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Chapter 5: Geometry (3 Weeks)

Utah Core Standard(s)

- Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. (6.G.1)
- Find the volume of a right rectangular prism with appropriate unit fraction edge lengths by packing it with cubes of the appropriate unit fraction edge lengths *(*for example, $3½ \times 2 \times 6$*)*, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (Note: Model the packing using drawings and diagrams.) (6.G.2)
- Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. (Standard 6.G.3)
- Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. (6.G.4)

Academic Vocabulary: area, square unit, polygon, quadrilateral, parallelogram, rectangle, square, kite, Rhombus, Trapezoid, length, width, triangle, right triangle, acute triangle, obtuse triangle, scalene triangle, isosceles triangle, equilateral triangle, base, height, irregular figure, horizontal, vertical, coordinate plane, ordered pair, vertices, volume, right rectangular prism, unit cube, cubic inch/cubic centimeter, fractional edge length, net, prism, pyramid, face, lateral face, edge, vertex, surface area

Chapter Overview:

In this chapter students extend previous work done with area and volume. Using shape composition and decomposition skills learned in previous grades, students develop and use formulas for the area of parallelograms and triangles. As they work with these polygons they investigate how they can choose any side to be the base of the parallelogram or triangle and that this choice determines the height. In turn, students learn that they can find the area of special quadrilaterals and other polygons by subdividing them into rectangles and triangles.

Next the focus is turned to developing their understanding of properties of two-dimensional shapes within the coordinate system. They graph and connect points to create polygons. They find vertical and horizontal side lengths of these polygons by counting out the distance between two points or by analyzing the difference between a corresponding *x* and *y* coordinates. Finally they use these techniques to find the area of polygons in the coordinate plane within the context of the real-world application.

Building on the knowledge of volume and spatial structuring abilities developed in previous grades students learn to find the volume of right rectangular prisms with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. They apply the formulas $V = lwh$ and $V = Bh$ to find the volumes of right rectangular prisms and other polyhedral solids by decomposition of their parts.

As students continue to work with polyhedral solids they learn to describe the shape of the faces, and identify the edges and vertices on a solid. This leads them to make and use drawings of nets to represent the surface area of the solid. They find the surface area of solids by decomposing their associate nets into rectangles and triangles of which they can find the area. As student's knowledge of surface area and volume evolve they solve problems that require them to apply strategies and formulas to find solutions of real-world and mathematical problems.

Connections to Content:

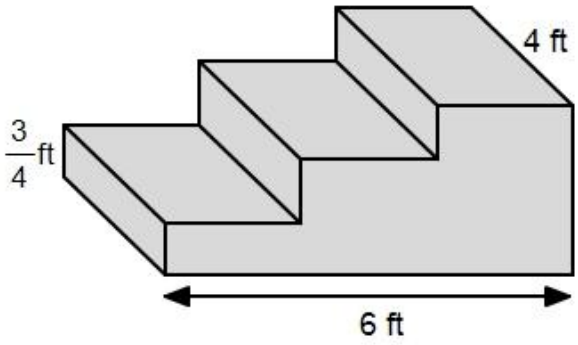
Prior Knowledge:

In previous grades students have investigated writing and solving simple equations. Working with area and volume provides a context for developing and using these equations. Students have also classified triangles and quadrilaterals and have developed an understanding of their properties and relationships. They have also learned how to graph points in the coordinate plane. In 3rd grade they recognize area as an attribute of plane figures and investigate concepts of area measurement. In 4th grade they apply area formulas to real-world and mathematical problems. Volume is studied in 5th grade where students learn to recognize it as an attribute of solid figures and investigate concepts of volume measurement. This prior work with area and volume help them to develop competencies in shape composition and decomposition and form a foundation for understanding the formulas for area and volume and the coordinate plane.

Future Knowledge:

Composition and decomposition of shapes is used throughout geometry in middle school and high school. In 7th grade will solve problems involving scale drawings of geometric figures. A good understanding and strong ability to calculate area will come to bear as they compute lengths and areas from scale drawings and reproduce a scale drawing at a different scale. Students will use their knowledge of finding distance of line segments in the coordinate plane when they study the distance formula. They will also work with three-dimensional figures as they learn about in investigate their cross sections. Lastly investigations involving area, volume, and surface area continue in 7th and 8th grade as students tackle more complex problems related to real-world applications.

MATHEMATICAL PRACTICE STANDARDS

	Make sense of problems and persevere in solving them.	Gloria is planning on pouring a set of concrete cement steps on the side of her front porch. She has drawn out a diagram of the steps below where the "rise" and "run" of each step is equal.  a. Determine the total amount of cement she will need for the steps. Assume that angles that appear to be right angles are right angles. *Students use the understanding that you need to subdivide the steps into composite figures that you can find the volume of as an entry point for solving this problem. The total volume is the sum of the composite volumes. This explanation acts as a "road map" for solving the problem, rather than just jumping right into making calculations that they do not understand.*
	Reason abstractly and quantitatively.	Describe in words and write a formula about how to the find area of any parallelogram. *Throughout this chapter students generalize methods for finding area and volume of polygons and 3D objects into formulas. As students develop these formulas they decontextulize as the move from finding the area or volume of one specific shape or object to finding the area or volume any of these given shapes or objects.*
	Construct viable arguments and critique the reasoning of others.	Olivia claims that $V = s^3$, where *s* is the side length of a cube is the formula you should use to find the volume of a cube. Harrison claims that the correct formula is $V = lwh$, where *l* is the length, *w* is the width, and *h* is the height of the cube. *This problem requires students to think critically about the claims made; they must draw upon previous knowledge about the composition of an algebraic expression and what the variables in the formulas represent. They use this knowledge to construct an argument as to who they think is correct in their reasoning.*

<table>
<tr>
<td>  </td>
<td>Look for and express regularity in repeated reasoning.</td>
<td>Find the length of each vertical segment, record your answers in the table below.
<table>
<tr><th>From Point</th><th>To Point</th><th>Length</th></tr>
<tr><td>(2, 5)</td><td>(2, 3)</td><td></td></tr>
<tr><td>(6, 5)</td><td>(6, 3)</td><td></td></tr>
<tr><td>(8, 1)</td><td>(8, 6)</td><td></td></tr>
</table>
What do you notice about the ordered pairs that line up vertically?
Is there are way that you could determine the length of these segments without plotting the points.
As students repeatedly observe the length of vertical line segments they begin to see a pattern emerge about the relationship between the difference between the x and y coordinates and the length of the line. Through these observations they can generalize a method or “shortcut” for finding the length without counting out the distance on a graph.</td>
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<tr>
<td>  </td>
<td>Model with mathematics.</td>
<td>A new park is being designed for your city. The plans for the design are being drawn on the coordinate plane below. The vertices given form a polygon that represents the location for 5 different features in the park. Plot each set of points in the order they are given to form a polygon. Label the feature that the polygon represents on the graph.
In the problem above students use the coordinate plane to model real world objects and their locations. By assigning these objects a location on the coordinate plane they can find lengths of different dimensions within an object and distances between objects. In turn they will use these measurements to find area and use this information to answer questions and make informed decisions about real-world applications.</td>
</tr>
<tr>
<td>  </td>
<td>Attend to precision.</td>
<td>A company that manufactures dice packages the dice in rectangular boxes that measure 7 inches by 2 inches by 5 inches as shown.
<ol type="a">
How many 1-inch by 1-inch dice can fit into one box?
What is the volume of the box in cubic inches?
The company also makes mini dice that measure $\frac{1}{2}$-inch by $\frac{1}{2}$-inch. How many $\frac{1}{2}$-inch dice can fit into a rectangular box that has the same dimensions? Use the figure above to help you answer.
What is the volume of the box when measured in ½-inch cubes?
Why do you get two different numeric values for the volume for parts b. and d. on the previous page? Does the volume of the box the change depending on whether it is packed with 1-inch dice or ½-inch mini dice?
How can you show or prove that the number of 1-inch dice that fit into the box takes up the same amount of space or has the same volume as the number of ½-inch mini dice that fit into the box.

Two different units of measure are being compared and contrasted in the problems above. It is imperative for students to attend to precision as they communicate about these problems and their solutions, because if they neglect units of measure while speaking, this is likely to lead to confusion.</td>
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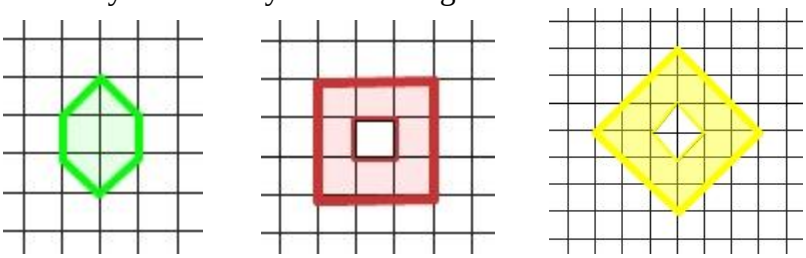
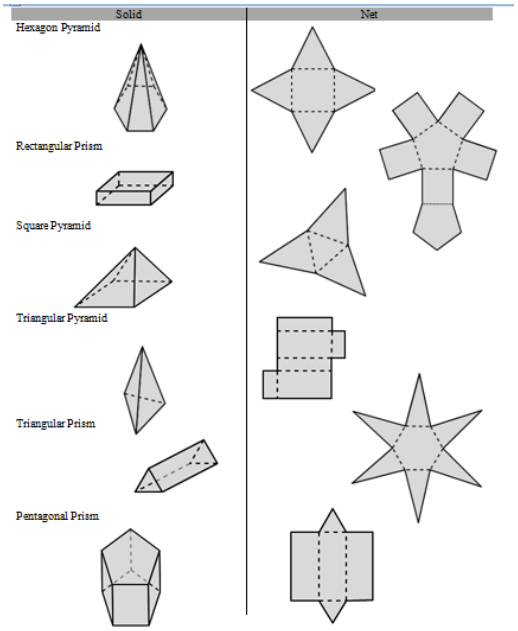
	Look for and make use of structure.	Find the area of each colored figure if each square represents one square unit. Be ready to discuss your reasoning.  *As students try to find the area of each figure, they must recognize that they can decompose different parts of the figure in order to recompose them into whole square units that they can count. As they do this they must step back and shift their perspective of how to view a square unit of area.*
	Use appropriate tools strategically.	Draw a line to match each solid with its net.  You can explore nets with online interactive manipulatives as you investigate the problem above. *Online tools for viewing 3D objects and their nets can really help students that have difficulty visualizing how an object and its net are related. These tools bring this relationship to life as students can see how the faces, edges, vertices, etc correspond to each other between a 3D object and its net.*

Section 5.1: Area of Polygons

Section Overview:
This section revolves around finding the area of right triangles, other triangles, special quadrilateral, and polygons. Students begin their work with area by finding area of figures on a graph and counting out unit squares. They learn that when confronted with irregular figures they can decompose and rearrange the figures into shapes that they can easily find the area of. A lot of work is done in this section with developing methods and formulas for finding the area of parallelograms, triangles, and trapezoids. Students do this by finding the area of several different examples of these shapes through transformations and then generalize their methods. Once students are familiar with finding area they turn to finding missing measurements. They use the relationship between a polygon's dimensions and its area to do so. They use their knowledge of finding area to answer question related to real-world applications and finally work with finding the area of irregular figures by decomposing these figures into triangles, rectangles, and/or trapezoids.

Concepts and Skills to Master in this Section:

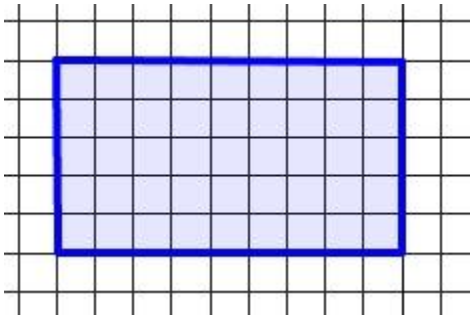
By the end of this section, students should be able to:

1. Find the area of a parallelogram including a rectangle and square.
2. Find the area of a triangle.
3. Find the area of a trapezoid.
4. Find the area of irregular figures by decomposing them into triangles, rectangles, and/or trapezoids.
5. Solve real-world and mathematical problems that involve finding the area of polygons.

5.1a Class Activity: Finding Area

Find the area of each colored figure if each square represents one square unit. Be ready to discuss your reasoning.

1.



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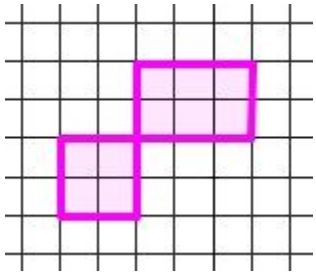

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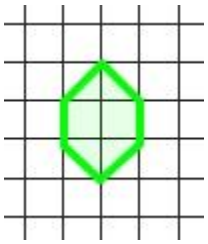

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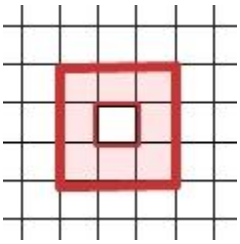

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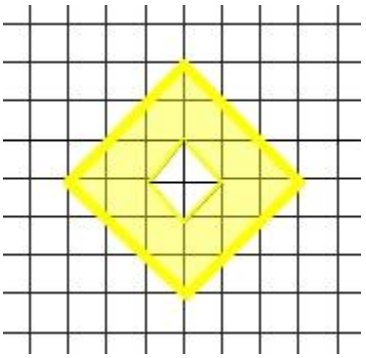
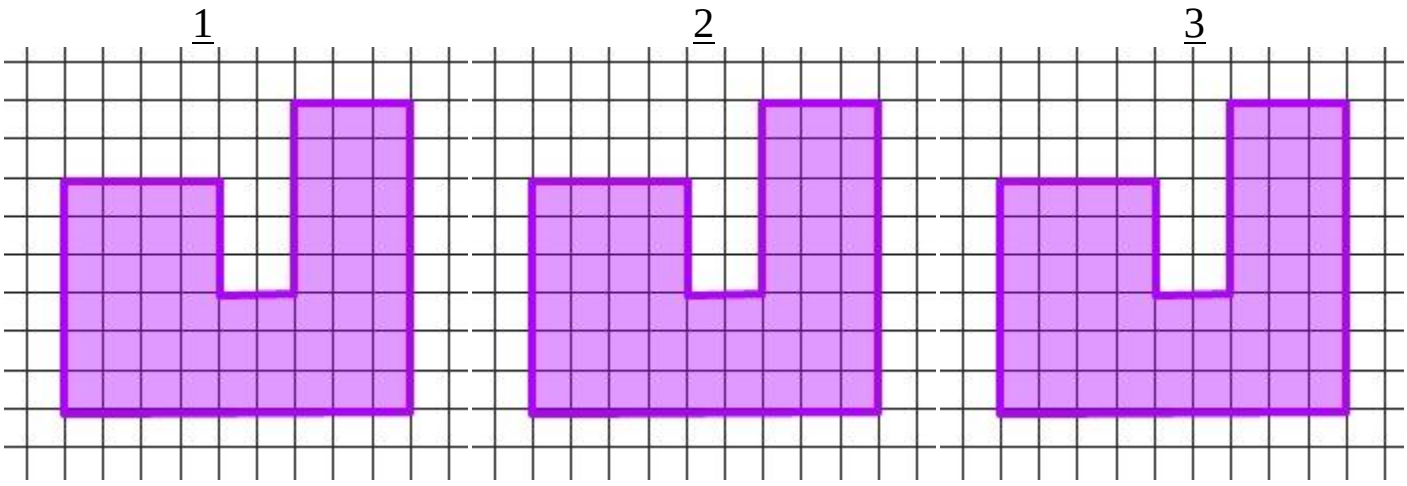


6. Three copies of the same figure are shown below. Each square represents one square unit. There are many ways to find the area of this figure.



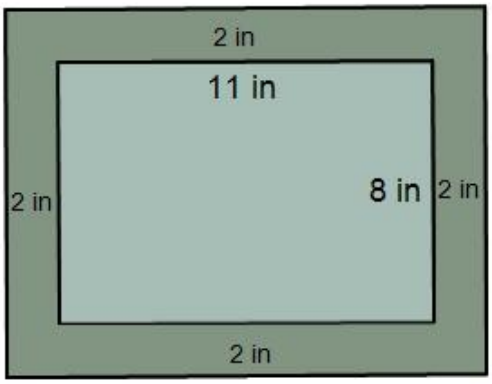
a. Split this figure into different non-overlapping regions in at least three ways. Show your work above.

b. For every figure that you split in part a write an expression that represents the area of the sum of the regions.

c. Find the total area of the figure.

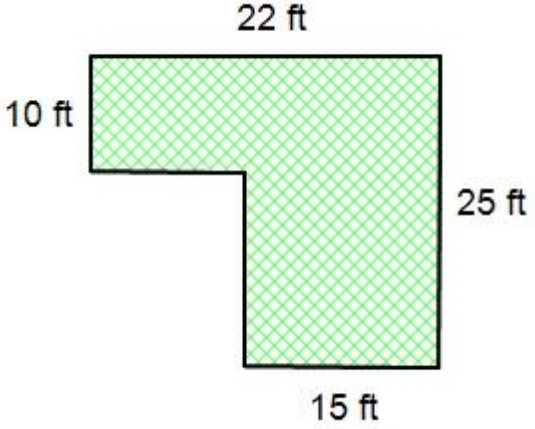
7. Gloria is painting a feature wall in her bedroom. The dimensions of the wall measure 14 feet by 11 feet. The gallon of paint that she purchased will cover 400 square feet. Does she have enough paint to do two coats on the wall? Justify your answer.

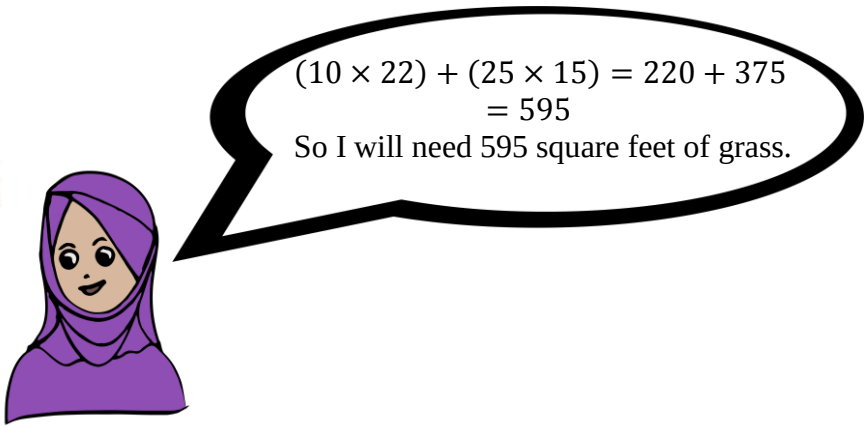
8. Steven is designing a rectangular frame to go around a drawing that measures 11 inches by 8 inches, he has budgeted $15.00 to cover the cost of the material for the frame. He would like the frame to be 2 inches wide around the perimeter of the drawing. The diagram below shows the dimensions of the drawing and the frame. The material for the frame costs $0.20 per square inch. Will Steven have enough money to cover the cost of the frame?



Find, Fix, and Justify

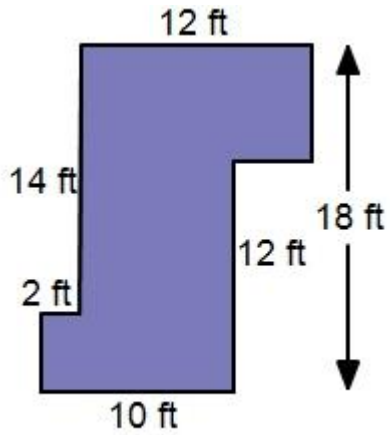
9. Antonia is laying grass sod in her back yard. She has drawn a diagram of where she would like to put the grass. Antonia's calculation to find the area of the yard that will need grass sod is shown. She has made a mistake. Find Antonia's mistake, explain what she did wrong, and then find the correct area. Assume that all angles that appear to be right angles are right angles.



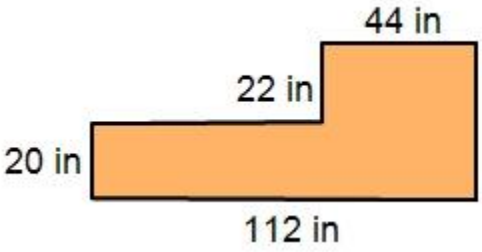


Directions: For numbers 9-12 find the total area of each figure by decomposing it into non-overlapping regions. Assume that all angles that appear to be right angles are right angles.

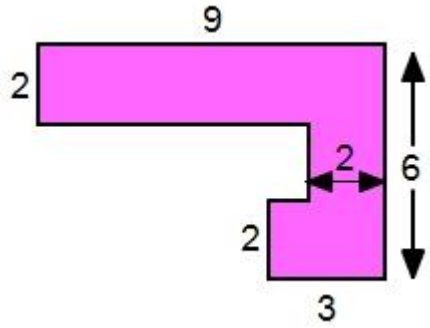
9.



10.



11.



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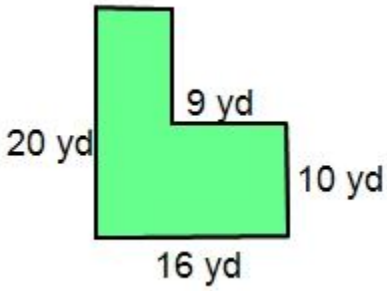
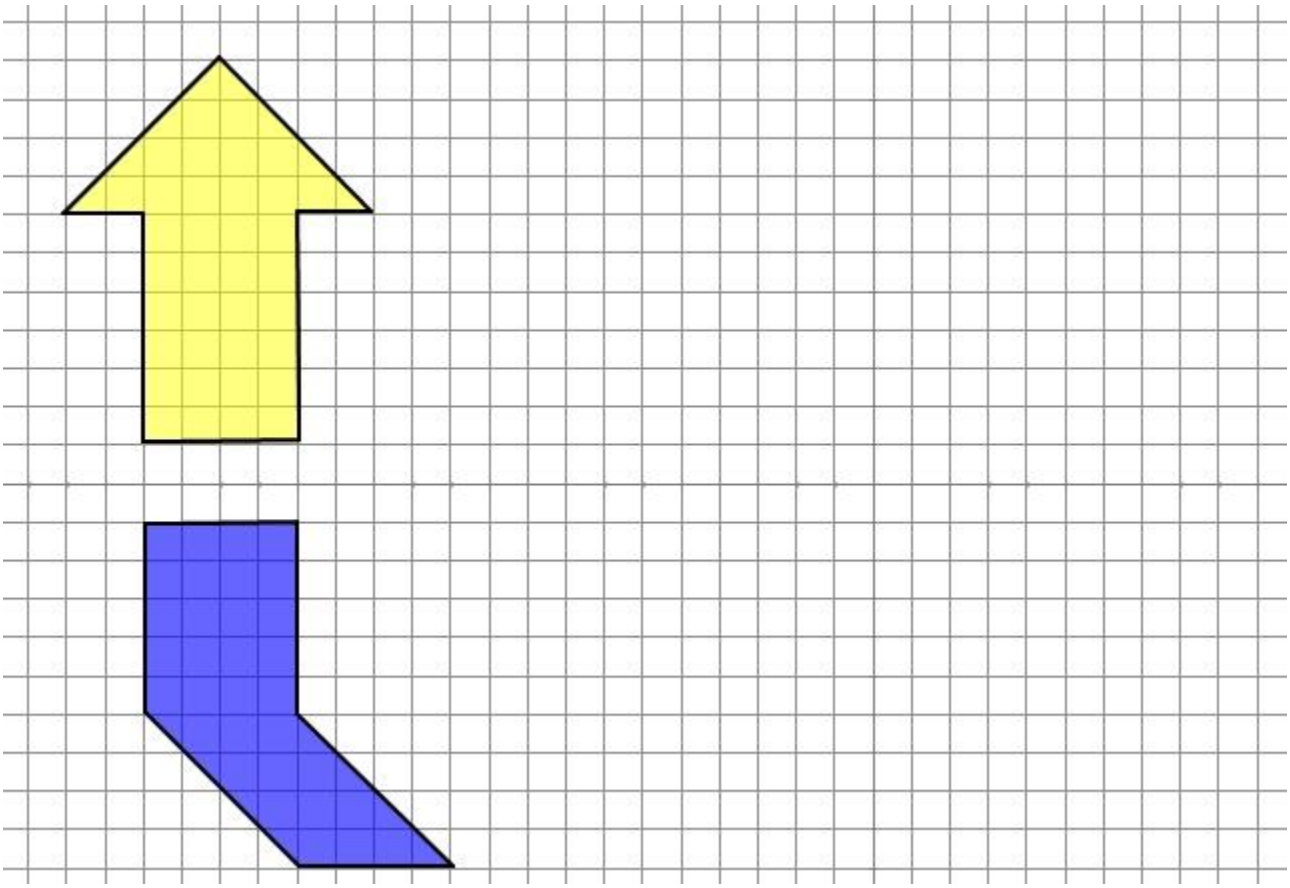


13. The area of a flat screen TV is 864 square inches. The width of the TV is 36 inches. What is the height of the TV? Draw and label a picture to help you solve.

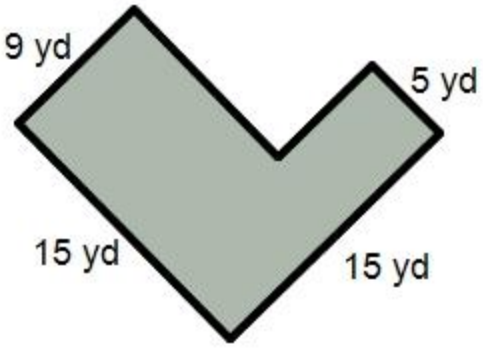
14. Find the area of the shapes given below if each square represents one unit. Be ready to discuss your reasoning.



5.1a Homework: Finding Area

Directions: Find the area of each figure. Assume that all angles that appear to be right angles are right angles.

1.



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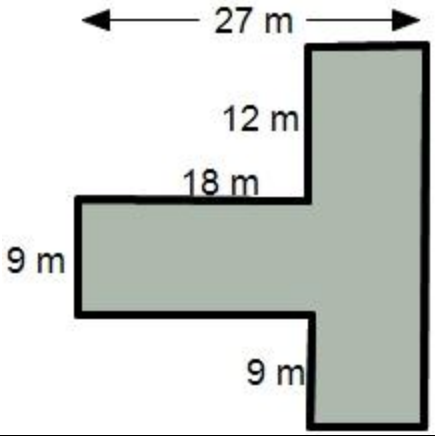

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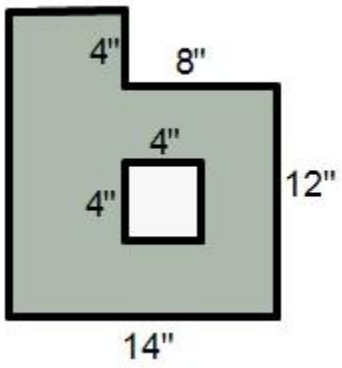

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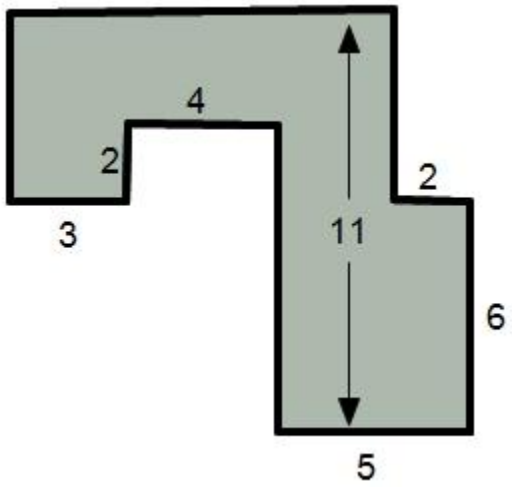


5. A painter is painting a wall in an office building. The wall measures 4 yards by 3.5 yards. The wall has two windows on it each that measure 1 yard by 1.5 yards.

   a. Draw a label a diagram that represents the wall.

   b. Find the amount of paint that is needed to do one coat of paint on the wall.

   c. One can of paint will cover 8 square yards. How many cans of paint does the painter need to paint the wall?

6. How many 4-inch square tiles are needed to cover a table that measures 24 inches by 40 inches? Draw and label a picture if needed.

7. Sandra is making a quilt that she wants to have an area of 51 square feet. The length of the quilt is 8.5 feet. What is the width of the quilt? Draw and label a picture if needed.

8. You are surrounding your backyard pool will patio pavers. The pavers come in two sizes, the 2 feet by 2 feet pavers costs $6 each and the 1 foot by 2 feet pavers cost $3 each. Your swimming pool measures 20 feet by 30 feet. You would like the patio to surround the pool extending 3 feet in all directions.

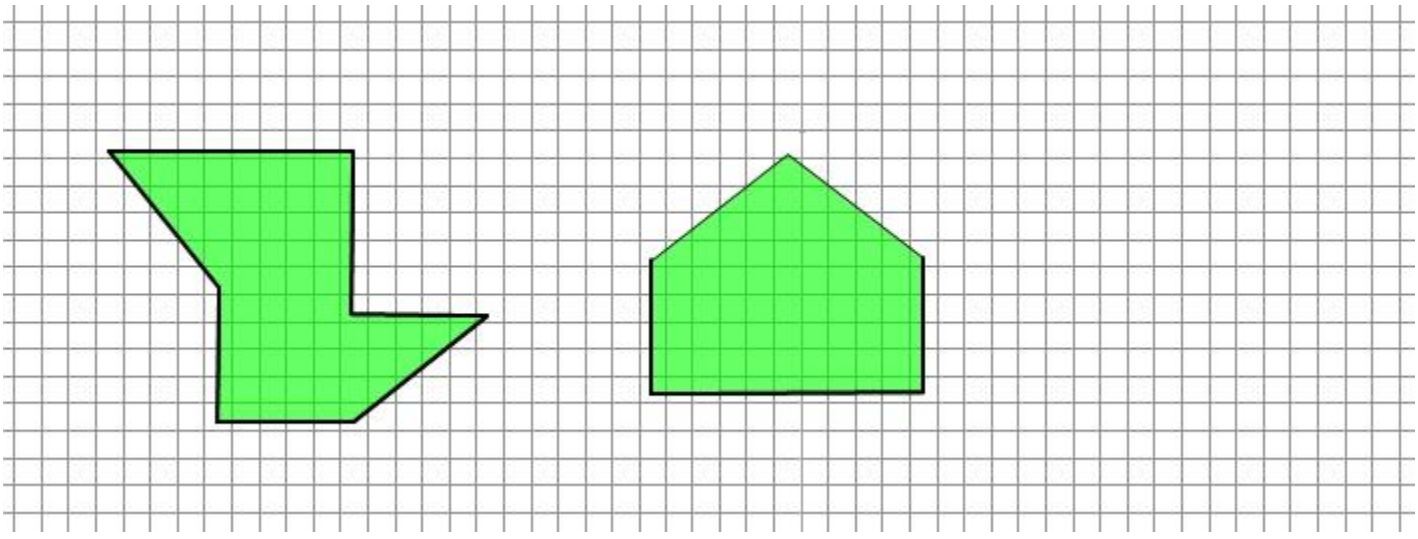
   a. Draw a diagram that represents the pool, patio, and the pavers.

   b. Find the total area that you want to cover in pavers?

   c. Your brother claims that it will cost less to use the $6 pavers because they are bigger. Do you agree with him? Why or why not. What pavers do you think you should use?

   d. How much money will is cost to buy the pavers?

8. Prove that the figures below have the same area. Then sketch a figure of your own that has the same area.



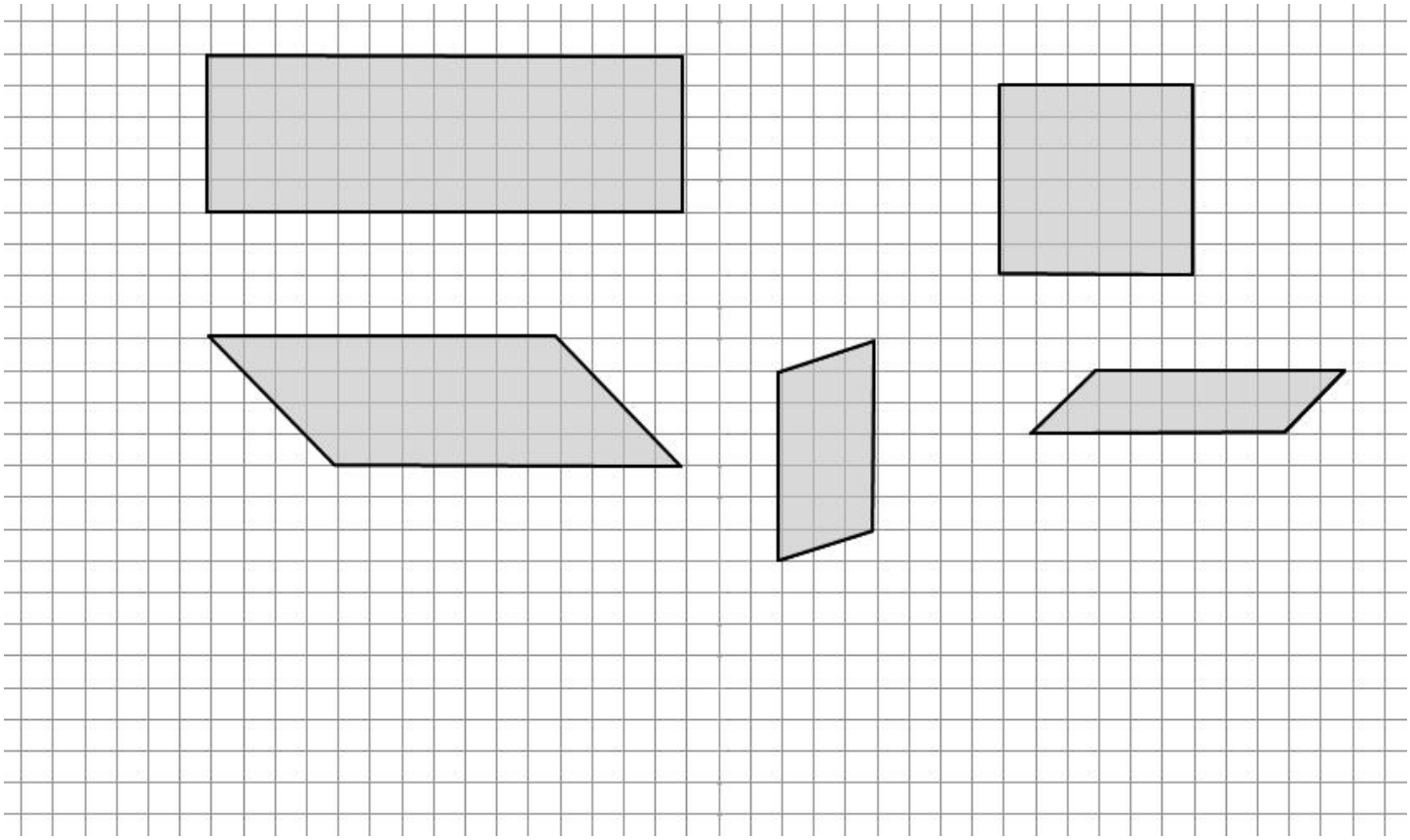
5.1b Class Activity: Area of Parallelograms

1. Draw a picture of each quadrilateral under its name. Then write a sentence that describes the quadrilateral.

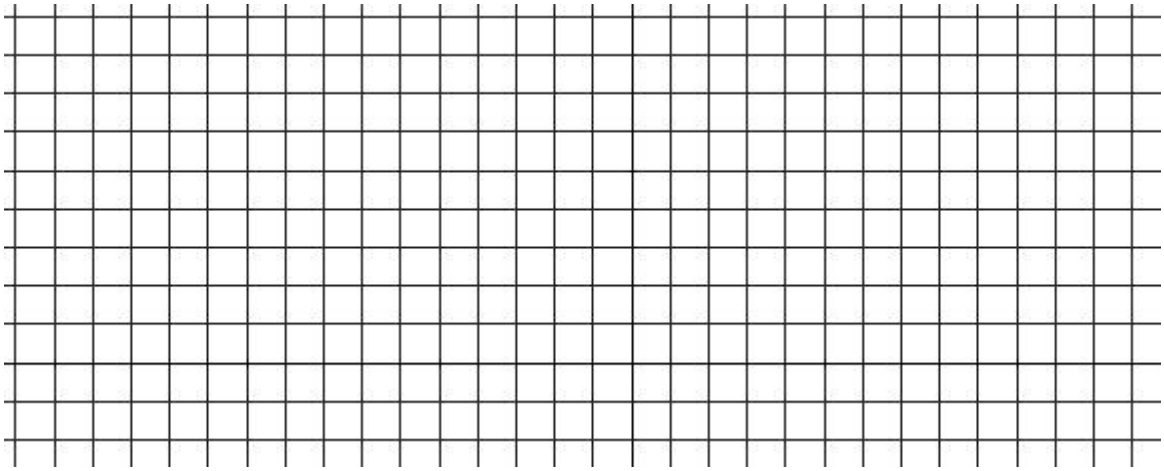
Quadrilateral:	Trapezoid:	Kite:

Parallelogram:	Rectangle	Square:	Rhombus:

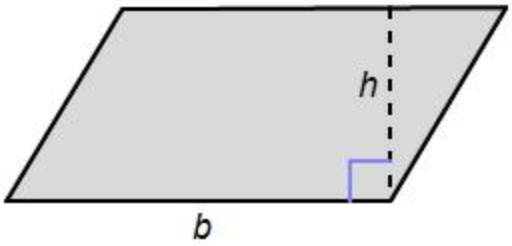
2. Find the area of each parallelogram below. Be ready to discuss how you found the area.



3. Will your method for finding the area work for any parallelogram? In the space provided draw any parallelogram and use your method to find the area. Answers will vary

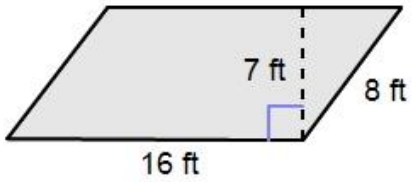


4. Describe in words and write a formula about how to the find area of any parallelogram. n#



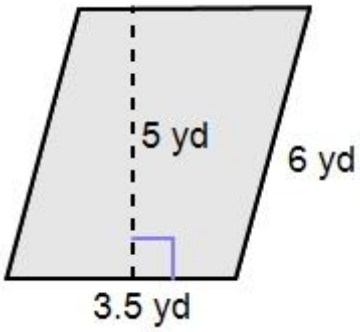
5. Find the area of each parallelogram.

a.

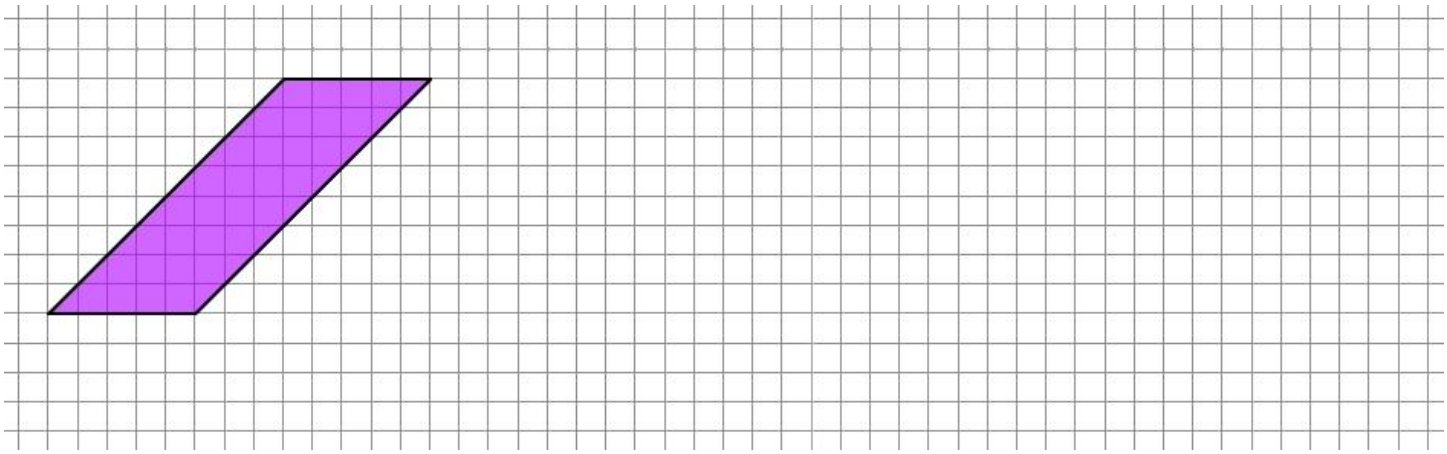


b.

c.

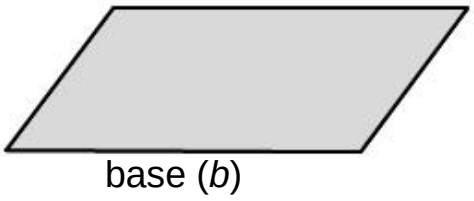


6. Find the area of the parallelogram below.



7. Draw and label the height for each parallelogram given the base.

a.



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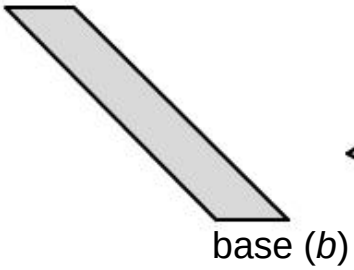
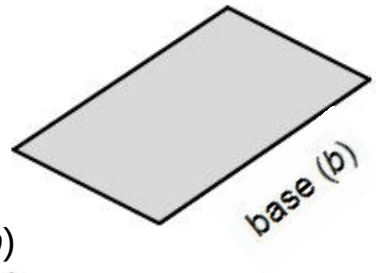
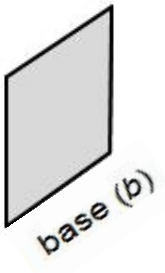


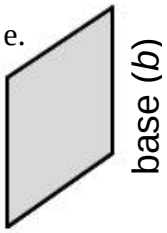
c.



d.

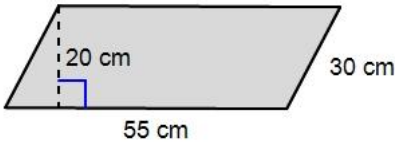


e.

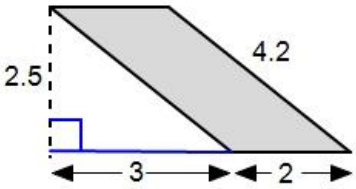


8. Find the area of each parallelogram.
*Figures are not drawn to scale

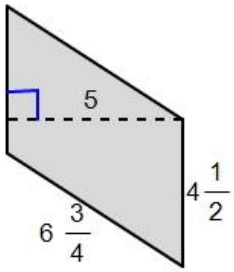
a.



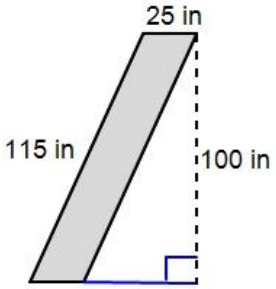
b.



c.

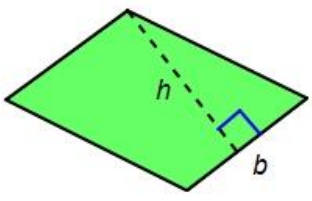


d.

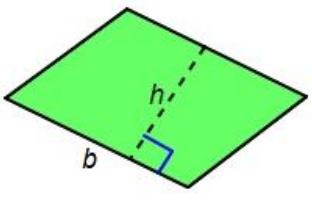


9. Chris, Susan, Tyler, and Harper have all labeled the height and base for the parallelogram below. Are they all correct? If not, who is correct and why?

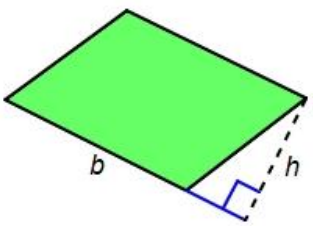
Chris



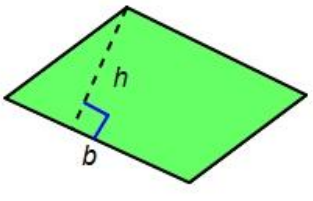
Susan



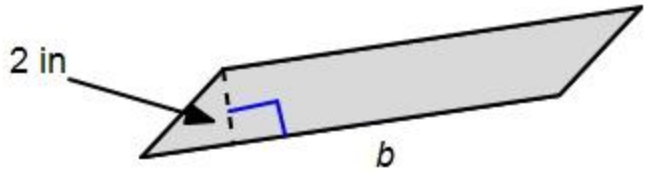
Tyler



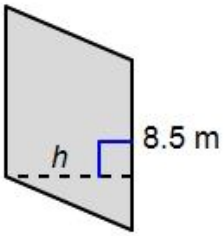
Harper



10. The area of the parallelogram below is 24 square inches, what is the length of the base?



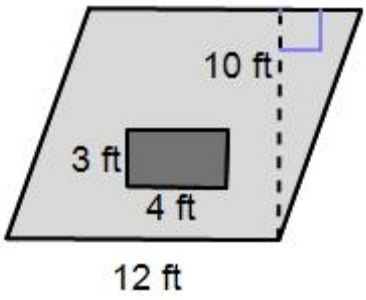
11. The area of the parallelogram below is 59.5 m^2, what is the height?



12. A parallelogram has a base length of 4.5 centimeters and a height of 2.75 centimeters. What is the area of the parallelogram? If needed draw and label a picture.

13. A staircase has 4 parallelogram shaped panels on the wall. The base of each panel is 2 ft and the height 1.5 ft. How much area do the 4 panels take up on the wall altogether?

14. Olivia is drawing up blue print designs for the floor space in a tree house. Her drawing is shown; the rectangular hole in the floor of the tree house is the opening to climb down the ladder. How much floor space does the tree house have?

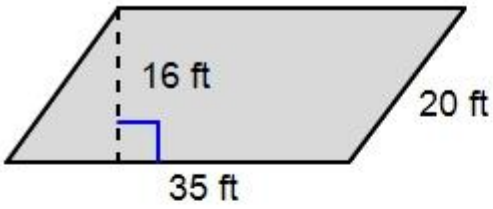


5.1b Homework: Area of Parallelograms

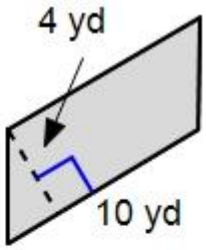
Find the area of each parallelogram

*Figures are not drawn to scale

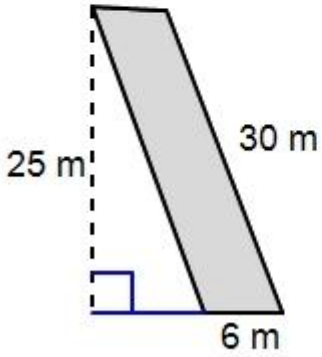
1.



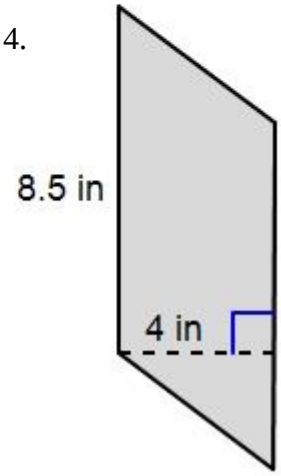
2.



3.



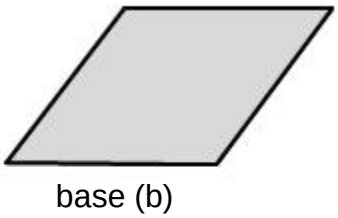
4.



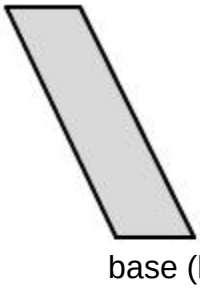
For each parallelogram given below the base is given, find and label the height.

*Figures are not drawn to scale

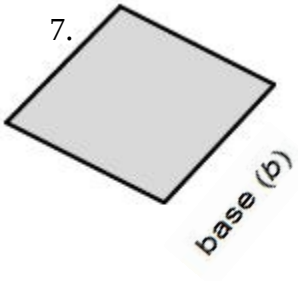
5.



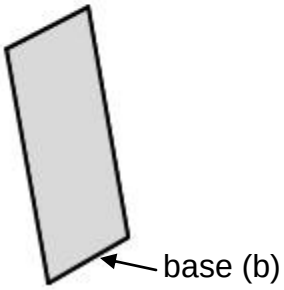
6.



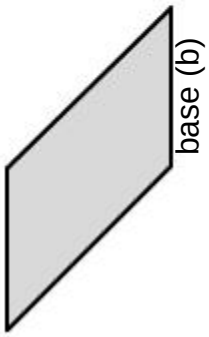
7.



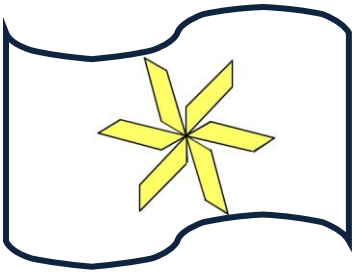
8.



9.



10. You are making a design for a flag. The design is created by rotating a parallelogram around a fixed point 6 times as shown. Find the total area that the design will take up on the flag if the parallelogram has a height of 3 inches and a base length of 8 inches.



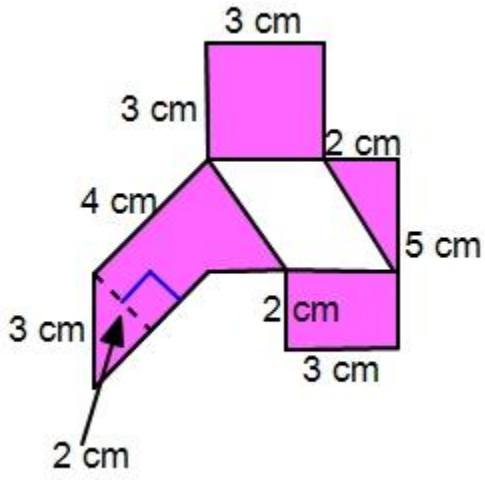
11. Bonnie is making a sign that is a parallelogram. Its base measures 14 feet and its height is 10 feet. She cuts a rectangular piece out of the middle of the sign that measures 6 feet by 2 feet.
    a. Draw and label a picture of Bonnie's sign.

    b. Find the total area of the sign after she cuts out the rectangle.

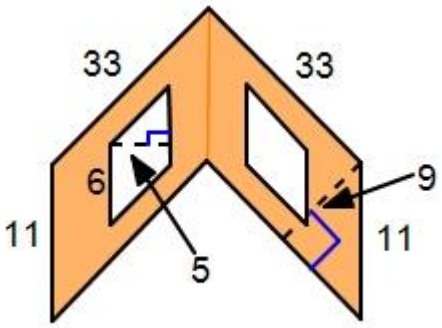
12. A tile that is the shape of a parallelogram has a base of 5 inches and a total area of 17.5 in^2. What is the height of the tile?

13. Find the area of each shaded region. Assume that angles that appear to be right angles are right angles.

a.

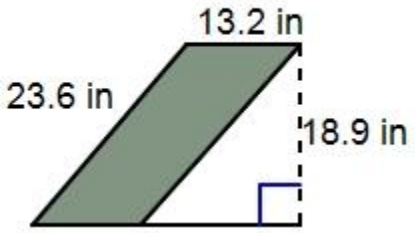


b. The two parallelograms inside the figure are the same size.

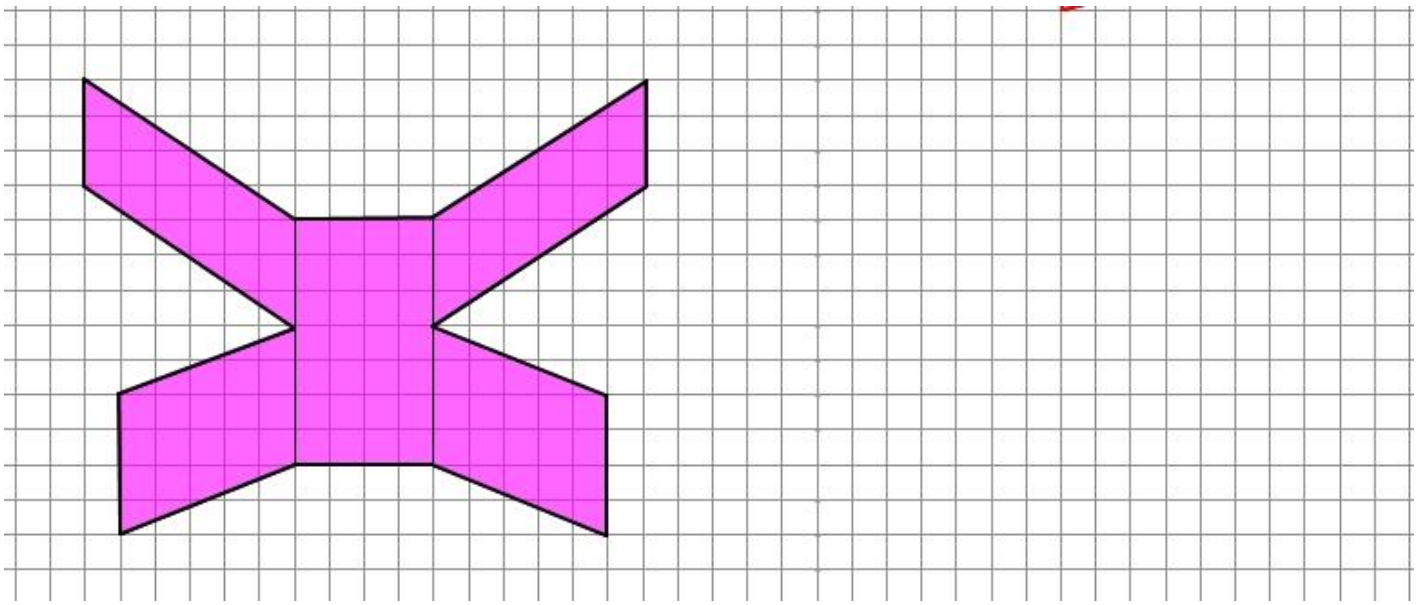


Find, Fix, and Justify

14. Zara has made a mistake on the two problems below, her work is shown. Find her mistake, explain what she did wrong, and then solve the problem correctly.

Find the area of the parallelogram below.	The area of a parallelogram is 36 square feet, the height is 4.5 feet. What is the length of the base of the parallelogram?
 $(23.6)(13.2) = 311.52$ The area of the parallelogram is 311.52 in².	$(36)(4.5) = 162$ The length of the base is 162 feet.

15. Find the area of the figure below and then draw a figure that has the same area.



5.1c Class Activity: Area of Triangles

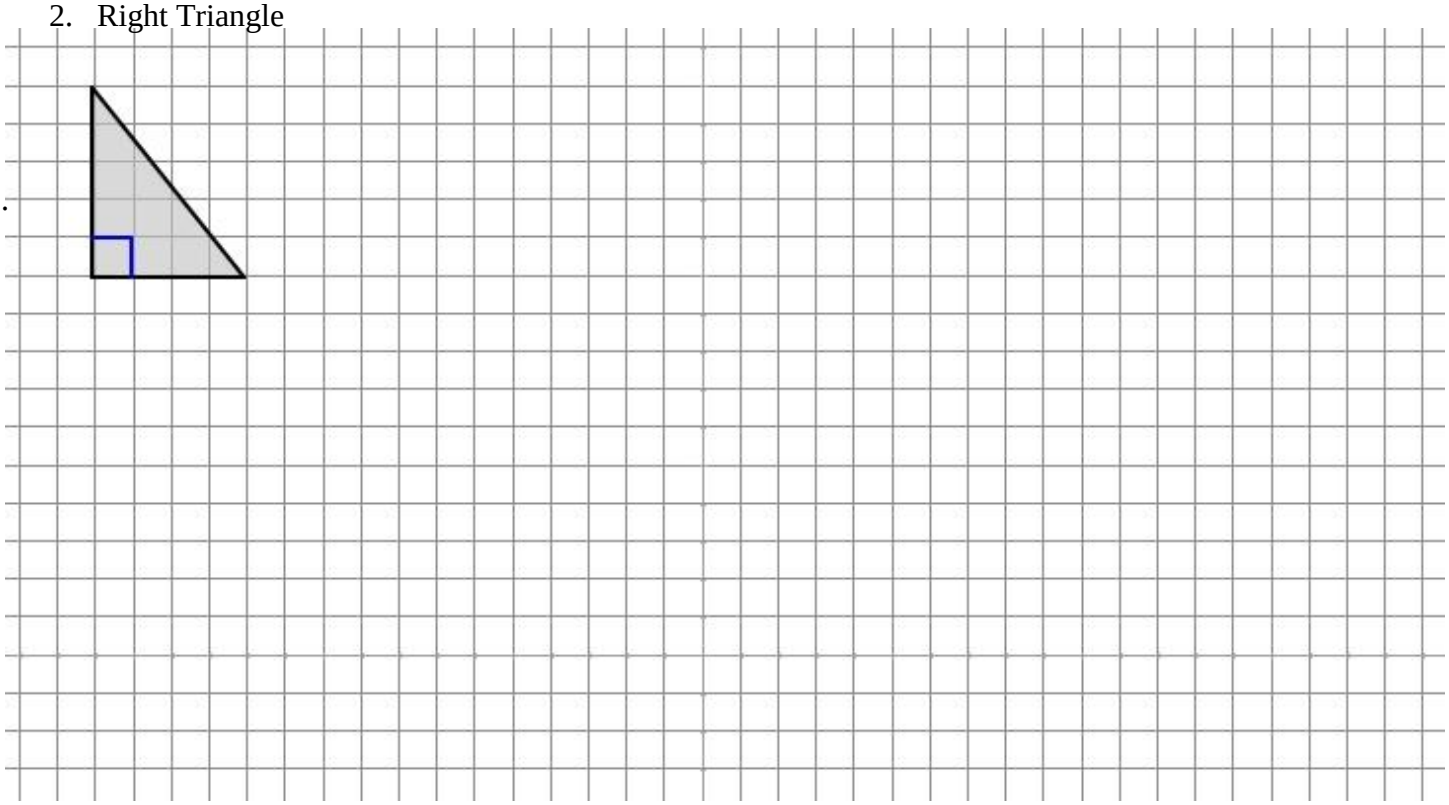
1. Draw a picture of each triangle under its name. Then write a sentence that describes the triangle.

Triangle:	Right Triangle:	Acute Triangle:	Obtuse Triangle:
	Scalene:	Isosceles:	Equilateral:

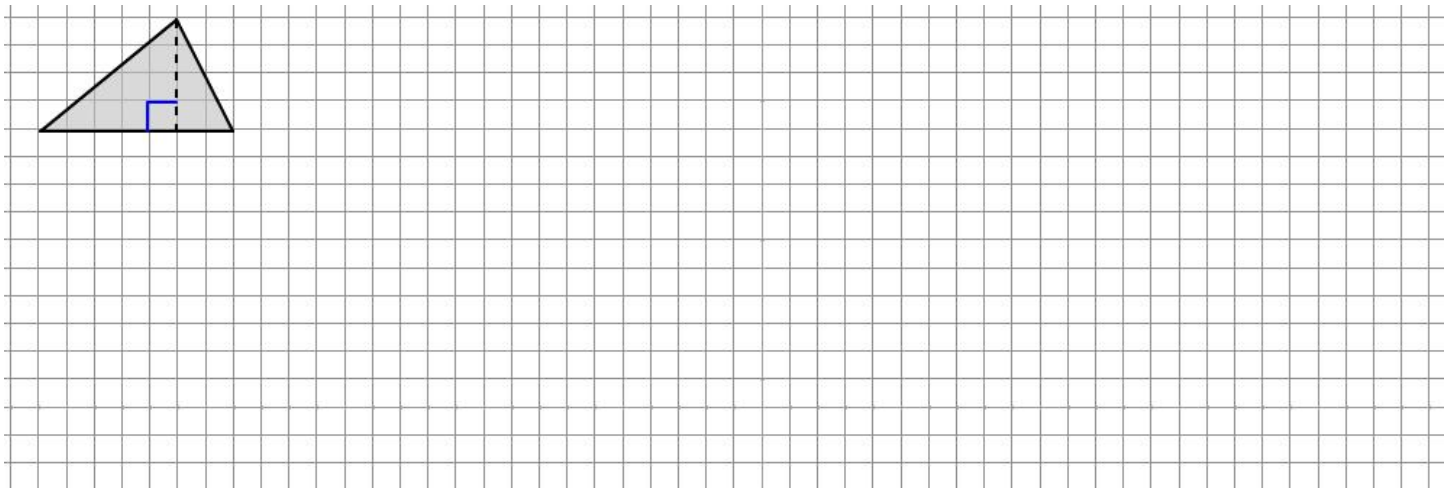
Directions: Find the area of each triangle if each square represents one square unit. Be ready to discuss how you found the area.

2. Right Triangle

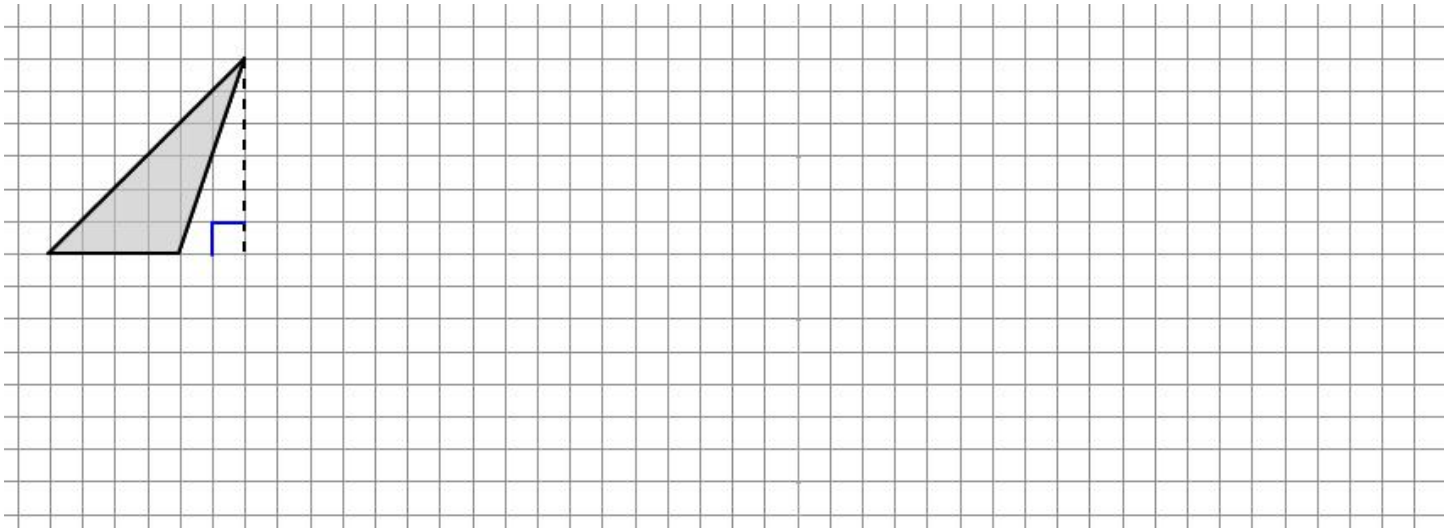
.



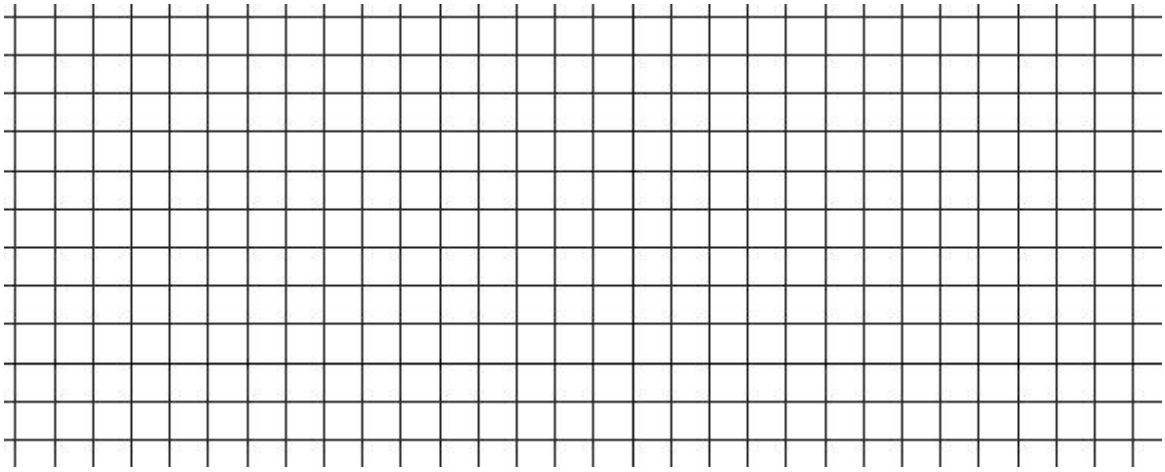
3. Acute Triangle



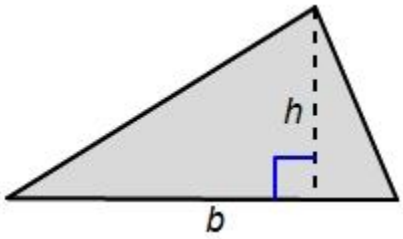
4. Obtuse Triangle



5. Will your method for finding the area work for any triangle? In the space provided draw any triangle and use your method to find the area.



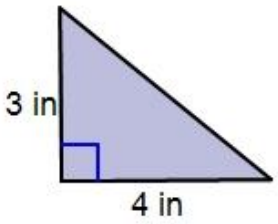
6. Describe in words and write a formula about how to find the area of any triangle.



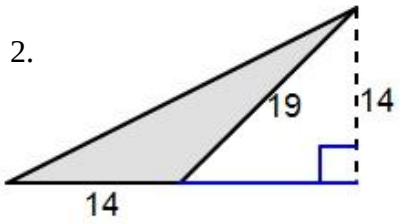
7. Find the area of each triangle.

*Figures are not drawn to scale

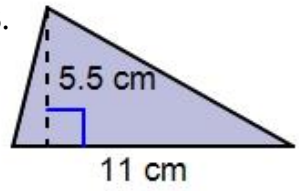
1.



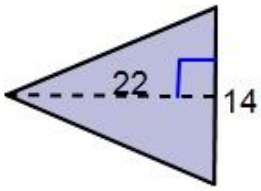
2.



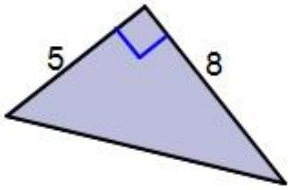
3.



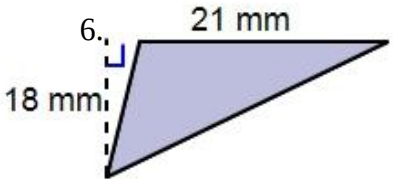
4.



5.

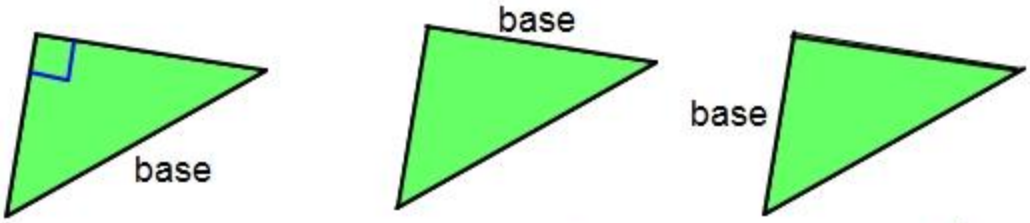


6.

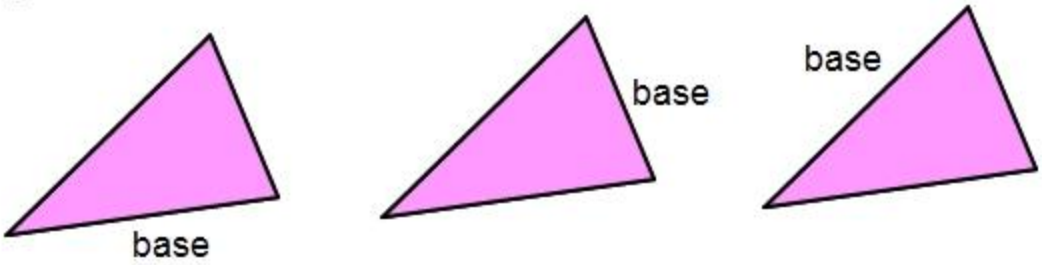


The same triangle is given three times. Draw and label the height from the given base.

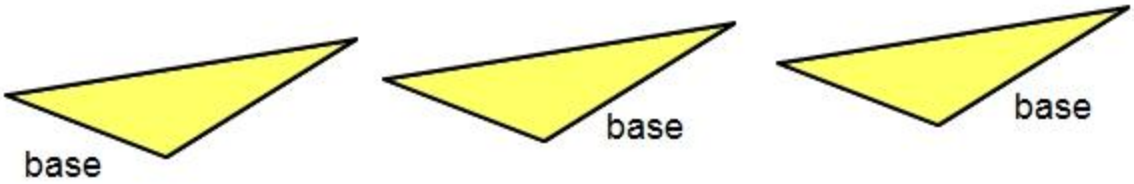
8.



9.

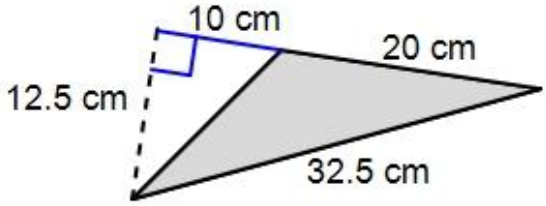


10.

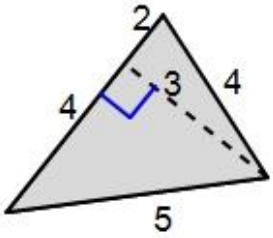


Directions: Find the area of each figure
*Figures are not drawn to scale

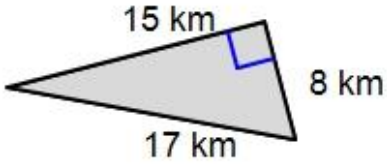
11.



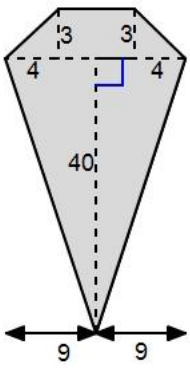
12.



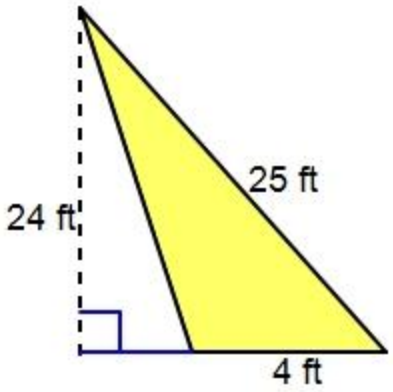
13.



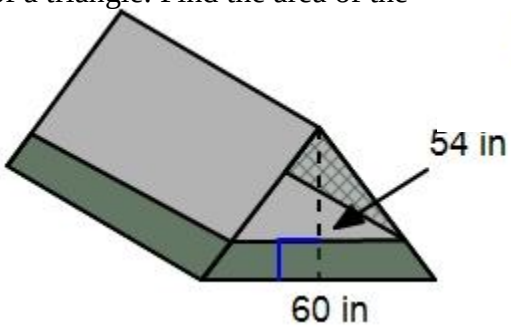
14.



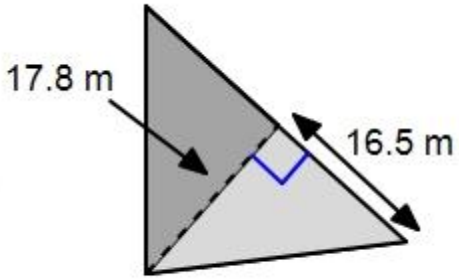
15. Olivia is replacing the material for the sail on her sail boat. The dimensions of the sail are given. How much money will Olivia spend on replacing the material if it costs $4 per square foot?



16. The door of a tent is the shape of a triangle. Find the area of the door.



17. A large triangle is made up of two small triangles as shown. The area of the small dark grey triangle is 124.6 m^2. Find the area of the large triangle.

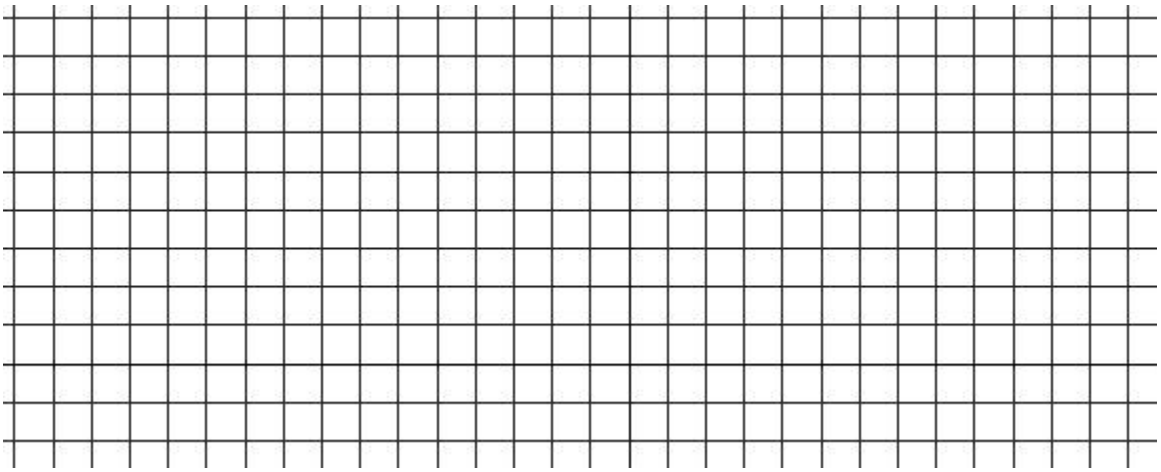


18. The area of a triangle is 23 yd^2. Its base measures 8 yards. What is the height of the triangle?

n#

19. The area of a triangle is 72 ft^2. Its height measures 12 ft. What is the length of the triangle's base?

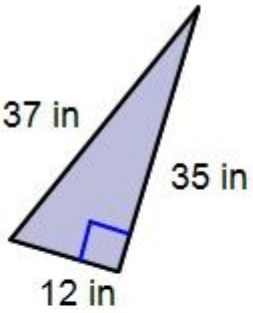
20. Create a right triangle, an acute triangle, and an obtuse triangle that all have the same area. Explain how you know that their areas are the same.



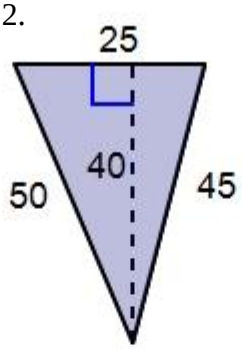
5.1c Homework: Area of Triangles

Directions: Find the area of each triangle

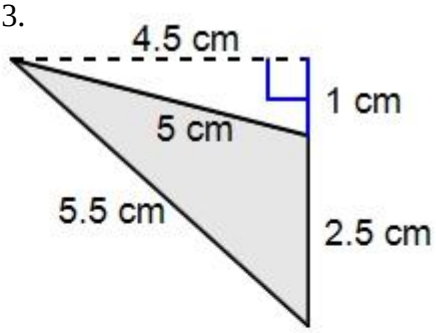
1.



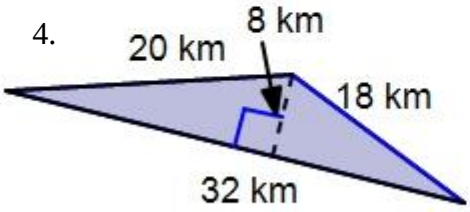
2.



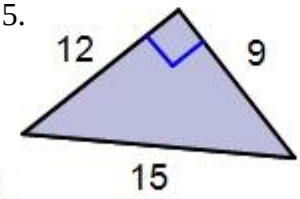
3.



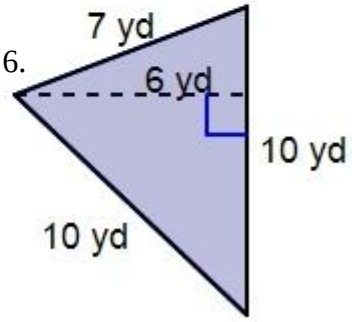
4.



5.

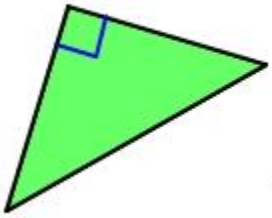


6.

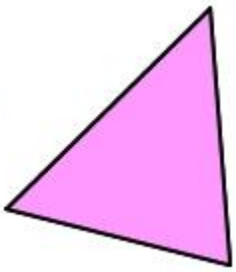


For each triangle below label the base, then draw and label the height based off of your chosen base.

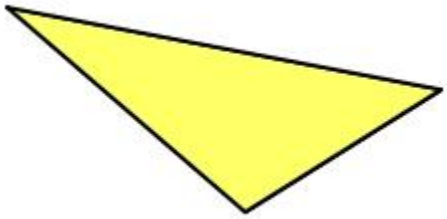
7.



8.



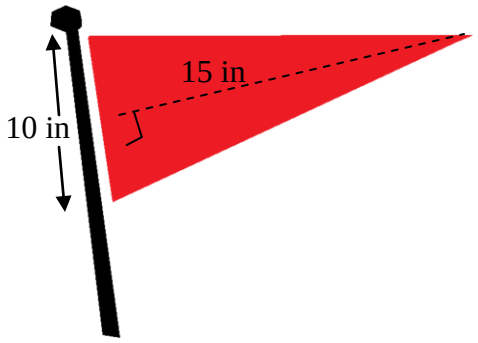
9.



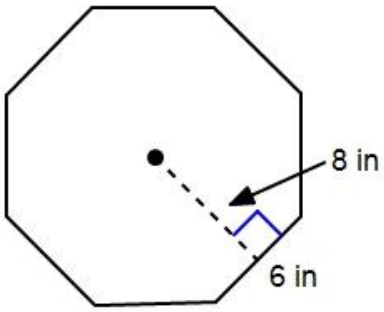
10. The wingspan of a triangular kite is 26 inches at its base, the height is 14 inches.
    a. Draw a label a picture of the kite.

    b. Find the area of the kite?

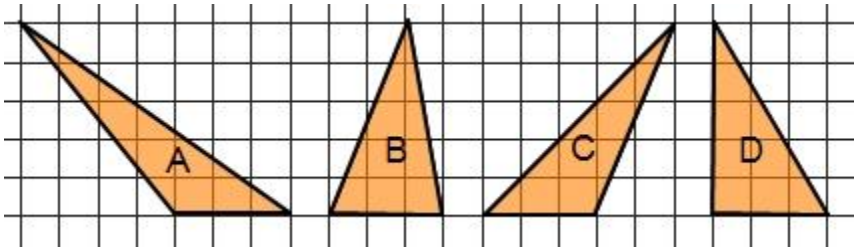
11. Find the area of the flag.



12. Find the area of the regular octagon below. A regular octagon has all sides that are equal.

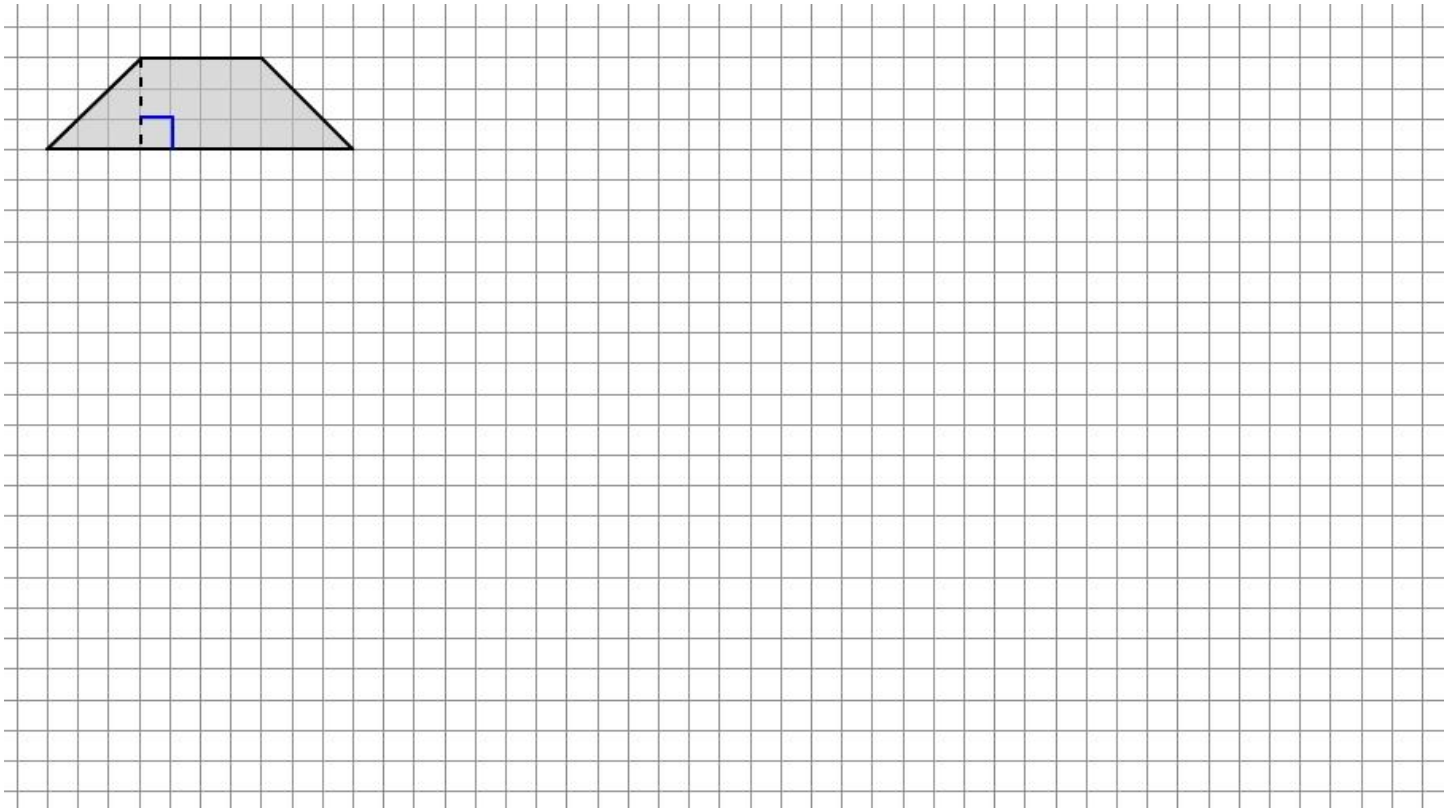


13. Without doing any calculations determine which triangle below has the largest area if each square represents one square unit? Explain how you know.



5.1d Class Activity: Area of Trapezoids

1. Find the area of each trapezoid if each square represents one square unit. Be ready to discuss how you found the area.



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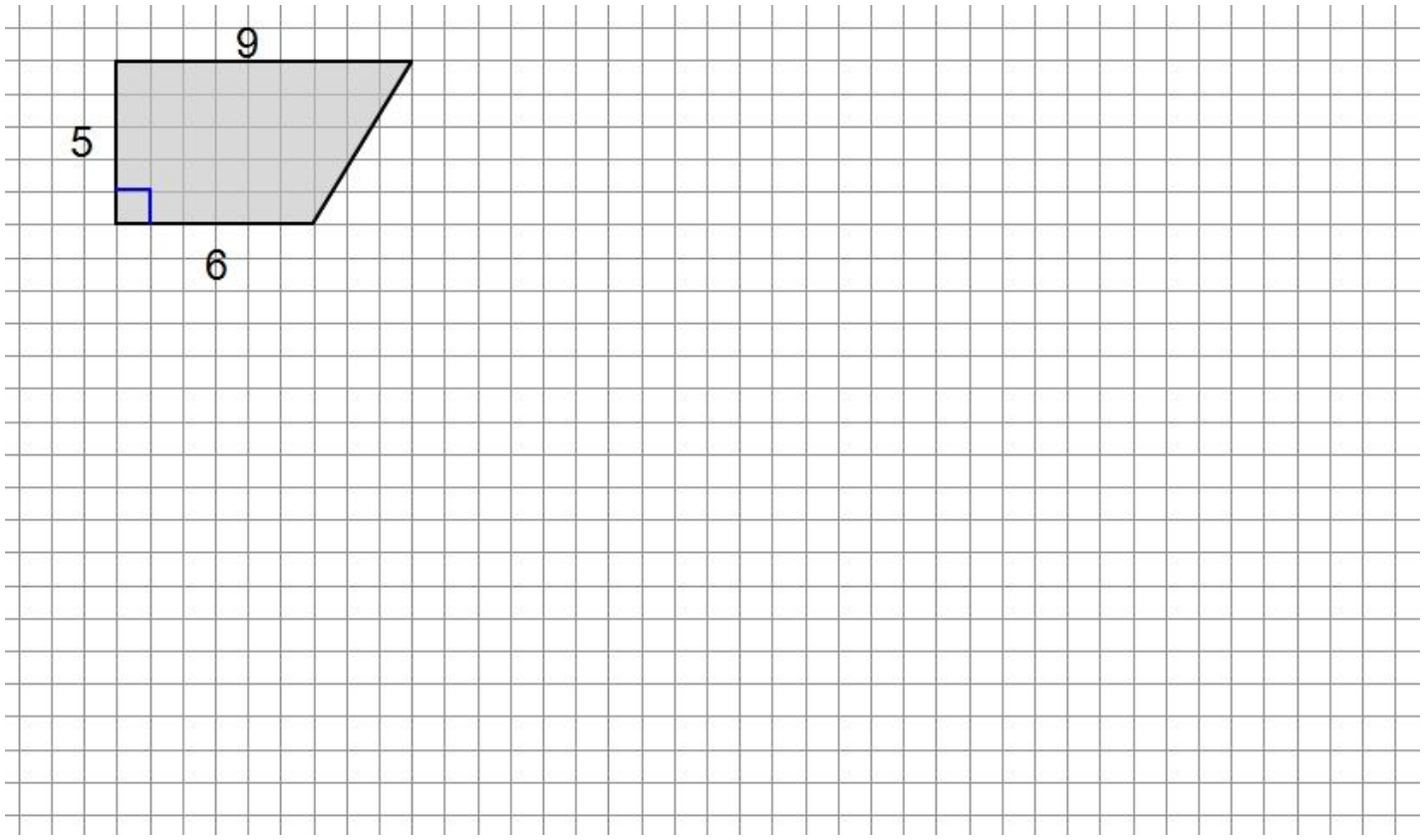
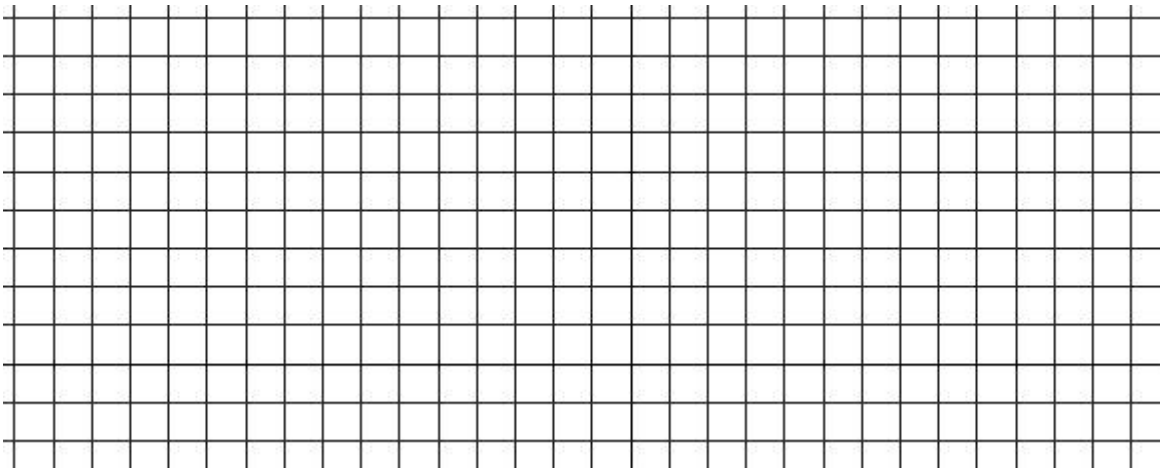
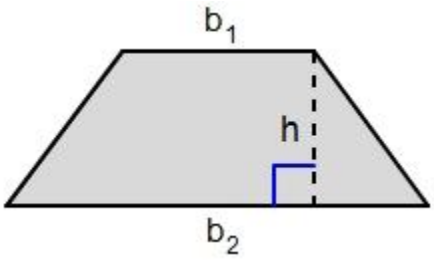
2.



3. Will your method for finding the area work for any trapezoid? In the space provided, draw any trapezoid and use your method to find the area.



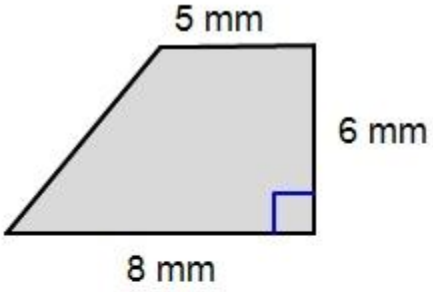
4. Describe in words and write a formula about how to find the area of any trapezoid.



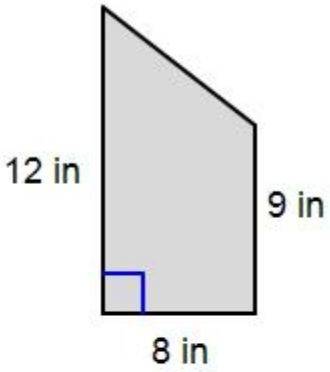
5. Directions: Find the area of each trapezoid.

*Figures are not drawn to scale.

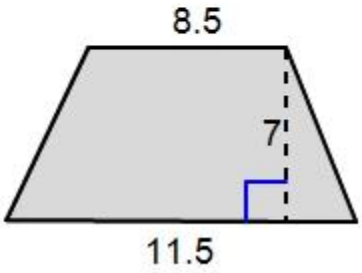
a.



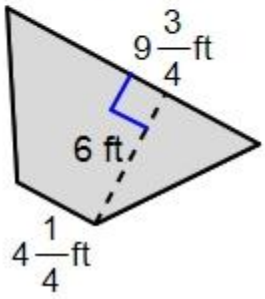
b.



c.



d.

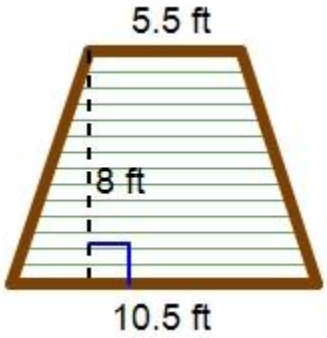


Find the area of each trapezoid. If needed draw and label a picture.

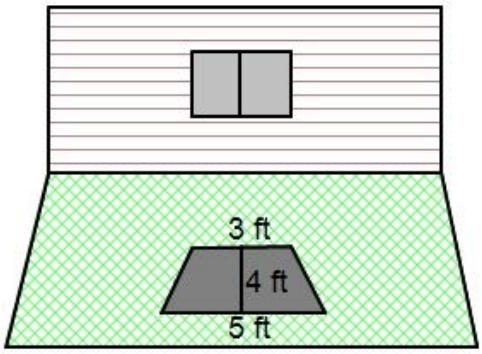
6. A trapezoid with base lengths of 4 meters and 8 meters and a height of 2 meters.

7. A trapezoid with base lengths of 12 inches and 6 inches and a height of 3 inches.

8. A garden plot is the shape of a trapezoid as shown. Find the total area that the garden plot takes up.



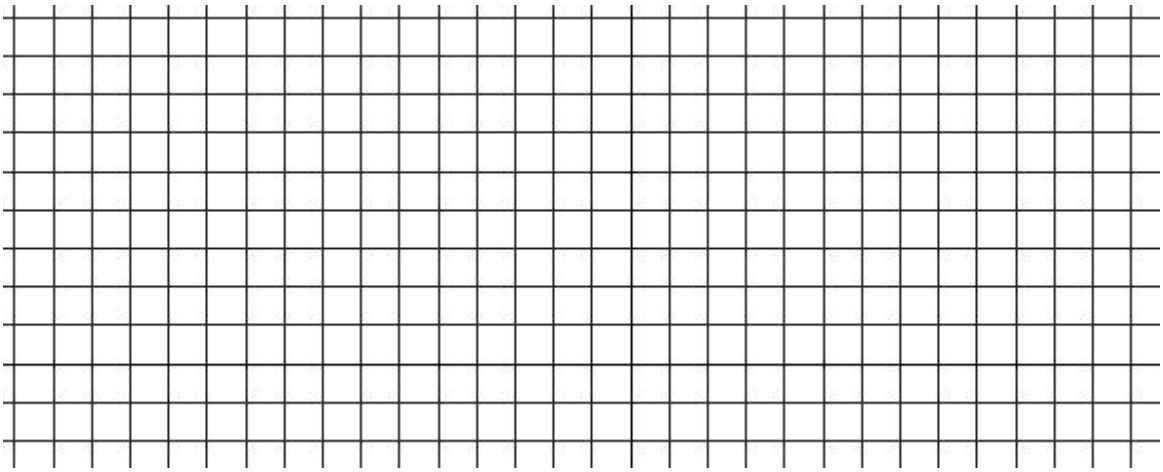
9. A window casts a shadow on the ground that is the shape of a trapezoid. Find the area that the shadow takes up.



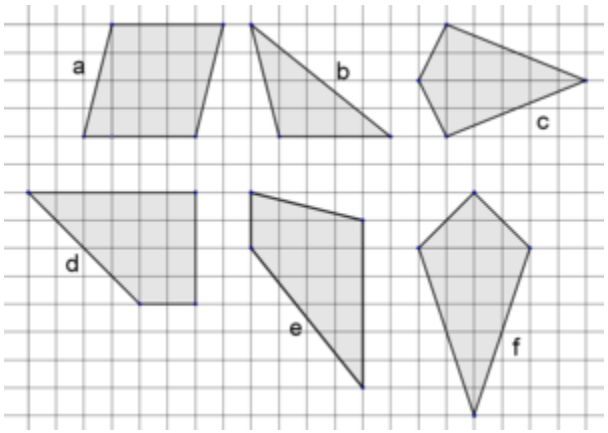
10. The area of a trapezoid is 600 square inches. One base of the trapezoid measures 10 inches and the other base measures 15 inches. Find the height of the trapezoid. Draw a label a picture if needed.

11. A traffic sign is the shape of a trapezoid; it has a total area of 7 square feet. The height of the sign is 2 feet. Find two possible values for the measurements of each base. Draw and label a picture to help you find the base measurements.

n#



12. Of the polygons shown, which have equal areas? Explain how you know.

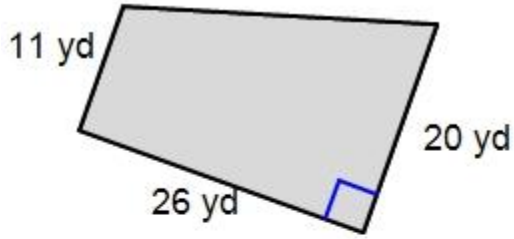


This is an Illustrative Mathematics Task

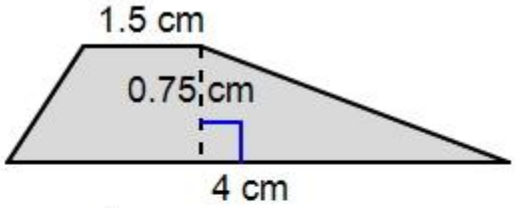
5.1d Homework: Area of Trapezoids

Directions: Find the area of each trapezoid.
*Figures are not drawn to scale.

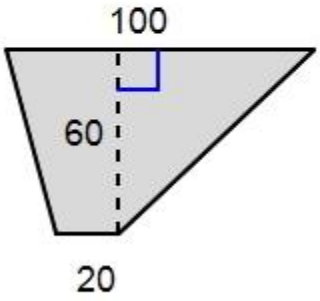
1.



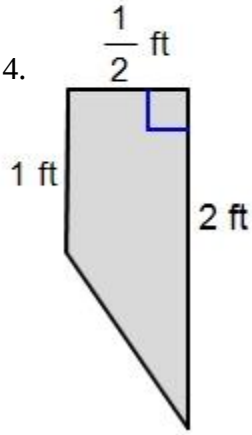
2.



3.



4.

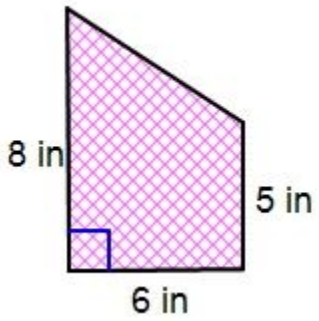


Find the area of each trapezoid, if needed draw and label a picture.

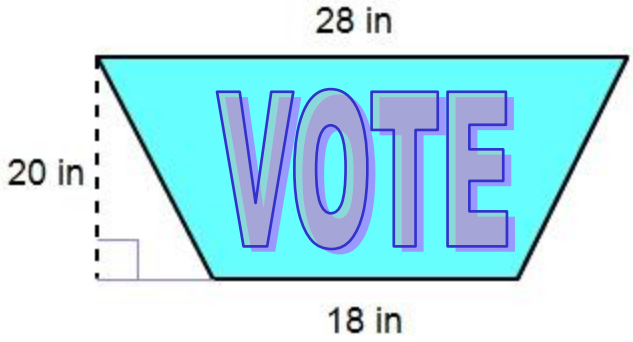
5. A trapezoid with base lengths of 14.8 yards and 20.3 yards and a height of 23.5 yards.

6. A trapezoid with base lengths of 105 cm and 80 cm and a height of 65 cm.

7. Lucy is cutting out fabric into trapezoids for a quilt that she is piecing together. She needs 8 trapezoids like the one pictured. How many square inches of fabric will she use to cut out all 8 trapezoids?

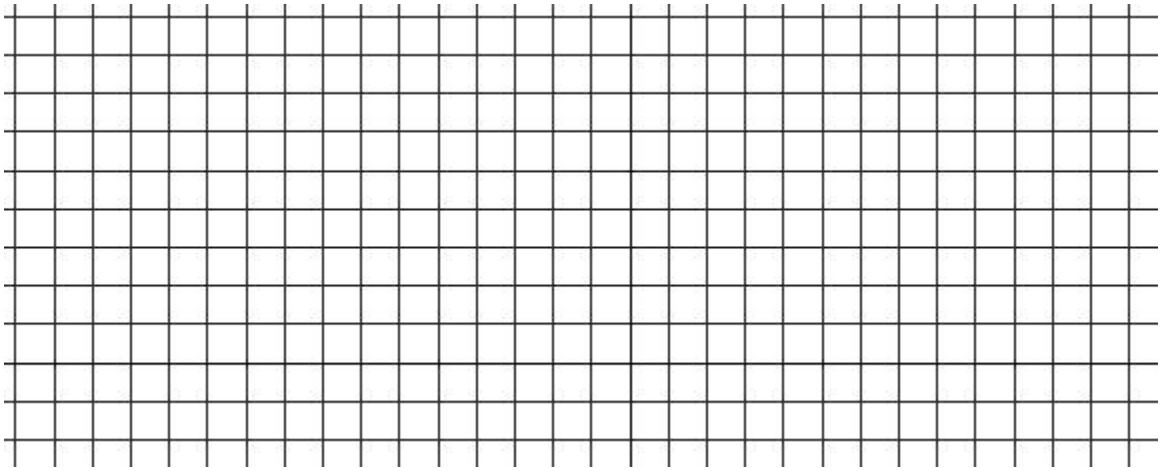


8. The sign for a student election campaign is shown. Find the total area of the sign.



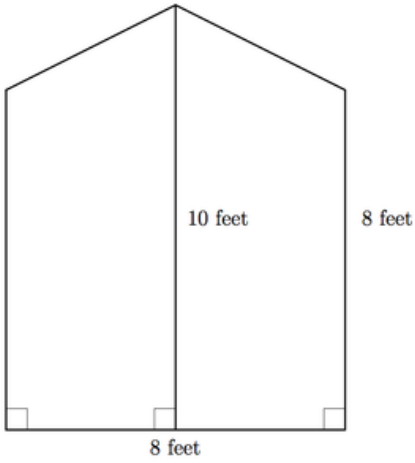
9. The area of a trapezoid is 1,125 square centimeters. One base of the trapezoid measures 35 cm and the other base measures 55 cm, find the height of the trapezoid. Draw and label a picture if needed.

10. The area of a trapezoid is 20 square meters. The height of the trapezoid is 10 meters. Find two possible values for the measurements of each base. Draw and label a picture to help you find the base measurements.



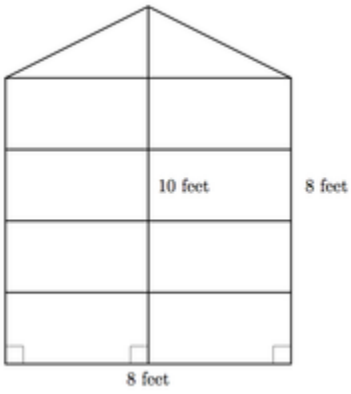
11. Jamie is planning to cover a wall with red wallpaper. The dimensions of the wall are shown.

a. How many square feet of wallpaper are required to cover the wall?



b. Wallpaper comes in long rectangular strips which are 24 inches wide. If Jamie lays the strips of wallpaper vertically, can she cover the wall without wasting any wallpaper? Explain.

c. If Jamie lays the strips of wallpaper horizontally, can she cover the wall without wasting any wallpaper? Explain.

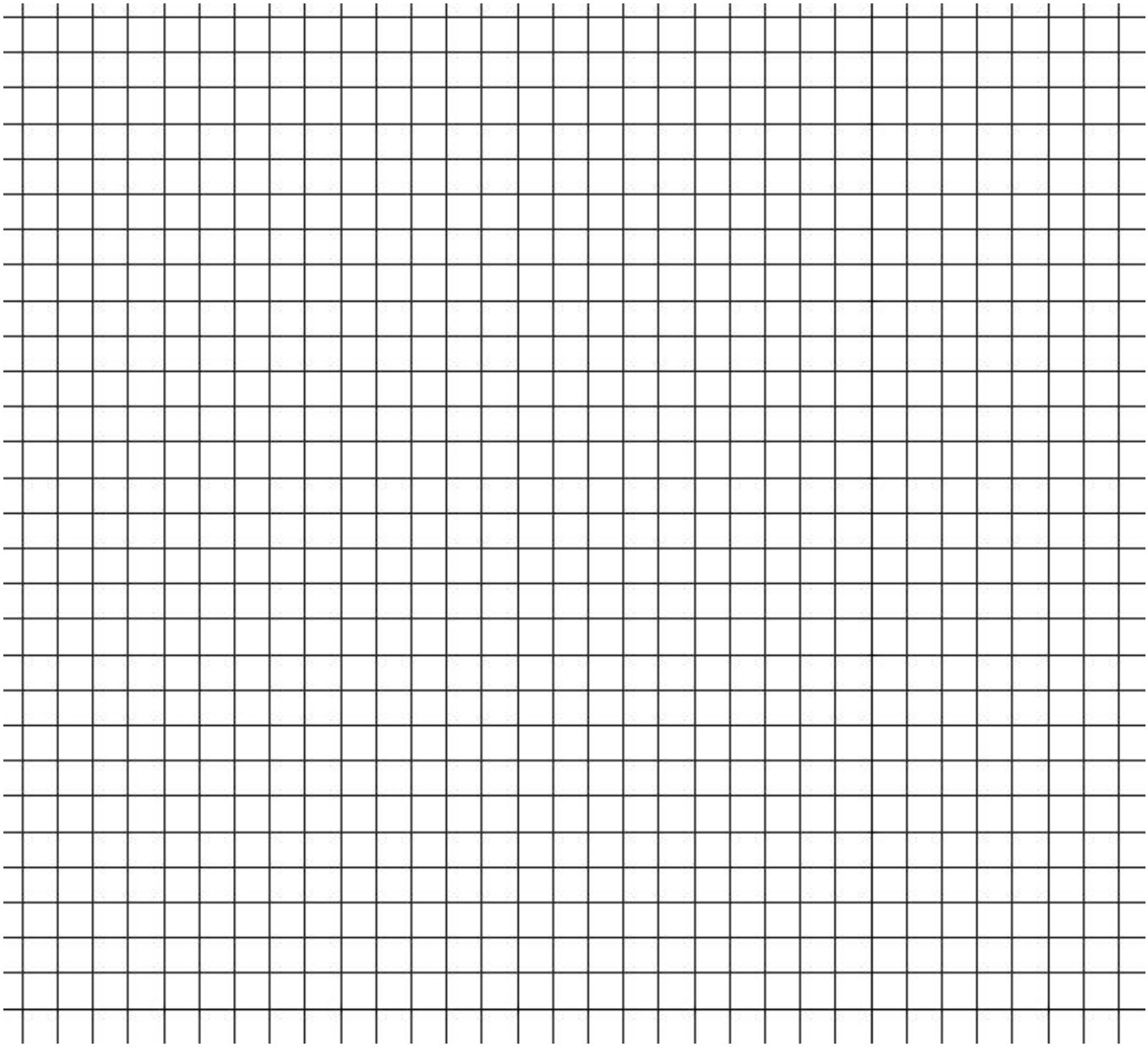


*[*This is an Illustrative Mathematics Task]*

5.1e Class Activity: Area of Irregular Figures

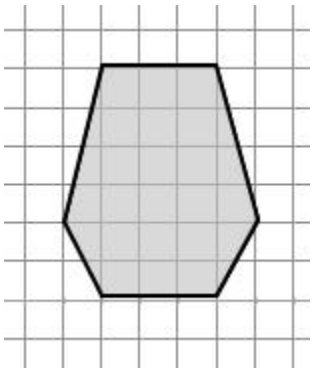
1. On the graph below one square represents one square unit. Draw one of each of the following;
   a. A rectangle with an area of 24 square units
   b. A parallelogram with an area of 24 square units
   c. A triangle with an area of 24 square units
   d. A trapezoid with an area of 24 square units
   e. A 5 sided polygon with an area of 24 square units
   f. A polygon with more than 5 sides with an area of 24 square units.

   Be ready to explain why each polygon has an area of 24 square units.

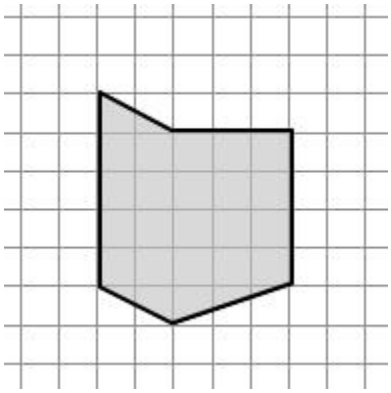


2. Find the area of each shaded figure. Show all your work.

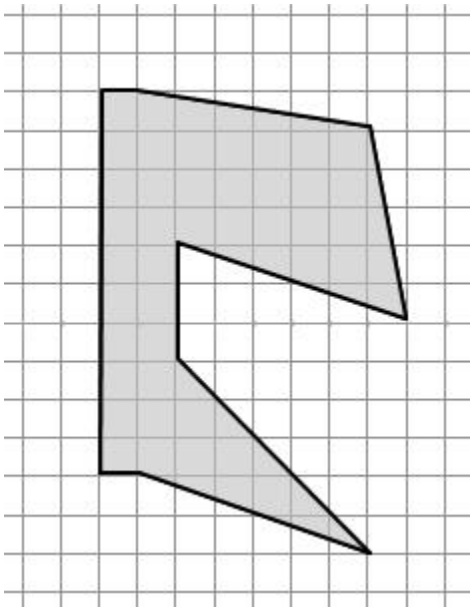
a.



b.

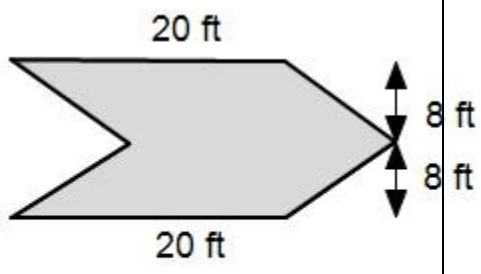


c.

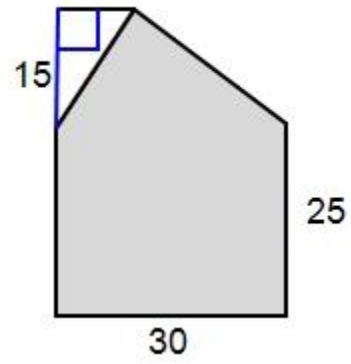


3. Find the area of each shaded figure. Assume that angles that appear to be right angles are right angles.

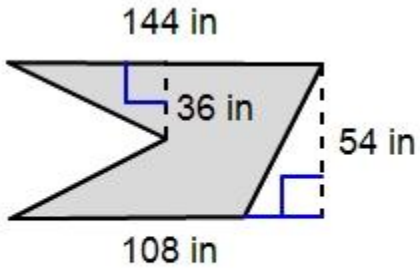
a.



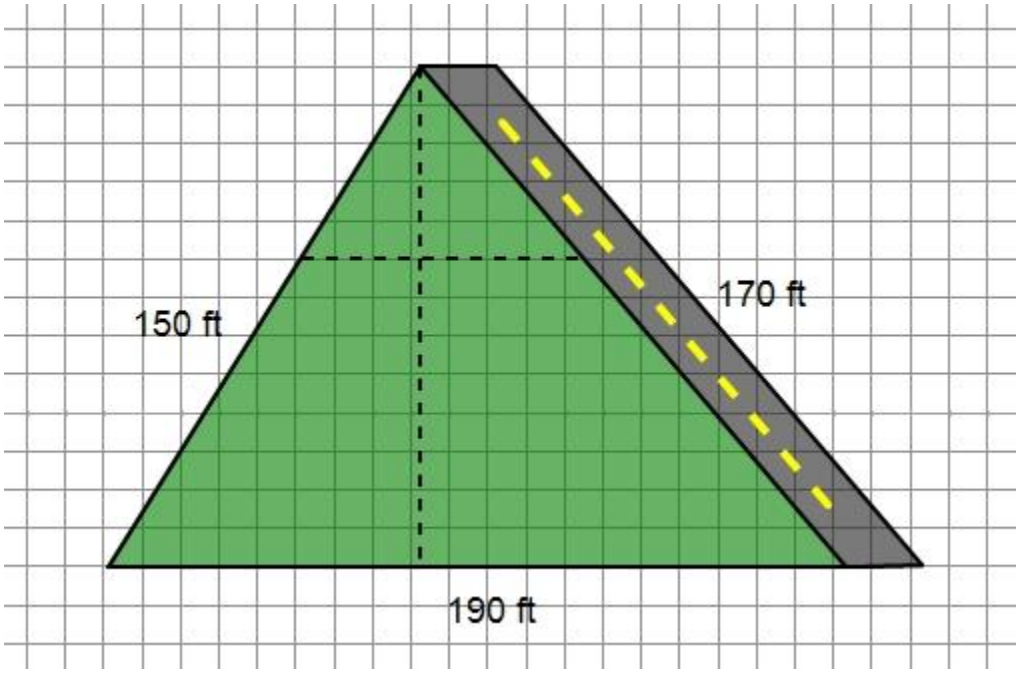
b.



c.



4. The Suderman family bought a triangular piece of property, with 170 feet of roadside frontage as shown below. The distance between grid lines is 10 feet, and thus each grid square has area 100 square feet. The lengths of the other two sides are (roughly) 190 feet (the bottom side of the figure) and 150 feet (the side to the left). Mr. Suderman decides to divide the property into two pieces with the dashed line as shown; on the top piece he will put his house and on the bottom piece he will create a garden.



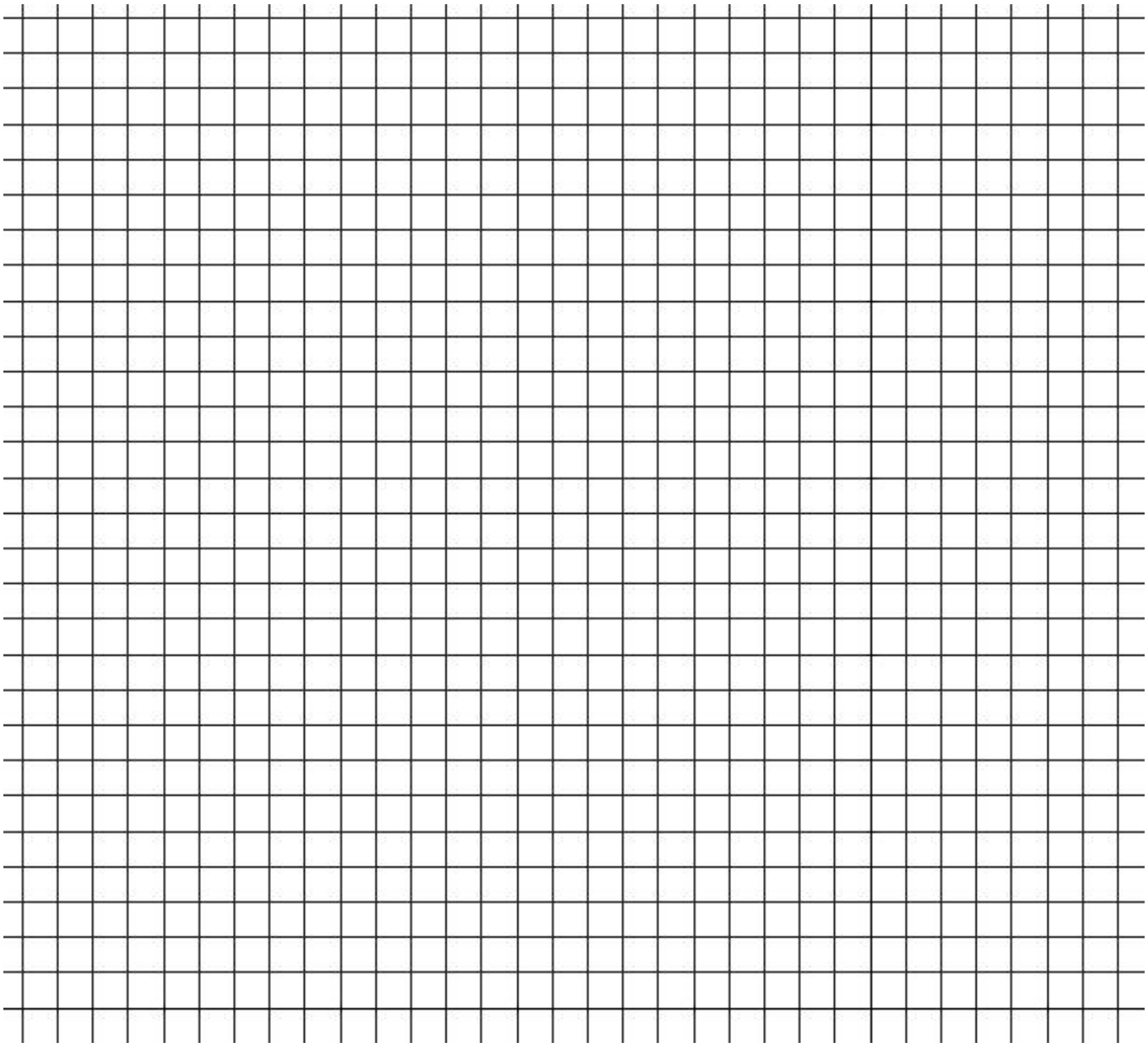
a. What is the approximate area of the house lot?

b. What is the approximate area of the garden?

5.1e Homework: Area of Irregular Figures

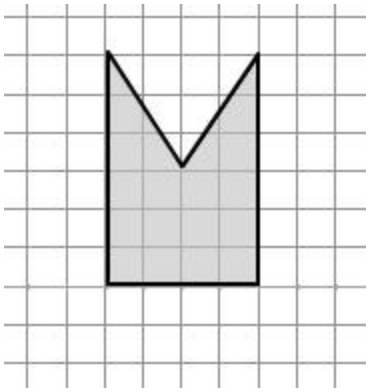
1. On the graph below one square represents one square unit. Draw one of each of the following;
   a. A blue 5 sided polygon with an area of 30 square units
   b. A green 6 sided polygon with an area of 40 square units.
   c. A yellow7 sided polygon with an area of 50 square units.

Show how you know that each polygon has the given area.

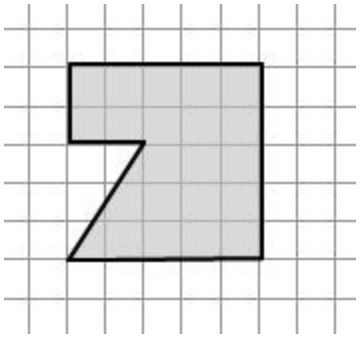


2. Find the area of each shaded figure. Show all your work.

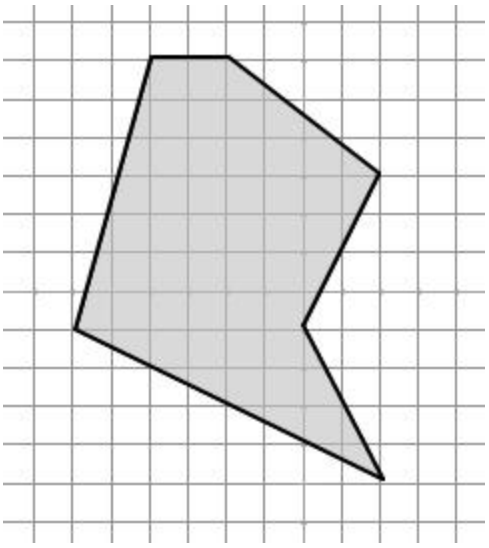
a.



b.

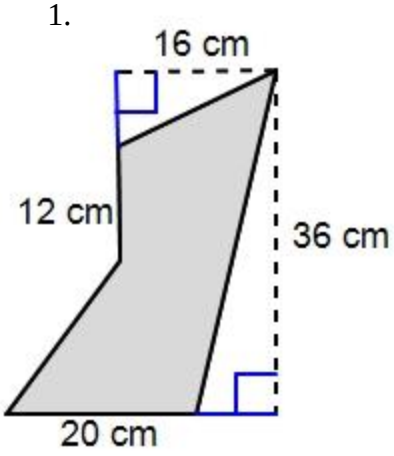


c.

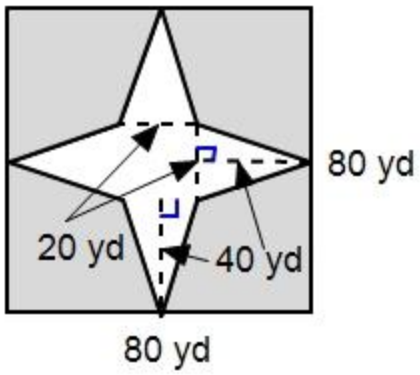


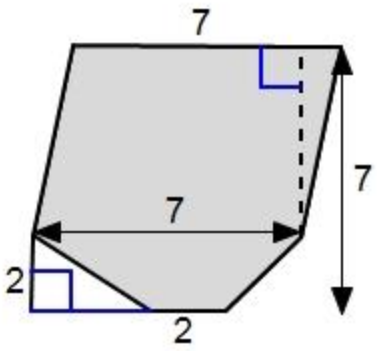
3. Find the area of each shaded figure. Assume that angles that appear to be right angles are right angle.

1.

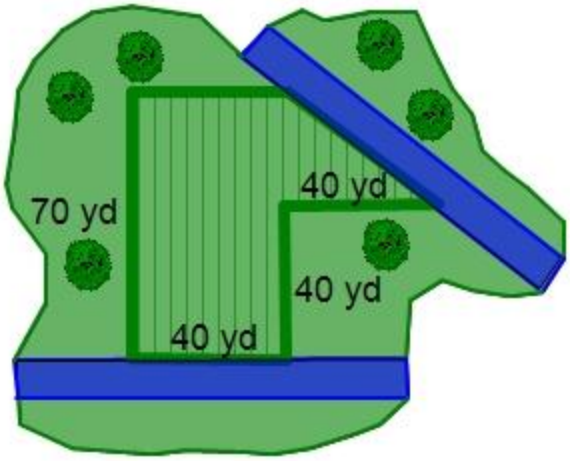


2.

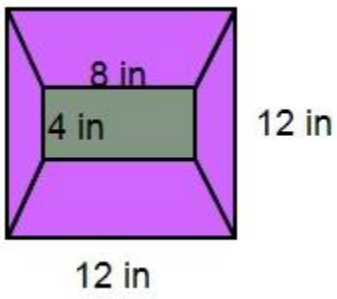




3. A fairway on a golf course is enclosed by two streams as shown. Find the area of the fairway.



4. Ruby is designing a quilt block like the one below.



   a. What is the area of the purple section of the block?

   b. How much purple fabric will Ruby need if she would like 8 of these blocks in a quilt?

Section 5.2: Polygons in the Coordinate Plane

Section Overview:
This section begins with students graphing and connecting coordinates to form polygons in the coordinate plane. Then they turn to finding the length of vertical and horizontal lines in the coordinate plane. They do this either by graphing points and then counting the number of spaces between the points or examining the difference between the set of *x*-coordinates or the set of *y*-coordinates. Once students are comfortable with graphing polygons and finding their vertical and horizontal side lengths they turn to using these measurements to find area. Often this includes shifting their perspectives and decomposing given polygons into triangles and rectangles of which they can easily find the area of. Finally students use the coordinate plane to represent real-world objects and locations and use their developed techniques of finding distance and area to answer questions and make conjectures about the relationship and attributes of these objects.

Concepts and Skills to Master in this Section:

By the end of this section, students should be able to:

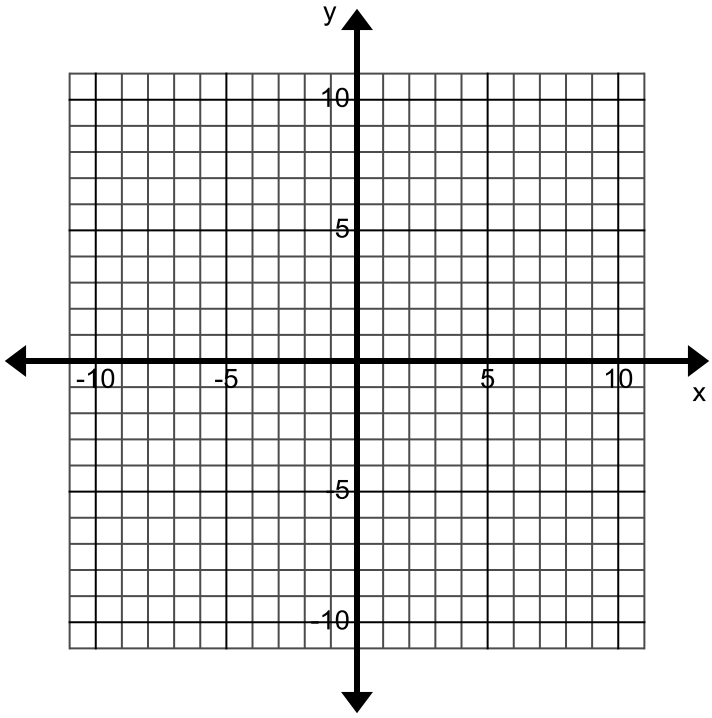
1. Draw a polygon in the coordinate plane from a given set of coordinates for the vertices.
2. Find the length of vertical and horizontal lines in the coordinate plane.
3. Find the area of a polygon in the coordinate plane.
4. Use polygons in the coordinate plant to solve real-world and mathematical problems.

5.2a Class Activity: Graphing Polygons in the Coordinate Plane

1. Plot the points on the coordinate plane, then connect the points in the order they are listed to form a polygon.

   Polygon 1: (2, 5), (6, 5), (6, 3), (2, 3)

   Polygon 2: (8, 6), (11, 1), (8, 1)



2. What polygons are formed by the points?

3. Find the length of each horizontal segment, record your answers in the table below

From Point	To Point	Length
(2, 5)	(6, 5)	
(6, 3)	(2, 3)	
(8, 1)	(11, 1)	

4. What do you notice about the ordered pairs that line up horizontally?

5. Find the length of each vertical segment, record your answers in the table below

From Point	To Point	Length
(2, 5)	(2, 3)	
(6, 5)	(6, 3)	
(8, 1)	(8, 6)	

6. What do you notice about the ordered pairs that line up vertically?

7. Is there are way that you could determine the length of these segments without plotting the points.

8. Find the area of each polygon if one square represents one square unit.

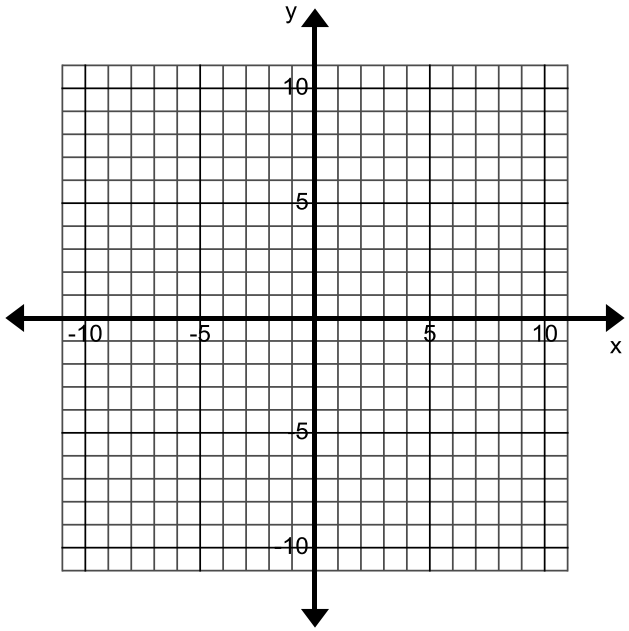
9. Use your method on the previous page to find the length of the segments created by connecting the following points. First determine if the segment formed is horizontal or vertical.

   Polygon 3: (3, 2), (4, 2), (4, 7), (3, 7)

   Horizontal line segment coordinates:

   Vertical line segment coordinates:

10. Plot the points on the coordinate plane, connect the points in the order they are listed. Check your answers above by counting out the distance for each line segment.



11. Will the subtraction method work for segments that extend into other quadrants? Try it by plotting the points below. Connect the points in the order they are listed. Count out the distance for each horizontal and vertical line segment. Use your method from above to find the distance and compare.

    Polygon 4: $(1, 2), (-3, 2), (-3, -4)$, and $(1, -4)$

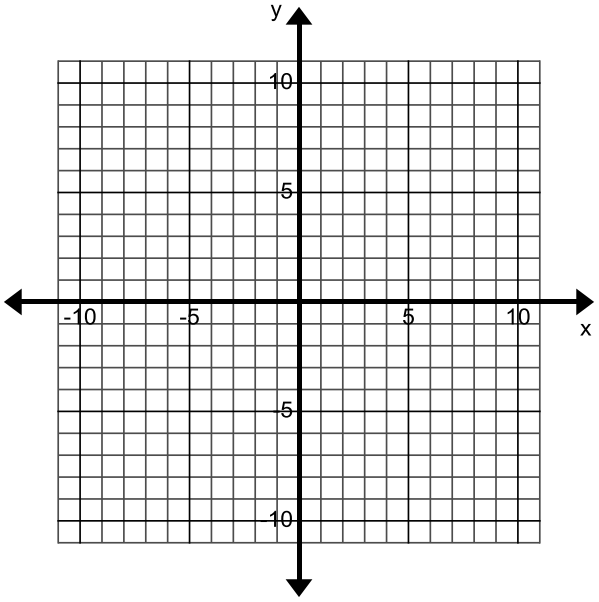
12. Find the area of each polygon

13. Without plotting determine whether the line segment that joins each pair of points is horizontal, vertical, or neither. Justify your answer.

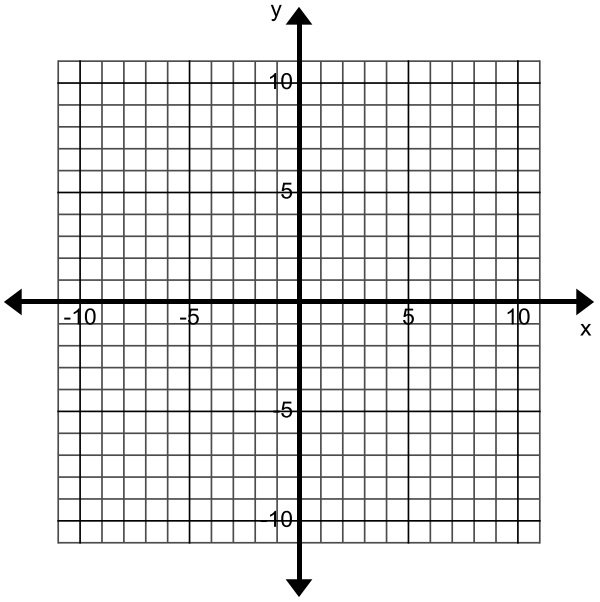
    a. $(-10, 3)\ (4, 3)$ b. $(0, 7)\ (4, 0)$ c. $(-3, 4)\ (-3, 5)$

14. Draw a polygon on the coordinate plane by plotting each set of vertices and connecting them in the order they are listed. Label the length of each vertical and horizontal line segment and find the area of the polygon if each square represents one square unit.

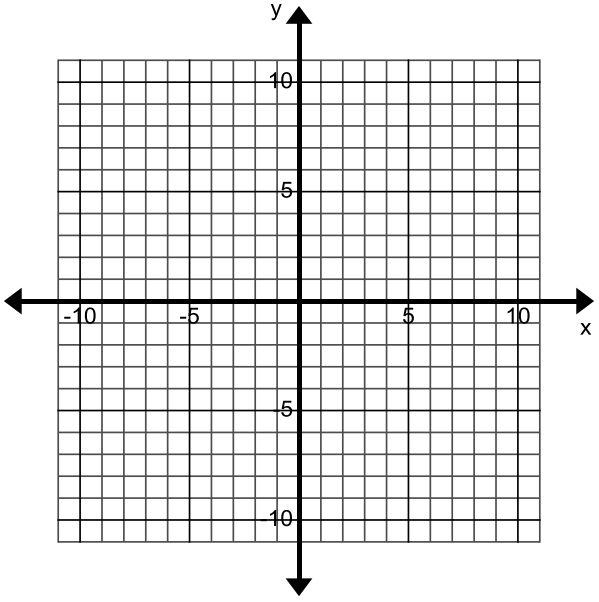
a. $(-2, 2), (4, 2), (4, -4), (-8, -4), (-8, -2), (-2, -2)$



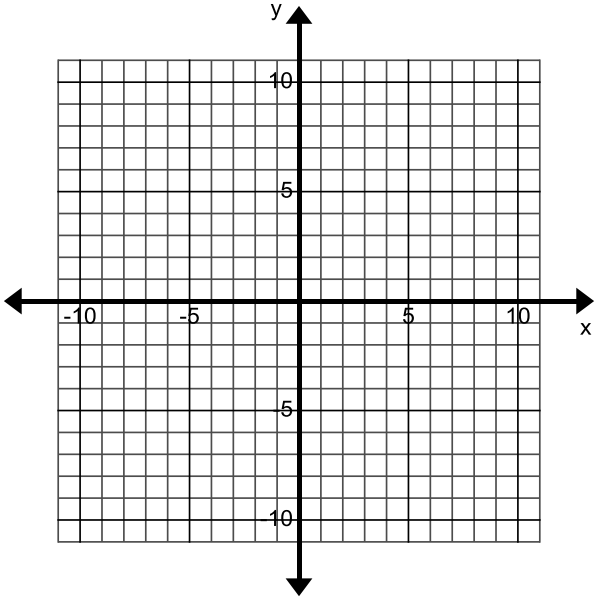
b. $(-2, -2)(2, -2), (2, 2), (6, 2), (6, -6), (-4, -6), (-4, -4), (-2, 4)$



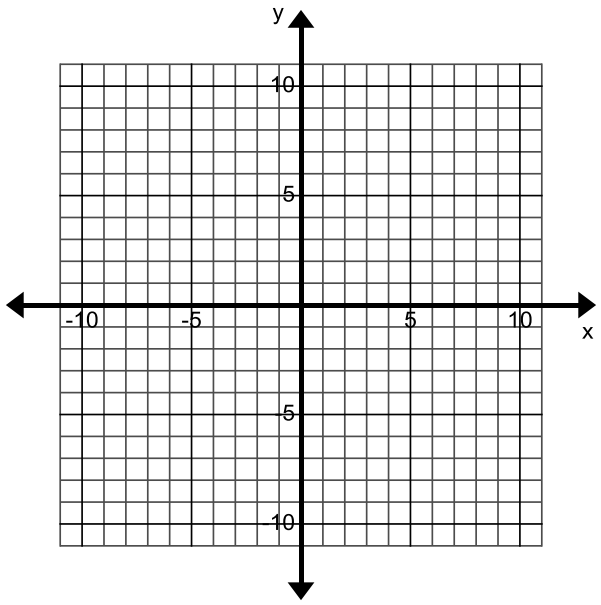
c. $(3, 5), (7, 2), (7, -5) (3, -2)$



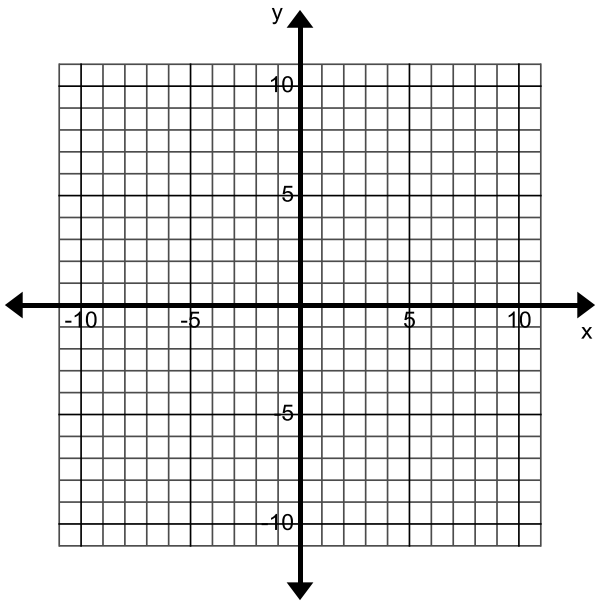
d. $(-2, 5)(-2, -2), (3, -2), (5, 5),$



e. $(-3, 10), (9, 4), (9, -10),$



f. $(-5, 8), (-8, 2), (-3, -2),$



5.2a Homework: Graphing Polygons in the Coordinate Plane

1. Without plotting determine whether the line segment that joins each pair of points is horizontal, vertical, or neither. Justify your answer.

   a. $(-4,-6)\ (-4,3)$ b. $(0,5)\ (0,10)$ c. $(-2,4)\ (-2,5)$

   d. $(3,-6)\ (-4,-6)$ e. $(14,4)\ (-14,7)$ f. $(8,8)\ (0,8)$

2. Name any two points that form a vertical line segment.

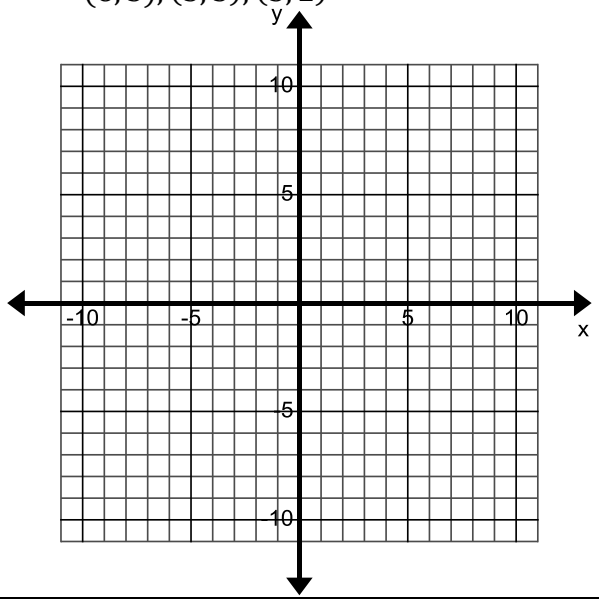
3. Name any two point that form a horizontal line segment

4. Name two points that form a vertical line segment that has a length of 8

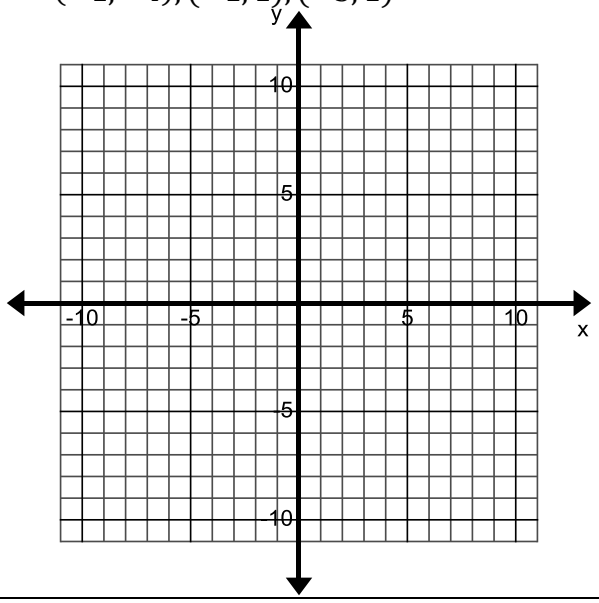
5. Name two points that form a horizontal line segment that has a length of 4.

Draw a polygon on the coordinate plane by plotting each set of vertices and connecting them in the order they are listed. Label the length of each horizontal and vertical line segment and find the area of the polygon if each square represents one square unit.

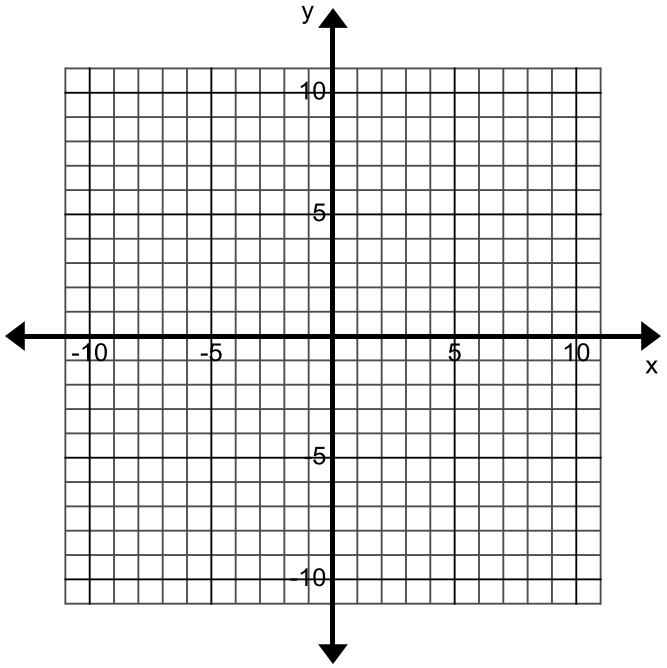
6. $(0,2),(0,-1),(5,-1),(5,-4),(8,-4),(8,5),(5,5),(5,2)$



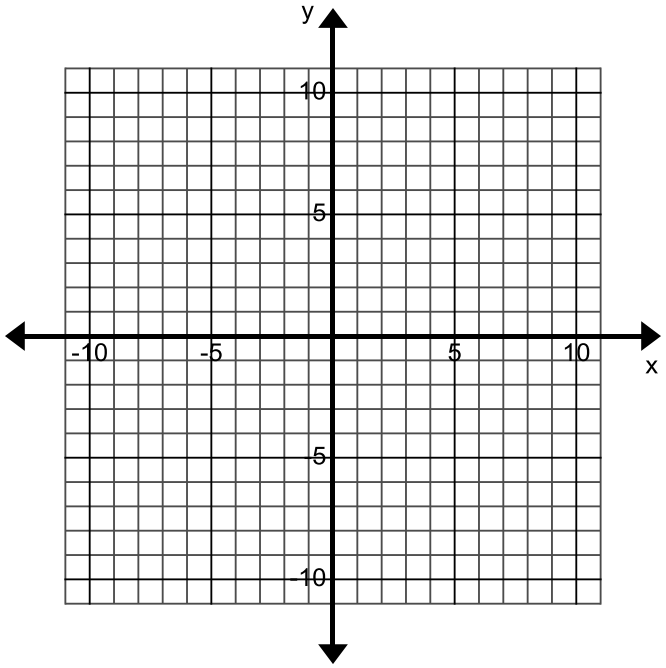
7. $(-5,7)(1,7),(1,2),(5,2),(5,-4),(-1,-4),(-1,1),(-5,1)$



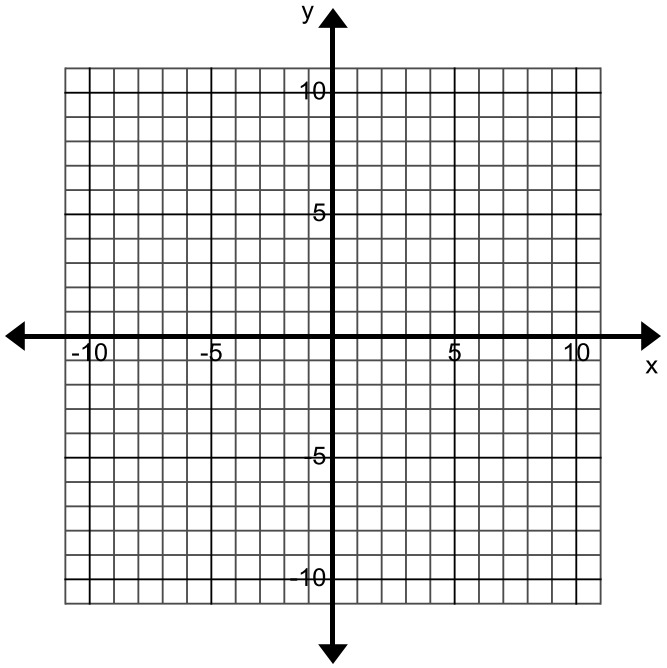
8. $(-3, 5), (1,2), (-3, -3)$



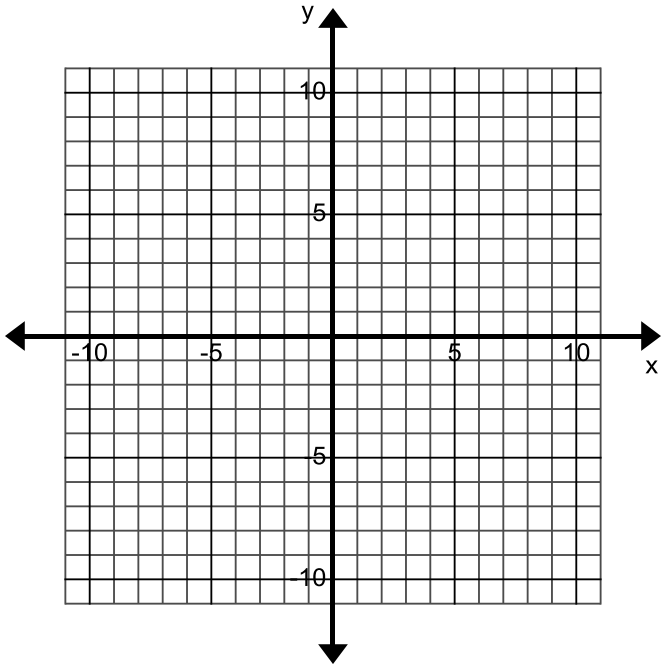
9. $(-5, -3), (4, -3), (5, -5), (-4, -5)$



10. $(7,9), (-1, 9), (-4, 4)$



11. $(-4, 4), (-9, -1), (-4, -6), (1, -1)$

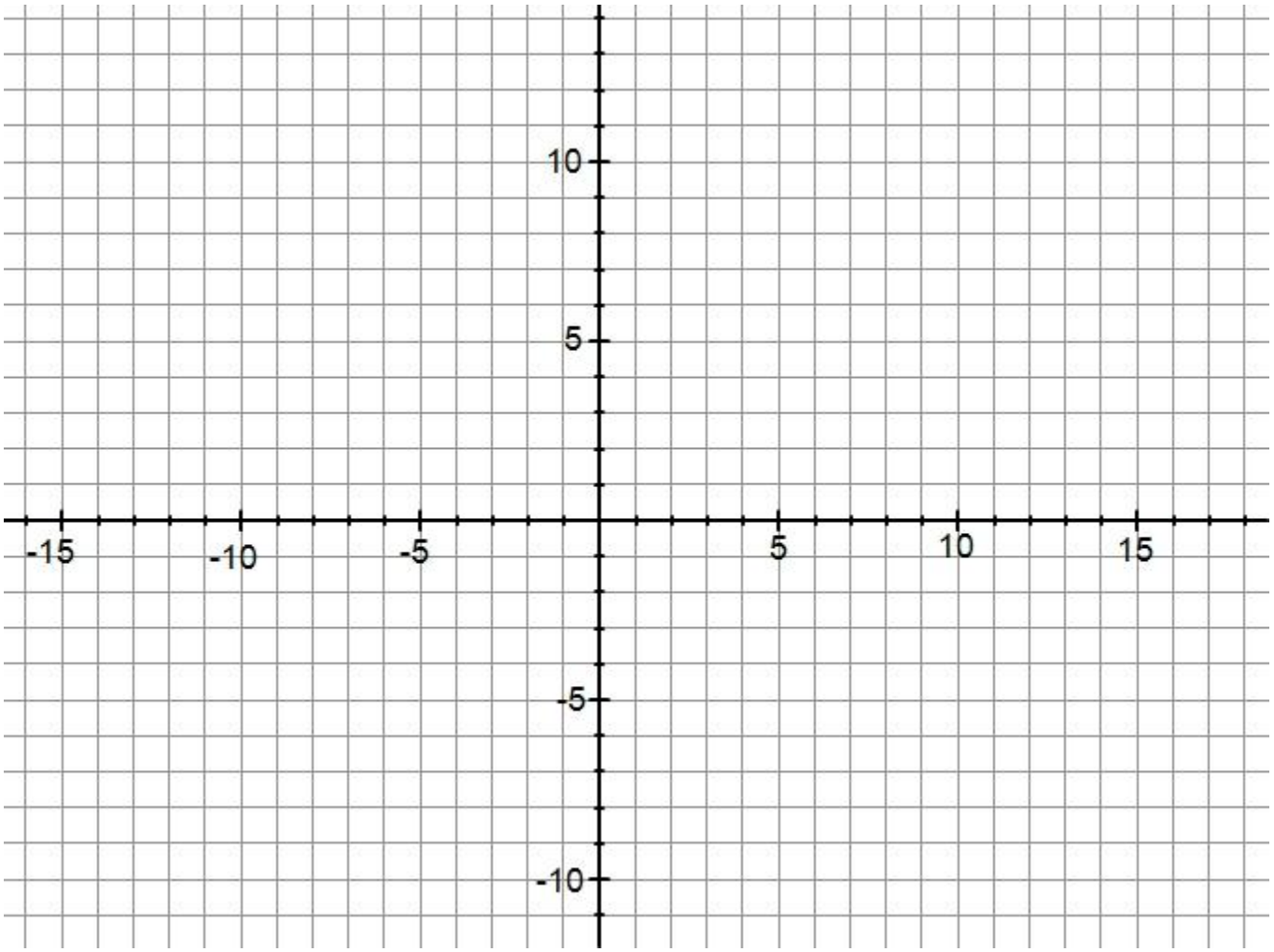


5.2b Class Activity: Finding Area of Polygons in the Coordinate Plane

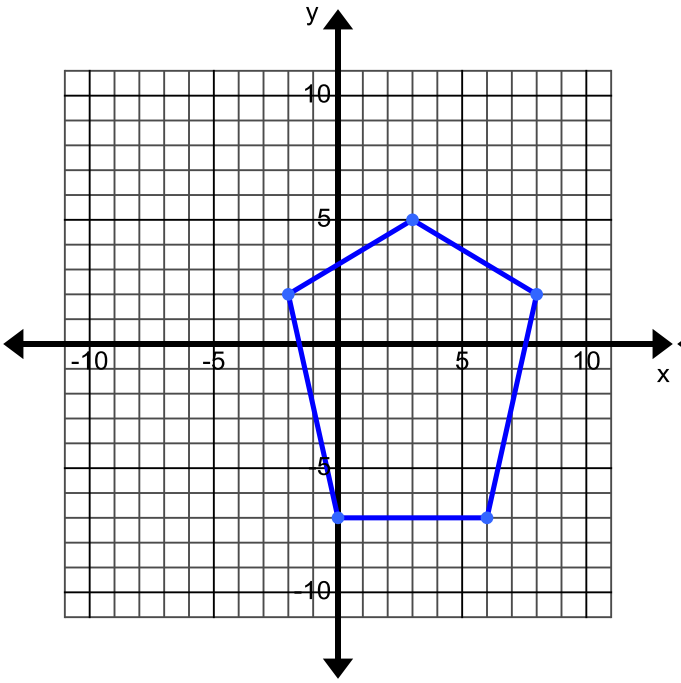
The vertices of five polygons are given below. For each polygon:

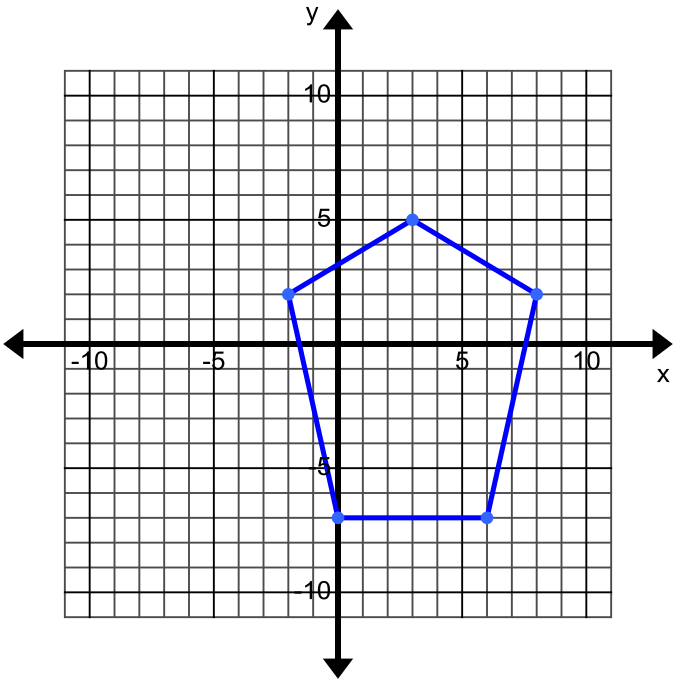
- Plot the points in the coordinate plane and connect the points in the order that they are listed.
- Color the shape the indicated color.
- Find the area of the polygon if one square represents one square unit. Be sure to show all your work

1. Blue Polygon: $(-4, 8), (-4, 3), (-7, 1), (-9,4), (-9, 6), (-7, 8)$
2. Pink Polygon: $(-3, 11), (0, 13), (5, 13), (4, 8), (0, 7)$
3. Green Polygon: $(10, 9), (13, 4), (8, 6)$
4. Orange Polygon: $(6, 5), (7,0), (9, -2), (9, -4), (3, -4), (3, 0)$
5. Yellow Polygon: $(-7, -2), (-5, -2), (-3, -4), (-5, -9), (-7, -9), (-9, -4)$



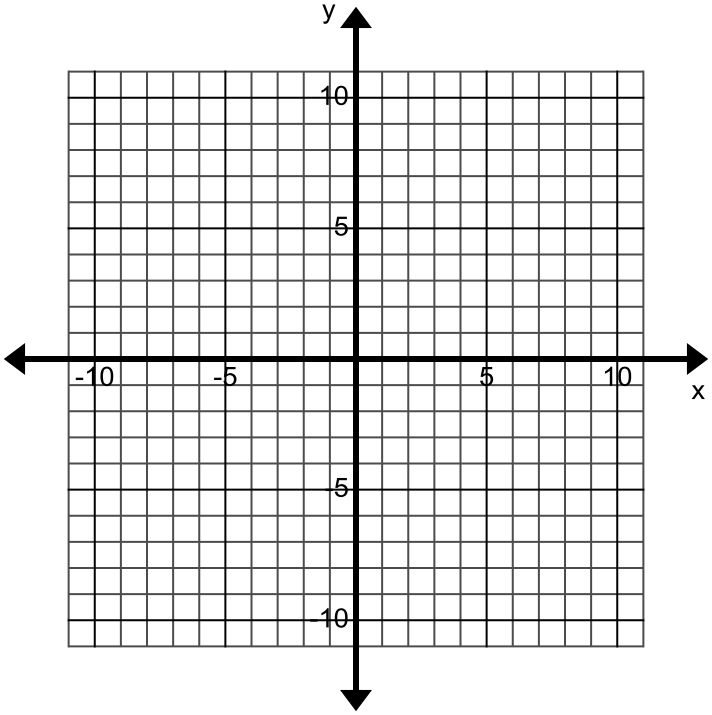
6. Show two different methods for finding the area of the polygon. Two copies are provided.





7. Two consecutive vertices (vertices that are next two each other) of a rectangle are $(5, 3)$ and $(5, 6)$, the area of the rectangle is 12. Name two possible locations of the other two vertices of the rectangle. Use the coordinate plane if needed.

8. Two vertices of a triangle are $(-3, 4)$ and $(-7, 4)$, the area of the triangle is 8. Name 3 possible locations of the other vertex of the triangle. Use the coordinate plane if needed.

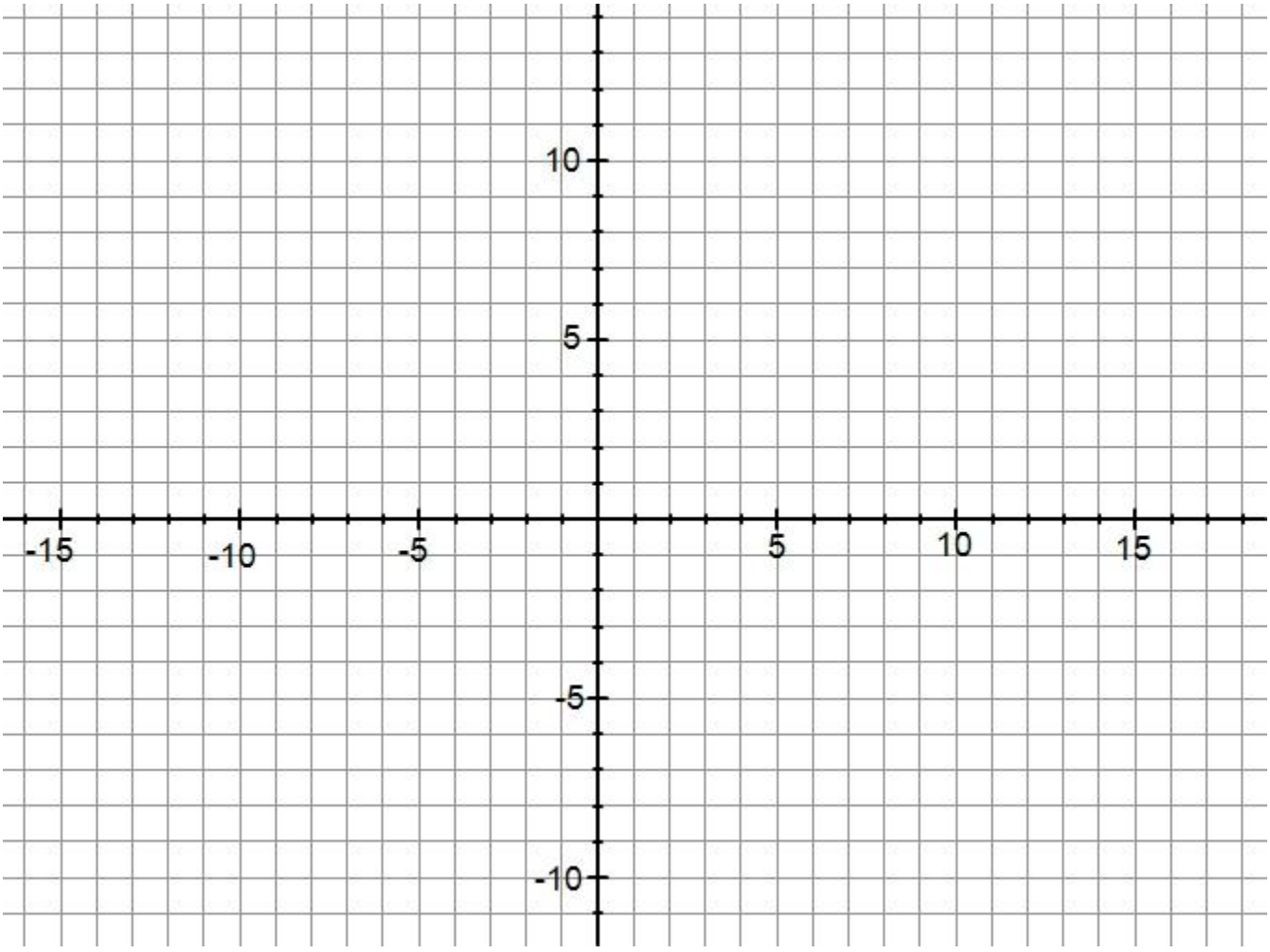


5.2b Homework: Finding Area of Polygons in the Coordinate Plane

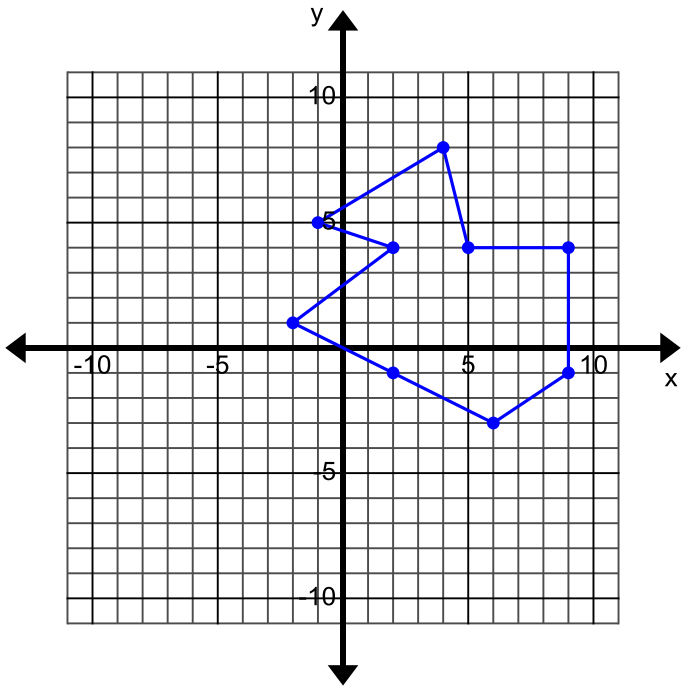
The vertices of four polygons are given below. For each polygon:

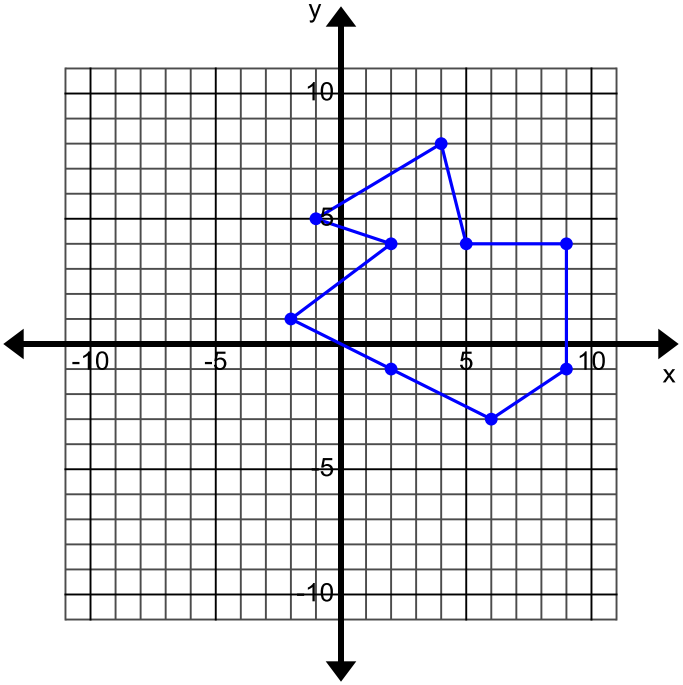
- Plot the points in the coordinate plane and connect the points in the order that they are listed.
- Color the shape the indicated color.
- Find the area of the polygon if one square represents one square unit. Be sure to show all your work.

1. Blue Polygon: $(0, 6), (4, 10), (6, 5), (6,4), (5, 4)$
2. Green Polygon: $(-7, 5), (-8, 8), (-5, 11), (-2, 8), (-6, 8)$
3. Orange Polygon: $(-5, 2), (-2, -2), (-2, -6), (-6, -6), (-8, -7), (-8, -2)$
4. Yellow Polygon: $(6,1,), (9, 1), (11, -2), (9, -4), (9, -8), (6, -8), (6, -4), (2, -2)$

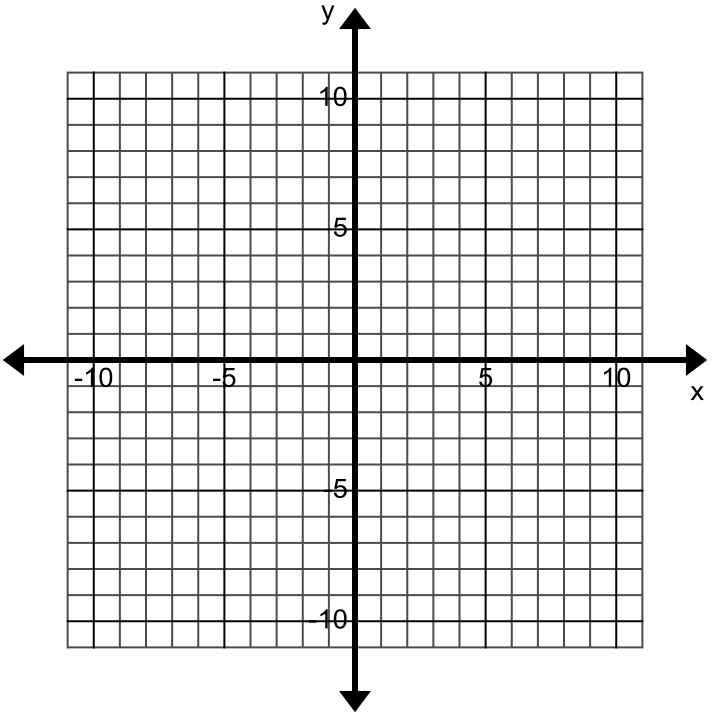


5. Show two different methods for finding the area of the polygon. Two copies are provided.





7. Draw one of the letters in your name using rectangles and/or triangles on the coordinate plane below. Then find the area of the letter.



5.2c Class Activity: Using Polygons in the Coordinate Plane to Solve Real World Probems

1. A new park is being designed for your city. The plans for the design are being drawn on the coordinate plane below. The vertices given form a polygon that represents the location for 5 different features in the park.

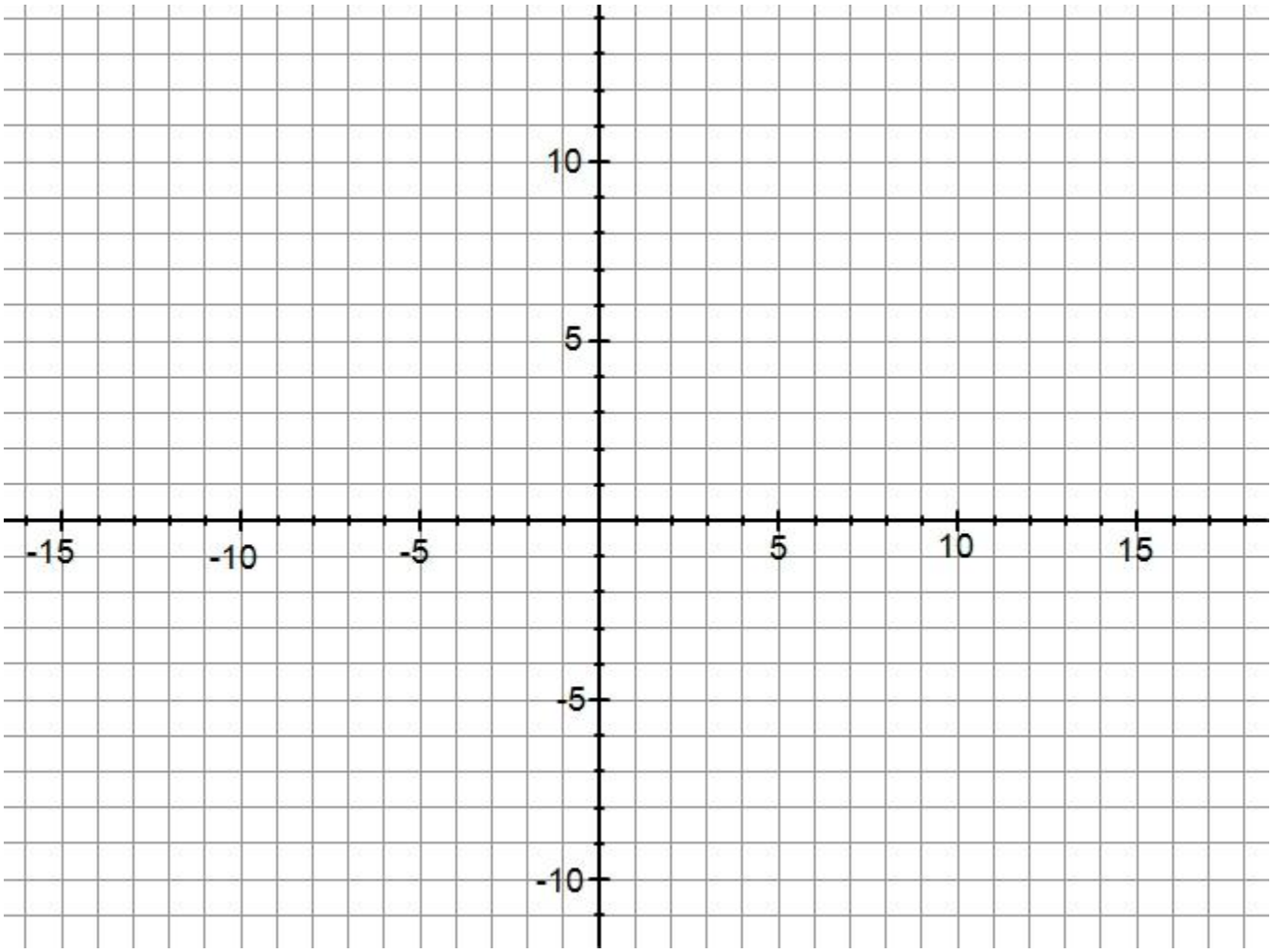
    a. Plot each set of points in the order they are given to form a polygon. Label the feature that the polygon represents on the graph.

Playground: $(-5,-2),(-8,-2),(-8,-5),(-11,-5),(-11,-8),(-8,-8),(-8,-11),(-5,-11)$

Splash Pad: $(10,4),(13,8),(6,8)$

Restrooms: $(-4,5),(-4,8),(-7,10),(-10,7),(-7,5)$

Picnic Pavilion: $(8,-1),(6,-3),(4,-3),(2,-1),(2,1),(4,3),(6,3),(8,1)$



If each square on the graph represents 1 square meter, answer the questions that follow.

b. Cement needs to be poured to lay the foundation for the bathroom and the splash pad. How many square meters of cement will the city need for the foundation of these two features?

c. The playground is going to be covered with wood chips, how many square meters of wood chips will the city need for the playground?

d. The citizens of the city have asked that the playground have a fence around its perimeter. How many meters of fencing will they need for the playground?

e. The foundation of the pavilion picnic area is going to be covered with special pavers. How many square meters of pavers will they need for the pavilion?

f. There are drinking fountains at the four corners of the walking path. The drinking fountains are at the points $(16, 12), (16, -12), (-13 - 12), (-13, -12)$. Draw these drinking fountains on the graph and find the distance a person will walk if they do three laps around the walking path.

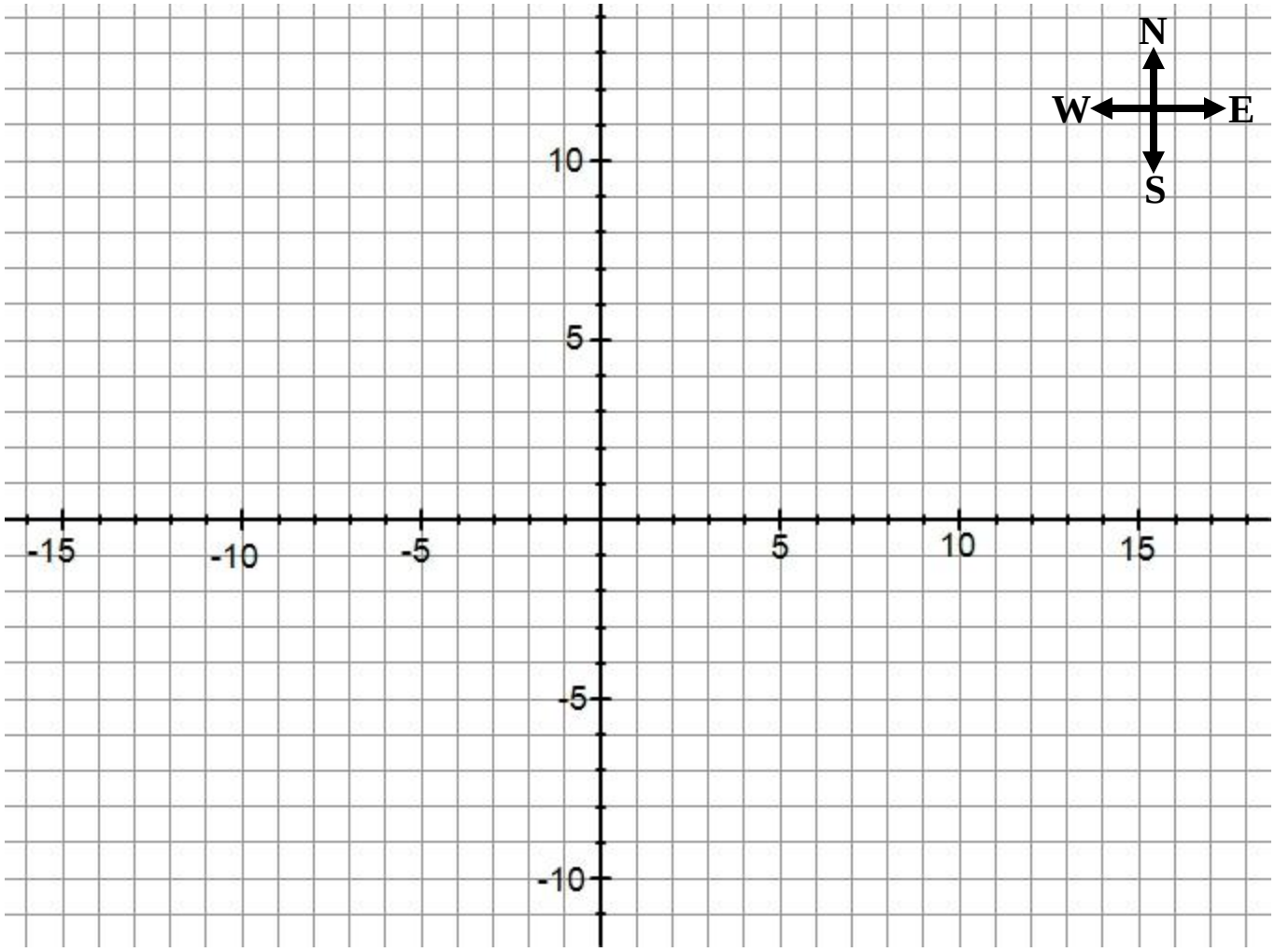
g. The city council has received word that they have enough money in the budget to build a rectangular sand volleyball court. They would like the dimensions of the court to be 4 meters by 7 meters. If two coordinates that connect to form one side of the court are $(-1, -5)$ and $(-1, -9)$ what are the other two coordinates that form the court? Keep in mind that you do not want the court to run into any other feature.

h. The youth city council would like to install a rock climbing wall in the park. They have been allotted 12 square meters to use for their wall. Draw a polygon that could represent the location of the rock climbing wall.

i. The remainder of the park is to be covered with grass. How many square meters of grass will the city need to plant?

j.

2. A coordinate grid represents the map of a city. Each square on the grid represents one city block.



a. Heather's apartment is at the point $(5, 7)$. She walks 4 blocks south, then 8 blocks west, then 4 blocks north, and then finally 8 blocks east back to her apartment. How many blocks did she walk total? Describe the shape of her path. Mark and label her apartment and highlight her walk.

b. Draw and describe in words least two different ways you could walk exactly 20 blocks and end up back where you started.

c. Carl lives at the point $(\mathbf{5}, \mathbf{-5})$. Find the distance between Heather's house and Carl's house, and then mark and label Carl's house.

d. Heather's work is directly due west of her house and is half the distance between Heather's house and Carl's house. Mark where Heather works on the graph and write the order pair.

e. Four corners of the city park are located at the points $(-8,-2), (-8,2), (-4,2)$, and $(-4,-2)$. Without graphing determine what kind of quadrilateral is made by the boundaries of the park? Explain how you know. Then plot and connect the ordered pairs to check your answer. Label the park on the graph.

f. What is the area of the city park?

g. The library is exactly 12 blocks west of Heather's apartment and 5 blocks north of the park. Mark the library on the graph and write the order pair that represents its location.

h. Lori lives at the point $(-2,1)$. She goes on a walk that starts and ends at her house. She states that the path that she traveled enclosed a polygon with an area of 12 square blocks and she cut through the park. Draw two possible shapes with a dotted line that her walk could have taken.

5.2c Homework: Using Polygons in the Coordinate Plane to Solve Real World Probems

1. You are drawing up plans for a map that is featured in a video game. The vertices given below form a polygon that represents the location for 5 different features on the map.

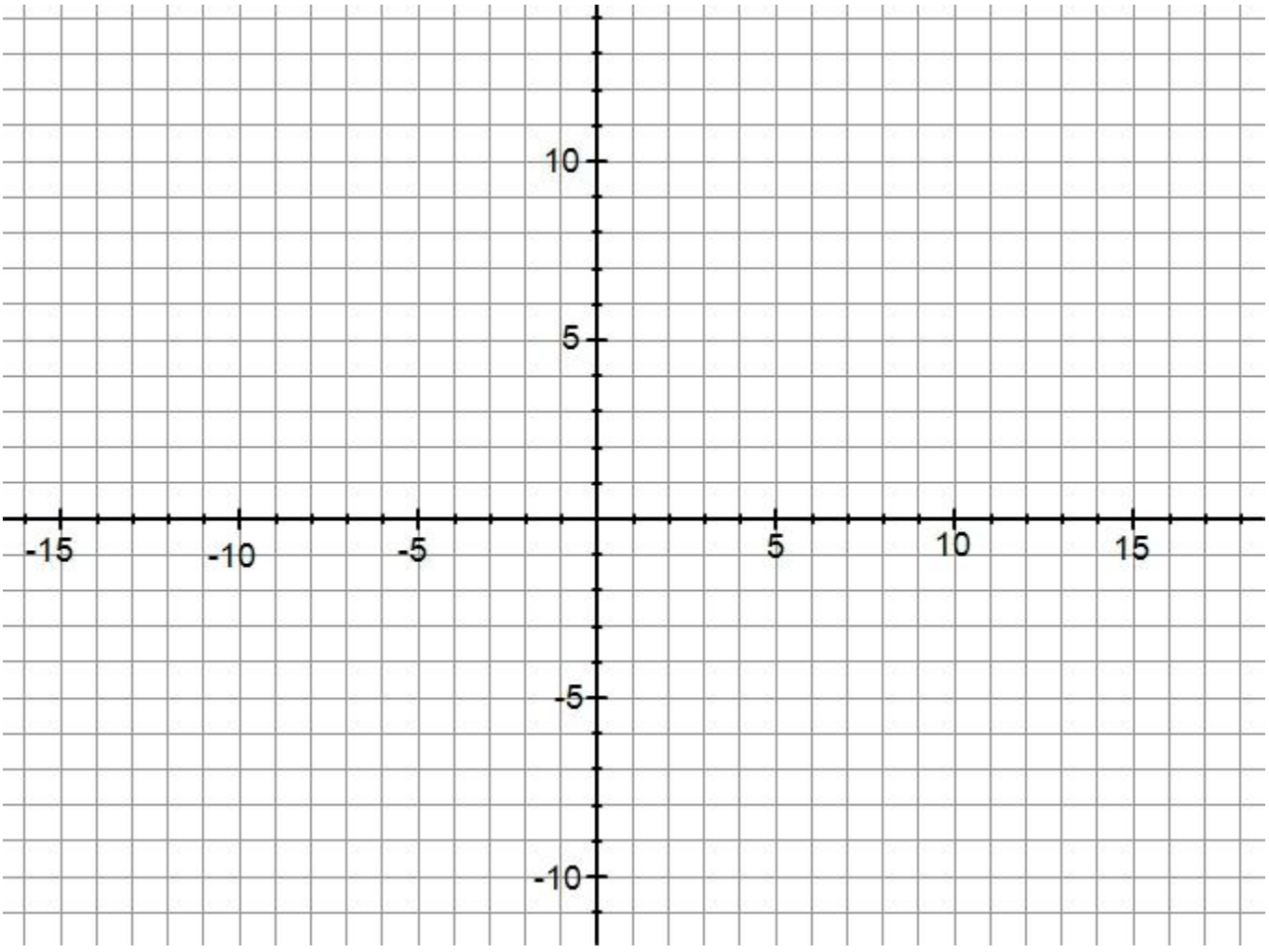
   a. Plot each set of points in the order they are given to form a polygon. Label the feature that the polygon represents on the graph.

   Crystal Palace: $(-5, 3), (-5, 6), (-7, 8), (-11, 9), (-10, 5), (-8, 3)$

   Battlefield Bunker: $(6,0), (10, 0), (10, 7), (7, 7), (7, 8), (5, 8), (5, 5), (8, 5), (8, 2), (6, 2)$

   Captain's Fortress: $(4, -9), (8, -7), (12, -7), (12, -2), (10, -5), (6, -3), (6, -7)$

   Lava Lake: $(1, 1), (3, -1), (0, -3), (-7, -1), (-9, -1), (-9, 1)$



Each square on the grid represents one square pixel unit.

b. You want the area of the Crystal Palace to be blue, how many square pixels of blue will you need?

c. You want the area of the Lava Lake to be red, how many square pixels of red will you need?

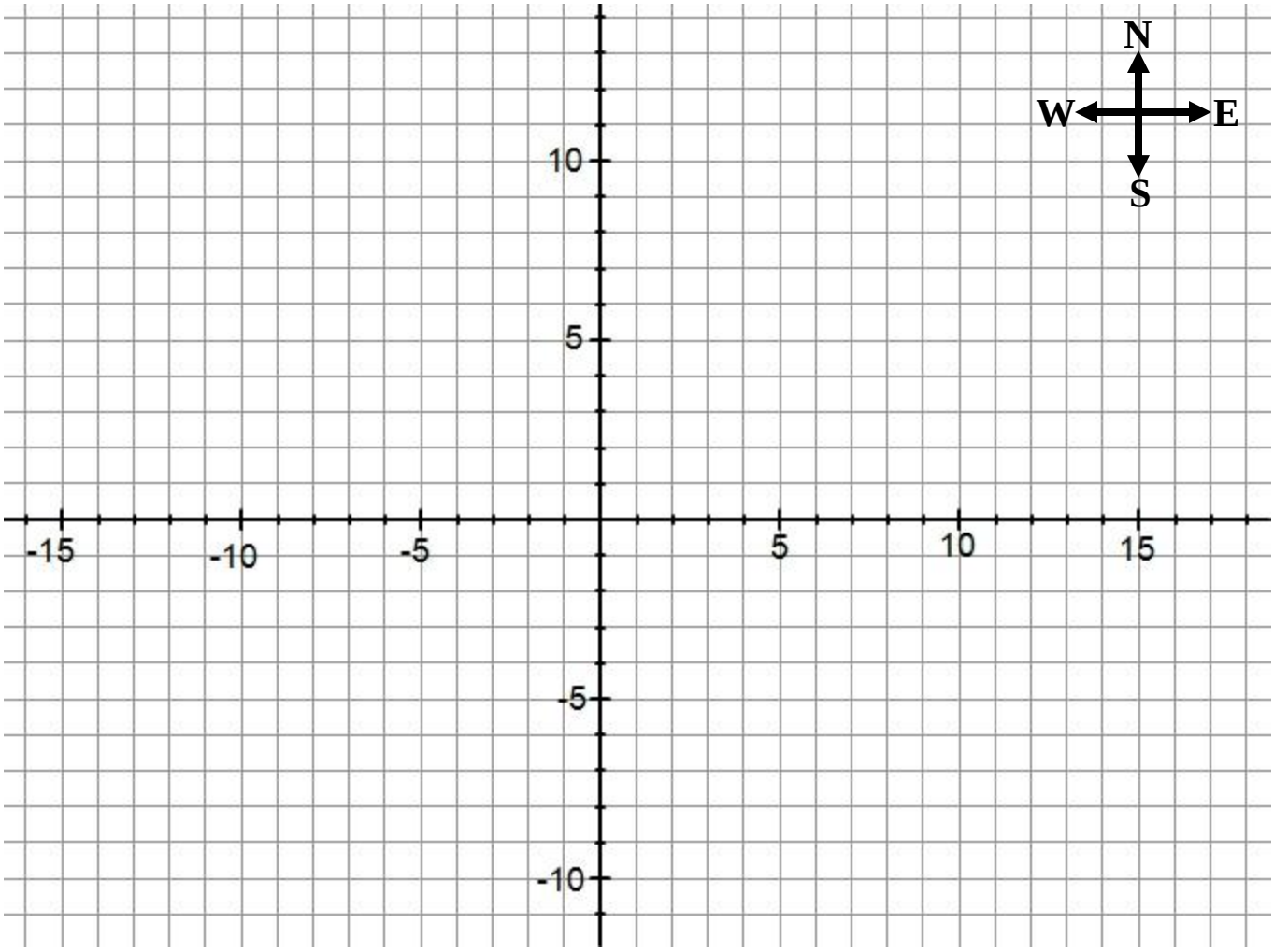
d. You want the area of the Battlefield Bunker and the Captain's Fortress to be brown, how many square pixels of brown will you need?

e. The Battlefield Bunker needs a steel wall built around it, how many pixels long will it take to show the wall?

f. There are four checkpoints that surround the Crystal Palace. They are at the points $(-4, 10), (-4, 2), (-13, 2), (-13, 10)$. Draw these checkpoint on the graph and find the distance in pixels a guard will walk if they do 2 laps around the checkpoints.

g. Finally you would like to add the Pit of Pythons. You want it to be triangular in shape and have an area of 27 square pixels that are green. Draw and color a possible triangle that could represent the Pit of Pythons.

2. A coordinate grid represents the map of a city. Each square on the grid represents one city block.



a. The fire station is located at the point $(3, 2)$. The school is exactly 8 blocks west of the fire station. What ordered pair represents the location of the school? Mark the locations of the fire station and school on the graph.

b. Aubrey's apartment is located at the point $(3, -1)$. Without, graphing determine the distance in blocks between Aubrey's house and the fire station. Then mark the location of Aubrey's apartment to check your answer.

c. If you were to connect the school, fire station, Aubrey's house, and the courthouse to form a polygon, a rectangle would be formed. What ordered pair represents the location of the courthouse? Draw the rectangle on the graph to check your answer.

d. Jordan lives the same distance away from the fire station as Aubrey but in the opposite direction. What ordered pair represents the location of Jordan's apartment? Mark Jordan's apartment on the graph to check your answer.

e. Draw and describe in words least two different ways you could walk exactly 16 blocks and end up back where you started.

f. Three corners of the city plaza are located at the points $(-2, 4), (-6, 4)$, and $(-2, 8)$. Without graphing determine what kind of polygon is made by the boundaries of the plaza? Explain how you know. Then plot and connect the ordered pairs to check your answer. Label the plaza on the graph.

g. What is the area of the city plaza?

h. Larry goes on a walk that starts and ends at school. He states that the path that he traveled enclosed a polygon with an area of 10.5 square blocks and he cut through the plaza. Draw two possible shapes with a dotted line that his walk could have taken.

Section 5.3: Volume of Three-Dimensional Shapes

Section Overview:
The first lesson of this sections reviews concepts of volume measurement. Such as a cube with side length of 1 unit is said to have "one cubit unit" of volume. A right rectangular prism can be packed without gaps or overlaps using unit cubes. Volume can be measured by counting unit cubes within the rectangular prism and is the same as multiplying the prisms edge lengths or equivalently the prisms height by the area of its base. Attention is then turned to right rectangular prisms with fractional edge lengths. Students use drawings and diagrams to pack these prisms with unit cubes of appropriate fractional edge lengths and find the volume by counting fractional unit cubes. They count these cubes by multiplying the number of fraction cubes along the length, width, and height of the prism. By changing the unit of measure from a fractional cube to a whole cube students see that they can obtain the same answer by multiplying the edge lengths of the prism. Once they have this understanding they apply the formulas for finding volume to find the volume of right rectangular prisms in a variety of mathematical problems and real-world applications.

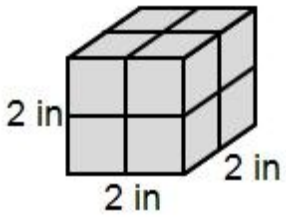
Concepts and Skills to Master in this Section:

By the end of this section, students should be able to:

1. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that it would be the same as would be found by multiplying the edge lengths of the prism.
2. Find the volume of right rectangular prisms with fractional edge lengths by applying the formulas $V = lwh$ and $V = Bh$.
3. Solve real-world and mathematical problems involving volume.

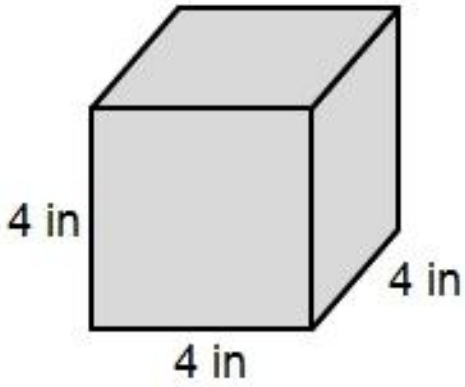
5.3a Class Activity: Finding Volume of Rectangular Prisms

1. Skyler wants to build a cube with sides that are 2 inches long using 1-inch cubes. How many 1-inch cubes will he need? Explain your reasoning.



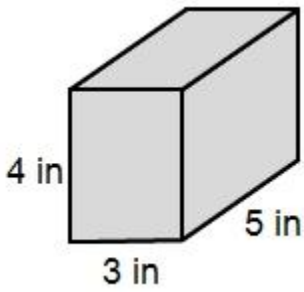
2. How many 1-inch cubes will he need to build a cube with 3-inch sides? Explain your reasoning.

3. How many 1-inch cubes will he need to build a cube with 4-inch sides? Explain your reasoning.

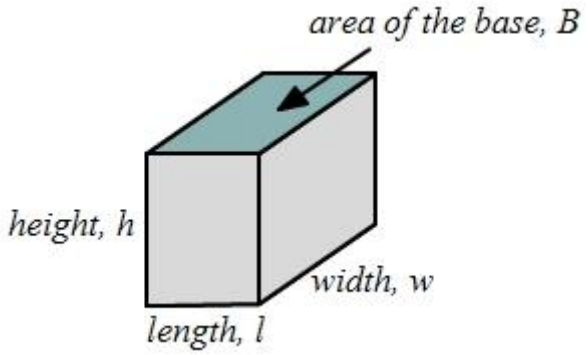


4. How many 1-inch cubes will he need to build a cube with 6-inch sides? Explain your reasoning.

5. How many 1-inch cubes will he need to build a rectangular prism that measures 3 inches by 5 inches by 4 inches? Explain your reasoning.

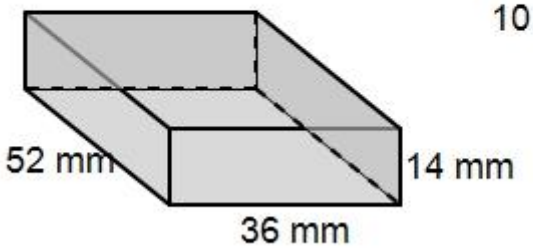


6. Explain how to find the volume of any rectangular prism. Relate your method to a formula for volume.

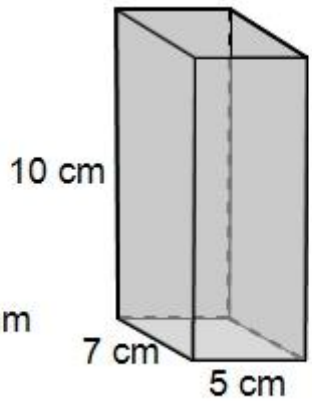


7. Find the volume of each rectangular prism given below.

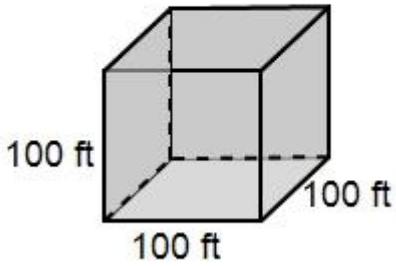
a.



b.

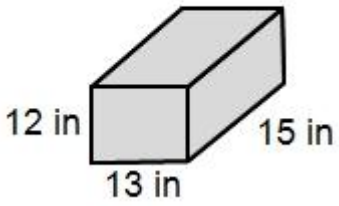


c.

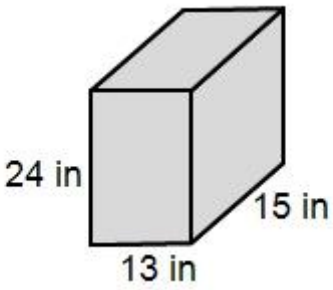


8. Three glass aquariums are rectangular prisms as shown below.

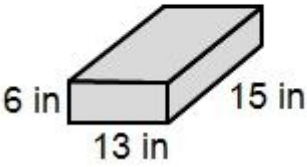
Aquarium A



Aquarium B



Aquarium C



a. Find the volume of each aquarium.

b. Order the aquariums from least to greatest according to the amount of water they will hold.

c. Is there any way to order the aquariums without calculating the volume?

d. Without using their volumes, determine the number of times you would have to fill the smallest aquarium with water and pour it into the largest aquarium to completely fill it.

e. What are the dimensions of an aquarium whose volume is 6 times that of the middle sized aquarium and takes up the same amount of floor space?

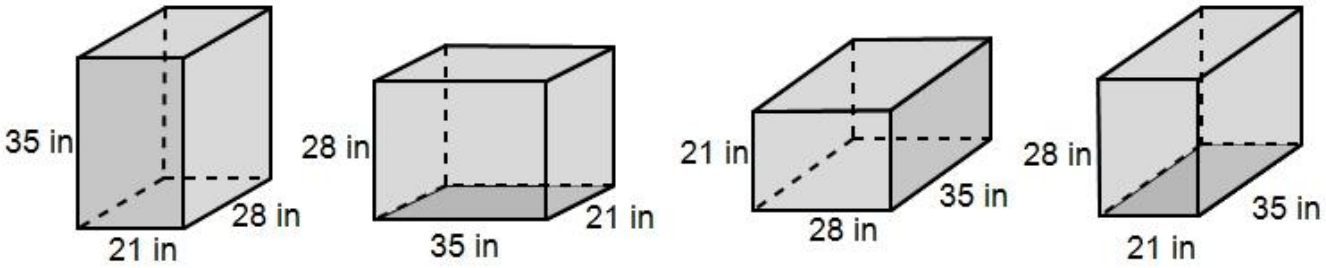
9. Use the table below to answer the questions that follow.

	Length	**Width**	**Height**
aquarium A	10 inches	4 inches	6 inches
aquarium B	8 inches	5 inches	12 inches

a. Without calculating the volumes determine which aquarium below will hold more water. Justify your answer.

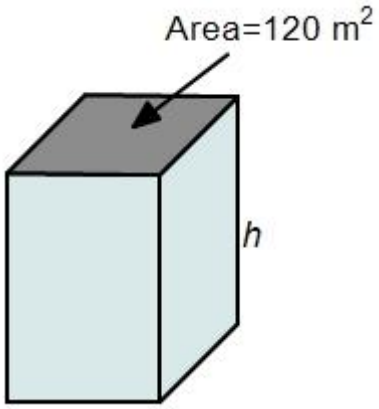
b. How many more cubic inches will the larger aquarium hold than the smaller aquarium?

10. Make a prediction about which rectangular prism below has the greatest volume, which one has the smallest volume? Find the volume of each prism to check your predictions.

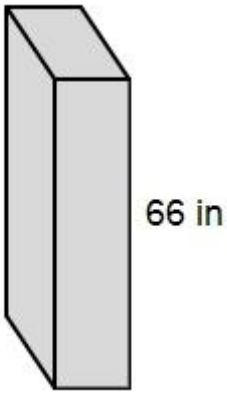


11. The base area of a rectangular prism is 56 cm^2 and its height is 9 cm, what is the rectangular prism's volume?

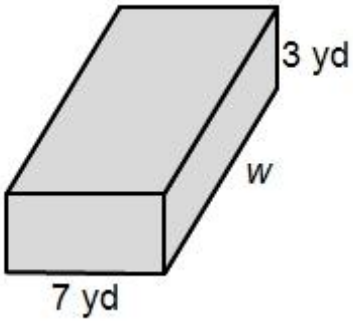
12. The volume of the rectangular prism given is 2,640 m^3 find the height.



13. The volume of the rectangular prism given is 23,958 in^3 find the area of its base



14. The volume of a rectangular prism given is 378 yd^3, find the width.



**The following three problems are [Illustrative Mathematics Tasks]*

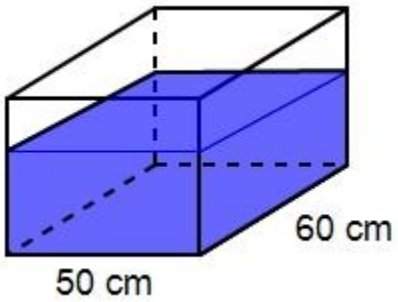
15. Amy has a fish tank shaped like a rectangular prism that is 20 cm by 20 cm by 16 cm.

    a. Draw and label a picture of the tank.

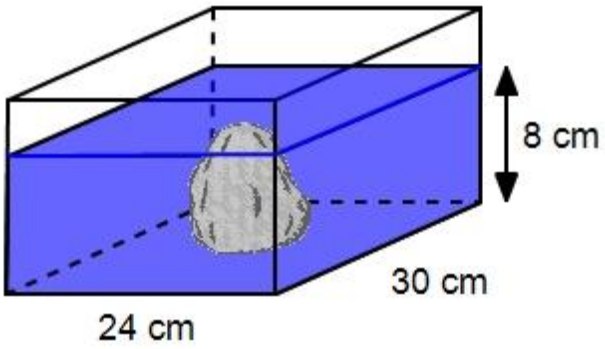
    b. What is the volume of the tank?

    c. If Amy only fills the tank $\frac{3}{4}$ of the way, what will be the volume of the water in the tank?

16. A rectangular tank is 50 cm wide and 60 cm long. It can hold up to 126 liters of water when full. If Amy fills $\frac{2}{3}$ of the tank as shown, find the height of the water in centimeters. (Recall that 1 liter =1000 cm^3.)



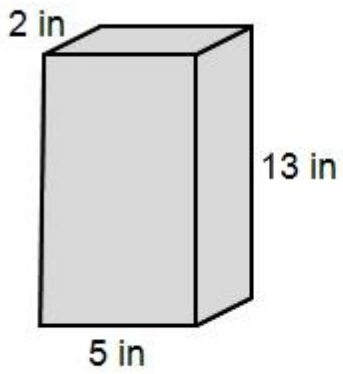
17. A rectangular tank is 24 cm wide and 30 cm long. It contains a stone and is filled with water to a height of 8 cm. When Amy pulls the stone out of the tank, the height of the water drops to 6 cm. Find the volume of the stone.



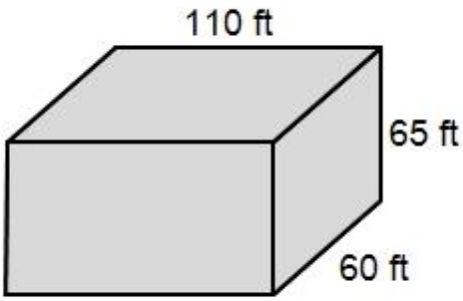
5.3a Homework: Finding Volume of Rectangular Prisms

1. Find the volume of each rectangular prism given below.

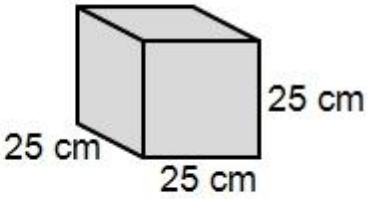
a.



b.



c.



2. Use the table below to answer the questions that follow.

	Length	**Width**	**Height**
aquarium A	90 inches	40 inches	45 inches
aquarium B	60 inches	60 inches	15 inches

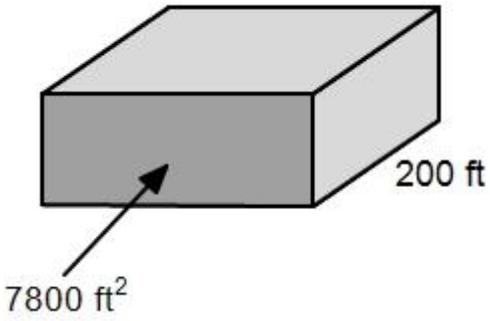
a. Without calculating the volumes determine which aquarium below will hold more water. Justify your answer.

b. How do the volumes of the aquarium compare? How many times bigger is the larger aquarium than the smaller aquarium?

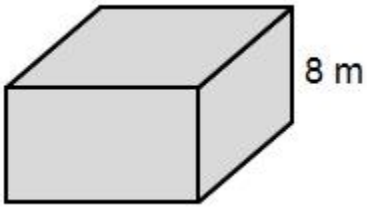
c. List a different set of dimensions that an aquarium could have if it has the same height and the same volume as aquarium A.

3. The base area of a rectangular prism is 280 in^2 and its height is 26 in, what is the rectangular prism's volume?

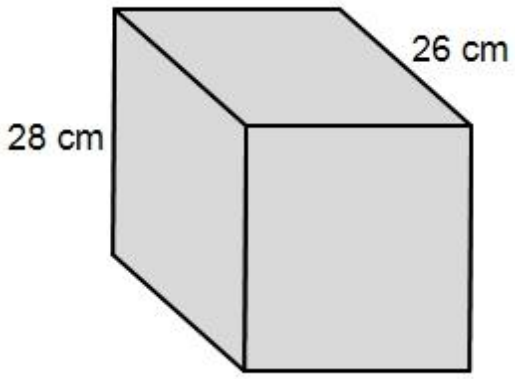
4. Find the volume of the rectangular prism given.



5. The volume of the rectangular prism given is 616 m^3 find the area of its base.



6. The volume of a rectangular prism given is 16,744 cm^3, find the length of the rectangular prism.



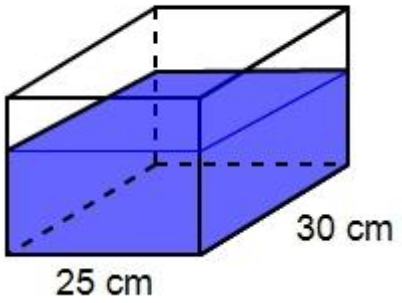
7. Andrew has a fish tank shaped like a rectangular prism that is 30 cm by 35 cm by 20 cm.

   a. Draw and label a picture of the tank.

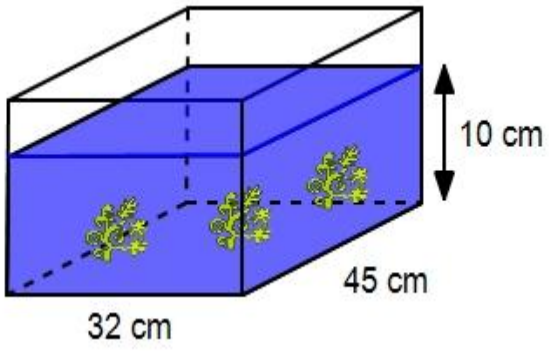
   b. What is the volume of the tank?

   c. If Andrew only fills the tank $\frac{1}{4}$ of the way, what will be the volume of the water in the tank?

8. A rectangular tank is 25 cm wide and 30 cm long. It can hold up to 18.75 liters of water when full. If Andrew fills $\frac{3}{5}$ of the tank as shown, find the height of the water in centimeters. (Recall that $1\ liter = 1000\ cm^3$.)



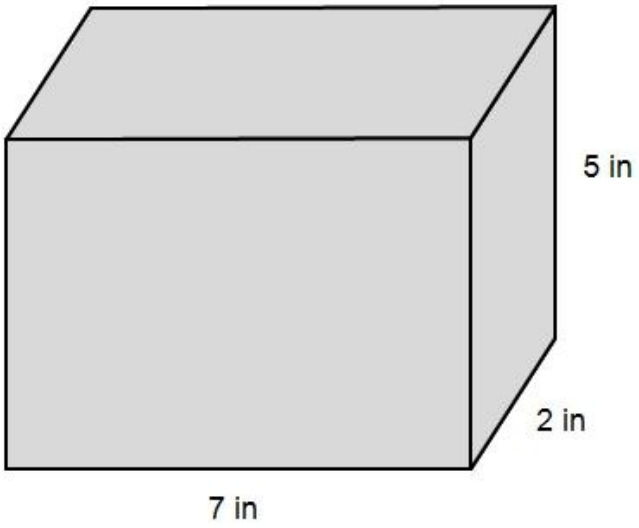
9. A rectangular tank is 32 cm wide and 45 cm long. It contains three identical plastic water plants and is filled with water to a height of 10 cm. When Amy pulls all three plastic water plants out of the tank, the height of the water drops to 8 cm. Find the volume of one plastic water plant.



5.3b Class Activity: Finding Volume of Rectangular Prisms with Fractional Edge Lengths

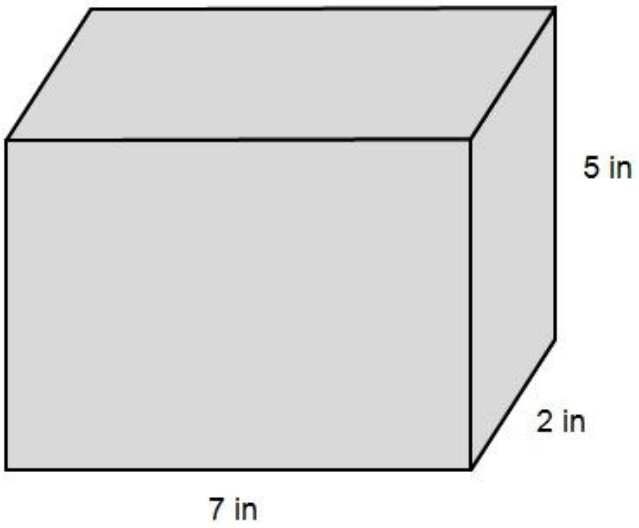
1. A company that manufactures dice packages the dice in rectangular boxes that measure 7 inches by 2 inches by 5 inches as shown.

   a. How many 1-inch by 1-inch dice can fit into one box?



   b. What is the volume of the box in cubic inches?

   c. The company also makes mini dice that measure $\frac{1}{2}$-inch by $\frac{1}{2}$-inch. How many $\frac{1}{2}$-inch dice can fit into a rectangular box that has the same dimensions? Use the figure above to help you answer.



d. What is the volume of the box when measured in ½-inch cubes?

e. Why do you get two different numeric values for the volume for parts b. and d. on the previous page? Does the volume of the box the change depending on whether it is packed with 1-inch dice or ½-inch mini dice?

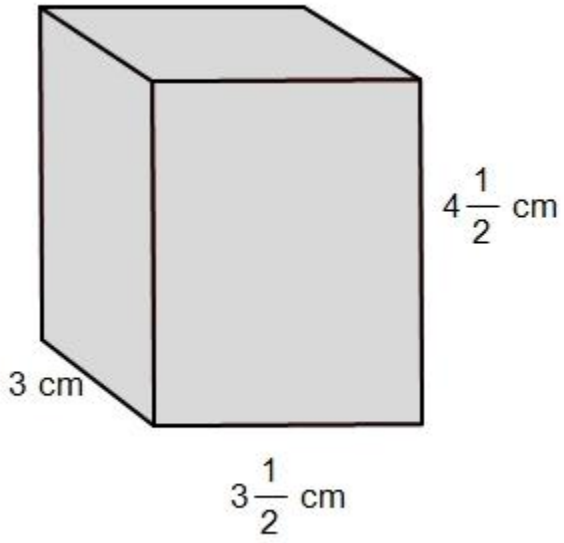
f. How can you show or prove that the number of 1-inch dice that fit into the box takes up the same amount of space or has the same volume as the number of ½-inch mini dice that fit into the box.

g. What if you were to fill the rectangular package with ¼ inch "mini-mini" dice? How many mini-mini dice will fit into the box?

h. Show how 4,480 ¼-cubic inches is the same volume as 70 cubic inches.

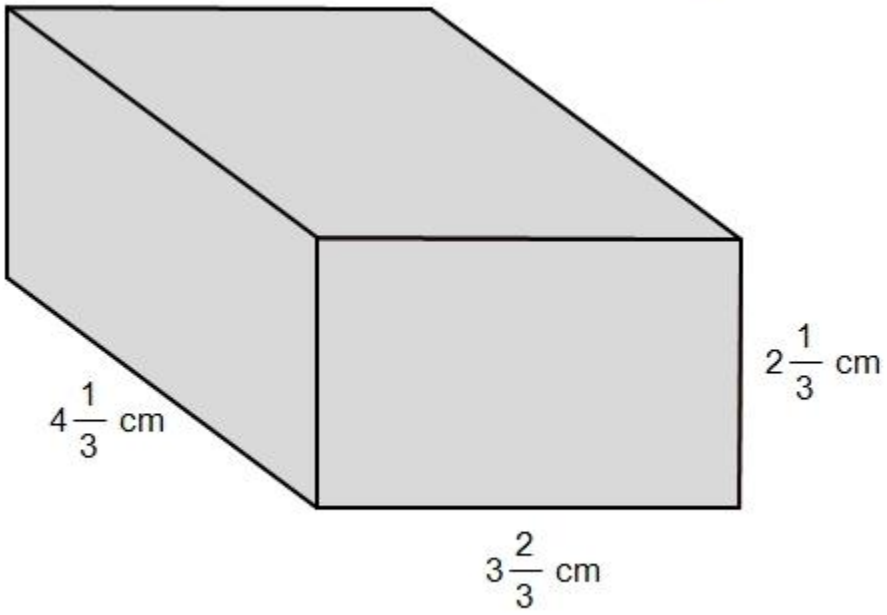
Why do we care about how many ½-inch mini dice and ¼-inch mini-mini dice will fit into a rectangular prism? Sometimes the edges of a rectangular prism will not have whole number lengths but rather fractional edge lengths. Will the volume formula work for finding the volume rectangular prisms with fractional edge lengths?

2. Find the volume of the rectangular prism below by packing it with fractional unit cubes.



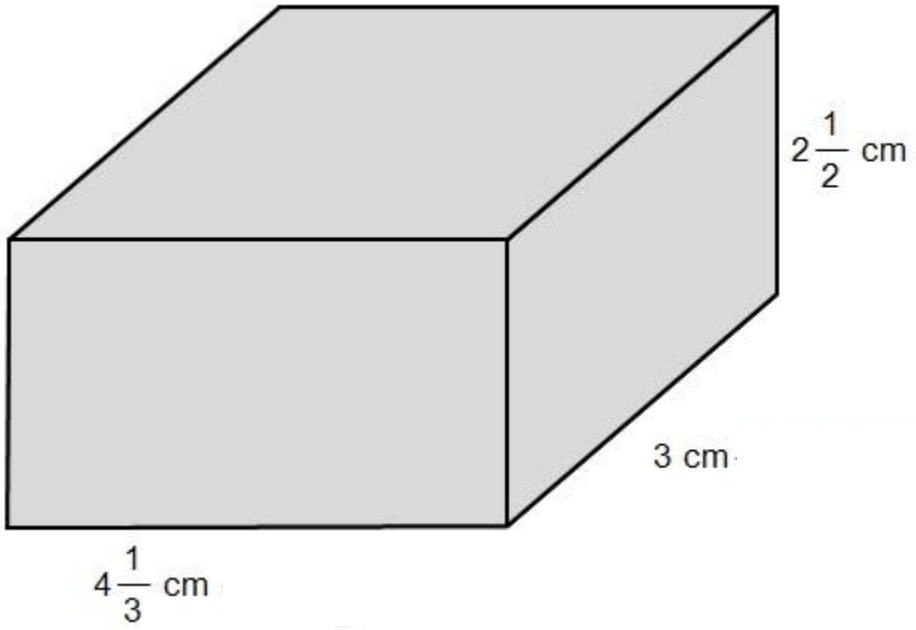
3. Find the volume of the rectangular prism by multiplying the edge lengths together; do you get the same answer as above?

4. Find the volume of the rectangular prism given by both packing it with fractional unit cubes and by using the formula. Then compare your answers.



5. Are there any other fractional unit cubes that would fit inside the prism perfectly without any space leftover?

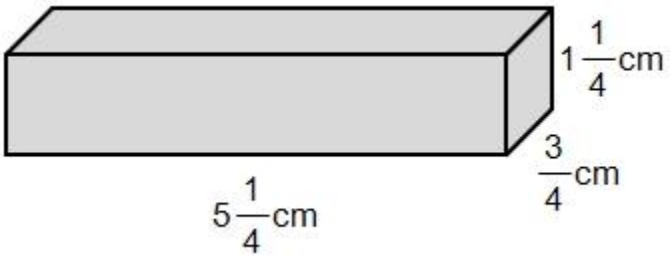
6. Find the volume of the rectangular prism given by both packing it with fractional unit cubes and by using the formula. Then compare your answers.



7. Which method do you prefer to use to find the volume of a rectangular prism with fraction edge lengths?

5.3b Homework: Finding Volume of Rectangular Prisms with Fractional Edge Lengths

1. Use the rectangular prism given to answer the questions that follow.



   a. Find the volume of the rectangular prism by using the volume formula.

   b. If you fill the prism with cubes whose sides lengths are less than 1 cm what size would be best?

   c. How many of these smaller cubes will fit into a single 1 cm cube?

   d. List the number of these smaller cubes that will fit along each edge length of the rectangular prism.
   length:

   width:

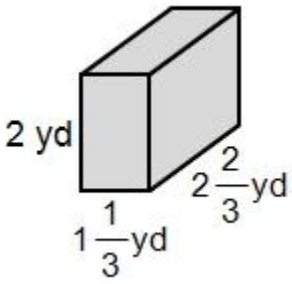
   height:

   e. How many of these smaller cubes will fill the prism?

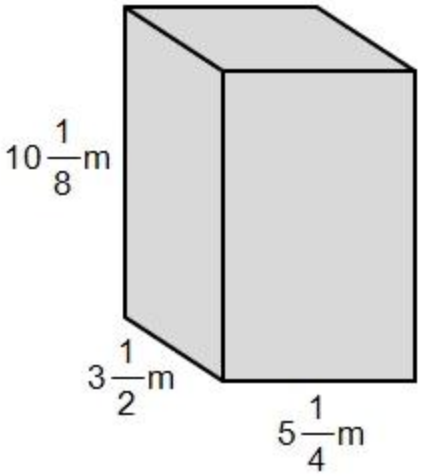
   f. Use the relationship between the number of cubes and the volume to prove that the volume that you found in part a. is correct.

2. Find the volume of each right prism using any method you want. Round your answer to the nearest hundredth.

a.



b.

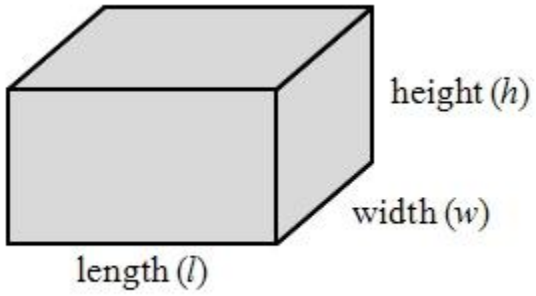


3. A rectangular prism is $\frac{5}{8}$ ft by $1\frac{1}{8}$ ft by $2\frac{1}{4}$ ft. Find the volume of the given prism using both methods. If needed, draw a picture. Round your answer to the nearest hundredth.

4. The volume of a rectangular prism is $109\ \frac{3}{8}\ m^3$, it has a width of $5\frac{1}{4}$ meters and a length of $6\frac{2}{3}$ meters. What is the height of the prism?

5.3c Class Activity: Finding Volume of Rectanuglar Prisms with Fraction Edge Lengths using Formulas

1. Write and explain the formulas for the volume of a rectangular prism. n#



2. Write and explain the formula for the volume of a cube. Explain how this formula relates to the formula for the volume of a rectangular prism.

3. Find the volume of each rectangular prism with the given dimensions. Be sure to show your work.

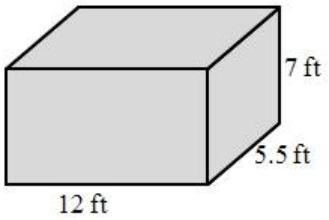
1. A rectangular prism that measures 12 meters by 2 ½ meters, by $3\frac{3}{5}$ meters.	2. A rectangular prism with a length of 1 ft, a width of $1\frac{1}{3}$ ft, and a height of $\frac{1}{2}$ ft.	3. A cube with a side length of 0.75 yards.

4. A rectangular prism has a base area of 170.5 ft^2. It has a height of $20\frac{1}{4}$ ft.
   a. Draw and label a picture of the rectangular prism.

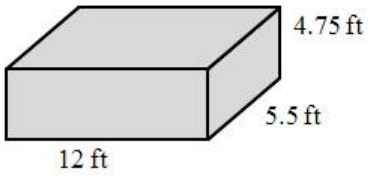
   b. Find the volume of the prism. Round your answer to the nearest hundredth.

5. Use the figures given to answer the questions that follow.

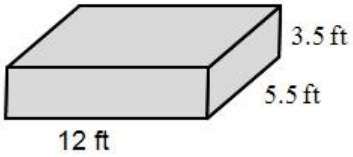
A



B



C



   a. Without calculating the volume, order the rectangular prisms from least to greatest based off of how many cubic centimeters they will hold. Explain your reasoning.

   b. How many times larger is the biggest prism than the smallest prism. Justify your answer.

5. A rectangular prism has a width that is $2\frac{1}{2}$ times its height and a length that is $\frac{3}{4}$ of its height. The height of the prism is 4 feet.
   a. Draw and label a picture with the information that you know.

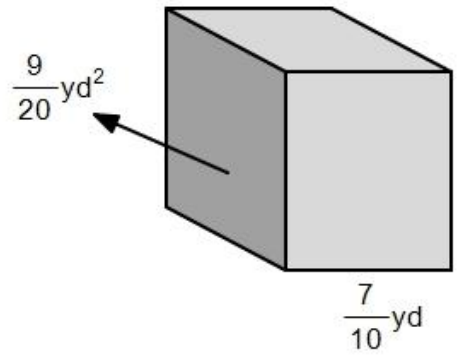
   b. Find the width and length of the prism.
      width:

      length:

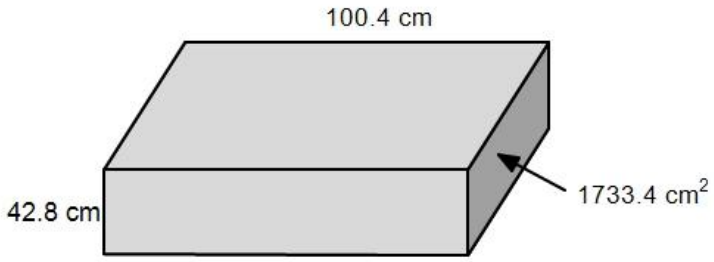
   c. What is the volume of the prism?

6. Find the volume of each rectangular prism given.

a.



b.



7. You are building a box to hold firewood in your shed. You have 2 ½ square feet of floor space in your shed for the box and would like the box to hold 6 cubic feet of wood.

   a. How high must the box be?

   b. List two possible sets of measurements you use for the length and width of the box.

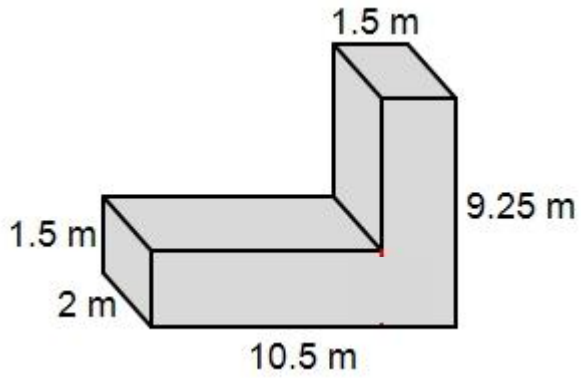
8. Olivia claims that $V = s^3$, where s is the side length of a cube is the formula you should use to find the volume of a cube. Harrison claims that the correct formula is $V = lwh$, where l is the length, w is the width, and h is the height of the cube.

   a. Who is correct? Why or why not.

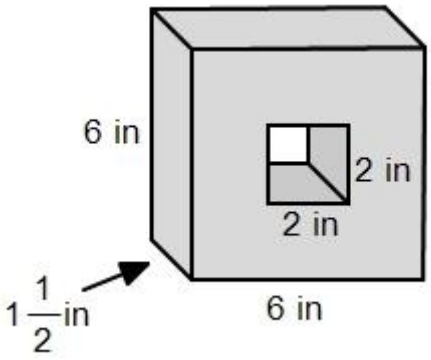
   b. Find the volume of a cube with a side length of $4\frac{3}{4}$ in. Round your answer to the nearest hundredth.

9. Find the volume of the composite figures given. Assume that angles that appear to be right angles are right angles.

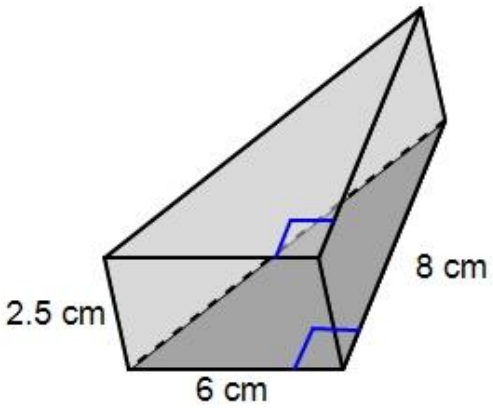
a.



b.

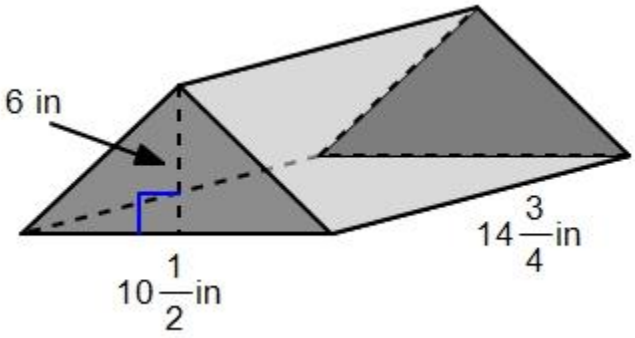


10. Consider the triangular prism with a base in the shape of a right triangle. The triangular base has a base length of 6 cm and a height of 8 cm. The height of the prism is 2.5 cm. Use what you know about how to find the volume of a rectangular prism to find the volume of the triangular prism.

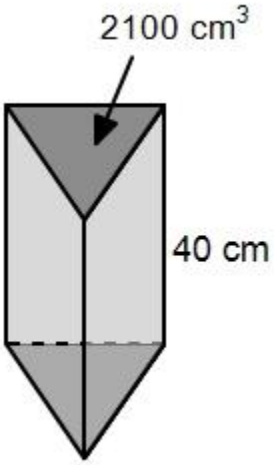


11. Find the volume of each right prism given; their bases have been shaded.

a.



b.



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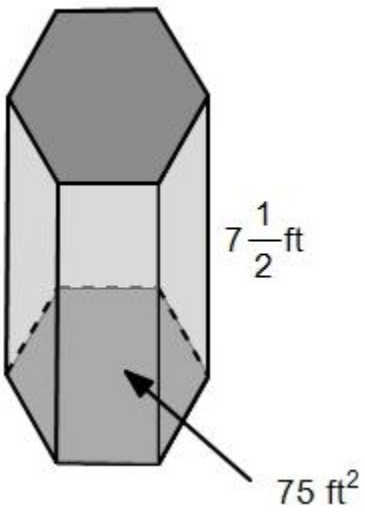


5.3c Homework: Finding Volume of Rectanuglar Prisms with Fraction Edge Lengths using Formulas

1. Find the volume of each rectangular prism with the given dimensions. Round your answers to the nearest hundredth.

a. A rectangular prism that measures 10 meters by 3 ½ meters, by $5\frac{1}{4}$ meters.	b. A rectangular prism with a length of 2 ft, a width of $3\frac{1}{3}$ ft, and a height of $\frac{2}{3}$ ft.	c. A cube with a side length of 1.45 yards.

2. A rectangular prism has a base area of 70 ft^2. It has a height of $13\frac{1}{3}$ ft.
   a. Draw and label a picture of the rectangular prism.

   b. Find the volume of the prism.

3. A rectangular prism has a width that is $3\frac{1}{2}$ times its height and a length that is $\frac{1}{3}$ of its height. The height of the prism is 9 feet.
   a. Draw and label a picture with the information that you know.

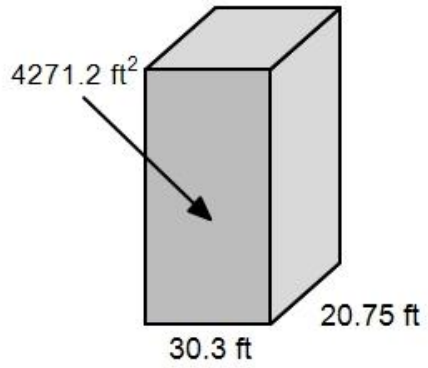
   b. Find the width and length of the prism.
   width:

   length :

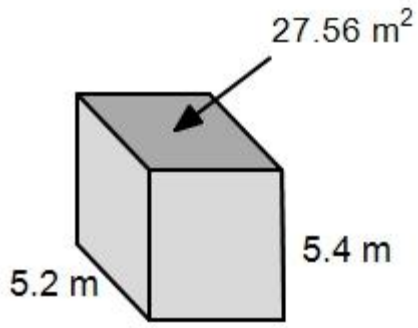
   c. What is the volume of the prism?

4. Find the volume of each rectangular prism given.

a.



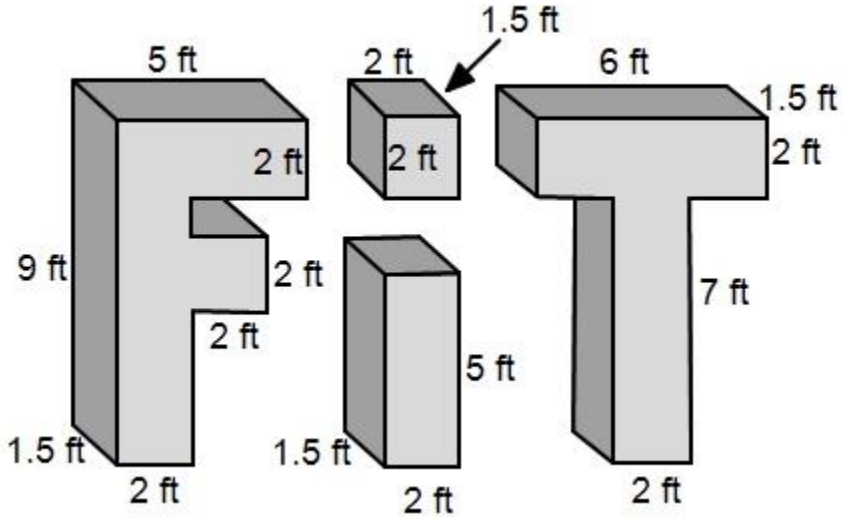
b.



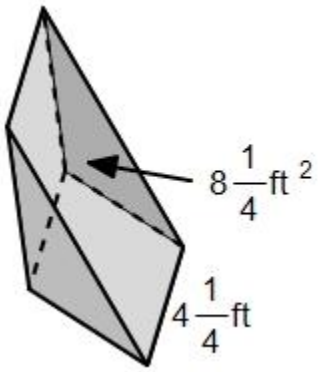
5. You have a garden box in your backyard that covers 28 square feet and has a height of $\frac{3}{4}$ of a foot.
   a. How much soil will you need to fill the box up completely?

   b. List two possible sets of measurements that could be the length and width of the garden box.

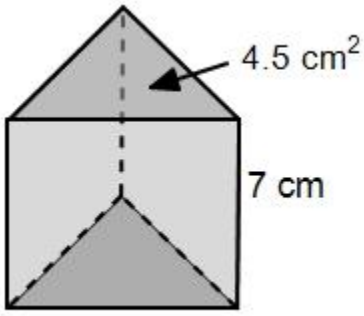
6. Find the volume of the block word. Assume that angles that appear to be right angles are right angles.



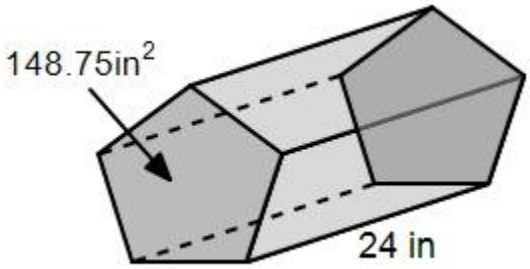
7. Find the volume of the right prism; its bases have been shaded.



8. Find the volume of the right prism; its bases have been shaded.

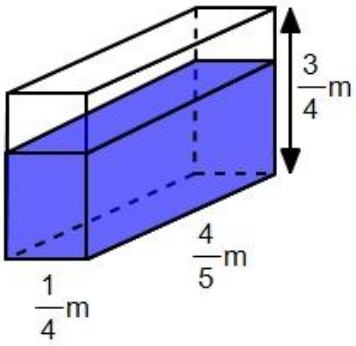


9. Find the volume of the right prism; its bases have been shaded.



5.3d Class Activity: Finding Volume in Real World Applications

1. You have just purchase a fish tank that is a rectangular prism.



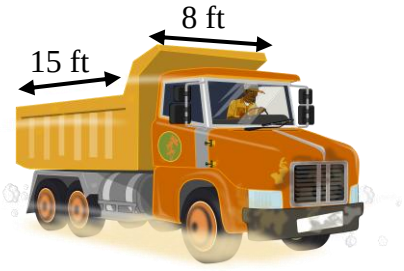
   a. How much water can the fish tank hold?

   b. If you fill the tank $\frac{2}{3}$ of the way full, how much water will be in the tank?

   c. You would like to add a rock feature to the tank. This feature has a volume of $\frac{1}{30} m^3$, do you have enough room in the tank to add the feature without any water spilling over?

2. A box of tissues has a volume of 1625 cm^3. Its length is 20 centimeters and its height is 6.5 centimeters. Find the width of the box of tissues.

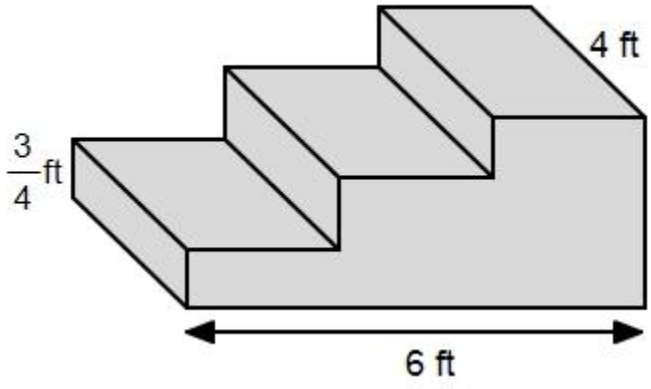
3. One cubic foot of gravel weighs 70 pounds.
   a. How many pounds of gravel can the dump truck haul?



   b. A sign posted in front of a bridge states that the bridge's weight capacity is 20 tons. One ton is equivalent to 2000 pounds. Can the dump truck safely cross the bridge with a full load of gravel?

4. Gloria is planning on pouring a set of concrete cement steps on the side of her front porch. She has drawn out a diagram of the steps below where the "rise" and "run" of each step is equal.

   a. Determine the total amount of cement she will need for the steps. Assume that angles that appear to be right angles are right angles.



   b. The cement costs $85 per cubic yard plus $60 delivery fee. Determine how many cubic yards of cement you will need and then find the total cost.

5. Leo's recipe for banana bread won't fit in his favorite pan. The batter fills the 8.5 inch by 11-inch by 1.75 inch pan to the very top, but when it bakes it spills over the side. He has another pan that is 9 inches by 9 inches by 3 inches, and from past experience he thinks he needs about an inch between the top of the batter and the rim of the pan. Should he use this pan?
   **This is an Illustrative Mathematics Task*

5.3d Homework: Finding Volume in Real World Applications

1. A sandbox is to be built in a children's museum exhibit to replicate an archeological digging site. The space allocated for the sandbox measures $6\frac{1}{2}$ ft by $5\frac{3}{4}$ ft . The museum has ordered $28\frac{1}{32}$ cubic feet of sand to fill the box.

    a. How high should the walls of the sandbox be so that the sand will fit into the box perfectly?

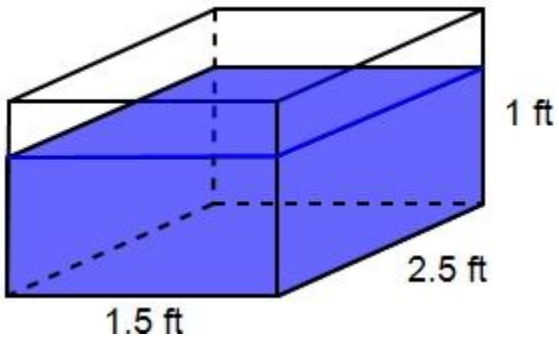
    b. After the first day of children digging in the sandbox, the sand is measured and it only goes up to a height of ½ ft. How much sand fell out of the box?

    c. What kind of adjustments, if any, do you think the museum should make to the archeological dig site exhibit?

2. A computer tower has a volume of 1728 in^3. Its width is 16 inches and its length is 6 inches. What is the computer tower's height?

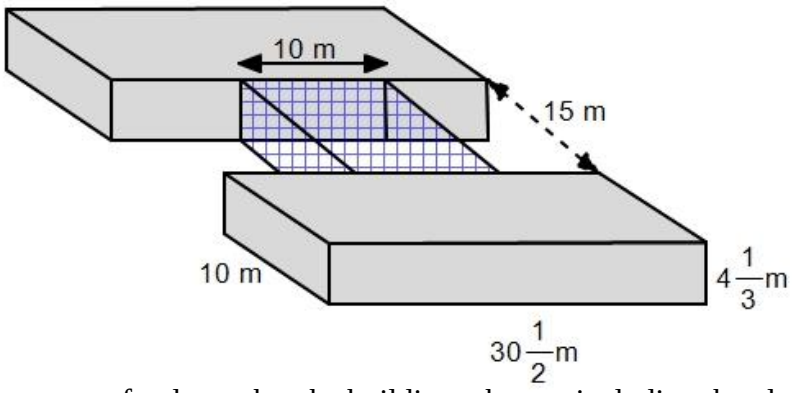
3. A fish tank in the shape of a rectangular prism is set on a folding table. The dimensions of the tank are shown.

    a. How much water can the tank hold?



    b. One cubic foot of water weights 64.4 pounds. The folding table is rated to hold 200 pounds. Can the table safely hold the fish tank when it is full of water?

4. The shape of a small office building is made up of two identical rectangular prisms connected by an enclosed glass walkway as shown.



   a. Find the total amount of volume that the building takes up including the glass walkway.
   The total volume is founding by summing the volumes of each composite part of the building.

   b. On average it costs $0.45 to air condition one cubic meter of space per month. How much will it cost the owner of the building to cool the building each month?

5. Roberto is making Jello to take to a family dinner. He has a 9 in $\times$ 13 in $\times$ 3 in glass dish with a layer of pineapple chunks in the bottom. He pours the Jello into the dish to a height of 2.5 inches. He then remembers that his mom does not like pineapple so he decides to spoon the pineapple out of the Jello before he puts it into the refrigerator to set up. The Jello drops to a height of 2 inches.

   a. How much pineapple did he take out of the Jello?

   b. He had 100 in^3 of whipped cream that he would like to spread across the top of the Jello after it has set up. Will the whipped cream fit inside the dish in order for Roberto to put a lid on top of it to transport to his mother's house without any spilling over?

Section 5.4: Surface Area

Section Overview:
In this section students investigate surface area and how nets can help you find surface area. They begin by constructing a net for a right rectangular prism. They learn how to decompose a three dimensional figure and represent it on a two dimensional surface by drawing its net composed of rectangles and triangles. Students learn that nets can also be used to identify the name of right prisms and pyramids by identifying the shape of the base. Once students are comfortable constructing nets they use them to find the surface area of its corresponding three-dimensional figure. Finally in the last lesson of this section students solve real-world and mathematical problems that relate to surface area and volume.

Concepts and Skills to Master in this Section:

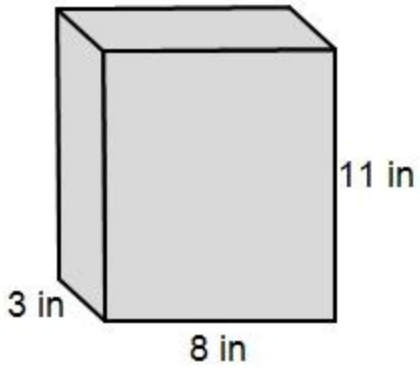
By the end of this section, students should be able to:

1. Identify and name right prisms and pyramids
2. Represent a three-dimensional figure using a net made up of rectangles and triangles.
3. Use a net to find the surface area of a three-dimensional figure.
4. Solve real-world and mathematical problems relating to surface area and volume.

5.4a Class Activity: Nets of 3-Dimensional Figures

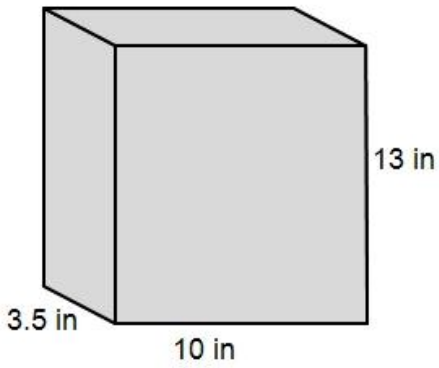
1. You have been asked by a popular breakfast cereal company to redesign the outside surface of their top selling cereal box. The dimensions of the box are shown.

   You need to send your ideas of what to put on each face of the box back to the company via email. Use the graph paper below and work with your group to figure out a way that you can show each side of the box on a flat two-dimensional surface.

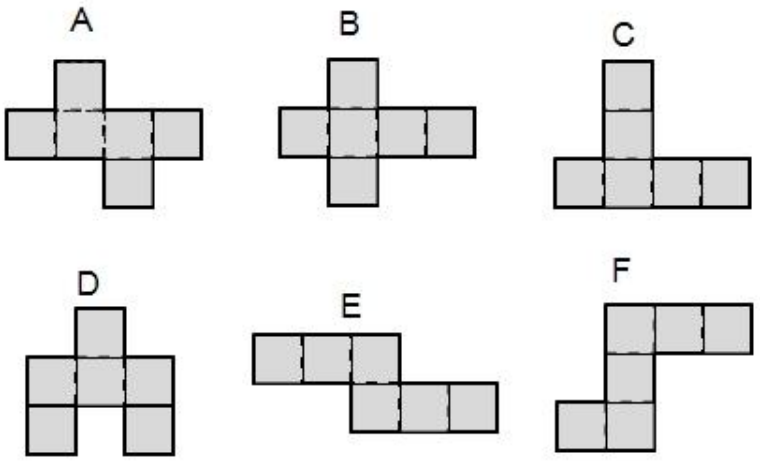


A two-dimensional drawing of a three-dimensional figure is called a **Net**. A net allows you to see all of the faces of a three-dimensional object on a two-dimensional surface.

2. Draw a net for the "Family Size" cereal box shown. Try to draw the net differently than you did for the cereal box described in number 1. Be sure to label all the dimensions.



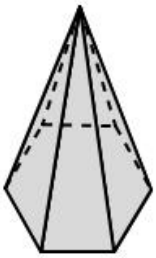
3. Which of the following Nets can be folded up to form a cube?

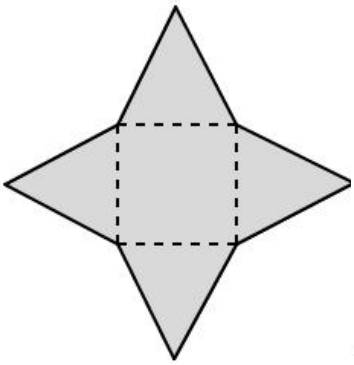


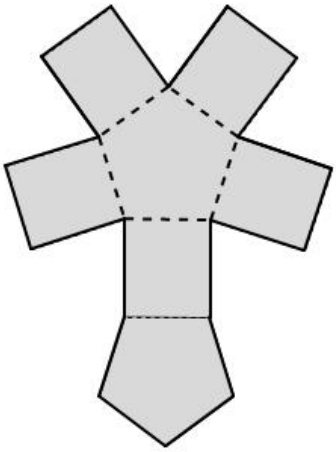
4. Draw an example of a prism and pyramid. Identify and label key features such as faces, edges, vertices, and bases based off of your discussion.

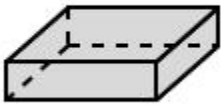
5. Draw a line to match each solid with its net.

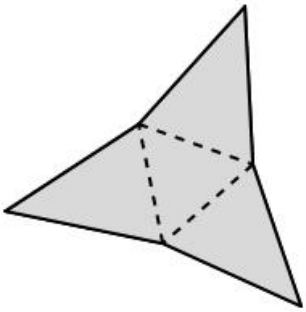
Solid	Net
Hexagon Pyramid	
Rectangular Prism	
Square Pyramid	
Triangular Pyramid	
Triangular Prism	
Pentagonal Prism	

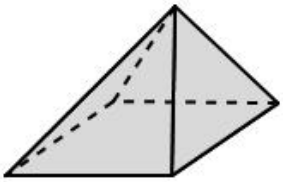


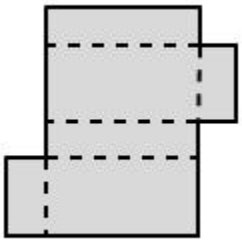


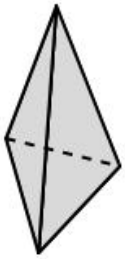


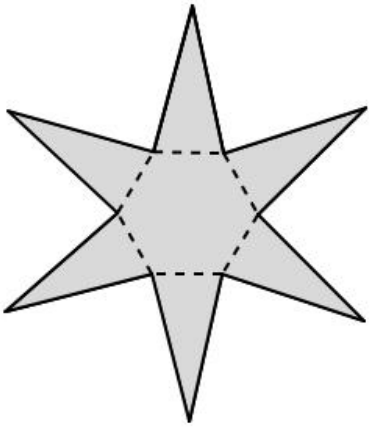


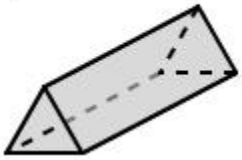


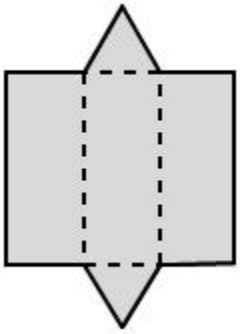


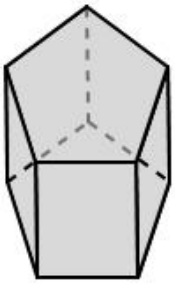




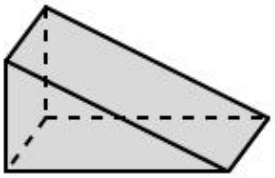
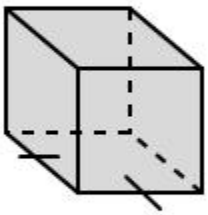
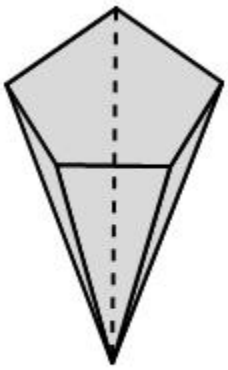








Sketch the net for each solid given. Then identify the shape of the base(s), whether it is a prism or pyramid, and write the name of the solid.

6. 	
7. 	
8. 	

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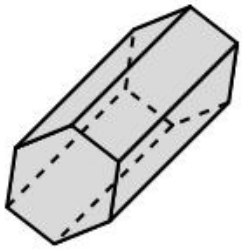

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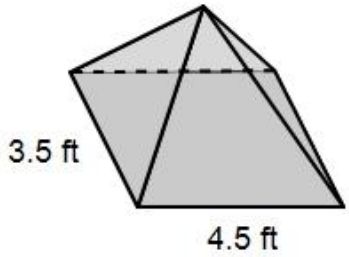

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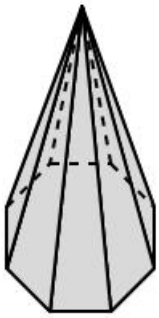
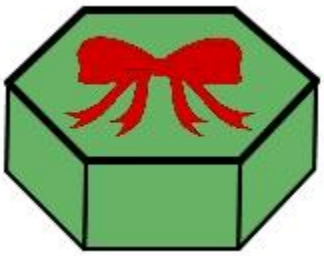
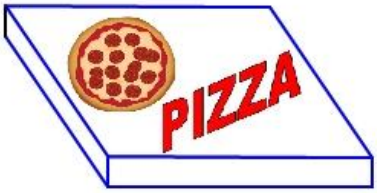
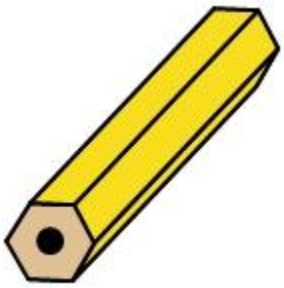
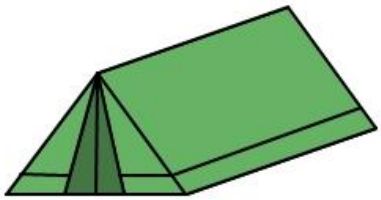
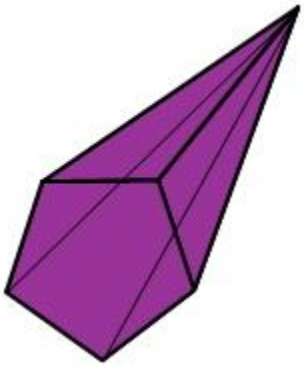
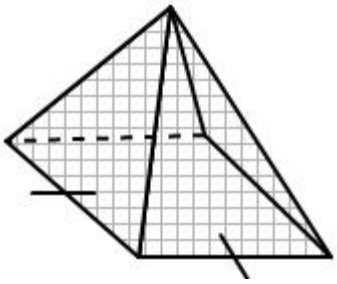


5.4a Homework: Nets of 3-Dimensional Figures

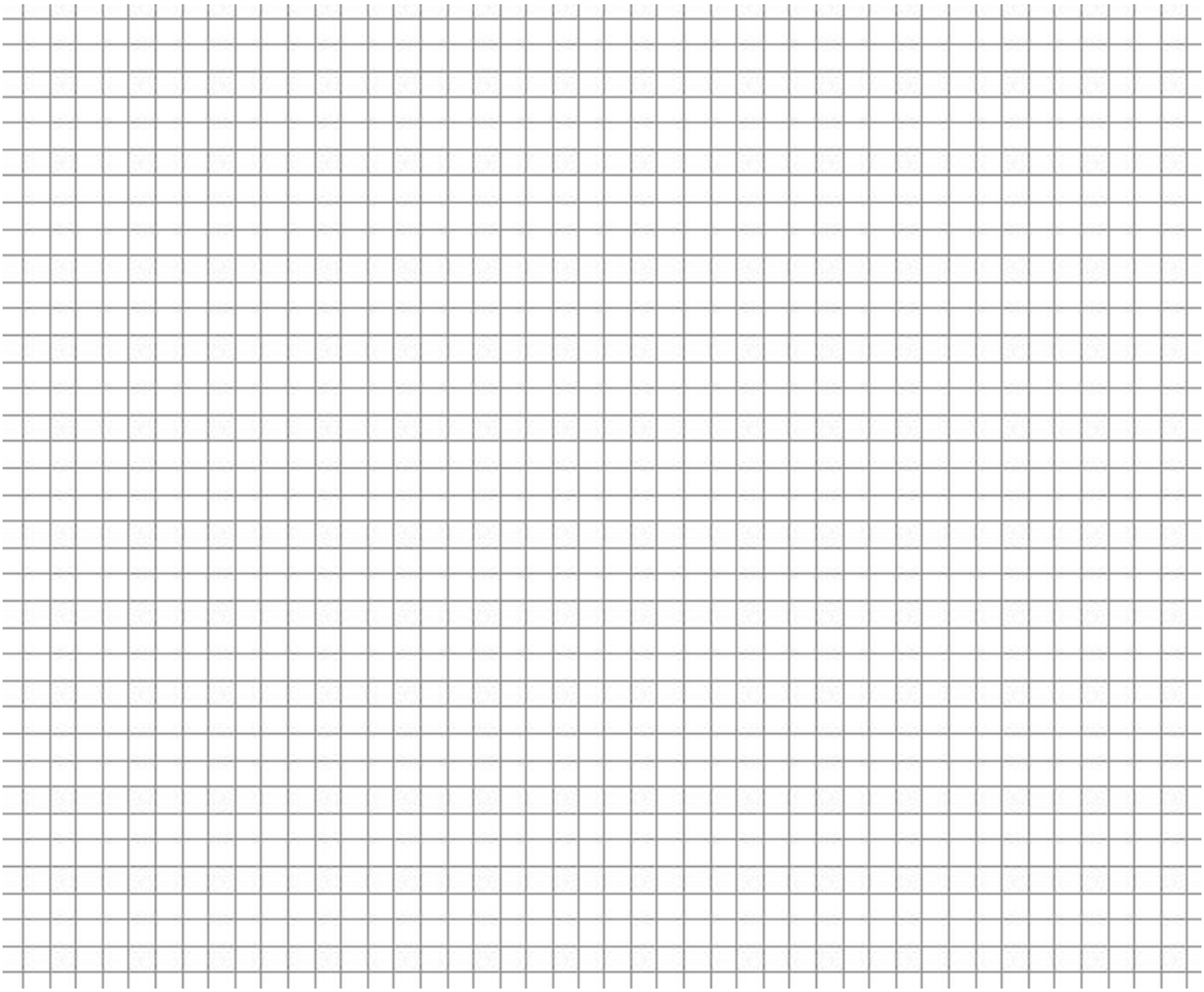
Sketch the net for each solid given. Then identify the shape of the base(s), whether it is a prism or pyramid, and write the name of the solid.

1. Holiday Chocolate Tin 	
2. Pizza Box 	
3. Unsharpened pencil 	

4. Tent 	
5. Crystal 	
6. The top of a stone monument. 	

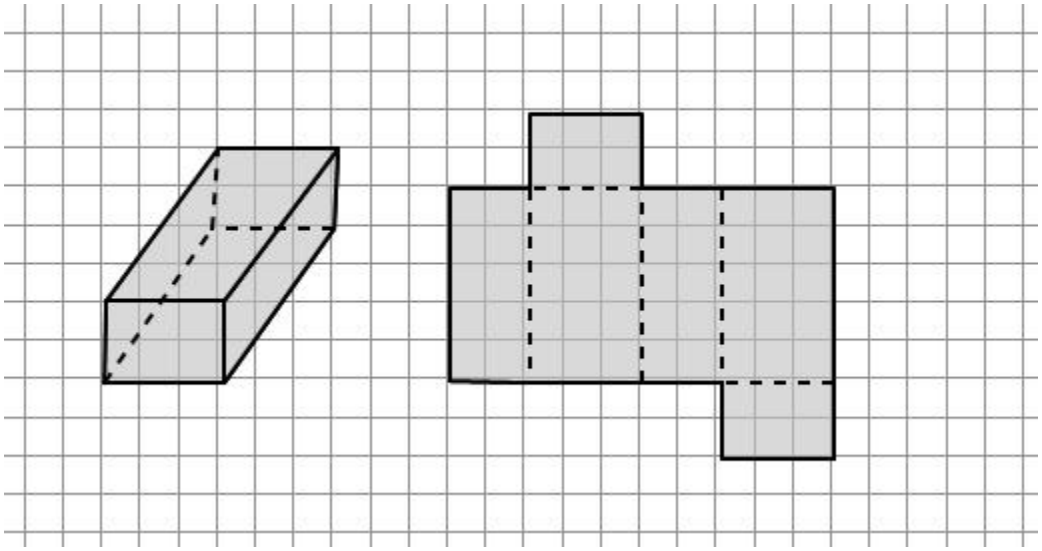
5.4b Class Activity: Finding Surface Area

1. Once again you are working on redesigning the packaging of a cereal box for a cereal company. Now they would like to know how much ink they will need to print your new designs on the outside of the box. In order to know how much ink they need they would like to know how much area the outside of the box takes up. Use the work that you did with the cereal box in the previous lesson to answer this question.

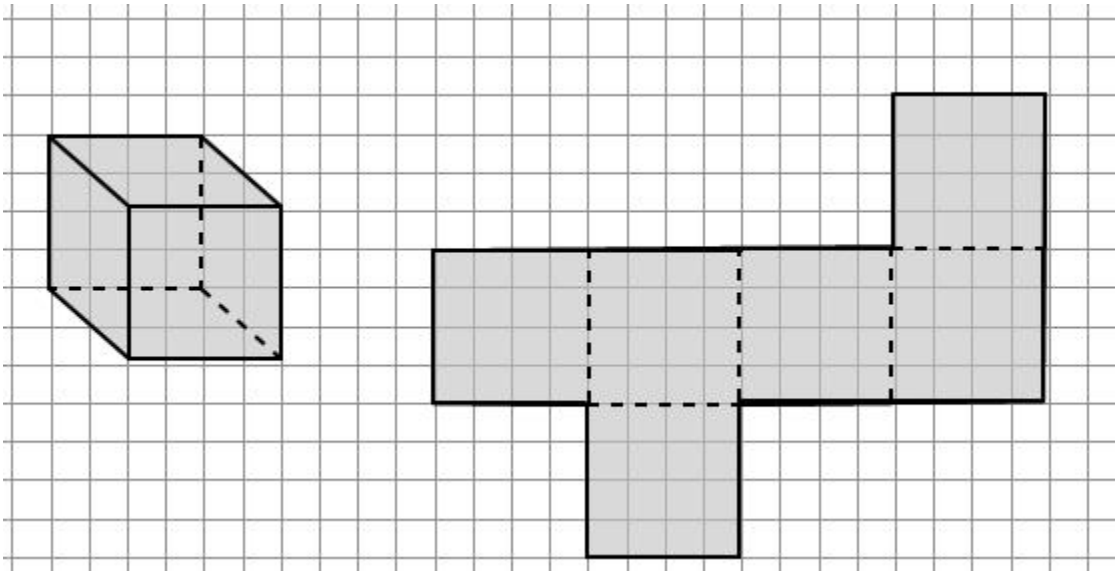


2. The nets for three different solids are shown. One square on the graph represents one square centimeter. Label any useful dimensions on each net, state the name of each solid, and find its surface area.

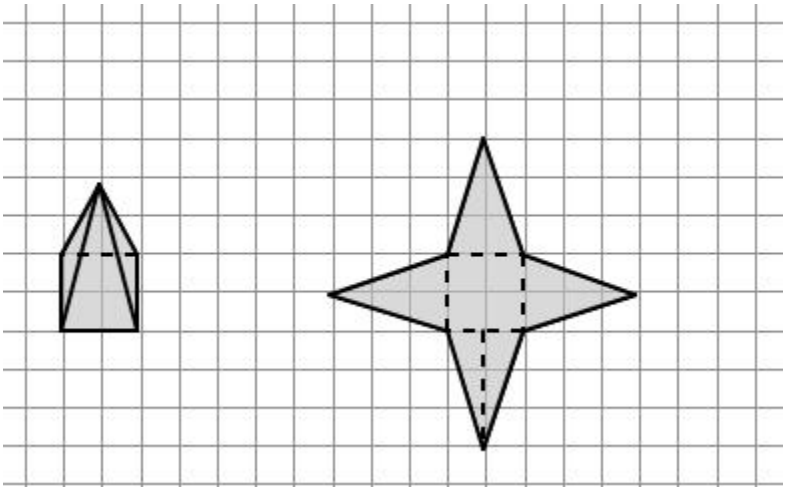
a.



b.

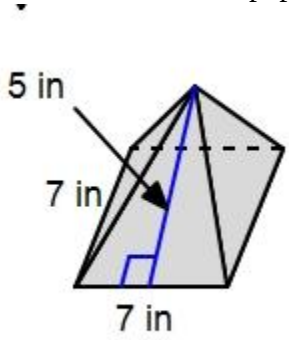


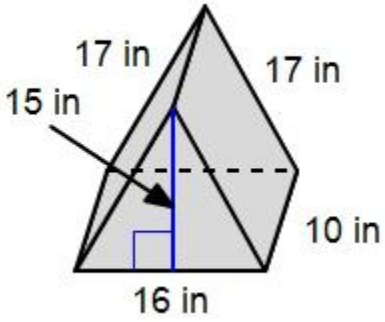
c.

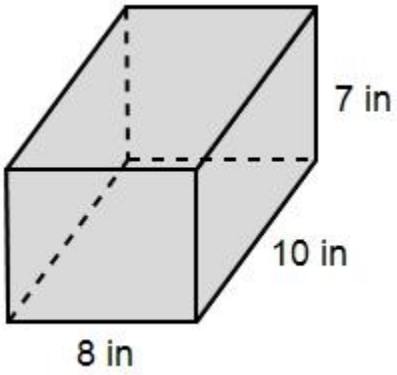


3. Explain in your own words what surface area is and how to find the surface area of a three-dimensional object.

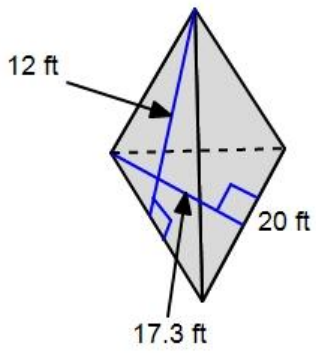
4. Julio and Peggy's teacher has asked them to make the 3D figures below out of one sheet of poster board. The poster board measures 36 inches by 24 inches, will they have enough paper to make one of each 3D figure? If so how much paper will be left over? If not how much more paper will they need?





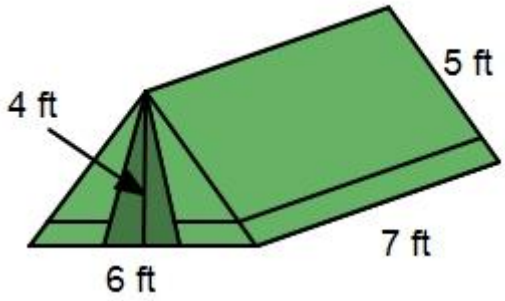


5. The top of a stone monument is the shape of a triangular pyramid; it is removed to be repainted.
    a. How much paint is needed to cover all of the faces of the pyramid? The side lengths of the base of the pyramid are all equal.



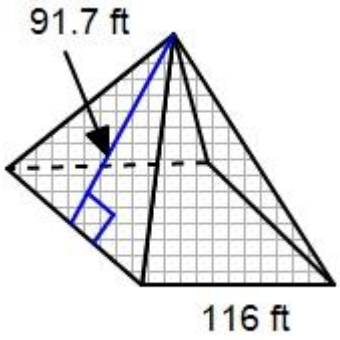
    b. Paint costs $2.50 per square foot, how much will it cost to paint the pyramid?

6. How much canvas fabric is needed to replace the canvas on the tent shown?

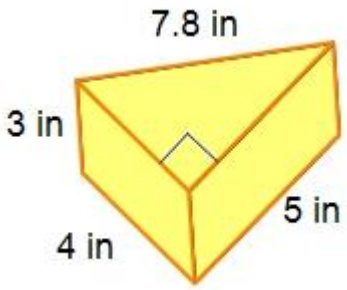


7. A small gift box measures 5 inches by 5 inches by 6 ½ inches. What is the least amount of paper needed to wrap the box? If needed draw and label a picture.

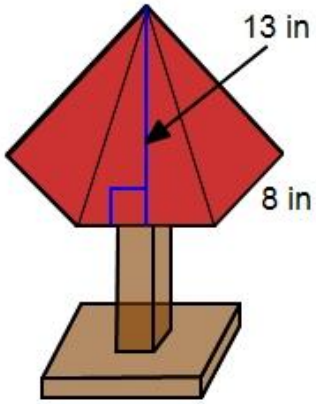
8. The entrance to the Louvre museum in Paris, France is a square pyramid made of glass. The side length of the base of pyramid is 116 feet and the height of one of the triangular faces is 91.7 feet. Find the surface area of it four triangular faces.



9. A cheese company produces wedges of cheese that are the shape of a triangular prism. Each wedge of cheese needs to be covered with a wax coating for preservation. How much wax coating will be needed to cover 100 wedges of cheese?



10. A lamp shade is shaped like a hexagonal pyramid. It does not have a bottom, how much fabric is needed to replace the fabric on the shade?



Challenge Extension: What is the surface area of a hexagonal pyramid with the same dimensions of the shade, if the perpendicular distance from the edge of the base hexagon to its center is 6.93 in?

11. Toni and Gardner are finding the surface area of a cube with a side measure of 14 inches. Toni writes the expression; $14^2 + 14^2 + 14^2 + 14^2 + 14^2 + 14^2$ to represent the surface area of the cube. Gardner claims that Toni's expression is not correct but rather that the surface area is represented by the expression; $6(14^2)$. Whose expression is correct? Explain your reasoning.

12. A juice box measures 4 inches by 2 inches by 5 inches.

    a. How much juice will the box hold?

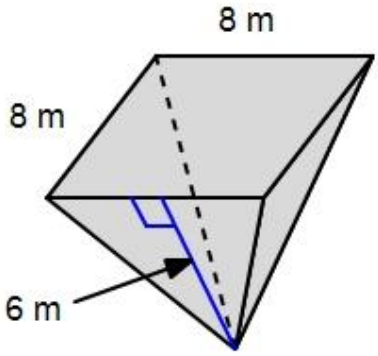
    b. What is the least amount of material needed to make one juice box?

    c. Explain the difference between volume and surface area.

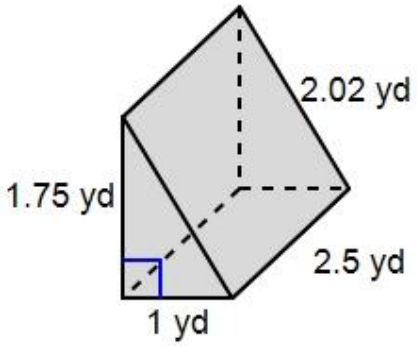
5.4b Homework: Finding Surface Area

Find the surface area of each 3D figure below. Be sure to show all calculations.

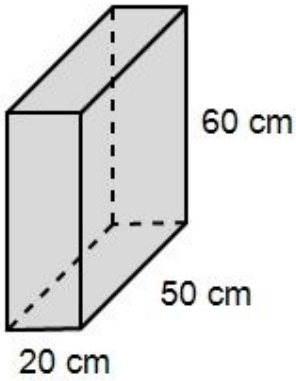
1.



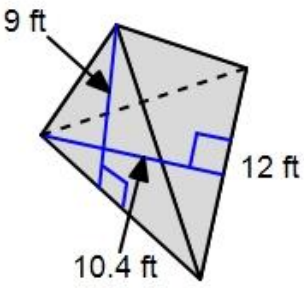
2.



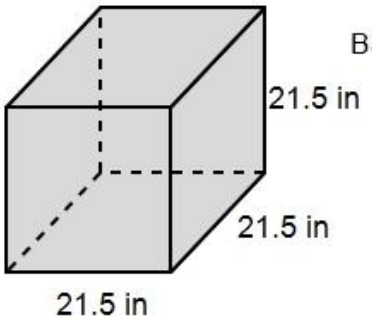
3.



4.



5.



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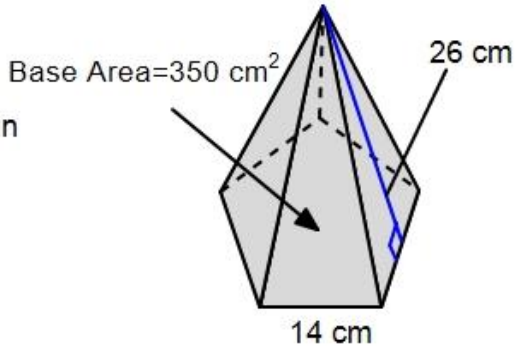
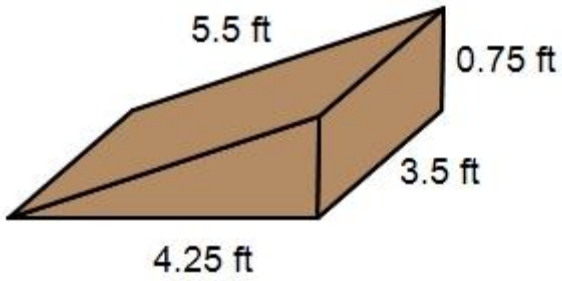


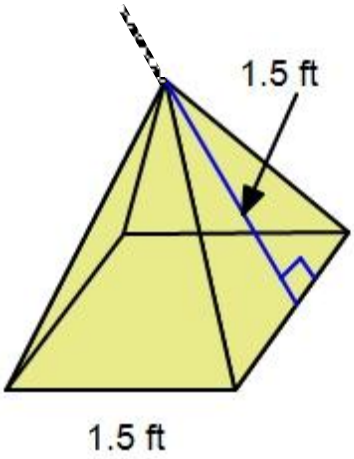
7. Carly and Nadia are painting their bike ramp. They would like to put two coats of paint on the entire ramp. They have one quart of paint which will cover 100 ft^2. Do they have enough paint to do the two coats? Justify your answer.



8. The material used to make a storage box costs $1.50 per square foot. The boxes have the same volume. How much money will a company save by choosing to make 100 of Box 1 over 100 of Box 2? Draw and label a net for each box if needed.

	Length	Width	Height
Box 1	20 in	5in	8 in
Box 2	25 in	4 in	8 in

9. A hanging light shade is the shape of a square pyramid that does not have a base or bottom. It is made of glass and hung by a chain that can hold 20 pounds of weight. One square foot of glass weighs 2.45 pounds. Can the chain support the light shade?



10. Chester is building a pool in his backyard. His only requirement is that the pool holds no more 3,000 ft^3 of water.
    a. Which pool option below should he choose based off of his requirement. Justify your answer.

       Option A: $20\text{ ft} \times 30\text{ ft} \times 6\text{ ft}$
       Option B: $15\text{ ft} \times 25\text{ ft} \times 8\text{ ft}$
       Option C: $20\text{ ft} \times 20\text{ ft} \times 8\text{ ft}$
       Option D: $16\text{ ft} \times 31.25\text{ ft} \times 6\text{ ft}$

    b. One gallon of paint covers 400 ft^2 and costs $30. Based off of this information which pool, option B or option D, will cost more to paint and by how much?

11. Helen is ordering popcorn bags for her movie theater. The bags cost $0.03 per square inch of paper used to make the bags. She can choose between a bag that measures $4 \times 9 \times 10$ inches or a bag that measures 6 x 6 x 10 inches. Which bag do you think she should choose if she wants to be able to fill the bag with the most amount of popcorn as possible and spend the least amount of money? Justify your answer.