## Number Flexibility

Summary
The students will be able to: use problem solving strategies in solving math problems and give opportunities to students to show different ways to solve problems.

## Main Core Tie

Mathematics Grade 2
Strand: NUMBER AND OPERATIONS IN BASE TEN (2.NBT) Standard 2.NBT. 7

## Additional Core Ties

Mathematics Grade 2
Strand: NUMBER AND OPERATIONS IN BASE TEN (2.NBT) Standard 2.NBT. 9
Materials
Materials for Activity:
A white paper plate that students can write on.
Punch two holes in the paper plate at the 10 and 2 positions.
Tie yarn through holes, making a loop big enough to fit over student's heads.
You will need the following for Adding and Subtracting Numbers, Making Tens:
Unifix cubes

- Ten frame template (pdf)

You will need the following for Number Flexibility:

- Number Flexibility Board (pdf) and Procedures (pdf)
Counters
Dry erase marker
- Starting Number Cards (pdf)
- Target Number Cards (pdf)

You will need the following for Numbers Incognito:

- Numbers Incognito Mat (pdf)
- Numbers Incognito Solutions page (pdf) (several for each student)
10 or more unifix cubes
Dry erase marker
Books:
- Mathterpieces: The Art of ProblemSolving
, by Greg Tang; ISBN 0439443881
- Mental Math in the Primary Grades by Jack A. Hope, Larry Leutzinger, Barbara Reys, Robert E. Reys
- Math for All Seasons
, by Greg Tang
- Grapes of Math
, by Greg Tang.
Media:
- Abbott and Costello Math (YouTube)


## Background for Teachers

Students need to have many experiences being flexible with numbers, such as composing, decomposing, and compensation. Exposure to these experiences increases their mental math capabilities. As adults we use mental math $85 \%$ of the time to compute practical, everyday problems. As elementary grade teachers, we have many priorities and objectives for our students as they progress through the early mathematics curriculum. These objectives include the learning of basic facts as well as standard paperandpencil computational procedures. Instruction dealing with each of these areas must include an emphasis on teaching for understanding. In particular, we must not limit instruction in computation to paperandpencil techniques, because that gives students the impression that such procedures (written algorithms) are necessary to compute all arithmetic problems.

## Intended Learning Outcomes

Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.
Reason logically, using inductive and deductive strategies and justify conclusion.
Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.

Instructional Procedures
Invitation to Learn:
Math-terpieces:
Read the book Mathterpieces The Art of Problem Solving by Greg Tang
Handout the recording sheet " Mathterpieces (pdf)."
Direct teachers to solve the first few "Mathterpiece" puzzles.
Instructional Procedures:
If students have not had the opportunity to take numbers apart to add problems, this may be difficult at first. Continued practice will increase mental math skills.
Adding Larger Numbers by "Making Tens" (Concrete Stage)
Write the addition combination $9+6$ on the board.
Ask the students to make ten and determine the answer.
Then write the combination 19+6 on the board.
Place 19 unifix cubes under the document camera. (Make sure you have a ten pack, then 9 connected also. Put next to the ten pack so it looks like a ten frame).
Explain that you are going to show the combination 19+6, and add 6 loose unifix cubes.
Ask students how many unifix cubes are needed to make 20 ?
Add one loose unifix cube to the one with nine.
Ask students how many we have in all now?
Explain that we have just used "making tens" to find the answer to the combination for $19+6$.
Continue this procedure until you feel students have a good grasp of the concept using the unifix cubes. This is the concrete stage.
Other numbers you can use are as follows: $19+8,19+5,19+3,28+4,28+6,28+7$.
After using the unifix cubes for the concrete stage, we move to the pictorial stage. A number bond template is a great graphic organizer for students to use for this stage. Start with easier numbers and move to more difficult numbers as students grasp this concept.
Adding Larger Numbers by "Making Tens" (Pictorial Stage)
Here is an example of how to use a number bond organizer for addition problems.
It doesn't matter which number students break apart, as long as they can explain. Make sure to plan which numbers you are going to use so you can encourage use of the strategy given. Today we will be focusing on the "making tens" strategy. "Making doubles" is another strategy that can be used on
a different day. After several strategies have been taught, mix it up. It doesn't matter which strategy students choose to use -- as long as they can explain their thinking! Here are some other number combinations that can be used for "Making Tens."
$8+5,28+5,9+6,49+6,3+8,53+8,7+9,87+9,8+4,28+4,9+3,69+3,5+9,15+9,7+8,77+8,7+6$, $57+6,4+7,84+7$.
After practicing as a class and explaining thinking this can be used in a math center. Dice can be used to generate the number combinations.
Adding Numbers by "Making Tens" (Abstract Stage)
Students also need practice doing this mentally no paper and pencil. Calculating in your head is a practical life skill. Many types of everyday computation problems can be solved mentally. In fact, practically speaking, many must be solved mentally, since we often need to make quick computation when we don't have a calculator or paper and pencil at hand. So be sure to give practice time for this same skill using mental math. When starting in this stage I would begin with having students still explain their thinking, then go on to answering orally.
You will need a ten frame template and unifix cubes or other counters to demonstrate the concrete stage.
Subtracting Numbers by "Making Tens" (Concrete Stage)
Show students the subtraction problem 137.
Using the "Ten Frame Template" -- start with the part 7 by placing 7 counters in the ten frame, and say, "We start with the part, 7 , and add to make ten. How many counters do we need to make ten?" (3)
Add these three to the ten frame.
"If we take 10 away from 13 we have 3 left. Now add three to the total. How many in all?" (6)
You can demonstrate with unifix cubes that adding three to both the total and the number you are subtracting does not change the difference.
Think of it this way
Start with the part.
Add on to make 10.
Now add on to the difference after you have subtracted 10 .
How many in all?
Continue demonstrating with ten frames and counters. Here are some combinations that can be used for this. 159, 135, 127, 128, 117, 127, 169, 149, 139. 159
Subtracting Numbers by "Making Tens" (Pictorial Stage)
This is done in the same way as addition, only using subtraction. Here is an example of how to do subtraction.
Subtracting Numbers "Making Tens" (Abstract Stage)
This is done the same as adding in the abstract stage. Lots of oral practice is needed. Once students have this down, combinations with larger numbers can be used. For example: 42 34. Other numbers that can be used are: $3123,4637,5142.2418,2214,4638,6657,4335,5349,2318,8177$.
The following are activities that could be used in centers:
Number Flexibility

## Materials:

Number Flexibility Board \& Procedures (these need to be run on cardstock, back to back \& laminated)
Counters to cover numbers or dry erase marker
Starting Number Cards
Target Number Cards

## Procedures:

Pass out one Number Flexibility Board to each student.

Pass out counters or a dry erase marker to each student.
Put students in groups of 24 players

## 4. Starting Number Cards

go face down next to Target Number Cards.
A Starting Number Card and Target Card are turned over.
On their boards students subtract numbers on the squares from their board from the starting number until they reach the target number. They have to in turn explain their process.
If a student does the computation wrong, they lose that turn and has to uncover the numbers he/she tried to use.
The first person to clear all the squares wins!
Numbers Incognito

## Materials:

## - Numbers Incognito Mat

(this needs to be printed on cardstock and laminated)
Several Numbers Incognito Solutions page
10 or more unifix cubes
Dry erase marker

## Procedures:

Give each student a Numbers Incognito Mat and some unifix cubes.
Give students a target number. Start with 10 and then do multiples of ten for a challenge.
Direct them to place a different number of manipulatives in each large shape (hexagon, triangle, and circle).
Tell the students to add the number of manipulatives in the hexagon to the number of manipulatives in the triangle and to record the sums in the overlapping squares.
Direct the students to do the same with the other shapes, recording the sums in the overlapping squares.
Remove the manipulatives, leaving only the recorded sums in the squares.
Have students trade their mats with other classmates. Challenge them to determine the missing manipulatives, or addends, based on the recorded answers. If their numbers add up correctly, they will have solved the problem.
Next have students records their solutions on the Numbers Incognito Solutions page.
Continue this process, having students exchange mats with different classmates.
Lesson and Activity Time Schedule:
Each lesson is 55 minutes.
EEach activity is 35 minutes.
ETotal lesson and activity time is 90 minutes.
Activity Connected to Lesson:
What's My Number?
Preparation: Using a dryerase marker, write a different number on each plate. Poke a hole in the top of the plate and attach a 14" length of yarn through the hole.
Directions:
Hang each plate around a students' neck so that the plate falls on his/her back and he/she can't see what's written on it.
Give each student a dry erase marker.
Have the students walk around the room while you play music. Tell them that when you stop the music, they're to stop moving.
When the music stops, explain that each student should pair up with the person closest to him/her. Next, tell the students that each of them is to write an equation on his/her partner's plate that equals the number on that plate. (i.e. if the number of Sue's plate is 25 , someone might write
$5 \times 5$ or 24+1, etc.).
Remind students that they're not to say anything --no clues or answers may be given! Then start the music again. Repeat this process as many times as you choose. Stop the music one last time, and ask students standing near each other to become partners and to read aloud the equations on each other's plates. After listening to the equations, the person tries to guess what his/her number is. Then the other person does the same.

## Extensions

For Numbers Incognito:
Students can be challenged to explore the nature of odd and even numbers. What happens when you add two odd numbers? Two even numbers? An odd and an even number?
Create a mat that has four shapes and repeat the activity.
Family Connections:
Students can use these strategies while in the car. For example, students can use the numbers on license plates. If parents are given some background, they could ask their students during any family times to add or subtract numbers.

Assessment Plan
Observations
Recording sheets
Any math assessment to see if strategies have been applied
Authors

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