3.2 Floating Down the River

A Solidify Understanding Task

Alonzo, Maria, and Sierra were floating in inner tubes down a river, enjoying their day. Alonzo noticed that sometimes the water level was higher in some places than in others. Maria noticed there were times they seemed to be moving faster than at other times. Sierra laughed and said “Math is everywhere!” To learn more about the river, Alonzo and Maria collected data throughout the trip.

<table>
<thead>
<tr>
<th>Time (in minutes)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
<th>110</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (in feet)</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>12</td>
<td>9</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

1. Use the data collected by Alonzo to interpret the key features of this relationship.

Maria created a graph by collecting data on a GPS unit that told her the distance she had traveled over a period of time.

2. Using the graph created by Maria, describe the key features (increasing, decreasing, domain, range, maximum, minimum, intercepts) of this relationship.
Part II: Interpreting data

3. Sierra looked at the data collected by her two friends and made several of her own observations. Explain why you either agree or disagree with each observation made.

a) The depth of the water increases and decreases throughout the 120 minutes of floating down the river.

b) The distance traveled is always increasing.

c) The distance traveled is a function of time.

d) The distance traveled is greatest during the last ten minutes of the trip than during any other ten minute interval of time.

e) The domain of the distance/time graph is all real numbers.

f) The y-intercept of the depth of water over time function is (0,0).

g) The distance traveled increases and decreases over time.

h) The depth of the water is never 11 feet.

i) The range of the distance/time graph is from [0, 15000].

j) The domain of the depth of water with respect to time is from [0,120]

k) The range of the depth of water over time is from [4,5].

l) The distance/time graph has no maximum value.

m) The depth of water reached a maximum at 30 minutes.
3.2 Floating Down the River – Teacher Notes

A Solidify Understanding Task

**Purpose:** The purpose of this task is to further define function and to solidify key features of functions given different representations. Features include:

- domain and range
- where the function is increasing or decreasing
- $x$ and $y$ intercepts
- rates of change (informal)
- discrete versus continuous

**Core Standards Focus:**

**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

**F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.*

**Related Standards:** F.IF.1, F.IF.6, A.REI.10

**Standards for Mathematical Practice of Focus in the Task:**

- SMP 2 – Reason abstractly and quantitatively
- SMP 3 – Construct viable arguments and critique the reasoning of others
The Teaching Cycle:

Launch (Whole Class): Read the scenario out loud and highlight the difference between the two representations (depth of water over time and distance traveled over time). Before students begin, access their background knowledge by asking “What are some of the key features of functions we look for when interpreting graphs (or tables)?” These features are still fairly new to students, with some being introduced in the prior task. After the key features are mentioned (domain, range, intervals where the function is increasing/decreasing, and intercepts), have students work on the task in small groups.

Explore (Small Group): As you monitor, listen for student explanations as they interpret key features of both the table and the graph. If students struggle with writing features based on the table, you may ask them to tell the story represented by the table values or prompt with a question such as “How deep is the water at 0 minutes? What do you know about the water level after this time?” Another prompt would be to ask students if using another representation would help them see the features. Look for students who make a visual connection to the table, students who use notation to identify where the water level is increasing/decreasing, and students who notice interesting features (such as we do not really know the depth of the water at 7 minutes). Encouraging students to visually connect the key feature described by the mathematical representation will help during the whole group discussion.

Note: If most students are struggling to name the features using the two representations (problem 1: table, problem 2: graph), bring the whole group together after all groups have had time to work on the table representation. Select a student who has visually connected the table to the context to describe where the water level is increasing and decreasing. Then ask the whole group what other features can be interpreted from the table. Include domain, range, and intercepts at this time. If a student has used interval notation, have them share. At some point, we want to bring up that while the table values are discrete, the function is continuous. If a student brings this up now, address it. Otherwise wait until the whole group discussion of the entire task.
Discuss (Whole Class): The goal of the whole group discussion is to highlight the connections between a given representation and the key features of that function. Be sure to use academic vocabulary throughout the whole group discussion. Before starting the whole group discussion, post the table and the graph so students can better communicate their observations about the feature they are describing. Begin the whole group discussion by going over a couple of the observation statements made by Sierra that will create an opportunity to have students communicate viable arguments. For example, the range of the depth of water conversation will bring out how it is important to look at output values when discussing the range, and not the beginning and ending point of the trip. After going over a couple of the observation statements, choose a student to share the key features of the graph (distance vs. time). Be sure this student highlights each feature on the graph while also writing the interpreted feature next to the graph. After the features are shared, ask the whole group about the other observation statements made by Sierra that relate to the graph. Then choose another student to share the key features of the table. After, go over the observation statements made by Sierra. If, at this time, the conversation has not come up about how this table of values is discrete but represents a continuous function, ask students about the domain. Be sure that at the end of this discussion, students understand that a table of values only shows some of the solution points for a continuous function.

Aligned Ready, Set, Go: Features 3.2
READY

Topic: Solve Linear Systems by Graphing

Graph each set of linear equations on the same set of axes. Name the coordinates of the point where the two lines intersect.

1. \[ \begin{align*} f(x) &= 2x - 7 \\ g(x) &= -4x + 5 \end{align*} \]
   Point of intersection:

2. \[ \begin{align*} f(x) &= -5x - 2 \\ g(x) &= -2x + 1 \end{align*} \]
   Point of intersection:

3. \[ \begin{align*} f(x) &= -x - 2 \\ g(x) &= 2x + 10 \end{align*} \]
   Point of intersection:

4. \[ \begin{align*} f(x) &= x - 5 \\ g(x) &= -x + 1 \end{align*} \]
   Point of intersection:

5. \[ \begin{align*} f(x) &= \frac{2}{3} x + 4 \\ g(x) &= -\frac{1}{3} x + 1 \end{align*} \]
   Point of intersection:

6. \[ \begin{align*} f(x) &= x \\ g(x) &= -x - 2 \end{align*} \]
   Point of intersection:
SET

Topic: Describing attributes of a functions based on graphical representation

For each graph state 1) the interval(s) where it is increasing, decreasing, or constant 2) if it has a minimum or maximum, and 3) identify the domain and range. Use interval notation.

7. Description of function

8. Description of function

9. Description of function
GO

Topic: Creating both explicit and recursive equations

Write equations for the given tables in both recursive and explicit form.

10. \[ n \quad f(n) \]
    \[
    \begin{array}{c|c}
    1 & 5 \\
    2 & 2 \\
    3 & -1 \\
    \end{array}
    \]

Explicit:

Recursive:

11. \[ n \quad f(n) \]
    \[
    \begin{array}{c|c}
    1 & 6 \\
    2 & 12 \\
    3 & 24 \\
    \end{array}
    \]

Explicit:

Recursive:

12. \[ n \quad f(n) \]
    \[
    \begin{array}{c|c}
    0 & -13 \\
    2 & -5 \\
    3 & -1 \\
    \end{array}
    \]

Explicit:

Recursive:

13. \[ n \quad f(n) \]
    \[
    \begin{array}{c|c}
    1 & 5 \\
    4 & 11 \\
    5 & 13 \\
    \end{array}
    \]

Explicit:

Recursive:

14. \[ n \quad f(n) \]
    \[
    \begin{array}{c|c}
    2 & 5 \\
    7 & 15,625 \\
    9 & 390,625 \\
    \end{array}
    \]

Explicit:

Recursive:

15. \[ n \quad f(n) \]
    \[
    \begin{array}{c|c}
    0 & -4 \\
    1 & -16 \\
    2 & -64 \\
    \end{array}
    \]

Explicit:

Recursive: