2.4 Getting Down to Business

A Solidify Understanding Task

Calcu-rama had a net income of 5 million dollars in 2010, while a small competing company, Computafest, had a net income of 2 million dollars. The management of Calcu-rama develops a business plan for future growth that projects an increase in net income of 0.5 million per year, while the management of Computafest develops a plan aimed at increasing its net income by 15% each year.

a. Create standard mathematical models (table, graph and equations) for the projected net income over time for both companies. (Attend to precision and be sure that each model is accurate and labeled properly so that it represents the situation.)

b. Compare the two companies. How are the representations for the net income of the two companies similar? How do they differ? What relationships are highlighted in each representation?
c. If both companies were able to meet their net income growth goals, which company would you choose to invest in? Why?

d. When, if ever, would your projections suggest that the two companies have the same net income? How did you find this? Will they ever have the same net income again?

e. Since we are creating the models for these companies we can choose to have a discrete model or a continuous model. What are the advantages or disadvantages for each type of model?
2.4 Getting Down to Business – Teacher Notes

A Solidify Understanding Task

Note: Use of technology tools such as graphing calculators is recommended for this task.

Purpose:
The purpose of this task is to compare the rates of growth of an exponential and a linear function. The task provides an opportunity to look at the growth of an exponential and a linear function for large values of $x$, showing that increasing exponential functions become much larger as $x$ increases. This task is a good opportunity to model functions using technological tools and to discuss how to set appropriate viewing windows for functions. The task also leads to a discussion of whether this particular situation should be modeled using discrete or continuous functions.

Core Standards Focus:

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

★For F.IF.7a, 7e, and 9 focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as \( y=3^n \) and \( y=100\cdot2^n \).

Standards for Mathematical Practice of Focus in the Task:

  SMP 4 – Model with mathematics.

  SMP 5 – Use appropriate tools strategically.

The Teaching Cycle

Launch (Whole Class):

Start with a short discussion of the context to be sure that students understand the problem situation. As part of this discussion, clarify the choice of units and scale. Students may choose to use 5 million with million being the unit or as 5,000,000 in their equations. (They will probably find it easier to use millions as a unit, but they will need to interpret the scale on their graphs and be consistent in their equations.) Be sure that students understand terms like “net income” so that they know what the problem is asking. When students understand the problem, set them to work on the task, starting with parts a, b, and c.
Explore (Small Group):

Monitor students as they work on the task. Be prepared to redirect students that may not think of one function as linear, based on the constant growth, and the other as exponential based on the 15% growth factor. Be sure that students have discussed their answers to “c” before returning to the whole group discussion. The discussion for “d” will follow later.

Discuss and Re-launch (Whole Class):

Have a group that has written the explicit and recursive equations correctly present their work. Ask the class which company has a linear model and which has an exponential model and how can they tell from both recursive and explicit forms of the function rules. Ask how the growth pattern shows up in the equations. Finally ask if the functions should be modeled as discrete or continuous. Ask why the companies might choose a discrete or continuous model. They may choose a continuous model because they feel that the net income is increasing on a steady basis across the year, so it makes sense to fill in all the points on the graph and use an explicit formula. They may choose a discrete model because there are fluctuations in income during the year, with the net income increasing. If they can’t predict the fluctuations, they may choose to use a discrete function, modeling with just one point each year.

Once students have discussed the equations, ask students to focus on the explicit equations and complete part “d” of the task. Encourage the use of technology, either graphing calculators or computers with programs with graphing capabilities.

When students have completed their work, ask a group to present their tables showing the projected net income of the two businesses. Ask how they could find where the net income of the two businesses would be the same using their tables. Then have a group present their graphs and demonstrate how to find the year where Computafest exceeds the net income of Calcu-rama. (You may ask how to use the equations to find where the net incomes will be equal, but students will not be able to find an analytic solution to the equation.)
Conclude the task with a discussion of the end behavior of the two functions. How much will each company be making in 10 years, 20 years, etc.? Trace the graphs and look at the difference between the net incomes over time. Ask why an exponential function becomes so much larger than a linear function over time. A big idea here is that exponential growth depends on the amount available at any given time, so the more available, the bigger the increase. In the early years when the company is small, an increase of 15% adds a small amount. As the company grows, 15% of the income becomes larger and larger, making the company grow by more each year. In contrast, linear growth has the same increase every time no matter how much is available.

**Aligned Ready, Set, Go: Linear and Exponential Functions 2.4**
READY

Topic: Comparing arithmetic and geometric sequences. The first and fifth terms of a sequence are given. Fill in the missing numbers if it is an arithmetic sequence. Then fill in the numbers if it is a geometric sequence.

Example:

<table>
<thead>
<tr>
<th></th>
<th>Arithmetic</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>+80</td>
<td>84</td>
<td>+80</td>
<td>164</td>
</tr>
<tr>
<td>Geometric</td>
<td>4</td>
<td></td>
<td>12</td>
<td></td>
<td>36</td>
</tr>
</tbody>
</table>

1. Arithmetic 3  
   Geometric 3

2. Arithmetic -6250  
   Geometric -6250

3. Arithmetic -12  
   Geometric -12

SET

Topic: Distinguishing specifics between sequences and linear or exponential functions. Answer the questions below with respect to the relationship between sequences and the larger families of functions.

4. If a relationship is modeled with a continuous function which of the domain choices is a possibility?
   A. \( x \in R, x \geq 0 \)  
   B. \( x \in W \)  
   C. \( x \in Z, x \geq 0 \)  
   D. \( x \in N \)

5. Which one of the options below is the mathematical way to represent the Natural Numbers?
   A. \( x \in R, x \geq 0 \)  
   B. \( x \in Q, x \geq 0 \)  
   C. \( x \in Z, x \geq 0 \)  
   D. \( x \in N \)
6. Only one of the choices below would be used for a \textit{continuous exponential} model, which one is it?
   
   A. \( f(x) = f(x-1) \cdot 4, f(1) = 3 \) \hspace{1cm} B. \( g(x) = 4^x(5) \)
   
   C. \( h(t) = 3t - 5 \) \hspace{1cm} D. \( k(n) = k(n-1) - 5, k(1) = 32 \)

7. Only one of the choices below would be used for a \textit{continuous linear} model, which one is it?
   
   A. \( f(x) = f(x-1) \cdot 4, f(1) = 3 \) \hspace{1cm} B. \( g(x) = 4^x(5) \)
   
   C. \( h(t) = 3t - 5 \) \hspace{1cm} D. \( k(n) = k(n-1) - 5, k(1) = 32 \)

8. What domain choice would be most appropriate for an arithmetic or geometric sequence?
   
   A. \( \{x \mid x \in R, x \geq 0\} \) \hspace{1cm} B. \( \{x \mid x \in Q, x \geq 0\} \) \hspace{1cm} C. \( \{x \mid x \in Z, x \geq 0\} \) \hspace{1cm} D. \( \{x \mid x \in N\} \)

9. What attributes will arithmetic or geometric sequences always have?
   (There could be more than one correct choice. Circle all that apply.)

   A. Continuous \hspace{1cm} B. Discrete \hspace{1cm} C. Domain: \( \{x \mid x \in N\} \) \hspace{1cm} D. Domain: \( \{x \mid x \in R\} \)
   
   E. Negative x-values \hspace{1cm} F. Something constant \hspace{1cm} G. Recursive Rule

10. What type of sequence fits with linear mathematical models?

    What is the difference between this sequence type and the overarching umbrella of linear relationships? (Use words like discrete, continuous, domain and so forth in your response.)

11. What type of sequence fits with exponential mathematical models?

    What is the difference between this sequence type and the overarching umbrella of exponential relationships? (Use words like discrete, continuous, domain and so forth in your response.)
GO

Topic: Writing explicit equations for linear and exponential models.

Write the explicit equations for the tables and graphs below. This is something you really need to know. Persevere and do all you can to figure them out. Remember the tools we have used. (#21 is bonus give it a try.)

12. \( \begin{array}{c|c} x & f(x) \\ \hline 2 & -4 \\ 3 & -11 \\ 4 & -18 \\ 5 & -25 \end{array} \) 
13. \( \begin{array}{c|c} x & f(x) \\ \hline -1 & 2/5 \\ 0 & 2 \\ 1 & 10 \end{array} \) 
14. \( \begin{array}{c|c} x & f(x) \\ \hline 2 & -24 \\ 3 & -48 \\ 4 & -96 \\ 5 & -192 \end{array} \) 
15. \( \begin{array}{c|c} x & f(x) \\ \hline -4 & 81 \\ -3 & 27 \\ -2 & 9 \\ -1 & 3 \end{array} \) 

16. [Graph 1]
17. [Graph 2]
18. [Graph 3]
19. [Graph 4]
20. [Graph 5]
21. [Graph 6]