Getting Serious About Cylinders-Surface Area and Volume

Summary

These activities ask students to find the surface area and volume of a cylinder.

Additional Core Ties

Social Studies - 6th Grade

Standard 1

Group Size

Small Groups

Materials

Invitation to Learn

- *Which Is Larger?*
  Measuring tapes
  Collection of cylinders

Instructional Procedures

Rolls of Lifesavers®

- *Net of Cylinder*
- *Surface Area of Cylinder*
- Surface Area Patterns
  Cardstock paper
  Scotch tape
  Filler materials
- *Volume Discoveries*
  Model of rectangular prism
- *Effects on Volume*
  Calculators
- *Will You Be Wet or Dry?*
  Water collection cans
  Measuring tapes
  Water
  Graduated cylinders
  Rain poncho
  Towels
  Paper
  Whiteboards
  Scissors

Additional Resources

Books

Ancient Egyptians and Their Neighbors, An Activity Guide, by Marian Broida; ISBN 1-55652- 360-2
An Ancient Greek Temple, by John Malam; ISBN 1904194680
Cubes, Cones, Cylinders, & Spheres, by Tana Hoban; ISBN 0-688-15326-7
How To Draw What You See, by Rudy de Reyna; ISBN 0-8230-2375-3

Background for Teachers

To find the surface area and volume of a cylinder, students must first be able to look at the parts of this geometric solid. A cylinder has two circular faces known as bases that are connected by a curved surface known as the lateral surface. The two circular bases are parallel and have the same area. The volume of the cylinder is the amount of space inside the cylinder. To find the volume, students will first need to know the area of the circular base. The formula for the area of the circle is the same as that used in surface area or r2. They next must know the height of the cylinder. To find the volume, they must multiply the area of the circle by the height of the cylinder. The formula is V = Bh where B = the area of the circular base or r2 and h = the height of the cylinder. Thus, the final formula is V = r2h. Volume is measured in cubic units.

It is helpful to consider the net of the cylinder to see how surface area is determined. When a cylinder is taken apart and looked at as a net, there are two congruent circles and a rectangle. The surface area includes the sum of the areas of each circle (the bases) and the rectangle (the lateral surface). Thus, students must have previously learned how to find the areas of circles and rectangles. The formula for the area of a circle is r2. The formula for the area of a rectangle is bh. When looking at the net, students will need to see that the dimensions of the rectangle are equal to the circumference of the circle and the height of the cylinder. The formula is developed as follows: S = 2r2 + 2rh where S is the surface area, r is the radius of the base, and h is the height of the cylinder. Surface area is measured in square units.

Intended Learning Outcomes

1. Develop a positive learning attitude toward mathematics.
2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.
3. Reason logically, using inductive and deductive strategies and justify conclusions.

Instructional Procedures

Invitation to Learn

Hold up a cylinder. Ask students to identify the geometric solid. Review the parts of a cylinder with the students, focusing on the two circular faces called bases and the curved surface. Note that the two circular bases are parallel and congruent. Point out the circumference of the circular bases and the height of the cylinder.

Tell students we are going to play a game with some cylinders. Each student will need a copy of the handout Which Is Larger? on which to mark their answers. The teacher will need to have a collection of ten cylindrical objects hidden from student view in a tub. Have each cylinder numbered one through ten. One at a time, hold up a cylinder and have students predict through their powers of observation which is larger, the height of the cylinder or the circumference of the base. After students have recorded their estimate for all ten objects, it is time to check their answers. One at a time, measure the height and circumference of each cylinder in front of the group. A possible discovery is that the circumference is often larger than the height. It is a common misconception that the reverse is true.

Instructional Procedures

(NOTE: The activities outlined in Instructional Procedures are intended to be taught sequentially. They will take several lessons/ days to complete with students.)

Surface Area

Give each student a roll of Lifesavers® candy. Discuss with students what the package/label of this cylindrically-shaped candy would look like if it were opened up and laid out flat. This would be called a net. Have students carefully open up the package and lay out the net. Discuss with students what the net of a closed cylinder (top and bottom included) looks like. Have students attach the wrapper/net of the cylinder in their journals and write about its parts.

Provide students with a copy of *Net of Cylinder*. Cut out the net and construct the cylinder, but do not tape it together. It is important for students to see how the construction works and how to lay it flat to see the components. Students may need to put the cylinder together and lay it flat several times as they are learning. Note how the circumference of the circle is actually the base of the rectangle. Write "circumference" along the base of the rectangle. Next, point out that the height of the cylinder is also the height of the rectangle. Write "height" along the height of the rectangle. This net can also be saved and placed in student's journals for future reference.
Tell students that knowing the parts of a cylinder can help to find its surface area. Have a discussion on the definition of surface area. It is the "wrapper" or "skin" of the cylinder because it includes the top, bottom, and curved side. Discuss why people might want to find the surface area of a cylinder. A possible answer might be to know how much tin is needed to make a tin can in order to calculate the material cost of production.
Give each student a copy of *Surface Area of Cylinder*. Ask students to determine the area of the cylinder by counting the squares. For the rectangle, students can multiply base times height. Have students record their answer for the rectangle on the net. For the circles, have students count the whole squares and then estimate the partial squares by putting them together to create whole squares. The answers will not be perfect, but should be close. Have students record their answers for each circle on the net. Have students add together the area of the rectangle plus the area of each circle to find the total surface area. They may record their answers on the handout.
Discuss whether counting squares is the most efficient method for finding the surface area of a cylinder. Make sure students see that the surface area is "2 x area of circle + area of rectangle." Guide students to use the formula for each part of the cylinder. First, remember that the top and bottom bases are both congruent circles. Review how to find the area of a circle. The formula for this is r2. Some students may present this as x radius x radius. Find the area of a circle on the *Surface Area of Cylinder* net together as a class using the formula and compare it to the students' estimates by counting squares. Remind students that there are two circles, so the first part of the formula for finding surface area is 2r2. Next, find the area of the rectangle. One edge of the rectangle is actually equal to the circumference of the circle which is2r. The other side is equal to the height of the cylinder. Thus, the formula for the rectangle is 2rh. Calculate the area of the rectangle together as a class using the formula and compare it to the students' estimates by counting squares. Help students to put together the final formula. The final formula is S = 2r2 + 2rh where S is the total surface area, r is the radius, and h is the height. All final answers on surface area are measured in square units.
Have students complete the handout *Surface Area Patterns*. Set 1 has cylinders that double in radius and height each time. The pattern students should discover is that the surface area quadruples each time. Set 2 has cylinders that triple in radius and height each time. The surface area is nine times larger in this pattern. Students must then determine the surface area of a fourth cylinder in each set.
Volume
Have students work in cooperative groups of four students to complete this activity. Give each group two sheets of cardstock paper of the same size. For example, one group will have two sheets of 8 1/2" x 11", another group will have two sheets of 8" x 9", another group will have two sheets of 6 12" x 10", and so forth. Roll the first sheet into a tall, thin open cylinder (has no top or bottom) and tape the sides together with no overlap or gap. Fill the cylinder with popcorn or other filler. Roll the second sheet the opposite way and tape it into a shorter, wider open cylinder with no overlap or gap. Place it around the first cylinder. Have students predict in their journals if it will have the same volume of the filled cylinder. Slowly, lift and remove the tall cylinder, allowing the filler to go into the second cylinder. Students will find that the contents of the first

cylinder do not completely fill the second cylinder. By completing this activity, students will be able to determine which cylinder has the larger volume. They should also discover that cylinders with the same lateral surface area (rectangle) do not always have the same volume. The size of the circular base affects the total surface area and, consequently, affects the volume of the cylinder. Students should discover that the shorter cylinder of the two has a larger volume. Discuss each group's findings as a class, and record the findings on an overhead transparency of *Volume Discoveries*. Have students write about their conclusions from this activity in their journals, including sketches as needed.

Explain to students that knowing the parts of a cylinder can help to find its volume. Remind students that the volume of the cylinder is the amount of space inside the cylinder. Show the students a rectangular prism. Ask them to remember how to find the volume of a rectangular prism. Have students discuss in groups how they might be able to find the volume of a cylinder. Have students share their ideas. If it has not been brought up in discussion already, remind students that the formula for volume is base times height or V = Bh. In this case, the base is a circle so students will need to review how to find the area of a circle. The formula for the area of the circle is r2. To find the volume, students must multiply the base by the height. Substituting r2 in the formula for B, the formula is thus V = r2h. All final answers on volume are measured in cubic units.

Have students answer the following question in their journals: "Which produces a greater effect on the volume of a cylinder-- changing the radius or changing the height?" After sharing student's ideas, have them complete *Effects on Volume* handout to justify their conclusions. Students should be able to use calculators to complete the computational work.

Explain the steps to the activity *Will You Be Wet or Dry?* to the class. Have each cooperative group of four students select a can from the teacher's collection. Students will need to find the measurements of the can and fill out the data required on the handout *Will You Be Wet or Dry?* as a group. After the volume is calculated, groups will take turns presenting their data to the teacher and other class members. The teacher will then fill the can with the volume of water determined by the group over a team member's head. If the calculation is too high, the water will overflow onto the student's head. If the calculation is too low, then all of the water will get poured on their head. An inexpensive rain poncho should be available for students to use if they would like. Be ready for some fun and have the towels handy.

Extensions

Curriculum Extensions/Adaptations/ Integration

Have students look for cylinders in their environment and make a list of their findings in their math journals.

Have students compare the parts of cylinders to the parts of cones and pyramids.

Have students learn how to draw cylinders and other geometric shapes.

Have students learn how the Mesopotamians used the cylindrical shape to invent cylinder seals, a method of marking property and signing documents in ancient times. Have students make cylinder seals from clay and put them on strings to wear as necklaces as did the Mesopotamians.

Have students learn how the ancient Greeks used the cylindrical shape to design columns as part of their architecture. Have students work in groups of five. Give each member of the group one sheet of 8 12" x 11" paper. Tell students that their task is to create cylinders of any size to support a 12" x 12" whiteboard that will balance as many textbooks as possible on top. Tell students you have also heard that it is possible to balance a person on the whiteboard placed on paper cylinders instead of textbooks. Students may use scissors to cut their paper, and they may use scotch tape to hold the cylinders closed.

Have students explore volume and surface area of other geometric solids.

Family Connections

Have family members look for cylinder shapes at home. Share how cylinders are used.
Have students look for cylindrical food cans. Compare the volumes listed on the labels. Explain to family members why certain cans have a greater volume.

Assessment Plan

Have students design a net of a cylindrical can for a new brand of peanut butter. After students have completed the writing and design work, they can put the cylinder together. Have students compute the surface area and volume of their can.
Have students place three different cylinders in order from least to greatest volume using estimation. Have students calculate the actual volumes of each cylinder to check their work.
Have students correct the *Surface Area Patterns* handout for an assessment grade.
Have students correct the *Effects on Volume* handout for an assessment grade.

Bibliography

Research Basis

Pierce, Rebecca L., & Adams, Cheryll M. Tiered lessons. *Gifted Child Today*, Spring 2004, Vol. 27, Number 2, p58-65.

Based on tenets of differentiated instruction supported by the NCTM, the authors of this article define tiered lessons and outline eight steps to designing them. The three main ways to differentiate a lesson are guided by student's readiness, interest, or learning profile. Grouping for differentiated instruction is designed to be flexible from one lesson to the next.

Pugalee, David K. Writing, mathematics, and metacognition: looking for connections through students' work in mathematical problem solving. *School Science and Mathematics*, May 2001, Vol. 101, Number 5, p236-245.

This study looked for evidence of a metacognitive framework based on students' writing about mathematical problem-solving processes. Students' writing was analyzed from the introduction of a topic through the execution of problems on the same topic. The findings proved that a metacognitive framework is established through the process of writing. Furthermore, the author emphasizes the importance of writing as an integral part of mathematics curriculum.

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