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Chapter 2 Operations with Rational Numbers (4 weeks)

UTAH CORE Standard(s)

Number Sense:

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **7.NS.1**
   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* **7.NS.1a**
   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. **7.NS.1b**
   c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **7.NS.1c**
   d. Apply properties of operations as strategies to add and subtract rational numbers. **7.NS.1d**

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **7.NS.2**
   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world concepts. **7.NS.2a**
   b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. **7.NS.2b**
   c. Apply properties of operations as strategies to multiply and divide rational numbers. **7.NS.2c**
   d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **7.NS.2d**

3. Solve real-world and mathematical problems involving the four operations with rational numbers. **7.NS.3**

CHAPTER OVERVIEW:

In Chapter 2, students extend and formalize their understanding of the number system, including arithmetic with negative rational numbers. In the first two sections of the chapter, students extend their understanding of the properties of arithmetic to operating with integers. Section 1 focuses on adding and subtracting integers, while section 2 focuses on multiplication and division with integers. By section 3, students operate with all rational numbers by applying the rules they learned in the previous two sections for working with integers. Throughout all three sections, students will increase their proficiency with mental arithmetic by articulating strategies based on properties of operations.

VOCABULARY:

additive inverse, difference, integers, multiplicative inverse, opposites, product, quotient, rational numbers, repeating decimal, sum, terminating decimal, zero pairs.

CONNECTIONS TO CONTENT:

Prior Knowledge

In 6th grade students understand that positive and negative numbers are used together to describe quantities having "opposite" directions or values. They use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. Students recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line and recognize that the opposite of the opposite of a number is the number itself. Student should come to 7th grade knowing how to locate integers on the real line as well as positive rational numbers. In Chapter 1, students have placed negative rational numbers on the number line and this chapter will expand on that knowledge.

Students also build on their prior understanding of addition (joining), subtraction (take-away or comparison), multiplication (repeated addition) and division (the inverse of multiplication). In previous grades students modeled these operations with manipulatives and on a number line (positive side only). They will now extend these strategies to working with integers and negative rational numbers. Note that 7th Grade will be students' first experience operating with negative numbers.

Future Knowledge

The development of rational numbers in 7th grade is a progression in the development of the real number system that continues through 8th grade. In high school students will move to extending their understanding of number into the complex number system. Note that through 7th grade students find points on the real line that correspond to quantities, e.g. students will locate $-3/8$ on the real line in 7th grade. In 8th grade, students do something different; they start with lengths and try to identify the location that corresponds to the length. For example, students might find that the length of the hypotenuse of a right triangle is $\sqrt{5}$, they then try to identify the number that corresponds to that length.

In later courses, students will also know and apply the properties of integer exponents to generate equivalent numerical expressions, for example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. Students will use the rules for operations on integers when solving linear equations with integer coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

In this chapter, attention is paid to understanding subtraction as the signed distance between numbers. This idea extends into geometric ideas in later mathematics such as subtraction of vectors.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

|		
	Make sense of problems and persevere in solving them	Students explain and demonstrate operations on integers using symbols, visuals, words, and real life contexts. Students demonstrate perseverance while using a variety of strategies (number lines, chips/tiles, drawings, etc.)
	Reason abstractly and quantitatively	Students demonstrate quantitative reasoning with integers by representing and solving real world situations using visuals, numbers, and symbols. They demonstrate abstract reasoning by translating numerical sentences into real world situations or for example reasoning that $a - (-b)$ is the same as $a + b$.
	Construct viable arguments and critique the reasoning of others	Students justify rules for operations with integers using appropriate terminology and tools/visuals. Students apply properties to support their arguments and constructively critique the reasoning of others while supporting their own position with models and/or properties of arithmetic.
	Model with mathematics	Students model understanding of integer operations using tools such as chips, tiles, and number lines and connect these models to solve problems involving real-world situations and rules of arithmetic. For example, students will be able to use a model to explain why the product of two negative integers is positive.
	Attend to precision	Students demonstrate precision by using correct terminology and symbols when working with integers. Students use precision in calculation by checking the reasonableness of their answers. Additionally, students will use properties of arithmetic to justify their work with integers, expressions and equations.
	Look for and make use of structure	Students look for structure when operating with positive and negative numbers in order to find algorithms to work efficiently. For example, students will notice that subtracting a negative integer is the same as adding its opposite or that when a product involves an even number of negative integers, the product is always positive.
	Use appropriate tools strategically	Students demonstrate their ability to select and use the most appropriate tool (paper/pencil, manipulatives, or calculators) while solving problems with integers. They also learn to estimate answers before using a calculators. Students should recognize that the most powerful tool they possess is their ability to reason and make sense of problems.
	Look for and express regularity in repeated reasoning	Students will use manipulatives to explore the patterns of operations with integers. Students will use these patterns to develop algorithms. They can use these algorithms to solve problems with a variety of problem solving structures.

2.0 Anchor Problem: Operations on the Number Line

A number line is shown below. The numbers 0 and 1 are marked on the line, as are two other numbers a and b.

$$b \qquad\qquad\qquad\qquad 0 \qquad 1 \qquad a$$
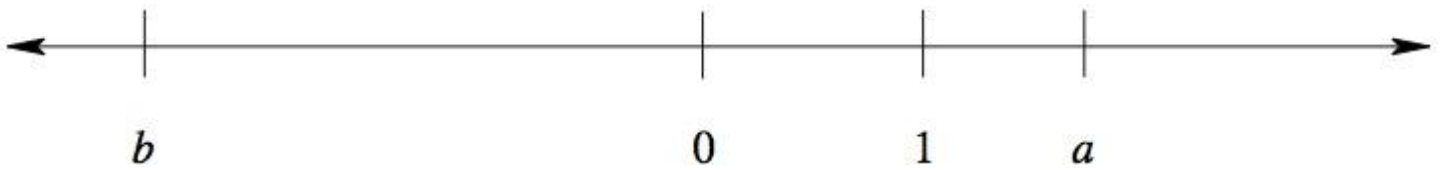
Which of the following numbers is negative? Choose all that apply. Explain your reasoning.

1. $a - 1$

2. $a - 2$

3. $-b$

4. $a + b$

5. $a - b$

6. $ab + 1$

Section 2.1: Add and Subtract Integers; Represent Using Chip/Tile Model and Number Line Model.

Section Overview:
Section 2.1 is the first time students add and subtract integers. For this reason, it is important to begin with hands-on manipulatives and number lines, gradually working towards developing the rules of arithmetic with integers and then ultimately adding and subtracting integers without models. The goal in this section is to build intuition and comfort with integer addition and subtraction so that by the end of the section students can *reason* through addition and subtraction of integers without a model.

Students start by working with "opposites" (additive inverses) to notice that pairs of positives and negatives result in "zero pairs." They then move to adding integers. Students know from previous grades that the fundamental idea of addition is "joining." Students should notice that joining positive and negative numbers results in one or more "zero pairs" and that the "left over" is the final sum. Students will develop this idea first with a chip or tile model and the number line.

Next, students move to subtraction. Students begin by reviewing from previous grades that there are two ways to think concretely about subtraction: i) "take-away;" Anna Maria has 5 gummy bears. Jose eats 3. How many gummy bears does she now have? $5 - 3$, thus 2 gummy bears. ii) "Comparison;" Anna Maria has 5 gummy bears, Jose has 3 gummy bears, how many more gummy bears does Anna Maria have than Jose? Again, the operation is $5 - 3$ resulting in 2 gummy bears. We build on the comparison conceptualization for a concrete way to think about subtraction with integers on the real line. We start by locating integers on the line and note that when comparing two integers, there is a directional or signed distance between them, e.g. when comparing 5 and 3 we can think "5 is two units to the right of 3," ($5 - 3$ is 2), or "3 is two units to the left of 5," ($3 - 5$ is -2). Another example, when comparing 5 and -3 and think, "5 is eight units to the right of -3," ($5 - (-3)$ is 8), but "-3 is eight units to the left of 5," ($(-3) - 5$ is -8.) In section 2.1e, students will examine subtraction exercises and notice that $a - b$ can be written as $a + (-b)$ and that $a - (-b)$ can be written as $a + b$. This is an essential understanding and one on which students can build fluency for working with integers.

Adding and subtracting integers can cause students a great deal of trouble particularly when students first confront exercises that have both addition and subtraction with integers. For example, in problems like $-5 - 7$ students are often unsure if they should treat the "7" as negative or positive integer and if they should add or subtract. For this reason, it is **extremely** helpful to make it a norm from the beginning of this chapter that students circle the operation before they preform it.

Rules for operating with integers come from extending rules of arithmetic. Students should understand that arithmetic with negative numbers is consistent with arithmetic with positive numbers. Students need to remember that subtraction in the set of integers is neither commutative nor associative. In other words, in general, $a - b \neq b - a$ and $(a - b) - c \neq a - (b - c)$. However, $a - b = a + (-b) = (-b) + a$.

Concepts and Skills to be Mastered (from standards)

By the end of this section, students should be able to:
1. Use a concrete model (chips/tiles or number line) to add integers.
2. Use a concrete model (chips/tiles or number line) to subtract integers.
3. Find the sums of integers accurately without a model.
4. Find the differences of integers accurately without a model.
5. Solve contextual problems involving adding or subtracting integers.

2.1a Class Activity: Additive Inverse (Zero Pairs) in Context and Chip/Tile Model

An *additive inverse* is an important mathematical idea that we will begin to informally explore in this activity. For now, we will use the informal term "zero pair" to refer to two things that are "opposite" or "undo each other." The sum of the two elements of a zero pair is zero. We see zero pairs in many contexts. For example, an atom with 5 protons and 5 electrons has a neutral charge, or a net charge of zero. Use the idea of "zero pairs" to complete the worksheet.
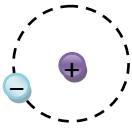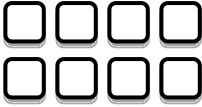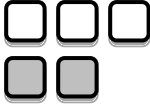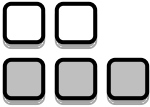
Context	Model/Picture	Net Result in Words	How Many Zero Pairs?
1. A Hydrogen atom has one proton and one electron.		The atom has a neutral charge.	1
2. I took 4 steps forward and 3 steps back.		I am one step ahead of where I started.	3
3. I got a check in the mail for $1,000,000, but then I got a bill that says I owe $1,000,000.	+$1,000,000 −$1,000,000	I will have no money.	1,000,000
4. I ate one candy bar with 200 calories. Then I ate four apples with 50 calories each.		I gained 400 calories. Healthy calories do not "undo" candy calories.	None
5. Joe found four quarters, but he had a hole in his pocket and lost one quarter.		Joe gained three quarters.	1
6. Lisa earned $8 then spent $6.	Student models should be similar to those shown above.	Lisa gained $2	6
7. George gained ten pounds, but then he went on a diet and lost twelve pounds.	Student models should be similar to those shown above.	George weighs 2 pounds less than at the start.	10
8. John took $15 out of his bank account then deposited $10.	Student models should be similar to those shown above.	John's account has $5 less than at the start.	10

The terms "additive inverse" and "zero pairs" can be used interchangeably. Move to using "additive inverse."

For the following exercises, use the key below:

☐ = 1

■ = −1
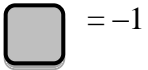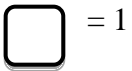
In pairs, look at the following models. Make a conjecture for what you think the model represents. Construct an argument to support your claim. **n#**

9. This model represents: 8 Argument: 8 positive squares	10. This model represents: −8 Argument: 8 negative squares
11. This model represents: 0 Argument: a zero pair	12. This model represents: 1 Argument: 2 zero pairs. 1 positive square
13. This model represents: −1 Argument: 2 zero pairs. A negative square	14. This model represents: −4 Argument: 1 zero pair, 4 negative squares

15. In general, explain how you can use a model to represent positive and negative numbers:
   Answers will vary.

Model the following expressions using integer chips/tiles. Draw pictures of your models using the above key to show what you did, then explain in words how you arrived at your answer.

16. Represent –5 with chips/tiles. Draw your representation in the space below:

Throughout these lessons you will be working with "zero pair." Note that this is related to the idea of an additive identity.

Explain your representation. Can you represent –5 in another way? Explain.
I have shown 5 gray (negative) tiles. I can also show 6 gray (negative) tiles and 1 while (positive) tile.

There are a variety of ways students might do this. All representations will have five negative chips and then a variety of zero pairs of chips. You might point out that no matter how many times we add zero to "something", we always just get the "something" back.

17. Represent 7 with chips/tiles. Draw your representation in the space below:
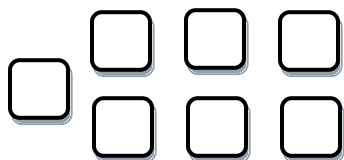
Explain your representation. Can you represent 7 in another way? Explain.

I have shown 7 white (positive) tiles. I could have shown 8 white and 1 gray.

18. Explain what it means to "add" numbers:
Addition means to combine or join. Help students notice that we combine "like" things and that it's useful to write things in the most concise manner possible.
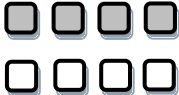
For #19 – 27, circle the operation, draw a representation, and then find the sum, explaining your reasoning.

19. Represent 5 + 6 with chips/tiles. Draw your representation and its sum in the space below: 11

Representation:	Explanation:
	There are a total of 11 positive chips, so I have positive 11.

20. Represent (–4) + 4 with chips/tiles. Draw your representation in the space below:

Representation:	Explanation:
	There are four zero pairs and nothing else. So I have 0.

21. Represent –4 + 3 with chips/tiles. Draw your representation in the space below:

Representation:	Explanation:
	There are three zero pair and then one negative tile left so I have –1.

22. Represent 8 + (–6) with chips/tiles. Draw your representation in the space below:

Representation:	Explanation:
	There are six zero pairs and then two positive tiles left so I have 2.

23. Why do you think #20 and #22 used "( )" around one of the numbers, but #21 did not?

Parentheses can lend clarity.
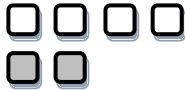
24. Represent –5 + (–8) with chips/tiles. Draw your representation and its sum in the space below:

Representation:	Explanation:
–13	There are five negative tiles and eight negative tiles for a total of thirteen negative tiles.

25. Represent (–3) + (8) with chips/tiles. Draw your representation and its sum in the space below:

Representation:	Explanation:
5	There are three zero pairs and five positive tiles left.

26. Represent 4 + –2 with chips/tiles. Draw your representation and its sum in the space below:

Representation:	Explanation:
2	There are two zero pairs and two positive tiles left.

27. Represent –5 + (–3) with chips/tiles. Draw your representation and its sum in the space below:

Representation:	Explanation:
–8	There are five negative tiles and three negative tiles for a total of eight negative tiles.

For positive numbers convention is to NOT write a sign in front of the number, however for negative numbers we use a "–" to signify that the number is located on the opposite side of the 0 on the real line.

2.1a Homework: Additive Inverse (Zero Pairs) in Context and Chip/Tile Model

Fill in the table below.

Context	Model/Picture	Net Result in Words	How Many Zero Pairs?
1. The Helium atom had two protons and two electrons.		0	2
2. The treasure map said to take ten steps north, then eight steps south.		2 steps north	
3. Jen earned $6 working in the garden, and then she spent $5 on a toy.			$5
4. The elevator started on the 8th floor, went up five floors, and then went down two floors.		11th floor	2
5. Seven students were added to Ms. Romero's class and then four students dropped it.			
6. The temperature fell twelve degrees between midnight and 6 a.m. The temperature rose twelve degrees between 6 a.m. and 8 a.m.		Same temperature	12
7. The home team scored a goal, which was answered with a goal from the visiting team.			1
8. The football team gained seven yards on the first down, and lost nine yards on the second down.			

Circle the operation, draw a chip/tile model for each sum, and then state the sum. The first one is done for you.

(Models may vary.)

For the following exercises, use the key below:

$\square$ = 1

$\blacksquare$ = –1

n#

9. 7 (+) (–4) 3	10. –3 + 8
11. 4 + 6 10	12. –5 + 8
13. –4 + (–2) –6	14. –6 + (–3)
15. 7 + (–6) 1	16. 4 + (–8)

Extension Questions:

Suppose $a + b = c$ and $c < 0$, give an example when this might happen.

What do you know about a and b relative to each other for $c < 0$? If a and b are negative, $c < 0$. If only one is negative, the one that's negative must have an absolute value greater than the one that's positive.

Spiral Review

1. Order the following rational numbers from least to greatest. $\frac{27}{3}$, 6.5, $\frac{18}{3}$, 0.99,

   0.99, $\frac{18}{3}$, 6.5, $\frac{27}{3}$

2. What is the opposite of -15?

   +15 or 15

3. What is 40% of 220? Use a bar model. **88**

4. Without using a calculator, determine which fraction is bigger in each pair. Justify your answer with a picture *and* words.

   a. $\frac{1}{3}$ or $\boxed{\frac{1}{2}}$
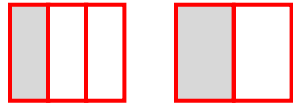
   1 piece out of a pie cut into 2 will be bigger than

   1 piece out of a pie cut into 3 pieces

   b. $\frac{3}{7}$ or $\boxed{\frac{3}{5}}$

5. Solve using a bar model. $\frac{1}{2} + \frac{3}{7} = $ _____ $\frac{13}{14}$
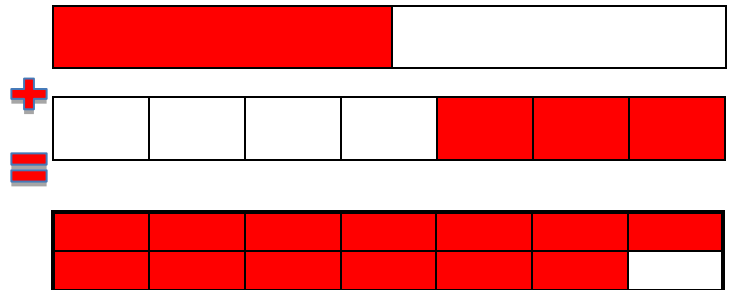
2.1b Class Activity: Add Integers Using a Number Line

Explore:
Answer each of the following questions. Using pictures and words, explain how you arrived at your answer.
Models will vary.

1. An osprey flies off the ground and reaches 35 feet above a river when he sees a trout. He then dives 37 feet down to get the trout. How many feet below the water does he end up?

    2 feet below the surface

2. Zach's football team moves the football 35 yards forward on the first down. On the next down, they lose 12 yards. On the down after that they go forward 8 yards. How many yards from the starting point did they move the football in the three down?

    31 yards.

3. You walk 3 miles from your house to the store. At the store you meet up with a friend and walk with her 1 mile back towards your house. How far are you from your house now?

    2 miles from your house

4. You ride your bike 12 miles and then get a flat tire! You turn around and walk the bike 4 miles before you mom is able to pick you up. How far are you from the house when your mom picks you up?

    8 miles

Review:

Place each of the following integers on the number line below. Label each point:

5. **A** = 4   **B** = −4   **C** = −15   **D** = 7   **E** = 18   **F** = −19
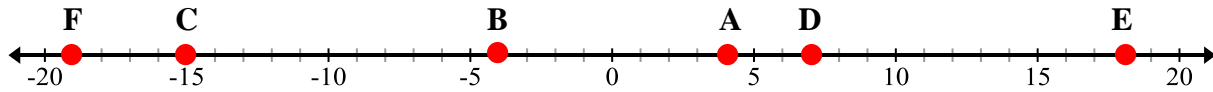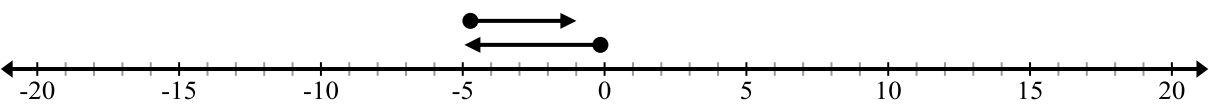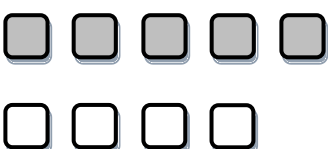
6. **A** = −20   **B** = −17   **C** = 7   **D** = 13   **E** = −6   **F** = 19
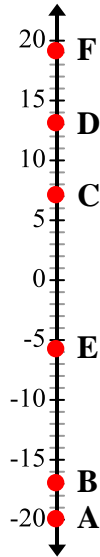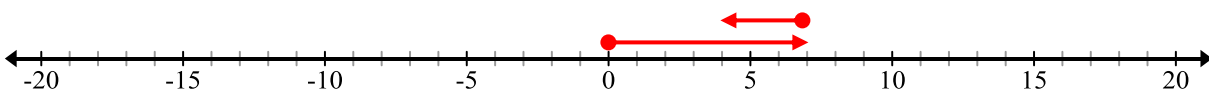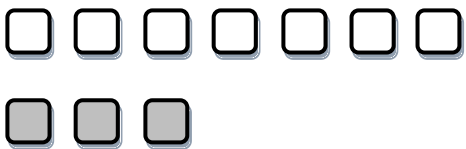
7. How did you locate 7 on the number line?
    7 is seven units RIGHT of 0 (or 7 units UP from 0).

8. How did you locate −15 on the number line?
    −15 is 15 units LEFT of 0

9. In general, how do you locate a positive or negative number on a number line? Positive numbers are to the right of 0 (up from 0) and negative numbers are to the left of 0 (below 0).

10. Brainstorm similarities and differences between a chip model and number line model for representing integers.
    Answers will vary, students will note many things. Highlight the zero pairs in both representations.

Previously in this section, you used a chip model for addition of integers. In this activity you will explore a number line model of addition of integers. Model each of the following with both a number line and chips. Start by circling the operation

Example: $-5 + 4$ Have students begin at zero, then draw the arrow in the appropriate direction to model the first number in the equation, then move on from there for the second number to arrive at the solution. You may also think about starting at the first number (-5) and from there add the next number, in this case, 4.

Number line:

$-5 + 4 = -1$

Chips

$-5 + 4 = -1$

11. Model $7 + -3$ on a number line and with chips: 4

Number line

Chips

12. Explain how the number line and chip models are related: You're moving students towards understanding, either with the chip or the number line model, that something like $7 + -3 = (4 + 3) + -3 = 4 + (3 + -3) = 4 + (0) = 4$. Students will likely not be ready to write this out, but they should be ready to start to explain it. Notice that you're using properties of arithmetic.

Circle the operation you are going to perform. Find the sum using a number line for the following addition exercises.

Note that the difference between an operation (add, subtract, multiply, or divide) and a positive or negative number; e.g. 6(−3) is not the same and 6 − 3.

13. 6 + (−3)   3
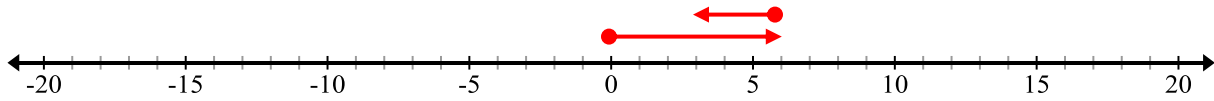
14. −3 + (−9) −12
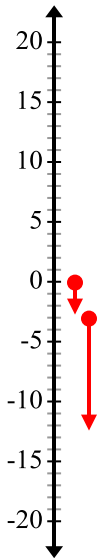
15. −4 + (−4) −8

Point out that there is NO "overlap" in #14 and #15, but in the next several problems there is. In #17, students may note: −9 + 8 = (−1 + −8) + 8 = −1 + (−8 + 8) = −1 + (0) = −1.
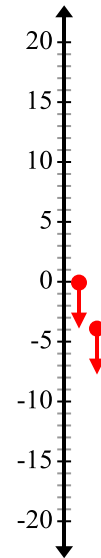
15. 6 + (−6)   0
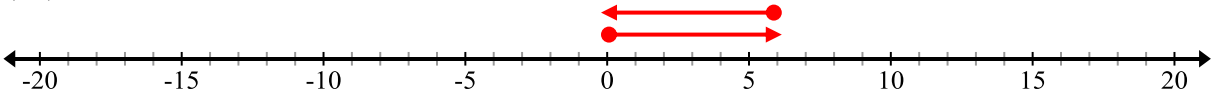
16. −9 + 8   −1
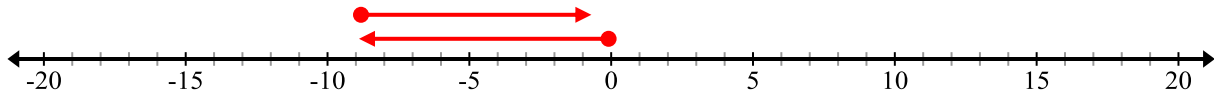
17. 20 + (−8) 12
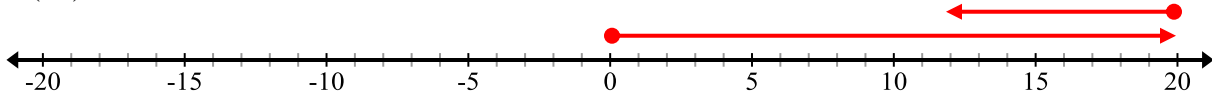
18. −12 + 14 2
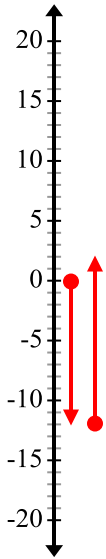
19. 9 + (−25) −16
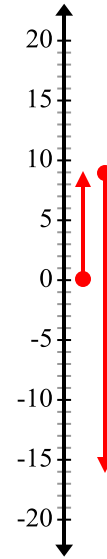
20. 12 + (−9) 3
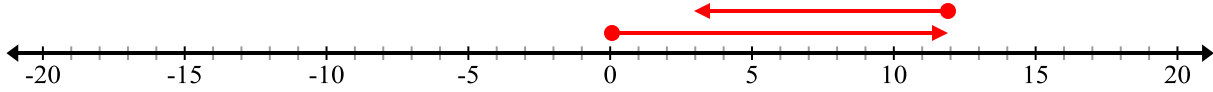
21. −8 + (−7) −15
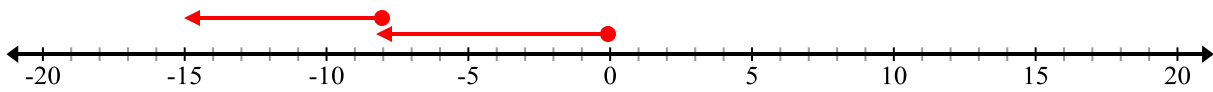
22. −7 + (−7) −14
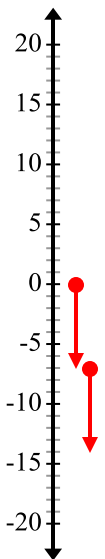
23. 13 + (−13) 0
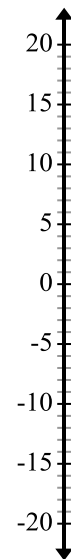
Below are the questions from the exploration at the beginning of Class Activity 2.1b. For each context, do the following:

    a. use a number line to model the problem situation
    b. answer the question in the context
    c. write an addition equation to show the sum
    d. explain how the model is related to how you found the answer when you answered the question

24. An osprey flies off the ground and reaches 35 feet above a river when he sees a trout. He then dives 37 feet down to get the trout. How many feet below the water does he end up?

a.
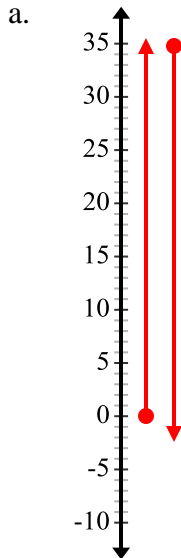
b. The osprey is 2 feet below the surface of the water.

c. $35 - 37 = -2$ or $35 + (-37) = -2$

d. Example: The model shows that I went from 0 up to 35 on the number line and then I went down 37 units from 35. This put me at –2. This represents the 2 feet below the surface when I found my answer.

25. Zach's football team moves the football 35 yards forward on the first down. On the next down, they lose 12 yards. On the down after that they go forward 8 yards. How many yards from the starting point did they move the football in the three downs?

a.
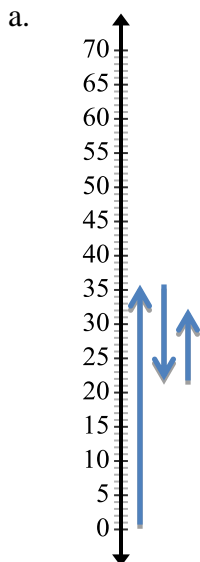
b. Zach's team is 31 yards farther down the field than when they started.

c. $35 + -12 + 8 = 31$

d. Answers will vary.

26. You walk 3 miles from your house to the store. At the store you meet up with a friend and walk with her 2 mile back towards your house. How far are you from your house now?

a.
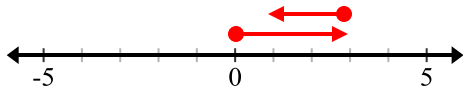
| b. I am 1 mile away from my house now. |
| --- |
| c. $3 - 2 = 1$ or $3 + (-2) = 1$ |
| d. Answers will vary. |

27. You ride your bike 12 miles and then get a flat tire! You turn around and walk the bike 8 miles before you mom is able to pick you up. How far are you from the house when your mom picks you up?

a.
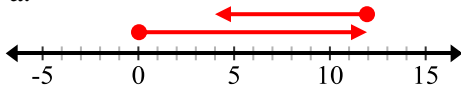
b. I am 4 miles away from my house.
c. $12 - 8 = 4$ or $12 + (-8) = 4$
d.   Answers will vary.

28. For the model below, write a numeric expression and then create a context.

$15 + -20$
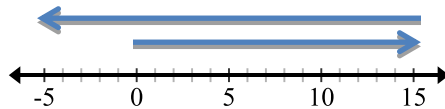
Contexts will vary.

2.1b Homework: Add Integers Using a Number Line

Circle the operation. Model the expressions using a number line and write the answer. The first one is done for you.

1. $12 + 5$ 17

2. $-7 + 5$
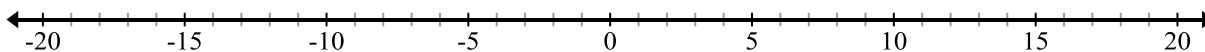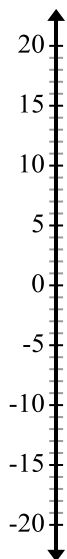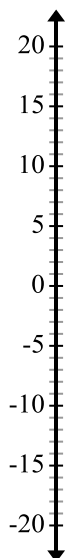
3. $9 + (-9)$ 0   4. $-11 + (-5)$ −   5. $8 + (-13)$ −
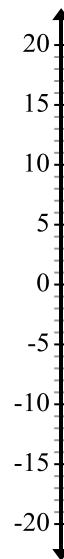
6. $-15 + (-4)$ −19
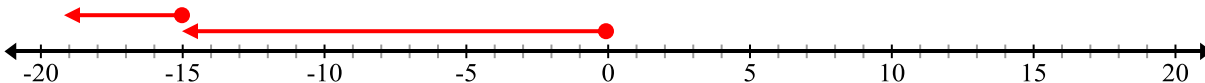
7. $11 + 9$
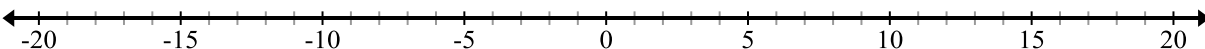
8. $7 + (-16)$
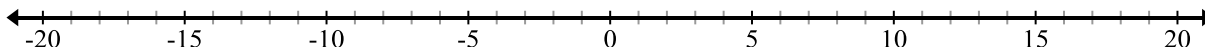
9. You borrowed $25 from your Mom to buy clothes. A week later you repaid her $13.

    a. Write an addition expression to show how much money you owe. (–25) + 13
       A numerical expression is a translation of the situation in words to a situation in numbers and/or symbols.

    b. Label the number line and model the expression. Circle the answer to the expression.

      –12
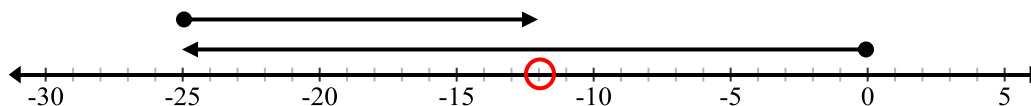
    c. Explain your expression and model.

      Answers will vary.

10. Write a context for the following the addition expression: $-15 + 25$

    a. Model the context using the number line.

11. Write a context and expression for the model; when state the answer.
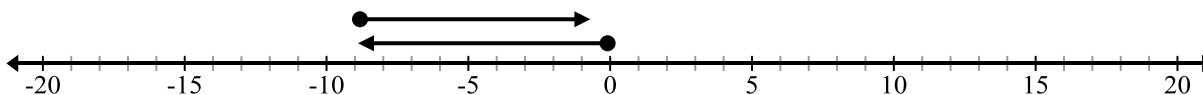
Spiral Review

1. Using bar notation, show $4\frac{1}{4} - 2\frac{1}{2}$.

   $1\frac{3}{4}$

2. Place the fractions on the number line below.

   $$\frac{6}{5}, \quad \frac{3}{10}, \quad \frac{1}{5}, \quad \frac{3}{2}$$
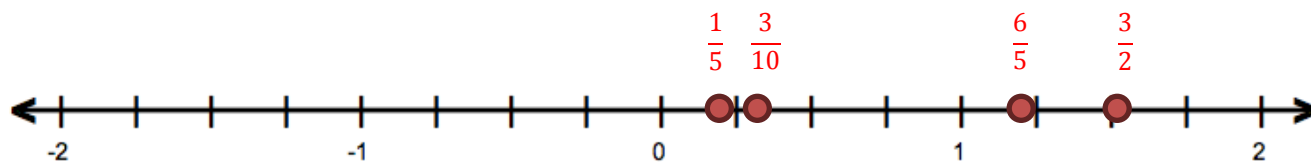
3. Write $\frac{1}{3}$ as a percent and decimal. $33.\overline{3}\%, \ 0.\overline{3}$

4. A spinner contains three letters of the alphabet.
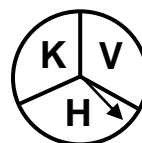
   a. How many outcomes are possible if the spinner is spun three times?
      27

   b. List all of the outcomes for spinning three times.
      KKK, KKV, KKH, KVK, KVV, KVH, KHK, KHV, KHH, VKK, VKV, VKH, VVK, VVV, VVH, VHK, VHV, VHH, HKK, HKV, HKH, HVL, HVV, HVH, HHK, HHV, HHH.
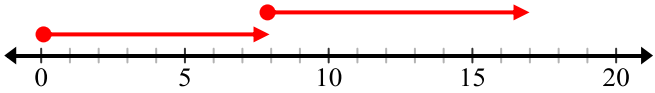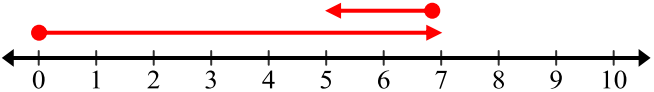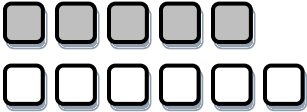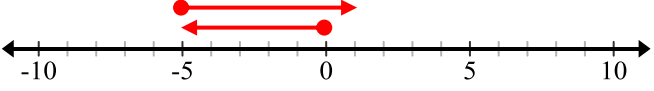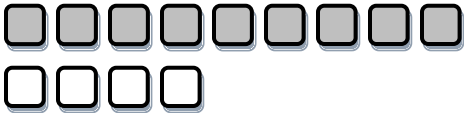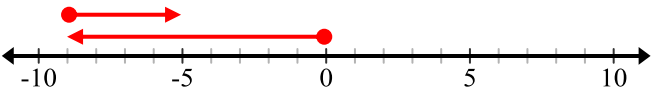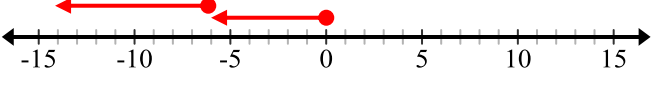
   c. What is the probability of getting exactly one H in three spins?
      $$\frac{12}{27} = \frac{4}{9} = 0.\overline{4} = 44.\overline{4}\%$$

2.1c Class Activity: Model Integer Addition with Chips/Tiles and Number Lines

Circle the operation you are going to perform. Model each integer addition exercise with chips/tiles (on the left) and with a number line (on the right). Record the sum.

Chips/Tiles	Number Line
1. 8 + 9 17	8 + 9
2. 7 + (−2) 5	7 + (−2)
3. −5 + 6 1	−5 + 6
4. −9 + 4 −5	−9 + 4
5. −6 + (−8) −14	−6 + (−8)

6. How are the models related? What do you notice about "zero pairs?"
   Students should explain that: $−9 + 4 = (−5 + −4) + 4 = −5 + (4 + −4) = −5 + (0) = −5$
7. Which model do you prefer and why? Students may use either model for thinking about addition of integers. However, the core explicitly states students must understand the number line.

Circle the operation you are going to perform. Use a model to find the sum for each problem # 8 - 15.

8. 5 + (–4) 1

5. –5 + 9 4

6. –2 + (–3) –5

7. 0 + (–9) –9

8. –1 + (–9) –10

9. –1 + 1 0

10. –2 + 6 4

11. –9 + (–7) –16

Circle the operation you are going to perform. Find the sum for each. Justify your answer.

12. 12 + (–9) 3

While students may not use models here, they should be able to justify their answers: 12 + (–9) = (3 + (9)) + (–9) = 3 + (9 + (–9)) = 3 + (0) = 3. Students should write a justification for some of these—either words, or as above.

13. –4 + (–7) –11

14. –3 + 8 5

15. 10 + (–11) –1

16. 7 + (–12) –5

17. –4 + (–9) –13

18. –5 + 9 4

19. 13 + 8 21

2.1c Homework: Compare Chips/Tiles and Number Line Models for Addition of Integers

Circle the operation you are going to perform. Model each integer addition problem with chips/tiles and with a number line. Record the sum.

Chips/Tiles	Number Line
1. $5 + 8$ 13	
2. $-4 + (-3)$	
3. $7 + (-9)$	

Circle the operation you are going to perform. Use the chip/tile or number line model to find the sum for each.	Circle the operation you are going to perform. Find the sum for each. Justify your answer.
4. $4 + (-4)$	12. $2 + (-7)$ –5
5. $-3 + 11$ 8	13. $-2 + (-5)$
6. $-3 + (-3)$	14. $-5 + 8$ 3
7. $0 + (-5)$ –5	15. $6 + (-12)$
8. $3 + 2$	16. $-5 + (-11)$
9. $-7 + (-4)$ –11	17. $3 + (-5)$ –2
10. $4 + (-3)$	18. $-4 + 6$
11. $7 + (-3)$ 4	19. $-9 + (-9)$ –18

Spiral Review

1. Order the following fractions from *least to greatest.*

   $$\frac{2}{3}, \quad \frac{3}{10}, \quad \frac{1}{2}, \quad \frac{3}{2}, \quad \frac{4}{3}$$

   $$\frac{3}{10}, \quad \frac{1}{2}, \quad \frac{2}{3}, \quad \frac{4}{3}, \quad \frac{3}{2}$$

2. Use a number line to solve $27 - 7$. **20**

   "Take-away" model

   OR

   "Difference" model
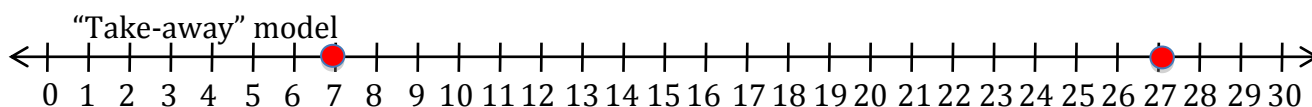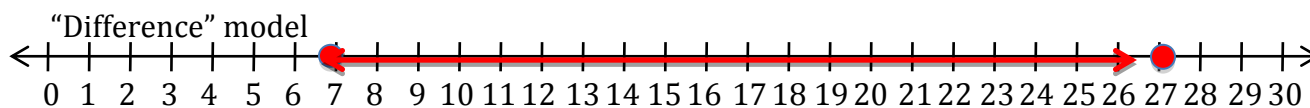
3. Write $\frac{27}{4}$ as a mixed number. Model to solidify understanding. $6\frac{3}{4}$
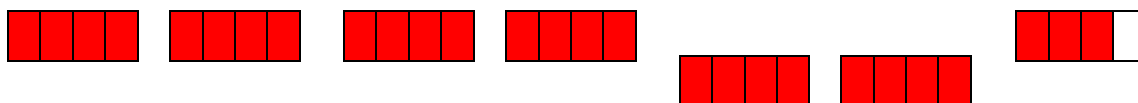
4. Write the fraction equivalent (in simplest form) for each decimal.

   a. 0.27 $\frac{27}{100}$ b. 0.35 $\frac{7}{20}$ c. 0.4 $\frac{2}{5}$ d. 0.125 $\frac{1}{8}$

5. Write 0.452 as a percent.

   45.2 %

2.1d Class Activity: Number Line Model for Subtraction

Plot each pair of points on the same number line.

1. 3 and 8
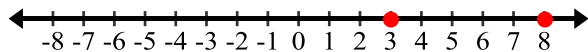
2. −4 and 3
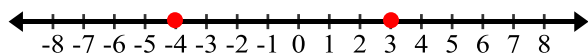
3. −1 and −7
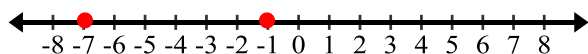
4. Circle the operation. Write a context for 8 − 3 and then state the difference. Stress that you are subtracting. Students will likely offer a "take away" context; e.g. "Maggie has 8 jelly beans. If Hugo eats 3 of her jelly beans, how many jelly beans will she have left?" This is a valid context, but push students to come up with a "comparison" context e.g. **"Maggie has 8 jelly beans and Hugo has 3 jelly beans, how many *more* jelly beans does Maggie have than Hugo?"** Point out that for both contexts the operation is subtraction. We will be using comparison in this section.

5. Circle the operation. Make a conjecture for 3 − 8. Ask students if this difference will be the same as 8 − 3, why/why not? Ask student to try to use the context from #4 for 3 − 8, e.g. **"Maggie has 8 jelly beans and Hugo has 3 jelly beans, how does the number of jelly beans Hugo has compare to that of Maggie?"** Refer to the number line. Help students see that the difference of both 8 − 3 and 3 − 8 have something to do with "5" because the *distance between the two numbers is 5 units*. The *direction* of the distance for 3 − 8 is opposite to that of 8 − 3. Thus the answers are opposite.

6. Circle the operation. Make a conjecture for the two differences below. Refer to the # 2 to help you.

$3 - (-4)$ 7 $(-4) - 3$ −7

Help students see that the distance between 3 and −4 is seven units. If we start at 3 and compare −4, we see that 3 (the start) is seven units to the ***right*** of −4, so the difference is positive 7. However, if we start at −4, and compare 3, while the distance is still 7 units, −4 (the start) is to the LEFT of 3, so the difference is −7.

7. Circle the operation. Make a conjecture for the following two differences below. Refer to #3 to help you.

$-1 - (-7)$ 6 $(-7) - (-1)$ −6

Subtraction can be thought of as the "*signed* distance" or "*directional* distance" between two points.

a. Model 7 − 2 on the number line.	b. Model 2 − 7 on the number line.
A number line from −8 to 8 with an arc from 2 to 7. Point at 2 labeled "f" and point at 7 labeled "s" with a blue arrow below.	A number line from −8 to 8 with an arc from 2 to 7. Point at 2 labeled "s" with a blue arrow below and point at 7 labeled "f".
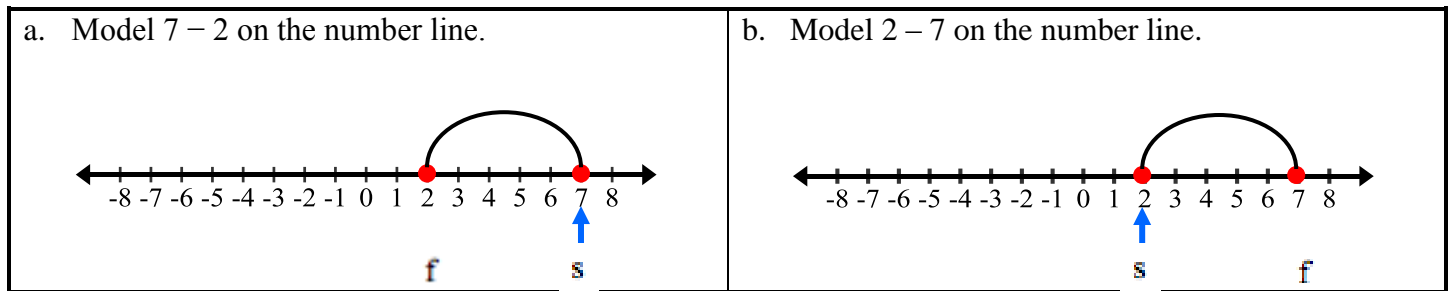
***The "s" is where we *start* and the "f" is where we *finish*.** The arrow is below the start to note if the start is bigger or smaller than the finish. For 7 − 2, the start is bigger (*to the right of*) the finish, so the answer is (positive) 5. For 2 − 7, the start is smaller, *to the left of* the finish, so the answer is −5.

For both **a** and **b** the difference is $|5|$. How can you use a number line to know if the difference is represented with a positive or negative integer?

Students studied absolute value in 6th grade. In 2.1e (the next class activity) you will formalize $a - b$ is the same as $a + -b$ and $a - (-b)$ is the same as $a + b$. The purpose of circling this operation is to focus students on what they are doing.

8. Circle the operation. Model 8 − 7 on the number line.
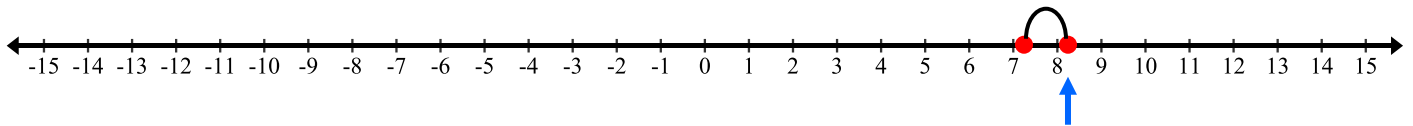
A number line from −15 to 15 with a small arc between 7 and 8; points at 7 and 8 with a blue arrow below 8.

With the subtraction circled, emphasize that we can think, "How far apart are 8 and 7?" 8 is the start, it's one unit from 7 and it's bigger than 7, so the difference is 1."

9. Circle the operation. Model 8 − (−7) on the number line. Circle the subtraction. Say, "8 is how far from −7? It is 15 units right. Thus the difference is 15."
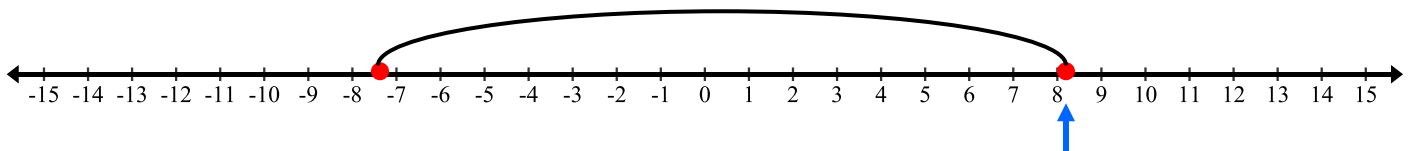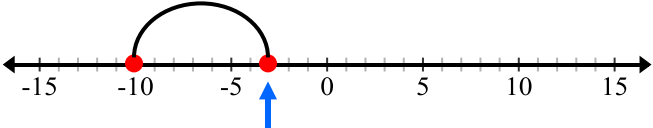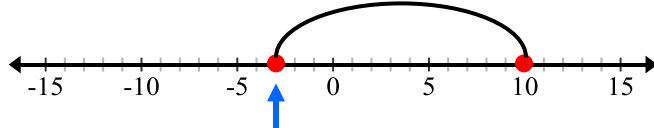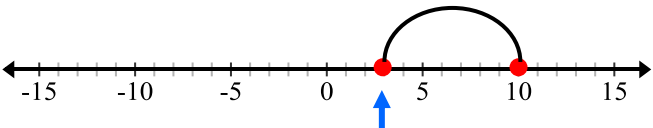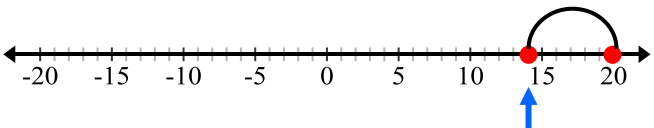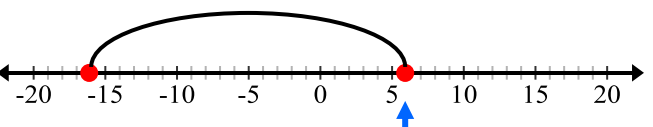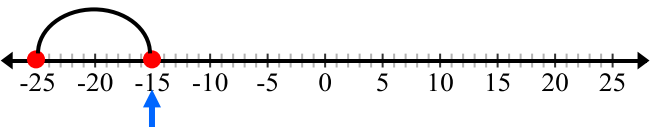
A number line from −15 to 15 with a large arc from −7 to 8; points at −7 and 8 with a blue arrow below 8.

With the subtraction circled, emphasize that we can think, "how far apart are 8 and −7?" 8 is the start, it's 15 units away from −7 and it's bigger than (to the right of) −7, so the difference is 15."

10. Explain why the answers for the two problems above are so different.

#8 we are comparing 8 and 7, they are only one unit apart and 8 is to the right of 7. For #9, we are comparing 8 and −7. These two numbers are 15 units apart and 8 is to the right of −7 so the difference is 15.

11. What is −8 − 7? Explain. −15. Emphasize that circling the operation here makes it clear we are subtracting (finding the directional distance). Talk about this relative to #10. −8 is fifteen units to the left of 7, thus the difference is −15.

Circle the operation. Locate each value in the difference expression on the number line and compute the difference.

−3 is **7 units** to the **right** of −10
12. −3 − (−10)   7

−3 is **13 units** to the **left** of 10
13. −3 − 10   −13

3 is **7 units** to the **left** of 10
14. 3 − 10   −7

14 is **6 units** to the **left** of 20
15. 14 − 20   −6

6 is **22 units** to the **right** of −16
16. 6 − (−16)   22

−15 is **10 units** to the **right** of −25
17. −15 − (−25)   10

Circle the operation. Draw a number line, locate each value in the difference expression on the number line, and compute the difference.

18. −14 − 1 −15

22. −13 − (−13)   0

19. 9 − 15 −6

23. 13 − 13   0

20. 14 − 18 −4

24. 15 − (−8)   23

21. −12 − (−9) −3

25. −10 − (−6)   4

2.1d Homework: Number Line Model for Subtraction

Circle the operation. The model is provided. Find the difference. The first one is done for you.

1. $4 - 17$ Answer: -13
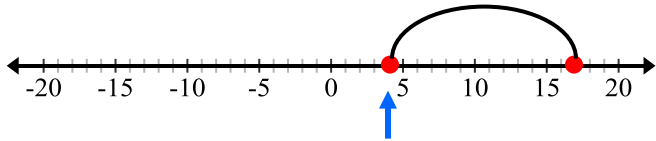
2. $5 - (-10)$
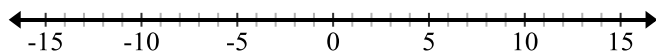
3. $-7 - 6$ -13
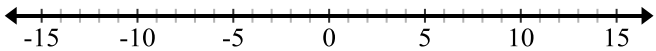
4. $-8 - (-9)$ 1
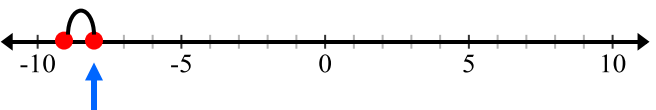
5. $9 - 4$
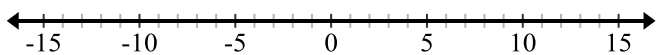
Circle the operation. Draw your own number line. Find the difference for each.

6. $7 - (-3)$ 10

7. $8 - (-4)$

8. $5 - 12$ -7

9. $6 - 18$

10. $14 - 3$ 11

11. $21 - 5$

12. $-13 - 17$ -30

13. $-8 - 3$

14. $-9 - (-4)$ -5

15. $-2 - (-6)$

Spiral Review

1. Use the chip model to find $-12 + 5$.

2. What percent of 80 is 60? 75%

80
20	20	20	20

$\leftarrow$ 75% $\rightarrow$

$\leftarrow$ 100% $\rightarrow$
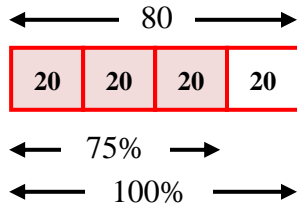
3. Camilla earned $160 over the summer. If she put 80% of her earnings into her savings account how much money did she have left over?
$32

4. Order the numbers from least to greatest.

17/7, 2.7, 2.15, 2.105   **2.105, 2.15, $\frac{17}{7}$, 2.7**

2.1e Class Activity: More Subtraction with Integers

1 – 20 Addition and Subtraction exercises. CIRCLE the operation. State the sum or difference.

1. $3 - 14$ −11	2. $3 + (-14)$ −11
3. $5 + (-13)$ −8	4. $5 - 13$ −8
5. $6 - 13$ −7	6. $6 + (-13)$ −7
7. $9 - 8$ 1	8. $9 + (-8)$ 1
9. $-6 - (14)$ −20	10. $14 + 6$ 20
11. $8 + 3$ 11	12. $8 - (-3)$ 11
13. $5 + 12$	14. $5 - (-12)$
15. $-3 + 18$ 15	16. $-3 - (-18)$ 15
17. $-7 + 4$	18. $-7 - (-4)$
19. $-15 - (-20)$ 5	20. $-15 + 20$ 5

Look back at the previous exercises. Which is easier to think about, addition or subtraction? What pattern do you notice with the subtraction exercises?

Students will likely be more comfortable with addition. Ask: is there any way to think about subtraction as addition? TAKE TIME here. Gradually move to: $a - b = a + (-b)$ or $a - (-b) = a + b$. You might want to examine with students numbers 5, 9, 12, and 14 as they begin to look for patterns. Have students write the pattern they see with examples, e.g. $6 - 13 = 6 + -13$ or $8 - (-3) = 8 + 3$.

n#

Write an equivalent addition expression for each subtraction expression and then simplify the expression.

Subtraction Expression	Addition Expression	Simplified
Example: $3 - 5$	$3 + (-5)$	-2
21. $-2 - 6$	$-2 + (-6)$	-8
22. $9 - 14$	$9 + (-14)$	-5
23. $7 - (-4)$	$7 + 4$	11
24. $-10 - (-7)$	$-10 + 7$	-3
25. $12 - 8$	$12 + (-8)$	4
26. $-5 - 3$		
27. $-1 - (-15)$		
28. $6 - (-8)$		

Write each term as two equivalent expressions both involving subtraction.
Example: -4; $-7 - (-3)$ or $3 - 7$

29. -2	30. -4	31. -1
Answers will vary. For example: $-5 - (-3)$ or $3 - 5$	$-7 - (-3)$ or $3 - 7$	

For these problems we want students to think about directional distance. They should identify two numbers that are 2 units apart and then write a difference expression so that the first term is to the left of the second: $2 - 4$; $-2 - 0$; $-7 - (-9)$.

Write each quantity below as two equivalent numeric expressions, one involving addition and the other involving subtraction. **Example:** -3; -3 can be written as $4 - 7$ or $4 + -7$; 5 can be written as $8 - 3$ or $8 + (-3)$

32. 4	33. -2	34. -4
Answers will vary. Example: $6 - 2$ or $6 + (-2)$	$2 - 4$ or $2 + (-4)$	

For these problems we want students to think about how directional distances is related to sums.

35. How will you determine if " $-$ " means subtract or negative in a numeric expression? Students will have a variety of responses. Help them notice that subtraction expressions can be rewritten as addition expressions. If students recognize they are working with subtraction (again emphasize the importance of circling the operation), they can continue to simplify with subtraction OR they might rewrite the difference expression as a sum expression: $a - b = a + (-b)$ or $a - (-b) = a + b$.

Answer the two questions below. Draw a model to justify your answer.

36. Louis picks up a football that has been fumbled by his quarter back 12 yards behind the line of scrimmage. He runs forward 15 yards before he is tackled. Where is he relative to the line of scrimmage?

    −12 + 15 or 15 − 12

    He is 3 yards ahead of the line of scrimmage

37. Leah owes her friend seven dollars. Leah's friend forgives three dollars of her debt. How much does Leah now owe her friend? Forgiving a debt is like removing a debt. This will likely be difficult wording for students. However, they will likely have an intuitive understanding that Leah now only owes 4 dollars. −7 − (−3) or − 7 + 3 or 3 + −7

    Leah owes her friend 4 dollars; e.g. −4

Circle the operation. Find the indicated sum or difference. Draw a number line or chip/tile model when necessary. You may change the expression to addition when applicable.

38. 2 − 8 −6	39. −3 + 7	40. 3 − (−2) 5
41. −2 − (−5) 3	42. −4 − 5 −9	43. −1 + −4
44. 7 − 3	45. 5 + −3 2	46. 6 − 9 −3

2.1e Homework: More Subtraction and Integers

Write an equivalent addition expression for each subtraction expression and then simplify the expression.

Subtraction Expression	Addition Expression	Simplify
1. −7 − 3	−7 + (−3)	−10
2. 6 − 11		
3. 12 − (−3)	12 + 3	15
4. −6 − (−8)		
5. 14 − 10	14 + (−10)	4
6. −2 − 7		
7. −8 − (−10)	−8 + 10	2
8. 5 − (−9)		

Circle the operation. You may change the expression to addition when applicable. Find the indicated sum or difference. Draw a number line or chip/tile model when necessary. Student models will vary.

9. 3 − (−9) 12	10. −4 + (−1)
11. −3 − (−7)	12. 8 − 5 3
13. 4 − 9	14. 5 − (−2) 7
15. −3 − 4 −7	16. −5 + (−2) −7
17. 3 − (−1)	18. 2 − 6
19. −5 + 10	20. −2 − (−9) 7

Spiral Review

1. What is 80% of 520?

   416

2. Suppose you were to roll a fair 6-sided number cube once, then flip a coin. List all the possible outcomes.

   1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T

Using the information in #2, answer the questions below. Write all probabilities as fractions, decimals and percentages.

3. What is the probability of getting a 2 and heads from the experiment above? $\frac{1}{12} = 0.08\overline{3} = 8.\overline{3}\%$

4. What is the probability that you would roll an even number and flip heads? $\frac{3}{12} = \frac{1}{4} = 0.25 = 25\%$

5. What is the probability that you would roll an even number <u>or</u> flip heads? $\frac{9}{12} = \frac{3}{4} = 0.75 = 75\%$

2.1f Class Activity: Applying Integer Operations

Use a chip or number line model to solve each of the following. Write a numerical expression that models your solution and then write a complete sentence stating your answer.

Student models and expressions will vary.

1. At the Masters Golf Tournament, Tiger Woods' score is –3 and Phil Mickelson's score is –5. Who is in the lead and by how many shots? (Note: In golf, the lower score wins.)

    –5 – (–3)
    Phil Mickelson leads by 2 shots.

2. Steve found a treasure map that said to take ten steps north, then ten steps south. Where is Steve in relationship to where he started?

    10 + (–10)
    Steve is standing where he began.

3. Tom left school and walked 5 blocks east. Charlie left school and walked the same distance west. How far apart did the two friends end up?

    5 – (–5)
    Charlie and Tom are 10 blocks apart.

4. Ricardo's grandmother flew from Buenos Aires, Argentina to Minnesota to visit some friends. When Ricardo's grandmother left Buenos Aires the temperature was 84°. When she arrived in Minnesota it was –7°. What was the temperature change for Ricardo's grandmother?

    84 – (–7)
    She has experienced a 91° change.

5. The elevator started on the 8th floor, went up 5 floors, and then went down 2 floors. What floor is it on now?

    8 + 5 – 2
    The elevator is on the 11th floor.

6. The temperature rose 13°F between noon and 5 p.m. and then fell 7°F from 5 p.m. to 10 p.m. If the temperature at noon is 75°F, what would the temperature be at 10 p.m.?

    75 + 13 – 7
    At 10 p.m. the temperature is 81°.

7. Paula was standing on top of a cliff 35 feet above sea level. She watched her friend Juan jump from the cliff to a depth of 12 feet into the water. How far apart are the two friends?

    35 – (–12)
    The two friends are 47 feet apart.

8. The carbon atom had 6 protons and 6 electrons. What is the charge of the atom?

    6 + (–6)
    The atom is neutral.

9. The Broncos got possession of the football on their own 20 yard line. They ran for an 8 yard gain. The next play was a 3 yard loss. What is their field position after the two plays?

    20 + 8 + (–3)
    The Broncos are on the 25 yard line.

10. Samantha earned $14 for mowing the lawn. She then spent $6 on a new shirt. How much money does she have now?

    14 – 6
    Samantha has $8.

2.1f Homework: Applying Integer Operations

Use a chip or number line model to solve each of the following. Write a numerical expression that models your solution and then write a complete sentence stating your answer. Student models will vary.

1. Chloe has $45 in her bank account. After she went shopping, she looked at her account again and she had –$12. How much did she spend shopping?

   $45 – (–12)$
   Chloe spent $57.

2. There is a Shaved Ice Shack on 600 South. Both Mary and Lina live on 600 South. Mary lives 5 blocks west of the shack and Lina lives 3 blocks east of it. How far apart do they live?

3. Eva went miniature golfing with Taylor. She shot 4 under par and Taylor shot 2 under par. By how many strokes did Eva beat Taylor?

   $–4 – (–2)$
   Eva won by 2 strokes (–2); in golf you want a lower score.

4. Mitchell owes his mom $18. He's been helping around the house a lot, so his mom decided to forgive $12 of his debt. How much does he owe now?

5. On the first play of a possession Drew Brees was sacked 4 yards behind the line of scrimmage. He then threw a pass for a 12 yard gain. Did the Saints get a first down?

   $–4 + 12$
   Drew Brees' team has made a gain of 8 yards. This is not enough for a first down.

6. Joe had $2. He found a quarter, but he lost a dollar in the vending machine. How much money does he have now?

Write two stories that you could model with positive and negative numbers, then model and solve both.
Answers will vary.

Spiral Review

1. What is the greatest common factor of 48 and 36?

   12

2. Fill in the equivalent fraction and percent for this decimal:

Fraction	Decimal	Percent
$\frac{4}{5}$	0.8	80%

3. Solve each of the following problems with or without a model.

   a) Wanda has 90 bracelets. She sells $\frac{1}{3}$ of the bracelets. How many bracelets did she sell?

   30

   $168

   b) Chad invested $840. If he earned 20% on his investment, how much interest did he earn?

   $168

   How much does he have now?

   $1,008

4. Write a context for the following numeric expression: $24(1) + 24\left(\frac{1}{4}\right)$

2.1f Integer Addition and Subtraction Practice

Solve.

1. $-4 + 9 =$ 5

2. $8 + (-8) =$

3. $3 - 4 =$ −1

4. $-7 + 8 =$

5. $4 - 2 =$

6. $-5 - 13 =$ −18

7. $8 + 12 =$

8. $6 - (-7) =$ 13

9. $-9 + (-10) =$

10. $-6 - (-8) =$ 2

11. $14 + 18 =$

12. $-4 - 4 =$

13. $-3 + (0 - 9) =$ −12

14. $5 - (-6) =$ 11

15. $4 + (-10) =$

16. $6 - 9 =$ −3

17. $8 + 3 =$

18. $10 - 11 =$ −1

19. $-12 + 7 =$ −5

20. $3 - 8 =$ −5

21. $9 - (-11) =$

22. $8 - (-2) =$ 10

23. $-7 - 8 =$ −15

24. $-8 + 10 =$

25. $-5 - 21 =$

26. $8 + (-10) =$ −2

27. $9 + (-1) =$

28. $8 - (-11) =$

29. $9 - 6 =$

30. $7 - (-4) =$ 11

2.1g Self-Assessment: Section 2.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Use a concrete model (chips/tiles or number line) to add integers. [1]	I can draw a model of integer addition. I struggle to use that model to add integers.	Most of the time when I use a model to add integers I get the right answer.	I always get the right answer when I add integers with a model.	I always get the right answer when I add integers with a model. I can explain how the model is related to the sum.
2. Use a concrete model (chips/tiles or number line) to subtract integers. [2]	I can draw a model of integer subtraction. I struggle to use that model to subtract integers.	Most of the time when I use a model to subtract integers I get the right answer.	I always get the right answer when I subtract integers with a model.	I always get the right answer when I subtract integers with a model. I can explain how the model is related to the difference.
3. Find the sums of integers accurately without a model. [3]	I struggle to add integers without a model.	Most of the time when I add integers without a model I get the right answer.	I always get the right answer when I add integers without a model.	I always get the right answer when I add integers without a model. I can explain how I got my answer.
4. Find the differences of integers accurately without a model. [4]	I struggle to subtract integers without a model.	Most of the time when I subtract integers without a model I get the right answer.	I always get the right answer when I subtract integers without a model.	I always get the right answer when I subtract integers without a model. I can explain how I got my answer.
5. Solve contextual problems involving adding or subtracting integers. [5]	I struggle to solve contextual problems involving integers.	I can usually write an expression to solve a contextual problem involving integers, but I struggle using that expression to get an answer.	I can solve contextual problems involving integers.	I can solve contextual problems involving integers. I can explain the solution in context.

Sample Problems for Section 2.1

1. Draw a model (chips/tiles or number line) to solve each of the following addition problems.
    a. $4 + 7$ c. $1 + (-7)$

    b. $4 + (-6)$ d. $-3 + 7$

2. Draw a model (chips/tiles or number line) to solve each of the following subtraction problems.
    a. $7 - 9$ c. $9 - (-2)$

    b. $-6 - (-3)$ d. $-4 - 7$

3. Find each sum without a model.
    a. $3 + 6$ c. $98 + (-1)$

    b. $-7 + (-5)$ d. $-6 + 7$

4. Find each difference without a model.
    a. $16 - 29$ c. $5 - (-3)$

    b. $-2 - (-8)$ d. $-90 - 87$

5. Solve each of the following contextual problems involving integers.

    a. Juan's football team gains 3 yards on one play. On the next play, the quarterback is sacked for a loss of 10 yards. What was the net change in their position?

    b. Kathryn, Loralie, and Madison are playing golf. Kathryn ends with a score of −8. Loralie's score is −4. Madison scores +5. What is the difference between the scores of Madison and Kathryn?

    c. Nantai is hiking in Death Valley. He starts out at Badwater Basin, the lowest point of Death Valley at an elevation of 282 feet below sea level. He walks northward towards Telescope Peak in the Panamints and reaches an elevation of 2400 feet. How much did his altitude change?

    d. Mr. O'Connor lives in North Dakota. When he leaves for work one wintry morning, the temperature is −4° C. By the time he comes home, the temperature has increased 25°. What is the temperature when he comes home?

Section 2.2: Multiply and Divide Integers; Represented with Number Line Model

Section Overview:

In Section 2.2 students encounter multiplication with integers. The goal for students in this section is two-fold: 1) fluency with multiplication and division of integers and 2) understanding how multiplication and division with integers is an extension of the rules of arithmetic as learned in previous grades.

This section starts with a review of concepts from elementary mathematics. The mathematical foundation more formally explains multiplication in this manner: "if *a* and *b* are non-negative numbers, then $a \cdot b$ is read as '*a* times *b*,' and means the total number of objects in *a* groups if there are *b* objects in each group." Students will apply this concept to integers. For example, students understand 3×5 as 3 groups of 5 or $5 + 5 + 5$ or 15. With the same logic students move to finding $3 \times (-5)$. Here they should reason: "this mean 3 groups of (–5) or (–5) + (–5) + (–5) or –15." Students will model these types of products on the real line. Next they move to products like: $(-3) \times 5$. There are two ways a student may attack this using rules of arithmetic: one way is to recognize that $(-3) \times 5$ is the same as $5 \times (-3)$ (e.g. the commutative property) and then apply the logic above: $-3 + -3 + -3 + -3 + -3$ or –15. The other is to rely on their understanding of integers from 6th grade. There they developed the idea of –3 by recognizing that it is simply the "opposite" of 3. Thus, $(-3) \times 5$ is the same as "the opposite of 3 times 5" or $-(3) \times 5$ or $-(3 \times 5)$ or $-(15)$ or –15 (here they are applying the associative property). Students then find products such as $(-3) \times (-5)$. Using the logic they developed with the previous example, student should reason: $(-3) \times (-5)$ can be thought of as "the opposite of 3 times –5" or $-(3) \times (-5)$ or $-(3 \times (-5))$ or $-(-15)$ or 15. This will all be modeled on the real line. Before students move to division, they will formalize that a product involving an even number of negatives is positive while a product involving an odd number of negatives is negative (assuming none of the factors is 0). There is a more thorough discussion of these ideas in the mathematical foundation.

Students begin the transition to division of integers by reviewing the relationship between multiplication and division:

$$(3)(5) = 15, \text{ so } \frac{15}{5} = 3 \text{ and } \frac{15}{3} = 5$$

The mathematical foundation describes this relationship as follows, "If *a* and *b* are any integer, where $b \neq 0$ then $a \div b$ is the unique (one and only one) integer (or rational number) *c*, such that $a = bc$." Since division and multiplication are inverse operations, it comes as no surprise that the rules for dividing integers are the same for multiplication.

Concepts and Skills to be Mastered (from standards)

By the end of this section, students should be able to:
1. Use a model to multiply integers
2. Explain why the product of a positive and negative integer is negative or the product of a negative and negative integer is positive.
3. Use rules for multiplying and dividing integers accurately.
4. Add, subtract, multiply and divide integers fluently.
5. Solve contextual problems involving integers.

2.2 Anchor Problem

In groups of 2-3 answer the following questions. Use words and pictures to provide evidence for your answer.

1. Miguel has $500 in the bank. Each week for 12 week he puts $50 in to his account. How much money will he have at the end of 12 weeks?

| $500 | | $50 | $50 | $50 | $50 | $50 | $50 |

| | $50 | $50 | $50 | $50 | $50 | $50 |

2. Rick borrowed $7 from his brother every week for 5 weeks. What is Rick's financial situation with his brother at the end of the 5 weeks?
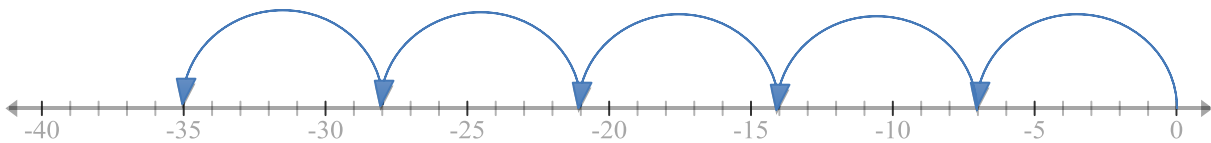
3. Lana wants to take a picture of her school. She stands on the sidewalk in front of her school, but she is too close to the school for her camera to capture the entire school in the frame. She takes 10 steps backwards. If each step is about 1.5 feet in length, where is she relative to where she started?

$$1.5 + 1.5 + 1.5 + 1.5 + 1.5 + 1.5 + 1.5 + 1.5 + 1.5 + 1.5$$

4. Lisa owes her mom $78. Lisa's mom removed 4 debts of $8 each when Lisa did some work for her at her office. What is Lisa's financial situation with her mom now?

$8 $8 $8 $8

$78
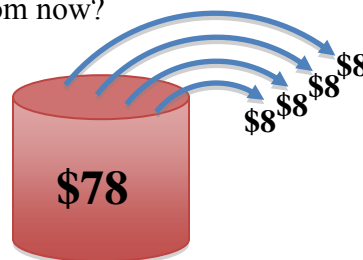
2.2a Class Activity: Multiply Integers Using a Number Line

Review:
In elementary school you modeled multiplication in several ways: arrays, skip counting, number lines, blocks, etc. You learned:

First factor tells you: How many groups	**X**	**Second factor** tells you: How many units are in each group	**=**	**Product** describes: The total with which you end
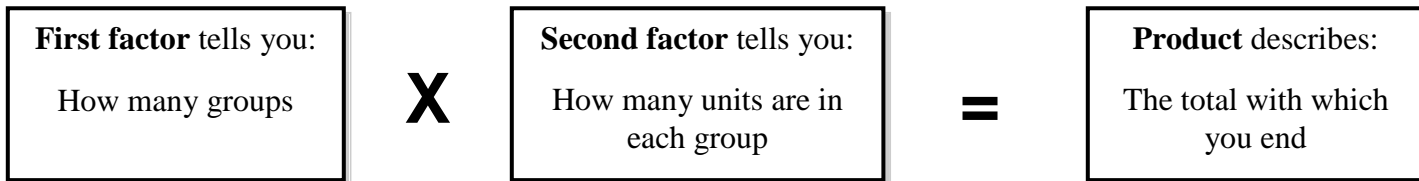
For example, you learned that 3×5 means three groups of 5 units in each group. The "product" is the total number of object. Three representations for 3×5 are:
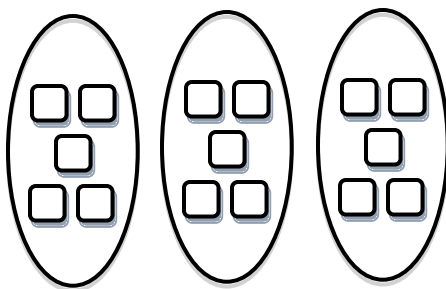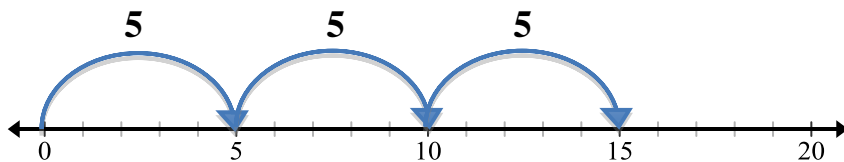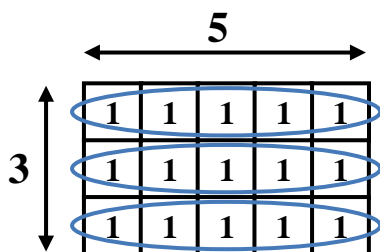
In this section we will apply this understanding to multiplication of integers

1. Show how you might use a model to do the following:

a. $3 \times (-5)$	b. $(-3) \times 5$	c. $(-3) \times (-5)$
		$(-3)(-5) = -(3)(-5)$. Recall that -3 is the "opposite of 3." Thus we want *the opposite* of 3 groups of (-5)

a.

$3 \times (-5)$

-5

-5

-5

-5

b.

$(-3) \times 5$

5

-1	-1	-1	-1	-1
-1	-1	-1	-1	-1
-1	-1	-1	-1	-1

Number line model with property: 5 x (-3) by the commutative property is -15.

c.

$(-3) \times (-5)$

3 groups of –5 is –15. Now apply "the opposite." The result is 15

See number line model below.

2. How are integers (positive and negative numbers) related to the models for multiplication?

opposite

-3   -3   -3
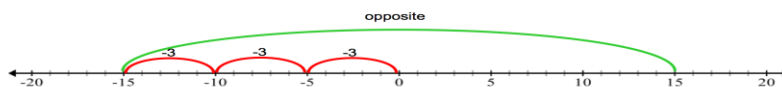
Write the meaning of each multiplication problem in terms of a number line model and then do the multiplication on the number line provided.

3. 2×3

   Means: Two groups of three.
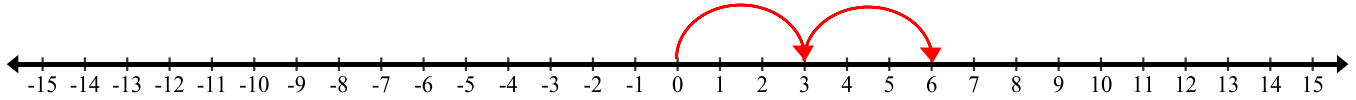
4. $2 \times (-3)$

   Means: Two groups of negative three.
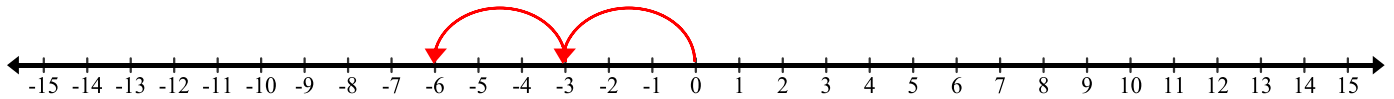
5. -2×3

   Means: The opposite of two groups of three.
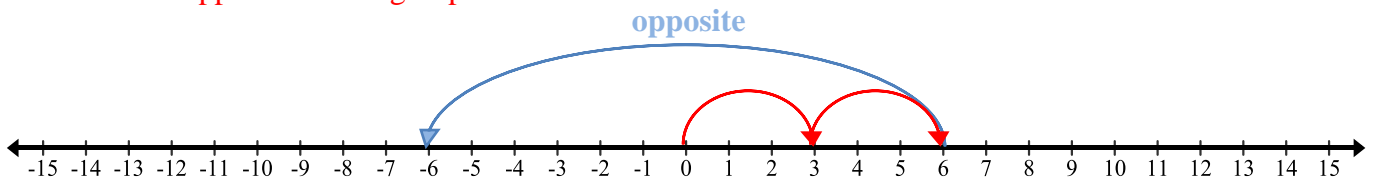
6. $-2 \times (-3)$

   Means: The *opposite* of two groups of negative three. $-2 \times (-3) = -(2 \times (-3))$..
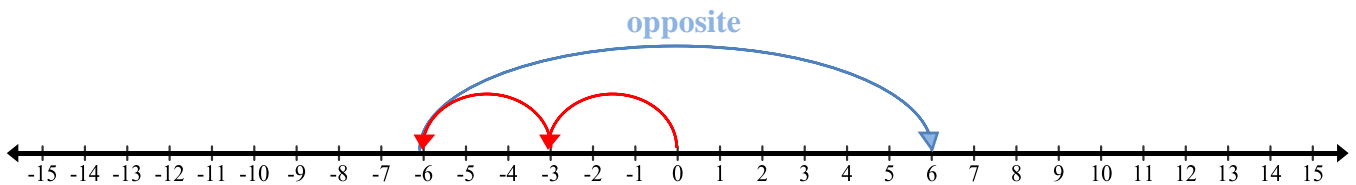
Model each multiplication problem using a number line. Write the product in the space provided.

7. $6 \times (-3) = -18$

8. $6 \times 3 = 18$

opposite

9. $-6 \times (-3) = 18$

$-(6 \times (-3)) = -(-18) = 18$

opposite

10. $-6 \times 3 = -18$

$-(6 \times 3) = -(18) = -18$

11. $2 \times (-5) = -10$

opposite

12. $-5 \times 2 = -10$

$-(5 \times 2) = -(10) = -10$

opposite

13. $-4 \times (-3) = 12$

opposite

14. $-4 \times 3 = -12$

opposite

15. $-6 \times (3) = -18$

16. $5 \times (-4) = -20$
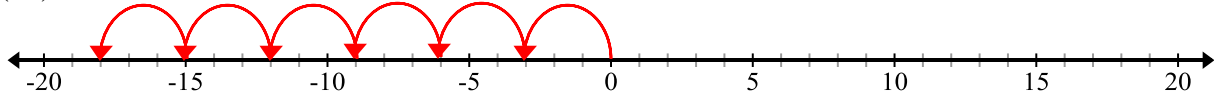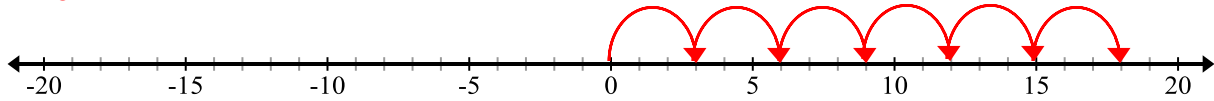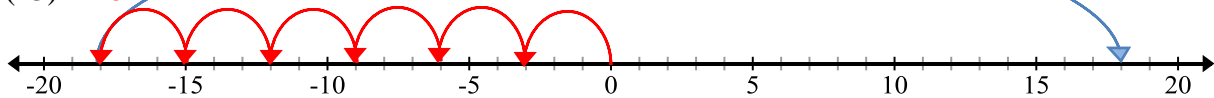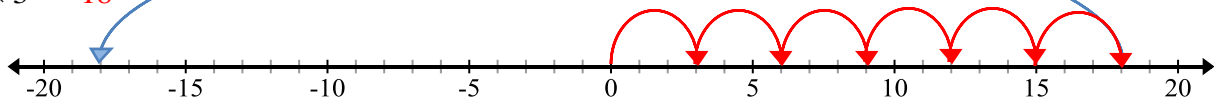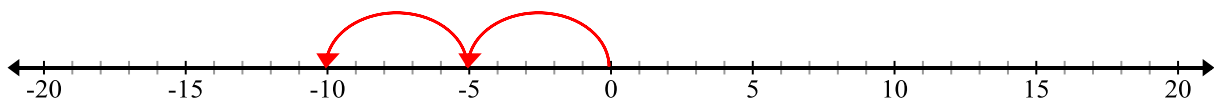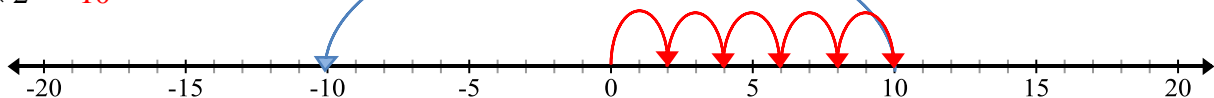
17. $0 \times (-4) = 0$
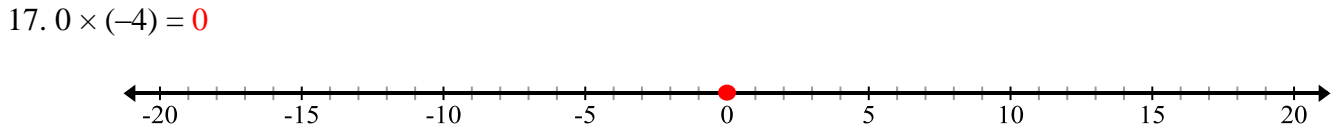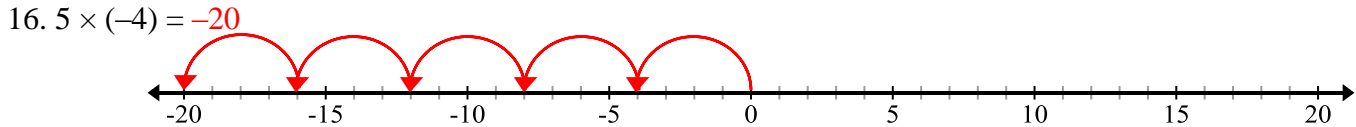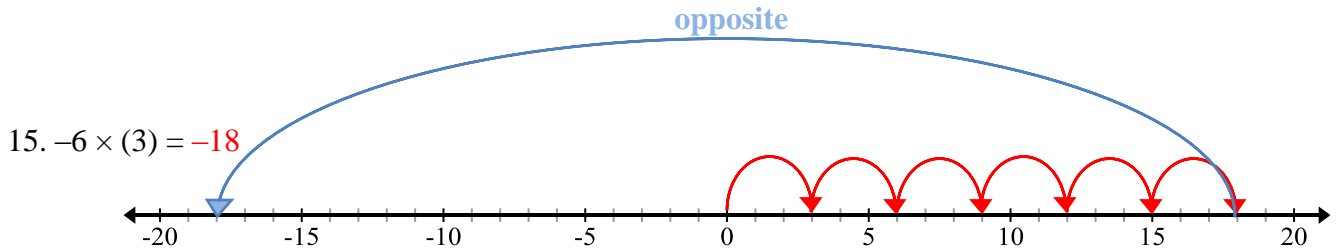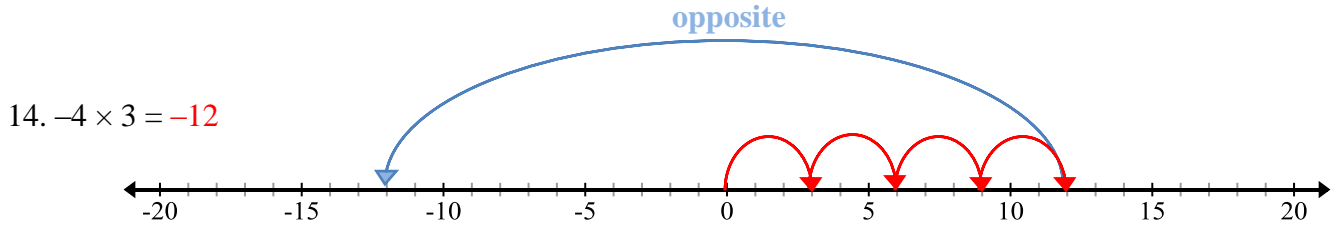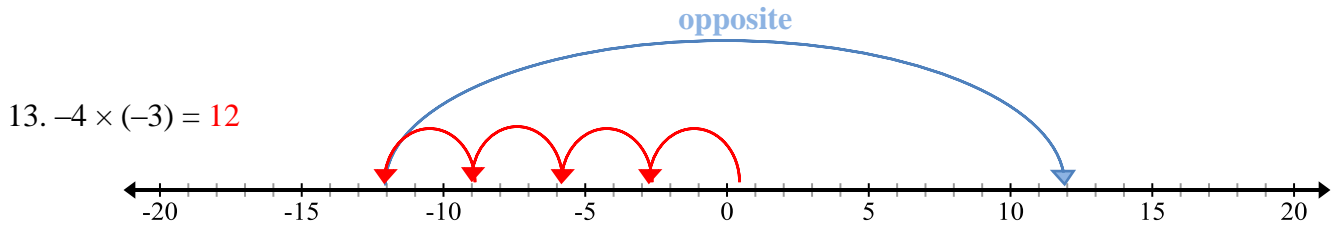
2.2a Homework: Multiply Integers

Write the meaning of each multiplication problem using a number line model. Then model the problem on the number line provided. Record the product. Look back at 2.2a Classroom Activity for assistance.

1. $3 \times 4 = 12$

   Means: Three groups of 4.

   Model:
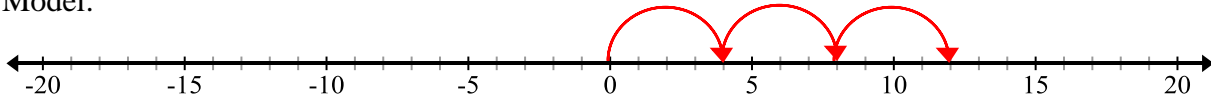
2. $3 \times (-6) =$

   Means:

   Model:
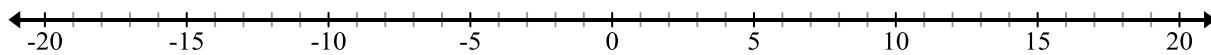
3. $-2 \times 6 = -12$

   Means: The opposite of two groups of 6.
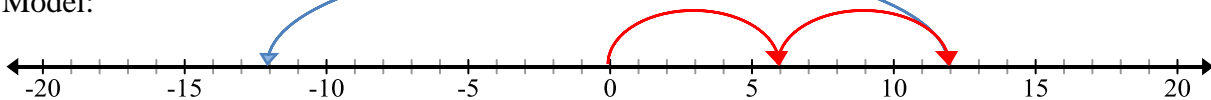
   Model:

4. $-5 \times (-3) = 15$
   Means: The opposite of five groups of negative 3.

   Model:
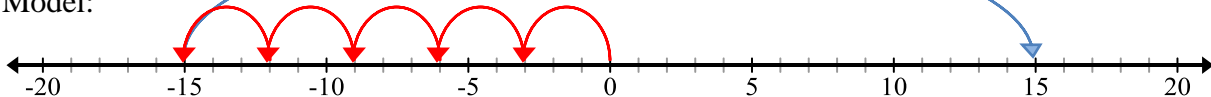
5. $-7 \times (-2) =$
   Means:

   Model:
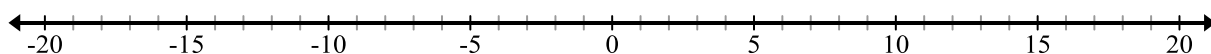
6. $-3 \times 4 =$

Means:

Model:
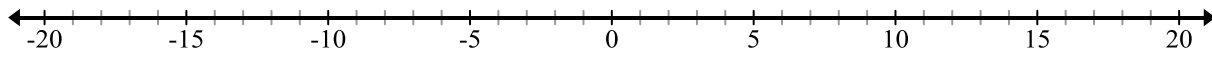
7. $-1 \times (-1) = 1$

Means: The opposite of one group of negative 1.

Model:
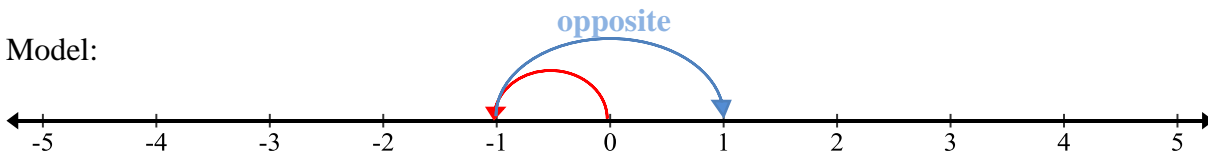
8. $-5 \times 0 =$

Means:

Model:
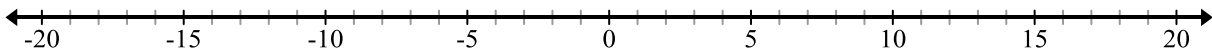
Spiral Review

1. Daniel left a $9 tip for the waiter at a restaurant. If the tip was 15% of the bill, how much was the bill? Solve with or without a model.

    $60

2.   What is the greatest common factor of 65 and 39?
        13

3.   In Mr. Garcia's 7th Grade Math class, $\dfrac{4}{5}$ of the students brought a pencil to class. If 6 people did not have

pencils, how many students are in Mr. Garcia's class?   30

4.   Order the numbers from least to greatest.   10/8, 4/3, 1.4, 1.08

    $1.08, \dfrac{10}{8}, \dfrac{4}{3}, 1.4$

5.   Express each fraction as a percent.
        a. 3/20 =   **15%**   b. 7/25 =   28%
Recall for "a" we can multiply by 5/5 and "b" by 4/4 to get a denominator of 100

2.2b Class Activity: Rules and Structure for Multiplying Integers

1. Complete the multiplication chart below. Be very careful and pay attention to signs.

2. Describe two patterns that you notice in the chart.
   Answers will vary.

3. Create a color code for positive and negative numbers (for example, you could choose yellow for positive and red for negative.) Shade the chart according to your color code.

	–5	–4	–3	–2	–1	0	1	2	3	4	5	
5	-25	-20	-15	-10	-5	0	5	10	15	20	25	**5**
4	-20	-16	-12	-8	-4	0	4	8	12	16	20	**4**
3	-15	-12	-9	-6	-3	0	3	6	9	12	15	**3**
2	-10	-8	-6	-4	-2	0	2	4	6	8	10	**2**
1	-5	-4	-3	-2	-1	0	1	2	3	4	5	**1**
0	0	0	0	0	0	0	0	0	0	0	0	**0**
–1	5	4	3	2	1	0	-1	-2	-3	-4	-5	**–1**
–2	10	8	6	4	2	0	-2	-4	-6	-8	-10	**–2**
–3	15	12	9	6	3	0	-3	-6	-9	-12	-15	**–3**
–4	20	16	12	8	4	0	-4	-8	-12	-16	-20	**–4**
–5	25	20	15	10	5	0	-5	-10	-15	-20	-25	**–5**
	–5	–4	–3	–2	–1	0	1	2	3	4	5	

4. What do you notice? Do you think this pattern is a rule?
   A positive multiplied by a positive is a positive. The product of a positive and a negative is negative. The product of two negatives is positive.

Review concept:

In 6th grade, you learned that opposite signs on a number indicate locations on opposite sides of 0 on the number line. For example, −3 is three units from 0, but on the opposite side of 0 than 3.

5. Use that logic to answer the following questions:

> a. What is the opposite of −1? 1
>
> b. Why is 1(−1) = (−1)(1) = −1 a true statement? Look back at #7 from 2.2a. You can change the order of multiplication. Students might also note that 1 times anything gives you back the "anything."
>
> c. What is the opposite of the opposite of −1? −1
>
> d. Why is (−1)(−1) = 1 a true statement? −1 is the opposite of 1; −(1)(−1) means "the opposite of the product of 1 and −1. 1 multiplied by −1 is −1, thus −(1)(−1) is −(−1) e.g. the opposite of −1 or 1.

6. Use your experiences above to generalize some rules for multiplication with integers:

> a. A positive times a positive is a positive.
>
> b. A positive times a negative is a negative.
>
> c. A negative times a positive is a negative.
>
> d. A negative times a negative is a positive.

Note: the core specifically asks that students be able to <u>justify why</u>, for example, a negative times a negative produces a positive product. Memorizing the rules is not enough. Students need to be able to explain them.

7. In groups, find each product. Be able to justify your answers for a – o.

8.

a. 2×8 16	b. $5 \times (-9)$ −	c. -7×7 −49
d. $-5(-3)$	e. $-5(3)$ −15	f. 11×7
g. $(4)(-3)$ −12	h. $(9)(-4)$	i. $(-6)(-4)$ 24

j. 0(–3) **0**	k. (–1)(1)(–3)	l. (4)(–2)(5) **–40**
m. (–2)(–2)(–2)(–2) **16**	n. (5)(–3)(–2) **30**	o. (–1)(–2)(3)(2)(–1)(2) **–24**

Note: k – o require students to multiply more than two factors.

8. In k – o you multiplied more than two factors together. Can you extend the rules you developed in #6 for this type of situation? *If you have an even number of negatives, your product will be positive, assuming no factor of 0. If there are an odd number of negatives, the products will be negative. Again assuming no 0 factors. Note: Students should have learned that 0 is even in early elementary. For students who struggle with this idea, you might point out numbers that end in 0 (10, 20, 30...) are all divisible by 2.*

29. Look at the following expressions. What is the operation for each? Explain how you identified the operation and then simplify each.
    Student explanations may vary.

a. $2(3)$ Multiplication   6
b. $2(-3)$ Multiplication   –6
c. $(2) - (3)$ Subtraction   –1
d. $-2(3)$ Multiplication   –6
e. $-2 - 3$ Subtraction   –5
f. $-2 - (-3)$ Subtraction   1
g. $2 - (-3)$ Subtraction   6

2.2b Homework: Rules and Structure for Multiplying Integers

Find each product.

1. $-2(3)$ -6

7. $15(-3)$ -45

13. -24×3 -72

2. $-4(-7)$

8. $15(4)$

14. $14(-12)$

3. $8(-4)$ -32

9. -1×4 -4

15. $3(-8)(-4)$ 96

4. $9(7)$

10. $15(-8)$

16. $-2 \times 5 \times 10$

5. $-3(7)$ -21

11. -4×13 -52

17. $-1 \times -3 \times -6$

6. $-12(-3)$

12. $-13(-3)$

18. $-8 \times 7 \times -3$ 168

Write a multiplication problem for each situation. Write the answer in a complete sentence.

19. Karla borrowed $5 each from 4 different friends. How much money does Karla owe her friends altogether?

Problem: $4 \times (-5)$ Answer: Karla owes $20 to her friends.

20. The temperature increased 2º per hour for six hours. How many degrees did the temperature raise after six hours?

Problem: Answer:

21. Jim was deep sea diving last week. He descends 3 feet every minute. How many feet will he descend in 10 minutes?

Problem: Answer:

Spiral Review

1. Express each percent as a fraction in simplest form.

   35%   35/100 = 7/20   22%   22/100 = 11/50

2 Solve the following expression by modeling with tiles, chips, or a number line $-6 + (-3)$ **−9**

3.   A snowboard at a local shop normally costs $400. Over Labor Day weekend, the snowboard is on sale for 60% off. What is the sale price of the snowboard?

   $(1 – 0.6) \times \$400 = \$160.$
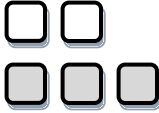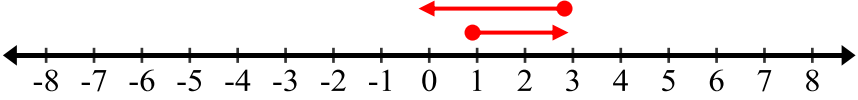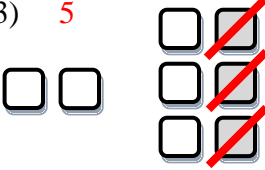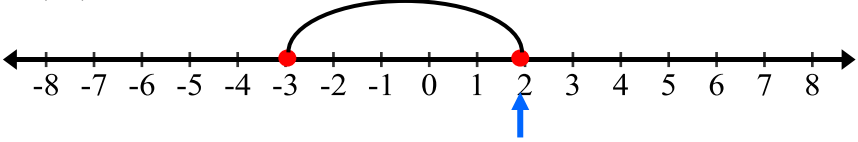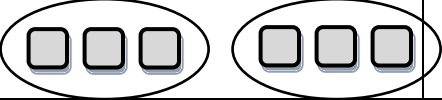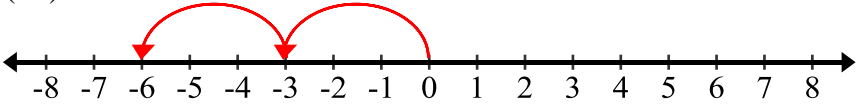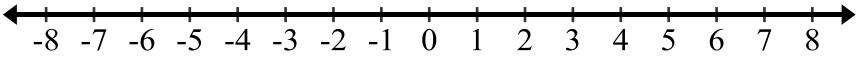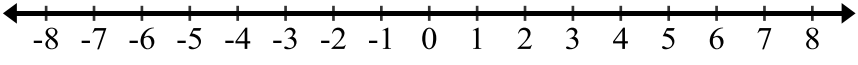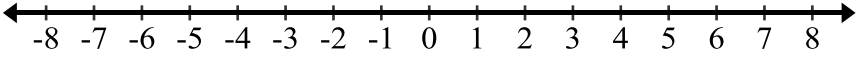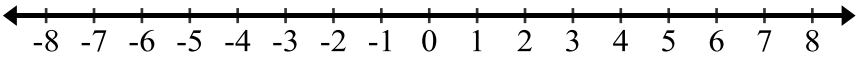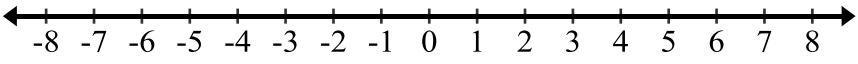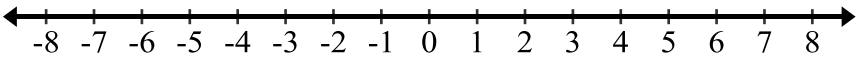
4.   Place each of the following integers on the number line below. Label each point:

   A = 4   **B** = −4   **C** = −15   **D** = 7   **E** = 18   **F** = −19

5. $8 – (–11) =$ **19**

Review: Operations with models. Draw models to find the answer to each problem.

Chips/Tiles	Number Lines
22. 2 + −3 −1	2 + −3
23. 2 − (−3) 5	2 − (−3)
25. 2(−3) −6	2(−3)
26. −4 + (−2) −	−4 + (−2)
27. −4 − (−2) −2	−4 − (−2)
28. −4(−2)	−4(−2)
29. −2 + 2	−2 + 2
30. −2 − 2	−2 − 2
31. −2(2)	−2(2)

2.2c Class Activity: Integer Division

Review concept:

In previous grades you learned that division is the inverse of multiplication.

Write the missing-factor in the multiplication equation. Then write the related division equation.

Missing Factor Multiplication Problems	can be thought of as . . .	Related Division Problems
1. $3 \times 4 = 12$	$\longleftrightarrow$	$12 \div 3 = 4$
2. $7 \times 6 = 42$	$\longleftrightarrow$	$42 \div 6 = 7$
3. $-9 \times 8 = -72$	$\longleftrightarrow$	$-72 \div -9 = 8$
4. $-7 \times -8 = 56$	$\longleftrightarrow$	$56 \div -7 = -8$
5. $-5 \times -3 = 15$	$\longleftrightarrow$	$15 \div -3 = -5$
6. $-12 \times 4 = -48$	$\longleftrightarrow$	$-48 \div 4 = -12$
7. $-9 \times 4 = -36$	$\longleftrightarrow$	$-36 \div -9 = 4$
8. $-11 \times -8 = 88$	$\longleftrightarrow$	$88 \div -8 = -11$

Make use of structure: How could you use related multiplication problems to find each quotient? Explain and then state the quotient.

9. $12 \div (-3)$ $-3 \times \underline{\quad} = 12$; -4

10. $-8 \div (-2)$ $-2 \times \underline{\quad} = -8$; 4

11. $40 \div (-10)$ $-10 \times \underline{\quad} = 40$; -4

12. $-96 \div (-12)$ $-12 \times \underline{\quad} = -96$; 8

13. What are the rules for dividing integers?

The rules for dividing integers are the same as multiplying integers.

2.2c Homework: Rules and Structure for Division of Integers/Application

Find each quotient.

1. $12 \div (-4)$ −3

2. $-36 \div (-6)$

3. $21 \div 3$ 7

4. $-45 \div 9$

5. $4 \div (-1)$ −4

6. $-52 \div (-4)$

7. $72 \div (-3)$ −24

8. $39 \div (-13)$

9. $-28 \div (-4)$ 7

10. $-63 \div 9$

11. $-36 \div (-3)$

12. $60 \div (-15)$ −4

13. $90 \div (-5)$

14. $160 \div 20$

Write a division expression for each situation. Answer the question in a complete sentence.

15. Keith borrowed a total of $30 by borrowing the same amount of money from 5 different friends. How much money does Keith owe each friend?

    Expression: $-30 \div 5$ Answer: Keith owes each friend $6.00.

16. The temperature fell 12º over 4 hours. What was the average change in temperature per hour?

    Expression: Answer:

17. Max lost 24 pounds in 8 weeks on his new weight-loss plan. What was his average change in weight per week?

    Expression: Answer:

18. Siegfried borrowed $4 a day until he had borrowed a total of $88. For how many days did he borrow money?

    Expression: $-88 \div -4$ or $88 \div 4$ Answer: Siegfried borrowed money for 22 days.

Spiral Review

1. Juan's football team gains 3 yards on one play. On the next play, the quarterback is sacked for a loss of 10 yards. What was the overall change in their position for the two plays?
–7

2. Lisa owes her mom $78. Lisa made four payments of $8 to her mom. How much does Lisa now owe her mother?
–78 + 4(8) = –46 so Lisa owes her mom $46.

3. Write $\frac{26}{4}$ as a mixed number. Model to solidify understanding. $6\frac{2}{4}$ or $\mathbf{6\frac{1}{2}}$
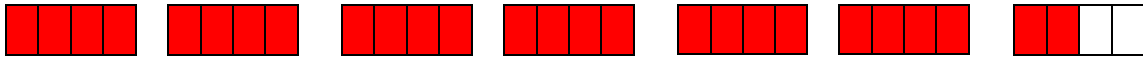
4. What is the greatest common factor of 24 and 56? **8**

5. $5 \times (-9)$ **–45**

2.2c Extra Integer Multiplication and Division Practice

Solve.

1. $3 \times 5 =$ 15

2. $96 \div (-12) =$

3. $-40 \div (-10) =$

4. $-6 \times 0 =$

5. $6 \times (-10) =$

6. $-55 \div 5 =$

7. $63 \div 9 =$

8. $-6 \div (-1) =$ 6

9. $-4 \times (-10) =$ 40

10. $10 \times (-3) =$

11. $-24 \div 12 =$ −2

12. $33 \div (-3) =$

13. $8 \times (-2) =$

14. $-21 \div (-3) =$

15. $9 \times (-4) =$ −36

16. $20 \times 4 =$

17. $10 \times (-4) =$ −40

18. $-5 \times (-9) =$

19. $-40 \div (-5) =$ 8

20. $1 \div (-1) =$

Write an equation and then solve. Write a sentence representing your answer.

11. Alicia owes \$6 to each of 4 friends. How much money does she owe?

12. A video game player receives \$50 for every correct answer and pays \$45 every time he gets a question incorrect. After a new game of 30 questions, he misses 16. How much money did he end up with?
    $(14 \times 50) + (16 \times (-45)) = 700 + (-720) = -20;$ Overall he lost \$20.

13. An oven temperature dropped 135° in 15 minutes. If the temperature dropped at a constant rate, how many degrees per minute did the temperature drop?

2.2d Extra Practice: Multiple Operations Review

1. The coldest place in Utah is Middlesink, near Logan. One day the temperature there was −17 degrees. You drive from Middlesink to Salt Lake where the temperature is 30 degrees. What is the change in temperature? Explain.

   $30 - (-17) = 47;$ You experience a temperature change of 47 degrees.

   Student explanations will vary.

2. You are playing a board game and you have to pay $50. Then you win $200. After that, you have to pay $800.

   a. Write the expression that represents this situation.

   b. What integer represents the amount of money you end up with?

3. You go into a business partnership with three other friends. Your business loses $65,000. You agree to share the loss equally. How much money has each of the four people lost?

   $-65{,}000 \div 4 = -16{,}250$ Each person loses $16,250.

Integers Review: Perform the indicated operation.

4. $5 + (-3)$ 2

5. $3 - 5$

6. -15×2 −30

7. $-6 \div (-2)$

8. $-14 - (-6)$ −8

9. $-5(3)$ −

10. $13 - (-5)$ 18

11. $-2(-3)$

12. $12 \div 12$

13. $(-6) - (-10)$

14. $15 \div (-3)$ −5

15. $-4 + 12$

16. $(-5) + (-4)$ −9

17. $-9(2)$

18. $\dfrac{-20}{4}$ −5

2.2e Self-Assessment: Section 2.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Use a model to multiply integers. [1]	I can draw a model for thinking about integers but I struggle to use models to multiply integers.	I can usually get the right answer when I multiply integers using a model.	I always get the right answer when I multiply integers using a model.	I always get the right answer when I multiply integers using a model. I can explain the relationship between the model and the product.
2. Explain why the product of a positive and negative integer is negative or the product of a negative and negative integer is positive. [2]	I don't know why the product of a positive and negative integer is negative or the product of a negative and negative integer is positive.	I know that product of a positive and negative integer is negative or the product of a negative and negative integer is positive, but I have a hard time showing why that's true with models and words.	I can *show* why the product of a positive and negative integer is negative or the product of a negative and negative integer is positive using models and words. But struggle to explain it with only words and symbols.	I can explain with models and words *and* with words alone why the product of a positive and negative integer is negative or the product of a negative and negative integer is positive.
3. Use rules for multiplying and dividing integers accurately. [3]	I know the rules for multiplying and dividing integers.	I know the rules to multiply and divide integers, but struggle to explain them with models and words.	I can accurately multiply and divide any integers using the rules and can show the rules with models and words.	I can accurately multiply and divide any integers using the rules. I can explain with models and words *and* with words alone why the rules are true.
4. Add, subtract, multiply and divide integers fluently. [4]	I sometimes get confused recognizing if the expression is asking me to add, subtract, multiply or divide.	I always know what operation I'm to preform, and most of the time I get the right answer when I add, subtract, multiply, and divide integers.	I always know what operation I'm to preform, and I always get the right answer when I add, subtract, multiply or divide integers.	I always know what operation I'm to preform, and I always get the right answer when I add, subtract, multiply or divide integers. I can explain my answer.
5. Solve contextual problems involving integers. [5]	I struggle to solve contextual problems involving integers.	I can usually write an expression to solve a contextual problem involving integers.	I can always solve contextual problems involving integers.	I can always solve contextual problems involving integers. I can explain the solution in context.

Sample Problems for Section 2.2

1. Draw a model (chips, array or number line) to solve each of the following multiplication problems.
    a. $-1(7)$ c. $8(-8)$

    b. $-3 \cdot (-6)$ d. $-2 \cdot 7$

2. Explain why the product of a positive and negative integer is negative or the product of a negative and negative integer is positive, using pictures, models, and words for your explanation.

3. Find each product or quotient.
    a. $9 \cdot 4$ d. $\dfrac{-80}{8}$

    b. $\dfrac{-14}{-7}$ e. $-9 \cdot -2$

    c. $5(-2)$ f. $\dfrac{-48}{2}$

4. Simplify each.
    a. $-6(-5)$ e. $-10 \cdot 31$

    b. $-16(-3)$ f. $-89 + (-6)$

    c. $\dfrac{-81}{-9}$ g. $6 - 9$

    d. $-4 + 21$ h. $\dfrac{-40}{8}$

5. Solve each of the following contextual problems involving integers.

    a.   David is in a submarine at 200 feet below sea level. Casey is above him in a helicopter at 5,900 feet altitude. How far apart are David and Casey?

    b.   Scuba diving, Elisa descends at a rate of 30 feet per minute. How long will it take her to reach a depth of 90 feet?

    c.   Rome was founded in 753 BCE. Rome fell in 476 ACE. How many years passed between Rome's founding and fall?

Section 2.3: Add, Subtract, Multiply, Divide Positive and Negative Rational Numbers (all forms)

Section Overview:

In this section students extend their understanding of operating with integers to operating with all rational numbers. Students start with an activity to help them think about: 1) a line as a way to represent numbers, 2) rational numbers as numbers that can be written in the form a/b, where $b \neq 0$, and 3) locating any number that can be written as a/b (any rational number) on the number line. Students then move to operating with rational numbers in all forms by applying the rules of arithmetic that they should have solidified in the previous two sections.

A central idea throughout this section is the importance of estimating quantities before one computes. This is not only an important life skill; it's also an important skill in making sense of answers and working with technology. Students should get in the habit of estimating answers before they compute them so that they can check the reasonableness of their results.

Concepts and Skills to be Mastered (from standards)

By the end of this section, students should be able to:
1. Estimate the sum, difference, product or quotient of positive and negative rational numbers.
2. Find the sum or difference of rational numbers fluently.
3. Find the product or quotient of rational numbers fluently.
4. Solve contextual problems involving positive and negative rational numbers.
5. Explain why division by 0 is undefined.

2.3a Class Activity: Rational Numbers/Number Line City—Add, Subtract Rational Numbers

Activity: Create your own number line with the length of string your teacher gave you. Pick a spot in the middle of your line for "0". Your "units" will be the length of your string. You will also find other points on your line. The mathematical ideas on which this activity is based can be found in the mathematical foundation page.

In Number Line School, there is one straight hallway. The numbers on the classrooms indicate the distance from the main office, which is at zero. Mark the location of the following classrooms on the map of Number Line School below. Students may benefit if you ask them to write an expression to find the difference on some/all of these problems. You may decide to assign this activity as homework if the above activity took too much class time.

Mr. Bowen: $-3\frac{1}{4}$ Ms. England: $-6\frac{1}{4}$ Mr. Francks: $-8\frac{1}{3}$

Ms. Abe: 3.5   Mrs. Chidester: 5.25   Mr. David: -5.2
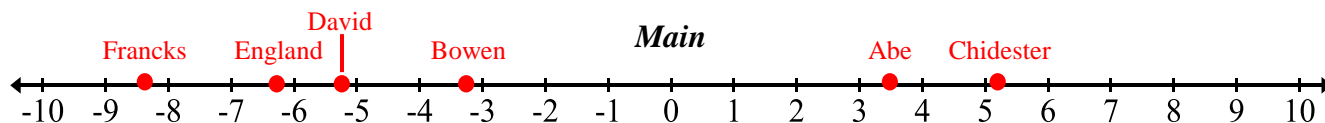
Work with your group to answer the following questions. Be sure to explain your reasoning!
Students explanations may vary.
1. Whose classroom is further from the Main Office, Ms. Abe, or Mrs. Chidester?
    Mrs. Chidester because she is more than five units away while Ms. Abe is less than four units away.

2. a. *Approximately* how far apart are Ms. Abe and Mrs. Chidester?   Approximately 2 units

    b. *Exactly* how far apart are Ms. Abe and Mrs. Chidester?   1.75 units

3. Joe has Mr. Bowen's class first hour and Ms. England's class second hour. How far must he walk to get from Mr. Bowen's class to Ms. England's class? Estimate your answer before you compute it.
    He must walk 3 units.

4. What is the distance between Mrs. Chidester's class and Mr. David's class? Estimate your answer before you compute it.
    Approximately 10 units
    The distance between their classes is 10.45 units.

5. What is the distance between Mr. Franck's class and Ms. England's class? Estimate your answer before you compute it.
    Approximately 2 units

    The distance between their classes is $2\frac{1}{12}$ units.

2.3a Homework: Rational Numbers/Number Line City

In Number Line City, there is one straight road. All the residents of number line city live either East (+) or West (–) of the Origin (0). All addresses are given as positive or negative rational numbers. Label the location of the following addresses on the map of Number Line City below with a point and the first letter of the resident (for example put a "B" above the point for Betsy.):

- ● Betsy: $-1\frac{3}{4}$
- ● Allyson: –2.5
- ● Chris: $5\frac{1}{2}$
- ● David: 6.0

- ● Frank: $\frac{10}{4}$
- ● Emily: 7.25
- ● Hannah: $6\frac{1}{5}$
- ● Jamee: –6.2

- ● Number Line School: $1\frac{1}{2}$
- ● Town Hall: 0
- ● Grocery Store: –4.8
- ● Post Office: –10
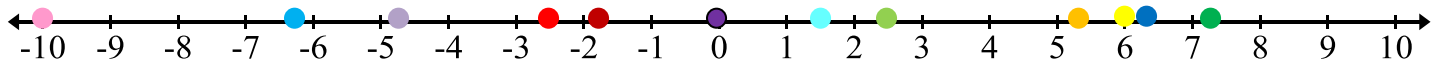
Now answer the following questions. Estimate your answer first, then compute it.

1. How far is it from Allyson's house to Betsy's house?

2. How far is it from Chris' house to Betsy's house?
   It is West 7.25 units from Chris' house to Betsy's house.

3. How far is it from David's house to Chris' house?

4. How far is it from Emily's house to Jamee's house?
   It is West 13.45 units to Jamee's house.

5. Who lives closer to the Grocery Store, Allyson or Jamee? Explain.

6. Who lives closer to Town Hall, Jamee or Hannah? Explain.
   Jamee and Hannah live the same distance from Town Hall.

7. The mailman leaves the post office and makes a delivery to Hannah, picks up a package from Jamee, and then delivers the package to Frank before returning to the post office. How far did the mailman travel?

8. Every morning, Betsy walks to Allyson's house and they walk together to school. How far does Betsy walk each morning?
Betsy walks 4.75 units each morning.

9. Iris moves into Number Line City. She finds out that she lives exactly 3.45 units from Emily. Where does Iris live?

10. Logan moves into Number Line City. He moves into a home 2¼ units from Chris' home. Where could Logan live? Where do you suggest he live and why?
Logan would live at 7.75 or 3.25.

Students' suggestions of where he lives will vary.

Spiral Review

1. Solve
   a. $16 - 25 =$

   b. $-2 - (-8) =$

   c. $5 + -11 =$

   d. $-81 - 19 =$

2. Model $\frac{4}{5} - \frac{2}{3} =$

   $\frac{2}{15}$

3. Jack and Jill went on their first date to Chili's. Their food bill totaled $32.00 plus the 15% tip they need to pay the waitress.

   a. How much will their 15% tip be? **$4.80**

   b. How much will Jack pay in all? **$36. 80**

2.3b More Operations with Rational Numbers

Review: Addition/Subtraction
In elementary school, you learned the following:
   Example 1: Adding is a way to simplify or collect together things that are the same units.

3 apples + 4 apples = 7 apples

3 apples + 4 bananas = 3 apples + 4 bananas

Example 2: Place value matters when you add whole numbers because you want to add units that are the same.

$24 + 7 \neq 94$ (you can't add the 2 to the 7 because the "2" represents the number of tens and the "7" represents the number of ones—they are not the same kinds of units.) $24 + 7 = 31$.
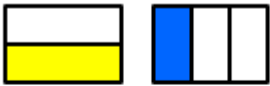
Example 3: Place value matters when you add decimals, again you want to add the same kinds of units.
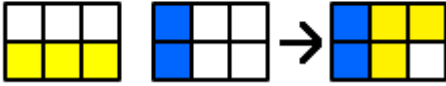
$3.2 + 4.01 \neq 4.3$ $3.2 + 4.01 = 7.21$

Example 4: To add or subtract fractions you must have the same units, thus you must have a common denominator.

$\frac{1}{2}$ and $\frac{1}{3}$ are not the same units. But, $\frac{3}{6}$ and $\frac{2}{6}$ can be added because the units $\left(\frac{1}{6}\right)$ are the same. There are a total of five one-sixth units or $\frac{5}{6}$.
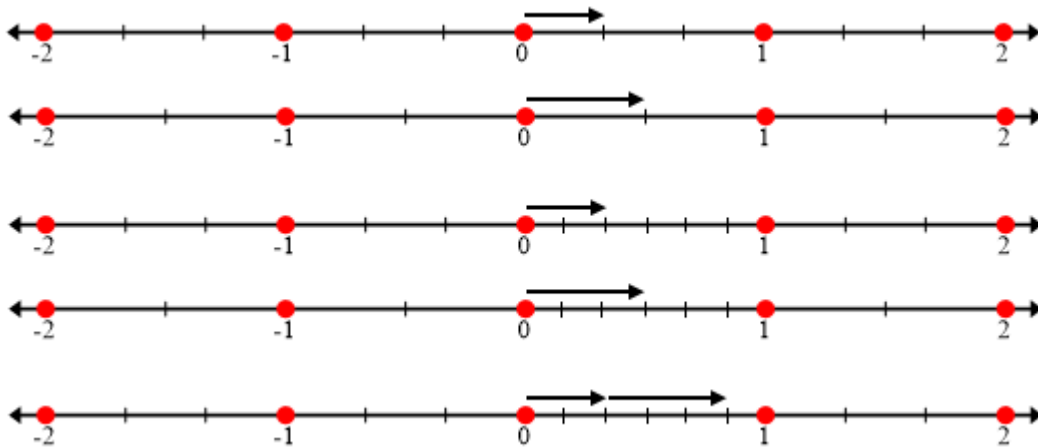
In this chapter we have learned how to add and subtract with integers first with models and then by applying rules. We will now apply these skills to all rational numbers.

1. Think back to the activity you did in 2.3a; what is a "rational number"?

Find the sum or difference for each.

2. $-\dfrac{2}{3}+\dfrac{1}{4}$ $\quad -\dfrac{5}{12}$	3. $\dfrac{5}{3}-\dfrac{3}{4}$ $\quad \dfrac{11}{12}$	4. $\dfrac{2}{3}-\left(-\dfrac{3}{2}\right)$ $\dfrac{13}{6}=2\dfrac{1}{6}$
5. $\dfrac{3}{4}-1\dfrac{2}{3}$ $\quad -\dfrac{11}{12}$	6. $-\dfrac{5}{4}-\left(-\dfrac{1}{3}\right)$ $\quad -\dfrac{11}{12}$	7. $-\dfrac{4}{3}+\left(-\dfrac{3}{4}\right)$
8. $-0.5-\dfrac{2}{3}$ $\quad -\dfrac{7}{6}$	9. $\dfrac{7}{4}+(-0.\overline{6})$	10. $-\dfrac{7}{3}-(-0.75)$ $\quad -\dfrac{19}{12}$
11. $-0.\overline{3}+1.4$	12. $-1.6-(-1.\overline{6})$ $\quad \dfrac{1}{15}$	13. $0-(-1.75)$

Review: Multiplication and Division

Review the general multiplication models in 2.2a Class Activity. Remember that multiplying means you have a certain number of groups with a certain number of units per group. You do NOT need a common denominator for multiplication.

Example 5: Whole number by fraction or fraction by whole number

$3 \times \dfrac{3}{4}$ means three groups of $\dfrac{3}{4}$, so $3 \times \dfrac{3}{4} = \dfrac{9}{4} = 2\dfrac{1}{4}$
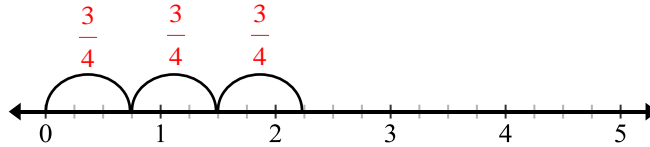
$\dfrac{2}{3} \times 6$ means two-thirds of a group of six, so $\dfrac{2}{3} \times 6 = 4$
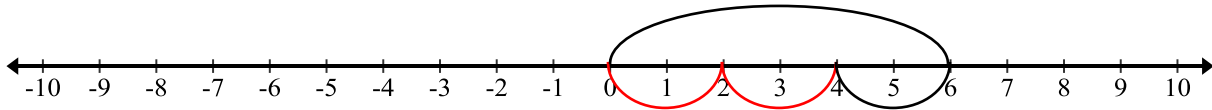
Example 6: Fraction by a fraction

$\dfrac{1}{2} \times \dfrac{1}{3}$ means "one-half groups of one-third."

Example 7: Division, whole number by a fraction

Recall that division is the inverse of multiplication. In other words, you're looking for a missing factor when you divide. $12 \div 3$ can mean that you're starting with 12 and you want to know how many groups of 3 units are in 12. Similarly, $4 \div \dfrac{2}{3}$ can mean starting with 4 and want to know how many groups of $\dfrac{2}{3}$ are in 4.
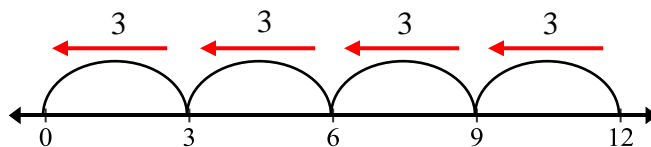
$12 \div 3 = 4$ because there are four groups of 3 units in 12.
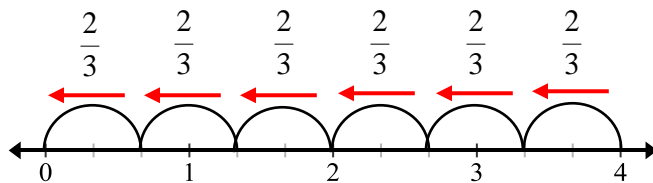
Similarly, $4 \div \dfrac{2}{3} = 6$ because there are 6 groups of $\dfrac{2}{3}$ in 4.

The other way to think about $12 \div 3$ is, you're starting with 12 and you want to know how many units are in each **whole** group if you have 3 groups. Similarly, $4 \div \frac{2}{3}$ can be thought of as starting with 4 and you want to know how many are in a **whole** group if you have $\frac{2}{3}$ of a group:
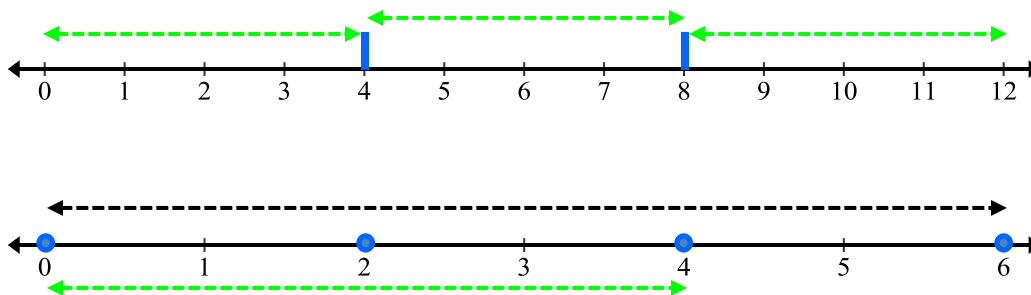
Use your understanding of multiplication of fractions and integers to find the following products:

13. $\left(-\frac{1}{2}\right)\left(-\frac{4}{5}\right)$ $\frac{4}{10} = \frac{2}{5}$	14. $\left(-1\frac{1}{2}\right)(-6)$ 9	15. $\left(-\frac{3}{4}\right)(12)$
16. $(-3)\left(\frac{5}{4}\right)$ $-\frac{15}{4} = -3\frac{3}{4}$	17. $\left(-\frac{2}{3}\right)\left(-\frac{1}{4}\right)$	18. $\left(\frac{2}{3}\right)\left(-\frac{3}{5}\right)$ $-\frac{2}{5}$

Use your understanding of division of fractions and integers to find the following quotients:

19. $(3) \div \left(-\frac{1}{4}\right)$ -12	20. $(-2) \div \left(-\frac{2}{3}\right)$	21. $\left(-\frac{2}{3}\right) \div \left(\frac{1}{2}\right)$ $-\frac{4}{3} = -1\frac{1}{3}$
22. $\left(-\frac{3}{5}\right) \div \left(-\frac{3}{2}\right)$	23. $-\frac{2}{10} = -\frac{1}{5}$	24. $-2\frac{3}{4} \div \left(-2\frac{3}{4}\right)$

2.3c Class Activity: Multiply and Divide Rational Numbers

Work in groups to extend your understanding of multiplying and dividing rational numbers.

1. Will the product of $\left(\dfrac{3}{4}\right)\left(\dfrac{1}{4}\right)$ be bigger or smaller than each of the factors? Explain

   It will be smaller than $\dfrac{3}{4}$ and $\dfrac{1}{4}$ because we are taking part (3/4) of a group of 1/4.
   Find the product to check your explanation.
   $\dfrac{3}{16}$

2. Explain the value of the product $\left(\dfrac{3}{4}\right)(2)$. Will it be bigger or smaller than each of the factors?

   It will be bigger than $\dfrac{3}{4}$ and smaller than 2.

   Find the product to check your explanation.
   $1\dfrac{1}{2}$

3. Explain the value of the product $\left(\dfrac{3}{4}\right)(0.5)$. Will it be bigger or smaller than each factor?

   It will be smaller than $\dfrac{3}{4}$ and 0.5.

   Find the product to check your explanation.
   0.375

4. Will the quotient of $\dfrac{3}{4} \div \dfrac{1}{4}$ be bigger or smaller than each term? Explain.

   It will be bigger than $\dfrac{3}{4}$ and $\dfrac{1}{4}$.

   Find the quotient to check your explanation.
   3

5. Explain the quotient $\dfrac{3}{4} \div 2$. Will it be bigger or smaller than each term?

   It will be smaller than $\dfrac{3}{4}$ and smaller than 2.

   Find the quotient to check your explanation.
   $\dfrac{3}{8}$

6. Explain the quotient $\dfrac{3}{4} \div 0.5$. Will it be bigger or smaller than each term?

   It will be bigger than $\dfrac{3}{4}$ and 0.5.

   Find the quotient to check your explanation.
   1.5

Estimate by rounding to the nearest integer. Discuss in your group if your answer is an over estimate or an under estimate of the actual value. Be able to justify your answer.

8. $3\frac{1}{3} \div \frac{7}{9} \approx$ _____ $\div$ _____ $\approx$ _____ $\approx 3 \div 1 \approx 3$

    3

9. $\left(6\frac{1}{3}\right)(2.2) \approx$ (____)(____) $\approx$ _____

    12

10. $4\frac{1}{9} \div 7.891 \approx$ _____ $\div$ _____ $\approx$ _____

    0.5

11. $\left(\frac{4}{5}\right)(-5.9)\left(4\frac{1}{8}\right) \approx$ (____)(____)(____) $\approx$ _____

    −24

Estimate and then confirm your estimation by finding the exact value with a calculator.

12. $-0.875 \div \frac{2}{7} \approx$ _____ $\approx (-1)/(1/4) \approx (-1)(4) \approx -4$ Exact value: _____ $-\dfrac{49}{16}$

13. $\left(3\frac{3}{5}\right)\left(2\frac{1}{10}\right) \approx$ _____ $\approx (7/2)\,(2) \approx 7$ Exact value: _____ $\dfrac{189}{25}$

14. $0.75\left(\frac{2}{11}\right)\left(\frac{11}{12}\right) \approx$ _____ $\approx (1)\,(2/10)\,(1) \approx 2/10$ Exact value: _____ $\dfrac{1}{8}$

15. $3\frac{4}{5} \div -19 \approx$ _____ $\approx 4 \div -20 \approx -1/5$ Exact value: _____ $-\dfrac{1}{5}$

16. $(-9)\left(-8\frac{1}{4}\right) \approx$ _____ $\approx (-9)(-8) \approx 72$ Exact value: _____ $74\dfrac{1}{4}$

    $9\frac{3}{4} \div 12\frac{1}{4} \approx$ _____ $\approx 9 \div 12 \approx 9/12$ or $3/4$ Exact value: _____ $\dfrac{39}{49}$

Exploration Activity:

Divide each

a. $12 \div 12 = 1$

b. $12 \div 6 = 2$

c. $12 \div 4 = 3$

d. $12 \div 3 = 4$

e. $12 \div 2 = 6$

f. $12 \div 1 = 12$

g. $12 \div 1/2 = 24$

h. $12 \div 1/3 = 36$

i. $12 \div 1/4 = 48$

j. $12 \div 1/6 = 72$

k. $12 \div 1/12 = 144$

What do you notice?
Students should notice that, for a given dividend, the smaller the divisor the larger the quotient and the larger the divisor the smaller the quotient.

Divide each:

l. $1/2 \div 12 = 1/24$

m. $1/2 \div 6 = 1/12$

n. $1/2 \div 4 = 1/8$

o. $1/2 \div 2 = ¼$

p. $1/2 \div 1 = ½$

q. $1/2 \div 1/2 = 1$

r. $1/2 \div 1/3 = 3/2$

s. $1/2 \div 1/4 = 2$

t. $1/2 \div 1/6 = 3$

u. $1/2 \div 1/12 = 6$

What do you notice?
Students should notice that, for a given dividend, the smaller the divisor the larger the quotient and the larger the divisor the smaller the quotient. Ask students if the pattern would hold if you were dividing by a negative.

What is your conjecture about $12 \div 0$ or $1/2 \div 0$? Justify your conjecture.
Discuss with students why these are undefined quotients. You may also want to go back to the discussion about division as the inverse of multiplication, e.g. if $12 \div 0 = 0$, then 0×0 would have to equal 12.

What do you think would happen if we took any (nonzero) number, *b*, and divided by 0?

2.3c Homework: Multiply and Divide Rational Numbers

Multiplying and dividing rational numbers. For #5 - 13 estimate first, then compute.

1. Will the product of $(2)\left(\dfrac{1}{3}\right)$ be bigger or smaller than each of the factors? Explain

   It will be bigger than $\dfrac{1}{3}$ and smaller than 2. As written, this means two groups of 1/3 which will be bigger than 1/3. However, using the commutative property of multiplication gives 1/3×2 or one third of a group of two. Therefore the product is smaller than 2.

2. Will the product of 2 and 3 be bigger or smaller than each factor? Explain.

3. Explain the quotient $2 \div \dfrac{1}{3}$: will it be bigger or smaller than each term?

4. Will the quotient of $\dfrac{1}{3} \div 2$ be bigger or smaller than each term?

5. $(2)\left(\dfrac{1}{3}\right) =$

   $\dfrac{2}{3}$

6. $5 \times \dfrac{1}{4} =$

7. $\dfrac{1}{2} \div 0 =$

8. $\dfrac{2}{3} \times 5 =$

9. $\left(2\dfrac{1}{2}\right)(3.5) =$

   8.75

10. $\dfrac{2}{7} \div -0.9 =$

    −20/63

11. $3 \div \dfrac{1}{2} =$

12. $\dfrac{5}{8} \div 3.7 =$

13. $\dfrac{1}{2} \div 4 =$

Spiral Review

Solve the following with or without a model

1. $(-12)(-10) =$ 120

2. $23 + (-23) =$ 0 (additive inverse)

3. $\frac{1}{6} + \frac{3}{7} =$ 7/42 + 18/42 = 25/42

4. $0.85 + \frac{1}{2} =$ 1.35 or 27/20

5. The temperature increased 2° per hour for six hours. How many degrees did the temperature raise after six hours? $6 \times 2 = 12$ so 12 degrees.

2.3c Extra Practice: Multiply and Divide Rational Numbers

Estimate each product or quotient and then compute.
Students should estimate first.

1. $-0.5 \times \dfrac{1}{3} = -\dfrac{1}{6}$

2. $= \dfrac{5}{9} \div \left(-\dfrac{5}{6}\right)$

3. $-0.375 \times \dfrac{2}{9} = -\dfrac{1}{12}$

4. $\left(-\dfrac{2}{7}\right) \times \left(-\dfrac{1}{3}\right) =$

5. $\dfrac{8}{9} \div 0.2 = \dfrac{40}{9}$

6. $-0.75 \times \dfrac{1}{3} =$

7. $\dfrac{4}{5} \times \dfrac{5}{8} = \dfrac{1}{2}$

8. $\dfrac{7}{8} \div \left(-\dfrac{2}{3}\right) =$

9. $\dfrac{2}{7} \div \dfrac{1}{7} =$

10. $\left(-\dfrac{2}{3}\right) \times -0.4 = \dfrac{4}{15}$

11.

12. $\left(-\dfrac{2}{3}\right) \div 0.24 = -\dfrac{25}{9}$

13. $0.25 \div \dfrac{1}{3} =$

14. $0.4 \times \left(-\dfrac{1}{2}\right) = -\dfrac{1}{5}$

15. $-0.75 \div \left(-\dfrac{3}{8}\right) =$

16. $\dfrac{3}{4} \div 0.125 =$

17. $\left(-\dfrac{1}{7}\right) \div \left(-\dfrac{1}{5}\right) =$

18. $(-0.2) \div \dfrac{3}{7} = -\dfrac{7}{15}$

19. $\dfrac{3}{7} \times \dfrac{7}{12} =$

20. $\left(-\dfrac{4}{11}\right) \div \left(-\dfrac{8}{11}\right) = \dfrac{1}{2}$

2.3d Class Activity: Multiply and Divide continued

For questions #1 – 5, do each of the following:
a. Write an equation or draw model.
b. Estimate an answer.
c. Find the exact answer.

1. Dory can run 3.25 miles every hour. How long will it take her to run a race that is $16\frac{1}{4}$ miles long?

$16\frac{1}{4} \div 3.25 = 5$ Note: Students might "count up" to get to the solution. In other words they might think; 3.25 + 3.25 + 3.25…until they get to 16.25. That works too. Allow students to think about the problems in ways that make sense to them rather than worry about algorithmic approaches. Have students share their thinking.

It will take Dory 5 hours to run the race.

2. From 6 p.m. to 6 a.m. the temperature dropped 1.4 degrees each hour. What was the total change in temperature between 6 p.m. and 6 a.m.?

$12 \times (-1.4) = -16.8$
The temperature dropped 16.8 degrees.

3. Bridgette wants to make 14 matching hair bows for her friends. Each hair bow requires $\frac{3}{4}$ of a yard of ribbon. How many yards of ribbon will Bridgette need to make all the hair bows?

$14 \times \frac{3}{4} = 10\frac{1}{2}$

Bridgette needs $10\frac{1}{2}$ yards of ribbon.

4. There are 18 apple trees in my orchard. If I can harvest the fruit from $1\frac{2}{3}$ trees each day, how many days will it take to harvest the entire orchard?

$18 \div 1\frac{2}{3} = 10\frac{4}{5}$

It will take $10\frac{4}{5}$ days to harvest the entire orchard.

5. I want to use a recipe for muffins that calls for 3 cups of flour and 2 eggs. If I only have 1 egg, how much flour should I use?

$\frac{1}{2} \times 3 = \frac{3}{2}$

I should use $1\frac{1}{2}$ cups of flour.

For problems # 6 – 15, first estimate, then compute.

Students' answers may be in decimal or fraction form. Estimations may vary.

6. $(-0.5)\left(\dfrac{5}{3}\right)=$

Estimate: $\approx (-0.5)(2) \approx -1$

Actual: $-\dfrac{5}{6}$

7. $(-0.75)\left(\dfrac{5}{7}\right)=$

Estimate: -0.5

Actual: $-\dfrac{15}{28}$

8. $-0.23\times\left(-\dfrac{2}{5}\right)=$

Estimate: 0.1

Actual: 0.092

9. $\dfrac{1}{2}\left(-\dfrac{1}{3}\right)=$

Estimate: $-\dfrac{1}{6}$

Actual: $-\dfrac{1}{6}$

10. $\left(\dfrac{2}{3}\right)(-0.8)=$

Estimate: -0.5

Actual: $-\dfrac{8}{15}$

11. $\dfrac{3}{4}\div 0.25=$

Estimate: 3

Actual: 3

13. $\dfrac{1}{4}\div\left(-\dfrac{1}{3}\right)=$

Estimate: -0.75

Actual: $-\dfrac{3}{4}$

14. $-\dfrac{2}{5}\div(-0.86)=$

Estimate: 0.5

Actual: $\dfrac{20}{43}$

15. $-\dfrac{1}{5}\div\left(-\dfrac{2}{3}\right)=$

Estimate: 0.3

Actual: $\dfrac{3}{10}$

16. Write a context (story) for the given problem:
$5\times\dfrac{3}{4}=$
Answers will vary.

17. Write a context (story) for the given problem:
$2\div\dfrac{1}{3}=$
Answers will vary. Bill has two pizzas. If each friend can eat 1/3 of a pizza, how many friends can Bill invite to eat pizza?

2.3d Homework: Problem Solve with Rational Numbers

Student explanations and work may vary.

Problem	Estimate the quantity, show how you arrived at your estimate	Calculate (show your work)
1. $2\frac{1}{3} - 4.8$	$2 - 5 \approx -3$	$2.\bar{3} - 4.8 = -2.4\bar{6} \ or -2\frac{7}{15}$
2. $(8.6)\left(-3\frac{3}{4}\right)$		
3. $-4.7 \div \frac{1}{4}$	$-5 \times 4 \approx -20$	$-4.7 \div 0.25 = -18.8$
4. $2\frac{1}{2} + \left(-\frac{3}{4}\right) + 0.75$		
5. $\frac{5}{8} - (-0.125)$		
6. $\frac{1}{2}\left(-\frac{3}{7}\right)(1.75)$	$\frac{1}{2} \cdot -\frac{1}{2} \cdot 2 \approx -\frac{1}{2}$	$\frac{1}{2}\left(-\frac{3}{7}\right)\left(\frac{7}{4}\right) = -\frac{3}{8}$
7. $\frac{1}{6} + \frac{1}{12} + (-0.25) + 1.5$	$0.2 + 0 + (-0.25) + 1.5 \approx 1.45$	$\frac{2}{12} + \frac{1}{12} + \left(-\frac{3}{12}\right) + \frac{18}{12} = \frac{18}{12} = 1\frac{1}{2}$
8. $-7\frac{1}{2} \div -2.5$		

Estimate first, then calculate. Finally, evaluate whether your answers are reasonable by comparing the estimate with the calculated answer. Correct your answers when appropriate.

9. Janice is ordering a new pair of eyeglasses. The frames she wants cost $98.90. High index lenses cost $76.00. Her insurance says that they will pay $100 of the cost, and 20% of the remaining amount. How much will Janice have to pay?
   **Estimate:** Cost $\approx$ $100 + $80. Insurance pays $100 plus 20% off of $80 remaining which is $16. Janice pays $64.
    Total: $98.90 + 76 = 174.90.
    Insurance pays: $100 + 0.2(74.90) = 114.98. So Janice pays: $174.90 - 114.98 = 59.92

10. Fernando entered a weight-loss contest. The first week he lost $5\frac{1}{5}$ pounds, the next week he lost 4.59 pounds and the last week he lost 2.31 pounds. What was his total change in weight?

11. A marathon is 26.2 miles. June wants to run the distance of a marathon in a month, but she thinks that she can only run $\frac{3}{4}$ of a mile each day. How many days will she need to run? Will she be able to finish in a month?
    Estimate: $26 \div 1 = 26$
    $26.2 \div 0.75 = 34.933$ \qquad June will need about 35 days. She will not be able to finish at that rate.

12. Reynold just got hired to paint fences this summer. He gets paid $0.60 for every foot of fencing that he paints. If Reynold paints an average of 45 yards of fencing a week, how much can he earn in $8\frac{2}{5}$ weeks?

13. A super freezer can change the internal temperature of a 10 pound turkey by $-2.4°$ F every 10 minutes. How much can the freezer change the temperature of the turkey in $3\frac{3}{4}$ hours?

14. Dylan is ordering 5 dozen donuts for his grandmother's birthday party. He wants $\frac{1}{4}$ of the donuts to be chocolate frosting with chocolate sprinkles, and $\frac{1}{3}$ of the donuts to have caramel frosting with nuts. The rest of the donuts will just have a sugar glaze. The donuts with the sugar glaze cost $0.12 each, and the other donuts cost $0.18 each. How much will Dylan have to pay for all the donuts?

    Estimate: $\$0.15 \times 60 = \$9.00$
    $0.18 \times ¼ \times 60 + 0.18 \times 1/3 \times 60 + 0.12 \times 25 = 9.30$ \qquad Dylan will have to pay $9.30

15. Penny and Ben want to buy new carpet for their living room. They measured the dimensions of the living room and found that it was $12\frac{1}{4}$ feet by $8\frac{5}{8}$ feet. They know that installation will cost $37.50 If Penny and Ben want to spend no more than $500 on carpet, how much can they afford to pay for each square foot?

Spiral Review

1. Use the chip model to solve $-14 + 5$.
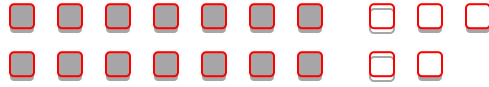
   -9

2. What percent of 80 is 60? **75%**

3. Parker earned $450 over the summer. If he put 70% of his earnings into his savings, how much money did he have left over?

   $450(1 - 0.70) = \$135$

4. Order the numbers from least to greatest.

   $-2.15, \dfrac{17}{7}, 2.7, -2.105$ **-2.15, -2.105, $\dfrac{17}{7}$, 2.7**

5. Max has $150. He spends 2/3 of his money on groceries. How much money did he have left over?

   $50

2.3e Self-Assessment: Section 2.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Estimate the sum, difference, product or quotient of positive and negative rational numbers. [1, 2]	I'm not very good at estimating the sum, difference, products or quotients with rational numbers.	Most of the time I can estimate the sum, difference, product or quotient of rational numbers.	I can always estimate the sum, difference, product or quotient of rational numbers.	I can always estimate the sum, difference, product or quotient of rational numbers. I can explain why my estimate is a reasonable.
2. Find the sum or difference of rational numbers fluently. [1]	I struggle to add and subtract rational numbers.	Most of the time I can find the sum or difference of rational numbers.	I can always find the sum or difference of rational numbers.	I can always find the sum or difference of rational numbers. I can explain why my answer is correct.
3. Find the product or quotient of rational numbers fluently. [2]	I struggle to multiply and divide rational numbers.	I usually get the right answer when I multiply and divide rational numbers.	I always get the right answer when I multiply and divide rational numbers.	I can multiply and divide positive and negative rational numbers fluently (quickly and accurately). I do not need to refer back to any rules or draw a model to find the answer.
4. Solve contextual problems involving positive and negative rational numbers. [3]	I struggle to solve contextual problems involving rational numbers.	I can usually solve contextual problems involving rational numbers.	I can always solve contextual problems involving rational numbers.	I can always solve contextual problems involving rational numbers. I can explain the solution in context.
5. Explain why division by 0 is undefined.	I forget if $0 \div 12$ is undefined or if $12 \div 0$ undefined.	I know which of these two statements is false: a) $0 \div 12 = 0$ b) $12 \div 0 = 0$	I can explain with examples and words why division by 0 is undefined.	I can explain with examples and words why division by 0 is undefined. This idea makes perfect sense to me.

Sample Problems for Section 2.3

1.   Estimate sum or difference then find the actual value.

    a. $-0.9 - \dfrac{4}{9}$

    b. $-0.2 + \left(-\dfrac{2}{5}\right)$

    c. $8 - (-0.8)$

    d. $-2.5 - \dfrac{3}{4}$

    e. $-\dfrac{5}{8} - \dfrac{5}{6}$

    f. $-0.9 - \dfrac{4}{9}$

    g. $-\dfrac{1}{2} + 4$

2.   Estimate each product or quotient. Then find the actual product or quotient.

    a. $-6\left(\dfrac{1}{6}\right)$

    b. $-1.6(-0.4)$

    c. $\dfrac{-8}{9}(0.5)$

    d. $-4(-0.2)$

    e. $-10 \div \dfrac{1}{2}$

    f. $-\dfrac{8}{9} \div \left(-\dfrac{3}{4}\right)$

    g. $\dfrac{6.3}{-0.7}$

    h. $\dfrac{-3}{8} \div -0.9$

3.   Solve each of the following contextual problems involving rational numbers. Express answers as fractions or decimals when appropriate.

    a.   Viviana buys a 500 mL bottle of water. She knows that she usually drinks 80 mL each hour. How many hours will it take her to finish her bottle of water?

    b.   The temperature at midnight was –2° C. By 8 am, it had risen 1.5°. By noon, it had risen another 2.7°. Then a storm blew in, causing it to drop 4.7° by 6 pm. What was the temperature at 6 pm?

    c.   Xochitl has saved $201.60. Yamka has saved $\dfrac{2}{3}$ the amount Xochitl saved. Zack has saved 1.5 times the amount Yamka saved. How much have they save all together?