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CHAPTER 3: Expressions and Equations Part 1 (4-5 weeks)

UTAH CORE Standard(s): Expressions and Equations

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. 7.EE.1

2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. 7.EE.2

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. 7.EE.3

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations to solve problems by reasoning about the quantities. 7.EE.4
   a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. 7.EE.4a

CHAPTER OVERVIEW:

The goal of chapter 3 is to facilitate students' transition from concrete representations and manipulations of arithmetic and algebraic thinking to abstract representations. Each section supports this transition by asking students to model problem situations, construct arguments, look for and make sense of structure, and reason abstractly as they explore various representations of situations. Throughout this chapter students work with fairly simple expressions and equations to build a strong intuitive understanding of structure (for example, students should understand the difference between $2x$ and x^2 or why $3(2x - 1)$ is equivalent to $6x - 3$ and $6x + -3$). Students will continue to practice skills manipulating algebraic expressions and equations throughout Chapters 4 and 5. In Chapter 6, students will revisit ideas in this chapter to extend to more complicated contexts and manipulate with less reliance on concrete models. Another major theme throughout this chapter is the identification and use in argument of the arithmetic properties. The goal is for students to understand that they have used the commutative, associative, additive and multiplicative inverse, and distributive properties informally throughout their education. They are merely naming and more formally defining them now for use in justification of mathematical (quantitative) arguments. See the Mathematical Foundation for more information.

Section 3.1 reviews and builds on students' skills with arithmetic from previous courses to write basic numerical and algebraic expressions in various ways. In this section students should understand the difference between an expression and an equation. Further, they should understand how to represent an unknown in either an expression or equation. Students will connect manipulations with numeric expressions to manipulations with algebraic expressions. In connecting the way arithmetic works with integers to working with algebraic expressions, students name and formalize the properties of arithmetic. By the end of this section students should be proficient at simplifying expressions and justifying their work with properties of arithmetic.

Section 3.2 uses the skills developed in the previous section to solve equations. Students will need to distribute and combine like terms to solve equation. In 7^{th} grade, Students only solve linear equations in the form of

$ax + b = c$ or $a(x + b) = c$, where a, b, and c are rational numbers. This section will rely heavily on the use of models to solve equations, but students are encouraged to move to abstract representation when they are ready and fluent with the concrete models.

Section 3.3 ends the chapter with application contexts. Contexts involve simple equations with rational numbers. Percent increase and decrease is revisited here. Time should be spent understanding the meaning of each part of equations and how the equation is related to the problem context. Note that the use of models is to develop an intuitive understanding and to transition students to abstract representations of thinking.

VOCABULARY: coefficient, constant, expression, equation, factor, integer, like terms, product, rational number, simplify, term, unknown, variable.

CONNECTIONS TO CONTENT:

Prior Knowledge:

Students extend the skills they learned for operations with whole numbers, integers and rational numbers to algebraic expressions in a variety of ways. For example, in elementary school students modeled 4×5 as four "jumps" of five on a number line. They should connect this thinking to the meaning of "$4x$" or "$4(x + 1)$." Students also modeled multiplication of whole numbers using arrays in earlier grades. In this chapter they will use that logic to multiply using unknowns. Additionally, in previous grades, students explored and solidified the idea that when adding/subtracting one must have "like units." Thus, when adding 123 + 14, we add the "ones" with the "ones," the "tens" with the "tens" and the "hundreds" with the "hundreds". Similarly, we cannot add 1/2 and 1/3 without a common denominator because the unit of 1/2 is not the same as a unit of 1/3. Students should extend this idea to adding variables. In other words, $2x + 3x$ is $5x$ because the unit is x, but $3x + 2y$ cannot be simplified further because the units are not the same (three units of x and two units of y.)

In 6th grade, students solved one-step equations. Students will use those skills to solve equations with more than one-step in this chapter. Earlier in this course, students developed skills with rational number operations. In this chapter, students will be using those skills to solve equations that include rational numbers.

Future Knowledge:

As students move on in this course, they will continue to use their skills in working with expressions and equations in more complicated situations. The idea of inverse operations will be extended in later grades to inverse functions of various types. Additionally, students in later grades will return to the idea of field axioms.

A strong foundation in simplifying expressions and solving equations is fundamental to later grades. Students will also need to be proficient at translating contexts to algebraic expressions and equations AND at looking at expressions and equations and making sense of them relative to contexts.

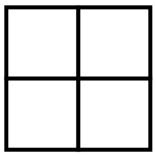
MATHEMATICAL PRACTICE STANDARDS (emphasized):

	Make sense of problems and persevere in solving them	Students will make sense of expressions and equations by creating models and connecting intuitive ideas to properties of arithmetic. Properties of arithmetic should be understood beyond memorization of rules.
	Reason abstractly and quantitatively	Students will, for example, note that $x + x + x + x + x$ is the same as $5x$. Students should extend this type of understanding to $5(x + 1)$ meaning five groups of $(x + 1)$ added together, thus simplifying to $5x + 5$. For each of the properties of arithmetic, students should connect concrete understanding to abstract representations.
	Construct viable arguments and critique the reasoning of others	Students should be able to explain and justify any step in simplifying an expression or solving an equation first in words and/or pictures and then with properties of arithmetic. Further, students should be able to evaluate the work of others to determine the accuracy of that work and then construct a logical argument for their thinking that involves properties of arithmetic.
	Model with mathematics	Students should be able to model situations with expressions and equations AND they should be able to translate expressions and equations to contexts. Further, they should be able to interchange models with abstract representations.
	Attend to precision	Students demonstrate precision by using correct terminology and symbols when working with expressions and equations. Students use precision in calculation by checking the reasonableness of their answers and making adjustments accordingly.
	Look for and make use of structure	Using models, students develop an understanding of algebraic structures. For example, in section 3.2 students should understand the structure of an equation like $3x + 4 = 5$ as meaning the same thing as $3x = 1$ or $x = 1/3$ when it is "reduced." Another example, in section 3.3, students will write equations showing that a 20% increase is the original amount plus 0.2 of the original amount or 1.2 of the original amount. They will note that in an expression that can be written in multiple ways: $x + 0.2x$ or $x(1 + 0.2)$.
	Use appropriate tools strategically	Students demonstrate their ability to select and use the most appropriate tool (paper/pencil, manipulatives, and calculators) while solving problems. Students should recognize that the most powerful tool they possess is their ability to reason and make sense of problems.
	Look for and express regularity in repeated reasoning	Students will study patterns throughout this chapter and connect them to both their intuitive understanding and the properties of arithmetic. Students will move to expressing patterns they notice to general forms.

3.0 Anchor Problem: Decorating a Patio.

Below are the first three steps of a pattern; on the next page we will investigate steps 10 and "x." Each square in the pattern is one unit.

Step 1



How many units are in this step of the pattern? Write down at least two methods that you could use to quickly "add up" the units.

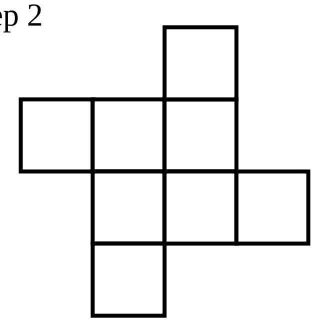
Method 1:

Method 2:

Method 3:

Method 4:

Step 2



How many units are in this step of the pattern? Write down at least two methods that you could use to quickly "add up" the units.

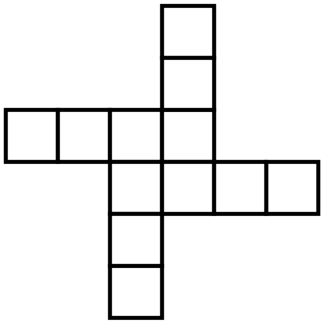
Method 1:

Method 2:

Method 3:

Method 4:

Step 3



How many units are in this step of the pattern? Write down at least two methods that you could use to quickly "add up" the units.

Method 1:

Method 2:

Method 3:

Method 4:

Step 10

Draw what you think step 10 would look like.

How many units are in this step of the pattern? Write down at least two methods that you could use to quickly “add up” the units.

Method 1:

Method 2:

Method 3:

Method 4:

Step *x*

Use this space to draw a model to help you think about the pattern.

How could you adapt the methods you used above to find the number of units for ***any*** pattern.

Method 1:

Method 2:

Method 3:

Section 3.1: Communicate Numeric Ideas and Contexts Using Mathematical Expressions and Equations

Section Overview: This section contains a brief review of numerical expressions. Students will recognize that a variety of expressions can represent the same situation. Models are encouraged to help students connect properties of arithmetic in working with numeric expressions to working with algebraic expressions. These models, particularly algebra tiles, aid students in the transition to abstract thinking and representation. Students extend knowledge of mathematical properties (commutative property, associative property, etc.) from purely numerical problems to expressions and equations. The distributive property is emphasized and factoring, "backwards distribution," is introduced. Work on naming and formally defining properties appears at the beginning of the section so that students can attend to precision as they verbalize their thinking when working with expressions. Through the section, students should be encouraged to explain their logic and critique the logic of others.

Concepts and Skills to be Mastered (from standards)

By the end of this section, students should be able to:

1. Recognize and explain the meaning of a given expression and its component parts.
2. Recognize that different forms of an expression may reveal different attributes of the context.
3. Combine like terms with rational coefficients.
4. Use the Distributive Property to expand and factor linear expressions with rational numbers.
5. Recognize properties of arithmetic and use them in justifying work when manipulating expressions.
6. Write numeric and algebraic expressions to represent contexts.

3.1a Class Activity: Naming Properties of Arithmetic

Naming Properties of Arithmetic
In mathematics, there are things called "properties;" you may think of them as "rules." There is nothing new in the properties discussed in this section. Everything you expect to work still works. We are just giving vocabulary to what you've been doing so that when you construct a mathematical argument, you'll be able to use language with precision.

Commutative Property
Examples:
The sum of both 8 + 7 + 2 and 8 + 2 + 7 is 17.
The sum of both 13 + 14 + (–3) and 13 + (–3) + 14 is 24.
The product of both $\left(\frac{1}{2}\right)$ (7)(8) and $\left(\frac{1}{2}\right)$ (8)(7) is 28.
The product of both (9) $\left(-\frac{1}{3}\right)$ (7) and $\left(-\frac{1}{3}\right)$(9)(7) is –21.

The word "commute" means "to travel" or "change." It's most often used in association with a location. For example, we say people ***commute*** to work.

Which pairs of expressions are equivalent?

1. equivalent 12 + 4 4 + 12	2. not equivalent 9.8 – 3.4 3.4 – 9.8
3. not equivalent 12 – 4 4 – 12 Ask students if there is a way to commute with subtraction. → 12 + (–4) = (–4) + 12	4. equivalent 5 · 4 4 · 5
5. equivalent 3 · 0.9 0.9 · 3	6. not equivalent 18 ÷ 6 6 ÷ 18

7. What pattern are you noticing? You can change the order of addition and multiplication and get an equivalent expression, but you cannot do that with subtraction and division.

8. In your own words, what is the Commutative Property?
$a + b = b + a$; $ab = ba$; Addition and multiplication are commutative (you can change the order of addition or multiplication) without affecting the result.

Associative Property

The word "associate" means "partner" or "connect." Most often we use the word to describe groups. For example, if a person goes to Eastmont Middle School and not Indian Hills Middle School, we would say that person is ***associated*** with Eastmont Middle School.

Examples:
The sum of both 3 + (17 + 4) + 16 and (3 + 17) + (4 + 16) is 40
The product of both (2×5)(3) and (2)(5×3) is 30

For each of the following pairs of expressions, the operations are the same, but the constants have been associated (grouped) in different ways. Determine if the pairs are equivalent; be able to justify your answer. Be sure to use the order of operations.

9. (12 + 4) + 6 equivalent 12 + (4 + 6)	10. (12 – 4) – 3 not equivalent 12 – (4 – 3) Ask how both could be written to make them equivalent→ 12 + (–4 + –3) = (12 + –4) + –3
11. (3 + 5) + 7.4 equivalent 3 + (5 + 7.4)	12. (20.9 – 8) – 2 not equivalent 20.9 – (8 – 2)
13. $(5 \cdot 4) \cdot \left(\frac{1}{2}\right)$ equivalent $5 \cdot \left(4 \cdot \left(\frac{1}{2}\right)\right)$	14. (18 ÷ 6) ÷ 3 not equivalent 18 ÷ (6 ÷ 3) Ask how both could be written to make them equivalent→(18 · 1/6) · (1/3) = 18 · (1/6 · (1/3))
15. (6 · 2) · 5 equivalent 6 · (2 · 5)	16. (24 ÷ 12) ÷ 3 not equivalent 24 ÷ (12 ÷ 3)

17. What patterns do you notice about the problems that were given?
Addition and multiplication can be grouped in different ways and still give the same result. This is not true of subtraction and division.

18. In your own words, what is the Associative Property? $(a + b) + c = a + (b + c)$; $(ab)c = a(bc)$; Addition and multiplication are associative (you can change the grouping), but subtraction and division are not.

Identity Property

The word "identity" has to do with "sameness." We use this word when we recognize the sameness between things. For example, you might say that a Halloween costume cannot really hide a person's true ***identity***.

Above we defined the Associative and Commutative Properties for both addition and multiplication. We need to do the same thing for the Identity Property.

19. What do you think the Identity Property for Addition should mean? Answers will vary. Look for something like "doesn't change the identity of the expression."

20. Give examples of what you mean:

21. In your own words, what is the Identity Property of Addition? $a + 0 = a$; you can add "0" to anything and it won't change the expression.

22. What do you think the Identity Property for Multiplication should mean? Answers will vary. Look for something like "doesn't change the identity of the expression."

23. Give examples of what you mean:

24. In your own words, what is the Identity Property of Multiplication: $a(1) = a$; you can multiply anything by 1 and it won't change the expression.

Inverse Properties

The word “inverse” means “opposite” or “reverse.” You might say, forward is the ***inverse*** of backward. There is an inverse for both addition and multiplication.

25. What do you think should be the additive inverse of 3?
-3

26. What do you think would be the additive inverse of –3?
3

27. What do you think would be the multiplicative inverse of 3?
$\frac{1}{3}$

28. What do you think would be the multiplicative inverse of $\frac{1}{3}$?
3

29. In your own words, what is the Inverse Property of Addition? $a + (-a) = 0$; discuss how this property is related to the additive identity

30. In your own words, what is Inverse Property of Multiplication?
$a(1/a) = 1$ for $a \neq 0$; discuss how this property is related to the multiplicative identity.

Generalizing Properties

Properties of Mathematics:

Name Property	Algebraic Statement	Meaning	Numeric Examples
Identity Property of Addition	$a + 0 = a$	Adding zero to a number does not change the number. "Zero" can take many forms.	$5 + 0 = 5$; $5 + (1 + (-1)) = 5$
Identity Property of Multiplication	$a(1) = a$	Multiplying a number by one does not change the number.	
Multiplicative Property of Zero	$a(0) = 0$	Multiplying a rational number by zero results in 0.	
Commutative Property of Addition	$a + b = b + a$	Reversing the order of addition does not change the result.	
Commutative Property of Multiplication	$ab = ba$	Reversing the order of multiplication does not change the result.	
Associative Property of Addition	$(a + b) + c = a + (b + c)$	Changing the grouping of addition does not change the result.	
Associative Property of Multiplication	$a(bc) = (ab)c$	Changing the grouping of multiplication does not change the result.	
Additive Inverse	$a + (-a) = 0$	A number added to its opposite will result in zero (zero being the additive identity).	
Multiplicative Inverse	$a(1/a) = 1$ for $a \neq 0$	Multiplying a number by its multiplicative inverse will result in one (one being the multiplicative identity).	

3.1a Homework: Naming Properties of Arithmetic

Complete the table below:

Identity Property of Addition: $a + 0 = a$	**1a.** Show the Identity Property of Addition with 2.17 2.17 + 0 = 2.17	**1b.** Show the Identity Property of Addition with –3
Identity Property of Multiplication: $a \cdot 1 = a$	**2a.** Show the Identity Property of Multiplication with 23	**2b.** Show the Identity Property of Multiplication with –3*b* $-3b \cdot 1 = 3b$
Multiplicative Property of Zero: $a \cdot 0 = 0$	**3a.** Show Multiplicative Property of Zero with 43.581	**3b.** Show the Multiplicative Property of Zero with –4*xy* $-4xy \cdot 0 = 0$
Commutative Property of Addition: $a + b = b + a$	**4a.** 4.38 + 2.01 is the same as:	
	4b. $x + z$ is the same as: $z + x$	
Commutative Property of Multiplication: $ab = ba$	**5a.** $\frac{5}{7} \cdot \frac{3}{8}$ is the same as:	
	5b. 6*k* is the same as: $k \cdot 6$	
Associative Property of Addition: $(a + b) + c = a + (b + c)$	**6a.** (1.8 + 3.2) + 9.5 is the same as: 1.8 + (3.2 + 9.5)	
	6b. $(x + 1) + 9$ is the same as:	
Associative Property of Multiplication: $(ab)c = a(bc)$	**7a.** (2.6 · 5.4) · 3.7 is the same as:	
	7b. $(wh)l$ is the same as:	
Use the listed property to fill in the blank.		
Multiplicative Inverse: $a\left(\frac{1}{a}\right) = 1$	**8a.** $3\left(\frac{1}{3}\right) = 1$	
	8b. $\frac{1}{4}(\quad) = 1$	
Additive Inverse: $a + (-a) = 0$	**9a.** $\frac{5}{9} + -\frac{5}{9} = 0$	
	9b. $\quad + -x = 0$	

Name the property demonstrated by each statement.

10.	$3 + -2 + 7 = 3 + 7 + -2$	Commutative Property of Addition
11.	$5 + (-5 + 4) + 6 = (5 + -5) + (4 + 6)$	
12.	$25 + (-25) = 0$	Additive Inverse Property
13.	$(2/5)(5/2) = 1$	Multiplicative Inverse
14.	$(x + 3) + y = x + (3 + y)$	
15.	$2.37 \times 1.5 = 1.5 \times 2.37$	
16.	$1 \cdot mp = mp$	Identity Property of Multiplication
17.	$9 + (5 + 35) = (9 + 5) + 35$	
18.	$0 + 6b = 6b$	
19.	$xy = yx$	Commutative Property of Multiplication
20.	$7x \cdot 0 = 0$	
21.	$4(3 \cdot z)=(4 \cdot 3)z$	Associative Property of Multiplication
22.	$\frac{2}{3} \cdot 4.9 = 4.9 \cdot \frac{2}{3}$	
23.	$x + 4 = 4 + x$	

Spiral Review

1. Order the following rational numbers from least to greatest. $\frac{27}{3}$, 6.5, $\frac{18}{3}$, 0.99

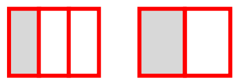
   0.99, $\frac{18}{3}$, 6.5, $\frac{27}{3}$

2. What is the opposite of -19? +19 or 19

3. What is 10% of 90? Use bar model to help you. 9

4. Without using a calculator, determine which fraction is bigger in each pair. Justify your answer with a picture *and* words.

   a. $\frac{1}{3}$ or $\frac{1}{2}$

   1 piece out of a pie cut into 2 will be bigger than 1 piece out of a pie cut into 3 pieces

   b. $\frac{3}{7}$ or $\frac{3}{5}$

5. Solve: $\frac{3}{4} + \frac{1}{8} = \mathbf{\frac{7}{8}}$

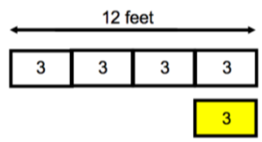
3.1b Class Activity: Translating Contexts to Numeric Expressions (Equivalent Expressions)

Review from Chapter 1: In 1.3, you drew models and then wrote arithmetic expressions to find fraction and percent decreases and increases. For example:

1.3a Class Activity 1a was a percent decrease of the original amount:

Larry has a piece of rope that's 12 feet long.

He cuts off 25% of the rope off. How long is the rope now?

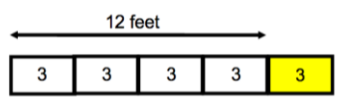


To write an expression for the length of the rope with the portion cut off, we can think about it two ways:

… we started with the 12 foot section and then subtracted 25% of 12: 12 – (0.25)12 12 – 3 9 feet of rope	…we can recognize that when we remove 25% of the rope, we're left with 75% (100% – 25%) of the rope: (1 – 0.25)12 (0.75)12 9 feet of rope

1.3 Class Activity 1b was a percent increase of the original amount:

Joe has a rope that is 25% longer than Larry's 12-foot long rope. How long is Joe's rope?



To write an expression for the percent increase we think about it two ways:

…we start with the 12 foot rope and then add 25% of 12 to the length: 12 + (0.25) 12 12 + 3 15 feet of rope	…we recognize that a 25% increase means we start with the whole length and then add 25%; thus we now have 125% (100% + 25%) of the original: (1 + 0.25) 12 (1.25) 12 15 feet of rope

In both examples above, we wrote ***equivalent expressions***. What do you think it means to say "two expressions are equivalent"?

For each context: a) write a numeric expression, b) justify an estimate for the answer, and then c) find the answer (a calculator will be helpful for these.)

1) The tree in Maria's yard has grown 27% in the last two years. If it was originally 12 feet tall two years ago, how tall is it now? 12(1) + 12(0.27) or 12(1.27) = 15.24 ft; estimate: 27% ≈ 25% and 25% of 12 is 3 so the tree should be about 15 feet tall.

2) Paulo can run a marathon in four hours and 20 minutes. If he's able to cut 18% off his time, how long will it take him to run a marathon? 260(0.82) or 260(1) – 260(0.18) or (1 – 0.18)(260) = 213.2 min

3) Joe's snowboard is 140 centimeters; Carly's is 14% longer. How long is Carly's snowboard?

140(1.14) or 140(1) + 140(0.14) or (1 + 0.14)(140) = 159.6 cm

4) Write a context involving a percent for the following numeric expression: 300 + 0.27(300)

Student responses will vary.

5) Write a context involving a percent for the following numeric expression: 0.78(144)

Student responses will vary.

6) Juan, Calista, and Angelo are working together on the following percent problem:

There are 400 fish in a large tank. Write a numeric expression to find the number of fish in the tank if the number of fish increases by 23%.

Juan says the expression is 1.23(400); Calista says it's 400 + 0.23; and Angelo says it's (1 + 0.23)(400). Are they all correct? Explain why each expression is correct or incorrect.

7) Below are 4 expressions, circle the three that are equivalent.
   a) (1 – 0.35)(423)
   b) (0.65)(423)
   c) 423 – 0.35(423)
   d) 423 – 0.35

For each of the five contexts below, there are four numeric expressions offered. Look at each expression offered and determine whether or not it is appropriate for the given context. Explain why the expression "works" or "doesn't work."

8. Josh made five 3-pointers and four 2-pointers at his basketball game. Write a numeric expression to represent how many points Josh scored.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	3 + 3 + 3 + 3 + 3 + 2 + 2 +2 + 2	23	Yes	Each basket is added together, giving the total.
b.	5 + 3 + 4 + 2	14	No	5+3 is not an accurate interpretation of "five 3-pointers"
c.	(5 + 3)(4 + 2)	48	No	5+3 is not an accurate interpretation of "five 3-pointers"
d.	5(3) + 4(2)	23	Yes	It is the same as A. 5(3) means 5 groups of 3 as above. 4(2) means 4 groups of 2.

9. Carlo bought two apples for $0.30 each and three pounds of cherries for $1.75 a pound. Write a numeric expression for how much money Carlo spent.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	2(1.75) + 3(0.30)			
b.	2(0.30) + 3(1.75)			
c.	0.30 + 0.30 + 1.75 + 1.75 + 1.75			
d.	(2 + 3)(0.30 + 1.75)			

10. Inez bought two apples for $0.30 each and three oranges for $0.30 each. Write a numeric expression for how much money Inez spent.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	(0.30 + 0.30 + 0.30) + (0.30 + 0.30)	$1.50	Yes	Added each purchase individually.
b.	3(0.30) + 2(0.30)	$1.50	Yes	This uses multiplication instead of repeated addition.
c.	5(0.30)	$1.50	Yes	This expression works because the cost for each fruit is always $0.30.
d.	0.60 + 0.90	$1.50	Yes	This sums the total cost of apples and the total cost of oranges

11. Aunt Nancy gave her favorite niece 3 dollars, 3 dimes, and 3 pennies. Write a numeric expression for how much money Nancy gave her favorite niece.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	3(1.00) + 3(0.10) + 3(0.01)			
b.	3(1.00 + 0.10 + 0.01)			
c.	3 + 1.00 + 0.10 + 0.01			
d.	3.00 + 0.30 + 0.03			

12. Grandma Nancy has 20 chocolate and 25 red velvet cupcakes. She gives each of her three grandchildren 1/3 of her cupcakes. Write an expression for how many cupcakes she gave each grandchild.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	3(20 + 25)	135	No	This triples the number of cupcakes Grandma has.
b.	(1/3)(20 + 25)	15	Yes	This combines the cupcakes and gives 1/3 to each grandchild.
c.	(20 + 25)/3	15	Yes	This combines the cupcakes and then divides the amount into 3 equal portions.
d.	(1/3)(20) + (1/3)(25)	15	Yes	This gives 1/3 of each kind of cupcake to each grandchild.

13. Mila bought a sweater for $25 and a pair of pants for $40. She had a 25% off coupon for her purchase, write an expression for how much Mila spent.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	0.25(25 + 40)			
b.	0.75(25 + 40)			
c.	0.75(25) + 0.75(40)			
d.	(3/4)(25 + 40)			

14. Paul bought two sandwiches for $5 each, a drink for $3.00, and a candy bar for $1.00. He had a 20% off coupon for the purchase. Write an expression to how much Paul spent for the food.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	0.20(2(5) + 3 + 1)	$2.80	No	This is 20% of the total amount (what he saved), not 20% off.
b.	0.80(2(5) + 3 + 1)	$11.20	Yes	20% discount is the same as paying 80% of the price of all items.
c.	0.80(2(5)) + 3 + 1	$12	No	You're only taking 20% off the sandwiches
d.	(4/5)(2(5) + 3 + 1)	$11.20	Yes	80% of an amount is the same as 4/5 of the amount

For each context: a) write a numeric expression for the context and b) answer the question.

15. Aunt Nancy gave each of her four nieces two dollars, 1 dime, and 3 pennies. How much money did Nancy give away? Answers will vary: 4(2(1) + 1(0.10) + 3(0.01)) $8.52
2(1) + 1(0.10) + 3(0.01) + 2(1) + 1(0.10) + 3(0.01) + 2(1) + 1(0.10) + 3(0.01) + 2(1) + 1(0.10) + 3(0.01)

16. Uncle Aaron gave 8 dimes, 2 nickels, and 20 pennies to each of his two nephews. How much money did he give away?

17. I bought 2 toy cars for $1.25 each and 3 toy trucks for $1.70 each. How much did I spend?
Answers will vary: 2(1.25) + 3(1.70) $7.60

18. The football team scored 1 touchdown, 3 field goals, and no extra points. How many points did they score in all? Hint: a touchdown is worth 6 points, a field goal worth 3, and an extra point worth one.
Answers will vary: 1(6) + 3(3) 15 points

19. I had $12. Then I spent $2.15 a day for 5 days in a row. How much money do I have now?

20. I earned $6. Then I bought 4 candy bars for $0.75 each. How much money do I have left?
Answers will vary: 6 – 4(0.75) $3

21. Cara bought two candles for $3 each and three books for $7 each. She had a 25% off her entire purchase coupon. How much did she spend on her purchases?

22. Mona and Teresa worked together to make $118 selling phone covers and $354 fixing computers. If they split the money evenly between the two of them, how much money did they each make?
(1/2)(118 + 354) $236

23. Dora bought three pair of shoes for $15 each and two pair of shorts for $20 each. If she had a 15% off coupon for her entire purchase, how much money did she spend?

3.1b Homework: Translating Contexts to Numerical Expressions

For each context: a) write a numeric expression, b) justify an estimate for the answer, and then c) find the answer (a calculator will be helpful for these).

1. Camila is allowed 1100 text messages a month on her plan. When she got her September bill it showed she had gone over the allowed text messages by 38%. How many text messages did she send in September?
   1100(1 + 0.38) = 1100 (1.38) = 1518

2. Leona made $62,400 in her first year as an engineer at IM Flash in Lehi. She's up for review and stands to get a 12.5% pay raise if her review is favorable. If she gets the pay raise, what will she be earning?

For each of the five contexts below, there are four numeric expressions offered. Look at each expression offered and determine whether or not it is appropriate for the given context. Explain why the expression "works" or "doesn't work."

3. I bought two toy cars for $5 each and three toy trucks for $7 each. Write a numeric expression for how much was spent.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	2(5) + 3(7)			
b.	2(3) + 5(7)	$41	No	This expression pairs the wrong quantities together.
c.	(2 + 3)(5 + 7)			
d.	(2 + 3) + (5 + 7)			

4. The football team scored three touchdowns, two field goals, and two extra points. Write a numeric expression for how many points were scored in all. (Hint: a touchdown is 6 points, a field goal is 3 points, and an extra point is 1 point)

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	3(6) + 2(3) + 2(1)			
b.	6 + 6 + 6 + 3 + 3 + 1 + 1			
c.	(6 + 6 + 6) + (3 + 3) + (1 + 1)			
d.	18 + 6 + 2	26	Yes	This is the same as a) or c) after the first step of simplification.

5. I earned $6. Then I bought 4 candy bars for $0.50 each. Write a numeric expression for how much money I have left.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	6 – 0.50 – 0.50 – 0.50 – 0.50			
b.	6 – 4(0.50)			
c.	6 – (0.50 – 0.50 – 0.50 – 0.50)			
d.	6 – (0.50 + 0.50 + 0.50 + 0.50)	$4	Yes	This subtracts the sum of 0.50 four times.

6. I earned $5. Then I spent $1 a day for 2 days in a row. Write a numeric expression for how much money I have now.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	5 − 1 + 1			
b.	5 – 1 – 1			
c.	5 – (1 – 1)	$5	No	We need to subtract the sum of the two dollars.
d.	5 – (1 + 1)			

7. Uncle Aaron bought three books for $12 each and a candy bar for $1.35. He had a coupon for 20% off his purchase. Write an expression for how much he spent.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	0.80(3)(12) + 1.35	$30.15	No	80% is applied only to the cost of the books.
b.	0.08(3(12) + 1.35)			
c.	0.80(3(12) + 1.35)			
d.	(4/5)(3(12) + 1.35)			

8. Leah bought four glue sticks for $0.50 each and three small cans of paint for $2.25 each at the craft store. She had a 25% off the entire purchase coupon she applied. Write an expression for how much Leah spent.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	4(0.50) + 3(2.25) – 0.25			
b.	0.25(4(0.50) + 3(2.25))			
c.	0.75(4(0.50) + 3(2.25))			
d.	0.75(4(0.50)) + 0.75(3(2.25))	$5.56 (rounded)	Yes	This is taking 75% of each part.
e.	3/4 (4(0.50) + 3(2.25))			

Write an expression of your own for each problem. Then evaluate the expression to solve the problem.
There are various accurate expressions for these problems. They should each result in the value given.

9. Josh made ten 3-pointers and a 2-pointer at his basketball game. How many points did he score?

10. I bought five apples for $0.30 each and 5 oranges for $0.30 each. How much money did I spend?
5(0.30) + 5(0.30) $3.00

11. I bought two apples for $0.50 each and four oranges for $0.25 each. When I got to the cash register, I got a 50% discount. How much money did I spend?

12. At the store I bought three sweaters for $25 each and two pair of pants for $30 each. I had a coupon for 25% off the total purchase. How much money did I spend?
0.75(3(25) + 2(30)) $101.25

Write a context similar to the ones in # 8 – 12 for each numeric expression given.
Answers will vary.

13. 10 – 3(0.75) – 2(0.50)

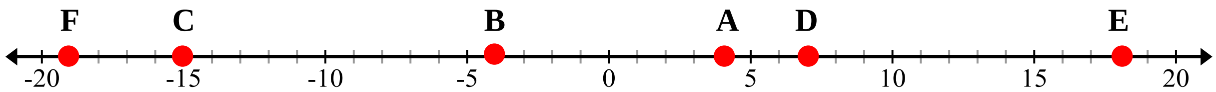
14. 3(2 + 5)

15. (1/2)(2(60) + 3(10))

16. (0.80)(25 + 15)

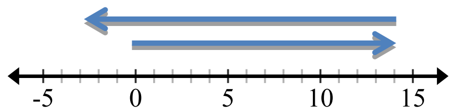
Spiral Review

1. Locate each of the following on the line below: **A** = 4 **B** = –4 **C** = –15 **D** = 7 **E** = 18 **F** = –19



2. For the model below, write a numeric expression and then create a context.

14 + –17= –3



3. Simplify: $2\frac{3}{5} - 1\frac{1}{2}$. $1\frac{1}{10}$

4. 18 ÷ (6 ÷ 3) = 6

5. $3 \div \frac{3}{4} =$ 4

3.1c Class Activity: Algebraic Expressions

Write two *different* numeric expressions for the context below:

Maria bought 5 apples for \$0.35 each.
$5(0.35)$or $0.35 + 0.35 + 0.35 + 0.35 + 0.35$
How would the expression change if she spent \$0.40 on each apple?
$5(0.40)$or $0.40 + 0.40 + 0.40 + 0.40 + 0.40$
What if you didn't know how much each apple cost, how could you write an expression?
$5x$ or $x + x + x + x + x$
For each context, four algebraic expressions are offered. Make a conjecture about the correctness of the expression. Then, evaluate it for the given value, and explain why the expression did or didn't work for the given context.

1. Ryan bought 3 CDs for x dollars each and a DVD for \$15. Write an expression of how much money Ryan spent.

	Expression	Correct expression?	*Evaluate* $x = 7$	Did it work?	Why or why not?
a.	$3 + x + 15$	-No-	\$25	No	"$3 + x$" does not translate "3 CDs for x dollars".
b.	$15x + 3$	-No-	\$108	No	"$15x$" does not represent "a DVD for \$15"
c.	$15 + x + x + x$	-Yes-	\$36	Yes	There is 15 for the DVD, then three CDs, each represented with an x.
d.	$3x + 15$	-Yes-	\$36	Yes	This uses multiplication to make expression C more concise.

2. I started with 12 jellybeans. Sam ate 3 jellybeans and then Cyle ate y jellybeans. Write an expression for how many jellybeans were left.

	Expression	Correct expression?	*Evaluate* $y = 6$	Did it work?	Why or why not?
a.	$12 – 3 – y$				
b.	$12 – (3 – y)$				
c.	$12 – (3 + y)$				
d.	$9 – y$				

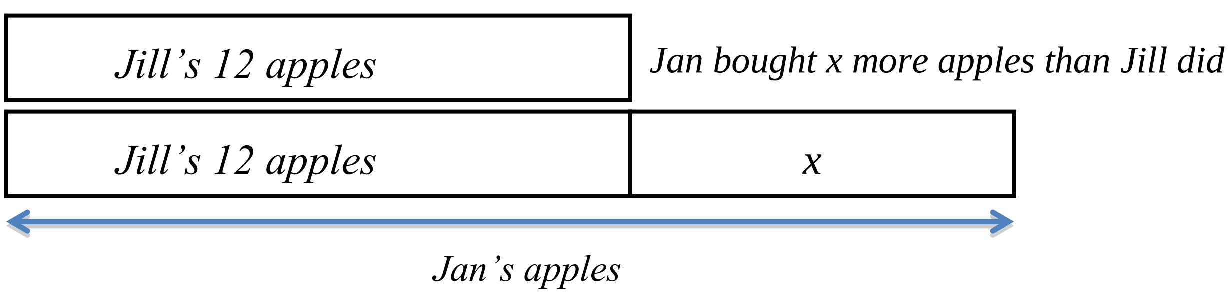
3. Kim bought a binder for \$5 and 4 notebooks for n dollars each. She received a 30% discount on the items. Write an expression for how much she spent.

	Expression	Do you think it will work?	*Evaluate* (use n = 3)	Did it work?	Why or why not?
a.	$0.70(5 + 4 + n)$	---	\$8.40	No	"$4 + n$" does not accurately represent "4 notebooks for n dollars each".
b.	$0.70(4n + 5)$	---	\$11.90	Yes	This sums 4 times the cost of a notebook with the other item and then 70% of that amount.
c.	$0.70(5n + 4n)$	---	\$18.90	No	This is 70% of the sum of \$5n AND 4n. We only need to multiply the 4 by n.
d.	$0.30(4n + 5)$	---	\$5.10	No	The $4n + 5$ is correct, but 30% is the discount, not the amount spent.

For each context below, draw a model for the situation, label all parts, and then write an ***expression*** that answers the question. The first exercise is done for you.

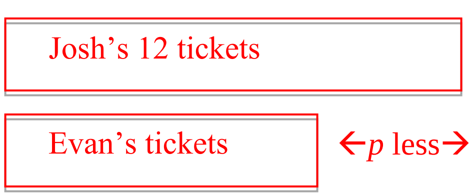
Example: Jill bought 12 apples. Jan bought x more apples than Jill. Write an expression to show how many apples Jan bought.

Jan bought $12 + x$ apples. This is not the only way to model these problems.



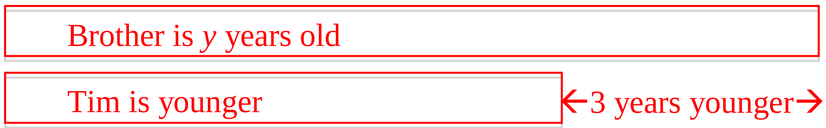
4. Josh won 12 tickets. Evan won p tickets fewer than Josh. Write an expression to represent the number of tickets Evan won.

$12 – p$



It is helpful to ask "who won more tickets?" or "Did Josh get more or less tickets than Evan?"

5. Tim is 3 years younger than his brother. If his brother is y years old, write an expression to represent Tim's age. $y - 3$



6. Carol washed 8 windows. Mila washed w windows fewer than Carol. If both Carol and Mila earn \$2 for each window they wash, write an expression to represent how much money Mila earned for washing windows.

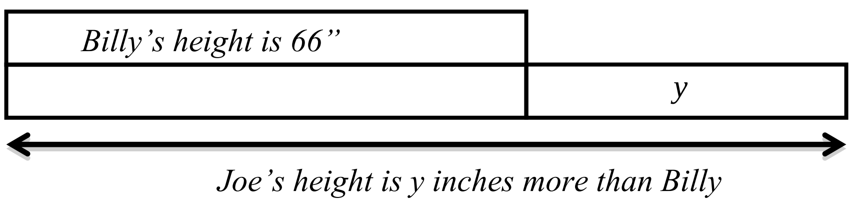
$\$2(8 - w)$ or $\$16 - \$2w$

7. Jan bought a more apples than Jill. Jill bought 4 apples. Each apple costs \$0.10. Write an expression to show how much money Jan spent on apples.

$\$0.10(4 + a)$

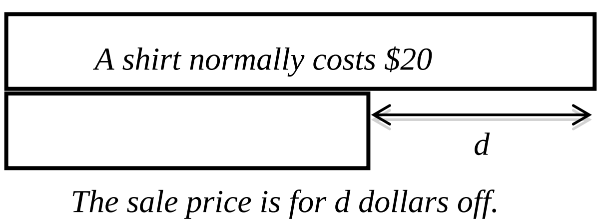
For the following models, write a context (with question) and algebraic expression that fits the given model.

8.



$66 + y$. Answers will vary: Billy and Joe measured themselves to see who was taller. Billy was 66" tall and Joey was y inches taller than Billy. Write an expression to represent Joe's height.

9.

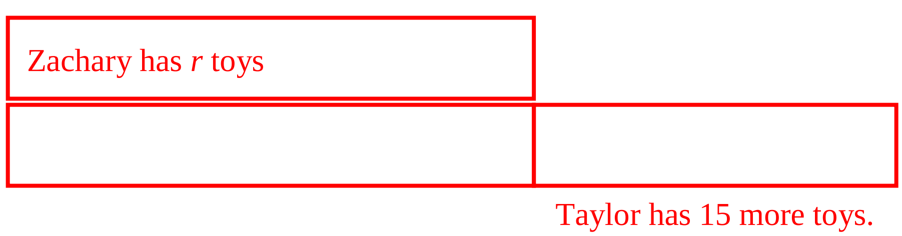


$20 - d$. Answers will vary: A shirt normally costs \$20, but the store is taking d dollars off the original price. Write an expression for the sale price of the shirt.

Create a context and draw a model for the following algebraic expressions.

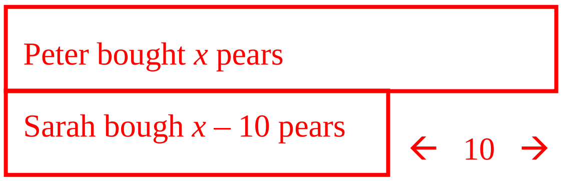
10. $r + 15$

Answers will vary: Taylor has 15 more toys than Zachary does. Zachary has r toys. Write an expression for the number of toys Taylor has.



11. $1.75(x - 10)$

Answers will vary: Sarah bought 10 fewer pears than Peter. Peter bought *x* pears and each pear cost $1.75. How much did Sarah spend? Alternative: During the first week of work Sarah earned *x* dollars and Peter earned $10 less than Sarah. The next week, Peter earned 75% more money. Write an expression for how much money Peter earned the second week of work.



Sarah bought 10 fewer pears than Peter.

Note that the same model can be used for the second context.

For each context involving a percent increase or decrease, write an algebraic expression for the questions. Use a model if desired:

12. Tanya had *x* number of marbles in her bag. She lost 25% of them when they spilled out by accident. Write an expression for the number of marbles she now has.

    $0.75x$ or $1x - 0.25x$

13. Bruno has an *m* inch vertical jump. He wants to increases it by 30%. Write an expression for how high he will be able to jump if he's able to increase his vertical jump by 30%.

    $1.3m$ or $1m + 0.3m$

14. It costs Guillermo *d* dollars to produce smartphone covers to sell. He wants to sell them for 45% more than it costs him to make them. Write an expression for how much he should sell the smartphone covers.

    $1.45d$ or $1d + 0.45d$

15. Juliana is training for a race. If she was able to reduce her time *t* by 17%, write an expression for how much time it will take to her run the race now.

    $0.83t$ or $1t - 0.17t$

3.1c Homework: Algebraic Expressions

Read each context. Make a conjecture about which expressions will work for the context given. Evaluate the expression for a given value. Explain why the expression did or didn't work for the given context.

1. Bob bought 5 books for x dollars each and a DVD for $12. Write an expression for how much money Bob spent.

	Expression	Do you think it will work?	*Evaluate* (use $x = 5$)	Did it work?	Why or why not?
a.	$5 + x + 12$	---	$22	No	"$5 + x$" does not accurately represent "5 books for x dollars each."
b.	$5(x)12$				
c.	$x + x + x + x + x + 12$				
d.	$5x + 12$				

2. Jim won 30 tickets. Evan won y tickets fewer than Jim did. Write an expression for the number of tickets Evan won.

	Expression	Do you think it will work?	*Evaluate* (use $y = 6$)	Did it work?	Why or why not?
a.	$30 - y$				
b.	$y - 30$				
c.	$y + 30$				
d.	$30 \div y$	---	5 tickets	No	The difference between their amounts is absolute; we must subtract.

Draw a model and write an expression for each context.

3. I did 4 more problems than Manuel. If I did *p* problems, write an expression for how many did Manuel did.

4. I bought *x* pair of shoes for \$25 each and 2 pairs of socks for \$3 each. Write an expression for how much money I spent.

   $\$25x + 2(\$3)$

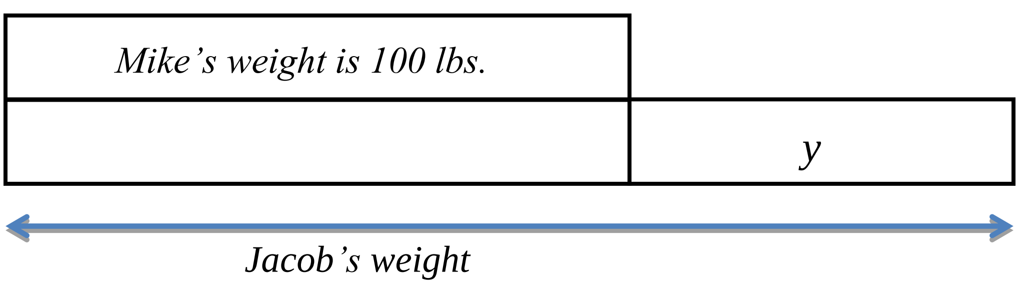
5. I bought *m* gallons of milk for \$3.59 each and a carton of eggs for \$2.24. Write an expression for how much money I spent.

6. Paul bought *s* sodas for \$1.25 each and chips for \$1.75. Write an expression for how much money Paul spent.

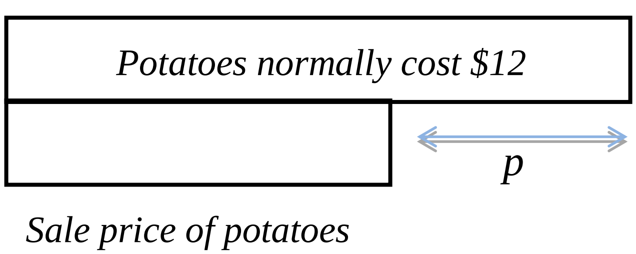
7. Roberto and Francisco went to the basketball game. Each bought a drink for *d* dollars and nachos for *n* dollars. Write an expression for the amount of money they spent all together.

For the following models, come up with a context and an expression that represents the model.

8.



9.



Create a model and write a context for the following expressions.
Answers will vary.

10. $2(q + 4)$

11. $5 - k$

For each context involving a percent increase or decrease, write an algebraic expression for the questions. Use a model if desired:

12. Marcia ran m miles on Monday. On Tuesday, she ran 18% farther than she ran on Monday. Write an expression for how far Marcia ran on Tuesday.

    $1.18m$ or $1m + 0.18m$

13. Jorge needs to reduce his expenses by 35%. If he currently spends q dollars a month, write an expression for how much he will be spending once he reduces his expenses.

14. Ruth normally orders p pounds of flour for her bakery every week. She's decided she needs to increase her order by 22%. Write an expression for how many pounds of flours she will now be ordering.

Spiral Review

1. Which number is larger? Justify your answer. -4.03 or -4.3 –4.03; it is further to the right on the real line.

2. Use long division to show how you can convert this fraction to a decimal and then a percent $\frac{2}{9}$
$0.\overline{2}$; $22.\overline{2}\%$

3. Which number is greater: –5.101 or –5.11? Justify. –5.101

3.1c Additional Practice

For each problem, draw a model, write out what the unknown stands for, and then write an expression modeling the situation.

1. Marina has \$12 more than Brandon. Represent how much money Marina has.
   b is the amount of money Brandon has. Then Marina has $b + 12$ dollars.

2. Conner is three times as old as Jackson. Represent Conner's age.

3. Diane earned \$23 less than Chris. Represent how much Diane earned.

4. Juan worked 8 hours for a certain amount of money per hour. Represent how much Juan earned.

5. Martin spent 2/3 of the money in his savings account on a new car. Represent the amount of money Martin spent on a new car.

6. Brianne had \$47 dollars. She spent \$15 on a new necklace and some money on a bracelet. Represent the amount of money Brianne has now.

7. For 5 days Lydia studied math for a certain amount of time and read for 15 minutes each day. Represent the total amount of time Lydia studied and read over the 5-day period.
   m is amount of time she studied math each day. This was repeated for 5 days so $5(m + 15)$.

8. Carlos spent \$8 on lunch, some money on a drink and \$4 on ice cream. Represent how much money Carlos spent.

9. Nalini has \$26 dollars less than Hugo. Represent the amount of money Nalini has.

10. Bruno ran four times as far as Milo. Represent the distance Bruno ran.

11. Christina earned \$420. She spent some of her earnings on her phone bill and spent \$100 on new clothes. Represent the amount of money Christina now has.
p is the amount spent on the phone bill. \$420 – p – \$100 or \$320 – p.

12. Camille has 4 bags of candy. Each bag has 3 snicker bars and some hard candy. Represent the amount of candy Camille has.

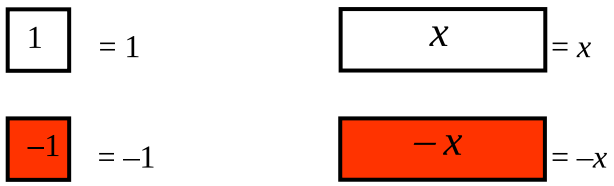
13. Heather spent ¼ of the money in her savings account on a new cell phone. Represent the amount of money Heather spent on the new cell phone.
a is the amount in her account. She spent ¼ of this or $(\frac{1}{4}) \times a$ or $(1/4)a$ or $a/4$ on the cell phone. Discuss these various representations with the class.

14. Miguel is 8 years older than Cristo. Represent Miguel's age.

3.1d Class Activity: Simplifying Algebraic Expressions with Models

In using Algebra tiles, every variable is represented by a rectangle, positive or negative and every integer is represented by a square, positive or negative.

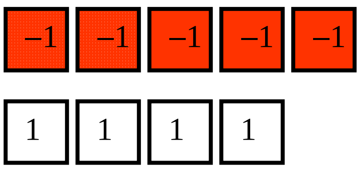
Key for Tiles:



Attention will have to be paid to: $a - (-b) = a + b$ and $a - b = a + (-b)$. For example, #1 can be expressed as $x - 2$ or $x + -2$; take time to discuss this. For problems #10 through 18, have students create the model in more than one way (e.g. $2x + 3$ can be modeled with $5x + -3x + 6 + -3$).

Write an expression for what you see and then write the expression in simplest form.

Example:

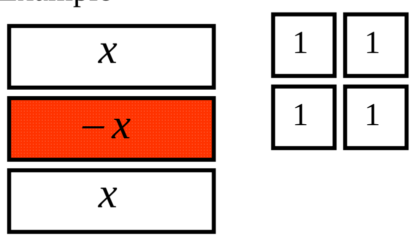


–5 + 4
–1

Remind students about "zero pairs."

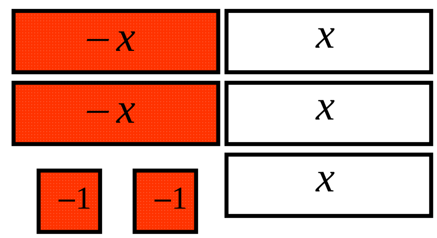
Attend to Precision

Example



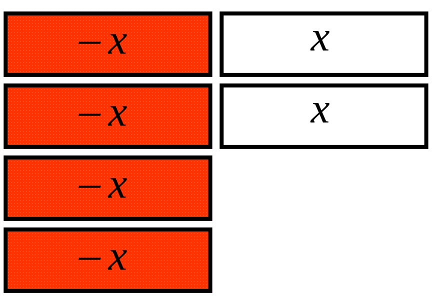
$2x - x + 4$ or
$2x + -x + 4$ or
$x + -x + x + 4$
$x + 4$

1.



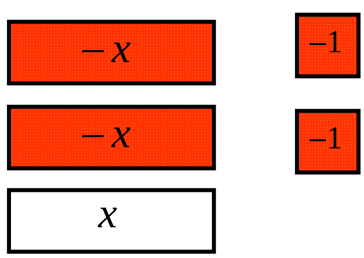
$x - 2$ or $x + (-2)$ talk about these two different ways to write the expression. This is an extension of what was dis-cussed in Chapter 2; remind students that $a - b = a + (-b)$ and $a - (-b) = a + b$.

2.



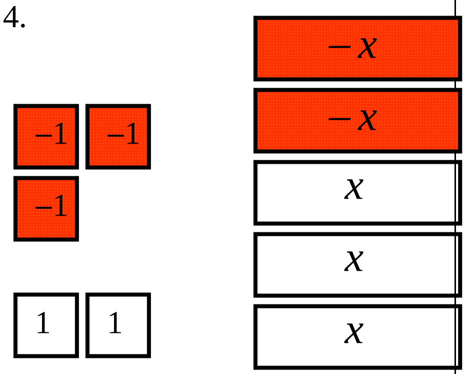
$-4x + 2x$ or $-2x$

3.



$-2x + x + -2$
$-x + (-2)$ or $-x - 2$

4.



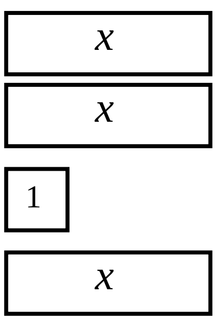
$-3 + 2 + -2x + 3x$
$x + (-1)$ or $x - 1$

Using algebra tiles, draw two different models or write two different expressions that can be simplified to the following expressions.

5. $x + 6$	6. $-3x + -2$	7. $4x - 1$

How many ways are there to model each of the expressions above? Justify. Discuss other possible representations (zero pairs in the representations).

Simplify each expression. In #10 remind students again that $x - 2$ is the same as $x + -2$.

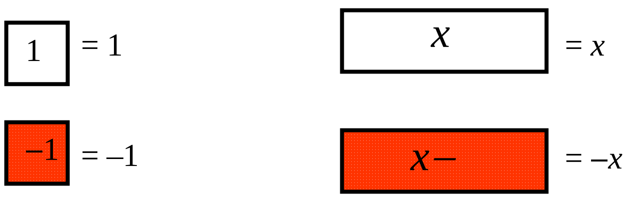
8. $2x + 1 + x$  $3x + 1$	9. $3x + 4 + (-2)$ $3x + 2$ Throughout exercises, point out that you are changing the order of the terms, using the commutative property, or grouping on addition (using subtract is add the opposite), using the associative property, but the value of the expression is not changing.	10. $2x + x - 2 + 3$ $3x + 1$
11. $-3x + 1 + -x$ $-4x + 1$	12. $2x + -3 + -2x$ -3	13. $-x + 3 + 4x$ $3x + 3$

14. $-2x + 4 + x - 7$ $-x - 3$ or $-x + (-3)$	15. $4x - 3 + 2 - 2x$ $2x - 1$	16. $-4x - 1 + 3x + 2 - x$ $-2x + 1$
17. $x + x + x + x$ $4x$	18. $x - x - x - x$ $-2x$	19. $-x - x - x - x$ $-4x$ Look at # 17, #18 and #19, note the differences.
20. $3 - 2x + x - 5$ $-x - 2$ or $-x + (-2)$	21. $4 - 2 - 4x - 2$ $-4x$	22. $2x - x - 3 + 5$ $x + 2$

3.1d Homework: Simplifying Algebraic Expressions with Models

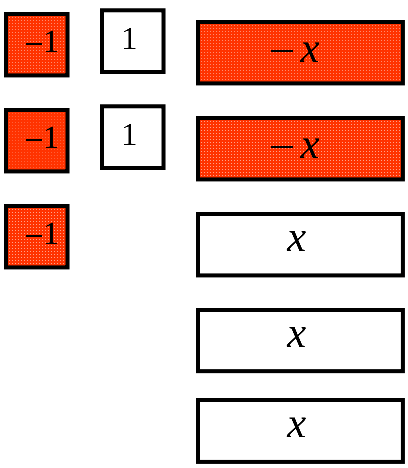
Use the key below to interpret or draw the algebraic expressions in your homework.

Key for Tiles:



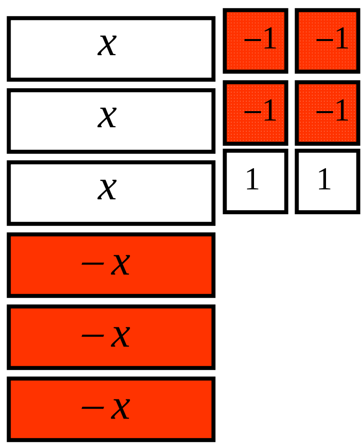
Write an expression for what you see, and then simplify if possible. The first one is done for you.

1.

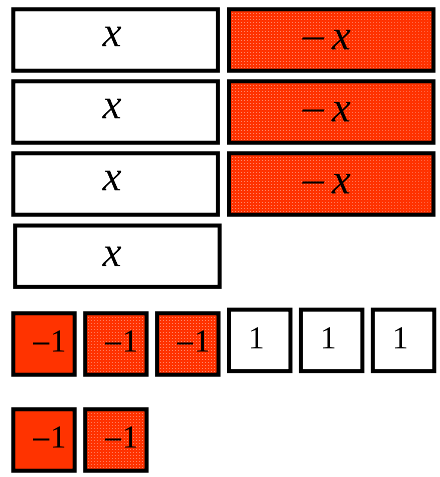


$-3 + 2 + -2x + 3x$, simplifies to $-1 + x$ or $x + -1$ or $x - 1$

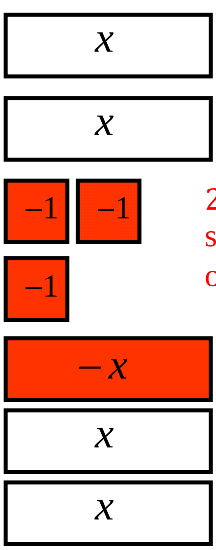
2.



3.

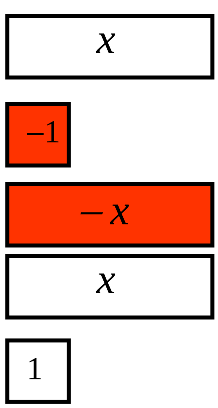


4.

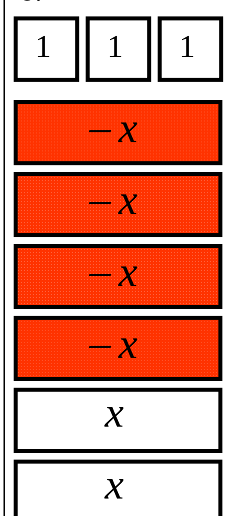


$2x - 3 - x + 2x$, simplies to $3x - 3$ or $3x + (-3)$

5.



6.



$3 - 4x + 2x$, simplifies to $2x + 3$

Simplify the following expressions. If needed, sketch a model in the space provided.

7. $2x + 4 – x$	8. $x – 5 + 3x$	9. $2x – 3 + 5$ $2x + 2$
10. $3x + 2 – 2x$	11. $2x + 1 + 3 – 5$	12. $–2x + 4 – 3$ $–2x + 1$
13. $–2x + 3 + 5x – 2$	14. $5x – 3 – 4 + x$	15. $–3x + 1 + 2x – 3$ $–x – 2$
16. $x + 4 + –3x – 7$	17. $–x – 3 + 2x – 2$	18. $4x – 3 – 7x + 4$ $–3x + 1$

Spiral Review

1. Mark has \$16 more than Becca. Represent how much money Mark has.
   If b is the amount of money Becca has, then Mark has $b + 16$ dollars.

2. What property is shown?
   8 + 7 + 2 and 8 + 2 + 7___commutative_________________
   18 + 0 and 0 +18 _______identity property of addition___

3. Find: $\frac{3}{5} + \frac{2}{3} =$

   $$\frac{19}{15} \ or \ 1\frac{4}{15}$$

4. Find: $-4 + -7 =$ -11

5. Find: $-3 - 8 = -11$

3.1e Class Activity: More Simplifying Algebraic Expressions

Activity 1:
Miguel saw the following two expressions:

$17 + 4 + 3 + 16$ $\qquad\qquad$ $43 - 8 - 3 + 28$

He immediately knew the sum of the first group is 40 and the sum of the second set is 60. How do you think he quickly simplified the expressions in his head?
Help students recognize that *commuting* terms make simplifying easier: $17 + 3 + 4 + 16 = 20 + 20 + 40$ and $43 + (-8) + (-3) + 28 = 43 + (-3) + 28 + (-8) = 40 + 20 = 60$. Emphasize again that the *commutative property* of addition allows us to change the order of addition.

Activity 2:
Abby has 3 turtles, 4 fish, a dog and 2 cats. Nia has 2 turtles, 2 fish, and 3 dogs. Mr. Garcia asks them, "how many pets do you two have all together?" How might Abby and Nia respond?
Students might first say, "Abby and Nia have a total of 17 animals." Push them to provide a more precise answer. Then students might say, "Abby and Nia have 5 turtles, 6 fish, 4 dogs and 2 cats." Emphasize that when adding (joining) we put together "like" items. To add emphasis, you might say out loud, " '2 fish plus 3 fish is 5 fish,' but when I say, '2 fish plus 3 dogs…' I can't add them because they are not the same 'units.' " The best I can say is that I have 5 animals."
Simplify the following expression with the Key provided below.

$x + y + 3x - 4y + 2$

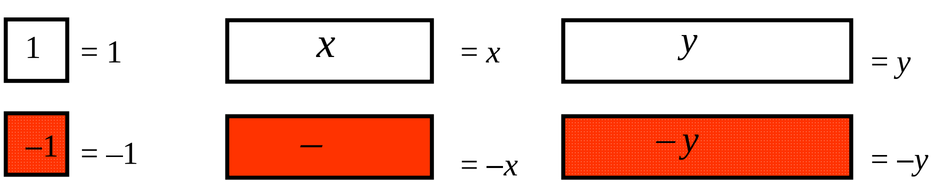
Emphasize that we add (join) "like" things: x's with x's and y's with y's. Point out that the "2" is neither an x, nor a y, it is its own unit. Thus it cannot go with either the x nor the y.

$4x + (-3y) + 2$

In the next problem, we introduce new variables. In the key below, we have made a new (larger) rectangle for the y unknown. Explain to students that x takes the place of a specific numer and y stands for some (possibly different) number. We can use other letters (such as z, a, b, etc.) to stand for other numers as well.

Justify your work:

Key for Tiles:



Explain how you would simplify:

$5x - 4 - 3x + 4y - z + 3z$

$2x + 4y + 2z - 4$

Again emphasize units. Point out that "x", "y" and "z" are different from each other, thus we cannot join them with addition.

For #1-16, model each expression (draw Algebra Tiles representations of the variables and numbers). Then simplify each expression.

1. $8x + 2x + 2y + 7y$ — $10x + 9y$

2. $3z + 2x + x$ — $3z + 3x$

3. $5x - 9x + 2w - w$ — $-4x + w$

4. $-6 + 4x + 9 - 2x + 6t - 2t$ — $3 + 2x + 4t$

5. $-3 + 3x + 11 - 5x - 2k - k$ — $-2x + 8 - 3k$

6. $9x - 12 + 12 - 9x$ — 0

7. $1 + x + 5x - 2 + 4k - 7k$ — $6x - 1 - 3k$

8. $-4x - 5x + 12m - 2m + 6q - 10q$ — $-9x + 10m - 4q$

Your friend is struggling to understand what it means when the directions say "simplify the expression." What can you tell your friend to help him? Answers will vary. Discuss "simplify" vs. "evaluate" vs. "solve" and "expression" vs. "equation". Also discuss why we simplify—when does it help and when is it easier to not simplify? You might refer back to Activity 2 above.

Your friend is also having trouble with expressions like problems #5 and #8 above. He's unsure what to do about the " –." What might you say to help him? Discuss $a - b = a + (-b)$; changing all subtract to "add the opposite" then allows us to use the commutative and associateive properties of addition.

Simplify each. Remind students about the work reviewing fractions in chapter 2. You may want to draw models for some of these. Note that (1/2)*x* is merely half of an *x*. Thus if one draws a representation of (1/2)*x* + (1/4)*x* they will see that joining the images gives (3/4) *x*

9. $\frac{1}{2}x + \frac{1}{2}x + \frac{1}{3} + \frac{1}{2}$

$x + \frac{5}{6}$

10. $\frac{2}{3}x + x + \frac{3}{4} - \frac{1}{4}$

$\frac{5}{3}x + \frac{1}{2}$

11. $\frac{1}{2}x + \frac{1}{3}y + \frac{1}{4}z$

Cannot be simplified any further.

12. $\frac{5}{3}x + \frac{4}{5} - \frac{2}{3}x - \frac{3}{5}$

$x + \frac{1}{5}x$

13. $\frac{1}{3}y - \frac{5}{3}y + \frac{1}{3}x + \frac{5}{3}x$

$-\frac{4}{3}y + 2x$

14. $3m + \frac{1}{4}m + 2y + \frac{3}{4}y$

$\frac{13}{4}m + \frac{11}{4}y$

15. $5x - 2y + 4 - 3x + y + x - 2$

$3x - y + 2$

16. $-5 + \frac{1}{2}x - \frac{1}{5}y - \frac{2}{10}y + \frac{3}{4}x - 7$

$\frac{5}{4}x - \frac{2}{5}y - 12$

3.1e Homework: More Simplifying Algebraic Expressions

For #1-16 model each expression, using Algebra Tiles if necessary. Then simplify each expression, combining like terms.

1. $5x + 3x + 5y$ $8x + 5y$

2. $1 + 3x + x$

3. $3x – 5x + 4y + 3y$ $–2x + 7y$

4. $7m – 2x – 9m + 4x$

5. $–4x – 5 + 2x$ $–2x – 5$ or $–2x + (–5)$

6. $5y – 4x + 5x + 10y$

7. $–4x – 5m + x + 7x – 12m$
$4x - 17m$ or $4x + (–17m)$

8. $x – 6x + 14y + 8y – 2x$

9. $2x + x + 2y + x + 1$

$4x + 2y + 1$

10. $x + 3y + x + 2 + y + 1$

11. $\frac{2}{3}t + \frac{2}{3}t + \frac{1}{6} + \frac{2}{3}$

$\frac{4}{3}t + \frac{5}{6}$

12. $\frac{5}{6}x + \frac{1}{6}x - \frac{2}{3}x + \frac{1}{2}y + \frac{1}{3}y$

13. $–2x + y – 3y + y + x$

$–x – y$ or $–x + (–y)$

14. $\frac{3}{2}y + \frac{5}{2}y - \frac{1}{2}x + \frac{1}{2}$

15. $5x – y + 2 – 4x + 2y + x + 2$
$2x + y + 4$

16. $\frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}y + \frac{1}{6}y + \frac{1}{5}z - \frac{2}{5}z$

3.1f Class Activity: Vocabulary for Simplifying Expressions

In groups of 2 or 3 students, consider the following expressions: a) $2x + 5 + 3y$, b) $2x + 5 + 3x$, and c) $2x + 5x + 3x$. How are these expressions similar? How are they different?

Students will note that a) cannot be simplified any further, b) can be simplified to two terms and c) can be simplified to one term. They may also note that the three are algebraic expressions (none are strictly numeric). Have them explain why some of the expressions can be simplified but others not.

Parts of an Algebraic Expression: Use the diagrams to create definitions for the following vocabulary words. Be prepared to discuss your definitions with the class.

Terms	Constants
$x-4+2y+3-5y$	$x-4+2y+3-5y$
There are five terms in this expression. The terms are x, –4, $2y$, 3, and $-5y$	The constants are –4 and 3. (Recall that subtracting is like adding a negative number.)

Coefficients	Like Terms
$x-4+2y+3-5y$	$x-4+2y+3-5y$
The coefficients are 1, 2, and –5	$2y$ and $-5y$ are like terms. –4 and 3 are also like terms.

Suggestions for what should be included in their definitions are below.

Terms: a part of an algebraic expression, either a product of numbers and variable(s) or simply a number. In each of the examples above, terms are separated by addition (when view subtract as add the opposite).

Constant: a "stand alone" number, not a variable

Coefficient: the numeric factor of the term

Like Terms: terms with the same variable(s)

2. Identify the terms, constants, coefficients, and like terms in each algebraic expression.

Expression	Terms	Constants	Coefficients	Like Terms
$4x-x+2y-3$	$4x$, $-x$, $2y$, –3	–3	4, –1, 2	$4x$, $-x$
$3z+2z+4z-1$	$3z$, $2z$, $4z$, –1	–1	3, 2, 4	$3z$, $2z$, $4z$
$2+3b-5a-b$	2, $3b$, $-5a$, $-b$	2	3, –5, –1	$3b$, $-b$
$a+b-c+d$	a, b, $-c$, d	none	1, 1, –1, 1	None

3. Identify the terms, constants, coefficients, and like terms in each algebraic expression.

Expression	Terms	Constants	Coefficients	Like Terms
a. $2y + 5y - 6x + 2$	$2y, 5y, -6x, 2$	2	2, 5, –6	$2y, 5y$
b. $-3x + 2x - (2/3)$	$-3x, 2x, -(2/3)$	$-(2/3)$	–3, 2	$-3x, 2x$
c. $\frac{4}{5}p + \frac{1}{5} - 3h + j$	$\frac{4}{5}, p, \frac{1}{5}, -3h, j$	1/5	4/5, 3, 1	None
d. $0.3x - 1.7 + 1.2 + 4.4y + 3.6y$	$0.3x, -1.7, 1.2, 4.4y, 3.6y$	–1.7, 1.2	0.3, 4.4, 3.6	–1.7 & 1.2; $4.4y$ & $3.6y$

Simplify the above expressions. Remember to show your work.

a. $7y - 6x + 2$

b. $-x - \frac{2}{3}$

c. $\frac{4}{5}p + \frac{1}{5} - 3h + j$

d. $0.3x - 0.5 + 8y$

3.1f Homework: Solidifying Expressions

Matching: Write the letter of the equivalent expression on the line.

1.	_e___ $3x - 5x$	a) $-8a$
2.	____ $4a - 12a$	b) $2x - 6$
3.	____ $3x + 5x$	c) $-x + y$
4.	____ $16a + a$	d) $9x - 4y$
5.	_f___ $2x - 2y + y$	e) $-2x$
6.	____ $2x - 2 - 4$	f) $2x - y$
7.	____ $x - y + 2x$	g) $x + y$
8.	____ $-y + 2y - x$	h) $3x - y$
9.	_g___ $5x + 4y - 3x - x + 3y - 6y$	i) $17a$
10.	_d___ $4x + 3y + 5x - 7y$	j) $8x$

Simplify each expression. For exercises #11 – 16, ***also*** list all the coefficients.

11. $13b - 9b$

12. $22x + 19x$

13. $44y - 12y$

14. $6a + 4a - 2b$ 6, 4, –2; $10a - 2b$

15. $16b - 4b + 2$

16. $\frac{2}{3}a - \frac{3}{4}a$ $\frac{2}{3}, -\frac{3}{4}, -\frac{1}{12}a$

17. $3.4 - 21.4x - 3.4y + 2.2$

18. $\frac{12}{20}b - \frac{1}{4}b + \frac{3}{4}$

19. $2x + 0.5x - 3y + 4.75y + 9.8$ $2.5x + 1.75y + 9.8$

20. $14w + 9 + 15 - 15w$

21. $4m + 6 + 2m - 5$

22. $6z - 16z - 9z + 18z + 2$ $-z + 2$

23. $16y - 12y - 20$

24. $2x - 2 + 3 + 8x$

25. $1\frac{2}{3}x + 3\frac{3}{4}x$ $5\frac{5}{12}x$

26. $11 - u - 14u + 13u$

27. $88.5z - 22.4y + 4.04z + 26.3$

$-22.4y + 92.54z + 26.3$

28. $4b - 3 + 2$

3.1f Additional Practice

Simplify each expression.

1. $2x - y + 5x$

   $7x - y$

2. $5y + x + y$

3. $2y + 8x + 5y - 1$

   $7y + 8x - 1$

4. $10b + 2 - 2b$

5. $8y + x - 5x - y$

   $-4x + 7y$

6. $9x + 2 - 2$

7. $-2x - 6 + 3y + 2x - 3y$

   -6

8. $6m + 2n + 10m$

9. $7b - 5 + 2b - 3$

   $9b - 8$

10. $2a - 3 + 5a + 2$

11. $8x + 5 - 7y + 2x$

    $10x - 7y + 5$

12. $4y + 3 - 5y - 7$

13. $6x + 4 - 7x$

    $-x + 4$

14. $2x + 3y - 3x - 9y + 2$

15. $-2b + a + 3b$

    $b + a$

16. $m - 5 + 2 - 3n$

17. $4 + 2r + q$

    $4 + 2r + q$

18. $5h - 3 + 2k - h + k$

19. $5t - 3 - t + 2$ $4t - 1$

20. $c + 2d - 10c + 4$

3.1g Class Activity: Multiplying Groups/Distributive Property

Review:
Show as many different ways to write 50 as you can.

Possible expressions:

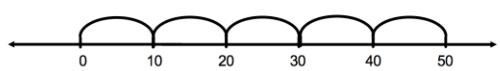
10+10+10+10+10 5(0+10), 5(1+9), 5(2+8), …

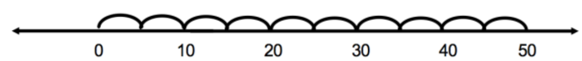
5(0)+5(10), 5(1)+5(9), 5(2)+5(8), … 0+50, 5+45, 10+40, …

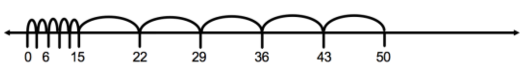
Also make sure to discuss with students the meaning of multiplication: 5(10) means 5 groups of size 10.
Examine the different ways to represent a product of 50 below. Explain how each is an accurate representation of 50.

Review:

NUMBER LINE MODELS for MULTIPLICATION (1 dimensional representation of multiplication)

<table>
<tr>
<td>

5 × 10</td>
<td>

10 × 5</td>
</tr>
<tr>
<td>

5(3 + 7)</td>
<td>Notice that in the number line to the left, first there are five groups of 3 then there are five groups of 7. This can also be represented as five groups of a length of (3 + 7) or 10.</td>
</tr>
</table>

Activity: On the number line below, show a length of 1 unit. Then make the length of 3 units. You may want to refer back to 2.3a for this discussion. To determine a length of three units, we must first agree on a length of ***one*** unit. Once we know how long "one" is, we can choose any point on the line as a starting point and then we can copy our "one" unit 3 times to find a length of 3.

On the number line below, show a length of x units. Then make the length of $3x$ units.

Discuss with students that a length of "x" is an *unknown* length. We simply choose a random point on the line as our starting point and another as the end point and call that length our length x. We then copy that length three times to find a length of $3x$. Thus, we can scale x in any way we want e.g. $2x$, $5x$, $0.25x$, etc.

AREA MODELS for MULTIPLICATION (2 dimensional representation of multiplication)

	25
2	**50**

2×25

	10
5	**50**

5×10

	3	7
5	15	35

$5(3 + 7)$

	5	5
5	25	25

$5(5 + 5)$ or $5^2 + 5^2$

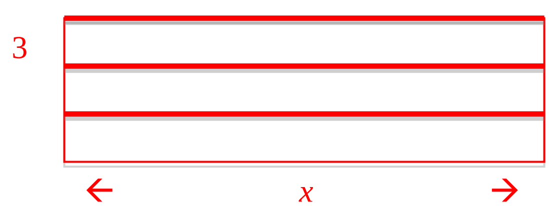
Create your own representation of 50:

(1/2)(100)

2.5×20

1×50

Can you draw a area model representation of $3x$?



Discuss this model with students. One dimension has a length of 3, and the other a length of x. In the model, we can see that there are 3 groups of rectangles that measure 1 by x. Remind student of the algebra tiles used earlier in the chapter. Point out that $3x$ is the same as (equivalent to) $x + x + x$. It is important that students clearly distinguish $3x$ from x^3.

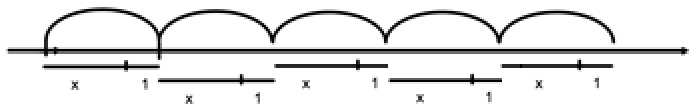
1. Model $5(x + 1)$ and then simplify. Justify your steps.

$5x + 5$

Students might do a one or two dimensional model (see below) OR write FIVE groups of $(x + 1)$: e.g. $(x + 1) + (x + 1) + (x + 1) + (x + 1) + (x + 1)$. Then, *using the commutative property and associative property,* combine like terms.

Students can also use technology to copy $x + 1$ and then paste it 5 times.

x	1
x	1
x	1
x	1
x	1



2. Model $4(x – 2)$ and simplify.

$4x – 8$

Take time to discuss the different ways that this expression can be written (see below).

An alternative is to have students write $(x – 2)$ or $(x + (–2))$ four times. This way they will see the four "x's" and four "–2's".

Write $4(x – 2)$ in three different ways. $4x – 8$; $4x + (–8)$; $4(x) – 4(2)$; $4(x) + (4(–2))$

What does the number in front of the parentheses tell you about the grouping? The number tells you how many groups of the amount in the "( )" there are.

3. a. Explain what $3(2x – 5)$ means, then simplify. $3(2x – 5)$ means three groups of $(2x – 5)$, which simplifies to $6x – 15$. Students might also write $3(2x + –5)$

b. Write $3(2x – 5)$ in three different ways. $3(2x) – 3(5)$; $6x – 15$; $6x + (–15)$; $(2x – 5) + (2x – 5) + (2x – 5)$

How can you use what we've learned about integers and what we know about writing expressions with parentheses to rewrite expressions that have " – " in the groupings?

$a(b – c) = a(b + (–c)) = ab + –ac$
Explanations will vary: There are a groups of $(b + –c)$ which gives you a groups of b and a groups of $–c$.

Write the expression below in an equivalent form. If necessary, draw a model.

4. $3(x + 2)$ $3x + 6$	5. $2(3x + 5)$ $6x + 10$	6. $3(x + 1)$ $3x + 3$
7. $4(3x – 1)$ $12x + (–4)$ or $12x – 4$	8. $2(3 + x)$ $6 + 2x$	9. $3(3x – 2)$ $9x + (–6)$ or $9x – 6$
10. $\frac{1}{2}(4x + 6)$ $2x + 3$	11. $\frac{3}{4}(2x - 5)$ $\frac{3}{2}x - \frac{15}{4}$ or $\frac{3}{2}x + (-\frac{15}{4})$	12. $0.2(7x – 9)$ $1.4x – 1.8$

In sixth grade, you talked about order of operations, what is the order of operations and how is it related to what you've been doing above?
PEMDAS—four things that should be discussed here: 1) generally, we simplify groupings first if we can. For example, $5(3 + 7)$ is the same as $5(10)$. This is more efficient than saying $5(3) + 5(7)$, finding the products and then adding—$15 + 35$ is 50. 2) The distributive property is particularly helpful when we cannot simplify a grouping. For example, in the expression $3(x + 2)$, we cannot combine x and 2, so the only way to rewrite the expression is to distribute the 3. In this context we understand "distribute" to mean there are 3 groups of $(x + 2)$. This allows us to reorder our terms and combine like terms. 3) Using $a – b = a + (–b)$ allows us to change order when there is subtraction. 4) "Simplifying" means that we write a expression in a manner that makes it more clear for our purposes. There will be times the expression $3(x + 2)$ is more simple for our purposes than the equivalent $3x + 6$.

3.1g Homework: Multiplying Groups/Distributive Property

Simplify each of the following. Draw a model or explain the meaning of the expression to justify your answer.

1. $5(x+1)$ $5x+5$	2. $2(3x+2)$	3. $4(x+3)$
4. $2(3x-1)$ $6x-2$ or $6x+(-2)$	5. $3(2x-3)$	6. $5(x-1)$
7. $\frac{1}{3}(3x+12)$ $x+4$	8. $\frac{3}{4}(5x-8)$	9. $-\frac{2}{5}(4x-7)$ $-\frac{8}{5}x+\frac{14}{5}$
10. $0.25(4x-8)$	11. $0.6(-2x+5)$	12. $-2.4(1.5x-0.9)$ $-3.6x+2.16$

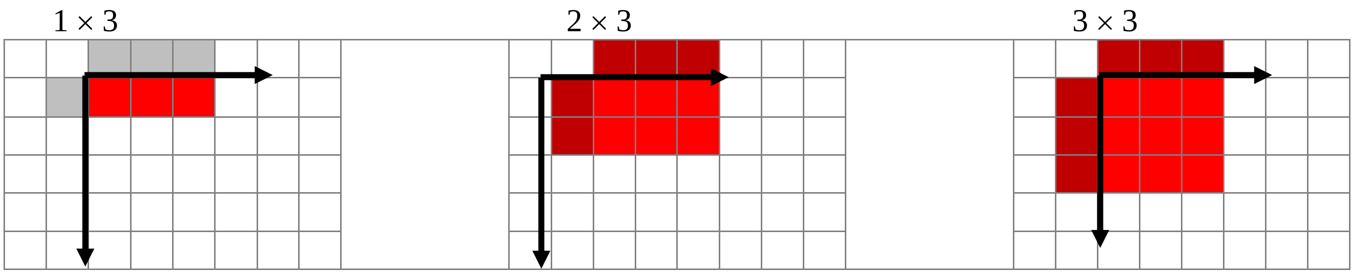
The expressions $2(5x-3)$ can be written and $10x-6$ OR $10x+(-6)$.
Write the following expressions in two different ways as the example shows:

15. $4(3x-5)$ $12x-20$ or $12x+(-20)$

16. $2(7x-3)$

3.1h Classwork: Modeling Backwards Distribution (Factoring)

Review: below is a review of modeling multiplication with an array.

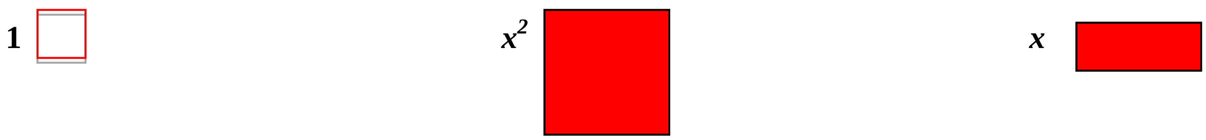


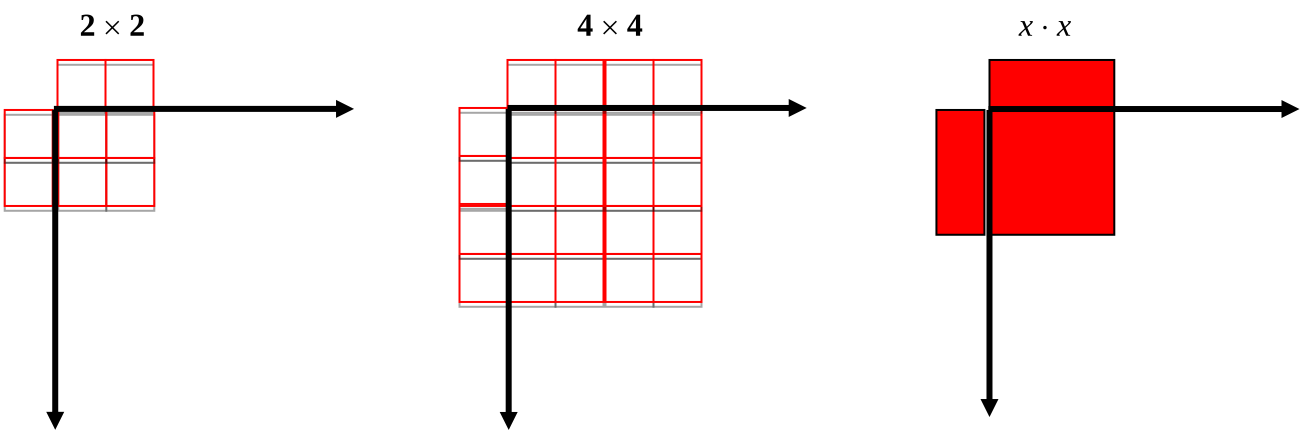
Factors: 1, 3
There is one group of 3
Product/Area: 3

Factors: 2, 3
There are two groups of 3
Product/Area: 6

Factors: 3, 3
There are three groups of 3
Product/Area: 9

Use the Key below to practice using a multiplication model. Use phrase x to second power leads to x squared.





Factors: 2, 2

Product/Area: 4

Factors: 4, 4

Product/Area: 16

Factors: x, x

Product/Area: x^2

Look at the three models above. What do you think 2^2 might be called? "two squared" Why? It is a square with side length of 2. What might 4^2 be called? "four squared" What do you think x^2 might be called? "x-squared" Why? Note the difference between 4 × 4, (4^2), 4 + 4, and 2(4).

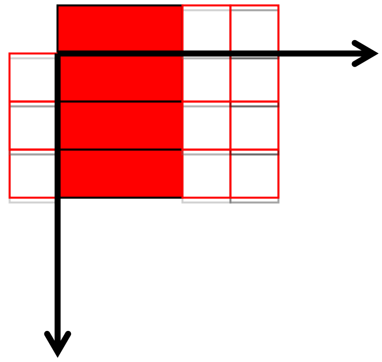
1. Build the factors for $3(x + 2)$ on your desk. Then build the area model. Draw and label each block below.

   What are the factors of the multiplication problem? 3, $x + 2$

   What is the area ? $3x + 6$

   What is the product of the multiplication problem? $3x + 6$

   Point out that when we multiply using a model, the factors are the dimensions of the rectangle that is created. When students factor, they will create a rectangle with the terms to find factors (dimensions).

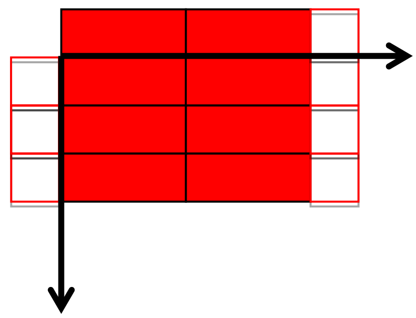


2. Build the factors for $3(2x +1)$ on your desk. Then build the area model. Draw and label each block below.

   What are the factors of the multiplication problem? 3, $2x +1$

   What is the area? $6x + 3$

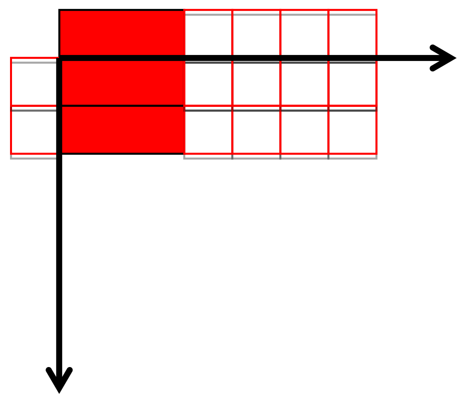
   What is the product of the multiplication problem? $6x + 3$



3. Build the factors for $2(x + 4)$. Build the area and draw.

   What is the area or product? $2x + 8$

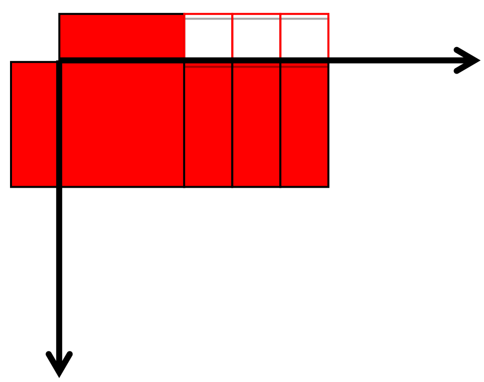
   What are the factors? 2 and $x + 4$.

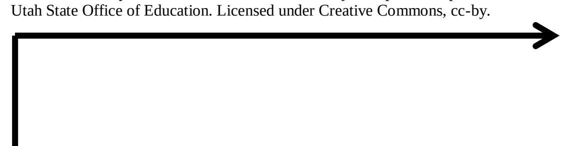


4. Build the factors for $x(x + 3)$. Build the area and draw.

   What is the area or product? $x^2 + 3x$

   What are the factors? x and $x + 3$



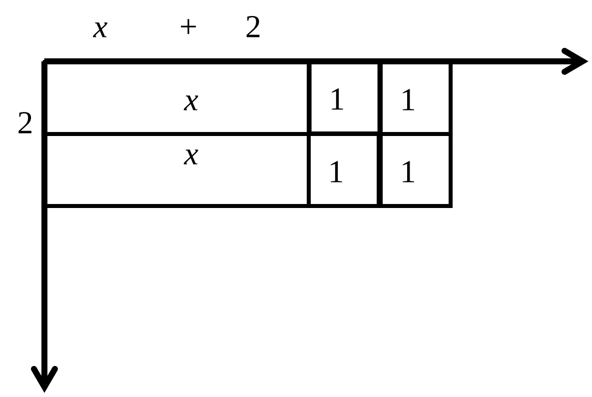


5. Build the factors for $x(2x + 5)$. Build the area and draw.

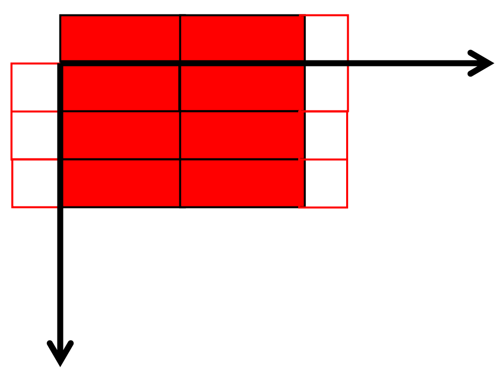
What is the area or product? $2x^2 + 5x$

What are dimensions or factors? x and $2x + 5$

Example: Factor $2x + 4$
We create a rectangle with the terms:
two "x" blocks and four "1" blocks.
Notice that the dimensions of the rectangle are
2 by $(x + 2)$
$2(x + 2) = 2x + 4$
$2(x + 2)$ is the factored form of $2x + 4$



6. Factor $6x + 3$ by creating a rectangle to see its dimensions.



What are the dimensions (factors) 3, $(2x + 1)$

Width: $2x + 1$ Height: 3

What is the area (product) of the rectangle? $6x + 3$

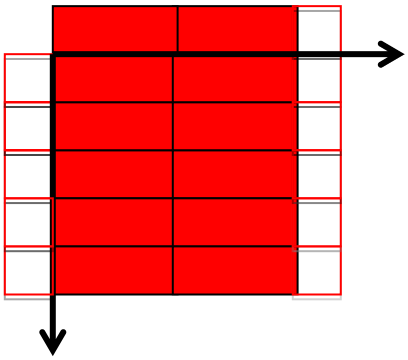
Write $6x + 3$ in factored form: $3(2x + 1)$

Also note that a rectangle can be described by its width and height, sometimes also referred to as length and width. Since a rectangle retains all of its properties even if it is rotated 90°, we can talk about the dimensions of a rectangle being length and width OR width and height. Both are acceptable common uses of ways to describe a rectangle.

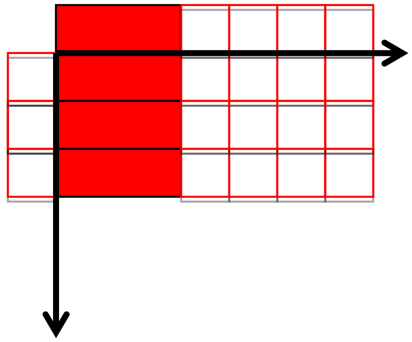
Remind students that factoring is the opposite of the distributive property. It might be helpful to draw this on the board.
$3(2x + 1) = 3(2x) + 3(1) = 6x + 3$
Distributing ------------------>
<---------------------Factoring

Draw a model to factor each of the following.

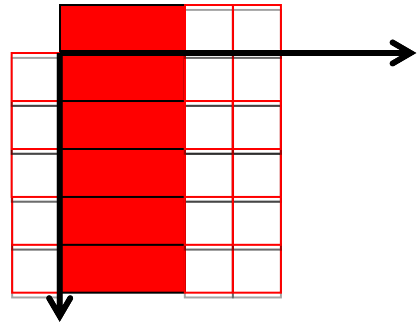
7) $10x + 5$



8) $3x + 12$

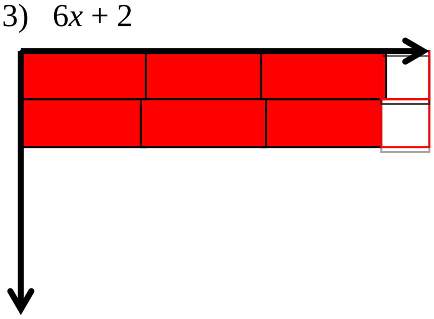


9) $5x + 10$

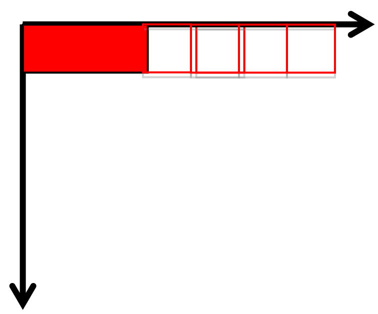


10) $10x + 5$	11) $3x + 12$	12) $5x + 10$
Dimensions of Rectangle: 5 and $2x + 1$ or Greatest Common Factor: 5 $10x + 5$ in factored form: $5(2x + 1)$	Dimensions of Rectangle: 3 and $x + 4$ or Greatest Common Factor: 3 $3x + 12$ in factored form: $3(x + 4)$	Dimensions of Rectangle: 5 and $x + 2$ or Greatest Common Factor: 5 $5x + 10$ in factored form: $5(x + 2)$

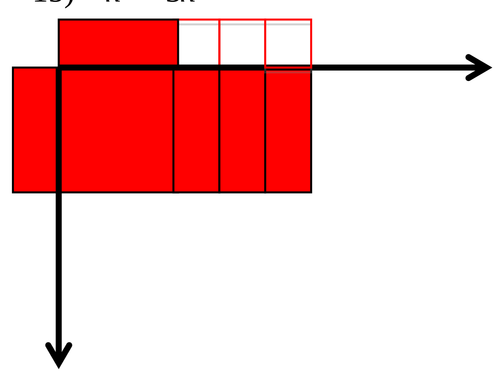
13) $6x + 2$



14) $x + 4$



15) $x^2 + 3x$



$6x + 2$	$x + 4$	$x^2 + 3x$
Dimensions of Rectangle: 2 and $3x + 1$ or Greatest Common Factor: 2 $6x + 2$ in factored form: $2(3x + 1)$	Dimensions of Rectangle: 1 and $x + 4$ or Greatest Common Factor: 1 $x + 4$ in factored form: $x + 4$ or $1(x + 4)$	Dimensions of Rectangle: x and $x + 3$ or Greatest Common Factor x $2x^2 + 4x$ in factored form: $x(x + 3)$

Practice: Write each in factored form. Be ready to justify your answer.

16. $30x + 6$ $6(5x + 1)$	17. $4b + 28$ $4(b + 7)$	18. $3m - 15$ $3(m - 5)$
19. $4n - 2$ $2(2n - 1)$	20. $25b - 5$ $5(5b - 1)$	21. $4x - 8$ $4(x - 2)$

Look at problems #16, 17, and 21. How else might these expressions be factored? #12 could be written as $2(15x + 3)$. Discuss "greatest common factor." Both $2(15x + 3)$ and $6(5x + 1)$ are factored forms of $30x + 6$. {We could even factor it as $7((30/7)x + 6/7)$.} However, 6 is the greatest common factor so the convention is to say $6(5x + 1)$ is factored completely because the greatest common factor was pulled out of each term. "Factoring" and "simplifying" are ways to write expressions to meet our purpose. For example, in 8th grade a student may be working with the linear equation $y = 25x - 5$ and want to know what the x intercept is. In that case, she might factor out the 25: $y = 25(x - 0.2)$ to reveal the x intercept.

Write the following in factored form. The first one is done for you.

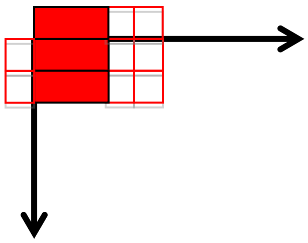
$\frac{2}{3}x + \frac{5}{3}$ Notice $\frac{1}{3}$ is a factor of both the coefficient and the constant. $\frac{2}{3} = \frac{1}{3} \cdot 2$ and $\frac{5}{3} = \frac{1}{3} \cdot 5$ so $\frac{2}{3}x + \frac{5}{3}$ can be written as $\frac{1}{3} \cdot 2x + \frac{1}{3} \cdot 5$ and factored as $\frac{1}{3}(2x + 5)$	22. $\frac{3}{2}x + \frac{7}{2}$ $\frac{3}{2} = \frac{1}{2} \cdot 3$ and $\frac{7}{2} = \frac{1}{2} \cdot 7$ so $\frac{3}{2}x + \frac{7}{2}$ can be written as $\frac{1}{2} \cdot 3x + \frac{1}{2} \cdot 7$ and factored as $\frac{1}{2}(3x + 7)$	23. $\frac{20}{3}x - \frac{4}{3} = \frac{20}{3}x + \left(-\frac{4}{3}\right)$ $\frac{20}{3} = \frac{4}{3} \cdot 5x$ and $-\frac{4}{3} = \frac{4}{3} \cdot (-1)$ $\frac{4}{3} \cdot 5x + \frac{4}{3} \cdot (-1)$ $\frac{4}{3}(5x + (-1))$ or $\frac{4}{3}(5x - 1)$
24. $\frac{3}{5}x - 6$ $6 = \frac{3}{5} \cdot 10$ $\frac{3}{5} \cdot x - \frac{3}{5} \cdot 10$ $\frac{3}{5}(x - 10)$	25. $2.3x + 6.9$ $6.9 = 2.3 \cdot 3$ $2.3x + 2.3 \cdot 3$ $2.3(x + 3)$	26. $1.6x - 2.4$ $1.6 = \frac{8}{5}$, $2.4 = \frac{12}{5}$ $\frac{8}{5} = \frac{4}{5} \cdot 2$ $\frac{12}{5} = \frac{4}{5} \cdot 3$ $\frac{4}{5} \cdot 2x + \frac{4}{5} \cdot 3$; $\frac{4}{5}(2x + 3)$ or $2.4/1.6 = 1.5$ --> $1.6(x - 1.5)$

3.1h Homework: Modeling Backwards Distribution

Write each in factored form. Use a model to justify your answer on problems #1 - 6.

1. $2x + 4$

$2(x + 2)$



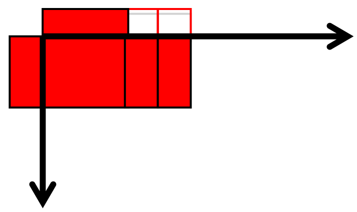
2. $3x + 12$

3. $2x + 10$

4. $3x + 18$

5. $x^2 + 2x$

$x(x + 2)$



6. $x^2 + 5x$

7. $\frac{4}{7}x + \frac{6}{7}$

8. $\frac{2}{3}x - 4$

$\frac{2}{3}(x - 6)$ or $\frac{2}{3}(x + (-6))$

9. $0.6x + 1.5$

$0.3(2x + 5)$ or $0.6(x + 2.5)$

Simplify each expression. Justify your answer with words or a model.

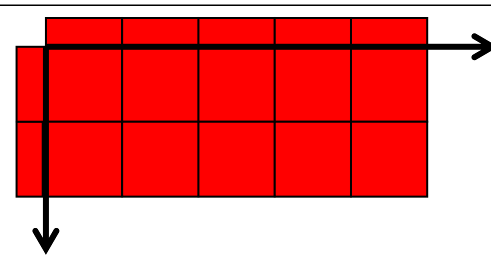
10. $2x + 3x$

11. $(2x)(3x)$

12. $5x - 2x$

13. $(5x)(-2x)$

$-10x^2$



3.1i Class Activity: More Simplifying

Review:

What is the *opposite* of "forward 3 steps"?
Backward 3 steps

What is the *opposite* of "turn right"
Turn left

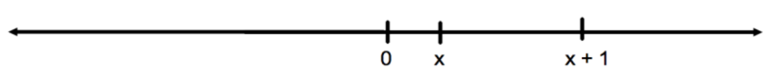
What is the *opposite* of "forward three steps then turn right"?
Backward 3 steips turn left

In math, what symbol do we use that means *opposite*?
Negative symbol

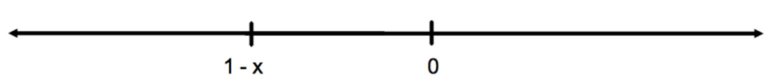
Using the logic above, what do you think each of the following means?

$-x$ the opposite of x	$-(x)$ the opposite of x—note that these first two mean exactly the same thing.
$-(x + 1)$ the opposite of $x + 1$. Talk about taking the opposite of ALL of $x + 1$. Relate to above. Thus it means $-x - 1$ or $-x + (-1)$. If students build this using models, they may note that it is also the same as $-1 - x$ or $-1 + (-x)$	$-(1 - x)$ the opposite of the entire quantity $1 - x$; $-1 + x$ or $x + (-1)$ or $x - 1$.

a) Consider the number line below. The location of x and $(x + 1)$ are identified on the line. Locate and label $-x$ and $-(x + 1)$. $-x$ is on the opposite side of 0 the same distance from 0 as x. $-(x + 1)$ is also on the opposite side of 0 and the same distance as $x + 1$.



b) Consider the number line below. The location of $(1 - x)$ is identified on the line. Locate and label $-(1 - x)$.

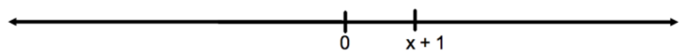


What does " – " in front of a set of parentheses tell us? Take the opposite of the "stuff" on the inside of the grouping. We can also think of the value of the quantity as a location on the real line and the "–" in front means we want the quantity on the opposite side of 0, the same distance away from 0.

On the line below the location of the quantity $x + 1$ is indicated.

c) Locate $3(x + 1)$

d) Locate $-3(x + 1)$



To find $3(x + 1)$ we need to measure the length of $x + 1$ as shown. That is ONE length of $x + 1$ from 0; to find $3(x + 1)$ we want three of those lengths from 0. $-3(x + 1)$ is the same distance from 0, but on the opposite side of 0. Students should make sense of both the 3 and the "–."

Activity:

Suppose Juanita has \$1200 in her savings account and once a week for 5 weeks she withdraws x dollars for bills and \$10 for entertainment. Write an expression for the amount of money she now has in her savings account.

\$1200 – $5(x + 10)$

Discuss this representation with your students. Talk about both the subtraction and why there is a "+10" inside the grouping.

In groups of 2 or 3 students, simplify each of the following. Be ready to justify your answer.

1. $4(x + 1)$ $4x + 4$	2. $-(x + 2)$ $-x - 2$ This can be thought of as "the opposite of $x + 2$."	3. $-(3x + 2)$ $-3x - 2$
4. $-2(x + 3)$ $-2x - 6$	5. $-3(x - 2)$ $-3x + 6$ This can be thought of as "the opposite of three groups of $x - 2$."	6. $2(3 - x)$ $6 - 2x$
7. $-2(5 - 3x)$ $-10 + 6x$	8. $-(4 - 3x)$ $-4 + 12x$	9. $-4(2x + 3)$ $-8x - 12$

In groups of 2 or 3 students, simplify the following exercises. Justify your answers.

10. $3x + 5 - x + 3(x + 2)$ $5x + 11$	11. $3x + 5 - x - 3(x + 2)$ $-x - 1$ or $-x + (-1)$ Compare problems #10 and 11
12. $2(x - 1) + 4x - 6 + 2x$ $8x - 8$	13. $-2(x - 1) + 4x - 6 + 2x$ $4x - 4$ Compare problems 12 and 13
14. $5 + 2(x - 3)$ $2x - 1$	15. $7x - 2(3x + 1)$ $x - 2$
16. $6x - 3 + 2x - 2(3x + 5)$ $2x - 13$	17. $-9x + 3(2x - 5) + 10$ $-3x - 5$
18. $(5 - 3x) - 7x + 4$ $-10x + 9$	19. $-(4x - 3) - 5x + 2$ $-9x + 5$
20. $9 - 8x - (x + 2)$ $-9x + 7$	21. $15 - 2x - (7 - x)$ $-x + 8$

3.1i Homework: More Simplifying

Simplify the following expressions:

1. $3(2x + 1)$	2. $-3(2x + 1)$ $-6x - 3$	3. $-3(2x - 1)$
4. $-(x + 4)$	5. $-(x - 4)$ $-x + 4$	6. $-(4 - x)$ $-4 + x$ or $x - 4$
7. $-2(4x - 3)$ $-8x + 6$	8. $-5(3x + 2)$	9. $-2.7(2x - 5)$
10. $5x + 2(x + 3)$ $7x + 6$ Note the subtle but important differences in problems # 10 – 13.	11. $5x + 2(x - 3)$	12. $5x - 2(x + 3)$
13. $5x - 2(x - 3)$ $3x + 6$	14. $3x + 2 - 4x + 2.1(3x + 1)$	15. $-7x + 4 + 2x - \frac{3}{2}(x + 2)$ $-\frac{13}{2}x - 1$
16. $10x - 4 - \frac{7}{2}x - \frac{3}{4}(2x - 3)$	17. $4.3x - 5(2.1x - 1.5) - 3x + 4$	18. $x - 7.8 - 2(3.4x - 3.7) + 6.1x$ $0.3x - 0.4$

3.1i Additional Practice: Simplifying

Matching: Write the letter of the equivalent expression on the line

1. ____	$3(x+6)$	a)	$-3x-3$
2. __i__	$3(x-6)$	b)	$3x+18$
3. ____	$3(x-1)$	c)	$2x-6$
4. ____	$-3(x+1)$	d)	$-2x-6$
5. ____	$6(x-3)$	e)	$-6x+18$
6. ____	$-(6x+18)$	f)	$-x+6$
7. __e__	$-(6x-18)$	g)	$6x-18$
8. __f__	$(x-6)(-1)$	h)	$-6x-18$
9. ____	$(x-3)2$	i)	$3x-18$
10. __d__	$(x+3)(-2)$	j)	$3x-3$

Practice: Simplify each expression.

11. $7(x+3)$
12. $3(y-3)$
13. $3(k+4)$
14. $\frac{2}{3}(3b-6)$
15. $7(y-z)$
16. $r(s+t)$
17. $2(3x+2)$
18. $1.6(2v-2)$
19. $(c-3)(-1)$
20. $8(x-8)$
21. $4(1-6n)$
22. $9.8(p+5.2)$
23. $-6(1+2n)$
24. $-\frac{1}{2}(2t+6)$
25. $-1(4+k)$
26. $-(3x-6)$
27. $3(5x+2)$
28. $14(j-12)$
29. $-(6x-6)$ $-6x+6$
30. $-0.8(0.5-0.7x)$ $-0.4+0.56x$
31. $-4(6-6p)$ $-24+24p$
32. $0.25(n-7)$ $0.25n-1.75$
33. $9(4-7m)$ $36-63m$
34. $-5\left(m+\frac{1}{3}\right)$ $-5m-\frac{5}{3}$
35. $-4(k-1)$ $-4k+4$
36. $-7(-3n+2)$ $21n-14$
37. $(6-t)(-4)$ $-24+4t$

3.1j Extra Practice: Simplifying Expressions

Simplify:

1. $-2x+5y+2x-4y-2z$

2. $7w-3q-5+8q-6-10w$

   $5q-3w-11$

3. $3p-2q+4p+4q-6-4$

4. $17.3v-2.6+12.8v-15.5$

   $30.1v-18.1$

5. $-3c+6c-5c+2d-4d-3d$

6. $-\frac{10}{3}y-\frac{10}{4}y-3$

   $-\frac{35}{6}y-3$

7. $31y+5x-4+12-13x-23y$

8. $-3.7y+2.9x-5.6y+5x-4.8x$

   $3.1x-9.3y$

9. $\frac{4}{3}(2x-5)$

10. $-6.4(5.1-2.5y)$

    $16y-32.64$

11. $(-8-6v)\left(-\frac{2}{5}\right)$

    $\frac{12}{5}v+\frac{16}{5}$

12. $(3-5h)(-3)$

13. $\frac{1}{2}(6a-8)$

14. $-1.8(x-2y)$

    $-1.8x+3.6y$

15. $-(3x+6)$

16. $-(-3k-5)$

    $3k+5$

17. $-(7h+2k)$

18. $\frac{7}{3}(-x+5q)$

    $-\frac{7}{3}x+\frac{35}{3}q$

19. $3+4(2x-5)$

20. $12-6(4-2y)$

    $12y-12$

21. $5+(-8-6v)$

22. $4+(3-5h)$ $-5h+7$

23. $2(6h-8)+10$

24. $5y-1(x-2y)+6$

$-x+7y+6$

25. $5x-(3x+6)+6$

26. $5k-(-3k-5)+8$

$8k+13$

27. $4h-(7h+2k)-5$

28. $7x-7(-x+5q)$

$14x-35q$

3.1k Class Activity: Modeling Context with Algebraic Expressions

Look back at the anchor problem at the beginning of the chapter. In particular, look back at your work for step 10 and step x. Your task was to express the number of units at each step of the pattern.

Look at your model and work for step 10. Use it to help you determine how many units the 100th step would contain.

Step 100

Draw a model if necessary for step 100:

How many units would the 100th step contain? Use your methods for the 10th step to help you determine the units in the 100th step.

Method 1: 4(100), 4 groups of 100 tiles

Method 2: 100(4), 100 groups of size 4

Method 3: 4(99) + 4, the four sides of the pattern plus the 4 in the middle

Method 4: Students might try to count, but they'll not be able to because of the size.

Step x

Draw a model if necessary for step x:

How could you adapt the methods you used before to find the number of units for ***any*** pattern.

Method 1: $4(x)$, 4 groups of x units

Method 2: $x(4)$, x groups of 4 units

Method 3: $4(x - 1) + 4$, the four sides of the pattern plus the 4 in the middle

Method 4: Students will not be able to "count" because it's abstract, but they should be able to explain the counting.

In the context above you wrote several expressions for each situation; often there is more than one equivalent way to algebraically model a context. Below are contexts; write two equivalent expressions for each situation. It may be helpful to draw a model. Discuss ideas of the associative and commutative properties of addition and multiplication throughout these exercises.

1. Marty and Mac went to the hockey game. Each boy bought a program for 3 dollars and nachos for n dollars. Write two different expressions that could be used to represent how much money the boys spent altogether.
   Throughout the exercises discuss properties as they are used naturally.
   $2(3 + n)$ or $(3 + n) + (3 + n)$ $\quad$ $2\cdot 3 + 2n$ or $(3 + 3) + (n + n)$

2. Members of the cooking club would like to learn how to make peach ice cream. There are 14 people in the club. Each member will need to buy 3 peaches and 1 pint of cream to make the ice cream. Peaches cost x cents each, and a pint of cream costs 45 cents. Write two different expressions that could be used to represent the total cost of ingredients for all 14 members of the club. Simplify each expression.
   $14(3x + 0.45)$
   $14\cdot 3\cdot x + 14\cdot(0.45)$
   $630 + 42x$ cents (most likely students will not use cents)

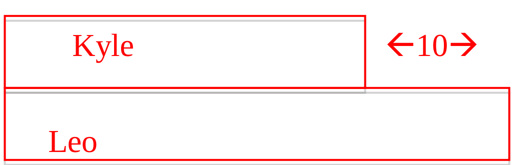
3. Martin's grandma gave him a \$100 gift certificate to his favorite restaurant. Everyday for four days Martin went to the restaurant and bought a drink for \$1.19 and fries for f dollars. Write two different expressions for how much money Martin has left on his gift certificate.

   $100 – 4(f + 1.19)$ or $100 – (f + 1.19) – (f + 1.19) – (f + 1.19) – (f + 1.19)$ or $100 – 4f – 4(1.19)$

4. Juab Junior High School wants to meet its goal of reading 10,000 books in three months. If 7^{th} grade reads x number of books every week, 8^{th} grade reads y number of books every week, and 9^{th} grade reads z number of books every week, write two different expressions for how many books the school will still need to read after 3 weeks?
   $10{,}000 – 3(x + y + z)$ or $10{,}000 – (x + y + z) – (x + y + z) – (x + y + z)$ or $10{,}000 – 3x – 3y – 4z$

For #5 – 10 sketch a model to help you answer the questions.

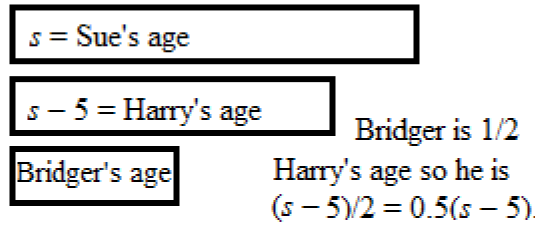
5. Leo and Kyle are training for a marathon. Kyle runs 10 fewer miles per week than Leo. Draw a model to help clarify contexts. Discuss what it means for something to be "in terms of" something else.
   a. Write an expression to represent the distance Kyle runs in terms of Leo's miles, L.
      $L - 10$
   b. Write an expression to represent the distance Leo runs in terms of Kyle's miles, K.
      $K + 10$
   c. Write an expression to represent the distance Kyle runs in 12 weeks in terms of Leo's miles.
      $12(L – 10)$ $\quad$ $12(L) – 12(10)$ $\quad$ $12L – 120$
   d. Write an expression to represent the distance Leo runs in 12 weeks in terms of Kyle's miles.
      $12(K + 10)$ $\quad$ $12(K) + 12(10)$ $\quad$ $12K + 120$



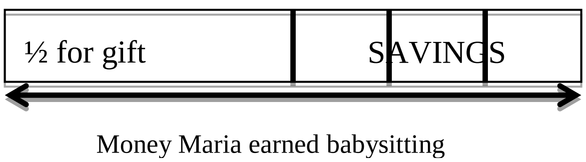
Kyle's distance is 10 miles per week FEWER than Leo's: $L – 10$
Leo's distance is 10 miles per week MORE than Kyle's: $K + 10$

6. Harry is five years younger than Sue. Bridger is half as old as Harry. Write **two** different expressions that could be used to represent Bridger's age in terms of Sue's age. Simplify each expression. (Hint: use a variable to represent Sue's age.)

$\frac{(s-5)}{2}$ $\frac{1}{2}(s-5)$ $\frac{s}{2}-\frac{5}{2}$ $\frac{s-5}{2}$



7. Maria earned some money babysitting. She spent ½ of her babysitting money on a gift for her mother and then put 2/3 of the rest into her savings account. Write at least one expression for how much money Maria put into her savings account. (A model is provided below.)



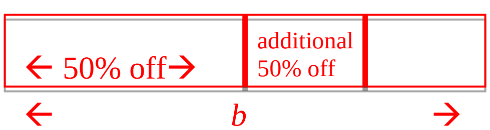
Let x be the amount Maria earned.
Savings: $(2/3)((1/2)x)$ or $(2/6)x$ or $(1/3)x$

8. Inez ran uphill for two-thirds of the marathon. Half of the rest was downhill and the other half was flat. Write at least one expression to represent the portion of the marathon Inez ran downhill.

$\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)x$ or $\frac{1}{6}x$

9. Cleon went to the bookstore where everything was 50% off. The store manager made an announcement that an additional 50% would be taken off the sale price at the cash register. Write an expression for the price of a book with both deductions taken.

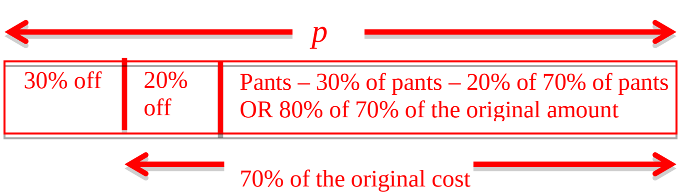
$b - 0.5b - 0.5(b - 0.5b)$ or $0.50(0.50\times b)$ or $0.25b$. Take time to examine the model for this problem. Show students both expressions are the same when simplified.



For #5 – 8, it is very important to have students justify their expressions.

10. Rico went to the store to buy a pair of pants because there was a 30% off all merchandise sale. When he got to the store he learned that purchases made before noon would receive an additional discount of 20% off the sale price. If the pants he picked out cost p dollars and he purchased them before noon, write an expression for the final cost of the pants.

$(p - 0.30p) - (0.20(0.70p))$ or $0.80(0.70)p$



3.1k Homework: Modeling Context with Algebraic Expressions

Write an algebraic expression for each context. In some cases you will be asked to write more than one expression. Be prepared to justify your expressions.

1. Marie would like to buy lunch for her three nieces. She would like each lunch to include a sandwich, a piece of fruit, and a cookie. A sandwich costs $3, a piece of fruit costs $0.50, and a cookie costs $1.

    a. Write two different expressions that could be used to represent the total price of all three lunches. Then simplify each expression that you wrote to find out how much total money Marie spent on lunch.
    $3(3 + 0.50 + 1)$ $\quad$ $3\cdot3 + 3\cdot0.50 + 3\cdot1$ $\quad$ $9 + 1.50 + 3$ $\quad$ $13.50

    b. Write two different expressions that could be used to represent the total price of all three lunches if a sandwich costs x dollars, a piece of fruit costs y dollars, and a cookie costs z dollars.
    $3(x + y + z)$ $\quad$ $3x + 3y + 3z$

2. Boris is setting up an exercise schedule. For five days each week, he would like to play a sport for 30 minutes, stretch for 5 minutes, and lift weights for 10 minutes.

    a. Write two different expressions that could be used to represent the total number of minutes he will exercise in five days. Then simplify each expression that you wrote.

    b. Write two different expressions that could be used to represent the total number of minutes he will exercise in five days if he plays a sport for x minutes, stretches for y minutes, and lifts weights for half the time he plays a sport.

3. James had $550 in his checking count. Once a week, he spends d dollars on music downloads and $5 on food. Write two expressions for how much money James has in his account after five weeks.

4. Five girls on the tennis team want to wear matching uniforms. They know skirts will costs $24 but are not sure about the price of the top. Write two different expressions that could be used to represent the total cost of all five skirts and five tops if x represents the price of one top. Simplify each expression.

5. Drake, Mike, and Vinnie are making plans to go to a concert. They have a total of $200 between the three of them. The tickets cost $30 each, and each boy plans to buy a t-shirt for *t* dollars. Write two different expressions to represent the amount of money the three boys will have left over. Simplify each expression.

   $200 - 3(30 + t)$ $200 - 30 - 30 - 30 - t - t - t$ $200 - 3(30) - 3t$

   Each expression should simplify to $110 - 3t$

6. Miguel has run three marathons. The first time he ran a marathon, it took him *m* minutes. The next time he ran he cut 10% off his time. The third time he ran a marathon, he was able to cut an additional 5% off his fastest time (the time he got on the second marathon). Write an expression for the amount of time it took Miguel to run the marathon the third time.

   $0.95(0.90x)$ or $(1x - 0.10x) - (0.05(1x - 0.10x))$

   This context and the ones in #7 & 8 are different than #1 – 5.

7. Ana earned *x* dollars at her summer job. She put 50% of her money into savings and spent 40% of the rest on a new outfit. Write an expression for the amount of money she has left.

8. Marcela earns money working in her dad's office over the summer. If she spent 40% of her money on a new phone and 50% of the rest on new clothes for school, write an expression for how much of her money she spent.

3.1l Self-Assessment: Section 3.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Recognize and explain the meaning of a given expression and its component parts. [1, 2, 3, 4]	I struggle to understand the meaning of parts of a given expression.	I can recognize the different parts of a given expression and match them with their meaning.	I can recognize the different parts of a given expression and can explain their meaning in my own words.	I can explain the meaning of a given expression and its parts in my own words. I can also write a meaningful expression given a context.
2. Recognize that different forms of an expression may reveal different attributes of the context. [5, 6]	I can't tell when two expressions are really different forms of the same expression.	I can recognize that two expressions are different forms of the same expression, but I struggle to see why I would write it in a different form.	I can recognize that two expressions are different forms of the same expression, and I can explain how the different forms reveal different attributes of the context.	I can write two expressions that are different forms of the same expression, and I can explain how the different forms reveal different attributes of the context.
3. Combine like terms with rational coefficients. [7 (rational c & d)]	I struggle to combine like terms.	I can combine like terms with a model.	I can combine like terms with integer coefficients without a model.	I can combine like terms with rational coefficients.
4. Use the Distributive Property to expand and factor linear expressions with rational numbers. [8 c,d]	I know what the Distributive Property is, but I struggle to use it.	I can use the Distributive Property to expand linear expressions with integers such as Problem 8a.	I can use the Distributive Property to expand linear expressions with rational numbers such as Problems 8a and 8b.	I can use the Distributive Property to expand and factor linear expressions.
5. Recognize properties of arithmentic and use them in justifying work when manipulating expressions. [10]	I know there are properties, but I have a hard time recognizing them in work.	I can name properties when I see them, but struggle to use them to justify manipulations.	I can use properties to justify manipulations with expressions.	I can use properties to justify manipulations with expressions and can use them to critique the arguments of others.
6. Write numeric and algebraic expressions to represent contexts.	I struggle to translate contexts to numeric and algebraic expressions.	Most of the time I can translate contexts to numeric and algebraic expressions.	I can translate contexts to numeric and algebraic expressions.	I have no trouble translating contexts to a variety of numeric and algebraic expressions.

Sample Problems for Section 3.1

1. Given the following situation, match the parts of the expression to the parts of the context.

   Luis went to a soccer game with some friends. He bought two sodas for $1.50 each and four giant candy bars for $2.25 each. How much did he spend?

   $2(1.50) + 4(2.25)$

2. Explain the different parts of the following expression:

   Matthew likes to buy movies from a DVD club. He pays $5 per month plus $2 per movie. If he buys six movies in June, how much did he pay for the month of June?

   $6(2)+5$

3. Write an expression to match the following situation. Explain the different parts of the expression.

   Nastas plays on the school basketball team. He scores five 3-pointers and 3 lay-ups (worth 2 points each) in one game. How many total points did he score in that game?

4. Write an expression to match the following situation. Explain the different parts of the expression.

   Millie bought two sweaters for $25 and three pair of pants for $30. She had a 25% off coupon for her entire purchase. Write an expression for the amount of money Millie spent.

5. Identify which expressions match the following situation. Explain how each different correct expression reveals different aspects of the situation.

   Ooljee goes to the movies with her two friends. They each pay for their own movie ticket at $8.50 each and each buys a soda for *d* dollars. How much money do the girls spend at the theater?

   $3(8.50)+3d$ $\quad$ $3(8.50)+d$ $\quad$ $3(8.50+d)$ $\quad$ $3(8.50)d$ $\quad$ $8.50+d+8.50+d+8.50+d$

6. Write at least two different expressions for the following situation. Explain how each different expression reveals different aspects of the situation.

Phoebe babysits every weekend. She puts 30% of each check into savings and spends 15% of the remainder on new books. How much does she have left?

7. Simplify the following expressions. Use a model if needed.

a. $9m - 3 + 4m$

b. $-71b - 4a + 4b - 4a$

c. $50q + 0.1t - 0.3t + 14q$

d. $\frac{9}{10m} - \frac{3}{5} + \frac{4}{5}m + \frac{1}{2}m$

8. Simplify the following expressions.

a.

$8(-1 - 7w)$ $-8(7w + (-6))$ $(10 + 7w)(-5)$

b.

$-6\left(\frac{1}{5}a - \frac{1}{6}\right)$ $\frac{1}{4}(-6a + 8b)$ $\frac{1}{3}\left(-7 - \frac{1}{6}a\right)$

9. Factor the following expressions.

$3x - 6$ $15 - 20y$ $\frac{5}{3}x + \frac{2}{3}$ $4.9t - 2.8$

10. Indentify the arithmetic property used in simplifying each expression.

a. $3x + 2 + x + -1$
$3x + x + 2 + -1$ Property:

b. $4x + (-4x)$
0 Property:

c. $(3x + 2y) + y$
$3x + (2y + y)$ Property:

Section 3.2 Solving Algebraic Equations ($ax + b = c$)

Section Overview:

This section begins by reviewing and modeling one- and two-step equations with integers. Students then learn to apply these skills of modeling and solving to equations that involve the distributive property and combining like terms. Students learn to extend these skills to solve equations with rational numbers and lok for errors in their own and other's work

Concepts and Skills to be Mastered (from standards)

By the end of this section, students should be able to:

1. Solve multi-step equations fluently including ones involving calculations with positive and negative rational numbers in a variety of forms.
2. Connect arithmetic solution processes that do not use variables to algebraic solution processes that use equations.
3. Use properties of arithmetic to create an argument or critique the reasoning of others in solving algebraic equations.

3.2 Class Discuss: Using Properties to Justify Steps for Simplify Expressions

At the beginning of 3.1, you named properties you've been using since elementary school. Without looking back or ahead, can you list the properties you've named to this point?

There is one last property to add to the list: Distributive Property of Multiplication over Addition.

The distributive property links addition and multiplication. Like the properties discussed at the beginning of 3.1, you've been using the distributive property informally since elementary school.

Examples:

Without a calculator, find the product 23 × 7. Explain your procedure.
Students are likely to say that 23 × 7 is 23 groups of seven. Since 23 = 20 + 3, 20 × 7 = 140, and 3 × 7 = 21, then 23 × 7 = 140 + 21 = 161. This mental math strategy is the distributive property of multiplication over addition.

Without a calculator, find the product: 4 × 3.1. Explain your procedure.
12.4; procedures will vary. However, it is likely someone will explain that 3.1 is 3 + 0.1, 4×3 + 4×0.1 is 12 + 0.4, thus the answer is 12.4. This mental math strategy is the distributive property.

Show two ways one might simplify: 2(3 + 4)
$2(3 + 4) = 2(7) = 14$
$2(3 + 4) = 2(3) + 2(4) = 6 + 8 = 14$

Use the distributive property to rewrite $a(b + c)$
$a(b + c) = a(b) + a(c)$

Review: Generalizing Properties

Students did this in 3.1. This form is the same as that one except that it adds the distributive property.

Properties of Mathematics:

Name Property	**Algebraic Statement**	**Meaning**	**Examples**
Identity Property of Addition	$a + 0 = a$	Adding zero to a number does not change the number. "Zero" can take many forms.	$5 + 0 = 5; 5 + (1 + (-1)) = 5$
Identity Property of Multiplication	$a(1) = a$	Multiplying a number by one does not change the number.	
Multiplicative Property of Zero	$a(0) = 0$	Multiplying any rational number by zero results in 0.	
Commutative Property of Addition	$a + b = b + a$	Reversing the order of addition does not change the result.	
Commutative Property of Multiplication	$ab = ba$	Reversing the order of multiplication does not change the result.	
Associative Property of Addition	$(a + b) + c = a + (b + c)$	Changing the grouping of addition does not change the result.	
Associative Property of Multiplication	$a(bc) = (ab)c$	Changing the grouping of multiplication does not change the result.	
Distributive Property of Multiplication over Addition	$a(b + c) = ab + ac$	"a" groups of $(b + c)$	$3(2 + 5) = 3(2) + 3(5)$ $3(2 - 5) = 3(2) + 3(-5)$
Additive Inverse	$a + (-a) = 0$	A number added to its opposite will result in zero.	
Multiplicative Inverse	$a(1/a) = 1$ for $a \neq 0$	Multiplying a number by its multiplicative inverse will result in one, the multiplicative identity.	

Use properties to justify steps:

Example: Jane wants to find the sum: 3 + 12 + 17 + 28. She uses the following logic, "3 plus 17 is 20, and 12 plus 28 is 40. The sum of 20 and 40 is 60." Why is this okay? The table below shows how to justify her thinking using properties for each step.

Statement	**Step**	**Justification**
3 + 12 + 17 + 28	*No change, this is where she started.*	*This expression was given.*
3 + 17 + 12 + 28	*The 17 and the 12 traded places.*	*Commutative Property of Addition*
(3 + 17) + (12 + 28)	*Jane chose to add the numbers in pairs first, which is like inserting parentheses.*	*Associative Property of Addition*
20 + 40	*Jane found the sums in the parentheses.*	*Jane is now following the Order of Operations.*
60	***Add, so . . . 3 + 12 + 17 + 28 = 60***	

1) The expression $3(x - 4) + 12$ has been written in five different ways. State the property that allows each change.

Expression	**Step**	**Justification / Property Used**
$3(x + (-4)) + 12$	No change	This expression was given, but rewritten using the idea that $a - b = a + (-b)$
$3x + 3(-4) + 12$	Three groups of $(x + (-4))$ means $3x + -12$	Distributive Property
$3x + (-12) + 12$	$3\times(-4) = -12$	Multiplication of integers
$3x + ((-12) + 12)$	Grouping for addition	Associative Property of Addition
$3x + 0$	12 and –12 combine to 0.	Additive Inverse Property
$3x$	$a + 0 = a$.	Additive Identity Property

2) The expression $2(3x + 1) + -6x + -2$ has been written in four different ways. State the property that allows each change.

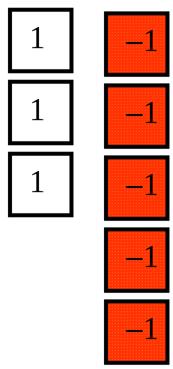
Expression	**Step**	**Justification**
$2(3x + 1) + -6x + -2$	No change	Given expression
$6x + 2 + -6x + -2$	Multiplied 2 by both $3x$ and 1	Distributive Property
$6x + -6x + 2 + -2$	Changed the order of the terms	Commutative Property of Addition/Addition is Commutative
$(6x + -6x) + (2 + -2)$	$6x + (-6x)$ and $2 + (-2)$ both sum to 0	Associative Property of Addition
$0 + 0$	$6x + (-6x)$ and $2 + (-2)$ both sum to 0	Additive Inverse

Review: Look back at Chapter 2 and review addition/subtraction with the chip/tile model.

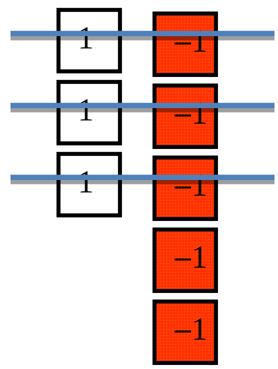
3) Model 3 + (–5) to find the sum

3 + (–5)

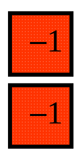
Step 1



Step 2



Step 3



Justify each step:

Step 1: this is a representation of the expression

Step 2: 1 + (–1) = 0, so each of these pairs are zero: –1 is the additive inverse of 1.

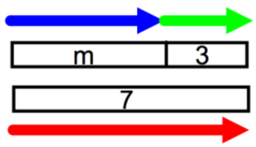
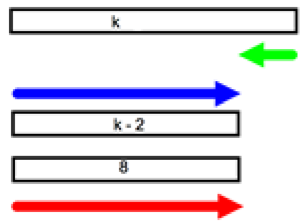
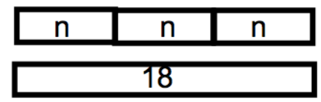
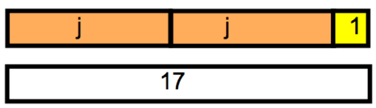
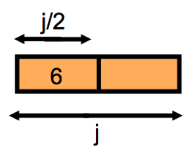
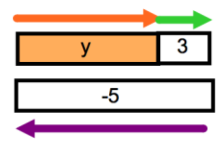
Step 3: this is a representation of what remains when the zero pairs are removed.

4) In chapter 2 you learned that a negative times a negative produces a positive product. We used models to discover why this is true. Below is a more formal proof. We start with the the Multiplicative Property of Zero (anything times zero is 0.)

Statement	Step	Justification
–1(0) = 0	Given	Multiplicative Property of Zero
–1(–1 + 1) = 0	0 was replaced with (–1 + 1)	Additive Inverse Property
–1(–1) + –1(1) = 0	–1 was multiplied by each term in parentheses	Distributive Property
–1(–1) + 1(–1) = 0	The –1(1) got switched to 1(–1)—changed the order of multiplication	Commutative Property of multiplication
–1(–1) + –1 = 0	1(–1) was replaced with –1	Identity Property of Multiplication
–1(–1) must equal 1 because if we get 0 when we add it to –1, it must be the additive inverse of –1	Additive Inverse Property	

3.2a Classroom Activity: Model and Solve Equations

Use any method you'd like to solve each of the following. Justify your answer with a model or words:

1. $m + 3 = 7$ 	2. $8 = k - 2$  To find k, we will need to "remove" –2 from both sides. We can also think about $k - 2$ as $k + (-2)$.	3. $3n = 18$ 
4. $17 = 2j + 1$ 	5. $j/2 = 6$ 	6. $y + 3 = -5$  To find y, we must "take away 3 from both the y +3 and the –5"

It may have been easy to solve some (or all) of the above in your head, that's good; that means you're making sense of the problem. In this section, we are going to focus on the structure of equations and how properties of arithmetic allow us to manipulate equations. Even though the "answer" is important, more important right now is that you understand the underpinnings of algebraic manipulations.

Evaluate the expression $2x + 1$ for each of the given values:

7. Evaluate $2x + 1$ for $x = 3$ 7 Discuss how the unknown in an expression can represent any number. Also, discuss "evaluate" vs. "solve."	8. Evaluate $2x + 1$ for $x = -2$ –3 Discuss how/why 7 – 9 each simplify to different values.	9. Evaluate $2x + 1$ for $x = -3$ –5

Solve each equation in any way you want. Be able to justify your answer with a model or words:

8. $2x + 1 = 5$ $x = 2$	9. $2x + 1 = 9$ $x = 4$	10. $2x + 1 = -9$ $x = -5$

Discuss that the expression $2x + 1$ was set equal to three different numbers, resulting in different values of the unknown x.

What do the terms "evaluate" and "solve" mean?

What is the difference between an equation and an expression?

For this activity use the following Key to represent variables and integers. Note: "x" or "$-x$" can be any variable.

x	$-x$
1	-1

11. Consider the equation: $x - 1 = 6$. What value of x makes this equation true? Justify:
Look for language that illustrates algebraic thinking. For example a student might say, "I know x has to be one-unit bigger than 6 because if you subtract 1 you get 6."

Draw a model of $x - 1 = 6$ below, then use it to illustrate the algebraic steps need

x -1 To isolate the "x," we want to add 1 to both sides to the equation.	=	1 1 1 1 1 1	$x - 1 = 6$
Throughout these exercises, students should discuss properties of arithmetic.		It will be very helpful to change the problem to $x + (-1) = 6$ and continue this structure throughout. In this way we are always ***adding*** the additive identity. As problems become more complex, students often become confused with problems like $5x - 7 = -3$; students will not know if they should add 7 or -7 or if they should subtract 7 or -7.	$x - 1 = 6$ $+1 \quad +1$ $x = 7$ additive inverse

12. Consider the equation: $x - 3 = 5$. What value of x makes this equation true? Justify:

Draw a model of $x - 3 = 5$ below, then use it to illustrate the algebraic steps needed to solve.

[x] [-1] [-1] [-1] This is the same as $x + (-3) = 5$	=	[1] [1] [1] [1] [1]	$x - 3 = 5$
Talk about "equality;" adding "3" to both sides of the equation maintains equality. 3 and (–3) are additive inverses.	=		$x - 3 = 5$ $+3 \quad +3$ $x = 8$ additive inverse

13. Use a model to solve $8 = 7 + m$. Write the algebraic procedure you followed to solve.

Students should be comfortable with the unknown on either side of the equal sign.	=	The format provide on these initial pages for solving equations is designed to help students think about equality. For this exercise, students will have "8" in the left box and $7 + m$ in the right box with the "=" in the middle. Emphasize that equality must be maintained. What ever we do to one side of the equation, we must do to the other	$8 = 7 + m$
	=		$8 = 7 + m$ $+(-7) \quad +(-7)$ additive inverse $1 = m$

14. Consider the equation: $6 = 3x$. What value of x makes this equation true? Justify:
Draw a model of $6 = 3x$ below, then use it to illustrate the algebraic steps need

[1] [1] [1] [1] [1] [1]	=	[x] [x] [x] Help students notice that each x corresponds to two units. To isolate the x in this situation we apply the multiplicative inverse.	$6 = 3x$
Students might say, " three times what is 6; well I know that 3 time 2 is 6." Push for how they know that.	=	Take time to distinguish between an additive inverse and a multiplicative inverse.	$(1/3)\cdot 6 = (1/3)\cdot 3x$ or $6/3 = 3/3\ x$ multiplicative inverse so $x = 2$

15. $8 = -2m$. Write the algebraic procedure you followed to solve.

Students might take the "opposite" of both sides: $-8 = 2m$, or they may work with the problem the way it is. Allow students time to Make Sense of the Problem.	=	Here the negative sign might throw students. You might ask, " –2 times what is positive 8?" Students will likely know the answer has something to do with 4. Have students test their conjectures.	$8 = -2m$
	=	LOOK FOR STRUCTURE. Students are trying to isolate the unknown. To isolate they need to use the "opposite" operation. In other words, they need to apply inverses, either additive or multiplicative.	$m = -4$

16. Consider the equation $-5 + 3n = 7$. What value of *n* mkes the equation true? Justify:

Draw a model of the equation and use it to solve the equation. Write the algebraic procedure you followed to solve.

This is the first two step equation students solve. The model allows students to "see" that the "–5" and "3*n*" are different from each other. Thus, when they add 5 to both sides of the equal sign, they will physically see the "zero pairs" (additive inverses). This becomes less clear if students work problems without a model.	=	Talk to students about the fact that there are two operations on the left, addition and multiplication. We need to "undo" both. Thus we need to apply the additive inverse to –5 and the multiplicative inverse to 3.	$-5 + 3n = 7$
	=	In applying the additive inverse, help students recognize that: $(-5 + 3n) + 5$ is $3n$ BUT $(-5 + 3n)(1/3)$ is $-5/3 + n$: link to the distributive property. We generally choose to apply the <u>additive inverse first</u> when solving not because there is any such rule, but rather for ease.	add 5 to both sides; additive inverse. $3n = 12$
	=		Multiply both sides by 1/3; multiplicative inverse; $n = 4$

17. Consider the equation $-4 = -3m + 8$. What value of m makes the equation true? Justify.

Draw a model of the equation and use it to solve the equation. Write the algebraic procedure you followed to solve.

	=		$-4 = -3m + 8$
	=		add –8 to both sides; additive inverse. $-3m = -12$
	=		Multiply both sides by –1/3; multiplicative inverse $n = 4$

Additional Practice Students may want to draw a model on a separate piece of paper.

18) $-7 = m - 9$ $m = 2$

19) $12 = w + (-4)$ $w = 16$

20) $-4k = -20$ $k = 5$

21) $-18 = 2n$ $n = -9$

22) $3x = -15$ $x = -5$

23) $40 = -5y$ $y = -8$

24) $5 = -2x + 9$ $x = 2$

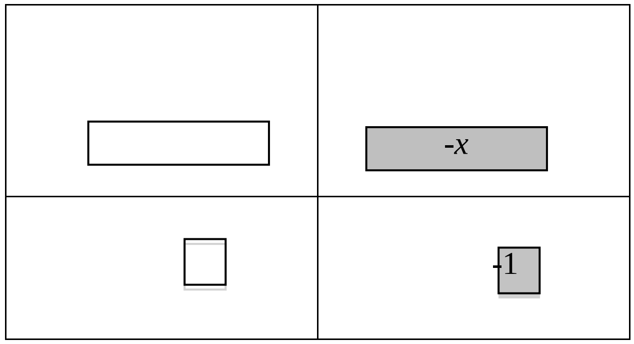
25) $4x + 7 = -5$ $x = -3$

26) $-8 = -3m + 10$ $m = 6$

27) $7z + 1 = 15$ $z = 2$

3.2a Homework: Model and Solve Equations

Model and solve each equation below. Use the key below to model your equations.



1. $x - 6 = -9$

x -1 -1 -1 -1 -1 -1	=	-1 -1 -1 -1 -1 -1 -1 -1 -1	$x - 6 = -9$ $x + -6 = -9$ $+6 \quad +6$ Additive Inverse
x	=	-1 -1 -1	$x = -3$

2. $-15 = x - 14$

	=		$-15 = x - 14$
	=		

3. $m + 2 = -11$

m 1 1	=	-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1	$m + 2 = -11$ $-2 = \ -2$ $m = -9$ Add negative 2 to both sides- Additive Inverse
m		-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1	$m = -9$

4. $4n = -12$

	=		
	=		

5. $-15 = -3m$

students may take the opposite of both sides to model: 15=3*m*	=		
	=		

6. $3t + 5 = 2$

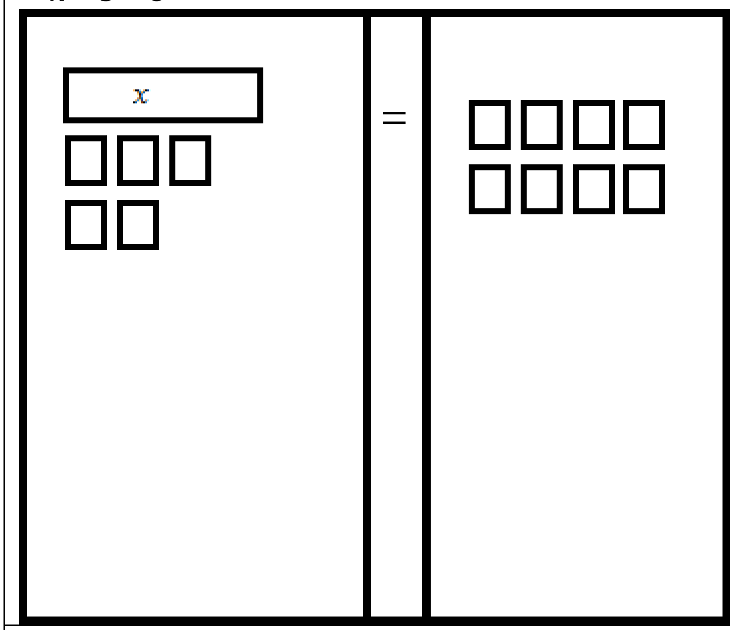
	=		
	=		
	=		

7. $8 = 2p - 4$

1 1 1 1 1 1 1 1	=	*p* *px* -1 -1 -1 -1	$8 = 2p - 4$ $+4 = \quad +4$ $12 = 2p$ Add 4 to both sides. Additive Inverse.
1 1 1 1 1 1 1 1 1 1 1 1	=	*p* *p*	$\left(\frac{1}{2}\right)12 = 2p\left(\frac{1}{2}\right)$ Multiply both side by ½. Multiplicative Inverse.
1 1 1 1 1 1	=	*p*	$6 = p$

3.2b Class Activity: More Model and Solve One- and Two-Step Equations

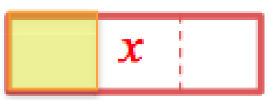
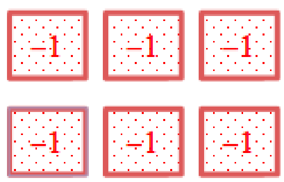
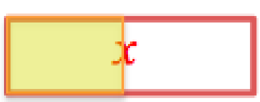
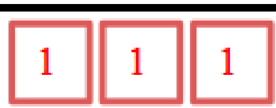
Draw a model, justify your steps and then check your answer. The first one is done for you.

Model/Draw the Equation	What are the solving actions? Record the steps using Algebra	Check solution in the equation.
1. $x + 5 = 8$ 	*Add –5 to both sides. –5 is the additive inverse of 5. This step isolates the x.* $x + 5 = 8$ $\quad -5 \quad -5$ $x \quad = 3$	$x + 5 = 8$ $(3) + 5 = 8$ True, so the solution is correct.
2. $5 = x + 8$	**Show work** $x = -3$	Show check

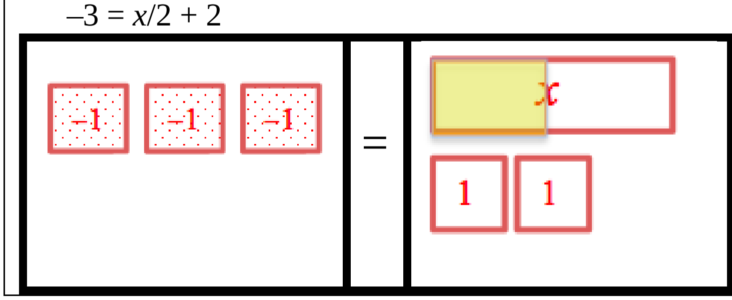
Look at # 1 and # 2. Why are the answers different?

	Show work	Show check
3. $3x = -6$ =	**Show work** $x = -2$	Show check

Explain the logic above in #3. The multiplicative inverse of 3 is 1/3, we multiply both sides by 1/3 to isolate the *x*. You might also talk about the model—three *x* rectangles are the same as six negative one units. Thus each *x* corresponds to two negative one units—we had to divide the units into 3 equal parts to see what each *x* represented. Remind students that multiplying both sides by 1/3 is the same as dividing both sides by three. How might you use related logic to model $x/3 = -6$? Ask students how they might rewrite the equation: $(1/3)x = -6$. Talk about 3 as the multiplicative inverse of (1/3). Also talk about whay $x/3$ means. e.g. if we have 1/3 of an *x*, we need to triple it to know how much one whole *x* represents. If we triple $x/3$ we need to triple –6 as well.

	Show work	Show check
4. $x/3 = -6$  1/3 of an *x* is –6, so if we triple both sides, a whole *x* is –18. = 	**Show work** $x = -18$	Show check
5. $(1/2)x = 3$  ½ of an *x* is 3, so a whole *x* is 6 = 	**Show work** $x = 6$	Show check

In problems # 3 and # 4, what happened to the terms on both sides of the equation? Both sides of the equation were multiplied by the multiplicative inverse of the coefficient of *x*.

6. $-9 = 2x - 5$	**Show work** $x = -2$	Show check
7. $7 = 3x - 2$	**Show work** $x = 3$	Show check
8. $-5 = -3 + 2x$	**Show work** $x = -1$	Show check
9. $7 + x/2 = -3$ Let students work with this. They might add -7 to both sides and then note that ½ of x is -10, so $x = -20$	**Show work** $x = -20$ Students might multiply both sides of the equation by 2 first and then subtract 14.	Show check
10. $-3 = x/2 + 2$ 	**Show work** $x = -10$ Students might multiply by 2 first and then add -4 to both sides.	Show check

3.2b Homework: More Model and Solve One- and Two-Step Equations

Model/Draw the Equation	What are the solving actions? Record the steps using Algebra	Check solution in the equation.
1. $2 = x + 5$ See model =	**Show work** $2 = x + 5$ $\underline{-5 = -5}$ $-3 = x$ $x = -3$	Show check $2 = (-3) + 5$ $2 = 2$
2. $-12 = 3x$ =		
3. $-\left(\frac{x}{4}\right) = -8$ =		
4. $-2 = \frac{1}{3}x$ See model =	**Show work** $-2 = \left(\frac{1}{3}\right)x$ $(3)(-2) = \left(\frac{1}{3}\right)x \cdot 3$ $-6 = x$ $x = -6$	Show check $-2 = \left(\frac{1}{3}\right)(-6)$ $-2 = -2$

	Show work	**Show check**
5. $-9=\frac{x}{2}-5$ See model =	$-9=\frac{x}{2}-5$ $+5=\frac{x}{2}+5$ $-4=\frac{x}{2}$ $(2)(-4)=\frac{x}{2}(2)$ $-8=x$ $x=-8$	$-9=\frac{-8}{2}-5$ $-9=-4-5$ $-9=-9$
6. $-3x+2=-13$ =		
7. $-11=-4x-3$ =		
8. $-2x-5 = 3$ =		

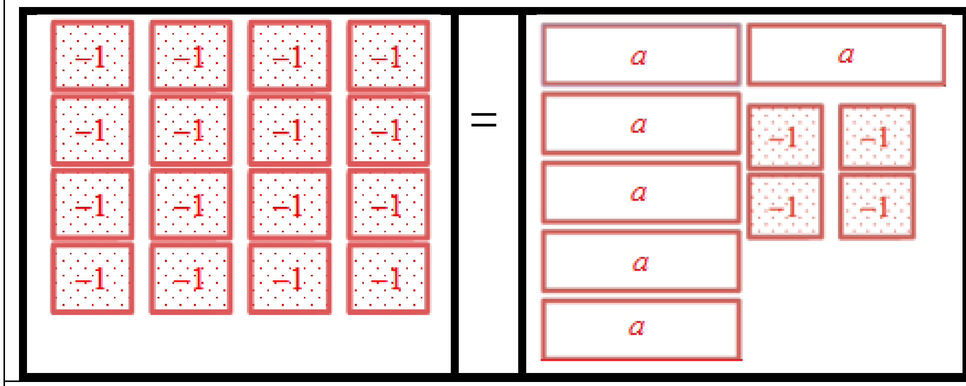
	Show work	Show check
9. $\frac{x}{3}-5=-2$ See model [ ] = [ ]	$\frac{x}{3}-5=-2$ $+5=+5$ $\frac{x}{3}=3$ $3\cdot\frac{x}{3}=3\cdot 3$ $x=9$	$\frac{9}{3}-5=-2$ $3-5=-2$ $-2=-2$
10. $2+5x=-8$ [ ] = [ ]		
11. $\frac{1}{2}x-5=-3$ [ ] = [ ]		

3.2c Class Activity: Model and Solve Equations, Practice and Extend to Distributive Property

Practice: Solve each.

Additional practice is available at 3.2g

1. $-16 = 6a - 4$



$a = -2$

2. $7 = 6 - \frac{n}{7}$

See model

=

$n = -7$

3. $-10 = -10 - 3x$

See model

=

$-10 = -10 - 3x$

$\underline{+10 = +10}$

$0 = -3x$

$\frac{0}{-3} = \frac{-3x}{-3}$

$0 = x$

$x = 0$

4. Review: Simplify each expression. Use words or a model to justify your answer.

a) $3(2x + 1)$ $6x + 3$; There are three groups of $2x + 1$	b) $-2(3x + 2)$ $-6x - 4$ or $-6x + (-4)$	c) $-4(2x - 3)$ $-8x + 12$

Solve each equation. In the space to the right, write a justification for your algebraic steps.

5. $2(x + 1) = -8$

Students may distribute first OR may chose to divide first. Discuss both approaches. "Plan A" $(1/2)(2(x + 1)) = (1/2)(-8)$ $x + 1 = -4$ $-1 \quad -1$ $x = -5$ Steps: multiply both sides of the equation by the multiplicative inverse of 2, then added the additive inverse of 1 to both sides of the equation.	=	"Plan B" $2x + 2 = -8$ $-2 \quad -2$ $2x = -10$ $(1/2)(2x) = (1/2)(-10)$ $x = -5$ Steps: Distribute 2 (there are two groups of $(x + 1)$). Add -2, the additive inverse of 2, to both sides of the equation. Then multiply both sides by the multiplicative inverse of 2.	$x = -5$

6. $6 = -3(x - 4)$

	=		$x = 2$

7. $-12 = -3(5x - 1)$

	=		$x = 1$

8. $-4(3 - 2m) = -12$

	=	

**Again, they may be unsure what to do when nothing is left on side of the equation. i.e. they will get $8m = 0$. Look back at Review #3.

$m = 0$

9. $-\frac{1}{2}(4x+2) = -5$

	=	

**Look for different strategies (like on #5): (a) doubling both sides OR (b) distributing the –1/2. Talk about both.

a). $(-2)\left(-\frac{1}{2}\right)(4x+2) = (-2)(-5)$

(b) $-\left(\frac{1}{2}\right)(4x+2) = -5$

$-2x - 1 = -5$

$x = 2$

10. $3(2x - 4) + 6 = 12$

	=	

$x = 3$

3.2c Homework: Model and Solve Equations, Practice and Extend to Distributive Property

Solve each equation. Justify your algebraic manipulations on the right.

1. $9 = 15 + 2p$

	=	

2. $-7 = 2h - 3$

See model	=	

$-7 = 2h - 3$

$+3 = \quad +3$

$-4 = 2h$

$\frac{-4}{2} = \frac{2h}{2}$

$-2 = h$

$h = -2$

3. $-5x - 12 = 13$

	=	

4. $6 = 1 - 2n + 5$

See model	=	

$6 = 1 - 2n + 5$

$6 = 6 - 2n$

$\underline{-6 = +6 - 2n}$

$0 = -2n$

$\frac{0}{-2} = \frac{-2n}{-2}$

$0 = n$

$n = 0$

5. $8x - 2 - 7x = -9$

	=	

6. $2(n - 5) = -4$

	=	

7. $-3(g-3)=6$

	=	

8. $-12=3(4c+5)$

See model	=	

$$-12=3(4c+5)$$
$$\frac{-12}{3}=\frac{3(4c+5)}{3}$$
$$-4=4c+5$$
$$\underline{-5=4c-5}$$
$$-9=4c$$
$$\frac{-9}{4}=\frac{4c}{4}$$
$$\frac{-9}{4}=c$$
$$c=-\frac{9}{4}$$

3.2d Class Activity: Error Analysis

Students in Mrs. Jones' class were making frequent errors in solving equations. Help analyze their errors. Examine the problems below. When you find the mistake, circle it, explain the mistake and solve the equation correctly. Be prepared to present your thinking.

Student Work	Explanation of Mistake	Correct Solution Process
1. $-6t=30$ $\frac{\quad}{6}$ $\frac{\quad}{6}$ $t=5$	The student forgot the negative. Multiplicative inverse is (–1/6)	$\frac{-6t}{-6}=\frac{30}{-6}$ $t=-5$
2. $\frac{3}{4}x=12$ $\frac{3}{4}\cdot\frac{3}{4}x=12\cdot\frac{3}{4}$ $x=9$	The Multiplicative inverse of 3/4 is 4/3	$\frac{4}{3}\cdot\frac{3}{4}x=\frac{4}{3}\cdot 12$
3. $8-5c=-37$ $-8 \quad -8$ $5c=-45$ $\frac{5c}{5}=\frac{-45}{5}$ $c=-9$	On the second line the student forgot to keep the negative. $a-b=a+(-b)$	$8-5c=-37$ $-8 \quad -8$ $\frac{-5c}{-5}=\frac{-45}{-5}$ $c=9$
4. $\frac{x+1}{3}=2$ $\frac{x+1-1}{3}=2-1$ $3\cdot\frac{x}{3}=1\cdot 3$ $x=3$		$3\cdot\left(\frac{x+1}{3}\right)=3\cdot 2$ $x+1=6$ $-1 \quad -1$ $x=5$

5. $4x-3=17$ $+3\quad +3$ $4x=20$ $-4\quad -4$ $x=16$	The student subtracted when division was necessary. Student applied additive inverse rather than multiplicative inverse.	$4x-3=17$ $+3\quad +3$ $\frac{4x}{4}=\frac{20}{4}$ $x=5$
6. $3(2x-4)=8$ $6x-4=8$ $+4\quad +4$ $\frac{6x}{6}=\frac{12}{6}$ $x=2$	The student did not distribute accurately. There are 3 groups, of (2*x* – 4). So there will be 6*x* and – 8.	$3(2x-4)=8$ $6x-12=8$ $+12\quad +12$ $\frac{6x}{6}=\frac{20}{6}$ $c=\frac{10}{3}$
7. $3x+2x-6=24$ $-2x\quad -2x$ $x-6=24$ $+6\quad +6$ $x=30$	The student subtracted 2*x* twice from the same side of the equation. The equality was not maintained.	$3x+2x-6=24$ $5x-6=24$ $+6\quad +6$ $\frac{5x}{5}=\frac{30}{5}$ $x=6$
8. $5x+1-(-2x)=-8$ $3x+1=-8$ $-1\quad -1$ $\frac{3x}{3}=\frac{-9}{3}$ $x=-3$	The student failed to note *a* – (–*b*) = *a* + *b*. Taking away –2*x* is the same as adding 2*x*.	$5x+1-(-2x)=-8$ $5x+1+2x=-8$ $7x+1=-8$ $-1\quad -1$ $\frac{7x}{7}=\frac{-9}{7}$ $x=-\frac{9}{7}$
9. $-2(x-2)=14$ $-2x-4=14$ $+4\quad +4$ $\frac{-2x}{-2}=\frac{18}{-2}$ $x=-9$	The student did not distribute correctly. Likely not accounting for the negatives.	$-2(x-2)=14$ $-2x+4=14$ $-4\quad -4$ $\frac{-2x}{-2}=\frac{10}{-2}$ $x=-5$

10. $3(2x+1)+4=10$ $6x+3+4=10$ $9x+4=10$ $-4 \quad -4$ $9x=6$ $\frac{9x}{9}=\frac{6}{9}$ $x=\frac{6}{9}=\frac{2}{3}$	The student added terms that were not like terms.	$3(2x+1)+4=10$ $6x+3+4=10$ $6x+7=10$ $-7 \quad -7$ $\frac{6x}{6}=\frac{-3}{6}$ $x=\frac{1}{2}$

3.2d Homework: Practice Solving Equations (select homework problems)

Solve each equation.

1. $8-t=-25$

2. $2n-5=21$

3. $3-y=13$

$-10 = y$

4. $12=5k-8$

$4 = k$

5. $-5-b=8$

6. $5=-6a+5$

7. $8=\frac{n}{-7}$

8. $8=\frac{x}{7}+5$

$21 = x$

9. $\frac{y}{3}+2=10$

10. $\frac{t}{3}+4=2$

11. $9=\frac{n}{-8}-6$

12. $\frac{y}{5}+4=-12$

$-80 = y$

13. $8+6=-p+8$

14. $-7+8x-4x=9$

$4 = x$

15. $8x-6-8-2x=4$

16. $6(m-2)=12$

17. $5(2c+7)=80$

18. $5(2d+4)=35$

$1.5 = d$

19. $3(x+1)=21$

$6 = x$

20. $7(2c-5)=7$

21. $6(3d+5)=75$

22. $4-14=8m+2m$

23. $-1=5p+3p-8-p$

$1 = p$

24. $5p-8p=4+14$

25. $2p-4+3p=-9$

$-1 = p$

26. $-8=-x+5-1$

27. $12=20x-3+4x$

$x = 15/24 = 5/8$

3.2

Solve One- and Two-Step Equations with Rational Numbers (use algebra to find solutions)

Before we begin… Additional practice is available at 3.2g

…how can we find the solution for this problem?	…do you expect the value for x to be larger or smaller than 4 for these problems? Explain.
$\frac{3}{5}x = 6$	$\frac{2}{3}x = 4$ $0.25x = 8$ …how can you figure out the solutions in your head? Review equivalent forms of rational numbers from Chp 1

Solve the equations for the variable. Show all solving steps. Check the solution in the equation (example #1 check: –13(3) = –39, true.). Be prepared to explain your work.

1. $-13m = -39$ $m = 3$ Students should divide both sides by – 13 to get m = 3. They should check their answer by substituting 3 for m in the original equation, so they will show –13(3) = –39, which is true.	2. $-2 = \frac{m}{16}$ $m = -32$	3. $y - 25 = 34$ $y = 59$	4. $-2y = 24$ $y = -12$ Compare #4 and #7. In #4) two "y"s are –24, in #7 half an x is 6. In both we want to know what one unknown is worth.
Check:			
5. $-3x = \frac{3}{4}$ $x = -\frac{1}{4}$	6. $-13 = -25 + y$ $12 = y$	7. $\frac{1}{2}x = 6$ $x = 12$	8. $\frac{3}{4}x = 6$ $x = 8$
Check:			

9. $\frac{2}{3}x=-5$ $x=-\frac{15}{2}$	10. $\frac{a}{1.23}=0.2$ $a=0.246$	11. $-3.25d=5.278$ $d=-1.624$	12. $8x=-\frac{1}{4}$ $x=-\frac{1}{32}$
Check:			

13. $\frac{3}{4}q=2$ $q=\frac{8}{3}$	14. $\frac{3}{4}r=\frac{2}{3}$ $r=\frac{8}{9}$	15. $-5.36a=\frac{67}{5}$ $a=-2.5$
Check:		
16. $9.2r+5.514=158.234$ $r=16.6$	17. $0.25x-2=8$ $x=40$	18. $-8.38v+10.71=131.382$ $v=-14.4$
Check:		
19. $\frac{n}{1.4}-2.9=-5.11$ $n=-3.094$	20. $2x-\frac{3}{4}=3.25$ $x=2$	21. $\frac{1}{4}x-2=3$ $x=20$
Check:		

3.2e Homework: Solve One- and Two- Step Equations (practice with rational numbers)

Solve the equations for the variable in the following problems. Use models if desired. **Show all solving steps. Check the solution in the equation.**

1. $22 = -11k$	2. $\frac{x}{7} = -7$	3. $x + 15 = -21$ $x + 15 = -21$ $-15 = -15$ $x = -36$
Check:		$-36 + 15 = -21$ $-21 = -21$
4. $-3y = -36$	5. $\frac{2}{5}x = -2$ $\frac{2}{5}x = -2$ $\frac{5}{2} \cdot \frac{2}{5}x = -2 \cdot \frac{5}{2}$ $x = -5$	6. $-54 = 16 + y$
Check:	$\frac{2}{5}(-5) = -2$ $-2 = -2$	
7. $\frac{1}{4}x = 3$	8. $0.25x = 3$	9. $\frac{3}{4}x = -6$ $\frac{3}{4}x = -6$ $\frac{4}{3} \cdot \frac{3}{4}x = -6 \cdot \frac{4}{3}$ $x = -8$
Check:		$\frac{3}{4}(-8) = -6$ $-6 = -6$
10. $\frac{m}{-3.68} = -26.9$	11. $23.45j = -469$	12. $-2x = \frac{4}{7}$ $-2x = \frac{4}{7}$ $-2x \cdot \left(-\frac{1}{2}\right) = \frac{4}{7}\left(-\frac{1}{2}\right)$ $x = -\frac{4}{14} \Rightarrow x = -\frac{2}{7}$
Check:		$-2\left(-\frac{2}{7}\right) = \frac{4}{7}$ $\frac{4}{7} = \frac{4}{7}$

13. $5b = 0.2$	14. $\frac{2}{3}r = \frac{2}{5}$	15. $2.5b = 1\frac{4}{5}$ $b = \frac{18}{25}$
Check:		
16. $-3.8 - 13.4p = -460.606$ $p = 34.09$	17. $0.4x + 3.9 = 5.78$	18. $\frac{m}{2.8} - 4.9 = -7.11$
Check:		
19. $0.4x - 2 = 6$	20. $3x - \frac{2}{3} = 5\frac{1}{3}$ $x = 2$	21. $\frac{1}{5}x - 3 = 2$
Check:		

3.2f Extra Practice: Equations with Fractions and Decimals

1. $x+\frac{1}{2}=5$ $x=\frac{9}{2}$	2. $v-\frac{3}{8}=\frac{1}{8}$ $v=\frac{1}{2}$	3. $\frac{2}{3}n=\frac{4}{9}$ $n=\frac{2}{3}$
Check:		
4. $k+\frac{2}{3}=\frac{4}{5}$ $k=\frac{2}{15}$	5. $\frac{1}{3}+n=\frac{7}{12}$ $n=\frac{1}{4}$	6. $\frac{5}{9}=u-\frac{2}{9}$ $\frac{7}{9}=u$
Check:		
7. $\frac{4}{5}n=0.625$ $n=0.78125$	8. $n+\frac{5}{7}=\frac{1}{2}$ $n=-\frac{3}{14}$	9. $x+0.5=4$ $x=3.5$
Check:		

10. $m - \frac{3}{4} = \frac{5}{6}$ $m = \frac{19}{12}$	11. $\frac{1}{4}x = 1\frac{1}{2}$ $x = 6$	12. $a + \frac{3}{4} = 5$ $a = 3.75$ or $a = 3\frac{3}{4}$ *or* $a = \frac{17}{4}$
Check:		
13. $\frac{q}{3.1} + 5 = 10$ $q = 15.5$	14. $-9.2 + \frac{k}{6} = 4$ $k = 79.2$	15. $-5 = \frac{h}{3} + 7$ $h = -36$
Check:		
16. $20 = \frac{w}{4} - 10$ $w = 120$	17. $\frac{2}{5}p = \frac{5}{8}$ $p = \frac{25}{16}$	18. $\frac{2}{3}x = \frac{5}{8}$ $x = \frac{15}{16}$
Check:		

3.2g Extra Practice: Solve Equation Review

1. $-6.2d = 124$	2. $k + 12\frac{1}{2} = -20$ $k = -32.5$	3. $\frac{a}{5} = 20$
Check:		
4. $g - 12.23 = 10.6$	5. $-\frac{1}{5}h = 3$ $h = -15$	6. $-\frac{2}{3}h = 5$
Check:		
7. $\frac{w}{-1.26} = -2.36$ $w = 2.9736$	8. $3d = \frac{2}{3}$	9. $\frac{c}{3} + 1 = 10$
Check:		

10. $28 = 8x - 4$	11. $-0.3x + 3 = 4.2$ $x = -4$	12. $\frac{s-4}{11} = 2$
Check:		
13. $-6(x+8) = -54$	14. $5(w-20) - 10w = 5$ $w = -21$	15. $\frac{2}{3} = 5(y - 0.2)$
Check:		
16. $\frac{1}{2}(4d+2) = \frac{2}{3}$ $d = \frac{-1}{6}$	17. $4\left(v + \frac{1}{4}\right) = 37$	18. $3(x-1) - 2(x+3) = 0$ $x = 9$
Check:		

19. $2\left(5a-\frac{1}{3}\right)=\frac{7}{3}$ $a=\frac{3}{10}$	20. $-2.4+0.4v=16$	21. $7(w+2)+0.5w=5$
Check:		

Solve Multi-Step Equations (distribution, rational numbers)

22. $49(x+2x)=-24$	23. $-5(5+x)=-65$	24. $-6(-4+6x)=24$ $x=0$
Check:		
25. $-3(-2x+3)=-57$	26. $2(4x-1)=42$ $x=5.5$	27. $-7(-2x+7)=105$
Check:		

28. $3(7x+8)=150$ $x = 6$	29. $5(7x+5)=305$	30. $5(1+7x)=320$
Check:		
31. $3(5x+6)=78$	32. $5(2x+3)=96$	33. $\frac{12.1+a}{4.9}=7.071$ $a = 22.5479$
Check:		
34. $-6\left(-2x-\frac{1}{2}\right)=123$	35. $5\left(-\frac{2}{7}x+2\right)=\frac{80}{7}$ $x = -1$	36. $\frac{1}{2}(6x+2)=-29$
Check:		

37. $\frac{-10.5+m}{11.57}=-2.748$	38. $-13.9+\frac{b}{12.8}=-13.306$ $b = 7.6032$	39. $\frac{n-12.9}{6.1}=-0.377$
Check:		

40. $2(4x+8)=-32$	41. $7(5x+8)=91$	42. $-2(-4x+2)=76$ $x = 10$
Check:		
43. $-7(3x+7)=175$ $x = -10\frac{2}{3}$	44. $4(-9+x)=-12$	45. $5(-7+6x)=175$
Check:		

46. $2(1x+4)=18$ $x = 5$	47. $3(7+4x)=33$	48. $3(10+6x)=84$
Check:		
49. $5(1x+7)=40$	50. $-8=-3(5-2x)+1$	51. $\frac{r-8.7}{3.6}=3.722$ $r = 22.0992$
Check:		
52. $2\left(\frac{2}{3}x+2\right)=8$	53. $5\left(-4x+\frac{3}{10}\right)=-10$ $x = \frac{23}{40}$	54. $-\frac{1}{2}(4+5x)=-7$
Check:		
55. $\frac{k-2.6}{5.2}=-0.418$	56. $-13.3+\frac{k}{11.796}=-0.296$ $k = 153.395184$	57. $\frac{-7.3+r}{9.2}=-0.739$
Check:		

3.2h Self-Assessment: Section 3.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Solve multi-step equations fluently including ones involving calculations with positive and negative rational numbers in a variety of forms. [1]	I can solve one and two-step equations, but I struggle to solve multi-step equations.	I can solve multi-step equations involving calculations with integers with a model.	I can solve multi-step equations involving calculations with integers without a model.	I can solve multi-step equations involving calculations with rational numbers.
2. Connect arithmetic solution processes that do not use variables to algebraic solution processes that use equations.	I do not understand the connection between arithmetic solution processes that do not use variables to algebraic solution processes that use equations.	I can identify the connection between arithmetic solution processes that do not use variables to algebraic solution processes that use equations.	I can explain the connection between arithmetic solution processes that do not use variables to algebraic solution processes that use equations.	I can explain the connection between arithmetic solution processes that do not use variables to algebraic solution processes that use equations using the vocabulary of mathematical properties as appropriate.
3. Use the properties of arithmetic to make an argument and/or critique the reasoning of others when solving algebraic equations. [3]	I struggle to understand the reasoning of algebraic equations.	I can tell if my reasoning or another's reasoning to a solution is correct or not, but I struggle to explain why.	I can tell if my reasoning or another's reasoning to a solution is correct or not, and I can use properties to explain why.	I can tell if my reasoning or another's reasoning to a solution is correct or not, and I can use properties to explain why. If a solustion is incorrect, I can also explain with properities how it needs to be changed to show a correct reasoning and solution.

Sample Problems for Section 3.2

1. Solve each of the following equations with or without a model.

a.

$-1 = x + 7$	$-2x + 3 = 7$	$30 = \frac{x}{7} - 20$

b.

$-3(x - 6) = 9$	$-3 = \frac{2x + 4}{2}$	$-3x + 5x + 4 = 4$

c.

$0.8 + (-8) = 0$	$\frac{1}{2}(-2x - 7) = \frac{3}{4}$	$-\frac{1}{4} = 2.4x + 10 - 0.35x$

2. Solve the following equations. Explain why you used fractions or decimals to get your final answer.

$-\frac{1}{7} = x - 4.7$	$-\frac{1}{2} = 2x + 0.8$

3. Francisca is asked to solve the following equation. Francisca's work is shown below.

$$3.5 = \frac{1}{3}(x - 1.5)$$

$$3.5 = \frac{1}{3}x - 1.5$$

$$+1.5 \qquad\qquad + 1.5$$

$$5 = \frac{1}{3}x$$

$$\cdot 3 \quad \cdot 3$$

$$15 = x$$

What mistake did Francisca make?

Help Francisca answer the question correctly.

Section 3.3: Solve Multi-Step Real-World Problems Involving Equations and Percentages

Section Overview:

In this section students learn how to solve percent problems using equations. They begin by drawing bar models to represent relationships in word problems. Then they translate their models into equations which they then solve. Students then use similar reasoning to move to problems of percent of increase and percent of decrease. Finally, students put all of their knowledge together to solve percent multi-step problems with various types of rational numbers.

Concepts and Skills to be Mastered (from standards)

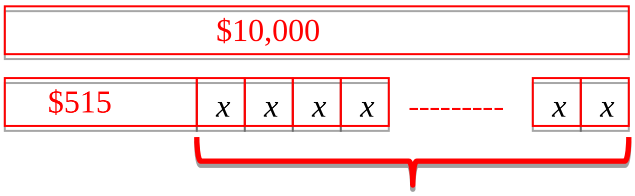
By the end of this section, students should be able to:

1. Use variables to create equations that model word problems.
2. Solve word problems leading to linear equations.
3. Solve multi-step real-life percent problems involving calculations with positive and negative rational numbers in a variety of forms.
4. Convert between forms of a rational number to simplify calculations or communicate solutions meaningfully.

3.3a Class Activity: Create Equations for Word Problems and Solve

For each each context, draw a model to represent the situation, write an equation that represents your model, solve the equation, and then answer the question in a full sentence. Check you answers in original problem.

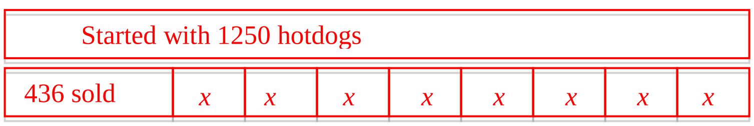
1. Today is Rosa's 12th birthday. She has a savings account with \$515 in it, but her goal is to save \$10,000 by the time she turns 18. How much money should she add to her savings account each month to reach her goal of \$10,000 between now and her 18th birthday?



6 years is 72 months (6×12)

$515 + 72x = 10{,}000$
$72x = 9485$
$x = 9485/72 \approx \$131.74$ per month

2. Eisenhower Junior High School made 1250 hotdogs to sell at the county fair as a fundraiser for new computers. They're only going to sell them for two days. On the first day, they sold 436 hot dogs. If they plan on selling for 8 hours on the second day, how many hot dogs per hour will they have to sell in order to sell the rest of their hotdogs?



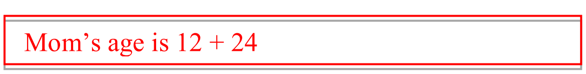
$8x + 436 = 1250$

$x = 101.75$

Make sense of the problem. You can't sell 0.75 of a hotdog. Talk about rounding and how to answer the question in a complete sentence.

3. Calisa's grandpa won't tell her how old he is. Instead he told her, "I'm twice your mom's age." Calisa knows her mom had her when she was 24 and Calisa is now 12. How old is Calisa's grandpa?

Mom was 24 when she had Calisa



Grandpa is twice (12 + 24)

G = 2(12 + 24)

Grandpa is 72 years old.

Grandpa is twice mom's age

4. Calisa's grandpa was really impressed that she was able to figure out his age, so he decided to give her another riddle. He said, "The sum of your great grandparents' age is 183. If great grandpa is 5 years older than great grandma, how old is each of them?"

g is Great Grandma's age	5 years
g is Great Grandma's age	+ 5

< ---------------Great Grandpa's age ------------ >

Great Grandpa's age *g* + 5	Great Grandma's age *g*

<---183--->

g = great grandma's age
$g + (g + 5) = 183$
$g = 89$
So Great Grandma is 89 and
Great Grandpa is 94

5. Peter tends a total of 87 sheep and goats. He had 15 more sheep than goats. How many sheep and goats does he have?

g is number of goats
g + 15 is the number of sheep

g + 15	g

< --87 --- >

$87 = g + (g + 15)$
$g = 36$
Peter has 36 goats and 51 sheep

6. Peter's neighbor raises chickens; both hens and roosters. Their family sells the eggs, so they like to have four times as many hens as roosters. If they currently have 45 birds, how many are hens?

r	Roosters; *r* = # roosters

r		*r*	*r*

Hens; four times as many hens as roosters

r	*r*	*r*	*r*	*r*

There are a total of 45 chickens

$45 = r + 4r$
$r = 9$
There are 9 roosters and 36 hens

7. James is five inches taller than twice his height when he was three years old. If he's now 5' 10" tall, how tall was he when he was three?

j

j; James' height at age 3

j	*j*

< ---5 inches----- >

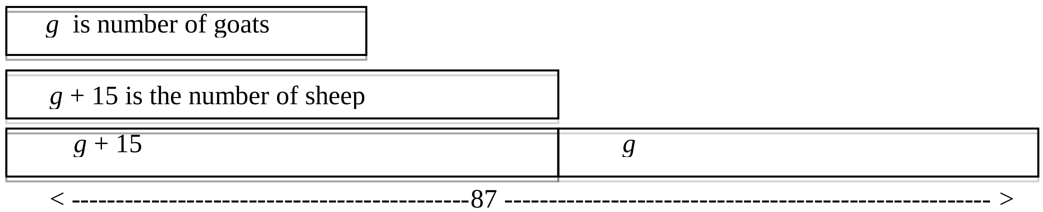
< -----------he is now 5'7" e.g. 67" ----------- >

$2j + 5 = 67$
$j = 33$
When James was 3 years old, he was 33" tall.

3.3a Homework: Create Equations for Word Problems and Solve

Example 1:

Peter tends a total of 87 sheep and goats. He had 15 more sheep than goats. How many sheep and goats does he have?



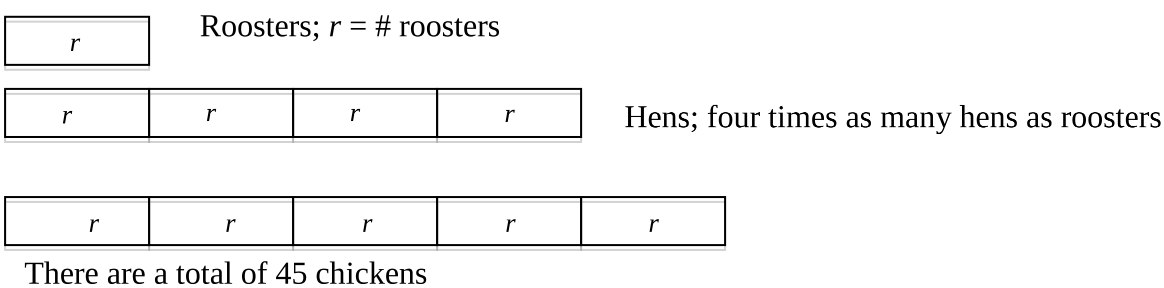
We know that there are more sheep than goat. We know that there are exactly 15 more sheep than goat, so the smaller rectangle represents g goat, and the larger is $g + 15$ sheep. There are total of 87 sheep and goat, so we can write the equation:

$g + (g + 15) = 87$

$g = 36$; thus we know that there are 36 goat and 51 sheep.

Example 2:

Peter's neighbor raises chickens; both hens and roosters. Their family sells the eggs, so they like to have four times as many hens as roosters. If they currently have 45 birds, how many are hens?



We know that there are more hens than roosters. Further, we know that there are four times as many hens as roosters. So if r is the numner of roosters, then $4r$ is the number of hen. All together there are 45 chickens, so we can write:

$r + 4r = 45$
$r = 9$
So there are 9 roosters and 36 hens.

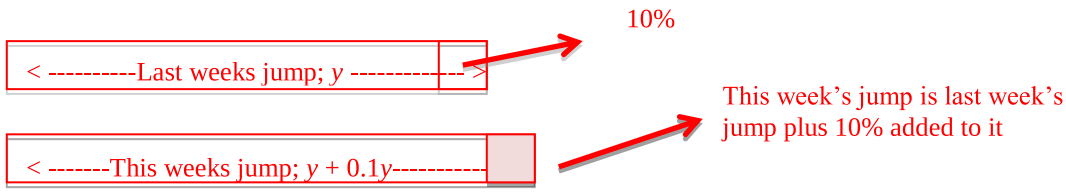
For each context, draw a model, write an equation, and then write a complete sentence to answer the question in the context.

1. The blue jar has 27 more coins than the red jar. If there are a total of 193 coins, how many coins are in each jar? *r* = number of coins in red jar *r* + 27 = number of coins in blue jar $193 = r + (r + 27)$ $r = 83$ There are 83 coins in the red jar and 110 coins in the blue jar	2. Leah's age is twice her cousin's age. If they add their ages together, they get 36. How old are Leah and her cousin?
3. There are a total of 127 cars and trucks on a lot . If there are four more than twice the number of trucks than cars, how many cars and trucks are on the lot? There are more trucks than cars on the lot. So, *c* = cars 2*c* + 4 = trucks $c + (2c + 4) = 127$ $c = 41$; there are 41 cars and 86 trucks.	4. Art's long jump was 3 inches longer than Bill's. Together they jumped 20 feet. How far did they each jump?
5. Juliana and Maria logged a total of 90 hours working for their dad. Juliana worked 2 hours more than three times as much as Maria. How many hours did Juliana work?	6. There are 17 more boys than girls at Juab Junior High School. If there are a total of 981 students there, how many students are boys?
7. A total of 81 points were scored at the basketball game between White Horse High School and Bear River High School. If White Horse scored 7 points more than Bear River, how many points did White Horse score? *b* = the number of points Bear River scored *b* + 7 = the number of points White Horse scored $b + (b + 7) = 81$ $b = 37$ White Horse scored 44 points.	8. Pedro spent 57 minutes doing his math and language arts homework. His language arts homework took him twice as long as his math homework. How much time did he spend on his language arts homework?

3.3b Class Activity: Writing Equations to Solve Percent Problems

Do the indicated for each context.

1. Last week, Dirk jumped y inches in the long jump. This week, he increased the length of his jump by 10%. If this week he jumped 143 inches, how far did he jump last week?
   a) Draw a model:



   b) Write an equation to answer the question: $143 = y + 0.10y$ or $143 = 1.1y$

   c) Solve for the unknown: $y = 130$

   d) Write a sentence stating the answer to the question: Dirk jumped 130 inches last week.

2. Consider these two contexts, write an equation for each and then solve:

Luis wants to buy a new smart phone. Because he's had another phone with the carrier, he will get an 18.25% discount. If the phone he wants is normally \$325.50, how much will he pay?	Mica bought a new smart phone for \$258.13. That was the price after his 18.25% discount. What was the original price of the phone?

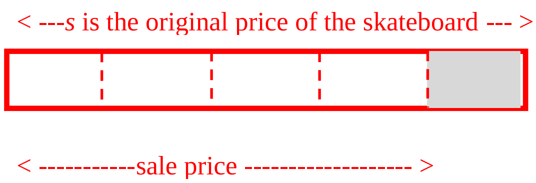
How are the equations similar? How are they different?

3. Consider the three equations below. Write a context for each and then state the answer to your context question.

a. $x - 0.28x = 252$ $x = 350$ The goal here is understanding structure of percent equations. e.g. understanding 28% off an amount is the same as 72% of the amount. Further it is not the same as 28% of the amount.	b. $0.72x = 252$ $x = 350$	c. $0.28x = 252$ $x = 900$

4. Drake wants to buy a new skateboard with original price of *s* dollars. The skateboard is on sale for 20% off the regular price; the sale price is $109.60.

   a. Draw a model, write an equation, and solve it to find the original price of the skateboard.

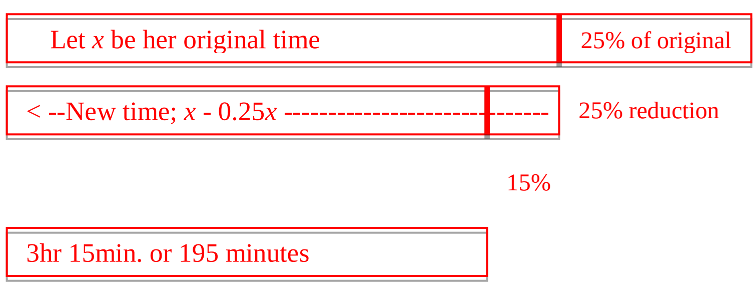


   $s - 0.2s = 109.60$ or $0.8s = 109.60$
   $s = 137$, the original price of the skateboard was $137.

   b. Drake decided to by the skate board for the sale price. When he got to the counter, he got an additional 10% off, but also had to pay 8.75% sales tax. What was the total amount Drake paid for the skateboard?
   Draw a model to help students see that the additional cut of 10% means he paid 90% of $109.60, but then had 8.75% of that amount in tax:
   $0.90(109.60) + 0.0875(0.90(109.60)) = 98.64 + 8.63 = 107.27$

5. Hallie has been training for a marathon for about a year. The first time she ran a marathon she had a really slow time. The next time she ran a full marathon, she cut about 25% off her original time. The third she recorded her time in a marathon, she reduced her time second time by approximately 15%. If newest time is 3 hours and 15 minutes, what was her original time?

   a. Draw a model to represent this situation.



   b. Write an equation to answer the question: $195 = 0.85(0.75x)$
   The original whole (x) is her first time running. The next time is 75% (a 25% reduction) of the original time ($0.75x$ or $x - 0.25x$). The next time it's 85% of the previous time of $0.75x$; e.g. $0.85(0.75x)$.

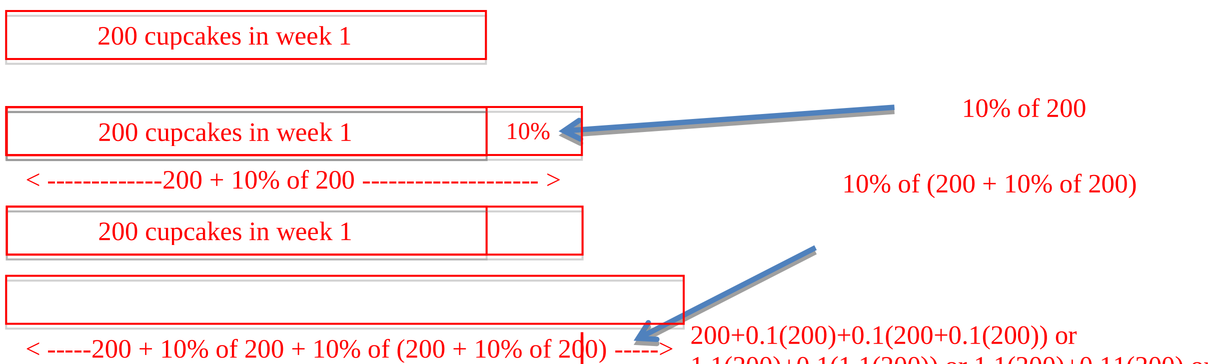
   c. Solve the equation:
   $x \approx 305.882$ this is in minutes so it's 305 minutes and 0.882 of a minute or about 53 seconds

   d. Answer the question in a sentence: Hallie's original time was almost 306 minutes, or about 5 hours, 6 minutes.

6. Alayna sells cupcakes. She currently sells 200 cupcakes a week, but now she's advertising on several local restaurant blogs and expects business to increase by 10% each week. If her business grows 10% from the previous week each week for two weeks in a row, how many cupcakes will she be selling in 2 weeks?

This will take lots of time for students to understand.

   a. Draw a model and write an expression to represent this situation:

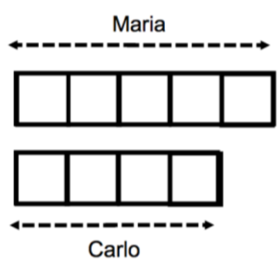


200+0.1(200)+0.1(200+0.1(200)) or 1.1(200)+0.1(1.1(200)) or 1.1(200)+0.11(200) or 1.21(200)

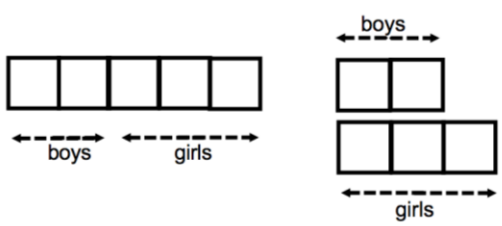
   b. Find the answer to the question. 242 cupcakes

Determine whether each statement below is true or false. Use words, models or an expression/equation to justify your answer. If the statement is false, write a correct statement with the given context.

7. If Maria has 25% more money than Carlo, then Carlo has 25% less money than Maria. False. Carlo has 20% less money than Maria.



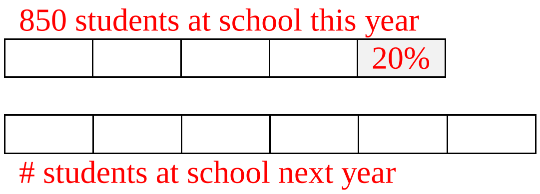
8. If 40% of Ms. Eischeid's class are boys and 60% are girls, then there are 20% more girls than boys in Ms. Eischeid's class. False. There are 50% more girls.



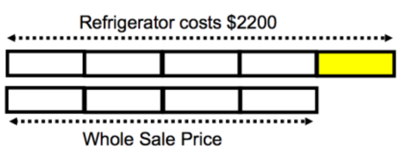
3.3b Homework: Writing Equations to Solve Percent Problems

For each context, write an equation for the context, solve the equation and then answer the question. Draw a model to help you understand the context where necessary.

1. There were 850 students at Fort Herriman Middle School last year. The student population is expected to increase by 20% next year. What will the new population be?

<table>
<tr><td>Draw a model to represent this situation.

</td><td>Write an equation to represent the new population.
$x = 1.2(850)$
Note that #5 and #6 are both “percent problems” but that the variable is in a different position. Compare and contrast the two situations. e.g. similar discussion as #2 in class activity 3.3b.</td></tr>
<tr><td colspan="2">What will the new population be? 1020</td></tr>
</table>

2. A refrigerator at Canyon View Appliances costs $2200. This price is a 25% mark up from the wholesale price. What was the wholesale price?

<table>
<tr><td>Draw a model to represent this situation.

</td><td>Write an equation to represent the whole sale price
$2200 = 1.25x$</td></tr>
<tr><td colspan="2">What was the whole sale price? $1760</td></tr>
</table>

3. Carlos goes to Miller’s Ski Shop to buy a snowboard, because there’s a 30% off sale. When he gets to the store, he gets a coupon for an additional 20% off the sale price. If he paid $252 for the snowboard, what was the original cost?

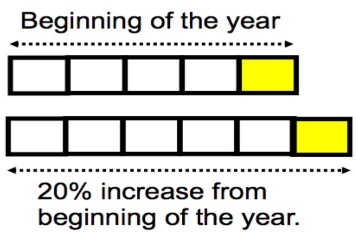
<table>
<tr><td>Draw a model to represent this situation.</td><td>Write an equation to represent the problem situation.</td></tr>
<tr><td colspan="2">What was the original cost of the snowboard?</td></tr>
</table>

For each context in #4 – 6, write an equation, solve it, and write and a complete sentence to answer the question. Draw a model to assist you in understanding the context.

4. Philip took a test and missed 35% of the questions. If he missed 28 questions, how many questions were on the test?

5. Dean took his friend to lunch last week. The service was really good, so he left a tip of 20% of the total bill, the meal and tax. If he spent a total of \$28.50, how much was the meal and tax?
   Let x = price of meal + tax. Then $1.2x = 28.50$; $x = 23.75$; So the meal with tax was \$23.75.

6. The size of Mrs. Garcia's class increased 20% from the beginning of the year. If there are 36 students in her class now, how many students were in her class at the beginning of the year?



x = number of students at the beginning of the year. $x + 0.2x = 36$ so $x = 30$. There were 30 students at the beginning of the year.

7. Write a context for each equation:

$0.40x = 450$	$0.60x = 450$	$x - 0.40x = 450$ $x = 750$ Contexts will vary.

8. Kaylee is training for a marathon. Her training regiment is to run 12 miles on Monday, increase that distance by 25% on Wednesday, and then on Saturday increase the Wednesday distance by 25%. How far will she run on Saturday?

9. Christina would like to give her brother new gloves for his birthday. She has a lot of coupons but is not sure which one to use. Her first coupon is for 50% off of the original price of one item. Normally, she would use this coupon. However, there is a promotion this week and gloves are selling for 35% off the original price. She has a coupon for an additional 20% off the sale price of any item. Which coupon will get her the lower price? She is not allowed to combine the 50% off coupon with the 20% off coupon.

   a. Draw a model to show the two different options.

      50% off coupon: 35% off sale with additional 20% off coupon:

   b. Let x represent the original price of the picture frame. Write two different expressions for each option.

      50% off coupon:

      $x(0.5)$; $x - x(0.5)$; $0.50x$

      35% off sale with additional 20% off coupon:

      $(0.65x)0.8$; $(x - 0.35x)0.8$; $0.52x$

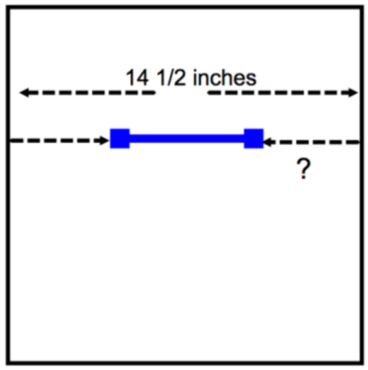
   c. Which coupon will get her the lowest price? Explain how you know your answer is correct.

      The 50% off coupon will get her the lowest price. The combination of the 35% off and then another 20% off means she would be paying 52% of the original price.

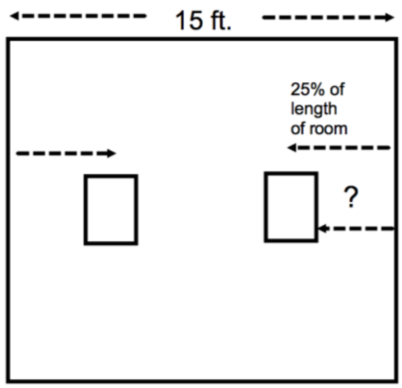
3.3c Class Activity and Homework: Word Problems with Various Forms of Rational Numbers

Use any strategy you want to answer each question. Be able to justify your answers.

1. Randy needs to hang a towel rack on a wall that's 50 ¾ inches wide. The towel rack is 14 ½ inches wide. How far in from the edge of the wall should Randy place the ends of the towel rack if he centers it?



2. Marcela has two 16 ¾ inch wide paintings to hang on her 15 foot wide wall. She wants the paintings to be centered 25% of the way in from the left and right edge of the wall. How many inches from the edge will (a) the hook need to be placed and (b) will the edge of the frame be from the edge of the wall?



3. Bruno earns $43,250 a year. Of this amount, he pays 17.8% to taxes. Of the remainer, 1/3 is for living expenses, 2/5 for food and entertainment, and 1/4 for other insurance and car expenses. What percent of the $43,250 does Bruno have left over for miscellanious expenses? How much money is left over?
17.8% for taxes means he has 82.2% left over: 0.822(43250) = $35,551.50
1/3 + 2/5 + 1/4 = 20/60 + 24/60 + 15/60 = 59/60 and this applies to the whole of $35,551.50. Bruno then has 1/60 of 35551.50 for misc. expenses: (1/60)×35,551.50 = 592.53.
592.53/43250 = 0.0137 so 1.37% of his total pay is available for misc. expenses.

4. There are 200 girls and 300 boys in 7th grade at Alpine Junior High. Cory learns that on Monday one-fifth of the girls and 30% of the boys in 7th grade bought school lunch.

a) How many boys and girls bought school lunch on Monday? 40 girls and 90 boys bought school lunch on Monday.

b) Did 25% of the 7th graders buy school lunch on Monday? Explain. No. A total of 130 students bought school lunch on Monday. This is 26% of the 7th grade students. We cannot "average" 20% and 30% of two different amounts. For a weighted average: $0.2(200) + 0.3(300) = x(500)$; $130 = x(500)$; $x = 0.26$

c) What fraction of the 7th graders bought school lunch? 13/50

5. Nicholas and Martin both have $2000. They both invest in two different mutual funds. Nicholas earns a 25% return on his investment the first year but then looses 25% of the new amount the next year. Martin looses 25% on his investment the first year, but then earns 25% on his new amount in the next year. At the end of two years, how much will each have? They both have the same amount of money, $1,875. Students will likely think they should both have $2000 again. Challenge that assumption and ask them to explain why both ended with less than they started. Also, ask what percent of their original investment do they now have; they have 93.75% of the original amount.
Would they have different ending amounts if they both started with $5000 or both strated with $35?
NO. For Nicholas we have 0.75(1.25(2000)) and with Martin we have 1.25(0.75(2000)). For any starting amount x, we have $0.75(1.25x)$ and $1.25(0.75x)$, both equal to $0.9375x$. We might say, by the commutative property, the amounts will be the same no matter what the starting amount is OR we might say that no matter the starting amount, x, we will have 93.75% of the original amount.
Suppose Martin lost 25% on his investment the first year. What percent increase of his new amount would Martin have to earn in order to be a back to his original $2000 investment?
$0.75(2000)x = 2000$; $x = 1.\overline{3}$; thus, Martin would need to make 33 1/3 % on his new amount to get back to his original $2000 investment.

6. Mario is taking College Algebra at the University of Utah. This final grade is weighted as follows:
14% Homework
18% Midterm 1
18% Midterm 2
18% Midterm 3
32% Final

If Mario earned the following grades so far:
92% Homework
83% Midterm 1
79% Midterm 2
91% Midterm 3

Mario earned 92% of the homework points which make up 14% of the overall grade: $0.92 \times 0.14 = 0.1288$.
Similarly, Midterm 1: $0.83 \times 0.18 = 0.1494$
Midterm 2: $0.79 \times 0.18 = 0.1422$
Midterm 3: $0.91 \times 0.18 = 0.1638$.
All together, this makes 0.5842 or 58.42% of the total points possible. Therefore we have the equation $0.5842 + x \cdot 0.32 = 0.90$ (for A–). Solving for x:
$0.32x = 0.90 - 0.5842 = 0.3158$. Dividing by 0.32,
$x = 0.9869$ or 98.7% (rounds to 99%).

What is the lowest score Mario needs to earn on the final to earn an A– in the course if the cutoff for an A– is 90%? (Note, all scores are rounded to the nearest whole number.)

7. Wayne and Tino are both selling lemonade. Wayne's lemonade is 70% water while Tino's is 80% water.

a. If Wayne has one gallon of lemonade and Tino has two gallons of lemonade and they mix it all together, what percent of the new mixture will be water?

$0.70(1) + 0.80(2) = x(3)$; $x = 0.7\overline{6}$; or 76 2/3%

b. How much lemonade would Wayne need to add to Tino's two gallons if they wanted the lemonade to contain 75% water?

One way students might reason: the amounts of lemonade need to be the same if we want to average 70% and 80%. Therefore, Wayne would have to contribute two gallons like Tino did. Here is a possible equation: $0.70x + 0.80(2) = 0.75(x + 2)$; $x = 2$.

8. A man owned 19 cars. After his death, his three children wanted to follow the instructions of his will which said: The oldest child will receive 1/2 my cars, the second child will receive 1/4 of my cars and the youngest will receive 1/5 of my cars. The three children didn't know what to do because there was no way to follow the will without cutting up cars. Not wanting to destroy any of the cars, they decided to ask their aunt (their father's sister) for help. She said she'd loan them one of her cars, that way they'd have 20 cars; 10 would go to the oldest, 5 to the second child and 4 to the youngest. Once they distributed the 19 cars, they could then return the car she lent them. Is this a good solution to the problem?

The above was adapted from http://mathafou.free.fr/pba_en/pb003.html *"Aegyptian fractions"*

Answers will vary.

9. Kevin is trying to understand percents, but often gets confused. He knows that in his school 55% of the students are girls and 45% are boys. It seems to him that there are 10% more girls than boys in his school. Is he correct? Explain.

No he is not correct. Suggest that student start by seeing what happens if there are 200 students in the school. In that case, there will be 90 boys and 110 girls; 20 more girls than boys. Twenty is $22.\overline{2}$% of 90, thus there are $22.\overline{2}$% more girls than boys. Now encourage students to generalize. There is no need to know how many students are in the school. We know that 55 is $122.\overline{2}$% of 45, thus the change is an increase of $22.\overline{2}$%.

Percents are relative to some whole. In the context, "55% of the students are girls," we mean 55% out of the *total* school population. Kevin was interested in looking at the portion of girls relative to *boys* NOT the total school population.

10. Ana Maria says 3/4 of the students in her class like rap music. Marco says that is 3/4% of the class. Is Marco correct?

No. ¾ of the class is the same as 75% of the class. ¾% is less than 1% of the class.

3.3d Self-Assessment: Section 3.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Use variables to create equations that model word problems. [1]	I struggle to begin writing an equation that models a word problem.	I can draw a model that represents a word problem. I struggle to use that model to create an equation.	I can write an equation that represents a word problem if I draw a model first.	I can write an equation that models a word problem.
2. Solve word problems leading to linear equations. [1]	I struggle to solve word problems leading to linear equations.	I can usually write an equation to solve a word problem leading to a linear equation, but I struggle using that equation to get a solution.	I can solve word problems leading to linear equations.	I can solve word problems leading to linear equations. I can explain the solution in context.
3. Recognize and explain the meaning of a given equation and its component parts when using percents. [2]	I struggle to understand the meaning of parts of a given equation when using percents.	I can recognize the different parts of given equation and match them with their meaning when using percents.	I can recognize the different parts of a given equation when using percents and can explain their meaning in my own words.	I can explain the meaning of a given equation and its parts in my own words. I can also write an equation given a context.
4. Solve multi-step real-life percent problems involving calculations with positive and negative rational numbers in a variety of forms. [3]	I struggle to solve real-life percent problems involving positive and negative rational numbers.	I can usually write an equation to solve a real-life percent problem, but I struggle using that equation to get a solution.	I can solve real-life percent problems involving positive and negative rational numbers.	I can solve multi-step real-life percent problems involving positive and negative rational numbers. I can explain the solution in context.
5. Convert between forms of a rational number to simplify calculations or communicate solutions meaningfully. [3c]	I struggle to convert between all forms of a rational number.	I can convert between forms of a rational number, but I struggle to know how to use that to solve equations or simplify calculations.	I can convert between forms of a rational number to simplify calculations, solve equations, and communicate solutions meaningfully.	I can convert between forms of a rational number to simplify calculations, solve equations, and communicate solutions meaningfully. I can explain why a particular form communicates the solution meaningfully.

Sample Problems for Section 3.3

1. Write an equation to represent each of the following word problems. Solve each problem. Express answers as fractions or decimals when appropriate.
   a. Chloe has twice as many cats as her sister has dogs. Her brother has two turtles. Together, they have five pets. How many of each pet do they have?

   b. Brian is three times older than Sydney. The sum of the ages of Brian and Sydney is eight. How old is Brian?

   c. Samantha is 8 and two-third years older than Jason. The sum of their ages is 23 years. How old is Jason?

2. Explain the meaning of the following equation that matches the given situation:

   I go to a department store with a coupon for 30% off any one item. The pants that I want are on sale for 40% off. What was the original price if I pay $21?

   $0.70(x - 0.40x) = 21$

3. Write an equation to represent each of the following real-life percent problems. Solve each problem. Express answers as fractions or decimals when appropriate.
   a. Victor is saving for retirement. He invested $500 plus the money his company invested for him. He earned 4% interest per year and had $780 at the end of one year. How much did his company invest for him?

   b. William and his family eat at a fancy restaurant that automatically charges 18% gratuity (tip). If his bill total is $53.02, how much was the bill before gratuity was included?

   c. Felipe saved some money in a CD with a rate of 1.5% per year. Ellie saved the same amount of money in a different CD with a rate of 0.75% interest per year. David saved $\frac{2}{5}$ the same amount of money in a box under his bed (no interest under there). If they had $969 total after one year, how much did Felipe save to start?