Cut - Stretch - Fold

Summary

This lesson will help students determine the area of a tangram piece without using formulas. After completing this activity students will use their knowledge to help them develop and use the formulas to determine the area of squares, rectangles, triangles, and parallelograms.

Materials

Geometry Journal
- *Grandfather Tang's Story*

Chinese costume
Poster board
Tangram sets
Overhead projector
9 x 12 art paper
Geoboards
Geobands
Graph paper
- *Origami Math*

Origami paper
Scissors
Ruler
- What's My Area? pdf

Additional Resources
Books
- *Grandfather Tang's Story*
  , by Ann Tompert; ISBN 0-517-88558-1
- *Dinah Zike's Foldables*
  , by Dinah Zike; ISBN 0-02-149593-9
- *Groovy Geometry*
  , by Lynette Long; ISBN 0-471-21059-5
- *Geometry for Every Kid*
  , by Janice VanCleave; ISBN-0-471-31141-3
- *Origami Math*
  , by Karen Baicker; ISBN-0-439-53992-7

Background for Teachers

Tangrams are ancient Chinese puzzles. A tangram begins with a square that is then cut into seven pieces. Each individual piece is called a tan. The pieces are used to form different shapes or pictures. They must touch, but not overlap when you put them together to form a tangram (shape).
The Chinese invented paper 2,000 years ago and along with it they also invented origami, or the art of paper folding. The Chinese brought paper folding to Japan in about the year 600 A.D. Once the Japanese people learned origami they became wonderful origami artists.
Working with both tangrams and origami has proven to have a beneficial impact on students' spatial reasoning and can also be used to teach other math skills and principles.
Before doing the tangram activities students need a basic understanding of what area is and the formula for finding the area of a rectangle. This lesson will help them determine the area of each

tangram piece without using formulas. After completing this activity students will use their knowledge to help them develop and use the formulas to determine the area of squares, rectangles, triangles, and parallelograms.

Intended Learning Outcomes

2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.
3. Reason logically, using inductive and deductive strategies and justify conclusions.

Instructional Procedures

Invitation to Learn

Dress up as Grandfather Tang with a Chinese hat, a long mustache, and a tunic. Then read the book *Grandfather Tang's Story*. As you read the story make the twelve tangram pictures from the story on a large poster or flannel board. Cut the twelve sets of tangram pieces from construction paper or foam board so they will attach easily to the poster or flannel board.

Instructional Procedures:

- Provide each student with a simple Geometry Journal to record definitions and activities in. It can be a student notebook or you can make a simple bound book for them to use. Dinah Zike's Foldables book has many different types of books that you could use as a journal.
- Use paper folding to make your own set of tangrams. As you decompose the square point out the relationships between the pieces. (e.g. A square cut in half on the diagonal forms two triangles; what two shapes can you see in the parallelogram.)
  - Fold a 9 x 12 piece of art paper to form a square. Cut off the extra piece at the bottom.
  - Cut the square in half on the diagonal fold to form two triangles.
  - Take one of the triangles and fold it in half to form two smaller congruent triangles. Cut along the fold.
  - Take the other large triangle and make a small pinch crease in the middle of the baseline to identify the center. Take the apex of the triangle and fold it to touch the center of the baseline. This forms a trapezoid.
  - Cut along the fold line. This gives you a trapezoid and a small triangle.
  - Fold the trapezoid in half (two congruent shapes) and cut along the fold line.
  - Take one half of the trapezoid and fold the pointed end to form a small square. Cut along the fold. This will give you a small square and a small triangle.
  - Take the remaining half of the trapezoid. Fold one of the corners of the square end to form a small triangle and a parallelogram. Cut along the fold.
- This should make a complete set of tangrams: One small square; two small congruent triangles; two large congruent triangles; a medium size triangle and a parallelogram.
- Give students a few minutes to experiment with making different animals from the tangrams like they saw in the Grandfather Tang book. Let them choose their favorite animal they made and glue it into their journals.
- Use Tangram Pieces to Compute Area
- Give each student a set of plastic tangrams to use for this activity. They are easier to trace than the paper ones.
- Review the definition of area: The measure of the number of square units needed to cover the surface of a plane figure. Write the definition in journals.
- We will be figuring the area of each of our tangram pieces by comparing it to the small square. We will call the small square "one square unit". The length and width are both one unit so we multiply 1 x 1 and get one square unit.
- Model each step on the overhead for the students, but give them time to try to determine how to

make the shapes before you show them. This thinking and experimenting process will help them develop their spatial reasoning. On each step have the students trace the shape into the journals, then after determining the area, write it next to the shape.

Find the two small congruent triangles and review the definition of congruent: same size, same shape. Put them together to make a square. What would the area of this shape be? Why?

Take just one of the small triangles. What would its area be? Why? Remember to relate it to the square.

Trace the parallelogram. How can we figure the area for a shape like this? We can also relate it to the square. Take your two small congruent triangles and lay them on the parallelogram. What do you discover? The two triangles make a congruent shape for the parallelogram. So what is the area of the parallelogram?

Trace the medium size right triangle. It is a right triangle because it has a square corner that is a right angle. Fit the two small congruent triangles on top of the right triangle. They form a triangle that is congruent with the right triangle you traced. Trace the two small triangles. What is the area of the right triangle? (1 square unit) How do you know? It has the same area as the two small triangles that we already know have an area of one square unit.

Make a triangle using the small square and the two small triangles. What will the area be? How do you know? 2 square units-the small square is one unit and the two small triangles together are another unit.

You can extend this activity to finding the area of any other polygons constructed from the tangram pieces.

Make a square using the medium size triangle and two small triangles. What will the area be? (2 square units) How do you know?

Make a rectangle with the parallelogram and the two small triangles. What is the area? (2 square units) How do you know?

Make a triangle congruent to one of the large triangles. Do not use the square. What is its area? (2) How do you know?

Construct a square using all seven tangram pieces. What is its area? (8) How do you know?

Construct a trapezoid. Determine its area.

Construct a parallelogram. Determine its area.

Make a pentagon. Determine its area.

Using Geoboards to Determine Formulas to Compute Area

Make a square with an area of nine square units. Determine its length (3) and its width (3). Remember that a square is also a rectangle, then review how we multiply length x width to determine the area of a rectangle so 3 x 3 = 9 square units. Divide the square in half diagonally. Ask what shapes they have made. (Two triangles) Is a triangle half of the rectangle? What would the area be? Half of nine, which is four and one- half. Make several more rectangles, figure the area, and then divide each of them in half to form triangles. Lead students to discover that a triangle's area is half of the rectangle. It is too difficult to try and count the square units so instead we can again use the formula "length x width"...if we divide it in half. Go back and multiply to find each rectangle's formula, then divide the area in half and they will get the triangle's areas. Have them write the formula for finding the area of triangles in their journals. The area of a triangle = 12 (length x width). Explain that when we are just working with triangles, we use the words base and height in place of length and width. The official formula to find the area of a triangle is 12 b x h.

Using Graph Paper to Determine Formulas to Compute Area for Parallelograms

There is also another strategy for determining the area of a parallelogram. Trace the parallelogram tangram onto a sheet of graph paper and cut it out. You must be very accurate!

Cut off one end of the parallelogram-follow a line on the graph paper and it will be easier to be accurate. Now take that piece and slide it to the other end, what shape do you have now? It is a rectangle. So we can now use the area formula of length x width which we use for rectangles. It is four squares long and eight squares wide so multiply 4 x 8 = 32 square units. Can you use this formula on a parallelogram without cutting it apart? Put your parallelogram back the original way. Count how many units tall it is (4) and how many units long it is (8). The formula for area works perfectly on parallelograms!

Origami

Use any of the activities from *Origami Math* by Karen Baiker to continue developing spatial reasoning and recognition of congruency and symmetry. You can also use other origami projects and simply adapt them yourself by looking for shapes formed as you fold, determining the area, lines of symmetry, and congruent shapes.

Formative Assessment can be done by observing students during the activities and evaluating their journals.

Final Summative Assessment *"What's My Area?"* is included in the activity.

Extensions

Use the Geoboards to play CopyCat. Pair students up, give each a geoboard. Have them sit back to back so they cannot see each other's boards. Have Student 1 make a polygon on their board. He then needs to tell Student 2 the area of his figure. Student 2 tries to "copycat" his figure. Switch roles. You can also do this activity using perimeter.

You can play a variation of this against your class. Say: I have made ______figures with a perimeter of _______. Give them 2 minutes to see how many figures they can come up with that have that perimeter. If they can make more than you did they win that round. Do the same with a fixed area.

For students with fine motor control difficulties do not have them trace the shapes. Just let them manipulate the tangrams. It might be helpful to give them two sets in different colors so they can more easily see the relationships. You can also make a copy of the tangram puzzle so they just have to cut and not fold to make the shapes.

Integration with language arts. Have them make a tangram animal and then write a legend about it.

Family Connections

Let students make a set of tangrams to take home to their families. Have them see how many figures their families can make. Remind them that every shape they make will have an area of 8 square units. Challenge them to explain that to their families.

Let the students take geoboards home and play CopyCat with their families.

Assessment Plan

Use "What's My Area?" pdf as a final assessment.

Bibliography

Research Basis Carter, J.A. (2003). Focus on Learning Problems in Mathematics: A survey of paper cutting, folding and tearing. Retrieved 12/14/2006 from www.findarticles.com

Origami or the art of paper folding receives substantial endorsement from current reform initiatives in mathematics education. Particularly, at the elementary school level, the National Council of Teachers of Mathematics in its Principles and Standards for School Mathematics recommends that students use paper folding for initial investigations in geometry. Students benefit from experimenting and exploring with physical materials and models, and learning opportunities that require students to visualize, draw, and compare figures that help them develop spatial sense. Silverman and Marzano

(1996) noted that what is accomplished by using origami is no less than the planting and nourishing of the seeds of geometric thinking.
Sutton, J., Krueger A. (2002). EDThoughts What We Know About Mathematics Teaching and Learning, (90).
Mathematics achievement is increased through the long-term use of concrete instructional materials and active lessons at various grade levels. The more avenues there are to receive data through the senses, the more connections the brain can make. The more connections that are made, the better a learner can understand a new idea. Teachers must intervene frequently to help students focus on underlying mathematical ideas and to help build bridges from students' active work to their corresponding work with mathematical symbols or actions.

Authors

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