triangle $ABC$ over the line of reflection and label the image $A'B'C'$.

12. Reflect parallelogram $ABCD$ over the line of reflection and label the image $A'B'C'D'$.

13. Given triangle $XYZ$ and its image $X'Y'Z'$ draw the line of reflection that was used.

14. Given parallelogram $QRST$ and its image $Q'R'S'T'$ draw the line of reflection that was used.