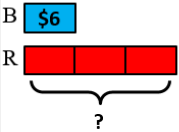
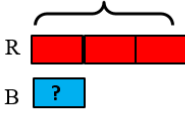
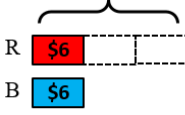


Use the four operations with whole numbers (addition, subtraction, multiplication, and division) to solve problems (Standards 4.OA.1–3).	
<b>Standard 4.OA.1</b> Interpret a multiplication equation as a comparison ( <i>for example, interpret <math>35 = 5 \times 7</math> as a statement that 35 is 5 times as many as 7 and 7 times as many as 5</i> ). Represent verbal statements of multiplicative comparisons as multiplication equations.	
Concepts and Skills to Master	
<ul style="list-style-type: none"> <li>• Distinguish between additive comparisons and multiplicative comparisons (8 can be compared to 2 additively, 8 is 6 more than 2 and can also be compared to 2 multiplicatively, 8 is 4 times as many as 2)</li> <li>• Recognize that any two factors and their product can be read as a comparison (8 is 4 times as many as 2, or 2 times as many as 4 is 8)</li> <li>• Write multiplication equations from multiplicative comparison situations</li> <li>• Represent multiplicative comparisons using a variety of models and strategies</li> </ul> <p>Teacher Note: Fourth grade is the first time that students view multiplication as a comparison. In first and second grade, students work with additive comparisons. Third grade students understand multiplication as groups or arrays of objects. In fourth grade, these understanding extend to multiplication as a comparison. Multiplicative comparison situations are more complex than equal groups and arrays, and must be carefully distinguished from additive comparison problems. This standard should be taught with Standard 4.OA.2 using the following multiplication and division situations. (See: TABLE 2. Common multiplication and division situations.)</p> <ul style="list-style-type: none"> <li>• <b>Compare/Larger Unknown</b> word problems (A blue hat costs \$3. A red hat costs 4 times as much as the blue hat. How much does the red hat cost?)</li> <li>• <b>Compare/Smaller Unknown</b> word problems (A red hat costs \$12 and that is 4 times as much as a blue hat costs. How much does a blue hat cost?)</li> <li>• <b>Compare/Multiplier Unknown</b> word problems (A red hat costs \$12 and a blue hat costs \$3. How many times as much does the red hat cost as the blue hat?)</li> </ul>	
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
<p><b>4.OA.2</b> Multiply and divide to solve word problems involving multiplicative comparisons</p> <p><b>4.OA.3</b> Solve multi-step word problems using whole numbers and having whole-number answers using the four operations</p> <p><b>4.MD.1</b> Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit</p> <p><b>4.NF.4</b> Apply and extend understanding of multiplication to multiply a fraction by a whole number</p>	<p><b>5.NF.3</b> Interpret a fraction as division of the numerator by the denominator</p> <p><b>5.NF.4, 5.NF.5</b> Apply and extend previous understandings of multiplication and division to multiply a fraction or whole number by a fraction</p> <p><b>6.RP.1</b> Understand the concept of a ratio</p>
Critical Background Knowledge from Previous Grade Levels	
<ul style="list-style-type: none"> <li>• Interpret products of whole numbers and whole-number quotients (3.OA.1, 3.OA.2)</li> <li>• Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (3.OA.3)</li> <li>• Use addition and subtraction to two-step solve word problems comparing unknowns in all positions (2.OA.1)</li> </ul>	
Academic Vocabulary	
multiply, compare, multiplicative comparison, additive comparison, equation, array, factor, product	

Suggested Models				Suggested Strategies
	<b>Unknown Product</b>	<b>Group Size Unknown</b> <small>How many in each group?</small>	<b>Number of Groups Unknown</b> <small>How many groups?</small>	<ul style="list-style-type: none"> <li>• Use bar models, number lines, equations, and context to represent multiplicative comparison</li> <li>• Use concrete models such as connecting cubes, Cuisinare Rods, etc.</li> </ul>
<b>Multiplicative Comparison</b>	 <p style="text-align: center;"><math>3 \times 6 = ?</math></p>	<p style="text-align: center;">\$18</p>  <p style="text-align: center;"><math>3 \times ? = 18</math> <math>18 \div 3 = ?</math></p>	<p style="text-align: center;">\$18</p>  <p style="text-align: center;"><math>? \times 6 = 18</math> <math>18 \div 6 = ?</math></p>	
General	$a \times b = ?$	$a \times ? = p$ , and $p \div a = ?$	$? \times b = p$ , and $p \div b = ?$	

Use the four operations (addition, subtraction, multiplication, and division) with whole numbers to solve problems (Standards 4.OA.1–3).	
<b>Standard 4.OA.2</b> Multiply or divide to solve word problems involving multiplicative comparison, <i>for example, by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.</i>	
Concepts and Skills to Master	
<ul style="list-style-type: none"> <li>Solve the following types of comparison multiplication and division situations             <ul style="list-style-type: none"> <li><b>Compare/Larger Unknown word problems</b> (A blue hat costs \$3. A red hat costs 4 times as much as the blue hat. How much does the red hat cost?)</li> <li><b>Compare/Smaller Unknown word problems</b> (A red hat costs \$12 and that is 4 times as much as a blue hat costs. How much does a blue hat cost?)</li> <li><b>Compare/Multiplier Unknown word problems</b> (A red hat costs \$12 &amp; a blue hat costs \$3. How many times as much does the red hat cost as the blue hat?)</li> </ul> </li> </ul> <p>Teacher Note: Fourth grade is the first time that students view multiplication as a comparison. In first and second grade, students work with additive comparisons. Third grade students understand multiplication as groups or arrays of objects. In fourth grade, these understanding extend to multiplication as a comparison. Multiplicative comparison situations are more complex than equal groups and arrays, and must be carefully distinguished from additive comparison problems. This standard should be taught with standard 4.OA.1.</p>	
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
<p><b>4.OA.3</b> Solve multi-step word problems using whole numbers and having whole-number answers using the four operations</p> <p><b>4.OA.1</b> Interpret multiplication as comparison</p> <p><b>4.MD.1</b> Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit</p> <p><b>4.NF.4</b> Apply and extend understanding of multiplication to multiply a fraction by a whole number</p> <p><b>4.NF.1</b> Extend understanding of fraction equivalence and ordering</p>	<p><b>5.NF.3</b> Interpret a fraction as division of the numerator by the denominator</p> <p><b>5.NF.4</b> Apply and extend previous understandings of multiplication and division to multiply a fraction or a whole number by a fraction</p> <p><b>5.NF.5</b> Interpret multiplication as scaling</p> <p><b>5.OA.2</b> Write and interpret numerical expressions</p> <p><b>5.NF.6</b> Extend previous understanding of multiplication and division to multiply and divide fractions</p> <p><b>6.RP.1</b> , <b>6.RP.2.</b> Understand the concept of a ratio and solve ratio problems</p>
Critical Background Knowledge from Previous Grade Levels	
<ul style="list-style-type: none"> <li>Interpret products of whole numbers and whole-number quotients (3.OA.1, 3.OA.2)</li> <li>Use addition and subtraction to word problems comparing with unknowns in all positions (2.OA.1)</li> </ul>	
Academic Vocabulary	
multiplicative comparison, additive comparison	
Suggested Strategies	
<ul style="list-style-type: none"> <li>Use bar models, number lines, equations, and context to represent multiplicative comparison</li> <li>Use concrete models such as connecting cubes, Cuisinare Rods, etc.</li> </ul>	

Suggested Models

**Tape diagram used to solve the Compare problem in Table 3**

$B$  is the cost of a blue hat in dollars

$R$  is the cost of a red hat in dollars

$\$6$

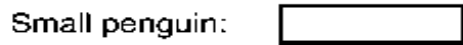
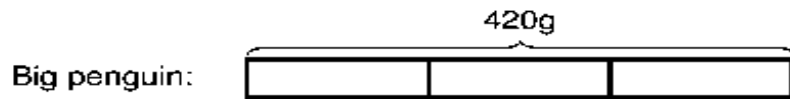
$$3 \times B = R$$

$\$6$   $\$6$   $\$6$

$$3 \times \$6 = \$18$$

**A tape diagram used to solve a Compare problem**

A big penguin will eat 3 times as much fish as a small penguin. The big penguin will eat 420 grams of fish. All together, how much will the two penguins eat?



$B$  = number of grams the big penguin eats  
 $S$  = number of grams the small penguin eats

$$3 \cdot S = B$$


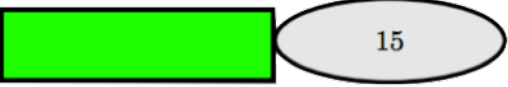
$$3 \cdot S = 420$$

$$S = 140$$

$$S + B = 140 + 420 = 560$$

	Unknown Product	Group Size Unknown <small>How many in each group?</small>	Number of Groups Unknown <small>How many groups?</small>
Multiplicative Comparison	$3 \times 6 = ?$	$3 \times ? = 18$ $18 \div 3 = ?$	$? \times 6 = 18$ $18 \div 6 = ?$
General	$a \times b = ?$	$a \times ? = p$ , and $p \div a = ?$	$? \times b = p$ , and $p \div b = ?$

Use the four operations with whole numbers (addition, subtraction, multiplication, and division) to solve problems (Standards 4.OA.1–3).	
<p><b>Standard 4.OA.3</b> Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.</p> <p><b>a.</b> Represent these problems using equations with a letter standing for the unknown quantity.</p> <p><b>b.</b> Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.</p>	
Concepts and Skills to Master	
<ul style="list-style-type: none"> <li>• Differentiate between one-step and two-step word problems (Two-step word problems may include any combination of two operations in the same problem)</li> <li>• Determine the operation(s) based on the actions in the context of multi-step word problems (avoid relying on keyword strategies)</li> <li>• Use numbers and symbols to represent word problems (+, -, ×, ÷, =, and a letter for unknowns)</li> <li>• Know that multiplication and division are performed (in the order they appear in the problem; from left to right) prior to addition and subtraction (in the order they appear in the problem; from left to right)</li> <li>• Interpret remainders when necessary.</li> <li>• Determine the reasonableness of the calculated answer using mental computation and estimation strategies</li> </ul>	
Related Standards: Current Grade Levels	Related Standards: Future Grade Levels
<p><b>4.OA.2</b> Multiply and divide to solve word problems involving multiplicative comparisons</p> <p><b>4.OA.3</b> Solve multi-step word problems using whole numbers and having whole-number answers using the four operations</p> <p><b>4.NBT.3</b> Use place value understanding to round multi-digit whole numbers to any place</p> <p><b>4.NBT.4</b> Fluently add and subtract multi-digit whole numbers using the standard algorithm</p> <p><b>4.NBT.5</b> Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers</p> <p><b>4.NBT.6</b> Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division</p>	<p><b>5.NF.2</b> Solve real-world problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers</p> <p><b>5.NF.6</b> Solve real-world problems involving multiplication of fractions and mixed numbers</p>
Critical Background Knowledge from Previous Grade Levels	
<ul style="list-style-type: none"> <li>• Interpret products of whole numbers and whole-number quotients (3.OA.1, 3.OA.2)</li> <li>• Determine the unknown whole number in a multiplication or division equations (3.OA.4)</li> <li>• Apply properties of operations as strategies to multiply and divide (3.OA.5)</li> <li>• Understand the relationship between multiplication and division (3.OA.6)</li> <li>• Fluently multiply and divide (3.OA.7)</li> <li>• Represent &amp; solve two-step problems using equations with a letter standing for the unknown (3.OA.8)</li> <li>• Use addition and subtraction within 100 to solve one-step and two-step problems (2.OA.2)</li> </ul>	
Academic Vocabulary	
multi-step word problem, mental math, estimation, rounding, remainder, variable, operations, equation, reasonableness, inverse operations, multiplicative comparison, additive comparison, symbol	

Suggested Models	Suggested Strategies
<p><b>A two-step problem with diagram showing problem situation and equations showing the two parts</b></p> <p>Carla has 4 packages of silly bands. Each package has 8 silly bands in it. Agustin is supposed to get 15 fewer silly bands than Carla. How many silly bands should Agustin get?</p> <p>Carla: </p> <p>Agustin: </p> <p><math>C</math> = number of Carla's silly bands <math>A</math> = number of Agustin's silly bands</p> $C = 4 \times 8 = 32$ $A + 15 = C$ $A + 15 = 32$ $A = 17$ <p><i>Students may be able to solve this problem without writing such equations.</i></p>	<ul style="list-style-type: none"><li>• Use drawings, objects, and equations</li><li>• Use a bar model</li><li>• Apply Part/Part/Whole</li><li>• Create student-generated word problems</li><li>• Skip count</li><li>• Use the relationship between multiplication and division</li></ul>
Image Source: <a href="https://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oe_k5_2011_05_302.pdf">https://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oe_k5_2011_05_302.pdf</a>	

**TABLE 2.** Common multiplication and division situations. <sup>1</sup>

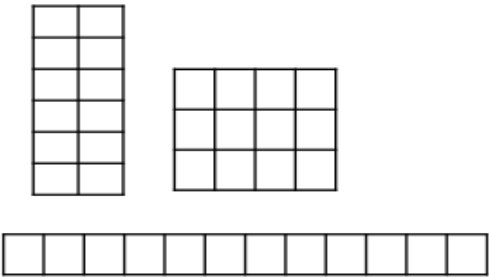

	Unknown Product	Group Size Unknown <i>("How many in each group?" Division)</i>	Number of Groups Unknown <i>("How many groups?" Division)</i>
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
<b>EQUAL GROUPS</b>	There are 3 bags with 6 plums in each bag. How many plums are there in all?  <b>Measurement example.</b> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?  <b>Measurement example.</b> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed?  <b>Measurement example.</b> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
<b>ARRAYS<sup>2</sup></b>	There are 3 rows of apples with 6 apples in each row. How many apples are there?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?
<b>AREA<sup>3</sup></b>	What is the area of a 3 cm by 6 cm rectangle?	A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
<b>COMPARE<sup>4</sup></b>	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?  <b>Measurement example.</b> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?  <b>Measurement example.</b> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?  <b>Measurement example.</b> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
<b>GENERAL</b>	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

<sup>1</sup> The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

<sup>2</sup> The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

<sup>3</sup> Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

<sup>4</sup> Multiplicative Compare problems appear first in Grade 4, with whole-number values in all places, and with the "times as much" language in the table. In Grade 5, unit fraction language such as "one third as much" may be used. Multiplying and unit fraction language change the subject of the comparing sentence, e.g., "A red hat costs A times as much as the blue hat" results in the same comparison as "A blue hat costs 1/A times as much as the red hat," but has a different subject.

Gain familiarity with factors and multiples (Standard 4.OA.4).	
<b>Standard 4.OA.4</b> Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	
Concepts and Skills to Master	
<ul style="list-style-type: none"> <li>• Understand factor pairs as two whole numbers that multiply together to get one product</li> <li>• Understand that prime numbers have exactly one factor pair</li> <li>• Understand that composite numbers have more than one factor pair</li> <li>• Understand multiples as a product of two given whole numbers.</li> <li>• List the multiples of the numbers 2 through 9 up to 100</li> <li>• Create a list or chart of factor pairs of whole numbers 1-100</li> <li>• Identify, from a list or chart, which whole numbers are prime or composite</li> </ul> <p>Teacher Note: The number 1 is neither prime nor composite. A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors.</p>	
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
<p><b>4.NBT.5</b> Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers</p> <p><b>4.NBT.6</b> Find whole-number quotients with up to four-digit divisors and one-digit dividends</p>	<p><b>6.NS.4</b> Find the greatest common factor and least common multiple of two whole numbers</p>
Critical Background Knowledge from Previous Grade Levels	
<ul style="list-style-type: none"> <li>• Determine unknown whole numbers in multiplication and division equations (3.OA.4)</li> <li>• Understand the relationship between multiplication and division (3.OA.6)</li> <li>• Fluently multiply and divide (3.OA.7)</li> <li>• Identify and explain arithmetic patterns in multiplication and addition tables (3.OA.9)</li> </ul>	
Academic Vocabulary	
factor, factor pairs, multiple, prime, composite, whole number	
Suggested Models	Suggest Strategies
<p>The number 12 can be made into several different rectangular arrays (<math>1 \times 12</math>, <math>3 \times 4</math>, <math>6 \times 2</math>) and is therefore a composite number.</p> 	<p>The number 7 can only be made into one rectangular array and is therefore a prime number.</p> 
	<ul style="list-style-type: none"> <li>• Use counters to build rectangular arrays</li> <li>• Use tools such as number lines, hundreds charts, arrays, or cubes to model relationships between factors and multiples</li> </ul>



Generate and analyze numeric and shape patterns (Standard 4.OA.5).

**Standard 4.OA.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Concepts and Skills to Master

- Understand that number and shape patterns follow a given rule
- Understand that there are sometimes features of the pattern that are not stated in the rule
- Complete a given number or shape pattern (e.g., 3, 6, 9, \_\_, \_\_, 18)
- Determine the rule of a given pattern (e.g., 3, 6, 9, 12, 15, 18, ... The rule is to skip count by 3 or multiples of 3)
- Generate a number or shape pattern that follows a rule and state the rule
- Identify and state any alternate features of the pattern that are not stated in the rule
- Extend a given number or shape pattern using manipulatives, skip counting, pictures, etc.

Teacher Note: Although students find the given rule to a pattern, they are not required to use variables to explain or write the related expression. Emphasis should be placed on patterns with one-step; although, students may be exposed to patterns with two-steps.

Related Standards: Current Grade Level

**4.OA.3** Solve multi-step word problems and represent these problems with equations that use variables

Related Standards: Future Grade Levels

**5.OA.3** Generate two numerical patterns using two given rules, identify relationships between corresponding terms, and form ordered pairs for graphing on a coordinate plane  
**6.EE.2** Write, read, and evaluate expressions in which letters represent numbers  
**6.EE.9** Use variables to represent two quantities that change in relationship to one another and write expressions to express one quantity in terms of the other quantity  
**6.RP.1** Understand the concept of a ratio and use ratio language to describe a relationship between two quantities

Critical Background Knowledge from Previous Grade Levels

- Identify and explain arithmetic patterns (3.OA.9)
- Determine whether a group of objects (up to 20) has an odd or even number of members (2.OA.3)

Academic Vocabulary

number pattern, shape pattern, pattern rule, sequence, input/output table

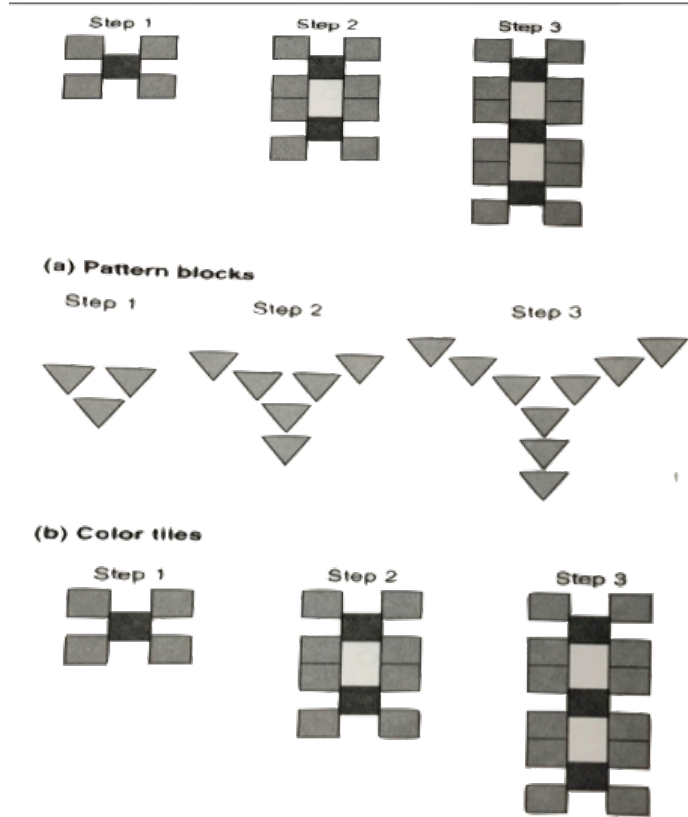
Suggested Models

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

What is the rule of the pattern?  
*Start at 4 and add 3*

What do you notice about this pattern?  
*even, odd, even, odd, even, odd, ...*  
*It looks like stairs with two white steps in between*

Write a context or story to match this pattern.  
*It costs \$4 to play a game and \$3 for each additional game*



Suggested Strategies

- Create and describe patterns in number charts
- Create and describe patterns using pattern blocks, colored tiles, cubes paper squares, etc.
- Create and work with tables
- Use input-output tables
- Write sequences forwards and backwards
- Predict terms that come later in given patterns