3.6 Interpreting Functions
A Practice Understanding Task

Given the graph of \( f(x) \), answer the following questions. Unless otherwise specified, restrict the domain of the function to what you see in the graph below. Approximations are appropriate answers.

1. What is \( f(2) \)?
2. For what values, if any, does \( f(x) = 3 \)?
3. What is the x-intercept?
4. What is the domain of \( f(x) \)?
5. On what intervals is \( f(x) > 0 \)?
6. On what intervals is \( f(x) \) increasing?
7. On what intervals is \( f(x) \) decreasing?
8. For what values, if any, is \( f(x) > 3 \)?
Consider the linear graph of $f(t)$ and the nonlinear graph of $g(t)$ to answer questions 9-14. Approximations are appropriate answers.

9. Where is $f(t) = g(t)$?
10. Where is $f(t) > g(t)$?
11. What is $f(0) + g(0)$?
12. What is $f(-1) + g(-1)$?
13. Which is greater: $f(0)$ or $g(-3)$?
14. Graph: $f(t) + g(t)$ from $[-1, 3]$

The following table of values represents two continuous functions, $f(x)$ and $g(x)$. Use the table to answer the following questions:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>44</td>
<td>-13</td>
</tr>
<tr>
<td>-4</td>
<td>30</td>
<td>-9</td>
</tr>
<tr>
<td>-3</td>
<td>20</td>
<td>-5</td>
</tr>
<tr>
<td>-2</td>
<td>12</td>
<td>-1</td>
</tr>
<tr>
<td>-1</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>19</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>23</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>27</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>31</td>
</tr>
</tbody>
</table>

15. What is $g(-3)$?
16. For what value(s) is $f(x) = 0$?
17. For what values does $f(x)$ seem to be increasing?
18. On what interval is $g(x) > f(x)$?
19. Which function is changing faster in the interval $[-5, -1]$? Why?
Use the following relationships to answer the questions below.

\[ h(x) = 2^x \quad f(x) = 3x - 2 \quad g(x) = 8 \quad x = 4 \quad y = 5x + 1 \]

20. Which of the above relations are functions? Explain.

21. Find \( f(2) \), \( g(2) \), and \( h(2) \).

22. Write the equation for \( g(x) + h(x) \).

23. Where is \( g(x) < h(x) \)?

24. Where is \( f(x) \) increasing?

25. Which of the above functions has the fastest growth rate?

Create a graph for each of the following functions, using the given conditions

26. This function has the following features: \( f(2) \) is positive; \( f(\cdot-2) = 0 \), \( f(x) \) is always increasing and has a domain of All Real Numbers.

27. This function has the following features: \( f(3) > f(6) \); \( f(1) = 0 \); \( f(2) = 4 \); \( f(x) \) is increasing from \([\cdot-5, 3]\); has a domain from \([\cdot-5, 10]\)

28. This function has the following features: \( f(x) \) has a constant rate of change; \( f(5) = 0 \)

29. Create your own conditions- have at least three and then create examples where the solution could be different graphs.
3.6 Interpreting Functions – Teacher Notes

A Practice Understanding Task

**Purpose:** Students have been using function notation in various forms and have become more comfortable with features of functions. In this task, the purpose is for students to practice their understanding of the following:
- Distinguish between input and output values when using notation
- Evaluate functions for inputs in their domains
- Determine the solution where the graphs of $f(x)$ and $g(x)$ intersect based on tables of values and by interpreting graphs
- Combine standard function types using arithmetic operations (finding values of $f(x) + g(x)$)
- Create graphs of functions given conditions.

**Core Standards Focus:**

**F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

**F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.*

**F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- Graph linear and quadratic functions and show intercepts, maxima, and minima.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

**F.BF.1b** Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations.

**A.REI.11** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

**A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

**Related Standards:** F.IF.1, A.REI.10, A.REI.11, N.Q.1, A.CED.2

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**Standards for Mathematical Practice of Focus in the Task:**

- **SMP 7** – Look and make use of structure
- **SMP 8** – Look for and express regularity in repeated reasoning

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**The Teaching Cycle:**

**Launch (Whole Class):**

Students should be able to get started on this task without additional support, since it is similar in nature to the work they did on “The Water Park” and “Pooling It Together”. If preferred, you may wish to sketch a graph on the board and ask students a couple of questions using function notation before having them begin the task. This would be a good task to have students start on their own, then have them pair up after most have completed the first set of questions.

**Explore (Small Group):**

Watch for students who confuse input/output values. Without context, keeping track of this is a common mistake. Encourage students to explain their reasoning to each other while working through solutions to problems. If students are incorrect in their thinking, be sure to redirect their
thinking. As you monitor, make note of the areas where students are struggling and select students who explain/clarify details.

**Discuss (Whole Class):**

Go over problems that seem to be common issues that students are still grappling with first. After this, choose students to share their method for graphing number 14. Compare students who used point by point to those who added on from one graph to the next. Last, choose students who have correct but different graphs for one of the following problems (either 26 or 27). Have students compare/contrast graphs and explain why both are appropriate given the conditions.

The goal of this whole group discussion is that ALL students can evaluate functions using notation, can interpret features of functions using a graph or table of values, can create a graph given conditions, and can combine two functions to make another function.

**Aligned Ready, Set, Go: Features 3.6**
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SECONDARY MATH 1 // MODULE 3
FEATURES OF FUNCTIONS – 3.6

READY, SET, GO!

REady

Topic: Solving Systems by Substitution

In prior work the meaning of \( f(x) = g(x) \) was discussed. This means to find the point where the two equations are equal and where the two graphs intersect. It is possible to find the point of intersection algebraically instead of graphing the two lines. Since \( f(x) = g(x) \), it’s possible to set each equation equal to the other and solve for \( x \).

Example: Find the point of intersection of function \( f(x) = 3x + 4 \) and function \( g(x) = 4x + 1 \).

Since, \( f(x) = g(x) \), let \( 3x + 4 = 4x + 1 \). Then solve for \( x \).

\[
3x + 4 = 4x + 1 \quad \text{Subtract } 3x \text{ and } 1 \text{ from both sides of the equation.}
\]

\[
-3x - 1 = -3x - 1
\]

\[
0x + 3 = 1x + 0
\]

\[
3 = 1x \quad \text{Now let } x = 3 \text{ in each equation to find } f(x) \text{ and } g(x) \text{ when } x = 3.
\]

\[
f(3) = 3(3) + 4 \rightarrow 9 + 4 = 13 \quad \text{and} \quad g(3) = 4(3) + 1 \rightarrow 12 + 1 = 13
\]

When \( x = 3 \), \( f(3) \) and \( g(3) \) both equal 13. The point of intersection is \( (3, 13) \).

Find the point of intersection for \( f(x) \) and \( g(x) \) using the algebraic method in the example above.

1. \( f(x) = -5x + 12 \) and \( g(x) = -2x - 3 \)
2. \( f(x) = \frac{1}{2} x + 2 \) and \( g(x) = 2x - 7 \)
3. \( f(x) = -\frac{2}{3} x + 5 \) and \( g(x) = -x + 7 \)
4. \( f(x) = x - 6 \) and \( g(x) = -x - 6 \)
SET
Topic: Describing attributes of a functions based on graphical representation

Use the graph of each function provided to find the indicated values.

5. $f(x)$

a. $f(4) = \underline{\hspace{2cm}}$

b. $f(-4) = \underline{\hspace{2cm}}$

c. $f(x) = 4, \ x = \underline{\hspace{2cm}}$

d. $f(x) = 7, \ x = \underline{\hspace{2cm}}$

6. $g(x)$

a. $g(-1) = \underline{\hspace{2cm}}$

b. $g(-3) = \underline{\hspace{2cm}}$

c. $g(x) = -4 \ x = \underline{\hspace{2cm}}$

d. $g(x) = -1, \ x = \underline{\hspace{2cm}}$

7. $h(x)$

a. $h(0) = \underline{\hspace{2cm}}$

b. $h(3) = \underline{\hspace{2cm}}$

c. $h(x) = 1, \ x = \underline{\hspace{2cm}}$

d. $h(x) = -2, \ x = \underline{\hspace{2cm}}$

8. $d(x)$

a. $d(-5) = \underline{\hspace{2cm}}$

b. $d(4) = \underline{\hspace{2cm}}$

c. $d(x) = 4, \ x = \underline{\hspace{2cm}}$

d. $d(x) = 0, \ x = \underline{\hspace{2cm}}$
For each situation either create a function or use the given function to find and interpret solutions.

9. Fran collected data on the number of feet she could walk each second and wrote the following rule to model her walking rate \( d(t) = 4t \).
   a. What is Fran looking for if she writes \( d(12) = \text{_____} \)?

   b. In this situation what does \( d(t) = 100 \) tell you?

   c. How can the function rule be used to indicate a time of 16 seconds was walked?

   d. How can the function rule be used to indicate that a distance of 200 feet was walked?

10. Mr. Multbank has developed a population growth model for the rodents in the field by his house. He believes that starting each spring the population can be modeled based on the number of weeks with the function \( p(t) = 8(2^t) \).
    Find \( p(t) = 128 \).  
    Find \( p(4) \).  
    Find \( p(10) \).  
    d. Find the number of weeks it will take for the population to be over 20,000.

    e. In a year with 16 weeks of summer, how many rodents would he expect by the end of the summer using Mr. Multbank’s model?

What are some factors that could change the actual result from your estimate?
GO

Topic: Describe features of functions from the graphical representation.

For each graph given provide a description of the function. Be sure to consider the following: decreasing/increasing, min/max, domain/range, etc.

11. Description of function:

12. Description of function:

13. Description of function: