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Chapter 6: Expressions and Equations

Utah Core Standard(s):

- Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express* $36 + 8$ *as* $4(9 + 2)$. (6.NS.4)
- Write and evaluate numerical expressions involving whole number exponents. (6.EE.1)
- Write, read, and evaluate expressions in which letters stand for numbers. (6.EE.2)
  a) Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*
  b) Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*
  c) Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length s =* $\frac{1}{2}$.
- Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* (6.EE.3)
- Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.* (6.EE.4)
- Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (6.EE.5)
- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (6.EE.6)
- Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. (6.EE.7)
- Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (6.EE.8)

Academic Vocabulary: numeric expression, equivalent numeric expressions, simplify, order of operations, grouping symbols (parentheses, brackets, fraction bar), middle dot ·, fraction bar as division: $\frac{a}{b}$ can be interpreted as $a \div b$, algebraic expressions, equivalent algebraic expressions, evaluate, sum, difference, product, quotient, simplified form of an algebraic expression, term, like terms, coefficient, constant, unknown, variable, Commutative Property of Addition/Multiplication, Associative Property of Addition/Multiplication, Distributive Property, Identity Property of Addition/Multiplication, perimeter, area, factor, exponent/power, exponential form, expanded form, base, squared, cubed, equation, solution, solve, substitute, Addition Property of Equality, Subtraction Property of Equality, Multiplication Property of Equality, Division Property of Equality, inverse operations, inequality, number line diagram, constraints, solution set, minimum, maximum, at least, no more than, at most, $<, >, \leq, \geq$

Chapter Overview: This chapter begins by helping students to develop an understanding of equivalent numeric expressions. Students write several expressions to represent a variety of real world problems and observe that the expressions all simplify to the same value. In the process, students are introduced to important notation used in algebra. For example, students come to understand that a middle dot can be used to represent multiplication. Following this, students determine whether two or more algebraic expressions are equivalent. They learn that equivalence can be shown for linear expressions by testing two different values for the variable. Students may begin to surface ideas about the ways expressions can be transformed to create different but equivalent expressions. Following this lesson, students write different but equivalent algebraic expressions to represent real world problems. The lessons are structured so that students start by writing numeric expressions with various numerical inputs. As the inputs change, students identify patterns and processes taking place. They identify the pieces of the expression that are changing and the pieces that are staying the same. They identify the structure of the expressions. This helps them to abstract the expression for any value of the input.

Throughout Section 1, students have been surfacing ideas about how expressions can be transformed to create different but equivalent expressions. In Section 2, students use the properties of operations to transform expressions. The emphasis in this section is on writing different but equivalent expressions, including the simplified form of an expression which takes the form $Ax + C$. While students learn how to write expressions in simplest form, the core underscores the importance of using the form that is best suited for the purpose at hand. This will become important for students in subsequent math courses. Students write equivalent expressions by combining like terms in an expression, distributing to write expressions without parentheses, and factoring to write expressions as the product of two factors. This work requires students to operate fluency with positive rational numbers. In the second part of the section, students review exponents as a short-hand way to represent repeated multiplication. They acquire a geometric understanding of expressions containing exponents. Representing expressions geometrically helps students to correctly evaluate expressions for given values of the variable(s) in the lessons that follow. This section concludes with students writing and evaluating algebraic expressions, including expressions arising from formulas, to represent real world problems.

In Section 3, students learn that an equation can be formed by stating an equivalence between two expressions. In Section 2, students evaluated expressions for different values of the variable. In Section 3, students consider the question, "For what values of the variable does an expression evaluate to a specific value?" Students use substitution to determine whether a given number is a solution to an equation. Next, students solve story problems in which they use the problem-solving strategy of working backward to arrive at the answer. This surfaces ideas about the solving process and the role of using inverse operations to find the unknown. In the lesson that follows, students construct equations using the properties of equality and realize that the same properties that allow us to construct an equation are the same properties that allow us to transform the equation back to its simplest form, revealing the solution. In the lessons that follow, students develop fluency with the process involved in solving equations that take the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. Next, students solve real-world and mathematical problems by writing and solving equations. Finally, students write inequalities to represent a constraint or condition in a real-world or mathematical problem. They compare inequalities to equations and realize that while equations and inequalities are solved using a similar process, inequalities have infinitely many solutions. Students learn to represent these solutions to inequalities on number line diagrams.

Connections to Content:

Prior Knowledge: In previous grades, students worked with the properties of operations with whole numbers, fractions, and decimals. In 5th grade, students learned how to use whole number exponents to represent powers of ten. Students have been writing numerical expressions throughout their elementary course work. Additionally, students have been writing and solving equations, representing the unknown with question marks, boxes, and letters. Many of the examples used in the problems in this Chapter rely on previous work done in 6th grade. For example, students learned about the area formulas for polygons in Chapter 5. Students also rely on their understanding of percent, statistics, and many other domains studied earlier in the year.

Future Knowledge: In 7th grade, students will encounter expressions with positive and negative rational numbers. As coursework progresses, students will write expressions to model different types of functions such as exponential and quadratic functions. Being able to examine numeric expressions and identify and abstract patterns is an important part of being able to write explicit rules to model a function. In later grades, students will see more complex equations. In 7th and 8th grade, students see equations that involve multiple steps to reach a solution and encounter equations with no solution and infinitely many solutions. Students also go on to solve different types of equations such as quadratic and exponential equations and rely on the understanding what it means to solve an equation and the role inverse operations play in the solving process. In 8th grade, students will be introduced to negative exponents and the exponent properties.

MATHEMATICAL PRACTICE STANDARDS

<table>
<tr>
<td></td>
<td>Make sense of problems and persevere in solving them.</td>
<td>Desiree is making a flag in the shape of a triangle for a parade float. To fit on the float, the area of the flag can be no more than 600 square inches. She has decided to make the height of the flag equal to $3\frac{1}{2}$ feet. Write an inequality to represent possible lengths for the base of the flag.

$b \leq 28.\overline{571428}$

When students are solving inequalities, they need to be able to interpret the solution. Mathematically, the solution above tells us that the base can be any length $\leq 28.\overline{571428}$; however, in the real world, it is not likely that someone would measure to this degree of precision. It is more likely that this person would measure to the nearest inch or half inch. In the problem above, students might reason that Desiree would make the base of the flag 28 inches or 28.5 inches. In similar problems, the quantities being considered are whole objects, such as people. Students take this into consideration when considering solutions to the problem.</td>
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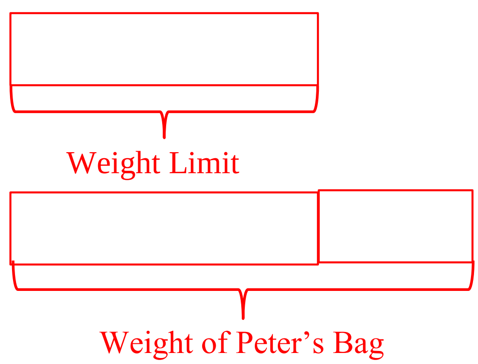
<tr>
<td></td>
<td>Reason abstractly and quantitatively.</td>
<td>Peter is checking his suitcase at the airport. He puts it on the scale and the person working at the counter tells him that his bag weighs $1\frac{1}{2}$ times the weight limit for checked baggage. If Peter's bag weighs 75 pounds, what is the weight limit for checked bags?



Let w = weight limit of checked bags
"If we multiply the weight of a bag by 1.5, we will get the weight of Peter's bag"
$1.5w = 75$
$w = 50$
The weight limit for checked bags is 50 pounds.
Throughout this chapter, students are transitioning to a more abstract way of thinking. In the problem above, students are asked to write and solve an equation for the problem given. Teachers and students are encouraged to use numeric examples and models to arrive at the equation or symbolic representation of the situation.</td>
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	Construct viable arguments and critique the reasoning of others.	Write three expressions that are equivalent to the expression $x + x + 4(x + 3)$. *Throughout the chapter, students are asked to justify the process used to write equivalent algebraic expressions. What properties allow them to transform the expression? How can they show the two expressions are equivalent? Students consider equivalent expressions written by classmates and determine whether they agree the expressions are equivalent. When solving equations, students must justify the properties of equality that allow them to transform the equation to its simplest form to reveal the solution.*
	Model with mathematics.	You must be at least 46 inches tall to ride the roller coasters at an amusement park. Owen is currently $43\frac{1}{2}$ inches tall. Write and solve an inequality to show the amount Owen must grow to be tall enough to ride the roller coasters at the amusement park. "Owen must be 46 inches or taller" $43\frac{1}{2} + g \geq 46$ $g \geq 46 - 43\frac{1}{2}$ $g \geq 2\frac{1}{2}$ where g is a rational number 46 or higher $43\frac{1}{2}$ = current height | Amount Owen needs to grow - g *Throughout the chapter, students are applying the math they are learning to solve real world problems. When considering a problem, they think, "What am I given? What do I need to know? Are there constraints I need to consider? Does my answer make sense?" They create pictures, models, numeric expressions, algebraic expressions, equations, and inequalities to represent and solve real world problems. For the problem above, students might start by testing values: What if Owen grows 2 inches taller? 3 inches taller? At what point is he just tall enough? What if he grows more than he needs to?*
	Use appropriate tools strategically.	*Students use area models as a tool throughout the chapter to better understand the structure of expressions. This understanding helps them to transform and evaluate expressions. For example, students explore the difference between the expressions* $3x + 2$, $3(x + 2)$, *and* $3(2x)$ *using area models. The first is a rectangle with dimensions of* 1 *and* $3x + 2$, *the second a rectangle with dimensions of* 3 *and* $x + 2$, *and the third a rectangle with dimensions* 3 *and* $2x$. *Examining these different area models helps students to see why the second expression can be written as* $3x + 6$ *and the third expression can be written as* $6x$.

	Attend to precision.	Evaluate the expressions when $r = 12, s = 2,$ and $t = 5$. $4t^2 + 3t^2$ $5r - s^3 + 2$ $\frac{t-s}{r}$ *When evaluating expressions with exponents there are many details for students to consider. In the first expression, what is the operation between the* 4 *and* t^2*? Do I multiply* t *by 4 and then square the result or do I square* t *first and then multiply the result by 4? What does it mean to square a number? Am I following the order of operations? Do I know and understand the different grouping symbols? Am I computing correctly, particularly when fractions and decimals are involved?*
	Look for and make use of structure.	*Students use structure throughout the chapter when they write algebraic expressions. For example, students consider the expression "three copies of the sum of twice a number and seven" in word form and translate it to a symbolic representation:* $3(2x + 7)$*. To write this symbolically, students consider the sum of twice a number and seven as an object held together with parentheses. This also helps students to understand why an equivalent expression is* $6x + 21$*. The expression* $3(2x + 7)$ *can be re-written as* $(2x + 7) + (2x + 7) + (2x + 7)$ *and then by the Commutative Property of Addition written as* $2x + 2x + 2x + 7 + 7 + 7$ *and finally* $6x + 21$. *Students create area models to understand the structure of the expressions with exponents such as* $2x^2$ *and* $(2x)^2$*. In the first expression, students create a square with a side length of* x*. Then, they duplicate that square to show two copies of it. In the second expression, students first double a length x and then create a square with a side length of 2x which they realize has an area of* $4x^2$*. Representing these expressions geometrically helps students to correctly evaluate the expressions for given values of x.*
	Look for and express regularity in repeated reasoning.	Marin has 50 tickets to spend on rides at a carnival. Each ride takes 6 tickets. Write different expressions to represent the number of tickets Marin has left based on the number of rides she goes on. *Students first consider this problem numerically. What if Marin goes on two rides? Three rides? Four rides? While writing numeric expressions, students begin to identify patterns in the expressions. What is changing? What is staying the same? This allows students to write an expression to represent the number of tickets Marin has remaining when she goes on r rides.*

6.0 Anchor Problem:

A local charity has a benefit to raise money. You are on the planning committee and have been tasked to determine the number of tickets that must be sold for the charity to raise at least $5,000 after all expenses have been covered.

Here is some other information that may be helpful to you while working on this task.

- Attendees purchase tickets to attend the event. The price of the ticket includes dinner, a selection of mini desserts, and dancing.
- The event is from 6:30 pm – 10:30 pm.
- The cost of a ticket to attend the event is $90. Included in the cost of the ticket is dinner and a selection of mini desserts.
- The caterer for the event charges $40 for each dinner. You need to purchase a dinner for everyone attending the event plus fifteen additional dinners for volunteers. Volunteers do not pay for their dinner.
- Based on experience, you have determined that each person takes on average 2.5 mini-desserts. Each dessert costs $1.50.
- Eight people can sit at each table. In addition to tables for the guests, the committee will set up two additional tables for the volunteers.
- The decorating committee has decided to put 2 vases of flowers and 4 votive candles on every table for decoration. Each vase of flowers costs $4.50 and each votive candle costs $0.90.
- You have chosen a DJ to play music at the event. The DJ charges $125 per hour to play music. The DJ will play music for the entire time the event is going on.
- The committee is buying party favors to give out as people leave the event. In the past, less than 75% of the guests have taken party favors. Based on this data, you have decided to buy party favors for exactly 75% of the guests. Each party favor costs $3.00.

6.0 Alternative Anchor Problem:

Part 1: The side length of a square is unknown.

a. Write an expression for the perimeter of the square.

b. Write an expression for the area of the square.

c. Write an expression to show the perimeter of 3 copies of the square. Assume the squares are not touching.

d. Write an expression to show the area of 3 copies of the square.

e. The side length of the square from above is tripled. Write an expression for the new perimeter of the square. Compare the perimeter of the new square to the perimeter of the original square.

f. Write an expression for the new area of the square. Compare the area of the new square to the area of the original square.

6.0a The Properties of Arithmetic Reference Sheets

Make a copy of each of the property sheets for students to keep as a reference sheet. At the start of the chapter, fill out the chart with the definition in words and symbols and any examples and notes that would be helpful at the time. Then, refer to the reference sheet and continue to add examples and notes when you come across a problem in the chapter that relies on one of the properties. You may also consider making poster size reference sheets to put up in your classroom.

Commutative Property

Definition in Words

Changing the order of the addends/factors in an addition/multiplication problem does not change the sum.

Tie to the word "commute" which means to move from one place to another.

Definition in Symbols

$$a + b = b + a$$

$$(a)(b) = (b)(a)$$

Examples

- $3 + 4 = 4 + 3$
- $2(5) = 5(2)$
- $20 + n = n + 20$
- $56 + 78 + 44 = 56 + 78 + 40 + 4 = 56 + 4 + 40 + 78 = 60 + 40 + 78 = 100 + 78 = 178$.
- Finding the area of a trapezoid with a height of 7 and bases equal to 10 and 8. $\frac{1}{2}(7)(10 + 8)$ We can change the order of these factors to make the expression easier to simplify. $\frac{1}{2}(10 + 8)(7) = \frac{1}{2}(18)(7) = 9 \cdot 7 = 63$
- 3 copies of $(x + 2)$: $(x + 2) + (x + 2) + (x + 2) = x + x + x + 2 + 2 + 2 = 3x + 6$
- $(2x)^3 = (2x)(2x)(2x) = 2 \cdot x \cdot 2 \cdot x \cdot 2 \cdot x = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x = 8x^3$
- $(x + 3)4 = 4(x + 3) = 4x + 12$

Notes

- Does not work for subtraction and division.
- Used in mental math computation.
- Used to write equivalent numeric and algebraic expressions.
- Can help us make sense of the structure of expressions. For example, why does $(2x)^3 = 8x^3$.

Definition in Words	**Definition in Symbols**
You can group addends/factors in different ways to add/multiply. Tie to the word "associate" which is to partner, connect, "hang out" with someone or something.	$(a+b)+c=a+(b+c)$ $(a\cdot b)\cdot c=a\cdot(b\cdot c)$

Associative Property

Examples	**Notes**
• $8+7+3=8+(7+3)=8+10=18$ • $682\cdot 8\cdot 2=682\cdot(8\cdot 2)=682\cdot 10=6{,}820$ • Finding the area of a trapezoid with a height of 7 and bases equal to 10 and 8. $\frac{1}{2}(7)(10+8)$ We can change the order of these factors to make the expression easier to simplify. $\frac{1}{2}(10+8)(7)=\left(\frac{1}{2}\cdot 18\right)(7)=9\cdot 7=63$ • 3 copies of $(x+2)$: $(x+2)+(x+2)+(x+2)=(x+x+x)+(2+2+2)=3x+6$ • $(2x)^3=(2x)(2x)(2x)=2\cdot x\cdot 2\cdot x\cdot 2\cdot x=(2\cdot 2\cdot 2)\cdot(x\cdot x\cdot x)=8x^3$ • $4(2x)=4\cdot 2\cdot x=8x$	• Does not work for subtraction and division. • Used in mental math computation. • Used to write equivalent numeric and algebraic expressions. • Can help us make sense of the structure of expressions. For example, why does $(2x)^3=8x^3$.

You will notice that the Commutative and Associative are often used in conjunction with one another when simplifying and evaluating expressions and making sense of the structure of expressions.·

Additive Identity Property of Zero

Definition in Words

When you add zero to a number/expression, the number/expression does not change.

Tie to the word "identity". Your identity is who you are. Dressing up in a Halloween costume does not change your identity. Or ask, "What can I add to a number that does not change the number's identity?

Definition in Symbols

$$a + 0 = a$$

Examples

- $8 + 0 = 8$
- $n + 0 = n$
- $x + 6 = 13$

$$\begin{aligned} -6 &= -6 \\ \hline x + 0 &= 7 \\ x &= 7 \end{aligned}$$

Notes

- Used when solving equations.
- Used to write equivalent numeric and algebraic expressions.

Multiplicative Identity Property of One

Definition in Words

When you multiply a number/expression by one the number/expression does not change.

Tie to the word "identity". Your identity is who you are. Dressing up in a Halloween costume does not change your identity. What number can you multiply by that does not change the number's identity?

Definition in Symbols

$$a \cdot 1 = a$$

Examples

- $3 \cdot 1 = 3$
- $1a = 1 \cdot a = a$
- A rectangle with dimensions $3x$ and 1 has an area equal to $3x \cdot 1 = 3x$.
- $\frac{7x}{7} = \frac{14}{7}$
  $1x = 2$
  $x = 2$
- 2.8 is what percent of 7?
  $\frac{2.8}{7} = \frac{n}{100}$
  Clear the decimal by multiplying by $\frac{10}{10}$ or 1 to get $\frac{28}{70}$ which simplifies to $\frac{4}{10}$ which is 40%.

Notes

- Used to write equivalent numeric and algebraic expressions.
- Used when solving equations.

Definition in Words	**Definition in Symbols**
When you multiply a sum (or difference) by a number, you multiply each term in the sum or difference by that number. Tie to the word "distribute" which means to give to or pass out.	$a(b+c) = ab + ac$ $a(b-c) = ab - ac$
Examples	**Notes**
• $8(53) = 8(50+3)$ $= (8)(50) + (8)(3)$ $= 400 + 24 = 424$ • $3x + 2x = (3+2)x = 5x$ • $3(2x+1) = 3 \cdot 2x + 3 \cdot 1 = 6x + 3$ • $24x + 18 = 6(4x+3)$ • Write "3 copies of the sum of x and 4" symbolically. $3(x+4) = 3x + 12$	• Used in mental math computation. • Used to write equivalent numeric and algebraic expressions. • Used when solving equations. • Can help us make sense of the structure of expressions.

Distributive Property

In bullet three of the examples above, we see the Distributive Property used in conjunction with the Associative Property.

Section 6.1: The Structure of Numeric and Algebraic Expressions

Section Overview:
This section begins with students writing several different but equivalent numeric expressions to represent real-world problems. This process surfaces ideas about equivalence. It also provides a nice opportunity to review several of the concepts studied throughout the year, including operations with positive rational numbers, ratio problems, percent problems, perimeter and area problems, and statistics problems. Additionally, students review and practice simplifying numeric expressions with grouping symbols, remembering to follow the order of operations. Students are then introduced to algebraic expressions and learn how to determine whether two or more algebraic expressions are equivalent. In the last lesson, students write algebraic expressions to represent real world problems. Identifying patterns and making sense of the structure of the expressions helps students as they transition from numeric to abstract representations of real world problems.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Write different but equivalent numeric expressions to represent a real-world problem using grouping and operator symbols correctly (parentheses, fraction bars, middle dot, etc.).
2. Determine whether two or more numeric expressions are equivalent by correctly simplifying the expressions (follow order of operations, recognize and interpret grouping symbols and operator symbols, etc.)
3. Determine whether two or more algebraic expressions are equivalent by evaluating the expressions for specific values of the variable.
4. Translate the word form of an algebraic expression to its symbolic representation and vice versa. Use and understand academic vocabulary.
5. Recognize and interpret patterns and structure in numeric expressions and use this understanding to write algebraic expressions.

6.1a Class Activity: Translating Contexts to Equivalent Numeric Expressions

Directions: Write as many expressions as you can to represent each context below. Then, simplify the expressions to determine whether they "work" or "don't work".

Give students time to generate and simplify their own expressions for each problem. If students are struggling to write expressions, ask them to verbalize their thinking out loud. Once students have come up with a few expressions on their own, call students to the board to present, sharing how their thinking about the problem corresponds to the expression they wrote and explaining the steps for simplifying the expression. Encourage students to consider the arguments of their classmates and ask clarifying questions if they think an expression is incorrect or they think the expression given does not correspond to the way the student thought about the problem. Students will see that correct expressions simplify to the same value while incorrect expressions do not simplify to correct value. Once students have given the different expressions they came up with, put up any expressions from the teacher manual key that students did not come up with (some are incorrect and highlight common errors) and ask students if they think the expressions are correct. Then, simplify the expressions to verify.

Introduce special operator notation: ( ) and · for multiplication; fraction bar for division

Discuss grouping symbols: parentheses, nested parentheses, fraction bar as a grouping symbol

1. Shannon makes $8 an hour babysitting. How much does Shannon make for 3 hours of babysitting?

	Expression	Simplified Form	Does it work?	Ideas
a.	$3 + 8$	11	No	This space is for students to brainstorm, draw models, jot down anything they learn while doing these problems, etc. In this problem, students may recall that multiplication is a way to represent repeated addition. Therefore, $3(8)$ and $8 + 8 + 8$ are both expressions we can use to represent this situation. Notice they both simplify to 24.
b.	$3(8)$	24	Yes	
c.	$8 + 8 + 8$	24	Yes	
d.	$3 + 3 + 3$	9	No	

2. Carmen has 420 tickets to spend at the prize counter at an arcade. She buys 3 packs of Nerds that cost 50 tickets each. How many tickets does Carmen have left?

	Expression	Simplified Form	Does it work?	Ideas
a.	$420 - 3 \cdot 50$	270	Yes	Be sure to discuss order of operations and the role of grouping symbols in this problem. Why do we need parentheses to force the repeated addition to take place first? Why don't we need parentheses when we express the repeated addition using multiplication?
b.	$420 - 50 + 50 + 50$	470	No	
c.	$420 - 50 - 50 - 50$	270	Yes	
d.	$420 - (50 + 50 + 50)$	270	Yes	

This is a great problem to discuss the role of order of operations. The first expression works because by the order of operations, we preform multiplication first. In the last expression, the parentheses force you to first find the sum of 3 copies of 50 which is the same as the first step in expression a.

3. The second-grade class at Granite Elementary sold 80 raffle tickets one week. The third-grade class sold three times as many tickets as the second-grade class. How many tickets did the third-grade class sell?

	Expression	Simplified Form	Does it work?	Ideas
a.	$80 + 3$	83	No	You may want to use this space to draw a bar model to represent this situation: Second Grade: [80] Third Grade: [80 | 80 | 80] The last expression shows a common error where students find the sum of the raffle tickets sold by both classes.
b.	$80 + 80 + 80$	240	Yes	
c.	$3(80)$	240	Yes	
d.	$80 + 80 + 80 + 80$	320	No	

4. Use the information from the previous problem to determine the **total** number of tickets sold by the second- and third-grade classes.

	Expression	Simplified Form	Does it work?	Ideas
a.	$80 + 80 + 80$	240	No	A common error is for students to think that expression a. works. Ask students how this problem is different than the previous one. If you drew a bar model for the previous problem, ask students how we can use that to find the total number of tickets sold by both classes. You may also want to highlight where we see the tickets sold by each grade in each expression.
b.	$80 + 80 + 80 + 80$	320	Yes	
c.	$4(80)$	320	Yes	
d.	$80 + 3(80)$	320	Yes	

5. Ariana and Yesenia are twin sisters selling girl scout cookies. Ariana sells 50 boxes of girl scout cookies. Yesenia sells half as many boxes as her sister. How many boxes of girl scout cookies did the girls sell all together?

	Expression	Simplified Form	Does it work?	Ideas
a.	$50 + 25$	75	Yes	Again, a bar model might be helpful here: Ariana: [50] Yesenia: [25] Reinforce the fact that taking one-half of a number is the same as dividing that number by two.
b.	$50 + \frac{1}{2}$	$50\frac{1}{2}$	No	
c.	$50 + \frac{1}{2}(50)$	75	Yes	
d.	$50 + \frac{50}{2}$	75	Yes	

6. A rectangle has a length of 30 inches and a height of 15 inches. What is the perimeter of the rectangle?

	Expression	Simplified Form	Does it work?	Ideas
a.	$30 + 15 + 30 + 15$	90	Yes	Students may want to start by drawing a picture of the rectangle. This is an excellent problem to surface ideas about the Distributive Property and to highlight the Commutative and Associative Properties of Addition and Multiplication.
b.	$2(30 + 15)$	90	Yes	
c.	$2(30) + 15$	75	No	
d.	$2(30) + 2(15)$	90	Yes	

7. One base of a trapezoid measures 10 centimeters. The other base measures 8 centimeters. The height of the trapezoid is 7 centimeters. What is the area of the trapezoid?

	Expression	Simplified Form	Does it work?	Ideas
a.	$\frac{1}{2}(7)(10 + 8)$	63	Yes	The expressions that work show that to find the area of a trapezoid you multiply three factors together: $\frac{1}{2}$, the sum of the bases, and the height of the trapezoid. Using the Commutative and Associative Properties of Multiplication, we can multiply these factors in any order. It is much easier and faster to multiply $\frac{1}{2}$ and 18 first and then multiply the result by 7. This problem also highlights the role of grouping symbols, including the fraction bar being used as a grouping symbol.
b.	$\frac{7(10 + 8)}{2}$	63	Yes	
c.	$\left(\frac{7}{2}\right)\left(\frac{10 + 8}{2}\right)$	31.5	No	
d.	$\frac{(10 + 8)}{2} \cdot 7$	63	Yes	
e.	$\frac{7 \cdot 10 + 8}{2}$	39	No	

8. Matt makes \$450 from mowing lawns over the summer and \$350 babysitting. Matt saves 30% of the money he makes. How much does Matt save over the summer?

	Expression	Simplified Form	Does it work?	Ideas
a.	$0.3(450 + 350)$	240	Yes	These expressions also work: $(0.3)(800)$ $\frac{3(800)}{10}$ $0.15(450 + 350) + 0.15(450 + 350)$
b.	$0.1(800) + 0.1(800) + 0.1(800)$	240	Yes	
c.	$\frac{3}{10}(800)$	240	Yes	
d.	$\frac{3(450 + 350)}{10}$	240	Yes	

9. Antony made two 3-pointers and six 2-pointers at his basketball game. How many points did Antony score?

	Expression	Simplified Form	Does it work?	Ideas
a.	$3+3+2+2+2+2+2+2$	18	Yes	
b.	$(8)(2+3)$	40	No	
c.	$2(3)+6(2)$	18	Yes	
d.	$6+12$	18	Yes	

10. Gary bought 3 pounds of turkey for $8.99 per pound and 3 pounds of ham for $5.99 per pound for a work picnic. How much did Gary spend on lunch meat for the picnic?

	Expression	Simplified Form	Does it work?	Ideas
a.	$8.99+8.99+8.99+5.99+5.99+5.99$	44.94	Yes	This is an excellent problem for utilizing mental math and estimation strategies and reviewing operations with decimals. Students can use mental math to see that we can think of the first expression as $9+9+9+6+6+6-0.06$ which they can then think of as $10+10+10+5+5+5-0.06$ which is equal to 44.94
b.	$3(8.99)+3(5.99)$	44.94	Yes	
c.	$3(8.99)(5.99)$	About 180	No – too high	
d.	$3(8.99+5.99)$	44.94	Yes	

11. Nancy gave her niece 4 dollars, 4 dimes, and 4 pennies. How much money did Nancy give her niece?

	Expression	Simplified Form	Does it work?	Ideas
a.	$4(1.00)+4(0.10)+4(0.01)$	4.44	Yes	This reinforces operations with decimals, including the reason we line up decimals when we add. Money shows the different values of the number four in the expressions 4, 0.4, and 0.04.
b.	$4(1.00+0.10+0.01)$	4.44	Yes	
c.	$(4+4+4)+(1.00+0.10+0.01)$	13.11	No	
d.	$4.00+0.40+0.04$	4.44	Yes	

12. At a neighborhood egg hunt, there are 25 solid color eggs and 35 decorative eggs. The eggs are divided evenly between the 10 children at the egg hunt. How many eggs does each child get?

	Expression	Simplified Form	Does it work?	Ideas
a.	$10(25 + 35)$	600	No	This reinforces that dividing by 10 is the same as multiplying by $\frac{1}{10}$. It again shows the fraction bar being used as a grouping symbol and that the fraction bar also signifies division. The last expression is interesting because a child cannot have part of an egg (i.e., 2.5 solid eggs and 3.5 decorative eggs). It is mathematically correct but does not make sense in the real world.
b.	$\frac{1}{10}(25 + 35)$	6	Yes	
c.	$\frac{25+35}{10}$	6	Yes	
d.	$\frac{1}{10}(25) + \frac{1}{10}(35)$	6	Yes	

13. Brian tips 20% on a bill of $45. How much does Brian leave for a tip?

	Expression	Simplified Form	Does it work?	Ideas
a.	$0.20(45)$	9	Yes	A double number line or partial table, like the ones used in Chapter 2, can be used in this problem if needed.
b.	$\frac{1}{5}(45)$	9	Yes	
c.	$2(0.10(45))$	9	Yes	
d.	$\frac{1}{10}(45) + \frac{1}{10}(45)$	9	Yes	

14. Lorenzo scores a 98%, an 89%, a 93%, and a 90% on his four science tests one quarter. What is the mean of Lorenzo's test scores?

	Expression	Simplified Form	Does it work?	Ideas
a.	$98 + 89 + 93 + 90$	370	No	Emphasize mental math strategies. To find the sum of the test scores, give 2 from the number 93 to 98 to make it 100 and give 1 from the 93 to the number 89 to make it 90. The expression for the sum then becomes: $100 + 90 + 90 + 90$.
b.	$98 + 89 + 93 + 90 \div 4$	302.5	No	
c.	$(98 + 89 + 93 + 90) \div 4$	92.5	Yes	
d.	$\frac{98 + 89 + 93 + 90}{4}$	92.5	Yes	

15. Xander buys a paddle board that costs $450 and a wetsuit that costs $190. The sales tax rate is 6%. How much will Xander pay in sales tax?

	Expression	Simplified Form	Does it work?	Ideas
a.	$0.6(450 + 190)$	384	No, way too high	Again, a great problem for mental math. How can we quickly add 450 and 190? Give 10 to 190 to turn it to 200 leaving $440 + 200 = 640$. Students should estimate first. What is 10% of 640? 64. That means 5% of 640 is 32; therefore, our answer should be slightly greater than 32. Students may also use mental math to find 10% of the total and then divide it by 2 and add that to 1% of the total as shown in the last expression in the table.
b.	$0.06(450 + 190)$	38.4	Yes	
c.	$0.1(640) - 4(0.01(640))$	38.4	Yes	
d.	$\frac{0.1(640)}{2} + 0.01(640)$	38.4	Yes	

16. Find 58% of 20.

	Expression	Simplified Form	Does it work?	Ideas
a.	$0.6(20) - 2(0.01)(20)$	11.60	Yes	
b.	$0.5(20) + 0.02(20) + 0.02(20) + 0.02(20) + 0.02(20)$	11.60	Yes	
c.	$0.58(20)$ $0.58(10 + 10)$	11.60	Yes	
d.	$0.5(20) + 4(2(0.01 \cdot 20))$	11.60	Yes	

Directions: Write a story to match the numeric expression given. Then, answer the question.

Stories will vary. A possible story is given for each expression. Have students share out their stories. Other students can challenge if they do not think the story matches the expression. Write the expression that would correspond with any incorrect stories.

17. $40(10)$ ___Chelsea makes $10 an hour working as a lifeguard at the pool. She works 40 hours a week. How much does she make in one week? $400

18. $125 - 2(50)$ ___Olivia takes $125 to the store to buy jeans. She buys two pairs of jeans that each cost $50. How much money does she have left? $25

19. $2(12) + 2(10)$ ___ The floor of a room has a length of 12 feet and a width of 10 feet. What is the perimeter of the floor? 44 feet

20. $0.8(45)$ ___Jameson got an 80% on his math test. If there were 45 questions on the math test, how many questions did Jameson get correct?

21. $\frac{2+0+1+2+3+1+1+1+0+1}{10}$ ___Ben asked ten of his friends how many siblings they have. What is the mean of the data? 1.2

22. $(0.1)(25) + \frac{0.1(25)}{2}$ ___Colton tipped 15% on a $25 bill. How much did Colton tip? $3.75

Spiral Review

1. Which property is being illustrated in each problem below?

a. $3 + 5 = 5 + 3$	b. $\left(\frac{1}{2}\right)(9)(14) = \left(\frac{1}{2}\right)(14)(9)$
c. $5 \cdot 0 = 0$	d. $3(14) = 3(10 + 4)$

2. Simplify the expressions.

a. $4 + 2 \times 3$	b. $(4 + 2) \times 3$
c. $20 \div 5(2)$	d. $20 \div (5 \cdot 2)$

3. What is the value of 10^3?

4. Write the prime factorization for 8.

6.1a Homework: Translating Contexts to Equivalent Numeric Expressions

Directions: Determine whether the expressions given for each problem "work" or "don't work". If an expression does not work, provide a justification for why it is incorrect. If you can come up with additional expressions that work, write them in the ideas column.

1. Lily had $150 in her savings account at the start of the month. She saved $60 a week for 4 weeks. How much does Lily have in her savings account now?

	Expression	Simplified Form	Does it work?	Ideas
a.	$150 + 60$			
b.	$150 + 60 + 60 + 60 + 60$			
c.	$150 + 4(60)$			
d.	$150 + 4 + 60$			

2. Max takes $100 to a sporting goods store to buy baseball pants. He buys two pairs of baseball pants that each cost $29.99. How much money does Max have left?

	Expression	Simplified Form	Does it work?	Ideas
a.	$100 - (29.99 - 29.99)$	100	No	
b.	$100 - (29.99 + 29.99)$	40.02	Yes	
c.	$100 - 29.99 - 29.99$	40.02	Yes	
d.	$100 - 2(29.99)$	40.02	Yes	

3. Gia is seven years younger than her cousin Nick. If Nick is thirteen years old, what is the sum of Nick and Gia's ages?

	Expression	Simplified Form	Does it work?	Ideas
a.	$13 + 6$			
b.	$13 + (13 - 7)$			
c.	$(13 - 7) + 13$			
d.	$13 + 7$			

4. The side length of a square is 6 inches. What is the perimeter of the square?

	Expression	Simplified Form	Does it work?	Ideas
a.	$6 + 6 + 6 + 6$			
b.	$4(6)$			
c.	$6 \cdot 6$			
d.	$2(6) + 2(6)$			

5. The base of a triangle measure 3 feet. The height of the triangle measures 6 feet. What is the area of the triangle?

	Expression	Simplified Form	Does it work?	Ideas
a.	$2(3)(6)$	36	No	
b.	$\frac{3(6)}{2}$	9	Yes	
c.	$\frac{1}{2}(3 \cdot 6)$	9	Yes	
d.	$\frac{1}{2} \cdot 6 \cdot 3$	9	Yes	

6. Giselle completes 20 math problems in 5 minutes. How many problems does Giselle complete in 3 minutes?

	Expression	Simplified Form	Does it work?	Ideas
a.	$3(\frac{20}{5})$			
b.	$\frac{20}{5}+\frac{20}{5}+\frac{20}{5}$			
c.	$4+4+4$			
d.	$3(4)$			

7. I bought two toy cars for $5 each and three toy trucks for $7 each. How much money did I spend?

	Expression	Simplified Form	Does it work?	Ideas
a.	$2(5)\ +\ 3(7)$			
b.	$2(7)\ +\ 3(5)$			
c.	$(2\ +\ 3)(5\ +\ 7)$			
d.	$(5+\ 5)\ +\ (7+7+\ 7)$			

8. A football team scored three touchdowns, two field goals, and two extra points. How many points did the football team score? (Hint: a touchdown is 6 points, a field goal is 3 points, and an extra point is 1 point)

	Expression	Simplified Form	Does it work?	Ideas
a.	$3(6)\ +\ 2(3)\ +\ 2(1)$			
b.	$6\ +\ 6\ +\ 6\ +\ 3\ +\ 3\ +\ 1\ +\ 1$			
c.	$(6\ +\ 6\ +\ 6)\ (3\ +\ 3)\ (1\ +\ 1)$			
d.	$18\ +\ 6\ +\ 2$			

9. 25% of the students in Marina's class play a musical instrument. If there are 32 students in Marina's class, how many play an instrument?

	Expression	Simplified Form	Does it work?	Ideas
a.	$0.25 + 32$	32.25	No	
b.	$0.25(32)$	8	Yes	
c.	$\frac{1}{4}(32)$	8	Yes	
d.	$\frac{32}{4}$	8	Yes	

Directions: Write a context to match the numeric expression given. Then, evaluate each expression to answer the question.

10. $4 \cdot 8$ ______________________________

11. $10 - 3(0.75) - 2(0.50)$ ______________________________

12. $0.2(32) + 0.2(28)$ ______________________________

13. $40 + 6(25)$ ______________________________

14. $\frac{(7)(4)}{2}$ ______________________________

15. $\frac{1}{2}(45 + 60)$ ____________________

____

____

16. $0.1(50) + (0.01)(50) + (0.01)(50)$ ____________________

____

____

17. $2(3 \cdot 12)$ ____________________

____

____

6.1b Class Activity: How Many Expressions Can You Make?

Directions: Working in a group, try to write as many different numeric expressions as you can to represent each context.

For students who struggle, it may help them to draw models. For example, in Round 1, they may draw rectangles to represent the cars and trucks.

Round 1: I bought 2 toy cars for \$1.25 each and 3 toy trucks for \$1.70 each. How much did I spend?

Sample Expressions: $1.25 + 1.25 + 1.70 + 1.70 + 1.70$ or $2(1.25) + 3(1.70)$

\$7.60

Round 2: I had \$12. Then I spent \$2.15 a day for 5 days in a row. How much money do I have now?

\$1.25

Round 3: I bought five apples for \$0.30 each and five oranges for \$0.35 each. How much money did I spend?

\$3.25

Round 4: A rectangular garden has a length of 10 feet and a width of 6 feet. What is the perimeter of the garden?

32 feet

Round 5: According to a study, 15% of adolescents get the recommended amount of sleep (8 – 10 hours) each night. At Jenna's middle school, there are one hundred twenty 6^{th} graders, ninety-four 7^{th} graders, and one hundred six 8^{th} graders. Using the number from the study, how many students at Jenna's school would we expect to get the recommended amount of sleep each night?

48 students

Round 6: Nathan is a waiter. He made $165 in tips on Saturday night and $80 in tips on Sunday night. He must pay 10% of what he earns in tips to the people who bus the tables. How much must Nathan pay out to the people who bus the tables at the end of the weekend?

$24.50

Round 7: Uncle Aaron gave 3 dimes, 2 nickels, and 5 pennies to each of his two nephews. How much money did he give away?
$0.90

Round 8: Jess and his two friends, Kyle and Jayden, are trick-or-treating. They have decided to split their candy evenly at the end of the night. Jess collects 63 pieces of candy, Kyle collects 57 pieces of candy, and Jayden collects 66 pieces of candy. How many pieces of candy will each person get?
62 pieces of candy

Spiral Review

1. Simplify.

a. $\frac{30}{5}$	b. $\frac{5}{30}$
c. $\frac{6}{10}$	d. $\frac{10}{6}$

2. Simplify the expressions.

a. $5 \cdot 8 - 10 \div 2$	b. $48 - 3 \cdot 5 + 2$
c. $\frac{8+4}{3}$	d. $9 - 32 \div 4$

3. What is the value of 2×10^3?

4. What are the common factors of 8 and 20? What is the greatest common factor of 8 and 20?

6.1b Homework: How Many Expressions Can You Make?

Directions: Come up with *at least two different* expressions to represent each context. Then, simplify the expressions.

1. I earned \$6. Then, I bought 4 candy bars for \$0.75 each. How much money do I have left?
   \$3
   $6 - 4(0.75)$

   $6 - 0.75 - 0.75 - 0.75 - 0.75$

   $6 - (0.75 + 0.75 + 0.75 + 0.75)$

2. Cara bought two candles for \$3 each and three books for \$7 each. How much did she spend on her purchases?

3. Mona and Teresa worked together to make \$118 selling phone covers and \$354 fixing computers. If they split the money evenly between the two of them, how much money did they each make?

4. Jack makes 60% of the free throws he shoots. If he shot 5 free throws in his first game, 8 free throws in his second game, and 2 free throws in his third game, how many free throws would you expect that he made?

5. Marcela bought 5 cheese pizzas and 5 pepperoni pizzas for a soccer team party. If the cheese pizzas each cost \$8.99 and the pepperoni pizzas each cost \$11.99, how much did Marcela spend on pizza for the team party?

6.1c Class Activity: Algebraic Expressions and Equivalence

Activity 1: Variables – What are They and How Do We Use Them?

a. **Letters to Represent Unknowns:** You have seen symbols being used to represent unknown quantities in expressions and equations for many years. Early on, unknown quantities were represented by question marks and boxes. Later, letters were used to represent unknown quantities. Do the following problems look familiar?

$5 + ? = 11$	$4 \times \square = 12$	$300 \div x = 3$
$? =$ ______	$\square =$ ______	$x =$ ______

b. **Variables to Communicate Mathematical Ideas:** You have also seen letters used to communicate mathematical ideas, such as the Properties of Addition and Multiplication. For example, we can state the Commutative Property of Addition using **words**:

Commutative Property of Addition: Changing the order of the addends in an addition problem does not change the result.

We can also illustrate the Commutative Property of Addition using **letters and symbols**:

Commutative Property of Addition: $a + b = b + a$

We also see letters in geometry formulas. For example, we can state the formula for the area of a triangle using **words**:

Area of a Triangle: Multiply the base and the height of the triangle and then divide the result by two.

We can also show this formula using **letters and symbols**:

$$A = \frac{b \times h}{2}$$

c. **Variables to Show Relationships Between Quantities:** We have also seen variables being used to represent relationships between two quantities. For example, we can express how to change feet into inches using **words**:

To find the number of inches in a specified number of feet, multiply the number of feet by 12.

We can also show this relationship using **letters and symbols**:

$i = 12f$

In this case, we may treat one of the variables as an unknown. For example, we may ask, "How many inches are in 3 feet?" In this case, i is an unknown number and we can determine that it is equal to 36. Alternatively, we can treat the variables as a set of numbers and examine how one variable changes as the other changes. I may want to know how many inches are in 1 foot, 2 feet, 3 feet, 4 feet, and 5 feet.

Activity 2: In the previous lesson, we saw different but equivalent ways to write numeric expressions to represent real world situations. Numeric expressions are a shorthand way to describe a process – they consist of numbers, operations, and grouping symbols. **Equivalent numeric expressions** simplify to the same number. For example,

- $3(8)$ and $8 + 8 + 8$ are equivalent numeric expressions because they both simplify to 24
- $420 - 3 \cdot 50$, $420 - 50 - 50 - 50$, and $420 - (50 + 50 + 50)$ are equivalent numeric expressions because they all simplify to 270
- $30 + 15 + 30 + 15$, $2(30) + 2(15)$, and $2(30 + 15)$ are equivalent numeric expressions because they all simplify to 90.

Again, discuss the role of order of operations and grouping symbols in the examples above.

What about **algebraic expressions**? Like numeric expressions, algebraic expressions are a shorthand way to describe a process and consist of numbers, operations, and grouping symbols. Algebraic expressions also contain one or more variables. So, how can we tell if two or more algebraic expressions are equivalent? For example, how do we know if the expression $x + 5$ is equivalent to $5 + x$ or if the expression $\frac{n}{4}$ is equivalent to $\frac{4}{n}$? It is worth mentioning to students that expressions do not have equal signs like equations which they will see in Section 3. Also, you cannot *solve* an expression. You can *transform* it to write equivalent expressions, including the *simplified form* of the expression and you can *evaluate* an expression for given values of the variable.

Equivalent algebraic expressions simplify to the same number regardless of which value is substituted into them. In the examples above, $x + 5$ and $5 + x$ are equivalent algebraic expressions. We can substitute in any value for x and the expressions will simplify to the same number. Substituting in a specific number for the variable in an expression and simplifying the expression is called **evaluating** the expression.

	$\boldsymbol{x + 5}$	$\boldsymbol{5 + x}$
Let $\boldsymbol{x = 2}$	$2 + 5 = 7$	$5 + 2 = 7$
Let $\boldsymbol{x = 13}$	$13 + 5 = 18$	$5 + 13 = 18$

You can see that when $x = 2$ both expressions simplify to 7 and when $x = 13$ both expressions simplify to 18.

Expressions that are NOT equivalent: What about the expressions $\frac{n}{4}$ and $\frac{4}{n}$? Let's test a few values to see if the expressions are equivalent. Just looking at these expressions, we might assume that the expressions are not equivalent because the dividend and divisor have been switched and we know that division is not commutative. Let's test a few values to verify our thinking:

	$\boldsymbol{\frac{n}{4}}$	$\boldsymbol{\frac{4}{n}}$
Let $\boldsymbol{n = 4}$	$\frac{4}{4} = 1$	$\frac{4}{4} = 1$
Let $\boldsymbol{n = 8}$	$\frac{8}{4} = 2$	$\frac{4}{8} = \frac{1}{2}$

When $n = 4$, the expressions evaluate to the same number; however, when $n = 8$ they do not. This example shows us the importance of **testing <u>at least two values</u> to determine equivalence**. **If we can find one number that causes the expressions to simplify to different values when substituted for the variable, then we can show that the expressions are *not equivalent.*** It is not practical to test for equivalence by substituting in *every* value for the variable. Since the problems in this lesson are linear expressions (expressions in the form $Ax + B$), it suffices to test two different values for the variable. Once we show that the expressions simplify to the same value for two *different* values for the variable, we have shown the expressions are equivalent. The reasoning for this is beyond the scope of Grade 6. See the Mathematical Foundation for further discussion on this.

Directions: In each of the problems shown, two algebraic expressions are shown. Determine whether the expressions are equivalent or not equivalent.

a. Equivalent or Not? ____Not equivalent____________________

Helpful Hint: Putting a number side by side with a variable, as in the expression $3x$, is a shorthand way of expressing multiplication. So, the expression $3x$ means 3 multiplied by x or $3 \times x$. Since the symbol $\times$ is often confused with the variable x, we can also use a ***middle dot*** *or* ***parentheses*** *to represent multiplication. So, $3x$ can also be written as $3 \cdot x$ or $3(x)$.*

	$x + 3$	$3x$
Let $x = 0$	$0 + 3$ 3	$3 \cdot 0$ 0
Let $x = 2$	$2 + 3$ 5	$3 \cdot 2$ 6

b. Equivalent or Not? ______Not equivalent__________________

	$3x$	$x \cdot x \cdot x$
Let $x = 0$	$3 \cdot 0$ 0	$0 \cdot 0 \cdot 0$ 0
Let $x = 2$	$3 \cdot 2$ 6	$2 \cdot 2 \cdot 2$ 8

Be careful on this problem. Students may think that because both expressions simplify to 0 when 0 is substituted in for *x* that these expressions are equivalent. However, if we substitute in 2 for *x*, we see that the expressions are not equivalent.

c. Equivalent or Not? ___Equivalent______________________

	$3x$	$x+x+x$
Let $x = 0$	$3 \cdot 0$ 0	$0+0+0$ 0
Let $x = 2$	$3 \cdot 2$ 6	$2+2+2$ 6

d. Equivalent or Not? _____Equivalent____________________

	$5+c+c+c$	$5+3c$
Let $c = 2$	$5+2+2+2$ 11	$5+3 \cdot 2$ $5+6$ 11 Remind students to follow the order of operations.
Let $c = 3$	$5+3+3+3$ 14	$5+3 \cdot 3$ $5+9$ 14

e. Equivalent or Not? _________Not equivalent________________

	$5+3c$	$8c$
Let $c = 2$	$5+3 \cdot 2$ $5+6$ 11 Remind students to follow order of operations.	$8 \cdot 2$ 16
Let $c = 3$	$5+3 \cdot 3$ $5+9$ 14	$8 \cdot 3$ 24

f. Equivalent or Not? ___Equivalent____________________

Helpful Hint: One way to interpret a fraction is to view it as the numerator divided by the denominator. So, the fraction $\frac{a}{b}$ *can be thought of as* $a \div b$.

	$\frac{x}{10} + 3$	$\frac{1}{10}(x + 30)$
Let $x = 20$	$\frac{20}{10} + 3$ $2 + 3$ 5	$\frac{1}{10}(20 + 30)$ $\frac{1}{10}(50)$ 5
Let $x = 50$	$\frac{50}{10} + 3$ $5 + 3$ 8	$\frac{1}{10}(50 + 30)$ $\frac{1}{10}(80)$ 8

g. Equivalent or Not? _____Equivalent__________________

	$\frac{1}{10}(x + 30)$	$\frac{1}{10}(x) + \frac{1}{10}(30)$
Let $x = 20$	$\frac{1}{10}(20 + 30)$ $\frac{1}{10}(50)$ 5	$\frac{1}{10}(20) + \frac{1}{10}(30)$ $2 + 3$ 5
Let $x = 50$	$\frac{1}{10}(50 + 30)$ $\frac{1}{10}(80)$ 8	$\frac{1}{10}(50) + \frac{1}{10}(30)$ $5 + 3$ 8

h. Equivalent or Not? _Equivalent______________________

	$\frac{1}{10}(x + 30)$	$\frac{x+30}{10}$
Let $x = 20$	$\frac{1}{10}(20 + 30)$ $\frac{1}{10}(50)$ 5	$\frac{20+30}{10}$ $\frac{50}{10}$ 5
Let $x = 50$	$\frac{1}{10}(50 + 30)$ $\frac{1}{10}(80)$ 8	$\frac{50 + 30}{10}$ $\frac{80}{10}$ 8

i. Equivalent or Not? ______Not equivalent____________________

	$4n + 2n$	$6 \cdot n \cdot n$
Let $n = 5$	$4(5) + 2(5)$ $20 + 10$ 30	$6 \cdot 5 \cdot 5$ $30 \cdot 5$ 150
Let $n = 10$	$4(10) + 2(10)$ $40 + 20$ 60	$6 \cdot 10 \cdot 10$ $60 \cdot 10$ 600

j. Equivalent or Not? _______Equivalent___________________

	$4n + 2n$	$(4 + 2)n$
Let $n = 5$	$4(5) + 2(5)$ $20 + 10$ 30	$(4 + 2)5$ $(6)5$ 30
Let $n = 10$	$4(10) + 2(10)$ $40 + 20$ 60	$(4 + 2)10$ $(6)10$ 60

k. Equivalent or Not? ___Equivalent______________________

	$4n + 2n$	$6n$
Let $n = 5$	$4(5) + 2(5)$ $20 + 10$ 30	$6 \cdot 5$ 30
Let $n = 10$	$4(10) + 2(10)$ $40 + 20$ 60	$6 \cdot 10$ 60

l. Equivalent or Not? _______Not equivalent_________________

	$2(x+3)$	$2x+3$
Let $x=1$	$2(1+3)$ $2(4)$ 8	$2(1)+3$ $2+3$ 5
Let $x=5$	$2(5+3)$ $2(8)$ 16	$2(5)+3$ $10+3$ 13

m. Equivalent or Not? _____Equivalent___________________

	$2(x+3)$	$2x+2\cdot 3$
Let $x=1$	$2(1+3)$ $2(4)$ 8	$2\cdot 1+2\cdot 3$ $2+6$ 8
Let $x=5$	$2(5+3)$ $2(8)$ 16	$2\cdot 5+2\cdot 3$ $10+6$ 16

n. Equivalent or Not? _______Equivalent_________________

	$2(x+3)$	$(x+3)+(x+3)$
Let $x=1$	$2(1+3)$ $2(4)$ 8	$(1+3)+(1+3)$ $4+4$ 8
Let $x=5$	$2(5+3)$ $2(8)$ 16	$(5+3)+(5+3)$ $8+8$ 16

Activity 3: Write an algebraic expression for each phrase.

Students will likely need a review of the following vocabulary: sum, difference, product, and quotient.

Have students underline key words. When a phrase starts out as "the sum of", "the difference of", "the product of", or "quotient of" students should look for the word and – it helps to identify the different parts of the expression - the addends (addition), the minuend and subtrahend (subtraction), the factors (multiplication) and the dividend and divisor (division).

a. The sum of a number n and twenty $n + 20$

b. The sum of twenty and a number n $20 + n$
Although we know that this expression can be written as $n + 20$ due to the Commutative Property of Addition, the literal translation is $20 + n$.

c. Four less than a number c $c - 4$
This often confuses students because they think that the first quantity in the phrase is always the first quantity in the symbolic representation. If students are struggling, put up a few numeric examples:
What is 4 less than 5? 1 Symbolic Representation: $5 - 4 = 1$
What is 4 less than 8? 4 Symbolic Representation: $8 - 4 = 4$
What is 4 less than 10? 6 Symbolic Representation: $10 - 4 = 6$

d. A number c less than four $4 - c$

e. The quotient of a number n and three $\frac{n}{3}$

f. The quotient of three and a number n $\frac{3}{n}$

g. The difference of twice a number x and five $2x - 5$

h. The difference of five and twice a number x $5 - 2x$

i. Twice the difference of a number x and five $2(x - 5)$
Work from the inside out. Start by writing the difference of a number x and five symbolically. Then, treat the difference as a single object, denoted with parentheses, that is doubled. Ask, "Twice, what?" *Twice the difference.* We can hold the difference together with parentheses. This relates directly to the order of operations – we want to take the difference first and then double it.
As an extension, ask students why this expression will always simplify to an even number regardless of the value that is substituted for x. Two multiplied by any number will result in an even number.

j. Four times the sum of a number n and six $4(n + 6)$
Again, represent the sum symbolically and then view the sum as a single object that is being quadrupled.

k. The sum of four times a number *n* and six $4n + 6$

l. The sum of eight and the product of two and a number *x* $8 + 2x$

m. Are the expressions you wrote for parts *a* and *b* equivalent? Justify your answer.
Yes. Students may justify their answer by stating that *a* and *b* are equivalent by the Commutative Property of Addition. Alternatively, they may test for equivalence by substituting in two different values for *n*.

n. Are the expressions you wrote for parts *c* and *d* equivalent? Justify your answer.
No. Students may again justify their answer by stating that subtraction is not commutative. Alternatively, they may test values for *c*. Even though students have not been introduced to operations with negative numbers, they should have an intuitive understanding that, depending on the value chosen for *c*, one of the expressions is going to be negative.

o. Are the expressions you wrote for parts *e* and *f* equivalent? Justify your answer.
No. Students should be strategic in the values they substitute in for *n*. For this problem, it would make sense to choose numbers that are divisible by 3.

p. Are the expressions you wrote for parts *g* – *i* equivalent? Justify your answer.
No. We can see this by various choices of *x* yielding different values.

q. Are the expressions you wrote for parts *j* and *k* equivalent? Justify your answer.
No, this problem surfaces ideas about the Distributive Property

Activity 4: Write a phrase for each algebraic expression.
Have students share out their phrases. Sample answers have been provided.

1. $x + 8$ "A number *x* plus eight." "The sum of a number *x* and eight." "A number *x* increased by 8." "Eight more than a number *x*."

2. $14 - n$ "The difference of fourteen and a number *n*." "Fourteen decreased by a number *n*." "A number *n* less than fourteen." "

3. $2(n + 3)$ "Twice the sum of a number *n* and three." "Two times the *quantity n* plus three."

4. $2n + 3$ "The sum of twice a number *n* and three." "Twice a number *n* increased by three."

5. $\frac{15}{y}$ "The quotient of fifteen and a number *y*." "Fifteen divided by a number *y*."

6. $\frac{y}{15}$ "The quotient of a number *y* and fifteen." "A number *y* divided by fifteen."

7. $\frac{x}{20}+4$ "Four more than the quotient of a number *x* and twenty."

8. $\frac{x+4}{20}$ "The quotient of the sum of a number *x* and four and twenty." "The quantity *x* plus four divided by twenty."

9. $3x+5$ "Three times a number *x* increased by five."

Spiral Review

1. Simplify.

a. $\frac{3+7}{9-4}$ $\frac{10}{5}=2$	b. $\frac{9-4}{3+7}$ $\frac{5}{10}=\frac{1}{2}$

2. Find the perimeter of a rectangle with a length of 7 inches and a width of 5 inches.
24 inches

3. Find the mean of the data set $\{12, 22, 16, 12, 14\}$.
15.2

4. Find the volume of a cube with a side length equal to 3 inches.
27 cubic inches

6.1c Homework: Algebraic Expressions and Equivalence

Directions: In each of the problems shown, two algebraic expressions are shown. Determine whether the expressions are equivalent or not equivalent.

1. Equivalent or Not? ____

	$x + x + x + x$	$4 \cdot x \cdot x \cdot x \cdot x$
Let $x = 1$		
Let $x = 2$		

2. Equivalent or Not? _____Equivalent___________________

	$x + x + x + x$	$4x$
Let $x = 1$		
Let $x = 2$		

3. Equivalent or Not? _______________________________________

	$4x$	$2x + 2x$
Let $x = 20$		
Let $x = 30$		

4. Equivalent or Not? _____Not Equivalent___________________

	$4 \cdot 2x$	$6x$
Let $x = 3$		
Let $x = 4$		

When students evaluate the expression $4 \cdot 2x$, encourage them to put the dot in for multiplication. This helps them to see that multiplication is the only operation and using the Associative Property of Multiplication, we can find the product of 4 and 2 and view this as $8x$. This will also help students in the next section when faced with problems like $4x^2$.

5. Equivalent or Not? _____Equivalent___________________

	$4 \cdot 2x$	$8x$
Let $x = 3$		
Let $x = 4$		

6. Equivalent or Not? ______________________________________

	$(2x) + (2x) + (2x) + (2x)$	$4 \cdot 2x$
Let $x = 3$	$(2 \cdot 3) + (2 \cdot 3) + (2 \cdot 3) + (2 \cdot 3)$ $6 + 6 + 6 + 6$ 24	$4 \cdot 2 \cdot 3$ 24
Let $x = 4$		

The expressions in this problem show the relationship between repeated addition and multiplication. Both are four copies of $2x$.

7. Equivalent or Not? ____

	$\frac{x}{2} - 8$	$\frac{1}{2}x - 8$
Let $x = 20$		
Let $x = 30$		

8. Equivalent or Not? ____

	$\frac{1}{2}x - 8$	$\frac{1}{2}(x - 16)$
Let $x = 20$		
Let $x = 30$		

9. Equivalent or Not? _____Equivalent____________________

	$\frac{1}{2}(x - 16)$	$\frac{x - 16}{2}$
Let $x = 20$		
Let $x = 30$		

#7 – 9 are a great preview of the Distributive Property. Additionally, they reinforce that multiplying by $\frac{1}{2}$ is the same as dividing by 2.

10. Equivalent or Not? ____

	$30 - n - n - n$	$30 - 3n$
Let $n = 2$		
Let $n = 5$		

11. Equivalent or Not? _____Equivalent____________________

	$30 - n - n - n$	$30 - (n + n + n)$
Let $n = 2$		
Let $n = 5$		

12. Equivalent or Not? ____

Evaluate	$30 - (n + n + n)$	$30 - n + n + n$
Let $n = 2$		
Let $n = 5$		

13. Equivalent or Not? ______Equivalent____________________

	$0.2(x)$	$0.1(x) + 0.1(x)$
Let $x = 50$		
Let $x = 100$		

14. Equivalent or Not? ____

	$0.2(x)$	$\frac{2x}{10}$
Let $x = 50$		
Let $x = 100$		

15. Equivalent or Not? ____

	$\frac{2x}{10}$	$\frac{x + x}{10}$
Let $x = 50$		
Let $x = 100$		

16. Equivalent or Not? ____

	$10y - 8y$	$2y$
Let $y = 5$		
Let $y = 8$		

17. Equivalent or Not? _____Not equivalent____________________

	$10y - 8y$	$10y - 4y + 4y$
Let $y = 5$		
Let $y = 8$		

18. Equivalent or Not? ____

	$10y - 8y$	2
Let $y = 5$		
Let $y = 8$		

Directions: Write an algebraic expression for each phrase.

19. A number *j* increased by four

20. The quotient of five and a number *x*

$\frac{5}{x}$

21. Ten less than three times a number *y*

$3y - 10$

22. The product of a number *n* and one-third

23. The quotient of the sum of a number *b* and seven and two

$\frac{b+7}{2}$

24. A number *k* decreased by four

25. A number four decreased by *k*

26. Four more than the quotient of a number *x* and three

27. The quotient of four more than a number *x* and three

28. Seventeen minus the product of seventy-five hundredths and a number *n*

29. Are the expressions in #26 and #27 equivalent? Justify your answer.
No, students can substitute in a number for *x* and show that when simplified the results are different. Alternatively, students may be use informal ideas about the Distributive Property to justify their answer.

30. Write an expression that is equivalent to the expression in #26?

31. Write an expression that is equivalent to the expression in #27? $\frac{x}{3}+\frac{4}{3}$; $\frac{1}{3}(x+4)$

Directions: Write a phrase for each algebraic expression.
Answers will vary. Sample answers are provided.

32. $x-7$

33. $7-x$

34. $15-2x$ "The difference of fifteen and twice a number *x*."

35. $2x-15$ "The difference of twice a number *x* and fifteen."

6.1d Class Activity: Transitioning from Numeric Expressions to Algebraic Expressions

We will re-visit many of the problems in this lesson in the lessons that follow.

Directions: For each situation, complete the table to show several different expressions to represent the situation.

The purpose of this lesson is to transition students from numeric expressions to algebraic expressions. A numeric expression helps students to identify patterns and see and interpret the structure of the expression. This provides scaffolding for the introduction of a variable. Additionally, having students write several expressions to represent a situation reinforces the learning about equivalent expressions and begins to surface ideas about how to simplify expressions which students will study in detail in Section 2. Be sure to let students know that there may not be three different expressions to represent each situation. If students generate more than three expressions, have them write the additional expressions in the space below each table. Also, you may wish to have calculators available for simplifying the expressions to ensure they are equivalent.

a. Maria bought 5 apples at the store. Write different expressions to represent the total cost of the apples based on the price of the apples.

Cost of Each Apple	Total Cost of Apples Expression 1	Total Cost of Apples Expression 2	Total Cost of Apples Expression 3
\$0.35	$0.35 + 0.35 + 0.35 + 0.35 + 0.35$	$5 \cdot 0.35$	$2(0.35) + 3(0.35)$
\$0.40	$0.4 + 0.4 + 0.4 + 0.4 + 0.4$	$5 \cdot 0.4$	$2(0.4) + 3(0.4)$
c	$c + c + c + c + c$	$5 \cdot c$ or $5c$	$2(c) + 3(c)$ or $2c + 3c$

It is important to remind students that when multiplication involves a variable, it is often written as a juxtaposition. In the example above, $5 \cdot c$ is written as $5c$. This makes it easier to interpret more complicated expressions and equations such as $(3x - 4)(2x + 4) = 54$ which would be difficult to interpret if written as $(3(x) - 4)(2(x) + 4) = 54$ or $(3 \cdot x - 4)(2 \cdot x + 4) = 54$ We cannot use this shorthand when the multiplication involves two numbers such as the expression $5 \cdot 0.35$. A juxtaposition of these would result in a different expression: 50.35.

b. Write different expressions to represent the perimeter of a square based on the side length of the square.

Side Length of the Square (units)	Perimeter of Square (units) Expression 1	Perimeter of Square (units) Expression 2	Perimeter of Square (units) Expression 3
5	$5 + 5 + 5 + 5$	$4(5)$	$2(5) + 2(5)$
8	$8 + 8 + 8 + 8$	$4(8)$	$2(8) + 2(8)$
s	$s + s + s + s$	$4(s)$ or $4s$	$2s + 2s$

c. Marin has 50 tickets to spend on rides at a carnival. Each ride takes 6 tickets. Write different expressions to represent the number of tickets Marin has left based on the number of rides she goes on.

Number of Rides Marin Has Gone On	Number of Tickets Remaining Expression 1	Number of Tickets Remaining Expression 2	Number of Tickets Remaining Expression 3
2	$50 - 6 - 6$	$50 - 2(6)$	$50 - (6 + 6)$
3	$50 - 6 - 6 - 6$	$50 - 3(6)$	$50 - (6 + 6 + 6)$
r	$50 - \underbrace{6 - 6 - 6 - 6 - 6 \ldots}_{r \text{ times}}$	$50 - r(6)$ or $50 - 6r$	$50 - \underbrace{(6 + 6 + 6 + 6 + 6 \ldots)}_{r \text{ times}}$

The second expression will likely bring up the discussion about whether we can write $50 - r6$. Tell students that the convention when we juxtapose a variable and a number is to put the number before the variable. Refer to the role order of operations plays in this problem. We can subtract r copies of 6 or add the r copies of 6 first (by inserting parentheses) and then subtract this sum from 50.

d. Tamara earns money babysitting and doing chores. She saves 25% of what she earns. Write different expressions to represent the amount Tamara saves based on the amount she makes babysitting and doing chores.

Tamara's Earnings ($)	Tamara's Savings ($) Expression 1	Tamara's Savings ($) Expression 2	Tamara's Savings Expression 3
Babysitting: $40 Chores: $20	$0.25(40) + 0.25(20)$	$0.25(40 + 20)$	$\frac{40 + 20}{4}$
Babysitting: $100 Chores: $80	$0.25(100) + 0.25(80)$	$0.25(100 + 80)$	$\frac{100 + 80}{4}$
Babysitting: b Chores: c	$0.25(b) + 0.25(c)$ $0.25b + 0.25c$	$0.25(b + c)$	$\frac{b + c}{4}$

e. Sam currently has $120 in his savings account. He has a summer job and has decided to save a certain amount of money each week for 6 weeks. Write different expressions to represent the total amount of money he will have in his account based on the amount he saves each week.

Amount Saved Each Week (in dollars)	Savings ($) Expression 1	Savings ($) Expression 2	Savings ($) Expression 3
$8	$120 + 8 + 8 + 8 + 8 + 8 + 8$	$120 + 6(8)$	$120 + 5(8) + 8$
$15	$120 + 15 + 15 + 15 + 15 + 15 + 15$	$120 + 6(15)$	$120 + 5(15) + 15$
s	$120 + s + s + s + s + s + s$	$120 + 6s$	$120 + 5s + s$

Students may not naturally gravitate to the third expression shown but it is a good one to show students. This expression makes it easier for students to use mental math to simplify the numeric expressions. It is also a great preview to show that $5s + s$ is equivalent to $6s$.

f. Mr. Johnson's class is helping clean up the garden at school. There are 52 students in his class. Four of the students will oversee putting weeds into garbage cans. The remaining students will be divided into groups to weed different areas of the garden. Write different expressions to represent the number of groups that can be made based on the number of students in each group.

Students in a Group	Number of Groups Expression 1	Number of Groups Expression 2	Number of Groups Expression 3
6	$(52-4) \div 6$	$\frac{52-4}{6}$	$\frac{1}{6}(52-4)$
8	$(52-4) \div 8$	$\frac{52-4}{8}$	$\frac{1}{8}(52-4)$
n	$(52-4) \div n$	$\frac{52-4}{n}$	$\frac{1}{n}(52-4)$

g. On Monday night, Aidan got 6 hours of sleep. On Tuesday night, he got 8 hours of sleep. On Wednesday night, he got 8 hours of sleep and on Thursday night he got 7 hours of sleep. Write different expressions to represent the average number of hours of sleep Aidan got Monday – Friday based on the number of hours of sleep he got on Friday night.

Hours of Sleep on Friday Night	Average Hours of Sleep from Monday – Friday Expression 1	Average Hours of Sleep from Monday – Friday Expression 2	Average Hours of Sleep from Monday – Friday Expression 3
5	$\frac{6+8+8+7+5}{5}$	$\frac{1}{5}(6+8+8+7+5)$	$\frac{6}{5}+\frac{8}{5}+\frac{8}{5}+\frac{7}{5}+\frac{5}{5}$
10	$\frac{6+8+8+7+10}{5}$	$\frac{1}{5}(6+8+8+7+10)$	$\frac{6}{5}+\frac{8}{5}+\frac{8}{5}+\frac{7}{5}+\frac{10}{5}$
h	$\frac{6+8+8+7+h}{5}$	$\frac{1}{5}(6+8+8+7+h)$	$\frac{6}{5}+\frac{8}{5}+\frac{8}{5}+\frac{7}{5}+\frac{h}{5}$

h. Approximately 32% of households own a cat. Write different expressions to represent the number of households with cats based on the number of households considered.

Number of Households Considered	Number of Households that Own a Cat Expression 1	Number of Households that Own a Cat Expression 2	Number of Households that Own a Cat Expression 3
90	$0.32(90)$	$0.1(90)+0.1(90)+0.1(90)+0.01(90)+0.01(90)$	$3(0.1(90))+2(0.01(90))$
120	$0.32(120)$	$0.1(120)+0.1(120)+0.1(120)+0.01(120)+0.01(120)$	$3(0.1(120))+2(0.01(120))$
n	$0.32n$	$0.1n+0.1n+0.1n+0.01n+0.01n$	$3(0.1n)+2(0.01n)$

Also have the students examine the expression $0.3(90)+0.02(90)$; $0.3(120)+0.02(120)$; $0.3n+0.02n$ and start to think about how this expression might come from expression 3. When students see an expression like $3(0.1n)$ they often don't realize that this is a multiplication problem with three factors. They view the parentheses as a grouping symbol rather than as multiplication. Therefore, they do not think that this expression can be simplified. It is important that they see that this is $3 \cdot 0.1 \cdot n$ and realize that the Commutative and Associative Properties of Multiplication allow us to multiply the factors in any order we want. Thus $3 \cdot 0.1 \cdot n$ can also be written as $0.3n$ and $2 \cdot 0.01 \cdot n$ can be written as $0.02n$.

i. A snack shack at a baseball stadium is buying cartons of hotdogs for the baseball season. Each carton of hotdogs contains 8 packages of hotdogs and each package of hotdogs contains 20 hotdogs. Write different expressions to represent the number of hotdogs based on the number of cartons purchased.

Cartons of Hotdogs Purchased	**Total Number of Hotdogs Expression 1**	**Total Number of Hotdogs Expression 2**	**Total Number of Hotdogs Expression 3**
3	$20(8)+20(8)+20(8)$	$3\cdot 8\cdot 20$	$160\cdot 3$
5	$20(8)+20(8)+20(8)+20(8)+20(8)$	$5\cdot 8\cdot 20$	$160\cdot 5$
c	$\underbrace{20(8)+20(8)+20(8)+20(8)+20(8)+20(8)+\cdots}_{c \text{ times}}$	$c\cdot 8\cdot 20$	$160c$

See the teacher notes on part h. regarding the importance of students seeing $20\cdot 8c$ as $20\cdot 8\cdot c$ and as $160c$.

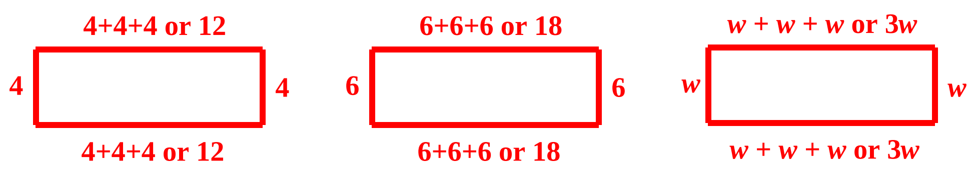
j. Chantelle is building a dog run for her dog, Otis. The length of the run will be three times longer than the width of the run. Write different expressions to represent the length of the run based on the width of the run.

Width of Dog Run (ft.)	**Length of Dog Run (ft.) Expression 1**	**Length of Dog Run (ft.) Expression 2**	**Length of Dog Run (ft.) Expression 3**
4	$4+4+4$	$3(4)$	
6	$6+6+6$	$3(6)$	
w	$w+w+w$	$3(w)$ or $3w$	

k. Use the information from the previous problem to write different expressions to represent the perimeter of the dog run Chantelle is building.

Width of Dog Run (ft.)	**Perimeter of Dog Run (ft.) Expression 1**	**Perimeter of Dog Run (ft.) Expression 2**	**Perimeter of Dog Run (ft.) Expression 3**
4	$4+4+12+12$	$2(4)+2(12)$	$2(4+12)$
6	$6+6+18+18$	$2(6)+2(18)$	$2(6+18)$
w	$w+w+3w+3w$	$2(w)+2(3w)$	$2(3w+w)$

A picture will really help with this problem, especially with expression 3.



This is an excellent problem to start to surface ideas about what it means to combine like terms.

l. Chantelle is planning to put a fence around her dog run. The fencing she is purchasing costs $10 a linear foot. Use the information from the previous problems to determine the cost of the fence.

Width of Dog Run (ft.)	**Cost of Fence ($) Expression 1**	**Cost of Fence ($) Expression 2**	**Cost of Fence ($) Expression 3**
4	$10(4+4+12+12)$	$10[2(4)+2(12)]$	$10(8(4))$
6	$10(6+6+18+18)$	$10[2(6)+2(18)]$	$10(8(6))$
w	$10(w+w+3w+3w)$	$10[2(w)+2(3w)]$	$10(8w)$ or $80w$

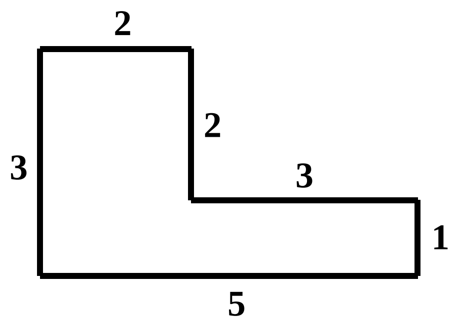
This is an excellent problem to start to surface ideas about distributing and combining like terms.

Spiral Review

1. Draw a number line model and an area model to represent $2 \cdot 5$.

Number Line Model	Area Model

2. Find the perimeter of the shape.



3. Simplify $2 \cdot 2 \cdot 2 \cdot 2$.

4. What are the common factors of 18 and 24. What is the greatest common factor of 18 and 24?

6.1d Homework: Transitioning from Numeric Expressions to Algebraic Expressions

1. Ali bikes 12 miles per hour. Write different expressions to represent the distance Ali bikes based on the number of hours she bikes for.

Number of Hours Ali Bikes	Distance Ali Bikes (in miles) Expression 1	Distance Ali Bikes (in miles) Expression 2	Distance Ali Bikes (in miles) Expression 3
2	$12 + 12$	$2(12)$	
5	$12 + 12 + 12 + 12 + 12$	$5(12)$	
h	$\underbrace{12 + 12 + 12 + 12 + 12 + 12 \ldots}_{h \text{ times}}$	$h(12)$ or $12h$	

2. Jason has $50 in his lunch account. Each lunch costs $2.25. Write different expressions for the remaining balance in Jason's account based on the number of lunches he buys.

Number of Lunches Jason Purchases	Remaining Balance ($) Expression 1	Remaining Balance ($) Expression 2	Remaining Balance ($) Expression 3
2	$50 - 2.25 - 2.25$	$50 - 2(2.25)$	$50 - (2.25 + 2.25)$
4	$50 - 2.25 - 2.25 - 2.25 - 2.25$	$50 - 4(2.25)$	$50 - (2.25 + 2.25 + 2.25 + 2.25)$
n	$\underbrace{50 - 2.25 - 2.25 - 2.25 - 2.25 - 2.25 - 2.25 \ldots}_{n \text{ times}}$	$50 - n(2.25)$ or $50 - 2.25n$	$50 - (\underbrace{2.25 + 2.25 + 2.25 + 2.25 + 2.25 + 2.25 \ldots}_{n \text{ times}})$

3. Ellen is buying packs of streamers and balloons for party decorations. Each pack of streamers costs $4.50 and each pack of balloons costs $5.25. Write different expressions for the amount Ellen spends on decorations depending on the number of packs of balloons and streamers she buys.

Packs of Decorations	Amount Ellen Spends ($) Expression 1	Amount Ellen Spends ($) Expression 2	Amount Ellen Spends ($) Expression 3
Streamers: 1 pack Balloons: 3 packs			
Streamers: 2 packs Balloons: 3 packs			
Streamers: x packs Balloons: y packs			

4. How would each of your expressions change above if Ellen gets a $5 discount on the items she purchases?

Packs of Decorations	Amount Ellen Spends ($) Expression 1	Amount Ellen Spends ($) Expression 2	Amount Ellen Spends ($) Expression 3
Streamers: 1 pack Balloons: 3 packs			
Streamers: 2 packs Balloons: 3 packs			
Streamers: *x* packs Balloons: *y* packs			

5. Write different expressions to represent the area of a triangle based on the lengths of the base and height of the triangle.

Base and Height (units)	Area of Triangle Expression 1	Area of Triangle Expression 2	Area of Triangle Expression 3
Base: 5 Height: 4	$\frac{5 \cdot 4}{2}$	$\frac{1}{2}(5 \cdot 4)$	$\frac{1}{2} \cdot 4 \cdot 5$
Base: 5 Height: 8	$\frac{5 \cdot 8}{2}$	$\frac{1}{2}(5 \cdot 8)$	$\frac{1}{2} \cdot 8 \cdot 5$
Base: *b* Height: *h*	$\frac{b \cdot h}{2}$	$\frac{1}{2}(b \cdot h)$	$\frac{1}{2} \cdot h \cdot b$

Again, this problem is a great illustration of the Commutative and Associative Properties of Multiplication. It is much easier to take half of an even number than an odd number. The Commutative Property of Multiplication allows us to switch the order of the factors to make the problem easier.

6. A study shows that 79% of teenagers enjoy cooking. Write different expressions to represent the number of teenagers you would expect to enjoy cooking based on the number of teenagers considered.

Number of Teenagers	Number of Teenagers Who Enjoy Cooking Expression 1	Number of Teenagers Who Enjoy Cooking Expression 2	Number of Teenagers Who Enjoy Cooking Expression 3
250	$0.79(250)$	$(0.8)(250) - 0.01(250)$	$0.75(250) + 4[0.01(250)]$
500	$0.79(500)$	$(0.8)(500) - 0.01(500)$	$0.75(500) + 4[0.01(500)]$
t	$0.79t$	$0.8t - 0.01t$	$0.75t + 4(0.01t)$

7. Isaac and Tatum are training for a marathon. On Monday, Tatum runs three times farther than Isaac. Write different expressions to represent the distance Tatum runs on Monday based on the distance Isaac runs.

Distance Isaac Runs (in miles)	**Distance Tatum Runs (in miles) Expression 1**	**Distance Tatum Runs (in miles) Expression 2**	**Distance Tatum Runs (in miles) Expression 3**
2			
4			
m			

8. Use the information from the previous problem to write different expressions to represent the total number of miles run by Isaac and Tatum based on the number of miles Isaac runs.

Distance Isaac Runs (in miles)	**Total Distance Run by Isaac and Tatum (in miles) Expression 1**	**Total Distance Run by Isaac and Tatum (in miles) Expression 2**	**Total Distance Run by Isaac and Tatum (in miles) Expression 3**
2			
4			
m			

9. Owen gave 7 dimes, 3 nickels, and 2 pennies to each of his cousins. Write different expressions to represent the total amount of money Owen gave away based on the number of cousins he has.

Number of Cousins Owen Has	**Total Amount Given Away by Owen ($) Expression 1**	**Total Amount Given Away by Owen ($) Expression 2**	**Total Amount Given Away by Owen ($) Expression 3**
4	$4(0.7 + 0.15 + 0.02)$	$4(0.87)$	$4[7(0.1) + 3(0.05) + 2(0.01)]$
7	$7(0.7 + 0.15 + 0.02)$	$7(0.87)$	$7[7(0.1) + 3(0.05) + 2(0.01)]$
c	$c(0.7 + 0.15 + 0.02)$	$c(0.87)$ or $0.87c$	$c[7(0.1) + 3(0.05) + 2(0.01)]$

6.1e Self-Assessment: Section 6.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Write different but equivalent numeric expressions to represent a real-world problem using grouping and operator symbols correctly (parentheses, fraction bars, middle dot, etc.).				
2. Determine whether two or more numeric expressions are equivalent by correctly simplifying the expressions (follow order of operations, recognize and interpret grouping symbols and operator symbols, etc.)				
3. Determine whether two or more algebraic expressions are equivalent by evaluating the expressions for specific values of the variable.				
4. Translate the word form of an algebraic expression to its symbolic representation and				

vice versa. Use and understand academic vocabulary.				
5. Recognize and interpret patterns and structure in numeric expressions and use this understanding to write algebraic expressions.				

Sample Problems for Section 6.1

Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. I had $5. Then I spent $1 a day for 2 days in a row. How much money do I have now? [1] [2}

	Expression	Simplified Form	Does it work?	Ideas
a.	$5 - 1 + 1$			
b.	$5 - 1 - 1$			
c.	$5 - (1 - 1)$			
d.	$5 - (1 + 1)$			

2. Inez bought two pencils for $0.30 each and two notebooks for $1.20 each. How much money did Inez spend? [1] [2]

	Expression	Simplified Form	Does it work?	Ideas
a.	$(2 + 2)(0.30 + 1.20)$			
b.	$2(0.30 + 1.20)$			
c.	$(2)(0.30) + 2(1.20)$			
d.	$0.60 + 2.40$			

6. Equivalent or Not? ________________________________ [3]

	$2x$	$x + x$
Let $x = 0$		
Let $x = 5$		

7. Equivalent or Not? ________________________________ [3]

	$5y + 2y + 4$	$11y$
Let $y = 2$		
Let $y = 3$		

8. Equivalent or Not? ________________________________ [3]

	$\frac{1}{2}(x + 16)$	$x + 8$
Let $x = 4$		
Let $x = 10$		

9. Write an algebraic expression to represent the phrase "The quotient of a number and three increased by 7." [4]

10. Write an algebraic expression to represent the phrase "One-third of the sum of a number and twenty-one." [4]

11. Are the expressions in #9 and #10 equivalent? Justify your answer. [3] [4]

12. Write the word form of the expression $5n + 4$. [4]

13. Write the word form of the expression $10 - 3x$. [4]

14. At Pizza King, a pizza costs $9.00 plus $1.25 for each additional topping. Write different expressions that represent the cost of a pizza based on the number of toppings on the pizza. [5]

Number of Toppings	**Cost of Pizza ($) Expression 1**	**Cost of Pizza ($) Expression 2**
1		
3		
5		
t		

15. Write different expressions to represent the perimeter of an equilateral triangle based on the side length of the triangle. [5]

Side Length (units)	**Perimeter of Triangle Expression 1**	**Perimeter of Triangle Expression 2**	**Perimeter of Triangle Expression 3**
4			
9			
y			

16. Antony and his family are driving 60 miles per hour on the interstate. [5]

   a. Complete the table below to show the distance Antony's family travels based on the number of hours they travel at a speed of 60 miles per hour.

Time (hours)	**Distance (miles)**
0	
1	
2	
3	
4	
5	
6	
t	

17. Chad owes his mom $150. He plans to pay back $15 each week. [5]

   a. Complete the table below to show the amount of money Chad still owes his mom based on the number of weeks that have passed.

Time (weeks)	**Amount Chad Owes ($)**
0	
1	
2	
3	
4	
5	
6	
t	

18. Augusto is saving money. He currently has $400 in his account and plans to save $25 each month for the next several months. [5]
    a. Complete the table below to show the amount of money Augusto will have in his account over time.

Time (months)	**Savings (dollars)**
0	
1	
2	
3	
4	
5	
6	
m	

19. Sachin has scored a 94%, 88%, and 85% on his first three math tests of the semester. [5]
    a. Complete the table below to show the average of Sachin's math test scores based on his score on the fourth math test.

Score on Fourth Math Test (percent)	**Average on Four Math Tests (percent)**
100	
95	
88	
85	
80	
s	

20. A store is offering a 25% discount on everything in the store. [5]

a. Complete the table below to show the amount of the discount based on the original price of the item. Then, determine the cost of the item after the discount.

Original Cost (dollars)	**Amount of Discount (dollars)**	**Cost of Item After Discount (dollars)**
\$20.00		
\$30.00		
\$40.00		
\$50.00		
\$60.00		
\$100.00		
c		

21. Christina tips 20% when she goes out to eat. [5]

a. Complete the table below to show the amount of money Christina leaves for tip and the total cost of the meal including tip depending on the cost of the meal before tip.

Cost of Meal Before Tip (dollars)	**Amount of Tip (dollars)**	**Total Cost of Meal Including Tip (dollars)**
\$15.00		
\$20.00		
\$55.00		
\$60.00		
\$74.00		
\$100.00		
c		

Section 6.2: Writing, Simplifying, and Evaluating Algebraic Expressions

Section Overview:

This section begins with students re-visiting problems from Section 1 of this chapter. Students examine different but equivalent expressions used to represent real world problems. Through a teacher-led discussion, students explain why the expressions are equivalent and what properties are being used to transform the expressions. Students learn and use academic vocabulary (terms, like terms, constants, coefficients) as they manipulate the expressions. Next students use models (number lines and tiles) to represent and transform algebraic expressions. During this process of writing equivalent expressions, students learn about a special form that an expression can take which is the expression's simplified form. The simplified form of a linear expression takes the form $Ax + C$ where A and C are numbers. The section then moves to exponents. In earlier grades, students learned that exponents can be used to represent powers of ten. This section extends this understanding to include bases other than ten. Students learn to move fluently between the expanded form, exponential form, and simplified form of an expression containing repeated multiplication. These concepts are explored both numerically and geometrically with connections to area and volume. From here, students learn to simplify expressions containing whole number exponents by following the Order of Operations. Then, they evaluate algebraic expressions and formulas containing exponents. In the last lesson of this section, students synthesize the ideas of the section by writing, simplifying, and evaluating expressions to represent real world problems.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Identify parts of an algebraic expression using mathematical language.
2. Apply the properties of operations to generate equivalent expressions, including the simplified form of an algebraic expression.
3. Simplify numeric expressions containing exponents using the Order of Operations.
4. Evaluate algebraic expressions for specific values of the variable, including expressions with exponents and expressions that arise from formulas.
5. Write algebraic expressions to represent real world problems.

6.2a Class Activity: Simplifying Algebraic Expressions Part I

Activity 1: Complete the statements below.

1. 5 apples + 2 apples = _____7_____ apples
2. 5 ones + 2 ones = ____7______ ones
3. 5 tens + 2 tens = ____7____ tens
4. $\frac{5}{8} + \frac{2}{8}$ = ______7______ eighths
5. $0.5 + 0.2$ = _____7_____ tenths
6. 5 feet + 2 feet = ___7____ feet
7. 5 miles + 2 miles = ______7______ miles

Point out to students that the problems above all have an answer of 7 but the 7 means very different things in each of the problems.

Throughout elementary school, students have come to understand that we can only combine things that are the same. We can add ones to ones, tens to tens, hundreds to hundreds, and so on, regrouping when necessary. We line up the decimal points when we add numbers with decimals. By doing this, we are adding tenths to tenths, hundredths to hundredths, thousandths to thousandths, and so on, again re-grouping as necessary. We find a common denominator when adding fractions and a common unit of measure when adding lengths or distances. These same ideas also apply to subtraction. This is a great launching point for learning how to simplify algebraic expressions by combining like terms.

It actually helps to say these statements out loud in the following way: "If I have 5 apples and I add 2 more apples, how many apples do I have? If I have 5 ones and 2 ones, how many ones do I have? If I have 5 tens and 2 tens, how many tens do I have? If I have five eighths and 2 eighths, how many eighths do I have?

If I have 5 x's and I add 2 more x's, how many x's do I have?

Try these…

$5x + 2x$ $7x$

$6a + 2a$ $8a$

$2b + 3b$ $5b$

$8y - 3y$ $5y$

$5f - 4f$ f

Activity 2: In this activity, you will re-visit some of the problems from 6.1d and examine the different algebraic expressions that were written to represent the problems. While doing this, think about the following questions: 1) What does it mean to say that two algebraic expressions are **equivalent**? How can you show two algebraic expressions are equivalent? 2) What does it mean to **simplify** an algebraic expression?

It is suggested you re-visit the following problems from the previous lesson:

Problem: Class Activity 6.1d part b. Perimeter of a square		
$s + s + s + s$	$4s$	$2s + 2s$

These expressions are equivalent. To show this, we can substitute in two different values for s and show that the expressions simplify to the same value. Ask students what they think the simplified form of the expressions is. Ask if they can explain how to get from $s + s + s + s$ to $4s$ or $2s + 2s$ to $4s$. As they start to formulate ideas about how to simplify expressions, help them to clarify their thinking and use correct vocabulary. For example, if they say, "You add the numbers and keep the letter the same," teach them that the numbers in front of the variable are called **coefficients** and the letters are called **variables** changing the statement to "You add the coefficients and keep the variable the same." Ask students what the coefficient of the s term is. "1" Why don't we need to write the 1? This goes back to the Identity Property of Multiplication – any number multiplied by 1 is just that number.

Problem: Class Activity 6.1d part e. Sam and his savings account		
$120 + s + s + s + s + s + s$	$120 + 6s$	$120 + 5s + s$

This problem will provide additional ideas about what it means to simplify an expression. Ask which expression they think is in simplified form and why. Again, ask how we can show these expressions are equivalent. Ask them to explain how you get from $120 + s + s + s + s + s + s$ to $120 + 6s$ and $120 + 5s + s$ to $120 + 6s$. Ask if the expression $126s$ is equivalent. Many students will think that it is. Show that it is not by substituting in values for s. Again, help them clarify their thinking about what it means to simplify an algebraic expression using correct vocabulary. This problem highlights that to simplify an algebraic expression, you combine like terms. **Terms** are either a single number or variable or numbers and variables multiplied together. **Like terms** are terms with the same variable and the variable has the same exponent. 120 does not have a variable attached to it. 120 is called a **constant**. Constants are also like terms and can be combined just as terms with the same variable can be combined.

Problem: Class Activity 6.1d part h. Households with a cat		
$0.32n$	$0.1n + 0.1n + 0.1n + 0.01n + 0.01n$	$3(0.1n) + 2(0.01n)$

This problem highlights that coefficients can be rational numbers. To simplify the second expression we would add $0.1 + 0.1 + 0.1 + 0.01 + 0.01$, remembering to line up the decimals. These are all n terms so our sum is also an n term. Another critical idea students learn in this problem is that when simplifying algebraic expressions, we follow the order of operations. In the third expression $3(0.1n) + 2(0.01n)$, we simplify the term $3(0.1n)$ by multiplying the 3 and 0.1 to get $0.3n$. Again, if students have a difficult time seeing the structure in this term, have them re-write it as $3 \cdot 0.1 \cdot n$ to see that this is *one term* and it can be simplified using multiplication. Some students will make the mistake of thinking that you cannot simplify $3(0.1n)$ because the 3 and $0.1n$ are not like terms. Remind students that the operation is multiplication and we can multiply things that are not the "same". Students have seen this for years when multiplying whole numbers, fractions, and decimals. We don't need to line up place values when we multiply numbers and we don't need a common denominator when multiplying fractions. Use a similar process to simplify $2(0.01n)$ to $0.02n$. Now students are ready to combine the like terms $0.3n$ and $0.02n$ resulting in $0.32n$.

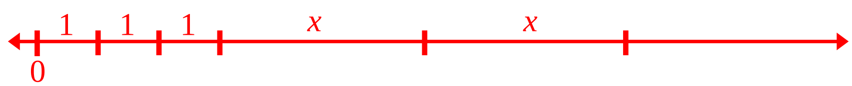
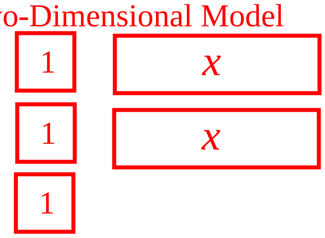
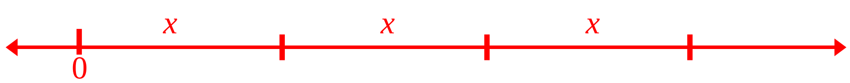
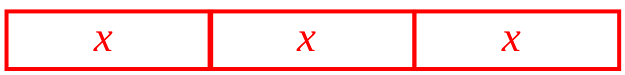
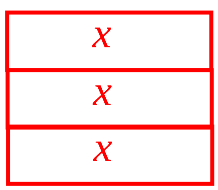
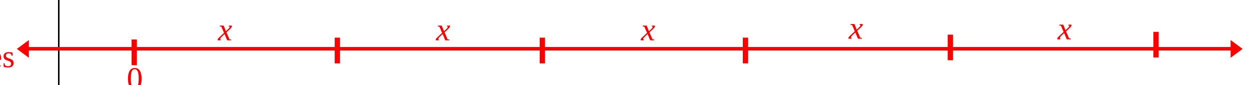
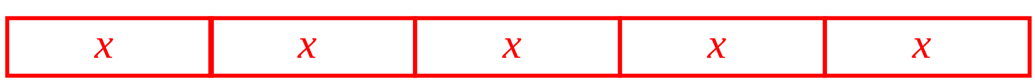
Problem: Class Activity 6.1d part j. Chantelle and the cost to put a fence around her dog run		
$10(w + w + 3w + 3w)$	$10[2(w) + 2(3w)]$	$10(8w)$ or $80w$

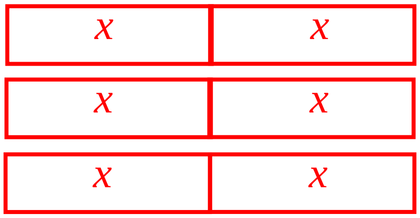
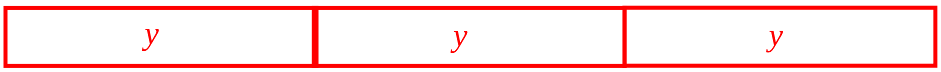
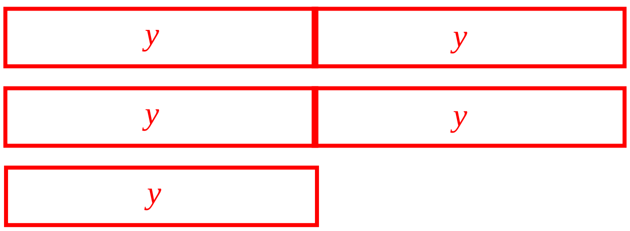
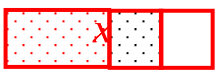
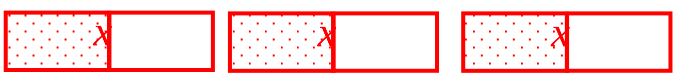
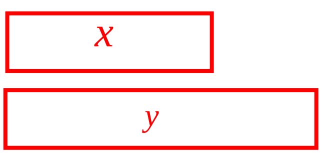
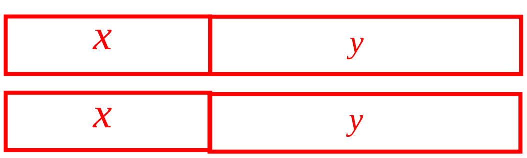
This is an excellent problem for synthesizing many of the ideas from the previous problems.

Activity 3: Draw a model to represent each expression. Then simplify the expression. If the expression is already simplified, write "already simplified". Students can draw either a one-dimensional model (a number line) *or* a two-dimensional model (an area model). Both models are shown for the first three problems in Activity 3, but it is suggested you choose one type of model to show your students to prevent confusion. If you are using Algebra tiles, you will want to introduce the tiles to the students before beginning. Templates to create your own Algebra tiles are widely available online. A typical set of Algebra tiles will have the following tiles:

Tile	Dimensions of Tile	Area of Tile
1	1×1	1
x	$1 \times x$	x
y	$1 \times y$	y
x^2	$x \times x$	x^2
y^2	$y \times y$	y^2

The tiles can be used in different ways. To construct lengths (one-dimensional models) with the tiles, use the side of the tile as a "ruler" to measure the desired length. Each tile also represents an area. Since each tile has a width of 1 unit, the number for the area of each tile (in square units) is equal to the length (in units) of the tile. It can sometimes be confusing to students that the area of the tile (a two-dimensional model) is equal to the side length of the tile (a one-dimensional model). Let them know that these are *special types of rectangles with a width of 1*. The area of the rectangles will be equal to the measure of the length of the rectangle (Identity Property of Multiplication).

<table>
<tr><th>Expression</th><th>Model</th></tr>
<tr><td>$1+1+1+x+x$
$3+2x$</td><td>One-Dimensional Model – A number line model shows length/distance from zero. In this model, we can see 3 copies of 1 and 2 copies of x. We can represent the 3 copies of 1 with the number 3 and the 2 copies of x as $2 \cdot x$ or $2x$. A common error is for students to try to simplify 3 and $2x$ to $5x$ or 5 copies of x. The models clearly show that $3 + 2x$ is not equivalent to 5 copies of x.

Two-Dimensional Model
</td></tr>
<tr><td>$x+x+x$
$3x$</td><td>This problem helps students to see that each x term has a coefficient of 1. In both models we see three copies of x or $3x$.


The rectangle above has dimensions of $3x$ and 1 which means it has an area of $3x$ (Identity Property of Multiplication). We can also arrange the tiles vertically:

This rectangle has dimensions of x and 3 so its area is $3 \cdot x$ or $3x$.</td></tr>
<tr><td>$3x+2x$
The models show 5 copies of x or $5x$</td><td>
</td></tr>
</table>

$3(2x)$ $6x$	Students often confuse this expression with the expression $3 + 2x$. Have students apply their understanding of multiplication to think about how to draw this model. This expression shows 3 copies of $2x$.  Students can also apply The Associative Property of Multiplication to simplify this expression. Have students re-write $3(2x)$as $3 \cdot 2 \cdot x$ so that they see that there are three factors. The Associative Property allows us to group the 3 and 2 and multiply them first resulting in $6 \cdot x$ or $6x$.
$2y + y$ $3y$	
$2(2y) + y$ $5y$	
$\frac{1}{2}x + \frac{1}{4}x$ $\frac{3}{4}x$	
$\frac{1}{2}(3x)$ $\frac{3}{2}x$	
$x + y$ Already simplified	
$(x + y) + (x + y)$ $2x + 2y$ This also shows 2 copies of $(x + y)$ or $2(x + y)$.	

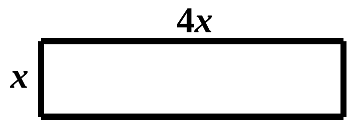
Activity 4: Simplify the expression. If the expression is already simplified, write "already simplified".

If students are still struggling, encourage them to continue to use tiles and/or draw pictures.

a. $3x + 4x$ $7x$	b. $a + a + a + a + a + a$ $6a$	c. $5(2x)$ $10x$ This is 5 copies of $2x$ which can be re-written as: $2x + 2x + 2x + 2x + 2x$
d. $5 + 2x$ Already simplified, compare this to part c.	e. $5(2x) + 10$ $10x + 10$	f. $f + 7 + 3f + 5$ $4f + 12$
g. $5(9x) + 5(7)$ $45x + 35$	h. $3(7x) - 10x$ $11x$	i. $3x + 6(4x)$ $27x$
j. $8(x + x + x)$ $24x$ Follow order of operations, grouping symbols first then multiplication.	k. $5(2d + d + d)$ $20d$	l. $6n + 10 + 5n - 7$ $11n + 3$
m. $a + b + c + 100$ Already simplified	n. $4x + 9x + 15 - 8$ $13x + 7$	o. $13 + b - 5$ $8 + b$
p. $12x - 5x + 3x$ $10x$ A common error is for students to add the $5x + 3x$ first. Remind students to follow the order of operations. Compare this problem to part q.	q. $12x - (5x + 3x)$ $4x$	r. $20x - (x + x + x)$ $17x$
s. $\frac{3}{8}x + \frac{2}{8}x$ $\frac{5}{8}x$	t. $1\frac{3}{4}a + \frac{2}{5}a$ $2\frac{3}{20}a$	u. $2\frac{3}{10}g - \frac{7}{10}g$ $1\frac{3}{5}g$
v. $0.5c + \frac{3}{5}c$ $1.1c$	w. $5p - 3.4p$ $1.6p$	x. $r + 0.54r + 2r$ $3.54r$
y. $n - n + 6$ 6	z. $\left(2\frac{3}{4}x + 9\right) + (2\frac{3}{4}x + 9)$ $5\frac{1}{2}x + 18$	

Activity 5: Write several expressions to represent the perimeter of each object. Simplify each expression to show they are equivalent.

a.



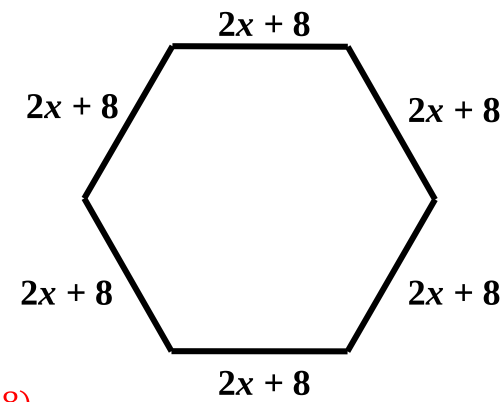
$x + 4x + x + 4x$

$2(x) + 2(4x)$

$2(x + 4x)$

All expressions simplify to $10x$

b.



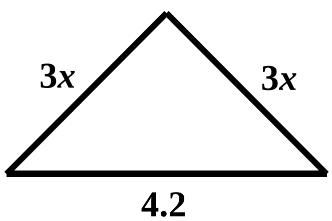
$6(2x + 8)$

$(2x + 8) + (2x + 8) + (2x + 8) + (2x + 8) + (2x + 8) + (2x + 8)$

$6(2x) + 6(8)$

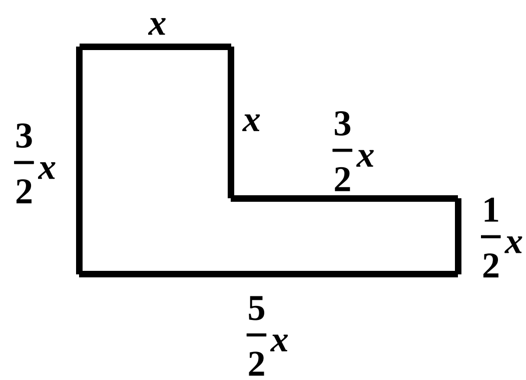
All expressions simplify to $12x + 48$

c.



$3x + 3x + 4.2$
$2(3x) + 4.2$
All expressions simplify to $6x + 4.2$

d.



$x + x + \frac{3}{2}x + \frac{1}{2}x + \frac{5}{2}x + \frac{3}{2}x$
$2(x) + 2\left(\frac{3}{2}x\right) + 1\left(\frac{5}{2}x\right) + 1\left(\frac{1}{2}x\right)$
All expressions simplify to $8x$

Activity 6:

For these, answers will vary. Sample answers are shown.

a. Write three different expressions that are equivalent to $6s$.
$3s + 3s$, $2(3s), s + 5s, 7s - s, 2.5s + \frac{1}{2}s + 3s, 6s + 7 - 7, \frac{12s}{2}$

b. Write three different expressions that are equivalent to $3.6y$.
$3y + 0.6y$, $12(0.3y), 4s - 0.4s$

c. Write three different expressions that are equivalent to x.
$2x - x$; $100x - 99x, \frac{1}{2}x + \frac{1}{2}x, \frac{11x}{11}$

d. Write three different expressions that are equivalent to $5x + 12y$.
$2x + 3x + 2(6y), 6x + 10y + 2y - x, 2.5x + 2.5x + 11y + y$

e. Write three different expressions that are equivalent to $6a + 18$.
$3(2a) + 3(6), a + a + a + a + a + a + 18, 5a + 20 + a - 2$

Activity 7:

a. Explain what each word means. Use examples and non-examples to support your ideas.

Consider using a Frayer Model to help students understand the following vocabulary.

Term: Terms are either a single number or variable or numbers and variables multiplied together.
Examples: $3, 3x, 3(2x), y$

Constant: A fixed value, a number. Constants are like terms that can be combined.
Examples: $14, 10, 1, \frac{2}{7}$

Coefficient: A number juxtaposed to a variable that is used to multiply the variable.
Examples: In the term $4x$, 4 is the coefficient. In the term c, 1 is the coefficient.

Like Terms: Terms with the same variable and the variable has the same exponent.
Examples: $4x$ and $2x$ are like terms, 2 and 3 are like terms

b. Write an expression with 3 terms. Make one of the terms a constant.
Answers will vary. A sample answer is $3x + 2x + 5$.

c. Write an expression with like terms. Then, simplify the expression.
Answers will vary. A sample answer is $3x + 2x + 5$.

d. Write an expression with one term that has a coefficient of 2 and another term with a coefficient of 1.
Answers will vary. A sample answer is $x + 2x + 5$.

e. Write an expression that is not in simplified form. Then, simplify the expression. Answers will vary.
A sample answer is $3x + 2x + 5$.

Spiral Review

1. Simplify.
    a. 5×6
    b. 5×60
    c. 5×600

2. Write the prime factorization of 80.

3. Find the volume of a rectangular prism with a length of 5 inches, a width of 7 inches, and a height of 4 inches.

4. Are the expressions $3x + 15$ and $3(x + 5)$ equivalent? Justify your answer.

6.2a Homework: Simplifying Algebraic Expressions Part I

1. Identify the terms, constants, coefficients, and like terms in each algebraic expression. The first one has been done for you.

Expression	Terms	Constants	Coefficients	Like Terms
a. $3x + x + 13 + 5$	$3x, x, 13, 5$	$13, 5$	$3, 1$	$3x$ and x 13 and 5
b. $2y + 10 + 3y + 4$	$2y, 10, 3y, 4$	$10, 4$	$2, 3$	$2y$ and $3y$ 10 and 4
c. $11 + 0.04c + c$				
d. $3x + 2y + 5x + 7y$				
e. $2(3x) + 7x + 13$	$2(3x), 7x, 13$	13	$2(3)$ or $6, 7$	$2(3x)$ and $7x$

Part e. may be confusing for students. Remind students that **terms** are either a single number or variable or numbers and variables multiplied together so $2(3x)$ is a term that we can also think of as $6x$.

2. Simplify the expressions from above.

a. $4x + 18$

b.

c.

d.

e. $13x + 13$

Directions: Simplify each expression. If the expression is already simplified, write "already simplified".

3. $y + y + y$	4. $15 + s + s + s + s$	5. $3a + 8 + 8a + 6$ $11a + 14$
6. $15x - 5$ Already simplified	7. $3x + 8 + 4x - 3$	8. $x + 3 + 3x + 3y$
9. $15 + 6b - 7$	10. $r + 2r + 3r$ $6r$	11. $2x + 1$
12. $x + y + z$	13. $4t + 10 + 9t - 1$ $13t + 9$	14. $9x - 4x + 2x$
15. $9x - (4x + 2x)$	16. $2(5x) + 3(4x)$ $22x$	17. $5(3x) + 5(9)$
18. $3(6x) - 2$	19. $7(2x + x + x + x)$	20. $4r + 2(7r)$
21. $3 + 15b - 8b$	22. $\frac{1}{6}y + \frac{2}{3}y$ $\frac{5}{6}y$	23. $1\frac{1}{5}a + \frac{3}{10}a + 2\frac{1}{4}$
24. $0.25x + \frac{3}{4}x$	25. $y - 0.1y$ $0.9y$	26. $0.45g + g$
27. $c - c$ 0	28. $\frac{3}{5}d + \frac{7}{10} + 0.1d + \frac{1}{20}$	29. $6\frac{2}{5}q - q$
30. $4(5x) - 20$	31. $(x + 1) + (x + 1)$ $2x + 2$	32. $(4x + 1) + (4x + 1)$
33. $y + 0.1y + 0.01y + 0.001y + 0.0001y$		
34. $5(d) + 4(0.1d) + 3(0.01d) + 2(0.001d) + 0.0001d$ $5.4321d$		

35. Write three expressions equivalent to the expression $4c$.

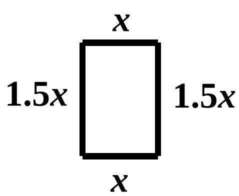
36. Write three expressions equivalent to the expression $6b + 5$.

37. Write three expressions equivalent to the expression $\frac{5}{8}k$.

$5\left(\frac{1}{8}k\right), \frac{2}{8}k + \frac{3}{8}k, 0.5k + 0.125k$

Directions: Write several expressions to represent the perimeter of each object. Simplify each expression to show they are equivalent.

38.



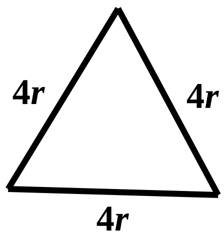
$x + 1.5x + x + 1.5x$

$2(x) + 2(1.5x)$

$2(x + 1.5x)$

All expressions simplify to $5x$

39.



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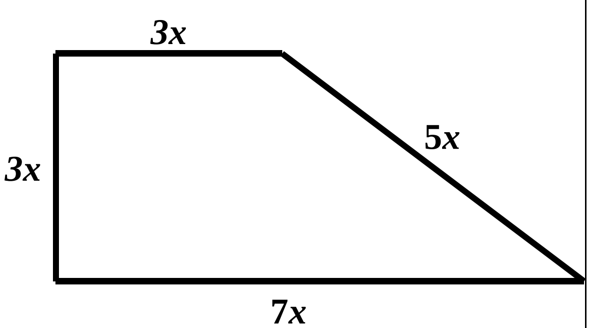

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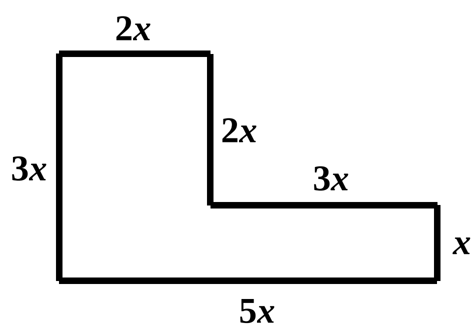


42. Write your own algebraic expression with four terms that is not simplified. Then, simply the expression.

Answers will vary. Sample answer: $x + x + x + 3$ which simplifies to $3x + 3$

6.2b Class Activity: Numeric Expressions and the Distributive Property

This lesson is a review of concepts learned in Grades 3 and 4: 3.OA.5, 3.MD.7, 4.NBT

Activity 1:

a. Luca is buying practice uniforms for soccer. Each jersey costs $20 and each pair of shorts costs $30. He purchases two practice jerseys and two pairs of practice shorts. Write an expression that can be used to represent the total amount Luca spends on practice uniforms. Then, simplify the expression to show the total amount Luca spends on practice uniforms.

Have students share the expressions they wrote. Some students may just add up the cost of each item, leading to the expression:
$20 + 20 + 30 + 30$

Some may find the cost of two jerseys and two pairs of shorts and then add the two, leading to the expression:

$2(20) + 2(30)$
$40 + 60$
100

Alternatively, a student may find the cost of one uniform (a practice jersey and a pair of shorts) and then multiply that quantity by 2 which would lead to the expression:

$2(20 + 30)$
$2(50)$
100

Draw connections between the different expressions. The first one shows repeated addition and the second shows multiplication (2 copies of 20 and 2 copies of 30). The third shows 2 copies of $(20 + 30)$. If we expand this out, we have $(20 + 30) + (20 + 30)$ which can be rearranged to get the first expression using the Commutative and Associative Properties of Addition.

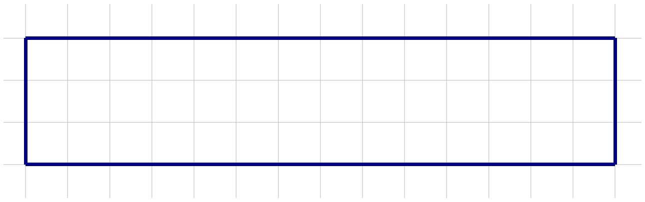
b. Hannah is purchasing CDs at a local music store. The CDs normally cost $12 each. The store is offering a discount of $2 off each CD if you buy 4 or more. Hannah is planning to buy 5 CDs. Write an expression to represent the cost before tax of Hannah's purchase.

Again, have students share the different expressions they create. Some students may use repeated addition and add the cost of each CD after the discount has been taken off:
$(12 - 2) + (12 - 2) + (12 - 2) + (12 - 2) + (12 - 2)$
Some students may first take the discount off and then multiply the new price by 5 leading to the expression:
$5(12 - 2)$
$5(10)$
50
Alternatively, a student may calculate the cost of 5 CDs before the discount and then take off the total discount, leading to the expression:
$12(5) - 2(5)$
$60 - 10$
50

Again, draw connections between the strategies. Ask students how the different expressions are related.

Activity 2:

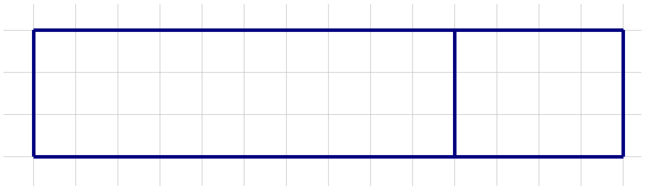
a. Xavier and Penelope were asked to find the area of the rectangle shown below.



Xavier found the height of the rectangle (3) and multiplied it by the base of the rectangle (14) and multiplied the two numbers together and found the answer to be 42 square units:

$3(14) = 42$

Penelope started by cutting the rectangle into two smaller rectangles:



Then, she found the area of each of the smaller rectangles:

$3(10 + 4)$
$3(10) + 3(4)$
$30 + 12$
42

Compare the strategies used by Xavier and Penelope. How does this problem relate to Activity 1?
Students can see that to find the area of the rectangle, we can multiply the base and height of the original rectangle, $3(14)$, or we can cut the larger rectangle into two smaller rectangles to make the problem easier: $3(10 + 4) \rightarrow 3(10) + 3(4) \rightarrow 30 + 12 \rightarrow 42$

b. The following are area models that can be used to represent the number 50. Under each area model, fill in the blanks to write an expression for the area model shown.

	10
5	**50**

$5(\underline{\ \ 10\ \ }) = 50$

	3	7
5	15	35

$5(\underline{\ \ 3\ \ } + \underline{\ \ 7\ \ }) = 50$

$5(\underline{\ \ 3\ \ }) + 5(\underline{\ \ 7\ \ }) = 50$

$15 + 35 = 50$

	5	5
5	25	25

$5(\underline{\ \ 5\ \ } + \underline{\ \ 5\ \ }) = 50$

$5(\underline{\ \ 5\ \ }) + 5(\underline{\ \ 5\ \ }) = 50$

$25 + 25 = 50$

Create your own area model to represent the number 50. Then, write the expressions for the area model you created.

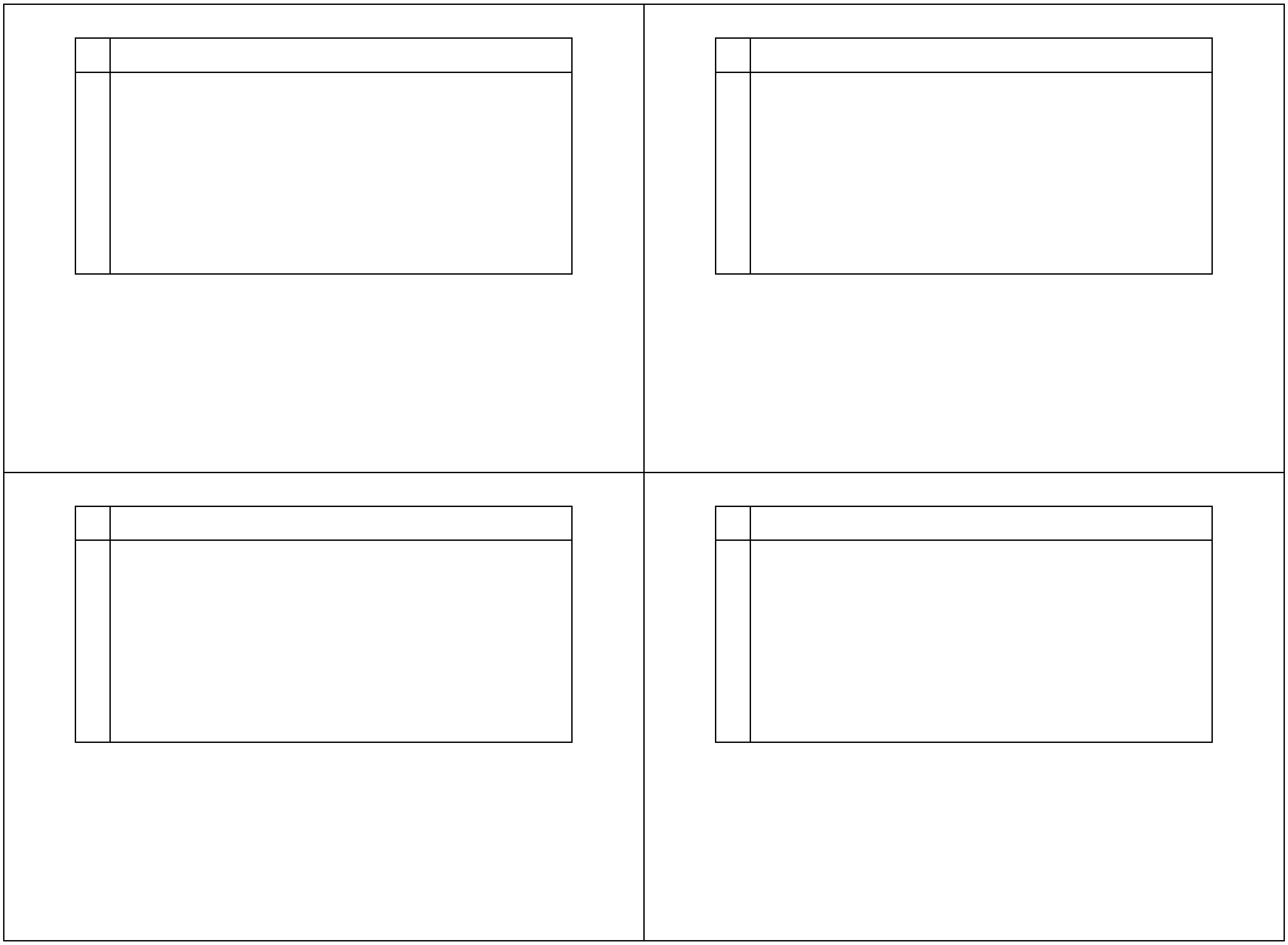
Answers will vary. Possible answers:

$2(20 + 5)$

$10(2 + 3)$

$10(10 - 5)$

c. Angela was asked to find the product of 6 and 28. Using the templates below, draw area models to show some of the different ways Angela can find the product of 6 and 28. Write the expressions that correspond to each area model. The templates have been provided. Students can cut them in a variety of ways $(6)(28)$, $(6)(20+8)$, $(6)(10+10+8)$, $(6)(25+3)$ $(6)(30-2)$. They can also use a different color, like red, to represent area that needs to be subtracted out.



Distributive Property

$$\boldsymbol{a(b+c)=ab+ac}$$

$$\boldsymbol{a(b-c)=ab-ac}$$

Activity 3: Simplify the following expressions using two different methods. The first one has been done for you.

a. $3(20+1)$ Method 1: $3(20)+3(1)$ $60+3$ 63 Method 2: $3(21)$ 63	b. $9(50+7)$ Method 1: $9(50)+9(7)$ $450+63$ 513 Method 2: $9(57)$ 513
c. $7(30-1)$ Method 1: $7(30)-7(1)$ $210-7$ 203 Method 2: $7(29)$ 203	d. $4(100-3)$ Method 1: $4(100)-4(3)$ $400-12$ 388 Method 2: $4(97)$ 388
e. $\frac{1}{5}(100+60)$ Method 1: $\frac{1}{5}(100)+\frac{1}{5}(60)$ $20+12$ 32 Method 2: $\frac{1}{5}(160)$ 32	f. Eight multiplied by the sum of thirty and six Method 1: $8(30+6)$ $240+48$ 288 Method 2: $8(36)$ 288

Activity 4: Use the Distributive Property to find the product using mental math strategies. The first one has been done for you. The purpose of these problems is to see that the Distributive Property can make multiplication problems easier.

a. $6(53)$ $6(50+3)$ $6(50)+6(3)$ $300+18$ 318	b. $8(104)$ $8(100+4)$ $8(100)+8(4)$ $800+32$ 832
c. $5(99)$ $5(100-1)$ $5(100)-5(1)$ $500-5$ 495	d. $12(53)$ $12(50+3)$ $12(50)+12(3)$ $600+36$ 636
e. $5(666)$ $5(600+60+6)$ $5(600)+5(60)+5(6)$ $3000+300+30$ $3,330$	f. $20(6\frac{1}{2})$ $20(6+\frac{1}{2})$ $20(6)+20\left(\frac{1}{2}\right)$ $120+10$ 130

g. $\frac{1}{4}(104)$ $\frac{1}{4}(100+4)$ $\frac{1}{4}(100)+\frac{1}{4}(4)$ $25+1$ 26	h. $\frac{1}{5}(29)$ $\frac{1}{5}(30-1)$ $6-\frac{1}{5}$ $5\frac{4}{5}$

Spiral Review

1. Write an algebraic expression for each phrase.
   a. Twice the sum of a number *n* and five.

   b. The sum of twice a number *n* and five.

   c. The sum of twice a number *n* and ten.

2. Which of the expressions in #1 are equivalent? Justify your answer.

3. Simplify the following expressions. If the expression is already simplified, write "already simplified".
   a. $2x + x + 5$.

   b. $4y + 6 + 3y - 4$.

   c. $6c - c + c$

   d. $3(5b)$

   e. $3 + 5b$

4. Justine sent 2,500 texts last month. Her brother sent half as many texts. How many texts did Justine and her brother send altogether?

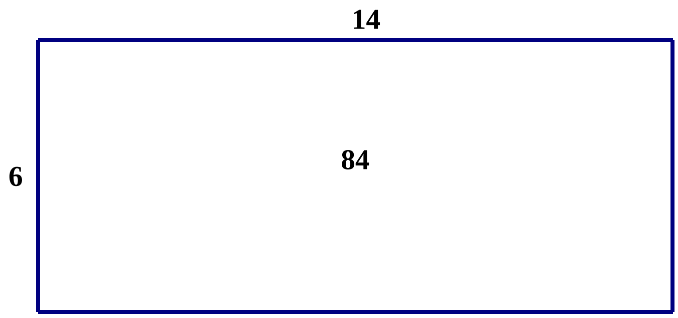
6.2b Homework: Numeric Expressions and the Distributive Property

Directions: Write two different expressions to solve each word problem. Then, simplify each expression to answer the question.

1. Mr. Green is purchasing four new computers for the computer lab. Each computer costs $350. In addition, Mr. Green is purchasing a protection plan for each computer. The cost of the protection plan is $25 per computer. Use the distributive property to write two different expressions that can be used to represent the cost of the computers with the protection plans. Then, simplify both expressions.

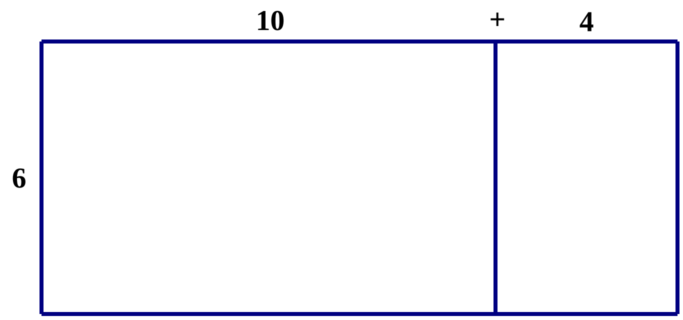
2. Ed burns 500 calories an hour running. He burns 650 calories an hour biking. Use the distributive property to write two different expressions that can be used to represent the difference between the number of calories Ed burns running for 2 hours vs. biking for 2 hours. Then, simplify both expressions.
   $2(650 - 500)$ or $2(650) - 2(500)$
   300 calories

3. There are several methods shown to find the product of 6 and 14 using area models. Show the area of each rectangle on the picture. Complete the expressions below each picture so that they correspond to the picture. The first one has been done for you.



6(14)

84

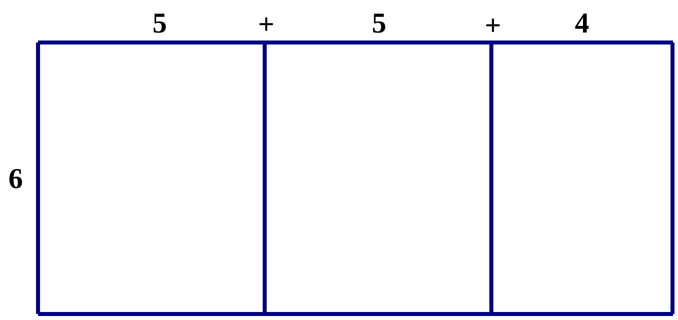


6(__10__ + __4__)

6(__10__) + 6(__4__)

__60__ + __24__

84

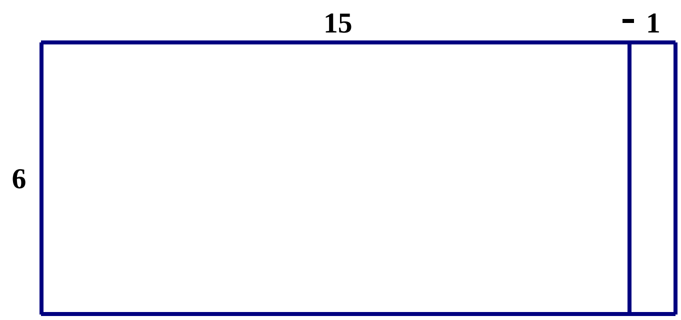


6(______ + ______ + ______)

6(______) + 6(______) + 6(______)

______ + ______ + ______

84



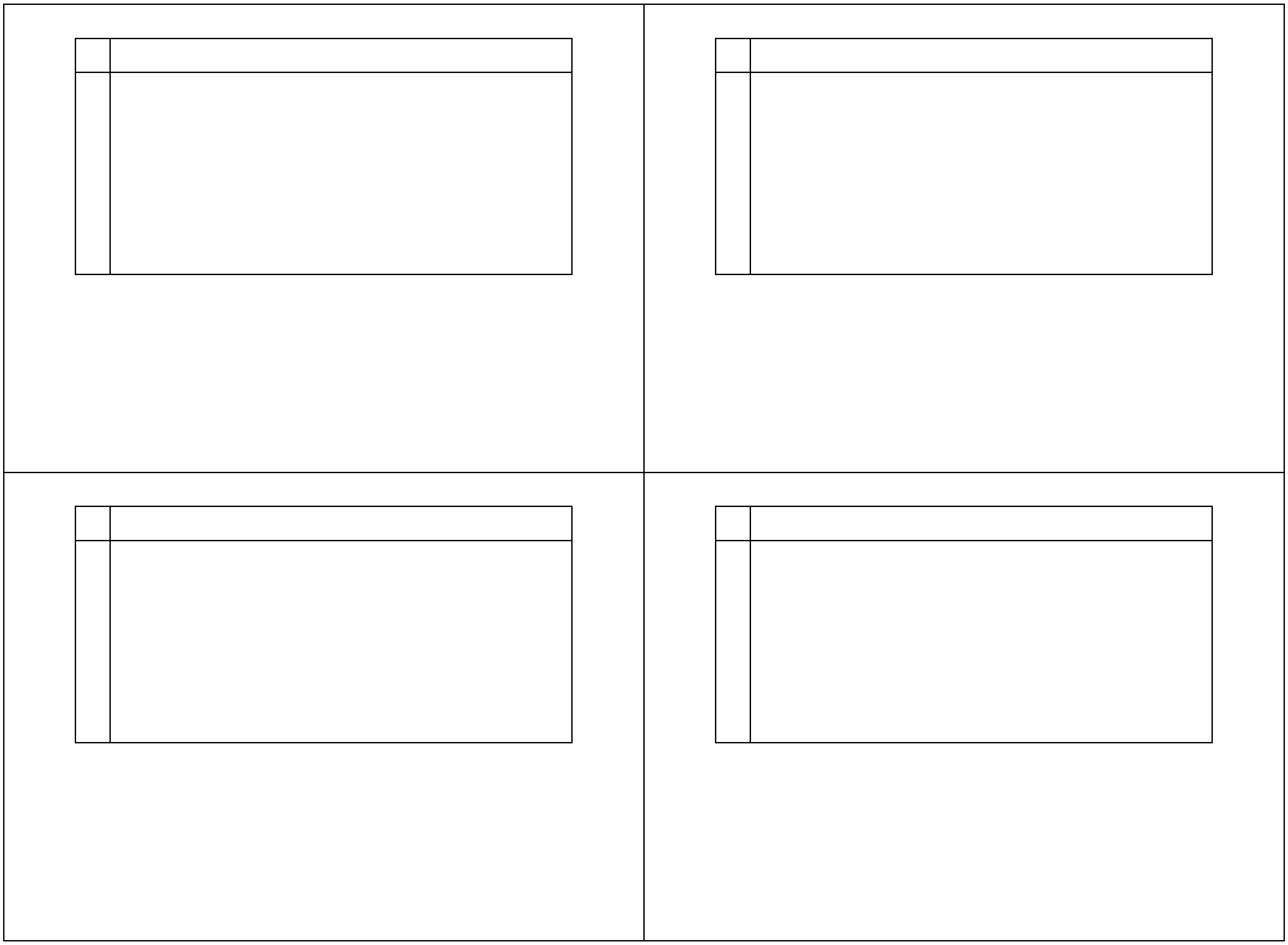
6(______ − ______)

6(______) − 6(______)

______ − ______

84

4. Using the templates below, draw area models to show different ways to represent the product of 5 and 68. Write the expressions that correspond to each area model. Answers will vary. Possible answers include:
5×68
$5 \times (60 + 8)$
$5 \times (70 - 2)$
$5 \times (30 + 30 + 8)$



Directions: Simplify the following expressions using two different methods.

5. $7(10 + 3)$		6. $8(30 - 1)$	
Method 1: $7(10) + 7(3)$ $70 + 21$ 91	Method 2: $7(13)$ 91	Method 1:	Method 2:

7. $9(100 + 10 + 1)$ Method 1: Method 2:	8. $\frac{3}{5}(60 + \frac{2}{3})$ Method 1 Method 2:
9. Four multiplied by the sum of thirty and six Method 1: Method 2:	10. Three multiplied by the difference of fifty and one Method 1: Method 2:

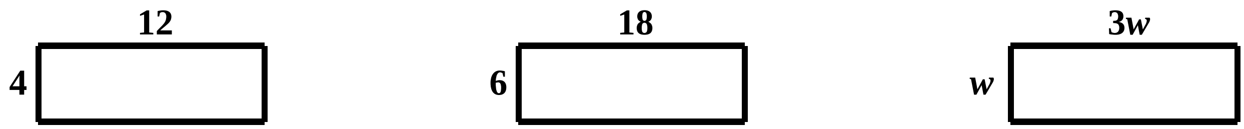
Directions: Use The Distributive Property to find the product. Methods will vary. Sample answers are shown.

11. $7(12)$	12. $4(105)$ $4(100 + 5)$ $4(100) + 4(5)$ $400 + 20$ 420
13. $3(24)$ $3(25 - 1)$ $3(25) - 3(1)$ $75 - 3$ 72	14. $6(47)$
15. $7(19)$	16. $5(222)$
17. $4(8\frac{3}{4})$	18. $\frac{1}{3}(24\frac{2}{3})$ $\frac{1}{3}(24 + \frac{2}{3})$ $\frac{1}{3}(24) + \frac{1}{3}\left(\frac{2}{3}\right)$ $8 + \frac{2}{9}$ $8\frac{2}{9}$

6.2c Class Activity: Simplifying Algebraic Expressions Part II

Activity 1:

a. Let's revisit part k. from the 6.1d Class Activity. It is about Chantelle and the dog run she is building. If you recall, Chantelle wants the length of her dog run to be three times longer than the width. Here are some pictures of possible dog runs Chantelle can build:



Complete the chart to show three different expressions to represent the perimeter of the rectangle based on the width of the dog run. Have students describe different ways to find the perimeter of each rectangle. If necessary, provide scaffolding so that students come up with the three different expressions shown in the table.

Width of Dog Run (ft.)	**Perimeter of Dog Run (ft.) Expression 1**	**Perimeter of Dog Run (ft.) Expression 2**	**Perimeter of Dog Run (ft.) Expression 3**
4	$4+4+12+12$	$2(4)+2(12)$	$2(4+12)$
6	$6+6+18+18$	$2(6)+2(18)$	$2(6+18)$
w	$w+w+3w+3w$	$2(w)+2(3w)$	$2(w+3w)$

Discuss the relationship between the different expressions. The first shows repeated addition. The second changes the repeated addition to a multiplication problem: 2 copies of the length plus 2 copies of the width. The third shows 2 copies of the sum of the length and width or $(w+3w)+(w+3w)$. This expression can be rearranged to the first expression using the Commutative and Associative Properties of Addition.

Ask students if they can see how to get from the third expression to the second expression and vice versa as this illustrates The Distributive Property.

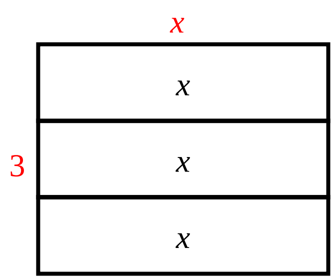
Activity 2:

a. Complete the table below. The first one has been done for you.

Multiplication Sentence	Describe Using Words	Related Addition Sentence	Simplified Form of Expression
$3(2)$	3 copies of two	$2+2+2$	6
$3x$	3 copies of x	$x+x+x$	$3x$
$3(2x)$	3 copies of $2x$	$2x+2x+2x$	$6x$
$3(x+2)$	3 copies of $(x+2)$	$(x+2)+(x+2)+(x+2)$ $(x+x+x)+(2+2+2)$ Commutative and Associative Properties of Addition at work.	$3x+6$

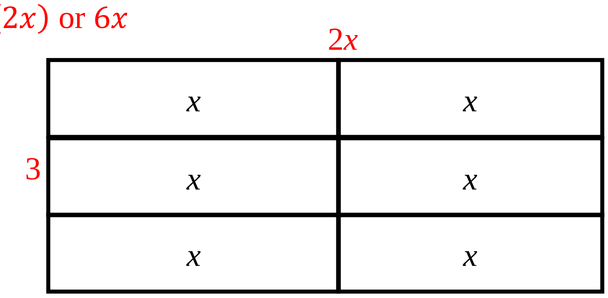
b. Four area models are shown below. Write the expression from part a. that corresponds to each area model.

$3x$

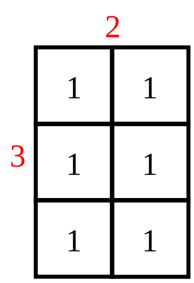


Have students label the dimensions of the rectangle as shown above. To find the area of the rectangle, we multiply the base and height: $3 \cdot x$ or $3x$.

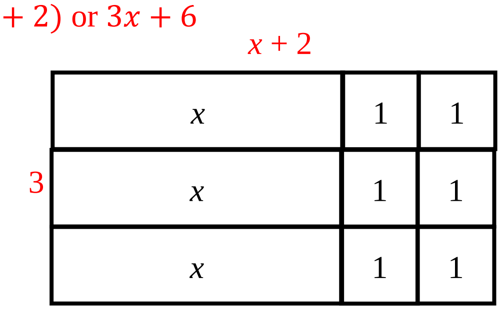
$3(2x)$ or $6x$



$3(2)$



$3(x+2)$ or $3x+6$



Ask students, which expression shows the Associative Property? $3(2x) \rightarrow 6x$ Which expression shows the Distributive Property? $3(x+2) \rightarrow 3x+6$

c. Complete the chart below.

Multiplication Sentence	**Describe Using Words**	**Related Addition Sentence**	**Simplified Form of Expression**
$4(6)$	four copies of six	$6+6+6+6$	24
$5x$	five copies of x	$x+x+x+x+x$	$5x$
$2(8x)$	2 copies of $8x$	$8x+8x$	$16x$
$4(x+1)$	4 copies of $(x+1)$	$(x+1)+(x+1)+(x+1)+(x+1)$	$4x+4$
$3(2x+7)$	Three copies of the sum of twice a number and seven	$(2x+7)+(2x+7)+(2x+7)$	$6x+21$
$2(4x+8)$	Two copies of the sum of four times a number and eight	$(4x+8)+(4x+8)$	$8x+16$

Activity 3: Write an equivalent expression without parentheses. Again, highlight the "simplified" form of the expression which takes the form $Ax + C$.

a. $4(x+7)$ $4x+28$	b. $9(x+2)$ $9x+18$
c. $6(x+4)$ $6x+24$	d. $5(10+x)$ $50+5x$
e. $8(x-7)$ $8x-56$	f. $12(x-5)$ $12x-60$
g. $\frac{1}{3}(x-9)$ $\frac{1}{3}x-3$	h. $\frac{3}{4}(x+12)$ $\frac{3}{4}x+9$
i. $0.6(x-4)$ $0.6x-2.4$	j. $0.5(7-x)$ $3.5-0.5x$

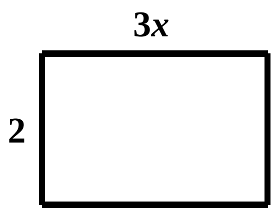
k. $8(7x+1)$ $56x+8$	l. $\frac{1}{5}(25x+45)$ $5x+9$
m. $2(x+3)+7x$ $9x+6$	n. $9(3x-4)-x$ $26x-36$
o. $10+3(x+6)$ $3x+28$	p. $6(x+3)-10$ $6x+8$
q. $4(2x+9)+7(x-3)$ $15x+15$	r. $5(a+b)+6a$ $11a+5b$
s. $3(2x+4)+3(2x\cdot 4)$ $30x+12$	t. $2(2x)+2(2+x)$ $6x+4$
u. $(x+3)4$ $4x+12$	v. $(3x-5)2+4x$ $10x-10$
w. $0.25(16x+4)-0.6x$ $3.4x+1$	x. $\frac{1}{8}x+0.5(\frac{1}{4}x+0.7)$ $\frac{1}{4}x+0.35$
y. $0.04(0.2x-800)+\frac{3}{10}x$ $0.308x-32$	z. $\frac{9}{10}\left(\frac{3}{2}x-80\right)+\frac{4}{5}x$ $2\frac{13}{20}x-72$

Talk to students about u. and v. They can use the Commutative Property to change the order of the factors: $(x+3)4 \rightarrow 4(x+3)$

In y. and z., a common error is for students to think that the subtraction sign goes with the x term when simplifying. For example, in y., they would distribute to get the expression $0.008x - 32 + \frac{3}{10}x$. When they go to combine like terms, they would subtract $\frac{3}{10}x$ from $0.008x$. One strategy you can use to prevent this error is to have students rearrange the terms first to $0.0008 + \frac{3}{10}x - 32$ or have them "blot out" the -32 with their finger to see that they are *adding* $\frac{3}{10}x$. Students will study this further in 7th grade when working with expressions that contain positive and negative rational numbers.

Activity 4:

a. Write expressions in simplified form to represent the area and perimeter of the rectangle.



Perimeter: $6x + 4$

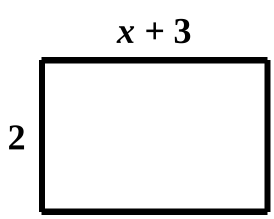
Area: $6x$

b. If the length and width of the rectangle in part a. are tripled, write expressions in simplified form to represent the area and perimeter of the new rectangle.

Perimeter: $18x + 12$

Area: $54x$

c. Write expressions in simplified form to represent the perimeter and area of the rectangle.



Perimeter: $2x + 10$

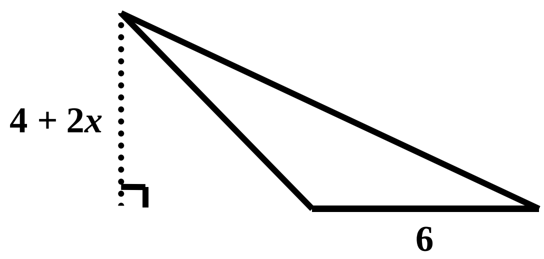
Area: $2x + 6$

d. If the length and width of the rectangle in part c. are doubled to create a new rectangle, write expressions for the perimeter and area of the new rectangle.

Perimeter: $4x + 20$

Area: $8x + 24$

e. Write an expression in simplified form to represent the area of the triangle.



Area: $6x + 12$

This problem is great for examining the structure of the expressions. Students should see this as the product of three factors and realize they can use the Commutative and Associative Properties of Multiplication to multiply the factors in any order:

$\frac{1}{2}(6)(4+2x) \to 3(4+2x) \to 12+6x$

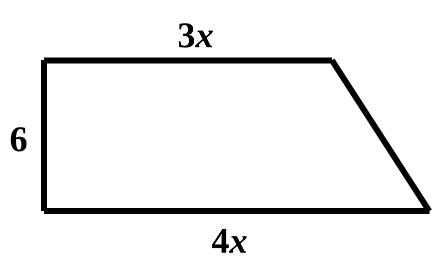
$\frac{1}{2}(4+2x)(6) \to (2+x)6 \to 12+6x$

$\frac{1}{2}(6)(4+2x) \to \frac{1}{2}(24+12x) \to 12+6x$

f. If the base and height of the triangle in part e. are halved, write an expression in simplified form to represent the area of the new triangle. Help students to see the connection between the expressions shown below in the answers

Area: $\frac{3x+6}{2}$, $\frac{1}{2}(3x+6)$, $\frac{3}{2}x+3$, $1.5x+3$

g. Write an expression in simplified form to represent the area of the trapezoid.



Area: $21x$

Spiral Review

1. Find 81% of 90.

2. Find the mean absolute deviation of the data set: $\{10, 12, 8, 12, 8\}$

3. Put the following numbers in order from least to greatest.

   a. $-2, -4, -10, 0$

   b. $-2.1, -2, -2.01, -2.11$

4. Complete the table below to show the relationship between meters and centimeters.

Meters	**Centimeters**
1	
2	
3	
10	
m	

Write an equation to represent the number of centimeters c based on the number of meters m.

6.2c Homework: Simplifying Algebraic Expressions Part II

Directions: In each of the following problems, an expression is given. Circle the expressions that are equivalent to the given expression. Justify your answer.

1. Select all the expressions that are equivalent to $2(x+5)$. Justify your answer.

   $2(x)+2(5)$

   $2x+5$

   $2x+10$

   $12x$

2. Select all the expressions that are equivalent to $3(y-1)+2y$. Justify your answer.

   $3(y)-3(1)+3(2y)$

   $3y-3+2y$

   $5y-3$

   $2y$

3. Select all the expressions that are equivalent to $\frac{1}{4}(x+12)$. Justify your answer.

   $x+3$

   $\frac{1}{4}x+\left(\frac{1}{4}\right)12$

   $\frac{1}{4}x+3$

   $\frac{x}{4}+\frac{12}{4}$

4. Select all the expressions that are equivalent to $4(2x+7)$. Justify your answer.

   $4(2x)+4(7)$

   $6x+28$

   $8x+28$

   $36x$

Directions: Match each expression on the left to its model on the right by writing the letter for the corresponding model under the expression. Then, simplify each expression.

<u>**Expression**</u>	<u>**Model**</u>

5. $2(2x + 2)$

   Matches to Model: __D___

   Simplified Expression:
   $4x + 4$

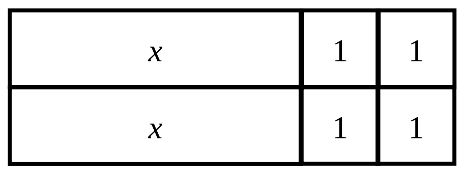
A

6. $2(2x) + 2$

   Matches to Model: _______

   Simplified Expression:

B

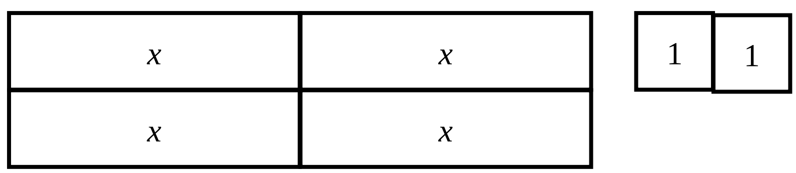


7. $2x + 2$

   Matches to Model: _______

   Simplified Expression:

C

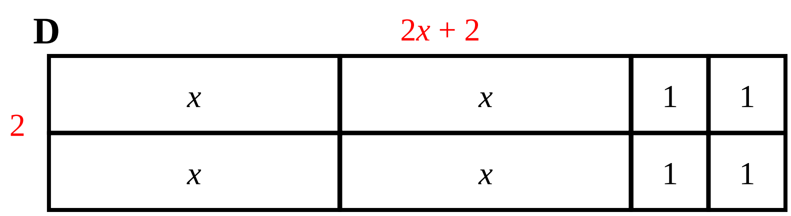


8. $2(x + 2)$

   Matches to Model: _______

   Simplified Expression:

D



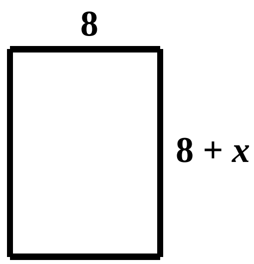
9. Complete the chart below.

Multiplication Sentence	Describe Using Words	Related Addition Sentence	Simplified Form of Expression
$2(8)$			
$4(7x)$	Four copies of the product of seven and a number x	$7x + 7x + 7x + 7x$	$28x$
$3(2x + 5)$	Three copies of the sum of twice a number and five	$(2x + 5) + (2x + 5) + (2x + 5)$	$6x + 15$
		$(x + 6) + (x + 6) + (x + 6) + (x + 6)$	
		$6x + 6x + 6x + 6x$	
	Two copies of the sum of three times a number n and twenty		

10. Simplify each of the following expressions.

a. $2(x + 9)$	b. $12(x - 3)$
c. $7(3x + 8)$ $21x + 56$	d. $11(5 + x)$ $55 + 11x$
e. $4(2x + 8)$	f. $9(x - 6) - 3x$ $6x - 54$
g. $2x + 8(3 + x)$	h. $15 + 2(3x - 4)$
i. $5(b + 3) + 6b$	j. $2(4y + 3) + 5(y + 1) + y$ $14y + 11$
k. $\frac{1}{8}(64x - 8)$ $8x - 1$	l. $\frac{1}{2}(x + 24) + 3x + 8$
m. $0.1(100 + 10x) + 0.2(100 + 10x)$	n. $\frac{1}{6}(2x + 42) + \frac{1}{4}x - 2$

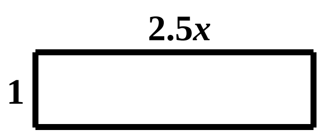
11. Write expressions in simplified form to represent the perimeter and area of the rectangle.



Perimeter: $2x + 32$

Area: $64 + 8x$

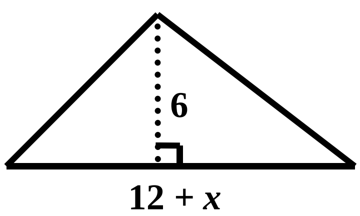
12. Write expressions in simplified form to represent the perimeter and area of the rectangle.



Perimeter:

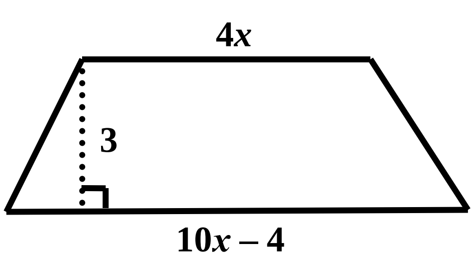
Area:

13. Write an expression in simplified form to represent the area of the triangle.



Area:

14. Write an expression in simplified form to represent the area of the trapezoid.

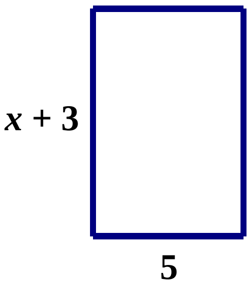


Area: $21x - 6$

6.2d Class Activity: Modeling Backwards Distribution (Factoring)

Activity 1:

a. The base of a rectangle measures 5 units. The height of the rectangle can be represented by the expression $(x + 3)$. Write an expression to represent the area of the rectangle.

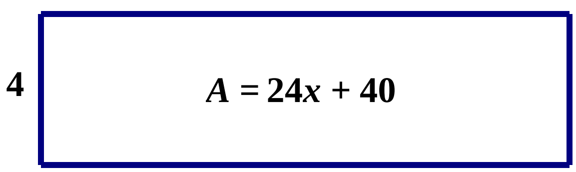


Area = $(x + 3)(5)$ or $(5)(x + 3)$

Area = $5x + 15$

b. The height of a rectangle measures 4 units. The area of the rectangle can be represented by the expression $24x + 40$ square units. Write an expression to represent the length of the rectangle.

Students use area models as a tool to understand factoring. They also rely on structure to factor expressions. If students have a difficult time jumping to the problem below, re-visit a numeric example. For example, show a rectangle with an area of 50 square units and a base of 5 units. What is the measure of the height?



Area = 4(?)

$24x + 40 = 4(?)$

$24x + 40 = 4(6x + 10)$

c. A rectangle has an area of $8x + 32$ square units. Complete the following table to show what the height of the rectangle is based on the base of the rectangle.

Base of Rectangle (units)	Height of Rectangle (units)	Area of Rectangle (square units)
2 units	$(4x + 16)$	$8x + 32$
4 units	$(2x + 4)$	$8x + 32$
8 units	$(x + 4)$	$8x + 32$

d. A rectangle has an area of $32x + 24$ square units. Complete the table to show the possible dimensions of the rectangle. Answers will vary

Base of Rectangle (units)	Height of Rectangle (units)	Area of Rectangle (square units)
2	$16x + 12$	$32x + 24$
4	$8x + 6$	$32x + 24$
8	$4x + 3$	$32x + 24$
$\frac{1}{2}$	$64x + 48$	$32x + 24$

These are all equivalent expressions. The equivalent expression that shows the GCF factored out is the third one: $8(4x + 3)$.

Activity 2: Write each expression as the product of two factors. Answers will vary. Some students will factor out the greatest common factor (GCF) and others will not. The goal of 6th grade is for students to be able to write equivalent expressions. Have students share out answers and discuss the expression that is completely factored (i.e. the greatest common factor has been factored out).

a. $5x + 20$ $5(x + 4)$	b. $4x - 18$ $2(2x - 9)$
c. $12x + 2$ $2(6x + 1)$	d. $20x + 30$ $10(2x + 3)$ or $2(10x + 15)$ or $5(2x + 6)$
e. $12x + 6$ $6(2x + 1)$	f. $12 - 2x$ $2(6 - x)$
g. $8 + 4x$ $4(2 + x)$ or $2(4 + 2x)$	h. $11x + 44$ $11(x + 4)$
i. $10x + 12$ $2(5x + 6)$	j. $20x + 12$ $4(5x + 3)$

Spiral Review

1. Jeremy can swim 3 laps in 5 minutes. How many laps does Jeremy swim in 1 minute?

2. Using the information from the previous problem, determine the number of laps Jeremy can swim in 8 minutes.

3. Write 10^2 in standard form.

4. Write (5×10^2) in standard form.

6.2d Homework: Modeling Backwards Distribution (Factoring)

1. Which of the following expressions are equivalent to $12x + 18$?

   $2(6x + 9)$

   $3(4x + 6)$

   $6(2x + 3)$

   $12(x + 18)$

2. A parallelogram has an area of $36x + 30$ square units. Complete the following table to show what the height of the parallelogram is based on the base of the parallelogram.

Base of Parallelogram (units)	Height of Parallelogram (units)	Area of Parallelogram (square units)
2 units	$18x + 15$	$36x + 30$
3 units		$36x + 30$
6 units		$36x + 30$

3. A rectangle has an area of $80x + 100$ square units. Complete the table to show the possible dimensions of the rectangle.

Base of Rectangle (units)	Height of Rectangle (units)	Area of Rectangle (square units)
10 units	$8x + 10$	$80x + 100$
		$80x + 100$
		$80x + 100$
		$80x + 100$

4. Show some different ways the expression $6x + 42$ can be written as the product of two factors.
   Answers will vary. Sample answers $6(x + 7)$ or $3(2x + 14)$

5. Show some different ways the expression $12x + 18$ can be written as the product of two factors.

Directions: Write each expression as the product of two factors. Answers will vary. Sample answers are shown.

6. $18r + 24$ $6(3r + 4)$ or $2(9r + 12)$ or $3(6r + 8)$	7. $9r + 12$
8. $30m - 6$	9. $8n + 32$
10. $42m + 66$	11. $6 + 66x$
12. $12x + 8$	13. $32x + 8$
14. $35x + 63$	15. $18x - 30$

6.2e Class Activity: Repeated Multiplication and Exponents

Before beginning, it would be a good idea to review the different symbols that can be used to represent multiplication: 3×5, $3 \cdot 5$, $3(5)$.

Activity 1: We know we can express **repeated addition** using **multiplication**. The table below shows the relationship between addition and multiplication:

Addition Sentence	Describe Using Words	Related Multiplication Sentence
$5 + 5 + 5$	3 copies of 5	$3(5)$
$6 + 6 + 6 + 6$	4 copies of 6	$4(6)$
$10 + 10 + 10 + 10 + 10 + 10$	6 copies of 10	$6(10)$

What if we want to express **repeated multiplication** using shorthand notation. For example, if you are **multiplying** the number four by itself nine times, it would be time-consuming to write:

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

We can use **exponents** to represent **repeated multiplication** in shorthand notation.

In earlier grades, you learned that numbers such as ten, one hundred, one thousand, ten thousand, etc. can be written using **powers of ten**. Study the chart below to review:

Number in Standard Form	Number in Expanded Form	Number Written as a Power of Ten
10	10	10^1
100	$10 \cdot 10$	10^2
1,000	$10 \cdot 10 \cdot 10$	10^3
10,000	$10 \cdot 10 \cdot 10 \cdot 10$	10^4
100,000	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	10^5

The third column shows repeated multiplication being written using exponents. These numbers are all written using **exponential notation** or in **exponential form**.

a. Apply what you learned about powers of ten to re-write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ using exponents.

4^9

Definition of an Exponent

$$a^n = \underbrace{a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a}_{n \text{ times}}$$

a is called the **base** and *n* is called the **exponent** or **power**.
The exponent tells you how many times to multiply the base by itself.
We read the expression as, "*a* to the power of *n*" or "*a* raised to the n^{th} power".

Activity 2: Complete the chart below. It may be good to have a calculator on hand to complete this chart.

Expanded Form	Exponential Form	Simplified Form
a. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	3^5	243
b. $11 \cdot 11$	11^2	121
c. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	10^5	100,000
d. $5 \cdot 8 \cdot 8 \cdot 8 \cdot 8$	$5^1 8^4$ or $5 \cdot 8^4$ or $5(8^4)$	20,480
e. $\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5}$	$\left(\frac{2}{5}\right)^4$	$\frac{16}{625}$
f. $2 \cdot 2 \cdot 2$	2^3	8
g. $3 \cdot 3$	3^2	9
h. $1 \cdot 1 \cdot 1 \cdot 1$	1^4	1
i. $3 \cdot 5 \cdot 5$	$3 \cdot 5^2$	75

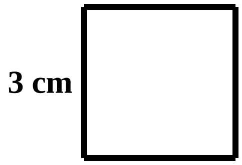
j. $(0.1)(0.1)(0.1)$	$(0.1)^3$	0.001
k. $r \cdot r \cdot r \cdot r$	r^4	r^4
l. $5 \cdot g \cdot g \cdot g \cdot g \cdot g \cdot g =$	$5g^6$	$5g^6$
m. $(2x)(2x)(2x)$	$(2x)^3$	$8x^3$
n. $a \cdot b \cdot b \cdot b \cdot b \cdot b$	ab^5	ab^5
o. $d \cdot d \cdot d \cdot d \cdot d \cdot d \cdot d$	d^7	d^7
p. $p \cdot p \cdot p \cdot p \cdot p$	p^5	p^5
q. $(4r)(4r)(4r)$	$(4r)^3$	$64r^3$
r. $4 \cdot r \cdot r \cdot r$	$4r^3$	$4r^3$
s. $a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b$	a^3b^4	a^3b^4
t. $m \cdot n \cdot n$	mn^2	mn^2
u. $(mn)(mn)$	$(mn)^2$	m^2n^2

Pay special attention to the following problems:

- d: A common mistake in d. is for students to write 58^4. Clarify with students what 58^4 means. Why would it be important in this case to either put the exponent of 1 with the 5 or separate the 5 and 8^4 using a multiplication symbol? The same ideas apply for part i.
- l: This expression is like d.; however, when you have a number next to a variable, multiplication is the implied operation. Students learned this in the geometry chapter. It will be helpful when students evaluate expressions later in the chapter, that they understand that, in the expression $5g^6$, only the g is being raised to the power of 6. We can expand this out to show: $5 \cdot g \cdot g \cdot g \cdot g \cdot g \cdot g$.
- m: To understand why $(2x)(2x)(2x)$ simplifies to $8x^3$, it will be very helpful for students to insert the symbol for multiplication between the 2 and the x giving $2 \cdot x \cdot 2 \cdot x \cdot 2 \cdot x$. From here, we can use the Commutative Property of Multiplication to rearrange the terms giving $2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x$. Then, the Associative Property of Multiplication allows us to group the twos and the x's giving $(2 \cdot 2 \cdot 2) \cdot (x \cdot x \cdot x)$ which we can then simplify to $8x^3$. Students will learn the exponent rules in Grade 8 but this type of thinking will lay the foundation for what is to come.
- n: This is like d. and l. from above. Again, emphasize that only the b is being raised to the power of 5 and then the result is being multiplied by a. We will emphasize this more in the next lesson on order of operations.

Structure plays a huge role in being able to represent expressions with exponents geometrically and to simplify expressions with exponents.

Activity 3: A square is shown below. Prior to doing this lesson, you may want to introduce students to the algebra tiles if you plan to use them in the lesson. Depending on your students, you may choose not to use the physical tiles and move right into having students draw pictures and models to represent the problems.



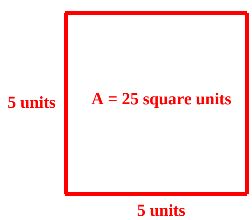
a. Write an expression in expanded form that can be used to find the area of the square.
   $3 \cdot 3$

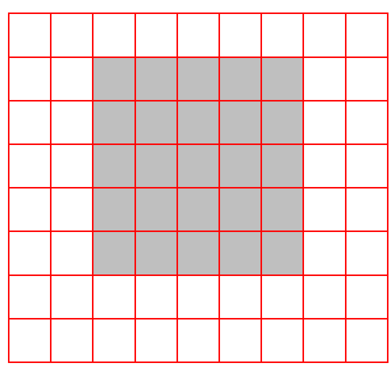
b. Write an expression in exponential form that can be used to find the area of the square.
   3^2

c. Write the simplified form for the area of the square.
   9 square centimeters

d. The expression 3^2 is read "3 to the power of 2" or "3 squared". Why do you think the power of two is also referred to as "squared"?
   The geometric representation of 3^2 is a square with a side length equal to 3 and an area equal to 9.

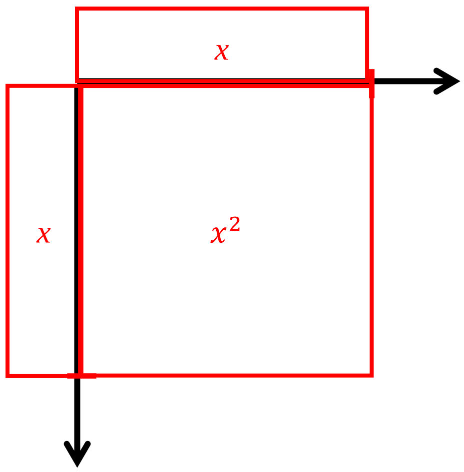
e. Draw a picture to represent 5^2.





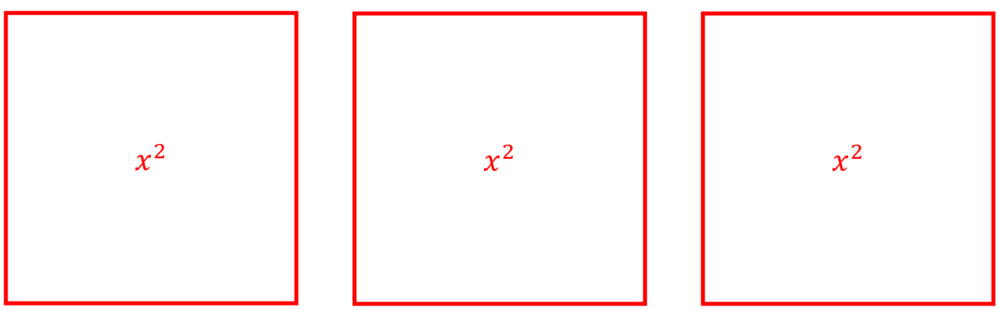
f. Draw a picture to represent $x \cdot x$ or x^2.

Have students use the *x* algebra tile to draw a square with a side length equal to *x* units. Students can use the side of the *x* tile to measure and mark off the lengths. Using the template below will help students to draw the sides of the square so that they are perpendicular. Students will see that the area of an *x* by *x* square is the square tile with a side length of *x*.

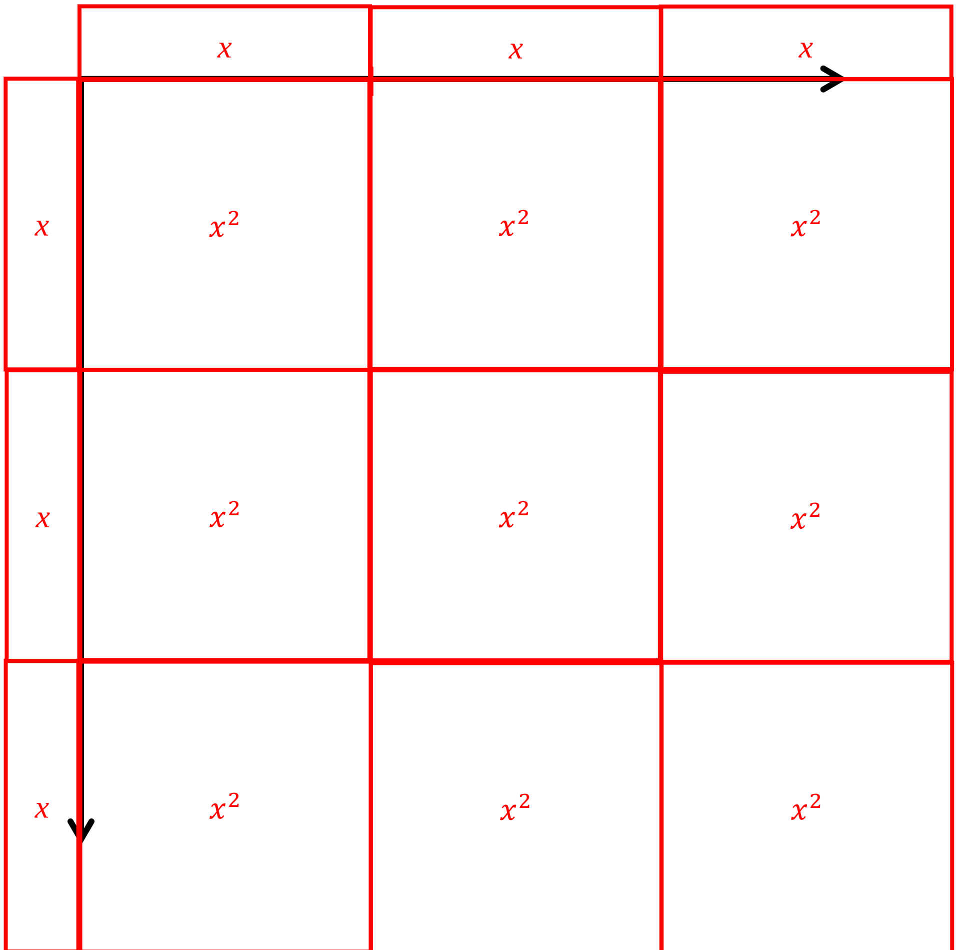


g. Draw a picture to represent $3x^2$.

It will help students if they re-write this using the multiplication symbol between the 3 and x^2 terms: $3 \cdot x^2$. What does this mean? We have three copies of x^2. Or, we can think of it as $x^2 + x^2 + x^2$. We can use our picture from above and copy it three times.

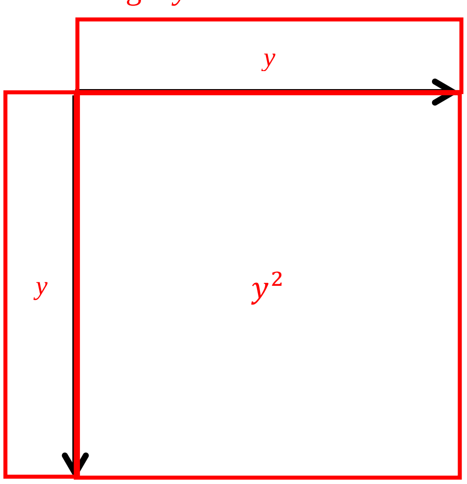


h. Draw a picture to represent $(3x)^2$. Students expand this out to $(3x)(3x)$ if needed. Prompt students, "How can we show $(3x)(3x)$ geometrically?" A square with a side length of $3x$. Students defined the length of x in part f. To show a side length of $3x$, students need to copy the length of x three times to construct the side of the square.

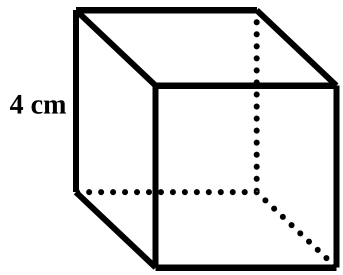


This process may seem time consuming but it really helps students to see why $(3x)^2$ simplifies to $9x^2$ and how it is different than $3x^2$.

i. Draw a picture to represent y^2. The purpose of this problem is to get students to think about x and y representing different numbers. So again, we can use any length that we want for y as long as it is different than the length we used for x above. If you are using algebra tiles, some kits have a tile to represent a length y.



Activity 4: A cube is shown below.



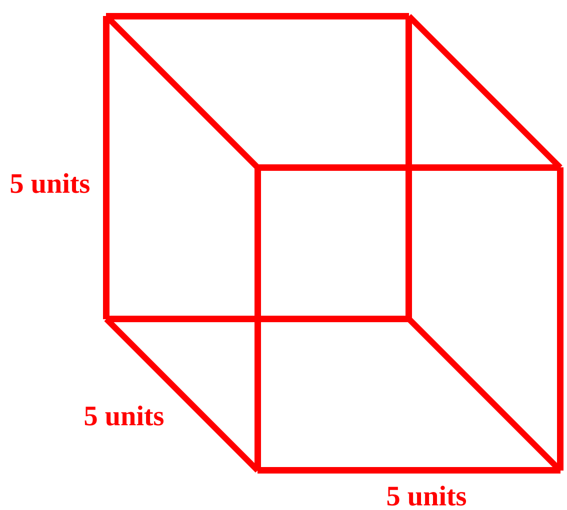
a. Write an expression in expanded form that can be used to find the volume of the cube.
$4 \times 4 \times 4$

b. Write an expression in exponential form that can be used to find the volume of the cube.
4^3

c. Write the simplified form for the volume of the cube.
64 cm^3

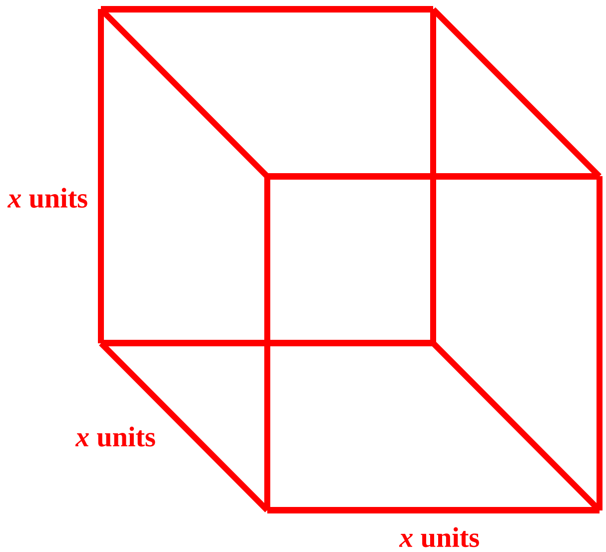
d. The expression 4^3 is read "4 to the power of 3" or "4 cubed". Why do you think the power of three is also referred to as cubed?
The geometric representation of 4 to the power of 3 is a cube.

e. Draw a picture to represent 5^3.



f. Draw a picture to represent x^3.

A cube with side lengths equal to *x*



g. Draw a picture to represent $5x^3$.

Parts g. and h. may be time consuming to draw so it might be easier to have students describe what the model would look like. This would be 5 copies of the cube from part f.

h. Draw a picture to represent $(5x)^3$.

A cube with side lengths of $5x$. Take the cube from part f. and make each of its side lengths 5 times longer. As an extension, ask students how the volume of this cube compares to the volume of the cube in part f. It is 125 times larger.

Activity 5: Reasoning A calculator would be helpful for this activity.

a. Is $2^3 = 3^2$? Justify your answer.
   3^2 is greater than 2^3. This addresses a common mistake which is that students will say the two expressions are equal.

b. Predict which is greater. $10 \cdot 2$ or 2^{10}. Then, test your prediction.
   2^{10} is bigger

c. Predict which is greater. 10^2 or 2^{10} Then, test your prediction.
   2^{10} is bigger

d. Compare 1^{49} and 1^{50}.
   These expressions are equal.

e. Tony thinks that $4x^3$ is the same as $(4x)^3$. Explain to Tony why his thinking is incorrect.
   The first expression represents three copies of a cube with a side length of *x*. The second expression represents one cube with a side length that is four times bigger than the side length of one of the cubes from the first expression.

f. Write as many expressions as you can that use exponents and simplify to the number 8.
   Answers will vary. 2^3, 8^1, $2^4 - 8$, $2^2 + 2^2$ Look for and address the common error of 2^4 or 4^2.

g. Write as many expressions as you can that use exponents and simplify to the number 16.
   Answers will vary.

h. Write as many expressions as you can that use exponents and simplify to the number 64.
   Answers will vary.

i. Simplify $x^3 + x^3 + x^3$
   This is 3 copies of a cube with a side length of *x*. The simplified form of this expression is $3x^3$.

As an extension, you can talk to students about perfect square numbers (1, 4, 9, 16, 25, 36, etc.) and why they are called perfect square numbers. Additionally, you can talk about perfect cube numbers (1, 8, 27, 64, 125, etc.) and again talk about why they are called perfect cube numbers. They will simplify square roots in Grade 8.

Spiral Review

Directions: Simplify the following expressions.

1. $b + b + b$	2. $5 + x + x + x + x$
3. $4x + 3y + x + 8y$	4. $4x + 4x + 4x$

6.2e Homework: Repeated Multiplication and Exponents

1. Represent the expression $9 + 9 + 9 + 9 + 9 + 9$ in shorthand notation. $6(9)$
2. Represent the expression $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$ in shorthand notation. 9^6
3. Explain how $4(2)$ is different than 2^4.

Directions: Complete the chart below.

Expanded Form	Exponential Form	Simplified Form
4. $5 \cdot 5$	5^2	25
5. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$		
6. $4 \cdot 4 \cdot 4$		
7. $3 \cdot 3 \cdot 3 \cdot 3$		
8. $10 \cdot 10 \cdot 10$		
9. $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$	$\left(\frac{1}{10}\right)^3$	$\frac{1}{1{,}000}$
10. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$		
11. $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$		
12.	12^3	
13.	8^2	
14. $(2)(2)(2)(2)(4)(4)$	$2^4 4^2$	256
15.	$2^3 3^3$	
16.	$2 \cdot 6^3$	

17.	$\left(\frac{2}{3}\right)^3$	
18.	$2 \cdot 3^2$	
19.	$3 \cdot 2^2$	
20. $(2.5)(2.5)(2.5)$	$(2.5)^3$	15.625
21. $d \cdot d \cdot d$	d^3	d^3
22. $x \cdot y \cdot y$		
23. $2 \cdot c \cdot c \cdot c \cdot c$	$2c^4$	$2c^4$
24. $(9a)(9a)$	$(9a)^2$	$81a^2$
25. $9 \cdot a \cdot a$		
26. $(7b)(7b)(7b)$	$(7b)^3$	$343b^3$
27. $7 \cdot b \cdot b \cdot b$	$7b^3$	$7b^3$
28. $(ab)(ab)(ab)(ab)$		

29. Draw or describe a picture to represent two cubed.
A cube with a side length of 2 and a volume of 8 cubic units.

30. Draw or describe a picture to represent ten squared.

31. Draw or describe a picture to represent x^2.
A square with a side length of *x*.

32. Draw or describe a picture to represent $4x^2$.

33. Draw or describe a picture to represent $(4x)^2$.
A square with a side length of 4*x* which has an area of $16x^2$.

34. Write as many expressions as you can that use exponents and simplify to the number 36. Answers will vary. Sample answers shown.
6^2
36^1
$(2 \cdot 3)^2$
$4 \cdot 3^2$
$2^5 + 4$

35. Write as many expressions as you can that use exponents and simplify to the number 100.

36. Give two numbers for *a* and *b* where $a^b \neq b^a$.

37. Give two numbers for *a* and *b* where $a^b = b^a$.

38. Give a number for *a* so that $a^{50} = a^{100}$.

39. Write $(x^3)(x^3)$ in exponential form.

40. Draw or describe a picture of $3^2 + 5^2$.

41. Draw or describe a picture of $(3 + 5)^2$.

6.2f Class Activity: Evaluating Algebraic Expressions

Activity 1: Simplify each expression.

a. $4 \cdot 3^2$ 36 If students struggle with this problem, have them write it out in expanded form as $4 \cdot 3 \cdot 3$.	b. $(4 \cdot 3)^2$ 144 Again, prompt students to think about the difference between this problem and the previous one.
c. $(2+3)^2$ 25	d. $(2 \cdot 3)^2$ 36
e. $4^2 + 6(8-3)$ 46	f. $2^3 \div 2 + 2(9-3)$ 16
g. $2 \cdot 4^2 - 3 \cdot 7$ 11	h. $(4+5)^2 - 11 \div 11$ 80
i. $8 - 2 \div \frac{1}{4}$ 0	j. $(\frac{4}{5} + \frac{3}{10})^2 \cdot 1\frac{9}{11}$ $\frac{11}{5}$ or $2\frac{1}{5}$

Activity 2: Evaluate each expression when $x = 3$.

Expression	**Value When $x = 3$**
a. $4x$	12
b. $3(4x)$	36
c. $4x^3$	108
d. $(4x)^3$	1,728
e. $4x + 4x + 4x$	36
f. $64x^3$	1,728

Which expressions from the table above are equivalent? Justify your answer.

b. and e. are equivalent

d and f. are equivalent

Have students share explanations/justifications. Sample answers: 1) They evaluate to the same number. 2) When you simplify the expressions, they are the same.

Activity 3: Evaluate the following expressions when $a = 2, b = 4,$ and $c = 10$.

a. $a + b$ 6	b. ab 8	c. $\frac{a}{b}$ $\frac{1}{2}$
d. $\frac{b}{a}$ 2	e. $b - a$ 2	f. $3(b + c)$ 42
g. $\frac{a+c}{b}$ 3	h. $\frac{2b}{c}$ $\frac{4}{5}$	i. $a \div 0.01$ 200
j. $a(b + c)$ 28	k. $4a - 2a + \frac{c}{b}$ 6.5	l. $2c - 2b$ 12
m. $\frac{c}{2} - 5$ 0	n. $\frac{c-5}{2}$ 2.5	o. $5b^2$ 80
p. $(5b)^2$ 400	q. c^2b^3 6,400	r. $2a^3 + 4a$ 24
s. $2(b - 1)^2 - 3$ 15	t. $5b - 2b$ 12 Some students may simplify first.	u. $2c + 4 - (a^4 + \frac{c}{a})$ 3
v. $2 + b^2$ 18	w. $2b^2$ 32	x. $(2 + b)^2$ 36
y. $\frac{1}{2}(2c + 4)$ 12	z. $0.35c + 0.01b^2$ 3.66	aa. $\frac{a^2}{c}$ $\frac{2}{5}$
bb. $\left(\frac{a}{c}\right)^2$ $\frac{1}{25}$	cc. $10(a + b)$ 60	dd. $10a + 10b$ 60

Activity 4: Evaluate the following expressions when $a = \frac{4}{5}, b = 5,$ and $c = \frac{9}{20}$.

This problem provides a nice opportunity for students to review operations with positive rational numbers. For problems like c. and f., it can help to re-write the division expression as shown so that students avoid creating a complex fraction (a fraction within a fraction).

a. $a + b$ $5\frac{4}{5}$	b. ab 4	c. $\frac{a}{b}$ $\frac{4}{5} \div 5$ $\frac{4}{5} \cdot \frac{1}{5} = \frac{4}{25}$
d. $a + c$ $1\frac{1}{4}$	e. ac $\frac{9}{20}$	f. $\frac{a}{c}$ $\frac{4}{5} \div \frac{9}{20}$ $\frac{4}{5} \cdot \frac{20}{9} = \frac{16}{9} = 1\frac{7}{9}$
g. $b + c$ $5\frac{9}{20}$	h. bc $2\frac{1}{4}$	i. $\frac{b}{c}$ $11\frac{1}{9}$
j. $\frac{c}{b}$ $\frac{9}{100}$	k. $b - a$ $4\frac{1}{5}$	l. $b \div a \cdot c$ $2\frac{13}{16}$
m. $b \div (a \cdot c)$ $13\frac{8}{9}$ or $13.\overline{8}$	n. $\frac{a+c}{b}$ $\frac{1}{4}$	o. $b(a + c)$ $6\frac{1}{4}$
p. $4a - 2a + b$ $6\frac{3}{5}$	q. $2b - 4c$ $8\frac{1}{5}$	r. $100a^2$ 64
s. $(100a)^2$ 6,400	t. $b - a^2$ $4\frac{9}{25}$	u. $(b - a)^2$ $17\frac{16}{25}$
v. $\left(\frac{c}{a}\right)^2$ $\frac{81}{256}$	w. $10(a + b)$ 58	x. $10a + 10b$ 58

Activity 5: Using Formulas

A marketing company is designing different sized boxes to ship clothes in. Each box will be in the shape of a cube.

a. Using the formula $6s^2$, determine the amount of material needed to make each box depending on the side length of the box. Have students show their work to the side of the table.

Side Length of Box (inches)	Surface Area of Box (inches^2)
8	384
12	864
16	1,536
20	2,400
24	3,456

As an extension, have students compare the area of the box with a side length of 8 in. to the area of the box with a side length of 16 in. As the side length doubles, the area quadruples. Use the formula to explain why this happens.

1. The expression $\frac{9}{5}C + 32$ can be used to determine the temperature in degrees Fahrenheit depending on the temperature in degrees Celsius.

b. Use the expression to complete the table below to determine the temperature in degrees Fahrenheit depending on the temperature in degrees Celsius.

Temperature (in degrees Celsius)	Temperature (in degrees Fahrenheit)
0	32
15	59
25	77
40	104
60	140
100	212

2. The expression $\frac{s^2 h}{3}$ can be used to determine the volume of a square pyramid where *s* represents the side length of the square base and *h* represents the height of the pyramid.
   a. Use the expression to complete the table to show the volume of a pyramid depending on the area of the base of the pyramid and the height of the pyramid.

Side Length of Base (inches)	**Height of Pyramid (inches)**	**Volume of Pyramid (inches^3)**
2	3	4 in^3
3	6	18 in^3
4	9	48 in^3
5	12	100 in^3
6	15	180 in^3

Spiral Review

1. Caleb claims that $x^2 = 2x$ using the following argument:

   If $x = 2$, then
   $2^2 = 2(2)$
   $4 = 4$
   Therefore, $x^2 = 2x$

   Give a counterexample to show that Caleb's thinking is incorrect.

2. The side length of a square is 5 meters. Select all expressions that can be used to find the area of the square. Find the area of the square.
   a. $2(5)$
   b. $5 + 5 + 5 + 5$
   c. $5 \cdot 5$
   d. 5^2
3. I earned $6. Then I bought 4 candy bars for $0.50 each. Select all the expressions that can be used to determine the amount of money I have left. Determine the amount of money I have left.

   a. 6 – 0.50 – 0.50 – 0.50 – 0.50
   b. 6 – 4(0.50)
   c. 6 – (0.50 – 0.50 – 0.50 – 0.50)
   d. 6 – (0.50 + 0.50 + 0.50 + 0.50)

4. I am thinking of a number. When I multiply my number by four, I get twenty-four. What number am I thinking of?

6.2f Homework: Evaluating Algebraic Expressions

Directions: Simplify each expression.

1. $(6+8) \div (12-5)$	2. $\frac{100}{5^2}$ 4
3. $\frac{12-4}{5+3}$	4. $5(8) + 24 \div 6 + 2 \times 9$ 62
5. $20 - 12 \div (1+3) \times 2$	6. $3 + 6(5+4) \div 3 - 7$
7. $3 + 1^2$	8. $(3+1)^2$

Directions: Evaluate each expression when $x = 3$.

Expression	**Value When $x = 3$**
9. $2x$	
10. $x + x$	
11. x^2	9
12. $(2x)^2$	
13. $2x + 2x + 2x + 2x$	
14. $8x^4$	648
15. $2x^2$	
16. $x^2 + x^2$	
17. $x \cdot x$	

18. Which expressions from the table above are equivalent? Justify your answer.

Directions: Evaluate each expression when $r = 12, s = 2,$ and $t = 5$.

19. $r + t$	20. st 10	21. $0.5r$
22. $\frac{t}{s}$ 2.5	23. $\frac{s}{t}$ $\frac{2}{5}$	24. $s(r - t)$
25. $r + st$	26. $r - s + t$	27. $r - (s + t)$
28. $10r \div 12t$	29. $3s + 12$	30. $60 - 48 \div r$ 56
31. $3(2s + 9)$	32. $4t + 8t$	33. t^2
34. 2^t	35. $3t^2$ 75	36. $(3t)^2$
37. $8t - 5t$	38. $\frac{t-s}{r}$	39. $\frac{r}{t-s}$
40. $\frac{r}{t} + \frac{s}{t}$	41. s^5	42. sr^2
43. $4t^2 + 3t^2$	44. $4(r - t)$	45. $4r - 4t$
46. $\left(\frac{s}{t}\right)^3$ $\frac{8}{125}$	47. $\frac{3}{5}(r - s)$	48. $5r - s^3 + 2$ 54

49. The expression $\frac{5}{9}(F-32)$ can be used to determine the temperature in degrees Celsius depending on the temperature in degrees Fahrenheit.

b. Use the expression to complete the table below to determine the temperature in degrees Celsius depending on the temperature in degrees Fahrenheit.

Temperature (in degrees Fahrenheit)	**Temperature (in degrees Celsius)**
32°	0°
50°	10°
77°	
86°	
104°	

50. The expression $\frac{d}{2}$ can be used to determine the length of the radius of a circle based on the length of the diameter of the circle.

a. Use the expression to complete the table to show the length of the radius of a circle based on the length of the diameter of the circle.

Length of Diameter (cm)	**Length of Radius (cm)**
6	
10	
15	
30	
57	
100	

6.2g Class Activity: How Many Expressions Can You Make Part II

Activity 1: Working in a group, write as many equivalent expressions as you can for the following expressions. Explain how you know the expressions are equivalent.
This activity synthesizes many of the ideas from this section. Answers will vary. You may also consider putting these on a bulletin board and having an "expression of the week", allowing students to put up their equivalent expressions.

Expression 1: $4x + 10$
$2x + 2x + 10$
$2(2x + 5)$

Expression 2: $18x$

Expression 3: $(2x + 3) + (2x + 3) + (2x + 3)$

Expression 4: $20x^3$

Expression 5: $(3x)^4$

Expression 6: $5y \cdot 5y \cdot 5y$

Expression 7: $18a + 24 + 2a$

Expression 8: $2x^2 + 5$

Expression 9: $12x$

Expression 10: $(4x)^3$

Expression 11: $4x^3$

Expression 12: $24a + 18b$

6.2h Class Activity: Writing Algebraic Expressions to Model Real World Problems

Problems #1 – 8 in this lesson are intended to provide scaffolding for students so that they can write their own algebraic expressions to represent real world problems starting in problem #9. It is recommended that you read the teacher note for #9 prior to starting this lesson.

1. Eight different classes at an elementary school are collecting canned goods for a food pantry. The eighth-grade class collected *c* cans. The statements below show the number of cans collected by the other grades. Match each statement to the correct expression.

The 7th grade class collected two more cans than the 8th grade class	$\frac{c-2}{2}$
The 6th grade class collected twice as many cans as the 8th grade class	$2c+10$
The 5th grade class collected 2 fewer cans than the 8th grade class	$2c$
The 4th grade class collected twice as many cans as the 7th grade class	$c+10$
The 8th grade class collected 10 fewer cans than the 3rd grade class	$2(c+2)$
The 2nd grade class collected 10 more cans than the 6th grade class	$c+2$
The 1st grade class collected half as many cans as the 5th grade class	$c-2$

2. If the 8th grade class collected 60 cans of food, how many cans of food did the school collect in all?

$60+62+120+58+124+70+130+29=653$

3. Clara is putting marbles in a jar. There are r red marbles in the bag. Match each statement about the other marble colors to the correct expression.

The number of orange marbles is twice the number of red marbles	$2r+3$
The number of yellow marbles is three times the number of orange marbles	r^3
The number of green marbles is the square of the number of red marbles	$(2r)^3$
The number of blue marbles is the cube of the number of red marbles	$2r^2$
The number of purple marbles is the cube of the number of orange marbles	$2r$
The number of black marbles is three more than the number of orange marbles	$3(2r)$
The number of white marbles is twice the number of green marbles	r^2

4. If there are 3 red marbles in the jar, how many marbles are there in all?

$3+6+18+9+27+216+9+18=306$ marbles

Simplify the expressions shown above to show that the simplified expression is equivalent to the original expression.

5. The baseball team is selling popcorn one week after school to raise money for uniforms. On Monday, they sold *b* bags of popcorn. Complete the table by writing expressions to represent the number of bags of popcorn they sold on the other days.

Number of Bags Sold Each Day In Words	**Number of Bags Sold Each Day Algebraic Expression**
On Monday they sold *b* bags of popcorn.	b
On Tuesday they sold 15 more bags of popcorn than on Monday.	$b+15$
On Wednesday they sold 20 more bags of popcorn than they did on Tuesday.	$b+35$
On Thursday they sold 5 fewer bags than on Monday.	$b-5$
On Friday they sold twice as many bags than they sold on Tuesday.	$2(b+15)$

6. Write an expression in simplified form to represent the total number of bags of popcorn the team sold on the five days.
$b+(b+15)+(b+35)+(b-5)+(2b+30)=6b+75$

7. If each bag of popcorn cost $1.50, write an expression in simplified form for the amount of money raised based on the number of bags sold on Monday.
$1.5(6b+75)$
$9b+112.5$

8. If the baseball team sold 45 bags of popcorn on Monday, how much money did they raise?
$517.50

9. Brad's age is unknown. Charlie is 5 years older than Brad. Devon is twice as old as Charlie.
There are many important concepts for students to understand when approaching problems like this. 1) This is a problem with unknowns. We can use variables to represent unknowns. Students may wonder, "Where do we start? What variable do we use?" We can start by assigning a variable to any of our unknowns. Often, we use the first letter of the unknown for our variable. We often see them italicized and if a letter can be confused with another mathematical symbol (e.g., l looks like the number one), we will write the variable in cursive. 2) Now that we have assigned a variable for one of the unknowns, what's next? Now, we need to write expressions to show the *relationship* between the unknowns. Often students will try to use different variables for each unknown (e.g., let b = Brad's age and c = Charlie's age. This is OK but it does not show the *relationship* between the variables. 3) Which quantity do we assign a variable to? In other words, which unknown will all of the expressions be written in relationship to? It does not matter. What we need to consider is which expressions will be easier to transform. In the example below, writing the expressions in terms of Brad or Charlie's ages leads to expressions that are "easier" for most students to manipulate. 4) Make sure to define your variable. For example, let b = Brad's age. This may seem redundant to some students as it is likely the case that students would assume b stands for Brad; however, what if you are using a variable such as x and it is not clear what x represents in the problem.

Students may also consider using a table as a tool to help them organize the information. See #5 for an example.

a. Write an expression in simplified form to represent each person's age.
The following expressions are written in relationship to Brad's age b:
Brad's age: b
Charlie's age: $b + 5$
Devon's age: $2(b + 5)$

The following expressions are written in relationship to Chad's age c:
Brad's age: $c - 5$
Charlie's age: c
Devon's age: $2c$

The following expressions are written in relationship to Devon's age d:
Brad's age: $\frac{d}{2} - 5$
Charlie's age: $\frac{d}{2}$
Devon's age: d

b. Write an expression in simplified form to represent the sum of the boys' ages.
If students wrote expressions in terms of Brad and b represents Brad's age:
$b + (b + 5) + 2(b + 5)$

c. If Brad is 7 years old, how old are Charlie and Devon? What is the sum of the boys' ages?
Charlie = 12 years old and Devon is 24 years old. The sum of the boys' ages is 43.

10. The measure of the smallest angle in a triangle is $x°$. The measure of the largest angle in the triangle is twice the measure of the smallest angle. The third angle measures 20 degrees less than the largest angle. Write expressions to represent the measures of the angles in the triangle.

Smallest Angle: ___$x°$________

Largest Angle: ____ $2x°$_______

Third Angle: ____ $(2x - 20)°$_______

11. If the measure of the smallest angle is 40°, find the sum of the angle measures in the triangle.

$40° + 80° + 60° = 180°$

12. Alexa bikes an unknown number of miles. Tom bikes 4 miles more than Alexa. Jesse bikes twice as far as Tom.

a. Write an expression to represent the total number of miles biked by Alexa, Tom, and Jesse.

Scaffold for students:
Alexa Miles + Tom Miles + Jesse Miles
$a + (a + 4) + 2(a + 4)$

b. If Alexa biked 10 miles, how far did Alexa, Tom, and Jesse bike in all?

$10 + 14 + 28 = 52$ miles

13. John takes the average of three numbers. The first number is unknown. The second number is the square of the first number. The third number is four times larger than the second number.

a. Write an expression to represent the average of the three numbers.

$$\frac{a + a^2 + 4a^2}{3}$$

b. If the first number is 6, what is the average of the three numbers?

$$\frac{6 + 36 + 144}{3} = 62$$

14. The length of a rectangular prism is unknown. The width of the prism is twice as long as the length. The height of the prism is three times as long as the width. Write an expression in simplified form for the volume of the rectangular prism in terms of the length.

$(l)(2l)(6l) = 12l^3$

15. Owen reads for an unknown number of minutes each night. His brother, Talen, reads 10 minutes longer. If Talen reads for 30 minutes each night, how many minutes will the two boys read together in the month of April? How many hours will the two boys read in the month of April?
Owen's Minutes in April + Talen's Minutes in April
(Number of Days in April)(Owen's Minutes Each Night) + (Number of Days in April)(Talen's Minutes Each Night)
$(30)(20) + (30)(30)$
$600 + 900$
1,500 minutes
25 hours

Spiral Review

1. I am thinking of a number. When I add 5 to my number, the result is 12. What number am I thinking of?

2. Use mental math to determine the value of *a*.
$a - 2 = 10$.

3. A number is less than 15. Give *three* possible values for the number.

4. Talen is playing a game with his brother. The box says that the game is recommended for children ages 8 and up. Give some possible values for the ages of kids that the game is recommended for.

6.2h Homework: Writing Algebraic Expressions to Model Real World Problems

1. Eight different classes at an elementary school are collecting Box Tops. The eighth-grade class collected b Box Tops. The statements below show the number of Box Tops collected by the other grades. Match each statement to the correct expression.

The 7th grade class collected three times as many Box Tops as the 8th grade.	$3b + 3$
The 6th grade class collected 10 more Box Tops than the 7th grade.	b
The 5th grade class collected 7 fewer Box Tops than the 6th grade.	$3b + 2$
The 4th grade class collected twice as many Box Tops as the 7th grade class	$3b$
The 3rd grade class collected the same number of Box Tops as the 8th grade class	$\frac{3b}{2}$
The 2nd grade class collected 2 more Box Tops than the 7th grade class	$3b + 10$
The 1st grade class collected half as many Box Tops as the 7th grade class	$2(3b)$

2. If the 8th grade class collected 100 Box Tops, how many Box Tops did the school collect in all?

3. A recipe for trail mix uses chocolate chips, raisins, almonds, and peanuts. The ratio of raisins to chocolate chips is 2:1. The ratio of almonds to chocolate chips is 3:1. The ratio of peanuts to chocolate chips is also 3:1. Complete the table by writing expressions to represent the relationship between the ingredients in the trail mix.

Ingredient In Words	**Ingredient In Relationship to Chocolate Chips**
Chocolate Chips	c
Raisins	$2c$
Almonds	$3c$
Peanuts	$3c$

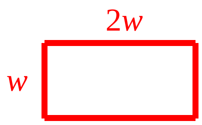
4. Lisa used $1\frac{1}{2}$ cups of chocolate chips in her batch of trail mix. How much of each of the other ingredients should she use if she is following the recipe above? How many total cups of trail mix will she have?
Chocolate Chips = 1.5 cups
Raisins = 3 cups
Almonds = 4.5 cups
Peanuts = 4.5 cups
Total Cups of Trail Mix = 13.5 cups

5. The cost of a shirt is unknown. A pair of jeans costs $20 more than a shirt. Maggie purchases 3 shirts and 2 pairs of jeans.
    a. Write an expression in simplified form to represent the total amount Maggie spent.

    b. If a shirt costs $25, how much did Maggie spend?

6. James is making building a garden to plant vegetables. He wants the length of the garden to be twice the size of the width.

   a. Draw a picture of the garden James is building.



   Students may also choose to write expressions for the length and width in relationship to the length (i.e. if l is the length, the width is $\frac{l}{2}$ or $\frac{1}{2}l$. Students often struggle to begin these problems. They can start by assigning a variable to either the length or the width. Writing everything in terms of the length introduces a fraction which can be difficult for students to manipulate.

   b. Write an expression in simplified form to represent the perimeter of the rectangle.

   $6w$ where w represents the width of the rectangle or $3l$ where l represents the length of the rectangle

   c. Write an expression in simplified form to represent the area of the rectangle.

   $2w^2$ where w represents the width of the rectangle or $\frac{1}{2}l^2$ where l represents the length of the rectangle

   d. James decides he has room to make the length of the garden 15 feet. What is the width of the garden?
   7.5 feet

   e. Based on the information given in part d., what is the perimeter of the garden James is building?
   45 feet

   f. Based on the information given in part d., what is the area of the garden James is building?
   112.5 square feet

7. Peter is purchasing movie tickets online. The cost of a movie ticket is $12.00 plus a service fee of $1.50 per ticket.

   a. Write an expression that represents the cost of movie tickets based on the number of movie tickets Peter purchases.

   b. If Peter purchases 4 movie tickets, how much will he spend?

6.2i Self-Assessment: Section 6.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Identify parts of an algebraic expression using mathematical language.				
2. Apply the properties of operations to generate equivalent expressions, including the simplified form of an algebraic expression.				
3. Simplify numeric expressions containing exponents using the Order of Operations.				
4. Evaluate algebraic expressions for specific values of the variable, including expressions with exponents and expressions that arise from formulas.				
5. Write algebraic expressions to represent real world problems.				

Sample Problems for Section 6.2

Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. Identify the terms, constants, coefficients, and like terms in the following algebraic expression. [1]

Expression	Terms	Constants	Coefficients	Like Terms
$2x + 4y + x + 3y + 9$				

2. Create an algebraic expression that has four terms and meets the following requirements: the expression has no like terms, the expression contains a constant, one of the coefficients is 3. [1]

3. Write three expressions that are equivalent to the expression $4x + 32$. [2]

4. Write three expressions that are equivalent to the expression $x + x + 4(x + 3)$. [2]

5. Write the expression $6x + 42$ as the product of two factors. [2]

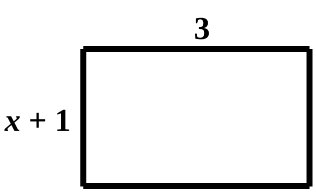
Directions: Simplify the following expressions. [2]

a. $4b + 3b$	b. $p + p + p + p + p + p$
c. $13f - 6f$	d. $9m + 3m + 8$
e. $9h + 8 + h + 2$	f. $5x + x + x + x + 4y$
g. $\frac{1}{2}x + \frac{1}{2}x$	h. $\frac{5}{8}y - \frac{1}{2}y$
i. $1.2x + 0.9x$	j. $a + 6\frac{3}{4}a + b - \frac{2}{3}b$
k. $9y - (2y + 3y)$	l. $9y - 2y + 3y$

m. $2(x+3)$	n. $9(3x+1)$
o. $6(x-7)$	p. $5(2+3x)$
q. $\frac{2}{3}(x-12)$	r. $\frac{1}{10}(50x+85)$
s. $4(x+6)+x$	t. $3(7x-5)-8x$
u. $4+2(x+9)$	v. $6(2x+1)+5(x+4)$

8. What does it mean to **simplify** an algebraic expression? Use examples to support your ideas. [2]

9. Use the figure below to complete the problem. [2]



a. Write an expression in simplified form to represent the perimeter of the rectangle.

b. Write an expression in simplified form to represent the area of the rectangle.

c. If $x = 4$, find the perimeter and area of the rectangle.

10. Complete the table below. [2]

Exponential Form	Expanded Form	Simplified Form
a. 4^3		
b. 5^2		
c. 3^4		
d.	$9 \cdot 9$	
e.	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	
f.	$2 \cdot 3 \cdot 3 \cdot 3$	
g. x^2		
h.	$x \cdot x \cdot x \cdot y \cdot y$	
i.	$2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x$	
j.	$4 \cdot 4 \cdot a \cdot a \cdot a \cdot a$	
k. $3x^3$		
l. $(3x)^3$		

11. Use the work of the three students shown below to answer the questions that follow. [3]

Kayla	Abe	Isaac
$6^2 - 2 \times 5 + 4$	$6^2 - 2 \times 5 + 4$	$6^2 - 2 \times 5 + 4$
$12 - 2 \times 5 + 4$	$36 - 2 \times 5 + 4$	$36 - 2 \times 5 + 4$
$12 - 10 + 4$	$36 - 10 + 4$	$36 - 10 + 4$
$2 + 4$	$36 - 14$	$26 + 4$
6	22	30

a. Which student simplified the expression $6^2 - 2 \times 5 + 4$ correctly?

b. Describe the error(s) made by the students who did not simplify the expression correctly.

12. Simplify each expression. [3]

a. $2 \cdot 4^2$	b. $(2 \cdot 4)^2$
c. $2^3 + 4(5 + 2)$	d. $4^3 \div 2 + 8$
e. $6 \cdot 2^2 + 10 - 8$	f. $(2 + 3)^2 - 10 \cdot 3$

13. Evaluate each expression when $r = 2, s = 8,$ and $t = 24$. [5]

a. $r + s$	b. rs	c. $\frac{1}{3}t$
d. $\frac{t}{s}$	e. $\frac{s}{t}$	f. $s(r + t)$
g. $t - (s + r)$	h. $10 + t \div 2$	i. s^2
j. $3r^3$	k. $(3r)^3$	l. $\frac{r}{t - s}$
m. $20 - 2r^3 + 5$	n. $10^2 + 2(t - r)$	o. $\left(\frac{24}{8}\right)^2$

14. Mrs. Haney is married and has four kids. Use the clues below to determine the ages of the members of the Haney family. Her kids' names are Ethan (oldest), Will (second oldest), Sam (third oldest), and Erik (youngest). [5]

Age of Mrs. Haney's Family Members In Words	**Age of Mrs. Haney's Family Members Algebraic Expression**
Will is y years old.	
Ethan is two years older than Will.	
Sam's age is half of Ethan's age.	
Mrs. Haney's age is eight more than three times Ethan's age.	
Mr. Haney is the same age as Mrs. Haney.	
Erik is three years younger than Sam.	

15. Write an expression in simplified form to represent the sum of the Haney family's ages in terms of Will's age. [5]

16. If Will is 8 years old, what is the sum of the ages of the Haney family?

17. Owen dumps out his piggy bank to see how much money he has. He has twice as many dimes as quarters. He has three times as many pennies as dimes. He has the same number of nickels as quarters. Complete the table by writing expressions to represent the number of each type of coin Owen has. [5]

Types of Coins In Words	**Number of Coins Algebraic Expression**
Quarters	
Dimes	
Nickels	
Pennies	

18. Write expressions for the *value* of the coins Owen has.

Types of Coins In Words	**Value of Coins Algebraic Expression**
Quarters	
Dimes	
Nickels	
Pennies	

19. If Owen has 22 quarters, what is the value of the coins in Owen's piggy bank?

20. Maria downloads an unknown number of apps on her tablet in the month of June. Her sister downloads 3 less apps than her that month. Each app costs $1.99. [5]

    a. Write an expression in simplified form for the total *number* of apps downloaded by Maria and her sister in the month of June.

    b. Write an expression in simplified form for the total cost of the apps downloaded by Maria and her sister.

    c. If Maria downloads 8 apps during the month of June, how much did Maria and her sister spend on apps in June?

Section 6.3: Equations and Inequalitities in One Variable

Section Overview:
In this section, students transition from expressions to equations and inequalities. The first lesson builds on student understanding of what an expression is. Students start by creating tables of values for a given expression. For example, students will create a table of values for the expression $x + 5$ when x = 0, 1, 2, 3, etc. This is followed up with the question, "For what value of x does the expression $x + 5$ evaluate to 7?" When students consider this question, they are informally creating and solving the equation $x + 5 = 7$. Here, an important connection is made between expressions and equations: When we evaluate an expression, we choose the values for x; however, when we solve an equation, we find the value for x in the expression that causes the expression to evaluate to a specific output or value. Students learn that the solution to an equation is the number that makes the equation true when substituted for the unknown. Next, students use substitution to determine whether a given number is the solution of an equation. From here, students numerically investigate the Properties of Equality to understand the "legal" moves when constructing and deconstructing equations and inequalities. Students use the Properties of Equality to construct equations and then learn that the same properties that allow us to build equations are the same properties that allow us to deconstruct equations back to their simplest form which reveals the solution. The connection is made to the problem-solving strategy "working backward". Given an equation, students identify: 1) What was done to the unknown to change it into a different number? 2) How do I "undo" what was done to the unknown? Students build fluency with solving one-step equations. These skills are then applied to solve real world problems. In the second part of the section, students build on their understanding of equations to learn about inequalities. First, they look at real world examples that can be modeled with inequalities. They realize that, unlike the equations they have been solving, inequalities often have infinitely many solutions and that a number line diagram can be an effective tool for representing the solutions to an inequality. Next, they learn how to solve inequalities, again relying on the understanding and skills gained when solving equations. Lastly, they write and solve inequalities to represent real world situations that have constraints. An important part of this process is being able to interpret the solution set in the context.

Note that linear equations can also have no solution or infinitely many solutions (all real numbers satisfy the equation). We do not mention this in the section overview because students will not see these types of equations until 8th grade. In sixth grade, students focus on equations that take the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Understand what the solution to an equation is.
2. Use substitution to determine whether a given number is a solution to an equation.
3. Solve one-step equations, including equations with rational numbers.
4. Write and solve equations to represent real-world problems.
5. Understand that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions and represent the solutions using a number line diagram.
6. Solve one-step inequalities, including inequalities with rational numbers.
7. Write and solve inequalities to represent real-world problems in which constraints are given. Interpret the solution set in the context of the problem.

6.3a Class Activity: Equations and their Solutions

Directions: Make a table of values to evaluate the expression for the values of x given. Then, answer the questions below the table.

1. $x + 5$

x	$x + 5$
0	5
1	6
2	7
5	10
10	15

2. What value for x makes the expression $x + 5$ evaluate to 7? When $x = 2$

3. What value for x makes the expression $x + 5$ evaluate to 15? When $x = 10$

4. $x - 2$

x	$x - 2$
2	0
3	1
4	2
8	6
15	13

5. What value for x makes the expression $x - 2$ evaluate to 0? When $x = 2$

6. What value for x makes the expression $x - 2$ evaluate to 13? When $x = 15$

7. $3x$

x	$3x$
0	0
1	3
2	6
3	9
10	30

8. What value for x makes the expression $3x$ evaluate to 3? When $x = 1$

9. What value for x makes the expression $3x$ evaluate to 9? When $x = 3$

10. $\frac{x}{4}$

x	$\frac{x}{4}$
0	0
4	1
8	2
12	3
16	4

11. What value for x makes the expression $\frac{x}{4}$ evaluate to 2? When $x = 8$

12. What value for x makes the expression $\frac{x}{4}$ evaluate to 4? When $x = 16$

13. You could have created an equation to represent many of the problems on the previous page. For example, in #2, you were asked:

What value for x makes the expression $x + 5$ equal to 7? In other words, when does the expression $x + 5$ equal 7? An **equation** is formed by setting two expressions equal to each other.

"is equal to"

$$\underbrace{x + 5}_{\text{Expression 1}} = \underbrace{7}_{\text{Expression 2}}$$

Expression 1 **Expression 2**

A **solution** to an equation is a number that makes the equation true when substituted for the unknown. Another way to think of this is to ask the question, "For what value of x does the expression $x + 5$ evaluate to 7?"

Directions: Determine whether the given number is the solution to the equation given. Justify your answer.

14. $x + 8 = 15$ Does $x = 7$? Yes	15. $x - 2 = 9$ Does $x = 7$? No	16. $3x = 24$ Does $x = 8$? Yes
17. $8x = 4$ Does $x = \frac{1}{2}$? Yes	18. $\frac{x}{2} = 12$ Does $x = 6$? No	19. $25 - x = 19$ Does $x = 6$? Yes
20. $\frac{42}{x} = 6$ Does $x = 7$? Yes	21. $x + \frac{2}{3} = 2\frac{1}{6}$ Does $x = 1\frac{1}{2}$? Yes	22. $50x = 5$ Does $x = 10$? No
23. $12 = x + 8$ Does $x = 4$? Yes	24. $35 = 5x$ Does $x = 30$? No	25. $1 = x - 9$ Does $x = 10$? Yes

Note that in #23 – 25, the variable is on the right side of the equation. Students will create and solve equations where the variable is on the right side of the equation. This can sometimes be challenging for students.

Directions: Solve the following equations using mental math. Justify your answer.

26. $x + 10 = 14$ $x = 4$	27. $x - 5 = 3$ $x = 8$	28. $4x = 20$ $x = 5$
29. $\frac{x}{7} = 3$ $x = 21$	30. $\frac{40}{x} = 5$ $x = 8$	31. $7 + x = 15$ $x = 8$
32. $9 - x = 8$ $x = 1$	33. $49 = 7x$ $7 = x$	34. $3 = \frac{x}{5}$ $15 = x$
35. $x + \frac{1}{2} = \frac{5}{2}$ $x = 2$	36. $0.1x = 10$ $x = 100$	37. $\frac{1}{4}x = 6$ $x = 24$

Spiral Review

1. Simplify the expression. If the expression is already simplified, write "already simplified".

a. $m + m + m + m$ $4m$	b. $4x + 5x + x$ $10x$
c. $5x + 5y$ Already simplified	d. $4x + 8 + x - 3$ $5x + 5$

2. Evaluate each expression for $x = 5$.

a. $3x$ 15	b. $3 + x$ 8
c. x^3 125	d. $2x^3$ 250

3. I am thinking of a number. When I divide the number by 3, I get 15. What is the number?
   The number is 45.
4. After Mario paid $2.75 for school lunch, he had $1.25 left. How much money did Mario have before he paid for school lunch. $4

6.3a Homework: Equations and Their Solutions

Directions: Make a table of values to evaluate the expression for the values of x given. Then, answer the questions below the table.

1. $x + 2$

x	$x + 2$
0	2
1	3
2	4
3	5
5	7

2. What value for x makes the expression $x + 2$ evaluate to 5? When $x = 3$

3. What value for x makes the expression $x + 2$ evaluate to 7? When $x = 5$

4. $x - 7$

x	$x - 7$
7	
8	
9	
10	
11	

5. What value for x makes the expression $x - 7$ evaluate to 1?

6. What value for x makes the expression $x - 7$ evaluate to 4?

7. $5x$

x	$5x$
0	
1	
2	
3	
10	

8. What value for x makes the expression $5x$ evaluate to 10?

9. What value for x makes the expression $5x$ evaluate to 50?

10. $\frac{x}{10}$

x	$\frac{x}{10}$
10	
20	
30	
50	
100	

11. What value for x makes the expression $\frac{x}{10}$ evaluate to 2?

12. What value for x makes the expression $\frac{x}{10}$ evaluate to 5?

Directions: Write an equation to represent each problem. Then, solve the equation.

13. For what value of x does the expression $x + 6$ evaluate to 8?
    $x + 6 = 8; x = 2$

14. For what value of x does the expression $x - 9$ evaluate to 3?

15. For what value of x does the expression $10x$ evaluate to 70?

16. For what value of x does the expression $\frac{x}{4}$ evaluate to 40?

Directions: Determine whether the given number is the solution to the equation given. Justify your answer.

17. $x + 10 = 45$ Does $x = 55$? No	18. $x - 6 = 12$ Does $x = 6$?	19. $10 = x + 4$ Does $x = 6$? Yes
20. $9x = 45$ Does $x = 9$?	21. $4x = 44$ Does $x = 11$?	22. $\frac{x}{8} = 9$ Does $x = 72$?
23. $x + 0.03 = 0.6$ Does $x = 0.3$?	24. $40 = \frac{x}{8}$ Does $x = 5$?	25. $\frac{3}{5}x = 36$ Does $x = 60$? Yes
26. $3 = x - 7$ Does $x = 4$?	27. $x + \frac{2}{3} = 1$ Does $x = \frac{1}{3}$?	28. $\frac{1}{3} = \frac{2}{3}x$ Does $x = \frac{1}{2}$?
29. $\frac{x}{0.1} = 30$ Does $x = 300$?	30. $16 = x \div \frac{1}{2}$ Does $x = 8$? Yes	31. $32x = 8$ Does $x = \frac{1}{4}$?

Directions: Solve the following equations using mental math. Justify your answer.

32. $x + 2 = 10$	33. $x - 8 = 7$	34. $8x = 32$ $x = 4$
35. $\frac{x}{10} = 40$	36. $0.5x = 3$	37. $\frac{x}{12} = 5$ $x = 60$
38. $4 + x = 9$	39. $\frac{x}{12} = 8$	40. $x - 7 = 8$
41. $x - 15 = 15$	42. $11x = 33$	43. $\frac{x}{3} = \frac{1}{3}$ $x = 1$
44. $x + \frac{1}{2} = \frac{9}{2}$	45. $0.1x = 10$ $x = 100$	46. $\frac{1}{4}x = 6$

6.3b Class Activity: Working Backwards to Solve Equations

Allow students to solve these equations in any way they wish. The idea is to get them thinking about working backward. One idea is to have students stand at the front of the room in a line and hold up index cards to show what has happened. Then, you can talk about how to undo each step to get back to the starting value. If students are ready, you can represent the problem and solving process symbolically by writing and solving an equation.

1. When Roberto left his house in the morning to walk to school, his mom gave him some money for lunch. On his walk to school, he found a $5 bill on the sidewalk. When he got to school, he counted his money and found he had $8. How much money did his mom give him when he left in the morning?
   $m + 5 = 8$
   His mom gave him $3 before he left for school.

2. Sam opened a bag of jellybeans and ate three of them. She had 12 jellybeans left in the bag. How many jellybeans were in the bag to start?
   $j - 3 = 12$
   There were 15 jelly beans in the bag to start.

3. Ted, Shannon, and Emilio raked leaves one Saturday for Ted's father. Ted's father gave the three kids some money to share evenly. When Shannon got home, she counted her money and she had $15. How much money did Ted's father pay the kids in total to rake the leaves?
   $m \div 3 = 15$
   Ted's father paid the kids $45 in total.

4. Sandia went up to the movie ticket counter and asked for 4 tickets to a movie. The cashier told her the total would be $32. How much is each movie ticket?
   $4x = 32$
   Each ticket is $8.

Directions: Solve the following number riddles.

Again, students do not need to write and solve an equation to do these problems; however, if they are ready for this step, it would be good to show their solving process symbolically through an equation.

5. I am thinking of a number. When I subtract 3 from my number, the result is 15. What number am I thinking of?
   $n - 3 = 15$
   The number I am thinking of is 18.

6. I am thinking of a number. When I add 6 to my number, the result is 13. What number am I thinking of?
   $n + 6 = 13$
   The number I am thinking of is 7.

7. I am thinking of a number. When I multiply my number by 4, the result is 24. What number am I thinking of?
   $4n = 24$
   The number I am thinking of is 6.

8. I am thinking of a number. When I divide my number by 5, the result is 8. What number am I thinking of?
   $\frac{n}{5} = 8$
   The number I am thinking of is 40.

9. How do you use the problem-solving strategy "work backward" to figure out the number riddles above?
   You "undo" what happened or do the inverse operation to the result to find the original number.

Directions: Write a number riddle to match each equation. Then, explain how to solve the number riddle.

10. $n + 5 = 9$
    I am thinking of a number. When I add 5 to the number, the result is 9. What is the number?
    To find the number, subtract 5 from 9. The number is 4.

11. $x - 1 = 8$
    I am thinking of a number. When I subtract 1 from the number, the result is 8. What is the number?
    To find the number, add 1 to 8. The number is 9.

12. $7n = 42$
    I am thinking of a number. When I multiply the number by 7, the result is 42. What is the number?
    To find the number, divide 42 by 7. The number is 6.

13. $\frac{x}{3} = 7$
    I am thinking of a number. When I divide the number by 3, I get 7. What is the number?
    To find the number multiply 7 by 3. The number is 21.

Directions: Write an equation to represent each number riddle. Describe what you need to do to find the unknown number and then find the unknown number.

14. When 5 is added to a number, the result is 7. What is the number?
$n + 5 = 7$

To find the unknown number, you need to subtract 5 from 7. The unknown number is 2.

15. When 8 is subtracted from a number, the result is 3. What is the number?
$n - 8 = 3$

To find the unknown number, you need to add eight to 3. The unknown number is 11.

16. When a number is multiplied by 6, the result is 48. What is the number?
$6n = 48$

To find the unknown number, you need to divide 48 by 6. The unknown number is 8.

17. When a number is divided by 10, the result is 5. What is the number?
$\frac{n}{10} = 5$

To find the unknown number, you need to multiply 5 by 10. The unknown number is 50.

Spiral Review

1. Simplify the expression. If the expression is already simplified, write "already simplified".

a. $13y - 5y + 2y$	b. $10g + 3$
c. $3 + 4x$	d. $3(4x)$

2. Show some different ways the expression $30x - 18$ can be written as the product of two factors.

3. You must be at least 54 inches to ride a roller coaster at an amusement park. Al is exactly 54 inches tall. Can he ride the roller coast?

4. To safely enter an underground parking lot, a car must be less than 8 feet tall. Christina's Honda Pilot is exactly 8 feet tall. Can she safely enter the parking garage in her car?

6.3c Class Activity: Constructing and Deconstructing Equations

Activity 1: Transforming an Equation – What are the "Legal" Moves?

a. Consider the equation $12 = 12$. Is this a true statement? Yes
b. Apply each operation to the equation $12 = 12$ and tell whether the resulting equation is still true. The first one has been done for you.

Starting Equation	Operation to Apply	Resulting Equation	Is the resulting equation true or false?
$12 = 12$	Add 4 to both sides.	$12 + 4 = 12 + 4$ $16 = 16$	Yes
$12 = 12$	Add 8 to both sides.	$12 + 8 = 12 + 8$ $20 = 20$	Yes
$12 = 12$	Add 4 to the left side of the equation only.	$12 + 4 = 12$ $16 = 12$	No
$12 = 12$	Multiply both sides of the equation by 3.	$3(12) = 3(12)$ $36 = 36$	Yes
$12 = 12$	Divide both sides of the equation by 3.	$\frac{12}{3} = \frac{12}{3}$ $4 = 4$	Yes
$12 = 12$	Multiply the left side of the equation by 3 and add 3 to the right side of the equation.	$3(12) = 12 + 3$ $36 = 15$	No
$12 = 12$	Multiply both sides of the equation by $\frac{1}{2}$.	$\frac{1}{2}(12) = \frac{1}{2}(12)$ $6 = 6$	Yes
$12 = 12$	Multiply the left side of the equation by $\frac{1}{2}$ and multiply the right side of the equation by 2.	$\frac{1}{2}(12) = 2(12)$ $6 = 24$	No

c. Look back through the chart. What are "legal" moves to apply to an equation to transform it into an equivalent equation?

Let students describe the legal moves in their own words and then formalize the ideas by introducing academic vocabulary: Addition Property of Equality - You can add the same number to both sides of an equation, Subtraction Property of Equality – You can subtract the same number from both sides of an equation, Multiplication Property of Equality – You can multiply both sides of an equation by the same number, and Division Property of Equality – You can divide both sides of an equation by the same number.

Activity 2: Constructing Equations

Directions: Start with the equation given. Apply the action to construct an equivalent equation.

Original Equation	Action	Transformed Equation
a. $x = 5$	Add 3 to both sides of the equation.	$x + 3 = 8$
b. $x = 10$	Subtract 8 from both sides of the equation.	$x - 8 = 2$
c. $x = 4$	Multiply both sides of the equation by 5.	$5x = 20$
d. $x = 12$	Divide both sides of the equation by 4.	$\frac{x}{4} = 3$

As students transform the equations above, ask whether the value of *x* changes. For example, in part a., when students add 3 to both sides of the equation, *x* is still equal to five.

Activity 3: Deconstructing Equations

Directions: The equations below are the equations you built in the previous activity. State the action you need to perform to deconstruct the equation. In other words, how can you get the equation back to what is was at the start of Activity 2 to determine the value of *x*?

Transformed Equation	Action	Original Equation
a. $x + 3 = 8$	Subtract 3 from both sides of the equation.	$x = 5$
b. $x - 8 = 2$	Add 8 from both sides of the equation.	$x = 10$
c. $5x = 20$	Divide both sides of the equation by 5.	$x = 4$
d. $\frac{x}{4} = 3$	Multiply both sides of the equation by 4.	$x = 12$

In Activities 2 and 3, students learn that the same properties that allow us to build equations are the same properties that allow us to deconstruct equations to determine the value of the unknown.

Activity 4: An equation is given. Select all the equations that are equivalent to the equation given. Justify your answer on the line provided.

a. Given: $x = 10$

☐ $2x = 12$ ________No; The left side was multiplied by 2 and the right sides was increased by 2. ____

☐ $x + 4 = 14$ _______Yes; 4 was added to both sides of the original equation (Addition Property of Equality)

☐ $\frac{x}{2} = 20$ _______No; the left side was divided by 2 while the right side was multiplied by 2______

☐ $3x = 30$ ___Yes; Both sides of the equation were multiplied by 3 (Multiplication Property of Equality

☐ $x - 6 = 4$ ____Yes____

Bonus: ☐ $x + x = 20$ _____Yes; Since $x = 10$ (given in the problem), 10 was added to both sides of the equation (Addition Property of Equality)

Spiral Review

1. Simplify the expression. If the expression is already simplified, write "already simplified".

a. $4(x - 5)$	b. $2(5x + 10) + 6x$
c. $\frac{3}{5}(25x + 10)$	d. $0.1(x - 90)$

2. Write three equivalent expressions for the expression $10x$.

3. What number is 20% of 60.

4. 60 is 20% of what number?

6.3c Homework: Constructing and Deconstructing Equations

Directions: Determine whether the following actions are "legal" moves when transforming an equation. Write "Y" for yes or "N" for no. For the moves that are "legal" moves, name the property of equality. The first two have been done for you.

Action	Is it a Legal Move When Transforming an Equation?	If Yes, Name the Property of Equality
1. Add 5 to both sides of the equation.	Yes	Addition Property of Equality
2. Add 5 to the left side of the equation and subtract 5 from the left side of the equation.	No	
3. Multiply one side of the equation by 7.		
4. Subtract 8 from both sides of the equation.		
5. Multiply one side of the equation by 10 and divide the other side of the equation by 10.		
6. Divide both sides of the equation by 4.		
7. **Bonus:** Divide the right side of the equation by $\frac{1}{3}$. Multiply the left side of the equation by 3.		
8. **Bonus:** Divide one side of the equation by 0.01 and multiply the other side of the equation by 100.		
9. **Bonus:** Multiply one side of the equation by 10. Multiply the other side of the equation by 5 and then by 2.		

Directions: In each problem below, a starting equation is given. In the middle column, the equation has been transformed into an equivalent equation. State the action that produced the equivalent equation. The first one has been done for you.

Original Equation	Transformed Equation	Action
10. $x = 6$	$x + 2 = 8$	Add 2 to both sides of the equation.
11. $x = 12$	$\frac{x}{3} = 4$	Divide both sides of the equation by 3 (or multiply both sides by $\frac{1}{3}$).
12. $x = 9$	$6x = 54$	
13. $x = 14$	$x - 3 = 11$	
14. $x = 7$	$x + 5 = 12$	
15. $5 = x$	$0 = x - 5$	
16. $48 = x$	$8 = \frac{x}{6}$	

Directions: The equations below are the equations you built in the table above. State the action you need to perform to deconstruct the equation back to the original equation. In other words, how can you get the equation back to what it was at the start? The first one has been done for you.

Transformed Equation	Action	Original Equation
17. $x + 2 = 8$	Subtract 2 from both sides of the equation.	$x = 6$
18. $\frac{x}{3} = 4$	Multiply both sides of the equation by 3.	$x = 12$
19. $6x = 54$		
20. $x - 3 = 11$		
21. $x + 5 = 12$		
22. $0 = x - 5$		
23. $8 = \frac{x}{6}$		

Directions: An equation is given. Select all the equations that are equivalent to the equation given. Justify your answer on the line provided.

24. Given: $x = 8$

☐ $x + 5 = 13$ _____Yes, Addition Property of Equality__________________________

☐ $3x = 11$ _____No____

☐ $\frac{x}{2} = 2$ ____

☐ $x - 8 = 0$ ____

☐ $4x = 2$ ____

25. Given: $x = 24$

☐ $x - 5 = 29$ ____

☐ $2x = 48$ ____

☐ $\frac{x}{3} = 8$ ____

☐ $x - 4 = 24$ ____

☐ $10 + x = 14$ ____

6.3d Class Activity: Solving Equations with Whole Numbers

Activity 1: Solving Equations by Working Backward

Directions: In each of the problems below, an operation has been performed to an unknown number to change it into a different number. Identify "what was done to the unknown number". Then, tell how you can "undo what was done". Show your solving actions in the last column. Verify the solution. The first one has been done for you.

There are many details that students need to be aware of when solving the equations that follow:

1) Many students will argue that they do not need to complete the chart because they can see what the answer is. Let them know that not only are we trying to solve for the unknown, we are also learning a *process*. Understanding this process and developing fluency with the process will help students to tackle problems with more difficult numbers (such as larger numbers, fractions, and decimals). It will also help students when the equations become more challenging in later grades.
2) In example a., teachers often skip the step of showing that 6 – 6 simplifies to 0 and just cross out the 6 – 6. It is encouraged that students show this interim expression of $x + 0$ initially. This should be followed by a discussion of the Identity Property of Addition. A similar discussion should take place in example h. Make explicit that $\frac{7}{7}$ simplifies to 1. When these steps are not made explicit, students cross out numbers haphazardly when they should not be crossed out and make unnecessary mistakes.
3) For problems like e., encourage students to attend to precision. A common error in a problem like this is for students to add 4 to both sides of the equation. Show students that when you add 4 to both sides of the equation, the resulting equation is $8 + y = 14$. This mistake is even more common in later grades when students see equations such as $4 - y = 10$. It may help students to re-write the left side of the equation using the Commutative Property of Addition.
4) For problems like f., students can become confused because the unknown is on the right side of the equation. The goal is to isolate the unknown, but it can be isolated on either side of the equation. Encourage students to *always start at the unknown* to determine how it was changed into another number. If students are confused when the variable is on the right side of the equation, they can transform the equation by picking up the right side of the equation and moving it to the left side and picking up the left side of the equation and moving it to the right side. You can show this on the whiteboard by writing the expressions on index cards, attaching them to the board with magnets, and then sliding each expression to the other side of the equation.
5) There are a few problems toward the end that require students to simplify prior to solving. This is not a requirement of 6th grade but a natural progression from Sections 1 and 2 of this chapter.

Equation	What was done to the unknown number?	How do you undo what was done?	Solving Actions
a. $x + 6 = 13$	Six was added to the unknown number	Subtract six from both sides of the equation	$x + 6 = 13$ $-6 = -6$ $x + 0 = 7$ $x = 7$
b. $x + 4 = 9$	Four was added to the unknown number	Subtract four from both sides of the equation.	$x + 4 = 9$ $-4 = -4$ $x + 0 = 5$ $x = 5$

c. $y - 5 = 18$	Five was subtracted from the unknown number	Add five to both sides of the equation.	$y - 5 = 18$ $+5 = +5$ $y + 0 = 23$ $y = 23$
d. $n - 8 = 3$	Eight was subtracted from the unknown number	Add eight to both sides of the equation.	$n - 8 = 3$ $+8 = +8$ $n + 0 = 11$ $n = 11$
e. $4 + y = 10$	Four was added to the unknown number	Subtract four from both sides of the equation.	$4 + y = 10$ $-4 \quad = -4$ $y = 6$
f. $6 = t - 8$	Eight was subtracted from the unknown number	Add eight to both sides of the equation.	$6 = t - 8$ $+8 \quad + 8$ $14 = t$
g. $11 = c + 5$	Five was added to the unknown.	Subtract five from both sides of the equation.	$11 = c + 5$ $-5 \quad - 5$ $6 = c$
h. $7x = 14$	The unknown was multiplied by 7.	Divide both sides of the equation by 7.	$\frac{7x}{7} = \frac{14}{7}$ $1x = 2$ $x = 2$
i. $8x = 48$	The unknown was multiplied by 8.	Divide both sides of the equation by 8.	$\frac{8x}{8} = \frac{48}{8}$ $1x = 6$ $x = 6$
j. $\frac{x}{5} = 4$	The unknown was divided by 5.	Multiply both sides of the equation by 5.	$5 \cdot \frac{x}{5} = 4 \cdot 5$ $1x = 20$ $x = 20$
k. $\frac{c}{6} = 3$			$c = 18$

l. $15 = 3x$			$x = 5$
m. $\frac{x}{11} = 6$			$x = 66$
n. $g - 12 = 39$			$g = 51$
o. $4g = 64$			$g = 16$
p. $\frac{x}{14} = 8$			$x = 112$
q. $7 = \frac{b}{8}$			$b = 56$
r. $x - 372 = 57$			$x = 429$
s. $18g = 270$			$g = 15$
t. $\frac{n}{54} = 15$			$n = 810$
u. $n + 840 = 1{,}574$			$n = 734$
v. $575 = 225 + x$			$x = 350$
w. $800 = 25x$			$x = 32$
x. $270 = x - 30$			$x = 300$

y. $181 + x = 190$			$x = 9$
z. $729 = \frac{f}{3}$			$f = 2{,}187$

Directions: Solve the following equations. Show the solving actions. Verify the solution. Students should verify the solution by substituting the value back in to the original equation and verifying that the resulting equation is a true statement.

1. $n + 8 = 28$ $-8 \quad -8$ $n = 20$	2. $x - 4 = 9$ $+4 \quad +4$ $x = 13$	3. $b + 6 = 15$ $b = 9$
4. $9x = 54$ $\frac{9x}{9} = \frac{54}{9}$ $x = 6$	5. $\frac{p}{20} = 4$ $20 \cdot \frac{p}{20} = 4 \cdot 20$ $p = 80$	6. $n - 10 = 21$ $n = 31$
7. $n + 10 = 21$ $n = 11$	8. $7y = 105$ $y = 15$	9. $\frac{g}{16} = 16$ $g = 256$
10. $5x = 5$ $x = 1$	11. $\frac{x}{5} = 1$ $x = 5$	12. $\frac{x}{5} = 5$ $x = 25$
13. $12x = 144$ $x = 12$	14. $x - 391 = 53$ $x = 444$	15. $\frac{g}{56} = 5$ $g = 280$
16. $4x = 128$ $x = 32$	17. $\frac{b}{8} = 307$ $b = 2{,}456$	18. $\frac{m}{11} = 16$ $m = 176$
19. $x + 402 = 9{,}140$ $x = 8{,}738$	20. $\frac{x}{20} = 50$ $x = 1{,}000$	21. $100x = 20{,}000$ $x = 200$

22. $45 = x - 90$ $x = 135$	23. $5 + t = 18$ $t = 13$	24. $16 = \frac{x}{2}$ $x = 32$
25. $17 = 6 + x$ $x = 11$	26. $48 = 6x$ $x = 8$	27. $12 = x - 36$ $x = 48$
28. $x + x + x = 15$ $3x = 15$ $x = 5$	29. $7x - x = 42$ $6x = 42$ $x = 7$	30. $5x + 3x = 64$ $8x = 64$ $x = 8$

Spiral Review

1. Which property is being illustrated in the equation below?
   $a + 5 = 5 + a$

2. Give *two* different values for *x* that make the following inequality true.
   $x < 10$

3. Find the sum. $\frac{3}{8} + \frac{7}{24}$

4. Find the quotient. $\frac{3}{8} \div \frac{7}{24}$

6.3d Homework: Solving Equations with Whole Numbers

Directions: Solve the following equations. Show the solving actions. Verify the solution.

1. $x + 2 = 5$	2. $y - 8 = 9$	3. $r - 3 = 11$
4. $8x = 40$	5. $x - 3 = 16$	6. $\frac{t}{5} = 2$
7. $q - 11 = 30$	8. $5y = 60$ $y = 12$	9. $\frac{x}{8} = 4$
10. $x + 45 = 60$	11. $9x = 108$	12. $\frac{t}{15} = 5$ $t = 75$
13. $4 + x = 10$	14. $108 = 6x$	15. $54 = \frac{t}{9}$
16. $12 = x + 3$	17. $120 = 40 + x$	18. $\frac{y}{14} = 14$
19. $15x = 300$	20. $600 + x = 1{,}932$ $x = 1{,}332$	21. $18 = \frac{d}{22}$
22. $6x = 1{,}032$	23. $8{,}054 = t - 728$ $t = 8{,}782$	24. $4{,}000 = 100x$
25. $4x + 2x = 48$	26. $x + x + x + x = 120$	27. $5x - 2x = 42$

6.3e Class Activity: Solving Equations with Rational Numbers

Activity 1: Solving Equations by Working Backward

Directions: In each of the problems below, an operation has been performed to an unknown number to change it into a different number. Identify "what was done to the unknown number". Then, tell how you can "undo what was done". Show your solving actions in the last column.

In this lesson, students are presented with equations that contain rational numbers or lead to rational number solutions. A strategy for tackling more difficult problems such as these is to refer to easier problems in the previous section. The process for solving these problems is still the same. This lesson also gives students the opportunity to review operations with rational numbers.

Equation	What was done to the unknown number?	How do you undo what was done?	Solving Actions
a. $n+\frac{2}{3}=\frac{7}{3}$	$+\frac{2}{3}$	$-\frac{2}{3}$	$n+\frac{2}{3}=\frac{7}{3}$ $-\frac{2}{3}=-\frac{2}{3}$ $n=\frac{5}{3}$
b. $b+1\frac{1}{2}=6$			$b=4\frac{1}{2}$
c. $x-\frac{3}{5}=7\frac{2}{5}$			$x=8$
d. $\frac{7}{8}=x+\frac{1}{2}$			$x=\frac{3}{8}$
e. $n-\frac{4}{9}=\frac{1}{4}$			$n=\frac{25}{36}$
f. $x+0.24=8.78$			$x=8.54$

g. $n - 1.2 = 8$			$n = 9.2$
h. $5.6 = x - 1.02$			$n = 6.62$
i. $0.2x = 4.8$			$x = 24$
j. $\frac{x}{0.2} = 4.8$			$x = 0.96$
k. $\frac{2}{3}x = 18$			$x = 27$
l. $6x = 12$			$x = 2$
m. $12x = 6$			$x = \frac{1}{2}$
n. $32 = \frac{4}{3}g$			$g = 24$
o. $6.4 = \frac{x}{4}$			$x = 25.6$

p. $4.5x = 90$			$x = 20$

Directions: Solve the following equations. Show the solving actions. Verify the solution.

1. $x + 1\frac{1}{4} = 3\frac{7}{8}$ $x = 2\frac{5}{8}$	2. $x - 8 = 4.03$ $x = 12.03$	3. $n + \frac{5}{12} = \frac{19}{24}$ $n = \frac{3}{8}$
4. $x + \frac{9}{10} = 5\frac{1}{10}$ $x = 4\frac{1}{5}$	5. $0.04t = 12$ $t = 300$	6. $5x = 1$ $x = \frac{1}{5}$
7. $\frac{7}{10}x = 42$ $x = 60$	8. $0.05x = 2$ $x = 40$	9. $\frac{5}{6}c = \frac{3}{5}$ $c = \frac{18}{25}$
10. $\frac{y}{0.03} = 320$ $y = 9.6$	11. $3x = 9$ $x = 3$	12. $9x = 3$ $x = \frac{1}{3}$
13. $n + \frac{7}{10} = 0.75$ $n = 0.05$	14. $n - \frac{2}{3} = 3\frac{4}{9}$ $x = 4\frac{1}{9}$	15. $x + 1.6 = 19.4$ $x = 17.8$
16. $n - \frac{5}{16} = \frac{7}{16}$ $n = \frac{3}{4}$	17. $x + 3\frac{1}{2} = 4\frac{2}{3}$ $n = 1\frac{1}{6}$	18. $y - 0.25 = 2.75$ $y = 3$
19. $x - 13 = 2.9$ $x = 15.9$	20. $y + 2.25 = 10$ $x = 7.75$	21. $\frac{y}{1.4} = 10$ $x = 14$
22. $1.2x = 9.6$ $x = 8$	23. $\frac{4}{9}y = 4$ $y = 9$	24. $0.01t = 5$ $t = 500$
25. $12.6 = x + 4.8$ $x = 7.8$	26. $144 = 1.2f$ $f = 120$	27. $20.5 = \frac{x}{4}$ $x = 82$

28. $8x = 100$ $x = 12.5$	29. $6x = 8$ $x = 1\frac{1}{3}$	30. $0.25x = 0.4$ $x = 1.6$
31. $0.1x + 0.4x = 7$ $x = 14$	32. $x + 4x = 4$ $x = \frac{4}{5}$	33. $\frac{1}{2}x + \frac{3}{8}x = 21$ $x = 24$

Spiral Review

1. Compare using $<, >$, or $=$.

   -3.1 _____ -3

   -3 _____ -3.1

2. Ray and his dad are fishing. The minimum length a catfish needs to be to keep it is 18 inches. They catch a fish that is 18.5 inches. Can they keep the fish or do they need to throw it back?

3. Find the area of a parallelogram with a base of 5 inches and a height of 8 inches.

4. Find the area of a triangle with a base of 5 inches and a height of 8 inches.

6.3e Homework: Solving Equations with Rational Numbers

Directions: Solve the following equations. Show the solving actions. Verify the solution.

1. $x + \frac{1}{8} = \frac{3}{8}$ $x = \frac{1}{4}$	2. $n - \frac{1}{3} = \frac{4}{9}$ $n = \frac{7}{9}$	3. $y + 1\frac{4}{5} = 5$
4. $y + 0.75 = 2.5$ $x = 3.25$	5. $0.5t = 10$ $t = 20$	6. $5x = 1.5$ $x = 0.3$
7. $18x = 9$ $x = \frac{1}{2}$	8. $\frac{x}{0.01} = 5{,}000$ $x = 50$	9. $\frac{3}{5}b = \frac{3}{5}$
10. $\frac{3}{4}a = 9$ $a = 12$	11. $\frac{5}{12} + x = \frac{35}{36}$	12. $7 = x - 8.2$
13. $20 = 0.2h$	14. $0.9n = 5.4$	15. $20 = \frac{5}{4}x$
16. $n - \frac{3}{4} = 2\frac{1}{2}$	17. $1.5 = \frac{y}{30}$	18. $1.2x = 60$
19. $21 = 5x$ $x = 4.2$	20. $0.2x = 0.15$	21. $\frac{y}{4} = 3.05$
22. $x - 2\frac{2}{3} = 3\frac{1}{4}$	23. $\frac{1}{4}y = 5$ $y = 20$	24. $4.5 = 1\frac{1}{4} + x$
25. $\frac{2}{3} + y = \frac{8}{9}$	26. $\frac{5}{4}x = 100$	27. $60 = \frac{y}{0.5}$
28. $12x = 100$ $x = 8\frac{1}{3}$	29. $2x = 15$	30. $x + 2.8 = 3\frac{3}{5}$
31. $\frac{5}{8}y - \frac{1}{2}y = 5$	32. $x + \frac{1}{2}x = 18$	33. $2x - 0.8x = 18$

6.3f Class Activity: Writing Equations to Solve Real World Problems

Directions: Write an equation to represent each problem. Then, solve the equation. Make sure to label your answer.

1. Clarissa is trying to save \$399 to buy a new gaming station. She has already saved \$140. Write and solve an equation to represent the amount Clarissa still needs to save.

It is encouraged that students use models to solve these problems. The models do not need to be drawn to scale but can help students who are struggling to come up with the equation. Sample models are shown in the first few problems.

Students need to attend to precision and define what the unknown in the equation stands for in the problem.

Let s = the amount Clarissa still needs to save:

Model for students how you can talk through what the model shows, "When I add what Clarissa has already saved to what she still needs to save, I will get the total amount she needs to save which is \$399."

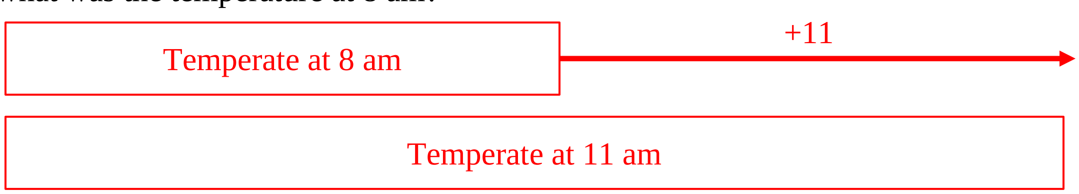
$140 + s = 399$ or

"If I subtract what Clarissa has already saved from the total amount she needs to save, I will be able to determine the amount she still needs to save."

$s = 399 - 140$ (encourage students to consider both equations)

$s = \$259$

2. From 8 am to 11 am this morning, the temperature rose 11°F. If the temperature at 11 am was 37°F, what was the temperature at 8 am?



Let t = the temperature at 8 am

$t + 11 = 37$ or

$t = 37 - 11$

$t = 26$ °F

3. Bianca needs $2\frac{1}{2}$ cups of flour to make a batch of cookies. If Bianca has put $1\frac{1}{4}$ cups of flour in the mixing bowl so far, how many more cups of flour does she still need to add to the mixing bowl?

$2\frac{1}{2}$	
$1\frac{1}{4}$	Amount Bianca still needs to add

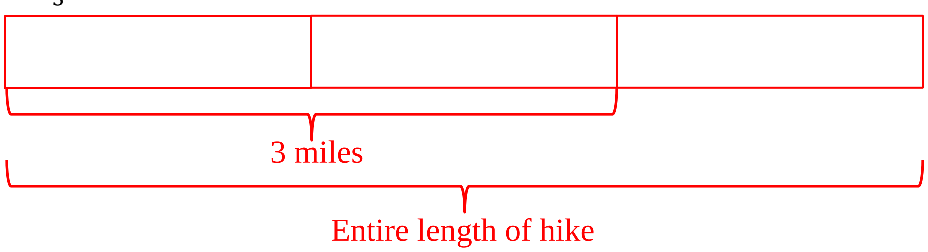
Let f = flour that still needs to be added

$f + 1\frac{1}{4} = 2\frac{1}{2}$ or

$f = 2\frac{1}{2} - 1\frac{1}{4}$

$f = 1\frac{1}{4}$ cups of flour still need to be added

4. Marco and his friends are on a hike. After they have hiked 3 miles, they reach a sign telling them they are $\frac{2}{3}$ of the way finished with the hike. How long is the hike?



Let h = length of the entire hike

$\frac{2}{3}h = 3$ or

$h = 3 \div \frac{2}{3}$ or

$h = 3 \cdot \frac{3}{2}$

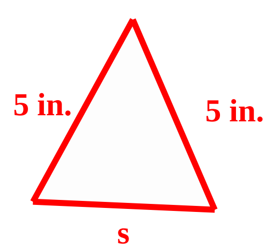
$h = 4\frac{1}{2}$ miles

Tie back to work done in Chapter 2 with division of fractions.

5. Two sides of an isosceles triangle measure 5 in. and 5 in. If the perimeter of the triangle is $13\frac{1}{2}$ in., what is the measure of the third side of the triangle?

Encourage students to draw a picture.

Let s = length of the third side



$s + 5 + 5 = 13.5$ or

$s = 13.5 - 5 - 5$

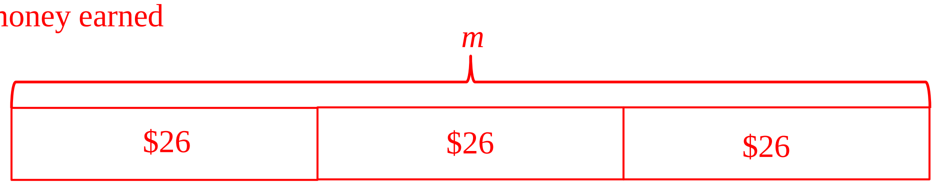
$s = 3.5$ inches

6. Maggie is buying movie tickets that cost \$9.50 each. If her total is \$47.50 before tax, how many movie tickets did she purchase.
Let t = number of tickets

$9.50t = 47.50$

$t = 5$ tickets

7. Eddie, Jax, and Amanda are raking leaves in their neighborhood. They split their earnings evenly. If each person received \$26, how much money total did the three kids earn raking leaves?
Let m = amount of money earned

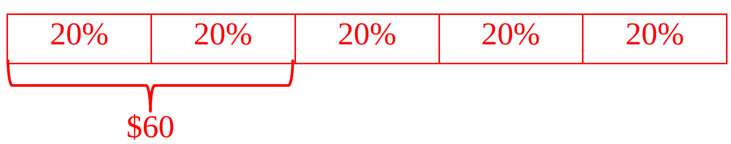


$\frac{m}{3} = 26$

Or

$m = 26(3)$
$m = \$78$

8. Trevor received money from friends and family for graduation. He put 40% of the money that was given to him in his savings account. If he put \$60 in his savings account, how much money was given to Trevor at graduation?



Talk through this model, "Trevor put 40% of the total amount given to him in savings. This is equal to \$60."
40% of the amount Trevor earned = \$60

Let m = amount of money Trevor received

$0.4m = 60$

$m = \$150$

9. A car is traveling 60 miles per hour. How long will it take the car to drive 210 miles?
Let t = time in hours

$60t = 210$
$t = 3.5$ hours

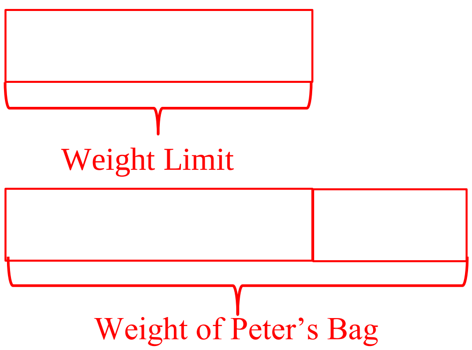
10. Hillary bought 3 shirts that each cost the same amount. If her total before tax is \$59.97, how much was each shirt.
Let c = cost of each shirt

$3c = \$59.97$
$c = \$19.99$

11. The area of a rectangle is 48 square centimeters. If the base of the rectangle measures 12 cm, what is the height of the rectangle?
Let h = height of the rectangle in centimeters
Any time we are dealing with a problem that requires a formula, encourage students to start by writing the formula:

$A = bh$
$48 = 12h$
$h = 4$ cm

12. Peter is checking his suitcase at the airport. He puts it on the scale and the person working at the counter tells him that his bag weighs $1\frac{1}{2}$ times the weight limit for checked baggage. If Peter's bag weighs 75 pounds, what is the weight limit for checked bags?



Let w = weight limit of checked bags
"If we multiply the weight of a bag by 1.5, we will get the weight of Peter's bag"
$1.5w = 75$
$w = 50$
The weight limit for checked bags is 50 pounds.

13. Seventy-five percent of the students in 6th grade voted to go to the planetarium on their next field trip. If 24 students voted to go to the planetarium, how many students are in 6th grade?
Let s = number of students in the 6th grade

$0.75s = 24$
$s = 32$ students

14. Steven got eighty-five percent of the questions on his final math exam correct. If there were 20 questions on the exam, how many did Steven answer correctly?
Let c = number of questions Steven answered correctly

$0.85(20) = c$
$17 = c$

Steven answered 17 questions correctly.

15. Mr. Romero has a big container of slime that he is distributing to his class for a science experiment. He needs $\frac{1}{3}$ cup of slime for each student to do the experiment. If he can give 16 students slime to do the experiment, how much slime was in the container to begin with?
Let s = amount of slime Mr. Romero started with.

$s \div \frac{1}{3} = 16$ or
$s = 16 \times \frac{1}{3}$
$s = \frac{16}{3} = 5\frac{1}{3}$ cups of slime to start

16. Meg is making a wooden frame in the shape of a rectangle to go around a picture. The base of the frame measures 10.5 inches and the height of the frame measures 13 inches. Meg has a long piece of wood that she is cutting the pieces of the frame from. After she cuts off the pieces she needs to make the frame, she has $1\frac{1}{2}$ feet of the wood board left. How long was the board at the start?
Let b represent the length of the board at the start.

$b - 10.5 - 10.5 - 13 - 13 = 18$

$b - 47 = 18$

$b = 65$ inches or $b = 5$ feet and 5 inches or $5\frac{5}{12}$ feet

17. The ratio of boys to girls entered in an upcoming spelling bee is 3 to 2. If there are 200 students competing in the spelling bee, how any are boys and how many are girls?

x	x	x	x	x

Three parts of the tape diagram shown are boys and 2 parts are girls:
$3x + 2x = 200$
$5x = 200$
$x = 40$

There are 120 boys and 80 girls.

18. The fifth-grade class at Lincoln Elementary collected half as many cans of food for a canned food drive as the sixth-grade collected. Together, the classes collected 132 cans of food. How many cans of food did each class collect.

Fifth Grade $\boxed{\frac{1}{2}x}$

Sixth Grade $\boxed{x}$

$\frac{1}{2}x + x = 132$

$1.5x = 132$

$x = 88$

The sixth grade collected 88 cans and the fifth grade collected 44 cans of food.

19. The perimeter of an equilateral triangle is 51 mm. What is the length of each side of the triangle?

$3s = 51$

$s = 17\text{ mm}$

20. Ali can bike ride 12 miles per hour. How long will it take her to bike 20 miles?

$d = rt$

$20 = 12t$

$1\frac{2}{3}\text{hours} = t$

21. The area of a triangle is 42 in^2. If the height of the triangle is 6 in., what does the base of the triangle measure?

$A = \frac{bh}{2}$

$42 = \frac{b \cdot 6}{2}$

$42 = 3b$

$14\text{ in.} = b$

22. The area of a trapezoid is 21 square feet. The height of the trapezoid is 6 feet. If one of the bases is 3 feet long, what is the length of the other base?

$$A = \frac{h(b_1 + b_2)}{2}$$

$$21 = \frac{6(3 + b_2)}{2}$$

b = 4 feet

This is a more complicated equation to solve. Students should reason that they need to make the top of the fraction equal to 42 so that when they divide by 2, they get 21. They can examine the structure of the expression to arrive at the answer or use guess and check.

Spiral Review

1. Compare using $<, >$, or $=$.

   10 _____ 0

2. Yvonne has a goal to spend no more than $75 on school supplies. Her total is exactly $75. Did she reach her goal?

3. Solve the following equation for *x*.

   $\frac{3}{4}x = 24$

4. Simplify the expression shown below.

   $\frac{5}{6}(30) + 9^2 - 15 \div 3$

6.3f Homework: Writing Equations to Solve Real World Problems

Directions: Write an equation to represent each problem. Then, solve the equation. Make sure to label your answer.

1. Charles and his 3 friends went out to dinner. They split the bill evenly. If each person paid $12.50, what was the total cost of the meal?
   $\frac{x}{4} = 12.50$
   $x = \$50$

2. Owen is buying a game that costs $21.99. If he gives the cashier $25, how much change will he get?

3. After buying school lunch, Terry has $4.40 left. If school lunch costs $2.50, how much did Terry have before she bought school lunch?
   $m - 4.40 = 2.50$
   $m = \$6.90$

4. Talen really wants to buy a new bike. His parents told him that if he covers $\frac{3}{4}$ of the cost of a bike, they will pay for the rest. If Talen has to pay $187.50, what is the total cost of the bike?

5. Penny and Ben are out picking apples. Penny picked 26 apples. Together, they picked 60 apples. How many apples did Ben pick?

6. Tim made 60% of the free throws he shot last season. If he made 24 free throws last season, how many free throws did he shoot?

7. A group of students were taking the bus home from school. At the first stop, 8 students got off the bus. Then, there were 33 students on the bus. How many students were on the bus when it left the school?

8. At a certain high school, 58% of the students take Spanish as their foreign language. If 493 students take Spanish, how many students attend the high school?
$0.58s = 493$
850 students

9. The height of a parallelogram is $3\frac{2}{3}$ feet. If the area of the parallelogram is 44 feet^2, what is the length of the base of the parallelogram?

10. The Eagles won 80% of the games they played last season. If they won 12 games, how many did they play?

11. One angle is three times larger than another angle. Together the angles sum to 90°. What is the measure of each angle?

12. A candy shop sells three types of ice cream: chocolate, vanilla, and strawberry. The ratio of chocolate to vanilla to strawberry cones sold in a week is 3:2:1. If the store sells 150 cones in a week, how many of each kind do they sell?

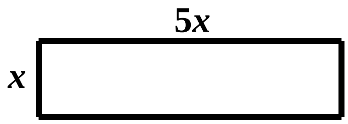
13. Lily makes $13 an hour babysitting. One month, she made $201.50 babysitting. How many hours did she babysit for?

14. Rachel needs $\frac{2}{5}$ yard of ribbon to make one bow for a cheerleading competition. If she has 15 yards of ribbon, how many bows can she make?

15. Miguel has $20 to spend at the candy store. Gummy bears cost $2.50 per pound and peppermint patties cost $4 a pound. If Miguel buys two pounds of gummy bears, how many pounds of peppermint patties can he buy?

16. A theater has 40 seats in a row. An elementary school is taking 500 students to the theater to see a production. How many rows of seats will the theater need to reserve for them?

17. The perimeter of the rectangle shown below is 54 square feet.



a. Find the value of *x*.

b. Find the dimensions of the rectangle.

18. Christina can bike 10 miles per hour. How long will it take her to bike 18 miles?

$10t = 18$

1.8 hours

6.3g Class Activity: Solving Percent Problems with Equations

In Chapter 2, you solved the following types of problems using a variety of strategies. In this lesson, we will solve different types of percent problems using equations.
In this lesson, we re-visit many of the percent problems from Chapter 2, lessons 2.1f – 2.1i. However, in this lesson, we solve these problems by setting up and solving an equation. Connect this work to work done in Chapter 2. Additionally, we show how students can use equivalent ratios to solve this problem. This work surfaces ideas about proportions which will be studied in 7th grade.

Activity 1 shows the three different types of percent problems and how they can be solved using equations. Following Activity 1 are several practice problems from Chapter 2. If students struggle, have them refer to the examples in Activity 1 for help.
Activity 1: Write and solve an equation for each problem. Verify your answer using estimation and mental math strategies.

a. What number is 80% of 40

$n = 0.80(40)$ Help students to decode the word representation of the problem to create the symbolic representation of the problem. Identify key words such as "is" and "of". "Is" translates to equals and "of" translates to multiplication.
$n = 32$

Some students might change 0.8 to its fraction equivalent which is $\frac{4}{5}$ and then take $\frac{4}{5}$ of 40.

Solving Using Equivalent Ratios (The Percent Proportion):

$$\frac{\text{part}}{\text{total}} = \frac{\text{percent}}{100}$$

$$\frac{n}{40} = \frac{80}{100}$$

What you are asking in the proportion above is, "What part of the whole do we need to have to create a ratio equivalent to $\frac{80}{100}$?" In other words, "If the whole changes from 100 to 40, how does the corresponding part change?"

Students can solve the equation above a variety of ways: 1) They may start by simplifying $\frac{80}{100}$ to $\frac{4}{5}$ and then using ideas about equivalent ratios:

$$\frac{n}{40} = \frac{4}{5}$$

We can see that $\frac{4}{5}$ is transformed into a ratio with a denominator of 40 by multiplying by $\frac{8}{8}$; therefore, $n = 32$.

Students may also solve this proportion by multiplying both sides of the equation by 40 to isolate n:

$$40 \cdot \frac{n}{40} = \frac{80}{100} \cdot 40$$

$$n = \frac{3200}{100} = 32$$

b. 40 is 80% of what number?

$40 = 0.80n$

$50 = n$

You can see that in the literal translation of this phrase, the variable would be on the right side of the equation.

Using equivalent ratios (the percent proportion):

$$\frac{\text{part}}{\text{total}} = \frac{\text{percent}}{100}$$

$$\frac{40}{\text{total}} = \frac{80}{100}$$

We can see that to create equivalent ratios, the unknown total would need to be 50.

c. 40 is what percent of 80?

$$\frac{40}{80} = \frac{n}{100}$$

If students simplify the left ratio to $\frac{1}{2}$, they can see that they need to multiply $\frac{1}{2}$ by $\frac{50}{50}$ to get the equivalent ratio $\frac{50}{100}$ which is 50%.

Directions: Solve the following problems by setting up and solving an equation.

1. What number is 25% of 40? $n = 0.25(40)$ $n = 10$ Solving Using Equivalent Ratios: $\frac{n}{40} = \frac{25}{100}$ Students may find it easier to solve this problem by taking $\frac{1}{4}$ of 40.	2. What number is 40% of 60? $n = 0.4(60)$ $n = 24$ Solving Using Equivalent Ratios: $\frac{n}{60} = \frac{40}{100}$
3. 48 is 60% of what number? $48 = 0.6n$ $80 = n$ $\frac{48}{\text{total}} = \frac{60}{100}$	4. 36 is 75% of what number? $36 = 0.75n$ $48 = n$ $\frac{36}{\text{total}} = \frac{75}{100}$

5. 14 is what percent of 35? $\frac{14}{35} = \frac{n}{100}$ Simplify $\frac{14}{35}$ to $\frac{2}{5}$ and create an equivalent ratio by multiplying $\frac{2}{5}$ by $\frac{20}{20}$ which equals $\frac{40}{100}$ or 40%.	6. What number is 10% of 120? $n = 0.1(120)$ $n = 12$ Solving Using Equivalent Ratios: $\frac{n}{120} = \frac{10}{100}$
7. 25% of a number is 4. What is the number? $0.25n = 4$ $n = 16$ $\frac{4}{\text{total}} = \frac{25}{100}$	8. 25% of 24 is what number? $0.25(24) = n$ $6 = n$ Note that the literal translation of this phrase would result in the variable being on the right side of the equation. Solving Using Equivalent Ratios: $\frac{n}{24} = \frac{25}{100}$ Some students might change 0.25 to its fraction equivalent which is $\frac{1}{4}$ and then take $\frac{1}{4}$ of 24 which is 6.
9. 7 is what percent of 20? $\frac{7}{20} = \frac{n}{100}$ Create an equivalent ratio by multiplying $\frac{7}{20}$ by $\frac{5}{5}$ which equals $\frac{35}{100}$ or 35%.	10. 30 is 120% of what number? $30 = 1.2n$ $25 = n$ $\frac{30}{\text{total}} = \frac{120}{100}$
11. 24 is 12% of what number? $24 = 0.12n$ $200 = n$ $\frac{24}{\text{total}} = \frac{12}{100}$	12. What number is 5% of 120? $n = 0.05(120)$ $n = 6$ Solving Using Equivalent Ratios: $\frac{n}{120} = \frac{5}{100}$
13. What number is 29% of 120? $n = 0.29(120)$ $n = 34.8$ Solving Using Equivalent Ratios: $\frac{n}{120} = \frac{29}{100}$ Note that the multiplier in the problem above is not as straight forward because it is not a whole number. Students can solve for n by multiplying both sides of the equation by 120.	14. 63.36 is 99% of what number? $63.36 = 0.99n$ $64 = n$ $\frac{63.36}{\text{total}} = \frac{99}{100}$

15. 25 is what percent of 40? $\frac{25}{40} = \frac{n}{100}$ Simplify $\frac{25}{40}$ to $\frac{5}{8}$. It is not as obvious what to multiply by to get to write an equivalent ratio with a whole of 100. Students may represent $\frac{5}{8}$ as a decimal which is 0.625 and then multiply by 100 resulting in 62.5%. Alternatively, students may solve for *n* by multiplying both sides of the equation by 100.	16. What number is 35% of 48? $n = 0.35(48)$ $n = 16.8$ Solving Using Equivalent Ratios: $\frac{n}{48} = \frac{35}{100}$ Again, the multiplier is not as obvious. Encourage students to think about how we can solve for *n*: multiply both sides of the equation by 48.
17. 2.8 is what percent of 7? $\frac{2.8}{7} = \frac{n}{100}$ A very helpful strategy for students is to learn how to clear decimals from fractions. We can multiply by 1 (expressed in a power of 10 divided by itself) to clear the decimal. So, in the example above, we would multiply $\frac{2.8}{7}$ by $\frac{10}{10}$ to get $\frac{28}{70}$ which simplifies to $\frac{4}{10}$ which is 40%. This is an example of the Identity Property of Multiplication because you are multiplying by 1.	18. What number is 200% of 75? $n = 2.0(75)$ $n = 150$ Solving Using Equivalent Ratios: $\frac{n}{75} = \frac{200}{100}$
19. 10% of a number is 4.2. What is the number? $0.1n = 4.2$ $n = 42$ Solving Using Equivalent Ratios: $\frac{4.2}{n} = \frac{10}{100}$	20. 52% of 180 is what number? $0.52(180) = n$ $93.6 = n$ Solving Using Equivalent Ratios: $\frac{n}{180} = \frac{52}{100}$
21. What number is 6.25% of 60? $n = 0.0625(60)$ $n = 3.75$ Solving Using Equivalent Ratios: $\frac{n}{60} = \frac{6.25}{100}$ A common error on this problem is for students to write 6.25% as 0.625 which would result in an answer of 37.5. If students make this mistake, ask them to use estimation to determine if their answer makes sense. 37.5 is over 50% of 60.	22. 120% of a number is 48. What is the number? $1.2n = 48$ $n = 40$ Solving Using Equivalent Ratios: $\frac{48}{n} = \frac{120}{100}$
23. 12.5% of 72 is what number? $n = 0.125(72)$ $n = 9$ Solving Using Equivalent Ratios: $\frac{n}{72} = \frac{12.5}{100}$	24. 4.5 is 15% of what number? $4.5 = 0.15n$ $30 = n$ Solving Using Equivalent Ratios: $\frac{4.5}{n} = \frac{15}{100}$

Some students might notice that 0.125 is $\frac{1}{8}$ and quickly find $\frac{1}{8}$ of 72 which is 9.	

Spiral Review

1. Compare using $<, >$, or $=$.

a. 3_____5	b. 10_____4	c. -15_____ -3

2. Compare using $<, >$, or $=$.

a. 5_____3	b. 4_____10	c. -3_____ -15

3. Compare problems #1 and #2. What is the same? What is different?

4. Give *two* different values for *x* that make the following inequality true.
$x + 4 < 10$

6.3g Homework: Solving Percent Problems with Equations

1. What number is 75% of 16?	2. 8 is what percent of 32?
3. 15 is 60% of what number?	4. What number is 18% of 50?
5. What number is 20% of 54? 10.8	6. 16 is 32% of what number? 50
7. 6 is what percent of 20? 30%	8. 15 is 50% of what number?
9. 4 is what percent of 20?	10. 15 is 75% of what number?
11. What number is 51% of 128?	12. What number is 120% of 40? 48
13. What number is 49% of 300?	14. What number is 76% of 32?

15. 30 is what percent of 80? 37.5%	16. What number is 250% of 50?
17. 2.8 is what percent of 20?	18. What number is 175% of 120?
19. 49 is what percent of 56?	20. What number is 150% of 32?
21. What number is 80% of 90?	22. 4.8 is 10% of what number? 48
23. 15 is 75% of what number?	24. 2.5 is what percent of 20?
25. What number is 90% of 48?	26. What number is 4.25% of 50? 2.125
27. 80 is what percent of 50?	28. What number is 0.5% of 40?
29. 3.6 is what percent of 18?	30. 87.6 is 30% of what number?

6.3h Class Activity: Understanding the Solution to an Inequality

Activity 1: Talen's mom told him he could spend no more than $10 at the arcade.

a. Give some values for the amount Talen can spend at the arcade.
   $0, $5, $8.50, $10

b. What is the maximum amount Talen can spend at the arcade? What is the minimum amount Talen can spend at the arcade?
   max = $10; min = $0

c. Describe in words the range of dollar amounts Talen can spend.
   Anything less than or equal to 10 and greater than or equal to zero

d. Write an inequality to represent the amount *a* (in dollars) Talen can spend at the arcade.
   We can list a multitude of values that make this equation true. We can also use words to describe this situation. An inequality is a symbolic way of representing this problem:

   $a \geq 0$ and $a \leq 10$
   We can combine these inequalities in the following way:
   $0 \leq a \leq 10$
   Students will study compound inequalities in high school courses. They will also learn more about restrictions on the domain of a function. The exploration of these ideas is very informal in 6th grade. You may ask probing questions such as, "Does it make sense for Talen to spend a negative amount of money?" It is very unlikely unless he was being paid to play the video games. "Can Talen spend a fractional amount?" Yes, he can.

e. Create a number line diagram to represent the amount Talen can spend at the arcade.
   A number line diagram is a visual tool that helps us to show the solutions to an inequality.

f. Talen spent $8 at the arcade. Did he follow his mom's instructions?
   Yes

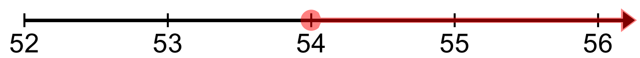
Activity 2: You must be at least 54 inches tall to drive the cars on the fastest race car track at an amusement park.

a. Give some values for the height you can be and drive the cars.
   54 inches, 55.5 inches, 60 inches, etc.

b. Give some values for the heights you *cannot* drive the cars.
   53.9 inches, 50 inches, 3 feet, etc.

c. Write an inequality to represent the height you need to be to drive the cars.
   Any height greater than or equal to 54 inches. $h \geq 54$

d. Create a number line diagram to represent the heights of the children who can play drive the cars.



Activity 3: You must be less than 48 inches tall to play in Kiddie Zone at an amusement park.

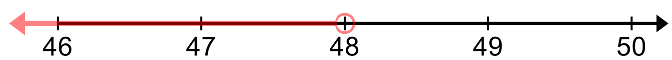
a. Give some values for the height you can be and enter the Kiddie Zone.
47 inches, 40 inches, 3 feet

b. Give some values for the heights that you *cannot* play in the Kiddie Zone.
48 inches, 40 inches, 3 feet

c. Write an inequality to represent the height of kids who can play in the Kiddie Zone.
$h < 48$

d. Stephan is 48 inches tall. Can he play in the Kiddie Zone?
No

e. Create a number line diagram to represent the heights of the children who can play in the Kiddie Zone.
A common mistake is for students to start the graph at 47 and shade to the left. Ask probing questions: "Can a child who is 47.9999 inches play in the Kiddie Zone?" "How do we show that the value cannot equal 48 inches?" We use an open circle on 48 to show that the height cannot be 48 but can be anything smaller than 48.



Activity 4: Create a number line diagram to represent each inequality. Then, circle the numbers that are solutions to the inequality. Students attend to precision when creating number line diagrams asking questions such as, "Should the circle be open or closed?" "Which direction should I shade the diagram?"

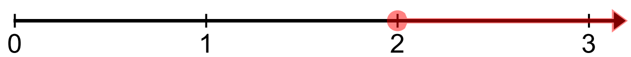
Students can have a difficult time determining how many numbers to show on the diagram and determining how to scale the diagram. There really is no right or wrong answer. Students should understand that the diagram they create should give a reader a clear picture of the solution set. It is common practice to show a few values to the left and right of the boundary number (number with circle on it). The scale chosen will vary depending on the problem.

a. $x > 2$

Which of the following are solutions to the inequality?

2   1.98   2.01 -2 10

b. $x \geq 2$



Which of the following are solutions to the inequality?

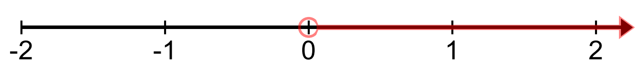
2   1.98   2.01 -2 10

c. $x \leq 10$

Which of the following are solutions to the inequality?

10   10.01   9   0 -4

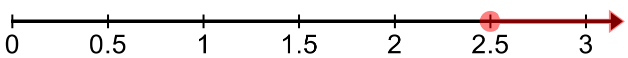
d. $x > 0$



Which of the following are solutions to the inequality?

$\frac{1}{100}$ -0.1 0   50 -1

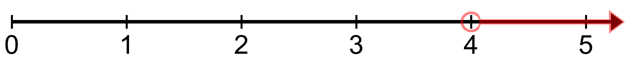
e. $x \geq 2\frac{1}{2}$



Which of the following are solutions to the inequality?

$2\frac{4}{5}$ 2   2.5   2.05   2.49

f. $4 < x$



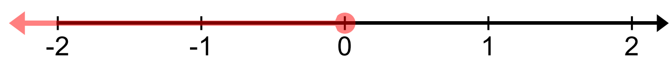
Which of the following are solutions to the inequality?

3   4   4.1   10   0

It can be challenging when the variable is on the right side of an inequality. Encourage students to always read from the variable, "I am looking for all the numbers that are greater than 4." Another way to think of this is, "If 4 is less than *x*, then that means that *x* is greater than 4."

Students should verify that they have shaded the correct direction by testing a point in the shaded region. If the point they choose makes the inequality true, then they have shaded correctly.

g. $0 \geq x$



Which of the following are solutions to the inequality?

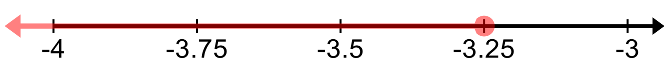
0   -1 1 $\frac{1}{2}$ -0.75

h. $h > -3$

Which of the following are solutions to the inequality?

0   -4 -2 -3.1 2

i. $-3\frac{1}{4} \geq n$



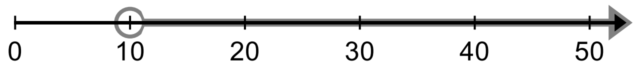
Which of the following are solutions to the inequality?

-3 -3.5 -3.3 -2 $-3\frac{1}{5}$

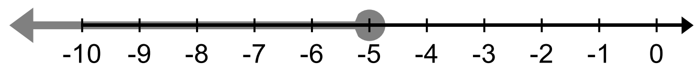
Activity 5: Write an inequality to match each number line diagram.

Once students write the inequality, they should test a point in the shaded region to confirm that they have written the inequality correctly.

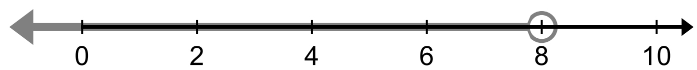
a. $x > 10$



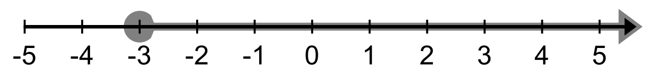
b. $x \leq -5$



c. $x < 8$



d. $x \geq -3$



Spiral Review

1. Solve the equation for *x*.

   $6x = 42.$

2. Solve the following equation for *x*.

   $\frac{x}{8} = 72$

3. Give two different values for *x* that make the inequality true.
   $x \geq 7$

4. Bodie saves 30% of what he earns. If he saved $54, how much did he earn?

6.3h Homework: Understanding the Solution to an Inequality

Directions: For each problem:
1) Write an inequality to represent the situation.
2) Create a number line diagram to represent the situation.
3) Give two values that make the inequality true. Make sure your values make sense in the problem. For example, it does not make sense to have $6\frac{1}{2}$ people.

1. A minimum of three people need to show up for a workout class for the instructor to hold the class.
$p \geq 3$

Sample answers: 3 or 4 people

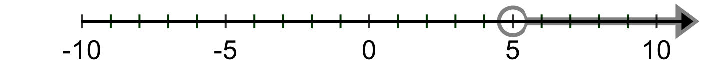
2. A maximum of 45 people can be in the school library at one time.

3. Water freezes at zero degrees Celsius or colder.

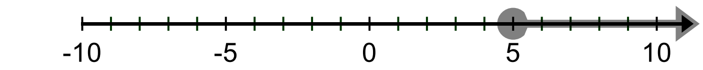
4. Shannon needs at least an 85% on her math test to get an A in math for the quarter.

Directions: Match each inequality to the corresponding number line diagram.

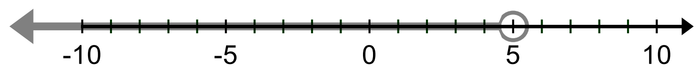
5. $x < 5$



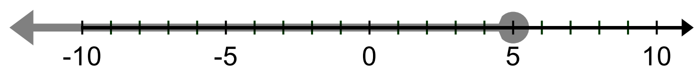
6. $x \leq 5$



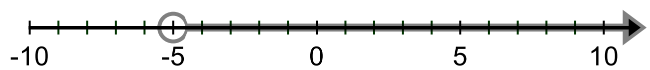
7. $5 < x$



8. $x \geq 5$

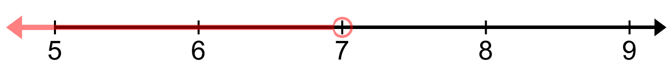


9. $-5 < x$



Directions: Create a number line diagram to represent each inequality. Then, circle the numbers that are solutions to the inequality.

10. $x < 7$



7 $\quad$ 0 $\quad$ $6\frac{3}{4}$ $\quad$ 10 $\quad$ -7

11. $x \leq 7$

7 $\quad$ 0 $\quad$ $6\frac{3}{4}$ $\quad$ 10 $\quad$ -7

12. $7 \leq x$

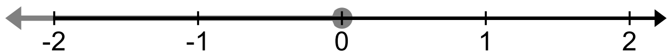
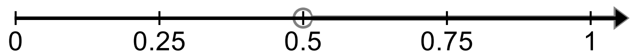
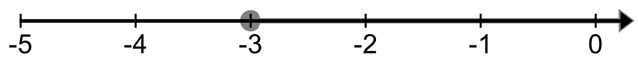
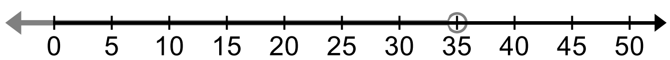
7 $\quad$ 0 $\quad$ $6\frac{3}{4}$ $\quad$ 10 $\quad$ -7

13. $-6 > v$

Which of the following are solutions to the inequality?

-7 -5.5 -6 0 -6.1

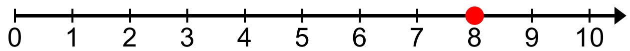
Directions: Write an inequality to match each number line diagram.

31. $x \leq 0$	
32.	
33.	
34.	

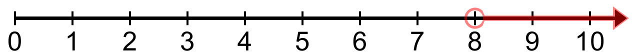
6.3i Class Activity: Solving Inequalities

Activity 1:

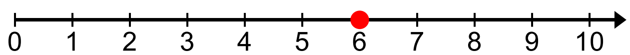
a. Solve the equation $x - 2 = 6$. Represent the solution on a number line diagram.
$x = 8$



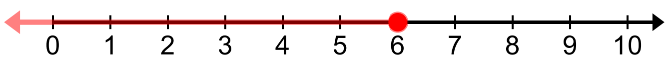
b. Solve the inequality $x - 2 > 6$. Represent the solution on a number line diagram.
$x > 8$



c. Solve the inequality $5x = 30$. Represent the solution on a number line diagram.
$x = 6$



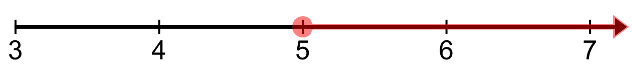
d. Solve the inequality $5x \leq 30$. Represent the solution on a number line diagram.
$x \leq 6$



e. How is solving an inequality like solving an equation? How is it different?
Same: You use the same process to solve an equation and inequality. Different: An equation has one solution (or no solution or all real numbers which students will learn in 8^{th} grade). An inequality often has infinitely many solutions.

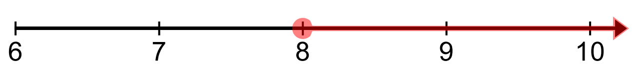
Activity 2: Solve each inequality. Represent the solution on a number line diagram. Check your solution by testing a point in the shaded region of the number line diagram.

a. $x + 7 \geq 12$
$x \geq 5$

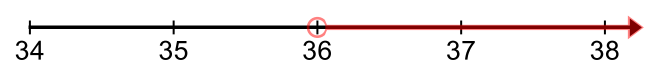


b. $n - 4 < 7$
$n < 11$

c. $7x \geq 56$
$x \geq 8$

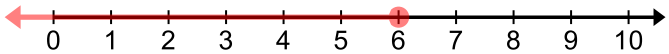


d. $\frac{y}{3} > 12$
$y > 36$



e. $9x \leq 54$

$x \leq 6$

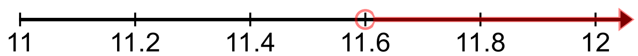


f. $0.1x \leq 8$

$x \leq 80$

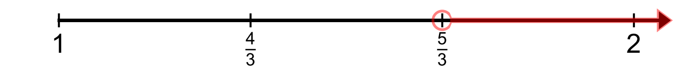
g. $x - 2.5 > 9.1$

$x > 11.6$



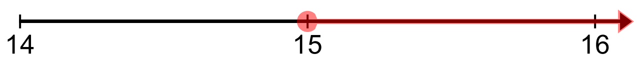
h. $9x < 15$

$x > 1\frac{2}{3}$



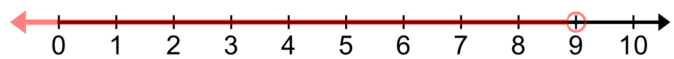
i. $\frac{c}{2} \geq 7.5$

$c \geq 15$



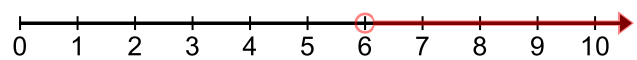
j. $2 > x - 7$

$9 > x$



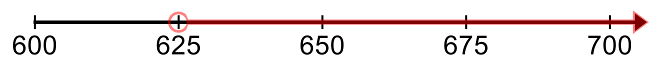
k. $18 < 3x$

$6 < x$



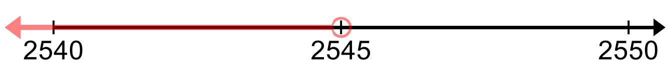
l. $125 < \frac{x}{5}$

$625 < x$



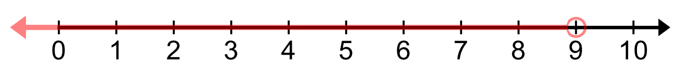
m. $x + 511 \leq 3{,}056$

$x < 2{,}545$



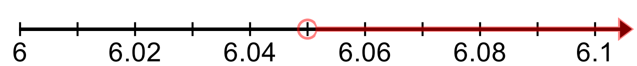
n. $6 + t < 15$

$t < 9$



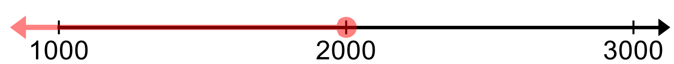
o. $r + 1\frac{3}{4} > 7.8$

$r > 6.05$



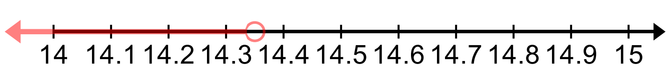
p. $0.1x \leq 200$

$x \leq 2{,}000$



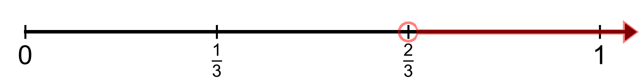
q. $x - 0.35 < 14$

$x < 14.35$



r. $3x > 2$

$x > \frac{2}{3}$



Spiral Review

1. Write and solve an inequality for the phrase "the sum of a number and five is greater than 12". Represent the solution on a number line diagram.

2. Write and solve an inequality for the phrase "the quotient of a number and seven is less than or equal to twelve". Represent the solution on a number line diagram.

3. Convert $5\frac{1}{4}$ hours to minutes.

4. Convert 3.5 feet to inches.

6.3i Homework: Solving Inequalities

Directions: Determine whether the number given is a solution to the inequality. Write yes if the number is a solution or no if the number is not a solution.

1. $x - 7 > 8; x = 15$ no	2. $3x \leq 15; x = 5$ yes
3. $\frac{3}{8}x \geq 1; x = 4$	4. $15 < x + 10; x = 4$

Directions: Solve each inequality. Represent the solution on a number line diagram.

5. $y - 5 \leq 12$	6. $b + 3 < 9$	7. $9x \geq 54$
8. $\frac{y}{7} < 6$	9. $10x \leq 120$	10. $0.5x > 4$ $x > 8$ Graph is an open circle at 8 shaded to the right.
11. $x + 4 > 5.2$	12. $8x \geq 2$	13. $9 < \frac{d}{3}$
14. $12 > x + 11$ $1 > x$ Graph is an open circle at 1 shaded to the left.	15. $\frac{4}{5}x \leq 36$	16. $1\frac{1}{2}y > 9$
17. $x - 57 \leq 429$	18. $8 + n < 11$	19. $r - 3\frac{1}{2} > 8.25$

6.3j Class Activity: Writing and Solving Inequalitites to Represent Real World Problems

In this lesson, students are making sense of problems, asking the following questions: What are the constraints in the given situation? When I solve for the inequality, what answers make sense in the context? For example, in problem 2 below, it does not make sense to order part of a pizza based on the way the problem is set up. In #10, students need to think realistically about the degree of precision someone will use when cutting fabric or paper.

Activity 1: Eva is buying scarves as Christmas presents for her sisters and nieces. Each scarf costs $15. If she spends at least $60, she will get a 25% discount on her purchase.

a. Write and solve an inequality to show the number of scarves Eva must purchase to get the 25% discount.
   Help students to talk through this problem, starting with words and slowly transforming the words into the symbolic representation (inequality)
   "We want the amount Eva spends to be at least $60."
   Amount Eva spends ≥ $60
   Let *s* be the number of scarves Eva buys.
   $15s \geq 60$
   $s \geq 4$

   If students struggle with writing the inequality, start asking them whether values work. "Will she spend enough if she buys 2 scarves? No. 3 scarves? No. 4 scarves? Yes, she will have spent exactly enough. 5 scarves? Yes, she has spent more than she needs to. This leads us to the inequality:
   $s \geq 4$ where *s* is a whole number

b. If Eva buys 4 scarves, will she get the discount? Explain.
   Yes, at 4 scarves she will have spent the exact amount needed to get the discount

c. Give some other values for the number of scarves Eva can buy and receive the discount.
   Discuss what types of answers are reasonable for this problem. For example, Eva cannot buy part of a scarf, so it would not make sense for students to give values such as 4.5.

Students will likely need support working through these problems.
Directions: For each problem, write and solve an inequality to represent the situation. Then, graph the inequality.

1. The area of a rectangular garden must be at least 120 square feet for Petunia to plant what she wants to plant.
   a. If the width of the garden is set at 15 feet, write and solve an inequality to represent the possible lengths for the garden.
      Again, help students to talk through this problem:
      "We want the area of the garden to be greater than or equal to 120."
      Area ≥ 120
      $lw \geq 120$
      $15l \geq 120$
      $l \geq 8$ where *l* is a rational number

b. If Petunia makes the garden 8 feet long, is she meeting the conditions for the area of the garden?
Yes

Ask students about other possible lengths for the garden. Can she make the garden 7 feet long? $10\frac{1}{2}$ feet long? Can the solution in this problem be a fraction or decimal? Yes, you can have a fractional part of a length.

2. Mr. Green is ordering pizzas for the middle school dance. Medium pizzas cost \$8. He can spend no more than \$60 on pizzas.

a. Write and solve an inequality to represent the number of pizzas Mr. Green can buy.

"Mr. Green has to spend \$60 or less"

Cost of Pizzas ≤ 60

$8p \leq 60$

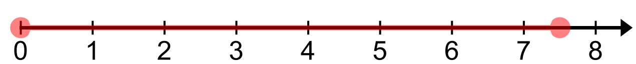
$p \leq 7.5$ where p is a whole number

Discuss the domain in this problem. Based on the information given in the problem, it is safe to assume Mr. Green is not buying a part of a pizza. So, he can buy 7 pizzas, 6 pizzas, 5 pizzas, etc.
Some students may argue that the inequality $p \leq 7$ where p is a whole number also describes this situation. Based on the context, this would be a correct inequality; however, if we are just solving the inequality $8p \leq 60$ out of context, $p \leq 7.5$ is the correct answer.

b. Give some possible values for the number of pizzas Mr. Green can buy.

Answers will vary but should be whole number less than or equal to 7.5. Technically, he cannot buy a negative number of pizzas either so the number of pizzas he buys should also be greater than or equal to zero. In an accelerated math class, you may wish to explore compound inequalities. This situation would be described by the following compound inequality:

$$0 \leq p \leq 7.5 \text{ where } p \text{ is a whole number}$$



3. You must be at least 46 inches tall to ride the roller coasters at an amusement park. Owen is currently $43\frac{1}{2}$ inches tall. Write and solve an inequality to show the amount Owen must grow to be tall enough to ride the roller coasters at the amusement park.

You can use models like the models used to write equations to help students.

"Owen must be 46 inches or taller."

$43\frac{1}{2} + g \geq 46$

$g \geq 46 - 43\frac{1}{2}$

$g \geq 2\frac{1}{2}$ where g is a rational number

46 or higher	
$43\frac{1}{2}$ = current height	Amount Owen needs to grow: g

If students struggle coming up with the equation, ask probing questions such as, "What if Owen grows 2 inches, will he be tall enough?" No. "What if Owen grows 3 inches, will he be tall enough?" Yes. "How many inches will Owen need to grow so that he is at the exact height requirement?" 2.5 inches. Therefore, 2.5 inches or higher.

4. Dane makes \$12 an hour babysitting his sister. He is trying to save over \$450 before the end of the summer. Write an inequality to represent the number of hours he needs to babysit to meet his goal.
"Dane's savings greater than \$450."
Let h represent the number of hours Dane babysits.
$12h > 450$
$h > 37.5$ where g is a rational number (you can have part of an hour)

Ask students to give plausible values for the number of hours Dane needs to work to meet his goal.

5. After spending \$5.75 on lunch, Dan has less than \$3 left. Write an inequality to represent the amount Dan had before he bought lunch.
"What Dan started with minus what he spent on lunch left him with less than \$3."
Let s be the amount Dan started with.
$s - 5.75 < 3$
$s < 8.75$ where s is a rational number

6. Devon can spend no more than \$200 on back-to-school clothes. He has already purchased 3 pairs of pants for \$19.99 each and 4 shirts for \$15.99 each. Write an inequality to represent the amount of money Devon has left to spend.
"Devon wants to spend less than or equal to \$200"
Spending ≤ 200
(What Devon has already spent) + (What he has left to spend) ≤ 200
$3(19.99) + 4(15.99) + s \leq 200$
$59.97 + 63.96 + s \leq 200$
$123.93 + s \leq 200$
$s \leq 76.07$ where s is a rational number

7. Kylie's teacher has asked her to read at least 15 minutes each night. Write an inequality to represent the number of minutes Kylie reads in one week if she is reading the amount she is supposed to.
$m \geq 15(7)$
$m \geq 105$ where m is a rational number

8. Kelly and two of her friends are washing cars. Their goal is to make at least \$40 *each*. How much money do they need to make together washing cars to meet their goal?
$\frac{e}{3} \geq 40$
$e \geq 120$ where e is a rational number

9. Clarissa saves 25% of what she earns. Write an inequality to represent the amount Clarissa must earn to have at least \$500 in savings.
$0.25e \geq 500$
$e \geq 2,000$ where e is a rational number

10. Desiree is making a flag in the shape of a triangle for a parade float. To fit on the float, the area of the flag can be no more than 600 square inches. She has decided to make the height of the flag equal to $3\frac{1}{2}$ feet. Write an inequality to represent possible lengths for the base of the flag.
Attend to precision: Note the need to make the units the same. In the solution shown, the units have been changed to inches.

$$\frac{bh}{2} \leq 600$$

$$\frac{b \cdot 42}{2} \leq 600$$

$$b \cdot 21 \leq 600$$

$$b \leq 28.\overline{571428}$$

Help students to make sense of this answer. What is reasonable in a situation like this? A person measuring paper or fabric to make a flag for a float is likely going to round to the nearest inch, half inch, or quarter inch. Present this to the students. What if Desiree wanted to make the biggest flag she could with a whole number measurement for the base? Then, she would make the base 28 inches. What if she was going to measure to the nearest half inch? Then she could make the base 28.5 inches long.

11. The sum of a number and fifteen is less than or equal to sixty-four.
$n + 15 \leq 64$
$n \leq 49$

12. The difference of a number and twelve is greater than 30.
$n + 12 > 30$
$n > 18$

13. The quotient of a number and 1.2 is at least 30.

$$\frac{n}{1.2} \geq 30$$

$$n \geq 36$$

The spiral review on the following page is a nice summary of expressions and equations and what you can "do" with each. When working with expressions, you can write equivalent expressions (including the simplified form of the expression) and you can evaluate expressions for a given value of the variable. When an expression with one variable is set equal to another expression to form an equation or inequality, you can solve for the unknown.

Spiral Review

1. Simplify.

a. $3x + 2x + x$	b. $3(x + 2) + 5x$
c. $8x - 5x + 2x$	d. $5 + 4(3x - 1) - x$

2. Evaluate when $x = 3$.

a. $4x^2$	b. $(4x)^2$
c. $6x + 3(x - 2) + x^3$	d. $\frac{2x}{x+7}$

3. Solve for x.

a. $4x = 26$	b. $\frac{x}{4} = 26$
c. $x - 347 = 800$	d. $x + 4.5 = 9\frac{3}{4}$

4. Solve for x. Draw a number line diagram to represent the solution.

a. $4x > 26$	b. $\frac{x}{4} \leq 26$
c. $x - 347 \geq 800$	d. $x + 4.5 < 9\frac{3}{4}$

6.3j Homework: Writing and Solving Inequalitites to Represent Real World Problems

1. Iya sells friendship bracelets for $4. Write and solve an inequality to represent the number of bracelets Iya needs to sell to make at least $150?
   Let b = number of bracelets
   $4b \geq 150$
   $b \geq 37.5$ where b is a whole number

2. A suitcase can weigh no more than 50 pounds to be checked on an airplane. Landon's suitcase currently weighs 37 pounds. Write and solve an inequality to represent the amount of weight Landon can add to his suitcase and still be able to check his suitcase.

3. You want to spend less than $20 on a birthday present for your friend. You have already bought a board game that costs $12.99. Write and solve an inequality to show the amount you have left to spend on your friend.

4. Owen can play no more than 1.5 hours of iPad a day. If he has already been playing iPad for 50 minutes today, write and solve an inequality to represent the number of minutes he can still play iPad.

5. You have at most $250 in your budget to decorate the faculty lounge. If you will be decorating the teacher lounge 8 times during the school year, about how much should you spend each time you decorate the lounge? Write and solve an inequality to represent this problem.

6. A librarian is packing books into boxes. She has 5 boxes and each box can hold no more than 30 books. Write and solve an inequality to represent the number of books she can pack into boxes.
   $\frac{b}{5} < 30$
   $b < 150$ where b is a whole number

6.3k Class Activity: Self-Assessment: Section 6.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Mastery 3**	**Substantial Mastery 4**
1. Understand what the solution to an equation is.				
2. Use substitution to determine whether a given number is a solution to an equation.				
3. Solve one-step equations, including equations with rational numbers.				
4. Write and solve equations to represent real-world problems.				
5. Understand that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions and represent the solutions using a number line diagram.				
6. Solve one-step inequalities, including inequalities with rational numbers.				
7. Write and solve inequalities to represent real-world problems in which constraints are given. Interpret the solution set in the context of the problem.				

Sample Problems for Section 6.1

Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. Consider the following equations: [1]

   $3x = 18$
   $x + 5 = 12$

   When you are asked to solve these equations, what are you being asked to do?

2. Determine whether the number given is a solution to the equation. Write yes or no. [2]

a. $7x = 14; x = 2$	b. $x + 9 = 14; x = 23$
c. $x - 7.01 = 11.99; x = 19$	d. $\frac{4}{3}x = 32; x = 18$

3. Solve. [3]

a. $x + 4 = 20$	b. $y - 7 = 4$	c. $3x = 24$
d. $\frac{x}{5} = 1$	e. $\frac{x}{8} = 9$	f. $35 = \frac{y}{5}$
g. $45 = x + 30$	h. $8 = x - 9$	i. $x - 8.9 = 7$
j. $2.5x = 100$	k. $x + 7\frac{5}{8} = 18.5$	l. $\frac{t}{0.2} = 25$
m. $7{,}340 + x = 10{,}397$	n. $310.8 = 37x$	o. $6\frac{2}{5} = g - 3.125$

Directions: For #4 – 13, write an equation to solve each problem. Then, solve the problem. [4]

4. Ramon is collecting donations for a charity. He is hoping to raise $250. If he has already collect $187 in donations, how much more does he need to collect

5. Gavin is out shopping. After he buys 2 pairs of pants for $27 each, he has $60. How much money did he have before he bought the 2 pairs of pants?

6. Augusto tips 20% on his meals. If he tipped $4.80 on a meal, how much was the meal?

7. Four families are renting a house together on vacation. If each family pays $75 per night to rent the house, what is the total cost of the house each night?

8. Sandy puts 6% of her income in a retirement account. If she made $1,500 in a pay period, how much of it will go into her retirement account?

9. The height of a parallelogram is 28.8 inches. If the area of the parallelogram is 155.52 inches^2, what is the length of the base of the parallelogram?

10. One angle is five times larger than another angle. Together the angles sum to 180°. What is the measure of each angle?

11. The ratio of 6th graders to 7th graders to 8th graders at a middle school is $3:1:1$. If there are 180 students at the middle school, how many of the students are in each grade?

12. A lunch table can hold 8 students. If there are 120 students who need to eat lunch at the same time, how many tables are needed?

13. A trapezoid has an area of 64 square centimeters. If the bases measure 4 cm and 12 cm, what is the height of the trapezoid?

14. Describe the similarities and differences between an equation and an inequality. Use examples to support your ideas. [5]

15. Circle the numbers that are solution(s) to the inequality? Then, give *two* more values that are also solutions to the inequality. [5]

$x \leq 3$

3	3.01	2.99	0	-3

16. Circle the numbers that are solution(s) to the inequality? Then, give *two* more values that are also solutions to the inequality. [5]

$3 < x$

3	3.01	2.99	0	-3

Directions: For #17 – 37 (all), solve each inequality. Represent the solution on a double number line. Then, write *two* solutions of the inequality. [5][6]

17. $x - 2 > 7$	18. $t + 3 \geq 9$	19. $32 \geq 8x$
20. $\frac{y}{11} < 66$	21. $42 \leq x - 36$	22. $\frac{2}{9}x < 36$
23. $x + 4 > 5.2$	24. $8x \geq 2$	25. $9 < \frac{d}{3}$
26. $12 > x + 11$	27. $\frac{4}{5}x \leq 36$	28. $1\frac{1}{2}y > 9$
29. $x \ - 57 \leq 429$	30. $8 + n < 11$	31. $r - 3\frac{1}{2} > 8.25$
32. $\frac{7}{8}t \geq 56$	33. $2 \geq 0.4b$	34. $\frac{x}{0.5} \leq 18$
35. $5x < 21$	36. $0.25x \geq 6$	37. $1.2 < x - \frac{9}{10}$

Directions: For #38 – 46 (all), write an inequality to represent each situation. Then, create a number line diagram to represent the solution set. Give at least one reasonable solution to the problem. [5][6][7]

38. The minimum grade point average required to play basketball is 2.3.

39. It is recommended that teenagers get at least 9 hours of sleep a night.

40. You must be at least 18 years old to vote

41. To qualify for a specific marathon, a runner's time had to be under 190 minutes.

42. Sally spent $60 on ingredients for a bake sale. She has sold $35 in baked goods. Write an inequality to represent the amount she still needs to sell to make a profit.

43. A bus can hold at most 40 people. If there are already 24 people on the bus, write an inequality to represent the number of people that can get on the bus at the next stop assuming no one gets off the bus.

44. An elevator has a 2,500-pound capacity. Write an inequality to represent the number of 75-pound boxes the elevator can hold.

45. Noah is at a video arcade earning tickets to buy prizes. He knows he wants a candy that costs 50 tickets. After he gets the candy, he hopes to have at least 525 tickets left. Write an inequality to represent the number of tickets he will need to win.

46. Write an inequality to represent the number of nickels Stephanie needs to have in order to have at least \$3.50.