Exploring Fractions

Summary

Activities involving a number line help students understand equivalencies among rational numbers.

Main Core Tie

Mathematics Grade 6

Strand: RATIOS AND PROPORTIONAL RELATIONSHIPS (6.RP) Standard 6.RP.1

Group Size

Large Groups

Materials

- *Fraction Cards*
  Beans
  Baggies
  Paper
- *Modeling Multiplication of Fractions Sheet*
- *Problem with a Dozen Sheet*
  Adding machine tape

Additional Resources

Books

Fraction Action, by Loreen Leedy; ISBN 0-8234-1244-X

Fraction Fun, by David A. Adler; ISBN 0-8234-1341-1

Piece = Part = Portion, by Scott Gifford; ISBN 1-58246-102-3

The Grizzly Gazette, by Stuart J. Murphy; ISBN 0-06-000026-0

Multiplying Menace: The Revenge of Rumpelstiltskin, by Pam Calvert; ISBN 1-27091-890-2

Organizations

National Council of Teachers of Mathematics 1906 Association Drive, Reston VA 20191-1502 (703) 620-9840 http://nctm.org/

National Council of Supervisors of Math 6000 E. Evans Ave. #3-205, Denver, CO 80222 (303) 758-9611 http://www.ncsmonline.org/

Utah Council of Teachers of Mathematics http://uctmonline.org/

Background for Teachers

Students should build their understanding of fractions as parts of a whole and as division. They need to see and explore a variety of models of fractions. By using an area model in which part of a region is shaded, students can see how fractions are related to a unit whole, compare fractional parts of a whole, and find equivalent fractions. It is necessary to develop strategies for ordering and comparing fractions, often using benchmarks, such as 12 and 1. Students should understand that between any two fractions, there is always another fraction (Adapted from NCTM, "Principles and Standards for School Mathematics, 2000).

The number line becomes an important model for representing the positions of numbers in relation to benchmarks like 14, 12, and 1. Number line models are helpful in allowing students to compare fractions. For instance, they can decide that 34 is greater than 2/5 because 2/5 is less than 12 while 34 is more than 12.

There are many ways to look at multiplying fractions beyond the traditional algorithm. These ideas will lead to deeper conceptual understanding taking students past the memorization of a rule. This lesson

will focus two key aspects. First, multiplying fractions can be looked at as repeated addition. For example:
14 x 12= 14 + 14 + 14 + 14 + 14 + 14 + 14 + 14 + 14 + 14 + 14 + 14 = 12/4 or 3
The second way of looking at multiplication is to see 14 X 12 as one fourth of twelve wholes. The diagram below is divided into four equal groups, one is shaded.

Intended Learning Outcomes

1. Develop a positive learning attitude toward mathematics.
6. Represent mathematical ideas in a variety of ways.

Instructional Procedures

Invitation to Learn
Let's Talk About Fractions
Have the following problem on an overhead transparency or written on the board ready to be uncovered or projected: _Today is Student Appreciation Day at I.M. Electronics. That means you get 12 off the marked prices on all items. You decide to buy the iPod of your dreams. The price tag reads $350. While waiting to pay for your iPod you are informed that you are the 100th shopper for the day and will receive an additional 1/10 off your purchase. What will you end up paying for the iPod?" Discuss with the students their thoughts about what 12 means. What does it represent? What are some everyday examples? (12 ton truck, 12 off a sale item, 12 dozen eggs, etc.). If students give the decimal and fraction equivalent, point out they are correct, but keep the focus more on the fractional representation. Project/uncover the problem and have students work through the problem alone or in small groups. When most students are finished, have a discussion about what they think the correct answer is and HOW they got it. Really focus on the "how". Write student answers on the board/overhead by the problem. Tell them we will be returning to this question at the end math time today. When ready, return to the answers given and discuss methods used to get the answer. Also have several students model their thinking. Conclude by having students write the problem and their method for solving in their math journals.
Instructional Procedures:
Part One: Multiplication of Fractions with a Model

State the content objective for the lesson: Students will be focusing on Standard I, looking at developing number sense with multiplication of rational numbers. Narrow the focus in on Objective 4: Model and illustrate meanings of multiplication. By the end of the lesson, students should be able to model multiplication of fractions with manipulatives and be able to explain the activity to a partner.

Ask what happens when you multiply a whole number by a fraction.

Pass out a sheet of 812" x 11" white copy paper and approximately 30 beans to each student. Walk through having the students folding it in half. Then, fold again. Now it is divided into fourths. Ask the students to predict the fractional parts when they fold it again. The paper needs to be folded 4 times. (It should have 16 boxes when opened up)

Tell the students they are going to explore twelve different problems by modeling each one with beans and show their work pictorially. Hand out a copy of Modeling Multiplication of *Fractions Sheet* to each student.

The first problem is 1/6 of 12 or 1/6 X 12. Model for the students on the overhead. We are trying to find 1/6 of the whole number twelve. Therefore, we need 12 beans. Count out 12. Now, take a closer look at the fraction, 1/6. The denominator represents how many ways we need to share 12, which is 6 shares. Divide the beans into 6 shares, like this: ++ ++ ++ ++ ++ ++. Now, we need to continue to examine the fraction by looking at the numerator, 1. This says how many shares we want, ++. Now students look at how many are in that share: 2.

Continue by looking at the next problem: look at whole number, then denominator, and finally the numerator.

Have students use beans to model each problem, and then pictorially record the solution.

After working on one or two problems together have the students try to model it on their own.

Before sharing whole group, have the students discuss strategies and solutions for modeling multiplication of fractions with beans.

Continue for the remainder of the problems circulating around the room, asking questions to solidify conceptual understanding:

a. Ask the students to explain the model and what each part of the model represents.

b. How did this model relate to the traditional algorithm?

c. What does it mean to multiply fractions by a whole number?

d. Can you make any generalizations?

e. Does this relate to any other operation?

After most students have completed the problems, ask the students to generate their own problems from the Modeling *Multiplication of Fractions Reference Sheet*. Have a whole class discussion based around their findings and focus on strategies that they used to solve the problems.

Part Two: Numbers, LINE UP!

Discuss that during this activity students will be focusing on sorting fractions, decimals, and percents on a number line using landmark strategies. If needed review what a "landmark" is.

Each student will receive an approximate three-foot length of adding machine tape. Each student will need to measure two and one fourth feet of adding machine tape. This will give students practice with measuring, especially lengths longer than a foot. Place strips horizontally on the desk. Write zero on the left end of the strip and one on the right end of the strip. Discuss briefly that the strip now represents one unit. This strip will be used to play a game in the next activity. Model the labeling as you go.

Have the students fold the right end of their strip over to the left end and crease. Have them open their strip and observe that the crease makes it divided into two equal parts. Have the students write "0/2" under the 0 on the left end, "1/2" on the crease, and "2/2" under the 1 on the right end.

Students will be adding percentages to the strip. Discuss what would be the appropriate percentages for zero, one half, and one whole. Now write 0% under the zero, 50% under the 12, and 100% under 2/2. Finally, add the decimal equivalents to 0, .5 and 1.

Explain that students need a strategy to facilitate examining fractions, decimals and percentages. Explain that they will be using the landmarks of 0%, 50%, and 100%, to approximate where numbers should be placed on a number line. Have students sort the decimals by the following criteria: closest to one, closest to 12 or closest to zero. Direct each group to discuss and then write about the method used to sort the cards. As the teacher, focus on what strategies and skills the students are using to place these decimals on the number line. For example, look the decimal .3, that is less than one half because it only has three tenths. It is two tenths away from .5 and 3 tenths away from zero. It is closer to one-half.

Now, students will sort the fractions into the same three groups with a small group (3 to 4 students): closest to one, closest to 12 or closest to zero. Direct each group to discuss and then write about the method used to sort the cards. As the teacher, focus on what strategies and skills the students are using to place these fractions on the number line. For example, look the fraction 1/5. One is less than half of five, so it will be less than 50%. It is 4/5 away from one whole and 1/5 away from zero. Therefore, it is closer to zero.

Continue with the percentages and pictures of fractions. See if students can see the connections between the cards.

Have the students discuss how they sorted the cards. Discuss which one was the easiest to sort: decimals, fractions, percentages, or fraction pictures. Now, talk to the students about the accuracy of the placement on the number line. Many students will have found decimals the easiest to order on the number line. Talk about this as another strategy to order numbers on the number line by converting all fractions to decimals. Model the algorithm of converting fractions to decimals.

Part Three: Fraction Number Line

This activity further expands students thinking on fractions, decimals, and percents and placing them on a number line. Play the game as a class as described below. Then divide into smaller groups to explore more in depth. Use the fractions, decimal, and percentage cards. Use your adding machine tape as labeled in activity two.

Mix up the sorted fractions and decimals. Deal out five cards to each player. Clarify that students are placing both fractions and decimals on the number line at the same time.

The goal of the game is place as many cards as you can on the number line. There are certain rules to the game: 1) Once a card is placed on the number line, it may not be moved. 2) Cards must be in increasing order from 0% to 100%.

Players must have five cards in their hands at all times until there are no more cards in the deck. On a turn, a player has three options: add a card in their hand to the number line, discard an unwanted card and draw another to see if they can play it, or pass if unable to play.

Play continues until no players can add to their number line. If you choose, you can have the kids keep track of points as this motivates most students. +1 point of each card placed on the line and -1 point for each card left in each player's hand.

Extensions

Curriculum Extensions/Adaptations/ Integration

Play *Fraction NIM* - See explanation on black-line master

Use the *Numbers, LINE UP!* adding machine strip to discuss probability and list the probability of different events on the same line using sticky notes (i.e. the sun will rise tomorrow- 100%).

Look at the timelines in the different Ancient cultures: Mesopotamia, Egypt, Greece, and Rome, place the major important events on a number line.

Examine how the Egyptians looked at fractions. The studies of rational numbers were integral to the building pyramids.

Family Connections

Have the students play the *Fraction NIM* game at home with the family. Students should explain their mental math strategies to their family.

Search for a recipe containing fractions. Bring to class to create a delicious fraction recipe book. Have each student take his or her recipe and double, triple and/or quadruple the recipe. Have the students write how much each recipe will serve. Have the students draw the original recipe amounts, then draw the new doubled recipe. For example: 14 cup of flour now is one-half of a cup.

Research the game of NIM on the Internet. Play the different versions of NIM, using whole numbers and objects.

Assessment Plan

Use *Fraction NIM* as a pre-assessment of student's ability to decompose and compose numbers.

Use the *Problem with a Dozen* activity to assess students understanding (see blackline).

Use a clipboard to record observations of students' strategies, fluency and ideas throughout the lesson.

Have students create their own number line and think of five fractions and/or decimals to place on

a number line. Exchange papers with a partner. Students discuss their strategies for placing numbers on an open number line.

Have students write in their journal about how they used the model to multiply fractions. They need to focus on: what does each part of the model mean, how to use the manipulatives, and how to check the accuracy of the answer.

Bibliography

Research Basis

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA.

The National Council of Teachers of Mathematics (NCTM) has been very outspoken about setting high standards and expectations for all students. All students can learn mathematics; just not all students learn in the same way. *The Principles and Standards for School Mathematics (PSSM)* by NCTM sets forth the ideal vision of all students to become mathematically powerful:

A major goal of school mathematics programs is to create autonomous learners, and learning with understanding supports this goal. Students learn more and better when they can take control of their learning by defining their goals and monitoring their progress. When challenged with appropriately chosen tasks, students become confident in their ability to tackle difficult problems, eager to figure thing out on their own, flexible in exploring mathematical ideas and trying alternate solution paths, and willing to preserve. (NCTM, 2000 p. 21).

Brooks, J. G., & Brooks, M. G. (1993). *In search of understanding: The case for constructivist classrooms*. Alexandria, VA: Association for Supervision and Curriculum Development.

According to the constructivist theorists, learning occurs when connections are made with prior knowledge. One tenet of the theory of constructivism focuses on connecting mathematical ideas to promote understanding so that students can apply that knowledge to new topics and to solve unfamiliar problems. Deeper understandings are developed through the construction of relationships like those found in fractions, decimals, and percentages. Only through making these connections in mathematical topics can students develop deeper conceptual understanding.

Authors

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