6.1 Leaping Lizards!

A Develop Understanding Task

Animated films and cartoons are now usually produced using computer technology, rather than the hand-drawn images of the past. Computer animation requires both artistic talent and mathematical knowledge.

Sometimes animators want to move an image around the computer screen without distorting the size and shape of the image in any way. This is done using geometric transformations such as translations (slides), reflections (flips), and rotations (turns), or perhaps some combination of these. These transformations need to be precisely defined, so there is no doubt about where the final image will end up on the screen.

So where do you think the lizard shown on the grid on the following page will end up using the following transformations? (The original lizard was created by plotting the following anchor points on the coordinate grid, and then letting a computer program draw the lizard. The anchor points are always listed in this order: tip of nose, center of left front foot, belly, center of left rear foot, point of tail, center of rear right foot, back, center of front right foot.)

Original lizard anchor points:
{(12,12), (15,12), (17,12), (19,10), (19,14), (20,13), (17,15), (14,16)}

Each statement below describes a transformation of the original lizard. Do the following for each of the statements:

- plot the anchor points for the lizard in its new location
- connect the pre-image and image anchor points with line segments, or circular arcs, whichever best illustrates the relationship between them
Lazy Lizard
Translate the original lizard so the point at the tip of its nose is located at (24, 20), making the lizard appears to be sunbathing on the rock.

Lunging Lizard
Rotate the lizard $90^\circ$ about point A (12,7) so it looks like the lizard is diving into the puddle of mud.

Leaping Lizard
Reflect the lizard about given line $y = \frac{1}{2}x + 16$ so it looks like the lizard is doing a back flip over the cactus.
6.1 Leaping Lizards! – Teacher Notes

A Develop Understanding Task

**Purpose:** This task provides an opportunity for formative assessment of what students already know about the three rigid-motion transformations: translations, reflections, and rotations. As students engage in the task they should recognize a need for precise definitions of each of these transformations so that the final image under each transformation is a unique figure, rather than an ill-defined sketch. The exploration and subsequent discussion described below should allow students to begin to identify the essential elements in a precise definition of the rigid-motion transformations, e.g., translations move points a specified distance along parallel lines; rotations move points along a circular arc with a specified center and angle, and reflections move points across a specified line of reflection so that the line of reflection is the perpendicular bisector of each line segment connecting corresponding pre-image and image points.

In addition to the work with the rigid-motion transformations, this task also surfaces thinking about the slope criteria for determining when lines are parallel or perpendicular. In a translation, the line segments connecting pre-image and image points are parallel, having the same slope. In a 90° rotation, the line segments connecting pre-image and image points are perpendicular, having opposite reciprocal slopes. Likewise, in a reflection, the line segments connecting pre-image and image points are perpendicular to the line of reflection.

Finally, this task reminds students that rigid-motion transformations preserves distance and angle measures within a shape—implying that the figures forming the pre-image and image are congruent. Students will be attending to two different categories of distances—the lengths of line segments that are used in the definitions of the transformations, and the lengths of the congruent line segments that are contained within the pre-image and image figures themselves. Students may determine that these lengths are preserved by counting units of “rise” and “run”, or by using the
Pythagorean Theorem. Ultimately, this work will lead to the development of the distance formula in future tasks.

Core Standards Focus:

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Related Standards: G.CO.2, G.CO.6, G.GPE.5

Teacher Note: Students’ previous experiences with rigid motions may have surfaced intuitive ways of thinking about these transformations, but such informal definitions will not support students in proving geometric properties based on a transformational approach. Experiences with sliding, flipping and turning rigid objects will have provided experimental evidence that rigid-motion transformations preserve distance and angle within a shape, such that,

- Lines are taken to lines, and line segments to line segments of the same length.
- Angles are taken to angles of the same measure.
- Parallel lines are taken to parallel lines.

Students who have used technology to translate, rotate or reflect objects may not have attended to the essential features that define such transformations. For example, a student can mark a mirror line and click on a button to reflect an object across the mirror line without noting the relationship between the pre-image and image points relative to the line of reflection. Consequently, research
shows that students harbor many misconceptions about the placement of an image after a transformation—erroneous assumptions such as:

- one of the sides of a reflected image must coincide with the line of reflection
- the center of a rotation must be located at a point on the pre-image (e.g., a vertex point) or at the origin
- a pre-image point and corresponding image point do not need to be the same distance away from the center of the rotation

Watch for these misconceptions as students engage in this task.

Standards for Mathematical Practice of focus in the task:

- **SMP 1** – Make sense of problems and persevere in solving them
- **SMP 5** – Use appropriate tools strategically
- **SMP 7** – Look for and make use of structure

Additional Resources for Teachers:

An enlarged copy of the image on the second page of the task can be found at the end of this set of teacher notes. This image can be printed for use with students who may be accessing the task on a computer or tablet.

The Teaching Cycle:

**Launch (Whole Class):**

Set the stage for the work of this learning cycle by discussing the ideas of computer animation as outlined in the first few paragraphs of this task. As part of the launch ask students why they think we need only keep track of a few anchor points, since the image of the lizard consists of infinitely many points, in addition to the eight points that are listed. The issue to be raised here is that rigid-motion transformations preserve distance and angle (properties that have been established in Math 8). Therefore a software animation program could draw features of the lizard, such as the toes on each of the feet, by starting at an anchor point and using predetermined angle and distance measures to locate other points on the toes. Make sure students pay attention to the order in which
each of the anchor points should be listed after completing each of the transformations. This will help students pay attention to individual pairs of pre-image and image points.

Provide multiple tools for students to do this work, such as transparencies or tracing paper, protractors, rulers, and compasses. The coordinate grid on which the images are drawn is also a tool for doing this work, but initially students may not recognize the usefulness of the grid as a way of carrying out the transformations, but rather just as a way of designating the location of the points after the transformation is complete. Technology tools may obscure the ideas being surfaced in the task, so it is best to use the tools described, which will allow students to pay attention to the details of their work.

It is intended that students should work on the transformations in the order listed in the task.

**Explore (Small Group):**

This task provides a great opportunity to pre-assess what students know about each of the rigid-motion transformations, so don’t worry if not all students are locating the final images correctly. Pay attention to the misconceptions that may arise (see teacher note).

If students use transparencies (or tracing paper) to copy the original lizard and then locate the image by sliding, turning or flipping the transparency, you will want to make sure they also think about these movements relative to the coordinate grid. Ask, “How could you have used the coordinate grid to locate this same set of points?” Focusing students’ attention on the coordinate grid will facilitate connecting the details that need to be articulated in the definitions of the rigid-motion transformations to coordinate geometry ideas, such as using slope to determine if lines are parallel or perpendicular. In this task, these ideas are surfaced and informally explored. In subsequent tasks these ideas are made more explicit and eventually justified.

Students should be fairly successful translating “Lazy Lizard”, since the point at the tip of the nose moves up 8 units and right 12 units, every anchor point must move the same. Watch for two different strategies to emerge: some students may move each point up 8, right 12; others may move
one point to the correct location, and then duplicate the relative positions of the points in the pre-image to locate points in the image—thereby preserving distance and angle between the points in the pre-image and those same points in the image.

To get started on “Lunging Lizard”, you may want to direct students’ attention to the point at the tip of the lizard’s nose, which lies on a vertical line, 5 units above the center of rotation. Ask students where this point would end up after rotating 90° counterclockwise. Watch for students who are attending to the 90° angle of rotation by drawing line segments from the center of rotation to the image and corresponding pre-image points. Also watch for how students determine that an image point is the same distance away from the center of rotation as its corresponding pre-image point: do they measure with a ruler, do they draw concentric circles centered at (12, 7), do they count the rise and run from (12, 7) to a point on the lizard and then use a related way of counting rise and run to locate the image point—in intuitively using the Pythagorean Theorem to keep the same distance, or do they ignore distance altogether?

For “Leaping Lizard”, watch for students who may have noticed that an image point and its corresponding pre-image point are equidistant from the line of reflection. Listen for how they justify that these distances are the same: do they measure with a ruler, do they fold the paper along the line of reflection, do they count the rise and run from the pre-image to the line of reflection and then from the line of reflection to the image point—in intuitively using the Pythagorean Theorem to keep the same distance. Also watch for students who notice that the line segments connecting the image points to their corresponding pre-image points are all parallel to each other—perhaps even noticing that all of these line segments have a slope of -2.

**Discuss (Whole Class):**

If students have not all located the same set of points for the images of the transformations, have students discuss whether this is reasonable or not. Inform students, “That transformations are like functions—any set of points that form a pre-image should have a unique set of points that form the image that is the result of the transformation. If we have not obtained unique images, then we have not recognized the precise nature of these transformations. That is the goal of our work today, to
notice what is important about each transformation so the images produced by the transformation are precisely defined.”

Discuss strategies for locating the images of the anchor points for each transformation. Here is a suggested list of a sequence of ideas to be presented, if available. While we will not be writing precise definitions for the transformations until the task Leap Year, it is important that the ideas of distance and direction (e.g., along a parallel line, perpendicular to a line, or along a circle) emerge during this discussion. If not all of the suggested strategies are available in the student work, at least make sure the debrief of each transformation does focus on both distance and direction. If either idea is missing, ask additional questions to prompt for it. For example, “How did you know how far away from the center point (or the reflecting line) this image point should be?” Also, be aware of the tasks that follow in this learning cycle—not everything needs to be neatly wrapped up in this discussion.

Debriefing the translation:

- Have a student present who used a transparency or tracing paper to get a set of image points that the whole class can agree upon.
- Next, have a student present who moved each anchor point up 8, right 12 units.
- Finally, have a student present who moved one anchor point up 8, right 12 units and then used the relative positions of the points in the original figure to locate related points in the image figure. Discuss that this is possible because translations preserve distance, angle and parallelism.

Debriefing the rotation:

- Have a student present who used a transparency or tracing paper to get a set of image points that the whole class can agree upon.
- Next, have a student present who used a protractor to measure 90° and a ruler to measure distances from the center of rotation. Draw in the line segments between (12, 7) and the corresponding image and pre-image points, using a different color for each image/pre-image pair. This will highlight the 90° angle of rotation, centered at (12, 7).
Next, have a student present who drew concentric circles (or arcs) to show that pairs of image/pre-image points are the same distance from (12, 7) because they lie on the same circle.

Finally, have a student present who showed that image/pre-image points are the same distance from (12, 7) by using the Pythagorean Theorem, or some strategy that is intuitively equivalent.

Debriefing the reflection:

- Have a student present who used a transparency or tracing paper to get a set of image points that the whole class can agree upon.
- Next, have a student present who used a ruler to measure distances from the line of reflection.
- If available, have a student describe how they determined these distances from the line of reflection using the Pythagorean Theorem, or some strategy that is intuitively equivalent.
- Next, have a student present who noticed that the segments connecting pairs of image/pre-image points are parallel, perhaps by pointing out that they have the same slope.
- Finally, have a student present who might argue that the segments connecting pairs of image/pre-image points are perpendicular to the line of reflection.

**Aligned Ready, Set, Go: Transformation and Symmetry 6.1**
Topic: Pythagorean Theorem

For each of the following right triangles determine the measure of the missing side. Leave the measures in exact form if irrational.

1. \( \sqrt{10} \)
2. \( \sqrt{17} \)
3. \( \sqrt{13} \)
4. \( ? \)
5. \( ? \)
6. \( ? \)
6.1 SET

Topic: Transformations.

Transform points as indicated in each exercise below.

7a. Rotate point A around the origin 90° clockwise, label as A’
    b. Reflect point A over x-axis, label as A’’
    c. Apply the rule \((x - 2, y - 5)\), to point A and label A’’’

8a. Reflect point B over the line \(y = x\), label as B’
    b. Rotate point B 180° about the origin, label as B’’
    c. Translate point B the point up 3 and right 7 units, label as B’’’
GO

Topic: Graphing linear equations.

Graph each function on the coordinate grid provided. Extend the line as far as the grid will allow.

9. \( f(x) = 2x - 3 \)

10. \( g(x) = -2x - 3 \)

11. What similarities and differences are there between the functions \( f(x) \) and \( g(x) \)?

12. \( h(x) = \frac{2}{3} x + 1 \)

13. \( k(x) = -\frac{3}{2} x + 1 \)

14. What similarities and differences are there between the equations \( h(x) \) and \( k(x) \)?

15. \( a(x) = x + 1 \)

16. \( b(x) = x - 3 \)

17. What similarities and differences are there between the equations \( a(x) \) and \( b(x) \)?