Getting It Right!

Summary

The Pythagorean Theorem is one of the most useful relations in mathematics. In the middle grades, an investigation of the lengths of the sides of right triangles and the area of squares drawn on those sides introduces students to irrational numbers, Pythagorean triples (derived from right triangles with integer sides), and methods of indirect measurement used for solving real-life problems.

Time Frame

4 class periods of 45 minutes each

Group Size

Small Groups

Materials

Software

Spreadsheet, The Geometer's Sketchpad (Key Curriculum Press) or Cabri Geometry (Texas Instruments)

Book

Bennett, D. (1995). *Pythagoras plugged in: Proofs and problems for The Geometer's Sketchpad*. Berkeley, CA: Key Curriculum Press.

Other

Trundle wheel (for large measurements), tape measures, cardboard, straight-edge ruler, scissors, paper, pencil

Background for Teachers

Students construct a variety of right triangles using a right-angled set square, cutting corners from pieces of paper or cardboard, or using dynamic geometry software. They measure the sides of these various right triangles and record measurements in a spreadsheet. Students use the spreadsheet to look for possible patterns in the measurements. They also use the spreadsheet to square the values of each measurement and look for possible relations among squared values.

Once the Pythagorean relation has been established, students generate visual proofs using duplicate cutouts of right triangles and the dynamic software. They search the Web for information on Pythagoras and many different visual proofs. They investigate the possible generalization of the theorem to other similar shapes drawn on the sides of right triangles using dynamic geometry software. As a culminating activity, students use the Pythagorean relation to find an estimate for the diagonal distance between two points on opposite sides of their school building.

Instructional Procedures

Working in groups of three or four, students create a variety of right-angled triangles by cutting corners from rectangular sheets of paper or cardboard using a straight edge. Each group measures the three sides of their triangles and enters the measurements into a spreadsheet. Students investigate relations between the long side of each triangle and the two shorter sides. Introduce the terms hypotenuse (long side) and legs (shorter sides).

Groups share with the whole class the patterns or relations they have found. Most likely, someone in the group suggested squaring the measures and summing the squares of the legs. If not, then suggest this as an exploration.

Use The Geometer's Sketchpad (Key Curriculum Press) or other dynamic geometry software to

extend the investigation and form generalizations. Students create a script for constructing right triangles, take measurements using Sketchpad, square these measurements, sum the squared legs, and see how this sum compares to the squared hypotenuse as they dynamically change the side measurements of their right triangles. Using a script for constructing squares, students construct a square on each side of their right triangle, measure the areas of these squares, and investigate relations among the areas. Students should also construct squares on the sides of non right triangles to see if the relationships hold for any triangle.

As an experiment, construct other polygonal figures on the sides of the right triangle. Students create scripts for various polygons (e.g., equilateral triangles and pentagons). With the constraint that the polygons on each side must be similar, have students measure the areas and investigate relations among these areas. The goal is to have students make their own conjectures about these relationships. They can also construct semicircles on the sides of the triangle and investigate the areas of the three semicircles.

Building from students' conjectures about the relationships of the three areas, introduce the Pythagorean Theorem - if students haven't already mentioned it themselves! Have students research Pythagoras and his contributions to mathematics. Hundreds of Web sites examine Pythagoras. Students can use these to find information.

Construct visual proofs of the Pythagorean Theorem using cardboard cutouts. Have groups find at least four different visual proofs they've learned from the Web sites. Students should demonstrate these proofs to the rest of the class using their own reasoning from the visual demonstrations.

Using Sketchpad, investigate the dynamic proofs illustrated in Pythagoras Plugged In (Bennett, 1995). Have groups brainstorm their own dynamic proofs of Pythagoras's Theorem using Sketchpad.

Use the Pythagorean Theorem to determine the distance between two points in a rectangular coordinate system. If the school has been plotted on a grid system, use the school map. To apply the Pythagorean Theorem, have groups calculate an estimate for the distance between two points on opposite sides of the school building. Using trundle wheels or tape measures, students can measure the legs of a right triangle that connect the two points indirectly by going around the school building; from these measurements, they can calculate the straight-line distance between the two points.

Using a spreadsheet, investigate integer values for the three sides of a right triangle (these are called Pythagorean Triples).

Extensions

The most powerful part of this learning activity, using The Geometer's Sketchpad or any other dynamic geometry software, lies in encouraging students to ask "What if?" questions. The software enables students to test their theories on their own. This exploration of conjectures and developing informal logical arguments encourages students to develop habits that are mathematically powerful. In addition, it is the classroom conversation or discourse about these conjectures and informal proofs that show their understanding and provides learning opportunities for the entire class. The spreadsheet and the dynamic software makes testing multiple theories easier, thus making mathematics thought more intriguing.

Assessment Plan

The following assessment points appear periodically within the learning activity. They can be used for formative performance assessment.

- Group reports of investigations
- Write-up on Pythagoras

Demonstration of visual proofs with rational explanations
Construction of dynamic proofs using the The Geometer's Sketchpad
Application of understanding to the problem of finding the distance between two points at school

Bibliography

The Utah Education Network received permission from ISTE (The International Society for Technology in Education) to share this lesson.
Written by: John Olive, University of Georgia, Athens James H. Wiebe, California State University, Los Angeles (idea adapted from the NCTM Standards, 2000)

Authors

Utah LessonPlans