SECONDARY MATHEMATICS II

THE FOCUS OF SECONDARY MATHEMATICS II is on quadratic expressions, equations, and functions and on comparing their characteristics and behavior to those of linear and exponential relationships from Secondary Mathematics I as organized into six critical areas, or units. The need for extending the set of rational numbers arises, and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

CRITICAL AREA 1: Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

CRITICAL AREA 2: Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

CRITICAL AREA 3: Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

CRITICAL AREA 4: Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

CRITICAL AREA 5: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

CRITICAL AREA 6: Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in Secondary Mathematics II describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

- **Standard SII.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard SII.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard SII.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard SII.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard SII.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard SII.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

- **Standard SII.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see* $5 - 3(x - y)^2$ *as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

- **Standard SII.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—The Real Number System (N.RN)

Extend the properties of exponents to rational exponents **(Standards N.RN.1–2).** Use properties of rational and irrational numbers **(Standard N.RN. 3).**

- **Standard N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*

- **Standard N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

- **Standard N.RN.3** Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational. Connect to physical situations (e.g., finding the perimeter of a square of area 2).

Strand: NUMBER AND QUANTITY—The Complex Number System (N.CN)

Perform arithmetic operations with complex numbers **(Standards N.CN.1–2).** Use complex numbers in polynomial identities and equations **(Standards N.CN.7–9).**

- **Standard N.CN.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

- **Standard N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Limit to multiplications that involve i^2 as the highest power of i.

- **Standard N.CN.7** Solve quadratic equations with real coefficients that have complex solutions.

- **Standard N.CN.8** Extend polynomial identities to the complex numbers. Limit to quadratics with real coefficients. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.

- **Standard N.CN.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Strand: ALGEBRA—Seeing Structure in Expression (A.SSE)

Interpret the structure of expressions **(Standards A.SSE.1–2).** Write expressions in equivalent forms to solve problems, balancing conceptual understanding and procedural fluency in work with equivalent expressions **(Standard A.SSE.3).**

- **Standard A.SSE.1** Interpret quadratic and exponential expressions that represent a quantity in terms of its context.★
    - **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
    - **b.** Interpret increasingly more complex expressions by viewing one or more of their parts as a single entity. Exponents are extended from the integer exponents to rational exponents focusing on those that represent square or cube roots.
- **Standard A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
- **Standard A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *For example, development of skill in factoring and completing the square goes hand in hand with understanding what different forms of a quadratic expression reveal.* ★
    - **a.** Factor a quadratic expression to reveal the zeros of the function it defines.
    - **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
    - **c.** Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Strand: ALGEBRA—Arithmetic With Polynomials and Rational Expressions (A.APR)

Perform arithmetic operations on polynomials. Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of *x* **(Standard A.APR.1).**

- **Standard A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Strand: ALGEBRA—Creating Equations (A.CED)

Create equations that describe numbers or relationships. Extend work on linear and exponential equations to quadratic equations **(Standards A.CED.1–2, 4).**

- **Standard A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- **Standard A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **Standard A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations; extend to formulas involving squared variables. *For example, rearrange the formula for the volume of a cylinder* $V = \pi r^2 h$.

Strand: ALGEBRA—Reasoning With Equations and Inequalities (A.REI)

Solve equations and inequalities in one variable **(Standard A.REI.4).** Solve systems of equations. Extend the work of systems to include solving systems consisting of one linear and one nonlinear equation **(Standard A.REI.7).**

- **Standard A.REI.4** Solve quadratic equations in one variable.
  - **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
  - **b.** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
- **Standard A.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.

Strand: FUNCTIONS—Interpret Functions (F.IF)

Interpret quadratic functions that arise in applications in terms of a context **(Standards F.IF.4–6).** Analyze functions using different representations **(Standards F.IF.7–9).**

- **Standard F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*★

- **Standard F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on quadratic functions; compare with linear and exponential functions. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

- **Standard F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

- **Standard F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

  a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

  b. Graph piecewise-defined functions and absolute value functions. Compare and contrast absolute value and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions.

- **Standard F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

  a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

  b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^{t}$, $y = (0.97)^{t}$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

- **Standard F.IF.9** Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored. *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Strand: FUNCTIONS—Building Functions (F.BF)

Build a function that models a relationship between two quantities **(Standard F.BF.1).** Build new functions from existing functions **(Standard F.BF.3).**

- **Standard F.BF.1** Write a quadratic or exponential function that describes a relationship between two quantities.★

  a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

- **Standard F.BF.3** Identify the effect on the graph of replacing *f(x) by f(x) + k, k f(x), f(kx), and f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Focus on quadratic functions and consider including absolute value functions. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Strand: FUNCTIONS—Linear, Quadratic, and Exponential Models (F.LE)

Construct and compare linear, quadratic, and exponential models and solve problems **(Standard F.LE.3).**

- **Standard F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Compare linear and exponential growth to quadratic growth.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Prove and apply trigonometric identities. Limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean Theorem and the distance formula **(Standard F.TF.8).**

- **Standard F.TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.

Strand: GEOMETRY—Congruence (G.CO)

Prove geometric theorems. Encourage multiple ways of writing proofs, such as narrative paragraphs, flow diagrams, two-column format, and diagrams without words. Focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning **(Standards G.CO.9–11).**

- **Standard G.CO.9** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

- **Standard G.CO.10** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

- **Standard G.CO.11** Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Strand: GEOMETRY—Similarity, Right Triangles, and Trigonometry (G.SRT)

Understand similarity in terms of similarity transformations **(Standards G.SRT.1–3).** Prove theorems involving similarity **(Standards G.SRT.4–5).** Define trigonometric ratios and solve problems involving right triangles **(Standards G.SRT.6–8).**

- **Standard G.SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor.
  - **a.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
  - **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- **Standard G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide whether they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- **Standard G.SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
- **Standard G.SRT.4** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally and conversely; the Pythagorean Theorem (proved using triangle similarity).*
- **Standard G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- **Standard G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- **Standard G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles.
- **Standard G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Strand: GEOMETRY—Circles (G.C)

Understand and apply theorems about circles **(Standard G.C.1–4).** Find arc lengths and areas of sectors of circles. Use this as a basis for introducing the radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course **(Standard G.C.5).**

- **Standard G.C.1** Prove that all circles are similar.

- **Standard G.C.2** Identify and describe relationships among inscribed angles, radii, and chords. *Relationships include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
- **Standard G.C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- **Standard G.C.4** Construct a tangent line from a point outside a given circle to the circle.
- **Standard G.C.5** Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G.GPE)

Translate between the geometric description and the equation for a conic section **(Standard G.GPE.1).** Use coordinates to prove simple geometric theorems algebraically. Include simple proofs involving circles **(Standard G.GPE.4).** Use coordinates to prove simple geometric theorems algebraically **(Standard G.GPE.6).**

- **Standard G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- **Standard G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*
- **Standard G.GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Explain volume formulas and use them to solve problems **(Standards G.GMD.1, 3).**

- **Standard G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Informal arguments for area formulas can make use of the way in which area scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
- **Standard G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Informal arguments for volume formulas can make use of the way in which volume scale under similarity transformations: when one figure results from

another by applying a similarity transformation, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k. ★

Strand: STATISTICS—Interpreting Categorical and Quantitative Data (S.ID)

Summarize, represent, and interpret data on two categorical or quantitative variables **(Standard S.ID.5).**

- **Standard S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and condition reltive frequencies). Recognize possible associations and trends in the date.

Strand: STATISTICS—Conditional Probability and the Rules of Probability (S.CP)

Understand independence and conditional probability and use them to interpret data **(Standards S.CP.1, 4–5).** Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standard S.CP.6).**

- **Standard S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

- **Standard S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

- **Standard S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are are a smoker with the chance of being a smoker if you have lung cancer.*

- **Standard S.CP.6** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.

SECONDARY MATHEMATICS II—HONORS STANDARDS

Strand: NUMBER AND QUANTITY—Complex Number System (N.CN)

Perform arithmetic operations with complex numbers **(Standard N.CN.3).** Represent complex numbers and their operations on the complex plane **(Standards N.CN.4–5).**

- **Standard N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- **Standard N.CN.4** Represent complex numbers on the complex plane in rectangular form, and explain why the rectangular form of a given complex number represents the same number.
- **Standard N.CN.5** Represent addition, subtraction, and multiplication geometrically on the complex plane; use properties of this representation for computation. *For example, (-1 + √3 i)3 = 8 because (-1 + √3 i) has modulus 2 and argument 120°.*

Strand: ALGEBRA—Reasoning With Equations and Inequalities (A.REI)

Solve systems of equations **(Standards A.REI.8–9).**

- **Standard A.REI.8** Represent a system of linear equations as a single-matrix equation in a vector variable.
- **Standard A.REI.9** Find the inverse of a matrix if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Analyze functions using different representations **(Standards F.IF.10–11).**

- **Standard F.IF.10** Use sigma notation to represent the sum of a finite arithmetic or geometric series.
- **Standard F.IF.11** Represent series algebraically, graphically, and numerically.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G-GPE)

Translate between the geometric description and the equation for a conic section **(Standards G.GPE.2–3).**

- **Standard G.GPE.2** Derive the equation of a parabola given a focus and directrix.
- **Standard G.GPE.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Strand: STATISTICS AND PROBABILITY—Conditional Probability and the Rules of Probability (S.CP)

Understand independence and conditional probability and use them to interpret data **(Standards S.CP.2–3).** Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standards S.CP.7–8).**

- **Standard S.CP.2** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- **Standard S.CP.3** Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of A and *B* as saying that the conditional probability of *B* given *A* is the same as the probability of *B*.
- **Standard S.CP.7** Apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A and B)*, and interpret the answer in terms of the model.
- **Standard S.CP.8** Apply the general Multiplication Rule in a uniform probability model, *P(A and B) = P(A)P(B|A) = P(B)P(A|B)*, and interpret the answer in terms of the model.