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Chapter 1: Ratio Relations (4 weeks)

Utah Core Standard(s):

- Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* (6.RP.1)
- Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* (6.RP.2)
- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (6.RP.3)
  a) Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  b) Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
  c) Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
  d) Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Academic Vocabulary: ratio, a:b, table, tape diagram, equivalent ratios, ratio statement, part to part, part to total, double number line, graph, coordinate plane, ordered pair, unit rate, rate unit, partition, iterate, equation, variable, dependent variable, independent variable

Chapter Overview: In this chapter, students extend their work with multiplication and division to solve ratio and rate problems about quantities. A ratio expresses a relationship between two or more quantities. The chapter begins with an introduction to ratio language and notation, a critical component to working with ratios. Students learn about different models that can be used to represent ratios, progressing from concrete to pictorial to abstract representations. Students reason about multiplication and division to find equivalent ratios and organize the equivalent ratios in tables and on graphs. Students learn about the different ways to represent and solve ratio problems (tiles, pictures, tape diagrams, double number lines, partial tables, graphs, and equations). They make decisions as to which representation might be best to use for a given problem, solve problems using a variety of strategies, and make connections between the strategies. Students learn that every ratio has as associated rate. They learn how to find unit rates and discover that unit rates can be a powerful tool for solving problems.

Connections to Content:
Prior Knowledge: In this chapter, students build on their understanding of multiplication and division from earlier grades. Models such as arrays and area models and an understanding of the distributive property, concepts from **3.OA**, are helpful tools for finding equivalent ratios. Students will rely on their fraction sense and operations with fractions, **5.NF**, to determine and iterate unit rates. In **5.G**, students graphed points in the first quadrant which will help them to plot equivalent ratios in this chapter. Work in **5.OA** with writing numerical expressions and analyzing patterns and relationships provides a good foundation for writing equations to show the relationship between two quantities.

Future Knowledge: Ratios and proportional relationships are foundational for further study in mathematics and science and useful in everyday life. Later in this course, students use ratios to build an understanding of percent and to solve problems involving percents such as tip, discount, and tax. They also use ideas about ratio in their study of descriptive statistics. In 7th grade, students will use ratios in their study of similar figures and probability. In 8th grade, students use ratios when studying slopes of lines and linear relationships.

MATHEMATICAL PRACTICE STANDARDS

<table>
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<td>Make sense of problems and persevere in solving them.</td>
<td>
The tables below show three different recipes for making orange juice using orange concentrate and water.
<table>
<tr><th colspan="2">Recipe 1</th></tr>
<tr><th>Cups of Orange Con.</th><th>Cups of Water</th></tr>
<tr><td>3</td><td>5</td></tr>
<tr><td>6</td><td>10</td></tr>
<tr><td>9</td><td>15</td></tr>
<tr><td>10</td><td>$16\frac{2}{3}$</td></tr>
</table>
<table>
<tr><th colspan="2">Recipe 2</th></tr>
<tr><th>Cups of Orange Con.</th><th>Cups of Water</th></tr>
<tr><td>2</td><td>3</td></tr>
<tr><td>4</td><td>6</td></tr>
<tr><td>6</td><td>9</td></tr>
<tr><td>8</td><td>12</td></tr>
</table>
<table>
<tr><th colspan="2">Recipe 3</th></tr>
<tr><th>Cups of Orange Con.</th><th>Cups of Water</th></tr>
<tr><td>5</td><td>8</td></tr>
<tr><td>10</td><td>16</td></tr>
<tr><td>15</td><td>24</td></tr>
<tr><td>20</td><td>32</td></tr>
</table>
Order the recipes from strongest orange flavor to weakest orange flavor. Justify your answer.
<i>To answer this problem, students must make sense of the quantities given and their relationship. There are a variety of strategies that students can use: find ratios with a common value, find the unit rate, examine the explicit relationship between the two quantities, etc. Students may use pictures and tape diagrams as tools while working toward an answer. They may utilize patterns of addition and multiplication. Students may rely on their fraction sense if they find and compare unit rates. Students will need to think critically about what the numbers are telling them. For example, if students find the unit rate of cups of water/cups of orange concentrate, a greater unit rate will actually indicate a weaker orange flavor. Students will be required to stop along the way and track their progress: “Which recipes have I compared?” “Which recipes do I still need to compare for my answer to be complete?” “How can I track my progress?”</i>
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<td>Reason abstractly and quantitatively.</td>
<td>Jeff’s soccer team loses 2 soccer games for every 3 that they win. Lauren’s soccer team loses 2 soccer games for every 3 games that they play. Compare these statements. Both have a ratio of 2:3. Do Jeff and Lauren’s teams have the same winning record? Why or why not?
<i>Throughout the chapter, students are required to make sense of the quantities in problem situations and their relationship to each other. They will construct models such as tape diagrams to represent ratio relationships and use the models to solve problems. A critical component of this process involves decoding ratio language. “What quantities are involved?” “What units are involved?” “Is this a part to part relationship or a part to total relationship?” “Does my model match the ratio statement given?”</i></td>
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<td>💬</td>
<td>Construct viable arguments and critique the reasoning of others.</td>
<td>Dominic and Emma are running for class president. For each vote that Dominic receives Emma receives four votes. List <u>five</u> possibilities for the total number of students who voted. Justify your answers. Write a rule that describes all the possible answers to this problem.</td>
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</table>

		Throughout the chapter, problems require students to justify their answers or explain the strategies used to solve a problem. Justification can take many forms. It may be an explanation in words given orally or in writing, a picture, model, or diagram, an illustration of patterns seen in tables and on graphs, writing an equation, etc. Students may solve this problem a variety of ways: using concrete manipulatives, drawing a picture, creating a tape diagram, using ideas about multiplication, creating a partial table, writing an equation, etc. Students should share out their strategies and conclusions in order to consider multiple approaches and make connections between the approaches. Since there are infinite answers to this question, students will begin to generalize and articulate a rule for total number of students who could have voted which will help to solidify and demonstrate their understanding of ratio relations.
	Model with mathematics.	At a baseball game, the snack bar sells 3 hotdogs for every 2 hamburgers they sell. Create a double number line to show the relationship between the number of hotdogs and the number of hamburgers sold at the baseball game. Organize the information in the table below. Complete the graph below. *Ratio relations can be modeled using several different representations (pictures, tape diagrams, double number lines, geometric models, tables, graphs, and equations). In this chapter, students have the opportunity to analyze and create these different representations to represent real world situations. They compare the representations, make connections between them, and determine when one representation may be better to use based on the problem and questions being asked.*
	Use appropriate tools strategically.	Chen makes 2 out of every 3 free throws that he shoots. If he shoots 18 free throws in a game, how many do you expect him to make? Solve using at least <u>two</u> strategies. *Throughout this chapter, students are exposed to a variety of tools that can be used to represent ratio relations and solve real world problems. These tools include concrete manipulatives such as tiles and chips, pictures, tape diagrams, tables, graphs, and equations. The chapter starts with more concrete tools then progresses into tools that are more abstract. For example, students start by modeling ratio relations with chips and tiles (concrete), then pictures (pictorial), and then tape diagrams (abstract). They understand that these all show the same relationship but more abstract representations can be more efficient and flexible for solving problems. They make connections between the different tools. For example, they consider patterns in tables and on graphs. They understand that an equation shows the explicit relationship between two variables given in a table. In the chapter, students are introduced to tools and then given the autonomy to decide which tools they want to use based on their comfort with the tools and the problem given.*

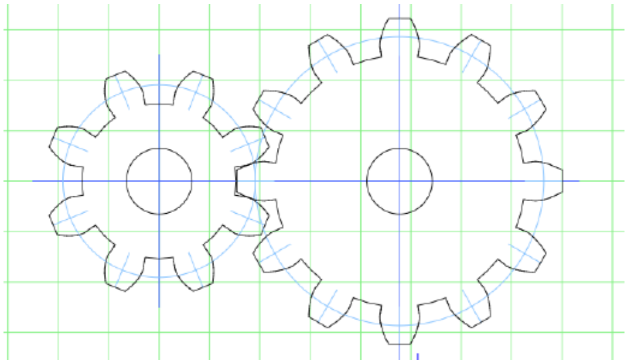
	Attend to precision.	Sophia's teacher asked her to create a pattern using only circles and squares so that the ratio of circles to total shapes is 2:5. Sophia created the following pattern. ○ ○ □ □ □ □ □ Is Sophia's pattern correct? Justify your answer. *Moving fluently between ratio statements and representations of ratio relations requires attention to precision. Students interpret and decode the language of ratio, asking questions such as, "What quantities are involved?" "What units are involved?" "Is this a part to part ratio or a part to total ratio?" "What does this notation mean?"*
	Look for and make use of structure.	A recipe for punch calls for 5 cups of lemonade for every 2 cups of fruit punch. Draw a model to represent this relationship. Then, interpret all of the following: 5 to 2 2:5 $\frac{2}{5}$ $\frac{5}{2}$ *Students use structure when moving fluidly between ratios and their associated rates. While the numbers given above all express the same relationship, they have different meanings. A model is helpful for revealing this relationship. In this example, students see a model that is 2 parts fruit punch and 5 parts lemonade. A multiplicative comparison between the two parts shows that fruit punch is* $\frac{2}{5}$ *the size of lemonade (or lemonade is* $\frac{5}{2}$ *or 2.5 times larger than fruit punch).*
	Look for and express regularity in repeated reasoning.	Marcus is training for an ultra-marathon where he will be running 100 miles. He can run 7 miles per hour. Complete the table below to show the relationship between time and distance for Marcus. Write an equation to show the relationship between time and distance for Marcus. Use *t* for time and *d* for distance. *Early on in the chapter, students use ideas about repeated addition and scalar multiplication to find equivalent ratios and complete ratio tables. They illustrate patterns they see on double number lines, tables, and graphs. These patterns begin to surface ideas about slope and linear relationships. Later in the chapter, students write equations to represent ratio relations. To help students arrive at the equation for the example given, students can start by writing numerical expressions:* *After 1 hour, distance = 1(7) = 7* *After 2 hours, distance = 2(7) = 14* *After 3 hours, distance = 3(7) = 21* *After t hours, distance = t(7) = 7t*

1.0 Anchor Problem: Connected Gears

This problem should help students to realize that just as the gears in the example are connected, so are the quantities in a ratio. When working with ratios, we are working with two quantities simultaneously and investigating the relationship between the two quantities. A value from one quantity is connected to a value from the other quantity to make a pair of values that describe an instance or event. Because the gears are connected, the number of revolutions one of them makes is dependent upon or affected by the number of revolutions the other one makes.

This anchor problem should be re-visited throughout the chapter. Students can experiment with part a. at the start of the chapter. You may wish to re-visit parts b. – f. at different points in the chapter. Many of the strategies and tools that can be used to solve this problem will develop as the chapter progresses. Encourage students to use multiple strategies to solve each part of the problem and explain the methods used.

Two gears are connected as shown in the picture below. The smaller gear has **8** teeth and the larger gear has **12** teeth.



You may want to show students animations of gears to make sure they know how gears work. Ask questions such as, “If the smaller gear is rotating clockwise, which direction will the larger gear rotate?” *The larger gear will rotate counterclockwise.* “If you turn the larger gear one full rotation, does the smaller gear make one revolution, more than one revolution, or less than one revolution?” *The smaller gear makes more than one revolution.*

a. Find a way to determine the number of revolutions the small gear makes based on the number of revolutions the large gear makes. Organize your results.
   You may want to give the students actual gears to experiment with (Lego gears or gears from a gear kit) or give the students cut-outs of the gears from above on cardstock so that they can manipulate the gears to help them solve the problem. They can tag the aligned teeth on the gears and find the number of revolutions it takes for the tagged teeth to re-align.

   As students start recording their data, encourage them to think of ways to organize their results. The students can organize their results in an ordered list or they may find it easier to organize their results in a table. Throughout this chapter, students will see that a table can be a very useful tool for organizing data. Sample values are given in the table below:

Larger Gear	2	4	6	8	10
Smaller Gear	3	6	9	12	15

b. If the larger gear makes **20** revolutions, how many revolutions will the smaller gear make?
If the larger gear makes 20 revolutions, the smaller gear will make 30 revolutions. Students can use concrete manipulatives, tape diagrams, partial tables, and numeric methods to solve this problem. Refer to **1.1c Activity 1** for examples of these different methods.

c. If the smaller gear makes **24** revolutions, how many revolutions will the larger gear make?
If the smaller gear makes 24 revolutions, the larger gear will make 16 revolutions. Again, students can use a variety of strategies including manipulatives, tape diagrams, partial tables, and numeric methods.

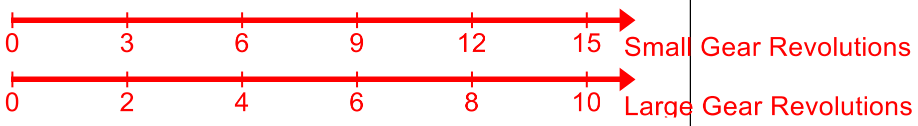
d. If the larger gear makes **1** full revolution, how many revolutions does the smaller gear make?
If the larger gear makes 1 full revolution, the smaller gear will make 1.5 or $\frac{3}{2}$ revolutions. Students can investigate this using the actual gears or draw pictures/models similar to those shown in **1.2d**.

e. If the smaller gear makes **1** full revolution, how many revolutions does the larger gear make?
If the smaller gear makes 1 full revolution, the larger gear will make $\frac{2}{3}$ of a revolution. Again, students can investigate this using the actual gears or draw pictures/models similar to those shown in **1.2d**.

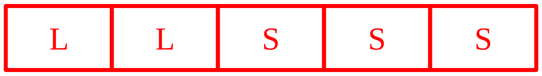
In parts d. and e. students are finding the unit rate associated with the ratio 3 to 2. There are two unit rates associated with this problem, the unit rate for revolutions of the large gear per revolution of the small gear and revolutions of the small gear per revolution of the large gear. In this chapter, students will see that these values are reciprocals. Additionally, students will realize that finding the unit rate associated with a ratio can be a very useful tool for solving real world problems.

f. Create <u>four</u> different representations of the relationship between the number of revolutions the large gear makes and the number of revolutions the small gear makes. Make up a question that can be answered using each representation.
Students can show the relationship by creating a tape diagram, double number line, table, graph, or equation. Have students share out, compare, and connect the different representations.

Representation 1: Double Number Line



Students may also choose to create a tape diagram.



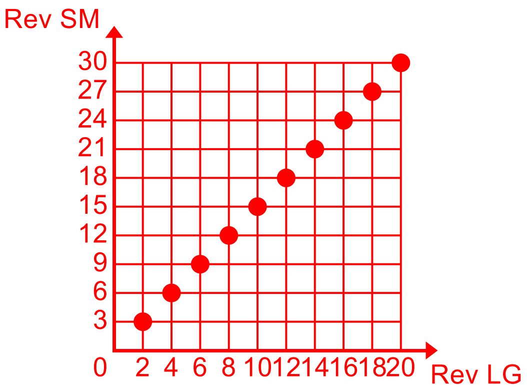
Question: Questions will vary. Sample: If the large gear makes 6 revolutions, how many revolutions will the small gear make?

Representation 2: Table

Larger Gear	2	4	6	8	10
Smaller Gear	3	6	9	12	15

Question: Questions will vary. Sample: If the gears together make a total of 20 revolutions, how many revolutions did the small gear make?

Representation 3: Graph – scales may vary



Question: Questions will vary. Sample: What does the ordered pair (4, 6) represent in the situation?

Representation 4: Equation

$l = \frac{2}{3}s$ or $s = \frac{3}{2}l$ where l = large gear and s = small gear

Question: Questions will vary. Sample: If the large gear makes 100 revolutions, how many will the small gear make?

This problem was adapted from a problem in <u>Developing Essential Understanding of Ratios, Proportions, & Proportional Reasoning</u> from the National Council of Teachers of Mathematics.

Section 1.1: Representing Ratios

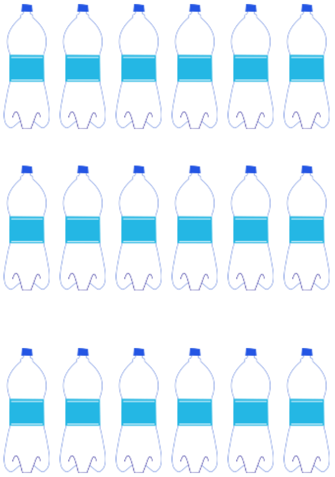
Section Overview:
In this section, students learn what a ratio is. Students create pictures and models (tape diagrams) to represent ratios given in words and they write ratio statements to describe relationships given in pictures and diagrams. Through the process, students develop flexibility and fluidity with ratio language and notation and familiarity with the models that can be used to represent ratios and solve ratio problems. Students also learn that there are different types of ratios (part to part and part to total) and because of this they must pay close attention to the quantities given in a problem and their relationship to each other. Students find equivalent ratios using concrete manipulatives, models, and numeric methods and connect this work to their knowledge of whole number multiplication and division. They organize equivalent ratios in tables. Lastly, students synthesize and apply the knowledge learned in the section to solve a variety of real world ratio problems.

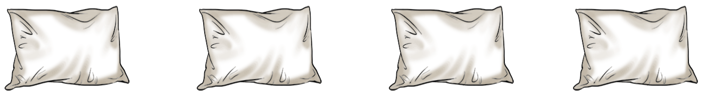
Concepts and Skills to Master:
By the end of this section, students should be able to:

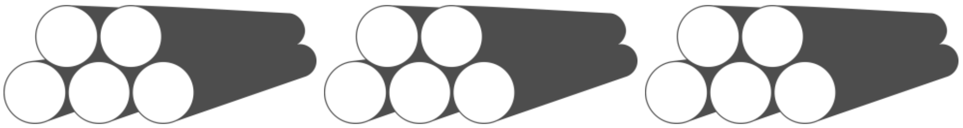
1. Understand what a ratio is.
2. Given a pictorial representation or model, use ratio language and notation to describe the relationship between two or more quantities.
3. Given a ratio statement, identify the important quantities and interpret the language and notation in order to create a pictorial representation or model of the relationship between the quantities.
4. Distinguish when a ratio is describing a part to part, part to total, or total to part relationship between quantities.
5. Determine equivalent ratios to describe a relationship between two quantities. Make and complete tables of equivalent ratios.
6. Use ratio reasoning and models (tape diagrams) to solve real world ratio problems.

1.1a Class Activity: The Language of Ratio

Activity 1:
Drew's family of four is going camping for three days. Here is the gear they have packed.







a. Make several comparisons between the items Drew's family has packed in relationship to other quantities (i.e. the number of people going, the number of days they are going, different items that are packed, etc.).

You may wish to have students do this in small groups or with a partner as a Think-Pair-Share. This problem is intended to elicit ratio language from students, which they use naturally every day. Listen to the language students are using and help students refine their language to show a *relationship between two quantities*. For example, if students say, "There are four sleeping bags" ask "How does that relate to the number of people camping?" "Each person needs one sleeping bag" or "They need one sleeping bag per person." Here are some of the comparison statements students may make about the different items. Key ratio language has been underlined.

- Tent – Each tent holds four people. There are four people per tent.
- Sleeping Bags and Pillows – There is one sleeping bag per person. Each person needs one sleeping bag. There is one pillow for every sleeping bag (or for every person).
- Water Bottles – There are six bottles of water per day. Each person needs $1\frac{1}{2}$ bottles of water per day (this may be more difficult for students to see but if they take one group of 6 water bottles and give one to each person, they will have two remaining. To divide these two remaining bottles among four people, each person would get another half bottle of water). Each person needs $4\frac{1}{2}$ bottles of water for the trip.
- Firewood – They need one bundle of wood per day. They need five logs of wood every day.
- Sunscreen – Each person needs $\frac{1}{2}$ bottle of sunscreen. They need $\frac{2}{3}$ bottle of sunscreen per day.

b. What else would you suggest Drew's family bring camping? How much/many should he bring? Relate the amount they should bring to another quantity (i.e. number of people going, number of days, quantity of another item). Answers will vary. Students may reason about the number of marshmallows needed, the number of flashlights, backpacks, first aid kits, meals, number of stakes, etc. Extension – What if another family of four was joining the trip? How many of each item would they need? What if a family of two was joining the trip? How many of each item would they need?

Activity 2: Marla, Chase, and Evelyn are putting red and blue marbles in a jar. Use the information below to draw a picture that represents the marbles that could be in each person's jar.

Marla:
Marla puts three blue marbles in the jar for every five red marbles she puts in the jar.

This problem is intended to surface several ideas about ratios:

1) **Equivalent Ratios:** As students start drawing the marbles, some students will naturally question, "Do I only draw 3 blue marbles and 5 red marbles or do I keep drawing sets of 3 blue marbles and 5 red marbles?" Ask them, "If you draw 6 blue marbles and 10 red marbles or 9 blue marbles and 15 red marbles, do you still have five red marbles for every three blue marbles?" Have students share their pictures so that they start thinking about equivalent ratios and how they express the same relationship between two quantities.
2) **The order of the quantities in the ratio statement matters.** This can be seen when comparing the marbles in Marla's jar to the marbles in Chase's jar.
3) **Part-to-Part and Part-to-Total:** The ratio statements that describe the marbles in Marla and Chase's jar are both part-to-part ratio statements while the ratio statement that describes the marbles in Evelyn's jar is a part-to-total ratio statement. Have a discussion with the students about the importance of attending to precision when reading a ratio statement: What quantities are involved? Is the ratio statement describing the parts or a part and the total? What order are the quantities written in? You may also begin to ask questions such as, "Is it possible for Marla to have 20 total marbles in her jar? 32 total marbles in her jar? Why or why not? Is it possible for Evelyn to have 20 total marbles in her jar? 32 total marbles in her jar? Why or why not?"
4) **Different Pictorial Representations of Ratios:** Some students may draw red and blue circles to represent the marbles. Others may use the letters "R" and "B" to represent the different colored marbles. As the chapter progresses, students will learn how to create tape diagrams to model ratios.

Chase:
Chase puts three red marbles in the jar for every five blue marbles he puts in the jar.

Students should have a picture with 3 red marbles and 5 blue marbles (or an equivalent ratio).

Evelyn:
Three out of every five marbles in Evelyn's jar are blue.

If students struggle with this, have them draw five marbles. Three of them are blue so that means that the rest are red; therefore 2 are red.

Note for Teachers: This problem touches on an important topic involving ratios. A common misconception is to treat all ratios as fractions. Since a fraction represents a part to a total, a part to part ratio is not considered a fraction. In this problem, Marla and Chase's ratio statements are not fractions as they are both part to part ratios; however Evelyn's is a part to total ratio and therefore a fraction. See the Mathematical Foundation for a more in-depth discussion of this.

The statements describing the marbles in Marla, Chase, and Evelyn's jars are **ratio** statements.

A **ratio** expresses a relationship between two or more quantities.

Activity 3: In one day, an ice cream shop sells 3 ice creams on a cone for each ice cream they sell in a cup.

a. Draw a picture to represent this relationship. This problem is intended to solidify concepts introduced in the previous problem – order matters, there are different types of ratio statements – part-to-part and part-to-total.

Additionally, introduce students to ratio notation (3 to 4 can be written 3:4).

b. Write several ratio statements about the ice creams sold at this shop. Tell whether the statement is a part-to-part, part-to-total, or total-to-part.

- There are 3 ice creams sold in a cone per ice cream sold in a cup.
- One out of every four ice creams sold is in a cup.
- For every four ice creams the shop sells, three are in a cone.
- The ratio of cones sold to cups sold is 3 to 1.
- The ratio of cups sold to cones sold is 1 to 3.
- The ratio of cups sold to cones sold is 1:3.
- The ratio of cups sold to total ice creams sold is 1:4.
- The ratio of total ice creams sold to cones sold is 4:3.

Have students underline key ratio language used in the statements above. Have students indicate whether the statement is a part-to-part (PP), part-to-total (PT), or total-to-part (TP) next to each statement.

c. Based on this ratio, give some possibilities for the total number of ice creams that are sold at this shop in one day. Answers will vary but should be multiples of 4.

You can use this space to have students summarize what they learned about ratios in these first three activities.

1. Mary is making a quilt. In her quilt, the ratio of stars to moons is 3 to 2. Sam is also making a quilt. In his quilt, the ratio of moons to stars is 3 to 2.

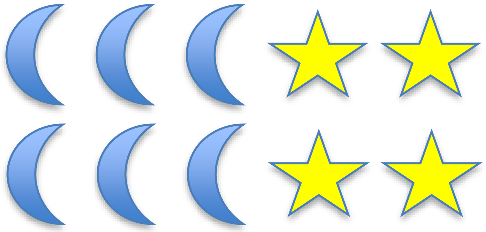
    1. Draw a picture to show the relationship between stars and moons in each person's quilt.

It helps students to write the numbers directly below the quantities as shown below:

For Mary's quilt, the ratio of stars to moons is 3 to 2: For Sam's quilt, the ratio of moons to stars is 3 to 2.

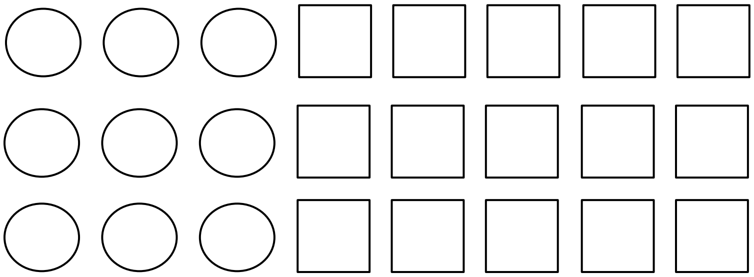
Stars to Moons
3 to 2

Moons to Stars
3 to 2



Pictures will vary. Students are starting to surface ideas about equivalent ratios. Some students may also use "S" for star and "M" for moon. They are transitioning from a pictorial representation to a more abstract representation.

2. Travis is drawing circles and squares on his paper using the pattern shown below.
   When reading and writing ratios, a common error is for students to mix up the order in which the quantities and their values are expressed. Students must attend to precision in the order in which the quantities are expressed in the ratio and notice the structure used in ratio language and notation.



    a. Complete the following statements:

    The ratio of ___circles_________________ to ____squares_______________ is 3 to 5.

    The ratio of ___squares_________________ to ____circles_______________ is 5 to 3.

3. At a carnival, there are 8 boys for every 3 girls waiting in line to climb a rock wall.
    a. Complete the following statements:

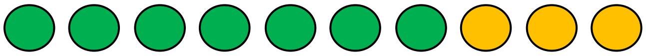
    The ratio of girls to boys standing in line is __3___:__8___.

    The ratio of boys to girls standing in line is __8___:__3___.

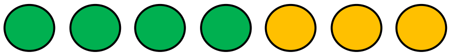
    The ratio of girls to total people standing in line is __3___:__11___.

4. Charlie and Teo are making pictures using yellow and green circles. In Charlie's picture, the ratio of yellow circles to green circles is 3:7. In Teo's picture, the ratio of yellow circles to total circles is 3:7.
    a. Draw a picture of the relationship between yellow and green circles for each boy.

Charlie's picture:



Teo's picture:



Students can write or say additional ratio statements to describe the relationship between green and yellow circles for each boy.

5. In a carnival game, a player selects a rubber duck out of a pool of water. If the rubber duck has a red sticker on it, the player wins. One out of four rubber ducks has a red sticker on the bottom. Encourage students to draw a picture.

    a. Complete the following statements:
    The ratio of winning ducks to losing ducks is __1:3____. PP

    The ratio of losing ducks to winning ducks is __3:1____. PP

    The ratio of winning ducks to total ducks is __1:4____. PT

    The ratio of losing ducks to total ducks is __3:4____. PT

    The ratio of total ducks to winning ducks is __4:1____. TP

    b. Next to each ratio above, indicate whether it is a part to part (PP), part to total (PT), or total to part (TP).

6. A deli uses 2 oranges for every 5 apples in their famous fruit salad recipe. If students are struggling, encourage them to draw a picture or use manipulatives to model the situation.

    a. Complete the following statements.

    The ratio of oranges to apples is __2__:__5__.

    The ratio of apples to total pieces of fruit is __5__:__7__.

    The ratio of total pieces of fruit to apples is __7__:__5__.

    The ratio of oranges to total pieces of fruit is __2__:__7__.

7. Leslie is making bouquets for a wedding. She uses 5 tulips for every 2 roses.
    a. Complete the following statements:

    The ratio of ___tulips___________ to __total flowers_______ is 5:7.

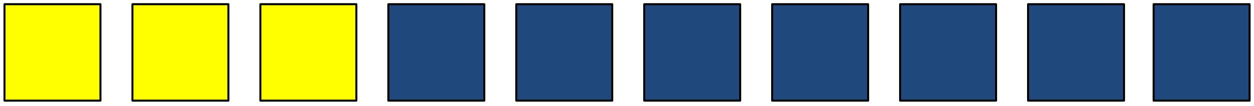
    The ratio of __total flowers_______ to ___roses___________ is 7:2.

8. In a herd of sheep, 1 out of every 40 sheep is black. The rest are white.
   a. Complete the following statements:
      The ratio of black sheep to white sheep in a herd is __1___:__39___.

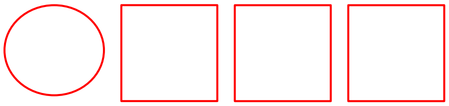
      The ratio of white sheep to black sheep in a herd is __39___:__1___.

      The ratio of white sheep to total sheep in a herd is ___39___:__40____.

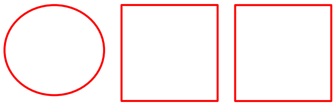
9. Felipe is drawing yellow and blue squares. The ratio of yellow squares to total squares in Felipe's picture is 3 to 10. Create Felipe's picture.



10. Kai is drawing circles and squares on his paper. The ratio of circles to squares in Kai's picture is 1:3. Create Kai's picture.

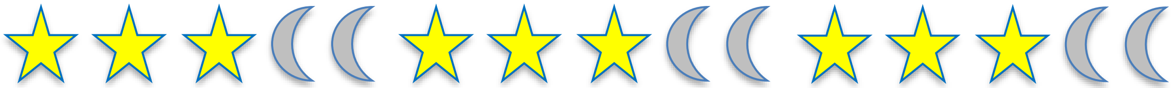


11. Ian is also drawing circles and squares on his paper. The ratio of circles to total shapes in Ian's picture is 1:3. Create Ian's picture.



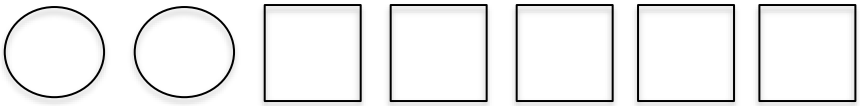
12. Both Kai's and Ian's pictures use the ratio of 1:3. Why are their pictures different?
    The quantities in the ratio are different. Kai's quantities are circles to squares and Ian's quantities are circles to total shapes.

13. Helena is drawing the following pattern on her paper.



   a. Write a ratio statement to describe Helena's picture.
      Answers will vary. This problem is intended to surface student thinking about iterating ratios. Students may note that the ratio of stars to moons 3 to 2 or that it is 6 to 4 or 9:6. They may say that for every 3 stars, there are 2 moons. All are correct ways of describing the relationship. Have students share out their statements.

14. Sophia's teacher asked her to create a pattern using only circles and squares so that the ratio of circles to total shapes is 2:5. Sophia created the following pattern.



   b. Is Sophia's pattern correct? Justify your answer. Sophia's pattern is not correct. Sophia created a pattern in which the ratio of circles to squares is 2:5 (or the ratio of circles to total shapes is 2:7).

Spiral Review

These problems are intended to help students connect their understanding of multiplication to solving ratio and rate problems later in the section. When iterating a ratio, students can think of it in two ways: 1) changing the **number of groups** or 2) changing the **number of items** in a group.

1. Marley is putting together bags of cookies for a bake sale. She puts 4 cookies into each bag. If she wants to put together 5 bags of cookies, how many cookies does she need? Draw a picture of this situation and write an equation to show the number of cookies Marley needs.
   Student pictures should show 5 bags of cookies with 4 items in each bag. She needs 20 cookies. The equation that matches this situation is $5 \times 4 = 20$, 5 groups with 4 items in each group.
   What if Marley wants to make 7 bags of cookies with 4 cookies in each bag? How many cookies will she need?
   Student pictures should show 7 bags of cookies with 4 items in each bag. Marley is changing the **number of groups** she has. The math sentence that represents this situation is $7 \times 4 = 28$ or 7 groups of 4.
2. Sierra is also putting together bags of cookies for the bake sale. If Sierra puts 5 cookies in each bag and wants to make 4 bags of cookies, how many cookies does she need? Draw a picture of this situation and write an equation to show the number of cookies Sierra needs.
   Sierra's picture would show 4 bags of cookies with 5 cookies in each bag. The math sentence that represents this situation is $4 \times 5 = 20$ or 4 groups with 5 items in each group.
   What if Sierra wants to make 4 bags of cookies with 7 cookies in each bag? How many cookies will she need?
   This picture would show 4 bags of cookies with 7 cookies in each bag. Sierra is changing the **number of items** in a group. The math sentence that represents this situation is $4 \times 7 = 28$ or 4 groups with 7 items in each group.
3. Compare problems #1 and #2. Both Marley and Sierra need the same number of cookies in the two different problems; however Marley is changing the number of groups and Sierra is changing the number of items in a group. This understanding of multiplication can be connected to ratio reasoning. When we iterate a ratio, we can either think about it as changing the number of groups or as changing the number of items in a group as we will see in **1.1c**.
4. Chloe made 3 cups of lemonade. If she wants to share the lemonade evenly with 6 people, how much lemonade will each person get? Draw a model and write a math sentence to represent this situation and solve the problem.
   Each friend will get ½ c. of lemonade. The math sentence that represents this situation is
   $3 \div 6 = \frac{1}{2}$.

1.1a Homework: The Language of Ratio

1. Simon just purchased six hamsters. Simon is trying to determine how many cages, hamster wheels, water bottles, food dishes, and bags of bark he needs to purchase with his hamsters. The owner of the pet shop told him the following:
   - Two hamsters can live in each cage.
   - You need one hamster wheel per cage.
   - You need one food bowl for every hamster.
   - You need one water bottle for each cage.
   - You need one bag of bark for every three cages.

   a. How many of each item should Simon purchase?

Cages: _3_ Wheels: _3___ Food Bowls: _6__ Water Bottles: _3__ Bags of Bark: _1_

2. Write <u>three</u> different ratio statements about the picture below. Use words like "to", "per", "for every", "each", and "ratio". Consider the relationship between birds and branches and also the relationship between birds and body parts of a bird. For example, "Each bird has two wings."



Answers will vary. Sample answers include: There are three birds per branch. The ratio of birds to branches is 3 to 1. Each bird has two legs. The ratio of legs to birds is 2:1. For every bird there are two wings. The ratio of beaks to eyes is 1:2.

3. Kara's necklace contains red and yellow beads. The ratio of red beads to yellow beads on Kara's necklace is 4 to 1.

   a. Draw a picture of the beads on Kara's necklace.

   b. Complete the following statements:

   The ratio of red beads to yellow beads is __4___: __1___.

   The ratio of yellow beads to red beads is __1___: __4___.

   The ratio of red beads to total beads is __4___: ___5__.

   c. Give a value for the total number of beads that might be on Kara's necklace. Give a value for the total number of beads that *cannot* be on Kara's necklace. Explain. Any multiply of 5 can be on the necklace. The necklace cannot have 12 beads, or 16, or 42.

4. A recipe to make Pumpkin Blondies uses 2 cups of Blondie cake mix for each cup of pumpkin puree.

   a. Draw a picture of this recipe.

Pictures will vary. Students may draw little cups or rectangles. Note that the cups/rectangles should be the same size. This work is leading up to a tape diagram which is a more abstract representation of a ratio and in a tape diagram the boxes are all the same size. Students may also just use the letters C for cake mix and P for pumpkin puree and represent the relationship symbolically.

   b. Complete the following statements:

   The ratio of __pumpkin puree_________ to ___cake mix________ is 1:2.

   The ratio of __cake mix___________ to _____puree______________ is 2:1.

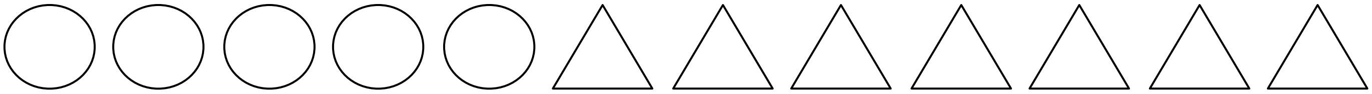
5. Create a pattern using green and orange circles where the ratio of green circles to orange circles is 5 to 3.
A picture with 5 green circles and 3 orange circles

6. Create a pattern using green and orange circles where the ratio of orange circles to green circles is 5 to 3.
A picture with 5 orange circles and 3 green circles

7. Create a pattern using green and orange circles where the ratio of total circles to orange circles is 5 to 3.
A picture with 3 orange circles and 2 green circles

8. Compare the patterns from the three problems above. They all use the ratio 5 to 3 but the patterns are different. Explain why.
   In the first two, the order of the quantities is different. The last one is a part to total.

9. George's teacher asked him to create a pattern where the ratio of triangles to circles is 5:7. George created the following pattern:



   a. Is George's picture correct? Why or why not?

   No, George's picture is not correct. George's picture should have 5 triangles and 7 circles (or an equivalent ratio). He mixed up the order of the quantities and their values.

10. In Lizzy's neighborhood, there are 2 cats for every 5 dogs.

   a. Mr. Beck asked his class to write a ratio statement to show the relationship between cats and dogs in Lizzy's neighborhood. These are the statements made by several students. Circle the names of the students who wrote a correct statement. For the ones that are incorrect, explain why they are incorrect in the space below.

      Eva's Statement: The ratio of cats to dogs is 2:5. Correct

      Mariah's Statement: The ratio of dogs to cats is 5:2. Correct

      Tom's Statement: Two out of every five animals are cats. Incorrect

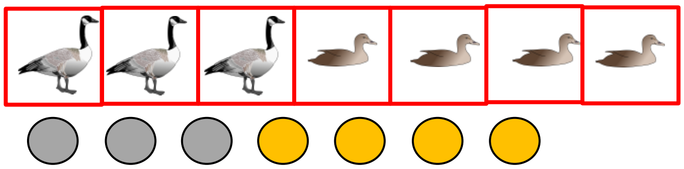
      Will's Statement: For every 6 cats, there are 15 dogs. Correct

1.1b Class Activity: Tape Diagrams

This lesson introduces tape diagrams while allowing students to continue practicing the concepts learned in the first lesson.

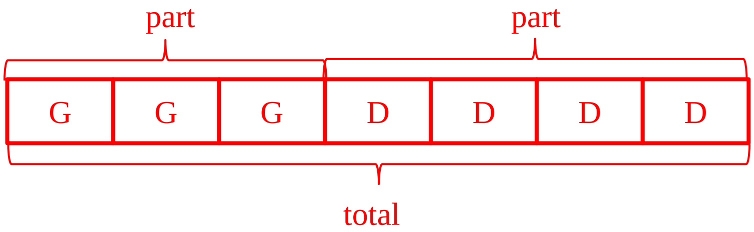
Activity 1: In a pond, there are 3 geese for every 4 ducks.

a. Draw a picture to represent this situation. See what students do. Have students share their pictures starting with the pictorial representation (someone who drew ducks and geese). Ask students, "Was it difficult to draw ducks and geese? Time-consuming? Did anyone make a different representation that was easier and faster to draw?" Then, move to one where a student moved toward a more abstract representation. Maybe they used different colored circles to represent each of the animals. Maybe they used the letter "D" to represent a duck and "G" to represent a goose.



G G G D D D D

Once students have shown their different representations, show a tape diagram as an overlay of the other representations – see above. A tape diagram is often faster to draw, more abstract, and more flexible than a picture. Start with the picture created in part a. and then show an overlay of the tape diagram as shown above. They need a total of 7 boxes, 3 of them need to be geese and 4 need to be ducks:



When drawing tape diagrams, help students attend to the following: 1) boxes should be the same size; 2) clearly distinguish different parts with a label or different color; 3) indicate parts and total.

Activity 2: Let's revisit the marble problem from the previous lesson. Draw a tape diagram to represent the marbles in each student's jar.

Marla: Make blue and red colored pencils available for students. They can also label with letters.
Marla puts three blue marbles in the jar for every five red marbles she puts in the jar.

B	B	B	R	R	R	R	R

Chase:
Chase puts three red marbles in the jar for every five blue marbles he puts in the jar.

B	B	B	B	B	R	R	R

Evelyn:
Three out of every five marbles in Evelyn's jar are blue.

B	B	B	R	R

A **tape diagram** is a model commonly used to represent ratios where the quantities have the same units. Tape diagrams can be useful tools for representing ratios and solving problems.

Directions: Draw a tape diagram to represent each of the following situations

1. The ratio of red paint to blue paint used to make a certain shade of purple paint is 2:3.

The tape diagram below shows the ratio 2 parts of red paint to 3 parts of blue paint. The parts can be gallons, cups, cans, etc.

R	R	B	B	B

2. The ratio of boys to girls standing in line is 3 to 5.

B	B	B	G	G	G	G	G

3. A bakery sells chocolate donuts and glazed donuts. The ratio of glazed donuts sold to total donuts sold is is 3:7.

G	G	G	C	C	C	C

4. The ratio of fruit punch to lemonade used in a recipe is 1 to 2.

FP	L	L

5. At a play, three out of every eight people in attendance are adults.

A	A	A	C	C	C	C	C

6. The tape diagram shows the ratio of red paint to yellow paint used to make a certain shade of orange paint. Write a ratio statement based on the diagram.

R	R	Y	Y	Y	Y	Y

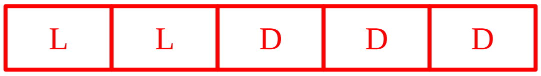
The ratio of red paint to yellow paint used to make orange paint is 2 to 5.

7. Bryce and Emma are running for class president. The tape diagram shows the ratio of votes Emma received to votes Bryce received. Write a ratio statement based on the diagram.

E	E	E	E	B	B	B

The ratio of votes for Emma to votes for Bryce is 4 to 3.
Four out of every seven votes are for Emma.

8. As people are leaving a movie, the theater manager polls them as to whether they liked the movie or not. For every 2 people that liked the movie, 3 did not like the movie.
   a. Draw a tape diagram of this situation.



   b. Complete the following statements:

The ratio of _people that did not like the movie__ to __people that did like the movie__ is 3:2.

The ratio of _people that liked the movie______ to _____total people_______________ is 2:5.

The ratio of __total people____________ to ____ people that did not like the movie_ is 5:3.

Spiral Review

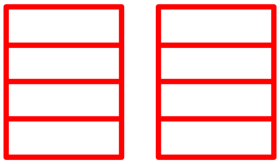
These problems are intended to help students when they begin simplifying ratios.

1. List the factors of 40.
   1, 2, 4, 5, 8, 10, 20, 40

2. List the factors of 16.
   1, 2, 4, 8, 16

3. What is the greatest common factor (GCF) of 40 and 16?
   8

4. Cole is making lemonade. Each lemon yields $\frac{1}{4}$ c. of lemon juice. How many cups of lemon juice will he get out of 8 lemons? Draw a picture to represent this situation and then write a math sentence to represent the problem and solution.



$8\times\frac{1}{4} = 2$

Cole will get 2 cups of lemon juice out of 8 lemons.

1.1b Homework: Tape Diagrams

1. At the skate park, Anthony notices that for every 3 kids that are on a bike, 2 kids are on a skateboard. Show some different ways you can represent this relationship using pictures or models.

2. Sixth graders at Mountain Crest Middle School are voting on where they want to go for their end-of-year field trip. They are choosing from the zoo and the planetarium. For each student that votes for the planetarium, two vote for the zoo.
   a. Draw a tape diagram to model this relationship.

P	Z	Z

   b. Complete the following statements:

   The ratio of zoo votes to total votes is __2:3____.

   The ratio of zoo votes to planetarium votes is ___2:1___.

   The ratio of planetarium votes to total votes is __1:3____.

   The ratio of total votes to planetarium votes is __3:1____.

3. A popular cereal brand is giving away baseball cards in some of its cereal boxes. Two out of every seven boxes of cereal contains a pack of baseball cards.
   a. Draw a tape diagram to represent this situation.

P	P	N	N	N	N	N

   b. Complete the following statements:
   The ratio of boxes with baseball cards to boxes without baseball cards is __2___:__5___.

   The ratio of boxes with baseball cards to total boxes is ___2___:___7___.

4. At Elsa's preschool, there is 1 adult in the room for every 10 children.
   a. Draw a tape diagram to represent this situation. It might help for students to have colored chips to lay out to represent this situation before drawing the tape diagram. A common error is for students to see this as a part to part ratio statement and draw a tape diagram with 1 adult and 11 children.

A	C	C	C	C	C	C	C	C	C	C

   b. Complete the following statements:
   The ratio of adults to children in the classroom is ___1___:__10___.

   The ratio of children to adults in the classroom is ___10___:___1___.

   The ratio of children to people in the room is ___10___:___11___.

5. Jeff's soccer team loses 2 soccer games for every 3 that they win.
   a. Draw a tape diagram to represent this situation.

L	L	W	W	W

   b. Write three different ratios to describe this picture, tell whether the ratios are part to part (PP), part to total (PT), or total to part (TP). Have students share out their ratio statements. Answers may vary. Sample answers include: The ratio of wins to losses is 3:2 (PP). For every five games, Jeff's team plays, they win 3 (TP). Jeff's team loses 2 out of every 5 games (PT).

6. Lauren's soccer team loses 2 soccer games for every 3 games that they play.
   a. Draw a tape diagram to represent this situation.

L	L	W

   b. Compare this problem to the previous problem. Both have a ratio of 2:3. Do Jeff and Lauren's teams have the same winning record? Why or why not?

No, the teams do not have the same winning record. Students need to pay attention to the quantities. In Jeff's problem, the quantities are losses to wins and in Lauren's the quantities are losses to total games. It should be obvious to students that Jeff's team has a better record.

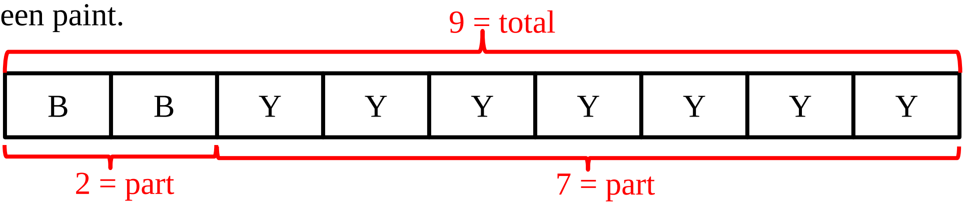
7. At Toby's school, students wear a uniform. Students can choose from a red shirt or a white shirt. The tape diagram below shows the ratio of white shirts sold to red shirts sold.

W	R	R	R	R

   a. Write three different ratio statements about the t-shirts sold. Tell whether your statement is a part to part (PP), part to total (PT), or total to part (TP).

Answers will vary. Sample answers include:
The ratio of red shirts sold to white shirts sold is 4:1. PP
The store sells four red shirts for each white shirt they sell. PP
Four out of every five shirts the store sells are red. PT

8. The tape diagram below shows the ratio of blue paint to yellow paint used to make a certain shade of green paint.



   a. Complete the following statements:

The ratio of ___yellow paint______ to _____blue paint________ is 7:2.

The ratio of ___blue paint________ to _____total paint________ is 2:9.

The ratio of ___yellow paint______ to _____total paint________ is 7:9.

The ratio of ____total paint_______ to _____blue paint________ is 9:2.

Fluency with the multiplication table is assumed at this point. Students are filling out the table below because tomorrow they will learn about equivalent ratios and equivalent ratios can be found using the multiplication table. Tables of equivalent ratios are “snippets” of the multiplication table. These ideas will be explored in the next lesson.

9. In preparation for tomorrow’s lesson, complete the multiplication table shown below.

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

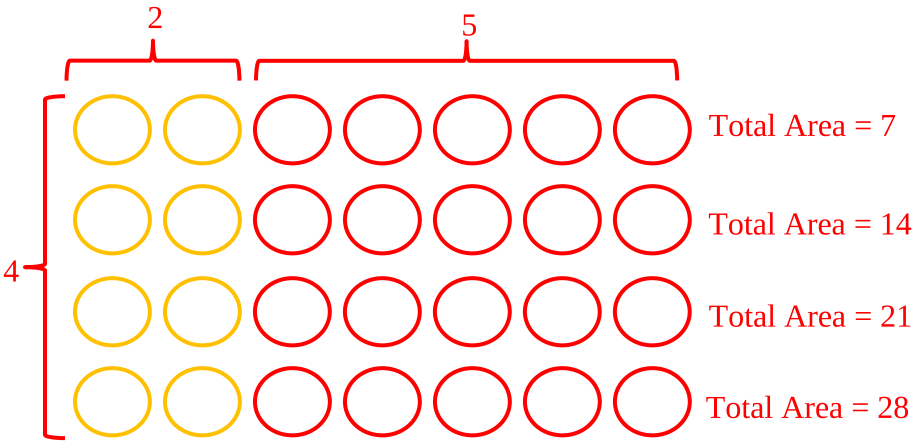
1.1c Class Activity: Equivalent Ratios and Tables

This lesson will likely take two days. It is suggested that you have colored tiles or chips available to students to represent the different quantities. You may also want to have a copy of the multiplication table available to students. There are many different tools that can be used to solve these problems as we will show in the teacher notes. Help students to make connections between the different strategies and to understand how to use the different tools to solve ratio problems using concrete manipulatives, models, and numerically.

Activity 1: A deli uses 2 oranges and 5 apples in their recipe for fruit salad. How many apples and oranges the deli should use if they want to double, triple, and quadruple their fruit salad recipe. How many apples and oranges should they use if they want to half the recipe? Organize your results in the table below.

What if they want to…	**Oranges**	**Apples**	**Total Pieces of Fruit**
Original Recipe	2	5	7
Double the recipe?	4	10	14
Triple the recipe?	6	15	21
Quadruple the recipe?	8	20	28
Half the recipe?	1	2.5	3.5

Concrete: Give students two different colored tiles to model this situation. The model below shows the number of apples, oranges, and total pieces of fruit needed to quadruple the recipe.



Help students to connect this model to work done with rectangular arrays/area models in 3.OA and to the distributive property 3.OA.5 (i.e. $7 \times 4 = (2 \times 4) + (5 \times 4)$ as we can see in the model above).

Tape Diagrams:

The model below represents a ratio of 2 oranges to 5 apples:

O	O	A	A	A	A	A

We can itererate the model several times (*change the number of groups*) to solve problems.

O	O	A	A	A	A	A
O	O	A	A	A	A	A
O	O	A	A	A	A	A
O	O	A	A	A	A	A

Or a simplified version of the tape model above:

2 oranges	5 apples
2 oranges	5 apples
2 oranges	5 apples
2 oranges	5 apples

We can also change the *number of items* in each group in the original model. Have students take their chips and stack them on top of each other.

4 O	4 O	4 A	4 A	4 A	4 A	4 A

In this model, each box represents 4 pieces of fruit and we see the equivalent ratio 8 oranges and 20 apples. What if each box had a value of 2 pieces of fruit? 3 pieces of fruit? 10 pieces of fruit? ½ a piece of fruit? Etc.

Numeric: Connect this work to the multiplication table that students completed in **1.1b Homework**. The table that we used to organize the information represents a "snippet" of the multiplication table – the equivalent ratios are derived from pairs of columns (or rows) in the multiplication table. Ask students what kind of patterns they see in the table such as the scalar multiplication shown below. Equivalent ratios can also be generated by repeated addition (+2, +2, +2; +5, +5, +5; +7, +7, +7).

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	1	2	3	4	5	6	7
②	2	4	6	8	10	12	14
③	3	6	9	12	15	18	21
④	4	8	12	16	20	24	28

Oranges	**Apples**	**Total Pieces of Fruit**
2	5	7
4	10	14
6	15	21
8	20	28
1	2.5	3.5

×4 ×3 ×2 ×2 ×3 ×4

We could have just as easily created a horizontal table to organize the information.

In this problem, you found **equivalent ratios**.

You will notice we use quotient notation in the answers below. Students may also express their answers using ratio notation (i.e. 2:5, 4:10, etc.). See the Mathematical Foundation for a discussion of ratios and fractions.

a. Write six equivalent ratios that show the relationship between apples and oranges in this recipe.

$\frac{\text{apples}}{\text{oranges}} = \frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} = \frac{12}{30} = \frac{1}{2.5}$ It may be helpful to have students organize their results in a table, illustrating the patterns of multiplication and addition that can be seen in the table.

Students already have 4 equivalent ratios from the work above. To find two more some students may continue using the tiles or iterating the model. Others may not need the manipulatives or models and may find the additional ratios numerically (using additive repeated reasoning, scalar multiplication, etc.)

$$\frac{2\text{ apples}}{5\text{ oranges}} \times \frac{2}{2} = \frac{4\text{ apples}}{10\text{ oranges}}$$

$$\frac{2\text{ apples}}{5\text{ oranges}} \times \frac{6}{6} = \frac{12\text{ apples}}{30\text{ oranges}}$$

$$\frac{4\text{ apples}}{10\text{ oranges}} \times \frac{2}{2} = \frac{8\text{ apples}}{20\text{ oranges}}$$

b. Write six equivalent ratios that show the relationship between oranges and total pieces of fruit in this recipe.

$$\frac{\text{oranges}}{\text{total pieces of fruit}} = \frac{2}{7} = \frac{4}{14} = \frac{6}{21} = \frac{8}{28} = \frac{10}{35} = \frac{12}{42} = \frac{1}{3.5}$$

c. Write six equivalent ratios that show the relationship between apples and total pieces of fruit in this recipe.

$$\frac{\text{apples}}{\text{total pieces of fruit}} = \frac{5}{7} = \frac{10}{14} = \frac{15}{21} = \frac{20}{28} = \frac{25}{35} = \frac{30}{42} = \frac{2.5}{3.5}$$

d. How many apples should the deli use if they use 16 oranges?

The deli would use 40 apples if they use 16 oranges in order to follow this recipe.

At this point, students may use a variety of strategies to solve this problem. Some students may need to continue using their chips/tiles to solve this. Others may notice that if we double the area model of 8 oranges and 20 apples shown on the previous page, we will have our 16 oranges and therefore would need 40 apples.

Others may solve the problem numerically. For example:

$$\frac{2\text{ apples}}{5\text{ oranges}} \times \frac{?}{?} = \frac{16}{?} \rightarrow \frac{2}{5} \times \frac{8}{8} = \frac{16\text{ apples}}{40\text{ oranges}}$$

or

$$\frac{8\text{ apples}}{20\text{ oranges}} \times \frac{?}{?} = \frac{16}{?} \rightarrow \frac{8}{20} \times \frac{2}{2} = \frac{16\text{ apples}}{40\text{ oranges}}$$

Have students share out the strategies they are using and connect the different strategies.

Activity 2: The tape diagram below shows the ratio of Ruiz's weekly allowance to his younger sister's weekly allowance. Determine at least five different allowances that Ruiz and his sister could be earning each week.

S	S	R	R	R

A common error is for students to look at this model and think that the only answer is that Ruiz earns $3 each week and his sister earns $2. Talk to students about the relationship – for every $2 Ruiz's sister gets, he gets $3. One option is to act out this situation in front of the class. Bring two students up to represent Ruiz and his sister and give them fake money to model what is happening. Or give students chips/tiles and allow them to model the problem concretely. Some students may also draw pictures. As students start listing possible allowances, it might make sense to organize the results in a table. A table is a tool that helps us to organize information.

Ruiz	**Ruiz's sister**
3	2
6	4
9	6
12	8
10	15

3	2
3	2
3	2
3	2
3	2

1. The ratio of children to adults in the swimming pool is 7:3.

   a. Draw a picture or tape diagram to represent this situation.

C	C	C	C	C	C	C	A	A	A

   b. Write five equivalent ratios that show the possible number of children and adults in the pool.
   7:3, 14:6, 21:9, 28:12, 35:15

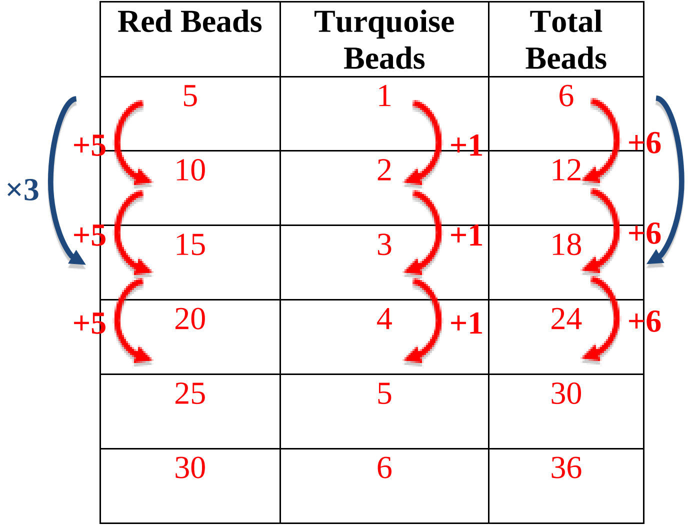
   c. Write five equivalent ratios that show the possible number of children to total people in the pool.
   7:10, 14:20, 21:30, 28:40, 35:50

   d. If there are 50 total people in the pool, how many adults are there in the pool?
   If there are 50 total people in the pool, 15 of them are adults.

   e. Is it possible for there to be 30 children in the pool with the given relationship? Explain.
   No, it is not possible. 30 is not a multiple of 7 and you cannot have part of a person.

   f. Is it possible for there to be 30 adults in the pool with the given relationship? Explain.
   Yes, 30 is a multiple of 3. There would be 30 adults and 70 children in the pool.

2. Suni is making a necklace. The ratio of red beads to turquoise beads on the necklace is 5:1.
   a. Determine several possibilities for the number of red and turquoise beads Suni could have used to make her necklace. Organize the information in the table below. Illustrate any patterns you see in the table. Sample answers shown.

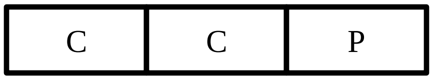


Red Beads	Turquoise Beads	Total Beads
5	1	6
10	2	12
15	3	18
20	4	24
25	5	30
30	6	36

It is important to understand that this table, and a ratio in general, shows how two or more quantities are related. In this example, there are pairs of values that correspond to each other. For example, the quantity 3 turquoise beads by itself has little meaning; however knowing that 3 turquoise beads pairs with 15 red beads reinforces the idea that we are examining a *relationship* between two or more quantities. Students may begin to notice many patterns in the table. For example, the pair 20 red beads and 4 turquoise beads is twice the pair 10 red beads and 2 turquoise beads or the pair (30, 6) is three times the pair (10, 2). Help them to connect these patterns to the pictorial representations and models and have them show the patterns on the table.

We have arbitrarily placed red beads in the first column of the table and turquoise beads in the second column. Over the next three years, students will learn where to place the quantities in the table. Conventionally, the independent variable is placed in the first column and the dependent variable is placed in the second column. The decision as to which quantity in a given problem is the dependent variable and which is the independent variable is usually driven by the question being asked. We will provide more discussion on this throughout this chapter.

3. Pizza Paradise sells cheese and pepperoni pizza. The tape diagram below shows the ratio of cheese to pepperoni pizzas sold in one day.



   a. With a partner, discuss three different ratio statements to describe this relationship. Tell whether your statements are part to part (PP), part to total (PT), or total to part (TP).
   Answers will vary
   b. Determine several possibilities of cheese and pepperoni pizza that Pizza Paradise may have sold in one day. Illustrate any patterns you see in the table. Have students use colored pencils to show the patterns of addition and multiplication we see in this table. Point out that a horizontal table is no different than a vertical table in terms of the relationship it shows and the patterns we can see, it is just oriented differently.

Cheese	2	20	50	100	200
Pepperoni	1	10	25	50	100

   c. Determine the number of cheese pizzas Pizza Paradise sold if they sold 25 pepperoni pizzas.
   50; Students should be able to reason through this problem using their understanding of multiplication. Some students may start to notice the explicit relationship between the two quantities which is that the number of cheese pizzas is twice the number of pepperoni pizzas (or the number of pepperoni pizzas is half the number of cheese pizzas). Others may rely on models.
   d. Determine the number of cheese and pepperoni pizzas sold if Pizza Paradise sold 12 total pizzas.
   8 cheese pizzas and 4 pepperoni pizzas. Students may use any of the strategies shown in **Activity 1** of this section.

4. The ratio of blue paint to red paint used to make a certain shade of purple paint is 2:3. Continue to reinforce with students the importance of attending to precision when reading and interpreting ratio statements: identifying the quantities in the problem, the order they are written, and whether the ratios are expressing part to part relationships or part to total relationships.
   a. Circle all ratios that would make the same shade of purple paint.

      6 cans of blue paint to 7 cans of red paint Not the same shade. This is a common error when finding equivalent ratios. Students think if you add the same amount of each color (in this case +4) you have created an equivalent ratio. Or they think that because the differences between 3 and 4 and 6 and 7 are both 1 these are equivalent ratios.

      8 cans of blue paint to 12 cans of red paint Same shade

      40 cups of blue paint to 60 cups of red paint Same shade

      18 cans of blue paint to 27 cans of red paint Same shade

      24 cans of red paint to 36 cans of blue paint Not the same shade. This is another common error. Students only look at the values and see that both numbers are multiplied by 12 so it is an equivalent ratio. They need to also pay attention to the quantities. It may be obvious to some students that because the number of cans of blue paint are greater than the number of cans of red paint in this problem this can't possibly be the same shade of purple.

      1 can of blue paint to $1\frac{1}{2}$ cans of red paint Same shade.

5. Maria is collecting donations to raise money for a food pantry. For every $6 that she collects, her parents will donate $1. The tape model below represents this situation. The white boxes represent dollars that Maria has collected and the gray box represents money her parents have contributed.
   a. Draw a tape diagram to represent this situation.

   b. Complete the table below to show several possible combinations of money that Maria collects and her parents donate. Illustrate any patterns you see in the table.

	Maria's Collections	**Parent's Contribution**		**Total Donation**	
+6	6	1	+1	7	+7
+6	12	2	+1	14	+7
+6	18	3	+1	21	+7
	24	4		28	
	30	5		35	

In this table, the ratios are consecutive. Students may use repeated addition to complete the table or scalar multiplication.

   c. How much does Maria need to collect if she wants her total donation to be $35?

      $30, Students will likely see this in the table but help them to also show the answer using multiplication: First, which two quantities are we interested in? Maria's contributions and total donations. The ratio of Maria's collections to total donations is 6 to 7. If Maria wants to earn $35, she would need to iterate the model 5 times (or make each box worth $5); therefore Maria's contributions would be $30. Numerically: $\frac{\text{maria'scollections}}{\text{total donations}} = \frac{6}{7} \times \frac{5}{5} = \frac{30}{35}$ Remember this is a pair of values, Maria's contribution of $30 corresponds to a total donation of $35.

6. A market research company is testing out a new cereal. Four out of every five people that try the cereal like it. Attention to precision, this is a part to total relationship.
   a. Draw a tape diagram to represent this situation.
   Tape diagram should have a total of 5 boxes with 4 boxes representing people who like the cereal and 1 box representing people who do not like the cereal.
   b. Complete the table below to show this relationship. Illustrate any patterns you see in the table.

Liked	Did Not Like	Tried
4	1	5
8	2	10
20	5	25
28	7	35
40	10	50

×10 ×5 ×5 ×10

In this table, the ratios are not consecutive. Some students may still need to use manipulatives and models to solve these problems. Others may be solving using scalar multiplication and by observing patterns in the table.

7. Lisa goes to the gym 5 out of 7 days each week.
   a. Draw a tape diagram to represent this situation.

Y	Y	Y	Y	Y	N	N

   b. Complete the following statements:
   The ratio of days Lisa went to the gym to days Lisa did not go to the gym is __5__ to __2__.

   The ratio of days Lisa does not go to the gym to total days is __2__:__7__.
   c. Determine the number of days Lisa did not go to the gym if she went to the gym 15 times.
   If Lisa goes to the gym 15 times, she will not go to the gym 6 times.
   Some students may use their tape model above and think about what they need to do to it to show that Lisa went to the gym 15 times (copy it 3 times or change the number of items in each group to 3).
   Some may solve the problem numerically:

$$\frac{5\text{ days at gym}}{2\text{ days not at gym}} \times \frac{?}{?} = \frac{15}{?} \rightarrow \frac{5}{2} \times \frac{3}{3} = \frac{15}{6}$$

   Or using a table:

Yes	No
5	2
15	6

×3 ×3

   d. Determine the number of days Lisa went to the gym in a 28-day period.
   Students need to recognize that 28 represents the total. In order to get 28 total days, we would need to iterate the model above 4 times (or change the number of days in each box to 4). Lisa would go to the gym 20 times in a 28-day period. Numerically:

$$\frac{5\text{ days at gym}}{7\text{ total days}} \times \frac{4}{4} = \frac{20}{28}$$

8. In the Lollipop Tree Game at a carnival, a player chooses a lollipop. If the lollipop is colored red on the end of the stick, the player wins the lollipop and an additional prize. If the lollipop is not colored red on the end, the player just gets to keep the lollipop. The ratio of lollipops with red ends to lollipops without red ends is 3:4.
   a. If there is space in the tree for 35 lollipops, how many will have red ends? 15

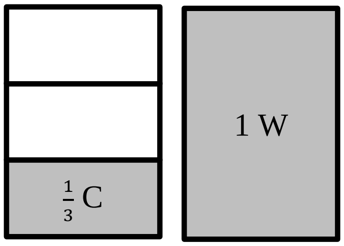
If students are struggling, have them model the problem with tiles or draw a tape diagram.

9. Camie is making a quilt with patches of red, white, and blue fabric. The ratio of red to white to blue patches in the quilt is 2:1:2.
   a. Write 5 possibilities for the number of red, white, and blue patches that could be on Camie's quilt. Organize your results in the table below.

Red Patches	**White Patches**	**Blue Patches**	**Total Patches**
2	1	2	5
4	2	4	10
6	3	6	15
10	5	10	25

   b. If Camie used 80 total patches, how many of each color did she use?
   32 red, 16 white, and 32 blue

10. To make orange juice, the ratio of concentrate to water is $\frac{1}{3}$ to 1 as shown in the picture.



   a. How many cups of concentrate would you need for two cups of water?
   $\frac{2}{3}$ cup concentrate, students can iterate this picture just as they have in previous examples. This is a review of **5.NF.4** where students used models to learn how to multiply a fraction by a whole number. If we have two cups of water, we are doubling the recipe; therefore we need to double the amount of concentrate:
   $2\times\frac{1}{3}=\frac{2}{3}$ - The model helps students to understand this problem.
   b. How many cups of concentrate would you need for three cups of water?
   1 c. concentrate. Again, we can copy the picture three times in order to solve this problem and then connect it to the algorithm:
   $3\times\frac{1}{3}=1$ cup
   c. How many cups of concentrate would you need for four cups of water?
   $\frac{4}{3}$ cups

As an extension, ask students how many cups of concentrate would be needed for ½ c. of water.

11. Blake is making a dog run in the shape of a rectangle. He plans to use 3 units of fencing for the length for each unit of fencing he uses for the width.
    a. Draw several possible combinations for Blake's dog run on the grid below. Note: Each horizontal or vertical segment represents 1 unit of fencing.

    b. Complete the table to show this relationship. Students may consider the dimensions of the dog run as shown in the table below or the total units of fencing needed for the length and width and if that is the case all the answers in the table would be doubled.

Units of Fencing for Width	1	2	3	4	5
Units of Fencing for the Length	3	6	9	12	15

You might point out that all of the rectangles on the grid above are the same shape, meaning the length is always three times larger than the width. Another way of thinking of this is that the rectangles are all dilations of each other. This is a foreshadowing of the study of similar figures which students will learn about in subsequent grades.

12. Dominic and Emma are running for class president. For each vote that Dominic receives Emma receives four votes. List <u>five</u> possibilities for the total number of students who voted. Justify your answers. Write a rule that describes all the possible answers to this problem.

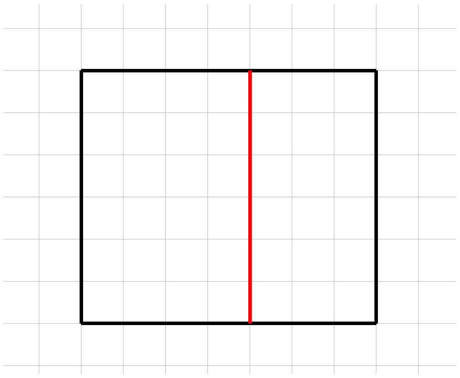
    All answers should be multiples of 5: 5, 10, 15, 20, 25, etc. Ask students, "Is it possible that 32 student voted? 100 students voted? 1,000 students voted? Why or why not?"

Spiral Review

The first 4 problems are intended to help prepare students for **1.1d** where they will be simplifying ratios. When students simplify ratios, they can either 1) create the same **number of groups** and determine how many items are in each group (partitive division) or they can 2) put the same **number of items** in each group and determine the number of groups (quotative).

1. Ryan is playing a card game with his friends. There are 40 cards in the deck and he deals them all out. If 5 people are playing, how many cards does each person get? Write the math sentence that models this problem.
   $40 \div 5 = 8$
   When the number of groups is known and you are trying to find the number of items in each group (or the measure of each group), then the problem is referred to as a partitive division problem. Partitive division is often thought of as "equal groups" or "sharing".
2. Peter is playing a different card game. In his game, there are 40 cards in the deck and each player needs 5 cards to play. How many people can play the game?
   $40 \div 5 = 8$
   When the number of items in each group is known and you are trying to find the number of groups, then the problem is referred to as a quotative division problem. It can be thought of as repeated subtraction, subtracting groups of items equal in number or measure to the divisor from the dividend.
3. Compare problems 1 and 2. What do you notice?
   Both problems can be modeled using the equation $40 \div 5 = 8$ but the equation has different meanings in each problem. In #1, we are taking 40 cards and dividing them evenly into 5 groups. The answer 8 represents the number of items in each group. In #2, we are taking 40 cards and creating groups with 5 items in each. The answer 8 represents the number of groups that can be made.
4. What are the common factors of 18 and 24? What is the greatest common factor of 18 and 24?
   It may help students to first list the factors of each number:
   18 – 1, 2, 3, 6, 9, 18
   24 – 1, 2, 3, 4, 6, 8, 12, 24
   The common factors are 1, 2, 3, and 6
   The greatest common factor is 6.

5. Use the grid below to answer the questions that follow.



   a. Find the area of the rectangle shown. Explain or show the strategy you used.
      42 square units; Students may count squares or multiply the length by the width.

   b. Ricky found the area by writing and simplifying the expression $(6\times4)+(6\times3)$. Explain and show on the grid the method Ricky used to find the area of the rectangle.
      Ricky broke the rectangle into two smaller rectangles and found the area of each. Make the connection that (6×7) is the same as $(6\times4)+(6\times3)$.

1.1c Homework: Equivalent Ratios and Tables

1. The ratio of sugar to flour used in a sugar cookie recipe is 1 cup sugar to 2 cups flour. Determine the amount of each ingredient needed to double, triple, quadruple, and half the recipe.

What if they want to…	**Sugar**	**Flour**
Original Recipe	1	2
Double the recipe?	2	4
Triple the recipe?	3	6
Quadruple the recipe?	4	8
Half the recipe?	0.5	1

2. The model below shows the ratio of chocolate to vanilla ice cream cones sold at the fair.

V	V	V	C	C	C	C	C

a. Write three different ratio statements to describe the relationship between vanilla and chocolate ice cream cones sold at the fair. Tell whether your statements are part to part (PP), part to total (PT), or total to part (TP).

Answers will vary

b. Write five equivalent ratios for the number of vanilla ice creams sold to the number of chocolate ice creams sold. Justify your answers.

3:5, 9:15, 30:50, 75:125, 300:500

c. Write five possible values for the total number of ice cream cones that could have been sold at the fair. Justify your answers.

Answers should be multiples of 8: 8, 16, 24, 80, 200, 800, etc.

3. Sydney polled students at her school to see whether or not they watch a certain TV show. The ratio of girls who watch the show to boys who watch the show is 3:1.

   a. Complete the following table to show different combinations of boys, girls, and total students who watch the TV show. Illustrate any patterns you see in the table.

Girls	**Boys**	**Total Students**
3	1	4
6	2	8
9	3	12
12	4	16
15	5	20

   b. If 16 total students watch the TV show, how many of them are boys? How many of them are girls? Draw a model to support your conclusion.

   Boys: __4_____   Girls: __12____

   c. If 15 girls watch the show, how many boys watch the show? Draw a model to support your claim.
   5

4. Alex makes 3 out of every 5 free throws he attempts.
   a. Draw a tape diagram to represent this situation.

Y	Y	Y	N	N

   b. Complete the following statements:

   The ratio of shots made to shots attempted is __3___ to __5___.

   The ratio of shots made to shots missed is __3___ to __2___.

   c. Use your model to determine several different combinations of makes and attempts. Organize the combinations in the table below. Illustrate any patterns you see in the table.

Makes	3	6	9	12	15
Misses	2	4	6	8	10
Attempts	5	10	15	20	25

   d. Use your model to determine how many free throws Alex attempted if he made 12 shots. How many shots did he miss if he made 12 shots?

   Attempts: ___20_____   Misses: ___8_____

5. Alli is training for a biathlon, a race where she will run and bike. For every three times that she goes for a run, she bikes once.
   a. Draw a tape diagram to represent this situation.

R	R	R	B

   b. Complete the table below to show this relationship. Illustrate any patterns you see in the table.

Runs	**Bikes**	**Total Workouts**
3	1	4
6	2	8
9	3	12
12	4	16
15	5	20
30	10	40

6. Students at a middle school are voting on whether or not their new mascot should be an eagle or a bobcat. The tape diagram shows the ratio of students who voted for the bobcat to students who voted for the eagle.

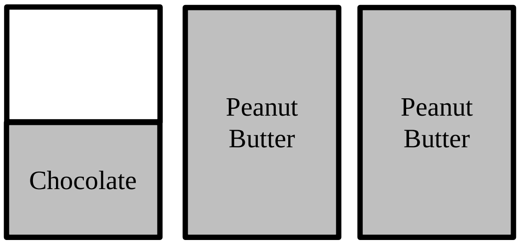
B	B	B	E	E	E	E

   a. Complete the table below to show this relationship. Illustrate any patterns you see in the table.

Bobcat	**Eagle**	**Total Votes**
3	4	7
12	16	28
24	32	56
30	40	70
120	160	280

   b. If there are 350 students in the school who voted, how many voted for the bobcat? How many voted for the eagle? Justify your answer.
   150 voted for the bobcat and 200 voted for the eagle. In order to get to 350 students, you would need to iterate the model above 50 times or make each box represent 50 students showing 150 votes for the bobcat and 200 votes for the eagle.

7. The ratio of chocolate to peanut butter used to make peanut butter cups is $\frac{1}{2}$ to 2 as shown in the picture below.



   a. Determine the number of cups of chocolate and peanut butter Reese should use if he wants to…

   Double the recipe: Cups of Chocolate: __1____ Cups of Peanut Butter: ___4___

   Triple the recipe: Cups of Chocolate: ___1½ __Cups of Peanut Butter: ___6___

   Quadruple the recipe: Cups of Chocolate: ___2___ Cups of Peanut Butter: ___8___

8. Determine whether each pair of ratios are equivalent ratios. Circle Yes or No. Justify your answers for b. c. and d. in the space below.

   a. 2 to 3 and 4 to 6 [✓ Yes] or No

   b. 4:5 and 8:9 Yes or [✓ No]

   c. $\frac{1}{2}$ and $\frac{5}{10}$ [✓ Yes] or No

   d. 2:5 and 8:20 [✓ Yes] or No

   e. $\frac{2}{5}$ and $\frac{4}{7}$ Yes or [✓ No]

Answers will vary. Students may justify their thinking using words, drawing pictures or models, or creating partial tables that show multiplication and addition patterns.

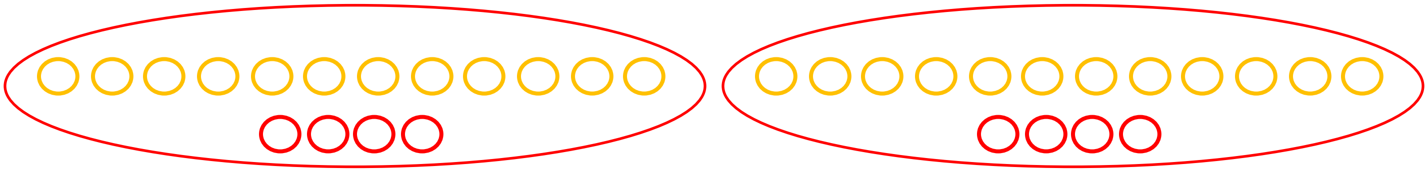
1.1d Class Activity: More Equivalent Ratios

Activity 1: Chandler’s class is at the zoo for a field trip. There are 24 students on the trip and 8 adult chaperones. The students and adults will break into smaller groups to walk around the zoo. Chandler’s teacher wants the ratio of adults to students to be the same in each group. What are some possibilities for the number of adults and students in each group? What is the simplified ratio of adults to students?

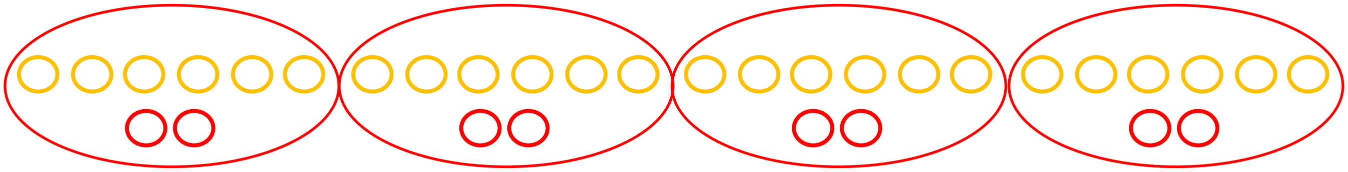
Up to this point, students have been iterating ratios to find equivalent ratios. This problem is asking them to think in the opposite direction, to simplify ratios. Just as in the previous lesson, students can solve this problem using a variety of methods. Here are a few examples of different strategies students may use.

Concrete:

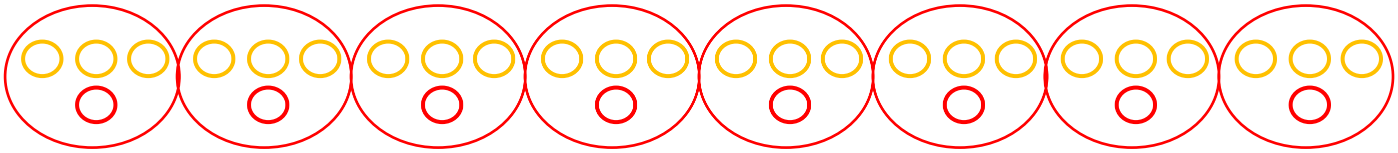
Students can use colored tiles. Students start by laying out 24 tiles to represent the students and 8 tiles to represent the adults. What if we halve the adults and the students?



12 students and 4 adults in each group. What if we halve the groups again?



6 students and 2 adults in each group. And if we halve them again?

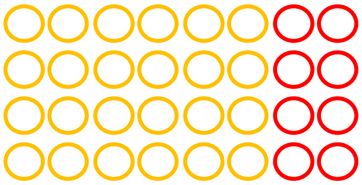


3 students and 1 adult in each

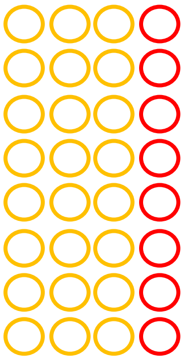
The method shown above relates to partitive division. We are breaking the children and adults into groups and determining the number of children and adults in each group. Another way of thinking about it is that we are determining how to “share” the adults evenly with the students.

Students may also determine the different rectangles they can make using the tiles:

12 students and 4 adults in each group



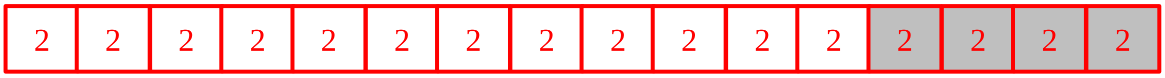
6 students and 2 adults in each group



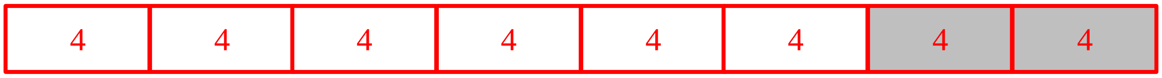
3 students and 1 adult in each group – this represents the simplified ratio of students to adults

Tape Diagram:

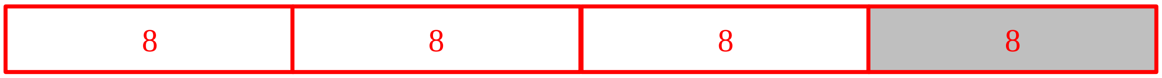
Putting 2 people in each group, we see the ratio 12 to 4:



Putting 4 people into each group, we see the ratio 6 to 2:



Putting 8 people into each group, we see the ratio 3 to 1:



Students may begin to notice that when we divide the two numbers by their GCF, this shows the simplified ratio.

Some students may view the tape model as an area model. The model is not drawn to scale but we can see the greatest common factor of 8 and the simplified ratio of 3 to 1.

Numeric:

Students may use the multiplication table to find equivalent ratios, realizing that in order to find simpler ratios, we need to divide both numbers by the same quantity. Students may also display the information in a table.

$$\frac{24\text{ children}}{8\text{ adults}} \div \frac{2}{2} = \frac{12\text{ children}}{4\text{ adults}}$$

$$\frac{24\text{ children}}{8\text{ adults}} \div \frac{4}{4} = \frac{6\text{ children}}{2\text{ adults}}$$

$$\frac{24\text{ children}}{8\text{ adults}} \div \frac{8}{8} = \frac{3\text{ children}}{1\text{ adult}}$$

Children	**Adults**
24	8
12	4
6	2
3	1

Make explicit for students that all of these representations (concrete model, tape diagram, table, numeric, etc.) are just different ways of representing the same relationship. All are valid approaches to solving ratio problems. Make connections between the different representations (i.e. Where do we see the patterns of multiplication and division in the table, concrete model, tape diagram, and numeric methods for solving?).

a. Nine additional students from Chandler's class arrive at the zoo late. How many more adults need to be added to the group to keep the ratio of adults to students the same? Justify your answer.
3 additional adults are needed to keep the ratio of adults to children the same. Each group of 3 students needs 1 adult so 9 students would need 3 adults. Students can show this using a model. They may also find the answer using a table and/or scalar multiplication.

b. Zoe's class is also going to the zoo. There are 30 students in Zoe's class. How many adults need to come to the zoo to have the same ratio of adults to students as there are for Chandler's class? Justify your answer.
10 adults need to come to the zoo

c. Noah's class is also at the zoo with the same ratio of adults to students as Chandler and Zoe's classes. There are 48 total people in Noah's group. How many of them are adults and how many are students? Justify your answer. Make sure students realize we are given the total in this problem and asked to find the parts.
12 adults and 36 students; students may find this and other answers by starting from a ratio they have already seen. For example, in part b. there are a total of 40 people in Zoe's class. Each group of 3 students and 1 adult that come on the trip increases the number of people on the trip by 4. If I add 2 groups of 4 (3 students and 1 adult) to 40, I will get the desired total of 48.

Refer students to the multiplication table so that they can observe all of the equivalent ratios for 3:1 that we saw in this problem.

Activity 2: A company that mass produces rice crispy treats uses 60 cups of marshmallows and 80 cups of crispy rice in their recipe. Jesse wants to follow the recipe to make rice crispy treats for her family but she wants to make a much smaller batch. How many cups of marshmallows and how many cups of crispy rice should Jesse use?

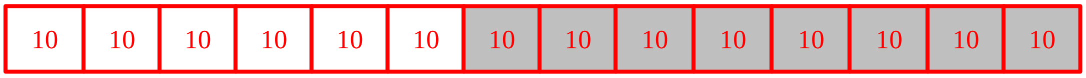
Some may start breaking the recipe into smaller batches:
Cut it in half - 30 cups of marshmallows and 40 cups of crispy rice
Cut in half again (or divide original recipe by 4) – 15 cups of marshmallows and 20 cups of crispy rice
Cut in half again (or divide the original recipe by 8) – 7.5 cups of marshmallows and 10 cups of crispy rice
Make 1/10 of the original recipe (or divide the original recipe by 10) – 6 cups of marshmallows and 8 cups of crispy rice

This problem does not have one correct answer, some students may reason that a batch made from 6 cups of marshmallows and 8 cups of crispy rice is reasonable for a family, others may reason that 3 cups of marshmallows and 4 cups of crispy rice seem reasonable. This gives students the opportunity to think about the size of these quantities.

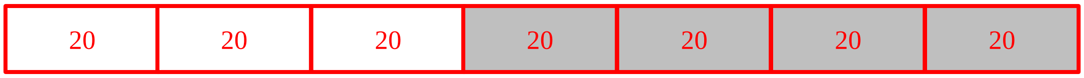
Some students may go back to the multiplication table created earlier and use this to find several combinations of marshmallows and crispy rice that can be used.

Some may use a tape model. Students can sketch out the bar model with the two different quantities. Then students can find a common factor.

Put 10 items in each group. Shows the ratio of 6 marshmallow to 8 crispy rice.



Put 20 items in each group. Shows the ratio of 3 marshmallow to 4 crispy rice.

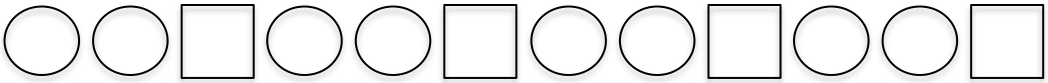


One way to attack this problem is to find the GCF of both numbers. This will give you the simplified ratio of marshmallows to crispy rice. In this problem, the GCF is 20. Dividing both quantities by the GCF, 20, we can see the simplified ratio of 3 parts marshmallow to 4 parts crispy rice.

If students are only finding whole number answers, ask if it makes sense in this context to have fractional answers. For example, "Can we make a batch that is smaller than 3 cups of marshmallows and 4 cups of crispy rice?" *Yes* "What if we only have 1 cup of crispy rice, how many marshmallows should we use?" $\frac{3}{4}$ *of a cup*
Compare this problem to Activity 1 where it did not make sense to have fractional answers.

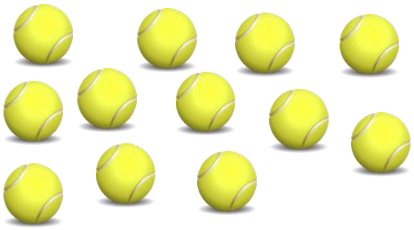
At the end, you may wish to have students organize the information in a table and observe the patterns of multiplication.

1. Danita drew the following picture on her paper.



   a. What is the simplified ratio of squares to circles in Danita's picture?
   1:2

2. Brendan is a tennis coach. He is gathering supplies for an upcoming lesson. The picture below shows the ratio of tennis racquets to tennis balls he needs for the lesson.



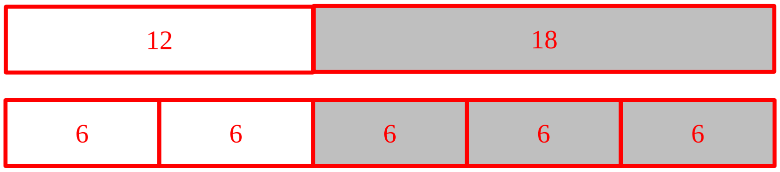
   a. What is the simplified ratio of tennis balls to tennis racquets?
   3:2 If students are struggling, give them chips/tiles to model the problem.

   b. If there are 6 tennis balls, how many racquets will Brendan need to keep the same ratio?
   4 tennis racquets

   c. If there are 20 tennis racquets, how many tennis balls will Brendan need to keep the same ratio?
   30 tennis balls

3. Sierra is making a big batch of fizzy fruit punch for a school carnival. She mixes 12 cans of Sprite with 18 cans of fruit punch.
   a. Find three ways to make a smaller batch of this fizzy fruit punch.

Some students may solve this problem numerically. Others may make a tape model as shown below. And some may prefer to model the problem using chips/tiles. Have students share out and connect their strategies.

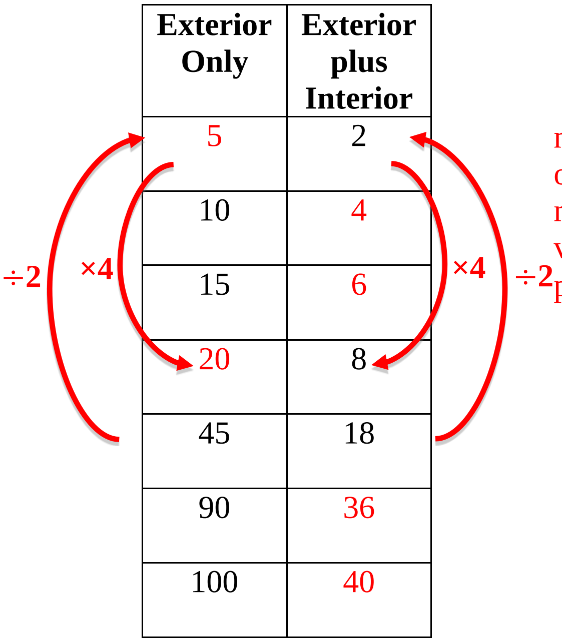


Dividing 12 and 18 by their GCF, 6, shows 2 parts Sprite for 3 parts fruit punch. Students can make the boxes above worth different values in order to determine different ratios. For example, if each box has a value of 1 can, we would mix 2 cans of Sprite with 3 cans of fruit punch. If each box has a value of 2 cans, we would mix 4 cans of Sprite with 6 cans of fruit punch. If each box has a value of $\frac{1}{2}$ a can, we would mix 1 can of Sprite with 1.5 cans of Fruit Punch. Etc.

Sprite	6	4	2	8	10	1
Fruit Punch	9	6	3	12	15	1.5

   b. If Sierra adds 8 more cans of Sprite, how many more cans of fruit punch should she add?
   Using the tape model, each box would have a value of 4 so we would add 12 cans of fruit punch

4. At a car wash, cars either come in for an exterior only wash or an exterior plus interior wash. On a particular day, Mandy observes that 45 cars come in for an exterior only wash and 18 cars come in for an exterior plus interior wash. The manager of the car wash tells her that this represents the ratio of exterior only washes to exterior plus interior washes.
   a. Complete the table below to show this relationship. Explain or show how you completed the table.



Exterior Only	Exterior plus Interior
5	2
10	4
15	6
20	8
45	18
90	36
100	40

Explanations may include showing patterns of multiplication (division) and addition (subtraction) on the table. Students may also solve using a model. Make sure that students understand that the values in the table are still just iterations of the picture/model.

5. Tinkerbell is gathering data on the number of girls and the number of boys that go on a particular ride at Disneyland. One morning, she observes that 300 girls go on the ride and 500 boys go on the ride. This represents the ratio of girls to boys that go on the ride.
   a. Complete the table below to show this relationship. Explain or show how you completed the table.

Girls	Boys	Total People
3	5	8
6	10	16
12	20	32
30	50	80
300	500	800
480	800	1,280
600	1,000	1,600

Problems 4 and 5 should help students to see that at times it is easier to find a simpler ratio in order to solve problems. For example, in #5 it may be difficult for students to determine how to get from the girl to boy ratio 300:500 to ?:800. It might be easier for them to use the ratio 6:10 or 12:20. It is important that students understand that in the ratios 300:500 and 480:800, the multiplicative relationship still holds true, the factor just happens to be a fraction (1.6) so can be more difficult to determine. Students may use patterns of repeated addition and multiplication to complete the table or models/pictures.

For #6 and #7 below, students may use a variety of strategies to solve the problems. They may draw tape diagrams, solve numerically, create tables, etc. #7c may be challenging for students as they need to consider a quantity whose value is not directly given in the problem (the amount Jason saves).

6. Tina is polling children ages 8 – 12 to determine whether they would rather go to Disneyland or Lego Land. Tina begins counting the votes and counts 60 votes for Disneyland and 12 votes for Lego Land. This represents the ratio of votes for Disneyland to votes for Lego Land.

   a. In the next batch, Tina counts 40 votes for Disneyland. How many votes for Lego Land would you expect to be in that batch? Justify your answer.
   You would expect there to be 8 votes for Lego Land. Students may find the answer numerically, realizing it is easier to first simplify the ratio 60:12.

$$\frac{\text{votes for Disneyland}}{\text{votes for Lego Land}} = \frac{60}{12} = \frac{5}{1} = \frac{40}{8}$$

   b. If Tina counts 48 total ballots, how many votes will be for Disneyland and how many for Lego Land? Justify your answer.
   8 votes for Lego Land and 40 votes for Disneyland

7. The ratio of the amount of money Jason saves to the amount of money he earns is always the same. When he makes $48 he saves $12. He spends the rest. Make sure students understand that this is a part to total ratio.

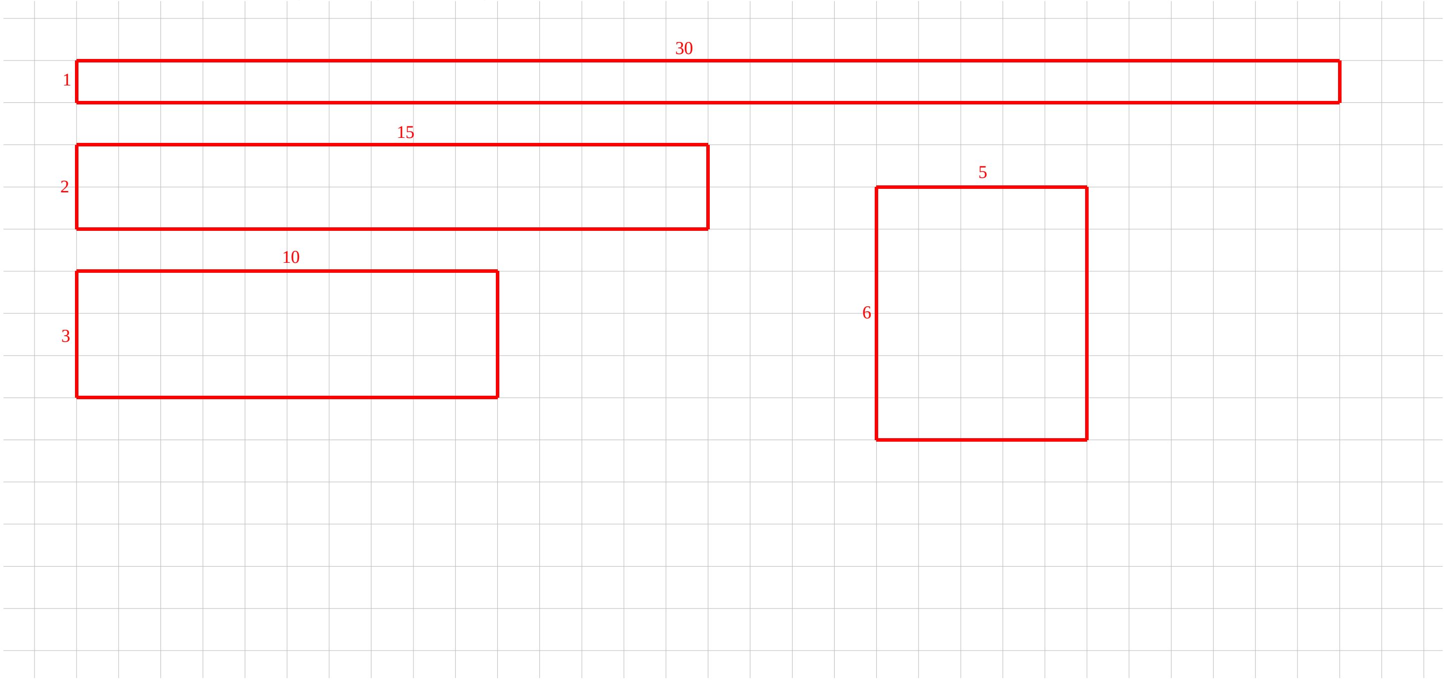
   a. If Jason saves $6, how much did he make? Justify your answer.
   $24

   b. If Jason makes $20, how much will he save? Justify your answer.
   $5

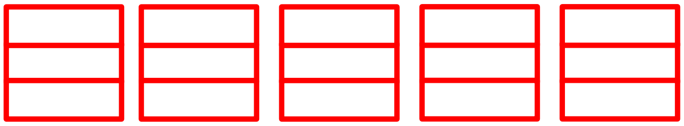
   c. If Jason spent $12, how much did he make? Justify your answer.
   $16

Spiral Review

1. On the grid below, draw as many rectangles as you can that have an area of 30. Write the dimensions (length and width) of the rectangles. Note: We have shown these with only whole number measurements – there are infinitely many rectangles that can be created that have an area of 30.



2. Jessica is making S'mores. For each S'more she needs $\frac{1}{3}$ of a chocolate bar. If Jessica has 5 chocolate bars, how many S'mores can she make? Draw a picture to represent this situation and then write a math sentence to model the problem and solution.



$$5 \div \frac{1}{3} = 15$$

3. Devon is taking 20 scouts on a camping trip. He has figured out that he needs 5 bars of chocolate for each scout to have one S'more.
   a. How many S'mores does one bar of chocolate make?
      One bar of chocolate makes 4 S'mores

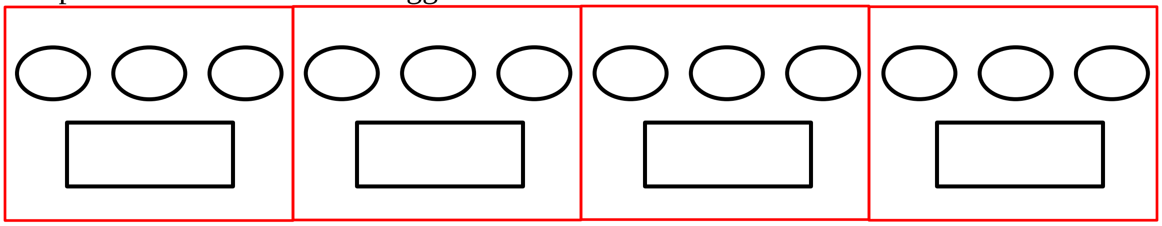
   b. How much of a chocolate bar does it take to make one S'more?
      It takes $\frac{1}{4}$ of a chocolate bar to make one S'more.

4. Amul has 4 pieces of licorice that he needs to divide evenly between 6 people. How much licorice does each person get? Draw a picture to represent this situation and then write the math sentence that goes with this problem.

This is an example of partitive division. The number of groups is known and we are trying to determine the measure of each group. Students may solve this problem a variety of ways. One way is to divide each piece of licorice into 6 equal pieces and see that each person would get 4 of these $\frac{1}{6}$ size pieces or $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{4}{6}$ or $\frac{2}{3}$. The math sentence is $4 \div 6 = \frac{2}{3}$.

1.1d Homework: More Equivalent Ratios

1. The picture shows the ratio of eggs to butter used to make a sauce.



   a. Write at least <u>three</u> equivalent ratios that describe the ratio of eggs to butter used in this recipe based on the picture.
   3:1; 6:2; 9:3; 12:4

   b. If Emeril is using 6 sticks of butter, how many eggs should he use? Justify your answer with words or by drawing a picture or model.
   18 eggs

   c. Paula has one egg. How much butter should she use for one egg? Justify your answer with words or by drawing a picture or model.
   $\frac{1}{3}$ stick of butter

   d. Bobby has two eggs. How much butter should he use for two eggs? Justify your answer with words or by drawing a picture or model.
   $\frac{2}{3}$ stick of butter

2. Jayden's soccer team wins 24 of the 30 games they play in a season.
   a. Find <u>two</u> equivalent ratios that are smaller than the one given in the problem. Explain how you found your smaller equivalent ratios.
   4:5, 8:10, 12:15 etc. Explanations may include a drawing or model or a numeric method such as dividing by a common factor.
   b. If Jayden's team won 16 games over a period of time, how many games do you expect they played? Justify your answer.
   You would expect that Jayden's team played 20 games. Students may use one of the simplified ratios from above to solve the problem numerically or they may draw a picture/model.
3. Sixth graders at a middle school can either choose art or music for their elective. Mrs. Benson, the counselor at the school, looks up 64 students and notes that 24 are enrolled in art and 40 are enrolled in music. This represents the ratio of the number of students in art to the number of students in music.
   a. In a group of 32 students, how many would you expect to be registered for art and how many for music? Justify your answer.
   12 in art and 20 in music
   b. What is the ratio of students enrolled in art to students enrolled in music? Find <u>two</u> equivalent ratios that are simpler than the one given in the problem.
   The ratio of students enrolled in art to students enrolled in music is 24:40. Some simpler ratios are 3:5; 6:10; 9:15
   c. If 15 students in a group are enrolled in art, how many would be enrolled in music?
   25
   d. Write <u>two</u> part to total ratios about this relationship.
   Three out of every eight (or an equivalent ratio) students choose art.
   Five out of every eight (or an equivalent ratio) students choose music.

4. In a certain city, it rains 20 out of the 30 days in April.
    a. How many days would you expect it to rain in this city over a 15-day period in April?
    10 days

    b. How many days would you expect it to rain in this city over a 6-day period in April?
    4 days

    c. What is the simplified ratio of days it rains to days it does not rain in this city in April?
    2:1

5. Juan and his older brother like to do push-ups together. The ratio of push-ups completed by Juan to push-ups completed by his older brother is always the same. One day, Juan does 18 push-ups and his older brother does 27 push-ups.
    a. Complete the table to show this relationship. Explain or show how you completed the table.

Juan	**Juan's Older Brother**
2	3
4	6
6	9
18	27
36	54
60	90

Students may use pictures/models to complete the table. They may also use patterns of repeated addition/multiplication to complete the table.

6. On her iPod, Emily has 50 pop songs she listens to when she works out and 25 classical songs she listens to when she studies.
    a. The ratio of pop songs to classical songs on Megan's iPad is the same as Emily's. If Megan has 20 pop songs on her iPod, how many classical songs does she have? Justify your answer.
    10 classical songs; Some students might start to notice the explicit relationship between the two quantities (i.e. the number of classical songs is always half the number of pop songs). We will examine the explicit relationship in **1.2g**.

    b. Theo also has the same ratio of pop songs to classical songs on his iPod. If Theo has 60 total songs on his iPod, how many are pop songs and how many are classical songs? Explain or show how you found your answer.
    40 pop songs and 20 classical songs

1.1e Class Activity: Solving Real World Ratio Problems

This lesson ties together all of the concepts studied so far in this section. You can use it as a formative assessment to see how students are coming along with the concepts. Use select problems from the Class Activity and Homework to meet the needs of your students.

Activity 1: Pam and Corinne are running for class president. The ratio of students who voted for Pam to students who voted for Corinne is 4:3.

a. If 60 students voted for Corinne, how many students voted for Pam? Solve using at least <u>two</u> strategies.

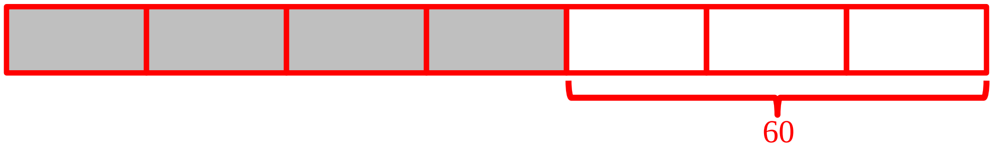
As we have seen in this section, students may solve this problem a variety of ways. One way to answer this question is to draw a tape model to represent this situation as show below. Another is to create a partial table.

First, we are given the following part to part ratio:

Students Who Voted for Pam	to	Students Who Voted for Corinne
4	:	3

In the tape model, the shaded boxes represent Pam's portion of the votes and the white boxes represent Corinne's portion of the votes.

Corinne's portion of the tape model represents 60 students.



In a tape model, each box represents the same number of items, in this case the items are students. We can see that each box in our tape model represents 20 students.

From our model, we see that 80 students voted for Pam if 60 students voted for Corinne.

We can double check our answer by making sure that the ratio of students who voted for Pam to students who voted for Corinne is equivalent to our original ratio:

Is 4 to 3 equivalent to 80 to 60? Yes, we can see that $\frac{4}{3}$ is equivalent to $\frac{80}{60}$:

$$\frac{4}{3} \times \frac{20}{20} = \frac{80}{60}$$

Help students to connect what is happening numerically with the model above.

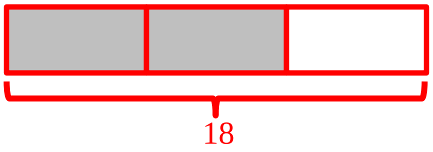
b. How many total students voted? Solve using at least <u>two</u> strategies.

If 80 students voted for Pam and 60 voted for Corinne, a total of 140 students voted. We also see this answer on our tape model – there are 7 groups of 20 students which equals 140 students.

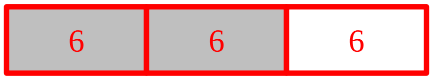
Activity 2: Chen makes 2 out of every 3 free throws that he shoots.

a. If he shoots 18 free throws in a game, how many do you expect him to make? Solve using at least <u>two</u> strategies.
Again, we can draw a tape diagram to represent this situation. In the diagram below, the gray boxes represent Chen's makes/successes.

In this problem, we know that he shoots 18 free throws so this represents the total on our tape model:



Therefore, each box represents 6 shots.



From the model, we can see that we would expect Chen to make 12 shots.

We can also look at this numerically:

$\frac{\text{successes}}{\text{attempts}} = \frac{2}{3} \times \frac{6}{6} = \frac{12}{18}$ If Chen attempts 18 shots, you would expect him to make 12.

Some students may also set up a partial table and use the patterns in the table to solve this problem:

	Successes	Attempts	
	2	3	
×6	?	18	×6

Make explicit for students that the tape diagram, numeric methods, and partial table all show the same thing.

b. If Chen shoots 18 free throws, how many do you expect him to miss? Solve using at least <u>two</u> strategies.

We can see from the model that Chen would miss 6 shots. And numerically,

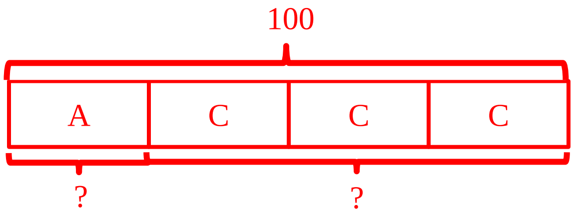
$\frac{\text{misses}}{\text{attempts}} = \frac{1}{3} \times \frac{6}{6} = \frac{6}{18}$ If Chen attempts 18 shots, you would expect him to miss 6.

c. What is the ratio of shots made to shots missed for Chen?
From the tape model, we can see that the ratio of shots made to shots missed is 2:1. We can also see this in a particular pair of values. When Chen shoots 18 shots, he makes 12 and misses 6. The ratio 12:6 is equivalent to 2:1:

$\frac{2}{1} \times \frac{6}{6} = \frac{12}{6}$

Directions: Solve each problem using at least <u>two</u> strategies. Make connections between your strategies. Students must attend to precision in these problems. What are they given – a part to part or part to total ratio? How do they draw the model? What does each part of the model represent and what is its value? How can we double check our answers using another method? How are the methods connected? There are several practice problems given – you may wish to jigsaw them out or allow students to choose problems to solve (i.e. choose 10 of the 18 to solve).

1. The ratio of adults to children at a movie is 1:3.
    a. If there are 100 total people at the movie, how many are children and how many are adults?



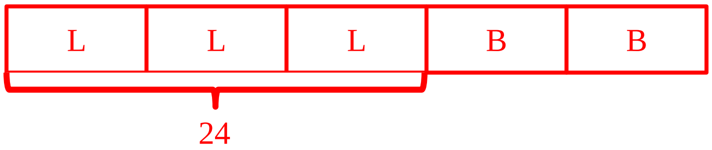
If there are 100 total people, each box represents 25 people; therefore there are 25 adults and 75 children. Numerically:

$\frac{\text{adults}}{\text{total people}} = \frac{1}{4} \times \frac{25}{25} = \frac{25}{100}$ 25 of the 100 people are adults; therefore 75 of the 100 people are children

    b. If 12 more people enter the theater, how many of them need to be adults and how many need to be children to keep the same ratio?

3 adults and 9 children

2. The ratio of Lance's allowance to his younger brother's allowance is 3 to 2. Over a four-week period Lance gets paid $24. How much does his younger brother get paid in the four-week period?



Each box represents $8; therefore Lance's younger brother makes $16 when Lance makes $24.

Is $\frac{24}{16}$ equivalent to $\frac{3}{2}$? Yes, $\frac{24}{16} \div \frac{8}{8} = \frac{3}{2}$

3. It rains five out of seven days in a certain part of Kauai. How many days you do expect it to rain in a 42-day period?

In a 42-day period, each box represents 6 days; therefore we would expect it to rain 30 days in a 42-day period.

4. In a group of 80 people, 8 people are left-handed.

    a. In a classroom of 30 students, how many would you expect to be left-handed?

In this problem, it would likely help students to find a simpler ratio first. In a classroom of 30 students, we would expect 3 students to be left-handed.

    b. In a school of 300 students, how many would you expect to be left-handed?

In a school of 300 students, we would expect 30 students to be left-handed.

    c. In a group of people, there are 5 left-handed people. How many right-handed people would you expect to be in that group? You would expect 50 total people so you would expect 45 right-handed people

5. At a certain middle school, four out of every five students prefer hip hop music to country music.

H	H	H	H	C

   a. If 30 students prefer country music, how many students prefer hip hop music?
   If 30 students prefer country music, 120 students prefer hip hop music.

   b. If 30 students prefer country music, how many students are there total at the school?
   If 30 students prefer country music, there are 150 students at the school.

6. Chloe surveyed adults to determine whether or not they prefer to watch football or basketball. Five out of every eight adults prefer to watch football. If 33 people in Chloe's survey prefer to watch basketball, how many adults took the survey?

F	F	F	F	F	B	B	B

If 33 people in the survey prefer basketball, then each box above represents 11 adults/votes; therefore 55 adults prefer to watch football. Therefore 88 adults were surveyed.

7. At an amusement park, Mike polled people coming off of a ride and asked whether they liked the ride or not. The ratio of people who liked the ride to those who did not like the ride is 5 to 2. In a group of 70 people who went on the ride, how many would you expect to dislike the ride?

L	L	L	L	L	NL	NL

In a group of 70 people, each box above would represent 10 people so we would expect 20 people to dislike the ride.

8. In Talen's class there are 24 students. Sixteen of the students go to daycare after school. In Sophia's class, the ratio of students who got to daycare after school is the same as in Talen's class. If there are 30 students in Sophia's class, how many go to daycare after school?
20 students in Sophia's class would go to daycare

9. Two out of twenty-five students have food allergies. In a school of 150 students, how many would you expect to have food allergies?
Students are welcome to draw a picture or model of this situation; however as numbers become larger, this can be time-consuming. They may choose to solve this numerically:
$\frac{\text{allergies}}{\text{total students}} = \frac{2}{25} \times \frac{6}{6} = \frac{12}{150}$ In a group of 150 students, you would expect 12 of them to have food allergies.

Students may also make groups of 25 and 2 until they get to 150 total students. They would have 6 groups and 6 groups of 2 equals 12.

10. A pizza store sells cheese and pepperoni pizzas for $5. The ratio of cheese pizza sold to pepperoni pizza sold is 2 to 1. If the store sells 90 pizzas, how many of them are cheese and how many of them are pepperoni?

C	C	P

If the store sells 90 pizzas, 60 of them are cheese and 30 of them are pepperoni.

11. Bianca is making a game for her friends to play. She is putting white and yellow pieces of paper into a bag. If her friends pick out a yellow piece of paper, they win. One out of every four pieces of paper are yellow.
    a. If there are 15 pieces of white paper in Bianca's bag, how many pieces of yellow paper are there in the bag?

Y	W	W	W

If there are 15 strips of white paper in the bag, each box represents 5 pieces of paper; therefore there would be 5 strips of yellow paper in the bag.

    b. Owen made the same game with the same chances of winning. Owen's bag has a total of 60 pieces of paper. How many pieces of paper are white and how many are yellow?

15 yellow and 45 white

12. For every $3 Tiffany donates to charity her parents donate $1. If Tiffany donates $27, how much do they donate all together?

T	T	T	P

If Tiffany donates $27, each box above represents $9; therefore together they would donate $36.

13. The ratio of cement to sand used to make concrete is 5 to 3. If Tim uses 25 pounds of cement, how much sand should he use?

C	C	C	C	C	S	S	S

If Tim uses 25 pounds of cement, each box above represents 5 pounds of material; therefore Tim should use 15 pounds of sand.

14. One out of every four students at Miley's school takes the bus home. If there are 200 students at Miley's school, how many take the bus home?

B	NB	NB	NB

If there are 200 people at Miley's school, then each box represents 50 students; therefore 50 students take the bus home.

15. The ratio of girls to boys in Mrs. Simpson's class is 4 to 3.

    a. If there are 16 girls, how many boys are there?

G	G	G	G	B	B	B

If there are 16 girls, each box represents 4 students; therefore there are 12 boys in Mrs. Simpson's class.

    b. If 8 more girls join the class, how many boys need to join to keep the same ratio?

6 more boys need to join

16. At Toby's school, students wear a uniform. Students can choose from a red shirt or a white shirt. For every white shirt that the school sells, they sell four red shirts. If the store sells 55 shirts one day, how many red shirts did they sell?

W	R	R	R	R

If the school sells 55 shirts each box above represents 11 shirts; therefore the school sold 44 red shirts.

17. The ratio of Greek yogurt to flour used to make pizza dough is 1 to 2. If Luigi uses 5 cups of flour, how many cups of Greek yogurt should he use?

Y	F	F

If Luigi uses 5 cups of flour, each box above is equal to $2\frac{1}{2}$ cups of ingredient; therefore Luigi should use $2\frac{1}{2}$ cups of yogurt.

18. A pancake recipe calls for 3 cups of pancake mix for every 2 cups of milk.

PM	PM	PM	M	M

    a. If Rhonda uses 5 cups of pancake mix, how much milk should she use?

    If Rhonda uses 5 cups of pancake mix, each box above would be $\frac{5}{3}$ or $1\frac{2}{3}$ c. of ingredients; therefore Rhonda should use $3\frac{1}{3}$ cup of pancake mix.

    If students have a difficult time figuring out how to divide the 5 cups evenly by 3, have them first give out one cup to each box. Then have them think about how to divide 1 cup – 1/3 would go into each box. Now, each box has $1\frac{1}{3}$ c. of ingredients. Next, divide up the final cup of pancake mix, again each box would get 1/3 of that cup. When we add it all together, each box would have $1\frac{2}{3}$ c. of pancake mix. If students have a difficult time determining the amount of milk, have them draw a picture.

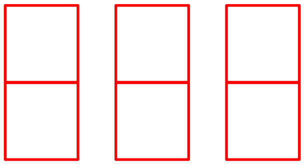
    b. If Rhonda uses 1 cup of milk, how much pancake mix should she use?

    If Rhonda uses 1 cup of milk, each box would be ½ c. of ingredients; therefore Rhonda should use $1\frac{1}{2}$ cups of pancake mix.

Spiral Review

1. Zoe has five cookies she has to share evenly with her brother. How many cookies will each child get? Draw a picture to represent this situation and write a math sentence to model this problem.
   Each child will get $2\frac{1}{2}$ cookies.

2. Mary is making pillows. Each pillow needs ½ yard of fabric. How many pillows can Mary make with 3 yards of fabric? Draw a picture to represent this situation and write a math sentence to model this problem.
   Pictures may vary. This picture shows that Mary can make 6 pillows if each requires ½ yard of fabric. This is an example of quotative division. We know the measure of each group and we are trying to determine the number of groups we can create.



$3 \div \frac{1}{2} = 6$

3. How many minutes are there in…

   a. 1 hour
   60 minutes

   b. 2 hours
   120 minutes

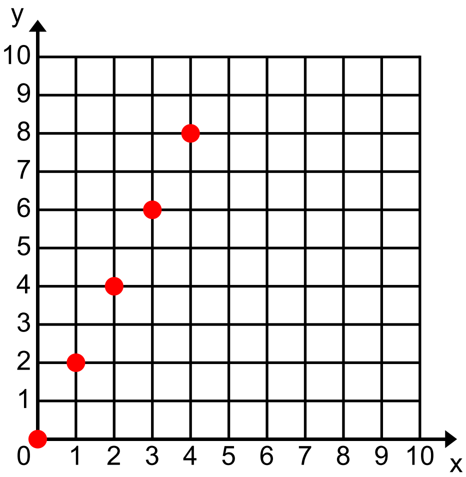
   c. 3 hours
   180 minutes

   d. $4\frac{1}{2}$ hour
   270 minutes

   e. $\frac{1}{3}$ hour
   20 minutes

   f. $\frac{1}{4}$ hour
   15 minutes

4. Plot the following ordered pairs on the coordinate plane: $(0, 0)$ $(1, 2)$ $(2, 4)$ $(3, 6)$ $(4, 8)$



Students learned to plot points in quadrant I in **5.G**. If students are struggling with this concept, you may want to give them additional practice plotting points in preparation for lesson **1.2b**.

1.1e Homework: Solving Real World Ratio Problems

1. Students in Mrs. Benson's gym class are voting on whether the next sport they learn how to play should be basketball or soccer. The tape model shows the results of the survey.

B	B	B	S	S	S	S

   a. If there are 35 students in Mrs. Benson's gym class, how many voted for soccer? Solve this problem using at least two different methods. Explain the methods you used and how they are related.
   If there are 35 students in Mrs. Benson's class, 20 students voted for soccer.

2. An illustrator and writer are submitting a children's book in a competition. They have agreed to split the cash prize in the following way: For every $3 the writer gets, the illustrator gets $2 as shown in the tape model below.

W	W	W	I	I

   a. If the winning prize in the competition is $500, how much will the writer and illustrator each get?

   Writer: __$300_______ Illustrator: __$200_______

   b. If the winning prize is $1,000, how much will the writer and illustrator each get?

   Writer: __$600_______ Illustrator: __$400_______

Students should notice that when you double the total, you also double the size of each piece and subsequently the parts are also doubled.

3. The ratio of students in band who play a wind instrument (like a flute) to students who play a percussion instrument (like a drum) is 5 to 1. If 30 people play a wind instrument, how many play a percussion instrument? Solve this problem using at least two different methods. Explain the methods you used and how they are related.

W	W	W	W	W	P

If 30 people play a wind instrument, 6 people play a percussion instrument.

4. In a pet store, there are 40 birds and 10 cats. If the store gets 12 new birds, how many new cats does the store need to get to keep the same ratio of birds to cats?
The store would need to get 3 new cats. Students may solve this a variety of ways. One way is to use a tape diagram to find the simplified ratio of 4 to 1.

40 B	10 C

10 B	10 B	10 B	10 B	10 C

If the store adds 12 new birds, each box above would have a value of 3; therefore they would need 3 new cats to keep the same ratio.

5. In an online game, Mario destroys blocks. Some of the blocks have power-ups in them and some do not. The ratio of blocks with power-ups to blocks without power-ups is the same in every level. On Level 1, there are 36 blocks and 9 of them have power-ups in them.
    a. On Level 2, there are 20 blocks. How many of them will have power-ups in? Justify your answer.
    5 blocks will have power-ups

    b. On Level 3, there are 7 blocks that have power-ups. How many total blocks are on this level? Justify your answer.
    There are 28 total blocks.

This would be an interesting problem to organize in a table to explore the repeated addition patterns.

6. The ratio of children to adults on a field trip is 5 to 1. If there are 60 children on the field trip, how many people total are there on the field trip?

C	C	C	C	C	A

There are 72 total people.

7. To make ice cream sandwiches, Ina uses 2 cookies for each scoop of ice cream. If Ina uses 60 cookies, how many scoops of ice cream does she need?

C	C	I

Ina needs 30 scoops of ice cream for 60 cookies.

8. An ice cream shop sells 3 ice creams in a cone for each ice cream they sell in a cup. If they sell 200 ice creams in a day, how many did they sell in a cone?

cone	cone	cone	cup

They sold 150 in a cone.

9. In a certain neighborhood there are 25 dogs and 10 cats.
    a. In a neighborhood across the way, the ratio of cats to dogs is the same. If there are 8 cats in the neighborhood, how many dogs are there?

There would be 20 dogs.

10. A popular cereal brand is giving away baseball cards in some of its cereal boxes. Two out of every seven boxes of cereal contain a pack of baseball cards. On a shelf with 49 boxes of this cereal, how many would you expect to have baseball cards in them?

C	C	NC	NC	NC	NC	NC

On a shelf with 49 boxes of cereal, you would expect 14 boxes to have cards in them.

11. A market research company is testing out a new cereal. Four out of every five people that try the cereal like it. If 80 people like the cereal, how many people tried the cereal?

L	L	L	L	D

If 80 people like the cereal, 100 people tried the cereal.

12. Camie is making a quilt using patches of red, white, and blue fabric. The ratio of red to white to blue patches in the quilt is 2:1:2. If she used 80 total patches to make the quilt, how many of each color patch does she have in her quilt?

R	R	W	B	B

Red: __32____   White: __16____   Blue: ___32___

13. The ratio of sugar to flour used in a sugar cookie recipe is 1 cup sugar to 2 cups flour. If Mrs. Smith uses 3 cups of flour, how many cups of sugar should she use to follow this recipe?

S	F	F

If Mrs. Smith uses 3 cups of flour, she should use $1\frac{1}{2}$ c. of sugar.

14. The ratio of blue paint to red paint used to make a certain shade of purple pain is 2:3. If John uses 1 gallon of red paint, how much blue paint should he use to make this shade of purple?

B	B	R	R	R

If John uses 1 gallon of red paint, he should use $\frac{2}{3}$ gallons of blue paint.

1.1f Self-Assessment: Section 1.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Understand what a ratio is.	I understand that a ratio involves two or more quantities.	I understand that a ratio expresses a relationship between two or more quantities. I recognize key ratio language.	I understand that a ratio expresses a relationship between two or more quantities. I recognize key ratio language. I can analyze ratios to identify the quantities involved and their relationship to each other and draw pictures to represent the relationship.	I understand that a ratio expresses a relationship between two or more quantities. I recognize key ratio language. I can analyze ratios to identify the quantities involved and their relationship to each other. I can move fluently between ratio statements and pictures/models of ratios.
2. Given a pictorial representation or model, use ratio language and notation to describe the relationship between two or more quantities.	I can write a ratio statement to show the relationship between two quantities given a picture or model using words but I sometimes mix up the order of the quantities and I am not sure about the different notations used in ratio statements.	I can write a ratio statement to show the relationship between two quantities given a picture or model using words but I often forget the different notations used when writing ratio statements.	I can accurately write a ratio statement to show the relationship between two quantities given a picture or model. I understand ratio notation and use it correctly.	I can accurately write several ratio statements (part to part and part to total) to show the relationship between two quantities given a picture or model. I understand ratio notation and use it correctly.
3. Given a ratio statement, identify the important quantities and interpret the language and notation in order to create a pictorial representation or model of the relationship between the quantities.	When given a ratio statement, I can identify the quantities involved but I don't always understand the notation being used so I have a difficult time drawing the pictures.	When given a ratio statement, I can identify the quantities involved and interpret ratio language and notation in order to draw a picture of the relationship. When drawing a picture, I sometimes mix up the quantities. I struggle to draw a tape diagram.	When given a ratio statement, I can identify the quantities involved and interpret ratio language and notation in order to accurately draw a picture of the relationship. I can also draw a tape diagram to represent the relationship between the quantities.	When given a ratio statement, I can identify the quantities involved and accurately draw a picture and tape diagram to represent the relationship between the quantities. I clearly label my pictures and diagrams so that the relationship is clear to others and shows the parts and total.

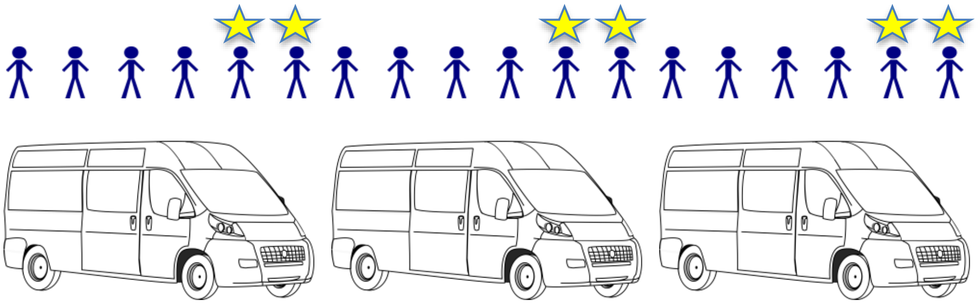
4. Distinguish when a ratio is describing a part to part, part to total, or total to part relationship between quantities.	I understand that there are different types of ratios (part to part and part to total) but I have a difficult time determining whether the quantities given are parts or totals.	I can identify the different quantities in a given ratio statement and tell whether the quantity represents a part or a total but I have difficulty describing the quantity not given (i.e. If I am given the ratio statement, “There are 3 ducks for every 2 geese”, I know one part is ducks and one part is geese but I have a difficult time determining that the total is birds).	I can identify the different quantities in a given ratio and determine whether the quantity represents a part or a total. I can name the hidden quantity (part or total) and use this information to draw models representing the ratios.	I can identify the different quantities in a given ratio and determine whether the quantity represents a part or a total. I can name the hidden quantity (part or total) and use this information to draw models representing the ratios. I can write additional ratio statements, moving fluently between using parts and totals in my statements.
5. Determine equivalent ratios to describe a relationship between two quantities. Make and complete tables of equivalent ratios.	I know that an equivalent ratio shows the same relationship between two quantities but I am not sure how to find one.	I can find equivalent ratios using concrete tools (tiles or chips) but have a hard time finding equivalent ratios without concrete tools. I can complete partially filled in tables that are in order but have a difficult time with tables that skip around.	I can find equivalent ratios using concrete tools, models such as tape diagrams, and numeric methods. I can complete partially filled in tables.	I can find equivalent ratios using concrete tools, models such as tape diagrams, and numeric methods. I can complete partially filled in tables and make tables of equivalent ratios from scratch. I understand the connection between equivalent ratios and multiplication and division.
6. Use ratio reasoning and models (tape diagrams) to solve real world ratio problems.	When given a problem that involves ratios, I can identify the quantities involved and key ratio language but I am not always sure on how to get started solving the problem.	When given a problem that involves ratios, I can identify the quantities involved and key ratio language. I can draw pictures or use concrete tools (tiles or chips) to solve the problem.	When given a problem that involves ratios, I can identify the quantities involved and key ratio language. I can use more than one strategy (picture, tape diagram, numeric, etc.) to solve the problem.	When given a problem that involves ratios, I can identify the quantities and key ratio language. I can use multiple strategies (picture, tape diagram, numeric, etc.) to solve the problem and make connections between the strategies.

Sample Problems for Section 1.1

Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. Explain ratio in your own words. Give several examples of ratios. [1]
   Answers will vary but it should be clear by their definitions and examples that students understand that a ratio expresses a relationship between two quantities.

2. In a running race, teams of runners compete in a 2-day running race that covers 120 miles. One team member runs at a time and the other teammates follow the runner in a van. They meet up at different points and runners swap out. The team runs the same number of miles each day and each runner runs the same distance each day. The picture below shows several teams lined up at the start of the race next to their vans. The people with the stars above their heads are the team captains. [2]



   Write several ratio statements about the race.
   Answers will vary. Sample answers include:
   There are 6 people on a team.
   Two out of every 6 people on a team are captains.
   The ratio of captains to non-captains is 2:4.
   There are 6 people in each van.
   The team runs 60 miles per day.
   Each person runs 10 miles per day.
   Each person runs 20 miles during the race.

3. Children in a summer camp are voting on whether or not they want their first activity every day to be swimming or tennis. The picture below shows the votes. S stands for swimming and T stands for tennis. [2] [4]

   T T T T S S S T T T T S S S T T T T S S S T T T T S S S

   Select all ratios that correctly describe the relationship between votes for tennis and votes for swimming.

   ☐ The ratio of votes for tennis to votes for swimming is 3 to 4. False

   ☐ The ratio of votes for swimming to total votes is 3:7. True

   ☐ The ratio of votes for swimming to votes for tennis $\frac{3}{4}$. True

   ☐ The ratio of votes for tennis to total votes is 4:28. False

   ☐ The ratio of votes for swimming to votes for tennis is 9:12. True

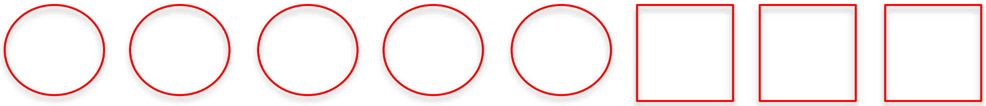
4. Talen’s dad is pitching the baseball to him. The picture below shows the ratio of hits to misses for Talen. The smiley faces represent hits and the frown faces represent misses. [2] [4]

Write several ratios to describe this situation. Tell whether your statements are part to part (PP), part to total (PT), or total to part (TP).
Answers will vary. Sample answers include:
The ratio of hits to misses is 2 to 1. Part to part
Talen hits two out of every three pitches. Part to total.

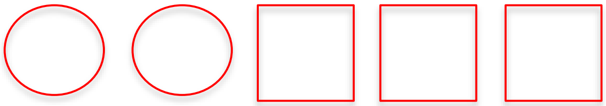
5. Using circles and squares, create pictures that represent the ratio given. [3] [4]

The ratio of circles to squares is 3:5.

The ratio of squares to circles is 3:5.



The ratio of squares to total shapes is 3:5.



6. For a science experiment, a salt water solution requires 1 cup of salt for every 2 cups of water. Select all ratios that are equivalent. [5]

☐ 2 cups of salt mixed with 3 cups of water Not equivalent

☐ 3 cups of salt mixed with 6 cups of water Equivalent

☐ 10 cups of salt mixed with 20 cups of water Equivalent

☐ ½ cup of salt mixed with 1 cup of water Equivalent

☐ 10 cups of salt with 5 cups of water Not equivalent

7. A recipe for trail mix calls for 1 cup of chocolate chips for every 3 cups of nuts. Determine the number of cups of chocolate chips and nuts someone would use if they want to…[5]

Double the recipe: Chocolate Chips: __2____ Nuts: ___6___

Triple the recipe: Chocolate Chips: ___3___ Nuts: ___9___

Quadruple the recipe: Chocolate Chips: ___4___ Nuts: ___12___

Half the recipe: Chocolate Chips: ___$\frac{1}{2}$___ Nuts: ___$1\frac{1}{2}$___

Quarter the recipe: Chocolate Chips: ___$\frac{1}{4}$___ Nuts: ___$\frac{3}{4}$___

8. A deck of cards has 8 wild cards and 40 numbered cards. Complete the following ratios to describe the relationship between wild cards and numbered cards in the deck. [5] [2] [4]

The ratio of ___wild cards_________________ to __numbered cards______________ is 1 to 5.

The ratio of ___wild cards_________________ to ____total cards_________________ is 1 to 6.

The ratio of __numbered cards_______________ to ____total cards_________________ is 5 to 6.

9. The table below shows the ratio of time Johnny spends playing outside to time spent watching TV each day. Complete the table to show this relationship. [5]

Time Spent Playing Outside (minutes)	**Time Spent Watching TV (minutes)**
15	9
20	12
40	24
80	48
100	60

10. The tape diagram shows the ratio of life guards to swimmers at the pool. S represents swimmer and L represents life guard. [6]

S	S	S	S	S	S	S	S	L

a. What is the ratio of swimmers to life guards at the pool? 8 to 1

b. If there are 56 swimmers at the pool, how many life guards are there? If there are 56 swimmers, there would be 7 life guards

11. The ratio of lemonade to Sprite in a punch recipe is 3:1. [6]
    a. Draw a tape diagram to represent the relationship between lemonade and Sprite in this recipe.

L	L	L	S

    b. Olivia needs 20 total cups of punch, how many cups of lemonade and Sprite should Olivia use?

Cups of Lemonade: ___15____ Cups of Sprite: ____5___

12. A cafeteria sells cartons of regular milk and chocolate milk. Three out of every five cartons of milk sold are regular milk. If the cafeteria sells 30 cartons of chocolate milk, how many total cartons of milk does the cafeteria sell? [6]

R	R	R	C	C

If the cafeteria sells 30 cups of chocolate milk, they sold 75 total cartons of milk.

13. The owners of a camp are required by law to have a certain ratio of camp counselors to children. There are 8 counselors at the camp for 48 children at the camp. [6]
    a. If 12 more children sign up for the camp, how many more counselors does the camp need to hire?

The camp would need to hire 2 more counselors.

14. Iya is making a blanket with yellow and blue patches of fabric. The patches of fabric are all the same size. She uses 48 patches of blue fabric and 32 patches of yellow fabric. [6]
    a. Hendrix is also making a quilt. He has 24 patches of blue fabric. How many patches of yellow fabric does he need to make a quilt that has the same ratio of blue to yellow fabric as Iya's quilt?

Hendrix would need 16 patches of yellow fabric. Students may see that they can just ½ the number of yellow patches.

$$\frac{\text{blue}}{\text{yellow}} = \frac{48}{32} \div \frac{2}{2} = \frac{24}{16}$$

    b. Kai is also making a quilt. Kai has 40 patches of yellow fabric. How many patches of blue fabric does Kai need to make a quilt that has the same ratio of blue to yellow fabric as the other quilts?

Kai would need 60 patches of blue fabric. It will likely help students to find the simplified ratio. A tape diagram may also help.

$$\frac{\text{blue}}{\text{yellow}} = \frac{48}{32} \div \frac{16}{16} = \frac{3}{2} \times \frac{20}{20} = \frac{60}{40}$$

15. Doug is mixing red and blue paint to make a certain shade of purple paint. He mixes 6 cups of red paint with 4 cups of blue paint. He realizes he needs more purple paint so he adds 2 more cups of blue paint. How much more red paint should he add to keep the same shade of purple paint? [6]

If Doug adds 2 more cups of blue paint, he should add 3 more cups of red paint. To solve this problem, students may determine the simplified ratio of red paint to blue paint which is 3 to 2. This ratio reveals the amount of red paint needed for 2 more cans of blue paint.

Students may solve the problem numerically:

$\frac{\text{red paint}}{\text{blue paint}} = \frac{6}{4} \div \frac{2}{2} = \frac{3}{2}$ or $\frac{\text{red paint}}{\text{blue paint}} = \frac{6}{4} \times \frac{1.5}{1.5} = \frac{9}{6}$ (He would have 6 total cups of blue paint so would need 9 total cups of red paint which would be an additional 3 cups of red paint. Students may do something similar using the simplified ratio of 3:2.)

Students may also use a model or picture to solve:

R	R	R	B	B

Initially, each of these boxes represents 2 cups of paint. If he adds two more cans of blue paint, each box will now represent 3 cans of paint so there will be 9 total cups of red paint – he has already added 6 cups of red paint so only needs to add 3 more cups.

Students may also draw a tape diagram that looks like the one below:

R	R	R	R	R	R	B	B	B	B

Initially, each of these boxes would represent 1 cup of paint. If he adds 2 cups of blue paint, each box will now represent $1\frac{1}{2}$ cups of paint; the result still being that he needs a total of 9 cups of red paint so 3 additional cups of red paint.

If students have a strong understanding of ratio and its multiplicative nature, this problem may be very easy for them. Encourage them to use multiple strategies to support their answer (model, numeric, picture, etc.)

A common error is for students to think that if Doug adds 2 more cups of blue paint, he should also add 2 more cups of blue paint. They incorrectly think that ratios generated by adding the same value to both quantities are equivalent ratios. If students make this error, they likely need additional support with their understanding of ratio.

Section 1.2: Rates, Graphs, and Equations

Section Overview:
In this section, students explore ratios that have different units of measure. They discover that a double number line is a valuable tool that can be used to represent ratios and solve real-world problems. They expand on their work with the coordinate plane, representing a collection of equivalent ratios on the coordinate plane. They examine features of the graph and use the graph to solve real world problems. Next, students learn what a unit rate is and use a variety of models to find the unit rates associated with a given ratio. They soon realize that the unit rate can be a valuable tool for solving real world problems. As the section progresses, students rely on many of the skills, concepts, and tools they have learned in this chapter in order to compare ratios. Lastly, students learn how to write equations to represent the relationship between two quantities.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Read double number lines, create them to show relationships between two quantities, and use them to solve real world problems.
2. Plot pairs of equivalent ratios on a coordinate plane.
3. Read and interpret graphs to solve real world problems involving ratios.
4. Understand what a unit rate is. Find the unit rates associated with a given ratio and use them to solve real world problems.
5. Use tables to compare ratios.
6. Write an equation to show a relationship between two quantities and use equations to solve real world problems.

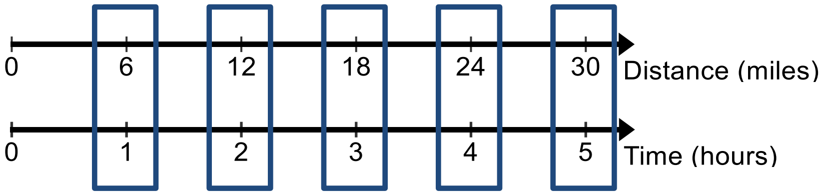
1.2a Class Activity: Ratios with Different Units

Activity 1: Harmony runs 6 miles per hour. How far can Harmony run in 1 hour? 2 hours? 3 hours? 4 hours? Organize your results in the table below. Up to this point, most of the ratio problems students have studied have had similar units (i.e. ratio of girls to boys – the unit is people, ratio of red paint to blue paint – the unit is cups, gallons, etc). In this lesson, students have the opportunity to examine ratios with different units. Some textbooks distinguish between ratios and rates, defining rates as ratios with different units of measure. The Common Core State Standards do *not* make this distinction. This problem, as with the others studied up to this point, expresses a relationship between two quantities and for this reason is a ratio. *All* ratios have associated rates which we will study later in this section.

Time (hours)	0	1	2	3	4	5
Distance (miles)	0	6	12	18	24	30

a. Write <u>three</u> equivalent ratios to show the relationship between time and distance.
The ratio of time to distance is 1 to 6, 2 to 12, 3 to 18.

A model that is commonly used to represent ratios where the units of measure are different is a **double number line diagram**. We can use the model below to represent the problem about Harmony. You may want to have a discussion with students about what the statement "the units of measure are different" means and why a tape diagram would not make sense in this situation. Also, students should start to see models such as the one below as tools that can help them to solve problems.



In a double number line, just as in a table, you can see that pairs of values are connected. The values on top are connected to the values directly below them.

Just as in a table, one can observe the patterns of repeated addition and the patterns of scalar multiplication on a double number line. Ask students what patterns they see in the double number line above.

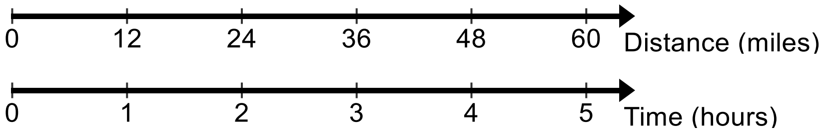
b. How far does Harmony run in 2 hours? Harmony runs 12 miles in 2 hours.

c. How long does it take Harmony to run 30 miles? It takes Harmony 5 hours to run 30 miles.

d. How far can Harmony run in 30 minutes? Justify your answer.
Harmony can run 3 miles in 30 minutes

e. How far can Harmony run in $4\frac{1}{2}$ hours? Justify your answer.
Harmony can run 27 miles in $4\frac{1}{2}$ hours. The repeated addition patterns will really help students to answer this question and the previous one.

1. Julia is riding her bike. The double number line shows Julia’s distance over time.



   a. Write three different ratios to describe this situation.
   12 miles in 1 hour, 24 miles in 2 hours, 36 miles in 3 hours, etc.

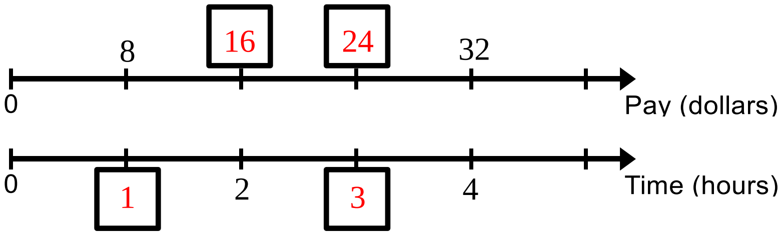
   b. How far does Julia ride in 1 hour? Julia rides 12 miles in one hour.

   c. How far does Julia ride in 2 hours? Julia rides 24 miles in two hours.

   d. How far does Julia ride in 4 hours? Julia rides 48 miles in 4 hours. Notice that when the time doubles, so does the distance.
   e. How long does it take Julia to ride 60 miles? It takes Julia 5 hours to ride 60 miles.

   f. How long does it take Julia to ride 4 miles? Justify your answer. It takes Julia 20 minutes to ride 4 miles. A common mistake is for students to just find the 4 on the double number line and in this case, they would answer 48 hours without thinking about the units or reasonableness of their answer.
   g. How far can Julia ride in $3\frac{1}{2}$ hours? Justify your answer. Julia can ride 42 miles in 3.5 hours.

2. The double number line below shows the amount Sean gets paid based on the number of hours he works. Complete the double number line by filling in the empty boxes. Then, answer the questions that follow. To complete this problem, students will use many of the same strategies they used in the previous section to complete tables. Some students will examine each pair of values and see that the pay is always 8 times the time. Others may use additive reasoning and skip count by 8s on the top number line and 1s on the bottom number line. Some may use multiplication/division to find the missing values.



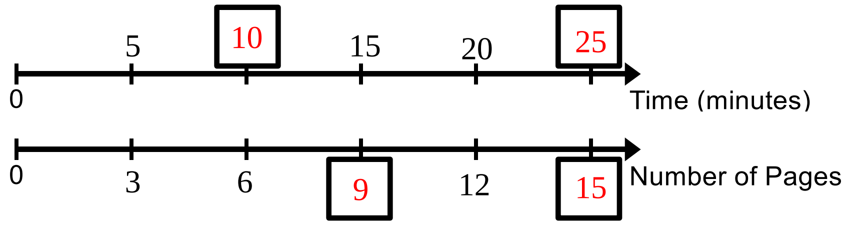
   a. How much does Sean get paid for 4 hours of work? Sean gets paid $32 for 8 hours of work.

   b. How many hours would Sean have to work to make $40? Justify your answer. Sean would have to work 5 hours to make $40.

   c. How much will Sean make if he works for 30 minutes? Justify your answer. Sean will make $4 if he works for 30 minutes.

   d. How much would Sean get paid for 8 hours of work? Justify your answer.
   Sean would get paid $64 for 8 hours of work. Students can solve this a variety of ways, including using the fact that he makes $8 each hour so $8(8) = 64$ or they can double the quantity $32 for 4 hours of work = $64 for 8 hours of work.

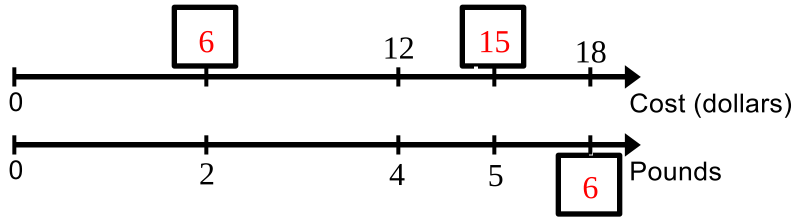
3. It takes Linda 5 minutes to read 3 pages in her book. Complete the double number line below by filling in the empty boxes to show the relationship between time and the number of pages Linda reads.



a. How many pages can Linda read in 40 minutes? Justify your answer. Linda can read 24 pages in 40 minutes.

b. How long will it take Linda to read 120 pages? Justify your answer. It will take Linda 200 minutes to read 120 pages.

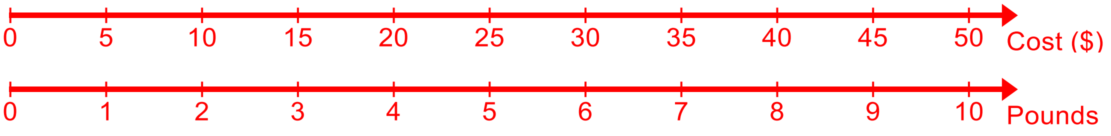
4. The number line below shows the cost per pound of jelly beans. Complete the double number line to show this relationship. Students may use a variety of strategies to solve this problem. Remind them that this problem is similar to the problems in the previous section where they were filling in missing values in a table. Students may use: 1) scalar multiplication (i.e. to get from 4 to 2, I would divide by 2 so I also divide 12 by 2 giving me 6); 2) they may fill in the missing marks and skip count, 3) they may use the explicit relationship between the two quantities – the cost per pound is $3 so the cost is always three times the number of pounds.



a. Ella purchased 3 pounds of jelly beans. How much did she pay? Justify your answer. $9, have students share the different strategies they used to solve this problem.

b. What is the cost of jellybeans per pound? Explain or show where you see this on the double number line.
$3 per pound. The cost per pound corresponds to the place on the number line where the number of pounds is equal to 1. Students can partition the interval from 0 – 2 pounds into two equal parts and do the same for the cost. Students may also find the cost per pound using ideas about division in each pair of values (i.e. $6 for 2 pounds = $3 per pound, $12 for 4 pounds = $3 per pound).

5. Dave paid $40 for 8 pounds of hamburger meat for a company picnic.
    a. Create a double number line to represent the relationship between cost, in dollars, and pounds of meat for 0 to 10 pounds of meat. Students may invert the number lines shown below, putting pounds on top and cost on the bottom. This double number line would also be correct. Have a discussion about how it doesn't matter because both double number lines show the same relationship so long as both number lines are labeled correctly.

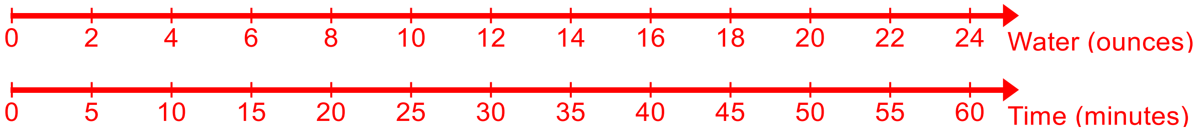


    b. Dave realizes that he needs more meat so he goes back to the store and buys 2 more pounds of meat. How much more will he spend on meat? He will spend $10 more.

    c. How much did Dave spend total on meat for the company picnic? He spent $50 total. Students may use their double number line to solve this problem or reason through the answer numerically.

    d. If Dave uses $\frac{1}{4}$ pound of meat for each hamburger, how much does the meat used for each hamburger cost? Justify your answer.
    The double number line should help students answer this question. They can see that the interval from 0 to 1 on the pounds number line needs to be divided into 4 equal parts. If we divide the interval from 0 to 5 on the cost number line into 4 equal parts, we can see that the cost of one hamburger is $1.25. If students struggle to answer this question have them draw a picture of 5 dollar bills and ask how they would divide them into 4 equal groups. You may also have the students first partition the number line into halves then into fourths.

6. A broken faucet is leaking 2 ounces of water every 5 minutes.
    a. Create a double number line to represent the relationship between amount of water leaking out, in ounces, and time, in minutes, for 0 to 60 minutes. It would also be fine to create a double number line with the order of the quantities switched (i.e. time on top and water on bottom).



    b. How much water will leak out of the faucet in $\frac{1}{2}$ an hour? 12 ounces of water will leak out of the faucet in $\frac{1}{2}$ hour.

    c. How much water will leak out of the faucet in 2 hours? 48 ounces of water will leak out of the faucet in 2 hours. Students should use precision given the fact that the number line is in minutes and these questions are written in hours. A common mistake is for students to say that the answer to this questions is 5 - they jump directly to the 2 on the number line without paying attention to which quantity the 2 corresponds to. Help students to accurately read the number line and pay attention to the quantities and their units.

    d. If there are 8 ounces in a cup of water, how many cups of water will leak out of the faucet in 2 hours? 6 cups of water will leak out of the faucet in 2 hours

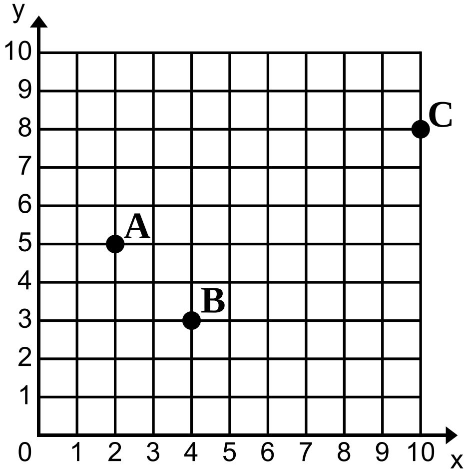
Spiral Review

1. Write the ordered pair that corresponds to the points on the graph below.

   Point A: $(2, 5)$

   Point B: $(4, 3)$

   Point C: $(10, 8)$



2. What is 108 ÷ 8?
   13.5
   These types of problems wils help students to freshen up on their division skills in preparation for the lessons on unit rate coming up in **1.2c – 1.2e**.

3. What is 2.75×4? Solve without a calculator (mentally) and show steps/thinking.
   Encourage students to use mental math to solve this problem, using the distributive property to create an easier problem:
   $(2.50\times 4) + (0.25\times 4) = 10 + 1 = 11$
   or
   $(3\times 4) - (0.25\times 4) = 12 - 1 = 11$

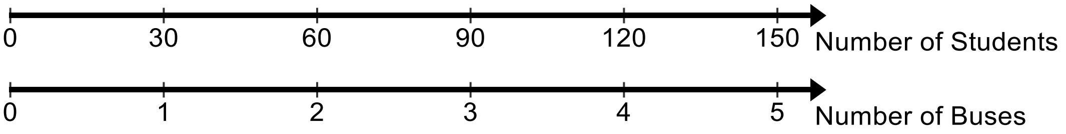
4. Find the least common multiple of the numbers given.
   a. 2, 4, and 6
      12

   b. 5, 8, 10
      40

Finding the LCM will help students compare ratios in lesson **1.2e**.

1.2a Homework: Ratios with Different Units

1. Alexis' class is taking a field trip. The double number line shows the relationship between the number of students going on the field trip and the number of buses needed.

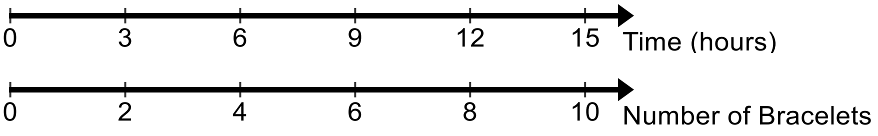


   a. How many students can ride on each bus? 30 students can ride on each bus

   b. How many students can ride on 3 buses? 90 students can ride on 3 buses

   c. How many buses will Alexis' class need if there are 120 students going on the field trip? Alexis' class will need 4 buses for 120 students.

2. Gabby is making friendship bracelets to sell at the state fair. The double number line shows the amount of time it takes her to make bracelets.



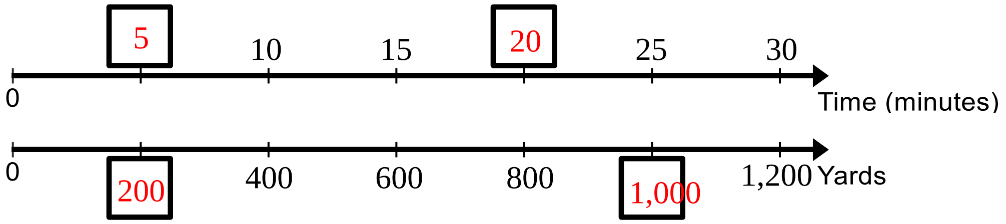
   a. How long does it take Gabby to make 8 friendship bracelets? It takes Gabby 12 hours to make 8 friendship bracelets.

   b. How many bracelets can Gabby make in 6 hours? Gabby can make 4 bracelets in 6 hours.

   c. How long does it take Gabby to make each bracelet? Justify your answer. It takes Gabby 1.5 hours to make each bracelet.

   d. Gabby plans to make 30 bracelets to sell at the state fair. How long will it take her? Justify your answer. It will take Gabby 45 hours to make 30 bracelets.

3. The double number line below shows the amount of time it takes Jonas to swim based on the number of yards he swims. Complete the double number line to show this relationship. Then, answer the questions that follow.

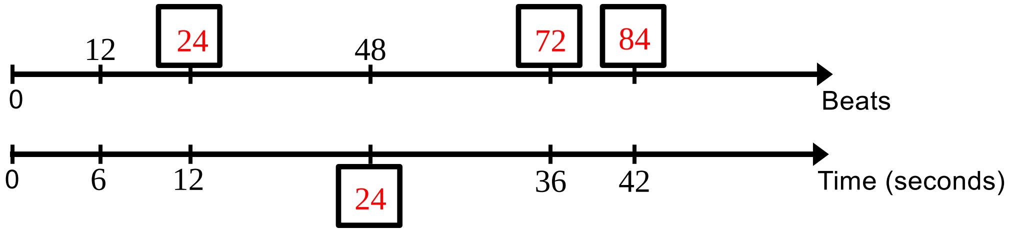


   a. How far can Jonas swim in one hour? Justify your answer. Jonas can swim 2,400 yards in one hour.

   b. How far can Jonas swim in 1 minute? Justify your answer. Jonas can swim 40 yards in one minute.

   c. How long would it take Jonas to swim 3,000 yards? Justify your answer. It would take Jonas 75 minutes or 1 hour and 15 minutes.

4. Stella is walking on the treadmill. The double number line shows the number of times Stella's heart beats over time. Complete the double number line to show this relationship. Then, answer the questions that follow.

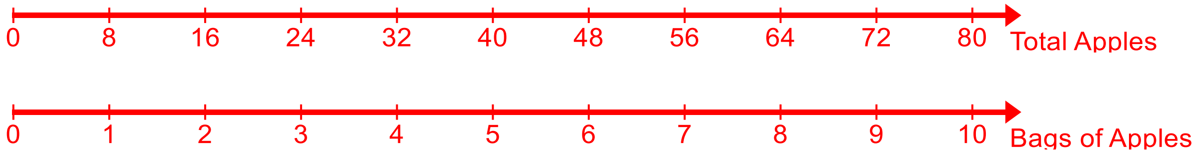


   a. How many times does Stella's heart beat in 30 seconds? Justify your answer.
   Stella's heart beats 60 times in 30 seconds.

   b. How many times does Stella's heart beat in 1 second? Justify your answer.
   Stella's heart beats 2 times per second.

   c. How many times does Stella's heart beat in 1 minute? Justify your answer. Remember that are 60 seconds in 1 minute.
   Stella's heart beats 120 times in 1 minute. In exercise classes, a common way to check heart rates is to count your heartbeat for 6 seconds and then multiply that number by 10.

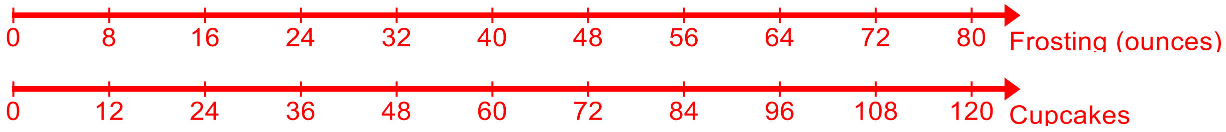
5. Holly is buying apples for an apple bobbing contest. Each bag of apples contains 8 apples.
   a. Create a double number line to represent the relationship between the number of bags of apples Holly purchases and the total number of apples she will have for 0 to 10 bags of apples. Hint: One of your number lines will show **Total Apples** and the other will show **Bags of Apples**.



   It is OK if the number lines are switched so that Bags of Apples are on top and Total Apples are on bottom.

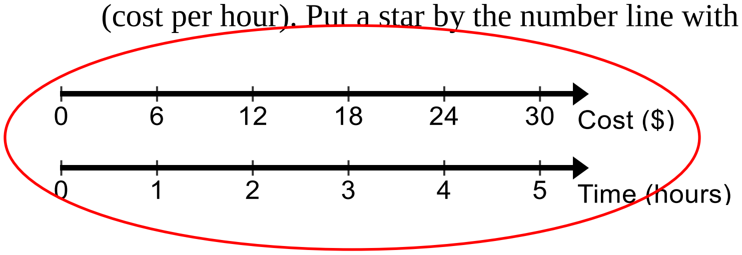
   b. If Holly needs 72 apples for the apple bobbing contest, how many bags of apples does she need to buy? Write your answer and circle the pair of values on your double number line that represents this answer. Holly needs 9 bags of apples for 72 total apples.

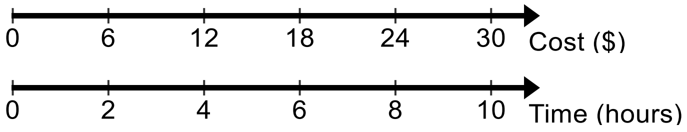
6. Rosa owns a cupcake shop. She uses 8 ounces of frosting for every 12 cupcakes.
   a. Create a double number line that shows the relationship between the number of cupcakes Rosa makes and the amount of frosting she will need. Double number lines may vary. Some students may use a simpler ratio (i.e. 4 to 6 or 2 to 3). Others may label their bottom number line as Cupcakes (dozens). Have students share out different double number lines and have discussions about whether they represent the same relationship between cupcakes and frosting and remind students to read the labels on the number lines carefully.

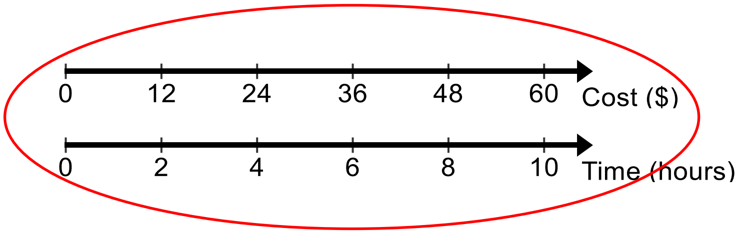


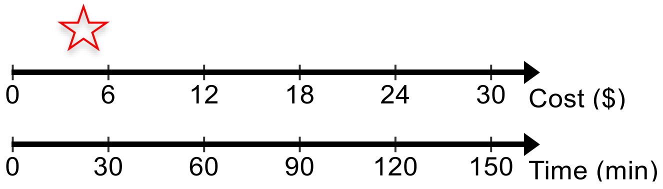
   b. If Rosa is planning to make 60 cupcakes one day, how many ounces of frosting will she need to make? Justify your answer. Rosa will need to make 40 ounces of frosting for 60 cupcakes.

7. The double number lines below show the prices at four different trampoline parks depending on how long you jump for. Circle the number lines that show the trampoline parks with the **same** hourly rate (cost per hour). Put a star by the number line with the highest hourly rate. Justify your answers.





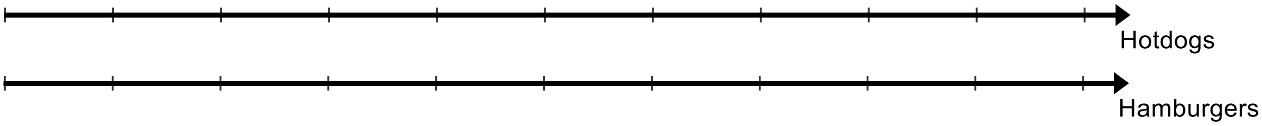




1.2b Class Activity: Graphs of Equivalent Ratios

Activity 1: At a baseball game, the snack bar sells 3 hotdogs for every 2 hamburgers they sell.

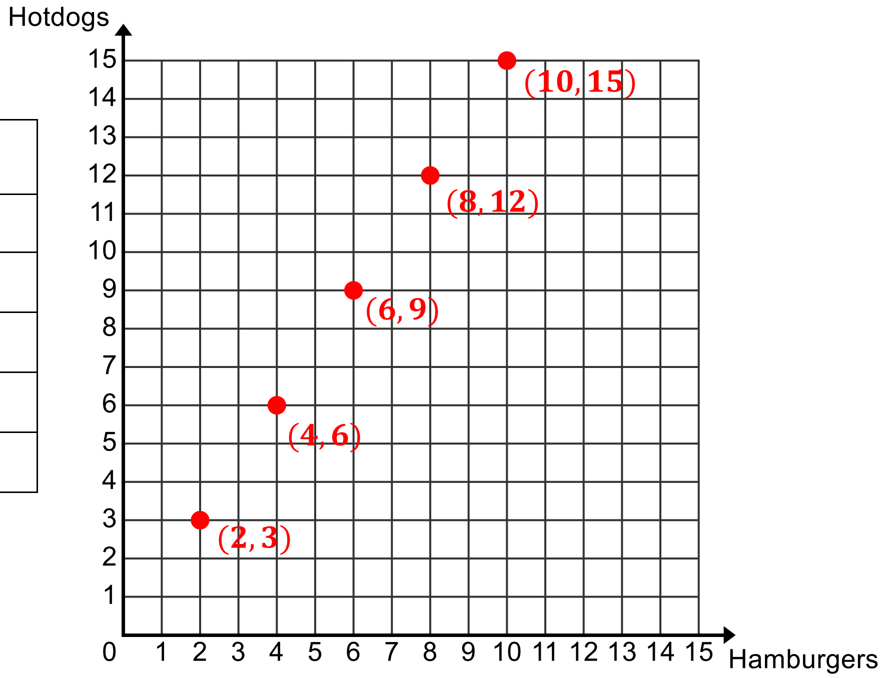
a. Create a double number line to show the relationship between the number of hotdogs and the number of hamburgers sold at the baseball game.



b. Organize the information in the table below. Students did graph points in 5th grade but they will likely need a review of the structure of a coordinate plane and an ordered pair: (x, y), in this case $(\text{hamburgers}, \text{hotdogs})$. One way to connect the graphical representation to the work they have already done with double number lines is to think about rotating the hotdog number line from above 90 degrees counterclockwise. You may want to cut these number lines out and physically show this rotation. This helps to solidify that each point on the graph corresponds to two quantities: number of hamburgers and number of hotdogs. Help students to see that the first column of the table (Hamburgers Sold) corresponds to the *x*-axis (by convention) and the second column of the table (Hotdogs Sold) corresponds to the *y*-axis. This will help students when they start constructing their own graphs in 7th grade.

Another way we can show the relationship between hotdogs and hamburgers sold is on a **graph**. Complete the graph below.

Hamburgers Sold	Hotdogs Sold	Ordered Pairs
2	3	$(2,3)$
4	6	$(4,6)$
6	9	$(6,9)$
8	12	$(8,12)$
10	15	$(10,15)$



c. What do you notice about the graph of this relationship?

Students should observe that the points fall on a line.

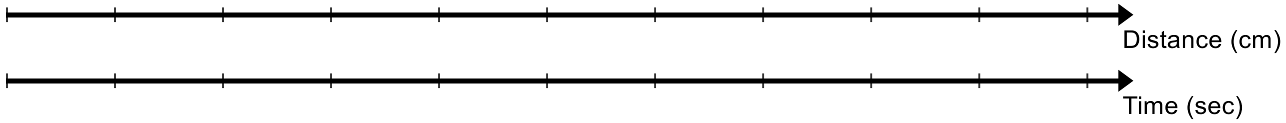
d. What does the ordered pair $(8, 12)$ represent in the situation?

The snack bar sold 8 hamburgers and 12 hotdogs.

e. Write the ordered pair that represents 30 hotdogs and 20 hamburgers being sold. $(20, 30)$

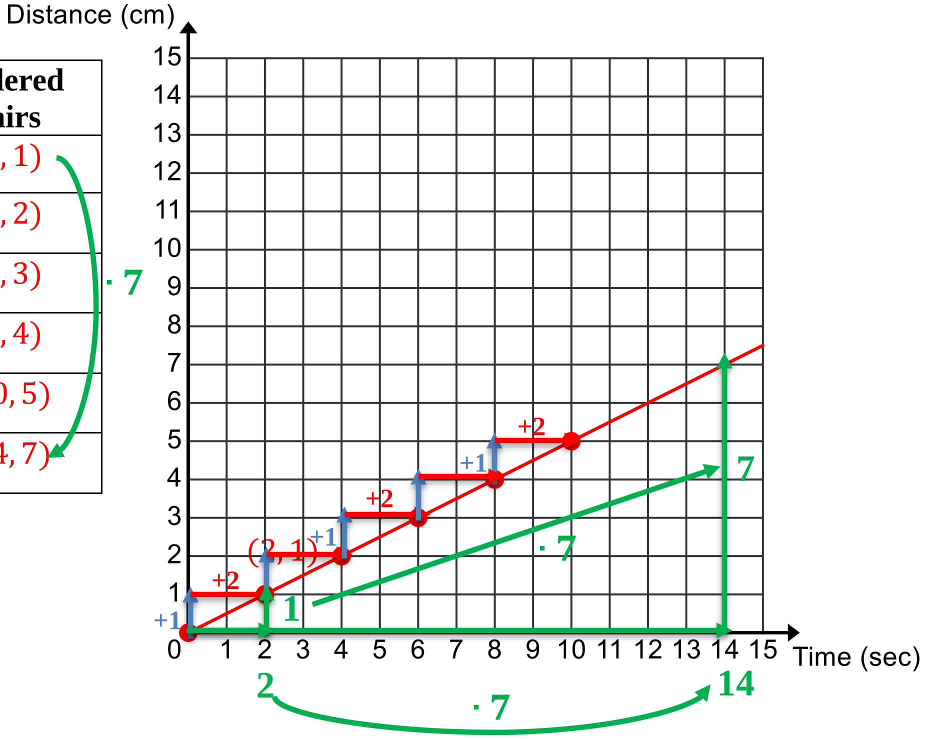
In this problem we created several different representations/models (double number line, table, graph) of the same relationship. We could have also created a tape diagram or double number line. In **1.2f**, we will see that we can also create an equation to represent this relationship.

1. A snail crawls 1 centimeter every 2 seconds. Create a **double number line**, **table**, and **graph** to show the relationship between time, in seconds, and distance, in centimeters, that the snail crawls.



Time (sec)	Distance (cm)	Ordered Pairs
2	1	(2, 1)
4	2	(4, 2)
6	3	(6, 3)
8	4	(8, 4)
10	5	(10, 5)
14	7	(14, 7)

+2 +2 +2 +1 +1 +1 · 7



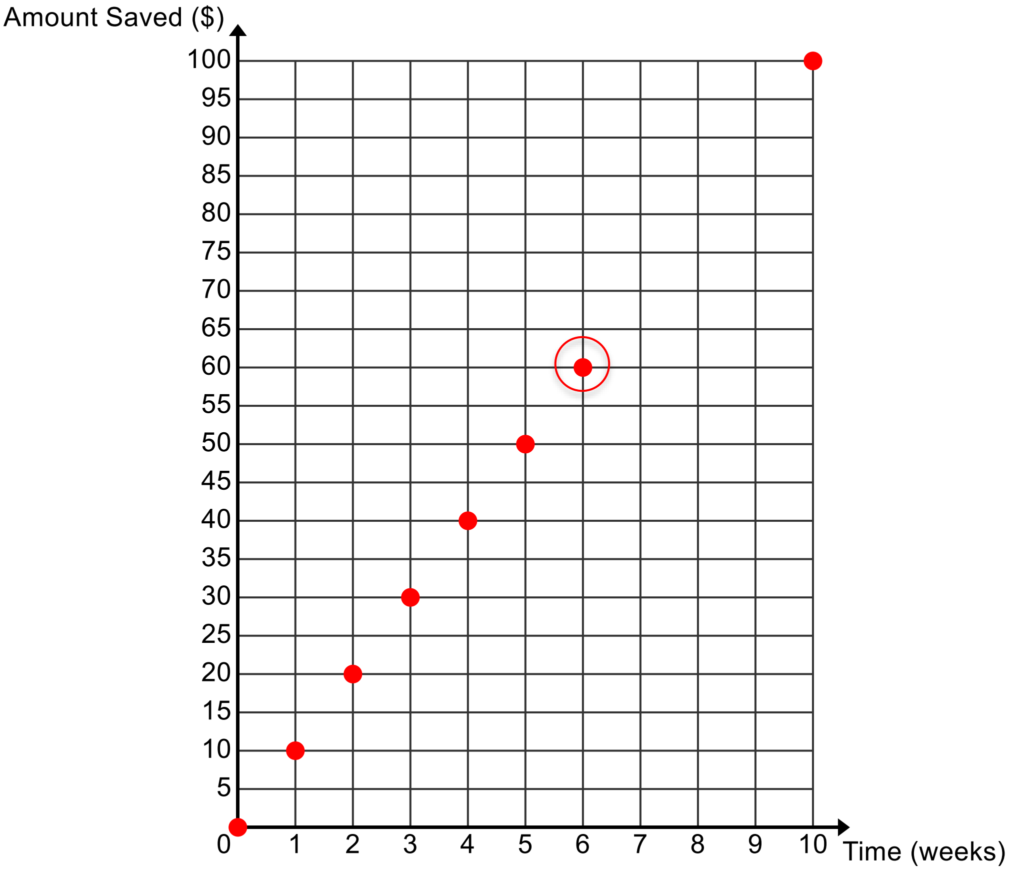
a. What does the ordered pair (6, 3) represent in the situation? The snail crawls 3 centimeters in 6 seconds or it takes 6 seconds for the snail to crawl 3 centimeters.

b. Use the graph to determine how far the snail crawls in 14 seconds. The snail crawls 7 cm in 14 seconds.

Help students observe repeated reasoning and structure in the graph and table. In the previous lesson, students learned that equivalent ratios could be generated from repeated addition. We can see this repeated addition on the graph as we move from one point to the next. We can also see the scalar multiplication if we draw slope triangles as shown in green. The larger triangle is a dilation by a factor of 7 of the smaller triangle. In fact, all slope triangles drawn from the line are dilations of each other. Students will study this more in 8th grade.

2. Jax is saving $10 per week.
   a. Complete the table and graph to show the relationship between time in weeks and the amount Jax has saved in dollars.

Time (weeks)	0	1	2	3	4	5
Amount Saved (dollars)	0	10	20	30	40	50

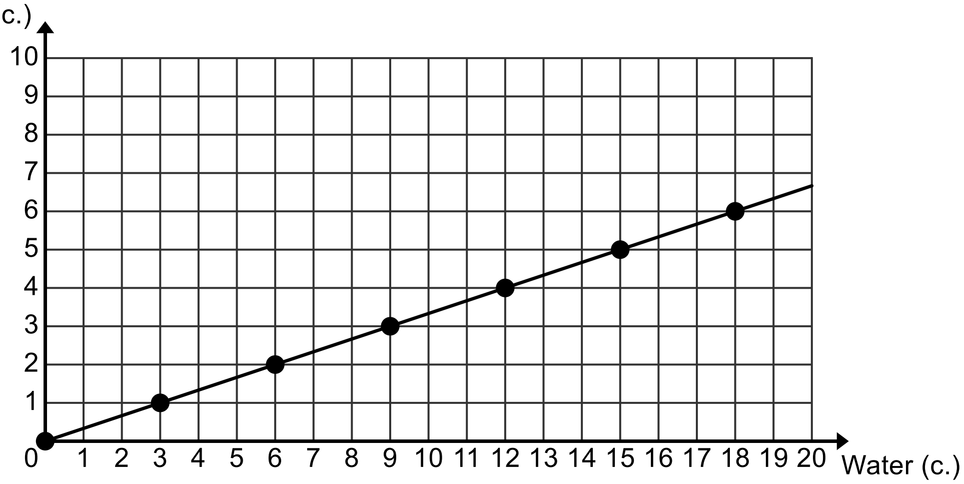


   b. Use the graph to determine the amount Jax will save in 6 weeks. Write your answer in the space below and circle the point on the graph that represents your answer.
      Jax will save $60 in 6 weeks.

   c. How long will it take Jax to save $100? Write your answer in the space below and graph the point that corresponds to your answer on the coordinate plane.
      It will take Jax 10 weeks to save $100.

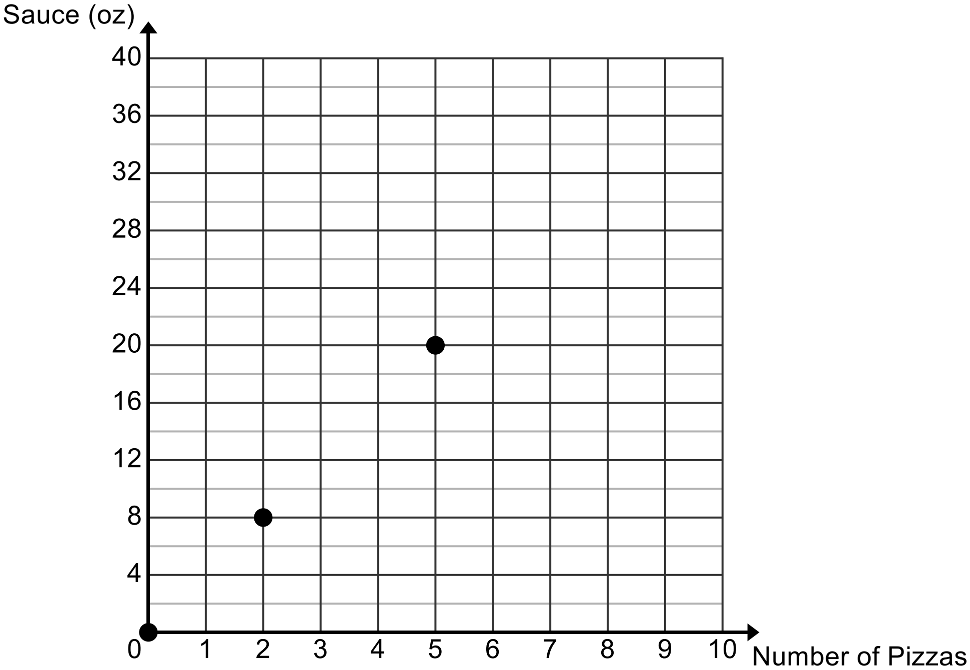
3. The graph below shows the ratio of orange concentrate to water used to make orange juice.
   a. Use the graph to complete the table and answer the questions that follow.

Water (c.)	**Concentrate (c.)**
3	1
6	2
9	3
12	4
15	5



   b. How much water is needed for 4 cups of concentrate?
   12 cups of water is needed for 4 cups of concentrate
   c. How much of each, water and concentrate, are needed to make 20 total cups of orange juice?
   15 cups of water and 5 cups of concentrate
   d. What is the ratio of concentrate to water used to make orange juice?
   1 to 3

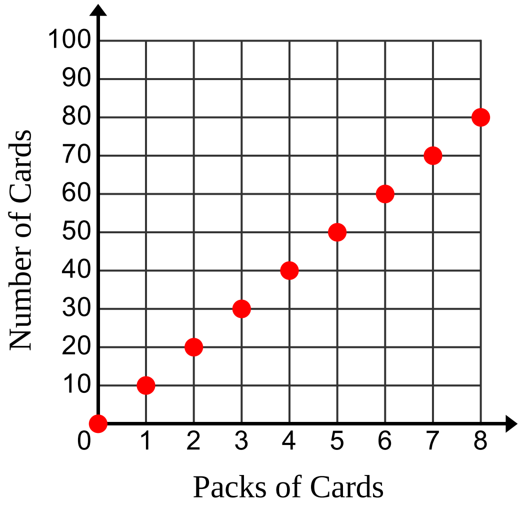
4. The graph below shows the amount of pizza sauce the Pizza Parlor uses on their medium pizzas. Use the graph below to answer the questions that follow.



   a. How much pizza sauce does the Pizza Parlor use to make 8 medium pizzas?
   The Pizza Parlor uses 32 ounces of sauce to make 8 medium pizzas.
   b. How many medium pizzas can the Pizza Parlor make with 24 ounces of sauce?
   The Pizza Parlor can make 6 medium pizzas with 24 ounces of sauce.
   c. How many ounces of pizza sauce does it take to make one medium pizza? Plot this point on the graph.
   It takes 4 ounces of sauce to make one medium pizza.

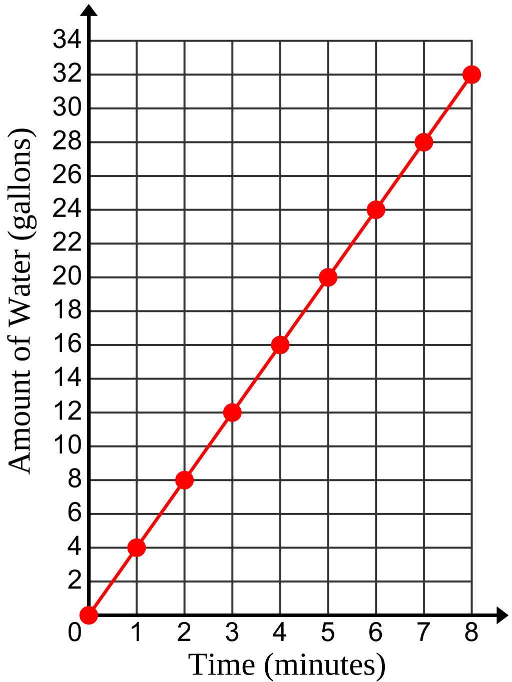
Directions: For each of the following situations, create a graph that shows the relationship between the two quantities. Some students may wish to create a table first. Once the graphs are created, ask students questions about the graphs or have them make up their own questions about the graphs and ask a neighbor to answer them. Students may wonder why some points are connected and some are not. This is not a focus of 6th grade; however feel free to discuss discrete (points not connected) and continuous (points connected) as appropriate.

5. There are 10 baseball cards in a pack of cards.



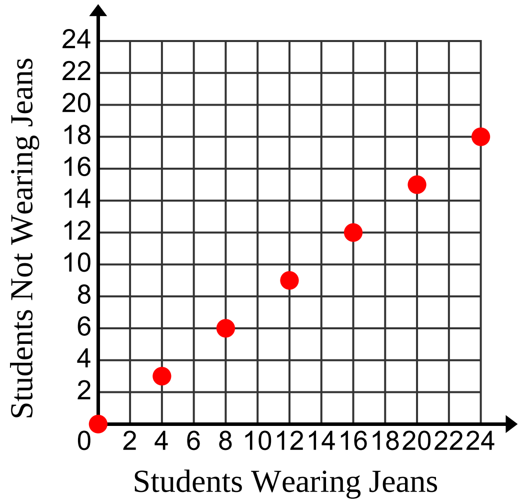
Sample Questions: 1) If you buy 6 packs of cards, how many cards will you have? 2) If you want 30 cards, how many packs should you buy?

6. The swimming pool is being filled at a rate of 4 gallons per minute.



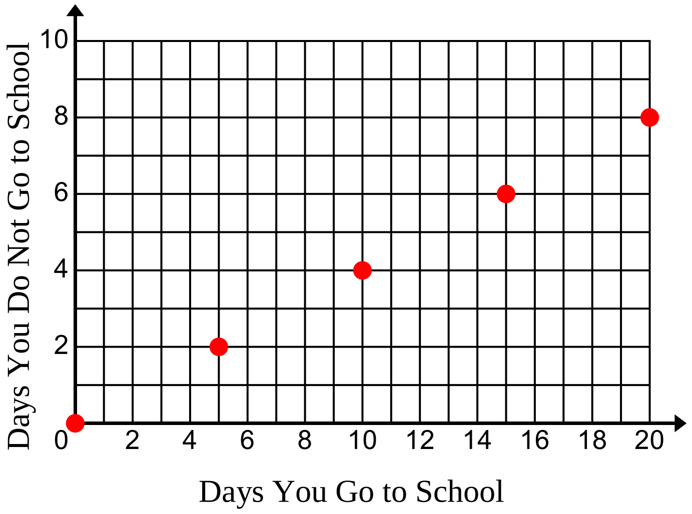
Sample Questions: 1) How much water will be in the pool after 3 minutes? 10 minutes? 4.5 minutes? 2) How long do you need to fill the pool if it holds 24 gallons of water?

7. The ratio of students wearing jeans to students not wearing jeans is 4 to 3.



Sample Questions: 1) If there are 12 students not wearing jeans, how many total students are in the class?

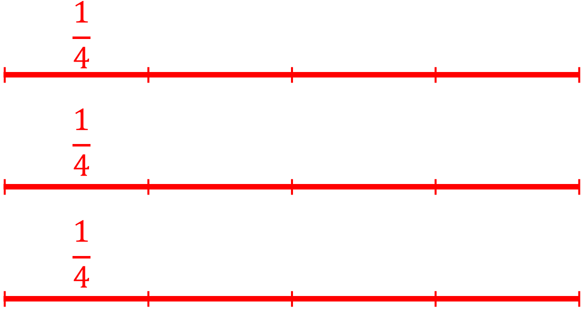
8. You go to school 5 out of 7 days a week.



Spiral Review

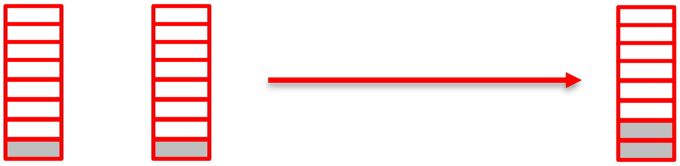
1. How would you share… It will really help students to draw pictures to solve these problems. This work will help students find fractional unit rates in lesson **1.2d**. Students did these types of problems in **5.NF.3**.

   a. 3 licorice ropes evenly between 4 friends? Each student would get $\frac{1}{4}$ of each licorice rope for a total of $\frac{3}{4}$ of a licorice rope.



   $\frac{3}{4}$ as $3\left(\frac{1`}{4}\right)$ or $3 \div 4$

   b. 2 candy bars evenly between 8 friends? If we divide each candy bar into 8 equal pieces, each friend will get $\frac{1}{8}$ of each candy bar for a total of $\frac{2}{8}$ or $\frac{1}{4}$ of a candy bar. Fraction bars may help students who are struggling.



   $\frac{1}{4}$ as $2\left(\frac{1`}{8}\right)$ or $2 \div 8$

2. Use mental math to solve the problem 3×4.99
   Students may use the distributive property to create an easier problem:
   $(3 \times 5) - (3 \times 0.01) = 15 - 0.03 = 14.97$

3. Solve the following problems:
   a. $400 \div 10$ 40

   b. $40 \div 10$ 4

   c. $4 \div 10$ 0.4

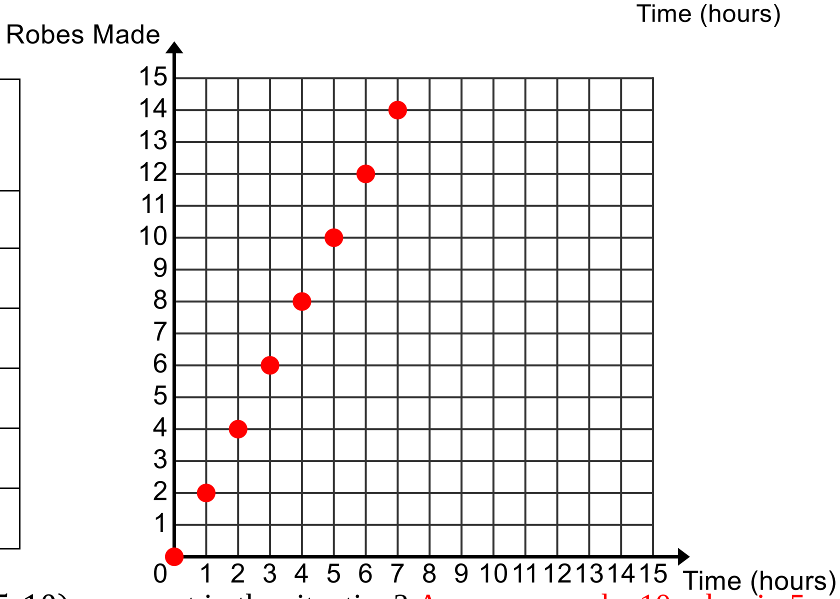
4. Shelley walks $\frac{3}{4}$ of a mile each day for 4 days. How far did she walk?

   $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{12}{4} = 3$ or $\frac{3}{4} \times 4 = 3$ These types of problems will also help students to solve unit rate problems in **1.2d**.

1.2b Homework: Graphs of Equivalent Ratios

1. Asmar is making Jedi robes for a birthday party. He can make 2 robes every hour. Create a **double number line**, **table**, and **graph** to show the relationship between time, in hours, and the number of Jedi robes Asmar makes.

Time (hours)	Number of Robes Made	Ordered Pairs
1	2	$(1, 2)$
2	4	$(2, 4)$
3	6	$(3, 6)$
4	8	$(4, 8)$
5	10	$(5, 10)$
6	12	$(7, 14)$



a. What does the ordered pair $(5, 10)$ represent in the situation? Asmar can make 10 robes in 5 hours.

b. Asmar needs to make 30 robes for the party. How long will it take him to make 30 robes? Justify your answer. It will take Asmar 15 hours to make 30 robes.

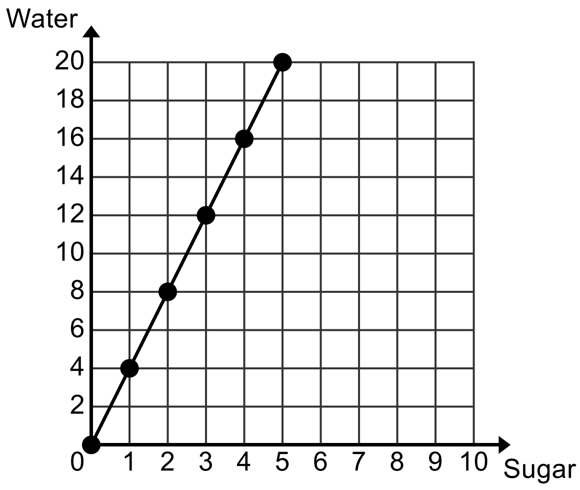
c. Write your answer from part b. as an ordered pair. $(15, 30)$

2. Louise is making a sugar solution for her hummingbird feeder. The graph below shows the relationship between the amount of sugar and amount of water used to make the solution.

a. What is the ratio of sugar to water used to make the solution for the hummingbird feeder? The ratio of sugar to water is 1 to 4.

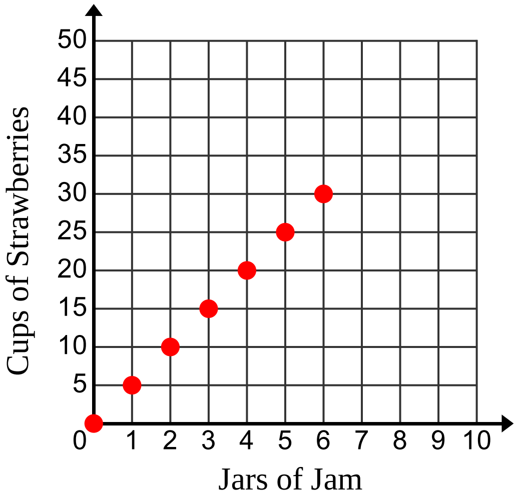
b. If Louise uses 4 cups of sugar, how much water should she use? Circle this point on the graph. She should use 16 cups of water for 4 cups of sugar.

c. If Louise uses 24 cups of water, how much sugar should she use? Justify your answer. She should use 6 cups of sugar for 24 cups of water.

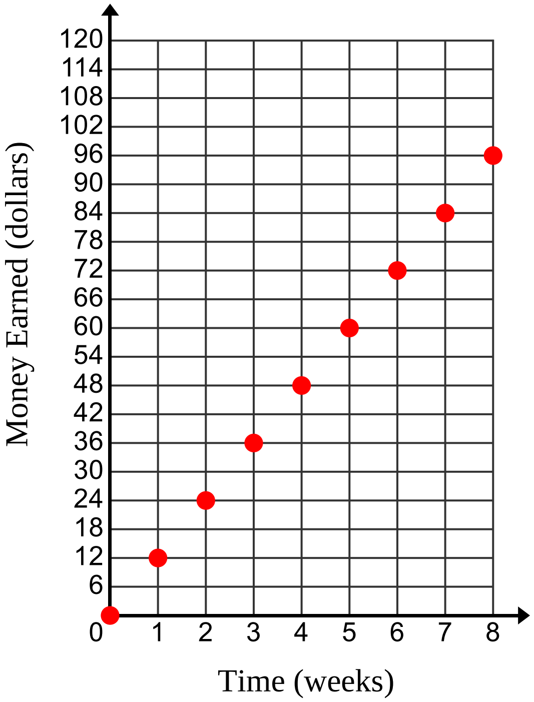


Directions: For #3 – 7 create a graph of the relationship given. Then, make up 1 question that can be answered using each graph in #3 - 7. Write your questions below the graph. For #8 and 9, use the graph to answer the questions. When reviewing homework, have students share their questions with the class for classmates to answer.

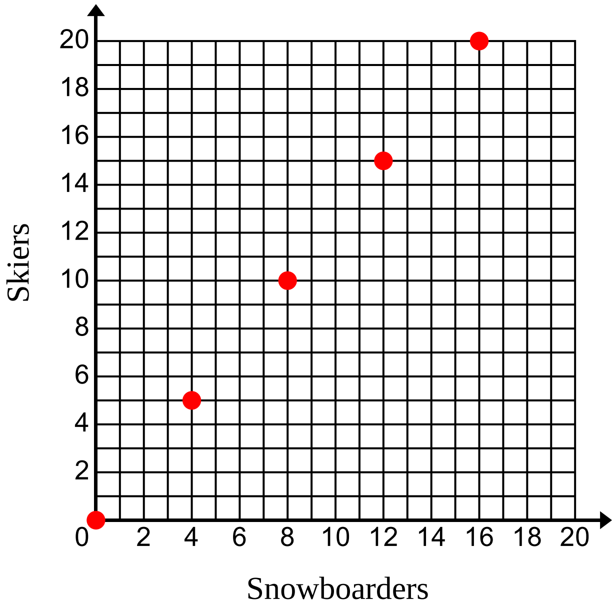
3. Marina uses 5 cups of strawberries for each jar of her strawberry jam.



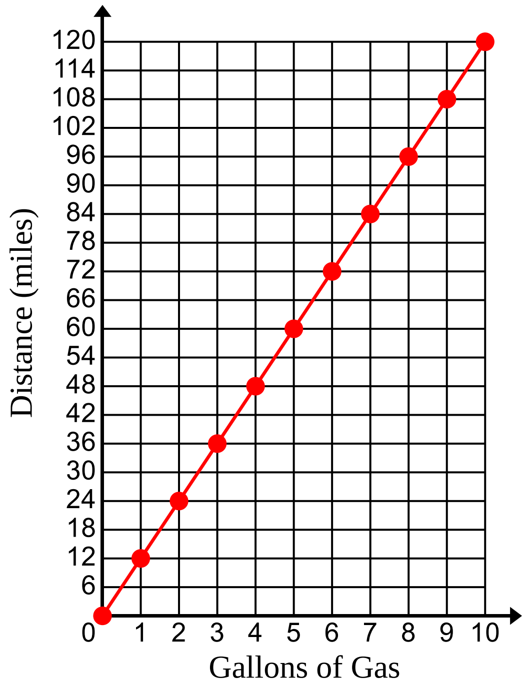
4. Darrius earns $12 per week for his allowance.



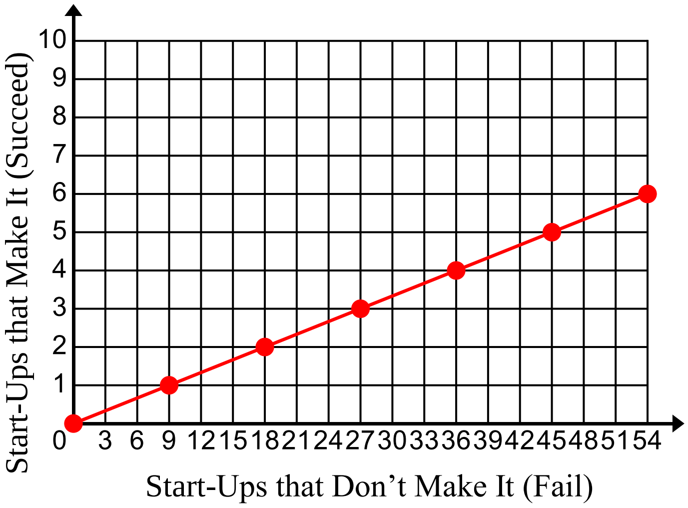
5. The ratio of snowboarders to skiers at a certain ski resort is 4:5.



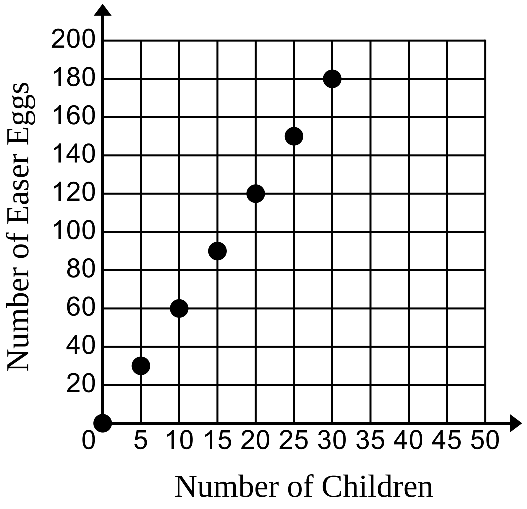
6. Dmity's truck can travel 12 miles on each gallon of gas.



7. Only 1 out of 10 start-up companies makes it (succeeds).



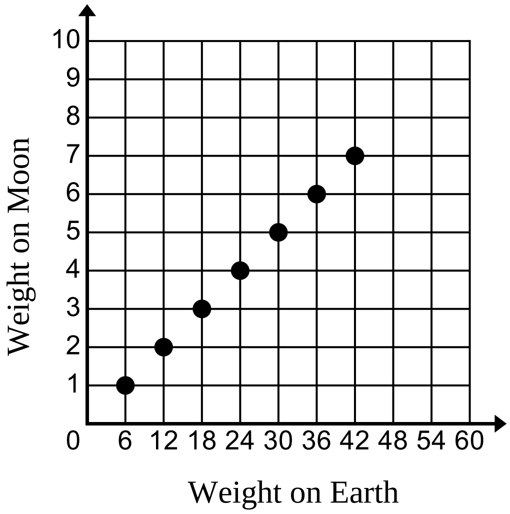
8. Chris is filling eggs for an egg hunt. The graph shows the number of eggs she needs to fill based on the number of children attending the egg hunt. Use the graph to answer the questions below.



a. How many eggs does Chris need to fill if 80 children are attending the hunt?
480 eggs

b. How many eggs does each child get?
6 eggs

9. The graph below shows the weight of an object on the Earth vs. the weight of the object on the Moon. Use the graph to answer the questions below.



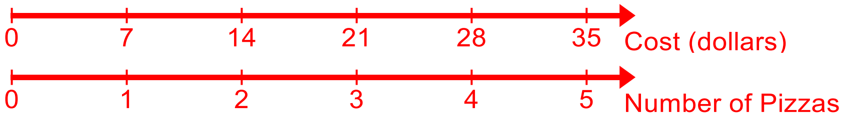
a. How much would an object that weighs 54 pounds on the Earth weigh on the moon?
It would weigh 9 pounds on the moon.

b. How much would a person who weighs 30 pounds on the moon weigh on Earth?
180 pounds

1.2c Class Activity: An Intro to Unit Rates

Activity 1: Samantha is buying pizzas for a school party. She pays $35 for 5 pizzas. She realizes she needs more and goes back to buy 2 more pizzas. How much more will she pay for the 2 additional pizzas? How much did she spend in all for pizza? Justify your answer.

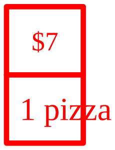
Students may solve this problem using a variety of strategies. Some may start by finding the cost of 1 pizza, or unit rate. In this lesson, students will likely begin to appreciate that finding the unit rate can be useful for solving problems. We will formally define unit rate in the next lesson.
A double number line may be helpful. Students start by drawing a double number line to represent the situation.



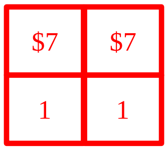
Students may think, "What do I have to do to determine the cost of 1 pizza?" Divide the bottom number line into 5 equal parts with each segment having a length of 1 unit. If we also divide the top number line into 5 equal parts, we see that each segment has a value of 7 units. The cost of 1 pizza is $7 so the cost of 2 pizzas is $14.

Some students may find it easier to draw boxes to represent the number lines. Each box represents one of the quantities and the boxes together correspond to an event:

They can then divide the boxes into 5 equal parts as we did above:



And then iterating this unit twice, we can find the cost of two pizzas:

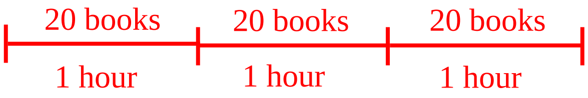


Students may not solve all of these problems by finding a unit rate. Have them share out different strategies they use. In this lesson and the one that follows, we show a few variations on models. Use the models that are easiest for your students. Even if students can solve these problems numerically, encourage them to also draw the models as the models will be very useful in **1.2d**.

1. The librarian can shelf 60 books in 3 hours.

   a. How long will it take him to shelf 100 books? Justify your answer. A slight variation of a double number line model is shown below. This model highlights that we are examining one event.

From here, we can partition the line into three equal parts:



From the model, we see that he can shelf 20 books in 1 hour so using ideas about multiplication, it would take him 5 hours to shelf 100 books.

   b. How many books can he shelf in 4 hours? Justify your answer.
   At 20 books per hour, he can shelf 80 books in 4 hours.

2. Marcus makes $72 in 8 hours.

   a. How much does he make in 3 hours? Justify your answer.
   Marcus makes $9 per hour so he makes $27 in 3 hours.

   b. How long will it take him to make $45? Justify your answer.
   It will take him 5 hours to make $45.

3. Ruby burns 800 calories in 2 hours of running.

   a. If Ruby runs for 3 hours, how many calories will she burn? Justify your answer.
   At 400 calories/hour, Ruby will burn 1,200 calories if she runs for 3 hours.

   b. If Ruby burned 200 calories, how long did she run for? Justify your answer.
   If Ruby burned 200 calories, she ran for ½ hour.

4. On a road trip, Ben's car can go 75 miles on 3 gallons of gas.
    a. If Ben's car holds 18 gallons of gas, how many miles can Ben drive with a full tank of gas? Justify your answer.
    Ben can travel 450 miles on a full tank. Students may find the unit rate of miles/gallon and multiply by 18 or they may multiply the quantities given in the original problem by 6. Help them to use mental math strategies to solve the multiplication problems (i.e. $75\times6 = (50\times6) + (25\times6) = 300 + 150 = 450$ – this is a problem students can easily relate back to money and quarters) OR $(25\times18) = (25\times16) + (25\times2) = 400 + 50 = 450$, again another problem that can be related back to money and quarters.

    b. If Ben is traveling 300 miles, how many gallons of gas will he need? Justify your answer.
    Ben will need 12 gallons of gas to travel 300 miles.

Spiral Review

1. At Jake's birthday party, there will be a treasure hunt. The picture below shows the number of children at the party and the number of maps needed.

    a. What is the ratio of children to maps? The ratio of children to maps is 16 to 4 or 4 to 1.

2. How would you share…
    a. 9 cups of lemonade evenly between 4 friends?
    $\frac{9}{4}$ cups of lemonade. Students may understand that $\frac{9}{4}$ represents $9 \div 4$. If they struggle, have them draw a picture of 9 cups of lemonade and 4 friends. They can start dividing it up. They might start by giving each friend 2 cups of lemonade. Then, there is one cup left to divide up so each friend would get another $\frac{1}{4}$ of a cup for a total of $2\frac{1}{4}$ cups of lemonade.

    b. 12 brownies evenly between 5 friends?
    $\frac{12}{5}$ or $2\frac{2}{5}$ brownies

This problem will help students find unit rates in **1.2d**. A unit rate can be thought of as a fair share.

3. What is $\frac{3}{2}+\frac{3}{2}+\frac{3}{2}+\frac{3}{2}$? 6

4. What is $3\times\frac{2}{5}$? $\frac{6}{5}$ or $1\frac{1}{5}$

Problems 3 and 4 will help prepare students for **1.2d** when they are iterating the unit rate.

1.2c Homework: An Intro to Unit Rates

1. Brooke receives 160 emails in 4 hours.
   a. How many emails will Brooke receive in 6 hours? Justify your answer.
   Brooke receives 240 emails in 6 hours.

   b. How long will it take for Brooke to receive 120 emails? Justify your answer.
   It will take Brooke 3 hours to receive 120 emails.

2. Olivia saves $60 in 4 weeks.
   a. At this rate, how much will Olivia have saved in 6 weeks? Justify your answer.
   Olivia will save $90 in 6 weeks.

   b. How long will it take Olivia to save $150? Justify your answer.
   It will take Olivia 10 weeks to save $150.

3. Elise's heart beats 18 times in 6 seconds after she takes a spin class.
   a. How many times will it beat in 10 seconds? Justify your answer.
   It will beat 30 times in 10 seconds.

   b. How many times will it beat in 1 minute (60 seconds)? Justify your answer. Her heart will beat 180 times in one minute.

4. Alex can do 120 pushups in 3 minutes.
   a. How many push-ups should Alex be able to do in 5 minutes?
   He can do 200 push-ups in 5 minutes.

   b. How long will it take Alex to do 100 push-ups?
   It will take $2\frac{1}{2}$ minutes for Alex to do 100 push-ups.

5. Jazmin works at a pizza place. She makes 80 pizzas in 4 hours.
   a. How many pizzas can Jazmin make in 6 hours?
   Jazmin makes 20 pizzas per hour; therefore Jazmin can make 120 pizzas in 6 hours.

   b. Jazmin just got an order for 5 pizzas. How long will it take her to make the 5 pizzas?
   It will take Jazmin $\frac{1}{4}$ hour to make 5 pizzas.

1.2d Class Activity: Ratios and Their Associated Rates

Activity 1: Chris and Danika are shopping for jelly beans to fill eggs.

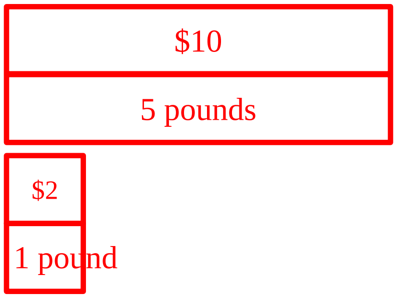
Jelly Beans on Sale!
5 pounds for $10

a. Danika needs to buy 8 pounds of jelly beans. How much will 8 pounds of jelly beans cost? Justify your answer.
8 pounds of jelly beans will cost $16. Watch to see the strategies students use. Some may reason that it costs $2/pound so 8 pounds would cost $16. Some may use multiplication and division to find other equivalent ratios (i.e. multiply by 8: 40 pounds is $80 then divide by 5: 8 pounds is $16). Others may notice that the cost is always twice the number of pounds.

b. Chris has $8. How many pounds of jelly beans can he buy for $8? Justify your answer.
Chris can buy 4 pounds of jelly beans for $8. Students may use strategies similar to those used above. Some may find how many pounds of jelly beans Chris can get for $1: ½ pound of jellybeans/dollar and then multiply by 8.

Often times, when solving word problems involving ratios, it is helpful to find the **unit rate**. The **unit rate** is the amount of one quantity that corresponds to 1 unit of the other quantity. In the example above, you can find the **unit rate** for cost per pound ($\frac{\text{dollars}}{\text{pound}}$). This tells us the cost for 1 pound. You can also find the **unit rate** for pounds per dollar ($\frac{\text{pounds}}{\text{dollar}}$). This tells us the number of pounds we can buy for $1.

c. What is the unit rate $\frac{\text{dollars}}{\text{pound}}$? In other words, how much will you pay for 1 pound of jelly beans?
The unit rate of dollars to pounds is 2. Teacher Note: The unit rate is the *value* of the ratio $\frac{A}{B}$ or $\frac{B}{A}$ depending on what is asked for in the question. The label $\frac{\text{dollars}}{\text{pound}}$ is the rate unit.
Students may draw a model to help them solve this:



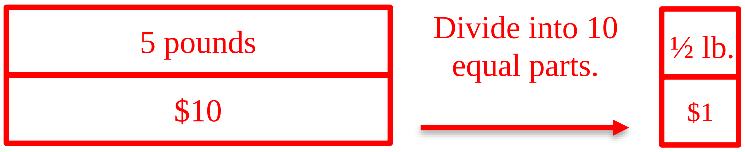
Divide the bar model into five equal parts, giving $2 for 1 pound.

Point out to students that the unit rate is just a special equivalent ratio, an equivalent ratio where one of the quantities has a value of 1.

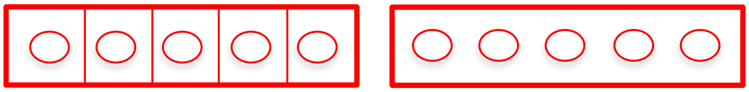
Or they may solve numerically:

$$\frac{\$10}{5\text{ pounds}} \div \frac{5}{5} = \frac{\$2}{1\text{ pound}}$$

d. What is the unit rate, $\frac{\text{pounds}}{\text{dollar}}$? In other words, how many pounds of jelly beans can you get for 1 dollar? The unit rate of pounds to dollars is 0.5. This means we can get ½ a pound of jellybeans for $1. It may help students to draw a picture of the bags of jelly beans or a model:



1. It costs \$2.00 for 10 erasers.
    a. How much is each eraser? Justify your answer.
    Each eraser costs \$0.20. The circles below represent the erasers and the rectangles represent the dollars. If we “distribute” the erasers evenly to the dollars, we see each eraser corresponds to 1/5 of a dollar or \$0.20.



    b. How many erasers can you buy for \$1? Justify your answer.
    You can buy 5 erasers for \$1.

    c. Calvin bought 4 erasers. How much did he spend? Justify your answer.
    \$0.80

    d. How many erasers can Hobbes buy with \$5? Justify your answer.
    25

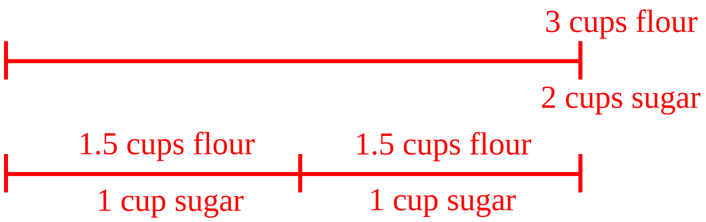
2. Eli can answer 30 math facts in 60 seconds.
    a. How long does it take Eli to answer 1 math fact? Justify your answer.
    It takes Eli 2 seconds to answer 1 math fact. Talk with students about which unit rate we found in this problem. Did we find…
    $\frac{\text{seconds}}{\text{fact}}$ or $\frac{\text{facts}}{\text{second}}$

    b. How many math facts can Eli solve in 1 second? Justify your answer.
    Eli can solve $\frac{1}{2}$ math fact in one second. While this is technically the correct mathematical answer, it does not really make sense in the context. Have a discussion with the students about this.

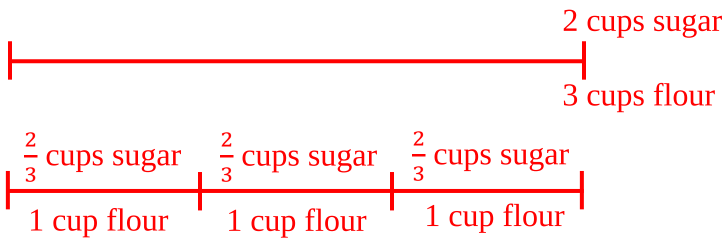
    c. How many math facts should Eli be able to complete in 8 seconds? Justify your answer.
    Eli should be able to complete 4 math facts in 8 seconds.
    $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$ or $8\times\frac{1}{2}$

    d. How long will it take Eli to complete 20 math facts? Justify your answer.
    It will take Eli 40 seconds to complete 20 math facts.

3. A cookie recipe calls for 3 cups of flour for every 2 cups of sugar.
   a. How many cups of flour should you use for 1 cup of sugar? Justify your answer.
   You should use 1.5 or $\frac{3}{2}$ cups of flour for each cup of sugar. A diagram may help:

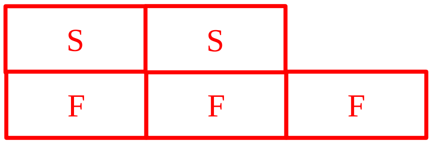


   Point out that we just found the unit rate of $\frac{\text{cups of flour}}{\text{cup of sugar}}$.
   b. How many cups of flour should you use for 1 cup of flour? Justify your answer.
   You should use $\frac{2}{3}$ cup of sugar for each cup of flour.



   Point out that we just found the unit rate of $\frac{\text{cups of sugar}}{\text{cups of flour}}$.

It may also be helpful to use linking cubes or to draw a tape diagram to show the multiplicative comparison between the two quantities:



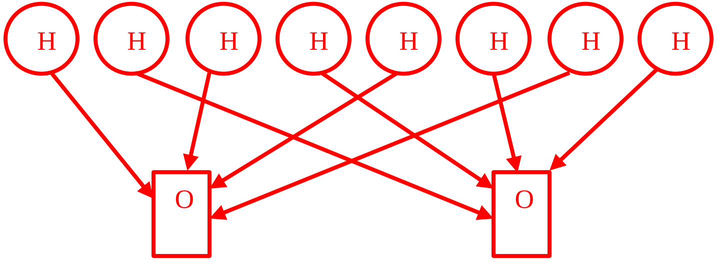
Sugar is $\frac{2}{3}$ as big as flour and flour is $1\frac{1}{2}$ times bigger than sugar.

At some point in this lesson, students may realize that the unit rates associated with a given ratio are reciprocals of each other (i.e. $\frac{\text{cups of flour}}{\text{cups of sugar}} = \frac{3}{2}$ and $\frac{\text{cups of sugar}}{\text{cups of flour}} = \frac{2}{3}$).

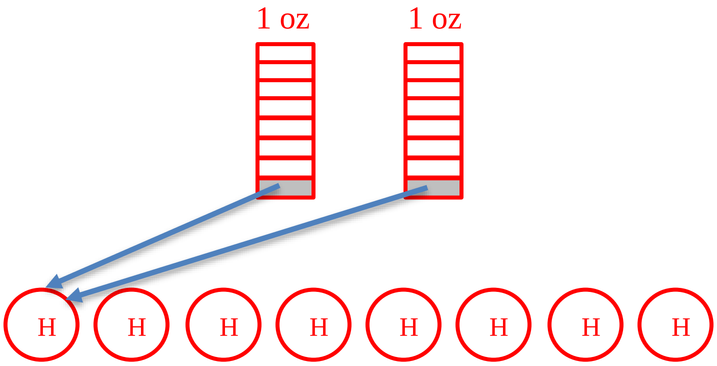
   c. If Jeremy uses 3 cups of sugar, how much flour will he need to use? Justify your answer.
   Jeremy will need to use 4.5 cups of flour. Students may solve this problem by iterating the unit ratio from part a. three times.
   d. If Sally uses 6 cups of flour, how much sugar will she need to use? Justify your answer.
   Many students may realize that this ratio is just double the ratio given in the original problem so she will need 4 cups of sugar. Others may iterate the unit ratio from part b. and get $\frac{12}{3}$ or 4 cups of sugar.
   e. If Bev uses 5 cups of flour, how much sugar will she need to use? Justify your answer.
   Some may notice that this is one less cup of flour than part d. so she will need 2/3 c. less of sugar or $3\frac{1}{3}$ cups of sugar. Others may iterate the unit ratio from part b. to get $\frac{10}{3}$ cups of sugar.

4. 2 ounces of water leaks out of a faucet every 8 hours

   a. What is the unit rate $\frac{\text{hours}}{\text{ounce}}$? It may help students to re-state this question as, "How many hours does it take for 1 ounce to leak out?"
   The unit rate of hours to ounce is 4. How can we divide 8 hours equally between 2 ounces?
   You can see in the picture below that 8 hours shared equally between 2 ounces is 4 hours/ounce.



   b. What is the unit rate, $\frac{\text{ounces}}{\text{hour}}$? In other words, how many ounces leak out in 1 hour?
   The unit rate of ounces to hours is $\frac{1}{4}$.



   From the picture above, we can see that each hour would get $\frac{1}{8}$ of an ounce from each ounce for a total of $\frac{2}{8}$ ounces or $\frac{1}{4}$ ounce.

   c. How many ounces of water will leak out in 5 hours? Show how you arrived at your answer.
   The unit rate and picture from part b. will likely help students answer this question. Students may use multiplication or repeated addition to solve this problem:
   If $\frac{1}{4}$ of an ounce leaks out every hour, then $5 \times \frac{1}{4}$ or $\frac{5}{4}$ ounces leaks out in 5 hours.

   d. How long will it take for 1 cup of water to leak out? Hint: There are 8 ounces in 1 cup of water. Show how you arrived at your answer.
   The unit rate and picture from part a. will likely help students answer this question. If it takes 4 hours for every ounce to leak out, it will take 32 hours for 8 ounces to leak out. Remind students that these are equivalent ratios: The ratio 4 to 1 is equivalent to 32 to 8:

$$\frac{4\text{ hours}}{1\text{ ounce}} \times \frac{8}{8} = \frac{32\text{ hours}}{8\text{ ounces}}$$

If appropriate you may teach students unit analysis:

$$\frac{4\text{ hours}}{\cancel{\text{ounce}}} \times 8\ \cancel{\text{ounces}} = 32\text{ hours}$$

5. A window washer can clean 12 window panes in 15 minutes.

   a. What is the unit rate, panes/minute? Show how you arrived at your answer.
   It may help students to first find a simpler ratio, such as 4 panes in 5 minutes.

4 panes
5 minutes

$\frac{4}{5}$ p	$\frac{4}{5}$ p	$\frac{4}{5}$ p	$\frac{4}{5}$ p	$\frac{4}{5}$ p
1 min.	1 min.	1 min.	1 min.	1 min.

   The unit rate of panes to minutes is $\frac{4}{5}$. This means that the window cleaner can clean $\frac{4}{5}$ of a window pane per minute.

   b. What is the unit rate, minutes/pane? Show how you arrived at your answer.

5 minutes
4 panes

$\frac{5}{4}$ min.	$\frac{5}{4}$ min.	$\frac{5}{4}$ min.	$\frac{5}{4}$ min.
1 pane	1 pane	1 pane	1 pane

   The unit rate of minutes to panes is $\frac{5}{4}$. This means it takes the window washer 1.25 minutes to wash each window.

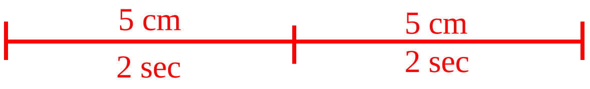
   c. How long will it take the window washer to clean 5 window panes? Show how you arrived at your answer.
   We already know it takes 5 minutes to wash 4 panes so another 5/4 of a minute for the next pane would give 6¼ minutes for 5 window panes.

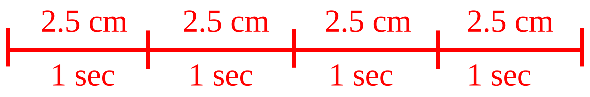
   d. How many windows can the window cleaner clean in 20 minutes? Show how you arrived at your answer.
   Students may multiply the unit rate of 4/5 panes/minute by 20 giving 16 panes. They may also multiply both quantities in the ratio 4 panes in 5 minutes by 4 to get 16 panes in 20 minutes.

6. An ant crawls 10 cm in 4 seconds.

   a. How far can the ant crawl in 5 seconds? Justify your answer.
   Students may find it helpful to determine how far the ant crawls in 1 second. In other words, what is the unit rate cm/sec?

Partition the diagram:

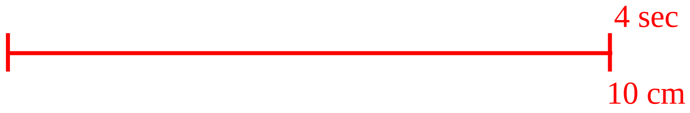


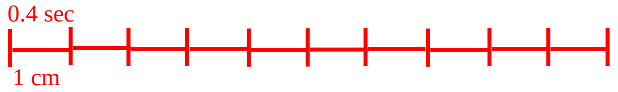


The unit rate of centimeters to seconds is $\frac{5}{2}$ or 2.5.

If we iterate this 5 times, we see that the ant can crawl 12.5 cm in 5 seconds.

   b. How long will it take the ant to reach a breadcrumb that is 8 centimeters away? Justify your answer.
   How long does it take the ant to crawl 1 cm?





The unit rate of seconds to centimeters is $\frac{2}{5}$ or 0.4.

So, it will take the ant $0.4 \times 8 = 3.2$ seconds to reach a breadcrumb that is 8 cm away.

7. Julius is making orange juice. He has determined that he needs 15 oranges to make 5 cups of orange juice. Complete the table below to show the relationship between oranges and cups of orange juice. Explain the process you used for completing the table.

Number of Oranges	**Cups of Orange Juice**
1	$\frac{1}{3}$
3	1
5	$\frac{5}{3}$
9	3
10	$\frac{10}{3}$
30	10
6	2

In the table, when the number of oranges is equal to 1, students are finding the unit rate $\frac{\text{cups of orange juice}}{\text{orange}}$. Similarly, when cups of orange juice is equal to 1, students are finding the unit rate $\frac{\text{oranges}}{\text{cup of juice}}$. These special ratios may prove useful for completing the rest of the table. Students may start by creating a double number line (or modified one as shown above) of this relationship and then partitioning and iterating the double number line to complete the table. As they do, help them make connections between the double number line and the table.

8. A pumpkin that weighs 4 pounds costs \$5. Complete the table below to show the relationship between the weight of the pumpkin and the cost. Explain the process you used for completing the table.

Weight (lbs.)	**Cost (dollars)**
$\frac{4}{5}$	1
1	\$1.25
5	\$6.25
8	10
12	15

Have students state in their own words what the first two entries in the table mean: Entry 1: “For \$1, I can buy a pumpkin that weighs $\frac{4}{5}$ of a pound.” Entry 2: “It will cost \$1.25 to buy a pumpkin that weighs 1 pound.”

Spiral Review

1. Solve the following problems.
   a. If $\frac{1}{2}$ a pound of walnuts costs \$3.25, how much is one pound of walnuts?
   \$6.50

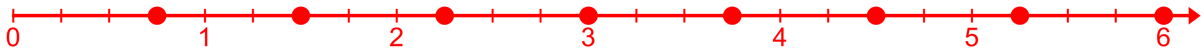
   b. If $\frac{1}{3}$ of a pound of taffy costs \$2.10, how much is one pound of taffy?
   \$6.30

2. What is 3.25 ÷ 5?
Encourage students to use mental math and relate to money: 1 dollar divided by 5 is 0.20 so 3 dollars divided by 5 is 0.60. A quarter or 0.25 divided by 5 is 0.05, so 3.25 divided by 5 is 0.65.

3. Solve the following problems.
   a. 1.2×3
   3.6

   b. 3×2.4
   7.2

4. If 6 yards of string are cut into pieces that are each ¾ yard long for a science experiment, how many strings can be made?
8 strings can be made. A number line is one model that can be used to solve this problem:



1.2d Homework: Ratios and Their Associated Rates

1. It costs $40 for 8 pounds of meat.
   a. How much does each pound of meat cost? Justify your answer.
   Each pound of meat costs $5.

   b. How many pounds of meat can you buy with $1? Justify your answer.
   You can buy 1/5 pound of meat with $1.

   c. How much will it cost for 5 pounds of meat? Justify your answer.
   $25

   d. How much meat can you buy with $4? Justify your answer.
   $\frac{4}{5}$ of a pound

2. Kara can make 15 bows with 5 yards of ribbon.
   a. How many bows can she make with each yard of ribbon? Justify your answer.
   Kara can make 3 bows with each yard of ribbon.

   b. How much ribbon is needed for each bow? Justify your answer.
   $\frac{1}{3}$ yard of ribbon is needed for each bow

   c. How much ribbon will Kara need to make 24 bows? Justify your answer.
   Kara will need 8 yards of ribbon to make 24 bows.

   d. If Kara has 3 yards of ribbon, how many bows can she make? Justify your answer.
   Kara can make 9 bows with 3 yards of ribbon.

3. Claudia can run 15 miles in 2 hours.
   a. How many miles can she run in 1 hour?
   Claudia can run 7.5 or $\frac{15}{2}$ miles per hour.

   b. How long does it take her to run 1 mile?
   It takes her $\frac{2}{15}$ of an hour or 8 minutes to run a mile.

4. For every 8 people at the party, Alecia is ordering 2 pizzas.
    a. What is the unit rate $\frac{\text{people}}{\text{pizza}}$? Restate this question in your own words and then solve.
    How many people can 1 pizza feed? The unit rate, people/pizza, is 4.

    b. What is the unit rate $\frac{\text{pizzas}}{\text{person}}$? Restate this question in your own words and then solve.
    How much pizza does one person get? The unit rate, pizzas/person is ¼.

    c. How many pizzas will Alecia need to get if there are going to be 28 people at the party?
    Alecia will need to get 7 pizzas.

    d. How many people can Alecia feed with 5 pizzas?
    Alecia can feed 20 people with 5 pizzas.

5. Rosa uses 8 ounces of frosting for 12 cupcakes.

    a. What is the unit rate $\frac{\text{ounces}}{\text{cupcake}}$? Restate this question in your own words and then solve.
    How much frosting does each cupcake need? The unit rate of ounces to cupcakes is $\frac{8}{12}$ or $\frac{2}{3}$.

    b. What is the unit rate $\frac{\text{cupcakes}}{\text{ounce}}$? Restate this question in your own words and then solve.
    How many cupcakes can you frost with 1 ounce of frosting? The unit rate of cupcakes to ounces is $\frac{12}{8}$ or $\frac{3}{2}$.

    c. How many ounces of frosting does Rosa need for 15 cupcakes?
    Rosa needs 10 ounces of frosting for 15 cupcakes.

    d. How many cupcakes can Rosa frost with 12 ounces of frosting?
    Rosa can frost 18 cupcakes with 12 ounces of frosting.

6. For every 12 eggs used to make a sauce, you need 2 sticks of butter. Complete the table to show the relationship between eggs and butter used to make this sauce. Explain the process you used for completing the table.

Eggs	**Sticks of Butter**
1	$\frac{1}{6}$
6	1
8	$1\frac{1}{3}$
30	5
31	$5\frac{1}{6}$
48	8

7. The ratio of blue paint to yellow paint used to make a certain shade of green paint is 2 to 5. Complete the table. Explain the process you used for completing the table.

Blue Paint (cups)	**Yellow Paint (cups)**
1	$\frac{5}{2}$
$\frac{2}{5}$	1
6	15
10	25
11	$27\frac{1}{2}$

1.2e Class Activity: Given the Unit Rate

In the previous lesson, students had the opportunity to find a unit rate given a ratio. In this lesson, students are given the unit rate and may find it helpful to convert the unit rate to a ratio.

Activity 1: A recipe for punch calls for 5 cups of lemonade for every 2 cups of fruit punch. Draw a model to represent this relationship. Then, interpret all of the following: Students use structure when moving fluidly between the unit rate and ratio for a given situation. They also reason abstractly and quantitatively, making sense of quantities and their relationships in problem situations.

5 to 2 The ratio of lemonade to fruit punch

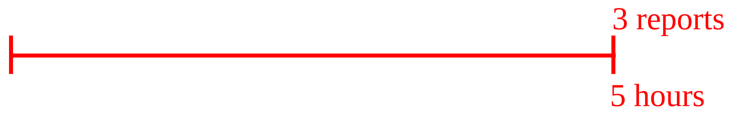
2:5 The ratio of fruit punch to lemonade

$\frac{2}{5}$ The unit rate of fruit punch to lemonade

$\frac{5}{2}$ The unit rate of lemonade to fruit punch

Activity 2: Hugo’s students wrote reports for their English final. Hugo can grade $\frac{3}{5}$ reports/hour.

a. Draw a model to represent this relationship.
Students can draw a picture or a model showing a ratio of 3 reports in 5 hours (or an equivalent ratio). Re-visit ideas from **Activity 1** in this lesson to help students create a model.



b. How many reports can he grade in 10 hours? Solve this problem using at least <u>two</u> strategies. Students can use their model to solve this problem. If we double the model shown above, we will see that Hugo can grade 6 reports in 10 hours. We can represent this numerically in the following way:

$$\frac{3\text{ reports}}{5\text{ hours}} \times \frac{2}{2} = \frac{6\text{ reports}}{10\text{ hours}}$$

In a partial table:

Hours	**Reports**
5	3
10	6

×2 ×2

Another strategy is to iterate the unit rate given in the problem 10 times.

$\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}$ or $10\times\frac{3}{5}=\frac{30}{5}=6$

All of these strategies are related. Try to help students make connections between the strategies.

Directions: For each problem, draw a model to represent the relationship. Then, answer the questions using at least two different strategies. Make connections between the different strategies.

1. Brett uses $\frac{3}{4}$ of a cup of blue paint for every cup of red paint to make a certain shade of purple paint. If he needs 21 cups of purple paint for an art project, how many cups of each, red and blue paint, does he need? Students may solve this problem using a variety of strategies. One way is to realize that the ratio of blue paint to red paint is 3 to 4. From here, students may draw a tape diagram:

B	B	B	R	R	R	R

If the total amount needed is 21 cups, then each box has a value of 3 cups of paint; therefore Brett would use 9 cups of blue paint and 12 cups of red paint.

Partial Table:

Blue Paint	**Red Paint**	**Total Paint**
3	4	7
9	12	21

×3 ×3 ×3

Additional strategies are explained in **Activity 2**.

2. It snows $\frac{2}{3}$ of an inch per hour. If 6 inches of snow fell, how long was it snowing for? The ratio associated with this rate is 2 inches in 3 hours.

In order for 6 inches of snow to fall, we would iterate the model above three times. It would take 9 hours for 6 inches to fall.

A common mistake in this problem is for students to multiply $\frac{2}{3}$ by 6 and say the answer is 4 hours. If students make this error, have them draw a picture to see that the answer is wrong. If it snows $\frac{2}{3}$ of an inch for 4 hours, only $\frac{8}{3}$ of an inch would have accumulated.

Some students may find the unit rate for hours/inch, realizing from the previous lesson that it is the reciprocal of the unit rate given in the problem. The unit rate of hours/inch is $\frac{3}{2}$. Then, solving numerically:

$$\frac{\frac{3}{2}\text{hours}}{\text{inch}} \times \frac{6\text{ inches}}{} = 9\text{ hours}$$

3. The unit rate of blue paint to white paint used to make sky blue is $\frac{1}{5}$. If Danny uses 4 pints of blue paint, how much white paint should he use?

This sky blue requires 1 part blue paint to 5 parts white paint:

B	W	W	W	W	W

If Danny uses 4 pints of blue paint, he should use 20 pints of white paint.

4. Todd can swim $\frac{7}{4}$ of a lap per minute. Use this information to complete the table below.

The ratio associated with this rate is 7 laps in 4 minutes. Drawing a model may help students complete the table below. There are many ideas that can be pulled from this table. Ask students what they see in the table. A few ideas: 1) The two rates associated with a ratio are reciprocals – we see this in the first two entries of the table. 2) What do the entries $\left(1, \frac{7}{4}\right)\left(\frac{4}{7}, 1\right)$ and $(4, 7)$ represent? How are they related? 3) What patterns do you see in the table?

Time (minutes)	**Laps Completed**
1	$\frac{7}{4}$
$\frac{4}{7}$	1
2	$\frac{14}{4}$ or $\frac{7}{2}$
4	7
12	21
60	105

5. The unit rate of an object's weight on Earth to its weight on Mars is approximately $\frac{5}{2}$. Use this information to complete the table below.

Weight on Earth (lbs.)	**Weight on Mars (lbs.)**
1	$\frac{2}{5}$
$\frac{5}{2}$	1
5	2
10	4
100	40
150	60
500	200

1.2e Homework: Given the Unit Rate

Directions: For each problem, draw a model to represent the relationship. Then, answer the questions using at least two different strategies. Make connections between the different strategies.

Refer to **Class Activity** for sample strategies.

1. On vacation, Kate reads $\frac{3}{4}$ of a book per day. If she is on vacation for 8 days, how many books will she read?
   This means that Kate can read 3 books in 4 days. On an 8-day vacation, Kate will read 6 books.

2. Charlie can run $\frac{3}{5}$ of a lap per minute. If he runs 12 laps, how long will it take him?
   This means that Charlie can run 3 laps in 5 minutes. If he runs 12 laps, it will take him 20 minutes.

3. The unit rate of salt to water used to make a saline solution is $\frac{1}{2}$. If Dawn uses 10 cups of water, how much salt should she put in?
   This solution uses 1 part salt to 2 parts water. If Dawn uses 10 cups of water, she should use 5 cups of salt.

4. It takes Elise $\frac{2}{5}$ of an hour to hike 1 mile. How long will it take her to hike 5 miles?

   It will take her 2 hours to hike 5 miles.

5. LaQuinta can type $\frac{1}{3}$ of a page in one minute. How long will it take her to type a 5-page report?
   This means that LaQuinta can type 1 page in 3 minutes. It will take her 15 minutes to type a 5-page report.

6. A recipe for a blueberry shake calls for $\frac{3}{8}$ cup of blueberries per cup of milk. Use this information to complete the table below.

Cups of Blueberries	**Cups of Milk**
$\frac{3}{8}$	1
1	$\frac{8}{3}$
$\frac{6}{8}$ or $\frac{3}{4}$	2
3	8
6	16

7. It takes a dog $\frac{4}{15}$ of a second to run 1 meter. Use this information to complete the table below.

Time (seconds)	**Distance (meters)**
1	$\frac{15}{4}$
4	15
20	75
60	225

1.2f Class Activity: Comparing Ratios

Activity 1: The tables below show three different recipes for making orange juice using orange concentrate and water.

Recipe 1	
Cups of Orange Concentrate	**Cups of Water**
3	5
6	10
9	15
10	$16\frac{2}{3}$

Recipe 2	
Cups of Orange Concentrate	**Cups of Water**
2	3
4	6
6	9
8	12

Recipe 3	
Cups of Orange Concentrate	**Cups of Water**
5	8
10	16
15	24
20	32

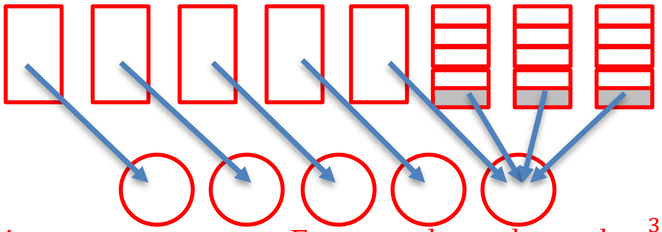
a. Order the recipes from strongest orange flavor to weakest orange flavor. Justify your answer.

From strongest orange flavor to weakest: Recipe 2, Recipe 3, Recipe 1. There are several strategies students can use to compare ratios. We have chosen to organize these types of problems in tables because standard **6.RP.3a** explicitly states that students should "Use tables to compare ratios."

Students may first notice that when comparing recipes 1 and 2, 6 cups of orange concentrate corresponds to 10 cups of water for recipe 1 and 9 cups of water for recipe 2. Students may need help realizing that *more* water actually dilutes the drink and makes the orange taste less strong. So far, recipe 2 has the strongest orange flavor. If we look at recipes 1 and 3, we see that for 10 cups of orange concentrate, recipe 1 uses $16\frac{2}{3}$ cup of water and recipe 2 uses 16 cups of water; therefore recipe 3 has a stronger orange flavor than recipe 1. Now, we need to compare recipe 2 to recipe 3. Students may try to find a ratio with a common value. For example, students may iterate recipe 2 one more time and see that 10 cups of orange concentrate corresponds to 15 cups of water. In recipe 3, 10 cups of orange concentrate corresponds to 16 cups of water so we see that recipe 2 has the stronger orange flavor.

Alternatively, students may find the unit rate of either $\frac{\text{concentrate}}{\text{water}}$ or $\frac{\text{water}}{\text{concentrate}}$. It should be easy to find the unit rate for $\frac{\text{water}}{\text{concentrate}}$ for recipe 1, as we increase from 9 to 10 cups of orange concentrate, the amount of water increases by $1\frac{2}{3}$, so this is our unit rate. For recipe 2, we can cut the ratio 2:3 in half to get $1:1\frac{1}{2}$. Recipe 3 may be a little trickier. A picture might help (the rectangles represent cups of water and the circles represent cups of orange concentrate:

Each cup of concentrate gets 1 cup of orange juice. Then if we divide each of the remaining three cups of water into 5 equal parts, each cup of concentrate will get another $\frac{3}{5}$ cup of water so one cup of concentrate corresponds to $1\frac{3}{5}$ cups of water.



Once students have the 3 unit rates, they can use fraction sense to compare. For example, we know that $\frac{3}{5}$ and $\frac{2}{3}$ are both greater than $\frac{1}{2}$ so $1\frac{1}{2}$ is less than both $1\frac{2}{3}$ and $1\frac{3}{5}$. Students can use pictures or models to compare $\frac{3}{5}$ and $\frac{2}{3}$. We determine that $1\frac{1}{2} < 1\frac{3}{5} < 1\frac{2}{3}$. Remember that these represent the unit rate of water to orange concentrate so the smaller number will actually have a stronger orange flavor.

Activity 2: Three pigs are racing in a pig race. The tables below show each pig's distance over time.

Bacon	
Time (sec)	**Distance (ft.)**
2	30
4	60
6	90

Jimmy Dean	
Time (sec)	**Distance (ft.)**
5	60
10	120
20	240

Pork Chop	
Time (sec)	**Distance (ft.)**
8	108
10	135
15	202.5

a. Order the pigs from fastest to slowest. Justify your answer.

In order from fastest to slowest: Bacon, Pork Chop, Jimmy Dean

Students may find the unit rate for each pig (Bacon: 15 ft./sec; Pork Chop: 13.5 ft./sec; Jimmy Dean: 12 ft./sec). Alternatively, students may compare ordered pairs with common values for one of the quantities. We can see that it takes Jimmy Dean 5 seconds to go 60 ft.; it only takes Bacon 4 seconds to go the same distance so Bacon is faster than Jimmy Dean. We also see that in 10 seconds, Jimmy Dean can go 120 ft. while Pork Chop can go 135 ft. Now we know that Jimmy Dean is the slowest. To compare Bacon to Pork Chop, students might find how far Pork Chip can go in 4 seconds: 54 ft. and realize that Bacon can go farther in 4 seconds (60 ft.). Alternatively, they may use repeated addition to determine how far Bacon can go in 8 seconds which is 120 feet. Have students share out their strategies so that they think about multiple ways of approaching this and similar problems.

The logic involved in solving these problems may be challenging for students. It may help to give students little post-it notes or slips of paper so that they can keep track of what they have already figured out or encourage them to record their results in a list as they go.

1. Three families leave Utah to drive to Disneyland. The tables below show each family's distance over time.

The Baker Family	
Time (hours)	**Distance (miles)**
2	120
4	240
6	360
1	60
3	180

The Sanchez Family	
Time (hours)	**Distance (miles)**
1	65
2	130
3	195
4	260

The Kim Family	
Time (hours)	**Distance (miles)**
3	165
6	330
9	495
1	55

a. Order the cars from fastest to slowest. Justify your answer.

Sanchez Family, Baker Family, Kim Family

2. Frank is comparing the cost of chicken at two different stores. Store A sells chicken for $3.50 per pound. Store B sells chicken in 5-pound bags for $16.25.

Store A

Pounds of Chicken	**Cost ($)**
1	3.50
2	7
3	10.50
4	14
5	17.50

Store B

Pounds of Chicken	**Cost ($)**
5	16.25
10	32.50
15	48.75
1	3.25

a. Which store has the better deal on chicken? Justify your answer.

Store B has the better deal on chicken. Students may continue to iterate the table for Store A to determine the cost of 5 pounds (or add the cost of 2 pounds and the cost of 3 pounds). They may find the cost of 1 pound of chicken for Store B. Help students with mental math strategies to solve the problem 16.25 ÷ 5. What if I divide $15 evenly between 5 people? Each person will have $3. Now, I have $1.25 left to divide between the 5 people. Each person will get another $0.25 giving a total of $3.25.

3. Cami is making pink paint for a Valentine's Day project in art class. She makes three different shades of pink by mixing red and white paint as shown in the tables below.

Shade A

Red (cups)	**White (cups)**
2	3
4	6
6	9
8	12
10	15
12	18

Shade B

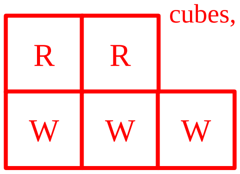
Red (cups)	**White (cups)**
3	4
6	8
9	12
12	16

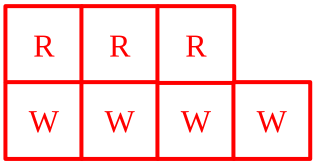
Shade C

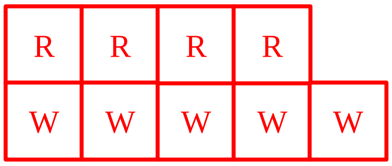
Red (cups)	**White (cups)**
4	5
8	10
12	15

a. Order the shades of pink from the lightest pink to the darkest pink. Justify your answer.

Shade A, Shade B, then Shade C. Space has been left in the table for students to iterate the ratio several times to find a common value for one of the quantities. Students may jump straight to the ratio with a common value of 12 using multiplication. Students may also solve this problem using several other strategies (unit rate, tape diagrams, linking cubes, number line, etc.).







We see in the models that for Shade A, red is 2/3 as big as white, for Shade B 3/4 as big and for Shade C 4/5 as big. We can also look at it from the other direction: for Shade A white is $1\frac{1}{2}$ times bigger than red, Shade B is $1\frac{1}{3}$ times bigger than red and for Shade C white is $1\frac{1}{4}$ bigger than red. From here, students should use fraction sense to order these fractions.

4. Olivia is making hot cocoa for the Halloween carnival at her school. She looks up three different recipes shown in the tables below.

Recipe 1	
Milk (ounces)	**Cocoa (T)**
10	4
5	2
60	24

Recipe 2	
Milk (ounces)	**Cocoa (T)**
4	2
12	6

Recipe 3	
Milk (ounces)	**Cocoa (T)**
12	5
60	25

a. Which recipe for hot cocoa will taste the most chocolaty? Which will taste the least chocolaty? Justify your answer. Recipe 2 will taste the most chocolaty. Recipe 1 is the least chocolaty. Comparing Recipe 1 to Recipe 2, we see that for 2 T of hot cocoa, Recipe 2 is chocolatier. Comparing Recipe 2 to Recipe 3 for 12 ounces of milk, we see that Recipe 2 is chocolatier. Comparing Recipe 1 to Recipe 3 for 60 ounces of milk shows that Recipe 3 is chocolatier.

At a quick glance, we can see that recipe 2 uses ½ as much cocoa as milk while recipes 1 and 3 use less than ½ (or Recipe 2 uses twice as much milk as cocoa while Recipes 1 and 3 use more than twice as much milk compared to cocoa). This quickly tells us that recipe 2 is the most chocolaty. From here, we only need to compare recipes 1 and 3.

5. The tables below show the number of calories in three different kinds of crackers based on the number of crackers you eat.

Cracker A	
Number of Crackers	**Calories**
16	140

Cracker B	
Number of Crackers	**Calories**
15	140

Cracker C	
Number of Crackers	**Calories**
4	30
16	120

a. Order the crackers from least number of calories per cracker to most number of calories per cracker. Justify your answer.
Least to greatest: C, A, B
Cracker A to Cracker B for 140 calories shows us that Cracker B has more calories than Cracker A (a person eats 1 less Cracker B and consumes the same number of calories so there are more calories in each Cracker B). Comparing A to C for 16 crackers, we see that A has more calories than C; therefore B has the most calories and C has the least.

6. At a carnival, there is a game with kiddie pools full of rubber ducks. Each player chooses a duck. If your duck has a red sticker on the bottom, you win a prize. There are three different kiddie pools, each with a different chance of winning as shown in the tables below. In Pool A, there are 6 losing ducks for every 2 winning ducks. In Pool B, 3 out of every 8 ducks is a winning duck. In Pool C, the ratio of winning ducks to losing ducks is 1:4.

Pool A	
Winning Ducks	**Losing Ducks**
2	6
1	3
6	18

8

Pool B	
Winning Ducks	**Losing Ducks**
3	5
6	10

8

Pool C	
Winning Ducks	**Losing Ducks**
1	4
6	24

a. Which pool should you play at if you want the best chances of winning? Justify your answer.

Pool B is the best pool to play at. There are many ways to solve this problem. If we compare Pool A to Pool C, we can find the number of losing ducks for 1 winning duck. We see that in Pool A, there are 3 losing ducks for each winning duck while in Pool C, there are 4 losing ducks for each winning duck. Another way to look at this is that the number of losing ducks in Pool A is only 3 times the number of winning ducks whereas for Pool C the number of losing ducks is 4 times the number of winning ducks. This makes Pool A the better of the two to play at. If we compare A to B, we could look at the total number of ducks. For 8 total ducks, 6 are losing ducks in Pool A while only 5 are losing ducks in Pool B. Another way to look at it is to see that in Pool A, the number of losing ducks is 3 times the number of winning ducks while in Pool B, the number of losing ducks is not even twice the number of winning ducks. This makes Pool B the best pool to play at.

7. Jen is buying Halloween candy for a school carnival. She researches the cost of Halloween candy in bulk at three different stores to see which has the best deal.

Store A	
Pounds of Candy	**Cost ($)**
$\frac{1}{2}$	$2.50
1	$5

Store B	
Pounds of Candy	**Cost ($)**
1	$5.10

Store C	
Pounds of Candy	**Cost ($)**
5	$25
1	$5

a. Which store has the best deal on Halloween candy? Justify your answer.
Stores A and C are the cheapest.

Spiral Review

1. Solve the following problems:

   a. 15,000 ÷ 3 5,000

   b. 15,000 ÷ 30 500

   c. 15,000 ÷ 300 50

   d. 15,000 ÷ 3,000 5

2. Solve the following problems:

   a. $24\times\frac{1}{3}$ 8

   b. $24\times\frac{2}{3}$ 16

   c. $24\times\frac{3}{3}$ 24

   d. $24\times\frac{4}{3}$ 32

3. Students at Snow Basin Elementary School had to stay inside for recess three out of five days this week due to bad weather.

   a. Draw a tape diagram to represent this situation.

   b. Write a part to part ratio to describe this relationship.
   The ratio of days inside to days outside is 3 to 2.

4. The tables below show Selena's scores on three different math quizzes. Complete the tables to show equivalent ratios.

Quiz 1	
Number of Questions Correct	**Total Number of Questions**
4	5
80	100

Quiz 2	
Number of Questions Correct	**Total Number of Questions**
9	10
90	100

Quiz 3	
Number of Questions Correct	**Total Number of Questions**
22	25
88	100

This problem is intended to surface ideas about ratios out of 100 (percent) which students will study in the next chapter.

1.2f Homework: Comparing Ratios

1. Mary and Tiffany are making paper hats for a Thanksgiving parade. The table below shows the number of hats each girl can make over time.

Mary	
Time (hours)	**Number of Hats**
1	5
2	10
3	15
4	20

Tiffany	
Time (hours)	**Number of Hats**
2	8
4	16
6	24
8	32

   a. If each girl needs to make 200 hats, who will finish first? Justify your answer.
   Mary will finish first. She makes hats at a faster rate.

2. Owen is racing three toy bugs across the floor. The tables below show each car's distance over time.

Red Bug	
Time (sec)	**Distance (cm)**
3	15
5	25
7	35
9	45

Blue Bug	
Time (sec)	**Distance (cm)**
2	9
4	18
6	27
8	36

Green Bug	
Time (sec)	**Distance (cm)**
3	12
6	24
9	36
12	48

   a. Order the bugs from fastest to slowest. Justify your answer.
   Red Bug, Blue Bug, Green Bug

3. Three friends are comparing how much they get in allowance. The tables show how much each person gets for allowance.

Harry	
Time (weeks)	**Allowance ($)**
1	6
2	12
3	18

Ron	
Time (weeks)	**Allowance ($)**
2	11
4	22
6	33

Hermione	
Time (weeks)	**Allowance ($)**
4	24
8	48
12	72

a. Which friend gets paid the most allowance? Justify your answer.
Harry and Hermione get paid the most. They both get paid $6/week while Ron only gets $5.50.

4. Three friends enter in a bike race. The tables below show each friend’s distance over time.

Moe	
Time (minutes)	**Distance (km)**
20	9
40	18
60	27

Larry	
Time (minutes)	**Distance (km)**
10	4.2
20	8.4
30	12.6

Curly	
Time (minutes)	**Distance (km)**
30	14
60	28
90	42

a. Order the bikers from fastest to slowest. Justify your answer.
Curly, Moe, Larry

5. Johnny is buying apples at the farmer’s market. The table below shows the cost of apples at three different stands.

Stand A	
Apples	**Cost ($)**
5	2.25
1	0.45
10	4.50

Stand B	
Apples	**Cost ($)**
10	4
1	0.40
10	4

Stand C	
Apples	**Cost ($)**
2	1
1	0.50
10	5

a. Which stand has the best deal on apples? Justify your answer.

Stand B has the best deal on apples.

6. Eva is making homemade peanut butter cups. She found two recipes shown in the tables below.

Recipe 1	
Ounces of Peanut Butter	**Ounces of Chocolate**
9	6
3	2

Recipe 2	
Ounces of Peanut Butter	**Ounces of Chocolate**
4	2
12	6

a. Which recipe is more peanut buttery? Justify your answer.

Recipe 2 is more peanut buttery. Students may quickly see that recipe 2 uses twice as much peanut butter as chocolate and in recipe 1 the amount of peanut butter is less than twice the amount of chocolate. Alternatively, students may find the unit rate of peanut butter to chocolate (1.5 for recipe 1 and 2 for recipe 2). Another method would be to create equivalent ratios with a common value for one of the quantities as shown in red in the table.

7. Pedro is buying bagels for soccer camp. He visits three stores to see how much they charge for bagels.

Store A	
Number of Bagels	**Cost ($)**
6	3.30
1	0.55

Store B	
Time (weeks)	**Allowance ($)**
12	5.40
6	2.70
1	0.45

Store C	
Time (weeks)	**Allowance ($)**
1	0.50
6	3.00
1	0.50

a. Which store has the best deal on bagels? Justify your answer.

Store B has the best deal on bagels.

8. The tables below show the amount of sugar in two popular sports drink.

Sports Drink A	
Ounces of Drink	**Sugar (tsp)**
20	7

Sports Drink B	
Ounces of Drink	**Sugar (tsp)**
32	14
16	7

a. Which sports drink is more sugary? Justify your answer.

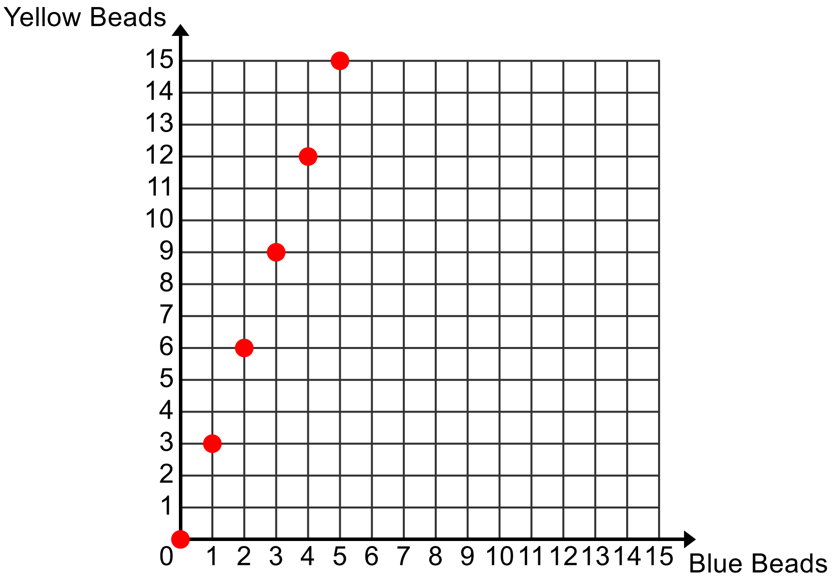
Sports Drink B is more sugary.

1.2g Class Activity: Equations to Represent Relationships Between Two Quantities

Activity 1: Chance is making a necklace with blue and yellow beads on it. The ratio of blue beads to yellow beads is 1 to 3.

a. Complete the table and graph to show the relationship between yellow and blue beads on Chance's necklace.

Blue Beads	Yellow Beads
1	3
2	6
3	9
4	12
10	30
100	300
b	$3 \times b$



b. Do your table and graph show all the possible values for the number of blue and yellow beads Chance can have on his necklace?
No, the table and graph show a limited number of ordered pairs due to space. Have students call out several more equivalent ratios that work.

c. Is it possible to list all the possible values for the number of blue and yellow beads that could be on Chance's necklace? Why or why not?
Students should recognize that there are an infinite set of equivalent ratios showing the relationship between yellow and blue beads on Chance's necklace.

It is *not* possible to list all the possible values for the number of blue and yellow beads on Chance's necklace. But we can write a rule that shows the relationship between blue and yellow beads on Chance's necklace. This rule is called an **equation** and it represents all the possible values for the number of blue and yellow beads on Chance's necklace. Every pair of values that makes the equation true is a possible pair of values for the number of blue and yellow beads on Chance's necklace. The equation represents the set of equivalent ratios such that each ratio of the y-value to its corresponding x-value is constant and in this case equal to 3.

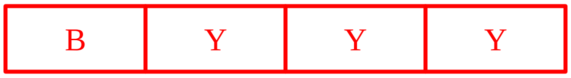
d. In your own words, state a rule that shows the relationship between yellow and blue beads on Chance's necklace. Answers will vary. Sample answers include, the number of yellow beads is always three times the number of blue beads, the number of blue beads is always 1/3 the number of yellow beads, the ratio of yellow beads to blue beads is always 3, etc.

e. Write an equation that shows the relationship between yellow beads and blue beads on Chance's necklace. Let y stand for yellow beads and b stand for blue beads.
$y = 3b$ or $b = \frac{1}{3}y$ or $b = \frac{y}{3}$; see notes for scaffolding writing equations on the next page

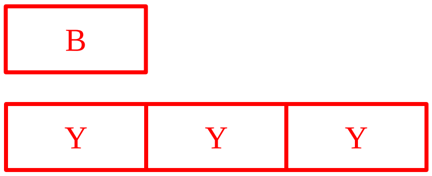
f. Chance has 12 blue beads to use on his necklace. How many yellow beads should he use?
He should use 36 yellow beads if he is using 12 blue beads. Students can substitute 12 in for b in their equation to solve this or they may just reason through the rule or use a tape diagram. Discuss different strategies being used.

Go back to the table and graph and discuss the connections between the equation, table, and graph. The equation shows the rule going across the table. In other words, how do I get from the input to the output for each pair or values in the table? In the graph, if I examine each point, the y-value is always three times larger than the x-value or, put another way, the ratio of y to x is always 3.

There are several things you can do to scaffold students who are struggling to come up with the equation. Having them state the rule in words first as we did in part d. is a great place to start. You may also go back to the bar model. Here they can physically see that the number of yellow beads is 3 times the number of blue beads. We can draw our bar model as we have in the previous lessons:



Alternatively, we can show the bars stacked. This gives a clearer picture that the number of yellow beads is three times the number of yellow beads.



The length of the yellow tape is three times the length of the blue tape. Viewing the ratio in this way shows the *multiplicative comparison* between the two quantities.

You can also give them a partially completed equation and have them fill in the rest:
The number of _______________ beads is three times the number of _________________ beads.
Yellow is three times *blue.*
y = 3b
Once students have their equations, have them substitute in values from the table to verify their equation.

The equation of a proportional relationship also connects back to work with unit rate. The unit rate of yellow beads to blue beads is 3:

$$\frac{\text{yellow beads}}{\text{blue beads}} = 3$$

$$\frac{y}{b} = 3$$

Rearranging this equation, we see that $y = 3b$.

Similarly, the unit rate of blue beads to yellow beads is $\frac{1}{3}$: $\frac{\text{blue beads}}{\text{yellow beads}} = \frac{1}{3}$ or $\frac{b}{y} = \frac{1}{3}$, rearranging this equation, we see that $y = \frac{1}{3}b$. Students will study this in 7th grade.

Activity 2: Marcus is training for an ultra-marathon where he will be running 100 miles. He can run 7 miles per hour.

a. Complete the table below to show the relationship between time and distance for Marcus.

Time (hours)	Distance (miles)	Expression for Distance
1	7	1×7
2	14	2×7
3	21	3×7
4	28	4×7
10	70	10×7
t	$7t$	$t\times7$

Writing out the expression for the distance Marcus travels can also help students arrive at the equation. Help students to see that 7 is the number of miles Marcus runs in 1 hour; therefore the total distance he runs is 7 miles for each hour that passes.

b. Write an equation to show the relationship between time and distance for Marcus. Use t for time and d for distance. Check your equation using values from the table.
$d = 7t$ or $t = \frac{d}{7}$

c. How far can Marcus run in 6 hours?
Marcus can run 42 miles in 6 hours.

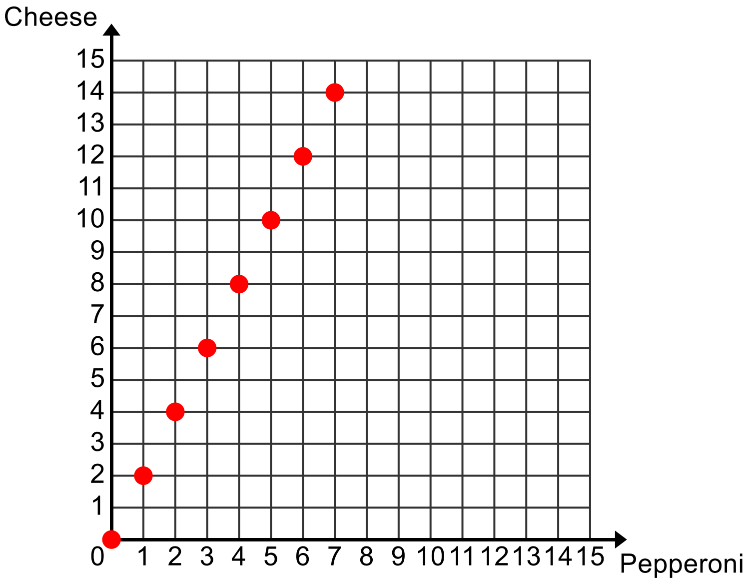
d. If Marcus plans to run 56 miles, how long will it take him?
It will take Marcus 8 hours to run 56 miles.

e. How long will it take Marcus to run the 100-mile ultra-marathon at this pace?
It will take him approximately 14.3 hours to run 100 miles.

When appropriate, introduce the vocabulary dependent variable and independent variable. The decision as to which quantity is the dependent variable and which quantity is the independent variable is usually interchangeable and is driven by *the question being asked*. When we ask a question about how far Marcus can run in a certain amount of time, as in part c. above, time is the independent variable and distance is the dependent variable. We would write the equation in the form $d = 7t$. In this case, we are interested in the number of miles Marcus can run based on how long he runs for. Alternatively, we may be interested in how long it takes Marcus to run a certain distance as was asked in parts d. and e. above. In this case, time is the dependent variable and distance is the independent variable and the corresponding equation is $t = \frac{d}{7}$.

1. Pizza Paradise only sells cheese and pepperoni pizzas. One out of every three pizzas sold is pepperoni. The rest are cheese.
   Encourage students to draw a bar model to represent this situation.
   a. Complete the table and graph to show the relationship between cheese and pepperoni pizzas sold at Pizza Paradise.

Pepperoni Pizzas Sold	**Cheese Pizzas Sold**
1	2
2	4
3	6
4	8
10	20
100	200
p	$2p$



   b. Write an equation that shows the relationship between cheese and pepperoni pizzas sold at Pizza Paradise. Let c stand for cheese and p stand for pepperoni.
   $c = 2p$ or $p = \frac{1}{2}c$

   c. If Pizza Paradise sells 200 pepperoni pizzas in one day, how many cheese pizzas will they sell?
   If they sell 200 pepperoni pizzas, they will sell 400 cheese pizzas.

   d. If Pizza Paradise sells 300 cheese pizzas in one day, how many pepperoni pizzas will they sell?
   If they sell 300 cheese pizzas in one day, they will sell 150 pepperoni pizzas.

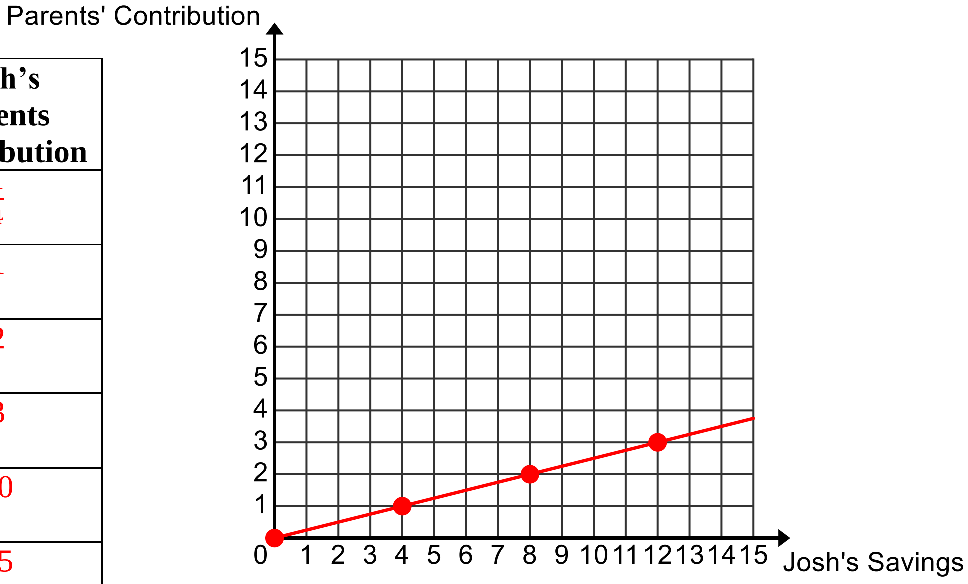
   e. If Pizza Paradise sells 900 total pizzas in one day, how many were cheese and how many were pepperoni? n#

It is important that students attend to the quantities in an equation. Our equation above relates cheese pizzas to pepperoni pizzas. We need a different equation to relate total number of pizzas to either cheese pizzas sold or pepperoni pizzas sold. In order to make sense of this, students may want to add another column to the table above labeled “Total Pizzas Sold” or make a tape diagram. From here, they might write an equation $t = 3p$ or $p = \frac{t}{3}$ where t represents total pizzas and p represents pepperoni pizzas. Using this equation, we see that if they sell 900 total pizzas, 300 would be pepperoni. That means 600 would be cheese. Similarly, they can write an equation that relates cheese pizzas sold, c, to total pizzas sold, t: $t = \frac{3}{2}c$.

Students may also use other strategies to solve this problem such as finding equivalent ratios - in part d. they sold a total of 450 pizzas so in order to sell 900 pizzas, the quantities would need to be doubled. Some students may draw a tape diagram to solve the problems. Make sure students understand that these are all just different ways of representing the same relationship and all are valid tools for solving real world ratio problems.

2. Josh is saving money to buy a tablet. For every \$4 he saves, his parents will contribute \$1.
   a. Complete the table and graph to show the relationship between the amount Josh saves and the amount his parents contribute.

Josh's Savings	**Josh's Parents Contribution**
1	$\frac{1}{4}$
4	1
8	2
12	3
80	20
100	25
j	$\frac{j}{4}$



   b. Write an equation that shows the relationship between the amount Josh's parents contribute, p, and the amount Josh saves, j.
   $p = \frac{j}{4}$

   c. If Josh saves \$200, how much will his parents need to contribute?
   \$50

   d. If the tablet Josh is saving for is \$450, how much will Josh need to save?
   Josh will need to save \$360. His parents will contribute \$90.

3. Zoe is saving \$5 each week.
   a. Write an equation that shows the relationship between time in weeks, t, and the amount Zoe saves in dollars, s.
   $s = 5t$
   Students may consider creating a table or double number line to help write the equation. It also really helps to have students write out the expression for amount saved. If Zoe saves for…
   1 week she will save $1(5) = \$5$
   2 weeks she will save $2(5) = \$10$
   3 weeks she will save $3(5) = \$15$
   t weeks she will save $t(5) = 5t$

   b. How long will it take Zoe to save \$35?
   It will take Zoe 7 weeks to save \$35.

   c. How much can Zoe save in 12 weeks?
   Zoe can save \$60 in 12 weeks.

4. Almonds cost $7.99 per pound.
    a. Write an equation that shows the relationship between cost in dollars, *c*, and pounds of almonds purchased, *p*.
    $c = 7.99p$

    b. How much will 3 pounds of almonds cost?
    $23.97

5. Teya is saving $400 per month.
    a. Write an equation that shows the relationship between time in months, *t*, and amount of money Teya saves, *s*.
    $s = 400t$

    b. Teya's goal is to have $12,000 saved. How long will it take her to save $12,000?
    30 months

6. There are 150 calories in 10 cheesy tortilla chips.
    a. Write an equation to show the relationship between number of calories, *c*, you consume based on how many cheesy tortilla chips, *t*, you eat.
    $c = 15t$

    b. Renee ate 6 cheesy tortilla chips. How many calories did she consume?
    Renee consumed 90 calories.

7. A car travels 135 miles in 3 hours at a constant rate. Consider showing what this graph looks like using a graphing calculator or online graphing software.
    a. Write an equation to show the relationship between time in hours, *t*, and the number of miles, *m*, the car travels at this rate.
    $m = 45t$

    b. How long will it take the car to reach a town that is 225 miles away?
    5 hours

8. In the Lollipop Tree Game at a carnival, a player chooses a lollipop. If the lollipop is colored red on the end of the stick, the player wins the lollipop and an additional prize. If the lollipop is not colored red on the end, the player just gets to keep the lollipop. The ratio of lollipops with red ends to lollipops without red ends is 3 to 4.
   a. Complete the table to show the relationship between lollipops with red ends and lollipops without red ends.

Lollipops with Red Ends	**Lollipops without Red Ends**
1	$\frac{4}{3}$
3	4
6	8
9	12
12	16
15	20
r	$\frac{4}{3}r$

   b. Write an equation to show the relationship between lollipops with red ends, *r*, and lollipops without red ends, *w*.

This equation may be more challenging for students since it involves a fraction. Even though the unit rate shown in the table does not really make sense in this context (it is not possible to have a part of the a lollipop), it may help students to arrive at the equation using ideas about multiplication. For each lollipop I have with a red end, I need $\frac{4}{3}$ of a lollipop without a red end. If I have…

1 lollipop with a red end, I have $1\left(\frac{4}{3}\right)$ without a red end.

2 lollipops with a red end, I have $2\left(\frac{4}{3}\right)$ without a red end.

3 lollipops with a red end, I have $3\left(\frac{4}{3}\right)$ without a red end.

12 lollipops with a red end, I have $12\left(\frac{4}{3}\right)$ without a red end.

r lollipops with a red end, I have $r\left(\frac{4}{3}\right)$ without a red end.

You may look at it as a rearranging of the unit rate:

We know that the ratio $\frac{\text{lollipops with red ends}}{\text{lollipops without red ends}} = \frac{3}{4}$

Rearranging, we see that lollipops with red ends = $\frac{3}{4}$ lollipops without red ends.

Similarly, $\frac{\text{lollipops without red ends}}{\text{lollipops with red ends}} = \frac{4}{3}$

Rearranging, we see that lollipops without red ends = $\frac{4}{3}$ lollipops with red ends

Alternatively, we may look at this using a tape diagram or linking cubes and make a multiplicative comparison. In the multiplicative comparison, we see that the lollipops without red ends are $1\frac{1}{3}$ times bigger than those with red ends (or the lollipops with red ends are $\frac{3}{4}$ the lollipops without red ends):

WR	WR	WR	WR
R	R	R	

c. If Lizzy has 24 lollipops with red ends, how many should she put in the tree without red ends?

$24 \cdot \frac{4}{3} = 32$ without red ends

9. The ratio of fruit punch to Sprite used to make Fizzy Fruit Punch is 3 to 2.

a. Complete the table to show the relationship between Sprite and fruit punch used to make Fizzy Punch.

Fruit Punch (cups)	**Sprite (cups)**
1	$\frac{2}{3}$
3	2
6	4
9	6
12	8
15	10
c	$\frac{2}{3}c$

b. Write an equation to show the relationship between cups of Sprite, *s*, and cups of fruit punch, *f*.

See strategies above for helping students find equation.

$s = \frac{2}{3}f$ or $f = \frac{3}{2}s$

c. If Jesse uses 27 cups of fruit punch for a school dance, how much Sprite should she use?

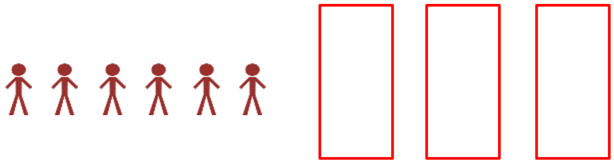
$27 \cdot \frac{2}{3} = 18$ cups of Sprite

Spiral Review

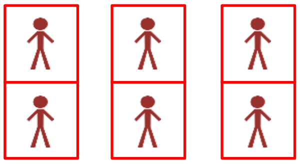
1. Mr. Longe is preparing for a science experiment. He has determined that he needs 15 cups of goo for 30 students to do the experiment.
   a. Mr. Longe teaches four class periods of science with 30 students in each class. How many cups of goo does Mr. Longe need for all four class periods?
   Students may use a variety of methods to solve this problem. They may draw a tape model and iterate it. They may figure out how much goo each student needs to do the experiment. They may also solve the problem numerically:
   $\frac{\text{cups of goo}}{\text{students}} = \frac{15}{30} \times \frac{4}{4} = \frac{60}{120}$; To do the experiment with 120 students, Mr. Longe needs 60 cups of goo.

   b. Each lab table in Mr. Longe's classroom holds 6 students. How many cups of goo does each lab table need?
   Each group needs
   $\frac{\text{cups of goo}}{\text{students}} = \frac{15}{30} \div \frac{5}{5} = \frac{3}{6}$
   Each lab table needs 3 cups of goo.

   c. How many cups of goo does each student need to do the experiment?
   A picture or model may be helpful for students to answer this question. From part b. students see that 6 students need 3 cups of goo.



6 students, 3 cups of goo

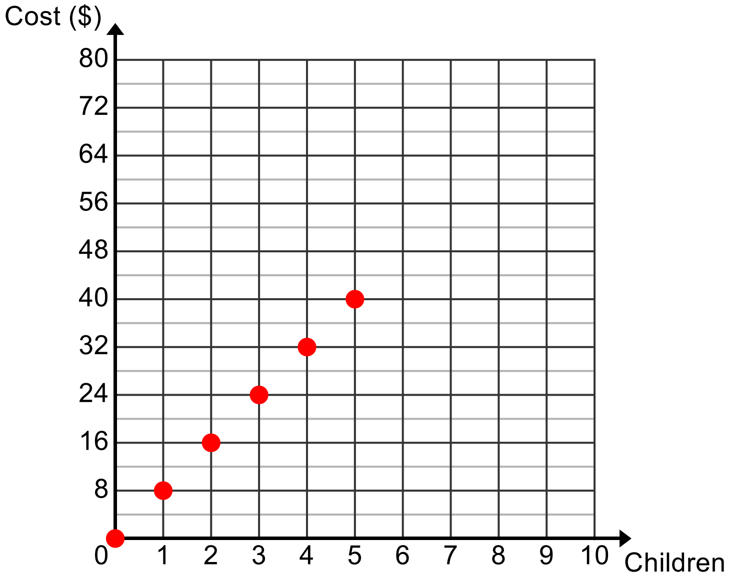


Each students needs ½ c. of goo

1.2g Homework: Equations to Represent Relationships Between Two Quantities

1. Stefan is planning a birthday party at a family fun center. It costs $8 for each person to attend.
   a. Complete the table and graph to show the relationship between the number of people that attend the party and the cost of the party.

Number of People	Cost (dollars)
1	8
2	16
3	24
4	32
10	80
p	$8p$



   b. Write an equation to show the relationship between number of people at the party, p, and total cost of the party, c.
   $c = 8p$

   c. If Stefan paid $160, how many children were at the party?
   If Stefan paid $160, there were 20 children at the party.

2. Jack and Carol both submitted drawings for the cover of the yearbook. The student body voted on whose drawing will be on the cover. For every vote that Jack receives, Carol receives three.
   a. Complete the table to show the relationship between votes for Jack, votes for Carol, and total votes.

Votes for Jack	Votes for Carol	Total Votes
1	3	4
2	6	8
3	9	12
10	30	40
25	75	100
100	300	400
j	$3j$	$4j$

   b. Write an equation that shows the relationship between votes for Jack, j, and votes for Carol, c.
   $c = 3j$ or $j = \frac{c}{3}$

   c. Write an equation that shows the relationship between votes for Jack, j, and total votes, t.
   $t = 4j$ or $j = \frac{t}{4}$

   d. If Jack received 60 votes, how many did Carol receive?
   If Jack received 60 votes, Carol received 180 votes.

   e. If 200 total people voted, how many voted for Jack and how many voted for Carol? 50 for Jack and 150 for Carol

3. Dessie is making Valentine's cards for her classmates. She can make 4 cards in 20 minutes.
   a. Complete the table below to show the relationship between time in minutes and the number of cards Dessie can make.

Time (minutes)	**Number of Cards**
20	4
40	8
60	12
120	24
200	40
t	$\frac{t}{5}$

   b. Write an equation to show the relationship between time Dessie works and the number of cards she can make. Use *t* for time in minutes and *c* for number of cards made.
   $c = \frac{t}{5}$

   c. If Dessie works on cards for 3 hours, how many can she complete?
   She can complete 36 cards in 3 hours.

4. It takes Sabina 1 hour to bike 12 miles.
   a. Write an equation to show how long it takes Sabina to bike based on how far she goes. Use *t* for time in hours and *d* for distance in miles.
   $t = \frac{d}{12}$
   b. If Sabina is biking 60 miles for a race, how long will it take her at this pace?
   It will take her 5 hours to bike 60 miles.

5. Charlie's school is selling wrapping paper for a fundraiser. The school makes $0.40 for each roll of wrapping paper they sell.
   a. Write an equation to show the relationship between the number of rolls of wrapping paper, *r*, the school sells and the amount of money they raise, *m*.
   $m = 0.40r$

   b. If the school sells 1,000 rolls of wrapping paper, how much money will they raise?
   $400

6. A car is driving at a constant speed. After 3 hours, the car has driven 195 miles.
    a. Write an equation that shows the relationship between time in hours, *t*, and distance, *d*, that the car travels at this rate.
    $d = 65t$

    b. How long will it take the car to drive 325 miles at this speed?
    5 hours

7. Ben is paid the same amount each hour that he babysits. When he works 6 hours, he is paid $36.
    a. Write an equation that shows the amount of money in dollars, *m*, Ben makes for *h* hours of work.
    $m = 6h$

    b. If Ben babysits for 8 hours, how much will he make?
    $48

8. Sandeep is buying hamburger meat for a company picnic. It costs $21.25 for 5 pounds of hamburger meat.
    a. Write an equation to show cost in dollars, *c*, for *p* pounds of hamburger meat.
    $c = 4.25p$

    b. If Sandeep buys 8 pounds of hamburger meat, how much will it cost him?
    $34

9. A new movie was just released. The movie theater is gathering data to see if boys or girls are more likely to see the movie. The ratio of girls to boys who come to see the movie is 5 to 3.
    a. Complete the table below to show the relationship between girls, boys, and total children who see the movie.

Girls	**Boys**	**Total Children**
5	3	8
10	6	16
15	9	24
20	12	32
g	$\frac{3}{5}g$	$\frac{8}{5}g$

b. Write an equation to show the relationship between number of boys *b* and number of girls *g* who see the movie.

$$b = \frac{3}{5}g$$

c. Write an equation to show the relationship between number of girls *g* and total number of children *t* who see the movie.

$$t = \frac{8}{5}g$$

d. If 50 girls are at a showing, how many boys would you expect to be at the showing?
30

e. If 100 girls are at a showing, how many total children would you expect to be at the showing?
160

1.1h Self-Assessment: Section 1.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

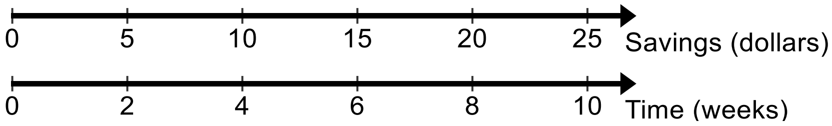
Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Mastery 3**	**Substantial Mastery 4**
1. Read double number lines, create them to show relationships between two quantities, and use them to solve real world problems.	I can read and answer questions about a double number line if the values are given on the number line but have a difficult time when the values are not given. I cannot complete or create double number lines.	I can read and answer questions about a double number line and complete a partially filled in double number line but have a difficult time creating my own double number line.	I can read and answer questions about a double number line and complete a partially filled in double number line. I can create my own double number line but sometimes need help with how to scale the number lines. I can iterate and partition a double number line to find additional values but sometimes struggle when the values are fractions.	I can read and answer questions about a double number line and complete a partially filled in double number line. I can create my own double number line choosing an appropriate scale for both number lines. I can iterate and partition a double number line to find additional values, including fractional values.
2. Plot pairs of equivalent ratios on a coordinate plane.	I know what an ordered pair is but I have trouble plotting ordered pairs because I often mix up the quantities.	I can plot ordered pairs on a coordinate plane that have been given in a table.	I can create tables of equivalent ratios and plot the ordered pairs on a coordinate plane. I understand that an ordered pair represents a pair of values that correspond with each other to show a relationship between two quantities.	I can create tables of equivalent ratios and plot the ordered pairs on a coordinate plane. I understand that an ordered pair represents a pair of values that correspond with each other to show a relationship between two quantities. I know what the graph of equivalent ratios should look like and I can use this information to find additional values and answer questions.
3. Read and interpret graphs to solve real world problems involving ratios.	I can identify the quantities on a graph but I have a difficult time reading the graph because I am not sure how to identify the ordered pairs that correspond to a point on a graph or what the ordered pairs mean in the context.	I can identify the quantities on a graph. I can read a graph to answer questions about points given on the graph and circle points on the graph that represent answers to questions.	I can identify the quantities on a graph. I can read a graph to answer questions about points given on the graph and circle points on the graph that represent answers to questions. I can write the ordered pairs that correspond to each point on the	I can identify the quantities on a graph. I can read a graph to answer questions about points given on the graph and circle points on the graph that represent answers to questions. I can write the ordered pairs that correspond to each

			graph, transferring this information to a table, and know what the ordered pairs represent in the context.	point on the graph, transferring this information to a table, and know what the ordered pairs represent in the context. I can use patterns in the graph to answer questions about values not shown on the graph.
4. Understand what a unit rate is. Find the unit rates associated with a given ratio and use them to solve real world problems.	I know that the unit rate is the amount of one quantity when the other quantity is equal to 1 unit but I am not sure how to find a unit rate given a ratio.	I know that the unit rate is the amount of one quantity when the other quantity is equal to 1 unit. I can find the unit rate for easy problems that do not involve fractions (i.e. Nicolas completes 100 math facts in 5 minutes. How many does he complete each minute?).	I know what a unit rate is. I can find the unit rates associated with a given ratio even those involving fractions. I can use the unit rates I find to answer additional questions about the situation.	I know what a unit rate is. I can find the unit rates associated with a given ratio even those involving fractions. I understand the relationship between the unit rates associated with a ratio. I know which unit rate to find to solve a real world problem based on the question that is asked.
5. Use tables to compare ratios.	I know to look for values in the table that are the same but I have a difficult time making sense of what the information is telling me in order to compare the ratios.	I can compare ratios given in two tables when at least one of the values in the input or output columns is the same.	I can compare ratios given in two or more tables by finding additional ordered pairs not given in the table.	I can compare ratios given in two or more tables by finding additional ordered pairs not given in the table. I can quickly and easily identify which additional ordered pairs would be helpful to find in order to compare the ratios. I can verify my results using additional strategies (i.e. finding the unit rate).
6. Write an equation to show a relationship between two quantities and use equations to solve real world problems.	I can express the relationship between two quantities using words (i.e. the cost is three times larger than the number of pounds) but have a difficult time writing the equation symbolically (i.e. $c = 3p$ where c is cost in dollars and p is pounds).	I can write an equation to show the relationship between two quantities in words and symbolically but I struggle when the equation involves fractions and I sometimes mix up where the variables go in the equation.	I can write an equation to show a relationship between two quantities in words and symbolically, including equations that contain fractions.	I can write an equation to show a relationship between two quantities in words and symbolically, including equations that contain fractions. I can write additional equations that show the same relationship (i.e. $c = 3p$ or $p = \frac{c}{3}$ where c is cost in dollars and p is pounds).

Sample Problems for Section 1.2

Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. Select all statements that are true about the double number line shown below. For the statements that are false, correct them to make them true. [1]



☐ Carl saves $5 each week. False; Carl saves $2.50 each week or Carl saves $5 every 2 weeks.

☐ In 10 weeks, Carl has saved $4. False; In 4 weeks, Carl has saved $10 or In 10 weeks, Carl has saved $25.

☐ In 8 weeks, Carl has saved $20. True

☐ Carl will save $30 in 15 weeks. False; Carl will save $30 in 12 weeks.

☐ It will take Carl 16 weeks to save $40. True

☐ In 5 weeks, Carl will have saved $12.50. True

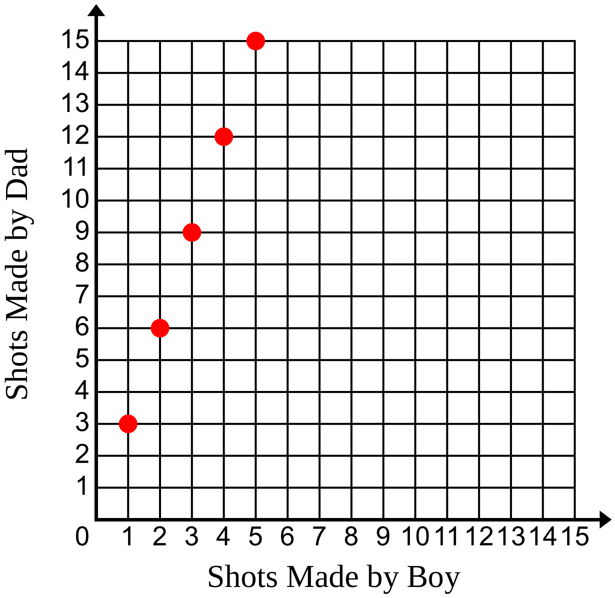
2. The Nest Egg Café charges $2 for 4 chocolate chip cookies. [1]
   a. Create a double number line to represent this situation.
   Double number lines may vary. Have students share with the class or a neighbor.

   b. Make up 3 questions that can be answered using your double number line. Then, answer them.
   Questions will vary. Have students share with the class or a neighbor.

3. A boy and his dad are taking turns shooting a basketball. For every shot that the boy makes, his dad makes three.

   a. Complete the table and graph to show this relationship. [2]

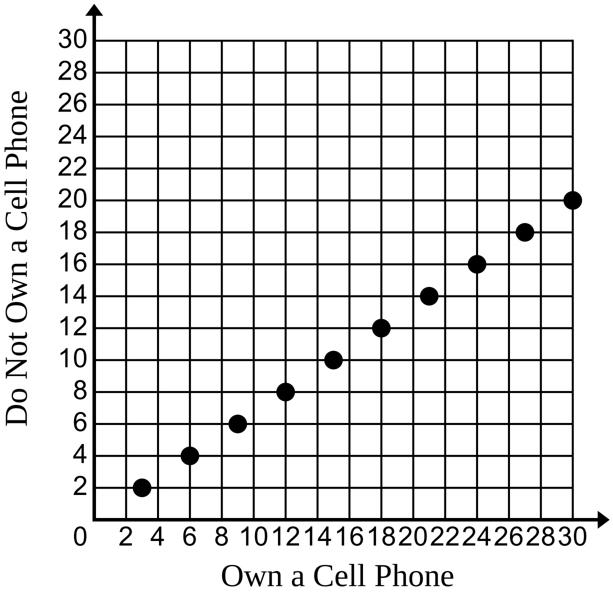
Shots Made by Boy	**Shots Made by Dad**
1	3
2	6
3	9
4	12
5	15



   b. What does the ordered pair $(8, 24)$ represent in this situation? [2]
   The boy makes 8 shots and his dad makes 24 shots.

   c. If the boy makes 4 shots, how many will his dad make? Circle this ordered pair on the graph. [3]
   If the boy makes 4 shots, his dad will make 12 shots.

4. Lance polled the 6th grade students at his school to determine whether or not they own their own cell phone. The graph shows the results of his survey. [3]



   a. In a homeroom of 6th graders at Lance's school, 16 students do not own a cell phone. Based on the graph, how many students in the homeroom own a cell phone?
   24 students

   b. Write a ratio statement to show the relationship between students who own a cell phone and students who don't own a cell phone.
   Answers will vary. The ratio of students who own a cell phone to those who don't is 3 to 2. Three out of every five students own a cell phone.

   c. If there are 200 6th graders in Lance's school, how many would you expect own a cell phone and how many would you expect do not own a cell phone? 80 do not and 120 do

5. Oscar is planning to buy 10 sub sandwiches for every 8 kids at a birthday party. [4]
   a. How much of a sub sandwich does each child get? Each child will get $1\frac{1}{4}$ sub sandwiches.

   b. If there are 12 kids at the birthday party, how many sub sandwiches does Oscar need to buy? Oscar should buy 15 sandwiches.

6. George can read 5 pages of his book in 20 minutes. [4]
   a. How long will it take him to read 8 pages of his book? At a rate of 4 minutes/page, it will take George 32 minutes to read 8 pages of his book.

   b. If he reads for an hour, how many pages will he be able to read? At a rate of $\frac{1}{4}$ page/minute, he will be able to read 15 pages in an hour, or 60 minutes.

You may want to review with students that these are all equivalent ratios and they all show George reading at the same rate:
5 pages in 20 minutes, 8 pages in 32 minutes, 15 pages in 60 minutes, ¼ page/minute, etc.

7. Tina jogs 5 meters every 2 seconds. [4]
   a. Find the unit rate $\frac{\text{meters}}{\text{second}}$. Write in your own words what this answer means. $\frac{5}{2}$; Tina can jog $\frac{5}{2}$ or 2.5 meters in one second.

   b. Find the unit rate $\frac{\text{seconds}}{\text{meter}}$. Write in your own words what this answer means. $\frac{2}{5}$; It takes Tina $\frac{2}{5}$ or 0.4 seconds to run 1 meter.

   c. How long will it take Tina to jog 100 meters? It will take her 40 seconds.

   d. How far can Tina jog in 30 seconds? She can jog 75 meters.

8. An elevator at a hotel travels 36 feet to get from floor 2 to floor 5. [4]

   a. If someone rides from floor 1 to floor 10, how many feet will they go? 108 feet

9. A recipe for pancakes calls for 1 cup of water for every $\frac{2}{3}$ cup pancake mix. [4]
   a. If Charlotte uses 10 cups of pancake mix, how much water does she need to use? 15 cups of water

10. Leo’s Landscaping is looking to hire another gardener. The tables below show the rate at which two different job candidates can mow lawns. [5]

Candidate 1	
Number of Lawns	**Time (hours)**
4	7
1	1.75
20	35
32	56

Candidate 2	
Number of Lawns	**Time (hours)**
5	8
1	1.6
20	32
35	56

a. Which gardener should Leo’s Landscaping hire if they want the candidate who mows lawns at the fastest rate? Justify your answer.
The table shows different equivalent ratios that students may find in order to help solve this problem. Leo’s Landscaping should hire Candidate 2. It takes him less time to mow each lawn. A common error is that students think the higher number indicates the faster candidate. Have a discussion about what the higher number means in this case – it takes that candidate more time.

11. There are 8 party hats in a package of party hats. [6]

a. Write an equation that shows the relationship between number of party hats h and packages of party hats p.
$h = 8p$

b. If Terry buys 3 packages of party hats, how many party hats will she have?
She will have 24 hats.

12. The table below shows the amount Hadley earns babysitting and the amount she saves. [6]

Amount Earned	**Amount Saved**
12	6
30	15
60	30
100	50

a. Write an equation that shows the relationship between the amount Hadley earns e and the amount she saves s.
$s = \frac{e}{2}$

b. If she earns \$45 on a babysitting job, how much will she save? \$22.5

13. Two pounds of macaroni salad at a deli costs \$7.40. Three pounds of the same macaroni salad costs \$11.10. [6]

If students struggle to write an equation, encourage them to make a table first.

   a. Write an equation that shows how much someone will pay in dollars, *c*, based on the number of pounds, *p*, of macaroni purchased.
   $c = 3.7p$

   b. If Ellie buys 5 pounds of this macaroni salad, how much will it cost her?
   \$18.50

14. A candy company just released a new pizza-flavored taffy. They are doing a taste test to see whether or not people like the taffy. Only one out of every five people that tries the taffy likes it. [6]

   a. Write an equation to show the relationship between the number of people who try the taffy *t* and the number of people who like the taffy *l*.
   $l = \frac{t}{5}$

   b. Write an equation to show the relationship between the number of people who try the taffy *t* and the number of people who don't like the taffy *d*.
   $d = \frac{4}{5}t$

   c. Write an equation between the number of people who like the taffy *l* and the number of people who don't like the taffy *d*.
   $d = 4l$