6.1 Photocopy Faux Pas

A Develop Understanding Task

Burnell has a new job at a copy center helping people use the photocopy machines. Burnell thinks he knows everything about making photocopies, and so he didn’t complete his assignment to read the training manual.

Mr. and Mrs. Donahue are making a scrapbook for Mr. Donahue’s grandfather’s 75th birthday party, and they want to enlarge a sketch of their grandfather which was drawn when he was in college. They have purchased some very expensive scrapbook paper, and they would like this image to be centered on the page. Because they are unfamiliar with the process of enlarging an image, they have come to Burnell for help.

“We would like to make a copy of this image that is twice as big, and centered in the middle of this very expensive scrapbook paper,” Mrs. Donahue says. “Can you help us with that?”

“Certainly,” says Burnell. “Glad to be of service.”

Burnell taped the original image in the middle of a white piece of paper, placed it on the glass of the photocopy machine, inserted the expensive scrapbook paper into the paper tray, and set the enlargement feature at 200%.

In a moment, this image was produced.

“You’ve ruined our expensive paper,” cried Mrs. Donahue. “Much of the image is off the paper instead of being centered.”

“And this image is more than twice as big,” Mr. Donahue complained. “One fourth of grandpa’s picture is taking up as much space as the original.”
In the diagram below, both the original image—which Burnell taped in the middle of a sheet of paper—and the copy of the image have been reproduced in the same figure.

1. Explain how the photocopy machine produced the partial copy of the original image.

2. Using a “rubber band stretcher” finish the rest of the enlarged sketch.

3. Where should Burnell have placed the original image if he wanted the final image to be centered on the paper?

4. Mr. Donahue complained that the copy was four times bigger than the original. What do you think? Did Burnell double the image or quadruple it? What evidence would you use to support your claim?

5. Transforming a figure by shrinking or enlarging it in this way is a called a dilation. Based on your thinking about how the photocopy was produced, list all of the things you need to pay attention to when dilating an image.
6.1 Photocopy Faux Pas – Teacher Notes
A Develop Understanding Task

**Purpose:** The purpose of this task is to develop a description of the essential features of a dilation:

a. Lines are taken to lines, and line segments to line segments of proportional length in the ratio given by the scale factor.
b. Angles are taken to angles of the same measure.
c. A line not passing through the center of dilation is taken to a parallel line, and lines passing through the center of dilation are unchanged.
d. To describe a dilation we need to specify a center of dilation and a scale factor. The center of dilation is a fixed point in the plane about which all points are expanded or contracted. It is the only invariant point under a dilation

e. Dilations create similar figures—the image and pre-image are the same shape, but different sizes (unless the scale factor is 1, then the image and pre-image are congruent).

Throughout the next few tasks students should formalize the definition of dilation: A dilation is a transformation of the plane, such that if \( O \) is the center of the dilation and a non-zero number \( k \) is the scale factor, then \( P' \) is the image of point \( P \) if \( O, P \) and \( P' \) are collinear and \( \frac{OP'}{OP} = k \).

**Core Standards Focus:**

**G.SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

**Mathematics II Note for G.GMD.1, G.GMD.3** Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor \( k \), its area is \( k^2 \) times the area of the first. Similarly, volumes of solid figures scale by \( k^3 \) under a similarity transformation with scale factor \( k \).

**Related Standards:** G.SRT.2, G.GMD.1

**Launch (Whole Class):**
Engage students in the context of this task by reading the scenario with Burnell and the Donahue’s, including both Mrs. Donahue and Mr. Donahue’s complaints about the ruined photocopy image. Then ask students to respond to questions 1-5. Allow students to work individually for a time on these questions before moving students into small groups to share and discuss their results.

You will need to show students how to make and use the rubber band stretchers called for in question 2. These are made by looping one rubber band through a second one of the same size to create a knot where the two bands meet. Hold one end of the rubber band stretcher at the center of
dilation and insert the point of a pencil at the other end. Let the knot in the middle of the rubber band stretcher trace over the pre-image while the pencil traces out the image.

**Explore (Small Group):**
Allow students to grapple with the questions without a lot of intervention. Make sure that students have identified that the center of dilation (by whatever name they refer to it) is located at the top left corner of the paper. Assist students who need help with the rubber band stretchers. It is not necessary for students to trace a perfect image of the Donahue’s grandfather with the rubber band stretcher, but they should at least identify where the four corners of the enlarged picture will be located. This will give them a sense of the area occupied by the enlargement. Have them also locate a few other key points on the image, such as the ends of the bow tie or the tip of the V in the collar.

Watch for where students decide they would locate the original picture in order for the copy to be centered on the paper and listen to their reasons for suggesting so. Identify any misconceptions students might have about dilations so these can be discussed as a whole class. Be aware of students who are thinking correctly about these concepts so you can call upon them to clarify any misconceptions.

**Discuss (Whole Class):**
Start the whole class discussion with question 4—did the image get doubled or quadrupled? Let students debate both perspectives. Evidence for quadrupling would come from the area of the large figure compared to the area of the original. Evidence for doubling would come from measuring distance between corresponding points on the pre-image and image, such as measuring the length of the bow tie in both figures or the width of the glasses. Assist students in recognizing that distances have been doubled, causing the area to be quadrupled since it is the product of length and width.

Next, discuss question 3. This question should surface the issue that scale factors between 0 and 1 shrink the image closer to the center of dilation while scale factors greater than 1 enlarge the image farther from the center.

Finally, discuss question 5. Make sure that the issues listed in a-e in the purpose statement above come out in this discussion.

**Aligned Ready, Set, Go:** Similarity and Right Triangle Trig 6.1
**Ready, Set, Go!**

**Ready**

Topic: Scale factors for similar shapes.

Give the factor by which each pre-image was multiplied to create the image. Use the scale factor to fill in any missing lengths.

1.

![Pre-image (2, 1.5) and Image (4, 3)]

2.

![Pre-image (1, 1) and Image (6, 4.5)]

3.

![Pre-image (2, 2.5) and Image (8)]

4.

![Pre-image (1.5, 4) and Image (4, 6)]

5.

![Image (10, 3.5) and Pre-image (2, 2)]

6.

![Image: Large Triangle (9, 3) and Pre-image: Small Triangle (4, 12)]
Set

Topic: Dilations in real world contexts

For each real-world context or circumstance determine the center of the dilation and the tool being used to do the dilation.

7. Fran walks backward to a distance that will allow her family to all show up in the photo she is about to take.

8. The theatre technician plays with the zoom in and out buttons in effort to fill the entire movie screen with the image.

9. Melanie estimates the height of the waterfall by holding out her thumb and using it to see how many thumbs tall to the top of the waterfall from where she is standing. She then uses her thumb to see that a person at the base of the waterfall is half a thumb tall.

10. A digital animator creates artistic works on her computer. She is currently doing an animation that has several telephone poles along a street that goes off into the distance.

11. Ms. Sunshine is having her class do a project with a rubber-band tracing device that uses three rubber bands.

12. A copy machine is set at 300% for making a photo copy.
Go

Topic: Rates of change related to linear, exponential and quadratic functions

Determine whether the given representation is representative of a linear, exponential or quadratic function, classify as such and justify your reasoning.

13. 

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<td>5</td>
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Type of function: 
Justification:

14.

Type of function: 
Justification:

15. \( y = 3x^2 + 3x \)

Type of function: 
Justification:

16. \( y = 7x - 10 \)

Type of function: 
Justification:

17.

Type of function: 
Justification:

18. 

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Type of function: 
Justification: