NCSM Great Tasks for Mathematics

Engaging Activities for

Effective Instruction and Assessment that Integrate

the Content and Practices of the

Common Core State Standards for Mathematics

A Resource from the
National Council of Supervisors of Mathematics

SAMPLE TASKS

SPRING 2012

LEADERSHIP IN MATHEMATICS EDUCATION
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Teacher Notes Proving Patterns

Task Title: Proving Patterns

Grade Level: High School Math 1 or Algebra

Task Overview:
Students will analyze quadratic patterns related to the difference of squares and use patterns with the number line. They will then prove the general rule with a sequence of calculations that model inductive reasoning.

Prerequisite Understandings:
Students should be proficient in multiplying a binomial by a binomial.

CCSSM Content Standards:
A.APR.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

CCSSM Mathematical Practices:
2. Reason abstractly and quantitatively.
4. Model with mathematics.

Supplies Needed: A number line and a table of squares for the launch.

Teaching Notes:

Launch activity:
Lead a class discussion on problem solving strategies such as make a list, make a table, look for patterns, or make a sketch. Ask students to list strategies that they have used in the past. The launch activity will also tune up their memorization skills and help with recognizing patterns later on.

Core task:
Students will develop an intuitive formula for the difference of squares and then prove that relationship using the distributive property to multiply two binomials.

Extension(s):
Teachers may want to get students to see an example of mathematical induction. If you know that $(x - n)(x + n) = x^2 - n^2$ when $= 1 \; and \; n = 2$, then to prove it works for all values of n, show that if $(x - n)(x + n) = x^2 - n^2$ then $\big(x - (n + 1)\big)\big(x + (n + 1)\big) = x^2 - (n + 1)^2$. Afterwards, discuss the geometric view of what is happening. If students rearrange 5 rows of 5, it is one less than 4 rows of 6. Try it! The rectangles are on the activity.

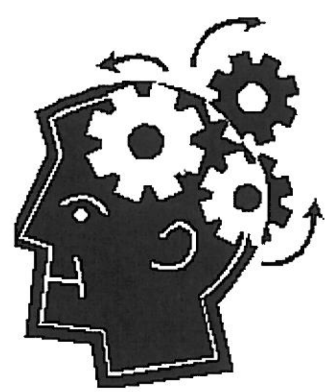
Launch Proving Patterns

n	n^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400
21	441
22	484
23	529
24	576
25	625

Memorizing some useful facts can help your ability to memorize, increase your understanding, make you faster at mental mathematics, and impress people. Memorizing in two different locations can help you remember better!

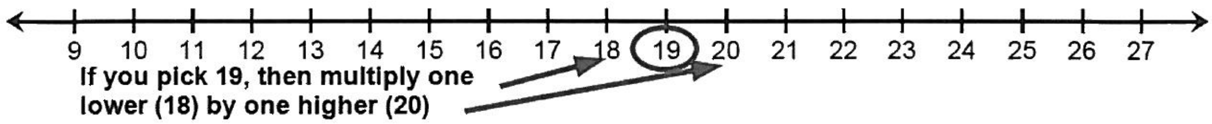
1. Take 10 minutes to memorize square numbers in the table to the left.
2. Cover the table and have a neighbor quiz you.
3. Quiz your neighbor.
4. Memorize this at home and have someone quiz you.

What strategies did you use to memorize this table? In the past, when you wanted to memorize something, what did you have to do to finally make that memory permanent in your mind?



Proving Patterns

Maria, who loves mental math, notices a strange pattern with the square numbers. She shared her findings with her friend, Ally. Maria asked Ally to select an integer on the number line. Then, Maria said, "Square that number. Now, look for the number that is one greater and one less than your original number. Find the product of the larger and smaller values. Check it out and see if you can discover what Maria found.



1. Copy and complete a table like the one below to find Maria's pattern for the first 20 counting numbers. Describe how the pattern of (n - 1)(n + 1) compares to the n^2.

n	1	2	3	4	5	…			20
n-1	0								
n+1	2								
(n-1)(n+1)	0								

2. Make a table for (n - 2)(n + 2) and describe how that pattern relates to n^2.

3. Repeat the process for (n - 3)(n + 3).

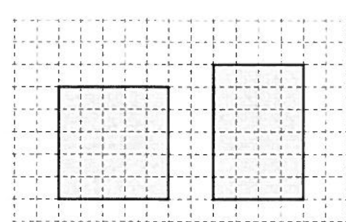
4. Predict what the results will be for (n - 4)(n + 4).

5. Check your pattern by expanding each of the expressions in 2 through 4 using the distributive property.

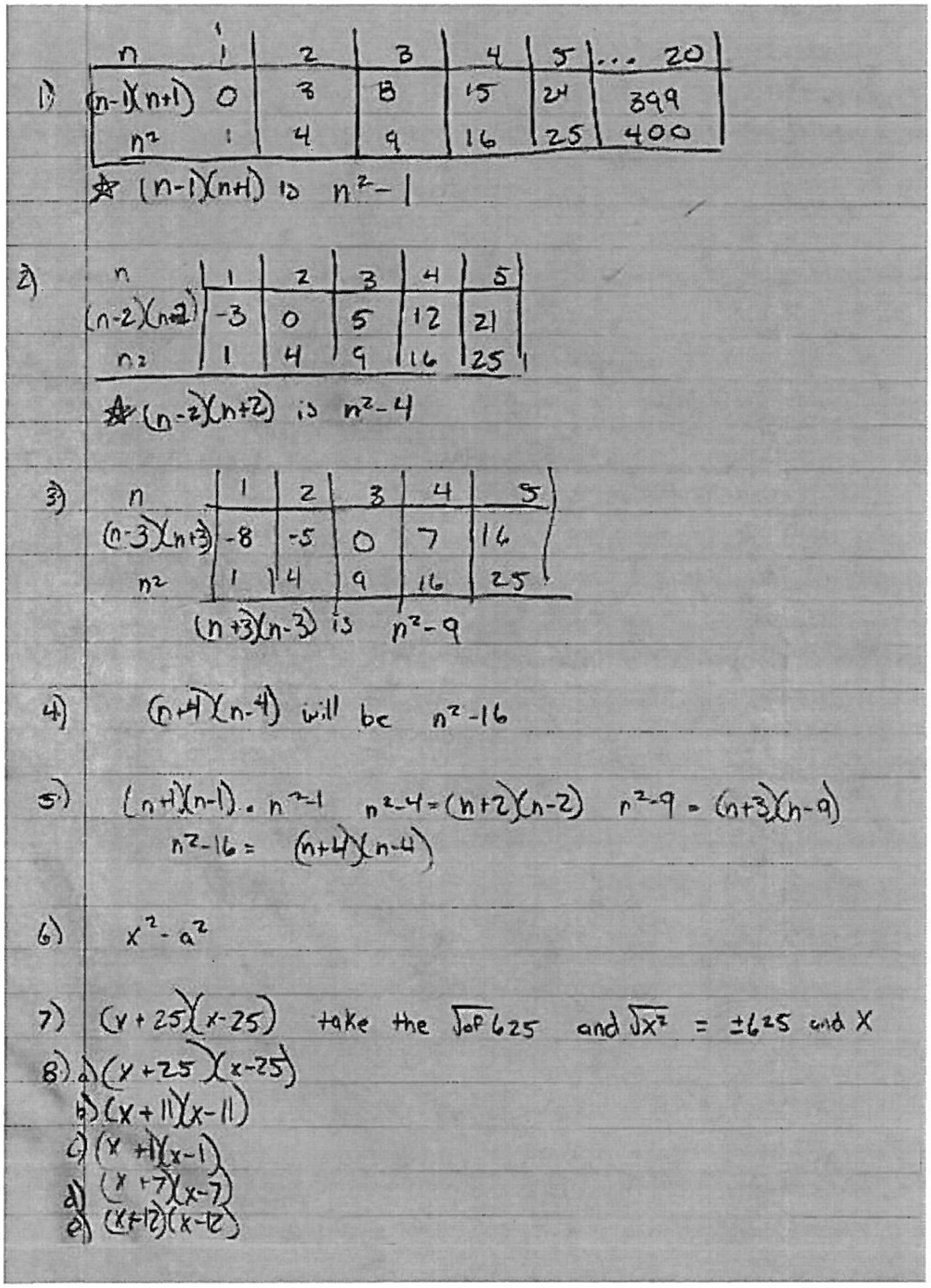
6. Expand (x - a)(x + a).

7. Use the property in number 5 to explain how you could factor $x^2 - 625$. Check your answer by multiplying it back out.

8. Factor the following:
   a. $x^2 - 64$
   b. $x^2 - 121$
   c. $x^2 - 1$
   d. $x^2 - 49$
   e. $x^2 - 144$



9. Check your answers (expand them) and tell what works in factoring squares.



1)

n	1	2	3	4	5	... 20
(n-1)(n+1)	0	3	8	15	24	399
n²	1	4	9	16	25	400

★ (n-1)(n+1) is n²-1

2)

n	1	2	3	4	5
(n-2)(n+2)	-3	0	5	12	21
n²	1	4	9	16	25

★ (n-2)(n+2) is n²-4

3)

n	1	2	3	4	5
(n-3)(n+3)	-8	-5	0	7	16
n²	1	4	9	16	25

(n+3)(n-3) is n²-9

4) (n+4)(n-4) will be n²-16

5) (n+1)(n-1) = n²-1 n²-4 = (n+2)(n-2) n²-9 = (n+3)(n-9)
n²-16 = (n+4)(n-4)

6) x² - a²

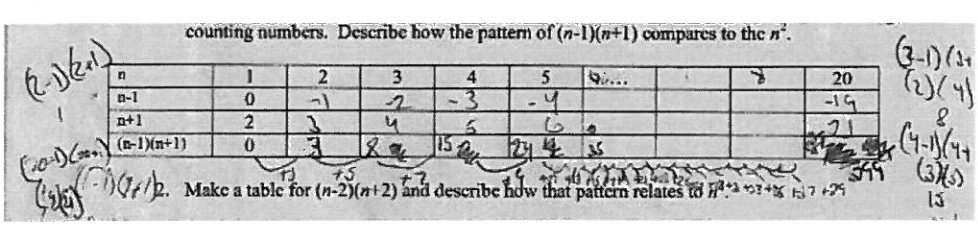
7) (x+25)(x-25) take the √of 625 and √x² = ±625 and x

8) a) (x+25)(x-25)
b) (x+11)(x-11)
c) (x+1)(x-1)
d) (x+7)(x-7)
e) (x+12)(x-12)

Lelah changed the table given when she was doing the problems. This bright student must have known where she was going and made all the patterns into simple comparison of the requested product to the values of n^2.

Her explanation in problem number 7 is pretty direct, and I would now want to ask her to frame the explanation in a sentence orally and ask her to reflect on how she could have written that.

Notice the quick error on problem 5. I think we could learn more of the thinking from a student who struggles more like Anthony.



counting numbers. Describe how the pattern of (n-1)(n+1) compares to the n².

n	1	2	3	4	5	6....			20
n-1	0	-1	-2	-3	-4				-19
n+1	2	3	4	5	6				21
(n-1)(n+1)	0	3	8	15	24	35			399

2. Make a table for (n-2)(n+2) and describe how that pattern relates to n².

Anthony filled out the table (left) and got lost in a difference pattern and missed the connection.