SECONDARY MATHEMATICS I

THE FUNDAMENTAL PURPOSE OF SECONDARY MATHEMATICS I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Secondary Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

CRITICAL AREA 1: By the end of eighth grade, students have had a variety of experiences working with expressions and creating equations. Students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

CRITICAL AREA 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. Students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs, and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas, as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

CRITICAL AREA 3: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This area builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

CRITICAL AREA 4: This area builds upon students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

CRITICAL AREA 5: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions (translations, reflections, and rotations) and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

CRITICAL AREA 6: Building on their work with the Pythagorean Theorem in eighth grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in Secondary Mathematics I describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

- **Standard SI.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard SI.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard SI.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard SI.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard SI.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard SI.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

■ **Standard SI.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see* $5 - 3(x - y)^2$ *as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

■ **Standard SI.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—Quantities (N.Q)

Reason quantitatively and use units to solve problems. Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions **(Standards N.Q.1–3).**

■ **Standard N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

■ **Standard N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

■ **Standard N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Strand: ALGEBRA—Seeing Structure in Expressions (A.SSE)

Interpret the structure of expressions **(Standard A.SSE.1).**

■ **Standard A.SSE.1** Interpret linear expressions and exponential expressions with integer exponents that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of* P *and a factor not depending on* P.

Strand: ALGEBRA—Creating Equations (A.CED)

Create equations that describe numbers or relationships. Limit these to linear equations and inequalities, and exponential equations. In the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs **(Standards A.CED.1–4).**

■ **Standard A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and simple exponential functions.

■ **Standard A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

■ **Standard A.CED.3** Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

■ **Standard A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's Law* V = IR *to highlight resistance* R.

Strand: ALGEBRA—Reasoning With Equations and Inequalities (A.REI)

Understand solving equations as a process of reasoning and explain the reasoning **(Standard A.REI.1).** Solve equations and inequalities in one variable **(Standard A.REI.3).** Solve systems of equations. Build on student experiences graphing and solving systems of linear equations from middle school. Include cases where the two equations describe the same line—yielding infinitely many solutions—and cases where two equations describe parallel lines—yielding no solution; connect to GPE.5, which requires students to prove the slope criteria for parallel lines **(Standards A.REI.5–6).** Represent and solve equations and inequalities graphically **(Standards A.REI.10–12).**

■ **Standard A.REI.1** Explain each step in solving a linear equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Students will solve exponential equations with logarithms in Secondary Mathematics III.

■ **Standard A.REI.3** Solve equations and inequalities in one variable.

a. Solve one-variable equations and literal equations to highlight a variable of interest.

b. Solve compound inequalities in one variable, including absolute value inequalities.

c. Solve simple exponential equations that rely only on application of the laws of exponents (limit solving exponential equations to those that can be solved without logarithms). *For example,* $5^x = 125$ *or* $2^x = 1/16$.

■ **Standard A.REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

■ **Standard A.REI.6** Solve systems of linear equations exactly and approximately (numerically, algebraically, graphically), focusing on pairs of linear equations in two variables.

■ **Standard A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

- **Standard A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately; e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear and exponential functions.★

- **Standard A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Strand: FUNCTIONS—Interpreting Linear and Exponential Functions (F.IF)

Understand the concept of a linear or exponential function and use function notation. Recognize arithmetic and geometric sequences as examples of linear and exponential functions **(Standards F.IF.1–3).** Interpret linear or exponential functions that arise in applications in terms of a context **(Standards F.IF.4–6).** Analyze linear or exponential functions using different representations **(Standards F.IF.7,9).**

- **Standard F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.

- **Standard F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- **Standard F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

- **Standard F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*★

- **Standard F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

- **Standard F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

- **Standard F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear functions and show intercepts.

e. Graph exponential functions, showing intercepts and end behavior.

■ **Standard F.IF.9** Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, compare the growth of two linear functions, or two exponential functions such as* $y=3^n$ *and* $y=100\cdot2^n$.

Strand: FUNCTIONS—Building Linear or Exponential Functions (F.BF)

Build a linear or exponential function that models a relationship between two quantities **(Standards F.BF.1–2).** Build new functions from existing functions (**Standard F.BF.3**).

■ **Standard F.BF.1** Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

■ **Standard F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions. ★

■ **Standard F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, for specific values of k (both positive and negative); find the value of k given the graphs. Relate the vertical translation of a linear function to its y-intercept. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Strand: FUNCTIONS—Linear and Exponential (F.LE)

Construct and compare linear and exponential models and solve problems **(Standards F.LE.1–3).** Interpret expressions for functions in terms of the situation they model. **(Standard F.LE.5).**

■ **Standard F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

- **Standard F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **Standard F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly.
- **Standard F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context. Limit exponential functions to those of the form $f(x) = b^x + k$.

Strand: GEOMETRY—Congruence (G.CO)

Experiment with transformations in the plane. Build on student experience with rigid motions from earlier grades **(Standards G.CO.1–5).** Understand congruence in terms of rigid motions. Rigid motions are at the foundation of the definition of congruence. Reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems **(Standards G.CO.6–8).** Make geometric constructions **(Standards G.CO.12–13).**

- **Standard G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **Standard G.CO.2** Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- **Standard G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- **Standard G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- **Standard G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Point out the basis of rigid motions in geometric concepts, for example, translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.
- **Standard G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use

the definition of congruence in terms of rigid motions to decide whether they are congruent.

- **Standard G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- **Standard G.CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
- **Standard G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Emphasize the ability to formalize and defend how these constructions result in the desired objects. *For example, copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
- **Standard G.CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Emphasize the ability to formalize and defend how these constructions result in the desired objects.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G.GPE)

Use coordinates to prove simple geometric theorems algebraically **(Standards G.GPE.4–5, 7).**

- **Standard G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*
- **Standard G.GPE.5** Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
- **Standard G.GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles; connect with The Pythagorean Theorem and the distance formula.★

Strand: STATISTICS AND PROBABILITY—Interpreting Categorical and Quantitative Data (S.ID)

Summarize, represent, and interpret data on a single count or measurement variable **(Standards S.ID.1–3).** Summarize, represent, and interpret data on two categorical and quantitative variables (**Standard S.ID.6).** Interpret linear models building on students' work with linear relationships, and introduce the correlation coefficient **(Standards S.ID.7–9).**

- **Standard S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).
- **Standard S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- **Standard S.ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Calculate the weighted average of a distribution and interpret it as a measure of center.
- **Standard S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
  - **a.** Fit a linear function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions, or choose a function suggested by the context. Emphasize linear and exponential models.
  - **b.** Informally assess the fit of a function by plotting and analyzing residuals. Focus on situations for which linear models are appropriate.
  - **c.** Fit a linear function for scatter plots that suggest a linear association.
- **Standard S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **Standard S.ID.8** Compute (using technology) and interpret the correlation coefficient of a linear fit.
- **Standard S.ID.9** Distinguish between correlation and causation.

SECONDARY MATHEMATICS I—HONORS STANDARDS

Strand: NUMBER AND QUANTITY: VECTOR AND MATRIX QUANTITIES (N.VM)

Represent and model with vector quantities **(Standards N.VM.1–3).** Perform operations on vectors **(Standards N.VM.4–5).** Perform operations on matrices and use matrices in applications **(Standards N.VM.6–13).**

- **Standard N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, $|v|$, $||v||$, v).

- **Standard N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

- **Standard N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors.

- **Standard N.VM.4** Add and subtract vectors.

  a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

  b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

  c. Understand vector subtraction $v – w$ as $v + (–w)$, where $–w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

- **Standard N.VM.5** Multiply a vector by a scalar.

  a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x , v_y) = (cv_x , cv_y)$.

  b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against vs (for $c < 0$).

- **Standard N.VM.6** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

- **Standard N.VM.7** Multiply matrices by scalars to produce new matrices, e.g., as when all of the pay-offs in a game are doubled.

- **Standard N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions.
- **Standard N.VM.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- **Standard N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
- **Standard N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
- **Standard N.VM.12** Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
- **Standard N.VM.13** Solve systems of linear equations up to three variables using matrix row reduction.