Represent and solve problems involving multiplication and division within 100 (Standards 3.OA.1–4 and Standard 3.OA.7).

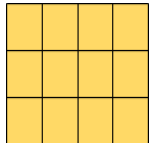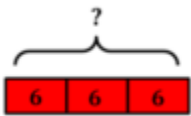**Standard 3.OA.1** Interpret products of whole numbers, such as interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

Concepts and Skills to Master

- Understand multiplication as combining equal groups of objects
- Model skip counting on a number line
- Understand that in a multiplication equation, the first factor equals the number of groups and the second factor equals the number in each group
- Find the total number of objects within equal groups (5 × 7 = 35; 5 groups of 7 is 35)
- Write multiplication expressions and equations to represent pictures
- Draw pictures to represent multiplication expressions and equations

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.2 Interpret whole-number quotients of whole numbers **3.OA.3** Use multiplication and division to solve word problems involving equal groups, arrays, and measurement quantities **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. **3.OA.5** Apply properties of operations as strategies to multiply and divide **3.OA.6** Understand both division as an unknown-factor problem and the relationship between multiplication and division **3.OA.7** Fluently multiply and divide within 100	**4.OA.1, 4.OA.2** Interpret and solve a multiplication equation as a comparison **4.NBT.5** Multiply multi-digit whole numbers **5.NBT.5** Fluently multiply multi-digit whole numbers **4.NF.4, 5.NF.4** Apply and extend previous understandings of multiplication to fractions

Critical Background Knowledge from Previous Grade Levels

- Use addition to find the total number of objects in an array (2.OA.4)
- Skip count by fives and tens (2.NBT.2)

Academic Vocabulary

equal groups, array, multiplication, factor, product, equation

Suggested Models	Suggested Strategies
Write an equation that can help you find the total number of points on the stars. 3 × 5 = 15 Frank bought six boxes of crayons. Each box of crayons has 8 crayons in it. How many crayons does he have? 	- Model equal groups with various counters - Discuss real-life situations where objects are in groups - Use and compare number lines, bar models, and area models

3.OA.1

Represent and solve problems involving multiplication and division within 100 (Standards 3.OA.1–4 and Standard 3.OA.7).

Standard 3.OA.2 Interpret whole-number quotients of whole numbers. *For example, interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into eight shares (partitive), or as a number of shares when 56 objects are partitioned into equal shares of eight objects each (quotative).*

Concepts and Skills to Master

- Understand that division may represent two different situations: partitive (fair sharing) and quotative (measurement)
- Understand division as repeated subtraction to find the number of equal groups
- Find how many equal groups can be made from a certain number of objects
- Find how many objects can be shared equally among a certain number of groups
- Solve and interpret division problems
- Model a division equations using pictures, objects, or numbers
- Use objects and drawings to represent equal groups
- Use objects, drawings, expressions, and equations to represent division situations

Teacher Note: This standard focuses on two distinct models of division: partitive and quotative. Partitive or fair share models provide students with the total number of objects and the number of groups. Students must solve for the number in each group. Quotative or measurement models provide students with the total number of objects and the number of objects in each group. Students must solve for the number of groups. Students are not expected to know or produce the terms partitive and quotative but should be exposed to them.

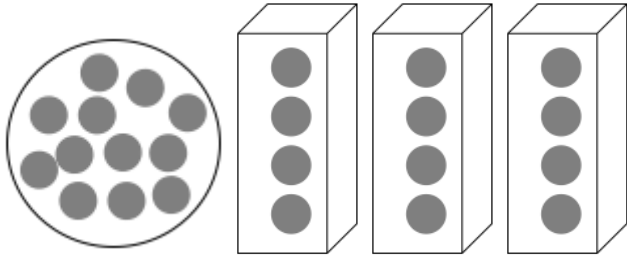| Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.1 Interpret the products of whole numbers **3.OA.3** Use multiplication and division to solve word problems involving equal groups, arrays, and measurement quantities **3.OA.4** Determine the unknown number in a multiplication or division equation relating three whole numbers **3.OA.5** Apply properties of operations as strategies to multiply and divide **3.OA.6** Understand both division as an unknown-factor problem and the relationship between multiplication and division **3.OA.7** Fluently multiply and divide	**4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison **4.OA.3** Solve multi step word problems with all operations **4.NBT.6** Find whole-number quotients with up to 4-digit dividends and 1-digit divisors **5.NBT.6** Find whole-number quotients with up to 4-digit dividends and 2-digit divisors **5.NBT.7** Solve equations involving decimals with all operations **5.NF.7** Apply and extend previous understandings of division to fractions

Critical Background Knowledge from Previous Grade Levels

- Add and subtract within 20 (2.OA.2)
- Use addition to find the total number of objects arranged in an array (2.OA.4)
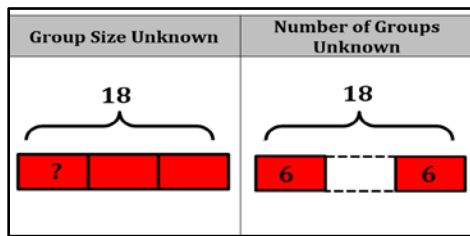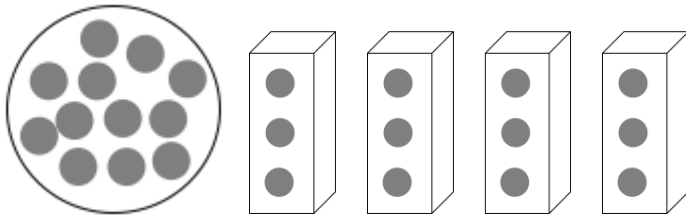
Academic Vocabulary

quotient, dividend, divisor, divide, equal groups, whole numbers

Partitive Division: There are 12 cookies. If you put them in three bags, how many cookies will be in each bag?

- Use manipulatives/objects or other models
- Use repeated subtraction
- Drawing pictures
- Model equal groups
- Model equal groups with various counters
- Discuss real-life situations where objects are in groups
- Use and compare number lines, bar models, and area models

Quotative Division: There are 12 cookies. If you give put 3 cookies in each bag, how many bags will you fill?

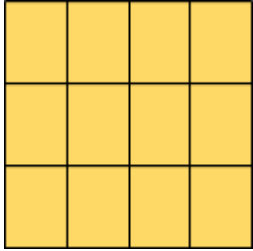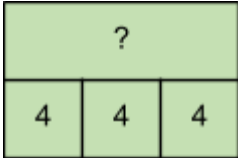| Group Size Unknown	Number of Groups Unknown
18	18
?	6		6
Partitive	Quotative

Represent and solve problems involving multiplication and division within 100 (Standards 3.OA.1–4 and Standard 3.OA.7).

Standard 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. *For example, use drawings and equations with a symbol for the unknown number to represent the problem.*

Concepts and Skills to Master

- Determine the operation based on the situation in the context of a word problem (avoid relying on keyword strategies)
- Use numbers and symbols to represent word problems (×, ÷, =, and a variety of symbols for unknowns)
- Solve the following multiplication and division situations. (See: TABLE 2. Common multiplication and division situations):
    - **Equal Groups of Objects/Product Unknown** word problems (There are 3 bags with 4 plums in each bag. How many plums are there in all?)
    - **Equal Groups of Objects/Group Size Unknown** word problems (24 plums are shared equally into 3 bags. How many plums will be in each bag?)
    - **Equal Groups of Objects/Number of Groups Unknown** word problems (24 plums are packed equally into some bags. 8 plums are packed into each bag. How many bags are needed?)
    - **Arrays of Objects/Product Unknown** word problems (The apples in the grocery window are in 3 rows and 4 columns. How many apples are there?)
    - **Arrays of Objects/Group Size Unknown** word problems (If 12 apples are arranged into an array with 3 rows, how many columns of apples are there?)
    - **Arrays of Objects/Number of Groups Unknown** word problems (If 12 apples are arranged into an array with 4 columns, how many rows are there?)

Teacher Note: In this standard emphasis should be placed in solving for products of two one-digit numbers. Students may also be expected to solve problems in which a two-digit number is multiplied by a one-digit with a product less than or equal to 100. Emphasis should be placed on one-digit numbers multiplied by one-digit numbers; however, students should be exposed to a variety of problems with products less than or equal to 100. Examples may include problems such as: 12 × 5 = 60, 25 × 4 = 100, 33 × 3 = 99, etc. Multiplicative comparison situations (35 is 5 times as many as 7 and 7 times as many as 5) should not be introduced in third grade. This concept will be introduced in fourth grade in Standards 4.OA.1 and 4.OA.2.

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.1, **3.OA.2** Interpret products of whole numbers and whole-number quotients **3.OA.4** Determine the unknown whole number in a multiplication or division equation **3.OA.5** Apply properties of operations as strategies to multiply and divide **3.OA.6** Understand the relationship between multiplication and division **3.OA.7** Fluently multiply and divide within 100 **3.OA.8** Solve two-step word problems **3.MD.2** Multiply and divide to solve measurement word problems **3.MD.7** Relate area to multiplication	**4.OA.2** Multiply and divide to solve word problems involving multiplicative comparisons **4.OA.3** Solve multi-step word problems using whole numbers and having whole-number answers using the four operations **4.NBT.5, 4.NBT.6** Multiply and divide with multi-digit numbers **4.NF.4** Extend understandings of multiplication to multiply a fraction by a whole number **5.NF.4, 5.NF.6, 5.NF.7** Extend understandings of multiplication and division to multiply and divide with fractions **5.NBT.5** Fluently multiply multi-digit whole numbers **5.NBT.6** Find whole-number quotients

Critical Background Knowledge from Previous Grade Levels
• Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends (2.OA.4)
• Partition a rectangle into rows and column of same-sized squares and count to find the total number of squares (2.G.2)
• Use addition and subtraction to solve word problems (1.OA.1, 2.OA.1)

Academic Vocabulary
equal groups, array, row, column, area model, multiply, product, factor, divide, quotient, divisor, dividend

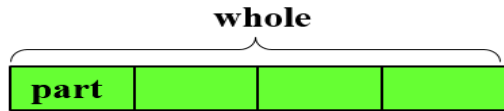| Suggested Models	Suggested Strategies
$3 \times 4 = 12$ 3 groups of 4 is 12 $4 + 4 + 4 + 4 = 12$ $4 \times 3 = 12$ 4 $4 \times 3 = 12$ 3 $4 \times 3 = ?$ $4 \times 3 = 12$? 4 4 4	• Use objects and drawings to represent equal groups and arrays; Describe factors, products, etc. in these models • Use bar models • Use counting all, skip counting, repeated addition to multiply • Write equations to represent drawings and objects; Explain connections between physical/visual models and equations • Use the relationship between multiplication and division to solve problems • Use a multiplication strategy (compensation, distributive property) to solve word problems • Apply the commutative or associative properties of multiplication • Students may create their own word problems • Use equal groups, arrays, area models, bar models to solve problems • Use repeated subtraction to divide

3.OA.3

Represent and solve problems involving multiplication and division within 100 (Standards 3.OA.1–4 and Standard 3.OA.7).

Standard 3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number— product, factor, quotient, dividend, or divisor—that makes the equation true in each of the equations $8 \times ? = 48$, $5 = ? \div 3$, $6 \times 6 = ?$.*

Concepts and Skills to Master

- Solve the following multiplication and division situations (See: TABLE 2. Common multiplication and division situations):
    - **Equal groups / unknown product** word problems (There are 3 bags with 6 plums in each bag. How many plums are there in all?)
    - **Equal groups / group size unknown** word problems (If 18 plums are shared equally into 3 bags, then how many plums will be in each bag)
    - **Equal groups / number of groups unknown** word problems (If 18 plums are to be packed 6 to a bag, then how many bags are needed?)
    - **Array or area / unknown product** word problems (There are 3 rows of apples with 6 apples in each row. How many apples are there?)
    - **Array or area / group size unknown** word problems (If 18 apples are arranged into 3 equal rows, how many apples will be in each row?)
    - **Array or area / number of groups unknown** word problems (If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?)
- Understand that equations involving multiplication and division relate three whole numbers in related facts ($3 \times$ __ $= 15$; $15 \div$ __ $= 3$; $15 \div 3 =$ __)
- Use a symbol to represent an unknown number
- Apply multiplication or division to solve for an unknown in an equation

Teacher Note: Comparison problem types are not introduced until 4th grade. Equations in the form of $a \times b = c$ and $c = a \times b$ should be used interchangeably, with the unknown in different positions. Examples: $24 = ? \times 6$, $72 \div$ __ $= 9$, or the following problem: Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? $3 \times 4 = m$

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.3 Use multiplication and division within 100 with symbols for the unknown number **3.OA.7** Fluently multiply and divide using the relationship between multiplication and division **3.MD.8** Solve real-world and mathematical problems involving perimeters	**4.NBT.5** Multiply a whole number of up to four digits **4.NBT.6** Find whole number quotients **4.OA.3** Solve multi-step word problems posed with whole numbers **4.OA.2** Multiply or divide to solve word problems **4.MD.3** Apply the area and perimeter formulas for rectangles; view the area formula as a multiplication equation with an unknown factor **5.NBT.5** Fluently multiply multi-digit whole numbers **5.NBT.6** Find whole digit quotients using the relationship between multiplication and division

Critical Background Knowledge from Previous Grade Levels

- Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends (2.OA.4)
- Partition a rectangle into rows and columns of same-sized squares and count to find the total number of squares (2.G.2)
- Use addition and subtraction within 20 to solve word problems involving situations with unknowns in all positions. (1.OA.1)
- Understand the meaning of the equal sign (1.OA.7)
- Determine the unknown whole number in an addition and subtraction equation relating three whole numbers (1.OA.8)

Academic Vocabulary	
symbol, equal, multiplication, product, factor, quotient, dividend, divisor, division	
Suggested Models	Suggested Strategies

Part Part Whole/Multiplication and Division
 	
whole
 	
part
 	
one part × number of parts = whole
whole ÷ number of parts = one part
	● Use a bar model to solve for the unknown whole number in an equation
	● Use counters to model the relationship between multiplication and division
	● Use base ten blocks to represent array and area models
	● When given an equation such as $4 \times ? = 40$, students explain their thinking, for example:
	  - 4 groups of some number is the same as 40
	  - 4 times some number is the same as 40
	  - I know 4 groups of 10 is 40 so the unknown number is 10
	  - • The missing factor is 10 because 4 times 10 equals 40

3.OA.4

Demonstrate understanding of the properties of multiplication and the relationship between multiplication and division (Standards 3.OA.5–6).

Standard 3.OA.5 Apply properties of operations as strategies to multiply and divide. *For example: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (commutative property of multiplication). 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30 (associative property of multiplication). Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (distributive property).* (Third grade students may, but need not, use formal terms for these properties.)

Concepts and Skills to Master

- Understand that multiplication is commutative and division is not commutative (the order of the factors does not change the product of an equation)
- Understand and apply the associative property of multiplication (factors can be grouped differently without changing the product)
- Understand and apply the distributive property of multiplication over addition (to support students in solving for products by breaking apart the numbers)
- Understand and apply the multiplicative identity property of one (8 × 1 = 8)
- Understand and apply the zero property of multiplication (8 × 0 = 0)
- Apply properties to simplify an expression into smaller problems (3 × 7 = (3 × 2) + (3 × 5); 3 × 8 = 3 × 2 × 4)

Teacher Note: Emphasis should be placed on understanding of the properties and why each property applies to a particular operation rather than memorizing names and definitions. Convention defines arrays as rows by columns, however students should be allowed flexibility in describing arrays as either rows by columns or columns by rows and should understand how rotating an array demonstrates the commutative property.
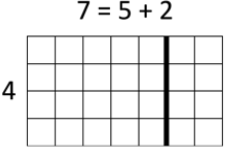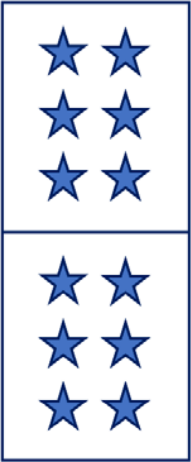
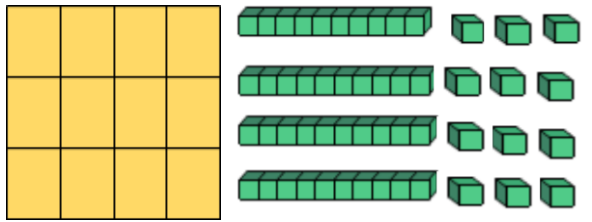| Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.1 Interpret the products of whole numbers **3.OA.2** Interpret whole-number quotients of whole numbers **3.OA.3** Use multiplication and division to solve word problems **3.OA.4** Determine the unknown whole number in a multiplication or division equation **3.OA.6** Understand division as an unknown-factor problem **3.OA.7** Fluently multiply and divide **3.MD.7** Relate area to the operations of multiplication and addition **3.OA.8** Solve two-step word problem **3.OA.9** Identify arithmetic patterns and explain them using properties of operations **3.NBT.3** Multiply one-digit whole numbers by multiples of 10 using strategies based on place value and properties of operations	**4.NBT.5** Multiply whole numbers using strategies based on the properties of operations **4.NBT.6** Find whole-number quotients and remainders based on the properties of operations **4.OA.3** Solve multi-step word problems **5.OA.1** Use parenthesis, brackets, and braces in numerical expressions **5.MD.5** Relate volume to the operations of multiplication and addition

Critical Background Knowledge from Previous Grade Levels

- Explain why addition and subtraction strategies work, using place value and the properties of operations (2.NBT.9)
- Use addition to find the total number of objects in a rectangular array (2.OA.4)
- Apply properties of operations as strategies to add and subtract (2.NBT.5, 1.OA.3, 1.NBT.4)

3.OA.5

Academic Vocabulary
multiplication, division, product, factor, dividend, divisor, quotient, commutative property of multiplication, associative property of multiplication, parentheses, distributive property of multiplication over addition, zero property of multiplication, multiplicative identity property of one, array

Suggested Models	Suggested Strategies
Commutative Property of Multiplication Model This array can be seen as rows by columns (2 x 4) or columns by rows (4 x 2). **Distributive Property of Multiplication Model** $4 \times 7 = 4 \times (5 + 2)$ $= (4 \times 5) + (4 \times 2)$ $= 20 + 8$ $= 28$ $7 = 5 + 2$ 4 $(4 \times 5) + (4 \times 2) = 4 \times 7$ $20 + 8 = 28$ **Associative Property of Multiplication Model** $(3 \times 2) \times 2$ $3 \times (2 \times 2)$ 2 groups of 3 × 2 arrays 3 groups of 2 × 2 arrays	• Use an array or grouping to model the commutative property • Model the distributive property of multiplication over addition (see Suggested Models) • Use base-ten blocks, multiplication charts, grid/graph paper, and area models

3.OA.5

Demonstrate understanding of the properties of multiplication and the relationship between multiplication and division (Standards 3.OA.5–6).

Standard 3.OA.6 Understand division as an unknown-factor problem. Understand the relationship between multiplication and division (multiplication and division are inverse operations). *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Concepts and Skills to Master

- Understand the relationship between multiplication and division as inverse operations, one operation can help solve the other
- Understand and solve unknown-factor problems
- Solve a division equation by using related multiplication facts

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.1 Interpret the products of whole numbers **3.OA.2** Interpret whole-number quotients of whole numbers **3.OA.3** Use multiplication and division within 100 to solve word problems **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers **3.OA.5** Apply properties of operations as strategies to multiply and divide **3.OA.7** Fluently multiply and divide **3.OA.8** Solve two-step word problem **3.OA.9** Identify arithmetic patterns and explain them using properties of operations **3.MD.7** Relate area to the operations of multiplication and addition	**4.NBT.6** Find whole-number quotients and remainders based on place value, the properties of operations, and the relationship between multiplication and division **5.NF.3** Interpret a fraction as division, solving real-world problems involving division of whole numbers **5.NF.6** Solve real-world problems involving multiplication of fractions and mixed numbers **5.NF.7** Apply and extend previous understandings of division to unit fractions and whole numbers

Critical Background Knowledge from Previous Grade Levels

- Use addition and subtraction with unknowns in all positions (2.OA.1)
- Use addition and subtraction within 20 to solve word problems involving situations with unknowns in all positions. (1.OA.1)
- Determine the unknown whole number in an addition and subtraction equation relating three whole numbers (1.OA.8)

Academic Vocabulary

related facts, multiplication, division, inverse operation, factor

Suggested Models	Suggested Strategies
 18 ÷ 6 = 3 3 × 6 = 18	• Use fact families and/or number bonds • Model arrays to show related multiplication and division equations (e.g., 3 × 2 = 6; 2 × 3 = 6; 6 ÷ 2 = 3; 6 ÷ 3 = 2). • Use equal groups, number lines, and area models

3.OA.6

Represent and solve problems involving multiplication and division within 100 (Standards 3.OA.1–4 and Standard 3.OA.7).

Standard 3.OA.7 Fluently multiply and divide.
a. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. *(For example, knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8).*
b. By the end of Grade 3, know from memory all products of two one-digit numbers.

Concepts and Skills to Master

- Apply multiplication and division strategies flexibly, accurately and efficiently
- Understand the inverse relationship of multiplication and division
- Understand and apply commutative and distributive properties
- Know from memory all products of two one-digit numbers

Teacher Note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms. Students develop fluency over time as they have repeated experiences that build conceptual understanding of multiplication (concrete and pictorial representations, patterns, context, etc.). Learning is enhanced when practice is organized to focus most heavily on understood but not yet fluent facts. Fluency may be reached by becoming fluent for each number (2s, 5s, etc. by noticing patterns, not through memorization) and then extending the fluency to several, then all numbers mixed together. To achieve fluency by the end of third grade, students must begin working toward fluency as early as possible. This is not a matter of instilling facts divorced from their meanings, but rather the outcome of a carefully designed learning process that heavily involves the interplay of practice and reasoning. (Adapted from: https://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf, p. 27)

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.1 Interpret the products of whole numbers **3.OA.2** Interpret whole-number quotients **3.OA.3** Use multiplication and division within 100 to solve word problems **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers **3.OA.5** Apply properties of operations as strategies to multiply and divide **3.OA.6** Understand division as an unknown-factor problem	**4.OA.4** Find all factor pairs for a whole number between 1-100 **4.NBT.5** Multiply up to four-digit numbers by one-digit numbers and two-digit numbers by two-digit numbers **4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors **5.NBT.5** Fluently multiply multi-digit whole numbers **5.NBT.6** Find whole-number quotients **4.OA.1–3, 4.NF.1–2 and 4, 5.NF.4 and 6–7** Fluency with multiplication is a foundation for extending strategies when multiplying and dividing multi-digit whole numbers, fractions, and decimals

Critical Background Knowledge from Previous Grade Levels

- See Related Standards: Current Grade Level
- Fluently add and subtract within 20 (2.OA.2)
- Work with equal groups (2.OA.4)
- Partition rectangles into squares (2.G.2)
- Apply properties of operations as strategies to add and subtract (1.OA.3)

Academic Vocabulary

product, factor, dividend, divisor, quotient, multiplication, multiply, division, divide, commutative property of multiplication, distributive property

Suggested Models	Suggested Strategies
See models listed in the Core Guide for 3.OA.3 as students work to build fluency. Area model for 3 × 4 Base ten blocks used to represent 4 × 13 **Patterns in multiples of 9** $1 \times 9 = 9$ $2 \times 9 = 2 \times (10 - 1) = (2 \times 10) - (2 \times 1) = 20 - 2 = 18$ $3 \times 9 = 3 \times (10 - 1) = (3 \times 10) - (3 \times 1) = 30 - 3 = 27,$ etc	• Model and/or count • Apply the Commutative Property • Find missing factors • Engage in number talk or math discourse • Play games for practice • Analyze multiplication by zeros and ones • Skip count (counting groups of __ and knowing how many groups have been counted) • Use doubles (2s), doubling twice (4s), doubling three times (8s) • Use tens facts (relating to place value, 5 × 10 is 5 tens or 50) • Use five facts (half of tens) • Recognize square numbers (e.g., 3 × 3) • Identify patterns in multiples of nines (10 groups less one group, e.g., 9 × 3 is 10 groups of 3 minus one group of 3) • Decompose into known facts (6 × 7 is 6 × 6 plus one more group of 6) • Use related facts (e.g., 6 × 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 × 6 = 24) • Recognize and use patterns in multiplication table

Fluently multiply and divide within 100

Knowing from memory all products of two one-digit numbers includes the following facts:

x	0	1	2	3	4	5	6	7	8	9	10
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Fluency involves a mixture of just knowing some answers, knowing some answers from patterns (for example, multiplying by one yields the same number), and knowing some answers from the use of strategies. It is important to push sensitively and encouragingly toward fluency of the designated numbers, recognizing that fluency will be a mixture of these kinds of thinking which may differ across students.

Emphasis should be placed on one-digit numbers multiplied by one-digit numbers; however, students should be exposed to a variety of problems with products less than or equal to 100. Students are expected to use concrete models and reasoning strategies to solve problems in which a two-digit number is multiplied by a one-digit with a product less than or equal to 100. Examples may include problems such as: 15 × 5 = 75, 25 × 4 = 100, 33 × 3 = 99, etc. The standard algorithm for multiplication is introduced in fifth grade in standard 5.NBT.5 and should not be taught in third grade.

Text Source: https://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf

Students use the four operations to identify and explain patterns in arithmetic (Standards 3.OA.8–9).

Standard 3.OA.8 Solve two-step word problems.
a. Solve two-step word problems using the four operations. Know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations). (Limit to problems posed with whole numbers and having whole number answers.)
b. Represent two-step problems using equations with a letter standing for the unknown quantity. Create accurate equations to match word problems.
c. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.

Concepts and Skills to Master

- Differentiate between one-step and two-step word problems (Two-step word problems may include any combination of two operations in the same problem)
- Determine the operation(s) based on the actions in the context of two-step word problems (avoid relying on keyword strategies)
- Use numbers and symbols to represent word problems (+, -, ×, ÷, =, and a letter for unknowns)
- Know that multiplication and division are performed (in the order they appear in the problem; from left to right) prior to addition and subtraction (in the order they appear in the problem; from left to right)
- Solve and apply the addition, subtraction, multiplication, and division situations listed in Standards K.OA.2, 1.OA.1, and 2.OA.1, and 3.OA.3

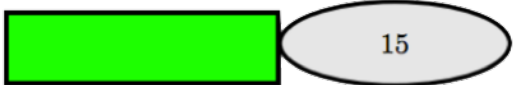| Related Standards: Current Grade Level	Related Standards: Future Grade Level
3.OA.1, 3.OA.2 Interpret products of whole numbers and whole-number quotients **3.OA.4** Determine the unknown whole number in a multiplication or division equations **3.OA.5** Apply properties of operations as strategies to multiply and divide **3.OA.6** Understand the relationship between multiplication and division **3.OA.7** Fluently multiply and divide **3.OA.8** Solve two-step word problems **3.MD.2** Multiply and divide to solve measurement word problems **3.MD.7** Relate area to multiplication	**4.OA.2** Multiply and divide to solve word problems involving multiplicative comparisons **4.OA.3** Solve multi-step word problems using whole numbers and having whole-number answers using the four operations **5.NF.4** Apply and extend previous understandings of multiplication and division to multiply and fraction or a whole number by a fraction

Critical Background Knowledge from Previous Grade Levels

- Interpret products of whole numbers and whole-number quotients (3.OA.1, 3.OA.2)
- Understand and use the associative and commutative properties
- Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends (2.OA.4)
- Partition a rectangle into rows and column of same-sized squares and count to find the total number of squares (2.G.2)
- Use addition and subtraction to solve word problems (1.OA.1, 2.OA.1)

Academic Vocabulary

Addends, sum, difference, round, estimate, equation, difference, multiplication, factors, product, array, multiples, division, divisor, dividend, quotient, reasonableness, symbol, ×, ÷, /

3.OA.8

Suggested Models	Suggested Strategies
A two-step problem with diagram showing problem situation and equations showing the two parts Carla has 4 packages of silly bands. Each package has 8 silly bands in it. Agustin is supposed to get 15 fewer silly bands than Carla. How many silly bands should Agustin get? Carla: [8 | 8 | 8 | 8] Agustin: [ ] (15) C = number of Carla's silly bands A = number of Agustin's silly bands $$C = 4 \times 8 = 32$$ $$A + 15 = C$$ $$A + 15 = 32$$ $$A = 17$$ *Students may be able to solve this problem without writing such equations.*	• Use drawings, objects, and equations • Use a bar model • Apply Part/Part/Whole • Create student-generated word problems • Skip count • Use the relationship between multiplication and division
Image Source: https://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf	

3.OA.8

Students use the four operations to identify and explain patterns in arithmetic (Standards 3.OA.8–9).

Standard 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that four times a number is always even, and explain why four times a number can be decomposed into two equal addends.*

Concepts and Skills to Master

- Recognize arithmetic patterns that can be found on a hundreds chart, a number line, an addition and a multiplication table
- Recognize multiplication patterns that can be found on a hundreds chart and a multiplication table
- Know that multiplication by an even number results in an even number
- Know that multiplication of an odd number by another odd number results in an odd number
- Know that multiplication of an odd number by an even number results in an even number
- Explain arithmetic patterns using properties of operations
- Find the products of the commutative property on the multiplication chart
- Model addition and multiplication patterns with a number line, hundreds chart, multiplication chart

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
3.OA.5 Apply properties of operations as strategies to multiply and divide	**4.OA.5** Generate number or shape patterns that follow a given rule
	5.OA.3 Generate two numerical patterns using two given rules

Critical Background Knowledge from Previous Grade Levels

- Determine whether a group of objects is odd or even (2.OA.3)
- Recognize patterns of skip counting with fives, tens, and hundreds (2.NBT.2)

Academic Vocabulary

sum, multiplication, multiples, factors, product, sequence, pattern , row, column

Suggested Models	Suggested Strategies
Highlight a given factor and discuss patterns noticed	- Use number lines - Use hundreds charts - Highlight and discuss patterns on multiplication and addition charts - Analyze patterns in basic facts **Patterns in multiples of 9** $1 \times 9 = 9$ $2 \times 9 = 2 \times (10-1) = (2 \times 10) - (2 \times 1) = 20 - 2 = 18$ $3 \times 9 = 3 \times (10-1) = (3 \times 10) - (3 \times 1) = 30 - 3 = 27,$ etc

3.OA.9

TABLE 2. Common multiplication and division situations. [1]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? ***Measurement example.*** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? ***Measurement example.*** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? ***Measurement example.*** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2]	There are 3 rows of apples with 6 apples in each row. How many apples are there?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?
AREA[3]	What is the area of a 3 cm by 6 cm rectangle?	A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE[4]	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? ***Measurement example.*** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? ***Measurement example.*** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? ***Measurement example.*** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[4] Multiplicative Compare problems appear first in Grade 4, with whole-number values in all places, and with the "times as much" language in the table. In Grade 5, unit fractions language such as "one third as much" may be used. Multiplying and unit fraction language change the subject of the comparing sentence, e.g., "A red hat costs A times as much as the blue hat" results in the same comparison as "A blue hat costs 1/A times as much as the red hat," but has a different subject.