1.4 Scott’s Workout

A Solidify Understanding Task

Scott has decided to add push-ups to his daily exercise routine. He is keeping track of the number of push-ups he completes each day in the bar graph below, with day one showing he completed three push-ups. After four days, Scott is certain he can continue this pattern of increasing the number of push-ups he completes each day.

1. How many push-ups will Scott do on day 10?

2. How many push-ups will Scott do on day $n$?
3. Model the number of push-ups Scott will complete on any given day. Include both explicit and recursive equations.

4. Aly is also including push-ups in her workout and says she does more push-ups than Scott because she does fifteen push-ups every day. Is she correct? Explain.
1.4 Scott’s Push-Ups – Teacher Notes

A Solidify Understanding Task

Purpose:

This task is to solidify understanding that arithmetic sequences have a constant difference between consecutive terms. The task is designed to generate tables, graphs, and both recursive and explicit formulas. The focus of the task should be to identify how the constant difference shows up in each of the representations and defines the functions as an arithmetic sequence.

Standards Focus:

F-BF: Build a function that models a relationship between two quantities.

1: Write a function that describes a relationship between two quantities.*
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus on linear and exponential only)

Construct and compare linear, quadratic and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.
5. Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:

Standards for Mathematical Practice of Focus in the Task:

SMP2 - Reason abstractly and quantitatively.

SMP7 – Look for and make use of structure.

The Teaching Cycle:

Launch (Whole Class): Remind students of the work they have done previously with Growing Dots and Growing, Growing Dots. Read Scott’s Workout with the students and ask a student to add the number of push-ups that Scott will do on the fifth day to the diagram for the class. Ask students what they are observing about the pattern. Allow just a few responses so that you know that students understand the task, but avoid giving away the work of the task.

Explore (Small Group): Monitor student thinking as they work by moving from one group to another. Encourage students to use tables, graphs, and recursive and explicit equations as they work on the task. Listen to students and identify different groups to present and explain their work on one representation each. If students are having difficulty writing the equation, ask them to be sure that they have the other representations first.

Discuss (Whole Class): When the various groups are prepared to present, start the discussion with a table. Be sure that the columns of the table are labeled. After students have presented their
table, ask students to identify the difference between consecutive terms and mark the table so that it looks like this:

<table>
<thead>
<tr>
<th>n Days</th>
<th>f(n)</th>
<th>Difference between terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>&gt; 2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>&gt; 2</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>&gt; 2</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>3 + 2(n − 1)</td>
<td></td>
</tr>
</tbody>
</table>

Ask if the sequence is arithmetic or geometric based upon the table. Students should be able to identify that it is arithmetic because there is a constant difference between consecutive terms.

Next, ask the students to present the graph. The graph should be labeled and look like this:

Ask students where they see the difference between terms from the table on the graph. Identify that for each day, the number of push-ups increases by 2, so for each increase of 1 in the x value, the y value increases by 2. The students should recognize this as a slope of 2. Ask why the points in the graph are not connected. Students should be able to answer that the push-ups are assumed to be
done all at once in the day. A continuous graph would suggest that the push-ups were happening for the entire time shown on the graph.

Next, ask students for their recursive equations. Students may have written any of these equations:

Number of push-ups today = Number of push-ups yesterday + 2

Or:

Next term = Previous term + 2

Ask how they see this equation in their table and their graph. On the table, they should point out that as you move from one row to the next, you add 2 to the previous term. They should be able to demonstrate a similar idea on the graph as you move from one y-value to the next.

Ask if anyone has written a recursive equation in function form. If no one has written the equation in function form, explain that the more formal method of writing the equation is:

\[ f(1) = 3, f(n) = f(n-1) + 2. \]

This form still denotes the idea that the current term is 2 more than the previous term. Ask students how they can identify that this is an arithmetic sequence using the recursive equation. The answer should be that the constant difference of 2 between terms shows up in the equation as adding 2 to get the next term. Also note that to use a recursive formula you have to know the previous term. That means that when you make a recursive formula for an arithmetic sequence, you need to provide the first term as part of the formula.

Conclude the discussion with the explicit equation, \( f(n) = 3 + 2(n - 1) \). Although this equation could be simplified, it is useful to consider it in this form. Ask students how they used the table to write this equation. How does this formula show the constant difference between terms? Also ask, “If you are looking for the 10th term, what number will you multiply by 2?” Help students to connect that the \((n - 1)\) in the formula tells them that the number they multiply by 2 is one less than the term they are looking for. Can they explain that using the table or graph?

Conclude the lesson by asking students to compare recursive and explicit formulas. What information do you need to use either type of formula? What are the advantages of each? What ideas about arithmetic sequences are highlighted in each?
Ready

Topic: Use function notation to evaluate equations.

Evaluate the given equation for the indicated function values.

1. \(f(n) = 5n + 8\)
   \[
   f(4) = \quad f(10) = \quad f(-5) = \quad f(9) =
   \]
   \[
   f(-2) = \quad f(-1) = \quad f(0) = \quad f(-11) =
   \]

2. \(f(n) = -2n + 1\)

3. \(f(n) = 6n - 3\)

4. \(f(n) = -n\)

5. \(f(n) = 5^n\)

6. \(f(n) = 3^n\)

7. \(f(n) = 10^n\)

8. \(f(n) = 2^n\)

Set

Topic: Finding terms for a given sequence.

Find the next 3 terms in each sequence. Identify the constant difference. Write a recursive function and an explicit function for each sequence. Circle where you see the common difference in both functions. (The first number is the 1st term, not the 0th term).

9. A) \(3, 8, 13, 18, 23, \ldots\)
   B) Common Difference: 
   C) Recursive Function: 
   D) Explicit Function:

10. A) \(11, 9, 7, 5, 3, \ldots\)
    B) Common Difference: 
    C) Recursive Function: 
    D) Explicit Function:

11. A) \(3, 1.5, 0, -1.5, -3, \ldots\)
    B) Common Difference: 
    C) Recursive Function: 
    D) Explicit Function:
GO

Topic: Reading a graph

Olaf is a mountain climber. The graph shows Olaf's location on the mountain beginning at noon. Use the information in the graph to answer the following questions.

12. What was Olaf's elevation at noon?

13. What was his elevation at 2 pm?

14. How many feet had Olaf descended from noon until 2 pm?

15. Olaf reached the base camp at 4 pm. What is the elevation of the base camp?

16. During which hour was Olaf descending the mountain the fastest? Explain how you know.

17. Is the value of $f(n)$ the time or the elevation?