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Chapter 2: Percent, Division with Fractions, and Measurement Conversion

Utah Core Standard(s):

- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (6.RP.3)
  c) Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
  d) Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Academic Vocabulary: percent, rational number, fraction, decimal, partial table, ratio, equivalent ratios, equivalent fractions, factor, multiple, greatest common factor (GCF), unit fraction, dividend, divisor, quotient, factor, product, metric system of measurement, customary system of measurement, conversion

Chapter Overview:

In this chapter, students use their work with ratio to understand a percent as a part to whole ratio with a whole equal to one hundred. They learn how to express parts of a whole using fraction, decimal, and percent notation and they convert fluently between these different but equivalent forms. Next, students learn about three different types of percent problems: 1) Finding a percent given a part and the whole. 2) Finding a part of a quantity given a percent and the whole. 3) Finding the whole given a part and a percent. While reasoning about and solving percent problems, students use a variety of models and strategies such as tape diagrams, double number lines, partial tables, unit rate, equations, etc. In Section two, students apply and extend previous understandings of multiplication and division to divide fractions by fractions. Using a variety of strategies and models, students solve mathematical and real-world problems that require an understanding of how to divide with fractions. They come to understand that dividing by a number is the same as multiplying by the number's reciprocal. In section three, students study measurement conversion. Measurement conversion provides another opportunity for students to apply their understanding of ratio and unit rate.

Connections to Content:

Prior Knowledge: In this chapter students draw on their work with ratio from Chapter 1 as they explore the meaning of percent – a part to whole ratio with a whole equal to 100. They express parts of a whole using fraction, decimal, and percent notation. To do this, they construct models learned previously (area models, including hundred grids, tape diagrams, double number lines, tables, etc.). Students know how to express a fraction as a decimal by creating an equivalent fraction with a denominator of 10 or 100 (4.NF). Students will rely on their ability to operate fluently with rational numbers (5.NF and 6.NS). They will also use their understanding of a rational number, $\frac{a}{b}$, as both a groups of $\frac{1}{b}$ (3.NF and 4.NF) and $a \div b$ (5.NF). Students have also used models to divide whole numbers by unit fractions and unit fractions by whole numbers (5.NF). They will build on this knowledge to divide fractions by fractions. Lastly, students have converted measurements within a single system of measurement (customary and metric) using ideas about multiplication and division (4.MD and 5.MD). They will connect these ideas to the ideas of ratio and use them to convert between systems of measurement.

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Future Knowledge: In Chapter 6 of this text, students will learn how to write and solve equations to represent the different types of percent problems studied in this chapter. In 7th grade, students will continue to focus on proportional relationships, learning how to set up and solve a proportion to solve percent problems, including problems involving discounts, interest, taxes, tips, and percent increase and decrease. They will also apply these skills and understandings to solve problems involving other types of proportional relationships (e.g., similar figures, scale drawings, probability and statistics, etc.). Students will examine the representations of a proportional relationship, a subset of linear relationships. In 8^{th} grade, students transition to linear relationships in general and proportions form the basis for understanding the concept of constant rate of change (slope). As they progress through high school coursework, they use ratios in algebra (functions), trigonometry (the basic trigonometric functions), and calculus (average and instantaneous rate of change of a function).

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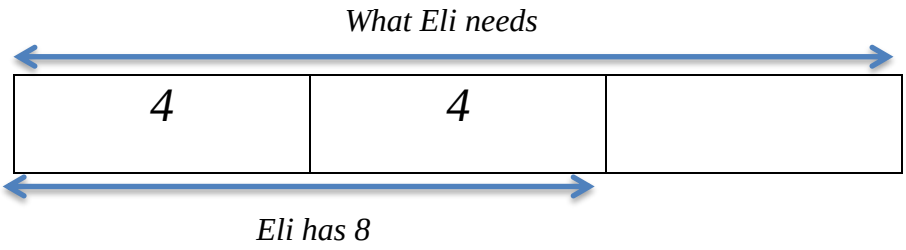
MATHEMATICAL PRACTICE STANDARDS

	Make sense of problems and persevere in solving them.	Flora's new baby has a birth weight of 8 pounds exactly. Her mother calls from London, England to ask about the baby and wants to know the baby's weight in kilograms. *Throughout the chapter, students will reason through the size of their answer. For this problem, students will use a conversion chart to find the conversion between pounds and kilograms. From here, students need to think about the size of their answer – should it be smaller or bigger than eight? This will help them determine a solution pathway and determine whether their answer makes sense.*
n#	**Reason abstractly and quantitatively.**	*Eli has 8 pints of ice cream. It's $\frac{2}{3}$ of what he needs. How much does he need? Draw a model of your choice to answer this question. Then, write number sentence to represent the problem.* *Students start by drawing a model to represent this problem.*  *The model shows that Eli needs 8 pints of ice cream. Students then connect this model to a division sentence:* $8 \div \frac{2}{3} = 12$. *The model reveals how to perform the division. First, eight must be distributed evenly into $\frac{2}{3}$ of the total (take half of 8), therefore each part must contain 4. There are 3 parts in the total, each with 4, so the total is 12 (multiply by 3):* $8 \div \frac{2}{3} \rightarrow 8 \times \frac{1}{2} \times 3 = 4 \times 3 = 12.$ *We see that dividing by $\frac{2}{3}$ is the same as multiplying by $\frac{3}{2}$.* *Students can also write a multiplication problem to solve this problem:* *$\frac{2}{3}$ of what equals 8* $\frac{2}{3} \times ? = 8$ $? = 8 \div \frac{2}{3}$ *This type of thinking forms the foundation for solving equations which students will study later in the year.*

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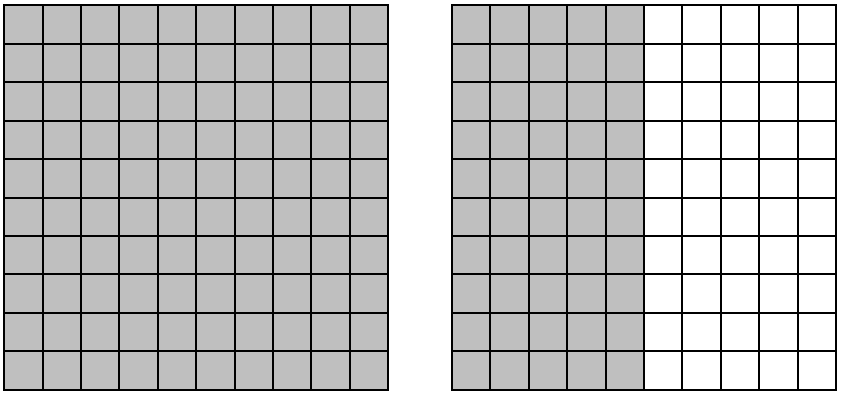
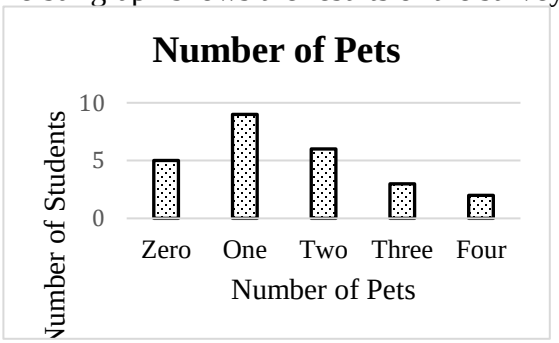
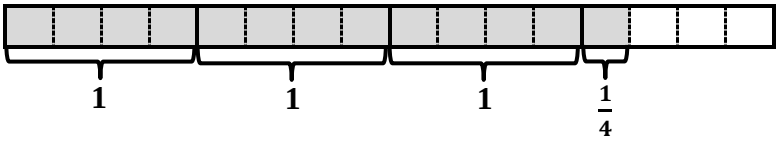
	Construct viable arguments and critique the reasoning of others.	Express the shaded portion as a fraction, decimal, and percent.  *Students may give either* $1\frac{1}{2}, 1.5, 150\%$ *OR* $\frac{3}{4}, 0.75, 75\%$ *as correct answers. The correctness depends on the "whole":* $1\frac{1}{2}, 1.5, 150\%$ *is correct if the student is interpreting the whole as ONE square, while* $\frac{3}{4}, 0.75, 75\%$ *is correct if the student is interpreting the whole as the TWO squares. Ask students to explain "150% of* ***what whole****"? Or "75% of* ***what whole****?"*
	Model with mathematics.	Renee surveyed the students in her class to see how many pets they have. The bar graph shows the results of the survey:  What percent of the students in Renee's class own two or more pets? What percent of the students in Renee's class own less than two pets? *Lesson 2.1d focuses on students solving real world problems involving percents. Students analyze and interpret a variety of graphs to draw conclusions about the data shown.*
	Use appropriate tools strategically.	Use the model below to solve the problem. $3\frac{1}{4} \div \frac{3}{4} =$ __$4\frac{1}{3}$__  *Students use a variety of models throughout the chapter to solve percent problems, divide fractions, and perform measurement conversions. They use these models to make sense of why dividing by a number is the same as multiplying by the number's reciprocal.*

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	Attend to precision.	Tia is putting red and blue marbles into different bags. In which of the bags are 25% of the marbles red? Justify your answers. **Bag 1:** One out of every four marbles in the bag is red. **Bag 2:** There are a total of 40 marbles in the bag. Of the 40 marbles, 10 are red. **Bag 3:** There are 25 red marbles and 75 blue marbles in the bag. **Bag 4:** The ratio of red marbles to blue marbles is 1:4. **Bag 5:** There are 400 marbles in the bag. Of the 400 marbles, 100 are red. **Bag 6:** For every three blue marbles Tia puts in the bag, she puts 1 red marble. **Bag 7:** The number of blue marbles is three times the number of red marbles. *A percent is a part to whole ratio with a whole equal to 100. As students convert ratios to percents throughout the chapter, they will need to make sense of what they are given, what they are looking for, and how the two are related. Models will be extremely useful in making sense of problems.*
	Look for and make use of structure.	Eli has 8 pints of ice cream. If a serving size is $\frac{2}{3}$ of a pint of ice cream, how many servings does he have? Estimate the answer. Draw a model to solve the problem. Then, write a number sentence to represent the problem. *Students construct a variety of models to divide with fractions. They connect these models to the number sense. Viewing the model and number sentence simultaneously helps students to understand why* $8 \div \frac{2}{3}$ *is equivalent to* $8 \times \frac{3}{2} = \frac{24}{2} = 12$. *We can create twenty-four thirds from 8 and then we pull them out two at a time.*
	Look for and express regularity in repeated reasoning.	Find 22% of 54. *There are a variety of strategies students can use to solve percent problems. One strategy is to use repeated reasoning. For the problem above, students may think that 10% of 54 is 5.4 so 20% is 10.8 and 1% is 0.54 so 2% is 1.08; therefore 22% of 54 is 10.8 + 1.08 or 11.88.* If 10% of a number is 8, what is… 20% of the number? 50% of the number? 100% of the number? *The students can apply this reasoning to a second type of percent problem they will encounter in the chapter - finding the whole given a part and a percent. If we are given a part and a percent, we can iterate that percent up and down until we reach 100% which represents the whole.*

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2.0 Anchor Problem:

Part 1:

Calvin's grandma, Maggie, is a math professor who loves to play math games with her grandson. One day, she said to him, "I am going to ask you some questions involving money. If you answer a question correctly, I will give you the amount of money equal to the answer."

Determine the amount of money Calvin can earn in each question.

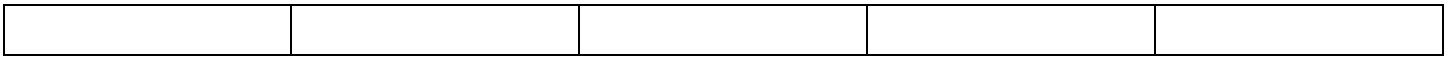
Directions: Use the tape diagram shown below to answer questions #1 and 2.

1. If the entire bar has a value of $1, what is the value of each box?
2. If the entire bar has a value of $100, what is the value of each box?

Directions: Use the tape diagram shown below to answer questions #3 and 4.

3. If the entire bar has a value of $1, what is the value of each box?
4. If the entire bar has a value of $100, what is the value of each box?

Directions: Use the tape diagram shown below to answer questions #5 and 6.



5. If the entire bar has a value of $1, what is the value of each box?
6. If the entire bar has a value of $100, what is the value of each box?

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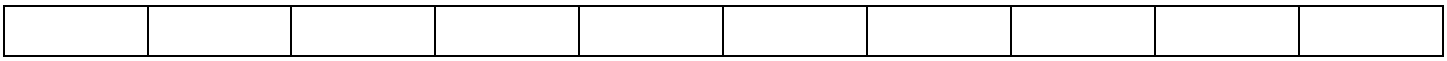
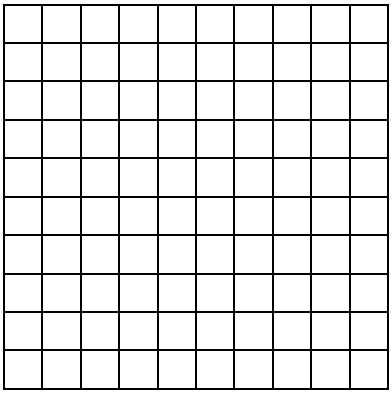
Directions: Use the tape diagram shown below to answer questions #7 and 8.



7. If the entire bar has a value of $1, what is the value of each box?

8. If the entire bar has a value of $100, what is the value of each box?

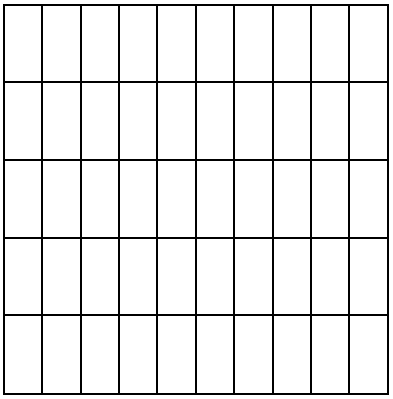
Directions: The grid below is a 10 by 10 grid (100 total squares). Use the grid to answer questions #9 and 10.



9. If the entire grid has a value of $1, what is the value of each box?

10. If the entire grid has a value of $100, what is the value of each box?

Directions: The grid below is a 10 by 5 grid (50 total squares). Use the grid to answer questions #11 and 12.



11. If the entire grid has a value of $1, what is the value of each small square in the grid?

12. If the entire grid has a value of $100, what is the value of each small square in the grid?

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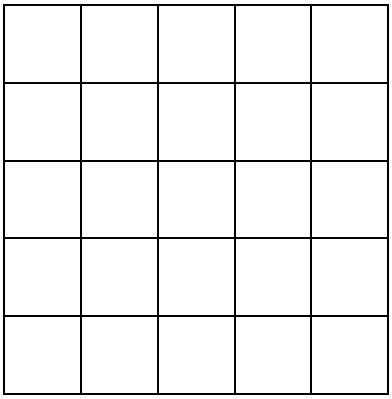
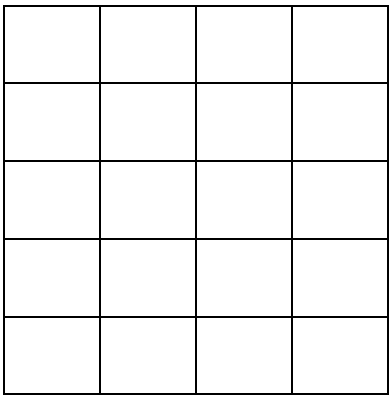
Directions: The grid below is a 5 by 5 grid (25 total squares). Use the grid to answer questions #13 and 14.



13. If the entire grid has a value of $1, what is the value of each small square in the grid?

14. If the entire grid has a value of $100, what is the value of each small square in the grid?

Directions: The grid below is a 4 by 5 grid (20 total squares). Use the grid to answer questions #15 and 16.



15. If the entire grid has a value of $100, what is the value of each small square in the grid?

16. If the entire grid has a value of $1, what is the value of each small square in the grid?

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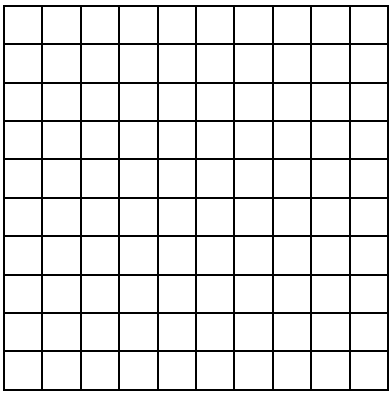
Part 2:



1. If the entire grid has a value of $100, what is the value of each small square in the grid?

2. If the entire grid has a value of $100, what is the value of 10 small squares in the grid?

3. If the entire grid has a value of $200, what is the value of each small square in the grid?

4. If the entire grid has a value of $200, what is the value of 10 small squares in the grid?

5. If the entire grid has a value of $80, what is the value of each small square in the grid?

6. If the entire grid has a value of $80, what is the value of 10 small squares in the grid?

7. If the entire grid has a value of $120, what is the value of each small square in the grid?

8. If the entire grid has a value of $120, what is the value of 10 small squares in the grid?

9. If the entire grid has a value of $24, what is the value of each small square in the grid?

10. If the entire grid has a value of $24, what is the value of 10 small squares in the grid?

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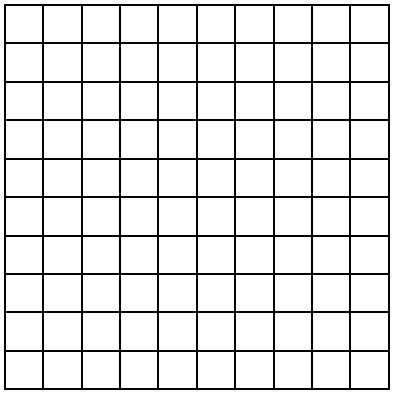
Part 3:



1. If 1 small square has a value of $0.3, what is the value of the entire grid?

2. If 1 small square has a value of $0.78, what is the value of the entire grid?

3. If 1 small square has a value of $2.1, what is the value of the entire grid?

4. If 2 small squares have a value of $5, what is the value of the entire grid?

5. If 2 small squares have a value of $1.1, what is the value of the entire grid?

6. If 10 small squares have a value of $9, what is the value of the entire grid?

7. If 10 small squares have a value of $6.4, what is the value of the entire grid?

8. If 80 small squares have a value of $88, what is the value of the entire grid?

9. If 16 small squares have a value of $32, what is the value of the entire grid?

10. If 75 small squares have a value of $30, what is the value of the entire grid?

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Section 2.1: Ratios out of 100

Section Overview:
In this section, students will learn to fluently transition between fractions, decimals, ratios, and percents. The section begins by introducing students to percent – a rate per 100. They explore problems that highlight the value of percent as a tool that can be used to compare ratios and fractional amounts of different quantities. The skills learned in Chapter 1 of building rates up to, or down to, rates out of 100 will be used continuously in this section. Students will be encouraged to use tables, tape models, and double line models to build their understanding. In the end, students will use the strategy they like best. The goal is that students make sense of problem situations rather than memorize algorithmic strategies.

In 2.1a through 2.1d the focus is exclusively on part to whole relationships—transitioning between fractions, decimals and percents. A primary goal will be on helping students convert representations to equivalent rates out of 100 using a model or partial table. For example, students will learn to see $\frac{13}{25}$ as $\frac{52}{100}$ or $\frac{17}{200}$ as $\frac{8.5}{100}$ and that $\frac{3}{5}$ of 200 is the same as 120 out of 200 or $\frac{60}{100}$, thus turning conversions to equivalences.

In 2.1e, students transition to converting part-to-part ratios to a percent. In other words, they will note that if a part to part relationship is 2 to 3, then two different part to whole relationships may be derived: $\frac{2}{5}$ or $\frac{3}{5}$. This means there is 40% of one quantity and 60% of the other.

Then in 2.1f – 2.1i, students build on these understandings to find a part, percent, or whole given the other two. Foundational in these sections is the relationship between quantities. Students will be encouraged to use models or tables to solve problems rather than algorithms.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Understand a percent as a part to total ratio with a whole equal to 100.
2. Represent fractional amounts of a quantity as a percent.
3. Fluidly transition between quantities represented as a percent, fraction, decimal or ratio.
4. Find a part of a quantity given a percent and the whole.
5. Find the whole given a part and a percent.
6. Solve real-world percent problems.

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2.1a Class Activity: Introduction to Percent as Rate Per 100

Activity 1:

a. Justin, Ariana, and Longar all have summer jobs and are saving part of the money they earn:
   - Justin saves $4 for every $10 he earns.
   - Ariana saves $9 for every $25 she earns.
   - Longar saves $7 for every $20 he earns.

If they all earn the same amount of money over the summer, who will save the most? Who will save the least? Justify your answer.

b. Stefan took three math tests last quarter. These are the scores he got on the tests:
   - Test 1: 23 correct out of 25
   - Test 2: 48 correct out of 50
   - Test 3: 18 correct out of 20

Which test did Stefan do the best on?

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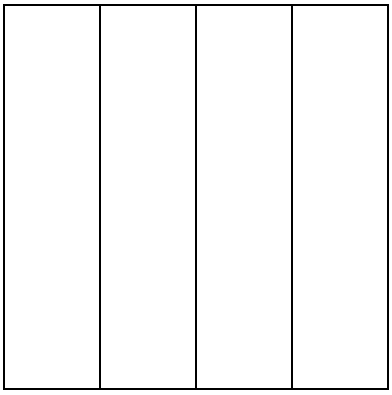
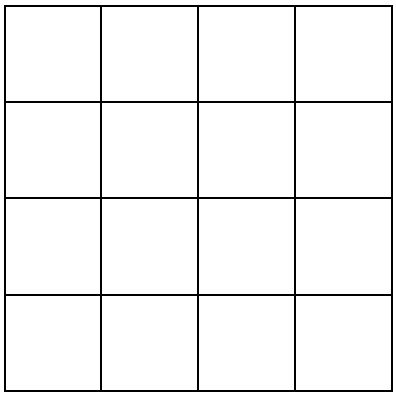
Activity 2: Each of the large squares below are the same size.

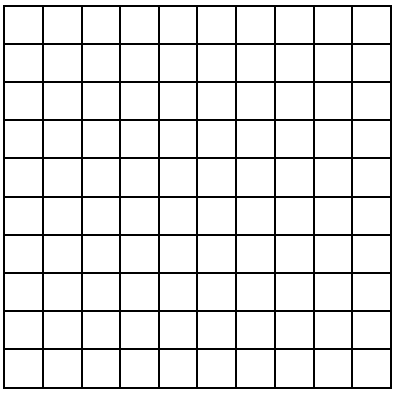
a. Shade each of the following models to represent $\frac{1}{4}$. Then write the fraction that represents the part that is shaded under the model.



Fraction: _____

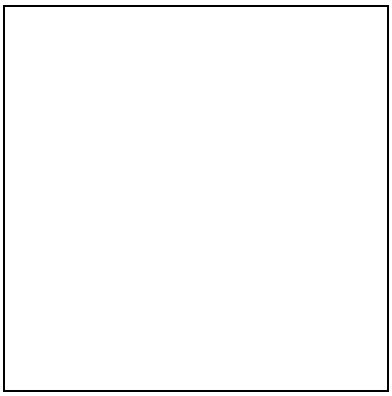


Fraction: _____



Fraction: _____

b. Cut up the square below in a way that is different than the three above and shade it to show $\frac{1}{4}$. Write the fraction that represents the shaded part under the model.



Fraction: _____

c. What percent of each of the grids in parts a. and b. above is shaded? Explain.

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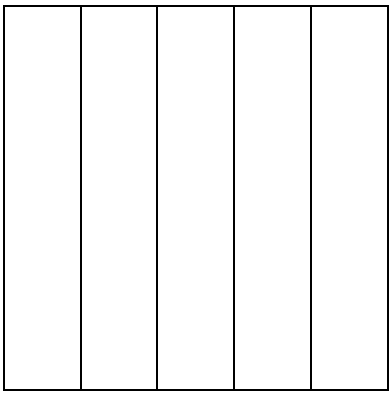
Activity 3: Each of the large squares shown below are the same size.

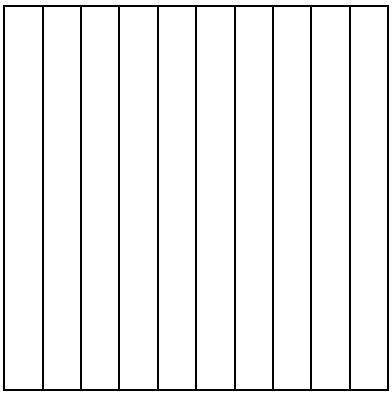
a. Shade $\frac{3}{5}$ of each model.

b. Express the shaded part of each model as a fraction and a percent.



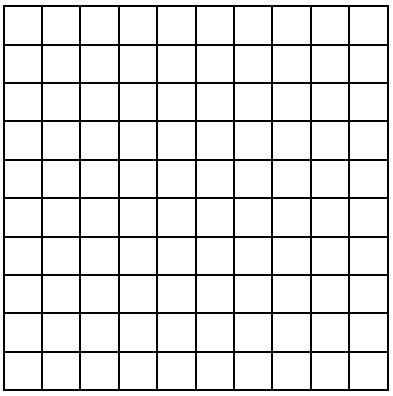
Fraction: _____

Percent: _____



Fraction: _____

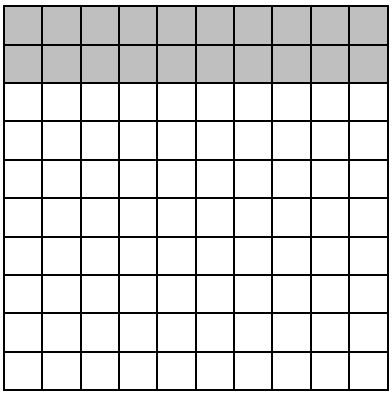
Percent: _____



Fraction: _____

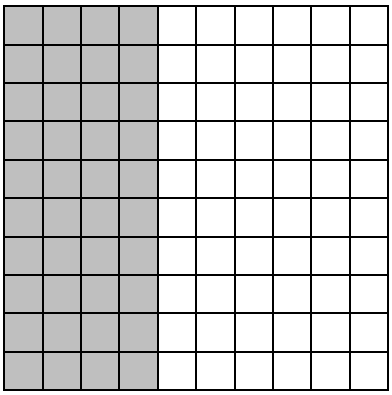
Percent: _____

Activity 4: Write the fraction and percent of each figure that is shaded.



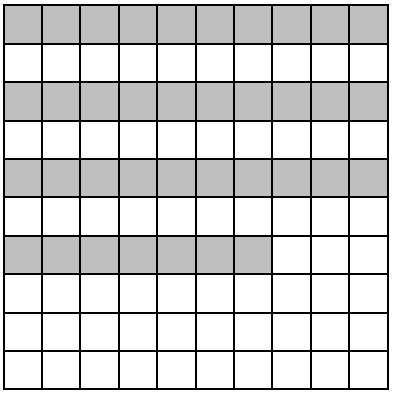
Fraction: _____

Percent: _____



Fraction: _____

Percent: _____



Fraction: _____

Percent: _____

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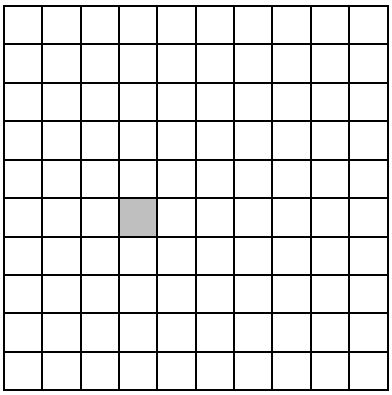
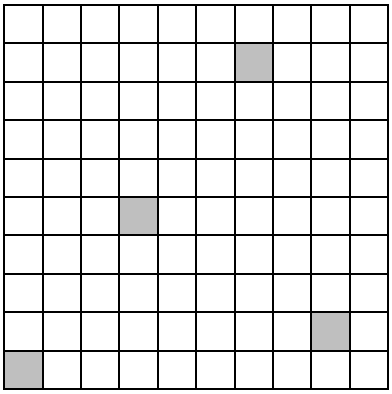


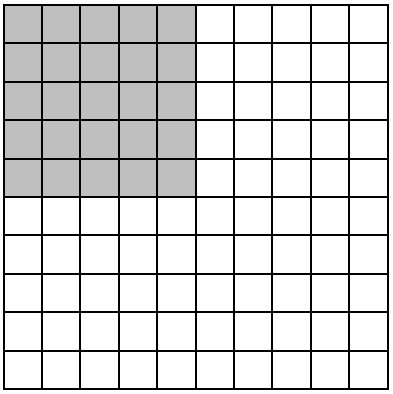
Fraction: _____

Percent: _____



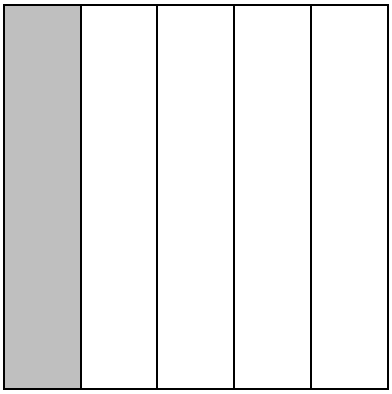
Fraction: _____

Percent: _____



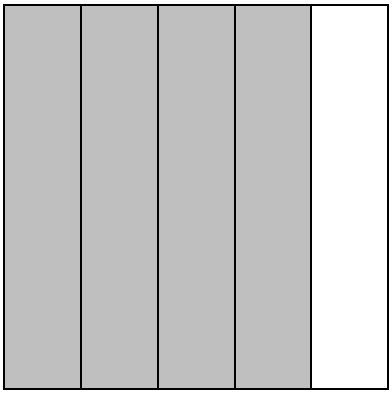
Fraction: _____

Percent: _____



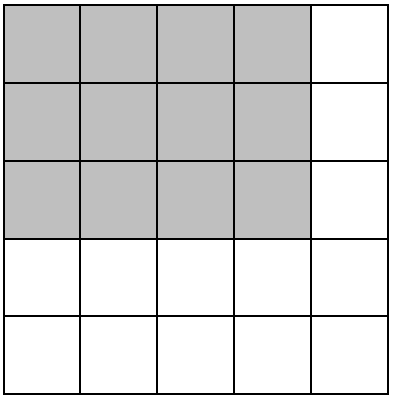
Fraction: _____

Percent: _____



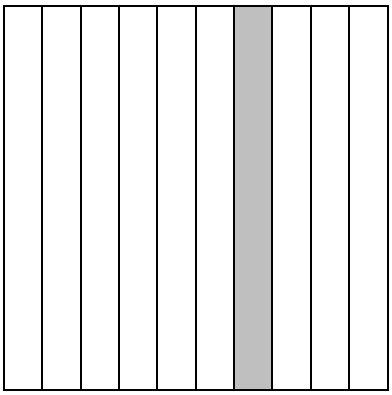
Fraction: _____

Percent: _____



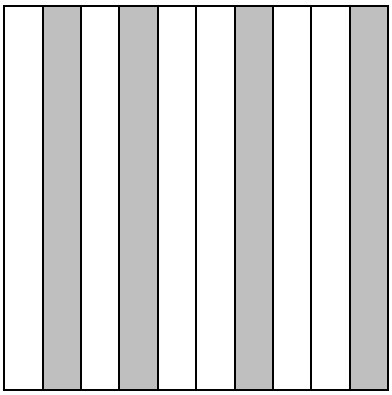
Fraction: _____

Percent: _____



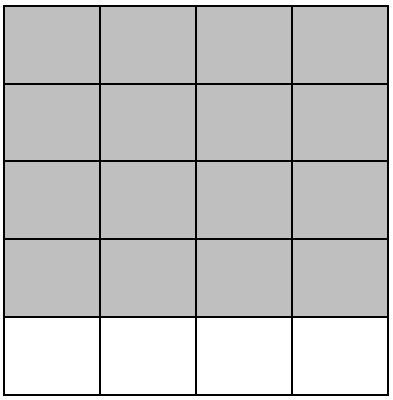
Fraction: _____

Percent: _____



Fraction: _____

Percent: _____



Fraction: _____

Percent: _____

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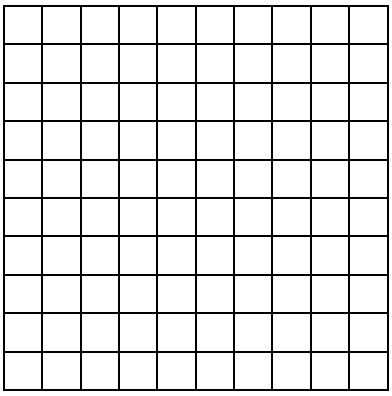
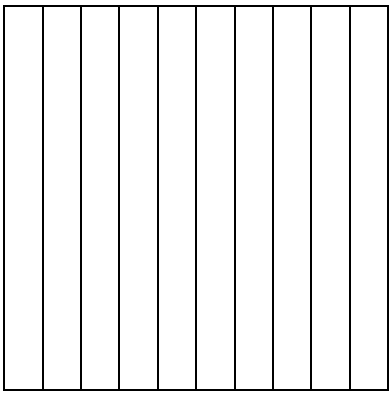
Activity 5:

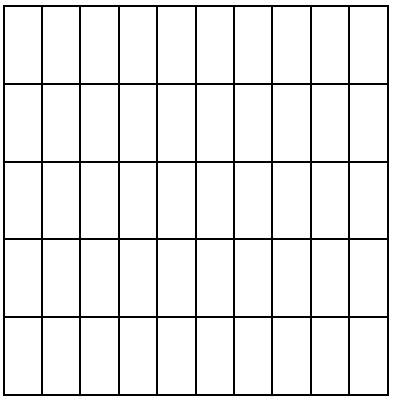
a. Shade 50% of each model. Then, write the fraction of the model that is shaded.



Fraction: _____

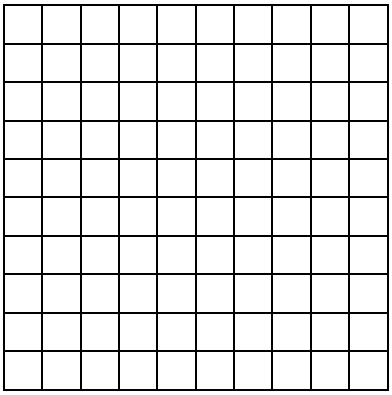


Fraction: _____

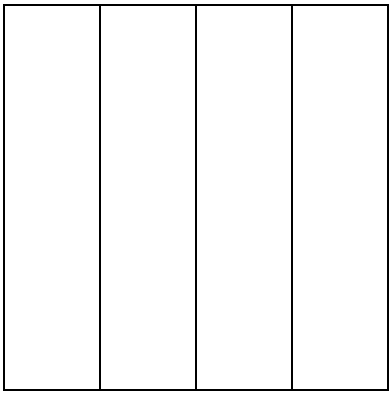


Fraction: _____

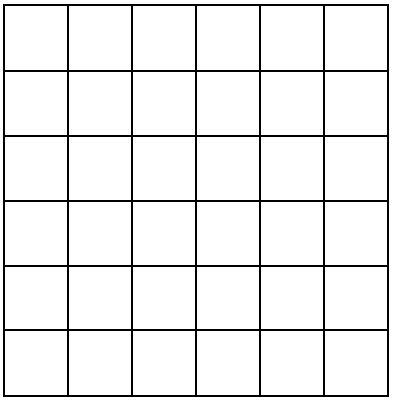
b. Shade 75% of each model. Then, write the fraction of the model that is shaded.



Fraction: _____



Fraction: _____



Fraction: _____

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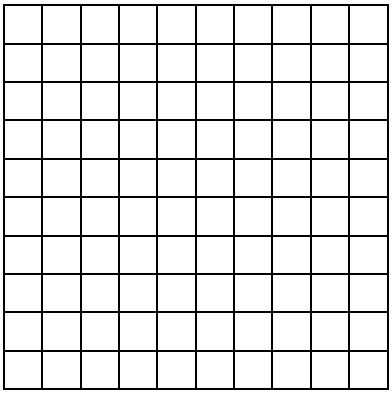
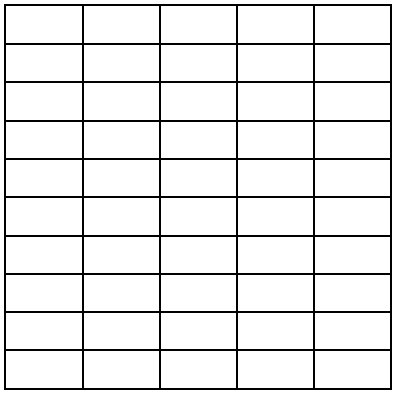
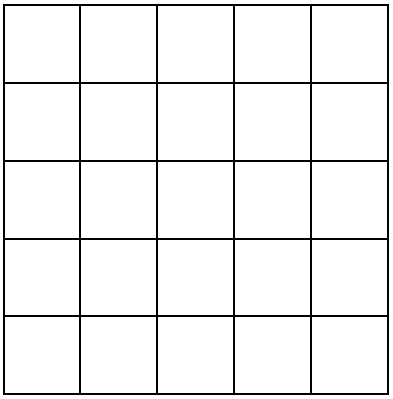
c. Shade 28% of each model. Then, write the fraction of the model that is shaded.



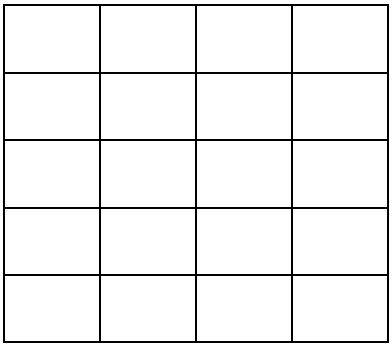
Fraction: _____



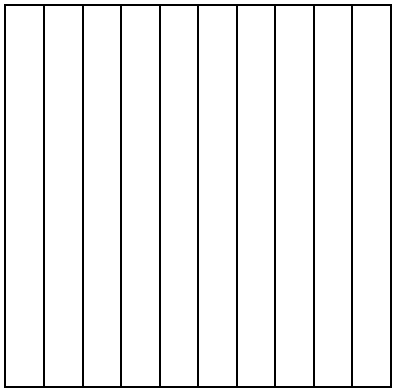
Fraction: _____



Fraction: _____



Fraction: _____



Fraction: _____

Fraction: _____

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Spiral Review

1. Write three fractions that are equivalent to $\frac{1}{2}$.

2. What number is $\frac{1}{2}$ of 30?

3. What number is $\frac{1}{2}$ of 50?

4. Express the following fractions as decimals.

a. $\frac{3}{10}$	b. $\frac{80}{100}$	c. $1\frac{9}{10}$
d. $\frac{45}{100}$	e. $\frac{150}{100}$	f. $\frac{2}{5}$
g. $\frac{1}{4}$	h. $\frac{9}{20}$	i. $1\frac{1}{2}$

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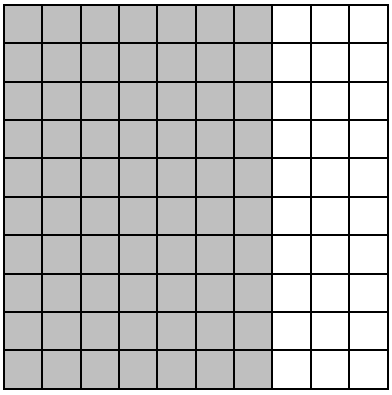
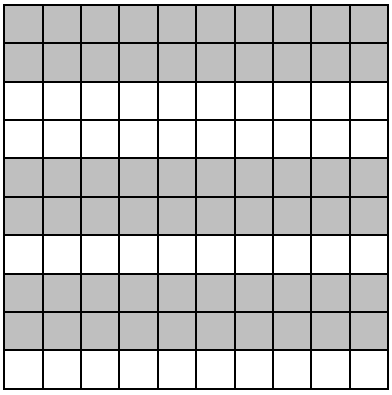
2.1a Homework: Introduction to Percent as Rate Per 100

1. Write the fraction and percent of each figure that is shaded.



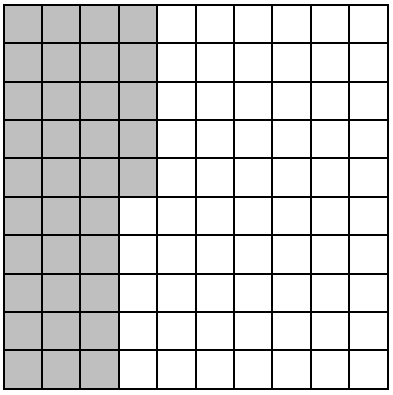
Fraction: _____

Percent: _____



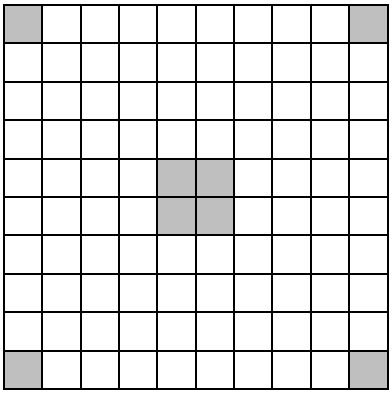
Fraction: _____

Percent: _____



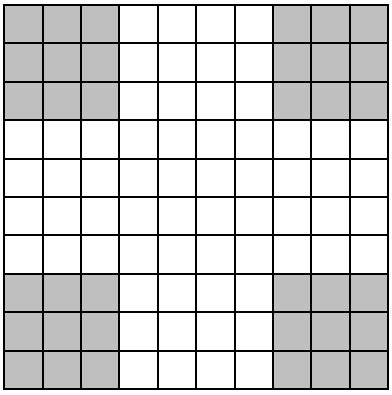
Fraction: _____

Percent: _____



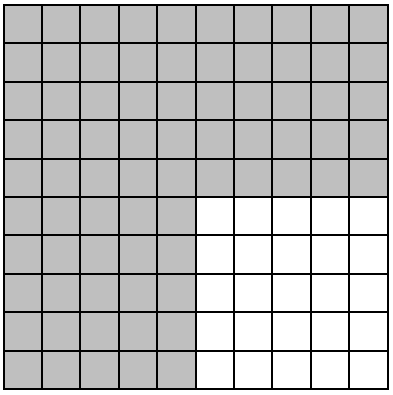
Fraction: _____

Percent: _____



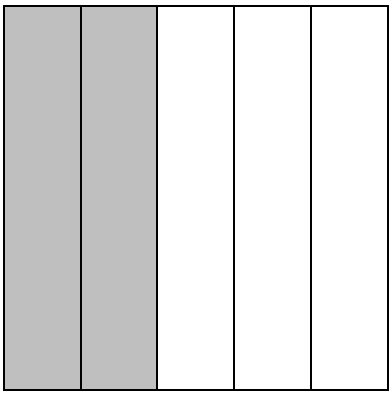
Fraction: _____

Percent: _____



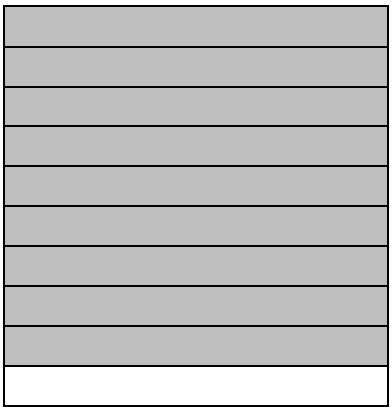
Fraction: _____

Percent: _____



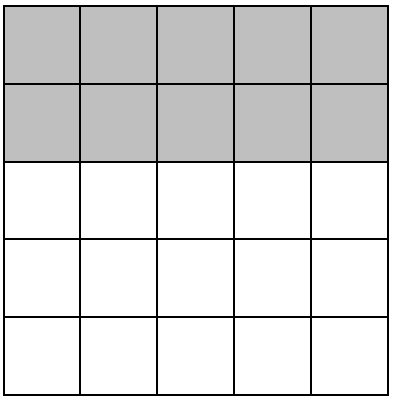
Fraction: _____

Percent: _____



Fraction: _____

Percent: _____



Fraction: _____

Percent: _____

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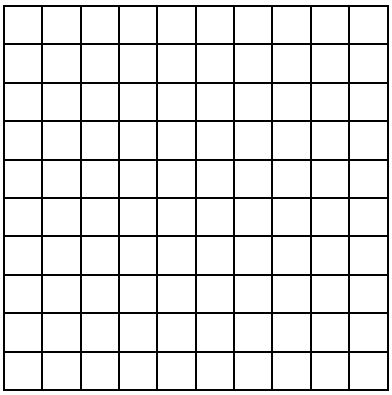
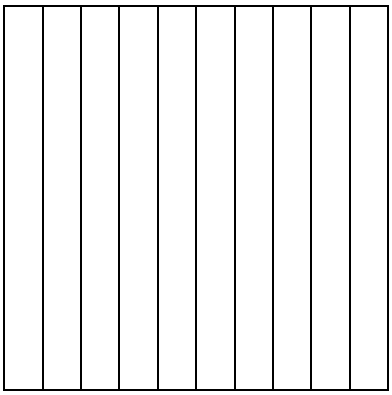
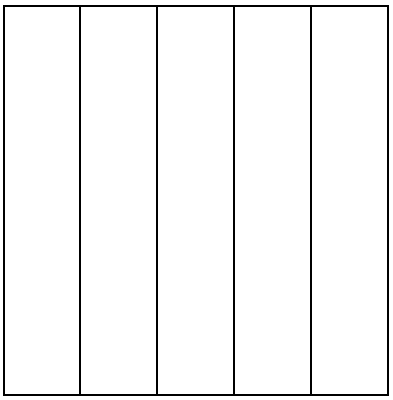
2. Shade 80% of each model. Then, write the fraction of the model that is shaded.



Fraction: _____

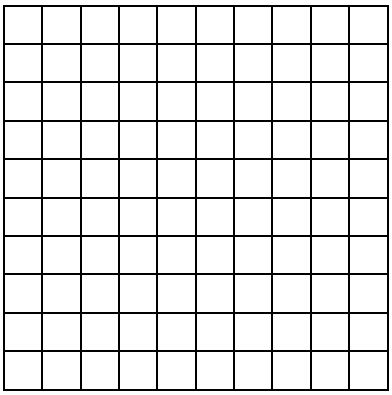


Fraction: _____

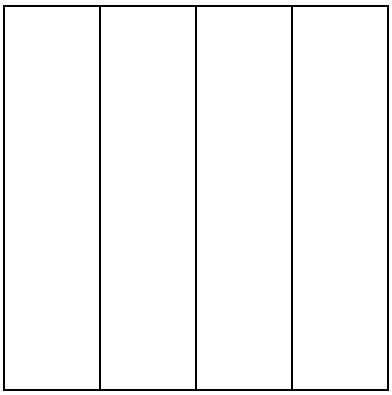


Fraction: _____

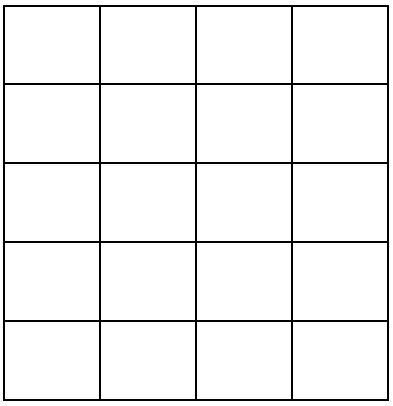
3. Shade 25% of each model. Then, write the fraction of the model that is shaded.



Fraction: _____

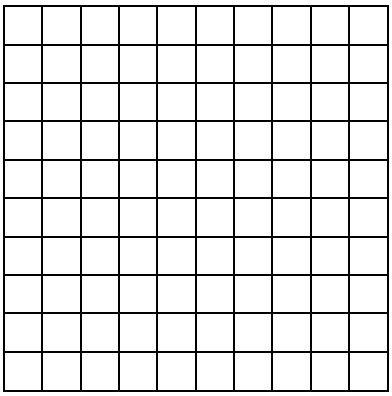


Fraction: _____

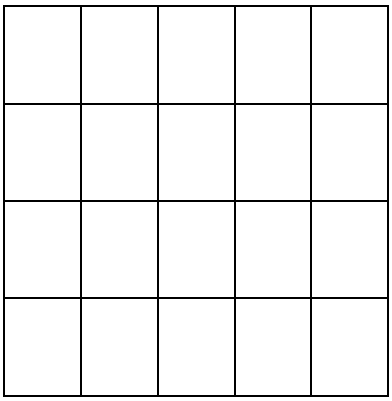


Fraction: _____

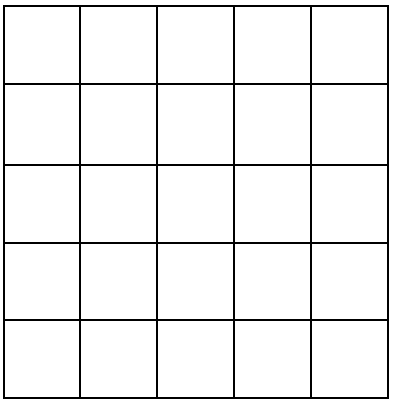
4. Shade 35% of each model. Then, write the fraction of the model that is shaded.



Fraction: _____



Fraction: _____



Fraction: _____

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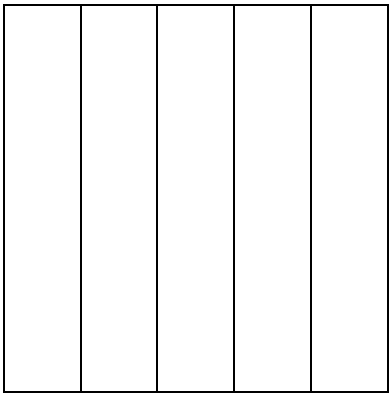
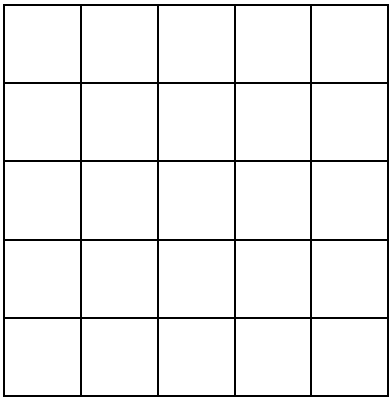
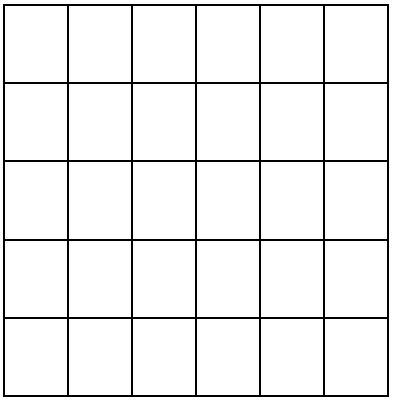
5. Shade 50% of each model. Then, write the fraction of the model that is shaded.



Fraction: _____



Fraction: _____



Fraction: _____

6. Draw three different models that represent 12%. Use grid or graph paper if needed.

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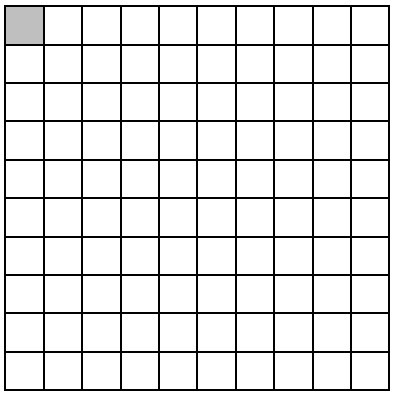
2.1b Class Activity: Fraction, Decimal, Percent Equivalences

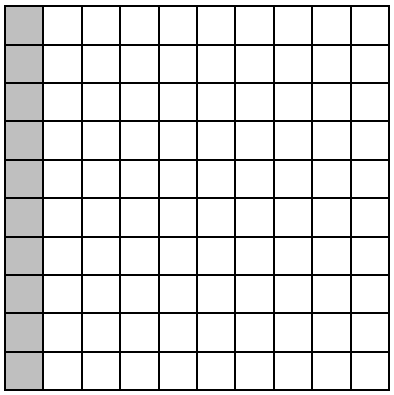
Directions: Express the shaded portion of each grid as a fraction, decimal, and percent. One large square represents the whole.

1.



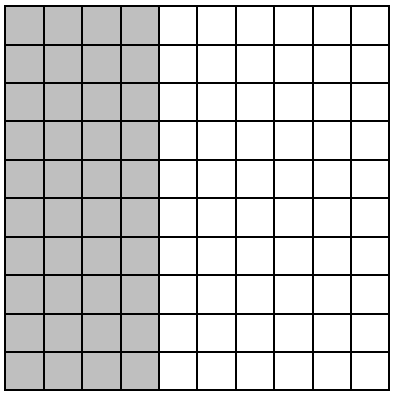
Fraction: _____ Decimal: _____ Percent: _____

2.



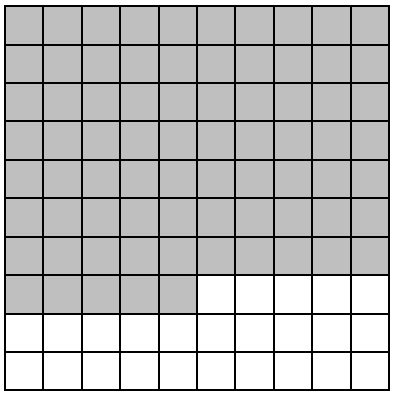
Fraction: _____ Decimal: _____ Percent: _____

3.



Fraction: _____ Decimal: _____ Percent: _____

4.



Fraction: _____ Decimal: _____ Percent: _____

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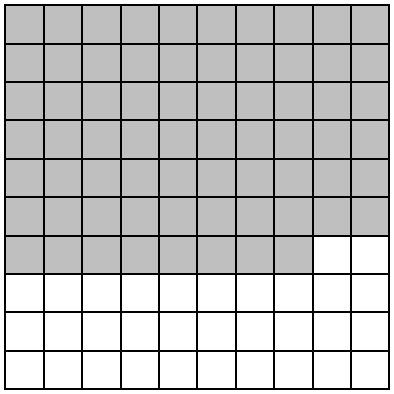

Fraction: _____ _ Decimal: _____ Percent: _____

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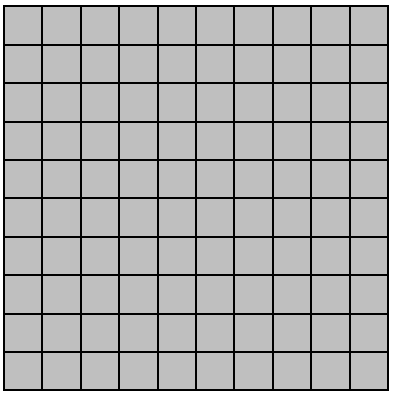

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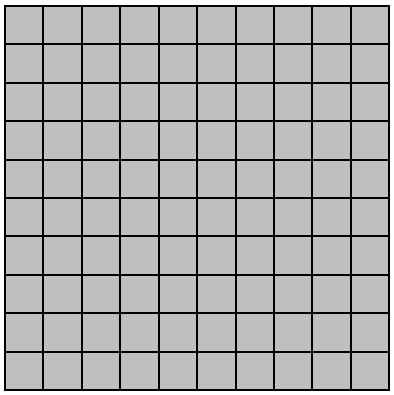

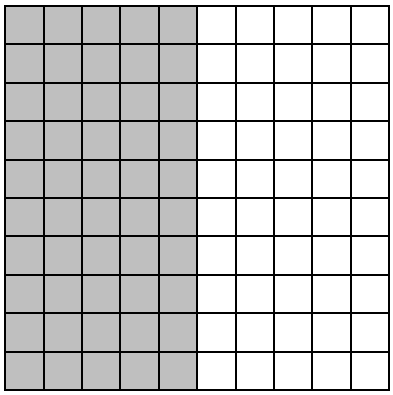

Fraction: _____ Decimal: _____ Percent: _____

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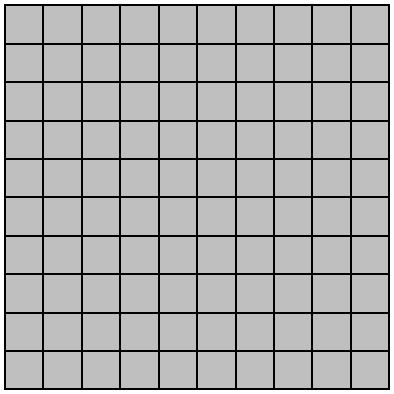

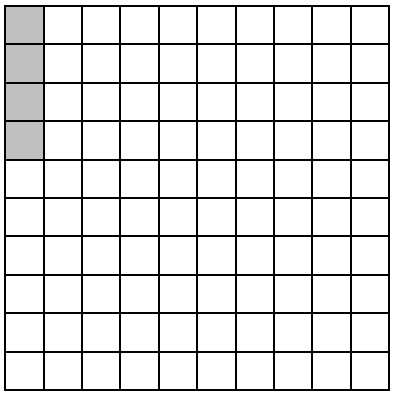

Fraction: _____ Decimal: _____ Percent: _____

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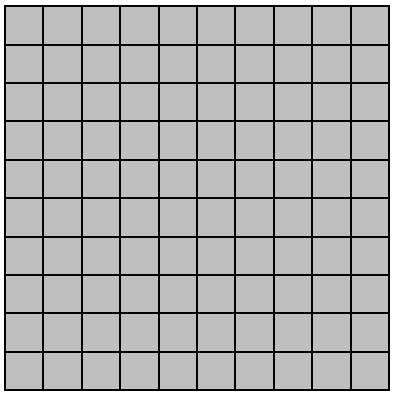
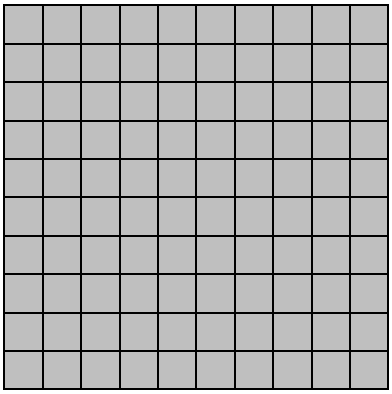
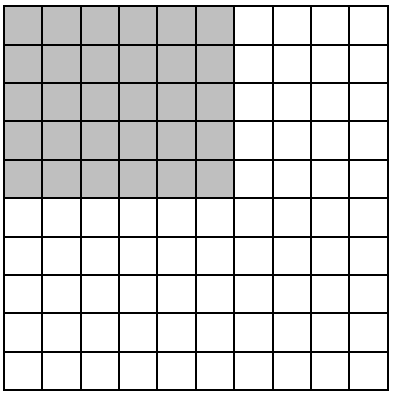
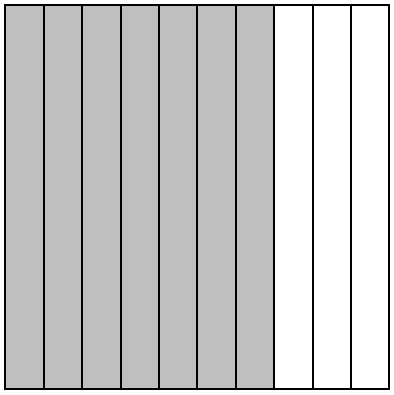






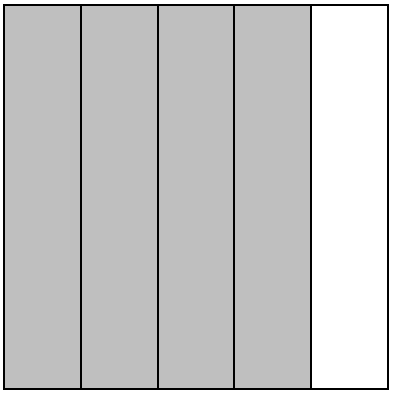
Fraction: _____ Decimal: _____ Percent: _____

10.



Fraction: _____ Decimal: _____ Percent: _____

11.



Fraction: _____ Decimal: _____ Percent: _____

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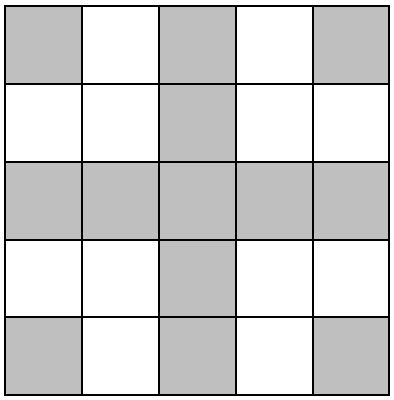
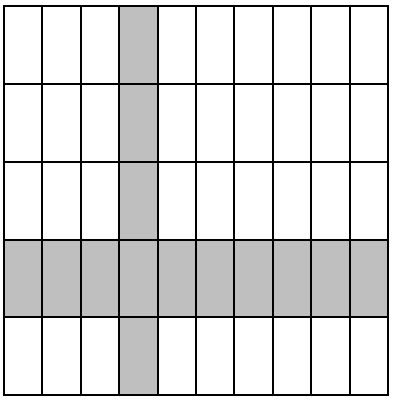
12.



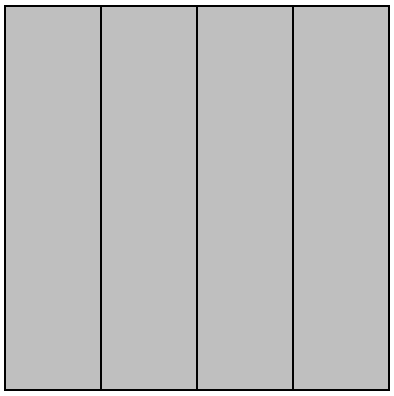
Fraction: _____ Decimal: _____ Percent: _____

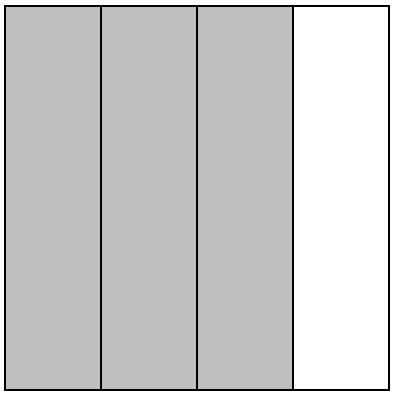
13.



Fraction: _____ Decimal: _____ Percent: _____

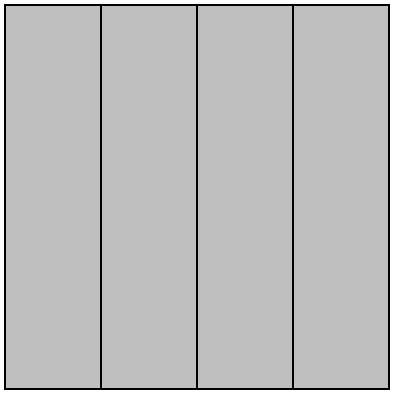
14.

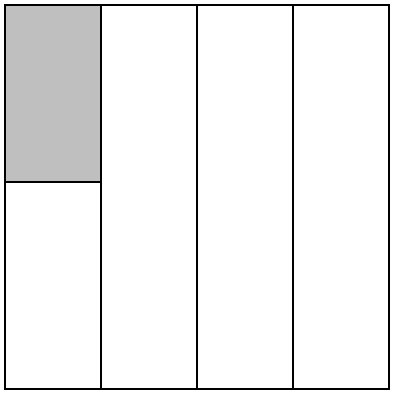




Fraction: _____ Decimal: _____ Percent: _____

15.





Fraction: _____ Decimal: _____ Percent: _____

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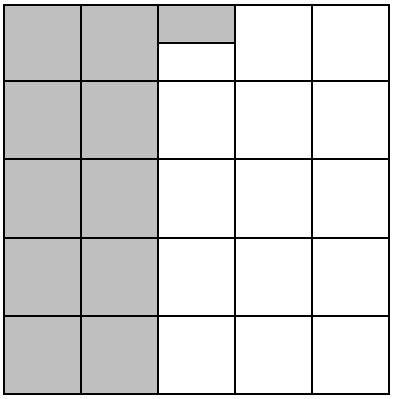
16.



Fraction: _____ Decimal: _____ Percent: _____

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Directions: Complete the table.

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	Fraction	Decimal	Percent
17.	$\frac{35}{100}$		
18.			80%
19.		0.75	
20.	$\frac{72}{100}$		
21.		0.06	
22.			25%
23.	$\frac{9}{10}$		
24.		0.5	
25.			36%
26.	$\frac{2}{5}$		
27.		0.92	
28.			8%
29.	$\frac{22}{25}$		
30.	$\frac{130}{100}$		
31.			145%
32.		1.5	
33.	$\frac{14}{200}$		
34.	$\frac{20}{500}$		
35.	$\frac{4}{400}$		
36.	$\frac{3}{12}$		
37.	$\frac{6}{8}$		
38.	$\frac{21}{35}$		

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Directions: How do the portions of the whole relate? Compare using $<, >$, or $=$.

39. $\frac{9}{10}$ _____ 90%

40. $\frac{6}{200}$ _____ 6%

41. 5% _____ 0.5

42. $\frac{14}{25}$ _____ 60%

43. $\frac{2}{3}$ _____ 75%

44. $\frac{3}{12}$ _____ 25%

45. 50% _____ $\frac{2}{5}$

46. $\frac{48}{50}$ _____ 48%

47. 125% _____ 1.3

48. $\frac{7}{4}$ _____ 175%

49. 1 _____ 100%

50. 1.01 _____ 99%

Directions: Put the following portions of a whole in order from smallest to largest. Justify your answer.

51. $\frac{9}{20}, 0.05, 50\%, \frac{11}{20}$

52. $118\%, 1\frac{1}{5}, 11.9\%, \frac{5}{4}$

Spiral Review

1. Find $\frac{1}{4}$ of 24.

2. Find $\frac{3}{4}$ of 24.

3. Simplify. Look for patterns.
   a. 100×420
   b. 10×420
   c. 1×420
   d. 0.1×420
   e. 0.01×420

4. Make a double number line to show the relationship between feet and inches.

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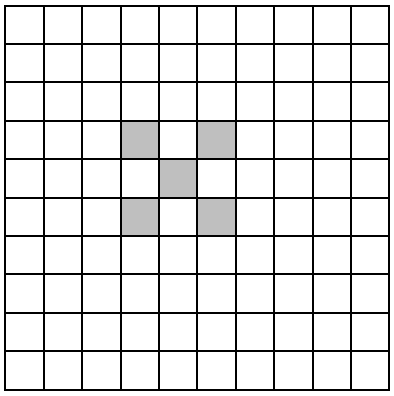
2.1b Homework: Fraction, Decimal, Percent Equivalences

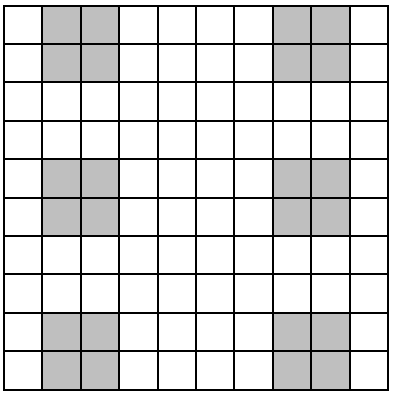
Directions: Express the shaded portion of each grid as a fraction, decimal, and percent. One large square represents the whole.

1.



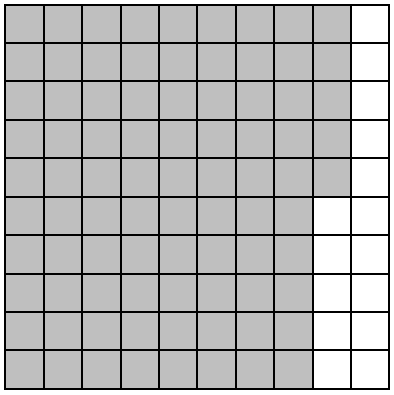
Fraction: _____ Decimal: _____ Percent: _____

2.



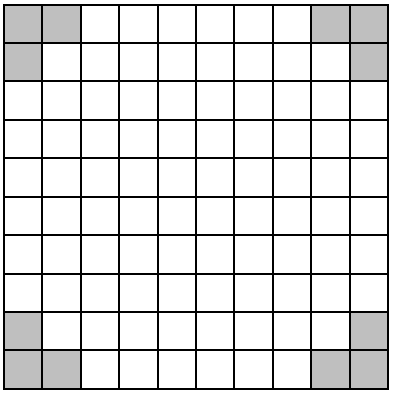
Fraction: _____ Decimal: _____ Percent: _____

3.



Fraction: _____ Decimal: _____ Percent: _____

4.



Fraction: _____ Decimal: _____ Percent: _____

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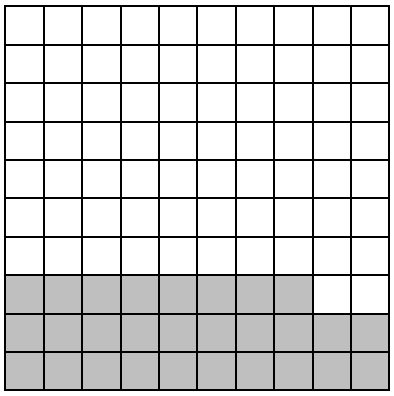
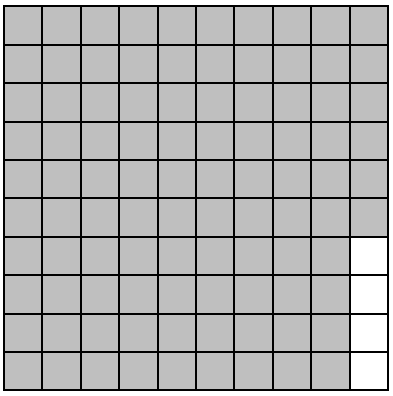
5.



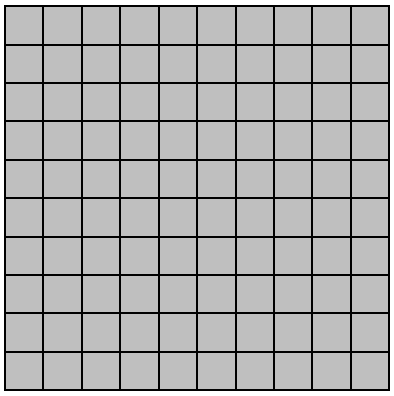
Fraction: _____ Decimal: _____ Percent: _____

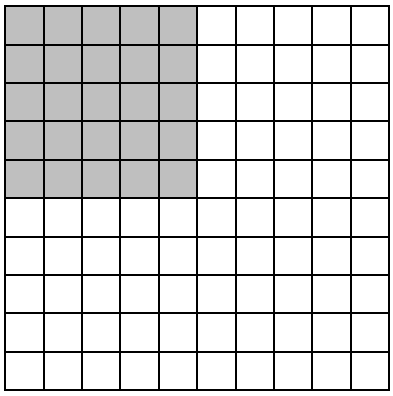
6.



Fraction: _____ Decimal: _____ Percent: _____

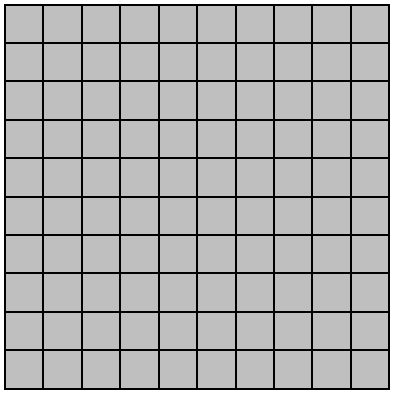
7.

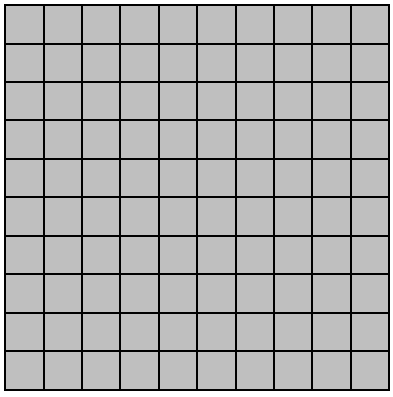




Fraction: _____ Decimal: _____ Percent: _____

8.





Fraction: _____ Decimal: _____ Percent: _____

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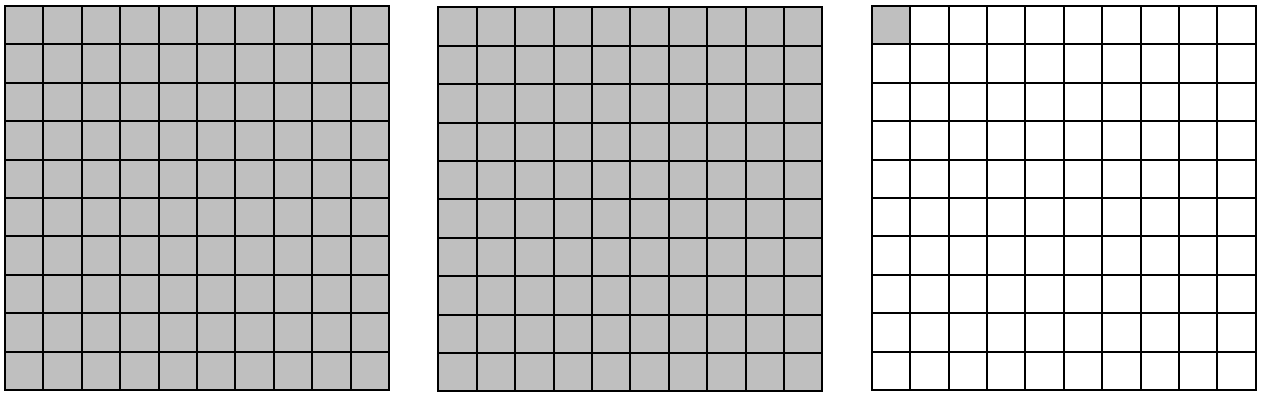
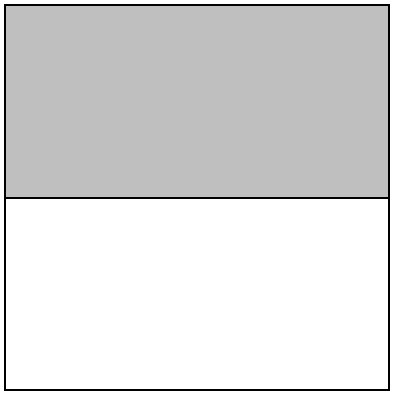
9.



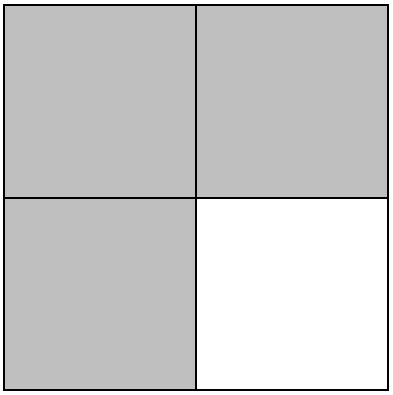
Fraction: _____ Decimal: _____ Percent: _____

10.



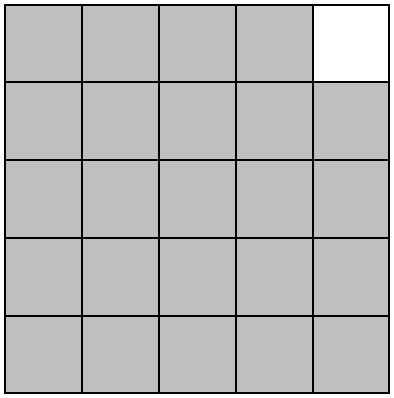
Fraction: _____ Decimal: _____ Percent: _____

11.



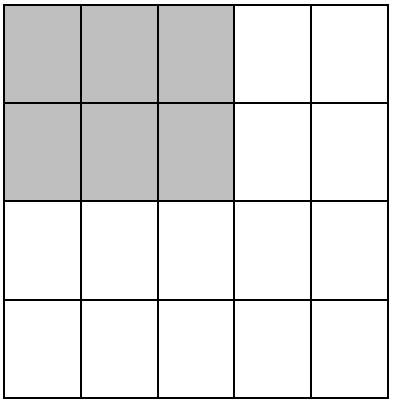
Fraction: _____ Decimal: _____ Percent: _____

12.



Fraction: _____ Decimal: _____ Percent: _____

13.



Fraction: _____ Decimal: _____ Percent: _____

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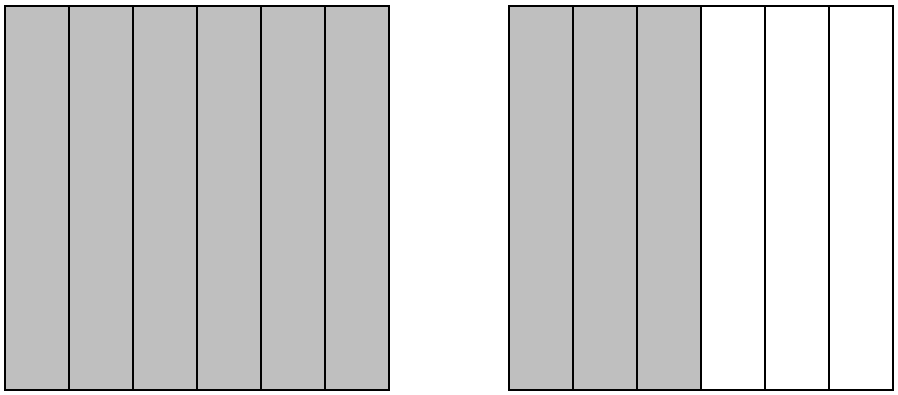
14.



Fraction: _____ Decimal: _____ Percent: _____

Directions: Complete the table.

	Fraction	Decimal	Percent
15.	$\frac{9}{100}$		
16.			88%
17.		0.45	
18.	$\frac{92}{100}$		
19.		0.01	
20.			75%
21.	$\frac{14}{20}$		
22.		0.84	
23.			56%
24.	$\frac{38}{50}$		
25.		0.2	
26.			2%
27.	$\frac{1}{10}$		
28.	$\frac{175}{100}$		

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29.			250%
30.		1.01	
31.	$\frac{100}{200}$		
32.	$\frac{33}{300}$		
33.	$\frac{100}{1000}$		
34.	$\frac{9}{12}$		
35.	$\frac{28}{40}$		
36.	$\frac{36}{90}$		

Directions: How do the portions of the whole relate? Compare using $<, >$, or $=$.

37. $\frac{8}{10}$ _____ 8%

38. $\frac{1}{5}$ _____ 20%

39. 45% _____ 0.045

40. 0.3 _____ 3%

41. $\frac{3}{20}$ _____ 16%

42. 0.39 _____ 40%

43. 33% _____ $\frac{8}{25}$

44. $\frac{1}{50}$ _____ 2%

45. 150% _____ 150

46. $\frac{5}{2}$ _____ 200%

47. 1.2 _____ 120%

48. $1\frac{1}{10}$ _____ 110%

Directions: Put the following portions of a whole in order from smallest to largest.

49. $\frac{7}{10}, 68\%, 0.08, \frac{36}{50}$

50. $\frac{9}{10}, 95\%, \frac{24}{25}, 0.099$

51. $\frac{13}{25}, 0.5, 49\%, \frac{50}{200}$

52. $199\%, \frac{200}{100}, 2\frac{1}{4}, 2.3$

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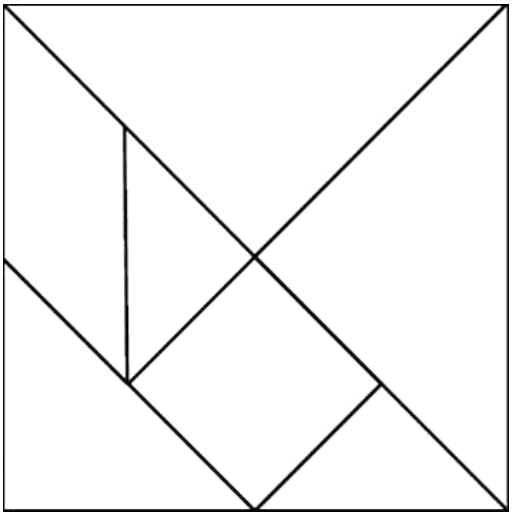
2.1c Class Activity: Tangrams and Percentages

Directions: Find the value of each shape relative to the entire square. Remember the large (entire) square represents 1 whole. Record your findings in the table below.



Name of Shape	Fraction	Decimal	Percent
Large triangle			
Medium triangle			
Small triangle			
Small Square			
Parallelogram			

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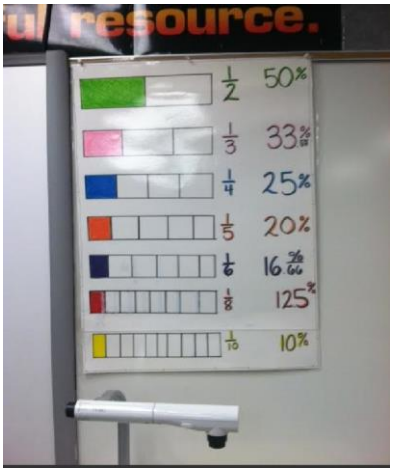
2.1c Homework: Benchmark Percentages

Directions: The bar models shown represent common percentages. Write the fraction, decimal, and percent that corresponds to each bar model. The first bar shown represents 1 whole or 100%. The first problem has been done for you.



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One Part: Fraction: $\frac{1}{1}$ or 1 Decimal: 1.0 Percent: 100%

One Part: Fraction: _____ Decimal: _____ Percent: _____

Two Parts: Fraction: _____ Decimal: _____ Percent: _____

One Part: Fraction: _____ Decimal: _____ Percent: _____

Two Parts: Fraction: _____ Decimal: _____ Percent: _____

Three Parts: Fraction: _____ Decimal: _____ Percent: _____

Four Parts: Fraction: _____ Decimal: _____ Percent: _____

One Part: Fraction: _____ Decimal: _____ Percent: _____

Two Parts: Fraction: _____ Decimal: _____ Percent: _____

Three Parts: Fraction: _____ Decimal: _____ Percent: _____

Four Parts: Fraction: _____ Decimal: _____ Percent: _____

Five Parts: Fraction: _____ Decimal: _____ Percent: _____

Six Parts: Fraction: _____ Decimal: _____ Percent: _____

Seven Parts: Fraction: _____ Decimal: _____ Percent: _____

Eight Parts: Fraction: _____ Decimal: _____ Percent: _____

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One Part: Fraction: _____ Decimal: _____ Percent: _____

Two Parts: Fraction: _____ Decimal: _____ Percent: _____

Three Parts: Fraction: _____ Decimal: _____ Percent: _____

Four Parts: Fraction: _____ Decimal: _____ Percent: _____

Five Parts: Fraction: _____ Decimal: _____ Percent: _____

One Part: Fraction: _____ Decimal: _____ Percent: _____

Two Parts: Fraction: _____ Decimal: _____ Percent: _____

Three Parts: Fraction: _____ Decimal: _____ Percent: _____

Four Parts: Fraction: _____ Decimal: _____ Percent: _____

Five Parts: Fraction: _____ Decimal: _____ Percent: _____

Six Parts: Fraction: _____ Decimal: _____ Percent: _____

Seven Parts: Fraction: _____ Decimal: _____ Percent: _____

Eight Parts: Fraction: _____ Decimal: _____ Percent: _____

Nine Parts: Fraction: _____ Decimal: _____ Percent: _____

Ten Parts: Fraction: _____ Decimal: _____ Percent: _____

One Part: Fraction: _____ Decimal: _____ Percent: _____

Two Parts: Fraction: _____ Decimal: _____ Percent: _____

Three Parts: Fraction: _____ Decimal: _____ Percent: _____

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2.1d Class Activity: Fractions, Decimals, Percents in the Real World

Directions: Solve the following problems.

1. $\frac{3}{5}$ of the 6th grade class at a certain school own a cell phone.
   a. Make a tape diagram to represent this situation.

   b. What percent of the students own a cell phone? _____

   c. What percent of the students do not own a cell phone? _____

   d. What is the ratio of students who own a cell phone to the ratio of students who do not own a cell phone? _____

   e. If there are seventy-five 6th graders at this school, how many own a cell phone? _____

2. 25% of the marbles in a bag are red.
   a. Make a tape diagram to represent this situation.

   b. What fraction of the marbles are red? _____

   c. What fraction of the marbles are not red? _____

   d. What is the ratio of marbles that are red to marbles that are not red? _____

   e. Give some possible pairs of values for the number of marbles that are red and the number of marbles that are not red. Organize your results in a table.

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3. Students at a certain high school can choose from three different language classes. $\frac{9}{20}$ of the students choose Spanish, 37% choose French, and $\frac{9}{50}$ choose Chinese.
    a. Order the languages from the one the most number of students take to the one the least number of students take. Justify your answer.

4. Owen has put several different colored marbles into a bag. The table below shows the different color marbles and how many of each color are in the bag:

Color of Marble	**Number of Marbles**
Red	40
Orange	10
Yellow	20
Green	10
Blue	20

    a. Make a tape diagram to represent this situation.

    b. What percent of the marbles in the bag are red? _____

    c. What percent of the marbles in the bag are not red? _____

    d. What percent of the marbles are yellow or green? _____

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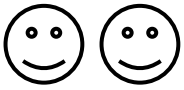
5. Hannah surveyed students at her school and asked what their favorite vegetable was. The pictograph shows the results of the survey.

= 25 students

Favorite Vegetable	Number of Students	Percent of Students
Tomatoes		
Green Beans		
Corn		
Carrots		
Broccoli		

a. Make a tape diagram to represent this situation.

b. Complete the table to show the percent of students who chose each vegetable.

c. What percent of students chose tomatoes or corn as their favorite vegetable? _____

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6. Noah surveyed the students in his class and asked how they got to school. Here are the results of the survey:

Mode of Transportation	Number of Students	Percent of Students
Car	16	
Bike	4	
Bus	8	
Walk	4	

   a. Make a tape diagram to represent this situation.

   b. Complete the table to show the percent of students who take each mode of transportation.

   c. Use the results of the survey to create a circle graph below. Be sure to include a key for your circle graph.

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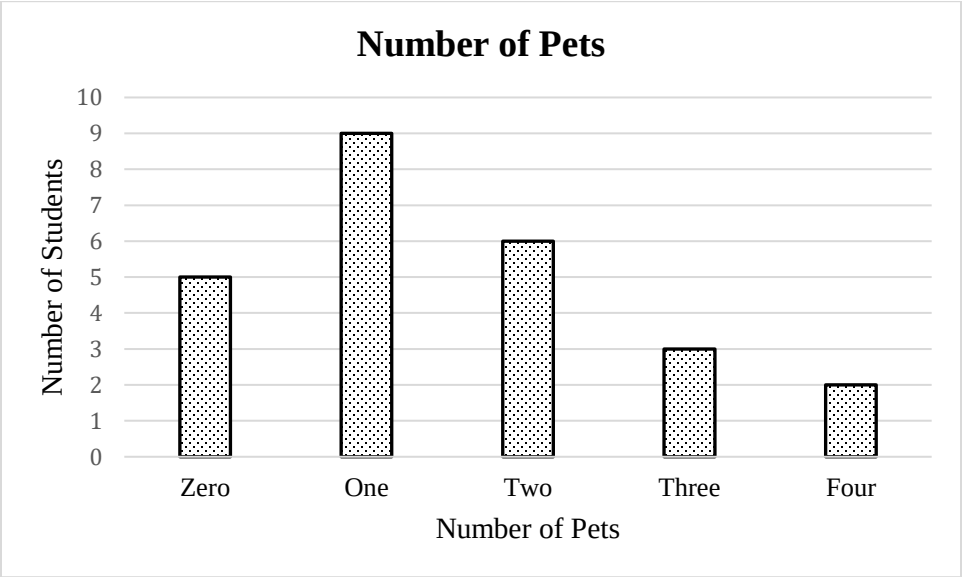
7. Renee surveyed the students in her class to see how many pets they have. The bar graph shows the results of the survey:



a. What percent of the students in Renee's class own two or more pets? _____

b. What percent of the students in Renee's class own fewer than two pets? _____

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Spiral Review

1. Carina is drawing circles and squares on her paper. The ratio of circles to squares in Carina's pattern is 2:5. Create Carina's pattern below.

2. Miguel is also drawing circles and squares on his paper. The ratio of circles to total shapes on Miguel's pattern is 2:5. Create Miguel's pattern below.

3. Simplify.

a. $\frac{1}{2}\times 20$	b. $\frac{1}{4}\times 20$	c. $\frac{1}{5}\times 20$	d. $\frac{1}{10}\times 20$

4. Complete the sentences.
   a. Multiplying by $\frac{1}{2}$ is the same as _____.
   b. Multiplying by $\frac{1}{4}$ is the same as _____.
   c. Multiplying by $\frac{1}{5}$ is the same as _____.
   d. Multiplying by $\frac{1}{10}$ is the same as _____.

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2.1d Homework: Fractions, Decimals, Percents in the Real World

Directions: Solve the following problems.

1. $\frac{1}{2}$ of the items in a bake sale are cookies.

    a. Draw a tape diagram to represent this situation.

    b. What percent of the items in the bake sale are cookies? _____

    c. What percent of the items in the bake sale are not cookies? _____

    d. What is the ratio of items that are cookies to items that are not cookies? _____

    e. If there are 150 cookies in the bake sale, how many items total are being sold? _____

2. $\frac{1}{5}$ of the students at Washington Middle School participate in the school band.

    a. Draw a tape diagram to represent this situation.

    b. What percent of the students participate in band? _____

    c. What percent of the students do not participate in band? _____

    d. What is the ratio of students who participate in band to students who do not participate in band? _____

    e. What is the ratio of students who participate in band to total students? _____

    f. If there are 200 students at Washington Middle School, how many participate in band? _____

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3. Sixty percent of the students in Ms. Serr's class are boys.

    a. Draw a tape diagram to represent this situation.

    b. What fraction of the class is boys? _____

    c. What fraction of the class is girls? _____

    d. If there are 30 students in Ms. Serr's class, how many are girls and how many are boys?

4. Seventy percent of iPhone users use the calculator application on their phone.
    a. Draw a tape diagram to represent this situation.

    b. What fraction of iPhone users use the calculator application on their phone? _____

    c. In a group of 60 people, how many would you expect use the calculator application on their phone?

5. Eli puts 75% of the money he earns working for his grandfather in the bank. Lucy puts $\frac{36}{50}$ of the money she earns working for her grandfather in the bank. If they earn the same amount, who puts more money in the bank? Justify your answer.

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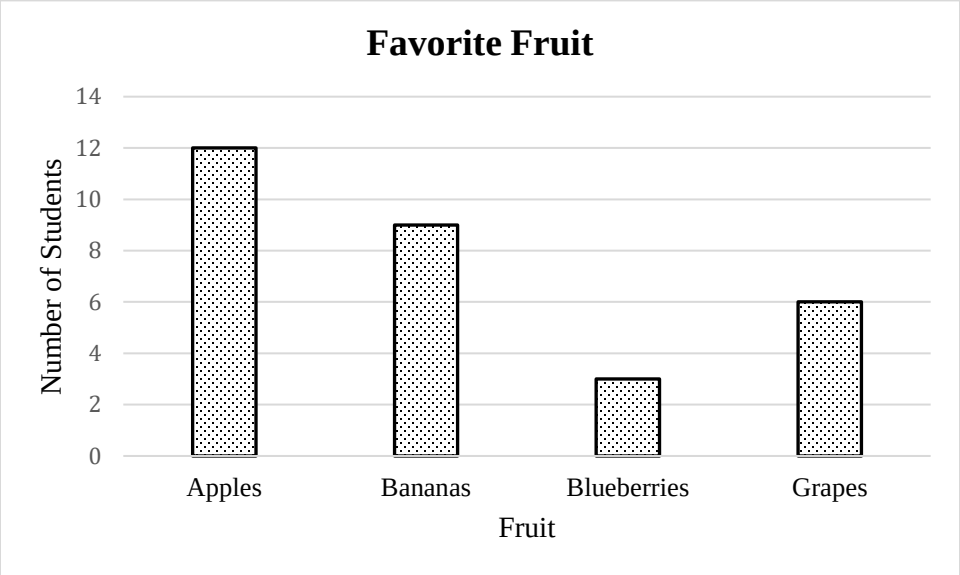
6. Jennifer surveyed the students in her class and asked what their favorite fruit is. The bar graph shows the results of the survey.



a. What percent of the students in Jennifer's class chose each type of fruit?

Apples: _____ Bananas: _____ Blueberries: _____ Grapes: _____

7. Zoe is on a competitive soccer team. The table below shows her team's record in the regular season:

Outcome	Number of Games
Wins	12
Losses	2
Ties	1

a. What percent of the soccer games did Zoe's team win? (A tie is not considered a win.)

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2.1e Class Activity: Percent as a Part to Total Ratio

Directions: Solve the following problems.

1. Tia is putting red and blue marbles into different bags. In which of the bags are 25% of the marbles red? Justify your answers.
    a. **Bag 1:** One out of every four marbles in the bag is red.

    b. **Bag 2:** There are a total of 40 marbles in the bag. Of the 40 marbles, 10 are red.

    c. **Bag 3:** There are 25 red marbles and 75 blue marbles in the bag.

    d. **Bag 4:** The ratio of red marbles to blue marbles is 1:4.

    e. **Bag 5:** There are 400 marbles in the bag. Of the 400 marbles, 100 are red.

    f. **Bag 6:** For every three blue marbles Tia puts in the bag, she puts 1 red marble.

    g. **Bag 7:** The number of blue marbles is three times the number of red marbles.

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2. Consider the following situations about five different basketball teams. Circle the letters of the teams that have the same winning percentage. Justify your answer.
    a. Team A wins four out of every 5 games it plays.

    b. For Team B, the ratio of games won to games lost is 4 to 5.

    c. For Team C, the ratio of games won to games lost is 4 to 1.

    d. Team D wins 24 out of 30 games.

    e. Team E wins three times as many games as it loses.

3. Cheryl is mixing red and white paint to make pink. To make the correct shade of pink, 60% of the mixture needs to be red. Which of the batches below will make the correct shade of pink?

    a. **Batch 1:** 40% of the mixture is white.

    b. **Batch 2:** The ratio of red paint to white paint is 6:10.

    c. **Batch 3:** Cheryl mixes 3 cups of red paint for every 2 cups of white paint.

    d. **Batch 4:** The amount of red paint is 1.5 times the amount of white paint.

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4. The following situations show the amount of money several different people save based on what they make. Find the percentage that each person saves.

   a. Jen saves $20 for every $100 she earns.

   b. For Brian, the ratio of dollars saved to dollars spent is 100 to 200.

   c. For every $9 Penelope spends, she saves $1.

   d. Each time Tiffany earns $50, she saves $25 of it.

   e. Drew spends three times more than he saves.

Spiral Review

1. What number is $\frac{1}{10}$ of 400? _____What number is $\frac{3}{10}$ of 400? _____

2. Find $\frac{4}{5}$ of 55. _____

3. Simplify. Look for patterns.

   a. 100×90 _____
   b. 10×90 _____
   c. 1×90 _____
   d. 0.1×90 _____
   e. 0.01×90 _____

4. Simplify. Look for patterns.

   a. 100×24 _____
   b. 10×24 _____
   c. 1×24 _____
   d. 0.1×24 _____
   e. 0.01×24 _____

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2.1e Homework: Percent as a Part to Total Ratio

1. Ricky is putting red and blue marbles into different bags. In which of the bags are 20% of the marbles red? Justify your answers.
    a. **Bag 1:** One out of every five marbles in the bag is red.

    b. **Bag 2:** There are 40 marbles in the bag. Of the 40 marbles, 8 are red.

    c. **Bag 3:** There are 20 red marbles and 80 blue marbles in the bag.

    d. **Bag 4:** The ratio of red marbles to blue marbles is 2 to 10.

    e. **Bag 5:** There are 400 marbles in the bag. Of the 400 marbles, 80 are red.

    f. **Bag 6:** There are five times as many blue marbles as red marbles.

2. Consider the following situations about four different people shooting free throws. Circle the letters of the people that have the same free throw percentage. Justify your answer.
    a. Piper makes 75% of her free throws.

    b. Mia makes 3 free throws for every 4 that she attempts.

    c. Evelyn makes 4 shots for every 1 that she misses.

    d. Fran makes 3 shots for every 1 that she misses.

    e. Harper makes two times more shots than she misses.

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3. Consider the following situations about how four different students did on an exam. Determine each student's score as a percentage if each problem is worth the same number of points.
    a. Erik missed 4 out of the 50 questions.

    b. For every 4 questions that Jon answered correctly, he missed 1.

    c. Malorie answered 4 out of every 5 questions correctly.

    d. For Dave, the ratio of correct answers to incorrect answers was 23 to 2.

    e. Trevor got three times more questions correct than incorrect.

4. Xander is mixing red and yellow paint to make orange paint. To make the correct shade of orange, 80% of the mixture needs to be red. Which of the batches below will make the correct shade of orange?

    a. **Batch 1:** 4 parts red to 1 part yellow

    b. **Batch 2:** $\frac{4}{10}$ of the mixture is red

    c. **Batch 3:** 1 out of every 4 cups is yellow

    d. **Batch 4:** For every 5 cups of paint, 4 are red.

    e. **Batch 5:** There is four times more red than yellow.

    f. **Batch 6:** There is five times more red than yellow.

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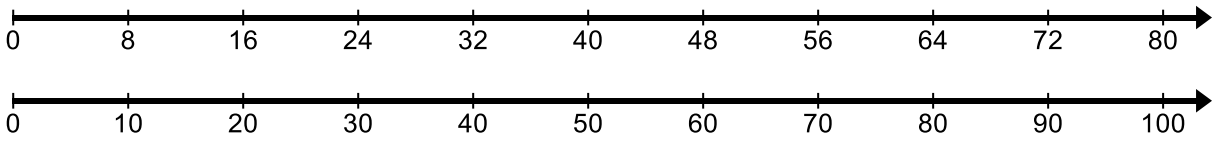
2.1f Class Activity: Types of Percent Problems

Activity 1:

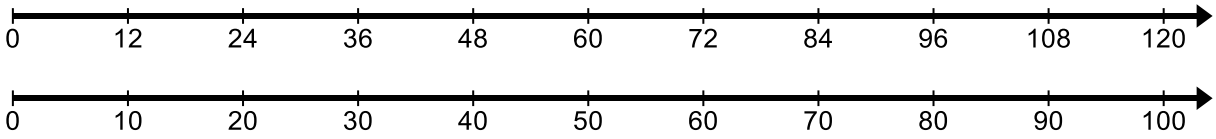
a. Write three statements that are true based on the vertical double number line shown.



b. Complete the following statements using the double number line shown in part a.

$\frac{50}{100} = \frac{?}{80}$ $\frac{30}{100} = \frac{?}{80}$ $\frac{?}{100} = \frac{72}{80}$ $\frac{16}{?} = \frac{20}{100}$

c. Compare the double number line below with the double number line from part a. Revise your statements from part a. so they describe the number line shown below.



d. Complete the following statements using the double number line shown in part c.

$\frac{50}{100} = \frac{?}{120}$ $\frac{30}{100} = \frac{?}{120}$ $\frac{?}{100} = \frac{72}{120}$ $\frac{20}{100} = \frac{24}{?}$

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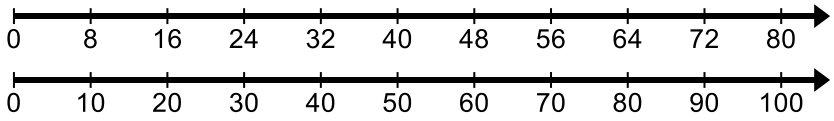
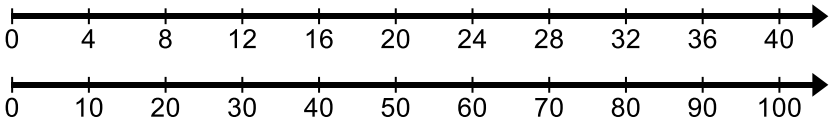
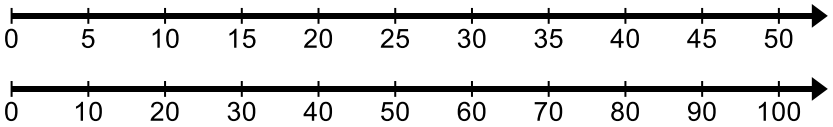
Activity 2: Draw lines to match each question to the double number line that can be used to solve the problem. Then, solve the problem.

What is 80% of 40?	
40 is 80% of what number.	
40 out of 80 is what percent?	

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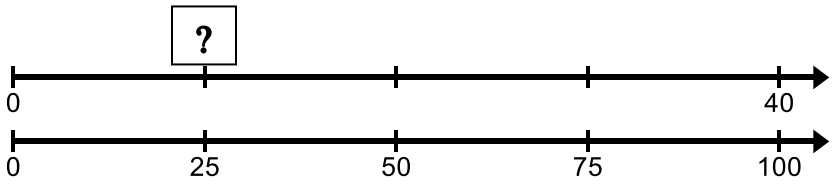
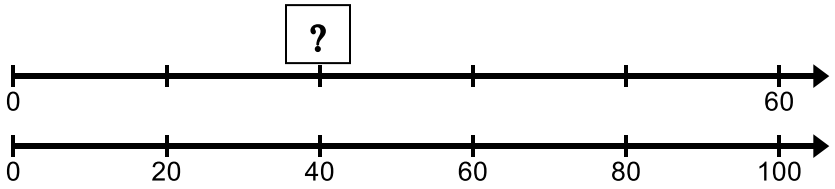
Activity 3: Write the question being asked by each model. Then, answer the question.

a. Question:



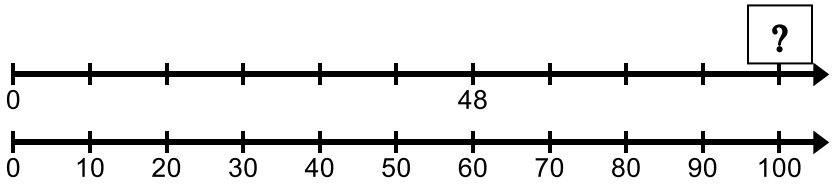
Answer:

b. Question:



Answer:

c. Question:



Answer:

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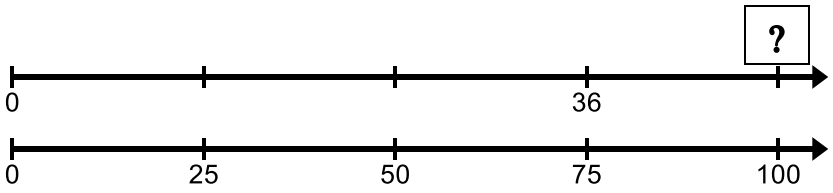
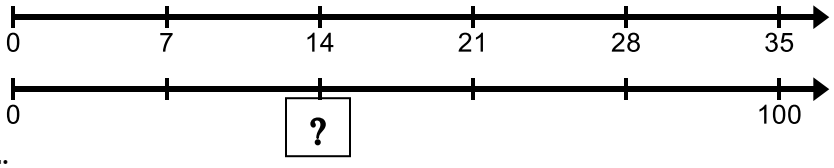
d. Question:



Answer:

Question:



Answer:

Spiral Review

1. What is $\frac{1}{10}$ of 30?

2. What is $\frac{1}{100}$ of 30?

3. Simplify. Look for patterns.
   a. $2{,}400 \div 10$
   b. $240 \div 10$
   c. $24 \div 10$
   d. $2.4 \div 10$

4. Simplify. Look for patterns.
   a. $8{,}000 \div 100$
   b. $800 \div 100$
   c. $80 \div 100$
   d. $80 \div 1{,}000$

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2.1f Homework: Getting Ready – Review Concepts

Directions: For each set of problems, draw a model. Then, answer the questions using the model and numeric strategies.

Set 1 Model:		
1. $\frac{1}{5}$ of 20	2. $\frac{3}{5}$ of 20	3. $\frac{4}{5}$ of 20

Set 2 Model:		
4. $\frac{1}{10}$ of 80	5. $\frac{5}{10}$ of 80	6. $\frac{9}{10}$ of 80

Set 3 Model:		
7. $\frac{1}{4}$ of 28	8. $\frac{2}{4}$ of 28	9. $\frac{3}{4}$ of 28

Directions: Simplify. Look for patterns.

10.

a. 100×30
b. 10×30
c. 1×30
d. 0.1×30
e. 0.01×30

11.

a. 100×78
b. 10×78
c. 1×78
d. 0.1×78
e. 0.01×78

12.

a. 0.1×80
b. 0.2×80
c. 0.3×80
d. 0.05×80
e. 0.01×80

13.

a. 0.1×50
b. 0.2×50
c. 0.3×50
d. 0.05×50
e. 0.01×50

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14.

a. $5000 \div 100$
b. $500 \div 100$
c. $50 \div 100$
d. $5 \div 100$

15.

a. $32{,}000 \div 100$
b. $3{,}200 \div 100$
c. $320 \div 100$
d. $32 \div 100$

Directions: Simplify each problem in a set. Then, think about the relationship between the problems in the set.

Set 1:

16. $40 \div 10$	17. $\frac{40}{10}$	18. $\frac{1}{10}$ of 40	19. 0.1×40

Set 2:

20. $40 \div 100$	21. $\frac{40}{100}$	22. $\frac{1}{100}$ of 40	23. 0.01×40

Set 3:

24. $12 \div 10$	25. $\frac{12}{10}$	26. $\frac{1}{10}$ of 12	27. 0.1 of 12

Set 4:

28. $32 \div 4$	29. $\frac{32}{4}$	30. $\frac{1}{4}$ of 32	31. 0.25 of 32

Directions: Simplify.

32. 0.5×16	33. 0.6×30	34. 0.75×28
35. 0.05×22	36. 0.34×200	37. 0.26×80
38. 0.99×50	39. 0.50×24	40. 0.05×24
41. 1.5×36	42. 2.1×60	43. 0.31×75

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2.1g Class Activity: Finding a Percent of a Quantity

Activity 1:

a. What number is 10% of 120?

b. What number is 30% of 120?

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c. What number is 5% of 120?

d. What number is 25% of 120.

e. What number is 1% of 120.

f. What number is 2% of 120.

g. What number is 26% of 120.

h. What number is 29% of 120.

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Activity 2:

a. If 10% of a number is 18, what is…

20% of that number?

5% of that number?

1% of that number?

2% of that number?

100% of that number?

99% of that number?

200% of that number?

If 25% of a number is 8, what is…

50% of that number?

10% of that number?

20% of that number

1% of that number?

100% of that number?

150% of that number?

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Activity 3: For the following problems: 1) Estimate the answer. 2) Explain or show a strategy for finding the answer. 3) Find the answer. 4) Check the answer with a calculator.

a. 60% of 30

b. 15% of 40

c. 76% of 32

d. 52% of 180

e. 90% of 48

f. 21% of 25

g. 200% of 75

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h. 120% of 40

i. 175% of 120

j. 11% of 16

k. 97% of 65

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Activity 4: Real-world Application – Statistics

Toby recently read an article and discovered the following facts:

- 15% of teens get the recommended amount of sleep each night (8 – 10 hours).
- 91% of teens have owned a pet
- 70% of teens own a cell phone
- 68% of teens enjoy cooking
- 58% of teens play an organized sport
- 29% of teens choose Summer as their favorite season
- 86% of teens say they enjoy school
- 63% of teens watch YouTube daily
- 41% of teens go Black Friday shopping

There are 400 students at Toby's high school. Based on the statistics above, how many students at Toby's school would you expect…

a. Get the recommended amount of sleep each night? _____

b. Have owned a pet? _____

c. Own a cell phone_____

d. Enjoy cooking? _____

e. Play an organized sport? _____

f. Would choose Summer as their favorite season? _____

g. Would say they enjoy school? _____

h. Watch YouTube daily? _____

i. Go Black Friday shopping? _____

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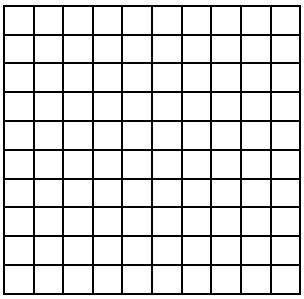
Spiral Review

1. Simplify.

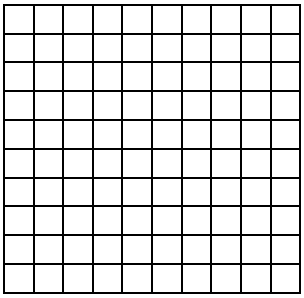
a. $50 \div 1$	b. $50 \div 0.1$	c. $50 \div 0.01$
d. $64 \div 32$	e. $64 \div 3.2$	f. $64 \div 0.32$
g. $8 \div 5$	h. $8 \div 0.5$	i. $8 \div 0.05$
j. $12 \div 15$	k. $12 \div 1.5$	l. $12 \div 0.15$

2. Give the value of each small square in the hundred grid below if the entire grid has a value of…



a. 100	b. 1	c. 80	d. 25	e. 120

3. Give the value of the *entire* grid in the hundred grid below if each small square has a value of…



a. 0.02	b. 0.3	c. 1.5	d. 0.24	e. 6

4. If one box on the tape diagram has a value of 7.9, what is the value of the entire tape diagram?

7.9									

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2.1g Homework: Finding a Percent of a Quantity

Directions: Draw a model to represent each set of problems. Use the model to answer the questions. Check your answers with a calculator.

Set 1 Model:		
1. 10% of 50	2. 60% of 50	3. 90% of 50

Set 2 Model:		
4. 25% of 36	5. 50% of 36	6. 75% of 36

3. Use mental math to find 10%, 15%, and 20% of 240.

4. If 1% of a number is 2, what is…
    a. 2% of the number?
    b. 10% of the number?
    c. 32% of the number?
    d. 5% of the number?
    e. 25% of the number?
    f. 100% of the number?
    g. 200% of the number.

5. Explain how you can find 19% of 450 using the following information. Check your answer with a calculator.
    - 20% of 450 is 90.
    - 1% of 450 is 4.5.

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Directions: For the following problems: 1) Estimate the answer. 2) Explain or show a strategy for finding the answer. 3) Find the answer. 4) Check with a calculator.

6. 50% of 21

7. 61% of 20

8. 22% of 54

9. 99% of 75

10. 26% of 44.

11. 150% of 16

12. 210% of 60

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13. Lavinia is friends with Toby (from the classroom activity) but goes to a different high school in town. She is also interested in determining the number of students at her school that would fit into each of the following categories based on the statistics below:

- 15% of teens get the recommended amount of sleep each night (8 – 10 hours).
- 91% of teens have owned a pet
- 70% of teens own a cell phone
- 68% of teens enjoy cooking
- 58% of teens play an organized sport
- 29% of teens choose Summer as their favorite season
- 86% of teens say they enjoy school
- 63% of teens watch YouTube daily
- 41% of teens go Black Friday shopping

There are 500 students at Lavinia's high school. Based on the statistics above, how many students at Lavinia's school would you expect…

a. Get the recommended amount of sleep each night? _____

b. Have owned a pet? _____

c. Own a cell phone? _____

d. Enjoy cooking? _____

e. Play an organized sport? _____

f. Would choose Summer as their favorite season? _____

g. Would say they enjoy school? _____

h. Watch YouTube daily? _____

i. Go Black Friday shopping? _____

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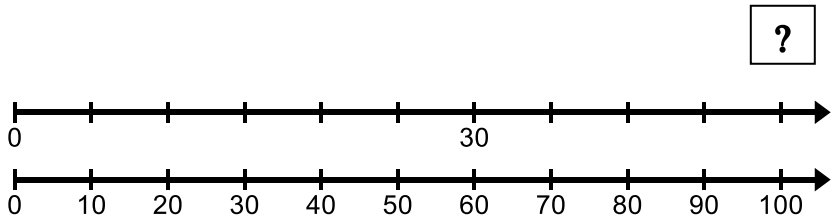
2.1h Class Activity: Finding the Whole Given the Percent and a Part

Activity 1: Use the double number line below for this activity.



a. Describe what the double number line shows. What question is being asked?

b. What is the answer to the question in part a.

c. Check your answer using ideas from the previous lesson.

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Activity 2:

a. If 10% of a number is 8, what is…

20% of the number?

50% of the number?

100% of the number?

b. If 1% of a number is 0.35, what is…

10% of the number?

20% of the number?

100% of the number?

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c. If 80% of a number is 64, what is…

50% of the number?

25% of the number?

100% of the number?

Activity 3: Solve the problems using multiple strategies (models, equivalent ratios, tables, equations, numeric reasoning, etc.). Then, check your answer.

a. 25% of a number is 4. What is the number?

b. 50% of a number is 11. What is the number?

c. 10% of a number is 4.2. What is the number?

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d. 40% of a number is 10. What is the number?

e. 120% of a number is 48. What is the number?

f. 1% of a number is 0.54. What is the number?

g. 5% of a number is 7. What is the number?

h. 30% of a number is 6.3. What is the number?

i. 11% of a number is 5.5. What is the number?

j. 15% of a number is 12. What is the number?

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k. 68% of a number is 34. What is the number?

l. 99% of a number is 79.2. What is the number?

m. 8% of a number is 6. What is the number?

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Spiral Review

1. Make a double number line to show the relationship between cups and pints.

2. Complete the table to show the relationship between meters and kilometers.

Kilometers	**Meters**
1	
2	
3	
10	

3. Tell whether the simplified form of the expression $50 \times a$ will be greater than 50 or less than 50 depending on the value of *a*.

a. $a = 2$	b. $a = \frac{1}{2}$	c. $a = 0.01$	d. $a = 1.01$

4. Tell whether the simplified form of the expression $50 \div a$ will be greater than 50 or less than 50 depending on the value of *a*.

a. $a = 2$	b. $a = \frac{1}{2}$	c. $a = 0.01$	d. $a = 1.01$

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2.1h Homework: Finding the Whole Given the Percent and a Part

Directions: Solve the problems using multiple strategies (models, equivalent ratios, tables, equations, numeric reasoning, etc.). Then, check your answer.

1. 10% of a number is 13. What is the number?

2. 20% of a number is 21. What is the number?

3. 4% of a number is 3. What is the number?

4. 75% of a number is 30. What is the number?

5. 80% of a number is 120. What is the number?

6. 150% of a number is 90. What is the number?

7. 1% of a number is 2. What is the number?

8. 65% of a number is 13. What is the number?

9. 20% of a number is 14.2. What is the number?

10. 26% of a number is 28.6. What is the number?

11. 12% of a number is 3. What is the number?

12. 40% of a number is 16. What is the number?

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2.1i Class Activity: Types of Percent Problems Mixed Review

Directions: The three problems below are examples of the three different types of percent problems we have studied in this chapter.

For each problem:

- Identify the type of percent problem. Are you…
  1) Finding a percent given a part and the whole?
  2) Finding a part of a quantity given a percent and the whole?
  3) Finding the whole given a part and a percent?
- Create a model to represent the problem.
- Solve the problem using a variety of methods (models, equivalent ratios, tables, equations, mental math, etc.).

1. 25% of 24 is what number?

2. 24 is what percent of 25?

3. 24 is 25% of what number?

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Directions: Solve the following problems using a variety of methods (models, equivalent ratios, tables, equations, mental math, etc.).

1. What is 25% of 32?	2. 32 is 25% of what number?
3. Find 75% of 16.	4. 7 is what percent of 20?
5. 25 is what percent of 40?	6. 30 is 120% of what number?
7. 35 is what percent of 100?	8. What percent of 16 is 8?
9. What number is 80% of 90?	10. 4.5 is 15% of what number?

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11. 1.2 is what percent of 10?	12. What number is 20% of 60?
13. Find 18% of 50.	14. 20 is what percent of 5?
15. Find 49% of 300.	16. Find 125% of 40.
17. 30 is what percent of 40?	18. What is 150% of 32?
19. Shelley got 22 questions correct on her math test. She got an 88%. How many questions were on Shelley's math test?	20. Charlie got 48 points out of 50 on his Science test. What percent did Charlie get on his Science test?

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Spiral Review

1. Evan has 4 cups of lemonade.
   a. How many 2-cup servings can he make?

   b. How many $\frac{1}{2}$-cup servings can he make?

2. At Theo's birthday party, $\frac{3}{4}$ of the cake was eaten. The next day, Theo's family of five shared the leftover cake. What part of the original cake did each family member get on the second day? Draw a picture and write an equation to model this situation. Then, solve the problem.

3. Solve for a in the equation $4 \div a = 20$.

4. Complete the table to show the relationship between centimeters and meters.

Meters	**Centimeters**
1	
2	
3	
8	

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2.1i Homework: Types of Percent Problems Mixed Review

1. What number is 50% of 80?	2. What number is 40% of 90?
3. 4 is what percent of 20?	4. Find 25% of 180.
5. What number is 15% of 200?	6. What number is 75% of 140?
7. 4.8 is 10% of what number?	8. What percent of 50 is 75?
9. What number is 51% of 128?	10. What number is 250% of 50?

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11. 15 is 60% of what number?	12. 16 is 32% of what number?
13. 20% of what number is 6?	14. 2.5 is what percent of 20?
15. What number is 110% of 150?	16. What number is 20% of 60?
17. Thirty-five percent of the students at Parker Junior High take the bus to school. If there are 200 students at Parker Junior High, how many take the bus?	18. Nine children on the swim team don't like ice cream. If this represents 18% of the children on the swim team, how many children are on the swim team?
19. Jennifer went to lunch. Her bill was $12. If she tips 20%, how much did she leave for tip?	20. A realtor makes 4% commission on the sale of a home. If she sold a home for $250,000, how much will she make in commission?

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2.1j Self-Assessment: Section 2.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Mastery 3**	**Substantial Mastery 4**
1. Understand a percent as a part to total ratio with a whole equal to 100.				
2. Represent fractional amounts of a quantity as a percent.				
3. Fluidly transition between quantities represented as a percent, fraction, decimal or ratio.				
4. Find a part of a quantity given a percent and the whole.				
5. Find the whole given a part and a percent.				
6. Solve real-world percent problems.				

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Sample Problems for Section 2.1
Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. Define percent in your own words. Use examples and or/models to support your definition. [1]

2. Eileen is making lemonade with water and lemon concentrate. What percent of each mixture is lemon concentrate? [1] [2]
    a. The ratio of water to lemon concentrate is 3 to 1

    b. Eileen mixes 4 parts water for each part of lemon concentrate

    c. 4 out of every 5 cups of the mixture is water

    d. There is twice as much water as lemon concentrate in the mixture

    e. The ratio of cups of water to cups of lemonade is 6:10.

3. The table below shows Miguel's score on several different math quizzes he took this quarter. Complete the table by determining the letter grade that Miguel earned on each quiz. The grading scale is shown. [1] [2]

Points Earned by Miguel	Total Possible Points	Percentage	Letter Grade
4	5		
9	10		
23	25		
16	20		
44	50		
32	40		

Percentage	Letter Grade
90 – 100%	A
80 – 89%	B
70 – 79%	C
60 – 69%	D
59% and below	F

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4. Complete the table below. [1][2][3]

Fraction	Decimal	Percent
$\frac{70}{100}$		
		25%
	0.32	
$\frac{5}{10}$		
	0.01	
$\frac{11}{20}$		
		180%
	1.25	
$\frac{41}{200}$		
$\frac{24}{40}$		

5. $\frac{4}{25}$ of the students at Clayton Middle School have food allergies. [6]
   a. What percent of the students at Clayton Middle School have food allergies? _____
   b. What percent of the students at Clayton Middle School do not have food allergies? _____
   c. If there are 36 students with food allergies at Clayton Middle School, how many total students are there? _____
6. Talen earns allowance each week. The pictograph below shows what he does with his money. Complete the table to show the percent of money that Talen spends, saves, and donates. [6]

Where Talen's Money Goes	Amount of Money	Percent of Money
Spends		
Saves		
Donates		

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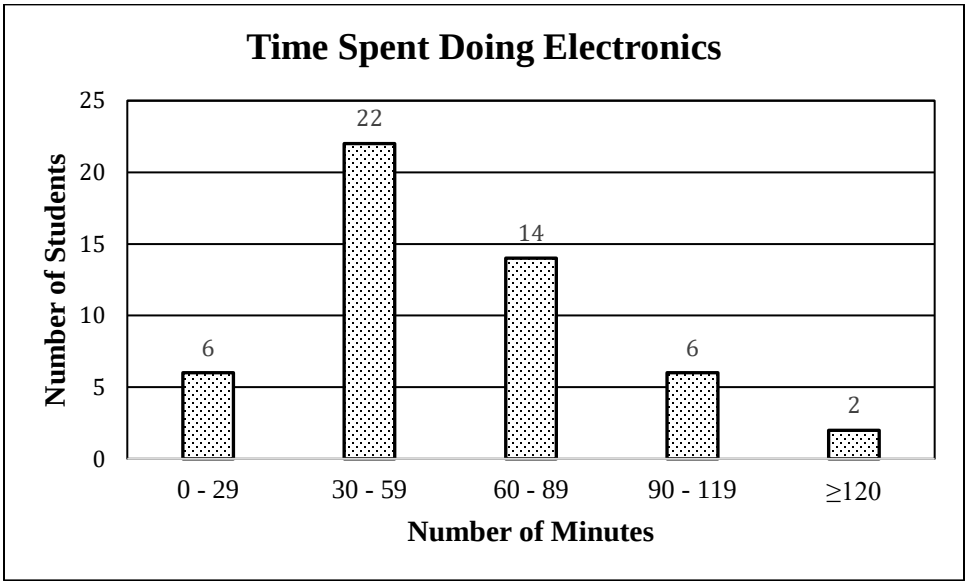
7. Monroe surveyed the students in his class and asked how much time they spend doing electronics each day. The bar graph shows the results of the survey: [6]



   a. What percentage of students in Monroe's class spend an hour or more each day doing electronics?

   b. What percentage of students in Monroe's class spend less than an hour each day doing electronics?

8. Use models to show the difference between the two problems below. Then, solve both problems. [4][5]

   42 is 60% of what number

   What is 60% of 42?

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Directions: Solve the following problems. [4][5]

9. 30% of what number is 120?

10. 30 is what percent of 120

11. What is 30% of 120?

12. What number is 120% of 30?

13. 120 is what percent of 30?

14. 30 is 120% of what number?

15. 25% of the students on the mock trial team are in 8^{th} grade. If there are 28 students on the mock trial team, how many are in 8^{th} grade? [6]

16. Shelly got 60 questions correct on her math test. If she got 75%, how many questions were on the test? [6]

17. There are 40 questions on a math test. Calvin got an 90%. How many questions did Calvin answer correctly? [6]

18. Maddie is buying a pair of jeans that cost $40. There is a 6% tax rate on the jeans. How much will she pay in tax? [6]

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Section 2.2: Division of Fractions

Section Overview:
The section begins with an optional review of the multiplication and division operation as well as a review lesson of standards in 5th grade (5.NF 3 – 7). It then moves to division of fractions in context (6NS.1). Students are encouraged to use various models (tape, double line, partial tables, etc.) to build conceptual understanding and connect ideas to ratio thinking. Students should make sense of problem contexts and the structure of the operations. For example, students should understand dividing by a fraction between 0 and 1 will produce a quotient larger than the dividend; or that multiplying by a fraction between 0 and 1 will produce a product smaller than the other factor. Making sense of problem structures and products and quotients with fractions will be very helpful for Section 3 (unit conversion). The section wraps up with a lesson to help students understand how $a \div \frac{b}{c}$ and $a \times \frac{c}{b}$ are related conceptually and that they produce the same answer.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Understand and explain the relationship between multiplication and division with fractions.
2. Solve division problems involving fractions using a variety of strategies, including models, related multiplication sentences, and the algorithm.
3. Create contexts for division of a fraction by a fraction.

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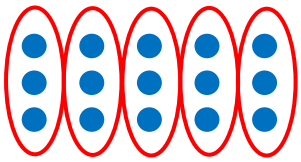
2.2a Class Activity: Division with Whole Numbers and Unit Fractions

In elementary school, **multiplication** is modeled in several ways: arrays, skip counting, number lines, blocks, etc. Each model emphasizes the idea that one factor tells you how many groups, the other how many items in a whole group, and the product is the total number of items:

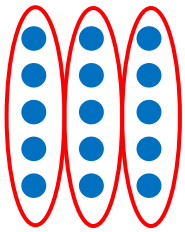
One factor tells you: GROUPS	X	**The other factor** tells you: ITEMS per group or the SIZE of the group	=	**Product** describes: TOTAL ITEMS or WHOLE SIZE

For example, 3×5 means three groups of 5 units in each group. The "product" is the total number of objects. Three representations for 3×5 and 5×3 are:

GROUPS and ITEMS MODEL

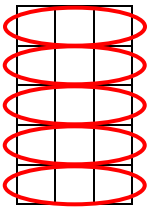


5 groups of 3

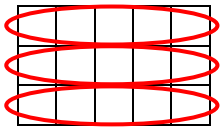


3 groups of 5

ARRAY MODEL

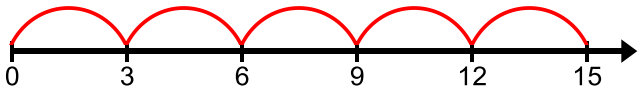


5 groups of 3



3 groups of 5

NUMBER LINE MODEL



5 groups of 3

3 groups of 5

Multiplication is commutative (order of the factors does not matter) and the product is always a total number of items or the whole size. In other word, for 3×5, we may have 3 bags (groups) with 5 candy bars (items) in each bag OR 3 candy bars (items) in each of 5 bags (groups) and in both cases, we get a total of 15 candy bars.

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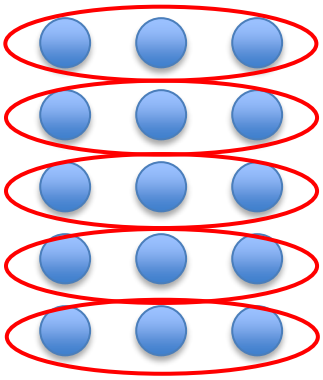
Division is the inverse of multiplication. We are looking for a missing factor, either the **number of groups** or number of **items per group/size of the group**. So, 15 ÷ 3 may mean either:

15 TOTAL ITEMS ÷ 3 ITEMS per GROUP = how many GROUPS? (Example 1)

OR

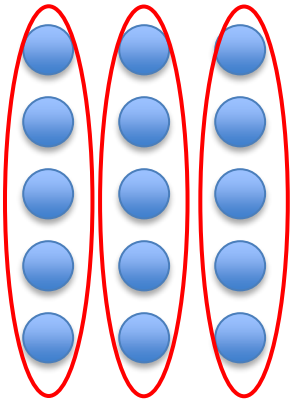
15 TOTAL ITEMS ÷ 3 GROUPS = how many ITEMS per GROUP? (Example 2)

Example 1:
Cora has 15 cookies, if she gives each friend 3 cookies, how many friends get cookies?



This is 15 ÷ 3 where we are looking for the "number of groups" factor. Thus, we are counting the number of groups with 3 items in each group. For the context, 5 friends (groups) get cookies. We can think 15 TOTAL ITEMS ÷ 3 ITEMS per GROUP = 5 GROUPS

Example 2:
Cora has 15 cookies to fair share among 3 friends. How many cookies can she give each friend?



Again, we have 15 ÷ 3, but now we are looking for the "number of items per group" factor. There are 3 groups with each group having 5 items in a whole group. So, for the context, each friend gets 5 cookies. We can think 15 TOTAL ITEMS ÷ 3 GROUPS = 5 ITEMS/GROUP

We can also think about the expression 15 ÷ 3 where 15 represents a WHOLE SIZE (for example, it might represent 15 feet). The 3 can represent either 1) the number of groups of 3 feet we can make OR 2) the size of a group if 3 groups are made.

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In 5th grade we extended these ideas to divide whole number by a fraction or a fraction by a whole number both conceptually and as the inverse of multiplication.

Activity 1: Cristina has 12 gallons of water to take on a hike. Complete the table below to answer each question.

Question	Model	Division Sentence	Related Multiplication Sentence
If she rations 6 gallons of water per person, how many people can go hiking?			
If she rations 4 gallons of water per person, how many people can go hiking?			
If she rations 3 gallons of water per person, how many people can go hiking?			
If she rations 2 gallons of water per person, then how many people can go hiking?			
If she rations 1 gallon of water per person, how many people can go hiking?			
If she rations $\frac{1}{2}$ a gallon of water per person, how many people can go hiking?			
If she rations $\frac{1}{3}$ of a gallon of water per person, how many people can go hiking?			

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Activity 2: Cal is having a party and serving pizza. Complete the table below to determine how much pizza Cal needs based on the different activities.

Question	Model	Division Sentence	Related Multiplication Sentence
Cal has 8 pizzas, it's four times as much as what he needs for his party. How many pizzas does he need?			
Cal has 8 pizzas, it's twice as much as what he needs for his party. How many pizzas does he need?			
Cal has 8 pizzas, it's exactly what he needs for his party. How many pizzas does he need?			
Cal has 8 pizzas, it's $\frac{1}{2}$ of what he needs for his party. How many pizzas does he need?			

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Cal has 8 pizzas, it's $\frac{1}{4}$ of what he needs for his party. How many pizzas does he need?			
Cal has 8 pizzas, it's $\frac{1}{8}$ of what he needs for his party. How many pizzas does he need?			

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Activity 3: Below are two contexts involving 8 and $\frac{1}{3}$. Solve both using models and equations and explain how the problems are similar and how they are different.

a. Eli has 8 pints of ice cream. If a serving size is $\frac{1}{3}$ a pint of ice cream, how many servings are in the 8 pints he has?

b. Eli has 8 pints of ice cream; he believes this is $\frac{1}{3}$ of what he needs. How much ice cream does he think he needs?

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Activity 4: You have $\frac{1}{2}$ a pizza left over from dinner last night. You invite two friends over, and the 3 of you will share the leftover pizza. What portion of a whole pizza will you each be getting? Solve using a model and an equation.

Activity 5: You have $\frac{1}{2}$ a cup of sugar but you need 3 cups to make a recipe. How much of the recipe can you make?

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Directions: For each context, a) draw a model of the context, b) write a number sentence showing the answer, and c) write your answer in a complete sentence.

1. Cora has 3 cups of sugar. The cookies she's making call for $\frac{1}{4}$ of a cup of sugar per batch. How many batches of cookies can Cora make with her 3 cups of sugar?

2. On average, Lucy needs a drink of water every $\frac{1}{2}$ mile. If Lucy runs 5 miles, how often will she take a drink of water?

3. Jose has run 3 miles, which is $\frac{1}{4}$ of his training for the day. How far is he planning on running?

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4. Eduardo is painting a mural depicting the beauty of the four seasons in his neighborhood. He wants to divide the spring portion ($\frac{1}{4}$ of the mural) into 4 equal parts showing graduation activities, planting of gardens, melting of snow, and children playing outside. What portion of the mural will depict graduation activities?

Spiral Review

1. Simplify.

a. $20 \times \frac{3}{5}$	b. $\frac{3}{8} \times \frac{1}{8}$	c. $\frac{4}{5} \times \frac{10}{3}$
d. $6 \times 1\frac{1}{2}$	e. $2\frac{1}{2} \times \frac{2}{11}$	f. $\frac{3}{8} \times 8$

2. Owen is making pancakes. The recipe calls for $\frac{2}{3}$ cups of water for each cup of pancake mix. If Owen only has $\frac{1}{2}$ cup of pancake mix, how much water should he use to follow the recipe?

3. Make a double number line to show the comparison between pounds and ounces.

4. Complete the table to show the relationship between days and hours.

Days	**Hours**
1	
2	
3	
7	

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2.2a Homework: Division with Whole Numbers and Unit Fractions

Directions: Solve each problem using a model of your choice. Check your work with the related multiplication sentence.

1. $5 \div \frac{1}{4}$	2. $3 \div \frac{1}{5}$
3. $\frac{1}{3} \div 4$	4. $\frac{1}{4} \div 2$

5. Write a context for either 1 or 2.

6. Write a context for either 3 or 4.

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2.2b Class Activity: Division with Rational Numbers - How Many Groups?

Activity 1:

a. Will the quotient $13 \div \frac{4}{5}$ be bigger or smaller than 13? Justify your answer.

b. Will the quotient $13 \div \frac{5}{4}$ be bigger or smaller than 13? Justify your answer.

c. Which quotient will be bigger $13 \div \frac{4}{5}$ or $13 \div \frac{2}{5}$? Justify your answer.

Activity 2: Eli has 8 pints of ice cream. If a serving size is $\frac{2}{3}$ of a pint of ice cream, how many servings does he have?

a. Estimate first.

b. Draw a model of your choice to answer this question.

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c. Write a division number sentence to represent the problem.

d. Check your work with the related multiplication sentence.

e. What if Eli has 9 pints of ice cream? How many servings would he have?

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Activity 3: Calvin wants to make as many batches of chocolate chip cookies as he can for a fundraiser bake sale. He has 6 cups of brown sugar. Each batch of cookies takes $\frac{3}{4}$ of a cup of brown sugar.

a. How many batches of cookies can he make with the 6 cups of brown sugar? Draw a model of your choice to answer this question. Then, write and solve a number sentence.

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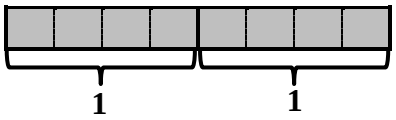
b. Eli comes over to help Calvin. He brings one more cups of brown sugar. Now how many batches of cookies can they make?

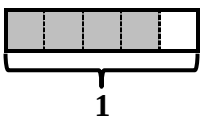
Activity 4: Mateo has $6\frac{1}{2}$ cups of sugar. How many batches of cookies can he make if each batch requires $\frac{3}{4}$ cups of sugar?

Activity 5: For each division problem, 1) Use the model to solve the problem and 2) Write a related multiplication sentence. The first one has been done for you.

a.

$2 \div \frac{1}{4} =$ __8__  	Related Multiplication Sentence: $? \times \frac{1}{4} = 2$ $\mathbf{8} \times \frac{1}{4} = 2$

b.

$\frac{4}{5} \div \frac{1}{5} =$ _____  	Related Multiplication Sentence:

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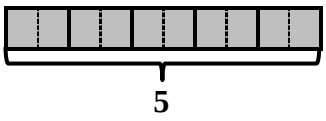
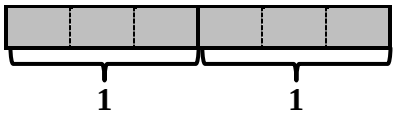
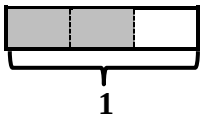
c.

$5 \div \frac{5}{2} =$ _____  	Related Multiplication Sentence:

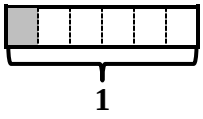
d.

$2 \div \frac{2}{3} =$ _____  	Related Multiplication Sentence:

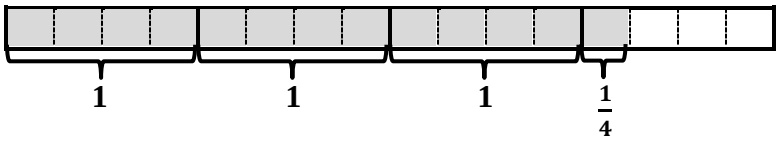
e.

$\frac{2}{3} \div \frac{1}{6} =$ _____  	Related Multiplication Sentence:

f.

$\frac{1}{6} \div \frac{2}{3} =$ _____  	Related Multiplication Sentence:

g.

$3\frac{1}{4} \div \frac{3}{4} =$ _____  	Related Multiplication Sentence:

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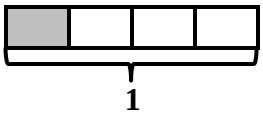
h.

$\frac{1}{4} \div 2 =$ _____  **1**	Related Multiplication Sentence:

Spiral Review

1. Simplify.

a. $1\frac{1}{2} \times \frac{1}{9}$	b. $\frac{3}{8} \times 2\frac{3}{4}$	c. $6\frac{2}{5} \times \frac{5}{8}$

2. Estimate whether the quotient will be bigger than, smaller than, or equal to the dividend.

a. $20 \div 2$	b. $20 \div 1$	c. $20 \div \frac{1}{2}$
d. $20 \div \frac{1}{4}$	e. $5 \div \frac{3}{4}$	f. $5 \div \frac{4}{3}$
g. $6\frac{1}{2} \div \frac{1}{2}$	h. $8 \div 1\frac{3}{4}$	i. $3 \div \frac{1}{100}$

3. Simplify.

a. $5 \div 0.1$	b. $5 \div 0.01$	c. $62 \div 0.1$
d. $3 \div 0.25$	e. $12 \div 0.3$	f. $22 \div 0.11$

4. Use your answers from #3 to simplify the following problems.

a. $5 \div \frac{1}{10}$	b. $5 \div \frac{1}{100}$	c. $62 \div \frac{1}{10}$
d. $3 \div \frac{1}{4}$	e. $12 \div \frac{3}{10}$	f. $22 \div \frac{11}{100}$

What is the relationship between the problems in #3 and #4?

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2.2b Homework: Division with Rational Numbers - How Many Groups?

Directions: Solve each problem with a model of your choice. Check your answer using multiplication.

1. $9 \div \frac{3}{4}$	2. $4 \div \frac{2}{5}$
3. $5 \div \frac{5}{2}$	4. $10 \div 1\frac{1}{3}$

5. Write a context for either 1 or 2.

6. Write a context for either 3 or 4.

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2.2c Class Activity: Division with Rational Numbers - How Big is the Whole?

Activity 1: Review of 2.2a Activity 2

a. Eli has 8 pints of ice cream. It's $\frac{2}{3}$ of what he needs. How much does he need? Draw a model of your choice to answer this question. Then, write number sentence to represent the problem.

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b. What if 9 pints of ice cream is $\frac{2}{3}$ of what Eli needs? Then, how much does he need?

Activity 2:

a. It takes 10 gallons of gas to fill Talen's gas tank $\frac{2}{3}$ of the way. How many gallons of gas does Talen's car hold? Draw a model of your choice to answer this question. Then, write and solve a number sentence.

b. Owen and Lucy also put 10 gallons of gas into their tanks. With ten gallons of gas, Owen's tank is $\frac{2}{5}$ of the way full. Ten gallons of gas fills Lucy's tank $\frac{3}{5}$ of the way. How many gallons of gas does each of their tanks hold?

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Activity 3:
Mauricio made 6 dozen cookies. Cora thinks it's one and a half times what he needs. How much does Cora think Mauricio should have made? Draw a model of your choice to answer this question. Then, write a number sentence to represent the problem.

Activity 4:
Write two different contexts for $4 \div \frac{2}{3}$: one where you are counting the number of $\frac{2}{3}$'s there are in 4 and the other where 4 is $\frac{2}{3}$ of a whole and you're looking for the whole. For each of the two contexts, a) draw a model showing how to solve the problem, and b) write a number sentence showing how the solution is related to the context.

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Spiral Review

1. At a school carnival, there is a dunk tank. Teachers will take turns sitting in the dunk tank for $\frac{1}{2}$ an hour at a time. If the carnival is 3 hours long, how many teachers will take a turn sitting in the dunk tank?

2. Write the reciprocal of each number.

a. $\frac{2}{3}$	b. $\frac{3}{4}$	c. $\frac{5}{2}$
d. $\frac{1}{2}$	e. 4	f. $2\frac{2}{3}$

3. There are 12 inches in a foot. How many inches are in…

   a. 3 feet?

   b. 4 feet?

   c. $3\frac{1}{2}$ feet?

4. There are 16 cups in a gallon. How many cups are in…

   a. $\frac{1}{2}$ gallon?

   b. 2 gallons?

   c. $2\frac{1}{2}$ gallons?

   d. 10 gallons?

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2.2c Homework: Division with Rational Numbers - How Big is the Whole?

Directions: Solve each problem using a model of your choice. Then, write a number sentence to represent the problem.

1. Lucy has 9 yards of string for her kite, but it's only $\frac{3}{4}$ of what she needs. How much string does she need?	2. Dina has 15 yards of string for her kite, but it's only $\frac{2}{3}$ of what she needs. How much string does she need?
3. Sasha has 18 yards of string for her kite; it's one and a half times what she needs. How much string does she need?	4. Kira has 25 yards of string for her kite; it's two and a half times what she needs. How much string does she need?

5. Write two different contexts for $6 \div \frac{3}{4}$: one where you are counting the number of $\frac{3}{4}$'s there are in 6 and the other where 6 is $\frac{3}{4}$ of a whole and you're looking for the whole. For each of the two contexts, a) draw a model showing how to solve the problem, and b) write a number sentence showing how the solution is related to the context.

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2.2d Class Activity: Mixed Division of Fractions

Activity 1: Explain your estimation for the following:

a. Will the quotient: $\frac{1}{2} \div \frac{1}{4}$ be bigger or smaller than $\frac{1}{2}$? Explain.

b. Will the quotient: $\frac{1}{4} \div \frac{1}{2}$ be bigger or smaller than $\frac{1}{4}$? Explain.

c. Will the quotient: $\frac{1}{2} \div \frac{5}{4}$ be bigger or smaller than $\frac{1}{2}$? Explain.

d. Which quotient will be bigger $\frac{1}{2} \div \frac{4}{5}$ or $\frac{1}{2} \div \frac{2}{5}$?

Activity 2:
Calvin, Eli, Lusy and Cora are making cookies. *Estimate* the number of *whole* batches of cookies each can make.

a. Lucy has $\frac{1}{2}$ of a cup of butter. She needs $\frac{3}{4}$ of a cup of butter to make a batch of cookies. How many batches of cookies can she make?

b. Calvin has $\frac{3}{4}$ of a cup of sugar. He needs $\frac{1}{3}$ of a cup of sugar to make a batch of cookies. How many batches of cookies can he make?

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c. Eli has $\frac{3}{5}$ of a cup of sugar. He needs $\frac{1}{4}$ of a cup of sugar to make a batch of cookies. How many batches of cookies can he make?

d. Cora has $3\frac{1}{2}$ cups of flour. She needs $\frac{3}{4}$ of a cup of flour to make a batch of cookies. How many batches of cookies can she make?

Activity 3:

Bernardo has $\frac{3}{4}$ of a cup of sugar and wants to make cookies that require $\frac{1}{2}$ of a cup of sugar. How many batches of cookies can he make?

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Activity 4: Create a model of your choice to answer this question. Then, write a number sentence to represent the problem. n#

a. Eva ran $\frac{2}{3}$ of a mile which is $\frac{1}{5}$ of her total route. How far is her route?

b. Penny has $4\frac{1}{2}$ quarts of ice cream. It's $\frac{3}{4}$ of what she needs for her party. How much ice cream does she need?

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c. Josephine has run 7.5 miles, which is $\frac{2}{5}$ of her training distance for the day. How far was she planning on running today?

d. Ben swam $\frac{3}{5}$ of a mile. It's $\frac{3}{5}$ of the distance he will swim today. How far will he swim today?

Activity 5: Cora has $6\frac{1}{2}$ cups of pancake syrup left. She estimates each person uses $\frac{2}{3}$ of a cup of syrup per serving. How many servings of syrup does she have?

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Activity 6:

a. Lucy put $\frac{2}{5}$ of a gallon of gas in the lawn mower. It's $\frac{2}{3}$ full. How much gas does the lawn mower hold?

b. Lucy double checked her measurements and realized she'd actually put $\frac{3}{5}$ of a gallon of gas in the lawn mower, which still filled it to $\frac{2}{3}$ full. How much gas does the lawn mower hold?

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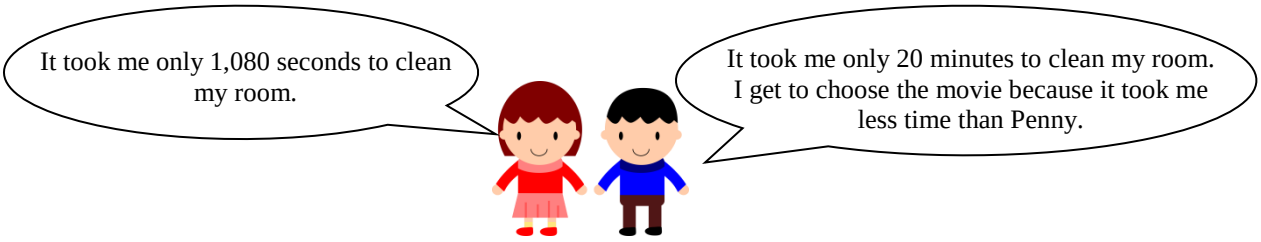
Spiral Review

1. Ben and Penny's dad has asked them to each clean their own rooms. He told them, whoever cleans their room the fastest gets to choose the movie they get to watch that night. After cleaning their rooms, they make the following statements:



   Do you agree with Ben's claim? Justify your answer.

2. There are 3 feet in 1 yard.
   a. How many feet are in 100 yards?

   b. How many yards are in 1,500 feet?

3. There are 1000 meters in 1 kilometer.
   a. How meters are in 5 km?

   b. How many kilometers are in 3,500 meters?

4. One pound is equal to 16 ounces.

   a. How many pounds are in 96 ounces?

   b. How ounces are in 12.5 pounds?

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2.2d Homework: Mixed Division of Fractions

Directions: Create a model of your choice to answer the questions below. Then, write a number sentence to represent the problem.

1. How many halves fit into three-fourths?	2. How many halves fit into two-fifths?
3. How many three-fifths fit into two-thirds?	4. Six is two-thirds of a group of what?
5. Two-thirds is six of a group of what?	6. Lisa has $\frac{3}{4}$ of a cup of sugar; it's $\frac{1}{2}$ of what she needs. How much sugar does she need?
7. Yolanda has $\frac{3}{4}$ of a cup of sugar; it's $\frac{3}{5}$ of what she needs. How much sugar does she need?	8. Gina has $\frac{3}{4}$ of a cup of sugar; it's $\frac{3}{2}$ of what she needs. How much sugar does Gina need?

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2.2e Class Activity: Dividing by Two or Multiplying by One-Half?

Activity 1: Lucy and Cora want to share $\frac{4}{5}$ of a pizza left over from a party the night before. They have the following conversation:

Lucy says, "That means we'll each get $\frac{4}{5}$ divided by 2 of the pizza."

Cora says, "Wait, doesn't it mean that we get $\frac{1}{2}$ of $\frac{4}{5}$ of a pizza?"

Lucy responds, "I'm confused, I think it's $\frac{4}{5} \div 2$, and you think it's $\frac{4}{5} \times \frac{1}{2}$. Aren't those two different expressions? Won't they give us two different answers?"

Answer Lucy's question with a model and a mathematical sentence.

Activity 2:
Cal and Eli have raised $450 to put a new educational app on the school's personal devices. Their teacher tells them it's 80% of what they need to buy the app for the school. Which of the following expressions can be used to determine how much Cal and Eli need to raise for the app? Justify your answer.

a. $450 \div \frac{5}{4}$	b. $450 \times \frac{4}{5}$
c. $450 \div \frac{4}{5}$	d. $450 \times \frac{5}{4}$

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Activity 3:
For each context below 1) State a number you know the answer is either greater or less than, 2) Write a division and multiplication sentence to solve the problem, and 3) Solve the problem.

a. Talen's paper airplane flew 10 feet which was $\frac{3}{5}$ as far as Owen's plane flew. How far did Owen's plane fly?

b. Geraldo has already run 4.5 miles. That's $\frac{8}{9}$ of the total distance he needs to run. How far is he planning to run in total?

c. Mario is selling artwork at a summer fair. He needs to pay the organizers 20% of his profit. If he made $220 at the fair, how much will he need to pay the organizers?

d. How many pieces of rope that are $\frac{3}{4}$ of a foot can be made from a rope that is 6 feet long?

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Activity 4:

Select all the contexts that can be solved with the number sentence $\frac{3}{4} \times \frac{2}{5} = ?$. If the context cannot be solved with $\frac{3}{4} \times \frac{2}{5} = ?$, write a number sentence that would work.

a. Aaron has $\frac{3}{4}$ of a cup of sugar. It's $\frac{2}{5}$ of what he needs. How much does he need?

b. Nina has $\frac{2}{5}$ of a pizza; her brother eats $\frac{3}{4}$ of it. How much of the pizza did he eat?

c. Patricio has $\frac{3}{4}$ of a cup of oil. It's $\frac{5}{2}$ as much as he needs. How much does he need?

Activity 5:

Select all the contexts that can be solved with the number sentence $\frac{3}{4} \div \frac{2}{3} = ?$. If the context cannot be solved with $\frac{3}{4} \div \frac{2}{3} = ?$, write a number sentence that would work.

a. Tasha has $\frac{3}{4}$ of a cup of sugar. It's $\frac{2}{3}$ of what he needs. How much does he need?

b. Tony has $\frac{3}{4}$ of a cup of sugar. If he spills $\frac{2}{3}$ of it, how much sugar is left?

c. Juan has $\frac{3}{4}$ cup of sugar, but he needs one and a half times that amount. How much does he need?

Spiral Review

1. There are 1000 milliliters in 1 liter.
   a. How many liters are in 1,500 ml?
   b. How many mL are in 13 liters?
2. What unit would you use to measure the length of a pencil?
3. What unit would you use to measure the length of your classroom?
4. What unit would you use to measure the amount of water a sink can hold?

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2.2e Homework: Dividing by Two or Multiplying by One-Half?

Directions: Show both a multiplication and division number statement for each problem. Then, state the solution.

1. Julia and Mateo want to split $\frac{3}{5}$ of a pizza. How much of the pizza will they each get?

2. Lincoln has 12 gallons of ice cream for a party. It's $\frac{3}{4}$ of what he needs. How much does he need?

3. Cora has $\frac{2}{3}$ of a gallon of gas in her tank. She needs double that amount to get to the end of the trail. How much gas does she need to get to the end of the trail?

4. Curtis has $4\frac{1}{2}$ cups of brown sugar. He needs $\frac{3}{4}$ of a cup brown sugar to make one batch of cookies. How many batches of cookies can he make?

5. There is $\frac{3}{4}$ of a gallon of milk to split among 3 people. How much milk will each person get?

6. Write a context for $3 \times \frac{3}{4}$

7. Write a context for $3 \div \frac{3}{4}$

8. Write a multiplication and division statement for: 8 students in Ms. Garcia's are doing biology science fair projects. That's $\frac{4}{5}$ as many as are doing physics projects. How many students are doing physics projects?

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9. Write a multiplication and division statement for: 8 students in Ms. Garcia's are going to space camp this summer. $\frac{3}{4}$ of them went last year. How many went last year?

Directions: Find the quotient by multiplying by the reciprocal.

10. $\frac{2}{5} \div \frac{2}{3}$	11. $\frac{6}{7} \div \frac{3}{5}$	12. $\frac{2}{3} \div \frac{4}{5}$
13. $\frac{1}{4} \div 5$	14. $5 \div \frac{1}{4}$	15. $6\frac{2}{3} \div \frac{2}{3}$
16. $\frac{8}{9} \div 9$	17. $2\frac{1}{2} \div \frac{5}{2}$	18. $\frac{3}{4} \div \frac{4}{3}$
19. $3 \div \frac{2}{5}$	20. $\frac{5}{8} \div \frac{3}{4}$	21. $5\frac{3}{5} \div 2$
22. $\frac{1}{2} \div \frac{1}{3}$	23. $\frac{1}{3} \div \frac{1}{2}$	24. $22\frac{1}{2} \div 11\frac{1}{4}$

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2.2f Self-Assessment: Section 2.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Mastery 3**	**Substantial Mastery 4**
1. Understand and explain the relationship between multiplication and division with fractions.				
2. Solve division problems involving fractions using a variety of strategies, including models, related multiplication sentences, and the algorithm.				
3. Create contexts for division of a fraction by a fraction.				

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Sample Problems for Section 2.2

Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. How much chocolate will each person get if 3 people share $\frac{1}{2}$ a pound of chocolate equally? Complete the following to answer this question. [1]

   $\frac{1}{2} \div 3 =$ _____ because $3 \times$ _____ $= \frac{1}{2}$

2. Carlos has run 10 miles. It is $\frac{2}{3}$ of the distance he is planning to run. How far is Carlos planning to run? [1]

   $10 \div \frac{2}{3} =$ _____because $\frac{2}{3} \times$ _____ $= 10$

3. Solve each problem using a method of your choice. [2]
   a. Peter bought $2\frac{1}{2}$ pounds of hamburger meat. He is planning to make $\frac{1}{4}$-pound hamburgers. How many $\frac{1}{4}$-pound hamburgers can he make?

   b. Christina needs $8\frac{3}{4}$ cups of tomato sauce to make her grandma's famous spaghetti sauce. She has 5 cups of tomato sauce. What portion of the tomato sauce that Christina needs does she have?

   c. How many $\frac{3}{4}$-cup servings are in $\frac{2}{3}$ of a cup of yogurt?

   d. A science teacher has 5 cups of solution for an experiment. Each lab table needs $\frac{2}{3}$ of a cup of solution for the experiment. How many $\frac{2}{3}$-cups of the solution can the science teacher make?

   e. Bo has raised $450 for a tablet. He realizes it is $1\frac{1}{2}$ times what he needs for the tablet. How much is the tablet?

4. Write a story for the equation $4 \div \frac{2}{3} =$?. Then, answer the question. [3]

5. Write a story for the equation $2\frac{1}{3} \div \frac{1}{6} =$?. Then, answer the question. [3]

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Section 2.3: Ratio Reasoning and Measurement Conversion

Section Overview:
Students have been converting between units of measurement throughout elementary school. The emphasis in 6th grade is to use ratio reasoning to convert between units of measurement. Students will continually be making sense of problems and using number sense, asking the questions, "Should my answer be bigger or smaller than the number I start with and what operations can I perform to produce the desired result?" Students will also apply their fluency with operations with rational numbers to solve many of the problems in this section. The section starts with a lesson that helps students to reason through the size of their answers and the operations that will get them there. The lessons proceed with conversion of units in the same measurement system and then conversion across the customary and metric systems of measurement. Students will use a variety of models from Chapter 1, including double number lines, tape diagrams, partial tables, equations, etc.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Reason about the size of an answer when performing a measurement conversion.
2. Use ratio reasoning to convert units in the same measurement system (metric to metric and customary to customary).
3. Use ratio reasoning to convert across measurement systems (metric to customary and customary to metric).

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2.3a Class Activity: Reasoning About Measurement Conversion

Activity 1: Without doing any calculations, tell whether the expression will simplify to a number that is bigger or smaller than the first number in the expression.

a. $48 \div 12$	b. $8 \times \frac{1}{4}$	c. $500 \div 0.1$
d. 0.8×100	e. $2{,}000 \div 1{,}000$	f. $12 \div 2.54$
g. 5×0.66	h. $80 \div 0.454$	i. $360 \times \frac{1}{60}$

Activity 2: Complete the following.

a. 2 ft = _____ in.

b. 5 yd = _____ ft

c. 5 gal = _____ qt

d. 3 T = _____ lb

e. 2 m = _____ cm

f. 5 L = _____ mL

Complete the following statement: When you convert from a larger unit of measure to a smaller unit of measure…

Activity 3: Complete the following.

a. 9 ft = _____ yd

a. 10 c = _____ pt

b. 180 min = _____ hr

c. 32 oz = _____ lb

d. 5,000 m = _____ km

b. 5,280 ft = _____ mi

Complete the following statement: When you convert from a smaller unit of measure to a larger unit of measure…

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Activity 4: Jenny was asked to change 42 quarts into gallons. Circle all the expressions that can be used to do this conversion?

a. $42 \div 4$	b. $42 \div 0.25$
c. $42 \times \frac{1}{4}$	d. $\frac{42}{4}$

Activity 5: Lloyd was asked to change 6 minutes to hours. Circle all the expressions that can be used to do this conversion.

a. 6×60	b. $6 \times \frac{1}{60}$
c. $\frac{6}{60}$	d. $6 \div \frac{1}{60}$

Activity 6: Owen was asked to change 500 meters to centimeters. Circle all the expressions that can be used to do this conversion.

a. $500 \div 100$	b. 500×100
c. $500 \times \frac{1}{100}$	d. $500 \div 0.01$

Spiral Review

1. Create a number line to show $75, 150$, and 125.

2. Create a number line to show $\frac{3}{4}, 1\frac{1}{2}$, and 1.25.

3. Find the area of the rectangle with a base of 5 inches and a height of 3 inches.

4. Find the area of the rectangle with a height of $4\frac{1}{2}$ feet and a base of $1\frac{2}{3}$ feet.

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2.3a Homework: Reasoning About Measurement Conversion

Directions: Answer the following questions.

1. Naoli is 13 years old. She wants to figure out how old she is in days. Will her answer be bigger or smaller than 13?

2. It took Tomaso 14 months to train for a marathon. If he wants to express his training time in years, will the number of years be bigger of smaller than 14?

3. A restaurant made 56 cups of salsa. If the restaurant wants to determine the number of gallons of salsa they made, will the number of gallons be bigger or smaller than 56?

4. Justin swam 2.5 km. If Justin wants to express the distance he swam in meters, will the number be bigger or smaller than 2.5?

5. A hockey puck weights approximately 150 grams. If Mike wants to express the weight of the hockey puck in kilograms, will the answer be bigger or smaller than 150?

6. Use a model of your choice to convert between feet and yards.

a. 6 yd = _____ ft

b. 24 ft = _____ yd

c. 10 yd = _____ ft

d. 1 yd = _____ ft

e. 1 ft = _____ yd

f. 10 ft = _____ yd

g. $4\frac{1}{2}$ yd = _____ ft

h. 20 ft = _____ yd

i. $2\frac{1}{3}$ yd = _____ ft

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2.3b Class Activity: Converting Within the Same System of Measurement

Activity 1: Create a model of your choice to convert between the units given. Then, write a numeric statement for the conversion.

Problem	Model	Numeric Statement
a. 6 in. = _____ ft		
b. 2.5 days = _____ hr		
c. 72 yd = _____ in.		

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d. $2\frac{1}{4}$ T = _____ lb		
e. $\frac{1}{2}$ c = _____ pt		
f. 55 km = _____ m		
g. 2.3 m = _____ cm		
h. 50 mL = _____ L		

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Activity 2: Compare the two measurements using $<, >,$ or $=$.

a. 2.1 ft _____ 24 in.	b. $\frac{4}{3}$ yd _____ 4 ft	c. 3 mo _____ $\frac{1}{4}$ yr
d. 3.5 c _____ 30 fl. oz.	e. 40 hr _____ $\frac{1}{5}$ wk	f. 299 mL _____ 0.3 L
g. 78 mm _____ 780 cm	h. 10 km _____ 10,000 m	i. 10 m _____ 1,000 mm

Activity 3: Find the sum or difference. Express your answer in the unit given.

1. 5 ft $+ 2$ in. = _____ ft	2. $1\frac{1}{2}$ min + 15 sec = _____ sec
3. 500 mL + 1 L = _____ mL	4. 3 km + 750 m = _____ km

Activity 4: A manager of a football team needs one quart of water for each of her 22 players. Her large orange cooler holds 5 gallons of water. Will it be big enough to hold all the water for her team? Justify your answer.

Activity 5: A catering business plans for a 3-oz serving of chicken for each guest. There are 25 people attending the party. If the catering business purchases $4\frac{1}{2}$ pounds of chicken, will they have enough chicken? Justify your answer.

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Activity 6: There are 8 ounces in 1 cup, 2 cups in a pint, and 2 pints in a quart.

a. How many cups are in 64 ounces?

b. How many ounces are in 5 quarts?

Activity 7:

Bethany and Beatrice are stacking dominos on end in rows to eventually knock over. In 40 seconds Bethany can stack 60 dominos and in 15 seconds Beatrice can stack 30 dominos.
Who is faster at stacking the dominos?

How many dominos can each girl stack in one minute?

How many dominos can each girl stack in one hour?

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Spiral Review

1. Create a number line to show $10, 50$, and 85.

2. Create a number line to show $\frac{1}{3}$, $1\frac{2}{3}$, and $\frac{1}{6}$.

3. The area of a rectangle is 50 square centimeters. If the base of the rectangle measures 10 centimeters, what is the height of the rectangle.

4. The area of a rectangle is $\frac{1}{8}$ of a square meter. If the base of the rectangle measures $\frac{1}{2}$ of a meter, what is the height of the rectangle?

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2.3b Homework: Converting Within the Same System of Measurement

Directions: Create a model of your choice to convert between the units given. Then, write a numeric statement for the conversion.

Problem	Model	Numeric Statement
1. 12 oz = _____ lb		
2. $5\frac{1}{2}$ pt = _____ c		
3. $\frac{2}{3}$ mi = _____ ft		
4. 500 sec = _____ min		

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5. 1.5 mi = _____ ft		
6. 500 mL = _____ L		
7. 100 g = _____ kg		
8. $5\frac{1}{2}$ L = _____ mL		

Directions: Compare the two measurements using $<, >,$ or $=$.

9. $\frac{3}{4}$ mi _____ 1,500 yd	10. 15 qt _____ 3.25 gal	11. 120 oz _____ 7.5 lb
12. $5\frac{1}{3}$ hr _____ 330 min	13. 240 cm _____ $2\frac{1}{2}$ m	14. 50 g _____ 0.5 kg

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Directions: Find the sum or difference. Express your answer in the unit given.

15. $4\frac{1}{2}$ hr + 45 min = _____ hr	16. 50 cm + 1 m = _____ m
17. $2\frac{1}{4}$ gal + 12 c = _____ gal	18. 1 kg + 1 g = _____ g

Directions: Solve the following problems.

19. A science teacher needs 50 mL of a solution for each lab table for a lab experiment. There are 15 lab tables in the room. The science teacher has 1 L of the solution. Does he have enough for all the lab tables?

20. Penny has 3 gallons of punch for a class party. She wants to have $1\frac{1}{2}$ cups of punch for each student in her class. There are 20 students in her class. Does she have enough punch?

21. There are 7 days a week, 24 hours in a day, and 60 minutes in an hour.
    a. How many minutes are in 2 weeks?

    b. How many days in 4,320 minutes?

22. Calvin, Eli, Lucy and Cora are preparing hygiene kits for the Red Cross. Calvin fills 4 in 3 minutes, Eli fills 75 per hour, it takes Lucy 20 minutes to fill 30, and Cora fills 140 every two hours. Who fills kits the fastest?

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2.3c Class Activity: Converting Across Systems of Measurement

Activity 1: Tell which unit of measure is larger.

1. Centimeters or inches

2. Miles or kilometers

3. Pounds or kilograms

4. Liters or gallons

Activity 2:

a. A pencil is 8 inches long. About, how long is the pencil in centimeters?

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b. If you have 2 gallons of milk, what is its approximate measure in liters?

c. Flora's new baby has a birth weight of 8 pounds exactly. Her mother calls from London, England to ask about the baby and wants to know the baby's weight in kilograms.

Activity 3: Compare the following rates to tell which is traveling faster.

a. Car A travels 40 miles per hour Car B travels 60 kilometers per hour

b. Snail A crawls 10 cm per minute Snail B crawls 4 inches per minute

c. Biker A is riding 22 feet per second Biker B is riding travels 400 meters per minute

d. Bird A flies 4,400 feet per minute Bird B flies 100 kilometers per hour

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Activity 4: Eli is using a cookbook to make a recipe, but he cannot find his measuring cups. He has, however, found a tablespoon. Inside the back cover of the cookbook, it says that 1 cup = 16 tablespoons. **This is an Illustrative Mathematics Task*

Explain how he could use the tablespoon to measure the following ingredients:

a. 2 cups of flour

b. $\frac{1}{2}$ cup sunflower seeds

c. $1\frac{1}{4}$ cup of oatmeal

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Activity 5:

Lucy and Cora are comparing the effects of a new plant food on Heirloom tomato plants. To run their experiment, they each had a total of 10 plants; 5 received the new plant food every week, and 5 others did not. Both groups of plants were planted in the same soil and got the same amount of water and sunlight. At the end of 8 weeks, Lucy and Cora averaged the height of the 5 plants in each of their two groups and recorded the following data:

Lucy's Data				**Cora's Data**			
Tomato plants that got plant food		Tomato plants that did NOT get plant food		Tomato plants that got plant food		Tomato plants that did NOT get plant food	
Week	Average Height of 5 plants	Week	Average Height of 5 plants	Week	Average Height of 5 plants	Week	Average Height of 5 plants
0	5 inches	0	6 inches	0	8 cm	0	7 cm
8	16 inches	8	14 inches	8	45 cm	8	40 cm

Lucy and Cora want to combine their data into one report, how would you advise them to put the data together?

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Spiral Review

1. Simplify $5 - 2 + 1$ and $5 - (2 + 1)$. Compare the problems.

2. Simplify $3 + \frac{1}{5} \times 10$ and $\left(3 + \frac{1}{5}\right) \times 10$. Compare the problems.

3. Simplify $8 \div \frac{1}{4} \times \frac{1}{2}$ and $8 \div \left(\frac{1}{4} \times \frac{1}{2}\right)$. Compare the problems.

4. Simplify $8 - 2 \times 0.5$ and $(8 - 2) \times 0.5$. Compare the problems.

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2.3c Homework: Converting Across Systems of Measurement

Directions: Answer the following questions using any method you wish. You may need to reference a conversion chart. Justify your answers.

1. How many kilometers are in 10 miles?

2. How many meters are in 100 yards?

3. How many feet are in 10 meters?

4. How many quarts are in 2 liters?

5. How many liters are in 5 quarts?

6. Which dog weighs more, a dog that weighs 10 kilograms or a dog that weighs 16 pounds?

7. Which running track is longer, a track that is 400 meters long or one that is $\frac{1}{3}$ a mile long?

8. Which container holds more soda, a container with a capacity of 2.5 liters or a container with a capacity of 0.75 gallons?

9. Which is heavier: 30 ounces or 1 kilogram?

10. Which hose pumps water at a faster rate, a hose that pumps water at a rate of 4 gallons per minute or a hose that pumps water at a rate of 0.25 liters per second?

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2.3d Self-Assessment: Section 2.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Mastery 3**	**Substantial Mastery 4**
1. Reason about the size of an answer when performing a measurement conversion.				
2. Use ratio reasoning to convert units in the same measurement system (metric to metric and customary to customary).				
3. Use ratio reasoning to convert across measurement systems (metric to customary and customary to metric).				

Sample Problems for Section 2.3

Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. There are 3 feet in 1 yard. Laurence is converting 125 feet into yards. [1]
   a. Should his answer be bigger or smaller than 125?

   b. To do this conversion, Laurence can…

2. There are 100 cm in 1 meter. Stephanie is converting 10 meters into centimeters. [1]
   a. Should her answer be bigger or smaller than 10?

   b. To do this conversion, Stephanie can…

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3. There are approximately 0.62 miles in 1 kilometer. Peter is converting 20 miles to kilometers. [1]
   a. Should his answer be bigger or smaller than 20?

   b. To do this conversion, Peter can…

4. Perform the following conversions. [2]

a. 12 ft = _____ in.	b. 4 yd = _____ ft	c. 3 yr = _____ mo
d. 22 c = _____ qt	e. 3.25 c = _____ pt	f. 55 g = _____ mg
g. 1 mm = _____cm	h. $5\frac{1}{2}$ km = _____ m	i. 60.2 kg = _____ g

5. Perform the following conversions. [3]

a. 20 cm ≈ _____ in.	b. 10 m ≈ _____ ft	c. 60 mi ≈ _____ km
d. 80 lb ≈ _____ kg	e. 10 gal ≈ _____ L	f. 1,500 m ≈ _____ mi

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