Extend understanding of equivalence and ordering of fractions (Standards 4.NF.1-2). Denominators for fourth grade are limited to $2,3,4,5,6,8,10,12$, and 100.

Standard 4.NF. 1 Explain why a fraction $a / b$ is equivalent to a fraction ( $n \times a) /(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

## Concepts and Skills to Master

- Understand equivalent fractions as the same quantity with different names based on the number and size of the parts
- Recognize and explain how multiplying the numerator and denominator of a fraction by the same number, $n$, corresponds physically to partitioning each unit fraction piece into $n$ smaller equal pieces (see model below)
- Explain how the principle of the multiplicative identity property of 1 transforms a fraction into an equivalent fraction and generate equivalent fractions using this principle (Students may, but need not, use the formal term for this property)

Teacher Note: Limit denominators to 2, 3, 4, 5, 6, 8, 10, 12, and 100.

## Related Standards: Current Grade Level

4 NF. 2 Compare and order fractions by generating equivalent fractions
4.NF.3c Replace mixed numbers with equivalent fractions
4.NF. 5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100

## Related Standards: Future Grade Levels

5.NF. 1 and 2 Add and subtract with unlike denominators
5.NF. 3 Interpret a fraction as division of the numerator by the denominator.
5.NF.5b Relate the principle of fraction equivalence
6.RP. 3 Generate equivalent ratios and compare ratios

## Critical Background Knowledge from Previous Grade Levels

- Represent whole numbers as equivalent fractions (3/3 = 1 and $4 / 1=4$ ) (3.NF.3)
- Understand equivalent fractions as the same quantity with different names (3.NF.3)
- Denominators are limited to 2, 3, 4, 6, and 8 in third grade


## Academic Vocabulary

Fraction, equivalent fraction, numerator, denominator,
Denominators new to fourth grade: fifths $(1 / 5)$, tenths $(1 / 10)$, twelfths $(1 / 12)$, hundredths $(1 / 100)$

## Suggested Models

Visual models may include: area of various shapes (circles, rectangles, etc.), bar models, number lines, and double number lines

$\frac{1}{2} \quad \frac{2}{4}=\frac{2 \times 1}{2 \times 2}$

$\frac{3}{6}=\frac{3 \times 1}{3 \times 2}$

$\frac{4}{8}=4 \times 1$
$\frac{4}{8}=\frac{4 \times 1}{4 \times 2}$


$\frac{4}{3}$ is 4 parts when each part is $\frac{1}{3}$, and we want to see that this is also $5 \times 4$ parts when each part is $\frac{1}{5 \times 3}$. Divide each of the intervals of length $\frac{1}{3}$ into 5 parts of equal length. There are $5 \times 3$ parts of equal length in the unit interval, and $\frac{4}{3}$ is $5 \times 4$ of these. Therefore $\frac{4}{3}=\underset{5 \times 4}{5 \times 4}=\frac{20}{15}$.

## Suggested Strategies

- Use a variety of visual area and linear fraction models to recognize and generate equivalent fractions.
- Explain connections between models and multiplying the numerator and denominator by the same number.
- Connect visual models to the multiplicative relationships of the numerators and denominators

Images Source: http://commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf
4.NF. 1

Extend understanding of equivalence and ordering of fractions (Standards 4.NF.1-2). Denominators for fourth grade are limited to $2,3,4,5,6,8,10,12$, and 100.

Standard 4.NF. 2 Compare two fractions with different numerators and different denominators, for example, by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1 / 2$. Recognize that comparisons are valid only when the two fractions refer to the same whole.
Record the results of comparisons with symbols $>,=$, or $<$, and justify the conclusions, for example, by using a visual fraction model.

## Concepts and Skills to Master

- Understand equivalent fractions as the same quantity with different names
- Understand comparisons are only valid when the two fractions refer to the same whole
- Compare fractions by reasoning that as the number of equal parts in a whole (denominator) increases, the size of the fractional parts decreases and that as the number of equal parts being considered (numerator) increases, the total amount being considered increases
- Compare a fraction to a benchmark fraction of $1 / 2$ and 1 whole
- Compare fractions through creating common numerators, multiplying at least one fraction by applying the multiplicative identity property of 1 (any number multiplied by one remains unchanged)
- Compare fractions through creating common denominators, multiplying at least one fraction by applying the multiplicative identity property of 1

Teacher Note: While denominators at this grade level are limited to $2,3,4,5,6,8,10,12$, and 100 , students may be exposed to other denominators based on strategies used to find common denominators.

Related Standards: Current Grade Level
4.NF. 1 Recognize and generate equivalent fractions
4.NF. 7 Compare two decimals to hundredths by reasoning about their
sizes
Critical Background Knowledge from Previous Grade Levels

- Compare two fractions with the same numerator or the same denominator by reasoning about their size (3.NF.3d)
- Denominators are limited to $2,3,4,6$, and 8 in third grade
- Understand that decomposing into more equal shares creates smaller shares (1.G.3)


## Academic Vocabulary

benchmark fraction (half, whole), numerator, denominator, equivalent fractions, compare, $>,=,<$, equal shares/parts, fraction greater than 1 , fractional form Denominators new to fourth grade: fifths (1/5), tenths (1/10), twelfths (1/12), hundredths (1/100)


Students reason using benchmarks such as $1 / 2$ and 1. For example, they see that $7 / 8$ is less than $13 / 12$ because $7 / 8$ is less than 1 (and is therefore to the left of 1 on a number line), but $13 / 12$ is greater than 1 (and is therefore to the right of 1 on a number line).

When using the benchmark fraction of $\frac{1}{2}$ to compare $\frac{4}{6}$ and $\frac{5}{8}$ you can use bar models such as these:

$\frac{4}{6}$ is $\frac{1}{6}$ larger than $\frac{1}{2}$, while $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}$. Since $\frac{1}{6}$ is larger than $\frac{1}{8}, \frac{4}{6}$ is the greater fraction.

Area model:
The first cake has more left over. The second cake has $5 / 12$ left which is smaller than $1 / 2$.


## Suggested Strategies

- Use a variety of visual fraction models, such as area models and number lines to compare fractions
- Use benchmark fractions to compare fractions
- If fractions are the same number of pieces from a whole, compare the size of the missing pieces. This creates opportunities to reason and create common numerators
- Create equivalent fractions that have common numerators, and then compare the denominators or the size of the fractional parts to compare the fractions (Example: Have students explore ways of comparing $2 / 3$ and $4 / 5$ by finding a common numerator.)
- Create equivalent fractions that have common denominators, and then compare the numerators or number of parts being considered (Example: have students explore ways of comparing $2 / 3$ and $4 / 5$ by finding a common denominator.)

Images Sources: http://commoncoretools.me/wp-content/uploads/2011/08/ccss progression nf 352013 09 19.pdf,
http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/4.pdf

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers (Standards 4.NF.3-4). Denominators for fourth grade are limited to $2,3,4,5,6,8,10,12$, and 100.
Standard 4.NF. 3 Understand a fraction $a / b$ with $a>1$ as a sum of fractions $1 / b$. In other words, any fraction is a sum of unit fractions.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, for example, by using a visual fraction model. For example, $3 / 8=1 / 8+1 / 8+1 / 8$;
$3 / 8=1 / 8+2 / 8 ; 21 / 8=1+1+1 / 8 ; 21 / 8=8 / 8+8 / 8+1 / 8$.
c. Add and subtract mixed numbers with like denominators, for example, by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
For example, $31 / 4+21 / 4=13 / 4+9 / 4=22 / 4 ; 31 / 4+21 / 4=(3+2)+(1 / 4+1 / 4)=5+2 / 4=52 / 4$, which is equivalent to 22/4.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, for example, by using visual fraction models and equations to represent the problem.
Concepts and Skills to Master

- Understand and represent that addition and subtraction of fractions with the same denominator is joining or separating parts referring to the same whole
- Understand a mixed number is a whole number and a fraction that can also be represented as a fraction greater than one
- Add and subtract fractions with like denominators by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction
- Solve word problems involving addition and subtraction of fractions with like denominators

Teacher Note: All concepts and skills may include fractions greater than one and mixed numbers.

Related Standards: Current Course
4.NF. 1 Recognize and generate equivalent fractions
4.NF. 4 Multiply a fraction by a whole number
4.NF. 5 Add fractions with denominators of 10 and 100
4.MD. 2 Solve word problems with fraction and decimal numbers
4.MD. 4 Make a line plot with measurements in fraction units

## Related Standards: Future Courses

5.NF. 1 and 2 Use equivalent fractions as a strategy to add and subtract fractions. 6.EE. 7 Solve real-world and mathematical problems by writing and solving equations of the form $x+a=b$ for cases in which $a, b$ and $x$ are all non-negative rational numbers 6.NS.2-4 Apply and extend previous understandings of numbers to the system of rational numbers

Critical Background Knowledge from Previous Grade Levels

- Understand a unit fraction is a fraction with a numerator of 1 (3.NF.1)
- Understand equivalent fractions (3.NF.3)
- Represent a fraction on a number line (3.NF.2)


## Academic Vocabulary

Compose (composition), decompose (decomposition), unit fraction, mixed numbers, fractional form (5/4), fraction greater than one, numerator, denominator

## Suggested Models

## Bar Model


$21 / 8=8 / 8+8 / 8+1 / 8$


Equations (Decompose Fraction)

$$
\frac{5}{3}=\frac{3}{3}+\frac{2}{3}=1+\frac{2}{3}=1 \frac{2}{3}
$$

Number Bond


## Number Line



Suggested Strategies

- Use visual fraction models such as number bonds, number lines, fraction strips, bar models, tape diagrams, area models, and rulers to add and subtract fractions with like denominators
- Connect equations to visual models

[^0]Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers (Standards 4.NF.3-4). Denominators for fourth grade are limited to $2,3,4,5,6,8,10,12$, and 100.
Standard 4.NF. 4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction $\mathrm{a} / \mathrm{b}$ as a multiple of $1 / \mathrm{b}$. For example, use a visual fraction model to represent $5 / 4$ as the product 5 x (1/4), recording the conclusion by the equation $5 / 4=5 x(1 / 4)$.
b. Understand a multiple of $a / b$ as a multiple of $1 / b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times(2 / 5)$ as $6 \times(1 / 5)$, recognizing this product as $6 / 5$. (In general, $n \times(a / b)=(n \times a) / b)$.
c. Solve word problems involving multiplication of a fraction by a whole number (for example, by using visual fraction models and equations to represent the problem). For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be five people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

## Concepts and Skills to Master

- Understand a non-unit fraction (a fraction with a numerator greater than one) as a multiple of a unit fraction and a whole number
- Represent a repeated addition expression with unit fractions as a multiplication expression with a whole number and a unit fraction
- Represent a non-unit fraction in an expression of a unit fraction multiplied by a whole number
- Understand a multiple of a non-unit fraction is equivalent to a unit fraction times a whole number ( $3 \times \frac{2}{5}$ as $6 \times \frac{1}{5}$ )
- Solve word problems involving multiplication of a fraction and a whole number


## Related Standards: Current Grade Level

4.NF. 2 Utilize benchmark fractions to check for reasonableness of an answer
4.NF. 3 Understand any fraction with a numerator greater than one is the sum of unit fractions
4.MD. 2 Use multiplication to solve word problems using measurement

Critical Background Knowledge from Previous Grade Levels

- Understand a unit fraction has a numerator of 1 and a non-zero denominator (3.NF.1)
- Understand and represent fractions on a number line (3.NF.2)
- Interpret the products of whole numbers as the total number of objects in all groups (3.OA.1)
- Determine the unknown whole number in a multiplication or division problem (3.OA.4)


## Academic Vocabulary

Unit fraction, multiple, fractional form (5/4), fraction greater than one, mixed number, factor, product, expression, equation, numerator, denominator, whole number


Bar Model


$$
\frac{3}{6}=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=3 \times \frac{1}{6}
$$

## Suggested Strategies

- Use a variety of visual fraction models with students when introducing and working with each substandard $a, b$, and $c$
- Build multiplication and repeated addition equations to represent problems
- Use contexts in word problems when making models to evaluate reasonableness of answers

Each person at a party eats $\frac{2}{5}$ of a pound of meat, and there are 3 people at the party. How many pounds of meat are needed? Between what two answers does you answer lie?

$3 \times \frac{2}{5}=\frac{2}{5}+\frac{2}{5}+\frac{2}{5}=6 \times \frac{1}{5}$

- Connect visual models to equations

Understand decimal notation to the hundredths and compare decimal fractions with denominators of 10 and 100 (Standards 4.NF.5-7). Denominators for
Standard 4.NF. 5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100 . For example, express $3 / 10$ as $30 / 100$, and add $3 / 10+4 / 100=34 / 100$.

## Concepts and Skills to Master

- Understand equivalent fractions as the same quantity with different names.
- Add fractions with denominators of 10 and 100
- Create equivalent fractions using Multiplicative Identity Property
- Visualize and create base-ten grids to express a fraction with a denominator of 10 or 100

Teacher Note: Student are first exposed to decimal numbers in fourth grade.

## Related Standards: Current Course

## 4.NF. 1 Equivalent fractions

4.NF. 3 Add and subtract fractions with like denominators
4.NF. 6 Use decimal notation for fractions with denominators 10 or 100
4.NF. 7 Compare two decimals to hundredths by reasoning about their size
4.MD. 2 Solve measurement word problems involving decimals

## Critical Background Knowledge

- Related Standards: Current Grade Level (see above)
- Explain equivalence and generate equivalent fractions (3.NF.3)

Academic Vocabulary
Base-ten fractions, common denominator, equivalent fraction, tenths, hundredths

4.NF. 5

Understand decimal notation to the hundredths and compare decimal fractions with denominators of 10 and 100 (Standards 4.NF.5-7). Denominators for fourth grade are limited to $2,3,4,5,6,8,10,12$, and 100 .
Standard 4.NF. 6 Use decimal notation for fractions with denominators 10 or 100 . For example, rewrite 0.62 as 62/100, describe a length as 0.62 meters; locate 0.62 on a number line diagram.

## Concepts and Skills to Master

- Understand that just as fractions represent part of a whole, decimals represent part of a whole
- Understand that a fraction represented with decimal notation holds the same value as the fraction
- Identify the digit to the right of the decimal point as the tenths place and the digit two places to the right of the decimal point at the hundredths place

Teacher Note: Being precise with language when saying decimals is important. For example, always say "five and two tenths", rather than "five point two." When teachers and students are precise with their language it will provide an opportunity for students to hear the connection between fractions and decimals (Van De Walle, 2014).

## Related Standards: Current Course

4.NF. 5 Express equivalent fractions with denominators of 10 and 100
4.NF. 7 Compare two decimals to hundredths by reasoning about their size
4.MD. 2 Solve measurement problems using simple decimals

## Critical Background Knowledge from Previous Grades

- Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (2.NBT.1)


## Academic Vocabulary

tenths, hundredths, decimal, equivalent fraction, decimal notation
Suggested Models

| Hundreds | Tens | Ones | $\bullet$ | Tenths | Hundredths |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  | $\bullet$ | 3 | 2 |

## Related Standards: Future Courses

5.NBT. 1 Recognize digits in places as 10 times as much as places to the right, and $1 / 10$ as much as places to the left
5.NBT. 3 Read, write, and compare decimals to thousandths
5.NBT.7 Add, subtract, multiply, and divide with decimals

## Suggested Strategies

- Relate fractions and decimals as the same value by using visual models such as base ten blocks and grids, number lines, meter sticks, place value chart and money

[^1]
## 4.NF. 6

Understand decimal notation to the hundredths and compare decimal fractions with denominators of 10 and 100 (Standards 4.NF.5-7). Denominators for fourth grade are limited to $2,3,4,5,6,8,10,12$, and 100 .
Standard 4.NF. 7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>,=$, or $<$, and justify the conclusions, for example, by using a visual model.

## Concepts and Skills to Master

- Understand comparisons are only valid when the two fractions or decimals refer to the same whole
- Compare decimals by reasoning that as the number of equal parts in a whole increases, the size of the fractional parts decreases (hundredths are smaller than tenths)
- Extend place value understanding to tenths and hundredths
- Create and compare base-ten grids to express a fraction with a denominator of 10 or 100


## Related Standards: Current Course

4.NBT. 2 Read, write, and compare multi-digit whole numbers
4.NF. 2 Compare fractions with different numerators and denominators
4.NF. 6 Use decimal notation for fractions with denominators 10 or 100

## Related Standards: Future Courses

5.NBT. 1 Recognize digits in places as 10 times as much as places to the right, and $1 / 10$ as much as places to the left
5.NBT. 3 Read, write, and compare decimals to the thousandths

Critical Background Knowledge from Previous Grade Levels

- Compare two fractions with the same numerator or the same denominator by reasoning about their size (3.NF.3d)
- Compare two three-digit numbers (2.NBT.4)


## Academic Vocabulary

decimal, tenth, hundredth, equivalent, >, =,<, compare,

## Suggested Models

Seeing that $0.2>0.09$ using a visual fraction model

The shaded region on the left shows 0.2 of the unit square, since it is two parts when the square is divided into 10 parts of equal area. The shaded region on the right shows 0.09 of the unit square, since it is 9 parts when the unit is divided into 100 parts of equal area.
Image Source: http://commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf
 .//commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf

Suggested Strategies

- Relate and compare fractions and decimals as the same value by using visual models such as base ten blocks and grids, number lines, meter sticks, place value chart and money


[^0]:    Images Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/4.pdf

[^1]:    Images Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/4.pdf

