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Chapter 3: Extending the Number Line (4 weeks)

Utah Core Standard(s):

- Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (6.NS.5)
- Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. (6.NS.6)
  a) Recognize opposites signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.
  b) Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
  c) Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
- Understand ordering and absolute value of rational numbers. (6.NS.7)
  a) Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret* $-3 > -7$ *as a statement that* -3 *is located to the right of* -7 *on a number line oriented from left to right*.
  b) Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write* $-3\ ℃ > -7\ ℃$ *to express the fact that* $-3\ ℃$ *is warmer than* $-7\ ℃$.
  c) Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of* -30 *dollars, write* $|-30| = 30$ *to describe the size of the debt in dollars.*
  d) Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than* -30 *dollars represents a debt greater than 30 dollars.*
- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (6.NS.8)

Academic Vocabulary: number line, whole number, positive number, negative number, integer, rational number, line symmetry, opposites, scale, quadrant, origin, *x*-axis, *y*-axis, absolute value, greater than, less than, deposit, withdrawal, debit, credit, ascend, descend, profit, loss

Chapter Overview:

In this chapter, students extend their knowledge of the number system to the set of all rational numbers, which includes negative rational numbers. Using string and a straightedge, students construct a ray that extends from zero to the right and they show the location of the counting numbers. Then, they partition their string to show the location of various positive rational numbers on the line. Using a similar process, students construct a ray that extends left of zero. They use the length they have defined as one unit to mark off the opposites of the counting numbers, realizing the set of numbers known as integers. Then, they partition their string to show the location of negative rational numbers, realizing the set of numbers known as rational numbers.

This activity surfaces several ideas for students, including the fact that all rational numbers can be represented by a point on the number line and that there are real-world and mathematical problems that have required us to expand the number system over time. For example, the need for negative numbers arose to describe shortages in shipments. Mathematically, the need for negative numbers arose to address problems such as $3 - 4$. Additionally, students discover the symmetry of the number line. They learn that every rational number has an opposite – a number that is equidistant from zero but located on the opposite side of zero. The opposite of a number can be found by reflecting the point across the vertical line passing through zero on a horizontal number line (a horizontal line passing through zero on a vertical number line).

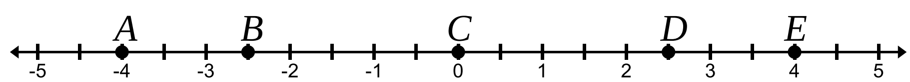
Understanding the structure of the number line plays a leading role for the remainder of the chapter. Students use ideas about symmetry to find and position rational numbers on number lines (and pairs of rational numbers on coordinate planes), to compare and order rational numbers, and to find the distance between two points on the same horizontal or vertical line. Students learn what absolute value is and distinguish between the value of a number relative to zero and the magnitude of a number. In the last section, students synthesize and apply these concepts as they investigate negative numbers in context.

Connections to Content:

Prior Knowledge: In elementary grades, students worked with the representation of numbers on the number line. In Grade 2 they learned how to put counting numbers at equally spaced intervals as tick marks on a straightedge. Then in Grade 3 students learned to represent a fraction $\frac{a}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts, recognizing that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line. They also learned to represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths of $\frac{1}{b}$ from 0, recognizing that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line. Throughout elementary grades, students have compared and ordered positive rational numbers. In Grade 5, students graphed ordered pairs in the first quadrant.

Future Knowledge: In Grade 7, students will learn to operate with positive and negative rational numbers. In Grade 8, the number system is expanded to include irrational numbers. Students come to understand that irrational numbers are points on the real number line even though they cannot be represented with an exact decimal value. Students will use rational approximations of irrational numbers to compare the size of irrational numbers and to estimate the value of expressions that contain irrational numbers. Lastly, students will rely on their knowledge of the coordinate plane when they study functions.

MATHEMATICAL PRACTICE STANDARDS

	Make sense of problems and persevere in solving them.	In this problem, you will order the *opposites* of the numbers from the previous problem, also from least to greatest. $$-0.45, -\frac{7}{8}, -\frac{5}{9}, -\frac{1}{10}, -0.9, -\frac{5}{8}, -\frac{2}{5}, -0.09, -\frac{1}{5}, -0.75$$ *Being asked to order several numbers can be overwhelming for students. The text provides scaffolding to make this problem more approachable. First, students are asked to order the opposites of the numbers shown above. The opposites of the numbers above are all positive rational numbers. At this point, students are likely more comfortable working with positive rational numbers. To order the list of positive rational numbers, students are encouraged to first categorize the numbers as closer to 0, closer to* $\frac{1}{2}$*, or closer to 1 and organize the information in a graphic organizer. Now, the students are tasked with ordering three subsets of the original set of numbers. Now, we turn back to the original problem above. If students have successfully ordered the opposites of the numbers shown above, they can use ideas about symmetry on the number line to order the negative numbers. Numbers that are bigger when positive become smaller when reflected over the vertical line passing through zero; therefore, the order of the negative numbers is the reverse of the positive numbers.*
	Reason abstractly and quantitatively.	Select all statements that are true based on the number line shown.  ☐ $-(D) = B$ ☐ $-(A) = -(E)$ ☐ $-(-(E)) = A$ ☐ $-(C) = C$ ☐ $B = -3\frac{1}{2}$ *Throughout the chapter, students use number line models to interpret abstract statements as shown in the example above. Students start the chapter by physically constructing a number line to show positive and negative rational numbers. Following the physical construction of a number line, students create and use models of number lines to investigate abstract concepts.*

Construct viable arguments and critique the reasoning of others.

Ms. Tucker tells her class that a and b are rational numbers and $a < b$. Describe what would have to be true about the values of a and b for the following statements to be true. Justify your answers.

a. The absolute value of a is larger than the absolute value of b.

b. The absolute value of a is smaller than the absolute value of b.

c. a is farther away from zero than b.

This problem requires students to apply what they know about the difference between the relative value of a number and the number's magnitude or distance from zero. Some students may create models to investigate the claims and use their models to justify their conclusions and communicate them to others. Others may test values for a and b to explore the problem and justify their claims.

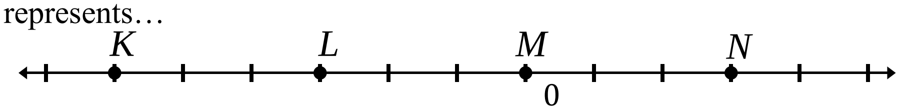
Model with mathematics.

The biggest temperature changes to occur in the same day occurred in Spearfish, South Dakota on January 22, 1943. The table below shows the temperature at different times during the day. Use this information to answer the questions below.

Time	**Temperature in Degrees Fahrenheit**
7:30 am	$-4°$
7:32 am	45°
9:00 am	54°
9:27 am	$-4°$

a. What was the change in temperature from 7:30 am to 7:32 am?
b. What was the change in temperature from 7:32 am to 9:00 am?
c. What was the change in temperature from 9:00 am to 9:27 am?
d. What was the warmest time of the day?
e. What was the coldest time of the day?

Section three of this chapter provides the opportunity for students to explore various real-world problems that necessitate the use of positive and negative numbers to represent quantities. While exploring these problems, students will likely create models such as number lines, and apply the skills learned previously in the chapter such as graphing, comparing, finding the distance between two points on the same horizontal or vertical line, etc.

	Use appropriate tools strategically.	*The tool used repeatedly in this chapter is a number line model. Students apply their understanding of the structure of the number line to identify and locate points on a number line, compare and order numbers, find the distance between two points on the same horizontal or vertical line, and represent real world quantities.*
	Attend to precision.	Graph the following sets of points and then find the distance between the two points. $(-2, -4)$ and $(4, -4)$ *Graphing points in the coordinate plane requires students to attend to precision. What does the first number in an ordered pair represent? The second number? If the first number is negative, do I move to the right or the left? Where do I start when I graph points? Once students have correctly graphed the points above, the next step is to find the distance between the two points. This task also requires precision. What are we counting to find the distance between the two points? Does it matter which point we start from? What do we know about our answer?*
	Look for and make use of structure.	*This chapter starts with students acquiring an understanding of the structure of the number line, specifically that the number line extends in both directions from zero and that we can create the left side of the number line (negative numbers) by reflecting the right side of the number line over the vertical line that passes through zero. Each point on the right side of the number line has a corresponding point on the left side of the number line, that is equidistant from zero but opposite in sign. The understanding of this structure plays an integral role throughout the chapter as students identify and locate rational numbers on the number line, compare and order rational numbers, and find distances between points. This structure also facilitates understanding of the coordinate plane as the intersection of a horizontal and vertical number line at the origin.*
	Look for and express regularity in repeated reasoning.	Give the values of *K*, *L*, *M*, and *N* on the graph below if each tick mark represents…  a. 1 unit b. 6 units c. 12 units d. $\frac{1}{2}$ of a unit e. $\frac{1}{3}$ of a unit f. $\frac{1}{6}$ of a unit *Students use repeated reasoning throughout the chapter when they scale graphs. In the problem above, each set of numbers is a multiple of another set of numbers. Students see that graphing 4 is like graphing* $\frac{4}{3}$ *because both numbers are 4 units to the right of zero on the number line but for the number 4 each interval has a length of 1 whereas for* $\frac{4}{3}$ *each interval has a length of* $\frac{1}{3}$*. This work ties back to 3.NF.2b: "Represent a fraction* a/b *on a number line diagram by marking off a lengths* $1/b$ *from* 0*. Recognize that the resulting interval has size* a/b *and its endpoint locates the number* a/b *on the number line."*

3.0 Anchor Problem:

There is a 136-mile long mountain bike ride where cyclists ride from Badwater Basin in Death Valley to Whitney Portal, a trailhead leading to Mount Whitney. Badwater Basin is the lowest place in North America with an elevation of 279 feet below sea level. Whitney Portal is 8,374 feet above sea level.

The riders start at Badwater Basin and pass through the places shown in the table in the order they appear in the table. The ride ends at Whitney Portal. Use this information to answer the questions that follow.

Places	**Elevation (to the nearest foot)**
Badwater Basin (start)	279 feet below sea level
Furnace Creek	190 feet below sea level
Beatty Junction	187 feet below sea level
Stovepipe Wells	10 feet above sea level
Panamint Springs	1,926 feet above sea level
Lone Pine	3,727 feet above sea level
Whitney Portal (end)	8,374 feet above sea level

a. Create a model to represent the elevations of the locations shown in the table.

b. Order the elevations from least to greatest.

c. Order the elevations from farthest from sea level to closest to sea level.

d. What is the change in elevation from Badwater Basin to Furnace Creek?

e. What is the change in elevation from Badwater Basin to Stovepipe Wells?

f. What is the change in elevation from Stovepipe Wells to Panamint Springs?

g. What is the change in elevation from the start of the ride (Badwater Basin) to the end of the ride (Whitney Portal)?

h. Whitney Portal is a trailhead that leads to Mount Whitney, the highest peak in the contiguous United States. Mount Whitney has an elevation of 14,505 feet. Following the bike ride, a rider decides to take the trail from Whitney Portal to the top of Mount Whitney. What is the change in elevation from Whitney Portal to the top of Mount Whitney?

i. What is the change in elevation from Badwater Basin to the top of Mount Whitney?

Section 3.1: The Symmetry of the Number Line

Section Overview:
Up to this point, students have worked with positive rational numbers. This section opens with a mathematical problem that serves as motivation for extending the number line to the left of zero, thus extending the number system to include all rational numbers. Students construct a number line, starting first with the portion of the number line to the right of zero and then, using a similar process, they extend the number line to the left of zero. Students discover that the number line is symmetric about the vertical line passing through zero and that every rational number has an opposite that is equidistant from zero but located on the opposite side of zero. Understanding of this structure will serve as a valuable tool throughout the chapter. As the section progresses, students write and interpret symbolic statements about opposites. Next, students explore scales of graphs. The ability to read and interpret the scale of a graph and to determine an appropriate scale to graph a given set of points or ordered pairs is a critical skill for students as they progress in mathematics. Next, students determine the value of rational numbers plotted on a number line and they represent rational numbers on a number line. Lastly, students see that we can intersect a horizontal and vertical number line at the origin to create a coordinate plane. They study the structure of the coordinate plane and find and position pairs of rational numbers on the coordinate plane.

Concepts and Skills to Master:
By the end of this section, students should be able to:
1. Understand a rational number as a point on the number line.
2. Construct a number line to show negative numbers using ideas about symmetry. Recognize opposite signs of numbers as indicating locations on opposite sides of zero on the number line.
3. Write and interpret symbolic statements about opposites (e.g., $-(-3)$ is read "the opposite of negative three" and can be simplified to 3).
4. Determine the value of rational numbers represented on a horizontal and vertical number line.
5. Represent rational numbers on horizontal and vertical number lines.
6. Represent pairs of rational numbers on the coordinate plane.

3.1a Class Activity: Constructing a Number Line

Activity 1 is intended to show students that there are mathematical problems that require us to expand our number system to include negative numbers, i.e., numbers to the left of zero on the number line.

In **Activity 2**, students construct a number line. They start by constructing the whole numbers and some positive rational numbers. Then, they use ideas about symmetry to construct the left side of the number line which is a reflection across zero of the right side they created.

Activity 1: Left of Zero

The purpose of this activity is to present a problem that provides motivation for extending the number line beyond the positive numbers. Students have been using number lines to model addition and subtraction problems since Grade 1. Many students have likely had a natural intuition about negative numbers since early on.

Step 1: Using a ruler, draw a horizontal line on the graph paper shown below.

Step 2: Mark a tick in the middle of the line and label it 0.

Step 3: Mark and label the numbers 1 – 10.

Step 4: Use your number line to show the location of $6 + 2$.

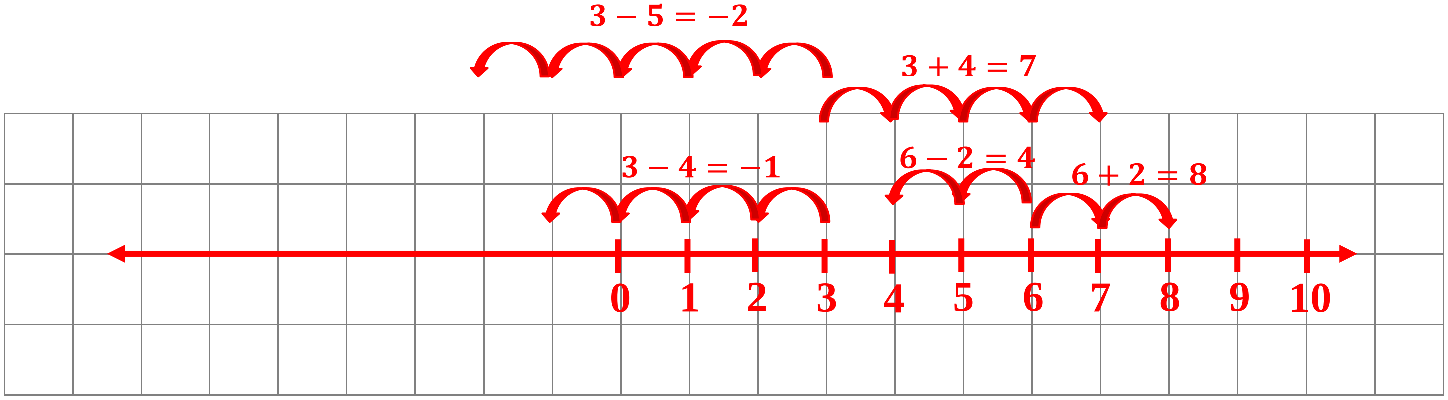
Step 5: Use your number line to show the location of $3 + 4$.

Step 6: Use your number line to show the location of $6 - 2$.

Step 7: Use your number line to show the location of $3 - 4$.

Step 8: Use your number line to show the location of $3 - 5$.

Step 9: Create your own problem that would show the location of -3. Answers will vary. Sample answers include $3 - 6, 0 - 3, 7 - 10$.



How did you use the Practice Standard "Use appropriate tools strategically?"

Student use a number line as a tool to simplify expressions.

This is an Illustrative Mathematics Task with minor modifications. The original task can be found at https://www.illustrativemathematics.org/content-standards/6/NS/C/6/tasks/1665.

Activity 2: Constructing the Number Line

In this activity, students physically construct a number line that includes positive and negative rational numbers. Students have been representing positive fractions on the number line since Grade 3:

3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

Materials Needed

- Strings to represent the unit interval.
- Three different colored markers.

Instructions

- Start by marking 0 in the middle of a long piece of butcher paper. Then, use a string to mark the unit intervals 1 – 5 (or more depending on space). The length of the string (or unit interval) can be arbitrarily chosen and students may have different lengths for their unit interval. Use the string as a tool to construct the number line. Start at zero and copy the string once to show the location of one. Continue this process to show the location of the whole numbers.
- Ask students how we can use the string to show the location of $\frac{1}{2}$. *Fold the string in half.* Mark off and label the $\frac{1}{2}$ unit intervals using a different color marker. Discuss equivalence as you mark off the $\frac{1}{2}$ unit intervals. We have already represented 1 unit using the number one but now we can also represent it with two halves or $\frac{2}{2}$ and 2 as $\frac{4}{2}$, etc. Have students write the equivalent forms for the number under each other on the butcher paper.
- Next, ask students how we can show the location of $\frac{1}{4}$. *Fold the string in half again.* Using a different colored marker, mark off and label the $\frac{1}{4}$ unit intervals. Again, discuss equivalence - (e.g., $\frac{7}{2}$ is the same as $3\frac{1}{2}$, $\frac{2}{4}$ is the same as $\frac{1}{2}$, $\frac{4}{4}$ is the same as $\frac{2}{2}$ is the same as 1).
- Stop here and discuss the following with students:
  - One can define the length of a unit interval and then use this unit interval to show the location of whole numbers and fractions such as $\frac{1}{2}$ and $\frac{1}{4}$.
  - Points on the line correspond to a number.
  - There is a one-to-one correspondence to each point on the line and a number. In other words, a point does not represent more than one number.
  - There are different, yet equivalent, ways to express the number associated with a point on the line (e.g., $\frac{7}{2}$ is the same as $3\frac{1}{2}$, $\frac{2}{4}$ is the same as $\frac{1}{2}$, $\frac{4}{4}$ is the same as $\frac{2}{2}$ is the same as 1).

Instructions continued…

- Using your string, find and label -1 (have students use the same color marker they used to mark the whole numbers). If students struggle, ask them to think about a parking garage. Point to zero and tell them zero is street level. Point to one student and tell them that they are parked one floor above street level in the parking garage. Have the student show you where they would be on the number line (+1). Point to another student and tell them that they are parked one floor below street level in the parking garage. Have the student show you where they would be on the number line (–1). Ask students where someone parked two floors below street level would be (–2). From here, they should be able to mark and label the negative integers. Have them label and mark the same number of negative integers as they did positive integers (i.e. if they marked from 1 to 5, have them mark from –1 to –5).
- Next, tell students to use the string and a different colored marker (the same color used to mark the positive $\frac{1}{2}$ intervals) to mark and label $-\frac{1}{2}$ and then continue to label the points that are $\frac{1}{2}$ unit apart on the left side of the zero. Some students will write $-\frac{1}{2}, -1, -1\frac{1}{2}, -2, -2\frac{1}{2}$, etc. while others will write $-\frac{1}{2}, -\frac{2}{2}, -\frac{3}{2}, -\frac{4}{2}, -\frac{5}{2}$, etc. Again, emphasize that there are different yet equivalent ways of expressing the number associated with a point on the line.
- Next, tell students to use the string and a different colored marker (the same color used to mark the positive $\frac{1}{4}$ intervals) to mark and label $-\frac{1}{4}$ and then continue to label the points that are $\frac{1}{4}$ unit apart on the left side of zero. Again, some students will write $-\frac{1}{4}, -\frac{1}{2}, -\frac{3}{4}, -1, -1\frac{1}{4}, -1\frac{1}{2}$, etc. while others will write $-\frac{1}{4}, -\frac{2}{4}, -\frac{3}{4}, -\frac{4}{4}, -\frac{5}{4}, -\frac{6}{4}$, etc.
- Rotate the number line 90° counterclockwise to show a vertical number line. What do students notice? *Positive numbers are above 0, negative numbers are below 0.*
- The following ideas should come up in the discussion:
  - You can construct the left side of a number line in the same way you constructed the right side of the number line. In both cases, you start from zero and work your way out. Positive numbers are to the right of zero on a horizontal number line and above zero on a vertical number line. Negative numbers are to the left side of zero on a horizontal number line and below zero on a vertical number line.
  - The value associated with a point is relative to zero (e.g., 1 is one unit to the right of zero while -1 is one unit to the left of zero); however, the length of the interval (or distance) from zero to a number is the same as the length of the interval (or distance) from zero to the number's opposite. This begins to surface ideas about the distinction between the relative value of a number and the magnitude of a number.
  - Just like there are different but equivalent ways of expressing positive numbers, there are also different but equivalent ways of expressing negative numbers.
- Define the following words with students using the number line as a visual aid. Provide examples to support the definitions.

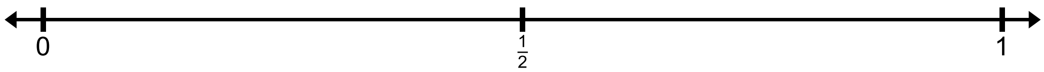
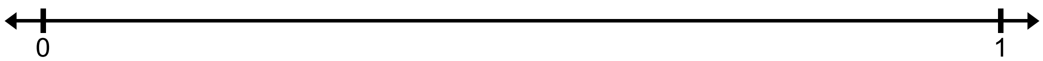
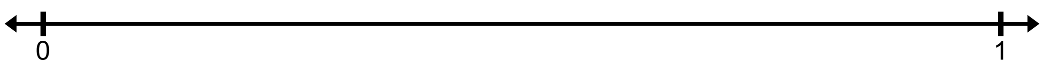
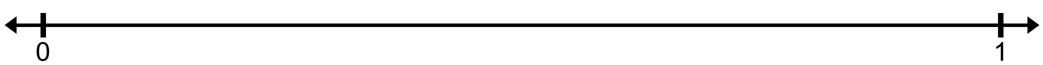
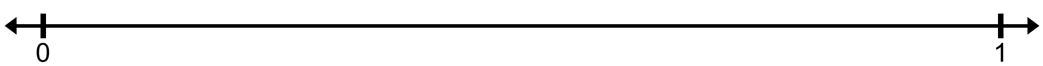
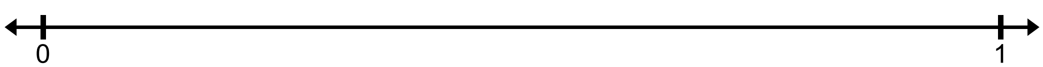
  Positive numbers: Numbers located to the right of 0. Examples include $0.01, \frac{1}{2}, 2, 100$

  Negative numbers: Numbers located to the left of 0. Examples include $-0.01, -\frac{1}{2}, -2, -100$

  Point out to students that zero is neither positive nor negative.
  Opposites: Two numbers that are on opposites sides of the number line and equidistant from zero.
  Whole numbers: $0, 1, 2, 3, 4, 5 \ldots$
  Integers: whole numbers and their opposites; $\ldots -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 \ldots$
  Rational Numbers: a number that can be written as one integer divided by a non-zero integer. Examples include $\frac{1}{2}, \frac{3}{4}, -1\frac{1}{2}, 3, -3$ We will examine the association between the different types of rational numbers at the end of section. Students will be introduced to irrational numbers in 8th grade.

Spiral Review

1. Partition the segment from zero to one to show the fractional parts. The first one has been done for you.

Partition the Number Line to Show…	**Number Line**	**Number of Cuts Between 0 and 1**
Halves		1
Thirds		
Fourths		
Fifths		
Sixths		
Eighths		
Ninths		
Tenths		

2. If you make 11 cuts between zero and one, how many segments will you create? What is the length of each segment?

3. How many cuts would you need to make between zero and one to create twentieths?

4. How many cuts would you need to make to create hundredths? Explain a process you can use to create hundredths.

3.1a Homework: Constructing a Number Line

1. In the space below, construct a number line showing the integers from -5 to 5.

2. Explain the process you used to create your number line. What steps did you take? What tools did you use?

3. How many points are 3 units away from 0 on your number line? Explain.
Two points are 3 units from 0. They are 3 and -3.

4. Explain a method for showing the location of $\frac{1}{3}$ on your number line.

5. Explain a method for showing the location of $-\frac{1}{3}$ on your number line.

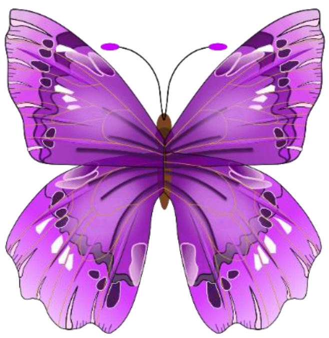
6. Explain a method for showing the location of $\frac{4}{3}$ on your number line. Mark $\frac{4}{3}$ on your number line.

7. Explain a method for showing the location of $-\frac{4}{3}$ on your number line. Mark $-\frac{4}{3}$ on your number line.

3.1b Class Activity: Symmetry on a Number Line

Activity 1:

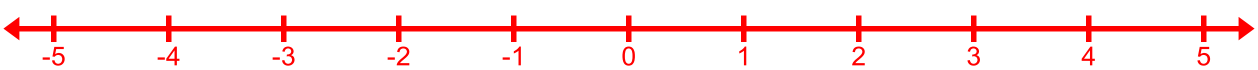
Start by asking students what they notice about the butterflies shown below. Students studied symmetry in 4.G.3 and will likely recognize that the butterflies exhibit line symmetry or bilateral symmetry. This is a form of symmetry where there is a vertical line of symmetry that divides an object into left and right sides. Body parts and shapes match up or correspond when the object is folded on the line of symmetry. Have students draw the lines of symmetry in the space below. Ask students for other objects that exhibit line symmetry. Peacocks, frogs, beetles, ladybugs, bats, humans, squares, rectangles, hexagons, the Batman logo, etc.



Students likely observed the symmetry of the number line in the previous lesson. In this activity, students solidify that understanding. Understanding of this geometric structure will help students to identify and graph points, compare and order rational numbers, and solve real world problems involving negative numbers. Guide students through this activity using the steps provided below. You will need patty paper or tracing paper for this activity. Patty paper can be purchased online and is relatively inexpensive.

Step 1: Using a ruler, draw a line on a piece of patty or tracing paper (include arrows on both ends to indicate that the line continues indefinitely in both directions). Mark a point 0 in the middle of the line.

Step 2: Define the unit interval using a ruler, side of an index card, etc. Make tick marks to show the location of 1 to 5. Then, using the same unit interval, mark and label -1 to -5.



Step 3: Fold the paper in half along the vertical line passing through 0. Ask students what they notice. *The number line is symmetric about the vertical line passing through zero. The fold shows the line of reflection. Opposites match up or correspond to each other when the paper is folded. Opposites are equidistant from zero.*

Step 4: Ask students how they can use these ideas to represent different negative rational numbers on the number line (e.g., $-\frac{1}{2}, -2\frac{2}{3}, -3\frac{1}{3}$, etc.). Since students have been representing positive rational numbers on the number line since early on, it may be easier for them to plot the opposite of these negative rational numbers and then reflect the point over the vertical line passing through zero. For example, to show the location of $-2\frac{2}{3}$, first plot $2\frac{2}{3}$. Then, fold the paper along the vertical line passing through zero. Make a tick mark where the point $2\frac{2}{3}$ hits the left sides of the number line. This point (the image) represents $-2\frac{2}{3}$.

Step 5: Have students rotate the number line they created 90° counterclockwise to show a vertical number line. What do they notice? *Still symmetric but now it is symmetric across a horizontal line passing through 0. Positive numbers are above zero and negative numbers are below zero. In section 3, students will combine a horizontal number line (x-axis) and a vertical number line (y-axis) when they represent ordered pairs on a coordinate plane in four quadrants.*

Step 6: Ask students how they would graph a negative rational number such as $-4\frac{3}{4}$ on a vertical number line using ideas about symmetry. Graph $4\frac{3}{4}$ and then reflect that point over the *horizontal* line passing through zero.

Step 7: In the space below, have students summarize what they learned in this activity.

- A horizontal number line is symmetric across a vertical line passing through zero. A vertical number is symmetric across a horizontal line passing through zero.
- If you fold a number line along its line of symmetry, opposites will match up or correspond.
- When you reflect a point (pre-image) the image will be the same distance from the line of reflection as the original point.
- Opposites are the same distance from 0 but one is to the right of 0 (positive number) and one is to the left of zero (negative number).
- If you reflect a point on the line of symmetry, it remains on the line of symmetry; therefore, the opposite of zero is zero. A point on the line of reflection is 0 units away from the line of reflection so its image will also be zero points away from the line of reflection.
- One way to represent negative numbers on a number line is to first represent the number's opposite (a positive number) and then reflect the point across the line of symmetry. This is particularly helpful for students who are just beginning to explore negative numbers. Graphing negative numbers can be confusing for some students so this structure will provide support for them.
- Define the following words with students using the number line as a visual aid. You may have students tape their number line into their notebooks so that they can label different parts of the number line (e.g., positive numbers, negative numbers, opposites, etc.) to support their definitions. Also, provide examples to support the definitions.

  Positive numbers: Numbers located to the right of 0. Examples include $0.01, \frac{1}{2}, 2, 100$

  Negative numbers: Numbers located to the left of 0. Examples include $-0.01, -\frac{1}{2}, -2, -100$

  Point out to students that zero is neither positive nor negative.

  Opposites: Two numbers that are on opposites sides of the number line and equidistant from zero.

  Whole numbers: $0, 1, 2, 3, 4, 5 \ldots$

  Integers: whole numbers and their opposites; $\ldots -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 \ldots$

  Rational Numbers: a number that can be written as a ratio of an integer and a non-zero integer. Examples include $\frac{1}{2}, \frac{3}{4}, -1\frac{1}{2}, 3, -3$

  We will examine the association between the different types of rational numbers at the end of the section.

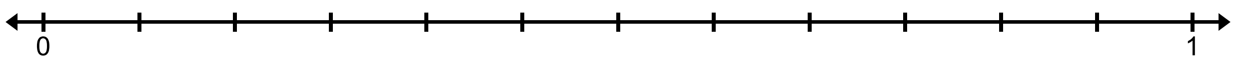
Spiral Review

1. Complete each statement to show the equivalent fractions.

a. $\frac{1}{2} = \frac{\quad}{12}$	b. $\frac{1}{3} = \frac{\quad}{12}$	c. $\frac{1}{6} = \frac{\quad}{12}$
d. $\frac{2}{3} = \frac{\quad}{12}$	e. $\frac{3}{4} = \frac{\quad}{12}$	f. $\frac{5}{6} = \frac{\quad}{12}$

2. Use the number line below to complete this activity.

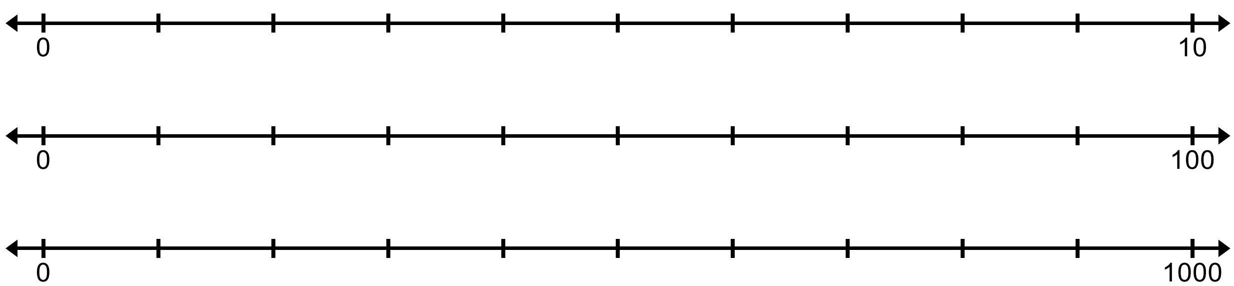
   a. Label the tick marks to show the scale of the graph.



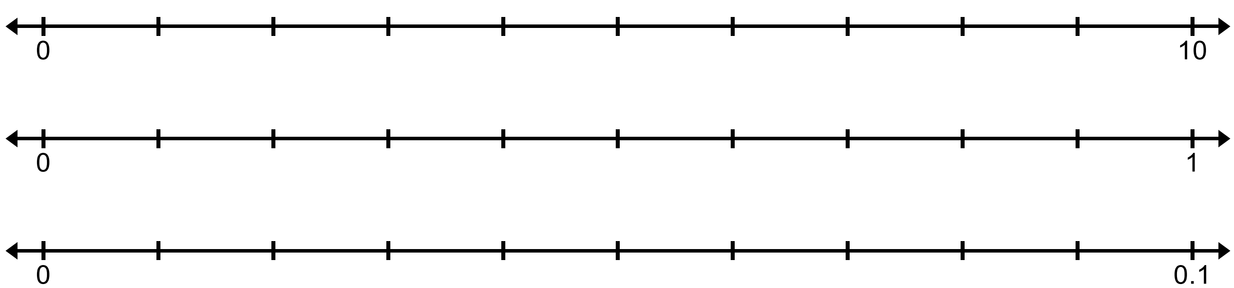
   b. Graph and label the following points on the number line:

A: $\frac{1}{2}$ B: $\frac{1}{3}$ C: $\frac{1}{6}$ D: $\frac{2}{3}$ E: $\frac{3}{4}$ F: $\frac{5}{6}$

3. Complete the scale on the number lines below:



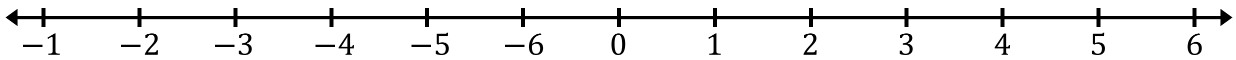
4. Complete the scale on the number lines below:



3.1b Homework: Symmetry on a Number Line

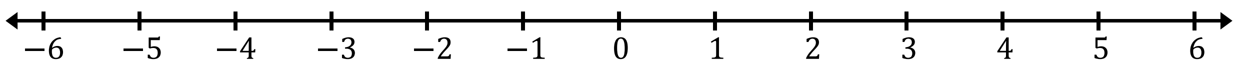
1. Mrs. Henderson asked her students to create a number line to represent the integers from -6 to 6. The work of five different students is shown below. Circle the names of the students who created a correct number line. For the number lines that are incorrect, explain the error.

Emma's Number Line:

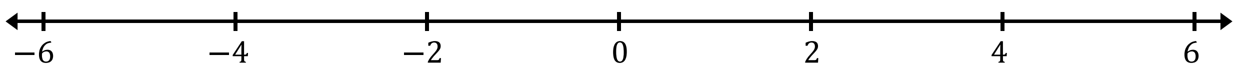


Sam's Number Line:

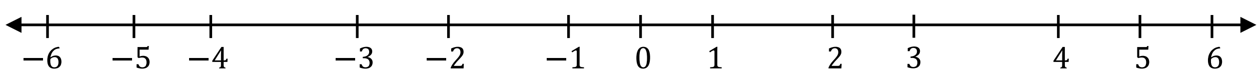
Tyson's Number Line:



Amy's Number Line:



Dom's Number Line:

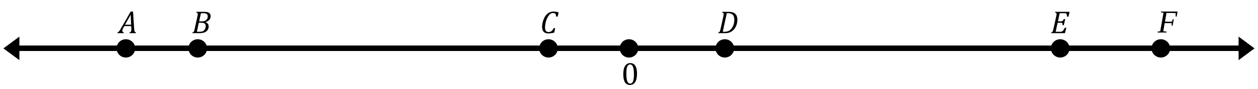


2. Select all the statements that are true about the number -5 on a horizontal number line.

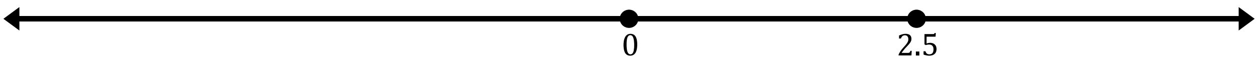
   - ☐ It is 5 units away from zero.
   - ☐ It is -5 units away from zero.
   - ☐ It is located to the left of zero.
   - ☐ It is 10 units away from its opposite.
   - ☐ It is 1 unit to the right of -4.

3. What is the relationship between 7 and -7 on the number line?
Answers will vary. They are opposites. They are the same distance from zero (have the same absolute value) but one is located to the left of zero and one is located to the right of zero. They are reflections of each other across the vertical line passing through 0.

4. Which set or sets of points are opposites on the number line shown below? Justify your answer.



5. Use the number line below to complete this question.



   a. The number line below shows the location of 2.5. Explain a method for representing -2.5 on the number line.

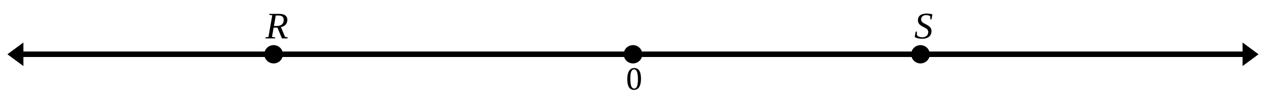
   b. Represent -2.5 on the number line.

6. Use the number line below to complete this question.

   a. Show the location of zero on the graph if points A and B are opposites.

   b. Explain the method you used to show the location of zero.

7. Two points R and S are graphed on the number line below.



Select all statements that you know are true about the *opposite* of R.

☐ It is a negative number. False

☐ It is the same distance from zero as R. True

☐ It is located to the right of S.

☐ It is a whole number.

Write your own statement that is true about the *opposite* of R.

8. How many numbers are $2\frac{1}{2}$ units away from 0 on a number line? Justify your answer.

9. For each statement below, answer either:

A: $\frac{1}{4}$ B: $-\frac{1}{4}$ C: $\frac{1}{4}$ and $-\frac{1}{4}$

a. Represents the number or numbers $\frac{1}{4}$ of a unit to the left of 0.

b. Represents the distance from 0 to $-\frac{1}{4}$.

c. Represents the number or numbers $\frac{1}{4}$ of a unit away from zero.

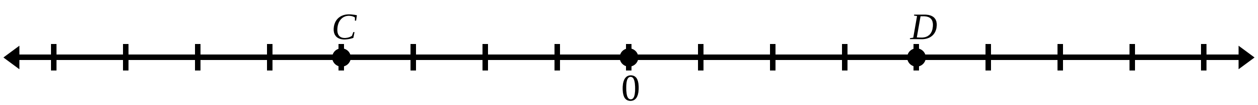
10. What number is equal to its opposite? Explain.

3.1c Class Activity: Exploring Scales of Graphs

A critical skill for success in subsequent math courses is to decide how to scale a graph to represent a set of points on the line and in the plane. Many students struggle with choosing an appropriate scale for a graph. This lesson is comprised of a set of problems aimed at helping students start to think about scales of graphs. They may use fraction sense, understanding of part to whole relationships, and ratio reasoning to work through these problems. In the lessons that follow, students will be asked to represent points on the number line and in the coordinate plane. When a scale is not given, students need to first choose a scale that makes sense for a given problem.

Activity 1:

Mrs. Potter put the following number line on the board and asked the students to give the values of C and D.



Here are the responses she got from six different students:

Shaun
$C = -4$ and $D = 4$

Nadia
$C = -8$ and $D = 8$

Daniella
$C = -200$ and $D = 200$

Gloria
$C = -1$ and $D = 1$

Jesse
$C = -\frac{4}{3}$ and $D = \frac{4}{3}$

Tabitha
$C = -\frac{2}{3}$ and $D = \frac{2}{3}$

a. Mrs. Potter told the class that all six students have given a correct answer. Shaun thought to himself, "How can we all have different answers and all be correct?" Help Shaun by explaining how all six students can be correct.

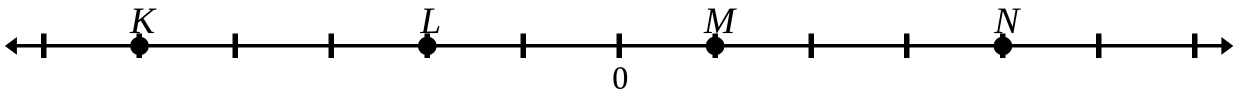
Since a scale was not given on the graph, all the students scaled the graph differently. Shaun scaled the graph by 1s, Nadia by 2s, Daniella by 50s, Gloria by $\frac{1}{4}$s, Jesse by $\frac{1}{3}$s, and Tabitha by $\frac{1}{6}$s. Examine the relationship between the values of C and D and the change in scale. When we double the scale (e.g., each tick mark represents a length of 2 instead of 1) the values of all the points double. When we quarter the scale (e.g., each tick mark represents a length of $\frac{1}{4}$ as opposed to 1) the values of all the points are quartered. Ask students how graphing 4 is like graphing $\frac{4}{3}$. *Both are 4 units to the right on the number line but for the number 4 each interval has a length of 1 whereas for* $\frac{4}{3}$ each interval has a length of $\frac{1}{3}$.
Students might struggle with Tabitha's scale. Ask how Tabitha's answers are related to Jesse's. They are half which means that Tabitha's scale is half of Jesse's. Jesse's scale is $\frac{1}{3}$ so Tabitha's is $\frac{1}{6}$. Have students show the scale for Tabitha on the graph $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}$ so that they can better understand where the $\frac{2}{3}$ comes from.

b. Give another possible set of values for C and D. Explain.

Answers will vary. Sample answers are $C = -400$ and $D = 400$; $C = -2$ and $D = 2$; $C = -0.4$ and $D = 0.4$

1. Give the values of K, L, M, and N on the graph below if each tick mark represents…

It may help students to complete the scale for the graph prior to giving the values of the points. They may choose to use a different colored pencil for each of the different scales.



Discuss equivalence (e.g., $\frac{3}{2} = 1\frac{1}{2}$; $\frac{6}{2} = 3$, etc.)

a. 1 unit

K: -5 L: -2 M: 1 N: 4

b. 6 units

K: -30 L: -12 M: 6 N: 24

c. 12 units

K: -60 L: -24 M: 12 N: 48

d. $\frac{1}{2}$ of a unit

K: $-2\frac{1}{2}$ L: -1 M: $\frac{1}{2}$ N: 2

e. $\frac{1}{3}$ of a unit

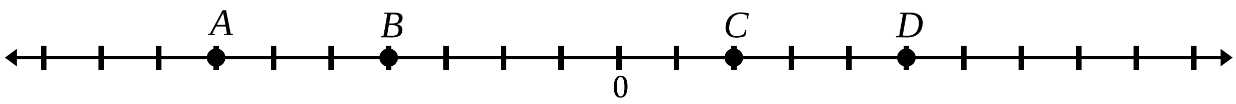
K: $-\frac{5}{3}$ L: $-\frac{2}{3}$ M: $\frac{1}{3}$ N: $\frac{4}{3}$

f. $\frac{1}{6}$ of a unit

K: $-\frac{5}{6}$ L: $-\frac{2}{6}$ or $-\frac{1}{3}$ M: $\frac{1}{6}$ N: $\frac{2}{3}$ or $\frac{4}{6}$

2. Mr. Tanner put the following number line on the board and asked his students to give the value of C. Mr. Tanner did not put a scale on the number line and these are the responses he got from six different students.

   a. Determine the scale that each student used.
   b. Use the same scale as each student to determine the values of A, B and D.



Eli's Response: A: -7 B: -4 C: 2 D: 5
Scale Eli Used: ___1____

Chelsea's Response: A: -14 B: -8 C: 4 D: 10
Scale Chelsea Used: ___2____

Martin's Response: A: -70 B: -40 C: 20 D: 50
Scale Martin Used: ___10____

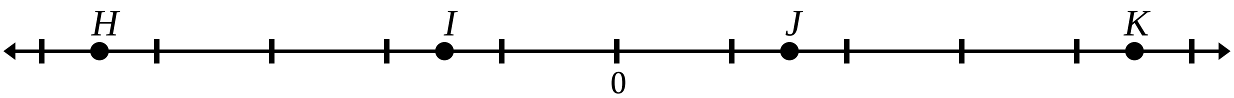
Hadley's Response: A: -0.7 B: -0.4 C: 0.2 D: 0.5
Scale Hadley Used: ___0.1____

Jordan's Response: A: $-\frac{7}{5}$ B: $-\frac{4}{5}$ C: $\frac{2}{5}$ D: 1
Scale Jordan Used: ___$\frac{1}{5}$____

Leila's Response: A: $-\frac{7}{8}$ B: $-\frac{1}{2}$ C: $\frac{1}{4}$ D: $\frac{5}{8}$
Scale Leila Used: ___$\frac{1}{8}$____

Ask students what they notice in this problem. Some ideas that may emerge are: 1) Each set of numbers is a multiple of another set of numbers. 2) A length can be equal to any number. This idea will become important when students are asked to graph sets of numbers on a number line and in the coordinate plane. They need to understand that they can define the scale depending on the numbers they are being asked to graph. 3) This problem also ties back to 3.NF.2b: "Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and its endpoint locates the number a/b on the number line." To explore this concept with students, refer to Eli and Leila's responses for D and ask students, "How is representing 5 on a number line like representing $5/8$ on a number line?" In both cases we move 5 lengths to the right (a is 5). Then ask, "What is the difference?" The difference is that in the case of 5, we are marking off 5 lengths of 1 while in the case of $5/8$, we are marking off 5 lengths of $1/8$. This idea may be helpful for students who struggle with representing fractions on a number line or coordinate plane. An alternative method for representing fractions on a number line is to define the interval from 0 to 1 as the whole and partition it into b equal parts (3.NF.2a). It is important for students to understand how to do both.

3. Give the values of H, I, J, and K on the graph below if each tick mark represents…



a. 1 unit

H: $-4\frac{1}{2}$ $\quad$ I: $-1\frac{1}{2}$ $\quad$ J: $1\frac{1}{2}$ $\quad$ K: $4\frac{1}{2}$

b. 6 units

H: -27 $\quad$ I: -9 $\quad$ J: 9 $\quad$ K: 27

c. 10 units

H: -45 $\quad$ I: -15 $\quad$ J: 15 $\quad$ K: 45

d. 50 units

H: -225 $\quad$ I: -75 $\quad$ J: 75 $\quad$ K: 225

One of the first things that students might notice in this problem is that H and K are opposites and I and J are opposites. **Strategy 1:** When the scale changes, students may use repeated reasoning to determine the value of the points. For example, for part b., to find J, students might count $6+\frac{1}{2}(6)=6+3=9$. To find K, students might count $6+6+6+6+\frac{1}{2}(6)=27$ or $4(6)+\frac{1}{2}(6)=24+3=27$. **Strategy 2:** Others might multiply the value of the points in part a. by the new scale. For example, for part b., to find J, students might multiply $1\frac{1}{2}$ by 6: $1\frac{1}{2}(6)=1(6)+\frac{1}{2}(6)=6+3=9$. To find K, students might multiply $4\frac{1}{2}$ by 6: $4\frac{1}{2}(6)=4(6)+\frac{1}{2}(6)=24+3=27$. Help students to draw connections between these two different strategies and any other strategies students might use.

Directions: Complete the scales on the number lines shown below. Use ideas about opposites to help you.

Students may use ideas about symmetry to complete the number lines.

4.

$-10 \quad -9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

5.

$-50 \quad -40 \quad -30 \quad -20 \quad -10 \quad 0 \quad 10 \quad 20 \quad 30 \quad 40 \quad 50$

6.

$-200 \quad -150 \quad -100 \quad -50 \quad 0 \quad 50 \quad 100 \quad 150 \quad 200$

7.

$-\frac{8}{4} \quad -\frac{7}{4} \quad -\frac{6}{4} \quad -\frac{5}{4} \quad -\frac{4}{4} \quad -\frac{3}{4} \quad -\frac{2}{4} \quad -\frac{1}{4} \quad 0 \quad \frac{1}{4} \quad \frac{2}{4} \quad \frac{3}{4} \quad 1 \quad \frac{5}{4} \quad \frac{6}{4} \quad \frac{7}{4} \quad \frac{8}{4}$

8.

$-1 \quad -0.9 \quad -0.8 \quad -0.7 \quad -0.6 \quad -0.5 \quad -0.4 \quad -0.3 \quad -0.2 \quad -0.1 \quad 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1$

9.

1.5, 1.25, 1, 0.75, 0.5, 0.25, 0, -0.25, -0.5, -0.75, -1, -1.25, -1.5

10.

15, 12, 9, 6, 3, 0, -3, -6, -9, -12, -15

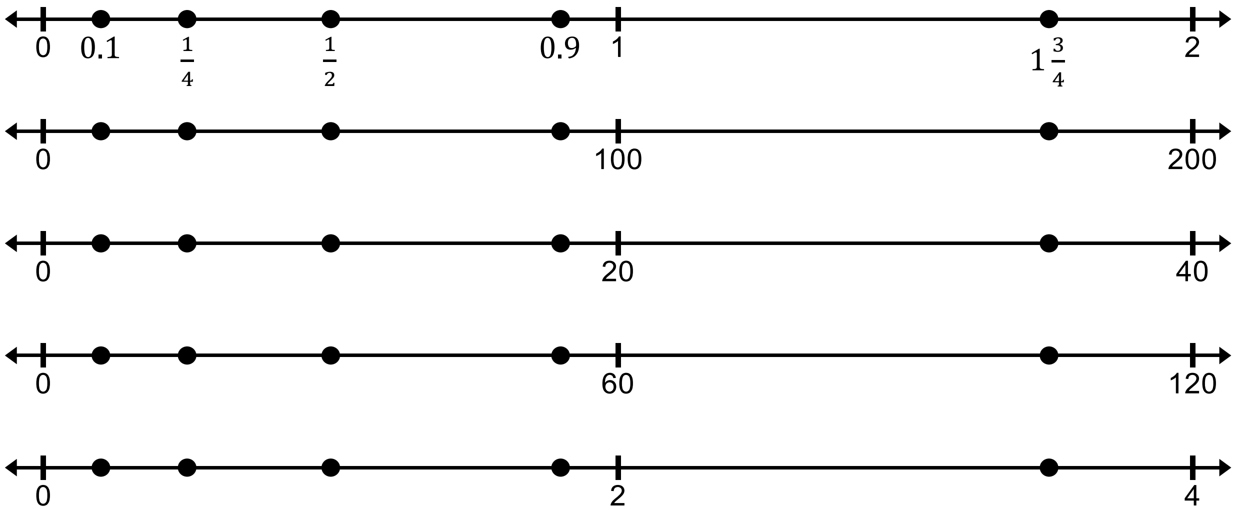
11.

$\frac{5}{10}$, $\frac{2}{5}$, $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{10}$, 0, $-\frac{1}{10}$, $-\frac{1}{5}$, $-\frac{3}{10}$, $-\frac{2}{5}$, $-\frac{5}{10}$

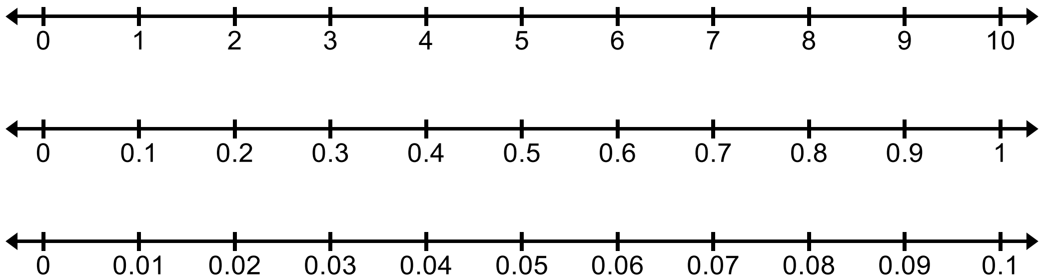
Discuss equivalence (e.g., for #9: $0.25 = \frac{1}{4}$; $1.25 = \frac{5}{4} = 1\frac{1}{4}$, for #11: $-\frac{2}{5} = -\frac{4}{10} = -0.4$; $\frac{5}{10} = \frac{1}{2} = 0.5$

Spiral Review

1. Use the first graph to determine the value of the points on the remaining four graphs. In the space below, explain the strategy you used to solve the problem.



2. Use the three number lines below to answer the questions below.



a. What number is half-way between 4 and 5 on the first number line? ______

b. What number is half-way between 0.4 and 0.5 on the second number line? ______

c. What number is half-way between 0.04 and 0.05 on the third number line? ______

d. What number is half-way between 0 and 1 on the first number line? ______

e. What number is half-way between 0 and 0.1 on the second number line? ______

f. What number is half-way between 0 and 0.01 on the third number line? ______

g. Give a number that is between 8 and 9 on the first number line. ______

h. Give a number that is between 0.8 and 0.9 on the second number line. ______

i. Give a number that is between 0.08 and 0.09 on the third number line. ______

3. Change each mixed number to an improper fraction.

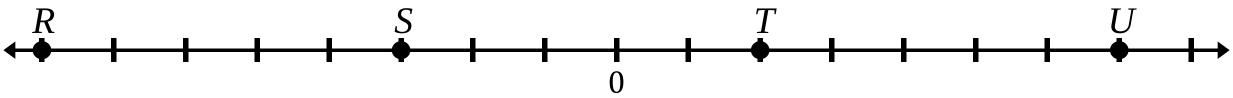
a. $3\frac{1}{2}$	b. $1\frac{2}{3}$	c. $2\frac{4}{5}$
d. $1\frac{3}{4}$	e. $8\frac{3}{10}$	f. $5\frac{3}{8}$

4. Change each improper fraction to a mixed number.

a. $\frac{9}{2}$	b. $\frac{10}{3}$	c. $\frac{15}{4}$
d. $\frac{35}{10}$	e. $\frac{22}{8}$	f. $\frac{32}{5}$

3.1c Homework: Exploring Scales of Graphs

1. Give the values of *R*, *S*, *T*, and *U* on the graph below if each tick mark represents…



a. 1 unit

R: *S*: *T*: *U*:

b. 2 units

R: *S*: *T*: *U*:

c. 5 units

R: -40 *S*: -15 *T*: 10 *U*: 35

d. 25 units

R: *S*: *T*: *U*:

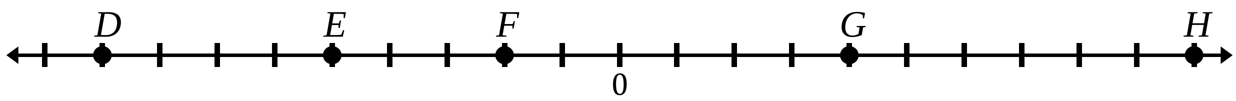
e. $\frac{1}{4}$ of a unit

R: $-\frac{8}{4}$ or -2 *S*: $-\frac{3}{4}$ T: $\frac{2}{4}$ or $\frac{1}{2}$ *U*: $\frac{7}{4}$ or $1\frac{3}{4}$

f. $\frac{1}{8}$ of a unit

R: *S*: *T*: *U*:

2. Mrs. Potter put the following number line on the board and asked her students to give the value of *D*, *E*, *F*, *G*, and *H*. Mrs. Potter did not put a scale on the number line and these are the responses she got from six different students. Determine the scale that each student used.



Trevor's Response: D: -9 E: -5 F: -2 G: 4 H: 10
Scale Trevor Used: ________

Kaitlyn's Response: D: -27 E: -15 F: -6 G: 12 H: 30
Scale Kaitlyn Used: ___3____

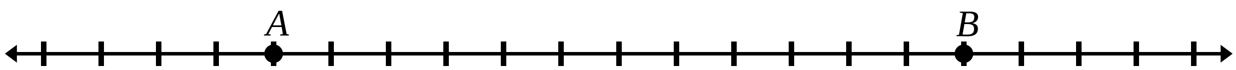
Jake's Response: D: -225 E: -125 F: -50 G: 100 H: 250
Scale Jake Used: ________

Marla's Response: D: -900 E: -500 F: -200 G: 400 H: $1{,}000$
Scale Marla Used: ________

Kai's Response: D: $-\frac{9}{4}$ E: $-\frac{5}{4}$ F: $-\frac{1}{2}$ G: 1 H: $\frac{5}{2}$
Scale Kai Used: ________

Iya's Response: D: $-\frac{9}{2}$ E: $-\frac{5}{2}$ F: -1 G: 2 H: 5
Scale Iya Used: ___$\frac{1}{2}$____

3. Use the number line below to answer the questions that follow.



a. Show on the graph where 0 would have to be for *A* and *B* to be opposites.

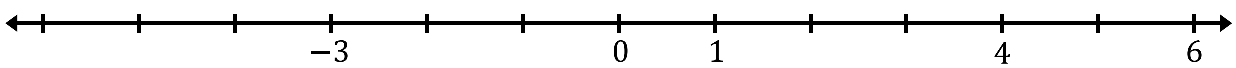
b. Give one possible set of values for *A* and *B*.
A: ____ *B*: ____

c. Give a different possible set of values for *A* and *B*.
A: ____ *B*: ____

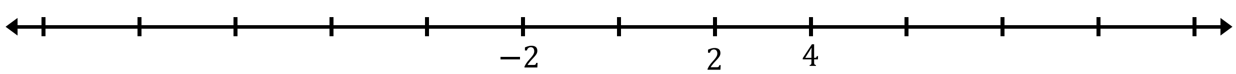
d. Cynthia's set of values were $A = -3$ and $B = 3$. How did Cynthia scale the graph?

Directions: Complete the scale on the number lines shown below. Use ideas about opposites to help you.

4.



5.



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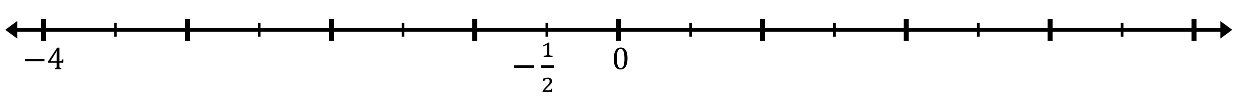

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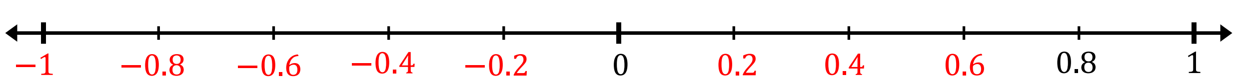

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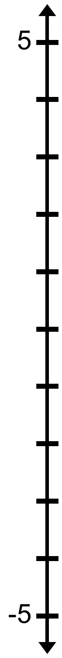

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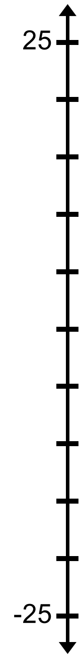

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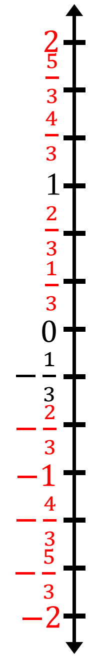


For #10, discuss equivalent ways of expressing the scale (e.g., $\frac{4}{3}$ is equivalent to $1\frac{1}{3}$; 2 is equivalent to $\frac{6}{3}$, etc.)

3.1d Class Activity: Opposites

Activity 1: Create a number line showing the integers from -5 to 5.

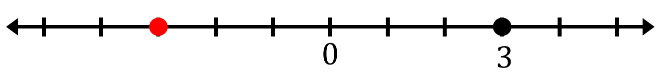
a. Graph 5 and its opposite on the number line you created.

b. Graph -3 and its opposite on the number line you created.

c. Graph $1\frac{1}{2}$ and its opposite on the number line you created.

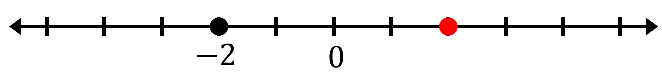
Directions: In each problem, a point is plotted. Plot the opposite of the point. Then, write the value of the original point and its opposite in the space provided.

1.



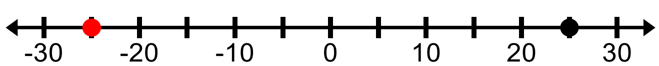
Original Point: __3____ Opposite: ___-3___

2.



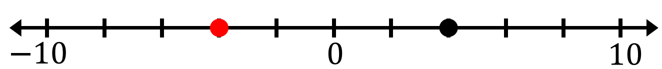
Original Point: __-2____ Opposite: __2____

3.



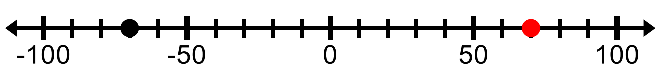
Original Point: __25____ Opposite: __-25____

4.



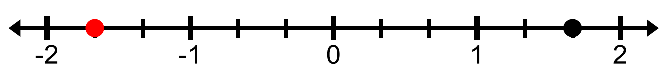
Original Point: __4____ Opposite: ___-4___

5.



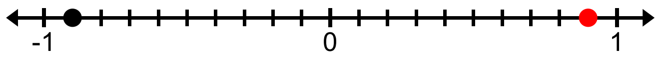
Original Point: __-70____ Opposite: ___70___

6.



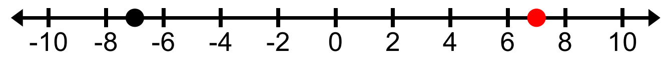
Original Point: __$1\frac{2}{3}$____ Opposite: ___$-1\frac{2}{3}$____

7.



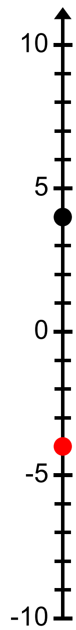
Original Point: __-0.9____ Opposite: __0.9____

8.



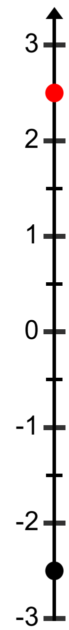
Original Point: ___-7____ Opposite: _____7___

9.



Original Point: ___4___ Opposite: ___-4___

10.



Original Point: ___$-2\frac{1}{2}$___ Opposite: __$2\frac{1}{2}$____

11. Tell whether the following statements are true or false. Justify your answers. Showing examples is a great way to justify when a statement is false.

   a. The opposite of a number is always a negative number.
   False; the opposite of a number is sometimes negative. For example, the opposite of 3 is -3; however, the opposite of -3 is 3, which is a positive number.

   b. The opposite of 0 is 0.
   True

   c. The opposite of a positive number is always a negative number.
   True

   d. The opposite of a number is always smaller than the original number.
   False; the opposite of a number is sometimes smaller than the original number. For example, the opposite of 5 is -5 and $-5 < 5$; however, the opposite of -5 is 5 and $5 > -5$. Another example, the opposite of 0 is 0 and $0 = 0$.

Activity 2:

In these problems, students translate between statements written using words and symbolic statements which are an abstract representation of the statements. Students also decode the structure of the numeric expression to simplify the expression.

a. What is the opposite of 15?
   Introduce the notation of using a negative sign to represent "the opposite of". In this problem, "the opposite of 15" can be written symbolically as $-(15)$ which is -15.

   Students are often confused by the symbol $-$. How do we know when it means opposite and how do we know when it means subtraction? See the mathematical foundation for more information on this. Students will discuss this further when they operate on rational numbers in Grade 7.

b. What is the opposite of -15?
   Again, show the notation. In this problem, "the opposite of -15" can be written symbolically as $-(-15)$ which is 15.

c. How do you read the expression $-(4)$? Simplify $-(4)$.
   the opposite of 4; -4.

d. How do you read the expression? $-(-2)$. Simplify $-(-2)$.
   the opposite of -2; 2.

e. Write "the opposite of -1.3" symbolically (using notation). Then, simplify.
   $-(-1.3) = 1.3$

12. How do you read the expression $-(-9)$? Simplify $-(-9)$.
    the opposite of negative nine; 9

13. How do you read the expression $-(10)$? Simplify $-(10)$.
    the opposite of ten; -10

14. Write "the opposite of $4\frac{1}{2}$" symbolically (using notation). Then, simplify.
    $-\left(4\frac{1}{2}\right)$; $-4\frac{1}{2}$

15. Write "the opposite of negative 12" symbolically (using notation). Then, simplify.
    $-(-12)$; 12

Activity 3: The Opposite of the Opposite

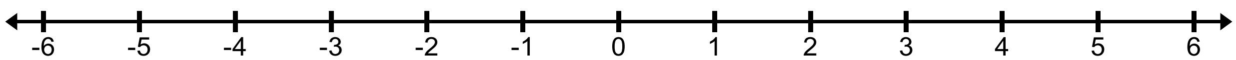
a. What is the opposite of being happy? Being sad

b. What is the opposite of the opposite of being happy? Being happy

c. What is the opposite of being on time? Being late

d. What is the opposite of the opposite of being on time? Being on time

Directions: Use the number line below to answer the questions that follow.



e. What is the opposite of 2?
-2

f. What is the opposite of the opposite of 2? Describe what is happening geometrically (on the number line).
The opposite of the opposite of 2 is 2. The opposite of the opposite of 2 is a double reflection across the vertical line passing through 0 (or horizontal line passing through zero if the number line is vertical).

g. Write "the opposite of the opposite of 2" symbolically. Then, simplify.
$-(-(2))$; Model for students how to break this problem down. A thought process might look like this, "First I take the opposite of 2 (write this down using symbols $-(2)$). Now, I need to take the opposite of $-(2)$." Cover $-(2)$ with your hand and say, "We have been asked to take the opposite of what is behind my hand. I can use parentheses to show I want to take the opposite of this entire quantity and then put the symbol for opposite in front." Keeping $-(2)$ covered, draw the parentheses and symbol for negative giving $-(-(2))$. When you cover the $-(2)$, you are treating it as an object that we have been asked to take the opposite of. This process makes the problem more abstract which is a great tie to the practice standard **Reason abstractly and quantitatively**. The parentheses provide the structure we need to represent the statement given symbolically. To simplify, work your way from the inside out. First, we take the opposite of 2 which is -2. Now we are left with $-(-2)$ which is 2.

h. How do you read the expression $-(-(-2))$? Simplify $-(-(-2))$.
Again, model how to break this down for students. Cover up the first negative sign and ask students what the expression $-(-2)$ means. Write "the opposite of negative two" below the expression. Then, uncover the first negative sign and cover $-(-2)$. Tell students, "We are being asked to take the opposite of whatever is behind my hand." The $-(-2)$ becomes an object that we are taking the opposite of. Write down "the opposite of" in front of where you wrote "the opposite of negative two". Your statement should now read, "the opposite of the opposite of negative two". We can simplify this expression in a similar manner. Cover up the first negative sign and tell students to focus on the expression $-(-2)$. This simplifies to 2. Now, we are left with $-(2)$ which is -2.

16. How do you read $-(-(-1))$? Simplify $-(-(-1))$.
the opposite of the opposite of negative one; -1

17. How do you read $-(-(4.5))$? Simplify $-(-(4.5))$.
the opposite of the opposite of four and five tenths; 4.5

18. Write "the opposite of the opposite of -150" symbolically (using notation). Then, simplify.
$-(-(-150))$; -150

19. Write "the opposite of the opposite of two tenths" symbolically (using notation). Then, simplify.
$-(-(0.2))$; 0.2

20. Will said, "The opposite of the opposite of a number is sometimes positive." Is Will's statement true or false? Explain.
True, the opposite of the opposite of a positive number is positive (e.g., $-\big(-(4)\big) = 4$) while the opposite of the opposite of a negative number is negative (e.g., $-\big(-(-4)\big) = -4$).

Spiral Review

1. Tell whether these fractions are closer to $0, \frac{1}{2}$, or 1.

a. $\frac{9}{10}$	b. $\frac{5}{8}$	c. $\frac{2}{9}$
d. $\frac{9}{12}$	e. $\frac{5}{11}$	f. $\frac{3}{5}$

2. Tell whether the fractions below are bigger than $\frac{1}{2}$ or smaller than $\frac{1}{2}$.

a. $\frac{3}{8}$	b. $\frac{11}{20}$	c. $\frac{5}{9}$
d. $\frac{5}{11}$	e. $\frac{5}{12}$	f. $\frac{3}{5}$

3. Tell whether these decimals are closer to $0, \frac{1}{2}$, or 1.

a. 0.5	b. 0.05	c. 0.19
d. 0.087	e. 0.45	f. 0.8

4. Compare the fractions using $>, <,$ or $=$.

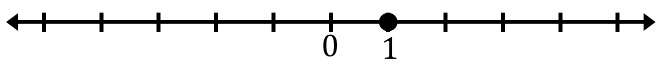
a. $\frac{3}{4}$ ____ $\frac{3}{8}$	b. $\frac{1}{8}$ ____ $\frac{1}{7}$	c. $\frac{3}{8}$ ____ $\frac{3}{7}$
d. $\frac{5}{11}$ ____ $\frac{11}{20}$	e. $\frac{7}{8}$ ____ $\frac{8}{9}$	f. $\frac{1}{3}$ ____ $\frac{6}{18}$

These problems present some comparisons that are often confusing for students. For a., when students see the 8 and 4, they think that because 8 is bigger than 4, $\frac{3}{4}$ must be bigger than $\frac{3}{8}$. Talk to students about the size of a fourth vs. an eighth. Would you rather share a candy bar with four people and get $\frac{1}{4}$ of the candy bar or share a candy bar with 8 people and get $\frac{1}{8}$ of the bandy bar? Once they understand that $\frac{1}{4}$ is bigger than $\frac{1}{8}$, it follows that $\frac{3}{4}$ (3 copies of $\frac{1}{4}$) is bigger than $\frac{3}{8}$ (3 copies of $\frac{1}{8}$) Parts b. and c. present a similar problem. For part d., both numbers are close to $\frac{1}{2}$ but one is smaller than $\frac{1}{2}$ and the other is bigger than $\frac{1}{2}$. Part e. can be very confusing for students. One way to think about part e., is to think about the fact that for each number, you are missing one "piece" to make a whole. If we think about the size of the piece that is missing, we can determine which number is bigger. For $\frac{7}{8}$, we are missing a piece that is $\frac{1}{8}$ in size. For $\frac{8}{9}$, we are missing a piece that is $\frac{1}{9}$ in size. Since $\frac{1}{8}$ is larger than $\frac{1}{9}$, $\frac{7}{8}$ is *missing* a *larger* piece of the whole, making it *smaller*.

3.1d Homework: Opposites

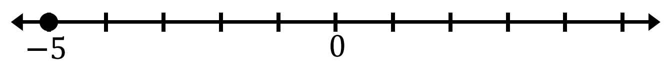
Directions: In each problem, a point is plotted. Plot the opposite. Then, write the value of the original point and its opposite in the space provided.

1.



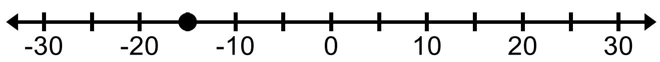
Original Point: ______ Opposite: ______

2.



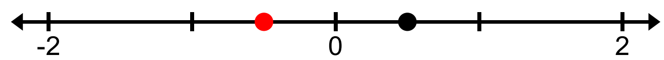
Original Point: ______ Opposite: ______

3.



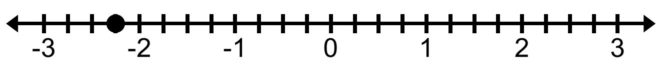
Original Point: ______ Opposite: ______

4.



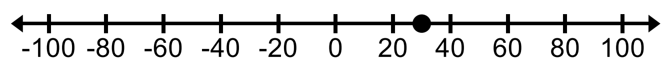
Original Point: ___$\frac{1}{2}$___ Opposite: ___$-\frac{1}{2}$___

5.



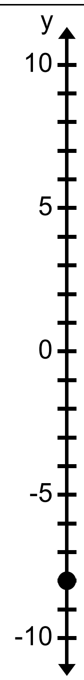
Original Point: ______ Opposite: ______

6.



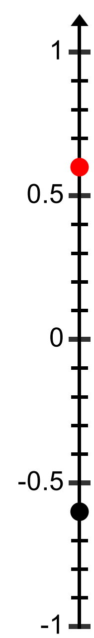
Original Point: ______ Opposite: ______

7.



Original Point: ______ Opposite: ______

8.



Original Point: __-0.6___ Opposite: __0.6____

9. How do you read the expression $-(-12)$? Simplify $-(-12)$.

10. How do you read the expression $-(12)$? Simplify $-(12)$.
    the opposite of 12; -12

11. How do you read the expression $-(-(-12))$? Simplify $-(-(-12))$.
    the opposite of the opposite of negative twelve; -12

12. How do you read the expression $-(-(12))$? Simplify $-(-(12))$.

13. Write "the opposite of 4" symbolically (using notation). Then, simplify.

14. Write "the opposite of the opposite of -1.25" symbolically (using notation). Then, simplify.
    $-(-(-1.25))$; -1.25

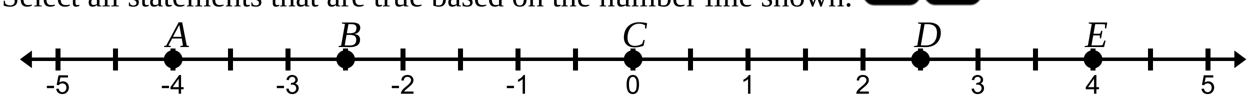
15. Simplify $-(-(7))$.

16. Simplify $-(0)$.

17. Simplify $-(-\frac{4}{3})$.
    $\frac{4}{3}$

18. Tell whether the following statements are true or false. Justify your answer.
    a. $-(10) = -10$
    b. $-\big(-(4)\big) = -4$ False
    c. $-(-3) = 3$
    d. $-\big(-(-20)\big) = -20$ True

19. Select all statements that are true based on the number line shown.



- ☐ $-(D) = B$ True
- ☐ $-(A) = -(E)$ False
- ☐ $-\big(-(E)\big) = A$
- ☐ $-(C) = C$
- ☐ $B = -3\frac{1}{2}$

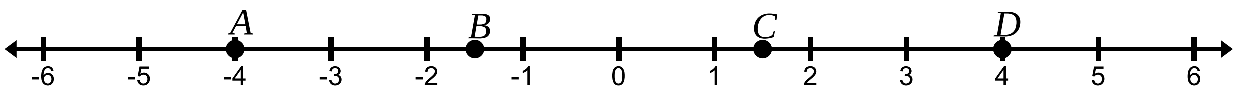
20. Make up your own statement that is true based on the number line in the previous problem.

3.1e Class Activity: Identifying Rational Numbers on the Number Line

In this lesson, students rely on several of the ideas studied so far in the chapter including the structure of the number line and different ways of scaling number lines. In the first few problems, students are determining the value of opposites. Finding the positive value first is usually the easiest way to approach these problems. As the problems progress, this scaffolding is taken away.

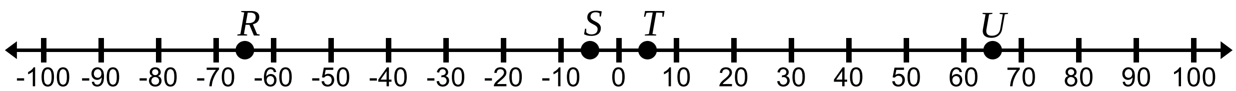
Activity 1:

a. Determine the values of *A*, *B*, *C*, and *D* on the number line shown below.



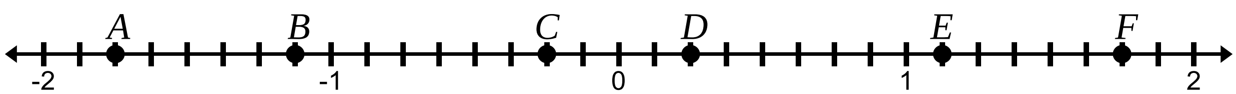
A: -4 *B:* $-1\frac{1}{2}$ *C:* $1\frac{1}{2}$ *D:* 4

b. Determine the values of *R*, *S*, *T*, and *U* on the number line shown below.



R: -65 *S:* -5 *T:* 5 *U:* 65

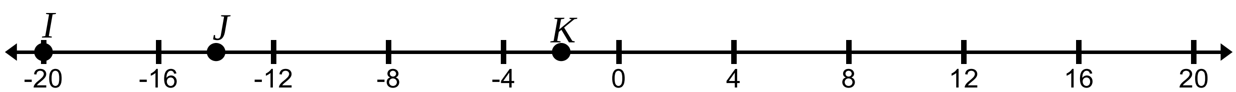
c. Determine the values of *A*, *B*, *C*, *D*, *E*, and *F* on the number line shown below.



A: $-\frac{14}{8}$ *B:* $-\frac{9}{8}$ *C:* $-\frac{2}{8}$ *D:* $\frac{2}{8}$ *E:* $\frac{9}{8}$ *F:* $\frac{14}{8}$

Student answers may vary but should be equivalent to those given (e.g., for *A*, correct answers include $-\frac{7}{4}, -1\frac{3}{4}, -1.75$).

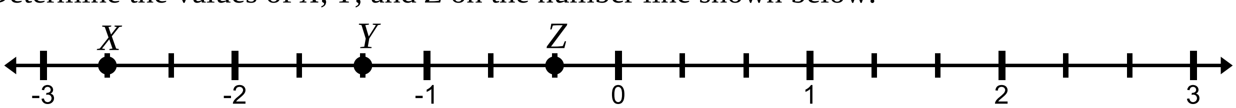
d. Determine the values of *I*, *J*, and *K* on the number line shown below.



I: -20 *J:* -14 *K:* -2

If students struggle with this problem, encourage them to first find the values of the opposites of *I*, *J*, and *K* by reflecting the points over 0. Students seem to have an easier time finding the value of positive numbers.

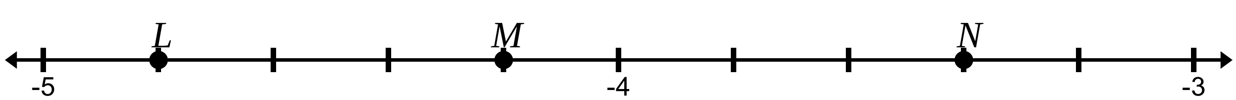
e. Determine the values of *X*, *Y*, and *Z* on the number line shown below.



X: $-2\frac{2}{3}$ *Y:* $-1\frac{1}{3}$ *Z:* $-\frac{1}{3}$

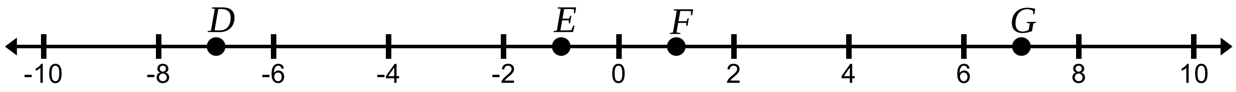
A common error for students is for them to say that *X* is $-3\frac{1}{3}$. Clarify what students are saying/thinking. They may in fact be stating that the value of the point is $-3\frac{1}{3}$ which is incorrect but they may also be stating that the value of the point is $-3+\frac{1}{3}$ which is a correct way of thinking about this problem.

f. Determine the values of *L*, *M*, and *N* on the number line shown below. If students struggle with this one, draw the reflection of this number line.



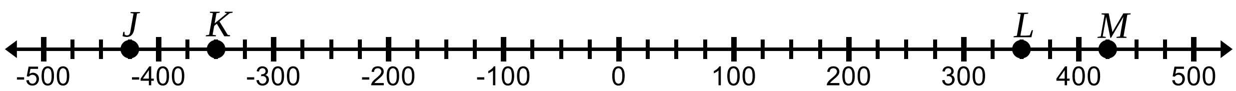
L: $-4\frac{4}{5}$ *M:* $-4\frac{1}{5}$ *N:* $-3\frac{2}{5}$

1. Determine the values of D, E, F, and G on the number line shown below.



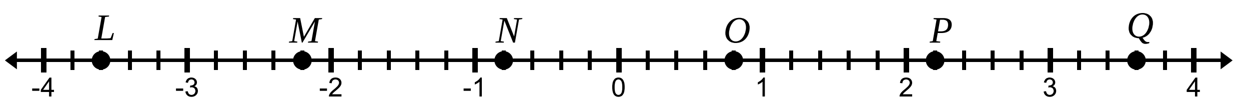
D: -7 E: -1 F: 1 G: 7

2. Determine the values of J, K, L, and M on the number line shown below.



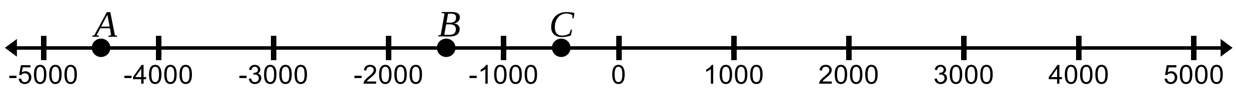
J: -425 K: -350 L: 350 M: 425

3. Determine the values of L, M, N, O, P, and Q on the number line shown below.



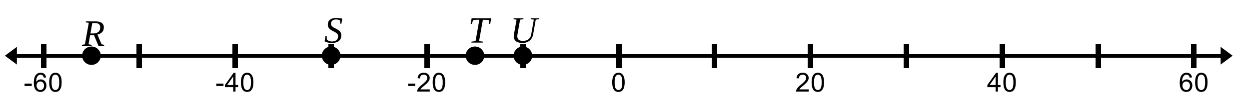
L: $-\frac{18}{5}$ M: $-\frac{11}{5}$ N: $-\frac{4}{5}$ O: $\frac{4}{5}$ P: $\frac{11}{5}$ Q: $\frac{18}{5}$

4. Determine the values of A, B, and C on the number line shown below.



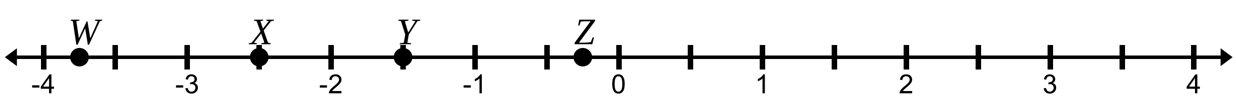
A: $-4{,}500$ B: $-1{,}500$ C: -500

5. Determine the values of R, S, T, and U on the number line shown below.



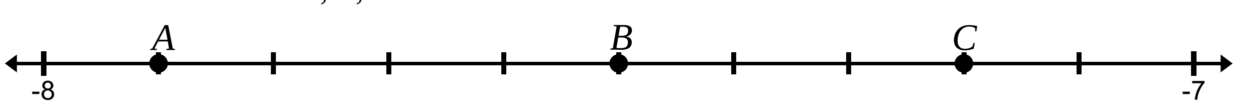
R: -55 S: -30 T: -15 U: -10

6. Determine the values of W, X, Y, and Z on the number line shown below.



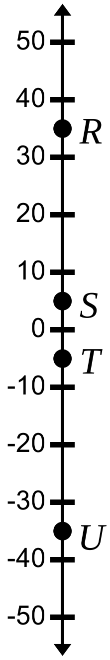
W: $-3\frac{3}{4}$ X: $-2\frac{1}{2}$ Y: $-1\frac{1}{2}$ Z: $-\frac{1}{4}$

7. Determine the values of A, B, and C.



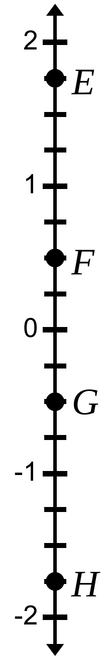
A: $-7\frac{9}{10}$ B: $-7\frac{1}{2}$ C: -7.2

8. Determine the values of R, S, T, and U.



R: 35 S: 5 T: -5 U: -35

9. Determine the values of E, F, G and H.



E: 1.75 F: 0.5 G: -0.5 H: -1.75

Spiral Review

1. Compare the decimals using $>, <$, or $=$.

a. 0.9 ____ 0.09	b. 0.49 ____ 0.5	c. 0.61 ____ 0.6
d. 0.1 ____ 0.11	e. 0.08 ____ 0.1	f. 0.30 ____ 0.3

2. Compare using $>, <$, or $=$.

a. $\frac{2}{3}$ ____ 0.5	b. $\frac{3}{10}$ ____ 0.35	c. $\frac{1}{4}$ ____ 0.25
d. $\frac{7}{4}$ ____ 1.25	e. 1.3 ____ $1\frac{3}{100}$	f. $\frac{1}{10}$ ____ 0.09

3. Compare using $>, <,$ or $=$.

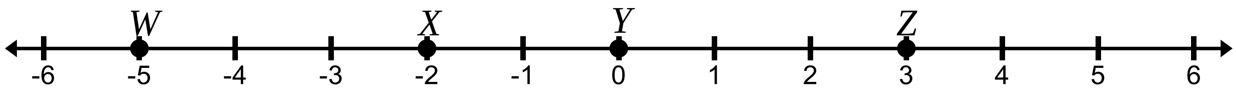
a. $\frac{2}{15}$ ____ $\frac{1}{8}$	b. $\frac{7}{22}$ ____ $\frac{1}{3}$	c. $\frac{5}{12}$ ____ $\frac{1}{2}$
d. $\frac{11}{4}$ ____ $2\frac{5}{8}$	e. $\frac{3}{16}$ ____ $\frac{1}{4}$	f. $\frac{1}{5}$ ____ $\frac{5}{20}$

4. Put the numbers in order from least to greatest.

a. $\frac{7}{12}, 0.5, \frac{9}{20}$	b. $\frac{1}{4}, \frac{5}{16}, \frac{5}{12}$
c. $\frac{7}{4}, 1.8, 1\frac{79}{100}$	d. $\frac{1}{5}, \frac{1}{10}, \frac{1}{4}$

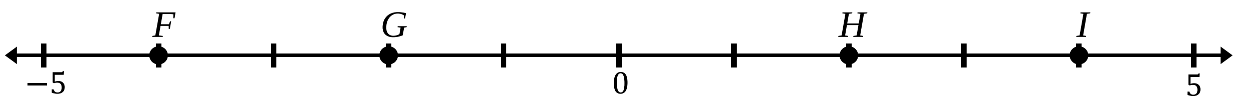
3.1e Homework: Identifying Rational Numbers on the Number Line

1. Determine the values of W, X, Y, and Z on the number line shown below.



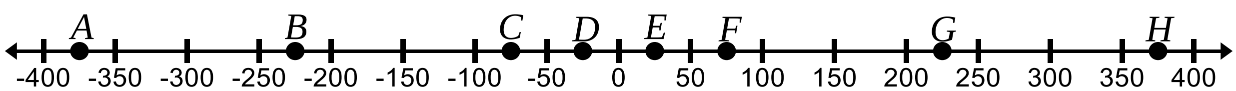
W: *X:* *Y:* *Z:*

2. Determine the values of F, G, H, and I on the number line shown below.



F: *G:* *H:* *I:*

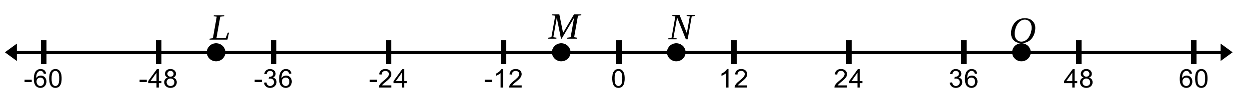
3. Determine the values of A, B, C, D, E, F, G, and H on the number line shown below.



A: −375 *B:* *C:* −75 *D:*

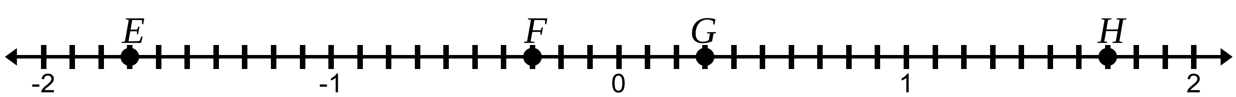
E: *F:* 75 *G:* *H:* 375

4. Determine the values of L, M, N, and O on the number line shown below.



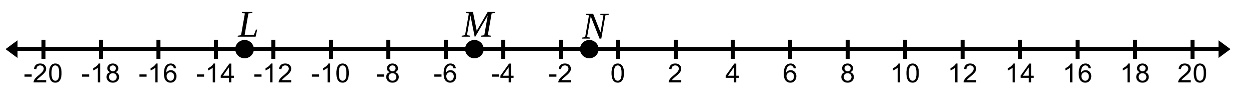
L: *M:* *N:* *O:*

5. Determine the values of E, F, G, and H on the number line shown below.



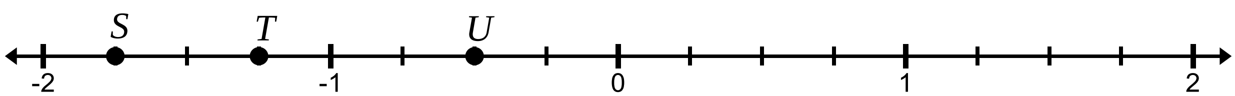
E: *F:* *G:* *H:* 1.7

6. Determine the values of *L*, *M*, and *N* on the number line shown below.



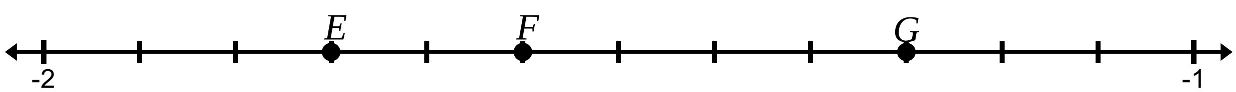
L: *M:* *N:*

7. Determine the values of *S*, *T*, and *U* on the number line shown below.



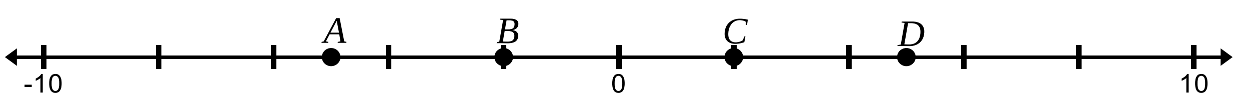
S: $-1\frac{3}{4}$ *T:* *U:*

8. Determine the values of *E*, *F*, and *G* on the number line shown below.



E: *F:* *G:*

9. **Find, Fix, and Justify:** Henry was asked to give the values of *A*, *B*, *C*, and *D*.



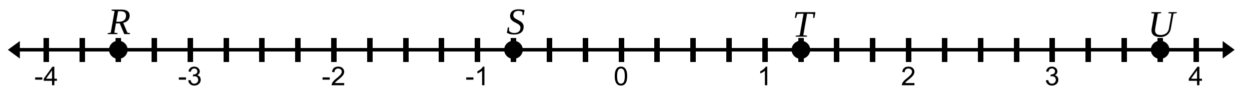
Henry's Answers:

A: $-2\frac{1}{2}$ *B:* -1 *C:* 1 *D:* $2\frac{1}{2}$

His teacher told him his answers are not correct. What common error did Henry make? What are the correct values of *A*, *B*, *C*, and *D*?

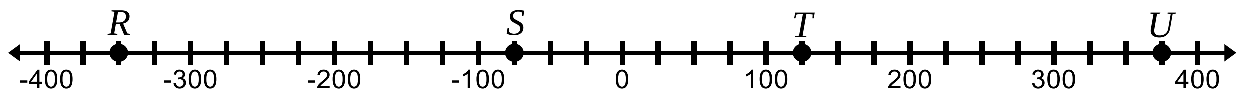
A: *B:* *C:* *D:*

10. Determine the values of *R*, *S*, *T*, and *U* on the number line shown below.



R: $-3\frac{1}{2}$ *S:* $-\frac{3}{4}$ *T:* $1\frac{1}{4}$ *U:* $3\frac{3}{4}$

11. Determine the values of *R*, *S*, *T*, and *U* on the number line shown below.

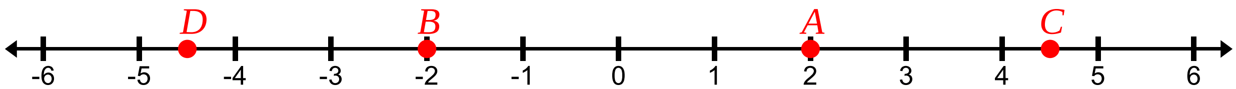


R: *S:* *T:* *U:*

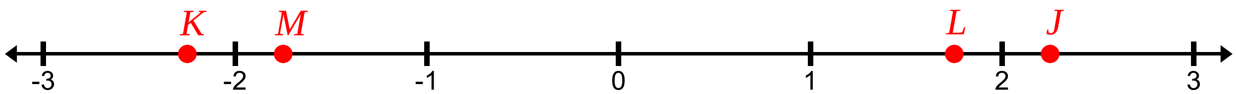
12. Compare problems #10 and #11. How are they similar? How are they different?

Activity 1: Graph and label the following points on the number lines provided. Use ideas about opposites to help you.

a. $A: 2$ $\quad B: -2$ $\quad C: 4\frac{1}{2}$ $\quad D: -4\frac{1}{2}$

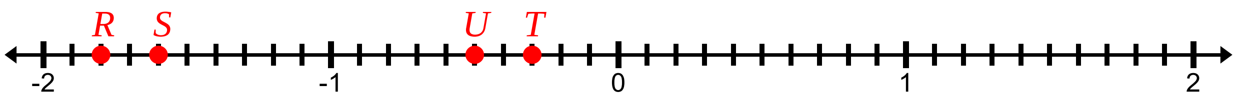


b. $J: 2\frac{1}{4}$ $\quad K: -2\frac{1}{4}$ $\quad L: 1.75$ $\quad M: -1.75$

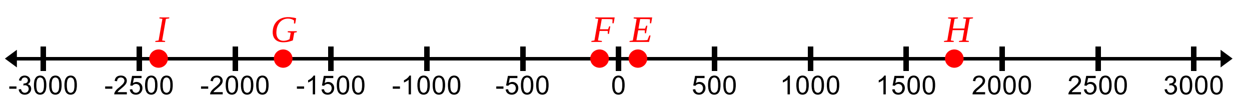


It might help students to first partition this number line into quarters.

c. $R: -1.8$ $\quad S: -1\frac{3}{5}$ $\quad T: -0.3$ $\quad U: -\frac{1}{2}$

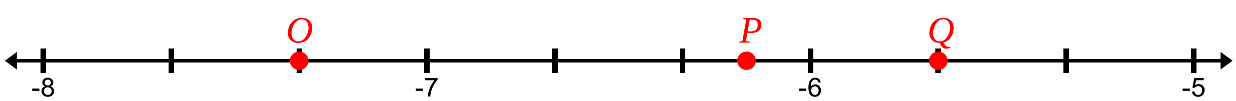


d. $E: 100$ $\quad F: -100$ $\quad G: -1{,}750$ $\quad H: 1{,}750$ $\quad I: -2{,}400$



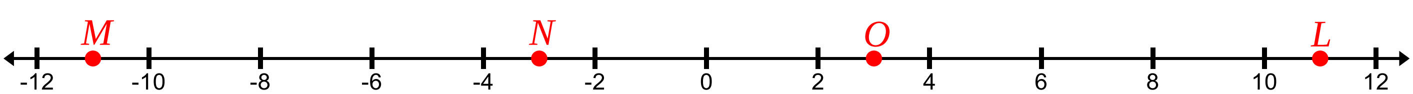
Students will likely rely on their fraction sense here to accurately graph these points. Increments of 100 are represented by $\frac{1}{5}$ of an interval. Increments of 250 are represented by $\frac{1}{2}$ of an interval.

e. $O: -7\frac{1}{3}$ $\quad P: -6\frac{1}{6}$ $\quad Q: -5\frac{2}{3}$

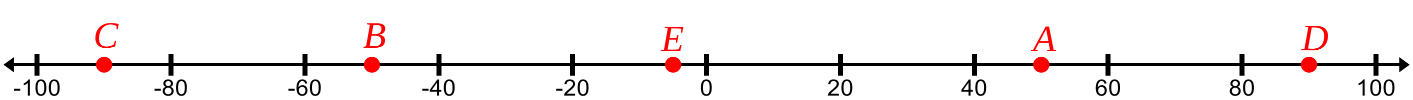


Directions: Graph and label the following points on the number lines provided. Use ideas about opposites to help you.

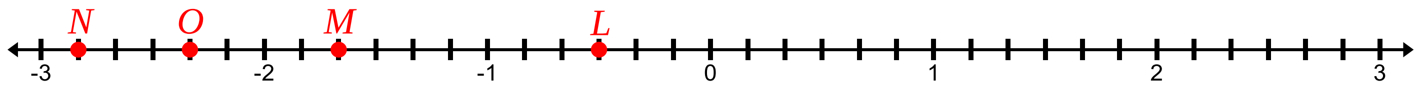
1. $L: 11$ $\quad M: -11$ $\quad N: -3$ $\quad O: 3$



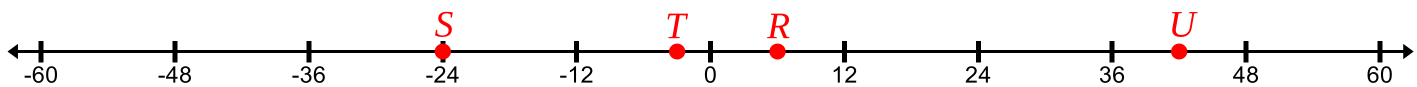
2. $A: 50$ $\quad B: -50$ $\quad C: -90$ $\quad D: 90$ $\quad E: -5$



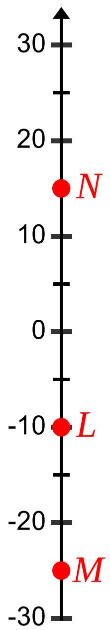
3. $L: -\frac{1}{2}$ $M: -1\frac{2}{3}$ $N: -2\frac{5}{6}$ $O: -2\frac{1}{3}$



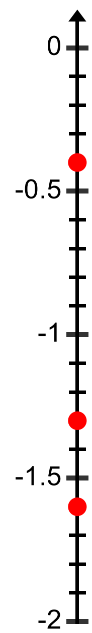
4. $R: 6$ $S: -24$ $T: -3$ $U: 42$



5. $L: -10$ $M: -25$ $N: 15$



6. $E: -0.4$ $F: -1\frac{3}{10}$ $G: -1\frac{3}{5}$

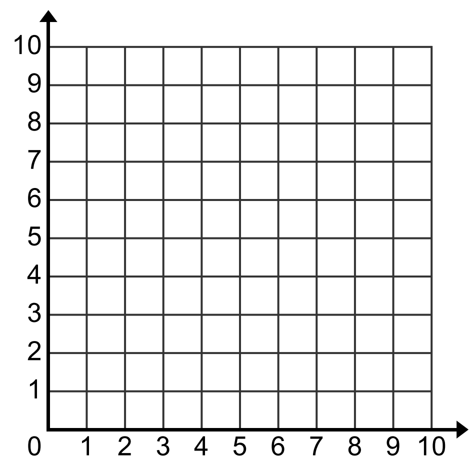


Spiral Review

1. A bakery uses two cups of cherries to make a cherry pie. Complete the table to show the relationship between number of pies and cups of cherries needed.

Number of Pies	**Cups of Cherries**
1	
2	
3	
8	

2. Create a graph to represent the relationship between number of pies and cups of cherries needed. Use Number of Pies as the independent variable and Cups of Cherries as the dependent variable.



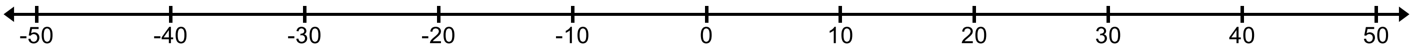
3. Write an equation to show the relationship between number of pies p and cups of cherries c.

4. If the bakery needs to make 25 pies for the county fair, how many cups of cherries will the bakery need?

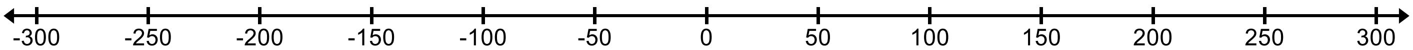
3.1f Homework: Representing Rational Numbers on a Number Line – Scale Given

Directions: Graph and label the following points on the number lines provided. Use ideas about opposites to help you.

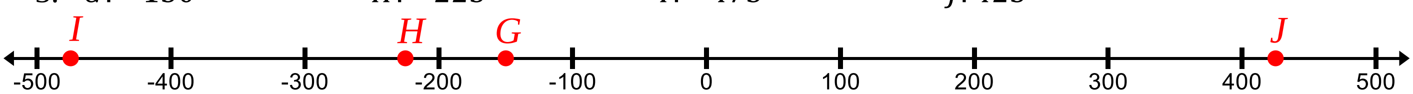
1. $Q: 10$ $\quad R: -10$ $\quad S: 25$ $\quad T: -35$



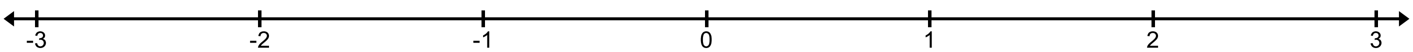
2. $L: 275$ $\quad M: -275$ $\quad N: 125$ $\quad O: -125$



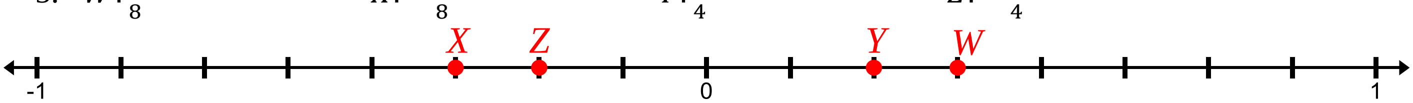
3. $G: -150$ $\quad H: -225$ $\quad I: -475$ $\quad J: 425$



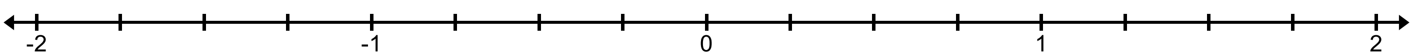
4. $L: \frac{2}{3}$ $\quad M: -\frac{2}{3}$ $\quad N: 2\frac{1}{3}$ $\quad O: -2\frac{1}{3}$



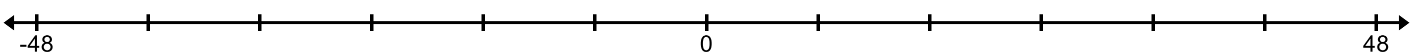
5. $W: \frac{3}{8}$ $\quad X: -\frac{3}{8}$ $\quad Y: \frac{1}{4}$ $\quad Z: -\frac{1}{4}$



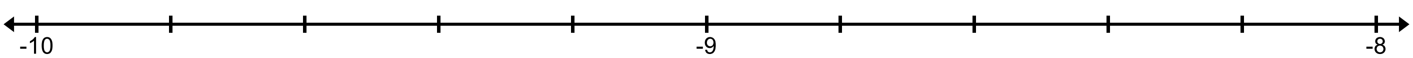
6. $J: -0.25$ $\quad K: -\frac{5}{4}$ $\quad L: -1.75$ $\quad M: -\frac{3}{2}$



7. $R: 32$ $\quad S: -32$ $\quad T: -4$ $\quad U: 4$



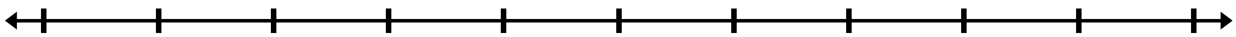
8. $E: -9.8$ $\quad F: -8.4$ $\quad G: -8\frac{3}{4}$ $\quad H: -9.5$



3.1g Class Activity: Representing Rational Numbers on a Number Line – Choosing a Scale

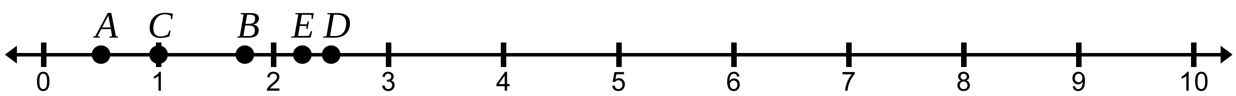
Activity 1: Mr. Frankel's class was asked to create a graph showing the location of the following numbers on the number line below:

A: $\frac{1}{2}$ $\quad$ B: $1\frac{3}{4}$ $\quad$ C: 1 $\quad$ D: $2\frac{1}{2}$ $\quad$ E: $\frac{9}{4}$

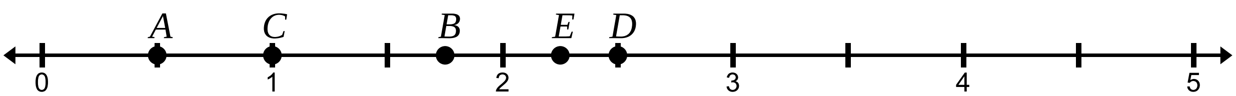


The following three graphs were created by the students.

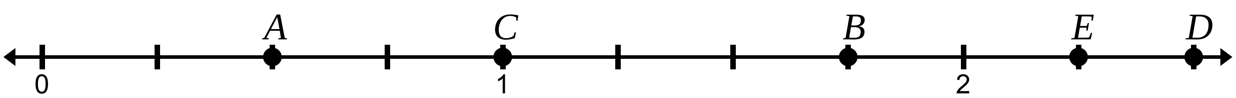
Kevin's Graph



Destiny's Graph



Malia's Graph



a. Which student(s) created a correct graph?
All students created a correct graph. They just scaled their graphs differently. It may be valuable to have students show the scale for each tick mark on Destiny and Malia's graphs.

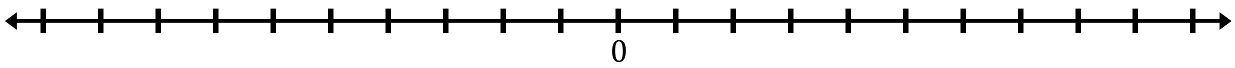
b. Compare the graphs. Is one better than the others? Explain.
All three graphs are correct. The graphs increase in precision from Kevin's to Destiny's to Malia's. Reading Kevin and Destiny's graphs, it may be difficult to determine the exact value of points that do not fall on a tick mark. In Malia's graph, each point falls on a tick mark so it is clear what the value of each point is. You can extend this question and ask students, "What if the students had also been asked to graph a sixth point $F = 4$?" Now, which scale would be best? In this case, it would be Destiny's or Kevin's because 4 would not fit on Malia's graph.

Activity 2:

a. Determine how to scale the number line to graph the points.

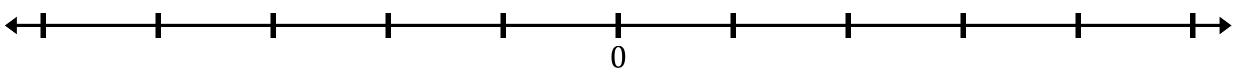
The purpose of this activity is for students to understand the factors we take into consideration when determining an appropriate scale for a graph. Often, a number line or coordinate grid are provided and students are tasked with determining an appropriate scale for what they have been given. When a number line or grid are not provided as in part c., students create their own, understanding that the ability to choose an appropriate scale is key to effectively and efficiently displaying the data.

$$A: 18, B: -30, C: -27, D: 12, E: -8, F: 25$$



Some students may suggest 2 as a scale for the graph. Encourage students to check whether all the points will fit if we scale by 2s. Students will see that we cannot fit all the points. What about a scale of 5? The points will fit but it is more difficult to estimate the location of a point like 18 or 27 (to find the location of these points, we need to visualize how to divide the interval into 5 equal parts). What about a scale of 3? This seems reasonable – all the points fit and it is easier to visualize how to divide the interval into thirds to plot points that are not multiples of 3. A scale of 4 is also reasonable – again the points fit. For many students, visualizing quarters is easier than visualizing thirds because they can cut the interval in half and then cut each half in half again to create quarters.

b. What if you have been given the number line below and asked to graph the same points as above? Would you use the same scale? Explain. Then, graph the points on the number line below.



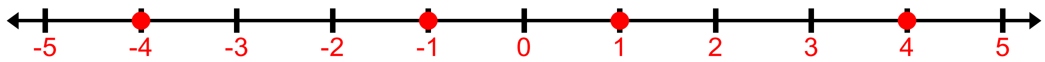
A scale of 10 would make sense here. A scale of 6 also makes sense and will be more precise. Teach students how to create sixths by first dividing the interval into thirds and then splitting each third in half again. Remind students that they are estimating where the points fall that do not hit at a tick mark but the more exact they can be with their estimating, the better.

c. If you were creating your own number line to display the data, what scale would you use? Explain. Create your own number line in the space below in display the points from part a.

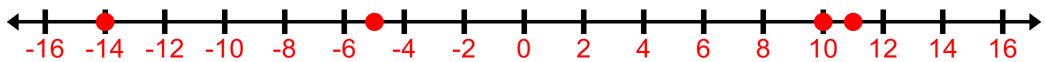
Using a scale of 1 is the most precise scale to display the data – every point will fall on a tick mark. However, this would be very time consuming and we would need to make the tick marks very small to fit all the data on the page we are given – everything would be squished together. A scale of 2 is reasonable and estimating where the odd numbers fall on the line is easy because you simply need to divide the interval in half. Some students may stick with the graph they created in part a.

Activity 3: Graph and label the following points on the number lines provided. First, determine a scale that makes sense for the problem given. Use ideas about opposites to help you. For many of these, we place zero in the middle of the number line; however, zero does not always need to be in the middle of the number line.

a. $A: 4$ $B: -4$ $C: 1$ $D: -1$

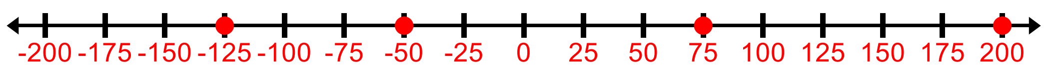


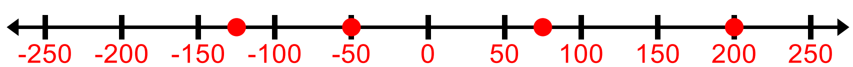
b. $P: -14$ $Q: 11$ $R: -5$ $S: 10$



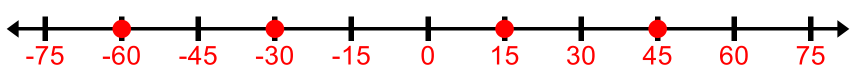
c. For this problem, determine how you would scale the number line if you were given the first number line. Then, determine how you scale the number line if you were given the second number line.

$R: 200$ $S: -125$ $T: 75$ $U: -50$



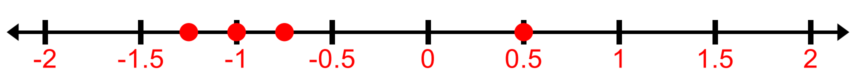


d. $P: 15$ $Q: -30$ $R: -60$ $S: 45$

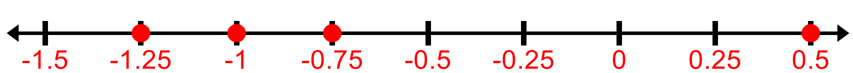


Some students may start by counting by 20s and realize that it can be challenging to estimate where some of the numbers fall. All the numbers are multiples of 15 so if we use 15 as the scale we can create a very precise graph.

e. $H: -\frac{3}{4}$ $I: -1$ $J: 0.5$ $K: -1.25$

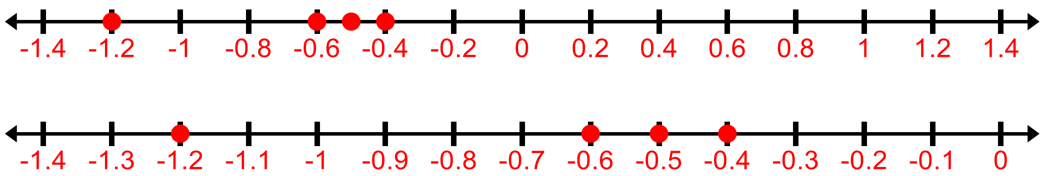


If students place zero in the middle of the number line, a scale of 0.5 makes sense. If students shift zero to the right, a scale of 0.25 would work and be more precise:

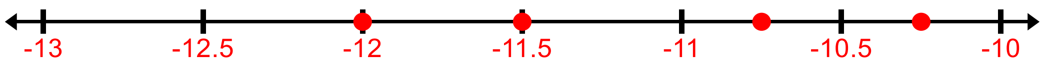


f. $A: -\frac{2}{5}$ $B: -0.6$ $C: -\frac{1}{2}$ $D: -\frac{6}{5}$

If students put zero in the middle of the graph, they see that the points only fall to the left of zero. What if we shift zero to the end of the number line and only focus on the portion of the number line to the left of zero? Now, we have room to scale by tenths.



g. $Q: -10.25$ $R: -12$ $S: -11.5$ $T: -10\frac{3}{4}$



Again, we can focus on one portion of the number line – the piece from negative 10 to negative 12. We do not always have to show zero. When students graph the solutions to inequalities in one-variable, they will often create number lines that zoom in on a specific portion of the number line.

Spiral Review

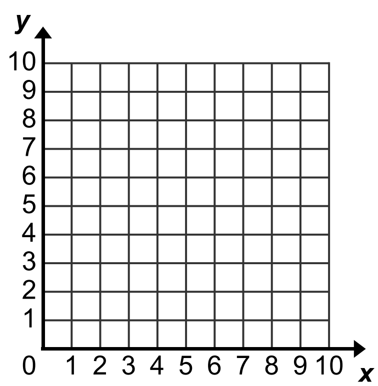
1. Graph and label the points on the coordinate grid.

A: $(1, 5)$

B: $(7, 3)$

C: $(9, 0)$

D: $(0, 9)$



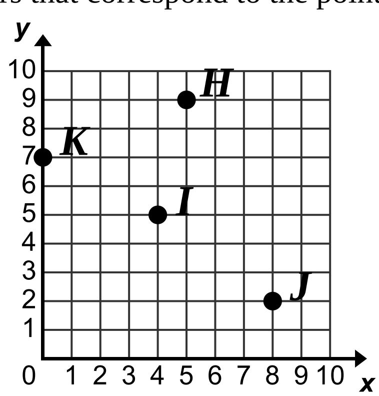
2. List the ordered pairs that correspond to the points on the coordinate grid.

H: ______

I: ______

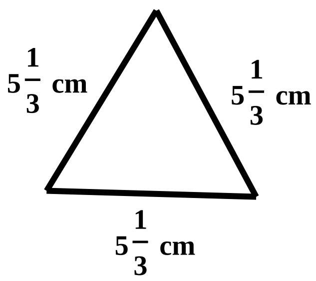
J: ______

K: ______

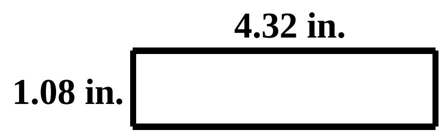


3. Find the perimeter of each shape. Shapes are not drawn to scale.

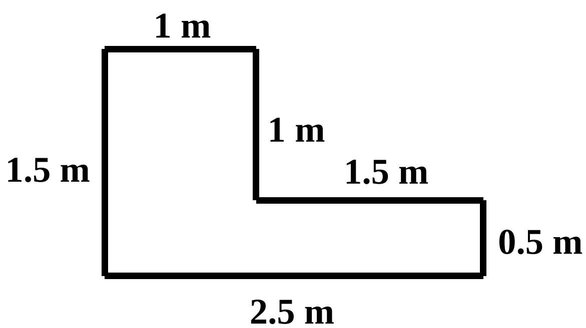
a.



b.



c.



d. A square with a side length of $3\frac{2}{3}$ feet.

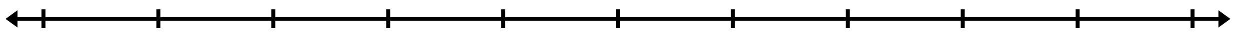
4. Complete the table below.

Fraction Representation	Decimal Representation
$\frac{1}{2}$	
$\frac{3}{4}$	
	0.25
	1.25
$\frac{1}{5}$	
	0.3
	0.8

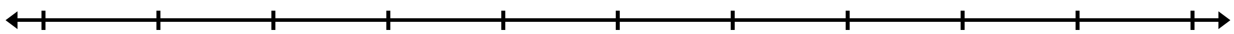
3.1g Homework: Representing Rational Numbers on a Number Line – Choosing a Scale

Directions: Graph and label each set of points on the number lines provided. First, determine a scale that makes sense for the points given in the problem.

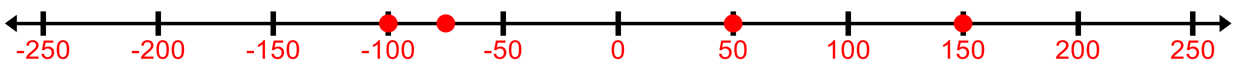
1. $C: -4$ $\quad$ $D: -2$ $\quad$ $E: 1$ $\quad$ $F: 5$



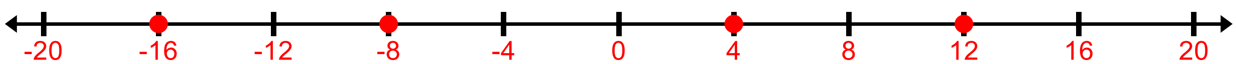
2. $E: -15$ $\quad$ $F: -40$ $\quad$ $G: 5$ $\quad$ $H: 20$



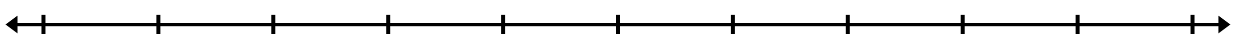
3. $E: 150$ $\quad$ $F: -100$ $\quad$ $G: -75$ $\quad$ $H: 50$



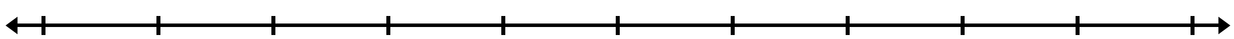
4. $P: -8$ $\quad$ $Q: 12$ $\quad$ $R: -16$ $\quad$ $S: 4$



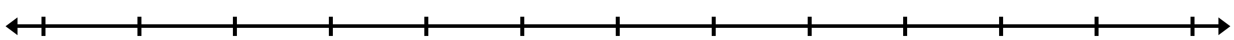
5. $A: 2$ $\quad$ $B: -2$ $\quad$ $G: -\frac{1}{2}$ $\quad$ $H: -1\frac{3}{4}$



6. $P: -5.2$ $\quad$ $Q: -5\frac{3}{10}$ $\quad$ $R: -5\frac{3}{5}$ $\quad$ $S: -5.9$



7. $H: 1$ $\quad$ $I: -1$ $\quad$ $J: \frac{1}{2}$ $\quad$ $K: -\frac{1}{2}$ $\quad$ $L: \frac{2}{3}$ $\quad$ $M: -\frac{2}{3}$

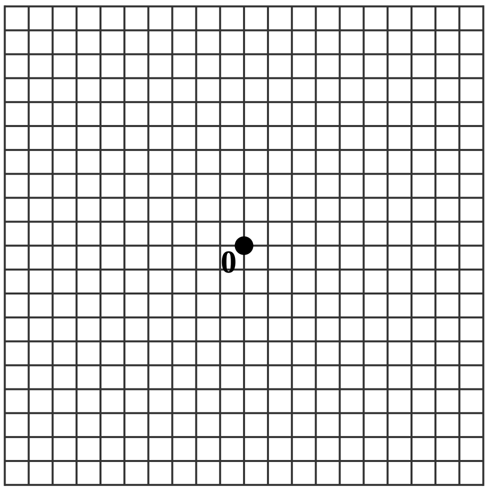


8. Construct your own number line to graph the points.
$E: -\frac{4}{3}$ $\quad$ $F: -\frac{2}{3}$ $\quad$ $G: \frac{1}{6}$ $\quad$ $H: 2$

3.1h Class Activity: Pairs of Rational Numbers in the Coordinate Plane

The goal of this lesson is for students to realize that the coordinate plane is an intersection of a horizontal and vertical number line that intersect at a point called the origin that allows us to graph in two-dimensions. The coordinate plane is an important tool for graphing relations between two variables.

Activity 1: The Intersection of a Horizontal and Vertical Number Line



a. Construct a horizontal number line that passes through the point shown on the grid.
b. Construct a vertical number line that passes through the point shown on the grid.
c. Label the points $R: (2, 4); S: (-2, 4);\ T: (2, -4); U: (-2, -4)$.
d. Highlight the ***x*-axis** in blue. What must be true about the values of the *x*- and *y*-coordinates of a point for it to be on the *x*-axis? The *y*-coordinate must be zero.

e. Highlight the ***y*-axis** in purple. What must be true about the values of the *x*- and *y*-coordinates of a point for it to be on the *y*-axis? The *x*-coordinate must be zero.

f. Shade **Quadrant I** in red. What must be true about the values of the *x*-and *y*-coordinates of a point for it to be in Quadrant I? The *x*- and *y*-coordinates both must be positive (e.g., greater than 0).

g. Shade **Quadrant II** in orange. What must be true about the values of the *x*-and *y*-coordinates of a point for it to be in Quadrant II? The *x*-coordinate must be negative (e.g., less than 0) and the *y*-coordinate must be positive (e.g., greater than 0).

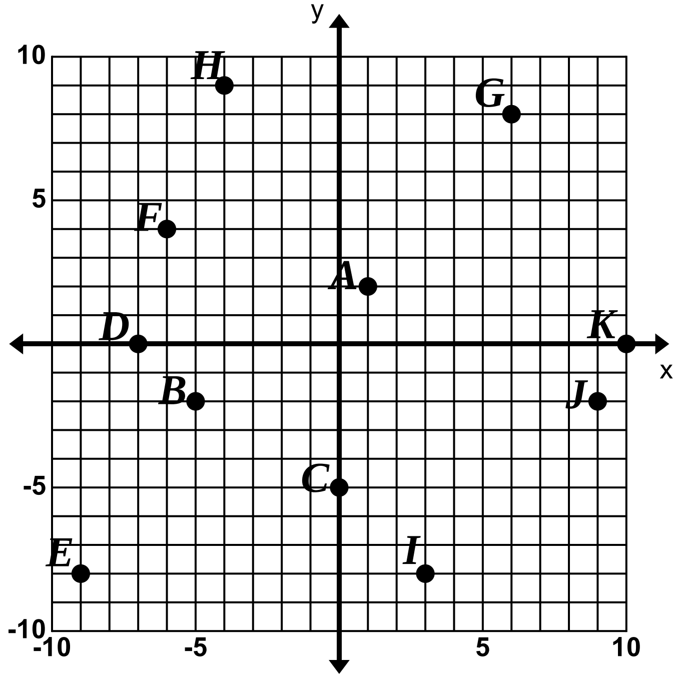
h. Shade **Quadrant III** in yellow. What must be true about the values of the *x*-and *y*-coordinates of a point for it to be in Quadrant III? The *x*- and *y*-coordinates both must be negative (e.g., less than 0).

i. Shade **Quadrant IV** in green. What must be true about the values of the *x* and *y*-coordinates of a point for it to be in Quadrant IV? The *x*-coordinate must be positive (e.g., greater than 0) and the *y*-coordinate must be negative (e.g., less than 0).

j. Mark the **origin** in black. Define origin in your own words. What is the ordered pair that corresponds to the origin? The origin is the point where the *x*- and *y*-axes intersect. The ordered pair (0,0) corresponds to the origin.

Directions: Use the coordinate plane below to answer #1 – 4.

1. Describe in words how to get from the origin to point *A*. Then, write the ordered pair for point *A*.
   "from the origin, go right one and up two"; $(1, 2)$



2. Describe in words how to get from the origin to point *B*. Then, write the ordered pair for point *B*.
   "from the origin, go left five and down two"; $(-5, -2)$

3. Describe in words how to get from the origin to point *C*. Then, write the ordered pair for point *C*.
   "from the origin, go over zero and down five"; $(0, -5)$

The convention in an ordered pair is to state left/right, then up/down. This often causes confusion for students in Grade 8 when they start talking about slope. When students determine slope (rise over run), they need to look for the *vertical* (up/down) change first and then the *horizontal* (left/right) change.

4. Write the ordered pair for points *D* – *K*.

   D: __$(-7, 0)$_______

   E: __$(-9, -8)$_______

   F: _$(-6, 4)$________

   G: __$(6, 8)$_______

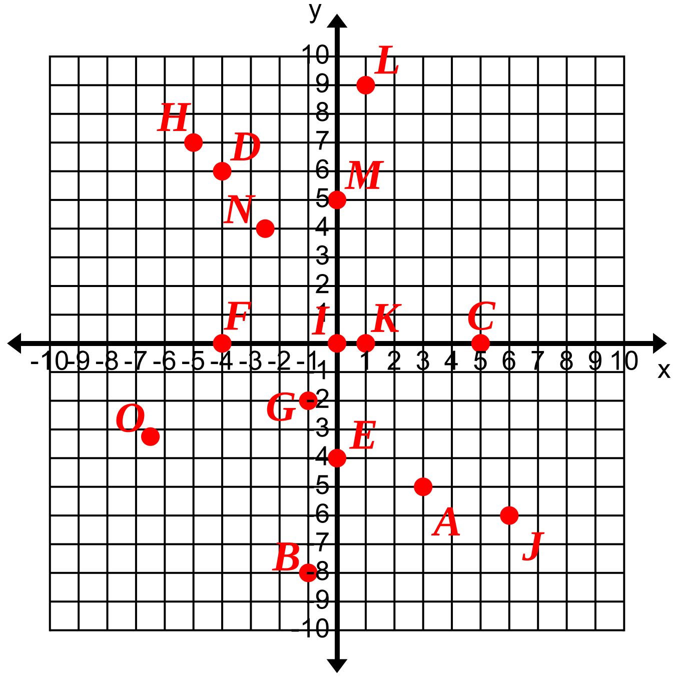
   H: _$(-4, 9)$________

   I: _$(3, -8)$________

   J: _$(9, -2)$________

   K: _$(10, 0)$________

Directions: Use the coordinate plane to answer #5 – 13.



5. Describe in words how to graph the point $A: (3, -5)$. Then, graph and label A.
   "from the origin, go right three and down five"
6. Describe in words how to graph point $B: (-1, -8)$. Then, graph and label B.
   "from the origin, go left one and down eight"
7. Describe in words how to graph point $C: (5, 0)$. Then, graph and label C.
   "from the origin, go right five and up/down zero"
8. Graph and label the following points.

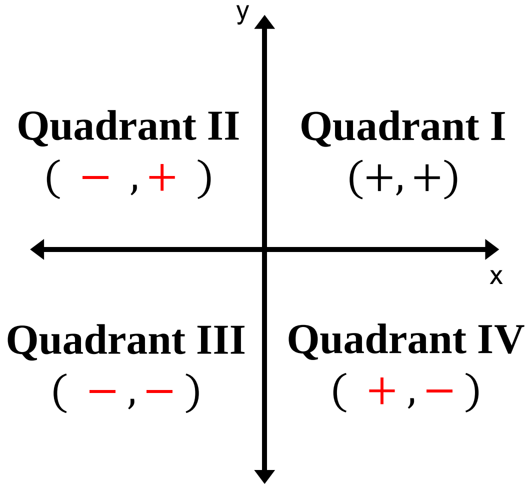
$D: (-4, 6)$	$E: (0, -4)$	$F: (-4, 0)$
$G: (-1, -2)$	$H: (-5, 7)$	$I: (0, 0)$
$J: (6, -6)$	$K: (1, 0)$	$L: (1, 9)$
$M: (0, 5)$	$N: \left(-2\frac{1}{2}, 4\right)$	$O: (-6.5, -3.25)$

9. List the points that are in…

Quadrant I	Quadrant II
L	*D, H, N*
Quadrant III	**Quadrant IV**
B, G, O	*A, J*

10. Are there any points you did not list in the table? Explain. *C, E, F, I, K, M* were not listed in the table. They are not in a quadrant; they are on an axis.
11. Plot a point in Quadrant II. Label your point *P*. Points will vary but the *x*-coordinate must be less than 0 and the *y*-coordinate must be greater than 0.
12. Plot a point that is not in any quadrant. Label your point *Q*. Answers will vary but must fall on an axis.
13. Plot a point with a positive *x*-coordinate and a negative *y*-coordinate. Label your point *R*. Answers will vary but the point should be in the fourth quadrant.
14. Plot a point with an *x*-coordinate equal to zero and a positive *y*-coordinate. Label your point *S*. Answers will vary but the point should be on the *y*-axis with a *y*-coordinate greater than zero.

15. Complete the table below to show the sign of the *x*- and *y*-coordinate in each of the Quadrants. Quadrant I has been completed for you.

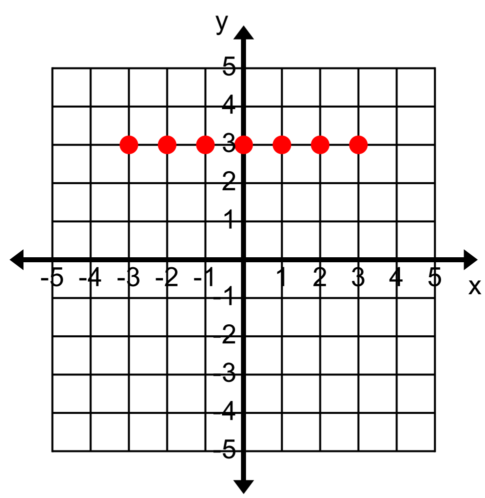


16. Graph the following points on the coordinate plane.
$\{(-3,3),(-2,3),(-1,3),(0,3),(1,3),(2,3),(3,3)\}$

a. Describe the shape of the graph. Why does the graph make this shape?

The points lie on the same horizontal line. This occurs when the points have the same *y*-coordinate.

b. Write another ordered pair that would continue the shape of the graph. Answers will vary but must have a *y*-coordinate equal to 3



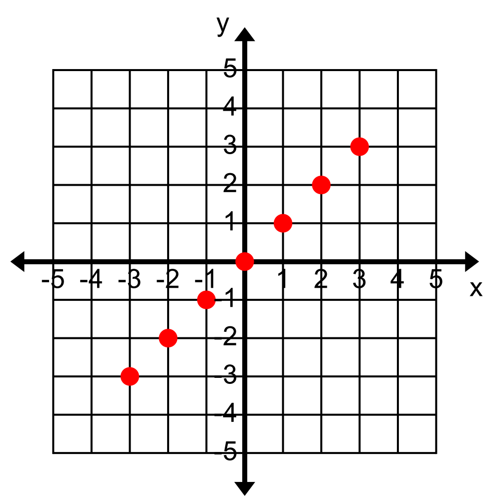
17. Graph the following points on the coordinate plane.
$\{(-3,-3),(-2,-2),(-1,-1),(0,0),(1,1),(2,2),(3,3)\}$

a. Describe the shape of the graph. Why does the graph make this shape?

The points lie on the same line – students will likely describe this line as slanted or diagonal.

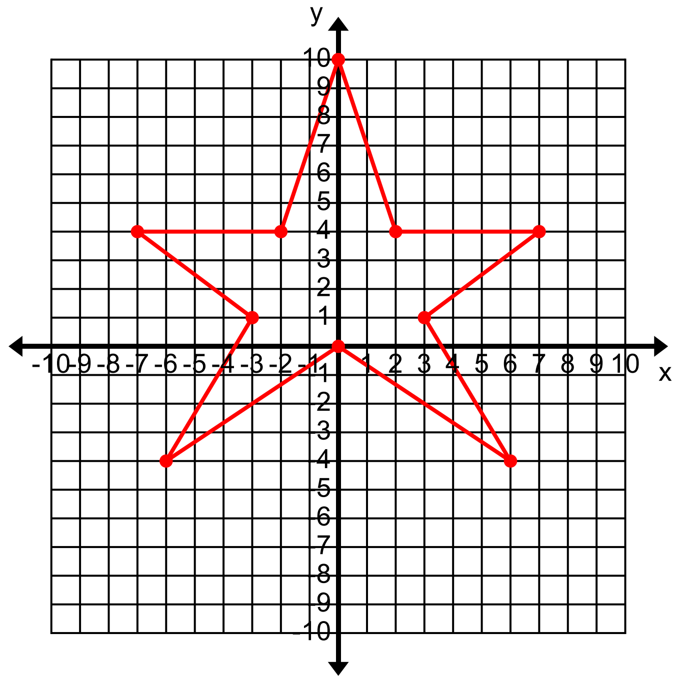
b. Write another ordered pair that would continue the shape of the graph.

Answers will vary but the *y*-coordinate must be equal to the *x*-coordinate.



18. Complete the table. Plot the points. Connect the points in the order they appear in the table.

x	y	(x, y)
0	10	$(0, 10)$
2	4	$(2, 4)$
7	4	$(7, 4)$
3	1	$(3, 1)$
6	-4	$(6, -4)$
0	0	$(0, 0)$
-6	-4	$(-6, -4)$
-3	1	$(-3, 1)$
-7	4	$(-7, 4)$
-2	4	$(-2, 4)$
0	10	$(0, 10)$



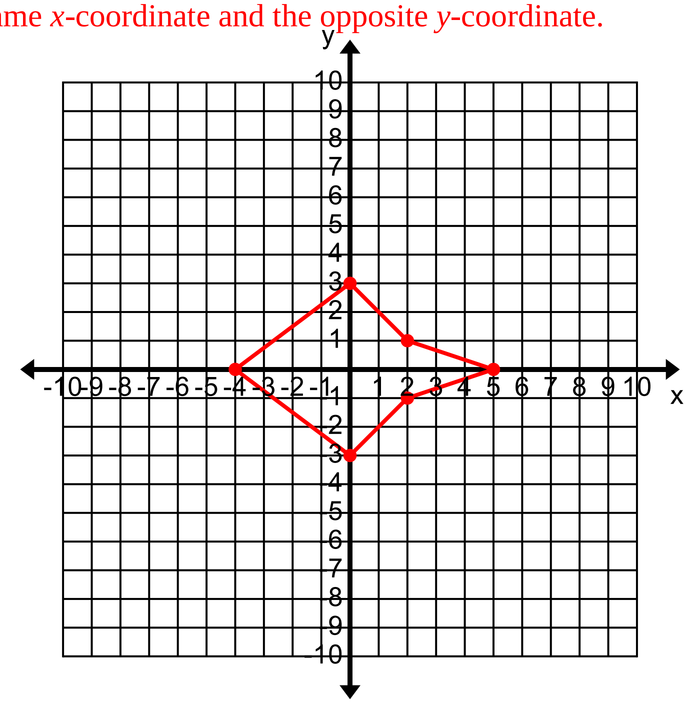
a. Does the figure exhibit symmetry? If yes, explain the symmetry and what causes it.

The object exhibits symmetry across the y-axis. Each point has a corresponding point with the same y-coordinate and the opposite x-coordinate: (x, y) and $(-x, y)$. This causes symmetry across the y-axis. Students will study this idea more in Grade 8 when they study reflections.

19. Create your own set of points that when connected are symmetric about the x-axis but not the y-axis.

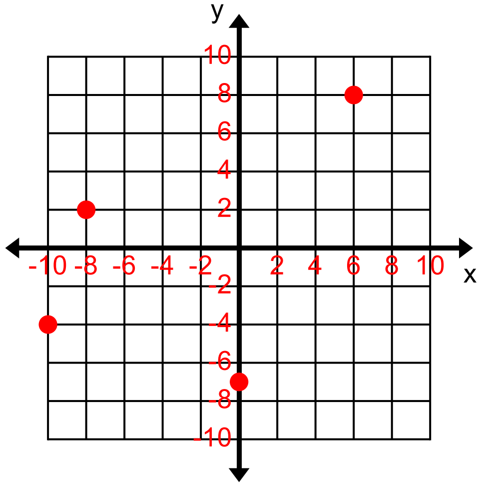
Answers will vary, a sample answer is shown. You will notice that for a figure to be symmetric across the x-axis, each point has a corresponding point with the same x-coordinate and the opposite y-coordinate.

x	y	(x, y)
0	3	
2	1	
5	0	
2	-1	
0	-3	
-4	0	



20. Graph the following points on the coordinate plane. First, determine a good scale for each axis.

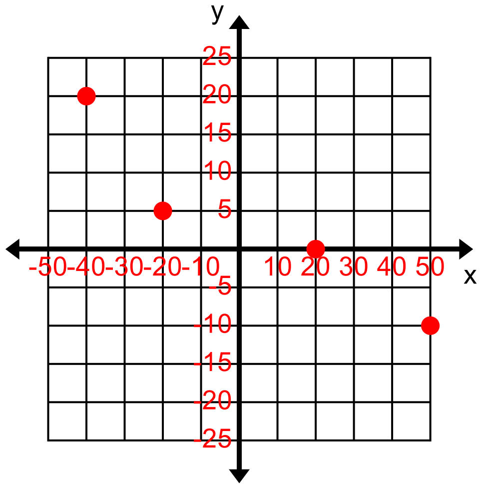
$\{(-10,-4),(6,8),(0,-7),(-8,2)\}$



21. Graph the following points on the coordinate plane. First, determine a good scale for each axis.

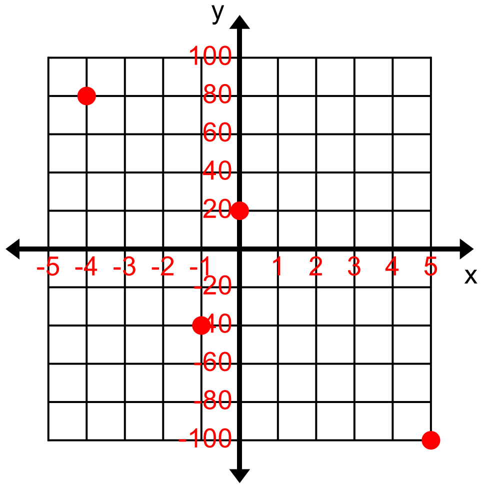
$\{(50,-10),(-20,5),(20,0),(-40,20)\}$

This problem highlights that we do not need to use the same scale on the *x*- and *y*-axis.



22. Graph the following points on the coordinate plane. First, determine a good scale for each axis.

$\{(-4,80),(-1,-40),(0,20),(5,-100)\}$



Spiral Review

1. Compare the decimals using $>, <,$ or $=$.

a. 8.58 ____ 8.6	b. $\frac{3}{8}$ ____ 0.5	c. $1\frac{3}{5}$ ____ 1.06
d. 0.25 ____ $\frac{3}{12}$	e. $\frac{5}{9}$ ____ 0.5	f. $\frac{12}{5}$ ____ 2.38

2. Use a model to compare $\frac{1}{3}$ to 0.3.

3. Use a model to compare $\frac{2}{3}$ to 0.6.

4. Solve and describe the difference between the two problems.

    a. Find 20% of 50.

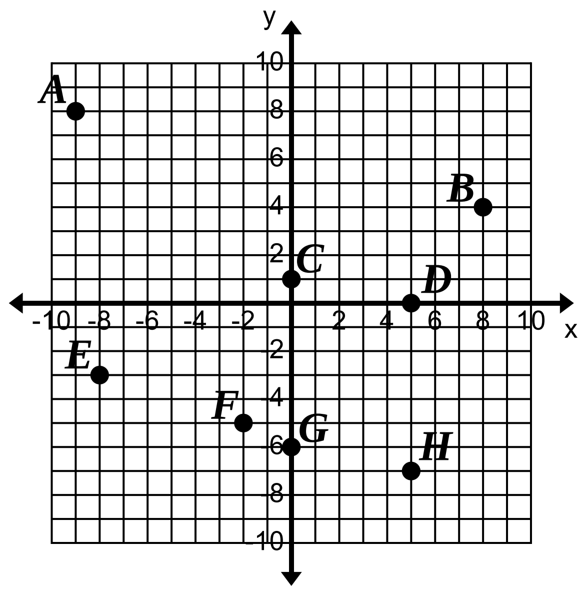
    b. 50 is 20% of what number.

3.1h Homework: Pairs of Rational Numbers in the Coordinate Plane

Answers will vary for #1 – 6. Sample answers are shown.

1. Write an ordered pair that would be in Quadrant I when graphed. ___$(3, 7)$_______
2. Write an ordered pair that would be in Quadrant II when graphed. __________
3. Write an ordered pair that would be in Quadrant III when graphed. __________
4. Write an ordered pair that would be in Quadrant IV when graphed. __________
5. Write an ordered pair that would be on the *x*-axis when graphed. __$(3, 0)$________
6. Write an ordered pair that would be on the *y*-axis when graphed. __________

7. Write the ordered pair for points *A* – *H*.



A: _$(-9, 8)$________

B: __________

C: __________

D: __________

E: _$(-8, -3)$________

F: __________

G: __________

H: __________

8. Graph and label the following points on the coordinate plane.

$L: (2, 5)$

$M: (-5, -3)$

$N: (7, -5)$

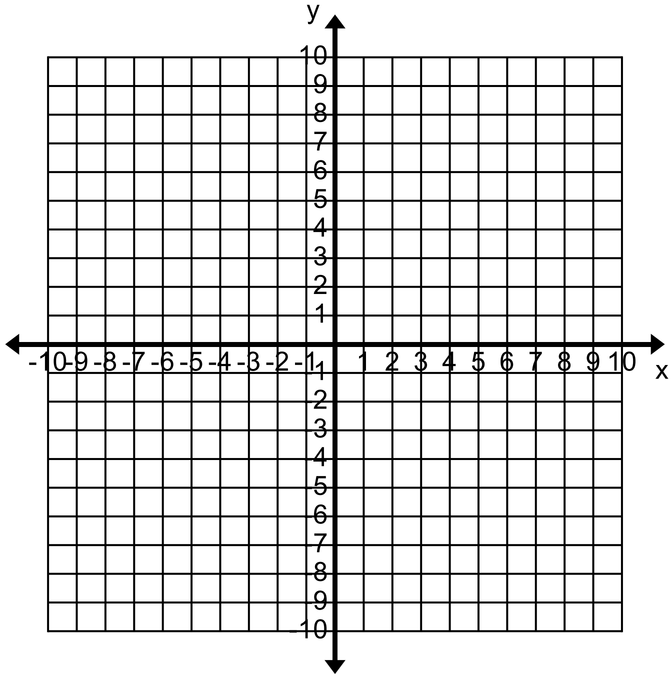
$O: (0, 4)$

$P: (-3, 9)$

$Q: (-7, 0)$

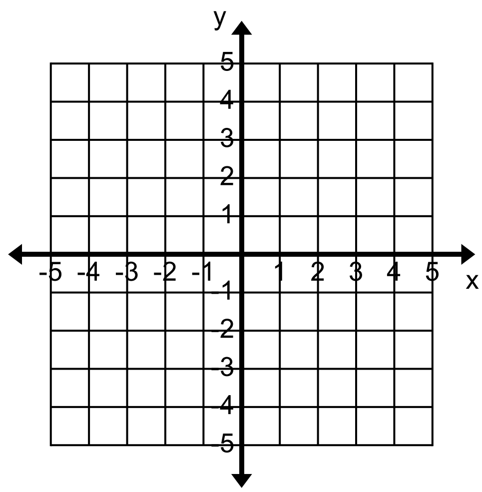
$R: (5\frac{1}{2}, -2)$

$S: (-3\frac{1}{2}, 4)$



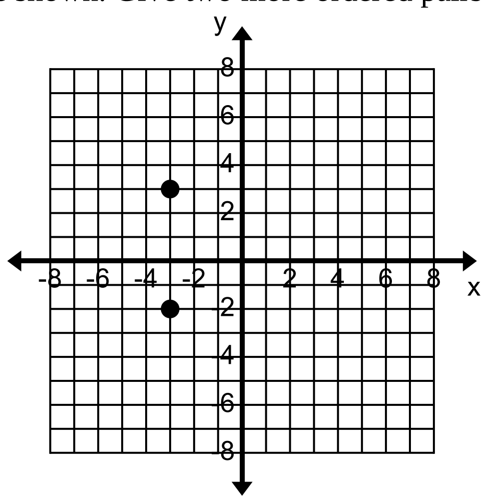
9. List 4 ordered pairs that when graphed would fall on the same **vertical** line.
Answers will vary but the points must have the same *x*-value and different *y*-values

10. Graph the following points on the coordinate plane.
$\{(-4, 3), (-4, 0), (0, 0), (0, 3)\}$

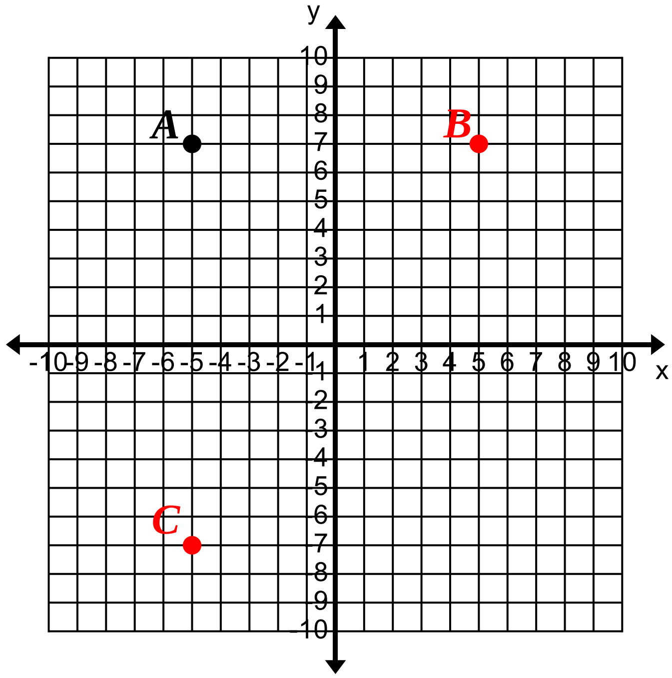


11. The points you graphed in the previous problem are vertices of a polygon. Classify the polygon.

12. Two vertices of a square are shown. Give two more ordered pairs that could be the other two vertices of the square.



13. Use the grid to answer the questions.



a. Plot the point that is symmetric about the *y*-axis to *A*. Label it *B*.

b. Write the ordered pairs for *A* and *B* next to the points on the grid. What do you notice about the ordered pairs of *A* and *B*?

A: $(-5, 7)$; *B*: $(5, 7)$; the points have the same *y*-coordinate but their *x*-coordinates are opposites: (x, y) and $(-x, y)$. When this is the case, the points will be symmetric about the *y*-axis.

c. Plot the point that is symmetric about the *x*-axis to *A*. Label it *C*.

d. Write the ordered pairs for *A* and *C* next to the points on the grid. What do you notice about the ordered pairs of *A* and *C*?

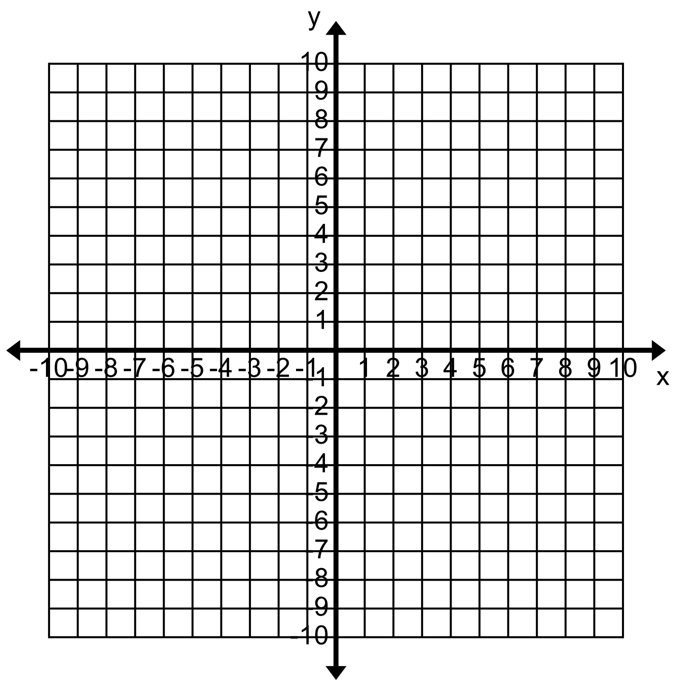
A: $(-5, 7)$; *C*: $(-5, -7)$; the points have the same *x*-coordinate but their *y*-coordinates are opposites: (x, y) and $(x, -y)$. When this is the case, the points will be symmetric about the *x*-axis.

e. What is the distance between *A* and *B*? 10 units

f. What is the distance between *A* and *C*?

14. Create your own set of points that when connected are symmetric about the *x*-axis *and* the *y*-axis.

x	y	(x, y)



3.1i Class Activity: Classifying Rational Numbers *(Optional)*

Activity 1: Nested Venn Diagram on Sports

a. Show the relationship between the words **exercise**, **team sports**, and **sports** by placing the words in the correct place in the Venn Diagram.

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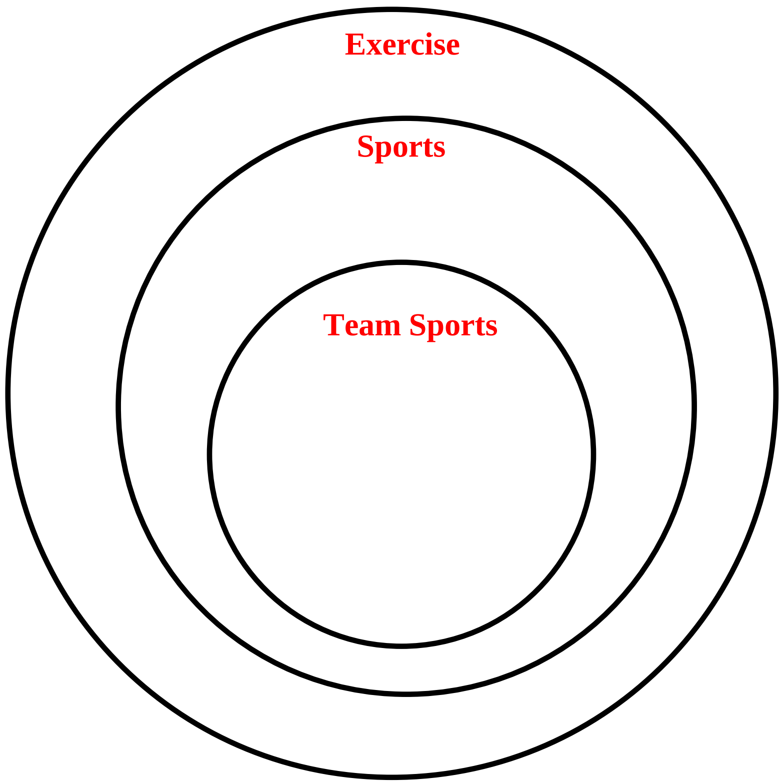


b. Place the following words in the diagram where they belong.
surfing, soccer, walking, tennis, aerobics, skiing, karate, basketball
A sample answer is shown. There are other possible correct answers. It all depends on how the students view these activities. For example, some students may view surfing as exercise and not a sport while others may consider it an individual sport. If student diagrams match their classifications of each of the activities, they are correct. You may also choose to decide as a class how you think of each of these activities before putting them into the diagram. The important thing is that they understand the structure of the diagram.

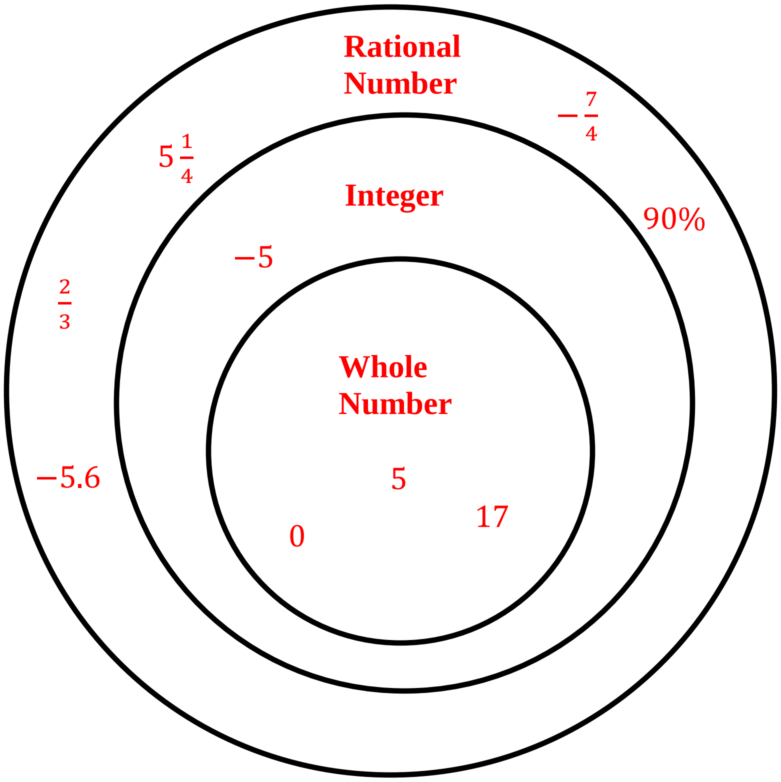
c. Add two of your own exercises to the diagram. Answers will vary.

d. Write <u>three</u> true statements based on the diagram. For example, "All team sports are sports."
Some exercises are sports. Not all sports are team sports. Some sports are team sports. All sports are exercise.

Activity 2: Nested Venn Diagram on Rational Numbers

Classifying numbers connects nicely to work students did in 3.G where they came to understand that shapes in different categories (e.g., rhombuses and rectangles) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). In a similar way, students come to understand that numbers in different categories (e.g., whole numbers and integers) may share attributes (e.g., can be expressed as a fraction), and that the shared attribute can define a larger category (e.g., rational numbers).

a. Show the relationship between the words **whole number**, **integer**, and **rational number** by placing the words in the correct place in the Venn Diagram.



b. Place the following numbers in the diagram where they belong.

$0, 5, -5, \frac{2}{3}, 5\frac{1}{4}, -\frac{7}{4}, 17, -5.6, 90\%$

c. Add two of your own numbers to the diagram using a different colored pencil.
Answers will vary

d. Tell whether the following statements are true or false. Justify your answers.

All whole numbers are integers. True

All integers are whole numbers. False

Some rational numbers are integers. True

Not all integers are rational numbers. False

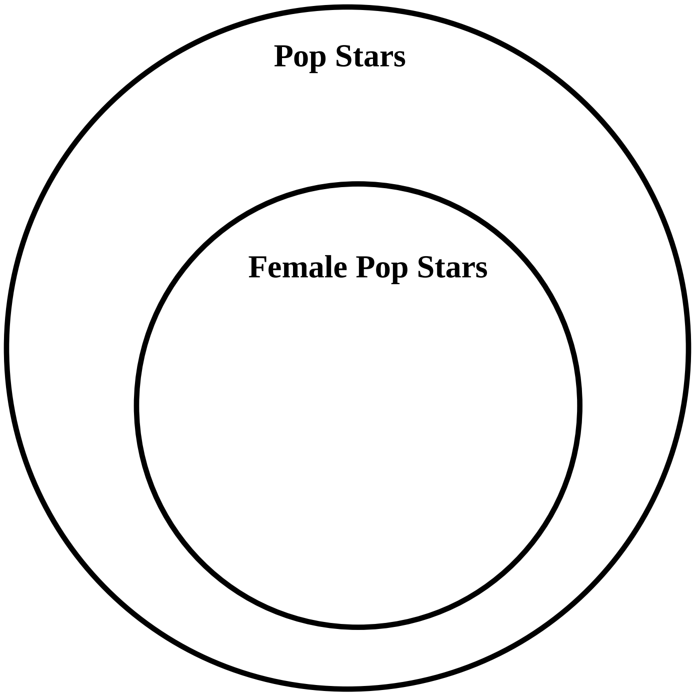
Some whole numbers are not integers. False

Activity 3: Complete the table below by writing a number in each row of the **Number** column that fits the classification given. Answers will vary except for the third row. Zero is the only answer for the third row. Sample answers are given for the other rows.

Number	**Positive Number**	**Negative Number**	**Whole Number**	**Integer**	**Rational Number**
5	X		X	X	X
$-\frac{2}{3}$		X			X
0			X	X	X
$\frac{5}{2}$	X				X
-2		X		X	X

3.1i Homework: Classifying Rational Numbers *(Optional)*

1. Put the following music artists into the Venn Diagram below where they belong: Taylor Swift, Justin Timberlake, Beyoncé, Selena Gomez, and Jason Derulo.



2. Add two additional music artists to the Venn Diagram in #1.

3. Tell whether the following statements are true or false. Justify your answer.

   All female pop stars are pop stars. True

   All pop stars are female pop stars. False

   Some pop stars are not female pop stars.

   Not all pop stars are female pop stars.

4. Classify each number by putting an X in the appropriate columns.

Point	Positive Number	Negative Number	Whole Number	Integer	Rational Number
-1		X		X	X
-2.3					
$1{,}000$					
$-3\frac{1}{3}$		X			X
0.25					
5%					

3.1j Self-Assessment: Section 3.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Understand a rational number as a point on the number line.				
2. Construct a number line to show negative numbers using ideas about symmetry. Recognize opposite signs of numbers as indicating locations on opposite sides of zero on the number line.				
3. Write and interpret symbolic statements about opposites (e.g., $-(-3)$ is read "the opposite of negative three" and can be simplified to 3).				
4. Determine the value of rational numbers represented on a horizontal and vertical number line.				
5. Represent rational numbers on horizontal and vertical number lines.				
6. Represent pairs of rational numbers on the coordinate plane.				

Sample Problems for Section 3.1

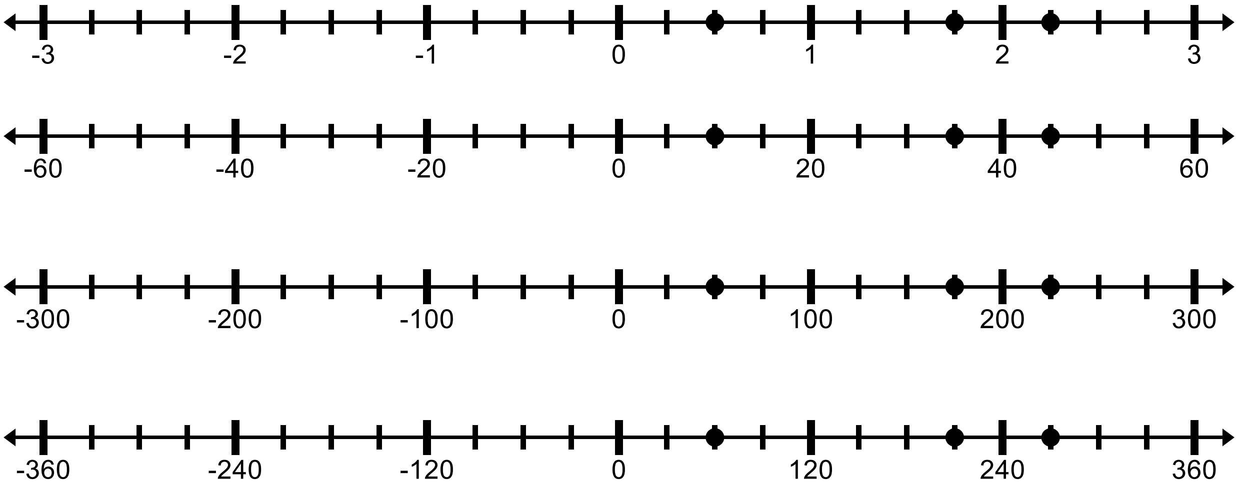
Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. Is the following statement true or false? Justify your answer.

   "All rational numbers can be represented by a point on the number line."

2. Construct a number line to show the location of the integers from -8 to 8. Explain how you used ideas about symmetry and opposites to construct your number line. [2]

3. Three points are plotted on each number line below. [2]



a. Write the value of each point under the points.

b. Compare the graphs. How are they similar? How are they different?

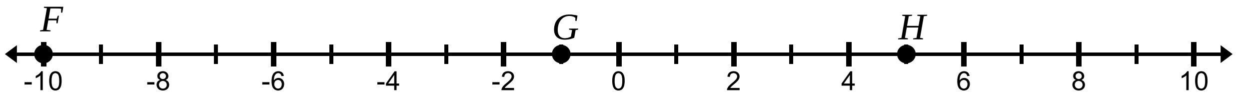
c. Graph the opposite of each point on each number line.

d. Write the value of the opposites under the points.

4. Complete the table. [3]

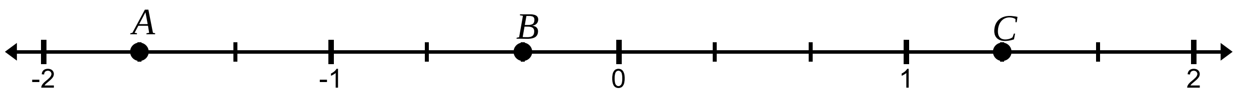
Statement in Words	**Symbolic Statement**	**Simplified Form**
The opposite of five		
The opposite of negative five		
	$-(-9)$	
	$-\big(-(8.5)\big)$	
The opposite of zero		
The opposite of the opposite of negative twenty		
		11
		-11

5. Determine the value of the points on the number line. [4]



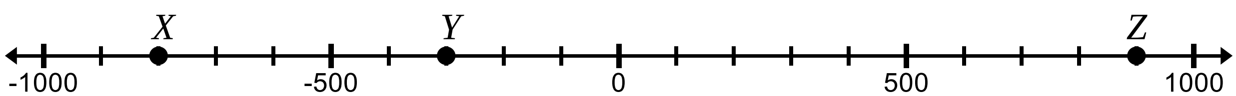
F: *G:* *H:*

6. Determine the value of the points on the number line. [4]



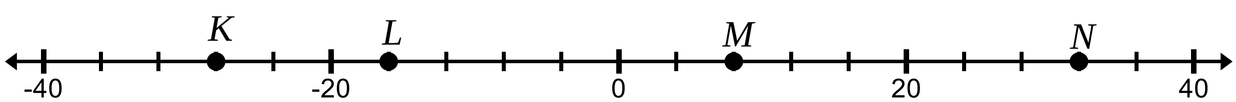
A: *B:* *C:*

7. Determine the value of the points on the number line. [4]



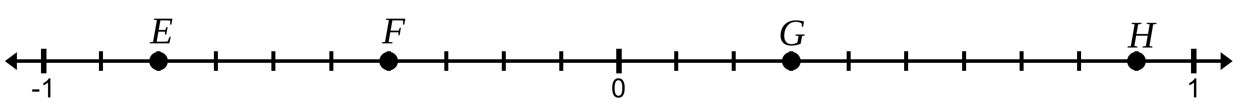
X: *Y:* *Z:*

8. Determine the value of the points on the number line. [4]



K: *L:* *M:* *N:*

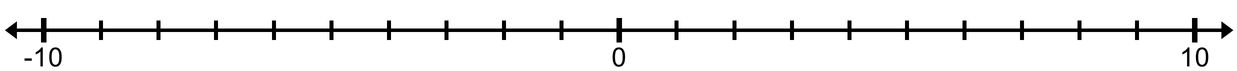
9. Determine the value of the points on the number line. [4]



E: *F:* *G:* *H:*

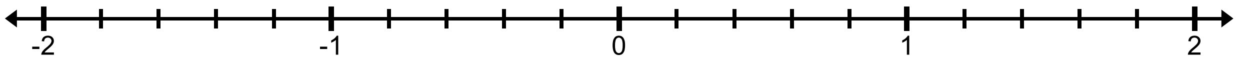
10. Represent the following points on the number line. [5]

$R: -6$ $S: -2$ $T: 4$ $U: 9$



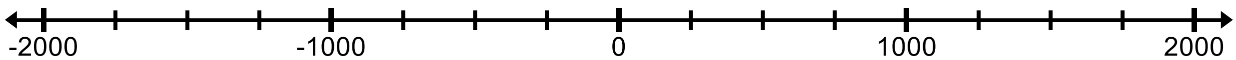
11. Represent the following points on the number line. [5]

$J: -1.8$ $\quad$ $K: -0.6$ $\quad$ $L: 0.8$ $\quad$ $M: 1.2$



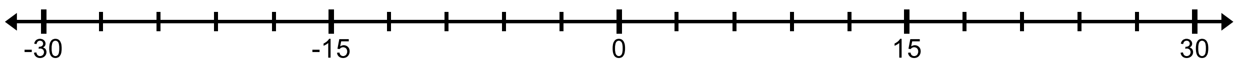
12. Represent the following points on the number line. [5]

$A: -1{,}750$ $\quad$ $B: -1{,}250$ $\quad$ $C: -500$ $\quad$ $D: 1{,}500$



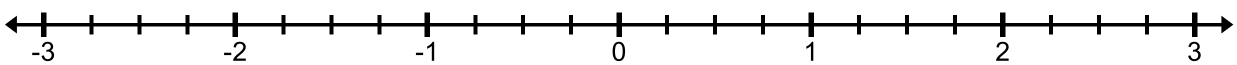
13. Represent the following points on the number line. [5]

$D: -24$ $\quad$ $E: -9$ $\quad$ $F: 6$ $\quad$ $G: 18$

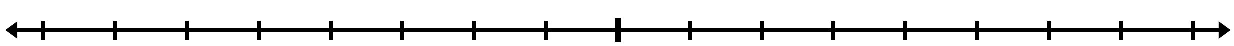


14. Represent the following points on the number line. [5]

$Q: -2\frac{3}{4}$ $\quad$ $R: -\frac{5}{4}$ $\quad$ $S: -0.5$ $\quad$ $T: 2.25$



15. Byron's teacher asked him to graph the points 120 and -120 on the number line shown below.

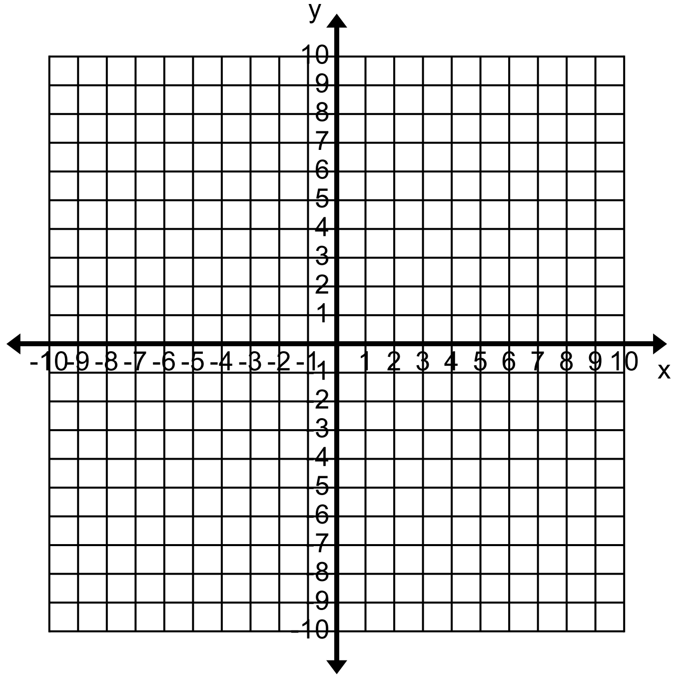


Byron thought about using a scale of 10 for his graph. Is this a good scale for this problem? Why or why not? Would you suggest a different scale to Byron? Explain. [5]

16. Construct a number line to represent the points $H: \frac{1}{2}, I: -\frac{1}{2}, J: \frac{3}{8}$, and $K: -\frac{3}{8}$. [5]

17. Represent the following ordered pairs on the coordinate plane. [6]

$A: (-1, 5)$	$B: (0, 8)$	$C: (-2, -7)$
$D: (-9, 0)$	$E: (6, 1)$	$F: (-5, 10)$
$G: (9, -2)$	$H: (4, -9)$	$I: (0, 0)$
$J: (1.5, 7)$	$K: (0, -6.5)$	$L: (-5.5, -3.5)$



Section 3.2: Absolute Value and Ordering

Section Overview:
In this section, students are formally introduced to the meaning of absolute value and start to distinguish between the value of a number relative to zero and the magnitude of the number (e.g., the number's distance from zero). They simplify absolute value statements and use ideas about absolute value to find the distance between two points on the same horizontal or vertical line. Then, students compare and order rational numbers, relying on their understanding of the structure of the number line and understanding that smaller numbers are located to the left of bigger numbers on a number line. Students come to realize that one way to compare negative numbers is to compare their opposites first. When you compare two positive numbers, the larger positive number becomes *smaller* when you reflect the numbers over the vertical line passing through zero. For example, if a student is asked to compare -11 to -12, they may start by comparing 11 to 12. Because 12 is farther to the right on the number line, it is bigger than 11; however, when you reflect both numbers over the vertical line passing through zero, -11 is now farther to the right than -12 and therefore -11 is bigger than -12. Students also rely on a great deal on number sense and estimation strategies when comparing and ordering rational numbers. For example, when comparing $\frac{3}{8}$ to $\frac{1}{2}$, students may reason that since $\frac{1}{2} = \frac{4}{8}$, $\frac{3}{8}$ is smaller than $\frac{1}{2}$. Then, when students are faced with the problem of comparing $-\frac{3}{8}$ to $-\frac{1}{2}$, they can reason that $-\frac{3}{8}$ is farther to the right on the number line and therefore bigger than $-\frac{1}{2}$.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Understand what the absolute value of a rational number represents. Simplify expressions containing absolute value.
2. Find the distance between two rational numbers on the same horizontal or vertical line.
3. Compare rational numbers.
4. Order rational numbers.

3.2a Class Activity: Absolute Value

Activity 1: Consider the numbers 5 and -5. Compare these numbers. How are they the same? How are they different?

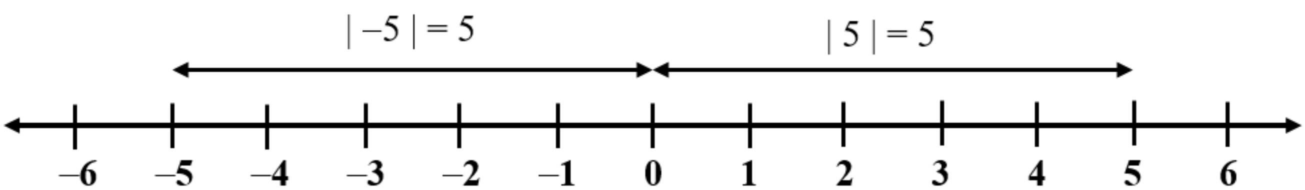
Students have been surfacing ideas about absolute value throughout the chapter so far. 5 and -5 are the same because they are both 5 units away from zero. They are different because one is to the left of zero (-5) and one is to the right of zero (5). Both numbers have a magnitude of five; however, their values relative to zero are different.

Absolute Value: Absolute value is distance from zero. Absolute value is denoted with the symbol $|\ \ |$:

$|5|$ is read "the absolute value of five"

$|-5|$ is read "the absolute value of negative five"

Both $|5|$ and $|-5|$ simplify to 5 because 5 and -5 are both five units away from zero on the number line.



Activity 2:

a. How many numbers have an absolute value equal to 8? Justify your answer. There are two numbers that are 8 units away from 0, eight and negative eight.

b. How many numbers have an absolute value equal to $\frac{2}{3}$? Justify your answer. Two, two-thirds and negative two-thirds

c. How many numbers have an absolute value equal to 0? Justify your answer. One, zero

d. How many numbers have an absolute value equal to -10? None, absolute value cannot be negative because it represents a distance

Activity 3: Simplify.

a. $\lvert -4 \rvert$ 4	b. $\lvert 4 \rvert$ 4	c. $\lvert 1 \rvert$ 1
d. $\left\lvert 3\frac{1}{2} \right\rvert$ $3\frac{1}{2}$	e. $\left\lvert -3\frac{1}{2} \right\rvert$ $3\frac{1}{2}$	f. $\lvert 0 \rvert$ 0
g. $-\lvert 20 \rvert$ -20	h. $-\lvert -20 \rvert$ -20	i. $-\lvert 1.1 \rvert$ -1.1

Have students read g. as "the opposite of the absolute value of twenty". The order of operations tells us to start with the grouping symbols, in this case the absolute value is the grouping symbol, and then take the opposite of our answer.

Activity 4: Determine whether the following statements are always true, sometimes true, or never true. Justify your answers. The statements below are abstract statements – encourage students to use examples to investigate the problems and to justify their answers.

a. The absolute value of a number is greater than the number. Sometimes true, this is true when the number is negative. For example, the absolute value of negative four is four and four is greater than negative four. This is not true when the number is greater than or equal to zero. For example, the absolute value of four is four and four is not greater than four. You may encourage students to write this statement symbolically while you explore the statement $|a| > a$.

b. The absolute value of a number is the number's opposite. Sometimes true. This is only true when the number is less than or equal to zero. Again, explore the statement symbolically $|a| = -(a)$.

c. The absolute value of a negative number is greater than the number. Always true.

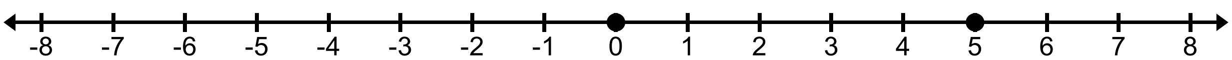
d. There are always two numbers with the same absolute value. Sometimes true. Zero is the only exception to this statement.

e. Opposites have the same absolute value. Always true

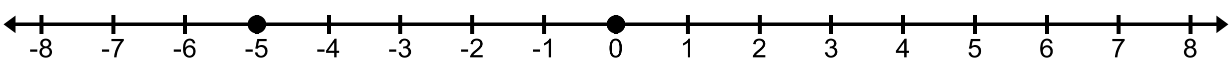
f. The absolute value of a positive number is a negative number. Never true

Activity 5: Use ideas about absolute value to find the distance between the two points shown on the number line. Students use the structure of the number line (symmetric about the vertical line passing through zero) to find the distance between two points on the number line. Rather than having students "count" how to get from one point to the other, have them think about how to get from each point to zero and then add those distances together.

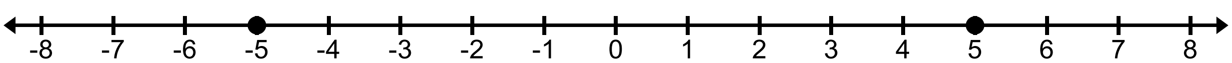
a. Distance between the two points: ___5 units_______



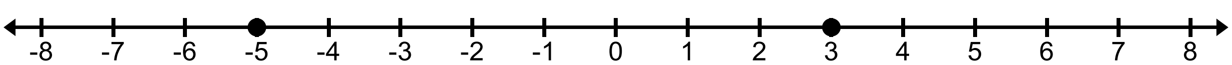
b. Distance between the two points: ___5 units _______



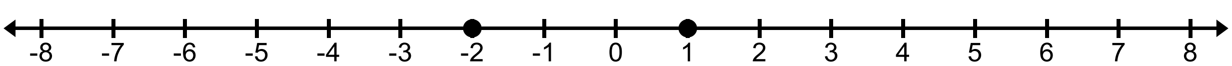
c. Distance between the two points: ___10 units _______



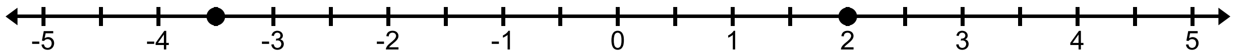
d. Distance between the two points: ___8 units _______



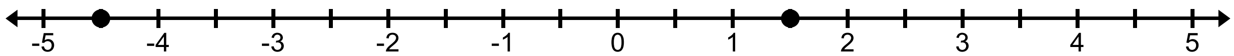
e. Distance between the two points: ___3 units _______



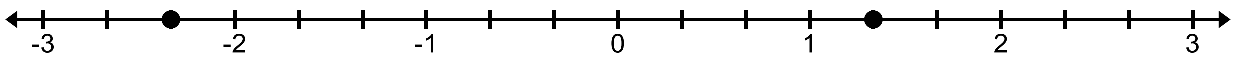
f. Distance between the two points: __5.5 units ________



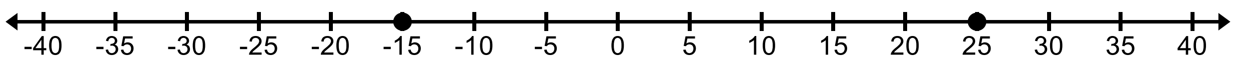
g. Distance between the two points: __6 units ________



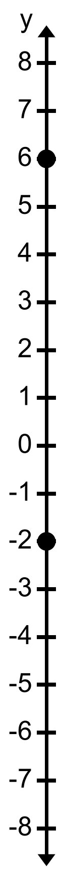
h. Distance between the two points: ___$3\frac{2}{3}$ units _______



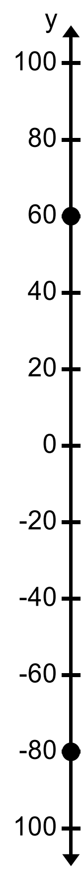
i. Distance between the two points: ___40 units _______



j. Distance between the two points: _8 units_

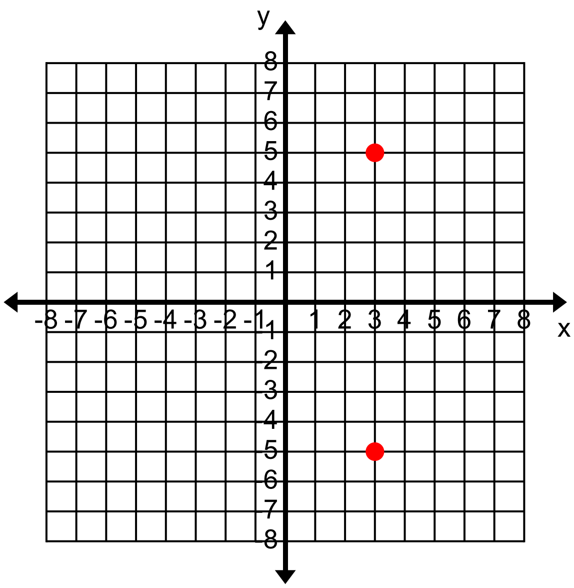


k. Distance between the two points: _140 units _



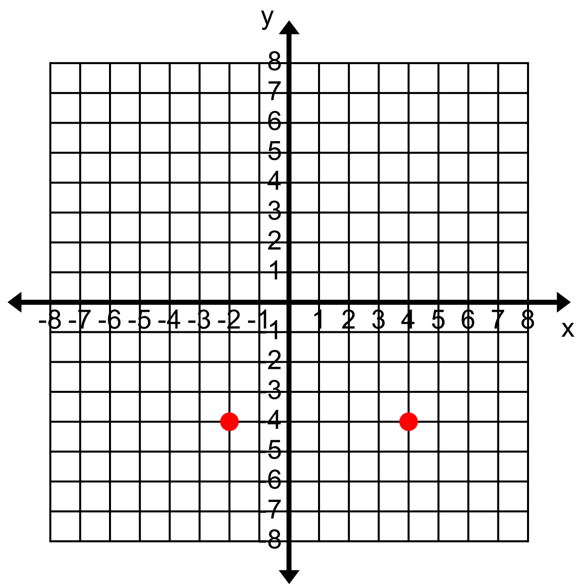
Activity 5: Graph the following sets of points and then find the distance between the two points.

a. $(3, 5)$ and $(3, -5)$



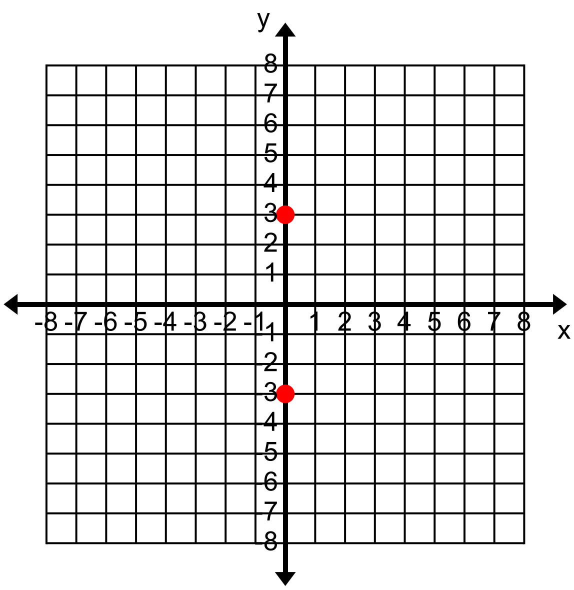
Distance between points: ___10 units________

b. $(-2, -4)$ and $(4, -4)$



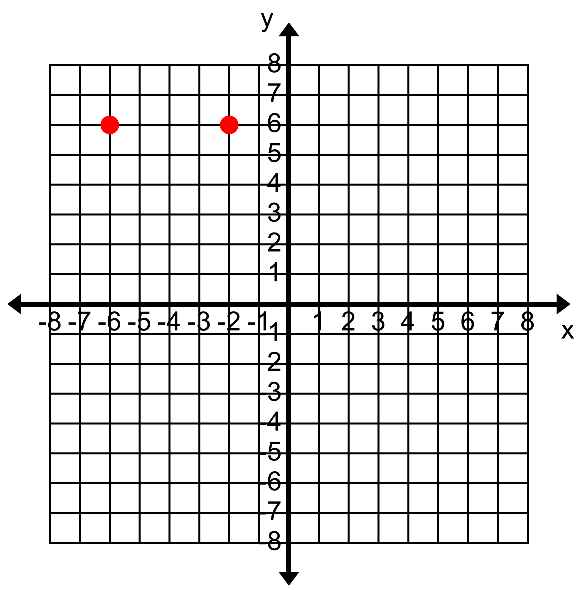
Distance between points: ___6 units________

c. $(0, 3)$ and $(0, -3)$



Distance between points: ___6 units________

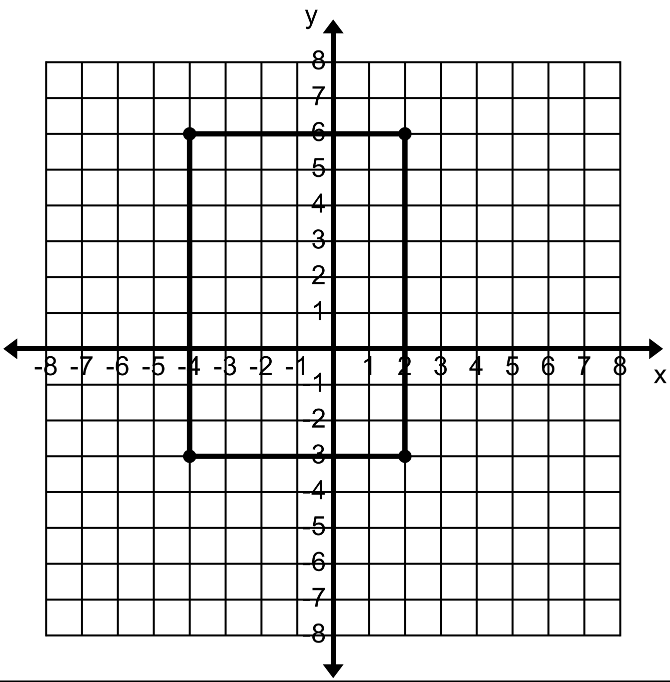
d. $(-6, 6)$ and $(-2, 6)$



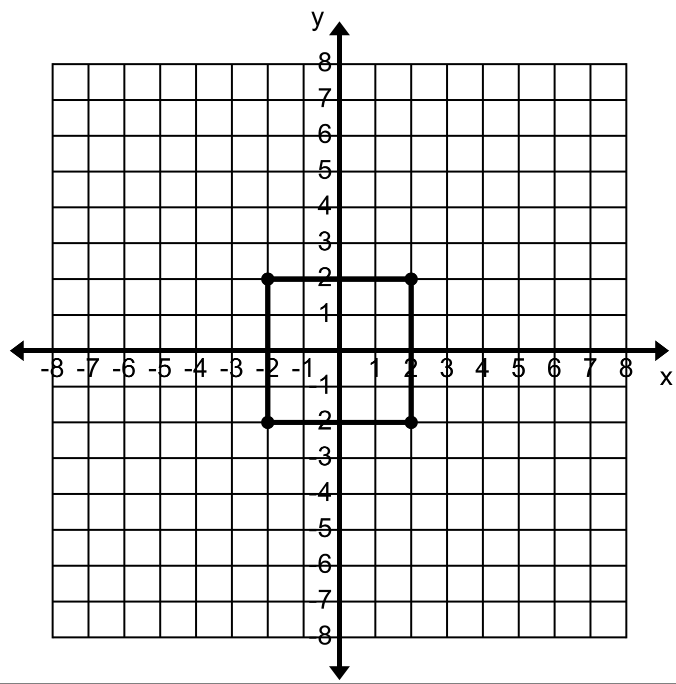
Distance between points: ___4 units________

Activity 6: Find the perimeter of each polygon. Any time students determine a length, they need to attend to precision when counting the number of segments that make up the length. A common error is for students to count the *points* including the one they start on. Encourage students to use ideas about absolute value and distance from zero to determine the lengths of the segments that make up the polygons. Students will explore standard 6.NS.8 (finding the distance between two points with the same first coordinate or the same second coordinate) further in Chapter 5 Geometry.

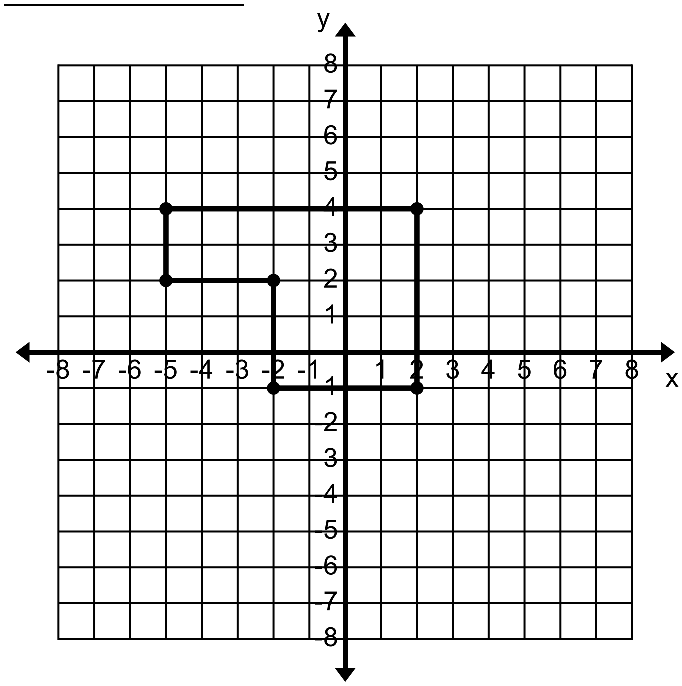
a. Perimeter: ______30 units__________



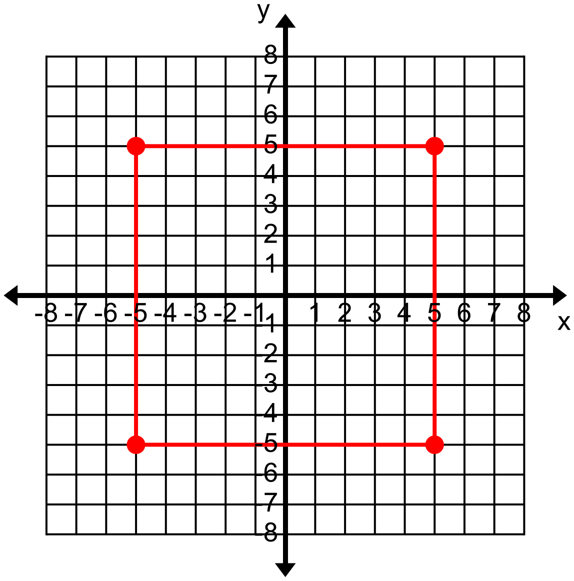
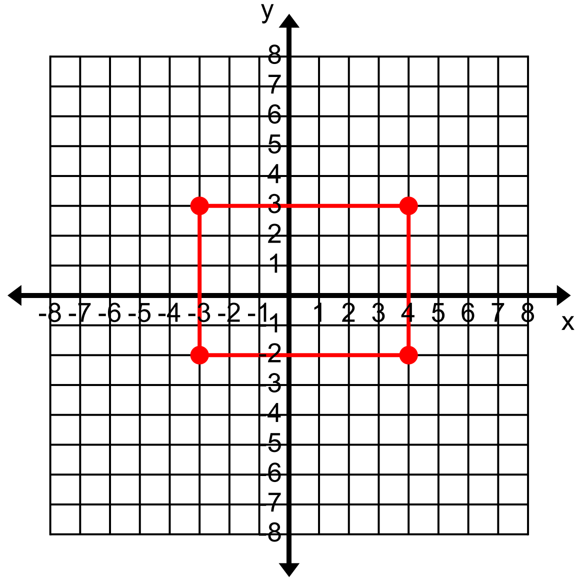
b. Perimeter: ____16 units ________________



c. Perimeter: ____24 units ________________



Activity 7: Graph and connect each set of points. Then, find the perimeter of the polygon.

a. $(5,5),(5,-5),(-5,-5),(-5,5)$  Perimeter of Polygon: _____40 units__________	b. $(4,3),(4,-2),(-3,-2),(-3,3)$  Perimeter of Polygon: _____24 units__________

Activity 8: Devise a strategy for finding the distance between the two points without graphing. Then, find the distance between the two points.

This problem requires students to make sense of the structure of the ordered pairs. What is it about a set of ordered points that causes the points to fall on the same horizontal line? The same vertical line? Once we determine whether the points are on the same horizontal or vertical line, we know which numbers we need to find the difference between to find the distance between the two points. If students struggle, have them sketch a graph of the points, without worrying about scale.

a. $(3,157)$ and $(3,84)$	b. $(-25,150)$ and $(174,150)$	c. $(20,-100)$ and $(20,-250)$
73 units	199 units	150 units

Spiral Review

1. Complete the table to show the area of each rectangle.

Length	**Width**	**Area**
6 in.	7 in.	
1.5 ft	24 ft	
$1\frac{1}{4}$ yd	$2\frac{1}{2}$ yd	

2. Complete the table to show the missing dimension of each rectangle.

Length	**Width**	**Area**
3 ft		36 ft^2
0.5 m		4 m^2
	$3\frac{3}{4}$ yd	$10\frac{5}{8}$ yd^2

3. Complete the table to show the volume of each rectangular prism.

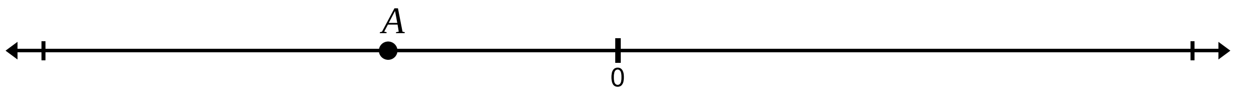
Length	**Width**	**Height**	**Volume**
10 cm	4 cm	7 cm	
0.8 m	2 m	1 m	
$\frac{1}{2}$ yd	$\frac{1}{2}$ yd	$\frac{1}{2}$ yd	

4. Complete the table to show the missing dimension of each rectangular prism.

Length	**Width**	**Height**	**Volume**
120 cm	50 cm		144,000 cm^3
5 ft		7 ft	0.35 ft^3
$\frac{2}{3}$ m	$\frac{3}{8}$ m		$6\frac{1}{4}$ m^3

3.2a Homework: Absolute Value

1. Use the number line below to answer the questions.



   a. Is the value of *A* positive or negative? Explain.

   b. Is the distance from zero to *A* positive or negative? Explain.
   Positive, distance is always positive.

2. What number or numbers, if any, make the statement true? $|x| = 16$

3. What number or numbers, if any, make the statement true? $|x| = -9$

4. What number or numbers, if any, make the statement true? $|x| = 0$

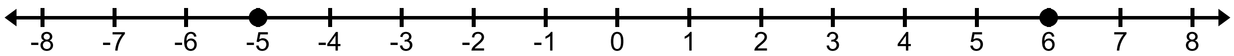
5. Give two numbers with an absolute value greater than three.

Directions: Simplify.

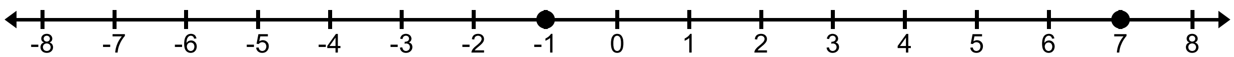
6. $\lvert 5\rvert$ 5	7. $\lvert -5\rvert$ 5	8. $\lvert -250\rvert$
9. $\left\lvert -3\frac{9}{10}\right\rvert$	10. $\lvert 11.06\rvert$	11. $\left\lvert \frac{11}{12}\right\rvert$
12. $-\lvert -18\rvert$	13. $-\lvert 18\rvert$ -18	14. $\lvert -18\rvert$

Directions: Find the distance between the two points.

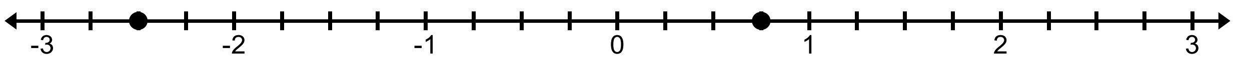
15. Distance between the two points: __________



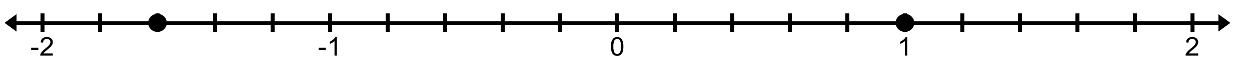
16. Distance between the two points: __8 units ________



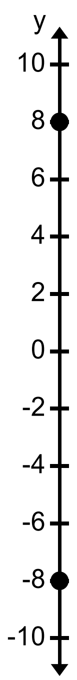
17. Distance between the two points: ___$3\frac{1}{4}$ units _______



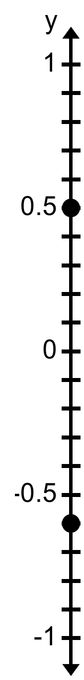
18. Distance between the two points: __________



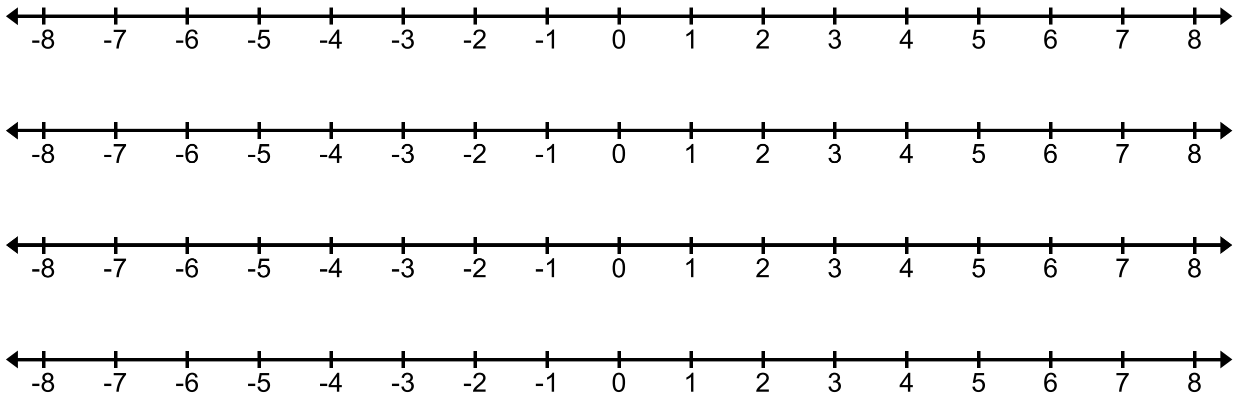
19. Distance between the two points: ___



20. Distance between the two points: __

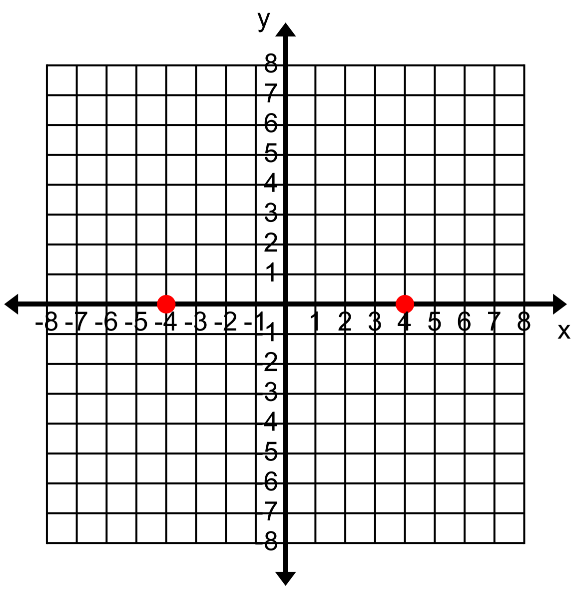


21. On the four graphs below, show four *different* sets of points that are 8 units apart.



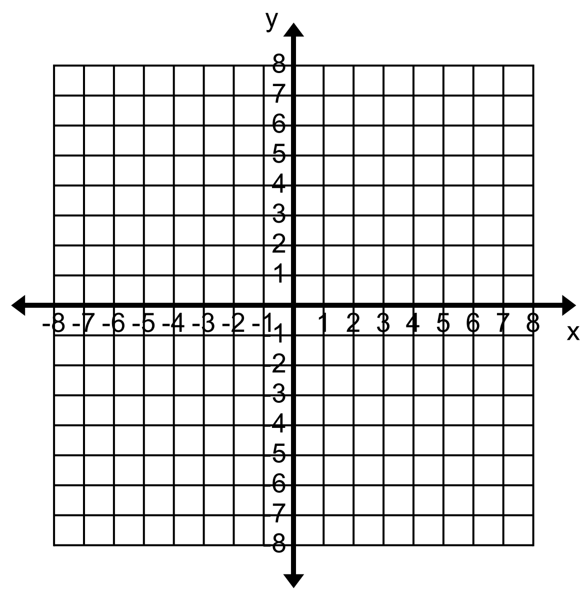
Directions: Graph the following sets of points and then find the distance between the two points.

22. $(-4, 0)$ and $(4, 0)$



Distance between points: ___8 units________

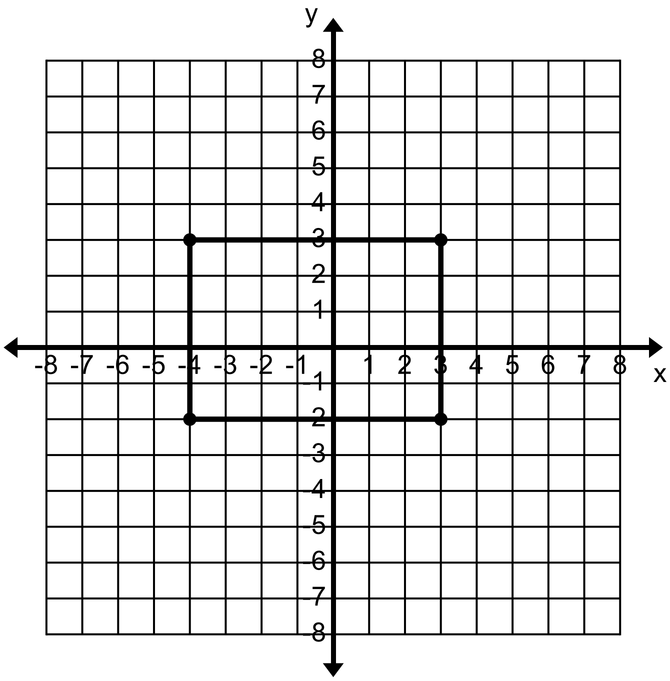
23. $(-4, 3)$ and $(-4, -6)$



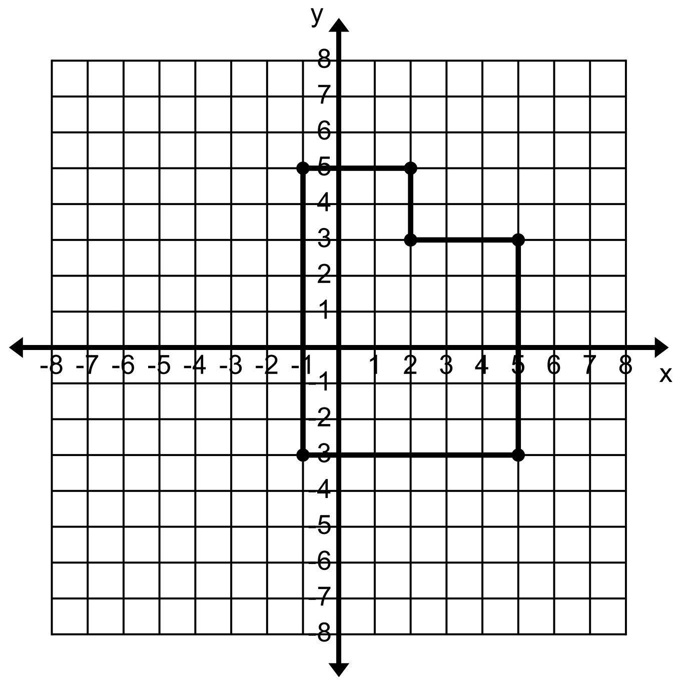
Distance between points: ____________

Directions: Find the perimeter of each polygon.

24. Perimeter: ________________

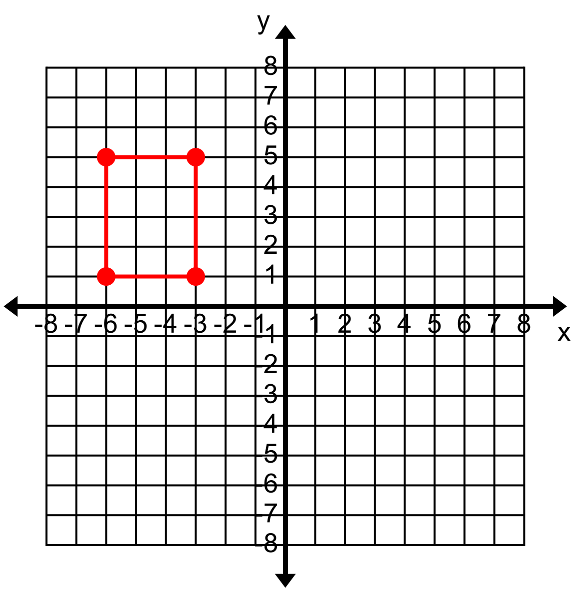


25. Perimeter: ____________________



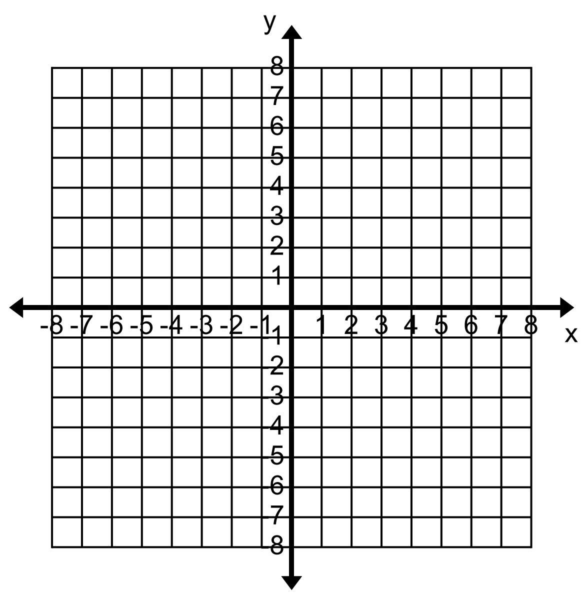
Directions: Graph and connect each set of points. Then, find the perimeter of the polygon.

26. $(-6,5),(-3,5),(-3,1),(-6,1)$



Perimeter of Polygon: _____14 units__________

27. $(-2,2),(2,2),(2,-2),(-2,2)$

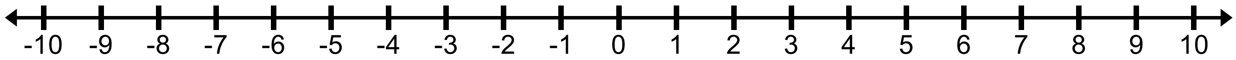


Perimeter of Polygon: ________________

3.2b Class Activity: Comparing Rational Numbers

Activity 1:

a. Graph the numbers 3 and 5 on the number line.



b. Compare 3 and 5 using <, >, or =.

3 __<__ 5

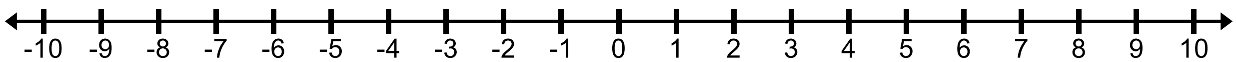
c. How does the number line help you determine which number is larger?
Numbers to the right are larger than numbers to the left (or numbers to the left are smaller).

d. Next, graph the opposites of 3 and 5 on the number line above.

e. Compare -3 and -5 using <, >, or =.

-3 __>__ -5
When you reflect 5 over the vertical line passing through zero, it becomes farther to the left than when you reflect 3 over the vertical line passing through zero, making -5 smaller than -3.

f. Graph the numbers 1 and 10 on the number line.



g. Compare 1 and 10 using <, >, or =.

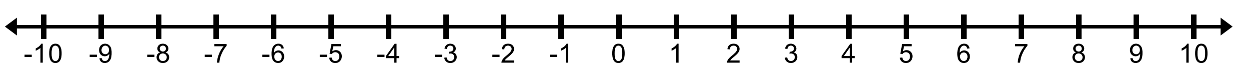
1 __<__ 10

h. Graph the numbers -1 and -10 on the number line above.

i. Compare -1 and -10 using <, >, or =.

-1 __>__ -10

j. Graph $\frac{3}{4}$ and $1\frac{1}{2}$ on the number line.



k. Compare $\frac{3}{4}$ and $1\frac{1}{2}$ using <, >, or =.

$\frac{3}{4}$ __<__ $1\frac{1}{2}$

l. Graph $-\frac{3}{4}$ and $-1\frac{1}{2}$ on the number line above.

m. Compare $-\frac{3}{4}$ and $-1\frac{1}{2}$ using <, >, or =.

$-\frac{3}{4}$ __>__ $-1\frac{1}{2}$

n. In your own words, explain how to compare numbers. Use examples to support your explanation.

Answers will vary. Key ideas to capture: 1) Numbers farther to the left on the number line are smaller; 2) One way to compare negative numbers is to compare their opposites. The number that is bigger when you compare the opposites becomes smaller when the numbers are reflected over the vertical line passing through zero. 3) When you compare two *negative* numbers, the number with the smaller absolute value is bigger. 4) When you compare two *positive* numbers, the number with the larger absolute value is bigger. 5) When you compare a positive number to a negative number, the positive number will always be bigger. Sometimes the absolute value of the positive number will be larger than the absolute value of the negative number and sometimes it will be smaller. Provide examples to help students understand.

Directions: Compare the numbers below using <, >, or =. Use a number line to help you.

2. $7 \underline{\;>\;} 5$ 3. $-7 \underline{\;<\;} -5$	4. $12 \underline{\;<\;} 15$ 5. $-12 \underline{\;>\;} -15$	6. $350 \underline{\;<\;} 400$ 7. $-350 \underline{\;>\;} -400$
8. $2 \underline{\;<\;} 2\frac{1}{2}$ 9. $-2 \underline{\;>\;} -2\frac{1}{2}$	10. $8.25 \underline{\;=\;} 8\frac{1}{4}$ 11. $-8.25 \underline{\;=\;} -8\frac{1}{4}$	12. $6.45 \underline{\;>\;} 6.4$ 13. $-6.45 \underline{\;<\;} -6.4$
14. $0.7 \underline{\;>\;} 0.07$ 15. $-0.7 \underline{\;<\;} -0.07$	16. $1.75 \underline{\;<\;} 1\frac{4}{5}$ 17. $-1.75 \underline{\;>\;} -1\frac{4}{5}$	18. $\frac{5}{8} \underline{\;>\;} 0.5$ 19. $-\frac{5}{8} \underline{\;<\;} -0.5$
20. $-3 \underline{\;>\;} -4$	21. $-54 \underline{\;<\;} -44$	22. $-\frac{827}{2,481} \underline{\;<\;} \frac{1}{4}$
23. $-3\frac{1}{3} \underline{\;<\;} -3\frac{1}{4}$	24. $-\frac{9}{20} \underline{\;>\;} -0.5$	25. $\frac{5}{3} \underline{\;>\;} -1\frac{2}{3}$
26. $-0.6 \underline{\;>\;} -\frac{2}{3}$	27. $-6.49 \underline{\;>\;} -6.5$	28. $-\frac{7}{8} \underline{\;<\;} -\frac{4}{5}$
29. $-\frac{1}{4} \underline{\;<\;} -0.23$	30. $-\frac{1}{8} \underline{\;>\;} -\frac{1}{4}$	31. $-101 \underline{\;<\;} -100$
32. $-\frac{5}{9} \underline{\;<\;} -0.5$	33. $-\frac{13}{25} \underline{\;<\;} -\frac{9}{20}$	34. $-0.25 \underline{\;<\;} -\frac{3}{16}$

35. Give a number between -6 and -7. Answers will vary, sample answers -6.5, -6.02

36. Give a number between $-2\frac{1}{2}$ and -3. Answers will vary, sample answers -2.6, -2.65

37. Give a number between -4 and -4.3. Answers will vary, sample answers -4.1, -4.2

38. Give a number between $-\frac{3}{10}$ and $-\frac{2}{5}$. Answers will vary, sample answers -0.35, -0.32

39. Ms. Tucker tells her class that *a* and *b* are rational numbers and $a < b$. Describe what would have to be true about the values of *a* and *b* for the following statements to be true. Justify your answers. It may help students to have a number line available to work through this problem.

   a. The absolute value of *a* is larger than the absolute value of *b*.
   a and *b* are both negative

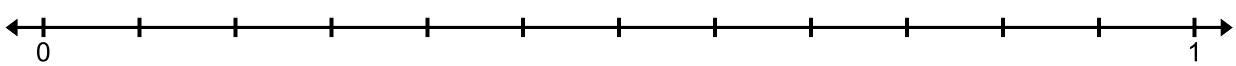
   b. The absolute value of *a* is smaller than the absolute value of *b*.
   a and *b* are both positive OR *b* is positive and *a* is negative and *a* is closer to zero than *b*.

   c. *a* is farther away from zero than *b*.
   a and *b* are both negative OR *a* is negative and *b* is positive but *b* is closer to 0 than *a*.

Spiral Review

1. Use the number line below to complete this activity.

   a. Label the tick marks to show the scale of the graph.



   b. Graph and label the following points on the graph:

A: $\frac{1}{6}$ B: $\frac{1}{3}$ C: $\frac{1}{2}$ D: $\frac{2}{3}$ E: $\frac{3}{4}$ F: $\frac{5}{6}$

2. Tell whether the number given is bigger than $\frac{1}{2}$ or smaller than $\frac{1}{2}$.

a. 0.45	b. 0.052	c. 0.501
d. $\frac{19}{40}$	e. $\frac{6}{11}$	f. $\frac{12}{25}$

3. In each problem, an attribute of a two-dimensional figure is given. Write the name(s) of the figure(s) that have that attribute.

   a. A quadrilateral with four right angles

   b. A triangle with three congruent angles

   c. A quadrilateral with four congruent sides

   d. A quadrilateral with two sets of parallel lines

4. Simplify.

a. 0.2×5	b. 0.01×345	c. 15×0.03
d. $5 \div 0.1$	e. $40 \div 0.8$	f. $3.6 \div 12$

3.2b Homework: Comparing Rational Numbers

1. Fill in the Blank: As you move to the right on the number line, the numbers __get bigger____________.

2. Paul's teacher asked him to compare -2 and -1 using $<$, $>$, or $=$. Paul's answer and thinking is shown below.

   Paul's Response:
   Since 2 is greater than 1, -2 is greater than -1.

   Is Paul's thinking correct? Justify your answer using a model.

Directions: Compare the numbers below using $<$, $>$, or $=$.

3. 19 ___<___ 20 4. -19 ___>___ -20	5. 50 ______ 45 6. -50 ______ -45	7. 8 ______ 8.1 8. -8 ______ -8.1
9. 1.45 ___<___ 1.5 10. -1.45 ___>___ -1.5	11. 1.2 ______ 1.02 12. -1.2 ______ -1.02	13. $\frac{7}{12}$ ______ $\frac{5}{12}$ 14. $-\frac{7}{12}$ ______ $-\frac{5}{12}$
15. -15 ______ -8	16. -200 ______ -225	17. -8.14 ______ -8.1
18. 8 ______ -2	19. $-1\frac{1}{4}$ ___=___ $-\frac{5}{4}$	20. $-\frac{3}{8}$ ______ $-\frac{5}{8}$
21. 1.5 ______ $-\frac{3}{2}$	22. -1 ______ $-\frac{3}{4}$	23. $-2\frac{1}{2}$ ______ -2.4
24. $-\frac{1}{4}$ ___>___ $-\frac{1}{2}$	25. 0 ______ $-\frac{1}{4}$	26. $-\frac{2}{3}$ ______ $-\frac{3}{4}$
27. -0.35 ______ $-\frac{1}{3}$	28. -0.8 ______ $-\frac{3}{4}$	29. -0.48 ___>___ $-\frac{1}{2}$

30. Write three numbers that are less than -1.

31. A is located to the right of B on the number line. Select all statements that you know are true.

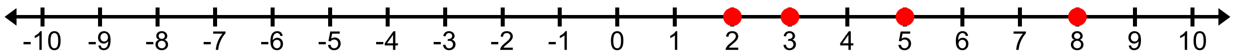
- ☐ $A > B$
- ☐ $|A| < |B|$
- ☐ $-A < -B$

32. If $a < b$ and $|a| = |b|$, what is true of a and b?

3.2c Class Activity: Ordering Integers

Activity 1:

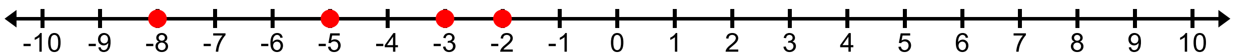
a. Graph the numbers $8, 2, 5, 3$ on the number line shown.



b. Put the numbers in order from least to greatest.

$2, 3, 5, 8$

c. Graph the numbers $-8, -2, -5, -3$ on the number line shown.



d. Put the numbers in order from least to greatest.
$-8, -5, -3, -2$

e. What do you notice? How does a number line help you to order numbers?
Have students share out ideas. A number line is a great tool to help order numbers. By plotting the points, students can see the location of the points relative to one another. The symmetry of the number line is valuable in understanding how to order negative numbers.

Directions: Order the numbers. Make sure students are paying attention to the directions in each problem. Are they being asked to order the numbers from least to greatest or greatest to least?

a. Put the numbers $1, 4, 7, -1, -4, -7$ in order from least to greatest.
$-7, -4, -1, 1, 4, 7$

b. Put the numbers $45, -45, 40, -40, 50, -50$ in order from greatest to least.
$50, 45, 40, -40, -45, -50$

c. Put the numbers $-11, -15, -2, -7$ in order from greatest to least.
$-2, -7, -11, -15$

d. Put the numbers $-1{,}000, -1{,}050, -1{,}500, -1{,}005$ in order from greatest to least.
$-1{,}000, -1{,}005, -1{,}050, -1{,}500$

Activity 2: The numbers below are listed in order from least to greatest. Give *two* possible *integer* values for the ? in each problem. Use a number line to help you.

a. $-10, ?, -4, -1$
Answers will vary, possible answers include -8 and -6.

b. $-7, -3, -1, ?$
Answers will vary, possible answers include 0 and 1.

c. $?, 0, 5, 8$
Answers will vary, possible answers include -1 and -6.

d. $-5, ?, 0, 2$
Answers will vary, possible answers include -4 and -3.

e. $?, -2, 5, 2$
Answers will vary, possible answers include -3 and -4.

Activity 3:

a. If $a < b < c$ and $|a| < |b| < |c|$, give some possible values for $a, b,$ and c.
Answers will vary, possible answer $a = 2, b = 5, c = 7$.

b. If $a < b < c$ and $|a| > |b| > |c|$, give some possible values for $a, b,$ and c.
Answers will vary, possible answer $a = -8, b = -6, c = -4$.

c. If $a, b,$ and c are to the left of zero and $|a| < |b| < |c|$, order $a, b,$ and c from least to greatest.
c, b, a

Activity 4: The following questions will help to prepare you for the next lesson.

a. Is $\frac{5}{12}$ greater than or less than $\frac{1}{2}$? Explain.

Less than. $\frac{6}{12}$ is equal to $\frac{1}{2}$ so $\frac{5}{12}$ is less than $\frac{1}{2}$

b. Is $\frac{4}{9}$ greater than or less than $\frac{1}{2}$? Explain.

Less than. $\frac{4.5}{9}$ is equal to $\frac{1}{2}$ so $\frac{4}{9}$ is less than $\frac{1}{2}$

c. Is $\frac{8}{23}$ greater than or less than $\frac{1}{3}$? Explain.

Greater than. $\frac{8}{24}$ is equal to $\frac{1}{3}$ so $\frac{8}{23}$ is greater than $\frac{1}{3}$. Remind students that $\frac{1}{23}$ is greater than $\frac{1}{24}$.

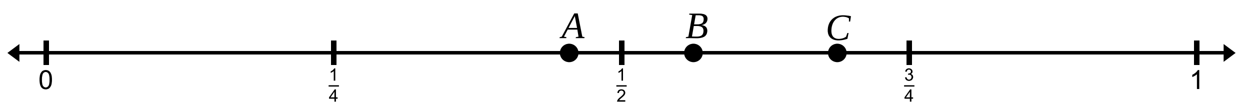
d. Replace the ? in the fraction $\frac{?}{16}$ with a number that would make the fraction greater than $\frac{1}{2}$ but less than $\frac{3}{4}$.

Since $\frac{8}{16} = \frac{1}{2}$ and $\frac{12}{16} = \frac{3}{4}$, anything greater than 8 and less than 12.

e. Replace the ? in the fraction $\frac{5}{?}$ with a number that would make the fraction greater than $\frac{1}{4}$ but less than $\frac{1}{2}$.

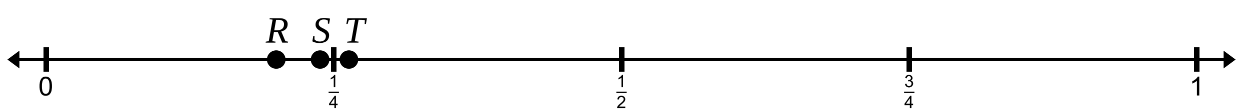
Since $\frac{5}{20} = \frac{1}{4}$ and $\frac{5}{10} = \frac{1}{2}$, anything greater than 10 and less than 20.

f. Which point best shows the location of $\frac{9}{16}$ on the number line below? Explain.



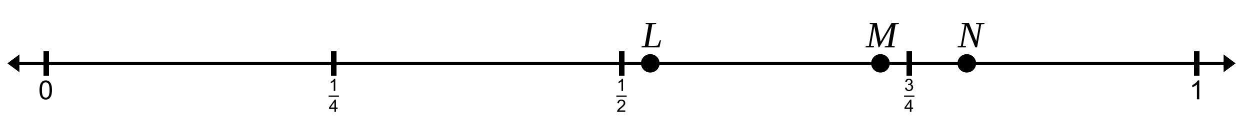
Point *B*. Since $\frac{8}{16}$ is equal to $\frac{1}{2}$, we know that the point is greater than $\frac{1}{2}$ so the answer is either *B* or *C*. We also know that $\frac{3}{4}$ is equal to $\frac{12}{16}$. If we look at the interval between $\frac{1}{2}$ or $\frac{8}{16}$ and $\frac{3}{4}$ or $\frac{12}{16}$, we see that *B* best shows the location of $\frac{9}{16}$.

g. Which point best shows the location of $\frac{5}{19}$? Explain.



$\frac{5}{19}$ is very close to $\frac{5}{20}$ which is equivalent to $\frac{1}{4}$. Since $\frac{5}{19}$ is greater than $\frac{5}{20}$, *T* best shows the location of $\frac{5}{19}$.

h. Which point best shows the location of $\frac{29}{40}$?



$\frac{29}{40}$ is very close to $\frac{30}{40}$ which is equivalent to $\frac{3}{4}$. Since $\frac{29}{40}$ is slightly smaller than $\frac{30}{40}$, *M* best shows the location of $\frac{29}{40}$.

Spiral Review

1. Complete the table below.

Fraction	Decimal	Percent
		50%
	0.25	
$\frac{3}{4}$		
		10%
$\frac{4}{5}$		
	1.2	

2. Simplify.

a. $5 \div \frac{1}{3}$	b. $\frac{1}{2} \div 3$	c. $\frac{2}{3} \div \frac{1}{6}$
d. $2\frac{1}{2} \div \frac{1}{4}$	e. $\frac{1}{16} \div \frac{1}{8}$	f. $\frac{3}{8} \div \frac{5}{8}$

3. Griffin rolled a six-sided die several times and recorded the number he rolled each time. He organized his results in the line plot below.

	X			X	
	X	X		X	X
X	X	X	X	X	X
X	X	X	X	X	X
X	X	X	X	X	X
1	2	3	4	5	6

Number on Dice

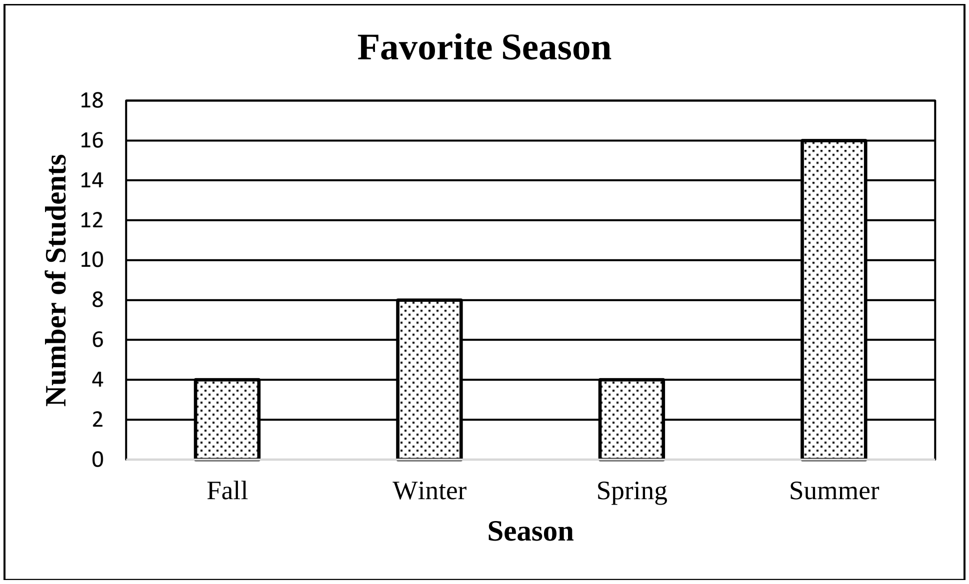
a. How many times did Griffin roll the die?

b. Which number did Griffin roll the most often?

c. What percent of the rolls did the die land on an even number?

d. What percent of the rolls did the die land on a number greater than 4?

4. Hannah surveyed the students in her class and asked what their favorite season is. She created a bar graph to display the results. Use the bar graph to answer the questions.



a. How many students were surveyed?

b. What season was chosen the most?

c. What season was chosen the least?

d. How many more students chose summer than spring?

e. What percentage of students chose each season?

Fall: ___________ Winter: ___________ Spring: ___________ Summer: ___________

3.2c Homework: Ordering Integers

1. Select all lists that show the numbers ordered from least to greatest.

   ☐ $-1, -2, -3, -4, -5$ No, this is a common error.

   ☐ $-5, -4, -3, -2, -1$ Yes

   ☐ $-5, -10, 0, 5, 10$

   ☐ $-20, -10, 0, 10, 20$

Read each problem carefully. Are you being asked to order from least to greatest or greatest to least?

2. Order the numbers $5, 9, 1, -5, -9, -1$ from least to greatest.

3. Order the numbers $175, -175, 150, -150, 190, -190$ from greatest to least.

4. Order the numbers $-15, -6, 0, -7, 3$ from greatest to least.

5. Order the numbers $-54, 2, -72, -10, 10$ from least to greatest.
   $-72, -54, -10, 2, 10$

Directions: The numbers below are listed in order from least to greatest. Give *two* possible *integer* values for the ? in each problem. Use a number line to help you.

6. $?, -10, 3, 8$ answers will vary, possible answers $-11, -12$

7. $-3, ?, 0, 5$

8. $-2, -1, 0, ?$

Directions: The following questions will help to prepare you for the next lesson.

9. Is $\frac{11}{20}$ greater than or less than $\frac{1}{2}$? Explain.

   Since $\frac{10}{20} = \frac{1}{2}$, $\frac{11}{20}$ is greater than $\frac{1}{2}$.

10. Is $\frac{4}{15}$ greater than or less than $\frac{1}{4}$? Explain.

Since $\frac{4}{16} = \frac{1}{4}$, $\frac{4}{15}$ is greater than $\frac{1}{4}$. Fifteenths are a bigger size piece that sixteenths.

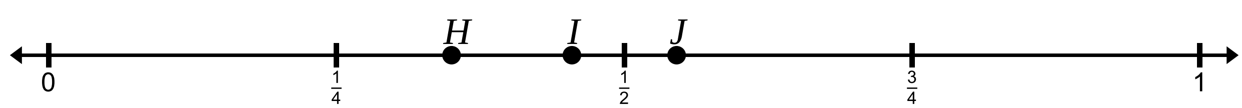
11. Is $\frac{4}{15}$ greater than or less than $\frac{1}{3}$? Explain.

12. Replace the ? in the fraction $\frac{?}{18}$ with a number that would make the fraction less than $\frac{1}{3}$.

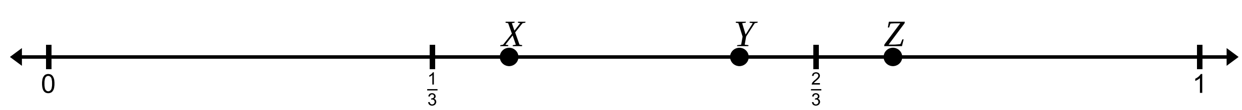
Since $\frac{6}{18} = \frac{1}{3}$ any number less than 6.

13. Replace the ? in the fraction $\frac{?}{12}$ with a number that would make the fraction greater than $\frac{2}{3}$ but less than 1.

14. Which point best shows the location of $\frac{5}{11}$? Explain.



15. Which point best shows the location of $\frac{11}{15}$? Explain.



3.2d Class Activity: Ordering Rational Numbers

Activity 1:

a. Put the following numbers in order from least to greatest. It is suggested that you put these numbers on pieces of paper or post-it notes that students can physically sort and move around. If you have a whiteboard, display the pieces of paper using magnets. You can also use technology to create cards that can be moved and sorted on a smartboard.

$0.45, \frac{7}{8}, \frac{5}{9}, \frac{1}{10}, 0.9, \frac{5}{8}, \frac{2}{5}, 0.09, \frac{1}{5}, 0.75$

$0.09, \frac{1}{10}, \frac{1}{5}, \frac{2}{5}, 0.45, \frac{5}{9}, \frac{5}{8}, 0.75, \frac{7}{8}, 0.9$

A problem that requires students to order several numbers can be overwhelming. Start by asking students what they notice about the set of numbers. Possible responses: Some are fractions and some are decimals. All the numbers are between 0 and 1. Ask if they notice any that are obviously bigger (or smaller) than the others. Hopefully, students will start to use number sense and estimation strategies, noticing that some numbers are close to 0 ($\frac{1}{10}$, 0.09, etc.) while others are close to 1 (0.9, $\frac{7}{8}$, etc.) and some are in between or closer to $\frac{1}{2}$ (0.45, $\frac{5}{9}$, etc.). We can use these ideas to organize the numbers.

Close to 0	**Close to $\frac{1}{2}$**		**Close to 1**
$\frac{1}{10}, 0.09, \frac{1}{5}$	$0.45, \frac{5}{9}, \frac{5}{8}, \frac{2}{5}$	0.75	$\frac{7}{8}, 0.9$

This type of graphic organizer simplifies the original problem into three smaller problems. You may consider passing out the cards to different students and ask them to come up and put the card into the appropriate category, explaining the reasoning they used for the placement of the card. If students are struggling to fit the numbers into these categories, money can be a great analogy to use with the decimals. For example, if you have \$0.09, do you have close to zero dollars, half a dollar, or one dollar? What about \$0.45? What about \$0.90? Moving on to $\frac{7}{8}$, students may consider that 1 is equal to $\frac{8}{8}$ and $\frac{1}{2}$ is equal to $\frac{4}{8}$ so $\frac{7}{8}$ is closer to 1. In examining $\frac{5}{9}$, students might think about the fact that half of 9 is 4.5 (or that half of 10 is 5) so $\frac{5}{9}$ is close to $\frac{1}{2}$. Students can use similar reasoning to classify the remaining numbers.

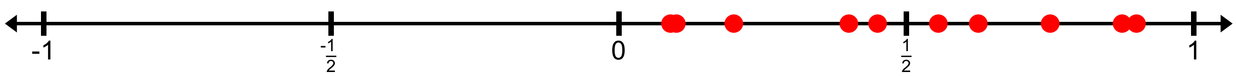
Once organized in this way, students can start to consider each set of numbers. Students will likely utilize their work with ratios and their knowledge of fraction/decimal/percent equivalence. Let's start with the numbers close to 0. Some students might immediately recognize that $\frac{1}{10}$ is half of $\frac{1}{5}$. Where does 0.09 fit in? From their work in chapters 1 and 2, students can change 0.09 and $\frac{1}{10}$ to percents, 9% and 10% respectively. Students have now ordered the first set of numbers: 0.09, $\frac{1}{10}$, $\frac{1}{5}$. Now, let's consider the numbers that are closest to $\frac{1}{2}$. We may want to subdivide the numbers again as Numbers that Are Less than $\frac{1}{2}$ and Numbers that Are Greater Than $\frac{1}{2}$. 0.45 and $\frac{2}{5}$ are smaller than $\frac{1}{2}$ while $\frac{5}{9}$ and $\frac{5}{8}$ are both greater than $\frac{1}{2}$. There are many strategies students can use to compare 0.45 and $\frac{2}{5}$. They can change 0.45 into a fraction $\frac{45}{100}$ and then find a common denominator for $\frac{2}{5}$ and $\frac{45}{100}$ by iterating the ratios up and down. Students may use a common denominator of 20 ($\frac{8}{20}$ and $\frac{9}{20}$). Another common denominator is 100. $\frac{2}{5}$ would become $\frac{40}{100}$ or 40% compared to 45%. They can now see that $\frac{2}{5}$ is smaller than 0.45. Other students may recognize that $\frac{1}{5}$ is equal to 0.2 so $\frac{2}{5}$ is equal to 0.4 and then compare 0.45 to 0.4. When comparing $\frac{5}{9}$ and $\frac{5}{8}$, ask students whether they would rather have 5 pieces of a pizza cut into 9 equal pieces or 5 pieces of pizza cut into 8 equal pieces. They should see that $\frac{5}{8}$ is greater than $\frac{5}{9}$. Students can use similar strategies to classify the numbers close to 1.

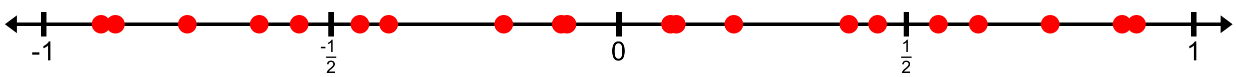
b. In this problem, you will order the *opposites* of the numbers from the previous problem, also from least to greatest.

$-0.45, -\frac{7}{8}, -\frac{5}{9}, -\frac{1}{10}, -0.9, -\frac{5}{8}, -\frac{2}{5}, -0.09, -\frac{1}{5}, -0.75$

$-\frac{9}{10}, -\frac{7}{8}, -0.75, -\frac{5}{8}, -\frac{5}{9}, -0.45, -\frac{2}{5}, -\frac{1}{5}, -\frac{1}{10}, -0.09$

Students will likely use ideas about symmetry and what they learned when comparing negative numbers in the previous lesson to determine the approximate location of these numbers on the number line. There will likely be students who immediately recognize that the order of the negative numbers will just be the reverse of their opposites from part a. To reinforce this idea, have the students create a human number line. Start with students holding up cards to represent $0, \frac{1}{2}$, and 1. Give the students the positive numbers from part a. and have them go and stand on the number line, showing the approximate location of the number they represent. Then, have two students come up to represent $-\frac{1}{2}$ and -1. Give another set of students the opposites given in this problem and have them determine where they should be standing on the number line using ideas about symmetry and reflections. If you do not have enough students to create the number line, you can also create the number line on the whiteboard or using technology (Smartboard, geometry software, etc.).



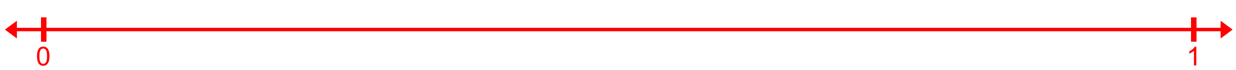


1. Put the following numbers in order from least to greatest.

   $\frac{1}{3}, 0.25, \frac{5}{18}, \frac{7}{10}, \frac{3}{16}, \frac{5}{8}, 0.75$

Ordered from least to greatest: $\frac{3}{16}, 0.25, \frac{5}{18}, \frac{1}{3}, \frac{5}{8}, \frac{7}{10}, 0.75$

Students may use strategies like those discussed in Activity 1, part a. to start to categorize the numbers (categories such as close to 0, close to $\frac{1}{2}$, close to 1, greater than $\frac{1}{2}$, less than $\frac{1}{2}$, etc.). It might also help students to start to put the numbers on a number line to show their approximate location. Again, we see that all the numbers are between 0 and 1 so this is a good place to start:



Students can start by showing the location of some of the easier number to plot (i.e. $\frac{1}{3}$, 0.25, and 0.75). After this, tackle some of the more challenging fractions. We know that $\frac{5}{18}$ is less than $\frac{1}{3}$. The next question to ask is whether $\frac{5}{18}$ is also less than 0.25. There are a few ways to think about this: 1) Half of 18 is 9 and half of 9 is 4.5 so one-fourth of 18 is 4.5 making $\frac{5}{18}$ greater than 0.25 or 2) $\frac{5}{20}$ is equal to $\frac{1}{4}$ so if you have $\frac{5}{18}$ your pieces are larger making $\frac{5}{18}$ greater than $\frac{1}{4}$. $\frac{3}{16}$ is less than 0.25 or $\frac{1}{4}$. It is not necessary to show the exact location of these numbers, just to make sure that the numbers are in the correct location relative to the other numbers in the list. When considering $\frac{5}{8}$, we know that it is greater than $\frac{1}{2}$ but smaller than 0.75 (it would take $\frac{6}{8}$ to make $\frac{3}{4}$ or 0.75).

2. Order the numbers from least to greatest.

$$-\frac{1}{3}, -0.25, -\frac{5}{18}, -0.4, -\frac{7}{9}, -\frac{3}{16}, -\frac{5}{8}, -0.75$$

See suggested strategies in Activity 1, part b.
Ordered from least to greatest: $-0.75, -\frac{7}{10}, -\frac{5}{8}, -\frac{1}{3}, -\frac{5}{18}, -0.25, -\frac{3}{16}$

3. Put the following numbers in order from greatest to least. Attend to precision – greatest to least, not least to greatest.
$7.05, 7.5, 7\frac{5}{24}, \frac{39}{5}, 7\frac{11}{20}, 7.25, 7\frac{1}{6}, 7\frac{9}{20}, 7.802$

Ordered from greatest to least: $7.802, \frac{39}{5}, 7\frac{11}{20}, 7.5, 7\frac{9}{20}, 7.25, 7\frac{5}{24}, 7\frac{1}{6}, 7.05$

At first glance, we see that these numbers are between 7 and 8 so students may choose to create a graphic organizer such as the one shown below. Some students may notice that since the whole number on all the numbers is 7, they can just ignore the whole number and only consider the fractions (you can think of the numbers in this problem as a translation 7 units to the right of a set of numbers between 0 and 1). For example, 7.05 is a translation 7 units to the right of 0.05, 7.5 is a translation 7 units to the right of 0.5, $7\frac{5}{24}$ is a translation 7 units to the right of $\frac{5}{24}$, etc. Students may need help with $\frac{39}{5}$ but since $\frac{40}{5}$ is equal to 8 and $\frac{35}{5}$ is equal to 7, we know that $\frac{39}{5}$ is between 7 and 8 and closer to 8 or they can change it to a mixed number $7\frac{4}{5}$.

Close to 7	**Close to $7\frac{1}{2}$**	**Close to 8**
$7.05, 7\frac{5}{24}, 7\frac{1}{6}$	$7.5,\ 7\frac{11}{20}, 7\frac{9}{20}$	$\frac{39}{5}, 7.802$
7.25		

From here, students can use several of the strategies mentioned in the previous problems to order the numbers.

4. Put the following numbers in order from greatest to least. Attend to precision – greatest to least, not least to greatest.
$-7.05, -7.5, -7\frac{5}{24}, -\frac{39}{5}, -7\frac{11}{20}, -7.25, -7\frac{1}{6}, -7\frac{9}{20}, -7.802$

A number line visual will be important here. Now that we have switched the problem to be greatest to least, students need to recognize that the negative numbers closest to 0 are the larger numbers.

Ordered from greatest to least: $-7.05, -7\frac{1}{6}, -7\frac{5}{24}, -7.25, -7\frac{9}{20}, -7.5, -7\frac{11}{20}, -\frac{39}{5}, -7.802,$

Spiral Review

1. Compare using $<$, $>$, or $=$.

a. $\lvert -2 \rvert$ ________ -2	b. $\lvert -2 \rvert$ ________ 2	c. $\lvert 6 \rvert$ ________ $\lvert -6 \rvert$
d. $\lvert 6.1 \rvert$ ________ 6.1	e. $\left\lvert \frac{3}{5} \right\rvert$ ________ $\left\lvert \frac{3}{4} \right\rvert$	f. $\lvert 0 \rvert$ ________ $\lvert -1 \rvert$
g. $-\lvert -2 \rvert$ ________ -2	h. $-\lvert -2 \rvert$ ________ 2	i. $-\lvert 2 \rvert$ ________ -2
j. $-(4)$ ________ $\lvert -4 \rvert$	k. $-(-4)$ ________ $\lvert -4 \rvert$	

2. Sammy's class collected canned food for a local food pantry for 10 days. The stem-and-leaf plot shows the number of cans collected over the 10-day period. Use the stem-and-leaf plot to determine the *total* number of cans Sammy's class collected over the 10-day period.

Stem	Leaf
0	7 8 8
1	0 0 2 5 8
2	1 5

3. Tell whether the following numbers are divisible by 2.

a. 68	b. 130	c. 121
d. 50	e. 11	f. 1,001

4. Tell whether the following numbers are divisible by 3.

a. 111	b. 304	c. 120
d. 123	e. 620	f. 1,000

3.2d Homework: Ordering Rational Numbers

1. Put the following numbers in order from least to greatest. Explain the process you used to order the numbers.

   $\frac{11}{15}, \frac{7}{8}, \frac{2}{3}, \frac{1}{5}, \frac{1}{7}, \frac{5}{6}$

See class notes for strategies students may use to solve these problems.

Ordered least to greatest: $\frac{1}{7}, \frac{1}{5}, \frac{2}{3}, \frac{11}{15}, \frac{5}{6}, \frac{7}{8}$

2. Put the following numbers in order from least to greatest. Explain the process you used to order the numbers.

   $-\frac{11}{15}, -\frac{7}{8}, -\frac{2}{3}, -\frac{1}{5}, -\frac{1}{7}, -\frac{5}{6}$

Ordered least to greatest: $-\frac{7}{8}, -\frac{5}{6}, -\frac{11}{15}, -\frac{2}{3}, -\frac{1}{5}, -\frac{1}{7}$

3. Put the following numbers in order from greatest to least. Explain the process you used to order the numbers.

   $1.5, 1\frac{3}{4}, 1\frac{3}{8}, 1.74, 1\frac{9}{10}, 1.09$

4. Put the following numbers in order from greatest to least. Explain the process you used to order the numbers.

   $-1.5, -1\frac{3}{4}, -1\frac{3}{8}, -1.74, -1\frac{9}{10}, -1.09$

5. Put the following numbers in order from greatest to least. Explain the process you used to order the numbers.

   $-4.5, -4\frac{3}{8}, -4\frac{13}{25}, -4\frac{13}{100}, -4.125$

6. Put the following numbers in order from least to greatest. Explain the process you used to order the numbers.

   $-\frac{5}{4}, -1.087, -1.2, -2, -1\frac{3}{5}$

7. Give a number between -1 and -2.

8. Give a number between -0.2 and $-\frac{1}{4}$.
   Answers will vary, sample answer -0.23

9. Give a number between -6.1 and -6.2.

10. Give a number between $-1\frac{3}{4}$ and -1.7.

3.2e Self-Assessment: Section 3.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Mastery 3**	**Substantial Mastery 4**
1. Understand what the absolute value of a rational number represents. Simplify expressions containing absolute value.				
2. Find the distance between two rational numbers on the same horizontal or vertical line.				
3. Compare rational numbers.				
4. Order rational numbers.				

Sample Problems for Section 3.2

Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. Describe absolute value in your own words. Use examples to support your explanation. [1]

2. Simplify. [1]

a. $\lvert 10 \rvert$	b. $\lvert -10 \rvert$	c. $\lvert 3.2 \rvert$
d. $-\lvert 1 \rvert$	e. $-\lvert -1 \rvert$	f. $-(-1)$

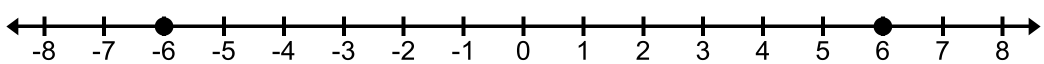
3. What number or numbers, if any, make the statement true? [1]
   $\lvert x \rvert = 12$

4. What number or numbers, if any, make the statement true? [1]
   $\lvert x \rvert = \frac{1}{3}$

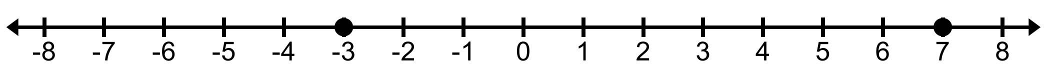
5. Write your own absolute value statement for which there is not a number that makes the statement true. [1]

6. Find the distance between the two points on the number line. [2]

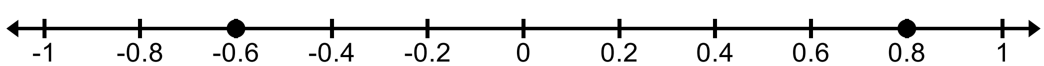
   a.



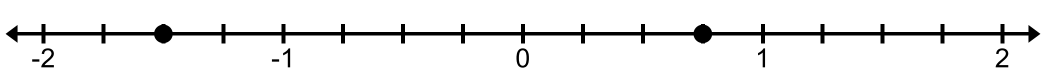
   b.



   c.



   d.



7. List three negative integers greater than -5. [3]

8. Compare the numbers below using $<$, $>$, or $=$. [3]

a. 0 _______ -3	b. -10 _______ -11	c. 5_______ -9
d. -6_______ -6.1	e. -0.65 _____ -0.6	f. $-\frac{1}{5}$ _______ $-\frac{1}{8}$
g. $-\frac{3}{100}$ _______ $-\frac{3}{10}$	h. -8.99 _______ -9	i. $-\frac{3}{4}$ _______ -1
j. $-\frac{1}{2}$ _______ -0.45	k. $-\frac{11}{8}$ _____ -1.375	l. -5 _______ 1

9. Order the numbers from least to greatest. [4]
   a. $-2.25, -2, -2.3, -2.09, -1.99$

   b. $-5.8, -5.75, -6, -5$

   c. $-0.3, -\frac{1}{3}, -0.48, -\frac{7}{12}, -1.3$

10. Order the numbers from greatest to least. [4]
   a. $-2.45, -2.5, -2, -2\frac{1}{4}$

   b. $-4.1, -4, -4.01, -4.11$

Section 3.3: Negative Numbers in the Real World

Section Overview:
This section only includes two lessons. The first lesson is focused on the vocabulary associated with positive and negative quantities. Students are also exposed to different real-world contexts that necessitate the use of positive and negative numbers to represent quantities. They explain the meaning of zero in each situation and what it means in the context to be to the left of zero versus being to the right of zero. The second lesson is a collection of activities that synthesizes the concepts learned in the chapter and provides the opportunity for students to apply the skills they learned in sections one and two in real-world situations.

Concepts and Skills to Master:
By the end of this section, students should be able to:
1. Use and interpret academic vocabulary used to describe situations with positive and negative quantities.
2. Represent real-world situations using positive and negative numbers.
3. Apply the skills learned in Sections 1 and 2 of this chapter to solve real-world problems involving positive and negative numbers.

This section is organized differently than the other sections in this textbook. The first lesson in this section, 3.3a, is in the typical format of a Class Activity followed by Homework that mirrors the lesson. The second lesson, 3.3b, is a collection of activities that synthesize the concepts of the chapter. You can decide how you want to break out this lesson, choosing some lessons to work on in class and others to work on at home. These activities might be mini-lessons that you work on over the course of several days or you can choose to cover the lesson in 2 – 3 days.

There is not a self-assessment at the end of this section. Students should self-assess as they work through the two lessons. You may wish to save one of the activities in lesson 3.3b as an assessment to use at the end of the lesson.

3.3a Class Activity: Using Negative Numbers to Represent Real World Quantities

Activity 1:

a. Give some examples of situations in the real world that people use negative numbers.
Answers will vary, sample answers include temperature, elevation, debt, loss, price drop, account balance, golf, football, etc.

b. Complete the table below. It is important that students can read, interpret, and use the correct vocabulary associated with positive and negative quantities found in the real world. This vocabulary can be challenging for students.

Word or Phrase	Opposite of Word or Phrase
Above Sea Level	Below Sea Level
Gain	Lose/Loss
Deposit	Withdrawal
Earn Money	Spend Money
Credit	Debit
Rise	Fall
Ascend	Descend
Profit	Loss

c. Which column contains quantities represented by positive numbers? Column 1

d. Which column contains quantities represented by negative quantities? Column 2

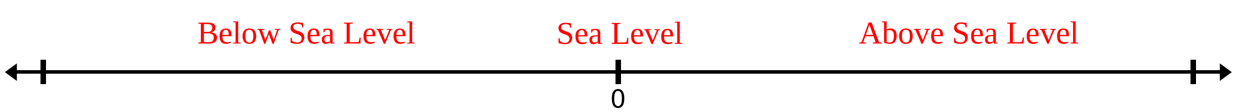
Activity 2:

For each of the following situations…

a. Write what zero represents.
b. Write what it means to be to the right of zero.
c. Write what it means to be to the left of zero.

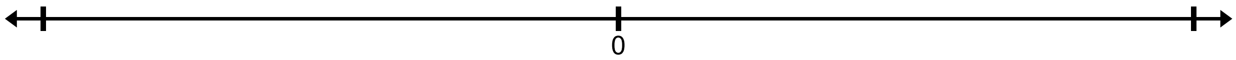
Be aware that many times we designate the "zero" in different real-world situations. This designation dictates what falls to the left of zero and what falls to the right of zero. For example, we may designate a passing score to test into an advanced math class as 90%. We may make 90% our zero. Scores below 90% are negative (did not make cut-off) scores above 90% are positive (made cut-off).

<u>Elevation:</u> When talking about elevation, zero is typically sea level. However, zero can also represent ground level or the surface of a body of water. For example, you can designate the surface of a lake as zero and anything below the surface of the lake is negative and anything above the surface of the lake is positive.



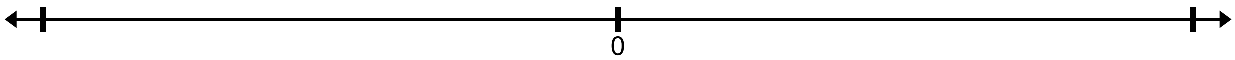
Temperature in Degrees Celsius

Below Freezing   Freezing Point of Water   Above Freezing



Football

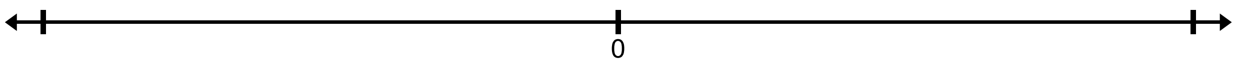
Loss of Yards   Line of Scrimmage   Gain of Yards



50-yard line could represent zero

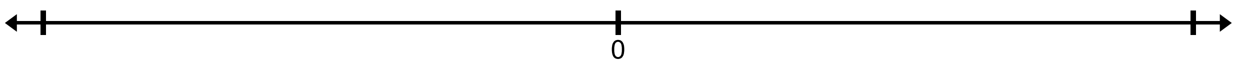
Money

Borrow Money   Even   Earn Money



Time

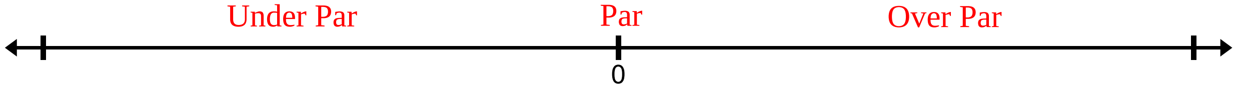
Past   Present   Future



Other possible answers for time include a countdown to an event, such as a liftoff (0), before liftoff, after liftoff; a big event such as a birth (0), before you were born, after you were born, the time since something started changing (0), etc.

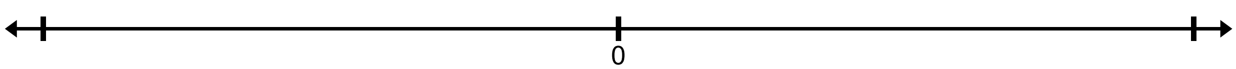
Golf

Under Par   Par   Over Par



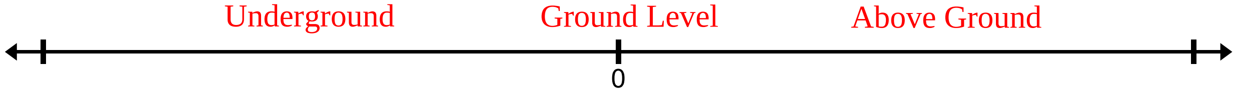
Business

Loss   Break Even   Profit



Buildings

Underground   Ground Level   Above Ground



Activity 3: Write a rational number to represent each of the following situations.

a. A debt of $10 ___−10_____

b. A gain of 15 yards ___+15 or 15_____

c. 20 feet below sea level __−20______

d. 2 points extra credit __+2 or 2______

e. A deposit of $200 __+200______

f. 4 degrees below freezing in degrees Celsius ___−4_____

g. A savings account earns $60.30 in interest __60.30______

h. Tyler earned $5 for cleaning his mom’s car ___5_____

i. A $20 credit on a gift card __20______

j. A stock price fell $0.33 ___−0.33_____

k. A loss of $150 ___−150_____

l. A fine of $50 __−50______

m. The line of scrimmage ___0_____

n. The freezing point of water in degrees Celsius ___0_____

o. The opposite of a debt of $25 ___25_____

p. The opposite of a loss of 7 yards __7______

q. The opposite of 5 degrees above zero ___−5_____

r. The opposite of a withdrawal of $100 ___100_____

s. The opposite of a rise of 20 feet ___−20_____

t. The opposite of 15 feet below sea level __15______

Activity 4: Write a situation that can be represented by each rational number. Answers will vary

a. -6

b. 11

c. -15.25

d. 150

e. -2

Students need to attend to precision when writing contexts that can be represented by the numbers shown. For example, a student might say that negative six represents a drop of six degrees. A common mistake is for a student to say that negative six represents a drop of negative six degrees which would not be correct.

Activity 5:

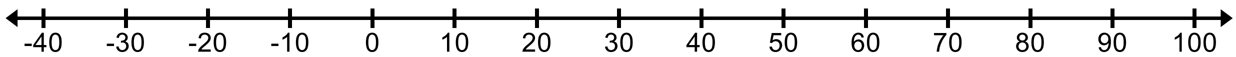
Write a story or draw a picture about a swimming pool where the surface of the swimming pool is zero. Include the location of at least four objects. Then, construct a number line showing the position of each object relative to the surface of the water. Stories and pictures will vary. A vertical number line makes sense in this situation as the objects are situated vertically in relationship to the surface of the water. Students need to attend to precision when writing their stories. For example, if they say that a swimmer dove down 10 feet below the surface of the water, this can be represented by the number -10 on the number line but in the story, the students would say, "A swimmer dove down 10 feet below the surface of the water." A common mistake is for students to say, "A swimmer dove down negative 10 feet below the surface of the water." Help students to distinguish between the value of a number relative to zero and the magnitude of the number. If a student says, there is an object 8 inches from the surface of the water, they should specify whether the object is *above* the surface of the water or *below* the surface of the water.

Spiral Review

1. Order the numbers $-0.2, -1.2, -0.02, -0.15$ from least to greatest.

2. Show the approximate location of each point on the number line.

   $D: -4$ $E: 32$ $F: 68$ $G: -17$



3. Compare using $<, >,$ or $=$.

a. -5 ______ $\|5\|$	b. 4 ______ $\|-7\|$	c. $\|-11\|$ ______ 11
d. $\|100\|$ ______ 100	e. $\|-4.2\|$ ______ $\|-5\|$	f. $\left\|\frac{3}{4}\right\|$ ______ $\left\|-\frac{4}{5}\right\|$
g. $-(4)$ ______ $-\|4\|$	h. $-(-(4))$ ______ $-\|4\|$	

4. Order the numbers from least to greatest.

a. $9.9, \|-10\|, -10.1, \|9.5\|$	b. $\frac{5}{4}, \|-1.5\|, \left\|\frac{1}{2}\right\|, -\frac{3}{4}$
c. $2, \|-3\|, -3, \|4\|$	d. $-1, \|-1.11\|, -1.11, \|-1.1\|$

3.3a Homework: Using Negative Numbers to Represent Real World Quantities

Directions: Write a rational number to represent each of the following situations.

1. 20 meters below sea level __-20______
2. 8 degrees above freezing in degrees Celsius ___8_____
3. A loss of seven yards __________
4. A $200 deposit into a savings account __________
5. A weight loss of 15 pounds __________
6. A stock price rose $0.25 __________
7. A profit of $2,400 __2,400______
8. The temperature dropped 12 degrees Fahrenheit __________
9. A withdrawal of $60 __________
10. Deduct 5 points for an assignment being late __________
11. Cheryl owes her dad $15 __________
12. A company reports a loss of $500,000 __________
13. Sea level ___0_____
14. The balloon ascended two meters per second __________
15. The opposite of a drop of 2 degrees ___2_____
16. The opposite of a profit of $1,500 __________
17. The opposite of a gain of 4 pounds __-4______
18. The opposite of a credit of $10 __________
19. Ground Level __________
20. The lowest known point on Earth is the Mariana Trench. The Mariana Trench is 10,911 m below sea level. Write an integer to represent the elevation of the Mariana Trench.
$-10{,}911$

Directions: Write a situation that can be represented by each rational number.

21. 2

22. 0

23. -10

24. $-15{,}000$

25. 3.25

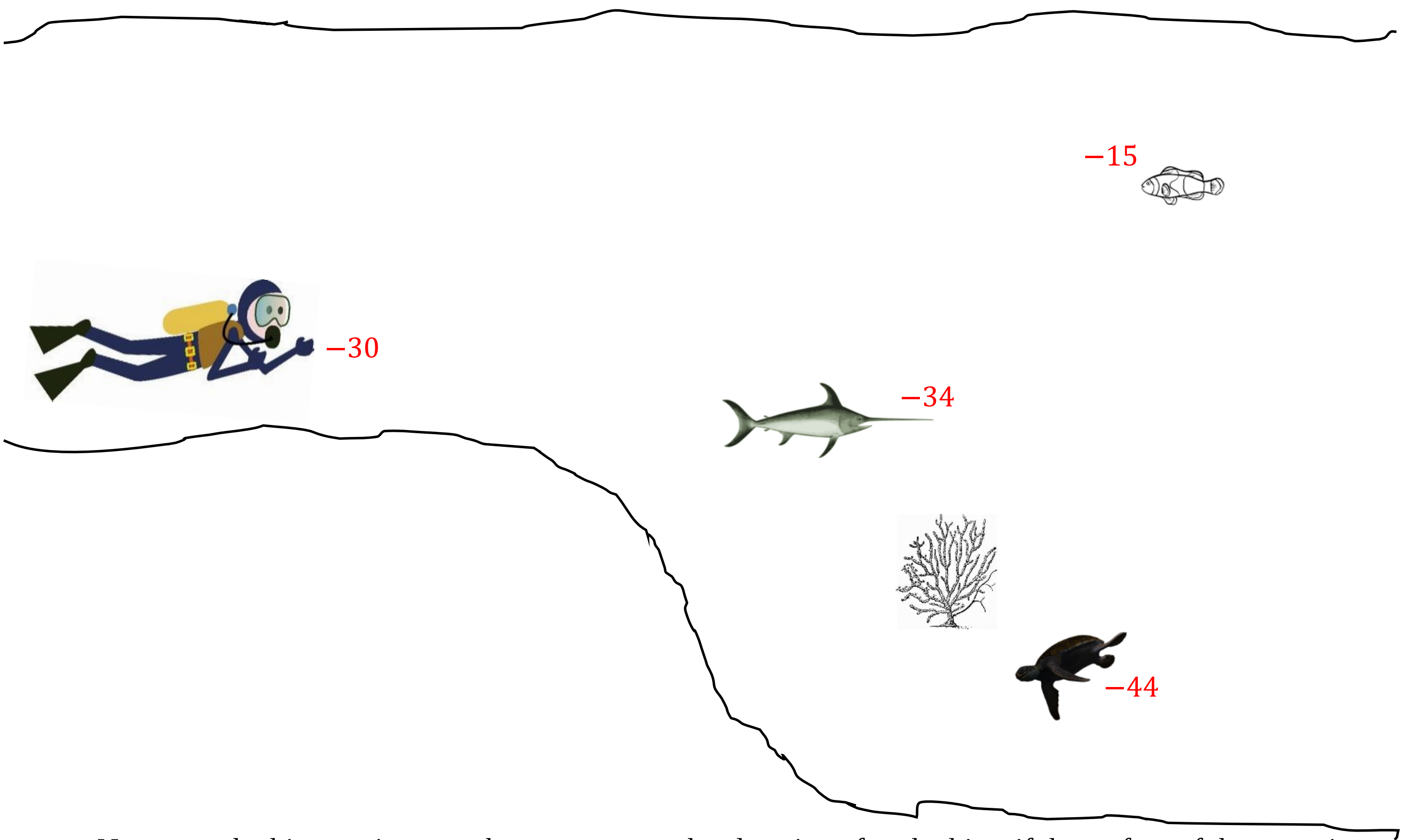
26. Write a story about your life where zero represents today. Include at least four events. Then, construct a number line showing the position of each event relative to today.

3.3b Class Activity: Applying What You've Learned

As you work through the activities in this lesson, reflect on how you use the following practice standards:

Activity 1: Use the following information about the picture below to answer the questions that follow:

- The bird is flying 15 feet above the surface of the water.
- The scuba diver is 30 feet below the surface of the water.
- The clown fish is 15 feet below the surface of the water.
- The swordfish is 34 feet below the surface of the water.
- The sea turtle is 44 feet below the surface of the water.



a. Next to each object, write a number to represent the elevation of each object if the surface of the water is at sea level. The students can create a vertical number line in the picture by shifting the objects horizontally until they are all on the same vertical line.

b. Which object is at the highest elevation? __bird ________The lowest elevation? __turtle________

c. Which object is closest to sea level? _bird and clown fish__ Farthest from sea level? __turtle______

d. Which object(s) are less than 32 feet from sea level? ___bird, clown fish, and diver___________

e. Which is farther from sea level, the bird or the swordfish? ___swordfish___________

f. Complete the following statements: The bird is ___15____ feet above sea level. The elevation of the bird can be represented by the number __15____.

g. Complete the following statements: The swordfish is __34_____ feet below sea level. The elevation of the swordfish can be represented by the number __-34____.

h. Which two objects are equidistant from the surface of the water? _____bird and clown fish________

i. Add your own object that is farther from sea level than any of the other objects shown. Add your own object that is closer to sea level than any of the other objects shown. Answers will vary, make sure that students understand that there are both positive and negative elevations that will work here

j. If the bird is directly above the fish, how far apart are the bird and fish? _____30 ft.______________

k. If the bird continues flying in a straight path, how far apart will the bird and scuba diver be when the bird is directly above the scuba diver? _____45 ft.______________

l. Write a comparison statement using words and symbols to compare the elevation of the sea turtle to the elevation of the scuba diver.
The turtle is 44 feet below the surface of the water while the scuba diver is 30 feet below the surface of the water. The turtle is at a lower elevation. $-44 < -30$

m. Write a comparison statement using words and symbols to compare the elevation of the swordfish to the elevation of the bird.
The swordfish is 34 feet below the surface of the water while the bird is 15 feet above the surface of the water. The bird is at a higher elevation. $15 > -34$

Activity 2: The table below shows the change in 5 different stock prices on a certain day.

Stock	Change in Stock Price (in dollars)
Stock A	+0.36
Stock B	−0.48
Stock C	+0.17
Stock D	−0.13
Stock E	−0.36

a. Which stock prices fell? __B, D, E__________ Which stock prices rose? ___ A, C ______________

b. Which stock price changed the most? _____B___________ By how much did it change? ___$0.48____
It changed by 0.48 – this is a question about the magnitude of the change. To account for the drop, we can say, "The price of Stock B *fell* by $0.48."

c. Which stock price changed the least? ________ D ________ By how much did it change? __$0.13____
Again, this question is about magnitude. Stock D changed by $0.13. It *rose* 0.13.

d. Which stock prices changed by more than 0.30? __A, B, E___________

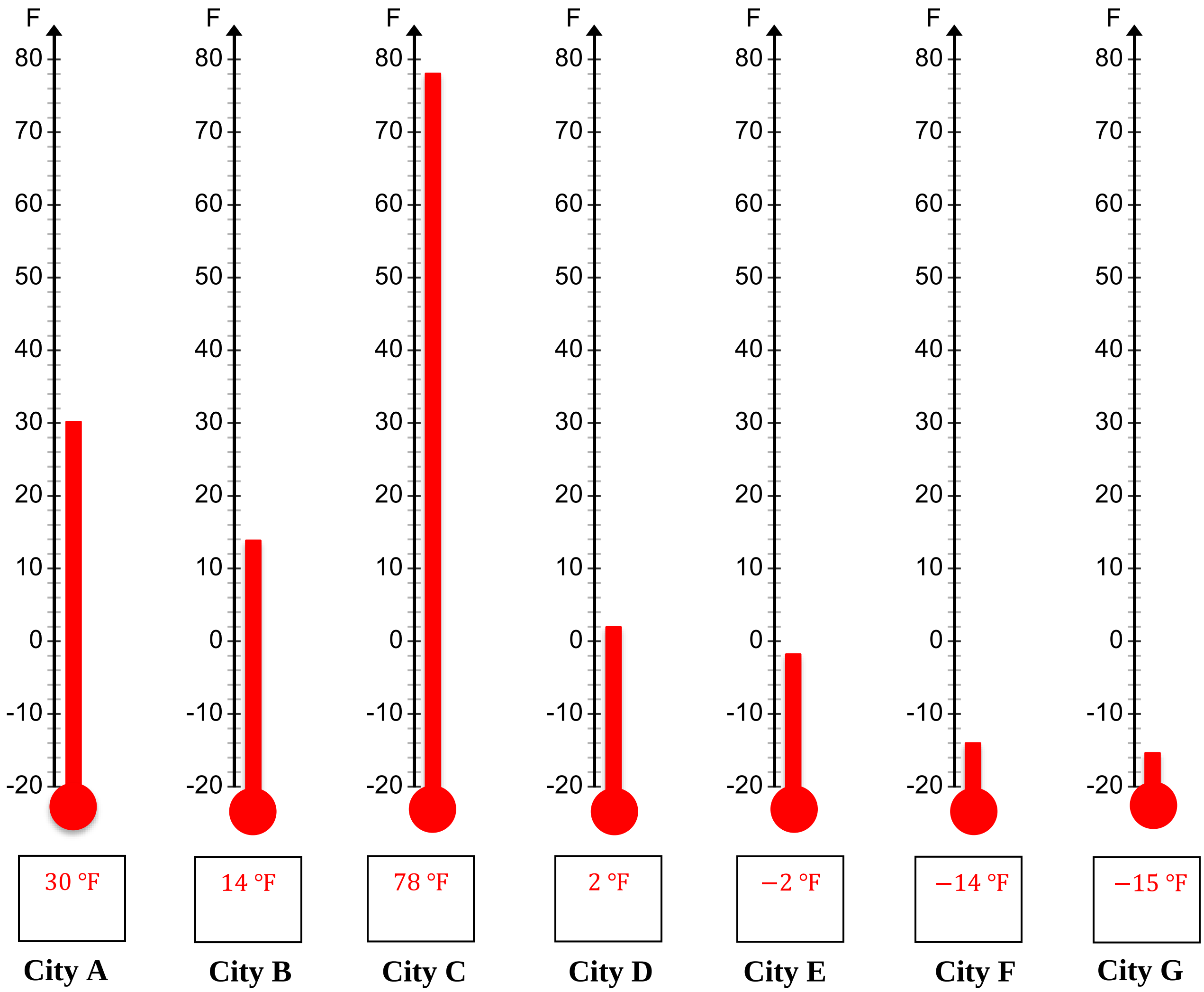
e. Complete the following statements: Stock A rose _____$0.36__________________. The change in the price of Stock A can be represented by the number ___0.36______.

f. Complete the following statements: Stock B fell ____$0.48___________________. The change in the price of Stock B can be represented by the number ___−0.48______.

g. Stock F, which is not listed in the table, fell more than $0.55. Write a possible number to represent the change in the price of Stock F. __−0.60________ answers will vary

h. Stock G, which is not listed in the table, changed by 0.07. Write two different numbers to represent the change in the stock price of Stock G. ___−0.07_________ or ____0.07________

Activity 3: The thermometers below show the temperatures of 7 different U.S. cities at noon on a certain day in February.



a. Write the temperature, to the nearest degree, shown on each thermometer. Write your answer in the box below the thermometer.

b. Order the temperatures from coldest to hottest.
$-15, -14, -2, 2, 14, 30, 78$

c. Give a temperature that is colder than the temperatures shown above. _sample answer −20°F__

d. Water freezes at 32° F. Put a star by the thermometers that show temperatures below freezing in degrees Fahrenheit. Cities A, B, D, E, F, and G

e. What is the range between the highest temperature and the lowest temperature? __93°F____

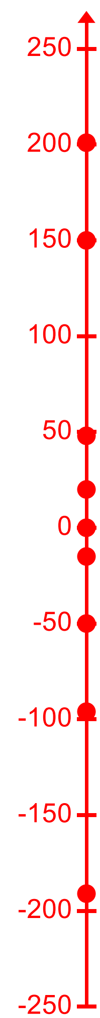
f. What is the difference between the temperature in City A and City F? __44°F_______

g. What is the difference between the temperature in City E and City F? __12°F_______

Activity 4: The table below is a table of distances from New York City to cities that lie directly north and directly south of New York City. The table starts with the city that is furthest north of NYC and ends with the city furthest to the south of NYC. The city locations are identified by their centers. Use this information to answer the questions that follow.

City	**Miles From New York City**
Great Falls	201
Albany	150
Peekskill	48
Yonkers	20
New York	0
Newark	15
Princeton	50
Philadelphia	96
Baltimore	191

a. Create a model to show the location of the cities relative to New York City.
See number line model to the right.



b. Which of these cities is farthest from New York City?
Great Falls

c. Which of these cities is closest to New York City?
Newark

d. Which of these cities are within 50 miles of New York City?
Peekskill, Yonkers, Newark, Princeton

e. What is the distance between Great Falls and Peekskill?
153 miles

f. What is the distance between Peekskill and Newark?
63 miles

g. What is the distance between Newark and Princeton?
35 miles

Activity 5: The table below shows the elevations of eight different cities in California.

Cities in California	**Elevation (in meters)**
Desert Shores	-61
Indio	-6
Imperial	-18
Salton Sea Beach	-67
El Centro	-12
Heber	-5
Thermal	-37
Coachella	-22

a. Put the cities in order from greatest elevation to least elevation in the table below. Start the table with the city with the greatest elevation.

Cities Ordered from Greatest Elevation to Least Elevation	
Cities in California	**Elevation (in meters)**
Heber	-5
Indio	-6
El Centro	-12
Imperial	-18
Coachella	-22
Thermal	-37
Desert Shores	-61
Salton Sea Beach	-67

b. Put the cities in order from the city that is farthest from sea level to the city that is closest to sea level. Start the table with the city that is farthest from sea level.

Cities Ordered from Farthest from Sea Level to Closest to Sea Level	
Cities in California	**Distance from Sea Level (in meters)**
Salton Sea Beach	67
Desert Shores	61
Thermal	37
Coachella	22
Imperial	18
El Centro	12
Indio	6
Heber	5

c. What is the difference in elevation between Heber and Indio? 1 meter

d. What is the difference in elevation between Heber and Salton Beach? 62 meters

Activity 6: The biggest temperature changes to occur in the same day occurred in Spearfish, South Dakota on January 22, 1943. The table below shows the temperature at different times during the day. Use this information to answer the questions below.

Time	**Temperature in Degrees Fahrenheit**
7:30 am	$-4°$
7:32 am	$45°$
9:00 am	$54°$
9:27 am	$-4°$

f. What was the change in temperature from 7:30 am to 7:32 am? $49°$

g. What was the change in temperature from 7:32 am to 9:00 am? $9°$

h. What was the change in temperature from 9:00 am to 9:27 am? $-58°$ or the temperature dropped $58°$

i. What was the warmest time of the day? 9:00 am

j. What was the coldest time of the day? 7:30 am and 9:27 am

The National Weather Service in Sioux Falls, S.D. describes the cause of the dramatic temperature changes:

"The wild temperature fluctuations were likely due to cold air and warm air sloshing back and forth along the plains at the base of the Black Hills. A similar effect would be to pour warm water into a shallow bowl of cold water. The water would slosh back and forth a few times before settling down. This is likely what happened with the warm and cold air along the Black Hills."

Source: weather.com

Activity 7: In February of 2011, Nowata, Oklahoma experienced a 110-degree rise in temperature over a 7-day period. On February 10, 2011, the low temperature in Nowata was $-31°$F, the coldest temperature ever recorded in Oklahoma. On February 17, 2011, the temperature at one point during the day was 110 degrees hotter than the temperature on February 10, 2011. What was the high temperature on February 17, 2011 in Nowata, Oklahoma?
Encourage students to sketch a model.
The high temperature on February 17, 2011 in Nowata was 79°F.

Source: weather.com

Activity 8:

a. Lance owes his dad money. His debt is greater than \$30. Give a possible number to represent Lance's debt. Answers will vary. Possible answer -45.

b. The temperature in Chicago is -4 degrees F. It is colder in Minneapolis. Give a possible temperature for Minneapolis.
Answers will vary. Possible answer -8 degrees F.

c. The temperature in Chicago is -4 degrees F. It is warmer in Milwaukee. Give a possible temperature for Milwaukee.
Answers will vary. Possible answer 2 degrees F.

d. While scuba diving, Cath descended more than 15 feet below sea level. Give a possible number to represent Cath's lowest elevation.
Answers will vary. Possible answer -20 feet.

e. One day in January, the temperature in Salt Lake City at 4:00 pm was 8 degrees Fahrenheit. Over the next 6 hours, the temperature fell more than 10 degrees. Give a possible temperature for Salt Lake City at 10:00 pm that night.
Answers will vary. Possible answer -3 degrees Fahrenheit

Activity 9: The table below represents the transactions that took place in Michael's bank account over a three-day period.

Day	**Transaction**	**Change to Account**
1	Purchased groceries for $109	-109
2	Wrote a check to pay off a $90 loan	-90
2	Made a $40 cash deposit	40
3	Withdrew $20 out of his account for lunch money	-20
3	Automatic deposit of paycheck totaling $150	150

a. Complete the table with the numbers that represent the change in the account balance for each transaction.

b. During which transactions did Michael's bank account change by more than $50?
Purchasing groceries, writing a check, automatic deposit

c. Which transaction caused the biggest change in Michael's bank account?
automatic deposit

d. Which transaction caused the smallest change in Michael's bank account?
Withdrawal of $20

e. After the automatic deposit on Day 3, the amount of money in Michael's bank account was positive. What does that tell you about the amount of money in Michael's account before he purchased groceries on Day 1? He started with more than $29 in his account.

Activity 10: Judy is a financial advisor who helps her clients get out of credit card debt. The following table shows the credit card debt carried by 5 of her clients:

Client	**Credit Card Debt**
Alex	$-8{,}500$
Barb	$-2{,}300$
Chad	$-1{,}800$
Derek	$-12{,}000$
Ellie	$-9{,}830$

a. Order the clients from the client with the least amount of credit card debt to the client with the most amount of credit card debt.
Chad, Barb, Alex, Ellie, Derek

b. Which of Judy's clients have a credit card debt greater than \$10,000? Derek

c. Which of Judy's clients have a credit card debt less than \$2,500? Barb and Chad

d. What is the difference between the credit card debt held by Derek and the credit card debt held by Alex?
\$3,500

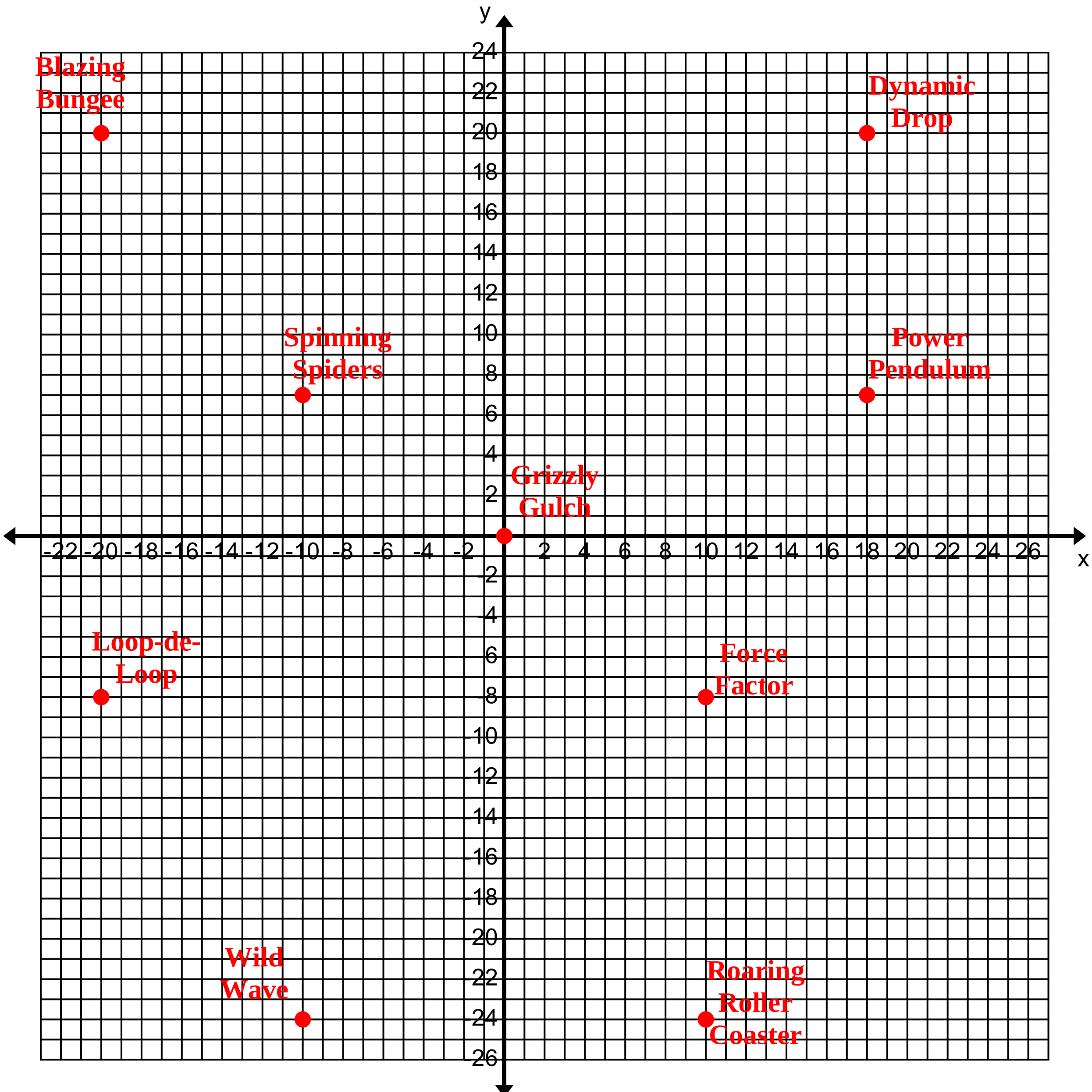
e. Ferdinand has more credit card debt that anyone listed in the table. Give a possible number to represent Ferdinand's credit card debt.
Answers will vary but must be more than 12,000 in debt. Sample answer $-15{,}000$.

f. Gwen has less credit card debt than anyone in the table. Give a possible number to represent Gwen's credit card debt.
Answers will vary but must be less than 1,800 in debt. Sample answer $-1{,}000$.

Activity 11: The ordered pairs show the location of the thrill rides at an amusement park.

a. Plot and label each ordered pair on the coordinate plane below.

Blazing Bungee $(-20, 20)$
Dynamic Drop $(18, 20)$
Force Factor $(10, -8)$
Grizzly Gulch $(0, 0)$
Loop-de-Loop $(-20, -8)$
Power Pendulum $(18, 7)$
Roaring Roller Coaster $(10, -24)$
Spinning Spiders $(-10, 7)$
Wild Wave $(-10, -24)$



If each unit on the grid represents 5 feet, find the distances between the following rides:

b. Blazing Bungee and Dynamic Drop 190 feet

c. Dynamic Drop and Power Pendulum 65 feet

d. Spinning Spiders and Wild Wave 155 feet

e. Loop-de-Loop and Force Factor 150 feet

f. Force Factor and Roaring Roller Coaster 80 feet

g. Wild Wave and Roaring Roller Coaster 100 feet

h. Blazing Bungee and Loop-de-Loop 140 feet

i. Spinning Spiders and Power Pendulum 140 feet

As an extension, students can convert the feet to yards.