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Chapter 4 Analyze Proportional Relationships and Use Them to Solve Real-World Problems (6 Weeks)

Common Core Standard(s)

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour*. 7.RP.1

2. Recognize and represent proportional relationships between quantities. 7.RP.2
   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. 7.RP.2a
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. 7.RP.2b
   c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*. 7.RP.2c
   d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. 7.RP.2d

3. Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. 7.RP.3

CHAPTER OVERVIEW:

This chapter focuses on extending understanding of ratio to the development of an understanding of proportionality in order to solve one-step and multi-step problems. The chapter begins by reviewing ideas from 6^{th} grade as well as 7^{th} grade chapters 1 – 3 and transitioning students to algebraic representations. Students will rely on knowledge developed in previous chapters and grades in finding unit rates, proportional constants, comparing rates and situations in multiple forms, writing expressions and equations, and analyzing tables and graphs. The goal is that students develop a flexible understanding of different representations of ratio and proportion to solve a variety of problems.

One important thing for teachers to note (and help students understand) is that a ratio can be written in many different ways, including part:part and part:whole relationships. Fractions represent a part:whole relationship.

VOCABULARY:

Bar/Tape model, comparison model, constant of proportionality (proportional constant), equation, part-to-part ratio, part-to-whole ratio, percent change, proportional relationship, proportion, rate, table, ratio, and unit rate.

CONNECTIONS TO CONTENT:

Prior Knowledge

Students should be able to draw models of part-to-part and part-to-whole relationships. From these models, students should be able to operate fluently with fractions and decimals, especially reducing fractions, representing division as a fraction, convert between mixed numbers and improper fractions, and solve percent problems. Many of these concepts were addressed in Chapters 1 and 3 and will be reviewed briefly in this chapter. Students worked extensively on ratio in 6^{th} grade where they learned to write ratios as fractions, using a colon, or using words.

At the beginning of this chapter, it is assumed that students can use models to solve percent, fraction and whole number ratio problems with models. Students will connect ratio and proportional thinking to a variety of multi-step problems.

Future Knowledge

A strong foundation in proportion is key to success throughout traditional middle and high school mathematics. Further, it is essential in Trigonometry and Calculus. In the next chapter, students will use proportions as a basis for understanding scaling. In 8^{th} grade, proportions form the basis for understanding the concept of constant rate of change (slope). Also in 8^{th} grade students will finalize their understanding of linear relationships and linear functions; proportional relationships studied in this chapter are a subset of these relationships. Later, in secondary math, students will solve rational equations (such as $\frac{x}{x+2} = \frac{3}{x^2-4}$), apply ratio and proportion to similarity and then to trigonometric relationships, and learn about change that is not linear.

MATHEMATICAL PRACTICE STANDARDS IN CHAPTER 4 CONTENT

	Make sense of problems and persevere in solving them.	Students use a variety of representations (tape and comparison models, tables, graphs and equations) for working with multi-step problems without a clear path to a solution or containing inconvenient numbers. The variety of tools used in this chapter should provide students with multiple entry points to situations. They will work to connect the different representations to each other and the context. Students should reflect on the reasonableness of their answer.
	Reason abstractly and quantitatively.	Students will reason abstractly throughout the chapter particularly when they begin to write proportion equations and solve them algebraically. Students should be encouraged to think about the relationship between equations, tables and graphs and the contexts they represent. Students should also be encouraged to reason quantitatively throughout the chapter with the emphasis on units. Students should note both vertical and horizontal patterns and relationships in tables and then how those relationships are expressed in the graph and the proportional constant $\left(\frac{y}{x}\right)$. When an answer is obtained, students should be able to state what the number represents. When students state a unit rate they should be able to give the units represented therein.
	Construct viable arguments and critique the reasoning of others.	Students should construct arguments and critique those of others throughout. With both, students should refer to representations of ideas and/or arithmetic properties in the construction of their arguments. Arguments should be made orally and in writing.
	Model with mathematics.	As students study proportional relationships, they begin to see examples in their lives: traveling on the freeway at a constant rate, swimming laps, scaling a recipe, shopping, etc. Most proportional relationships from the real-world can be modeled using the representations students learn in this chapter. Appropriate models include bar/tape models (either part-part-whole or comparison) or other similar representations, tables, graphs, or equations. Models selected should help to reveal relationships and structure. Further, students should be able to connect different representations to their algebraic representation.
	Attend to precision.	Students should be encouraged throughout the chapter to express unit rates exactly with appropriate units of measure by using fractions and mixed numbers rather than rounded decimals. When graphing or creating tables, labeling is crucial as it is when making an argument. Vocabulary should also be used appropriately in communicating ideas. When setting up a proportion equations precision must be accounted for when using units to dictate where to place given quantities.

	Look for and make use of structure.	Students should make use of structure as they identify the unit rate and/or the constant of proportionality from tables by looking at the relationship of values both vertically and horizontally. Additionally, students will discover that graphs of proportional relationships cross through the origin and are linear. When examining proportional relationships, students should note the structure of the equation and how it relates to the table and the steepness of the line and in turn examine how structure plays a role in setting up a proportion equation. Lastly, students should solidify their understanding of the difference between ratio and fraction in this chapter. This is an important distinction and one that will allow them to think more flexibly with either.
	Use appropriate tools strategically.	Students continue their transition in this chapter to more abstract representations of ideas. As such, students will be working towards using symbolic representation of ideas rather than models exclusively. This transition will occur at different rates for different students. Students should be encouraged to use mental math strategically throughout. For example, if students are given a rate of 3 pounds per $2, there is no need for a calculator to determine that the unit rate is 3/2 pounds per $1 or $0.67 per pound. Tables, graphs, equations, models, and the proportion equation are all tools students can use to solve real-world and mathematical problems.
	Look for and express regularity in repeated reasoning.	Students should note repeated reasoning in tables and graphs throughout the chapter. They will also use repeated reasoning to uncover the unit rate associated with percentages of increase and decrease and in the relationship of ratios and fractions.

4.0 Anchor Problem: Tasting Lemonade

You want to sell lemonade by the park. You have found several different recipes for making lemonade:

Recipe A:
2 cups concentrate,
3 cups water

Recipe B:
4 cups water mixes
with 1 cup concentrate

Recipe C:
$\frac{3}{5}$ cups concentrate,
1 cup water

Recipe D:
$\frac{1}{2}$ cups concentrate,
2 cups water

Recipe E:
8 cups water for 4
cups of concentrate

1. You want to sell the lemonade that tastes the most "lemony." Which recipe will be the most "lemony"? Order the recipes from least lemony to most lemony. Use multiple strategies to solve this problem and explain the strategies you used.

2. If you are planning to use 10 cups of water, how many cups of concentrate would you need for each recipe? Use multiple strategies to solve this problem and explain the strategies you used.

3. If you need a total of 50 cups of lemonade, how much water and concentrate should you use according to each recipe?

Section 4.1: Understand and Apply Unit Rates

Section Overview:

The purpose of this section is to extend a student's understanding of the concept of unit rate and constant of proportionality. This includes finding unit rates from contexts and tables. Students will visualize ratios with like units (i.e. girls to boys) and unlike units (i.e. miles per hour) using models—both part-part-whole and comparison tape models, with particular attention to ratios with unlike units as they begin to see a rate as a unit in and of itself. Students will identify the unit rate for *both* quantities(i.e. miles per hour *and* hours per mile). Students should then be able to move away from using models and be able to recognize the reciprocal relationship of the two unit rates to solve problems.

Tables will be introduced as students begin to look at a collection of ratios as a proportional relationship. Student will identify patterns and structures within the tables including the proportional constant. They will see that the proportional constant is the same value as the unit rate. In later sections students will connect the ratios in tables to graphs and equations.

Along the way, students will practice operations with rational numbers, with particular attention paid to precision with division problems containing fractions. Students should be encouraged to find exact unit rates; i.e. they should be encouraged to use fractions to divide numbers, rather than using long division or a calculator and getting decimal approximations. Students should understand that using a rounded unit rate will give imprecise results when finding missing quantities.

Domain(s) and Standard(s) from CCSS-M

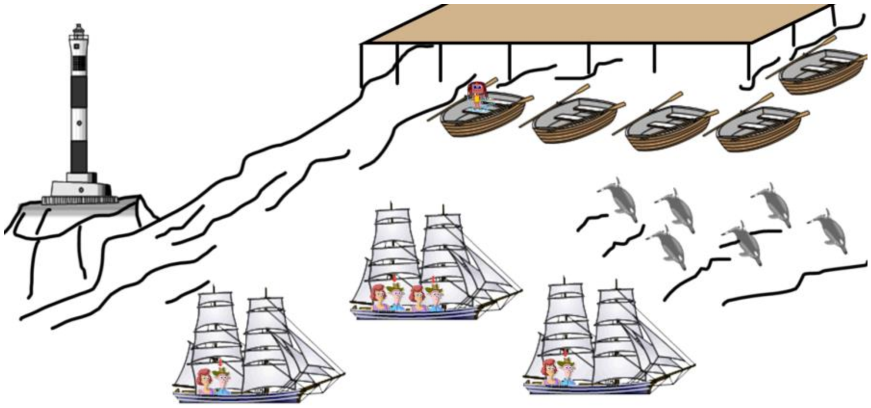
1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour*. 7.RP.1

Primary Concepts and Skills to Master in this Section:

By the end of this section, students should be able to:

1. Compute a unit rate from a context.
2. Determine the unit rate and constant of proportionality from a table of values.
3. Compare two rates to determine equivalence or to contrast differences.
4. Find the unit rate for BOTH quantities (i.e. miles per hour and hours per mile).
5. Use unit rate to find a missing quantity.
6. Determine if a set of ratios form a proportional relationship.

4.1a Class Activity: Equivalent Ratios, Fractions, and Percents (Review from 6th grade)



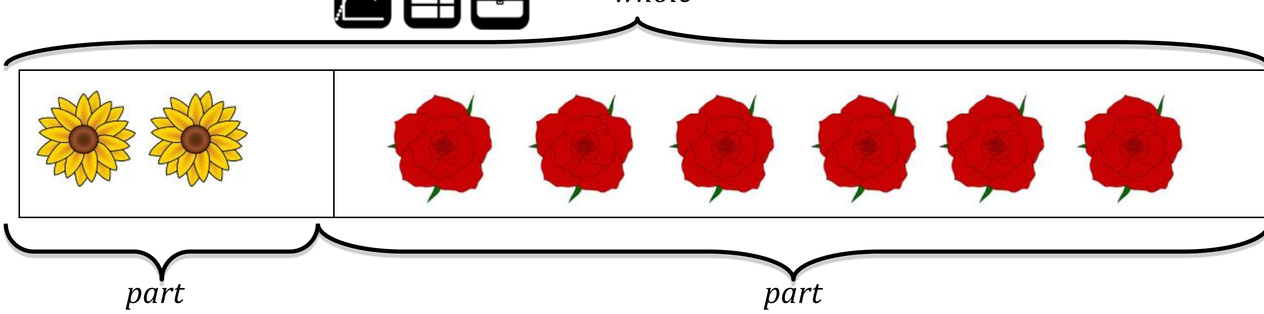
What is a ratio? A ratio expresses a numerical relation between two quantities. Quantities can be compared in part:part or part:whole ratios (examples given below). This section will review this concept using models and connect to fractions, percents, and rates.

In the space below, write as many ratios as you can find in the picture shown above. Be sure to label what you are comparing. Examples: 1:2 (1 sailboat to 2 dolphins), 3:5 (3 sailboats to 5 row boats), 1:6 (1 lighthouse to 6 dolphins). One possible example of a part:part ratio is $\frac{3\text{ sail boats}}{5\text{ row boats}}$. A possible example of a part:whole ratio is $\frac{3\text{ sail boats}}{8\text{ total boats}}$. Labeling quantities is very important. See student responses for other possibile answers.

Review Reducing Ratios and Relating Ratios to Fractions and Percents This is the first time in the 7th grade book that students work with part-to-part ratios. We will start by modeling ratios with *part + part = whole* tape models to help students see how ideas with ratio are related to fractions and percents. Tape models are usually used when the ratio has like units (i.e. girls to boys). In Example 3, we will also use a comparison model, which is usually used with ratios that have unlike units (i.e. gallons per mile).

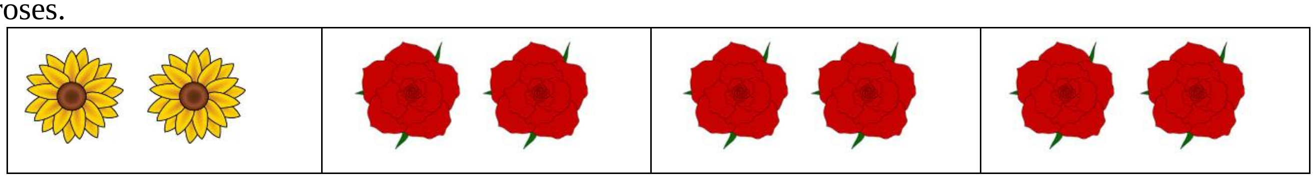
Example 1: Use a **tape or bar model** to find ratios that are equivalent to the ratio 2 sunflowers to 6 roses. Write each ratio from the model as a fraction. This example will be revisited later in this chapter. It shows us how models are a tool that we can use to solve problems. The structure within the tape model reveals how the quantities are related.

2 sunflowers to 6 roses

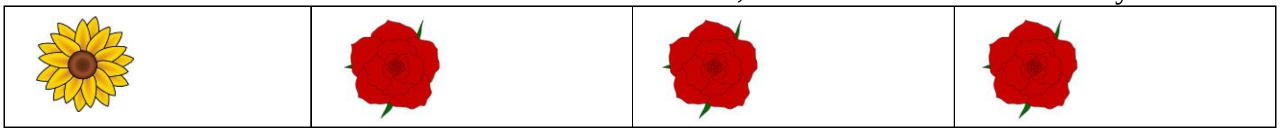


$\frac{2\text{ sunflowers}}{6\text{ roses}}$

We can place two flowers into each of four groups; this reveals 1 (group of 2) sunflowers to 3 (groups of 2) roses.



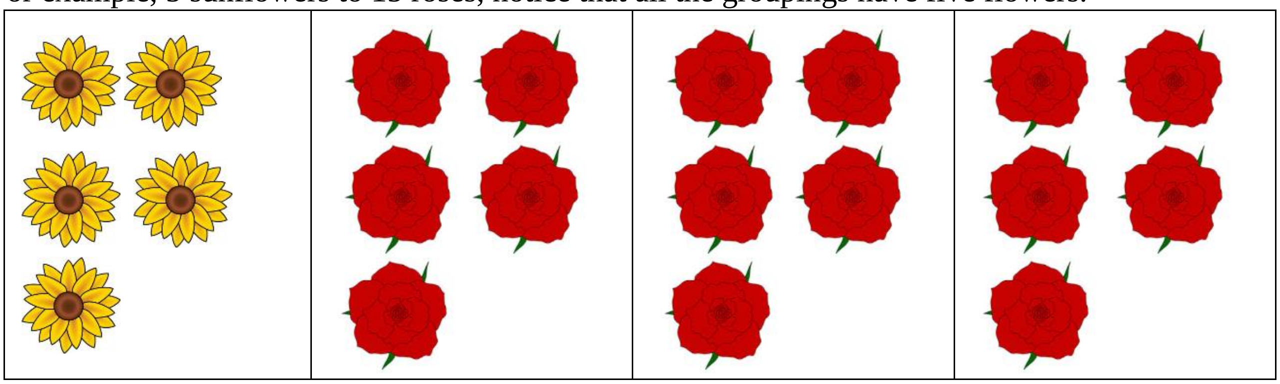
The reduced ratio is 1 sunflower to 3 roses. In other words, there is 1 sunflower for every 3 roses.



$\frac{\text{1 sunflower}}{\text{3 roses}}$

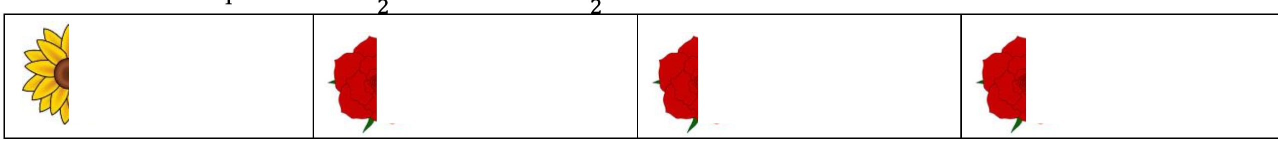
This is a tape model that shows the ratio 2:6 where we put the same number of flowers in each group. We can use the GCF(Greatest Common Factor) of the two given quantities to determine the number of items that go into each box/group. Then we can "reduce" by eliminating the same number of items from each box or "increase" by adding the same number of items into each box (as shown below). Be careful, adding one to each box of the 1:3 ratio gives us a ratio of 4:12 which is equivalent. This is NOT the same as adding one to each term, e.g. 1:3 is not equal to 1+1:3+1 or 2:5.

We can write other equivalent ratios by increasing the number of flowers in each group as long as all the groups have the same number of flowers.
For example, 5 sunflowers to 15 roses, notice that all the groupings have five flowers.



$\frac{\text{5 sunflowers}}{\text{15 roses}}$

The ratio is also equivalent to $\frac{1}{2}$ sunflower to $1\frac{1}{2}$ roses.



$\frac{\frac{1}{2}\text{ sunflowers}}{1\frac{1}{2}\text{ roses}}$

List all the ratios that you have found with the models.

$\frac{\text{2 sunflowers}}{\text{6 roses}}$, $\frac{\text{1 sunflower}}{\text{3 roses}}$, $\frac{\frac{1}{2}\text{sunflowers}}{1\frac{1}{2}\text{roses}}$, $\frac{\text{5 sunflowers}}{\text{15 roses}}$

Notice that these ratios all reduce to $\frac{\text{1 sunflower}}{\text{3 roses}}$. Recall that to create equivalent ratios, multiply the numerator and denominator by the same number.

This is a collection of equivalent ratios. When you have a collection of pairs of values in equivalent ratios it is called a **proportional relationship.** Another way of saying this is that equivalent ratios are **proportional** to one another.

1. Find two other ratios that are proportional to the ratios above.
   Answers can include any ratio equivalent to $\frac{1}{3}$.
2. A vase holds 6 sunflowers and 18 roses. Is this grouping proportional to the ratio of sunflowers to roses discussed above?
   This ratio is proportional. You can show this by scaling the model or writing it as a fraction and simplifying.
3. Another vase holds 9 sunflowers and 21 roses. Is this grouping proportional to the ratio of sunflowers to roses discussed above?
   This ratio is not proportional. Students can use the model to answer this. Additionally, when written as a fraction it does not simplify to an equivalent ratio.

The word **percent** means "per 100". Percents are a way to write a ratio as a part-to-whole, where the whole is 100. In order to find a percent you must write a ratio as part to whole. See section 1.2 and 1.3 for a review of percents.

Example 2: What percent of the flowers in Example 1 are sunflowers?

There are a total of 4 parts (1 part sunflowers and 3 parts roses) and sunflowers are 1 part of *all* 4 flowers. Written as a fraction this is $\frac{1}{4} = 25\%$. This means that the ratio 1 sunflower to 4 total flowers is equivalent to 25 out of 100.

Directions: Use a model to answer each question.

Students should be analyzing these situations more abstractly at this point. Models are important in making that leap.

1. The high school has a new ski club this year. In the club there are 4 girls and 16 boys.
   a. Use a model to find the simplest ratio of girls to boys in the ski club.

g g g g	b b b b b b b b b b b b b b b b

g g g g	b b b b	b b b b	b b b b	b b b b

g	b	b	b	b

   1 girl:4 boys or 1 girl to 4 boys

   b. What percent of the students in the ski club are girls?
   $\frac{1}{5} = 20\%$

   c. If there are 2 girls, how many boys are there?
   8 boys; one way to solve is to use the model and replace each box in the bottom model with 2; also notice that $\frac{2}{8} = \frac{1}{4}$

   d. Use the model to find another equivalent ratio.
   Answers will include any ratio equivalent to 1:4

   e. In the drama club, there are 3 girls and 14 boys. Is the ratio of girls:boys in the drama club the same as the ratio of girls:boys in the ski club?
   No, using the model, replace each box in the bottom model with 3, the boys would be 12. Fractions do not reduce to the same fraction. For the ratio $\frac{1}{4}$ if you multiply the numerator by 3, you would also have to multiply the denominator by 3 which would yield 12, not 14.

2. A cat has given birth to a litter of kittens. There are 3 black cats and 6 striped cats.
   a. Use a model to find the simplest ratio of black cats to striped cats.

b b b	s s s s s s

b b b	s s s	s s s

b	s	s

You do not have to draw all three models separately. You can simply draw the first model and then subdivide it into the parts shown below and then simplify by crossing off the same number of items in each box. The models are drawn separately here to show the different steps.

1 black kitten to 2 striped kittens or 1:2 black to striped cats.

   b. Write three equivalent ratios. How do you know they are equivalent?
   $\frac{4 \text{ black}}{8 \text{ striped}}$; 10:20; 5 to 10
   c. What fraction of the kittens are striped?
   $\frac{2}{3}$ of the kittens are striped.
   d. What percentage of kittens in the litter are black?
   $\frac{1}{3} \approx 33.3\%$
   e. Is the percentage of kittens in the litter that are black a part-to-part or part-to-whole ratio? Explain your answer.
   It is a part-to-whole relationship because you are comparing the number of black kittens to the total number of kittens.

3. There are 16 girls and 12 boys in Ms. Garcia's class.
   a. What is the simplest ratio of girls to boys in Ms. Garcia's class?

16 girls	12 boys

4 girls	4 girls	4 girls	4 girls	4 boys	4 boys	4 boys

1 girl	1 girl	1 girl	1 girl	1 boy	1 boy	1 boy

4 girls to 3 boys or 4:3 girls to boys

   b. Is the ratio of girls to boys in Ms. Garcia's class a part-to-part or part-to-whole ratio? Explain your answer.
   It is a part-to-part relationship because you are comparing the girls to boys or the parts of the class.

   c. What is the ratio of girls to students in her class?
   4 girls to 7 students

   d. Approximately what percent of the students in Ms. Garcia's class are boys?
   Approximately 43%

   e. What fractional portion of the students in Ms. Garcia's class are boys?
   $\frac{3}{7}$

f. Is the fractional portion of the students in Ms. Garcia's class that are boys a part-to-part ratio or a part-to-whole ratio? Justify your answer.
It is a part-to-whole relationship because you are comparing the number of boys to the total number of kids in the class.

g. Suppose Ms. Garcia had a huge class of 63 students at the same ratio as above. How many students would you expect to be boys and girls?

9 girls	9 girls	9 girls	9 girls	9 boys	9 boys	9 boys

36 girls and 27 boys.

h. In Mr. Lloyd's class, there are 10 girls and 12 boys. Is the ratio of girls to boys the same as in the other two classes? Explain. No, if we make the boy boxes in the model equal to 12, each box would be 4; therefore there would be 16 girls.

4. It takes you 3 hours to read 69 pages in your novel.

a. Create a model to find how many pages per hour you read?

b. If you maintain the same rate, how many pages can you read in 9 hours?

c. How many pages can you read in 2 hours?

Equivalent ratios can also be shown using a **comparison model**. A comparison model is helpful when the units are not the same.

Example 3: The comparison model for Example 4 above is shown.

It takes you 3 hours to read 69 pages in your novel.

69 pages
3 hours

To find the number of pages that you can read in one hour, divide the model into three one-hour pieces.

23 pages

$\frac{69}{3} = 23$ pages	$\frac{69}{3} = 23$ pages	$\frac{69}{3} = 23$ pages
1 hour	1 hour	1 hour

1 hour

Thus you can read 23 pages per hour.

The structure of the models reveal how a unit group iterates; e.g. here the ratio is 69 pages to 3 hours, a one-third iteration (or dividing each by 3) gives $69/3 = 23$ pages to $3/3 = 1$ hour. If we iterate the ratio 3 times we get 207 pages to 9 hours.

To find how many pages you can read in 9 hours, iterate the model 3 times.

207 pages

$\frac{69}{3} = 23$ pages	$\frac{69}{3} = 23$ pages	$\frac{69}{3} = 23$ pages	$\frac{69}{3} = 23$ pages	$\frac{69}{3} = 23$ pages	$\frac{69}{3} = 23$ pages	$\frac{69}{3} = 23$ pages	$\frac{69}{3} = 23$ pages	$\frac{69}{3} = 23$ pages
1 hour	1 hour	1 hour	1 hour	1 hour	1 hour	1 hour	1 hour	1 hour

9 hours

Thus you can read 207 pages in 9 hours.

Finally, the model below shows the number of pages you can read in 2 hours.

46 pages

$\frac{69}{3} = 23$ pages	$\frac{69}{3} = 23$ pages	$\frac{69}{3} = 23$ pages
1 hour	1 hour	1 hour

2 hours

Thus you can read 46 pages in 2 hours.

Students may want to know when to use a part-part-whole model and when to use a comparison model. Both are valid models and either can be used in most situations. However a tape model usually works best when you are comparing like units and a comparison model usually works best when you are comparing unlike units. Note: the goal is to transition to an algebraic representation of ideas; models are simply a tool in that transition.

Martin can read 100 pages in 4 hours. Is this proportional to the number of pages that you can read? If you iterate the model 4 times you get 92 pages in 4 hours, so this is not proportional to the number of page you can read. Also if you write the ratio as the fraction $\frac{100}{4}$ and simplify you get $\frac{25}{1}$, this is not equivalent to the ratio of 23 pages per hour.

5. On average Roman's car can go 450 miles on a full tank of gas. His gas tank can hold 15 gallons of gas.
   a. Use a model to determine the number of miles he gets per 1 gallon of gas at this rate.

450 miles
15 gallons

30 miles

30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile
1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal

1 gallon

Roman's car can go 30 miles on 1 gallon of gas.

   b. Use the model to determine how many miles he can go on 8 gallons of gas.

240 gallons

30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile
1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal

8 gallons

Roman's car can go 240 miles on 8 gallons of gas.

   c. Roman is going on a trip where there will not be any gas stations available. He completely fills his tank before he leaves and packs an additional 10 gallons of gas. How many total miles will he be able to travel before he completely runs out of gas?

450 miles

30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile
1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal

15 gallons

300 miles

30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile	30 mile
1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal	1 gal

10 gallons

Roman will be able to go $450 + 300 = 750$ miles.

d. Use the models you have drawn above to determine if driving 120 miles on 4 gallons of gas is proportional to Roman's gas mileage.
If you iterate the model 4 times you get 120 miles on 4 gallon of gas. This is proportional to Roman's gas mileage.
You can also show that the ratios are proportional by writing the ratios as fractions and simplifying.

e. Use the models you have drawn to determine if driving 15 miles on $\frac{1}{2}$ gallon of gas is proportional to Romain's gas mileage.
Yes, it is, cut each box into half.

f. Use the models you have drawn above to determine if driving at rate of 200 miles on 6 gallons of gas is proportional to Roman's gas mileage.
If you iterate the models 6 times you get 180 miles on 6 gallons of gas. This is not proportional to Roman's gas mileage.
You can also show that the ratios are not proportional by writing the ratios as fractions and simplifying.

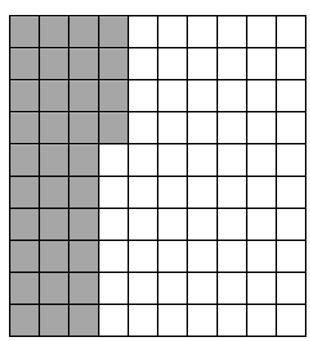
Spiral Review

1. Model $-18 + 6 =$

2. Simplify the following expression. Use a model if needed.
$9m - 3 + 4m$

3. Which number is larger? Justify your answer.
$\mathbf{-6}$ **or** $\mathbf{-4}$

4. Write this model in fraction form then simplify.



4.1a Homework: Equivalent Ratios, Fractions, and Percents (Review from 6th grade)

Directions: For each problem use a model (tape or comparison) to represent the situation. Then answer the questions that follow.

1. In a closet, there are 10 shirts and 15 skirts.
   a. Use a model to find the simplest ratio of shirts to skirts.

10 shirts	15 skirts

5 shirts	5 shirts	5 skirts	5 skirts	5 skirts

1 shirts	1 shirt	1 skirt	1 skirt	1 skirt

   2:3 shirts to skirts

   b. If I have 8 shirts, how many skirts would I need to have to keep the same ratio?
   12

   c. Allison has 4 shirts and 5 skirts in her closet. Is the ratio of shirts to skirts in Allison's closet the same as the ratio above? Explain.
   No, a common error is for students to add the same value to the ratio and think that produces an equivalent ratio. 2:3 add 2 to both is 4:5. Help students to see why this is not equivalent using the model and numerically.

   d. Use the same model to find the percent of the items in the closet that are shirts.
   $\frac{2}{5}$ =40% of the items are shirts.

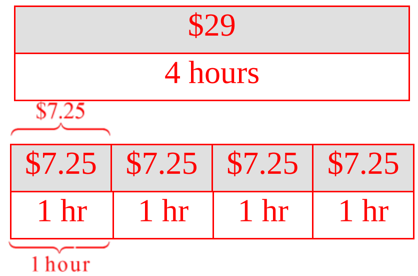
2. In my pocket, I have 8 quarters and 6 dimes.
   a. Use a model to find the simplest ratio of quarters to dimes.

   b. If I have 30 dimes in my pocket, how many quarters would I need to keep the same ratio as above?

   c. Is 32 quarters and 24 dimes an equivalent ratio? Explain.

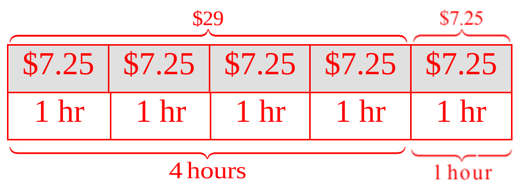
   d. What fractional portion of the coins in my pocket are dimes?

3. Marcus gets paid $29 for 4 hours of work.
   a. Use a model to determine how much Marcus gets paid per hour.



   Marcus makes $7.25 per hour.

   b. How much money will Marcus make if he works 5 hours?



   Marcus will make $36.25

   c. Leo makes $14.25 for 2 hour of work. Do the boys make the same amount per hour? Explain.
   No, If you cut the original model in half you get $14.50 for 2 hours of work.

4. In a banquet hall 19 chairs take up 76 square feet.
   a. How many square feet of space does 1 chair cover?

   b. How many square feet of space do 9 chairs cover?

5. In the toolbox, there are 4 nails and 30 screws.
   a. Use a model to find the simplified ratio of nails to screws.

4 nails	30 screws

2 n	2 n	2 s	2 s	2 s	2 s	2 s	2 s	2 s	2 s	2 s	2 s	2 s	2 s	2 s	2 s	2 s

1 n	1 n	1 s	1 s	1 s	1 s	1 s	1 s	1 s	1 s	1 s	1 s	1 s	1 s	1 s	1 s	1 s

   2:15 nails to screws

   b. If I have 60 screws, how many nails would I need to keep the same ratio?
   8

   c. What fractional portion of the items are nails?
   $\frac{2}{17}$ of the items are nails.

6. In a classroom, there are 12 girls and 8 boys.
   a. Find the simplified ratio of girls to boys. Use a model if needed.

   b. Is a classroom with 6 girls and 5 boys in the same ratio? Explain.

   c. What fraction of the students in the class are girls?

   d. What percent of the class are girls?

7. In the fridge, there are $2\frac{1}{2}$ bananas and 10 apples.
   a. Use a model to find the simplified ratio of apples to bananas.

   b. If I have 5 bananas, how many apples would I need to keep the same ratio?

   c. What percent of the fruit is bananas?

   d. Is the percent of the fruit that are bananas a part-to-part or part-to-whole ratio? Explain your answer.

8. A package of Skittles has 2 red skittles, 4 green skittles, and 4 purple skittles.
   a. Use a model to find the simplified ratio of red : green : purple candies.

2 red	4 green	4 purple

2 red	2 green	2 green	2 purple	2 purple

1 red	1 green	1 green	1 purple	1 purple

   1:2:2 red: green: purple skittles

   b. What percent of the skittles are purple?
   40% purple

   c. If the ratio of red: green: purple is always the same and there are 120 skittles in a package, how many of each color do you expect to have? Draw a model if needed.(Note: 120 divided by 5 is 24)

24 red	24 green	24 green	24 purple	24 purple

   24 red, 48 green and 48 purple

9. The ratio of rabbits to birds at the pet store is 5:2.
   a. Find three equivalent rabbit to bird ratios.

   b. What fraction of the rabbits and birds at this pet store are birds?

10. Char can type 225 words in five minutes.
   a. How many words can she type in 1 minute?

   b. How many words can she type in 7 minutes?

4.1b Class Activity: Equivalent Ratios and Proportional Relationships

Ratios have associated **rates** that compare the way the two quantities change. We can show this change in a model and a table.

In this section we transition from thinking about ratio in bar models to iterating on a table. We will be scaling up and down throughout this section. To start the transition, we will use models. Continue to focus on the structure models reveal about iterating unit groups. We can show this with comparison models, but tables make it even easier.

Example 1:

a. Use a model to determine how much money per hour you make if you make $42.30 in 5 hours. Notice we are using a comparison model here. Both 5 hours and $42.30 are attributes of the same events; e.g. 5 hours and $42.30 are two measures of the same "thing".

$42.30
5 hours

$42.30

$8.46	$8.46	$8.46	$8.46	$8.46
1 hour	1 hour	1 hour	1 hour	1 hour

5 hours

You can use the same idea to scale up or scale down, as seen in parts b and c.

b. How much will you make if you work 10 hours?

$84.60

$8.46	$8.46	$8.46	$8.46	$8.46	$8.46	$8.46	$8.46	$8.46	$8.46
1 hour	1 hour	1 hour	1 hour	1 hour	1 hour	1 hour	1 hour	1 hour	1 hour

10 hours

Thus, in 10 hours you will make $84.60.

c. Use the model to determine how much you would make in 3 hours.

$25.38

$8.46	$8.46	$8.46
1 hour	1 hour	1 hour

3 hours

In 3 hours you will make $25.38.

d. The table below has been filled in with the equivalent ratios found in parts a, b, and c. Find more equivalent ratios by filling in the remaining cells in the table using the rate. Observe and discuss the many patterns that are in table.

Notice the multiplicative structure going across the table (shown in blue). This shows the proportional constant. In the next section we show that the proportional constant has the same value as the unit rate.

The additive structure still holds true as you move down the table. You are just skipping numbers.

Hours	Dollars
1	8.46
2	16.92
3	25.38
4	33.84
5	42.30
8	67.68
10	84.60
15	126.90

(Annotations on the table: •8.46 across rows; +1 down the Hours column; +8.46 down the Dollars column; •2 and •5 down the columns.)

Observe the structure that exists looking down the columns of the table: 1) the additive structure (shown in red) and 2) the multiplicative structure (shown in green). The additive structure helps us to think about the unit rate (discussed in detail in the next lesson) as the amount of increase in your second quantity (output) as your first quantity (input) increases by 1. This can be seen in a table by looking at the change going down the columns.

This table is a collection of equivalent ratios. Recall when you have a collection of pairs of values in equivalent ratios it is called a **proportional relationship.**
You can express the proportional relationship many ways. In this example we have expressed this relationship with a model and in the table above.
In section 4.2 students will express a proportional relationship with a graph and equation and explain the correspondence between all the representations.
A proportional relationship is defined by a constant number that associates the quantities in a proportional relationship. This number is called the **constant of proportionality** or the **proportional constant.** It relates one quantity to another with their quotient.

In the example above the proportional constant is 8.46 because you multiply the number of hours by 8.46 to obtain the number of dollars. It is found by relating the two quantities with the quotient $\frac{dollars}{hours}$. It can be expressed as a rate by saying that you make money at a rate of $8.46 per hour. You can see the proportional constant in the table.

To summarize: A collection of equivalent ratios forms a proportional relationship or are said to be proportional. A proportional relationship is defined by a proportional constant or unit rate. In contrast, a proportion is an equation stating that two ratios are equivalent. Proportions will be addressed in section 4.3.

1. For your family reunion, you are in charge of planning a day at the lake. You want to choose a kayak rental shop near the lake, but you're not sure how many kayaks you'll need or how long you'll need them. You call around to get some prices and find two shops (Paddle Paradise and Floating Oasis) close to the lake. Both shops will charge you for the exact amount of time you have the kayaks, using a constant rate. Below are the rates each shop quoted you:

Paddle Paradise
$92.50 every 5 hours

Floating Oasis
$38 every 2 hours

Draw a model for each kayak shop's rate. Use the model to help you fill out the following tables. Assume that the hours and dollars keep a constant ratio.

$92.50
5 hours

$18.50	$18.50	$18.50	$18.50	$18.50
1 hour	1 hour	1 hour	1 hour	1 hour

$38
2 hours

$19	$19
1 hour	1 hour

Paddle Paradise:

Hours	Dollars
1 (•18.50 →)	18.50
5 (•18.50 →)	92.50
10	185
20	370
100	1850

Floating Oasis:

Hours	Dollars
2 (•19 →)	38
1 (•19 →)	19
10	190
20	380
100	1900

The Floating Oasis table is not "in order" to notice that the proportional constant can still be seen by multiplying it by the input to get the output. Likewise, the additive structure moving down the table corresponds with the difference in the numbers in the first column.

a. What is the proportional constant for each rental shop? Show where you see the proportional constant in the table.
Paddle Paradise: 18.50, Floating Oasis: 19

b. Which company is cheaper? Justify your answer.
Paddle Paradise is cheaper Price per hour for Paddle Paradise is $18.50 and $19 for Floating Oasis.

c. You decide to use Floating Oasis because it is closer to your family's campground. How much would it cost if you rented one kayak for 33 hours?
$627

d. Challenge: If you only had $1, approximately how long would you be able to kayak from either shop?
Approximately $\frac{1}{19}$ of an hour, or about 3 minutes.

e. Challenge: If you budgeted $150 for kayaks, approximately how long would you be able to kayak from either shop?
A little less than 8 hours.

2. A hiker backpacking through the mountains walks at a constant rate. Below is a table with how many miles she walked and the time it took.

Hours	8 •3	2 •3	$\frac{1}{3}$	1
Miles	24	6	1	3

a. Use the table to write as many equivalent ratios as you can.

24 miles:8 hours; 1 mile per $\frac{1}{3}$ hour; the ratio of miles to hours is 3:1; 6 miles to 2 hours

Students may write the ratio with a colon, "to" or as a fraction. Emphasize units, equivalence and that the ratio is a relationship between given quantities.

b. Write the rate of miles per hour in simplest form.

$\frac{3 \text{ miles}}{1 \text{ hour}}$

c. What patterns do you see in the table?

The number of miles is 3 times the number of hours, this is the proportional constant. Whatever number in the 1st column was multiplied by to go down to the next row, all other numbers in the 1st row where also multiplied by that number.e.g. $8 \times 3 = 24$ and $2 \times 3 = 6$. A student may also point out that the number of hours is $\frac{1}{3}$ the number of miles. $24 \times \frac{1}{3} = 8$ and $6 \times \frac{1}{3} = 2$.

d. Paul hiked 12 miles in 16 hours. Is he traveling at the same rate?

No, the ratio 12:16 is not equivalent to the ratio $\frac{3 \text{ miles}}{1 \text{ hour}}$.

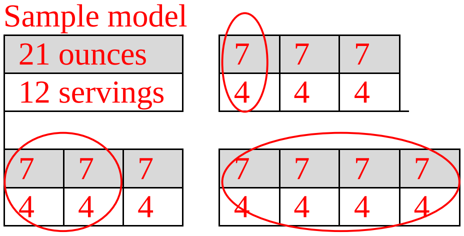
Directions: For each table below one ratio is given. Draw a model to represent the ratio and use the model to scale up or down to find equivalent ratios that form a proportional relationship. Then find the proportional constant.

3.

Servings	Ounces
12	21
4	7
16	28
8	14

Proportional Constant: $\frac{7}{4}$

Sample model



4.

Children	Cars
6	2
	1
	5
30	

Proportional Constant:

5.

Battalions	Soldiers
3	1500
1	500
4	2000
2	1000

Proportional Constant:500

See model

6. **Find, Fix, and Justify:**

This problem allows students to critique Darcy's reasoning and then make their own conjectures about the proportional constant.

The values in the table below represent the lengths of corresponding sides of two similar figures. The side lengths are proportional to one another. Darcy filled in the remaining values in the table and has made a mistake. Find her mistake and fix it by filling in the correct values in the table on the right. Then provide an explanation as to what she did wrong.

Darcy's Work

Figure A side lengths	Figure B side lengths
4	6
5	*7*
8	10
2.5	4.5

Figure A side lengths	Figure B side lengths
4	6
5	15/2
20/3	10
3	4.5

Instead of multiplying the values for Figure A by $\frac{3}{2}$ to get the values for Figure B Darcy added 2 to each value. Likewise she subtracted 2 from the values for Figure B to get the values for Figure A. She also may have added 1 the same number to each ordered pair to obtain the next (i.e. (4, 6) → $(4+1, 6+1) = (5, 7)$ or (4 + 4, 6 + 4) = (8, 10).

Spiral Review

1. Order the following rational numbers from least to greatest. $\frac{12}{3}$, 4.5, $\frac{14}{3}$, 0.94,

2. What is the opposite of 7?

3. What is 35% of 120? Use a bar model.

4. Without using a calculator, determine which fraction is bigger in each pair. Justify your answer with a picture *and* words.

   a. $\frac{5}{7}$ or $\frac{3}{5}$

   b. $\frac{6}{12}$ or $\frac{8}{14}$

   c. $\frac{10}{8}$ or $\frac{12}{10}$

4.1b Homework: Equivalent Ratios and Proportional Relationships

Directions: For each table below one ratio is given. If needed draw a model to represent the ratio, state the proportional constant, and fill in the missing values.

1.

Rose bushes	Square feet of garden
60	40
30	
	100
	1

2.

Scoops of corn	Number of deer fed
1.5	1
	5
	10
	30

3.

Hours	Distance travelled (km)
$\frac{1}{2}$	15
1	30
$\frac{1}{3}$	10
$\frac{10}{3}$	100

Proportional Constant: 30

4.

Hours worked	Lawns mowed
6	8
0.75	1
3.75	5
5.25	7

Proportional Constant: $\frac{4}{3}$

5.

Minutes	Gallons in the pool
90	54
10	6
5	3
$\frac{5}{3} = 1\frac{2}{3}$	1

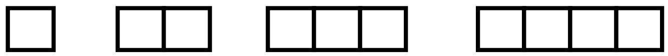
Proportional Constant: $\frac{3}{5}$

6.

Acres	Houses
$\frac{1}{4}$	1
1	
	100
100	

For #4, accept answers as fractions. Talking about the rate as hours/minutes may help us to better comprehend the amount of time, for example "3 hours and 45 minutes." For #5 consider how the context of a problem helps you decide the best representation of the answer. Although the answer of 5/3 minutes is mathematically correct, it would be more natural to say $1\frac{2}{3}$ minutes, or 1 minute and 40 seconds. Discourage students from answering in a repeating decimal, i.e. 1.666666… minutes.

7. Study the block pattern below.



Record the missing information in the table.

Number of Blocks	Perimeter	Area
1		
2		
	8	3
	10	
5		5
6		

a. Determine if the relationship between perimeter and the number of blocks is proportional.

b. Determine if the relationship between the area and the number of blocks is proportional.

8. **Find, Fix, and Justify:**
The table below shows the ratio of green candies to blue candies in a bag. Liam filled in the remaining values in the table and has made a mistake. Find his mistake and fix it by filling in the correct values in the table to the right. Then provide an explanation as to what he did wrong.

Liam's Work

Number of Blue Candies	Number of Green Candies
6	3
3	6
6	12
1	2

Number of Blue Candies	Number of Green Candies
6	3
3	
	12
1	

4.1c Class Activity: Model and Understand Unit Rates

Activity 1: Fill in the missing information for each table and then compare the tables.

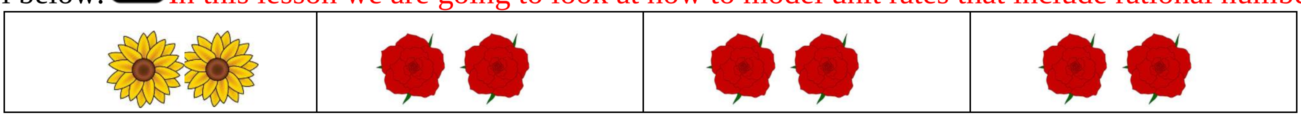
Hours	Miles
48	60
7	8.75

Hours	Miles
1	$1\frac{1}{4}$
7	8.75

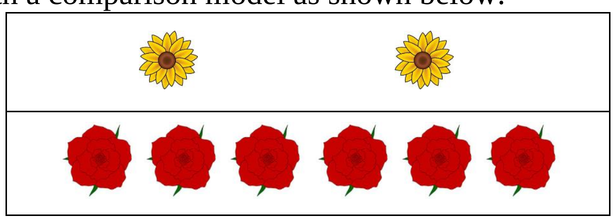
Each table represents the same relationship between miles and hours, i.e. the ratio of miles:hours is 5:4 They have the same proportional constant (5/4). However the second table has a clearer path to finding the missing value. In the second table, if you scale up the time to be 7 times as long, you'll also need to scale the miles to be 7 times as long. This activity shows how finding the unit rate helps to find other equivalent ratios. The row with the 1 hour represents a unit rate for miles per hour.

What do you notice?

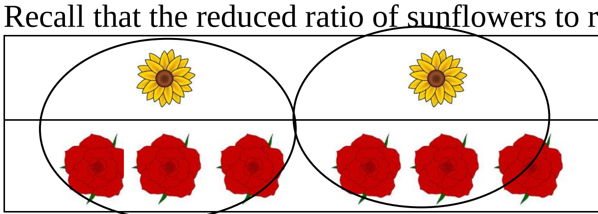
Recall the first example in the previous section. It stated the ratio of sunflowers to roses is 2 to 6 as shown in the model below. In this lesson we are going to look at how to model unit rates that include rational numbers.



We can also model this ratio with a comparison model as shown below.



Recall that the reduced ratio of sunflowers to roses is 1 sunflower to 3 roses.

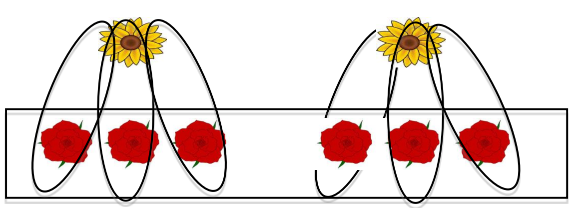


The relationship between sunflowers and roses here is 1 sunflower to three roses, or $\frac{1}{3}$ sunflower to each rose. The ratios 1:3 and $\frac{1}{3}$:1 are equivalent.

What if we wanted to know the ratio of sunflowers to 1 rose?

$$\frac{2 \text{ Sunflowers}}{6 \text{ Roses}} = \frac{1 \text{ Sunflowers}}{3 \text{ Roses}} = \frac{\frac{1}{3} \text{ Sunflower}}{1 \text{ Rose}}$$

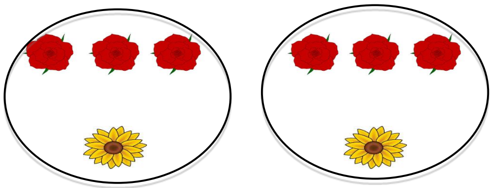
You are also extending the idea of ratio to rate more explicitly from here on out. In other words, the ratio $\frac{1}{3}$:1 for sunflowers to roses tells us that for every increase of 1 rose, there will be a $\frac{1}{3}$ increase in sunflowers. Thus if there are 30 roses, there will be 10 sunflowers.



The ratio of 2 sunflowers to 6 roses has a rate of $\frac{1}{3}$ of a sunflower per 1 rose. Sometimes we emphasize rates with the term unit rate. A **unit rate** is the ratio of the first quantity per one of the second quantity. It is the same value as the **constant of proportionality.** Thus the unit rate of sunflowers to roses is $\frac{1}{3}$ of a sunflower per 1 rose. The term unit rate is the numerical part of the rate; the "unit" is used to highlight the 1 in "per 1 unit of the second quantity."

We can also find the unit rate for roses to sunflowers.

$$\frac{6 \text{ Roses}}{2 \text{ Sunflowers}} = \frac{3 \text{ Roses}}{1 \text{ Sunflowers}}$$



Here, the unit rate is 3 roses per 1 sunflower.

Below are tables that show the **sunflower to roses** ratio and the **roses to sunflowers** ratio. Use each unit rate or the constant of proportionality to fill in the missing information in each table. Then discuss the similarities and differences between the tables.

Roses	Sunflowers
1	$\frac{1}{3}$
2	$\frac{2}{3}$
3	1
6	2
10	$3\frac{1}{3}$

(Roses column: +1, +1, +1; Sunflowers column: $+\frac{1}{3}$, $+\frac{1}{3}$; across: $\cdot\frac{1}{3}$)

Sunflowers	Roses
1	3
2	6
3	9
6	18
10	30

(Sunflowers column: +1, +1, +1; Roses column: +3, +3, +3; across: $\cdot 3$)

The labeling of the columns in the table may initially be counter-intuitive. For example, when making the table for the rate of miles per hour a student may want to automatically put the first quantity listed in the first column (in this case miles). However, when creating a table convention suggest the input (x) values go in the first column and the output (y) values go in the second column and the unit rate is represented by $\frac{y}{x}$. Thus, for the ratio of miles per hour or $\frac{\text{mile}}{\text{hr}}$ the miles are represented with the y values and the hours are represented with the x values.

The way a ratio or rate is phrased often suggests which values are the independent (x) and dependent (y) variables. Most often the y value occurs first and the x values second in a statement. However, this is not always the case and does not mean that we can't switch the quantities and look at the reciprocal unit rate. In the given example we may be interested in how long it takes to travel one mile or the rate of hours per mile. The unit rate would be hours/mile and in the table we would put miles (x) in the first column and hours (y) in the second column. It is important for students to understand that determining what values are the inputs and outputs directly correlates to what you want to find and what questions you are asking.

Activity 2: John walks at a constant rate of 8 miles every 3 hours. Answer the following:

a. What is the **rate** written as miles per hours? __8___miles :___3___hours

b. Use a bar model to find the *unit rate* of miles per hour.__$\frac{8}{3}$ ___miles : 1 hour or $\frac{\frac{8}{3}\text{ miles}}{1\text{ hour}}$

8 miles
3 hours

$\frac{8}{3}$ mile	$\frac{8}{3}$ mile	$\frac{8}{3}$ mile
1 hour	1 hour	1 hour

c. How far can John walk in 5 hours? 5 hours × ($\frac{8}{3}$ miles per hour) is $\frac{40}{3}$ miles or 13$\frac{1}{3}$ miles. In later sections, unit analysis will be discussed more (in this case, hour × miles/hour = miles). For now, simply help students understand that if John travels $\frac{8}{3}$ miles in one hour, we can multiply this quantity by 5 to find the distance traveled in 5 hours.
Help students "count" by $\frac{8}{3}$ i.e. $\frac{8}{3}, \frac{16}{3}, \frac{24}{3}$ etc. It may be helpful to show the jumps on a number line.

d. Fill in the following table.

Hours	0	1	2	3	4	5	6	10	20	*h*
Miles	0	$\frac{8}{3}$	$\frac{16}{3}$	$\frac{24}{3}$	$\frac{32}{3}$	$\frac{40}{3}$	$\frac{48}{3}$	$\frac{80}{3}$	$\frac{160}{3}$	$\frac{8}{3}(h)$

e. Where on the table can you find the unit rate for miles per hour?
At 1 hour, he travels $\frac{8}{3}$ miles

f. Suppose John wants to know how long it would take to walk 1 mile. What is this rate written as hours per mile? ___3__hours :___8___miles

g. Use a bar model to find the *unit rate* of hours per mile.__$\frac{3}{8}$___hours : 1 mile or $\frac{\frac{3}{8}\text{ hours}}{1\text{ mile}}$

3 hours
8 miles

⇒

$\frac{3}{8}$ hour	$\frac{3}{8}$ hour	$\frac{3}{8}$ hour	$\frac{3}{8}$ hour	$\frac{3}{8}$ hour	$\frac{3}{8}$ hour	$\frac{3}{8}$ hour	$\frac{3}{8}$ hour
1 mile	1 mile	1 mile	1 mile	1 mile	1 mile	1 mile	1 mile

John travels 8 miles in 3 hours. That means that in 1 mile he travels $\frac{3}{8}$ of an hour.

h. Fill in the following table:

Miles	0	1	2	3	4	5	6	7	8	9	10	20	m
Hours	0	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{9}{8}$	$\frac{12}{8}$	$\frac{15}{8}$	$\frac{18}{8}$	$\frac{21}{8}$	$\frac{24}{8}$	$\frac{27}{8}$	$\frac{30}{8}$	$\frac{60}{8}$	$\frac{3}{8}m$

i. Where on the table can you find the unit rate for hours per mile?
At 1 mile, it takes him $\frac{3}{8}$ of an hour.

j. Complete each sentence:

The number of miles is equal to ___________the number of hours. $\frac{8}{3}$

The number of hours is equal to ___________the number of miles. $\frac{3}{8}$

k. What is the relationship between miles per hour and hours per mile? How might this help you?
They are reciprocals. Go back to the $\frac{1}{3}$ sunflowers : 1 rose and 3 roses : 1 sunflower example at the beginning of this section for discussion.

l. Suppose it takes John 4 hours to go 9 miles. Is this proportional to the rate that John walked previously?
No, the ratios are not proportional. They do not have the same unit rate or proportional constant.

Directions: Use models to write two possible **unit rates** for each given situation. Be sure to include units of measure in your unit rate.

Models have been encouraged here because many students struggle with fractions. Help students connect models to algorithms for multiplying and dividing fractions.

1. The sandwich shop uses 24 slices of meat for every six sandwiches.

Remind students that the unit rates will be reciprocals of each other.

24 slices
6 sandwiches

4 slices	4 slices	4 slices	4 slices	4 slices	4 slices
1 sandwich	1 sandwich	1 sandwich	1 sandwich	1 sandwich	1 sandwich

4 slices of meat per sandwich.

6 sandwiches
24 slices

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s	1s

$\frac{1}{4}$ of a sandwich per slice of meat.

a. Use the models that you have created to determine how many slices of meat you will need for 5 sandwiches.
You will need 20 slices of meat for 5 sandwiches.

b. Use the model to determine how many sandwiches you can make from 15 slices of meat.
You can make $3\frac{3}{4}$ sandwiches from 15 slices of meat.

2. It took $\frac{3}{4}$ of a cake to feed $\frac{1}{3}$ of the class.

   a. Use models to write two possible **unit rates** for each given situation. Be sure to include units of measure in your unit rate.

$\frac{3}{4}$ of a cake
$\frac{1}{3}$ of a class

$\frac{9}{4}$ cake

$\frac{3}{4}$ of a cake	$\frac{3}{4}$ of a cake	$\frac{3}{4}$ of a cake
$\frac{1}{3}$ of a class	$\frac{1}{3}$ of a class	$\frac{1}{3}$ of a class

1 class

2.25 cakes per class or $\frac{9}{4}$ cakes per class.

$\frac{1}{3}$ of a class
$\frac{3}{4}$ of a cake

1/3 class

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	= $\frac{4}{9}$ of a class
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	= 1 cake

¾ cake

$\frac{4}{9}$ (0.44…) of the class per cake.

   b. How many cakes will you need to feed 2 classes?
   You will need 4.5 cakes to feed two classes.

   c. What percentage of the class will 1 cake feed?
   1 cake will feed approximately 44.4% of a class.

   d. It takes $3\frac{3}{4}$ of a cake to feed $1\frac{2}{3}$ of a class, is this proportional to the rate above?
   Yes, if you iterate the model 5 times you get $1\frac{2}{3}$ classes and $3\frac{3}{4}$ cakes.

3. Each $\frac{1}{2}$ hour a three-dimensional printer prints $\frac{1}{5}$ of a heart valve model.

$\frac{1}{2}$ hour
$\frac{1}{5}$ h.v.

$\frac{5}{2}$=2.5 hours

$\frac{1}{2}$ hour	$\frac{1}{2}$ hour	$\frac{1}{2}$ hour	$\frac{1}{2}$ hour	$\frac{1}{2}$ hour
$\frac{1}{5}$ h.v.	$\frac{1}{5}$ h.v.	$\frac{1}{5}$ h.v.	$\frac{1}{5}$ h.v.	$\frac{1}{5}$ h.v.

1 heart valve

$\frac{5}{2}$ = 2.5 hours per heart valve.

$\frac{1}{5}$ h.v.
$\frac{1}{2}$ hour

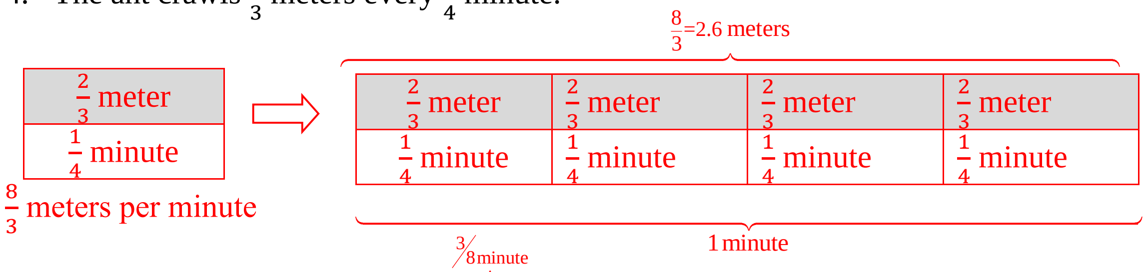
$\frac{2}{5}$ heart valve

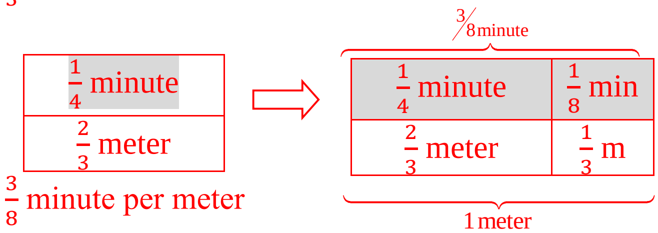
$\frac{1}{5}$ h.v.	$\frac{1}{5}$ h.v.
$\frac{1}{2}$ hour	$\frac{1}{2}$ hour

1 hour

$\frac{2}{5}$ = 0.4 heart valves per hour

4. The ant crawls $\frac{2}{3}$ meters every $\frac{1}{4}$ minute.





*To some students it may make more sense to talk about $\frac{8}{3}$ meters crawled every 1 minute. Talk to students about flexibility in thinking.

5. How can you find the unit rate without making a model?

The unit rate for a ratio a to b can be found by dividing the two quantities or by finding the quotient $\frac{a}{b}$. Go back through numbers 1-4 and find the unit rate by dividing to see if they yield the same answer.

Spiral Review

1. Mark has $16 more than Becca. Represent how much money Mark has if b is the amount of money that Becca has.

2. What Algebraic Property is shown?

   $8 + 7 + 2 = 8 + 2 + 7$ ____________________

   $18 + 0 = 18$ ______________________

3. Add $-\frac{3}{5} + \frac{2}{3} =$

4. Add $-4 + -7 =$

4.1c Homework: Model and Understand Unit Rates

Directions: Use models to help you write the two possible unit rates for each given situation. Be sure to label the units in the unit rate. See note to the right for an explanation on how students must attend to precision when expressing unit rates.

1. Tony Parker scored 6 points in every quarter of a game.
   
   24 points per game
   $\frac{1}{24} = (0.041\overline{6})$ of the game per point.
   (Talk about what this means; about 1 point every 2 minutes)
   It is reasonable that a student may infer that 6 points per quarter is the unit rate. In this case they would state that the unit rate is 6 points per quarter and $\frac{1}{6}$ of a quarter per point.

Here you could begin the conversation about whether every numeric form of a unit rate is meaningful in every context. A decimal unit rate is helpful for getting a sense of the size of the number, but answers will be inaccurate if a rounded decimal is used as a unit rate when scaling up or down. A fraction will always scale accurately.

2. A car traveled 18 miles on $\frac{3}{4}$ of a gallon of gas.
   
   24 miles per gallon.
   $\frac{1}{24} = 0.041\overline{6}$ gallons per mile.

   Note: #1 and #2 have the same numeric rates, but the units are different. Also point out that sometimes units are not explicitly pointed out like when saying the ratio of girls:boys is 5:4, we understand the units (girls and boys) so we don't say it. Here, though, unit is not apparent so we must clarify because it does not make sense to say distance:gas is 24:1—what unit of distance (miles? feet? kilometers?) and unit of gas (gallons? quarts? liter?).

3. A computer company can inspect 12 laptops in 8 hours.
   

Notice the reciprocal/inverse relationship when they write both their answers as fractions. This is a great way to check your answer.

4. Samantha can type 14 sentences in $3\frac{1}{2}$ minutes.
   

5. John can mow $\frac{2}{5}$ of a lawn in $\frac{2}{3}$ of an hour.

4.1d Class Activity: Finding Unit Rates

Activity 1: Mauricio sometimes swims laps at his local recreation center for exercise. He wants to check whether he is swimming laps faster over time. When he first starts swimming, he can swim one lap in 90 seconds. Each week he records the number of laps that he swims and the amount of time that it takes him. Fill in his unit rate for laps per second (which could be a fraction) and his unit rate for seconds per lap (which could be a mixed number).

Number of laps	1 lap	8 laps	5 laps	10 laps
Time spent swimming	90 seconds	600 seconds	390 seconds	720 seconds
Laps per second	$\frac{1}{90}$ (≈0.0111)	$\frac{1}{75}$ (≈0.01333)	$\frac{1}{78}$ (≈0.0128)	$\frac{1}{72}$ (≈0.0139)
Seconds per lap	90	75	78	72

Which set of laps had the fastest pace? Explain your answer.

Notice that this is not a table that shows a proportional relationship, since each pair of numbers has a different rate. This is meant to show that a unit rate makes it easier to compare which rate is faster. The last is the fastest because he can complete each lap in the shortest time. Also note that he swims more of a lap in one second.

Directions: Find each unit rate for the given situation by using a model, and then check your answer by dividing each quantity. Encourage students to write the desired units as a fraction before finding the unit rate by dividing. This will help them to determine which number goes on top. For example in number 1 part *a* before calculating they would determine that they are looking for $\frac{\text{cans}}{\text{dollar}}$ and in part *b* they are looking for $\frac{\text{dollars}}{\text{can}}$. As students progress through this lesson they should be able to reason more abstractly and become more comfortable with dividing the units and move away from the models.

1. At the store, you can get 15 cans of peaches for $3.

a. Find the unit rate for $1.

15 cans
$3

⇨

5 c	5 c	5 c
$1	$1	$1

$\frac{15}{3}$ =5 cans per dollar.

b. Find the unit rate for 1 can.

$1
5 cans

⇨

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
1	1	1	1	1

$\frac{3}{15} = \frac{1}{5}$ = $0.20 per can.

2. Traveling between countries means exchanging currencies (money). Suppose the exchange rate is 3 dollars to 2 Euros.

a. How many Euros would you receive in exchange for 1 dollar (the unit rate for $1)?

2 Euros
$3

⇨

$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$1	$1	$1

$\frac{2}{3}$ Euro per dollar.

b. How many dollars would a person receive in exchange for 1 Euro (the unit rate for 1 Euro)?

$3
2 Euros

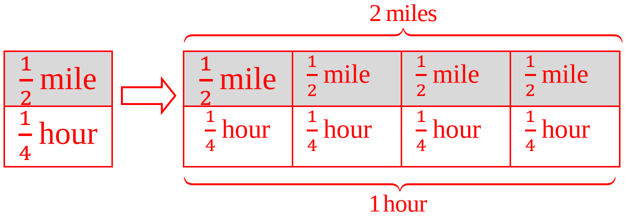
⇨

$\frac{3}{2}$	$\frac{3}{2}$
1 E	1 E

$\frac{3}{2}$ dollar per Euro.

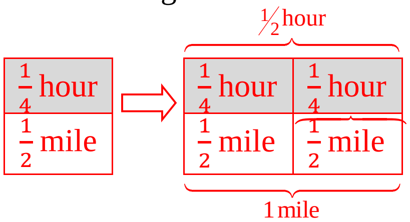
3. Emina can walk $\frac{1}{2}$ mile in $\frac{1}{4}$ hour.

a. How many miles can she walk in one hour?



$\frac{1/2}{1/4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$ miles per hour.

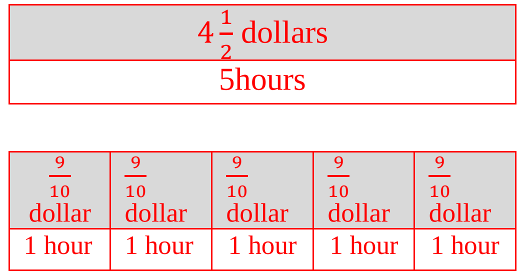
b. How long does it take her to walk 1 mile?



$\frac{1/4}{1/2} = \frac{1}{4} \times \frac{2}{1} = \frac{2}{4} = \frac{1}{2}$ hour per mile.

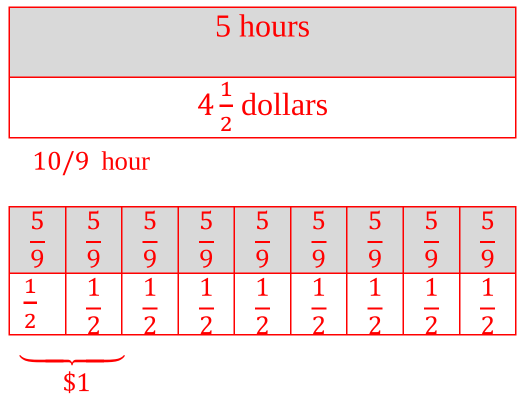
4. Napoleon earns $4\frac{1}{2}$ dollars in 5 hours.

a. How much does he make per hour?



$\frac{4^1/_2}{5} = \frac{9}{2} \times \frac{1}{5} = \frac{9}{10} =$\$0.90 per hour.

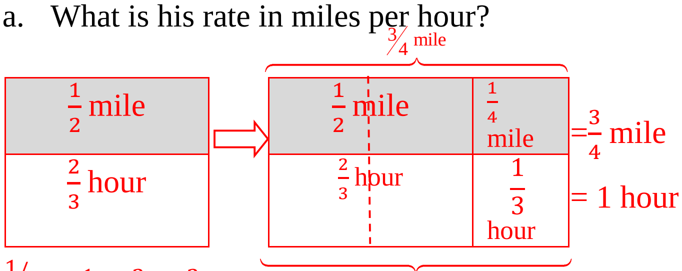
b. How long will he have to work to earn one dollar?



$\frac{5}{4^1/_2} = 5 \times \frac{2}{9} = \frac{10}{9} = 1\frac{1}{9}$ hours per dollar.

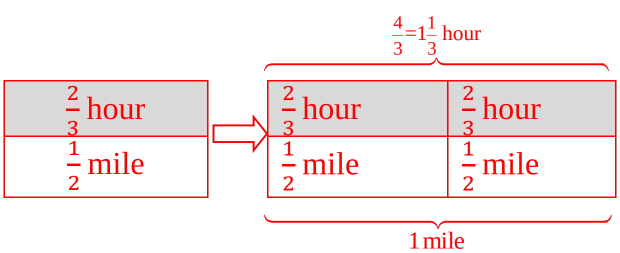
5. Diego can swim $\frac{1}{2}$ of a mile in $\frac{2}{3}$ of an hour.

a. What is his rate in miles per hour?



$\frac{1/2}{2/3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$ mile per hour

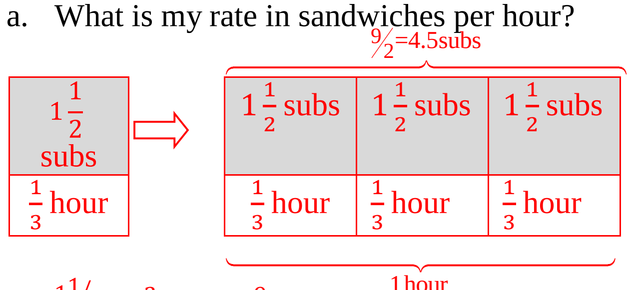
b. What is his rate in hours per mile?



$\frac{4}{3} = 1\frac{1}{3}$ hours per mile

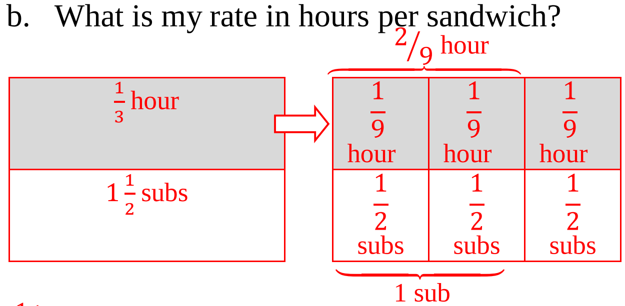
6. In 20 minutes, I ate $1\frac{1}{2}$ sub sandwiches.

a. What is my rate in sandwiches per hour?



$\frac{1^1/_2}{1/_3} = \frac{3}{2} \times 3 = \frac{9}{2} = 4.5$ subs per hour

b. What is my rate in hours per sandwich?



$\frac{1/_3}{1^1/_2} = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$ hour per sandwich.

7. Henry ate $2\frac{3}{4}$ slices of pizza in 20 minutes.

a. What is his average number of minutes per slice?

$\frac{20}{2^3/_4} = \frac{20}{11/_4} = 20 \times \frac{4}{11} = \frac{80}{11}$ =7.27…minutes per slice

b. Make a table that represents the rate of time per slice.

+1 +1

Number of slices	1	2	3	4.5	10
Time in minutes	$\frac{80}{11}$ =7.27…	$\frac{160}{11}$ =14.54…	$\frac{240}{11}$ =21.81…	$\frac{360}{11}$ =32.72…	$\frac{800}{11}$ =72.72…

•80/11 •80/11

$+\frac{80}{11}$ $+\frac{80}{11}$

c. Describe or show where you see the unit rate and the proportional constant in the table?
The proportional constant is $\frac{80}{11}$. This is the number that you multiply the number of slices by to get the time in minutes. You can also see the unit rate each time you add 1 slice of pizza, the time increases by $\frac{80}{11}$. The unit rate and proportional constant are the same value.

d. If Henry continues to eat at this rate how long will it take him to eat 12 pieces of pizza?
It will take Henry approximately 87.72 minutes to eat 12 pieces of pizza.

e. What is his average number of slices per hour?
$\frac{2^3/_4}{1/_3} = \frac{11/_4}{1/_3} = \frac{11}{4} \times \frac{3}{1} = \frac{33}{4}$ =8.25 slices per hour.

f. Why is this unit rate not a reciprocal of the unit rate of the number minutes per slice?
This unit rate was found using different units, we converted minutes to hours.

g. Make a table that represents the rate of number of slices per hour?

+1 +1+8.46

Time in hours	1	2	3	0.5	10
Number of slices	$\frac{33}{4}$ =8.25	$\frac{66}{4}$ =16.5	$\frac{99}{4}$ =24.75	$\frac{33}{8}$ =4.125	82.5

•8.2 •8.2

+8.25 +8.25

h. Describe or show where you see the unit rate and proportional constant in the table?
The proportional constant is 8.25. This is the number that you multiply the time by to get the number slices. You can also see the unit rate each time you add 1 hour, the number of slices increases by 8.25.

i. At this rate how many slices of pizza will Henry eat after 12 hours?
Henry will be able to eat 99 pieces of pizza.

j. Is it very likely for Henry to continue to eat at this rate?
No, he will most likely slow down because his stomach will get full.

8. Determine if each set of ratios below form a proportional relationship by finding the unit rate of each ratio.

a. $\frac{7}{9}$ $\frac{21}{27}$ yes	b. $\frac{10}{12}$ $\frac{15}{30}$ no	c. 1:2 3:13
d. 2 to 3 3 to 8	e. $\frac{9}{12}$ $\frac{49}{50}$ no	f. 7:3 28:12 yes
g. $\frac{15}{35}$ $\frac{24}{52}$	h. 4 to 16 2 to 8 yes	i. 6:9 8:4 no
j. 3.4 to 8.5 4.8 to 12 yes	k. $\frac{3}{5}$ to $\frac{1}{3}$ $1\frac{3}{4}$ to $\frac{7}{8}$ no	l. $\frac{1/5}{3/5}$ $\frac{1/4}{3/4}$ yes

Challenge: Can you think of other ways to determine proportionality? See student responses. If a student has been previously taught cross multiplication, or if a student notices the pattern of cross products, push them to use proportional reasoning to explain why that is true, but **don't** emphasize that (cross multiplication) as the way to solve these problems.

Spiral Review

1. You can buy 12 cans of soda for $2.40.
   a. Find the unit rate for 1 can.
   b. Find the unit rate for $1.
2. Use long division to show how you can convert this fraction to a decimal and then a percent.

$$\mathbf{\frac{2}{7}}$$

3. Solve: $\frac{3}{4} - \frac{7}{8} =$
4. Simplify: $x + 3x - 7x + 2x^2$

4.1d Homework: Finding Unit Rates

Directions: Find each unit rate by either drawing a model or dividing. Don't forget to label your answers.
Some students may still prefer to use a model, that's ok. See class activity for examples of models.

1. You can buy 5 oranges for $2.00.
   a. What is the unit rate for 1 orange?

   b. What is the unit rate for $1?

2. For three people, there are 5 candy bars.
   a. What is the unit rate for the number of candy bars for 1 person?

   b. What is the unit rate for the number of people for 1 candy bar?

3. In $12\frac{1}{2}$ minutes, Dulce read 50 pages.
   a. On average, how many pages did she read per minute?
   4 pages per minute.

   b. On average, how many minutes does it take to read one page?
   $\frac{1}{4}$ minute per page.

4. Yazmin's heart rate was measured at 19 beats in a $\frac{1}{4}$ minute.
   a. How many beats per minute?

   b. How many minutes per beat?

   .

5. In $\frac{1}{10}$ of an hour, Jane can clean $\frac{5}{8}$ of a window.
   a. What is her unit rate in windows per hour?
      $\frac{25}{4} = 6.25$ windows per hour.
   b. Describe a situation where finding this unit rate would be useful.
      Jane might want to know how many windows she could clean in a certain number of hours.

   c. Make a table to show this relationship.

Time(hours)	1	2	3.5	10
Number of Windows	$\frac{25}{4} = 6.25$	$\frac{50}{4} = 12.5$	$\frac{175}{8} = 21.875$	$\frac{250}{4} = 62.5$

   d. What is her unit rate in time per window?
      $\frac{4}{25} = 0.16$ hours per window.
   e. Describe a situation where finding this unit rate would be useful.
      Jane might to know how long it will take her to clean a certain number of windows.

6. Traveling between countries means exchanging currencies (money). Look up the current exchange rate for Europe (or replace Euros with the currency of a country of your choice).
   a. How many Euros would you receive for 1 dollar?

   b. How many Euros will you receive for $10?

   c. How many dollars would a European receive for 1 Euro?

   d. How many dollars would they receive for 10 Euros?

7. Determine if each set of ratios below form a proportional relationship by finding the unit rate for each ratio.

a. $\frac{3}{7}$ $\frac{6}{14}$	b. $\frac{11}{5}$ $\frac{33}{16}$	c. 1:5 1:15
d. 6 to 10 3 to 7 no	e. $\frac{8}{12}$ $\frac{78}{100}$ no	f. 9:5 36:20 yes
g. $\frac{20}{45}$ $\frac{24}{50}$	h. 9 to 15 3 to 5	i. 5:4 4:3
j. 16.3 to 10 18.2 to 12 No	k. $\frac{1}{3}$ to $\frac{1}{2}$ 1 to $1\frac{1}{2}$ yes	l. $\frac{2/3}{2}$ $\frac{3/4}{3}$ no

4.1e Class Activity: Comparing Unit Rates

Part I

Directions: Find a unit rate for each situation for both quantities. Pick the most useful unit rate for each situation (choose out of the two possibilities) and explain. For example choose between "pounds in one bag of flour" or "bags of flour in one pound." At this point students should be comfortable finding the unit rate by finding the quotient of the two quantities.

	Unit rate a	Unit rate b	Pick the more useful unit rate, if you think one is more useful than the other. Explain your choice.
1. The bakery has 8 bags of flour weighing a total of 40 pounds. 	5 lbs per bag.	$\frac{1}{5}$ bag per pound.	See student responses.
2. In four tennis ball cans, there are 12 tennis balls. 	3 balls per can.	$\frac{1}{3}$ can per ball.	See student responses.
3. Your dad bought 4 gallons of gas for $13. 			
4. In ½ hour, you can walk 2 miles. 	4 miles per hour.	$\frac{1}{4}$ hour per mile.	See student responses.
5. Marco went 300 miles in 5 hours on his road trip. 			
6. Estefania can run $\frac{3}{4}$ the race in $\frac{2}{3}$ of an hour. 	$\frac{9}{8}$ of the race per hour.	$\frac{8}{9}$ hour per race.	See student responses.

Part II

Directions: Find a unit rate for each situation to answer each question. Justify your answer.

It is important for students to attend to precision as the calculate unit rates and compare them. Some of the quantities are very close in value. Talk to students about how these small quantities may not seem like they make much of a difference when comparing but when added up the difference becomes more apparent.

7. Jean sells M&Ms to friends at 4 for 5 cents. The machine at the store sells 9 for 25 cents. Which is the better deal? Jean has the better deal her M&Ms cost 1.25¢ each. At the store the M&Ms are approximately 2.78¢ each. Students may also argue the reciprocal unit rate for each problem. For example: You get 0.8 M&Ms per cent from Jean and 0.36 M&Ms per cent from the machine. Thus you get more M&Ms for every cent from Jean than the Machine.	8. Frosted Flakes has 11 grams of sugar per ounce and Raisin Bran 13 grams of sugar per 1.4 ounces. Which cereal will taste more sugary? Frosted Flakes will taste more sugary because it has 11 grams of sugar per oz. Rasin Bran has approximately 9.3 grams of sugar per oz.	9. Tom sells baseball cards at 10 for 35 cents. Is that a better deal than 12 for 40 cents?
10. The hardware store sells 4th of July sparklers for 19 cents each. The fireworks stand charges 85 cents for four. Where would you purchase sparklers? The hardware store has the better deal. It cost 19¢ per sparkler. At the stand it is approximately 21¢ per sparkler.	11. Ribbon: 5 m for $6.45 or 240 cm for $3.19. Which ribbon is cheaper per meter?	12. An 11 ounce can of condensed soup (add 1 can of water), costs $1.45. A 20 ounce can of ready to serve soup costs $1.29. Which can of soup has the best price per ounce?
13. The flight from Oakland to Salt Lake City was 592 miles and took 180 minutes. The flight from Denver to Salt Lake City was 372 miles and took 124 minutes. On which flight were you traveling faster? On the flight from Oakland to SLC you are traveling at a rate of about 3.3 miles per minute. On the flight from Denver to SLC you are traveling at a rate of 3 miles per minute. The Oakland flight is faster because you traveled farther per minute.	14. At the beginning of the month it took Raul 8 days to paint 17.6 offices. At the end of the month is took Raul $2\frac{1}{2}$ days to paint $6\frac{1}{2}$ offices. Is Raul faster at the beginning of the month or the end of the month? Raul is faster at the end of the month, he can paint 2.6 offices per day. At the beginning of the month he can only paint 2.2 offices per day.	15. In 9 minutes, Guillermo reads $7\frac{1}{2}$ pages of the novel. In 15 minutes Courtney reads 18 pages of the same novel. Who is the slower reader? Guillermo is the slower reader, he read 0.83 pages per minute and Courtney reads 1.2 pages per minute.

Spiral Review

1. Luis went to a soccer game with some friends. He bought two sodas for $1.50 each and four giant candy bars for $2.25 each. Write an expression to represent how much Luis spent?

2. Juliana bought 3 bags of chips and 3 sodas for herself and two friends. The chips were $0.85 a bag. If she spent a total of $6, how much was each can of soda?

3. Simplify the following expression. Use a model if needed.

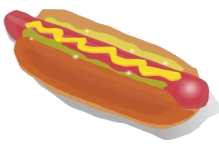
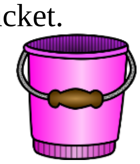
   $71b - 4a + 4b - 4a$

4. Write a context for the following expression: $4(x - 3)$

4.1e Homework: Comparing Unit Rates

Part I

Directions: Find a unit rate for each situation for both quantities. Pick the most useful unit rate for each situation (choose out of the two possibilities) and explain.

	Unit rate a	Unit rate b	Pick the more useful unit rate, if you think one is more useful than the other. Explain your choice.
1. In 8 hours, Mayra can mow 5 lawns. 			
2. George can eat 30 hotdogs in 2 hours. 			
3. A tree grows 15.5 feet in 10 years. 	$15.5 \div 10 =$ 1.55 feet per year.	$10 \div 15.5 =$ $\frac{2}{31} \approx 0.065$ years per foot.	See student responses.
4. Over five years, Luz has collected 25 nesting dolls. 			
5. After a $2\frac{1}{2}$ hour rainstorm, there are 5 inches of water in a bucket. 	$5 \div 2\frac{1}{2} = 2$ inches per hour.	$2\frac{1}{2} \div 5 = \frac{1}{2}$ hour per inch.	See student responses.

Part II

Directions: Find a unit rate for each situation to answer each question. Justify your answer.

Note that students may also argue the reciprocal unit rates for each problem.

6. You can buy applesauce at 24 oz for \$1.25 or 16 oz for \$0.89. What applesauce is the better deal?	7. You can buy $\frac{1}{2}$ gallon of grape juice for \$2.80 or 20 oz. for \$0.75. (Note: there are 128 fluid oz. in a gallon). What grape juice is the better deal? 20 oz for 75¢ is the better deal; this is approximately 3.8¢ per oz. The ½ gallon container is approximately 4.4¢ per oz.	8. You can buy 3 L of olive oil for \$16.99 or 500 ml of olive oil for \$3.09. What is olive oil is the better deal?
9. You can buy 4.3 kg of ground beef for \$21.70 or 1.6 kg for \$7.85. What is the better deal for ground beef?	10. You can order 3 boxes of 12 greeting cards for \$30 or 4 boxes of 15 for \$49. What is the better deal? 4 boxes for \$49 is the better deal; this is approximately 82¢ per card. The 3 box set is approximately 83¢ per card.	11. Stan typed 90 pages in 7.5 hours. Jan typed 110 pages in 9.4 hours. Who is the faster typist?
12. In 30 days Bob lost 8 ½ pounds. Kevin lost 8 pounds in 40 days. Who is losing weight at a faster rate?	13. On Monday it took Henry 10 hours to harvest $3\frac{1}{2}$ fields of corn. On Tuesday it took him $7\frac{1}{2}$ hours to harvest 3 of the same size fields of corn. On what day was Henry working slower? Henry was working slower on Monday because he worked at a rate of 0.35 fields per hour. On Friday he worked at a rate of 0.4 fields per hour.	14. Oscar's frosting recipe calls for $2\frac{3}{4}$ cubes of butter and 6 cups of powdered sugar. Marla's recipe calls for 7 cups of powdered sugar and 3 cubes of butter. Whose frosting will taste more buttery?

4.1f Class Activity: Using Unit Rates to Solve Problems

Directions: Find a unit rate from the information given. Then use the unit rate to answer the additional questions. Models may be useful. In this lesson student are using unit rate as a "tool" to answer questions. At the beginning of this lesson, the students are prompted to find a specific unit rate to answer the questions. As the lesson progresses less scaffolding is provided and the students must determine what unit rate will be most helpful in obtaining the information they need to answer a question.

1. The recipe that makes 24 cookies calls for 4 cups of flour.

   a. What is the unit rate of cups of flour per cookie?
   $\frac{1}{6}$ cup of flour per cookie.

   b. How many cups of flour would be used for 20 cookies?
   $3\frac{1}{3}$ cups of flour.

   c. What is the unit rate of cookies per cup of flour?
   6 cookies per cup of flour.

   d. How many cookies could be made with 5 cups of flour?
   30 cookies.

   e. How many cookies could be made with 0 cups of flour?
   0 cookies.

2. The table shows the total number of inches of snowfall for each ½ hour.

Inches of Snowfall	Hours
1	$\frac{1}{2}$
2	1
3	$\frac{3}{2}$
4	2
5	$\frac{5}{2}$

   a. What is the unit rate of hours per inch?
   ½ hour per inch.

   b. At this rate, if there is $7\frac{1}{2}$ inches of snow, how long has it been snowing?
   $3\frac{3}{4}$ hours.

   c. What is the unit rate of inches per hour?
   2 inches per hour.

   d. At this rate, how many inches of snow would have fallen in $2\frac{1}{3}$ hours?
   $4\frac{2}{3}$ inches.

3. The table shows the cost of a plumber for the amount of hours worked.

Hours	0	1	2	3	4
Plumbing Cost	$45	$70	$95	$120	$145

a. What is different about this table?
It has a value of $45 for 0 hours.

b. Is there a unit rate or proportional constant? Justify your answer.
No: $\frac{45}{0} = undefined, \frac{70}{1}, \frac{47.50}{1}, \frac{40}{1}, \frac{36.2}{1}$

c. Do you think this data is is proportional? Why or why not?
No. See student responses. May include $\frac{70}{1} = 70, \frac{95}{2} = 47.5$ so not the same rate throughout.

4. The side lengths of Triangles A and B are proportional. Triangle A has a side length of 4 inches, the corresponding side length on Triangle B has a length of 5 inches.

a. Fill in the table of values that represents the side length of the triangle.

Triangle A	Triangle B
4	5
3	$3\frac{3}{4}$
5	$6\frac{1}{4}$

b. What is the unit rate for the side lengths of Triangle B to A?
$\frac{5}{4}$ of an inch to 1 inch.

5. Arturo can run $1\frac{3}{4}$ miles in $\frac{1}{4}$ hour.

a. What is his speed in miles per hour?
7 miles per hour.

b. How many miles can Arturo run in 3 hours?
Arturo can run 21 miles in 3 hours.

6. In 2.5 hours, Daniel's brother can ride 40 miles on his bike.
    a. How many miles can Daniel's brother ride on his bike in 4 hours?

    b. How long will it take Daniel's brother to ride 152 miles?

7. There were 7 inches of rain in 24 hours. If the rain continues to fall at the same rate how many inches of rain will have fallen in 30 hours?
    There will be 8.75 inches after 30 hours.

8. The times for three runners in three different races are given in the table.

Runner	Distance	Time
A	5 kilometers	34 minutes
B	10 kilometers	62 minutes
C	21 kilometers	155 minutes

    a. Find the unit rate of kilometers per minute for each runner.
    $A \approx 0.15\frac{km}{min}, B \approx 0.16\frac{km}{min}, C \approx 0.14\frac{km}{min}$
    b. Which runner finished the race at the fastest average rate?
    Runner B finished at the fastest rate.

9. CD Express offers 4 CDs for $60. Music Place charges $75 for 6 CDs. Which store offers the better deal? Justify your answer.

10. Tia is painting her house. She paints $34\frac{1}{2}$ square feet in $\frac{3}{4}$ hour. At this rate, how many square feet can she paint each hour?
    46 square feet per hour.

4.1f Homework: Using Unit Rates to Solve Problems

Directions: Find a unit rate from the information given. Then use the unit rate to answer additional information. Models may be useful.

1. The table below shows the exchange rate for Pesos to Dollars.

Dollars	3	4.50	6	7.50
Pesos	1	1.5	2	2.5

   a. What is the unit rate of pesos per dollar?
   ⅓ peso per dollar.
   b. At this rate, how many pesos could you get for 8 dollars?
   2⅔ pesos.
   c. What is the unit rate of dollars per peso?
   3 dollars per peso.
   d. At this rate, how many dollars could you get for 18.60 pesos?
   $55.80.
   e. If you have 0 pesos, how many dollars is that worth?
   0 dollars.

2. The table below shows the rate of the number of homeruns to the number of swings.

Homeruns	1	2	3	4	5
Swings	7	14	21	28	35

   a. What is the unit rate of homeruns per swing?

   b. At this rate, if the batter swings 30 times, estimate the number of homeruns he would have gotten.

   c. What is the unit rate of swings per homerun?

   d. At this rate, if the batter has gotten 9 home runs, how many times did he swing?

3. In $1\frac{1}{2}$ hours, Sebastian can walk $4\frac{1}{2}$ miles.
   a. Find his average speed in hours per mile.
   $\frac{1}{3}$ of a hour per mile.
   b. At this rate how many hours will it take Sebastian to walk $\frac{3}{4}$ of a mile?
   It will take Sebastian 0.25 hours to walk ¾ of a mile.

4. The table below shows the number of parts produced by a worker on an assembly line.

# of Parts	24	36	42	54
Hours	4	6	7	9

   a. How many parts can the worker produce in 15 hours?

   b. How many hours will it take the worker to produce 132 parts?

5. Allyson measures her plant every year. It's now 16 years old and it measures 12 inches.
   a. What is the unit rate for the plant height per year?

   b. If the plant continues to grow at the same rate how tall will it be at 20 years old?

   c. Is it probable for the plant to continue to grow at this rate?

6. Mrs. Rossi needs to buy dish soap. There are four different sized containers.

Dish Soap Prices	
Brand	**Price**
Lots of Suds	$0.98 for 8 ounces
Bright Wash	$1.29 for 12 ounces
Spotless Soap	$3.14 for 30 ounces
Lemon Bright	$3.45 for 32 ounces

   a. Which brand costs the least per ounce? Justify your answer.
   12¼¢ per oz, 10¾¢ per oz, ≈10½¢ per oz and ≈10.78¢ per oz.
   Spotless Soap is the least expensive.
   b. Which brand gives you the most ounces for one dollar? Justify your answer.
   8.16 oz per dollar, 9.3 oz per dollar, 9.6 oz per dollar, 9.28 oz per dollar
   Spotless Soap gives you the most ounces for a dollar.

7. The Jimenez family took a four-day road trip. They traveled 300 miles in 5 hours on Sunday, 200 miles in 3 hours on Monday, 150 miles in 2.5 hours on Tuesday, and 250 miles in 6 hours on Wednesday. On which day did they average the greatest speed (miles per hour)? Justify your answer.

8. Paola is making pillows for her Life Skills class. She bought $2\frac{1}{2}$ yards of green fabric and $1\frac{1}{4}$ yards of purple fabric. Her total cost was $15 for the green fabric and $8.50 for the purple fabric. Which color fabric is cheaper? Justify your answer.
The green fabric is cheaper because it costs $6 per yard. The purple fabric costs $6.80 per yard.

9. Make up your own story that exhibits a ratio or rate.

a. Write your story here.	b. Make a table that shows the rate in your story. Find at least three values
c. Find the unit rate from your story.	d. Ask a question that you would use the unit rate to solve.

4.1g Self-Assessment: Section 4.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Compute unit rate from a context.	I struggle to understand how to compute a unit rate from a context.	I can compute a unit rate from a context that uses whole numbers.	I can compute a unit rate from a context that uses whole numbers and decimals.	I can compute a unit rate from a context that uses rational numbers in any form.
2. Determine the unit rate and constant of proportionality from a table of values.	I struggle to understand how to compute a unit rate from a table of values.	I can compute a unit rate from a table of values that uses whole numbers and/or fractions as long as the values aren't out of order.	I can compute a unit rate from a table of values that uses whole numbers and decimals regardless of the order of the values in the table.	I can compute a unit rate from a table of values that uses rational numbers in any form and values are in any order. Further, I can explain what the rate means for the context.
3. Compare two rates to determine equivalence or to contrast differences.	I can find rates, but I struggle to know how they compare.	I can find rates to determine equivalence or to determine which is greater.	I can change rates to a form that allows me to compare the two to determine equivalence or to determine which is greater. I can also use the different rates to make predictions.	I can change rates to a form that allows me to compare the two to determine equivalence or to contrast differences. I can express in my own words similarities and differences between the two and use the information to draw conclusions.
4. Find the unit rate for BOTH units (i.e. miles per hour and hours per mile).	I struggle to understand how to compute a unit rate.	I can find the unit rate for one unit but often not the other.	I can always find a unit rate for BOTH units.	I can find a unit rate for BOTH units. I can determine which unit rate would be more helpful in a given situation.
5. Use unit rate to find a missing quantity.	I struggle to know how to use unit rate to find a missing quantity.	I can use unit rate to find a missing quantity involving whole numbers.	I can use unit rate to find a missing quantity involving whole numbers and decimals.	I can use unit rate to find a missing quantity involving whole numbers, decimals, and fractions and I can explain the process I used.
6. Determine if a set of ratios form a proportional relationship.	I struggle to understand how to determine if a set of ratios form a proportional relationship.	I can determine if a set of ratios forms a proportional relationship involving whole numbers.	I can determine if a set of ratios forms a proportional relationship involving whole numbers and decimals.	I can determine if a set of ratios forms a proportional relationship with quantities that involve whole numbers, decimals, and fractions and I can explain the process I used.

Sample Problems for Section 4.1

Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. Given the following situations, find the indicated unit rate for each relationship.[1]
   a. Corrin has twin boys. She buys a box of 10 toy cars to share evenly between the boys.

      _______________ cars per boy

   b. Belle works at a donut shop. They sell a box of donut holes for $1.80. There are 20 donut holes in the box.

      _________ cost per donut hole

   c. Abby runs $14\frac{1}{4}$ laps in $42\frac{3}{4}$ minutes.

      ______________minutes per lap

2. Given the following tables, find the indicated unit rate and/or constant of proportionality for each relationship.[2]

   a.

Days	Total Push-ups
2	30
4	60
29	435
58	870

__________ push-ups per day

   b.

Minutes	Gallons of Water in Swimming Pool
2	12.4
3	18.6
17	105.4
27	167.4

___________ gallons per minute

   c.

Kilometers	Hours
5	$\frac{1}{3}$
15	1
7.5	$\frac{1}{2}$
$\frac{1}{4}$	$\frac{1}{60}$

_________ hours per kilometer

3. The following situations each describe two rates. Compare and contrast the two rates. State equivalence or contrast the differences in context of the situation.[3]
   a. In a two-minute typing test, Stacey types 100 words. In a five minute typing test, Sarah types 250 words.

   b. A 20 oz. bottle of ketchup is 99¢. A 38 oz. bottle of ketchup is $2.30.

4. For each of the following situations, find the unit rate for BOTH quantities.[4]
   a. Ivonne drove 357 miles on 10 gallons of gasoline.

   b. In the World Cup, Chile scored 8 goals in 4 games.

5. For each of the following situations, use the unit rate to answer the question. [5]
   a. Katherine is visiting patients in a hospital. She visits 12 patients in 6 hours. At the same rate, how many patients will she visit in 9 hours?
   b. If 4 gallons of gas cost $14.60, how much does 10 gallons of gas cost?
   c. Joaquin runs $2\frac{1}{2}$ kilometers in $\frac{1}{6}$ hours. If he continues running at the same pace, how long will it take him to run 11 kilometers?

6. For each of the following situations determine if the set of ratios forms a proportional relationship. Justify your answer. [6]

   a. $2\frac{3}{4} : 8\frac{1}{4}$ $\quad$ $\frac{5}{6} : 3\frac{1}{3}$

   b.

x	y
0.8	1.2
1.5	2.25
3	4.5

   c. 8:3 $\quad$ 32:10

Section 4.2: Construct and Analyze the Representations of Proportional Relationships

Section Overview:

In this section, students will represent proportional relationships between quantities using tables, graphs, and equations. Students will build on ideas about unit rate from 4.1 in order to construct the representations of a proportional relationship and they will identify the unit rate in the different representations. The section starts with students creating tables for relationships that are proportional and then transferring the data points onto a graph. In order to create these graphs, students must attend to precision while labeling axes and plotting points in order to accurately show the correspondence between the quantities. In addition, students must reason about how to scale the axes on a graph. Students will interpret graphs, explaining what a point (x, y) means in terms of the situation, paying special attention to the point $(1, r)$ where r is the unit rate. During this process, students will surface ideas about the graph of a relationship that is proportional, realizing that it is a straight line emanating from the origin. While students are creating the tables and graphs of proportional relationships, they will be asked to informally think about the equation that shows the correspondence between the two variables. Students then move into writing equations to represent proportional relationships, connecting their equations to work done with unit rates, diagrams, tables, and graphs. At this point, students should be able to move fluently between the different representations of a proportional relationship and draw correspondences between the representations. They use the representations to solve problems, realizing when it might make sense to use one representation over another to solve a problem. In the last few lessons, students explore the representations of relationships that are not proportional and compare these to the representations of relationships that are proportional, solidifying understanding of the representations of relationships that are proportional.

Domain(s) and Standard(s) from CCSS-M

1. Recognize and represent proportional relationships between quantities. 7.RP.2
    a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. 7.RP.2a
    b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. 7.RP.2b
    c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*. 7.RP.2c
    d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. 7.RP.2d

Primary Concepts and Skills to Master in this Section:

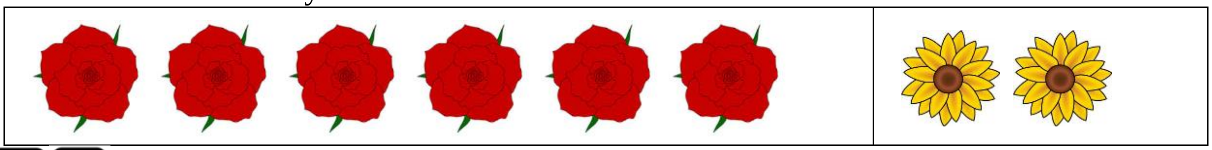
By the end of this section, students should be able to:

1. Create a table of values to represent a relationship that is proportional.
2. Create a graph to represent a relationship that is proportional.
3. Explain the meaning of a point (x, y) in the context of a relationship that is proportional.
4. Explain the significance of the points $(0, 0)$ and $(1, r)$ in the graph of a proportional relationship, where r is the unit rate.
5. Write an equation to represent a relationship that is proportional.
6. Identify the unit rate from a table, graph, equation, diagram, or verbal description of a relationship that is proportional.
7. Analyze the representations of a proportional relationship to solve real-world problems.
8. Determine whether two quantities are in a proportional relationship given a verbal description (context), table, graph, or equation. Explain why a relationship is or is not a proportional relationship.

4.2a Class Activity: Graphs of Proportional Relationships

Example 1: Let's revisit the flower problem from the previous section.

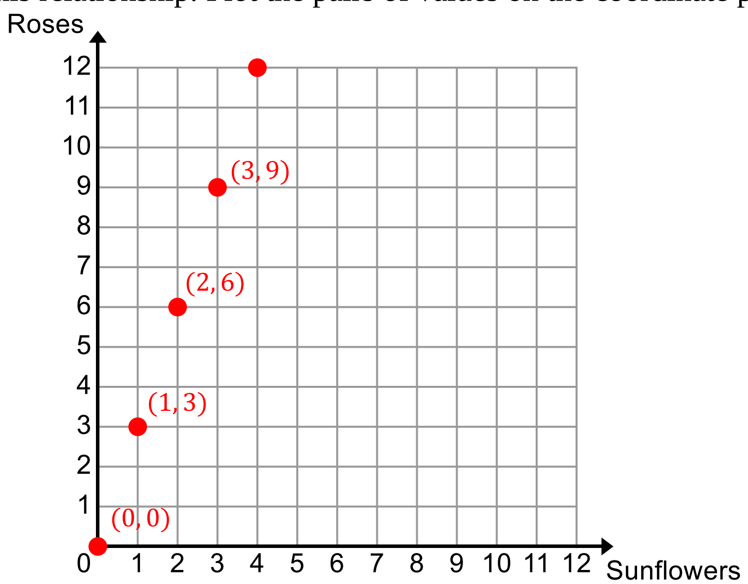
There are 6 roses to every 2 sunflowers.



In this section, students are modeling with mathematics when they create the representations that show the relationships between two quantities. Students have plotted points on the coordinate plane in 6th grade but will likely need a review of how to create graphs, including which column in the table corresponds to which axis, which axis is the *x*-axis and which is the *y*-axis, and how to plot an ordered pair. It is good practice to have students write the ordered pair on the graph that corresponds to a point as shown below. A common mistake is for students to put the quantity that comes first in the ratio in the first column of the table. Be sure that students are attending to precision and paying attention to the labels in the table and on the graph. The purpose of these first two problems is to help students to remember how to graph and to remind them to attend to the quantities when graphing.

a. Complete the table to show this relationship. Plot the pairs of values on the coordinate plane.

Number of Sunflowers	Number of Roses
0	0
1	3
2	6
3	9
4	12
5	15



b. What does the point $(4, 12)$ represent in the situation?
A bouquet with 4 sunflowers and 12 roses

c. Write the ordered pair that represents 9 roses and 3 sunflowers. $(3, 9)$

d. Which point represents a total of 8 flowers?
$(2, 6)$ a bouquet with 2 sunflowers and 6 roses

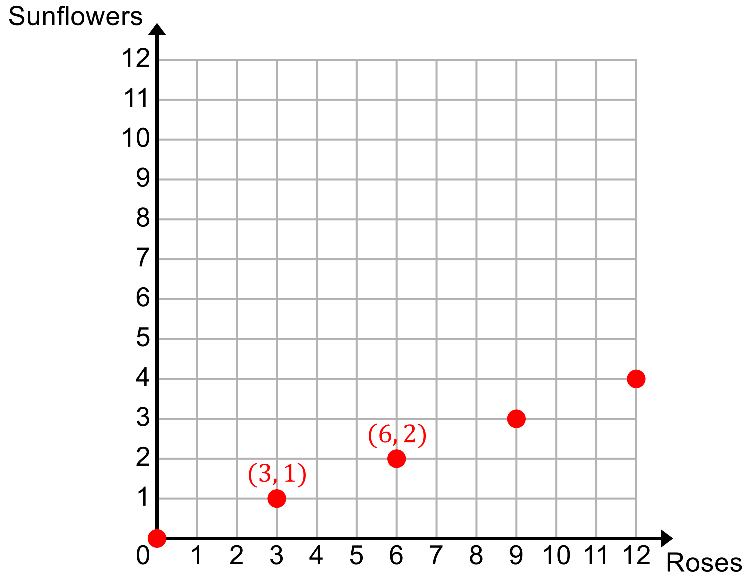
e. If I have 100 sunflowers, how many roses should I have to keep the same ratio? Write your answer as an ordered pair. $(100, 300)$
Students start to informally use the equation and notice the correspondence across the table.

f. Shelley made a bouquet with 8 roses and 4 sunflowers. Is Shelley's bouquet in the same ratio? Explain. No, the unit rate for Maria's bouquet is 2 roses for each sunflower. In a bouquet with 4 sunflowers, one should have 12 roses. Observe that when this point is graphed, it does not fall on the line.

2. Let's look at the flower problem again but this time we will put roses on the x-axis and sunflowers on the y-axis. Again, be sure to attend to precision, being mindful of the labels in the table and on the graph.

   a. Complete the table and plot the pairs of values on the coordinate plane.

Number of Roses	**Number of Sunflowers**
0	0
3	1
6	2
9	3
12	4
15	5



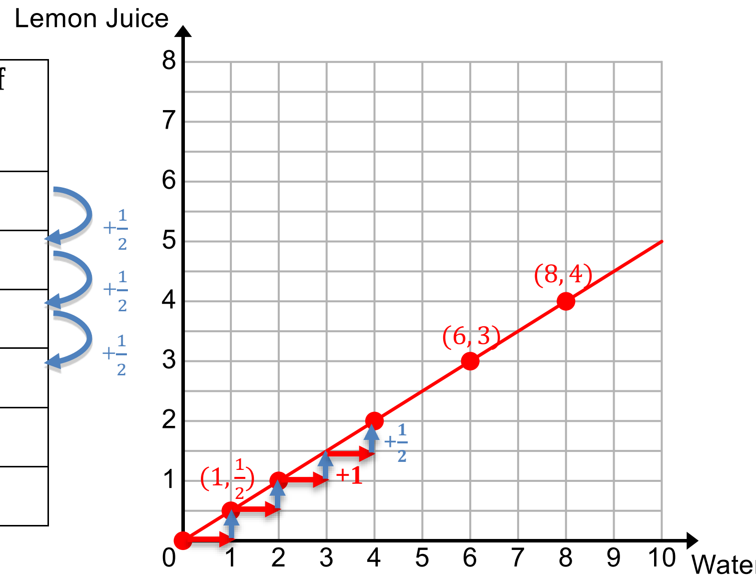
   b. What does the point $(12, 4)$ represent in the situation? A bouquet with 12 roses and 4 sunflowers

   c. Which point represents a total of 20 flowers?
   $(15, 5)$ A bouquet with 15 roses and 5 sunflowers

   d. If I have 45 roses, how many sunflowers should I have to keep the same ratio? Write your answer as an ordered pair. 15, $(45, 15)$ students should continue to recognize the correspondence across the table as we will ultimately move students toward writing an equation to show the relationship between the two quantities. If we look back at question 1, we can ask "If I know the number of sunflowers, how can I determine the number of roses?" Multiply by 3. Referring to question 2, "If I know the number of roses, how can I determine the number of sunflowers?" Divide by 3 (or multiply by $\frac{1}{3}$).

3. A recipe calls for ½ cup lemon juice for each cup of water. Complete the table and plot the pairs of values on the coordinate plane.

Cups of Water	Cups of Lemon Juice
0	0
1	0.5
2	1
3	1.5
4	2
30	15



a. What does the point $(4, 2)$ represent in this situation?
4 cups of water and 2 cups of lemon juice

b. If I use 100 cups of water, how many cups of lemon juice should I use to keep the same ratio? Write your answer as an ordered pair.
50; $(100, 50)$

c. If I have used 60 cups of liquid, how many cups of each, water and lemon juice, did I use? Write your answer as an ordered pair.
40 cups of water and 20 cups of lemon juice; $(40, 20)$

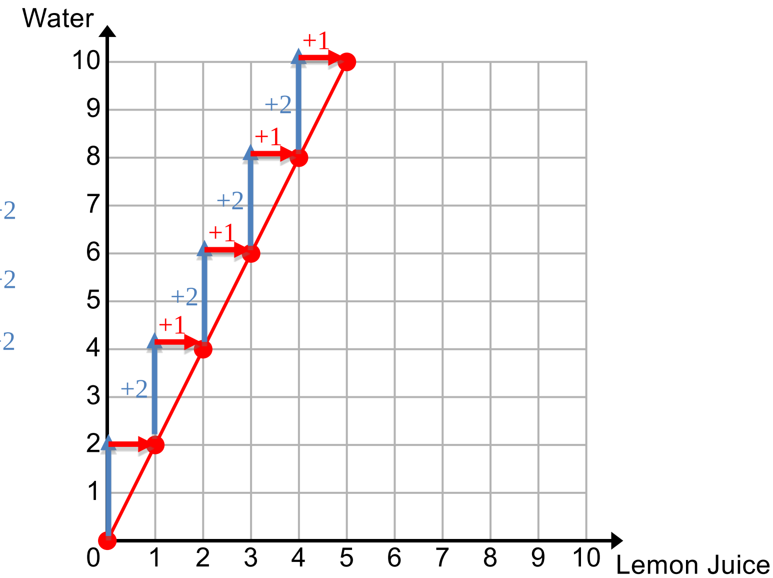
d. Hamid mixed 2 cups of lemon juice and 3 cups of water. Did Hamid follow this recipe? Explain.
No, students can show this on the graph or show that the ratios and/or unit rate are not equivalent

e. What is the unit rate (cups of lemon juice/cups of water)? Where do you see the unit rate on your table and graph? $\frac{\frac{1}{2}\text{ cup lemon juice}}{1\text{ cup water}}$; see teacher note below and help students annotate the table and graph to show the unit rate.

Throughout this section, students should be looking for structure in the tables and graphs of proportional relationships and using this structure to explain correspondences between the table, graph, and equation. Students may observe the additive structure that exists looking down the columns of the table (shown in red and blue). This additive structure helps students to think about the unit rate as the amount of increase in y as x increases by 1. This can be seen on a graph by looking at the vertical increase in a "unit rate triangle (slope triangle with a horizontal side length of 1)". Students will not formally learn about slope until 8th grade; however in 7th grade students will experience the repeated reasoning of linear relationships and begin to surface ideas about slope. We can also observe the unit rate on the graph in the ordered pair $(1, r)$ where r represents the unit rate. This point shows the amount of y for one unit of x.
Students should also observe the structure/correspondence going across the table (shown in green). To get the output, the input is multiplied by the unit rate/proportional constant. Students should also observe that the quotient $\frac{y}{x}$ is constant for each pair of values in the table and on the graph by writing the ordered pairs that correspond with different points as shown above.

4. Let’s look at the lemon juice problem again but this time we will put lemon juice on the *x*-axis and water on the *y*-axis. Complete the table and plot the pairs of values on the coordinate plane.

Cups of Lemon Juice	Cups of Water
0	0
1	2
2	4
3	6
4	8
30	15



a. What does the point $(2, 4)$ represent in this situation?
2 cups of lemon juice and 4 cups of water. Tie back to part a. in the previous problem. When we switch the quantities on the axes, the relationship between the variables does not change; however our input and output have changed.

b. What is the unit rate (cups of water/cups of lemon juice)? 2 cups of water/1 cup of lemon juice; have students show on the table and graph where they see the unit rate.

c. Where do you see the unit rate in each of the representations?
In the table, the unit rate can be seen:
- Where the input = 1
- In the quotient $\frac{y}{x}$ for each pair of input/output values.
- In the difference in *y* as *x* increases by 1 unit

In the graph, the unit rate can be seen:
- Where the input = 1: $(1, r)$ where r is the unit rate
- In the vertical change in *y* as *x* increases by 1 unit
- The quotient $\frac{y}{x}$ for each ordered pair

d. Compare the unit rate, table, and graph in this problem to the unit rate, table, and graph in the previous problem. What do you notice? Students should notice that the unit rates are inverses of each other.

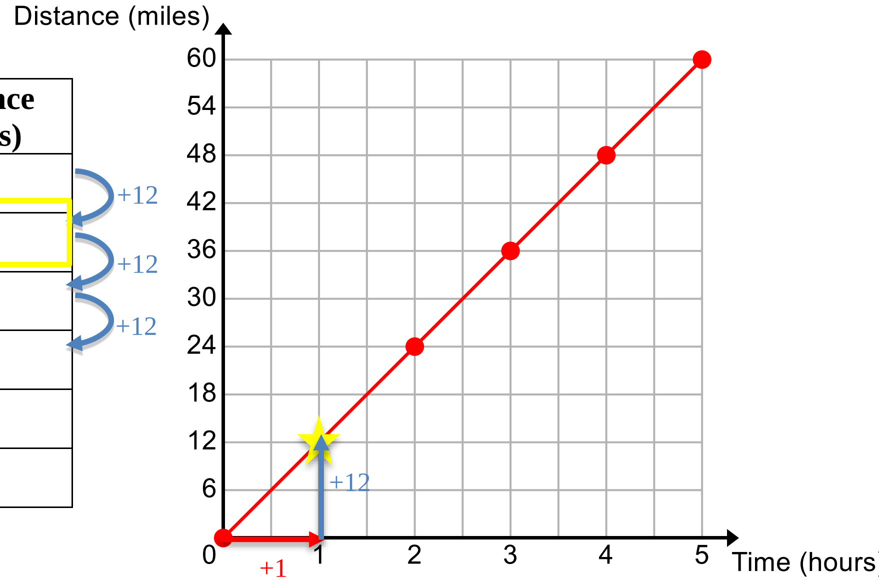
In a proportional relationship, the **unit rate** shows the amount of y that corresponds to one unit of x. We can also think of this as the amount of increase in y as x increases by 1 unit. On a graph, the unit rate is illustrated in the point $(1, r)$ where r is the unit rate. The unit rate can also be determined by finding the quotient $\frac{y}{x}$. In a proportional relationship, $\frac{y}{x}$ is constant. This is also called the **constant of proportionality** or **proportional constant**.

As the section continues, students will be asked to give the unit rate and the units will not be specified in the question. Students should know that the unit rate is equal to $\frac{y}{x}$ and they should include the correct labels in their answer.

5. Jon is riding his bike at a speed of 12 mph. Complete the table and graph to show this relationship.
Be sure to discuss the scale on the graph with students.

Time (hours)	Distance (miles)
0	0
1	12
2	24
3	36
4	48
5	60

+1 +1 +1 +12 +12 +12 •12



a. What does the point $(4, 48)$ represent in the context?
Jon rides 48 miles in 4 hours

b. What does the point $(0, 0)$ represent in the context?
Jon rides 0 miles in 0 hours

c. What is the unit rate? Annotate your table and graph to show the unit rate. 12 mph

d. If Jon bikes for 10 hours at this rate, how far will he go? Explain how you got your answer.
120 miles

e. Mary can bike 46 miles in 4 hours. Who can bike faster? One way to do this problem is to calculate the unit rate for Mary. She can bike 11.5 mph so Jon is faster. Alternatively, students can graph the point $(4, 46)$ and see that it is below the line representing Jon's distance or students may observe that Jon traveled 48 miles in 4 hours.

You will notice that we don't ask students for points that represent totals (i.e. which point represents 60 total cups of liquid). We cannot ask questions such as these in this problem because the quantities have different units.

6. The table below shows the ratio of lemon juice to water for a recipe. The graph shows the ratio of lemon juice to water used in the problem above. Plot the points in the table on the same coordinate plane using a different color.

Cups of Water	Cups of Lemon Juice
0	0
3	2
6	4
9	6
12	8

+3 +3 +3 +2 +2 +2

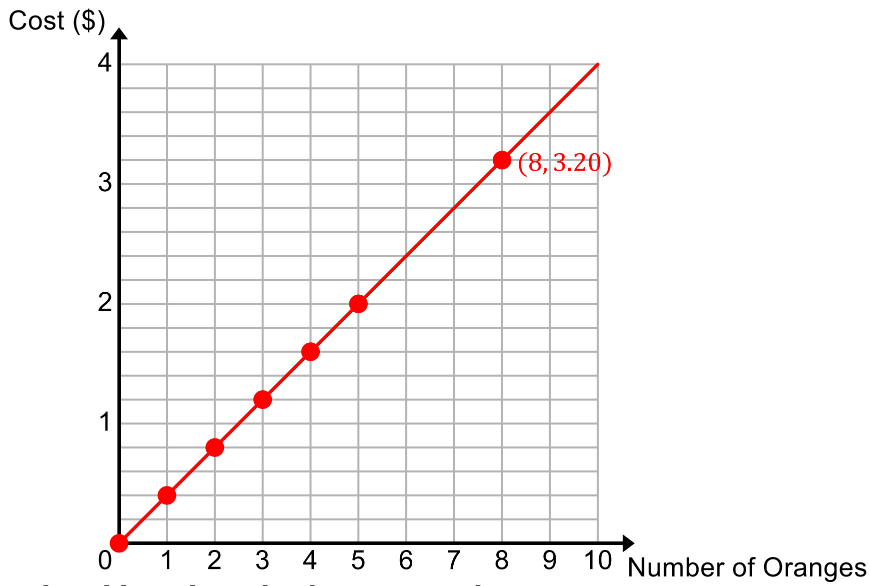
a. Are the recipes the same? How do you know?
No, see student responses

b. What is the unit rate for each of the recipes?
The unit rate shown in the table is $\frac{\frac{2}{3}\text{ lemon juice}}{1\text{ water}}$. Students can calculate the unit rate by finding the quotient $\frac{y}{x}$. The unit rate shown on the graph is $\frac{\frac{1}{2}\text{ lemon juice}}{1\text{ water}}$.
We want to help students to see the unit rate on the table and graph. Again, we are encouraging students to think about the amount of y that corresponds to one unit of x. Looking down the column in the table, we see that the inputs increase by 3 each time and the outputs increase by 2. Students can make the inputs go up by 1 by partitioning the interval into 3 equal segments. If we partition the difference in the output column into 3 equal parts, we see that each part has a value of $\frac{2}{3}$ units. A model similar to those used in 4.1 will help students to see this. Students can also observe on the graph that an input of 1 corresponds to an output of $\frac{2}{3}$ for one recipe and $\frac{1}{2}$ for the other recipe.

c. Aria is using 5 cups of lemon juice. How many cups of water should she use if she follows each recipe? Explain how you got your answers.
In the recipe shown on the table, she would use $3\frac{1}{3}$ cups of water. In the one given on the graph, she would use $2\frac{1}{2}$ cups of water. There are several methods a student may use to solve this problem, including using a model from 4.1, multiplying the input by the proportional constant, or using the graph to find the value of y when $x = 5$.

7. It costs \$2.00 for 5 oranges. Complete the table and graph to represent this situation.

Number of Oranges	Cost (\$)
1	0.40
2	0.80
3	1.20
4	1.60
5	2.00



a. What is the unit rate? Show on the table and graph where we see the unit rate.
   \$0.40 per orange; $(1, 0.40)$
b. If you buy 8 oranges, how much will you spend? Label this point on the grid.
   \$3.20
c. Owen paid \$2.45 for 7 oranges. Did he pay more or less? Explain and use your graph to support your explanation. Owen paid less. He paid \$0.35 per orange. If we look at the graph, 7 oranges at the rate in the problem would cost \$2.80.

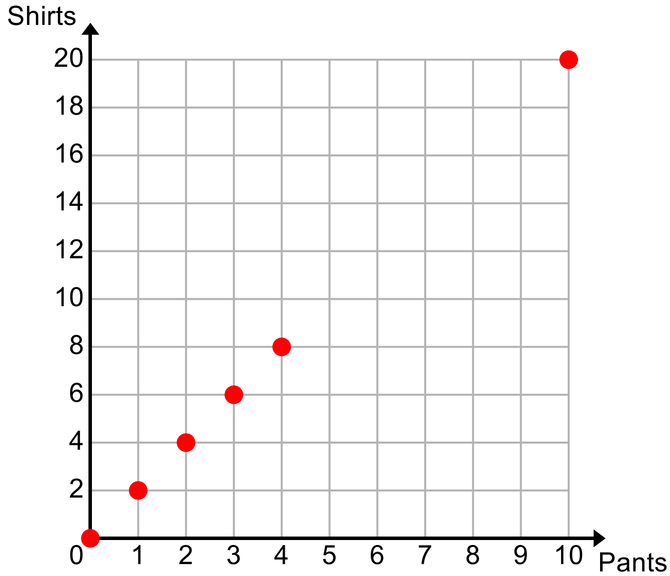
Spiral Review

1. Use a model to solve:
   a. $-14 = 3x - 2$
   b. $-8 = -3m + 10$
2. Evaluate $-2(x+1) - x$ for $x = -3$. 7
3. Write $\frac{17}{20}$ as a percent and decimal (without a calculator.)
4. Malory got a 75% on her math test. To earn that score, she got 27 questions correct. How many questions were on the test?
5. If 4 gallons of gas cost \$14.60, how much does 10 gallons of gas cost?

4.2a Homework: Graphs of Proportional Relationships

1. Jane has 2 shirts for every pair of pants in her closet. Complete the table to show this relationship. Plot the pairs of values on the coordinate plane.

Pairs of Pants	Shirts
0	0
1	2
2	4
3	6
4	8
10	20



a. What does the point $(4, 8)$ represent in the situation? 4 pants and 8 shirts

b. What does the point $(1, 2)$ represent in the situation? 1 shirt and 2 pants; the *y*-value is the unit rate.

c. If Jane has 8 pairs of pants, how many shirts does she have? Write the ordered pair that represents this combination. 16; $(8, 16)$

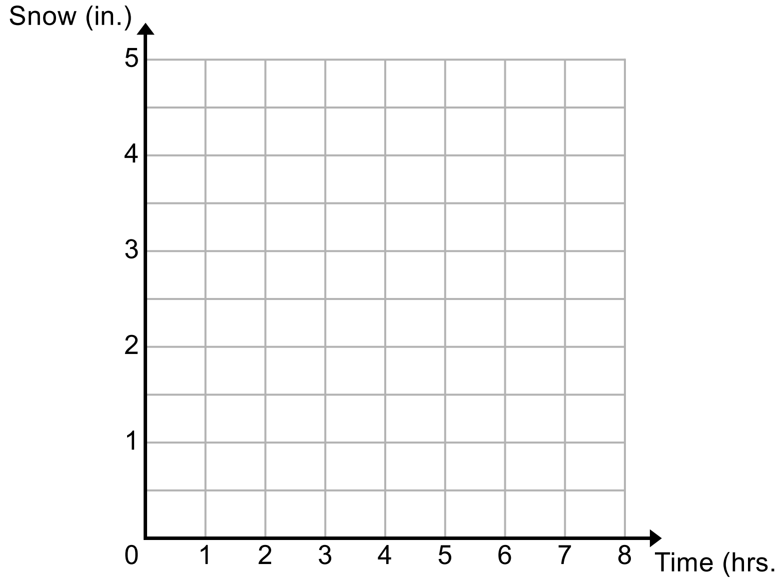
d. Complete the following ordered pairs to show this relationship: (7, _14_) (15, _30_) (__9_,18)

e. Which point represents a total of 24 articles of clothing?
$(8, 16)$ 8 pairs of pants and 16 shirts

f. What is the unit rate? Show on the table and in the graph where you see the unit rate. $\frac{\text{2 shirts}}{\text{1 pant}}$

2. During a snowstorm, it snowed $\frac{1}{2}$ inch every hour. Complete the table and graph to show this relationship.

Time (hrs.)	Snow (in.)
0	
1	
2	
3	
4	
5	



a. What does the point $(8, 4)$ represent in the context?

b. What is the unit rate in this situation? Show on the table and graph where you see the unit rate.

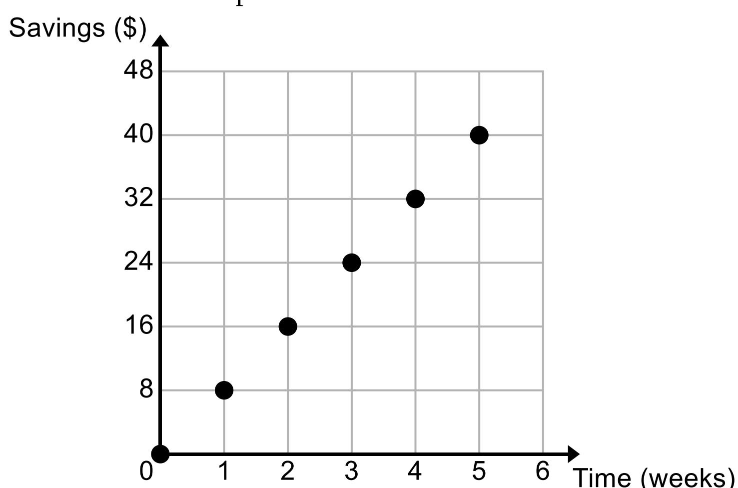
c. Put a star on the point on the grid that shows the unit rate. Write the ordered pair that represents the unit rate.

d. If it snows at this rate for 24 hours, how much snow will fall? Explain how you got your answer.

e. In a nearby town, it snowed $\frac{1}{4}$ inch every $\frac{1}{2}$ hour. Compare the rates of snowfall.

3. Toby is saving money to buy a gaming system. The graph below shows the amount of money Toby saves over time. Complete the table to show this relationship.

Time (weeks)	Savings ($)
0	0
1	8
2	16
3	24
4	32
5	40



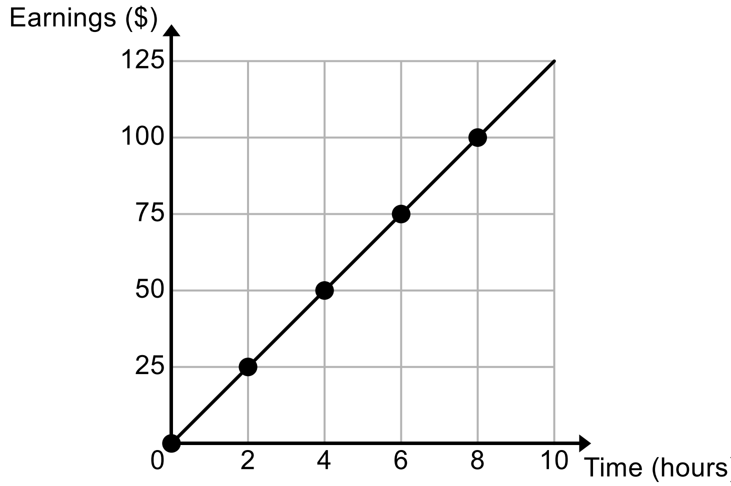
a. How much does Toby save each week? $(1, 8)$

b. The gaming station that Toby wants to buy costs $200. If Toby continues saving at this rate, how long will he have to save for? Explain how you got your answer.

25 weeks, Students may divide 200 by 8. Alternatively, they may use ratio reasoning – it takes 5 weeks to save $40, to get to $200 I multiply 40 by 5 so I would also multiply my time by 5 which is 25 weeks

4. The graph below shows the amount Gabby makes based on the number of hours she works. Complete the table to show this relationship.

Time (hours)	Earnings ($)
1	
2	
3	
4	
5	
6	



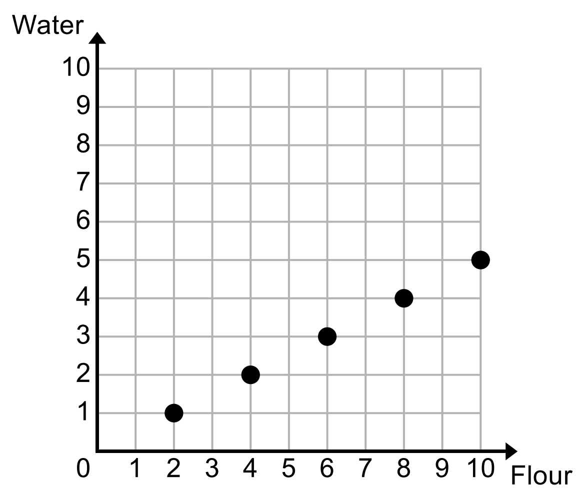
a. What is Gabby's hourly wage? Write the ordered pair that represents this point.

b. If Gabby works for 7 hours, how much money will Gabby earn? Explain how you got your answer.

c. How many hours does Gabby need to work to earn $150? Explain how you got your answer.

5. **Find, Fix, and Justify:** Rakesh was asked to plot the points shown in the table below. Rakesh made a common mistake. Explain the mistake that Rakesh made and graph the points correctly.

Flour	Water
1	2
2	4
3	6
4	8
5	10

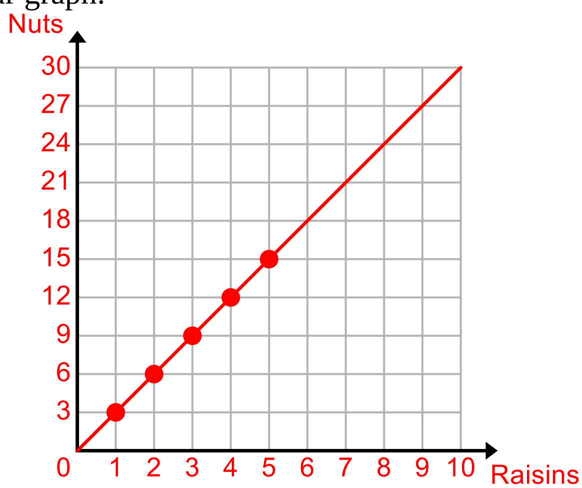


4.2b Class Activity: More Graphs of Proportional Relationships

Graphs are a tool that allow students to analyze the relationship between two quantities. In order to utilize this tool, students need to know how to proficiently construct graph, including scaling and labeling the graph. In this lesson and the ones that follow, we focus on building these skills for students.

1. In a bag of trail mix, there are 3 cups of nuts for every 1 cup of raisins. Complete the table and graph to show this relationship. Be sure to label your graph.

Raisins (cups)	Nuts (cups)
1	3
2	6
3	9
4	12
5	15



As a class, you may choose to use a different scale on the *y*-axis (possibly a scale of 1). Either way, be sure that students pay attention to the scale on both axes when creating and reading the graph.

a. What does the point $(3, 9)$ represent in the situation?
   A bag with 3 cups of raisins and 9 cups of nuts

b. What is the unit rate? $\frac{\text{3 cups of nuts}}{\text{1 cup of raisins}}$

c. What does the point $(1, 3)$ represent in the situation? 1 cup of raisins and 3 cups of nuts, the *y*-value represents the unit rate

d. If I use 7 cups of raisins, how many cups of nuts should I use? Write the ordered pair that represents this ratio. 21; $(7, 21)$

e. If I have a total of 60 cups of ingredients, how many cups of each ingredient did I use? Write your answer as an ordered pair. $(15, 45)$

f. Miguel mixed 4 cups of raisins with 7 cups of nuts. Did he follow this recipe? Explain.
   No, the unit rate is 1.75 cups nuts/cup raisins. Students can also show on the graph that this ordered pair does not fall on the line.

2. **Find, Fix, and Justify:** For the problem above, Darcy was asked what the ordered pair $(5, 15)$ represents in the situation. Darcy made a common mistake and said that the point represents 5 cups of nuts and 15 cups of raisins. Explain the mistake that Darcy made and write the correct answer.
   Darcy may not have realized that the first quantity in an ordered pair corresponds to the *x*-axis (horizontal axis) and the second quantity corresponds to the *y*-axis (vertical axis). Alternatively, she may have mixed up which axis was the *x*-axis and which axis was the *y*-axis. The correct answer is that this point corresponds to 5 cups of raisins and 15 cups of nuts.

3. Cherries cost $1.50 per pound. Complete the table and graph to show this relationship.

Cherries (lbs.)	Cost ($)
0	0
1	1.50
2	3.00
3	4.50
4	6.00
5	7.50

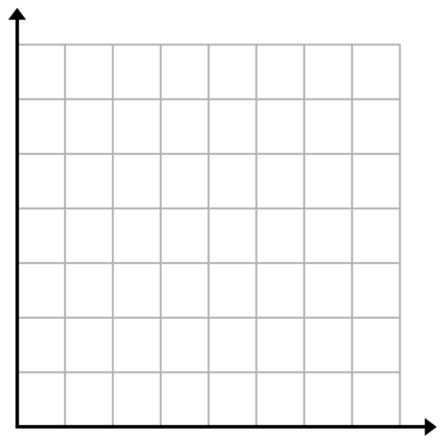
a. What does the point $(8, 12)$ represent in the context?
8 pounds of cherries cost $12

b. Put a star on the point on the grid that shows the unit rate in this situation. Write the ordered pair that shows the unit rate.
$(1, 1.50)$ Be sure to have students observe that the quotient $\frac{y}{x}$ for each pair of values remains constant and is equal to the unit rate. Students can use a calculator to demonstrate this.

c. If you buy 3 pounds of cherries, how much will you spend? Write the ordered pair that corresponds to this point. $4.50; $(3, 4.50)$

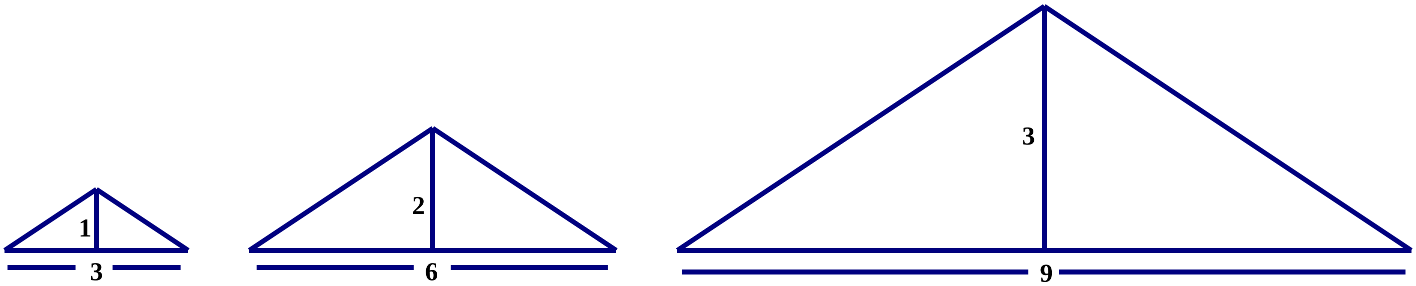
4. There are 150 calories in $\frac{1}{4}$ lb. of meat. Complete the table and graph to show this relationship. Be sure to label your graph.

Meat (lbs.)	Calories
0	
$\frac{1}{2}$	
1	
$\frac{3}{2}$	
2	
$\frac{5}{2}$	



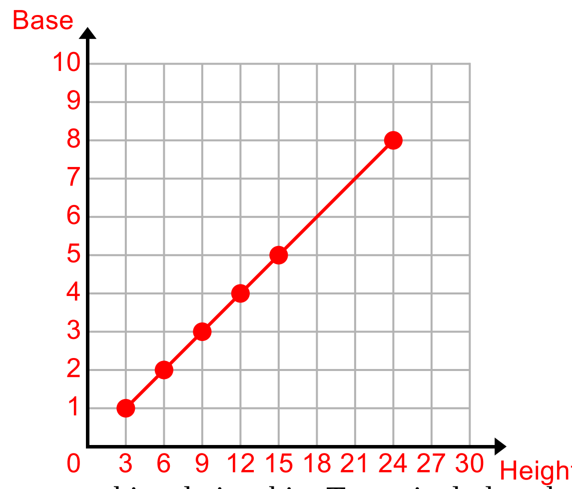
a. Put a star on the point on the grid that shows the unit rate in this situation. Write the ordered pair that shows the unit rate. What does it represent in the situation?

5. Gabriela is making several triangles that are the same shape but different sizes as shown in the picture below.



a. Complete the table and graph to show the ratio of the height to the base for the triangles.

Base	**Height**
3	1
6	2
9	3
15	5
24	8

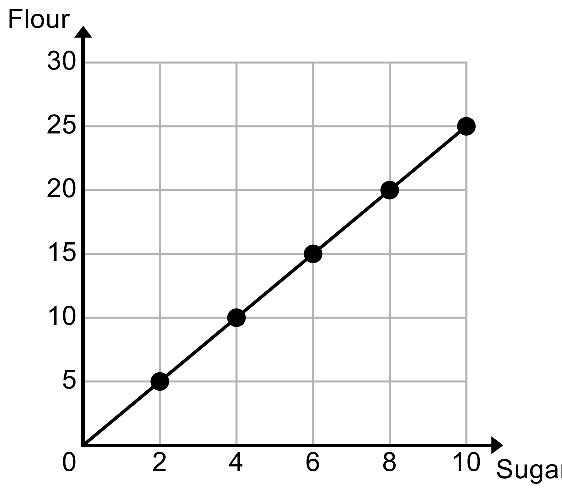


b. Write 3 additional ordered pairs that represent this relationship. Try to include at least one fraction in your ordered pairs.

$(33, 11)$ $(45, 15)$ $(8, 2\frac{2}{3})$

6. The graph below shows the ratio of flour to sugar used in a cookie recipe. Use the graph to complete the table.

Cups of Sugar	**Cups of Flour**

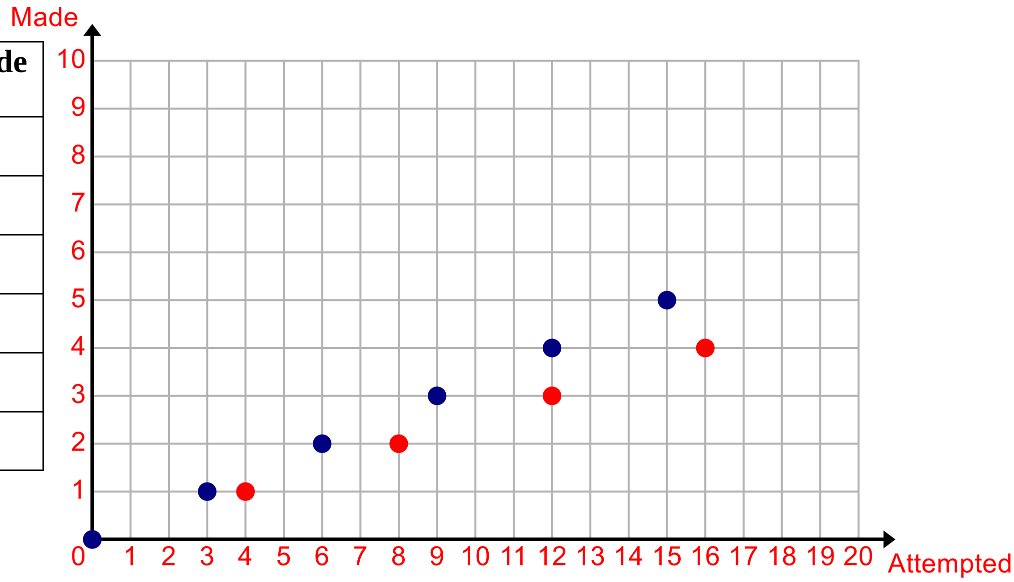


a. What does the point $(6, 15)$ represent in this situation?

b. For this recipe, if I use 1 cup of sugar, how many cups of flour should I use? Write your answer as an ordered pair. Put a star on the approximate location of this point on the graph.

7. Tim is on the basketball team. His ratio of shots made to shots attempted is 1:4. Complete the table and graph to show this relationship.

Shots Attempted	Shots Made
0	0
4	1
8	2
12	3
16	4
28	7



a. What is the unit rate $\left(\frac{\text{shots made}}{\text{shots attempted}}\right)$? $\frac{\frac{1}{4}\text{ shots made}}{1\text{ shots attempted}}$; Be sure to show this on the table – what is the change in *y* when *x* goes up by 1?

b. Complete the following ordered pair: (1, $\frac{1}{4}$). What does this point represent in the situation? Tim makes $\frac{1}{4}$ shot for every shot he attempts. The *y*-value represents the unit rate.

c. If Tim attempts 40 shots, how many would you expect him to make?
10

d. Alli's ratio of shots made to shots attempted is $\frac{1}{3}$: 1. Complete the table and graph to show the relationship for Alli. Use a different color on the graph.

Shots Attempted	Shots Made
0	0
3	1
6	2
9	3
12	4
21	7

e. Do Alli and Tim have the same scoring percentage? If not, who is better? Explain using at least two pieces of evidence. This problem may be counter-intuitive for many students due to the fact that 3 is smaller than 4; however Alli is better. She makes 1 shot for every 3 attempts while Tim makes 1 shot for every 4 attempts (or you could think about the fact that Tim only makes $\frac{3}{4}$ of a shot for every 3 attempts compared to Alli's 1). Students may reason through this problem or use the different representations. On the graph, we see that Alli's line is steeper indicating that she has a higher shooting percentage. If we focus on an input of 12, Alli would make 4 shots while Tim would only make 3.

8. Mario will pay you 360 gold coins for 2 hours of racing Go-Karts. Luigi will pay you 420 gold coins for 3 hours of racing Go-Karts. Find the best offer using three different methods (table, rate and graph.)

a. Complete the table.

Mario's Offer	
Hours	Payment
1	180
2	360
3	540
4	720
5	900

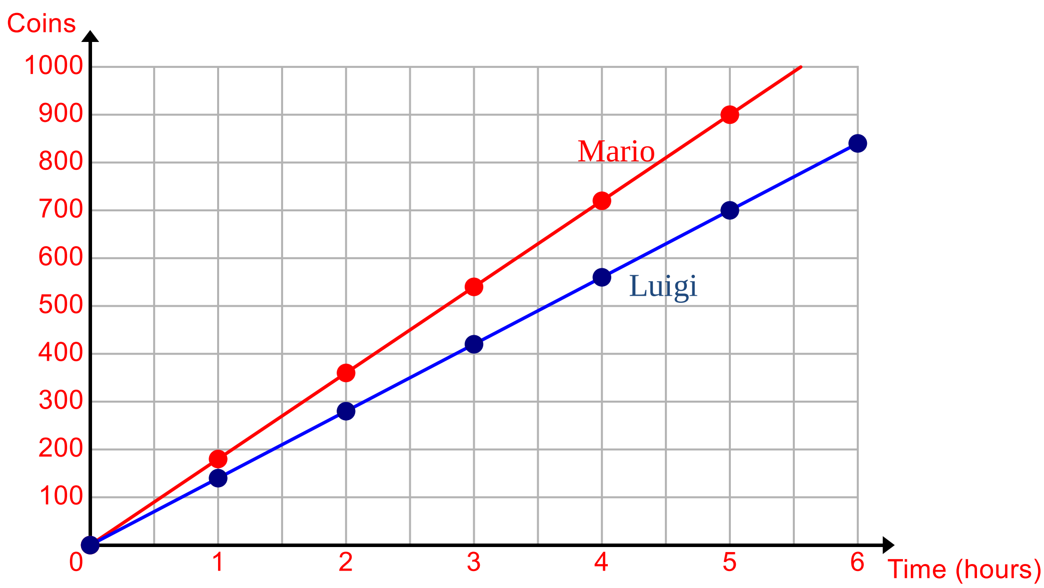
Luigi's Offer	
Hours	Payment
1	140
2	280
3	420
4	560
5	700

b. Find the unit rate for each offer, including labels.

Mario's: 180 coins per hour

Luigi's: 140 coins per hour

c. Graph each offer in a different color on the same coordinate plane. Label your axes.
Since many of the points in this problem do not fall on a corner of the gridlines, strategize with students about a quick and efficient way to create these graphs. Since two points determine a line, students can graph the origin and a point that falls on the corner of the gridlines, then go back and check a few other points to verify that the line is correct.



d. Which is the best offer? Why? (In your explanation, explain how the table, unit rate and graph help you to see the best offer.) Mario's offer is better because he will pay you more coins per hour than Luigi. You can see it on the top row of each table. On the graph, Mario's line is steeper than Luigi's. You can see the difference in the offers by examining the vertical gap between the two lines.

Spiral Review

1. Order the following fractions from least to greatest.

$$-\frac{9}{10}, -\frac{3}{7}, -\frac{3}{4}, -\frac{6}{5}, -\frac{4}{3}$$

2. I go to a department store with a coupon for 20% off any one item. The shoes that I want are on sale for 40% off. What was the original price if I paid $48?

3. Use the distributive property to rewrite the following expression $-4(5x - 1)$.

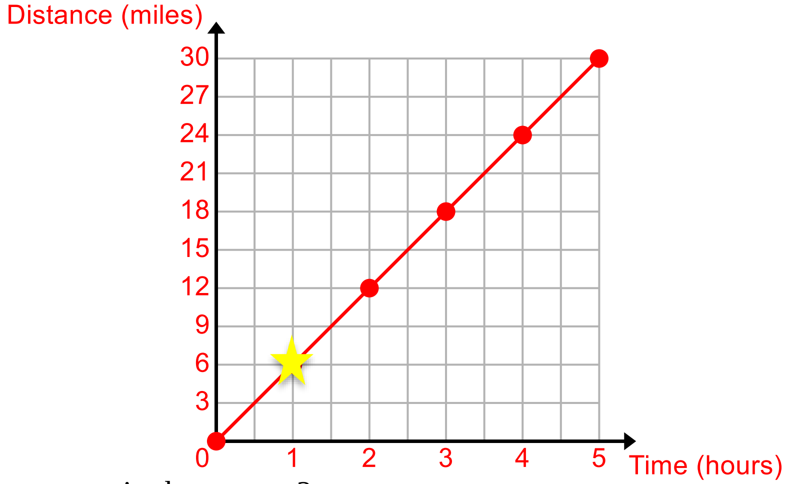
4. Find each sum without a model.
   a. $-27.2 + \frac{4}{5}$
   b.
   c. $98.1 + (-1.1)$

5. A mouse can travel 1.5 miles in $\frac{3}{4}$ of an hour.
   a. At that pace, how far can it travel in 1 hour?
   b. At that pace, how long does it take it to travel one mile?

4.2b Homework: More Graphs of Proportional Relationships

1. In a marathon, Toni ran 6 mph. Complete the table and graph to show this relationship. Be sure to label your graph and think about how to scale the graph.

Time (hours)	Distance (miles)
0	0
$\frac{1}{2}$	3
1	6
$\frac{3}{2}$	9
2	12
$\frac{5}{2}$	15



a. What does the point $(1.5, 9)$ represent in the context?
In 1.5 hours, Toni runs 9 miles

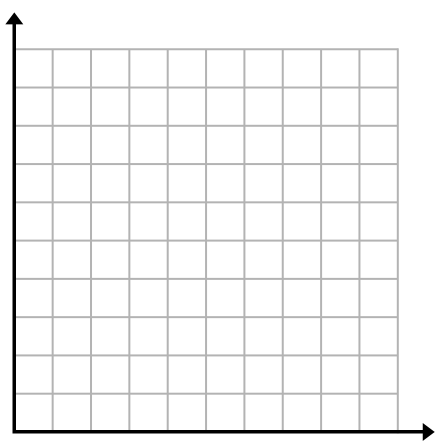
b. What is the unit rate? Write the ordered pair that represents the unit rate. Highlight the unit rate in your table and on your graph. 6 mph $(1, 6)$

c. If a marathon is 26.2 miles, approximately how long will it take Toni to finish? Approximately 4.4 hours (4 hours and 24 minutes);

d. Maggie was also in the marathon. She ran 20 miles in 3.5 hours. Which runner was going faster?
Toni is running faster; students can show that Maggie's rate is approximately 5.7 mph. They may also reason that Toni can run 21 miles in 3.5 hours.

2. Spencer can run one mile in 8 minutes. Complete the table and graph to represent this situation.

Time (min.)	Distance (miles)
0	
4	
8	
12	
16	
20	



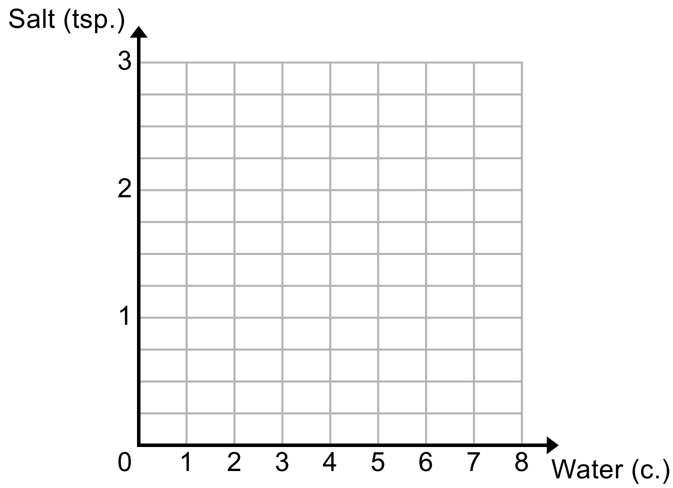
a. Complete the ordered pair $(1, \_\_)$. What does this point represent in the situation?

b. If Spencer runs for one hour, how far can he run?

c. Emily can run one mile in 10 minutes. Who is faster? Explain.

Amelia is doing a science experiment with salt water solutions. Solution A calls for $\frac{1}{4}$ tsp. salt for every $\frac{1}{2}$ c. of water. Complete the table and graph to represent this situation.

Water (c.)	**Salt (tsp.)**
0	
1	
2	
3	
4	



a. Solution B calls for $\frac{1}{4}$ tsp. salt for every $\frac{1}{3}$ c. of water. Complete the table to represent Solution B. Graph Solution B on the same graph above using a different color.

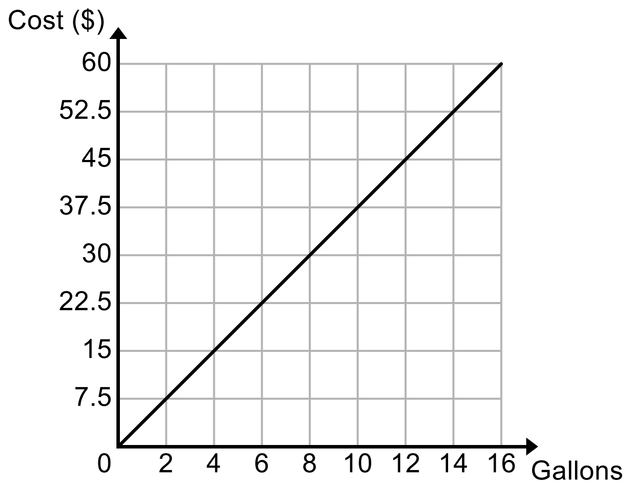
Water (c.)	**Salt (tsp.)**
0	
1	
2	
3	
4	

b. Which solution is saltier, Solution A or Solution B? Justify your answer with at least two pieces of evidence.

c. Solution C calls for $\frac{1}{8}$ tsp. salt for every $\frac{1}{4}$ c. of water. Which solution is saltier, Solution A or Solution C? Describe what the graph of Solution C would look like.

3. The graph below shows the cost of gas based on the number of gallons of gas pumped. Complete the table to show this relationship.

Amount (gallons)	Cost ($)
0	0
1	3.75
2	7.50
3	11.25
4	15.00
5	18.75



a. How much does gas cost per gallon? Write the ordered pair that represents this point.
$3.75 per gallon; (1, 3.75)

b. If Byron pumps 22 gallons of gas, how much will it cost? Explain how you got your answer.
$82.50 – multiply 22 by $3.75

c. At a different gas station, Hugo paid $28.80 for 8 gallons of gas. Did Hugo pay more or less per gallon? Explain. Use your graph to support your explanation. Hugo paid less. He paid $3.60/gallon. On the graph, it shows that it would cost Byron $30 for 8 gallons of gas. Since Hugo only paid $28.80, he paid less.

4. **Find, Fix, and Justify**: Jerry earns $10 an hour babysitting. James was asked to complete the table below to show this relationship. James made a common error when completing the table. Describe the error James made and fix the table so that it is correct.

Time (hours)	Earnings ($)
0	10
1	20
2	30
3	40
4	50

Time (hours)	Earnings ($)
0	
1	
2	
3	
4	

5. The Jones Family drives 200 miles in 5 hours. The Grant Family drives 360 miles in 6 hours. Compare the car trips. Assume they both traveled at a constant rate.

a. Complete the table.

Jones Family	
Hours	Miles
1	
2	
3	
4	
5	

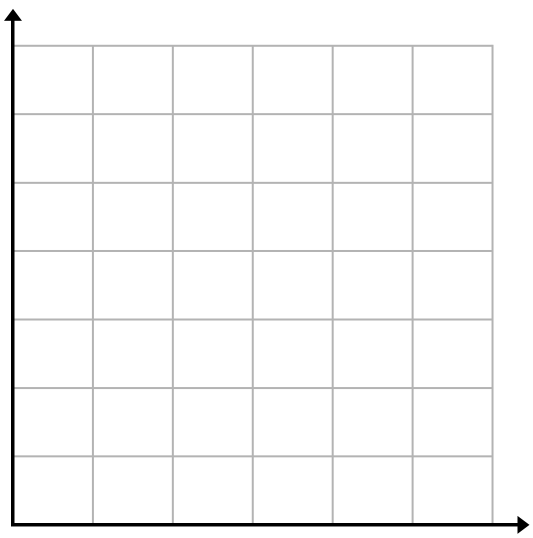
Grant Family	
Hours	Miles
1	
2	
3	
4	
5	

b. Find the unit rate for each trip, including labels.

Jones:

Grant:

c. Graph each family's trip in a different color on the same coordinate plane. Label your axes. This about a good scale for the data.

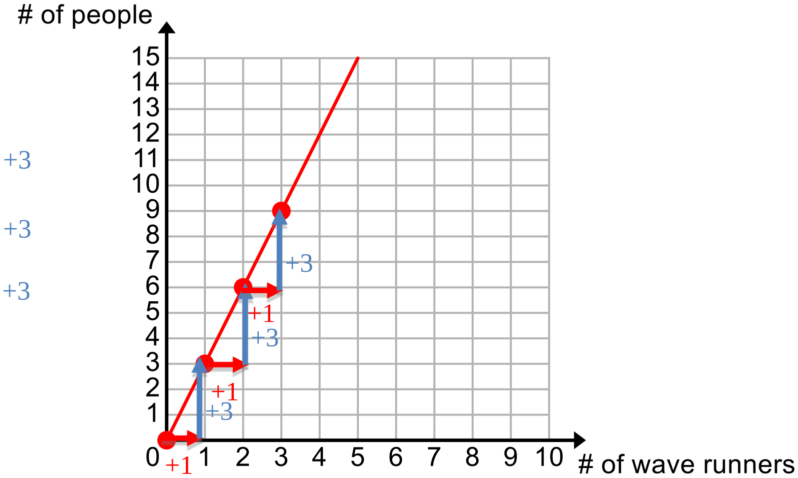


d. Compare the driving trips of the two families. In your comparison, explain how the table, unit rate, and graph help you to see the difference in the two trips.

4.2c Class Activity: Equations of Proportional Relationships

1. Peter is renting wave runners for a company party. Each wave runner holds 3 people. Complete the table and graph to show this relationship.

# of Wave Runners	# of People
0	0
1	3
2	6
3	9
8	24
w	$3w$



a. Complete the table below.

# of Wave Runners	Expression for Number of People	Number of People
1	1 • 3	3
2	2 • 3	6
3	3 • 3	9
4	4 • 3	12
5	5 • 3	15
w	w • 3	$3w$

One way for students to arrive at the equation is to first think about the equation numerically and then move into the symbolic (abstract) representation of the equation. The table shown allows students to tie the additive structure we have examined in the tables and the graphs to the equation:
For each one unit you move to the right on graph, move up 3 units.
When you go 2 units to the right, you go up 2 • 3 units.
When you go 3 units to the right, you go up 3 • 3 units.
When you go 4 units to the right, you go up 4 • 3 units.
When you go w (or x) units to the right, you go up 3 • w (or 3 • x) units.
Starting from $(0, 0)$, to get to a point (x, y), go x units to the right, so go up x • 3 units. Therefore $y = 3x$.

b. Write an equation that shows the relationship between the number of people p that can ride and the number of wave runners w.

By this point, students have been using the equation informally in order to complete the table. If students need scaffolding, you may provide blanks for the students to fill in:

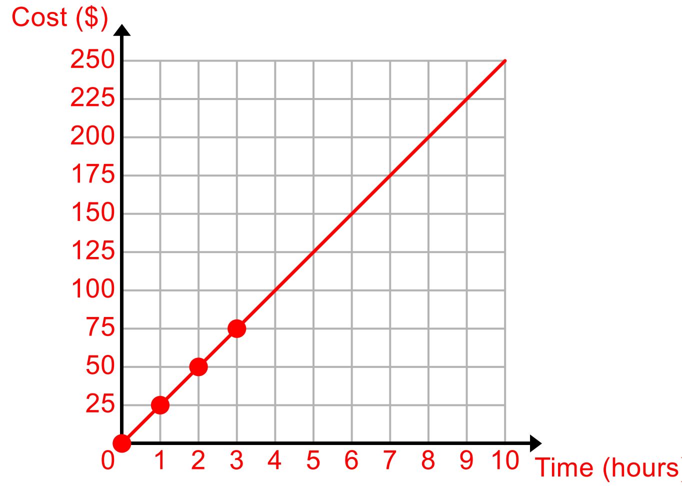
[# of people] = 3 • [# of wave runners]

Then, they can replace the phrases with the variables given.

c. Use your equation to determine the number of people that can ride if Peter rents 20 wave runners. 60

2. Bubba's Body Shop charges $25 per hour to fix a car. Complete the table and graph to represent this situation.

Time (hours)	Cost (dollars)
0	25
1	50
2	75
3	75
8	200
t	25*t*



a. Complete the table below.

# of Hours	Expression for Cost	Cost
1	1 • 25	25
2	2 • 25	50
3	3 • 25	75
4	4 • 25	100
5	5 • 25	125
t	*t* • 25	25*t*

Connect the expression that represents cost to the table and graph.

b. Write an equation that models the relationship between the cost *C* Bubba charges to fix your car based on the number of hours *t* he works on it.

$C = 25t$

c. Where do you see the unit rate in the equation?

In the equation, the unit rate is the coefficient in front of *t* (or the input/independent variable).

d. Use your equation to determine how much Bubba would charge if it takes him 12 hours to fix your car.

$300, students should substitute in 12 for *t* and solve

e. Use your equation to determine the number of hours Bubba worked on your car if he charged you $175.

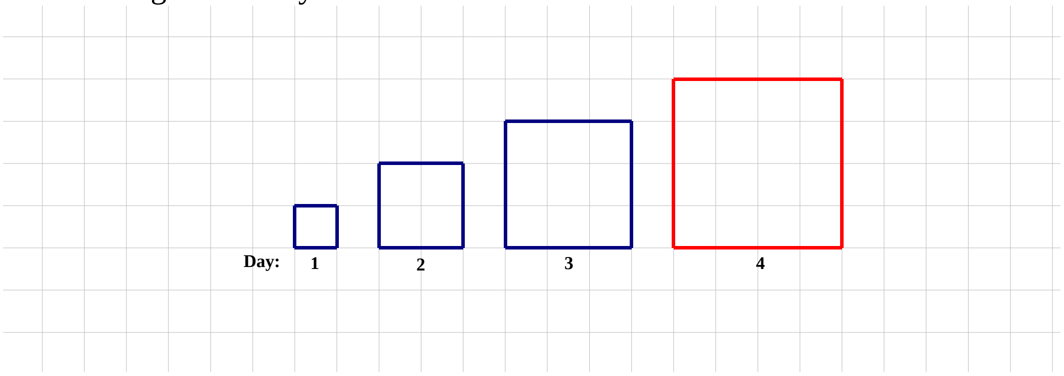
7 hours, students should substitute in $175 for *y* and solve

f. How do you think equations are useful?

An equation is a tool in math that models the relationship between two quantities. Once we have an equation, we can use the equation to determine outcomes. Equations are a quick and efficient way to determine outcomes for larger values of *x* or for inputs that are fractions (or result in fractional outputs) that may be difficult to identify on a graph.

3. Roberto is recovering from an injury. He increases the number of blocks he walks each day in the following pattern. The arrow represents one street block.

   a. Draw the figure for Day 4 and write the number of blocks Roberto will walk.



   b. Complete the table.

Day *(d)*	1	2	3	4	5	8	14	16	23	*d*
Blocks *(b)*	4	8	12	16	20	32	56	64	92	4*d*

   c. What is the unit rate? 4 blocks/day

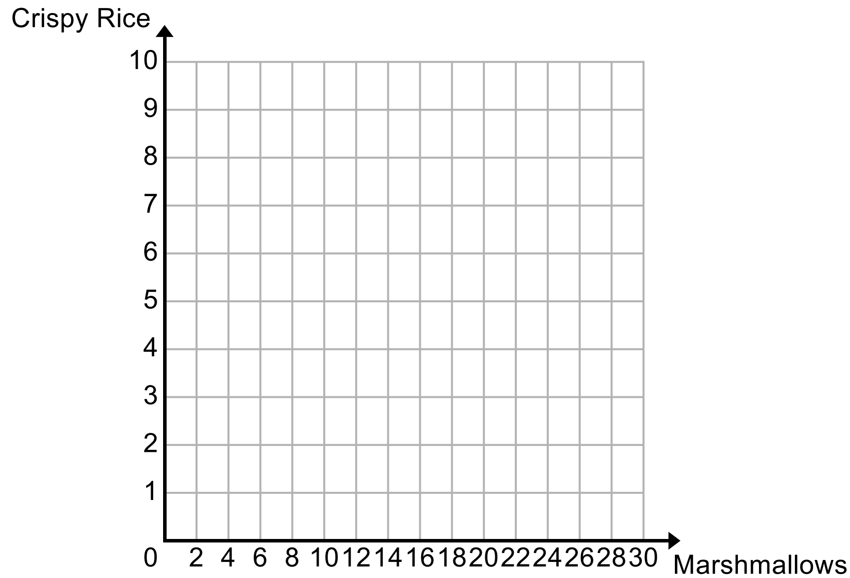
   d. Write an equation that shows the relationship between day *d* and the number of blocks *b* Roberto walks.

   $b = 4d$

Students should see that in order to get the output, we multiply the input by the constant of proportionality (unit rate). Some students may write the equation in the form $d = \frac{b}{4}$ which is OK. This equation does show the relationship between the two quantities. It will be more difficult for them to connect the unit rate to the equation so you may wish to help them transform the equation to the one shown above. This relates back to concepts studied in 4.1 – if we switch our inputs and outputs, the unit rates are inverses.

4. Carmen is making rice crispy treats for an upcoming bake sale. Carmen uses 6 cups of crispy rice for 36 jumbo marshmallows. Complete the table and graph to show this relationship. Be sure to label the axes of your graph.

Number of Marshmallows	Cups of Crispy Rice
0	
6	
12	
30	
48	
1	
m	



a. Complete the table.

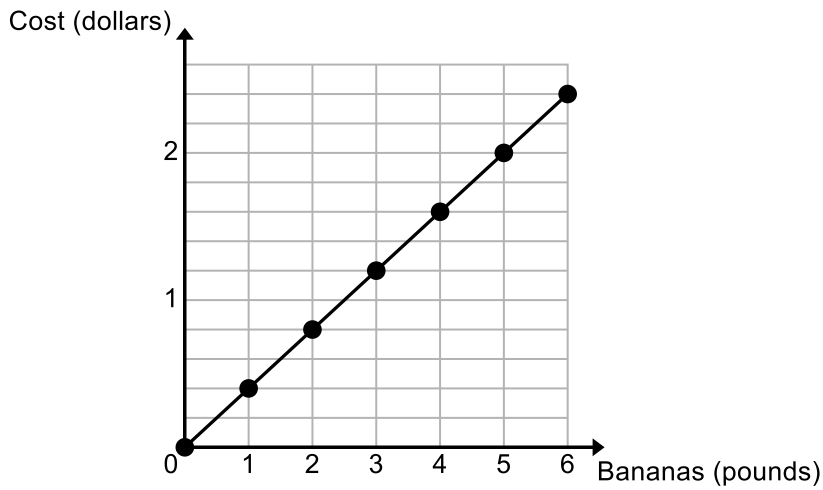
# of Marshmallows	Expression for Cups of Crispy Rice	Cups of Crispy Rice
6	$6 \bullet \frac{1}{6}$	1
12	$12 \bullet \frac{1}{6}$	2
30		
48		
m		

b. Write an equation that shows the relationship between the number of marshmallows *m* and the number of cups of crispy rice *r* needed to make rice crispy treats.

c. What is the unit rate in this situation? Where do you see the unit rate in your equation?

5. The graph shows the cost of bananas based on the number of pounds of bananas purchased. Complete the table to show the relationship between pounds of bananas purchased and cost.

Bananas Purchased (pounds)	Cost (dollars)
0	0
1	0.40
2	0.80
3	1.20
4	1.60
b	$0.4b$

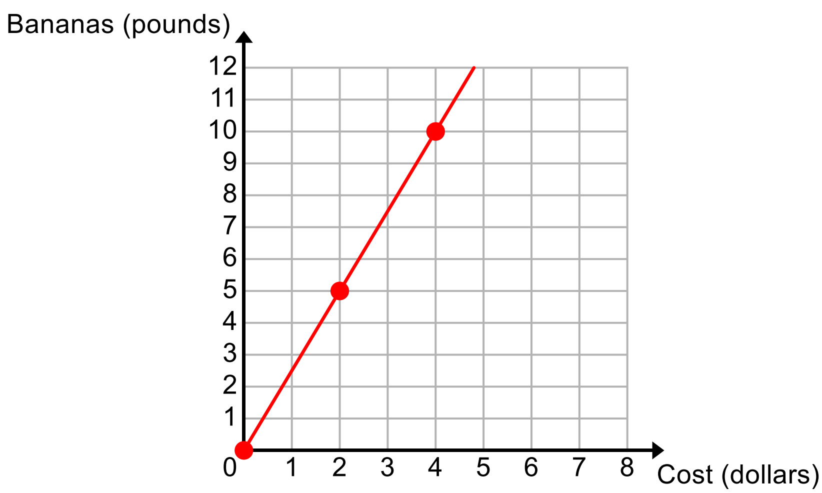


At this point, students should start to become fluent with writing equations, knowing that, the output = unit rate (proportional constant) multiplied by the input. They should also be able to identify the unit rate in each of the representations.

a. Complete the equation below to show the relationship between the pounds of bananas purchased b and the cost c.
$c =$ _0.4_ $\cdot b$

6. Emeril looks at the previous problem a little differently. He is interested in determining how many pounds of bananas he can purchase based on the number of dollars he has. Complete the table and graph to show the relationship. Assume the bananas cost the same amount as in the previous problem.

Cost (dollars)	Bananas Purchased (pounds)
0	0
1	2.5
2	5
3	7.5
4	10
c	$2.5c$



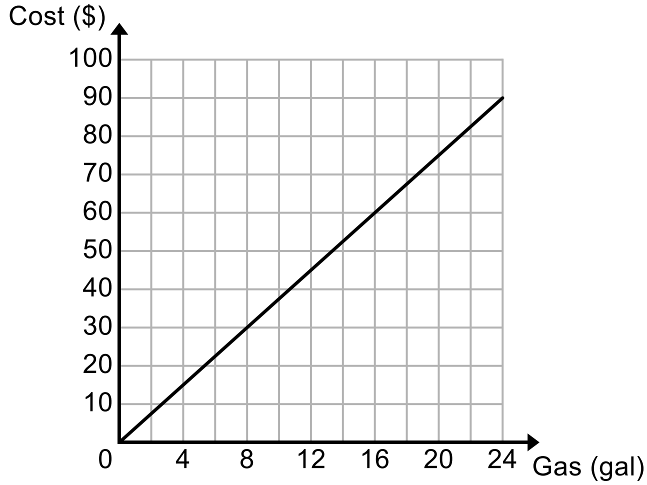
a. Complete the equation below to show the relationship between the amount of money Emeril has d and the number of pounds of bananas b he can purchase.
$b =$ _2.5_ $\cdot d$

b. Compare these two banana problems. How are they similar? How are they different?

In the first, the equation shows the cost of the bananas based on how many your purchase (your input is pounds of bananas and your output is dollars). In the second, the input and output are switched. The input is now dollars and the output is now pounds of bananas. The equation shows how many pounds of bananas can be purchased based on the number of dollars you have. Notice that the unit rates are inverses.

7. The graph shows the relationship between gallons of gas purchased and cost. Use the graph to complete the table.

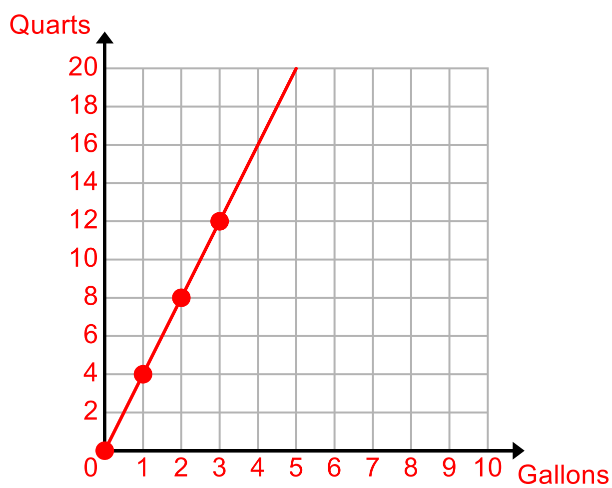
Gas (gallons)	Cost ($)
0	
1	
2	
10	
20	
g	



a. Write an equation that shows the amount spent d for g gallons of gas.

8. There are 4 quarts in a gallon. Complete the table and graph to show this relationship.
Since the table is not labeled, students may determine where to put the quantities in the table. The graph should follow the designation made in the table. You may wish to have half the class do it one way and the other switch the quantities in the table. Students should recognize that the relationship between the quantities does not change. Discuss the connections between the equations and the unit rate. The unit rates will be inverses. The graphs are reflections across the line $y = x$ – it is not expected that students understand this in 7th grade – rigid motion will be studied in 8th grade; however it may be something you consider exploring with honors' classes.

Gallons	Quarts
0	0
1	4
2	8
3	12
4	16



a. Write an equation that show the relationship between gallons g and quarts q.
$q = 4g$

b. What is the unit rate? 4 quarts/gallon

Spiral Review

1. Zach invested $1500. He earned 23.2% on his investment.
   a. Write an expression for the amount of money he has now.

   b. Determine how much money he has now.

2. Lara has $1,425 in her bank account. How much money did she start with if that amount reflects a 14% increase on her original amount?

3. There are 36 red and 44 blue marbles in a bag. What is the probability of randomly drawing a red marble?

4. Express each percent as a fraction in simplest form.

   44%   17.5%

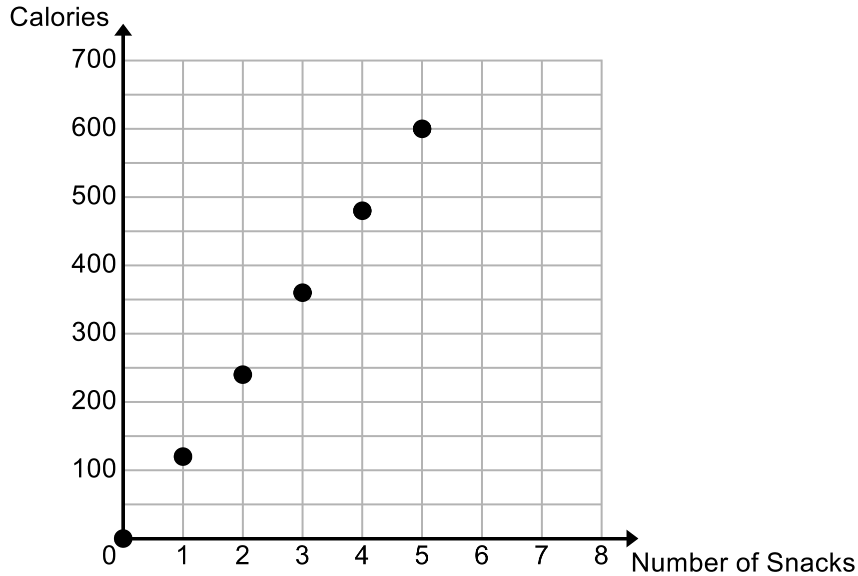
5. Estimate by rounding to the nearest integer.

   $3\frac{1}{3} \div \frac{7}{9} \approx$ _____ $\div$ _____ $\approx$ _____

   Is your answer an over estimate or under estimate, explain?

4.2c Homework: Equations of Proportional Relationships

1. The graph below shows the relationship between snacks and calories. Use the graph to complete the table.



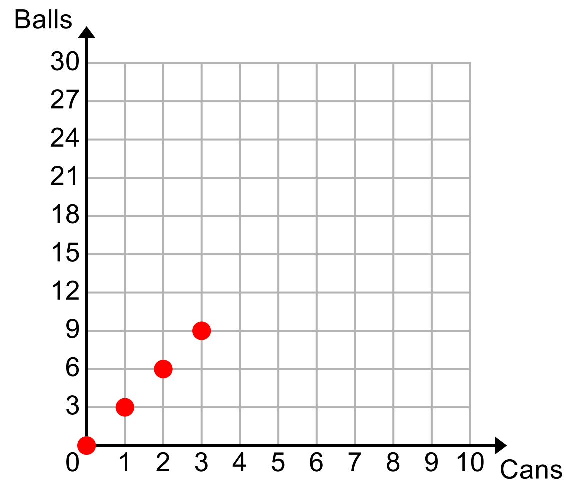
Number of Snacks	Calories	Ordered Pair	Write a complete sentence describing the meaning of this point on the graph.
0	0	$(0, 0)$	Zero snacks have zero calories.
1			
2			
3			
4			
s			

a. If c represents the number of calories and s represents the number of snacks, write an equation that shows the relationship between c and s.

b. What is the unit rate? Highlight the unit rate in your table, graph, and equation.

2. There are 3 tennis balls in a can. Complete the table and graph to show this relationship.

Number of Cans	Number of Tennis Balls
0	0
1	3
2	6
3	9
4	12
c	$3c$

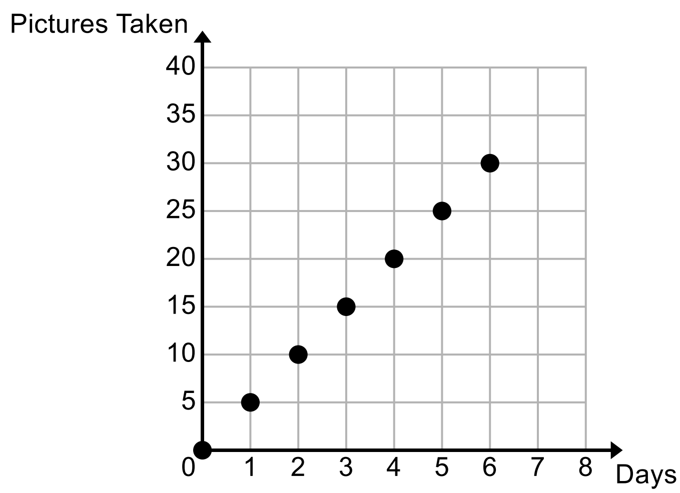


a. What is the unit rate? 3 balls/can

b. Write an equation that shows the number of balls b in c cans.
$b = 3c$

3. The number of pictures taken on a family vacation is shown in a graph. Complete the table.

Days	Pictures Taken

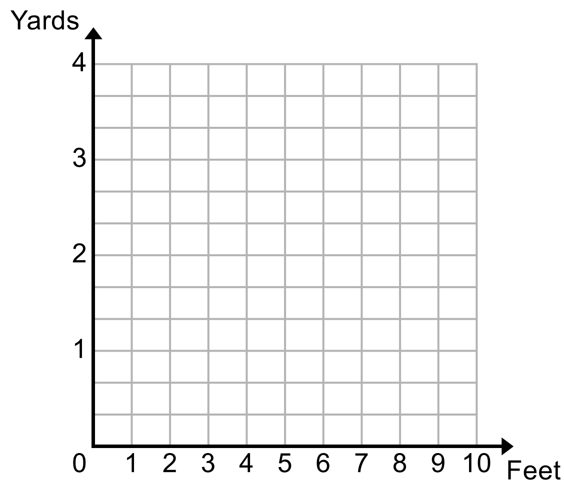


a. Write an equation that shows the number of pictures p taken on d days of vacation.

b. What is the unit rate in this situation? Where do you see the unit rate in the table, graph, and equation?

4. One yard contains 3 feet. Complete the table and graph below to show this relationship.

Feet	Yards
0	0
1	
2	
3	
6	



a. Write an equation to model this relationship where *f* represents feet and *y* represents yards.

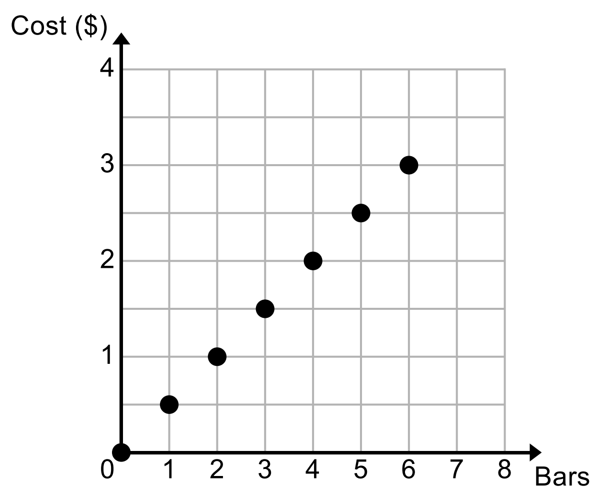
b. Use your equation to determine the number of yards in 15 feet.

c. Use your equation to determine the number of yards in 40 feet.

d. What is the unit rate in this situation? Where do you see the unit rate in your table, graph, and equation?

5. The graph shows the cost of ice cream bars depending on how many you purchase.

Bars	**Cost ($)**
0	0
1	0.50
2	1.00
3	1.50
4	2.00
5	2.50



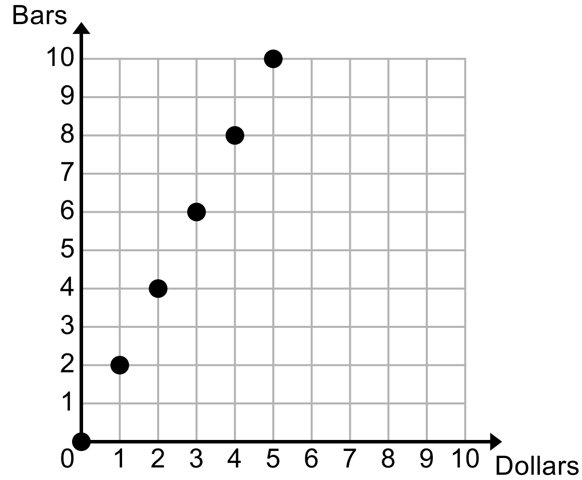
a. Write an equation that shows the cost *c* of *b* ice cream bars.
$c = 0.50b$

b. Complete the following ordered pair (1, _0.50__). What does this point represent in the situation? It costs $0.50 per ice cream bar.

c. How many bars can Barry buy with $10? 20

6. The graph shows the number of ice cream bars you can purchase depending on how much money you have.

Dollars	Bars
0	
1	
2	
3	
4	
5	



a. Write an equation that shows the number of bars *b* that can be purchased for *d* dollars.

b. Complete the following ordered pair (1, ___). What does this point represent in the situation?

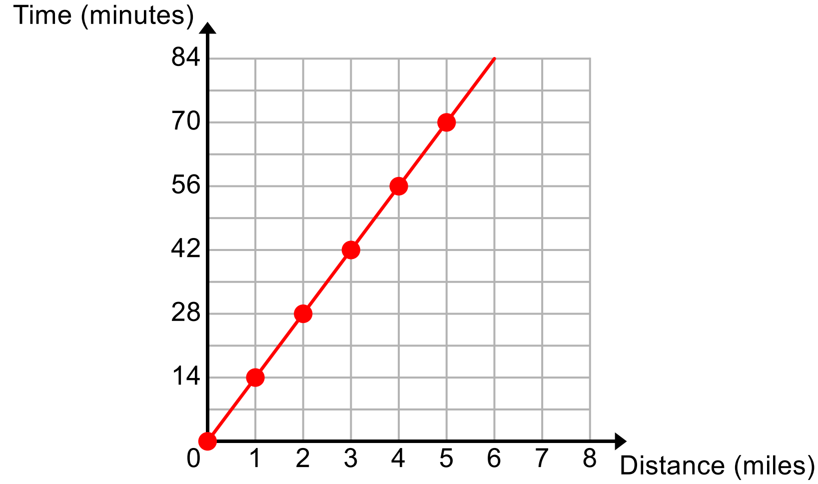
c. Compare this problem with the previous problem. 

4.2d Class Activity: More Equations of Proportional Relationships

In this section, we transition students into using the variables *x* and *y* to represent the quantities.

1. It takes Mara $3\frac{1}{2}$ minutes to walk $\frac{1}{4}$ mile. Complete the table and graph to show this relationship.

Distance (miles)	Time (minutes)
0	0
1	14
2	28
3	42
4	56
5	70



a. Write an equation that represents the amount of time *t* it takes Mara to walk *d* miles.
$t = 14d$

b. Now, re-write your equation from above using *x* and *y* to represent your quantities.
$y = 14x$

c. Complete the following statement. The proportional constant $\frac{y}{x}$ = __14__? Connect this equation to your equation in part b. Students should see that the equation for a proportional relationship in the form $y = kx$ is just a rearranged form of the equation $\frac{y}{x} = k$.

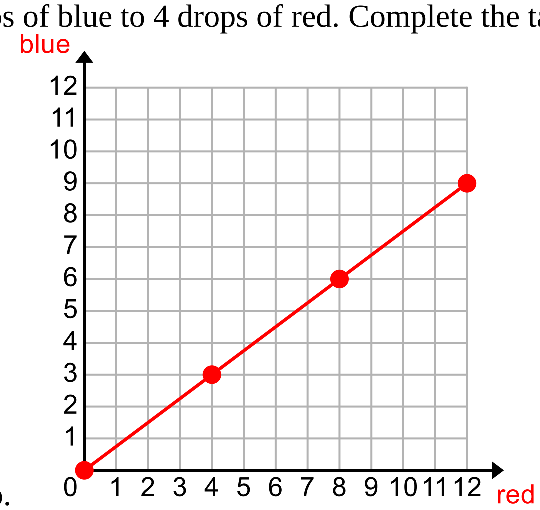
d. Complete the following ordered pair: (1, _14_). What does this point represent in the situation? Where do you see this point in your table and equation?
It takes Mara 14 minutes to walk one mile.

e. If Mara walks 7 miles, how long will it take her?
98 minutes or 1 hour and 38 minutes

f. If Mara walks for 49 minutes, how far can she walk?
3.5 miles

2. Ita is making purple dye. She is using a ratio of 3 drops of blue to 4 drops of red. Complete the table and graph to show this relationship.

Red	Blue
4	3
8	6
12	9
16	12
20	15



a. Write an equation to represent this relationship.
$y = \frac{3}{4}x$; if students switched where the variables are in the table and on the graph, their equation will be $y = \frac{4}{3}x$.

3. A tortoise can walk $\frac{1}{2}$ a mile in $\frac{1}{4}$ of an hour. The hare can run $1\frac{1}{2}$ miles in $\frac{1}{2}$ hour.

a. Complete the table.

Tortoise	
Hours	Miles
0	0
$\frac{1}{2}$	1
1	2
$1\frac{1}{2}$	3
2	4

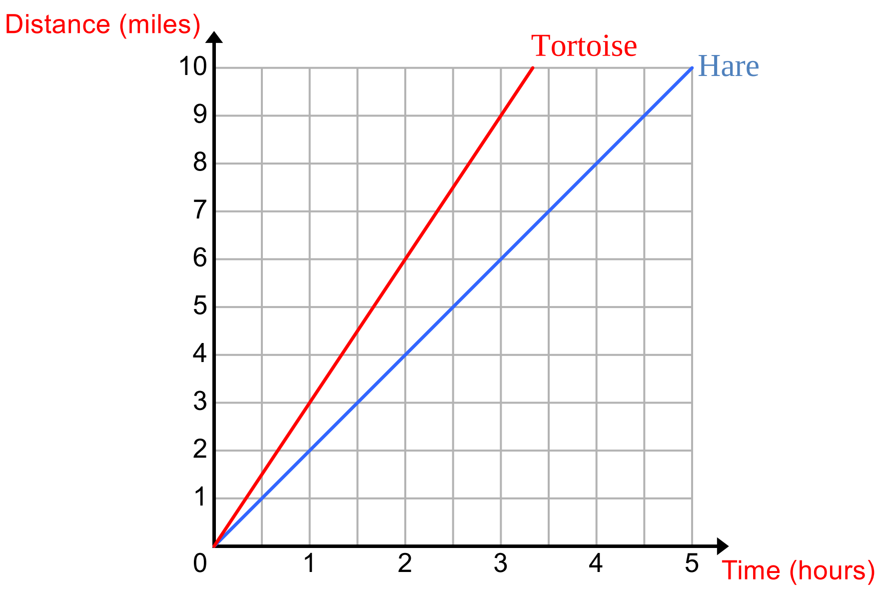
Hare	
Hours	Miles
0	0
$\frac{1}{2}$	$1\frac{1}{2}$
1	3
$1\frac{1}{2}$	$4\frac{1}{2}$
2	6

b. Find the unit rate for each animal. Be sure to include labels.

Tortoise: 2 mph

Hare: 3 mph

c. Graph each animal's trip in a different color on the same coordinate plane. Label your axes.



d. Write an equation for the distance y traveled after x hours for each animal.

Tortoise: __$y = 2x$__________________________

Hare: ____$y = 3x$____________________________

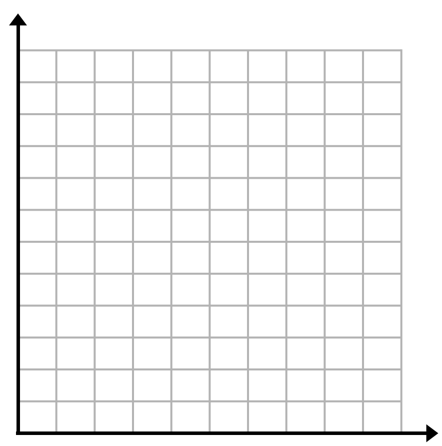
e. Compare the trips of the two animals. In your comparison, explain how the table, unit rate, graph, and equation help you to see the difference in the two trips.
See student responses.

4. Felipe's family is taking a road trip. The distance they travel can be modeled by the equation $y = 60x$ where y represents distance traveled in miles and x represents time in hours. Complete the table and graph to show this relationship. A sample table and graph are shown. Students will likely need help determining what to choose for the domain (inputs) while creating their table. Ask what increments of time make sense to examine in this situation. If they were taking a road trip, how often would they ask how far they have driven?

Time (hours)	**Distance (miles)**
0	0
1	60
2	120
3	180
8	480
10	600

Talk to students about a quick way to graph by plotting two points and connecting the line.

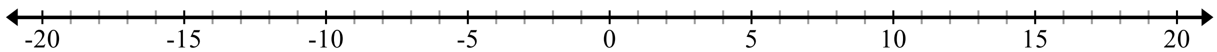
5. The cost of scrapbooking paper can be modeled by the equation $y = 0.25x$ where y represents the cost of the paper and x represents the number of sheets. Complete the table and graph to show this relationship.



Spiral Review

1. There are a total of 127 cars and trucks on a lot. The number of cars is four more than twice the number of trucks. How many cars and trucks are on the lot?

2. Use the number line below to show why $(-1)(-1) = 1$



3. If the ratio of girls to boys in a class is 3 to 4 and there are 35 students in the class, how many students are girls?

4. Fill in the equivalent fraction and percent for this decimal:

Fraction	Decimal	Percent
	0.95	

5. David is in a submarine at 200 feet below sea level. Casey is above him in a helicopter at 5,900 feet altitude. How far apart are David and Casey?

4.2d Homework: More Equations of Proportional Relationships

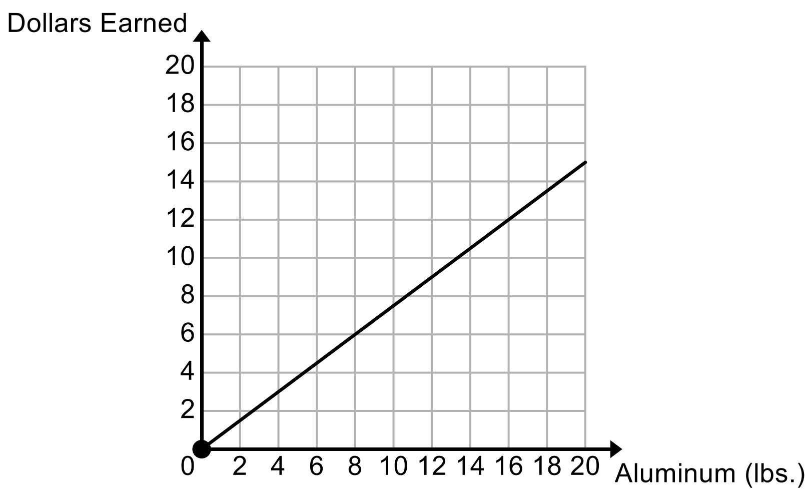
1. Bianca spends the same amount of money each day she is on vacation. The table below shows the amount she spends based on the number of days she is on vacation.

Time (days)	Money Spent (dollars)
0	0
2	30
3	45
8	120
10	150

   a. Write an equation that shows the amount of money *y* Bianca spends in *x* days.
      Type equation here.

   b. Use your equation to determine how much Bianca will spend in 12 days.

2. The graph shows the amount Jayden earns for recycling aluminum cans.

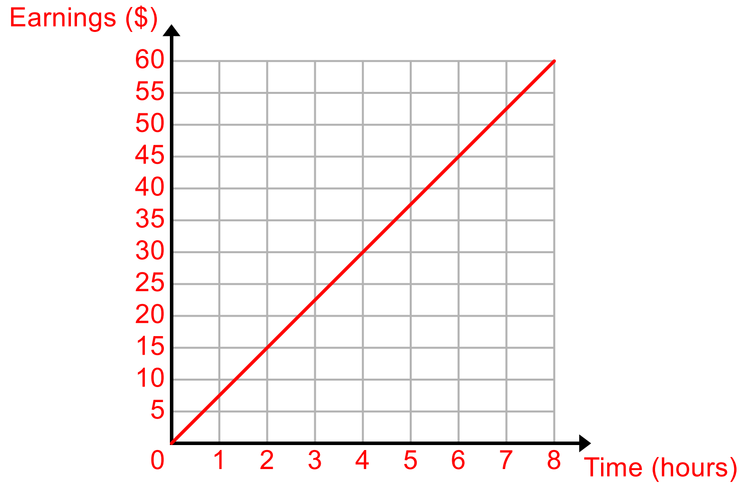


   a. Write an equation for the amount *y* Jayden earns based on the number of pounds *x* of aluminum he recycles. $y = 0.75x$ or $y = \frac{3}{4}x$

3. Erin swims $\frac{1}{2}$ mile in $\frac{1}{3}$ hr. Write an equation for the distance *y* Erin can swim in *x* hours.

4. The amount of money y Edward makes for x hours of work can be modeled by the equation $y = 7.50x$. Complete the table and graph to show this relationship.

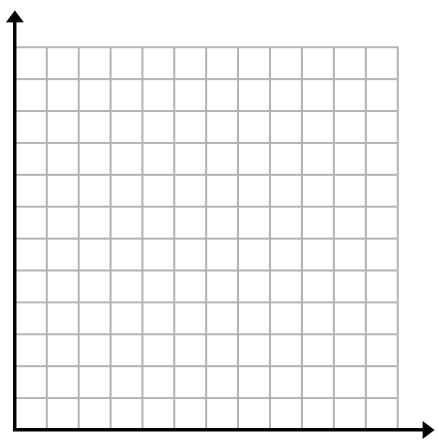
Time (hours)	Earnings ($)
0	0
1	7.5
2	15
5	37.50
8	60
16	120



a. Use the equation to determine how Edward makes for 40 hours of work. Write your answer as an ordered pair. $(40, 300)$

5. The number of pizzas y needed to feed x students can be modeled by the equation $y = \frac{1}{4}x$. Complete the table and graph to show this relationship.

0	
1	
2	
3	
4	
12	



a. What is the unit rate?

b. If you have to feed 200 students, how many pizzas do you need?

c. If you have to feed 500 students, how many pizzas do you need?

6. Milo and Sera each bought chocolate cinnamon bears from different candy stores. Milo paid \$3.00 for 2 pounds and Sera paid \$5.25 for 3 pounds.

a. Complete the table.

Milo	
Pounds	Cost
0	
1	
2	
3	
4	

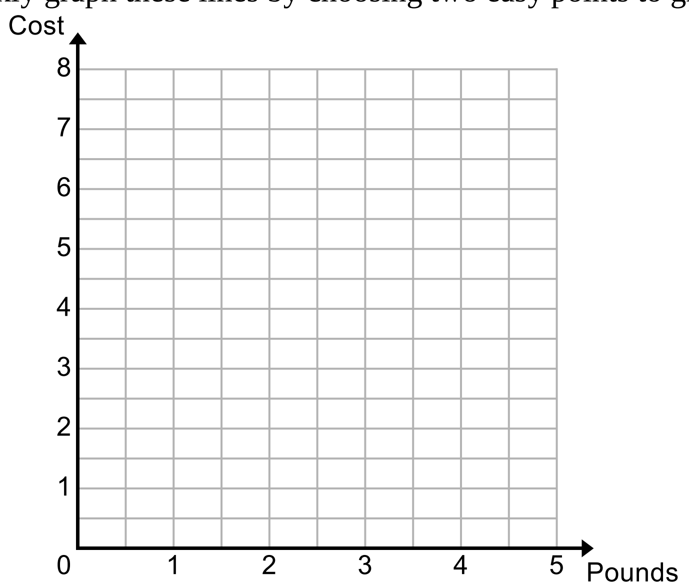
Sera	
Pounds	Cost
0	
1	
2	
3	
4	

b. Find the unit rate for each girl. Be sure to include labels.

Milo:

Sera:

c. Graph the cost of each girl's bears in a different color on the same coordinate plane. Think about how you can quickly graph these lines by choosing two easy points to graph.



d. Write an equation that relates the cost y to the pounds purchased x for each girl.

Milo: ______________________

Sera: ______________________________

e. Who got the best deal on cinnamon bears? How do each of the representations show who got the better deal?

4.2e Class Activity: Equations of Proportional Relationships $y = kx$

1. For each of the following tables, x and y are in a proportional relationship. Write an equation for y in terms of x.

This lesson should really solidify the connections between the constant of proportionality/unit rate and the equation of a proportional relationship. Students should be able to fluently write the equation for a proportional relationship in the form $y = kx$ using several strategies to determine the constant of proportionality.

a.

x	y
1	4
2	8
3	12

Equation: $y = 4x$

b.

x	y
40	8
35	7
30	6

Equation: $y = \frac{x}{5}$

Students may find the quotient $\frac{y}{x}$ or they may think about the difference in the x and y columns – how much would y increase if x increases by 1?

c.

x	y
1	$\frac{1}{3}$
2	$\frac{2}{3}$
3	1

Equation: $y = \frac{x}{3}$

Students should see that they have been given the unit rate in the table.

d.

x	y
$\frac{1}{2}$	3
1	6
$\frac{3}{2}$	9

Equation: $y = 6x$

e.

x	y
-2	-4
-1	-2
1	2

Equation: $y = 2x$

f.

x	y
-2	6
-1	3
1	-3

Equation: $y = -3x$

g.

x	y
0.75	1
3.75	5

Equation: $y = \frac{4}{3}x$

h.

x	y
90	54
10	6

Equation: $y = 0.6x$ or $y = \frac{3}{5}x$

i.

x	y
-1	$\frac{1}{2}$
1	$-\frac{1}{2}$

Equation: $y = -\frac{1}{2}x$

2. The variables x and y are proportionally related. Write an equation for y in terms of x. Write the ordered pair that corresponds to the unit rate $(1, r)$. Write one more ordered pair that is proportional to the one given.

a. $(2, 8)$ Equation: $y = 4x$ Unit Rate Ordered Pair: $(1, 4)$ Additional Ordered Pair: $(4, 16)$ Answers may vary for the additional ordered pair.	b. $(15, 3)$ Equation: $y = \frac{x}{5}$ Unit Rate Ordered Pair: $(1, \frac{1}{5})$ Additional Ordered Pair: $(20, 4)$ Answers may vary for the additional ordered pair.
c. $(1, -20)$ Equation: $y = -20x$ Unit Rate Ordered Pair: $(1, -20)$ Additional Ordered Pair: $(2, -40)$ Answers may vary for the additional ordered pair.	d. $(2, 25)$ Equation: $y = 12.5x$ Unit Rate Ordered Pair: $(1, 12.5)$ Additional Ordered Pair: $(3, 37.5)$ Answers may vary for the additional ordered pair.
e. $(3, 2)$ Equation: $y = \frac{2}{3}x$ Unit Rate Ordered Pair: $(1, \frac{2}{3})$ Additional Ordered Pair: $(9, 6)$ Answers may vary for the additional ordered pair.	f. $(2, \frac{2}{3})$ Equation: $y = \frac{1}{3}x$ Unit Rate Ordered Pair: $(1, \frac{1}{3})$ Additional Ordered Pair: $(15, 5)$ Answers may vary for the additional ordered pair.
g. $(0.25, 1)$ Equation: Unit Rate Ordered Pair: Additional Ordered Pair:	h. $(1, -\frac{2}{5})$ Equation: $y = -\frac{2}{5}x$ Unit Rate Ordered Pair: $(1, -\frac{2}{5})$ Additional Ordered Pair: $(10, -4)$ Answers may vary for the additional ordered pair.
i. $(90, 180)$ Equation: Unit Rate Ordered Pair: Additional Ordered Pair:	j. $(60, 48)$ Equation: $y = \frac{4}{5}x$ Unit Rate Ordered Pair: $(1, \frac{4}{5})$ Additional Ordered Pair: $(2, 1.6)$ Answers may vary for the additional ordered pair.

3. Use the equation given to complete the table.

a. $y = 3x$

x	y
6	18
8	24

b. $y = \frac{1}{2}x$

x	y
20	10
32	16
9	4.5

c. $y = \frac{5}{2}x$

x	y
10	25
1	$\frac{5}{2}$
5	12.5

d. $y = -4x$

x	y
-2	8
2	-8
$-\frac{1}{4}$	1

e. $y = \frac{3}{4}x$

x	y
1	
16	
	9

f. $y = 10x$

x	y
0	
	10
30	

g. $y = 0.1x$

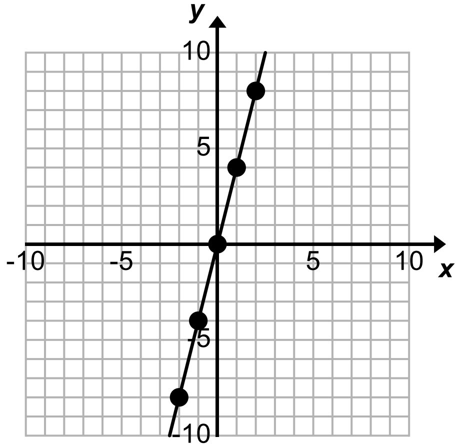
x	y
1	0.1
5	$\frac{1}{2}$
400	40

h. $y = -x$

x	y
0	0
4	-4
-4	4

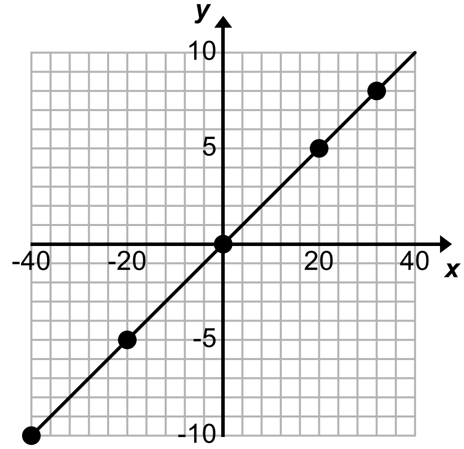
4. For each of the following graphs, write an equation for *y* in terms of *x*.

a.



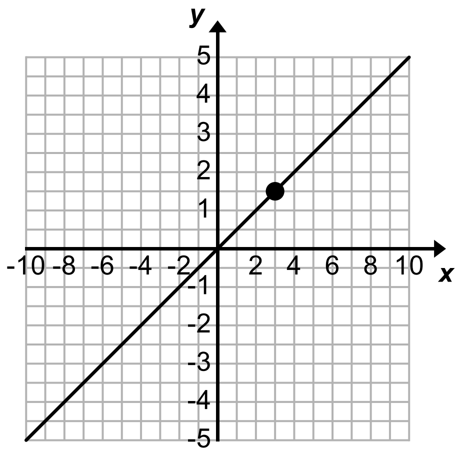
Equation: $y = 4x$

b.



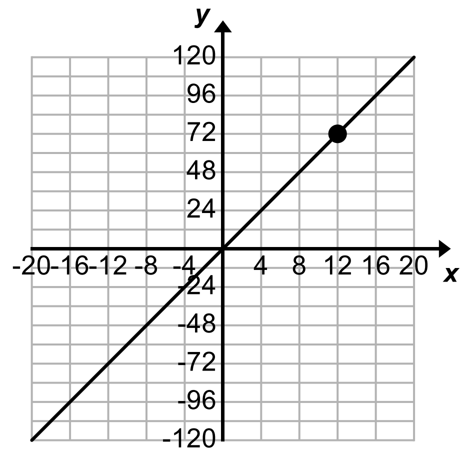
Equation: $y = \frac{x}{4}$

c.



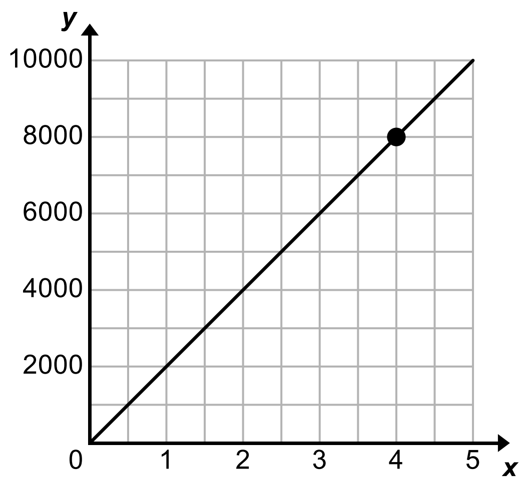
Equation: $y = \frac{x}{2}$

d.



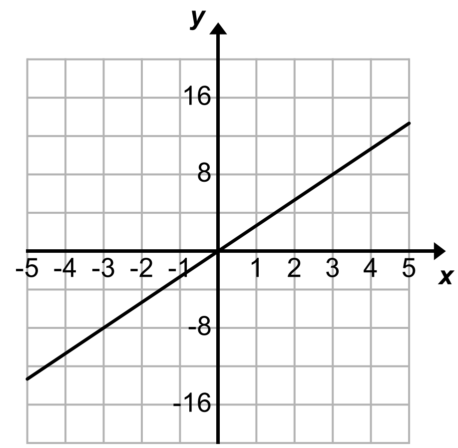
Equation: $y = 6x$

e.



Equation: $y = 2000x$

f.



Equation: $y = \frac{8}{3}x$

Spiral Review

1. Luke has 90 baseball cards. He sells $\frac{1}{3}$ of the cards. He stores 20% of the rest in a safety deposit box and the rest in his dresser drawer. How many baseball cards are in his dresser drawer?

2. Find the product without a calculator: $15.5(-8)$

3. Suppose you were to roll a fair 6-sided number cube once, then flip a coin. List all the possible outcomes.

4. Using the information in question 3, what is the probability of getting a heads and an even number?

4.2e Homework: Equations of Proportional Relationships $y = kx$

1. For each of the following tables, x and y are in a proportional relationship. Write an equation for y in terms of x.

a.

x	y
1	10
2	20
3	30

Equation:

b.

x	y
4	$\frac{1}{2}$
8	1
12	$\frac{3}{2}$

Equation:

c.

x	y
0	0
1	0.6
2	1.2

Equation: $y = 0.6x$

d.

x	y
3	2
6	4
9	6

Equation:

e.

x	y
-3	$-\frac{3}{2}$
-1	$-\frac{1}{2}$
1	$\frac{1}{2}$

Equation:

f.

x	y
-1	$\frac{1}{2}$
0	0
1	$-\frac{1}{2}$

Equation: $y = -\frac{1}{2}x$

g.

x	y
$\frac{3}{4}$	18
$1\frac{1}{2}$	36

Equation:

h.

x	y
12.5	50
18.75	75

Equation:

i.

x	y
$2\frac{1}{2}$	5
4	8

Equation: $y = 2x$

2. The variables x and y are proportionally related. Write an equation for y in terms of x. Write the ordered pair that corresponds to the unit rate $(1, r)$. Write one more ordered pair that is proportional to the one given.

a. $(4, 16)$ Equation: $y = 4x$ Unit Rate Ordered Pair: $(1, 4)$ Additional Ordered Pair: $(8, 32)$ Answers may vary for the additional ordered pair.	b. $(1, 1.5)$ Equation: Unit Rate Ordered Pair: Additional Ordered Pair:
c. $(16, -4)$ Equation: $y = \frac{x}{-4}$ Unit Rate Ordered Pair: $(1, -\frac{1}{4})$ Additional Ordered Pair: $(-20, 5)$ Answers may vary for the additional ordered pair.	d. $(\frac{1}{2}, \frac{1}{3})$ Equation: Unit Rate Ordered Pair: Additional Ordered Pair:
e. $(1\frac{1}{4}, 1)$ Equation: $y = \frac{4}{5}x$ Unit Rate Ordered Pair: $(1, \frac{4}{5})$ Additional Ordered Pair: $(10, 8)$ Answers may vary for the additional ordered pair.	f. $(-2, -\frac{2}{3})$ Equation: Unit Rate Ordered Pair: Additional Ordered Pair:
g. $(3.5, 14)$ Equation: $y = 4x$ Unit Rate Ordered Pair: $(1, 4)$ Additional Ordered Pair: $(2, 8)$ Answers may vary for the additional ordered pair.	h. $(1, \frac{2}{7})$ Equation: Unit Rate Ordered Pair: Additional Ordered Pair:
i. $(12, 8)$ Equation: $y = \frac{2}{3}x$ Unit Rate Ordered Pair: $(1, \frac{2}{3})$ Additional Ordered Pair: $(3, 2)$ Answers may vary for the additional ordered pair.	j. $(12, 20)$ Equation: Unit Rate Ordered Pair: Additional Ordered Pair:

3. Use the equation given to complete the table.

a. $y = 4x$

x	y
0	0
8	32

b. $y = -\frac{1}{2}x$

x	y
20	
	16
9	

c. $y = \frac{3}{2}x$

x	y
4	6
12	18
10	15

d. $y = -3x$

x	y
-1	
0	
	9

e. $y = 1.5x$

x	y
0	0
9	13.5
30	45

f. $y = x$

x	y
1	
80	
	10

g. $y = \frac{x}{3}$

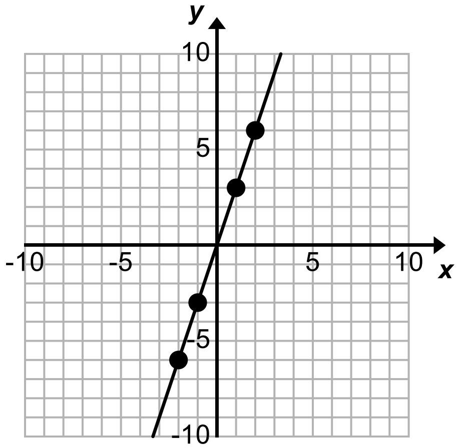
x	y
0	0
9	3

h. $y = 1.25x$

x	y
16	
	11.25

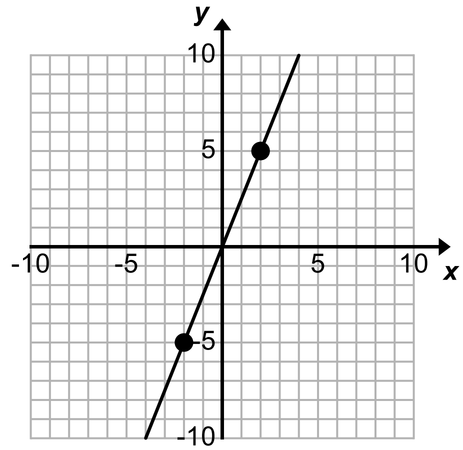
4. For each of the following graphs, write an equation for y in terms of x.

a.



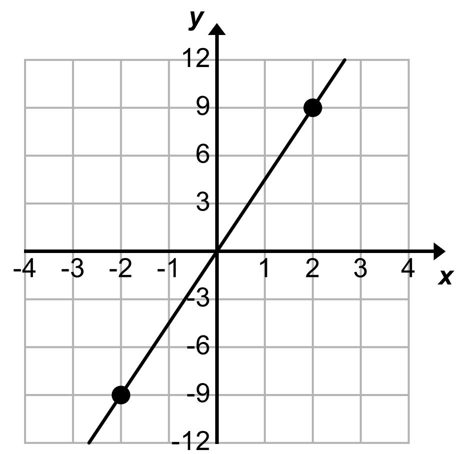
Equation:

b.



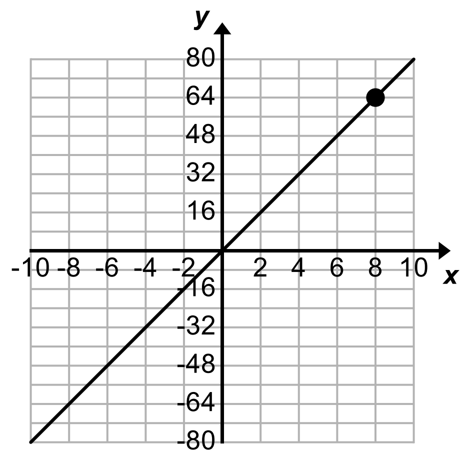
Equation: $y = \frac{5}{2}x$

c.



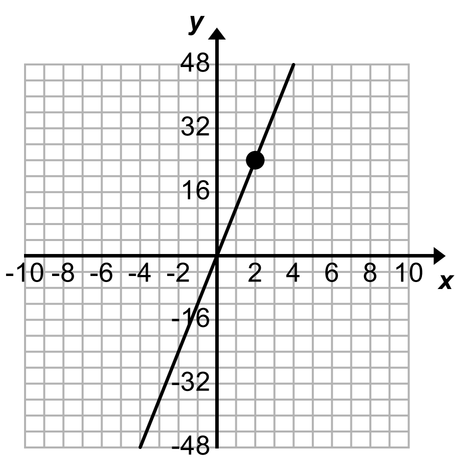
Equation:

d.



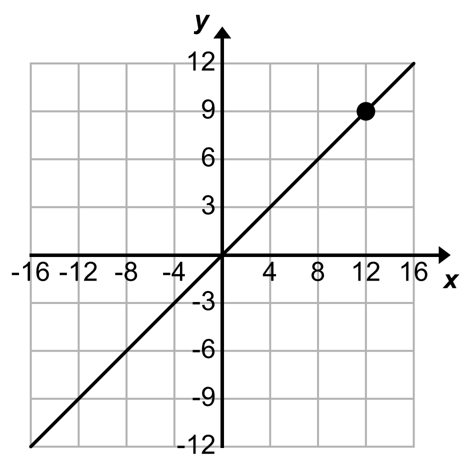
Equation: $y = 8x$

e.



Equation:

f.



Equation: $y = \frac{3}{4}x$

4.2f Class Activity: The Representations of Proportional Relationships

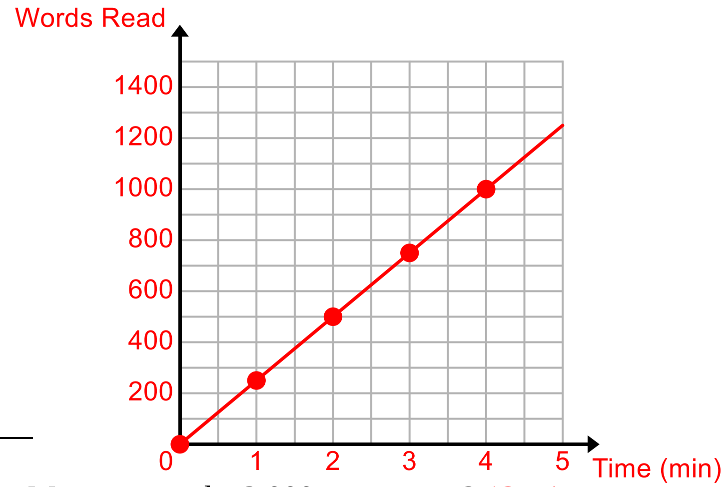
In this lesson, students move fluently between the different representations of a proportional relationship and make connections between the representations. They then use the representations to answer questions, determining when it makes more sense to use one representation over another.

Directions: In the following problems, one representation of a proportional relationship is given. Complete the other representations. Use your representations to answer the questions that follow. Then, create your own question that can be answer using the representations and answer your question.

1. Megan can read 125 words in 30 seconds.

Time (min.)	Words Read
0	0
1	250
2	500
3	750
4	1000

Equation: $y = 250x$



a. At this rate, how long will it take Megan to read a 3,000-page essay? 12 minutes

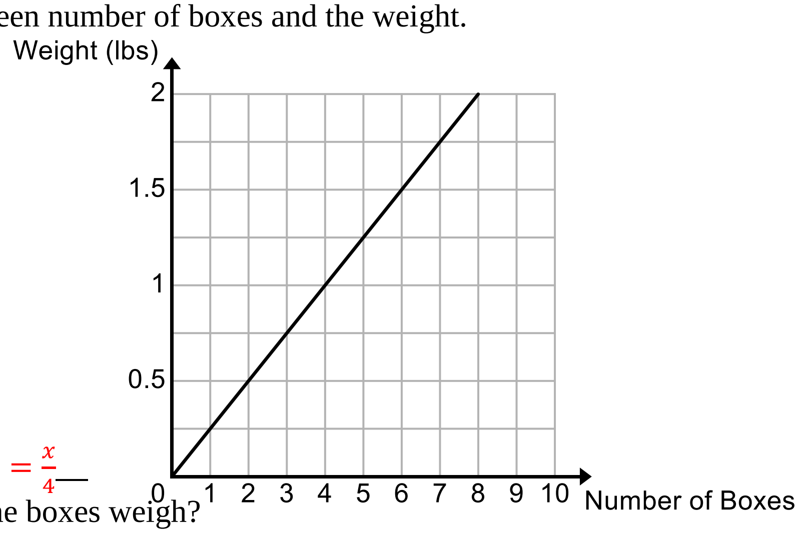
b. Nicholas can read 200 words per minute. Who reads faster? How long would it take Nicholas to read the 3,000 page essay? Megan can read faster. Have students use the representations to justify this. It would take Nicholas 15 minutes to read the essay.

c. Your Question: Composing questions requires students to make sense of the quantities in the problem, including considering the units involved, and to understand the relationship between the quantities. This is also a really nice opportunity to differentiate learning. Students who really understand the concepts may create more advanced questions while students who are struggling may pose easier questions.

2. The graph shows the relationship between number of boxes and the weight.

Boxes	Weight (lbs.)
1	0.25
2	0.5
3	1.5
4	1
5	1.25

Equation: $y = 0.25x$ or $y = \frac{1}{4}x$ or $y = \frac{x}{4}$

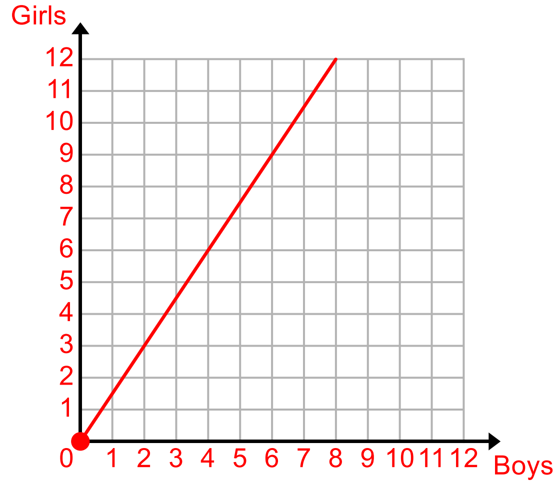


a. How much will 10 of these same boxes weigh?
2.5 pounds

b. Your Question:

3. The relationship between girls and boys in Mrs. Jimenez's class can be modeled by the equation $y = \frac{3}{2}x$ where y is the number of girls and x is the number of boys.

Boys	Girls
0	0
2	3
4	6
6	9
8	12



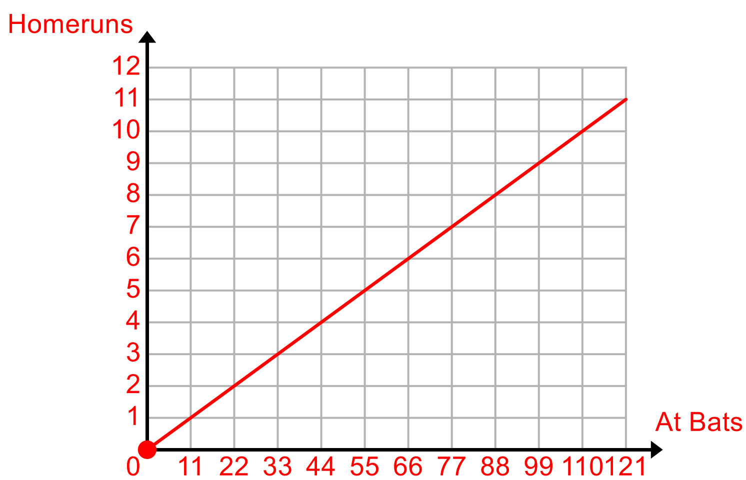
a. Use your representations to determine the number of girls if there are 4 boys. 6
Here it likely makes sense to use the table or graph to find the answer.

b. Use your representations to determine the number of girls if there are 60 boys. 90
Here it likely makes sense to use the equation to find the answer or to think about what y would need to be for the proportional constant to be $\frac{3}{2}$

c. Your Question:

4. The table shows the number of homeruns Mark hits based on the number of at-bats he has.

At Bats	Homeruns
0	0
11	1
22	2
33	3
44	4



Equation: _____ $y = \frac{1}{11}x$ ____________

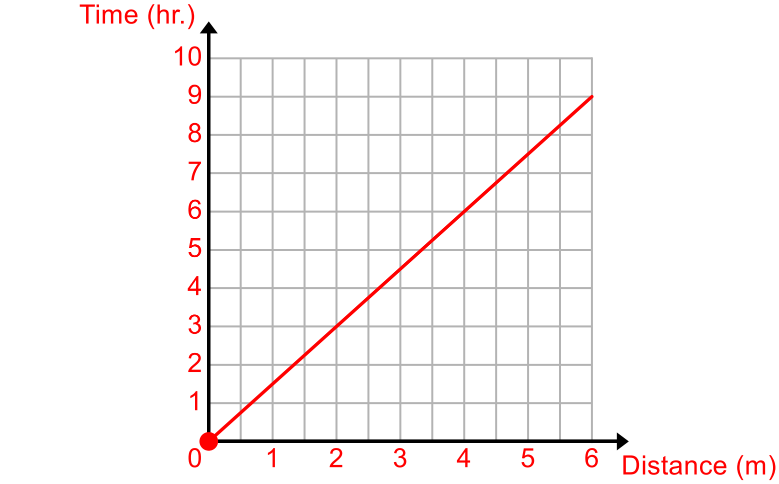
a. How many homeruns would you expect Mark to have if he is at bat 50 times? Approximately 4.5
In this context, you cannot have 0.5 of a homerun. Discuss with students that an answer of 4 or 5 makes sense for the context.

b. Mickey hits a homerun for every 15 times he is at bat. Who has a higher homerun percentage? Explain.
Mark has a higher homerun percentage, student explanations may vary.

c. Your Question:

5. The table below shows the amount of time it takes a snail to travel a certain distance. Complete the table assuming the snail's pace remains constant.

Distance (meters)	Time (hours)
0	0
$\frac{1}{3}$	$\frac{1}{2}$
1	1.5
2	3
3	4.5
4	6



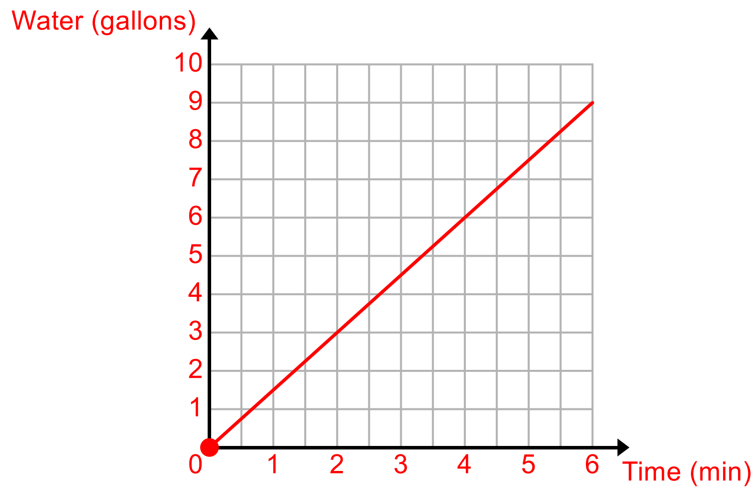
Equation: ___$y = 1.5x$_______________

a. How long will it take the snail to crawl 10 meters? 15 hours

b. Your Question:

6. Water flows out of a kitchen faucet at a rate of $\frac{3}{4}$ gallon in $\frac{1}{2}$ minute. In this problem, it is OK for students to designate water as the x-variable and time as their y-variable. If they do that, their graph would be reflections across the line $y = x$ and their equation would be $y = \frac{2}{3}x$.

Time (minutes)	Water (gallons)
0	0
1	1.5
2	3
3	4.5
4	6
5	9



Equation: ____$y = 1.5x$_______________

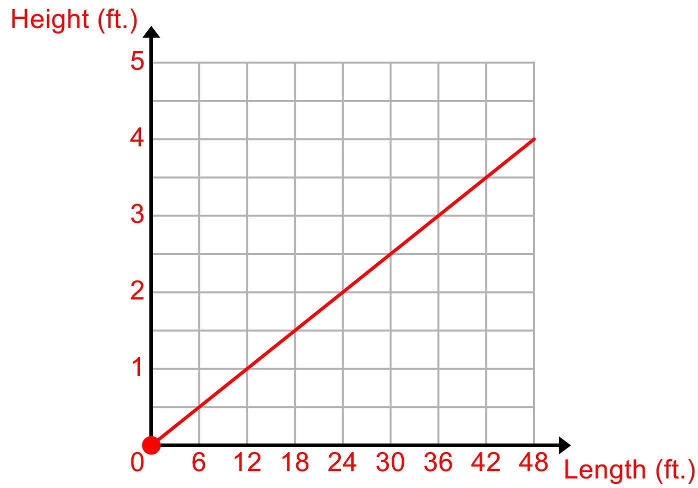
a. If this particular sink holds 30 gallons of water, how long will it take to fill the sink? 20 minutes

b. How much water will be in the sink after 9 minutes? 13.5 gallons

c. Your Question:

7. Ashley is building a wheelchair ramp. By code, the ratio of the height to length needs to be 1 to 12.

Length (feet)	Height (feet)
6	$\frac{1}{2}$
12	1
18	$1\frac{1}{2}$
24	2
30	$2\frac{1}{2}$



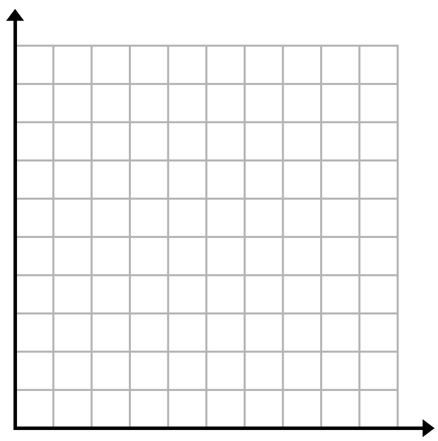
Equation: ______$y = \frac{1}{12}x$_________

a. If the ramp Ashley is building has to reach to a height of 3 feet, how long does it need to be?
36 feet long

b. Your Question:

8. Susannah is a designer making a blueprint for a kitchen remodel. The table shows the relationship between the actual measurements and the measurements on the blueprint.

Actual Length (feet)	Length on Blueprint (inches)
0	0
1	$\frac{1}{4}$
5	$\frac{5}{4}$
8	2
12	3



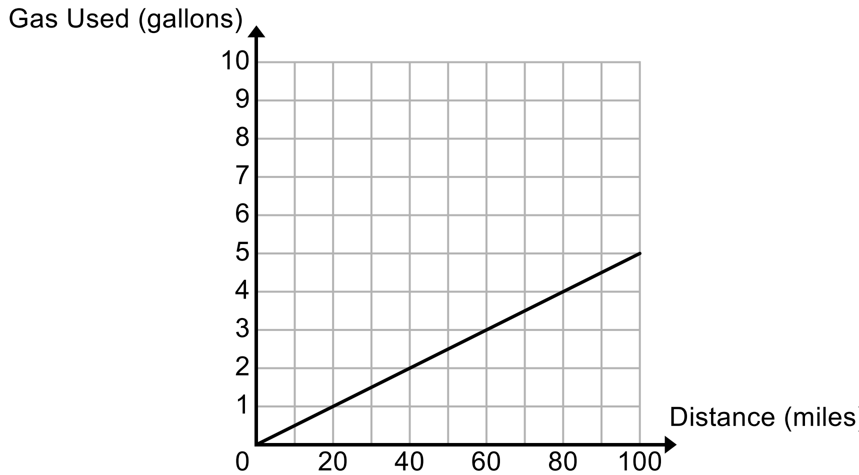
Equation: _________________

a. If the actual length of a wall is 15 feet, how long will that wall be on the blueprint?

b. Your Question:

9. The graph shows the amount of gas a car uses based on the number of miles it has been driven.

Distance (miles)	Gas Used (gallons)
20	1
40	2
60	3
80	4



Equation: ____$y = \frac{1}{20}x$____________

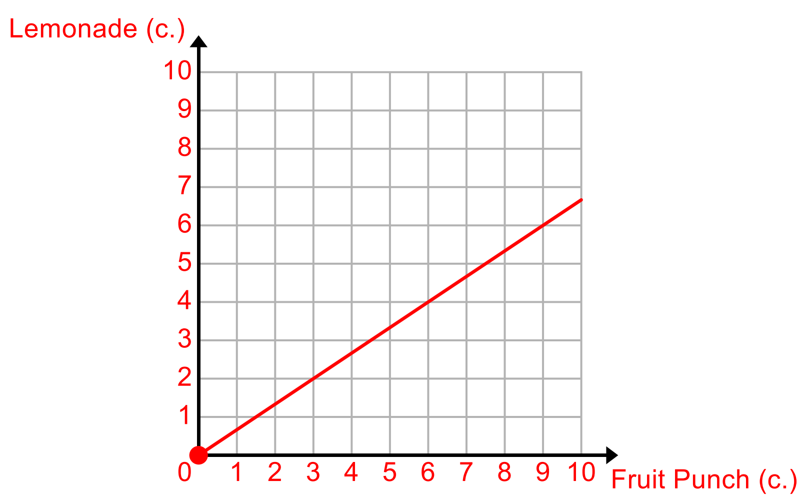
Discuss the unit rate in this problem with students. $\frac{\frac{1}{20}\text{ gallons of gas}}{1\text{ mile}}$. Another way to think about this is that 1 gallon of gas can drive 20 miles

a. Lauren is driving to a town 330 miles away. How many gallons of gas will she need?
16.5 gallons of gas

b. Your Question:

10. A recipe calls for 2 cups of lemonade for every 3 cups of fruit punch.

Fruit Punch (cups)	**Lemonade (cups)**
0	0
$\frac{1}{2}$	$\frac{1}{3}$
1	$\frac{2}{3}$
2	$\frac{4}{3}$
3	2
6	4



Equation: _____$y = \frac{2}{3}x$__________

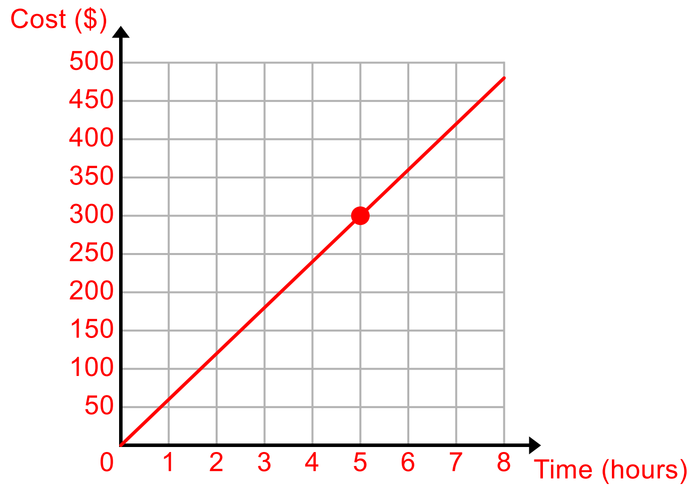
a. How many cups of lemonade will you need if you use 6 cups of fruit punch?
4

b. Eva used 6 cups of lemonade and 4 cups of fruit punch. Did she follow this recipe? Explain.
No, explanations will vary. A common error is for students to think this is in the same ratio. Encourage students to attend to precision.

c. Your Question:

11. The amount, in dollars, y a plumber charges based on the number of hours worked x can be modeled by the equation $y = 60x$.

Time (hours)	Charge ($)
1	60
2	120
3	180
4	240
5	300



In this problem, encourage students to choose 2 easy points to graph and connect them. For the graph above, the points (0, 0) and (5, 300) were used. The decision on how to scale the graph was also made with these points in mind. Scaling the y-axis by 30s or 60s also makes sense in this problem.

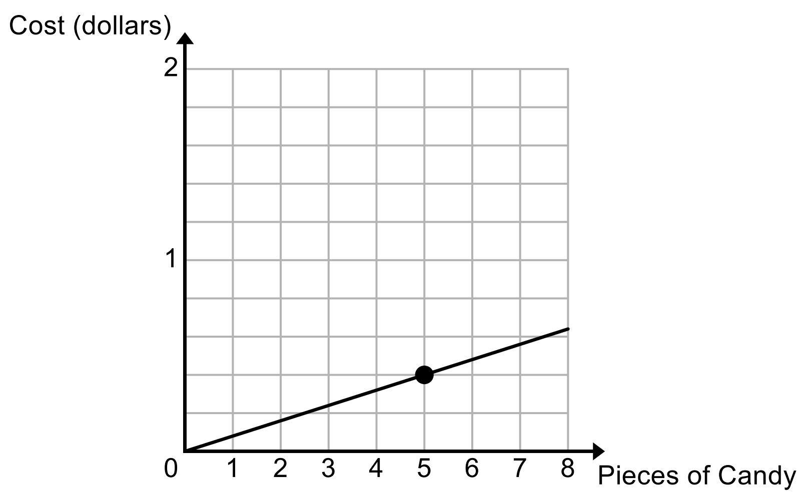
a. How much will the plumber charge for 4 hours of work?
$240

b. How much will the plumber charge for 8 hours of work? $480; Note students can use the equation or just double the answer in part a.

c. Your Question:

12. The graph below shows the relationship between the cost and number of pieces of candy Carmen buys.

Pieces of Candy	Cost ($)
1	
2	
3	
4	
5	



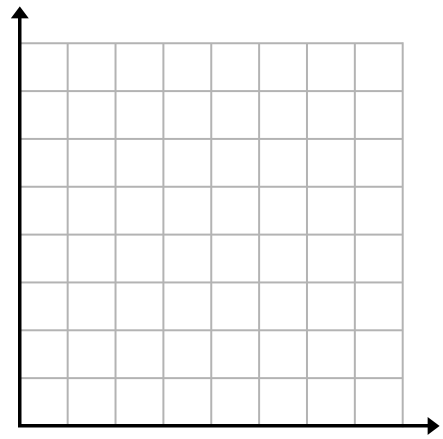
Equation: ___________

a. How much will Carmen pay for 30 pieces of candy?

b. Your Question:

13. The table below shows the approximate relationship between miles and kilometers.

Kilometers	Miles
0	0
1	0.62
2	1.24
3	1.86
4	2.48



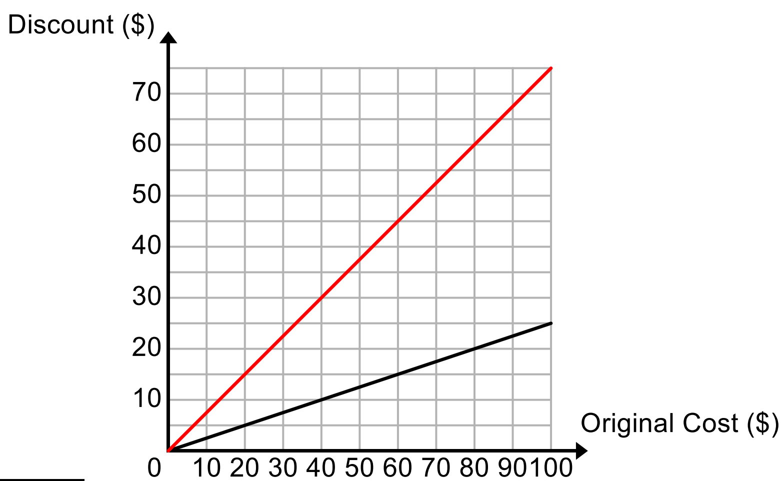
Equation: ________________

a. Will is running a 10-km race. How many miles is Will running?

b. Your Question:

14. Everything at Scooter's Skate Shop is on sale. The amount of discount based on the original cost of the item is shown on the graph. Students study 7.RP.3 in Chapter 2 using models to solve multi-step percent problems. We revisit percent problems in this chapter to help students view them as proportional relationships.

Original Cost ($)	Discount ($)
10	2.50
20	5
30	7.50
40	10
100	25



Equation: _______$y = 0.25x$____________

a. Chantelle is buying a sweater that originally cost $45, how much of a discount will she receive on the sweater? How much will she pay for the sweater before tax?
$11.25; $33.75

b. If Landon paid $60 for an item before tax, what was the original price of the item?
$80; it is interesting to think about this using the graph. Students may realize that they need to identify a point where the difference between the original cost and the discount is $60. This occurs at the point $(80, 20)$.

c. Write an equation that shows the relationship between the sale price y of an item before tax and the original cost of the item x. $y = 0.75x$

d. Create a graph of the relationship in part c) on the same grid above using a different color.
Students may opt to create a table to graph this relationship. Alternatively, students may reason that if the original cost of an item is $20 and I can see the discount is $5, I know I will pay $15 for the item. Similarly, if an item is $40, I can see the discount is $10, so I will pay $30.

Spiral Review

1. Maria is setting up a game for a school fundraiser carnival. The game will be a fishing pond with 300 rubber ducks. On the bottom of each rubber duck will be either a red, blue or white sticker to indicate the type of prize the player will win. A red sticker will earn the player the top prize. Maria wants the probability of getting a red sticker to be 5%. On how many of the ducks should Maria attach a red sticker?

2. Factor the following expressions.

   $3x - 6$ $\qquad$ $15 - 20y$

   $\frac{5}{3}x + \frac{2}{3}$ $\qquad$ $4.9t - 2.8$

3. Use the distributive property to rewrite $a(b + c)$

4. Use a model to solve $7 = 3x - 2$

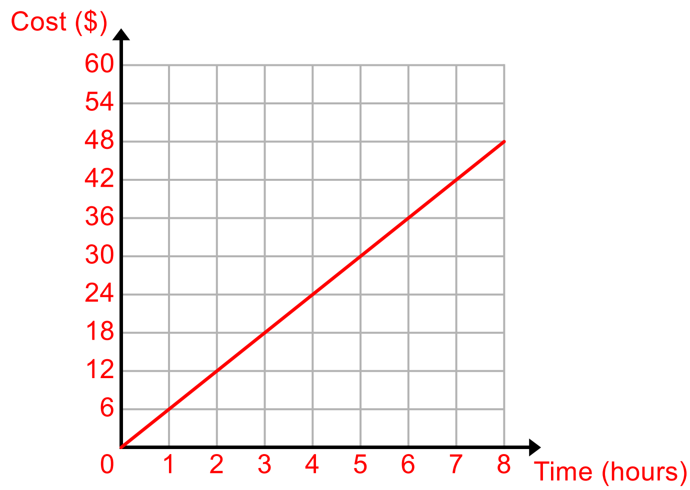
5. Chloe has twice as many cats as her sister has dogs. Her brother has 3 turtles. Together, they have six pets. How many of each pet do they have?

4.2f Homework: The Representations of Proportional Relationships

Directions: In the following problems, one representation of a proportional relationship is given. Complete the other representations. Use your representations to answer the questions that follow.

1. Rhonda earns $24 for 4 hours of babysitting.

Time (hours)	Amount Earned (dollars)
0	0
1	6
2	12
3	18
4	24

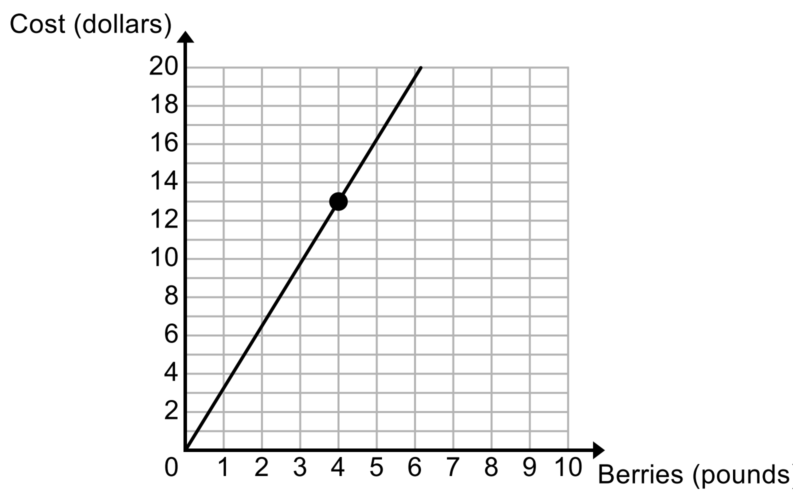


Equation: ____$y = 6x$______________

a. What is the unit rate in this situation? Highlight the unit rate in your table, graph, and equation.
$6 per hour

b. How much will Rhonda make if she babysits for 8 hours?
$48 – students can use the equation or double the values for $(4, 24)$

2. The cost of blackberries at the Berry Mart is shown in the graph.



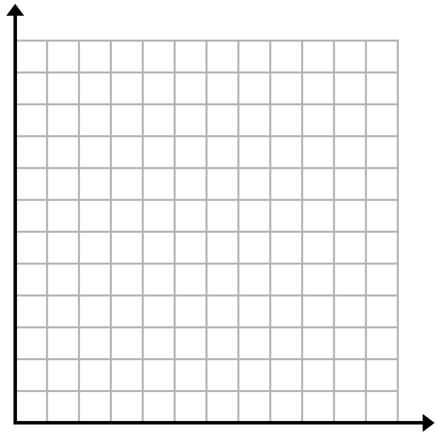
Equation: ____________

a. How much will 8 pounds of blackberries cost?

b. At a store across the way it costs 8 dollars for 2.5 pounds of blackberries. Which store has the better deal? Explain.

3. The table below shows the relationship between the number of houses that can be built in a new subdivision and the acres of land available.

Acres	Houses
$\frac{1}{4}$	1
1	
	100
100	



Equation: ________________

a. If the subdivision has an area of 15 acres, how many houses can be built? Write your answer as an ordered pair.

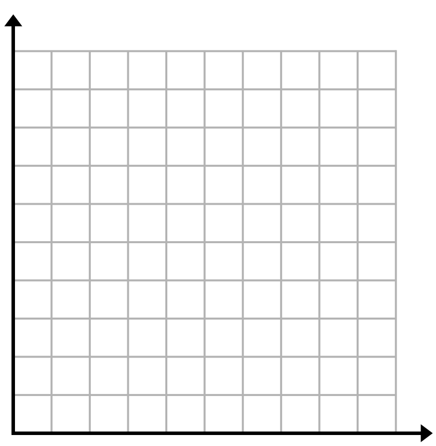
4. The equation $y = 8x$ can be used to find the number of ounces y in x cups.

Cups	Ounces
0	0
1	8
2	16
3	24
4	32

a. How many ounces are in 8 cups of water?
64

5. There are 5 inches of water in a bucket after a 2.5 hour rainstorm.

Time (hours)	Water (in.)
0	
$\frac{1}{2}$	
1	
2	
3	

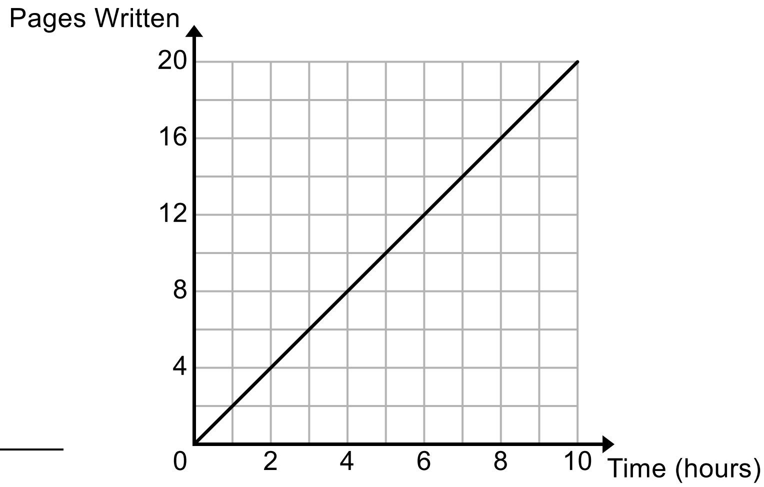


Equation: _____________

a. At this rate, how much water will there be in the bucket after 8 hours?

6. The graph shows the number of pages Lanna can write based on how long she writes for.

Time (hours)	Pages Written
0	0
0.5	1
1	2
1.5	3
2	4



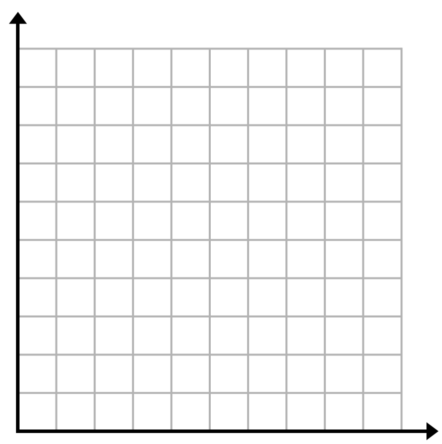
Equation: ___$y = 2x$___________

a. How many pages can Lanna write in 8 hours? 16

b. Lanna's book was 300 pages long. How long did it take her to write the book? 150 hours

7. The table shows the relationship between cost and number of apples purchased.

Number of Apples	Cost (dollars)
0	0
1	0.20
2	0.40
3	0.60
4	0.80
5	1

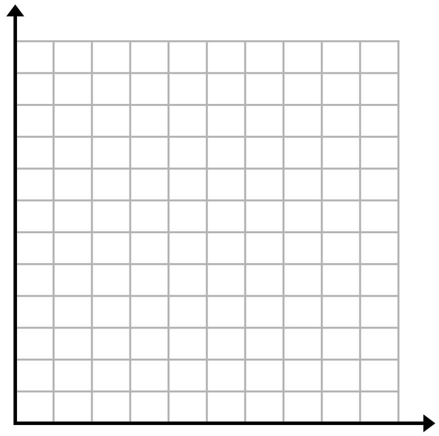


Equation: _______________

a. What is the unit rate? What does it represent in the situation?

b. How much will Snow pay for a dozen apples?

8. Alejandra bought $\frac{1}{4}$ of a pound of gummy bears for $2.

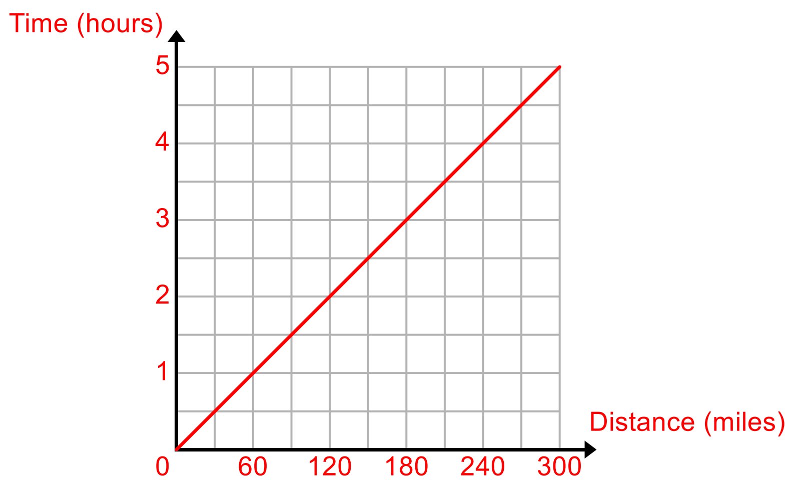


Equation: ________________

a. How many pounds of gummy bears can Alejandra buy with $5?

9. It takes Itzel a half hour to drive 30 miles.

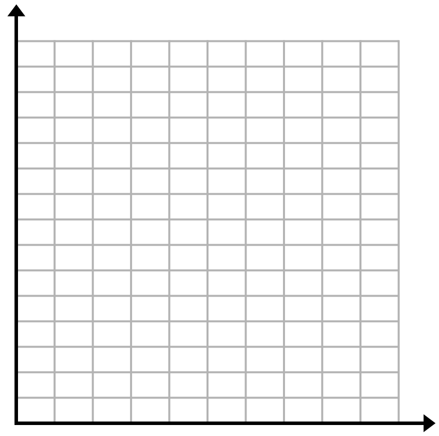
Distance (miles)	Time (hours)
30	0.5
60	1
90	1.5
120	2
150	2.5



Equation: __$y = \frac{x}{60}$______________

a. At this rate, how long will it take Itzel to drive 300 miles?
5 hours

10. Lorenzo always tips the same percentage of the total bill. On a bill that was $50, Lorenzo tipped $7.50. Complete the table, graph, and equation to show the relationship between the cost of a bill and the amount that Lorenzo tips.



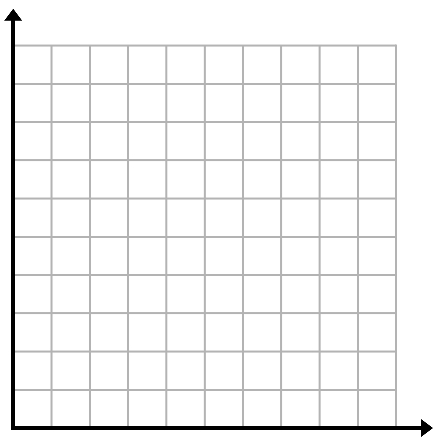
Equation: ______________

a. What percent does Lorenzo tip?

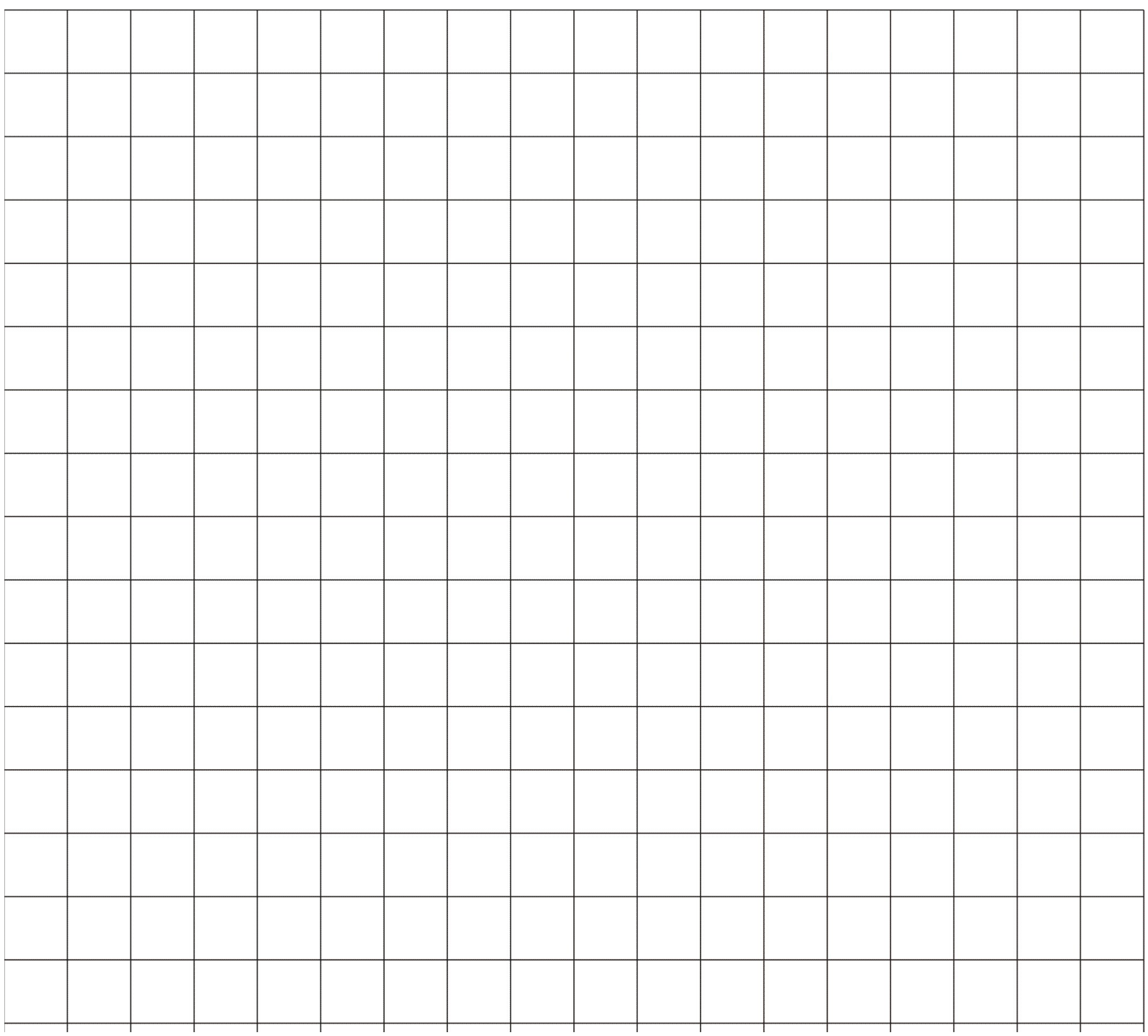
b. If the bill is $65, how much would you expect Lorenzo to leave in tip?

c. If Lorenzo left $6 in tip, how much was the bill?

11. Darien is making different sized rectangles for an art project. He wants all of the rectangles to be the same shape but different sizes. The equation $y = 2x$ shows the relationship between the length y in centimeters and the width x in centimeters of the rectangles.



a. Sketch a few of these rectangles on the grid below. Assume each segment measures 1 cm.



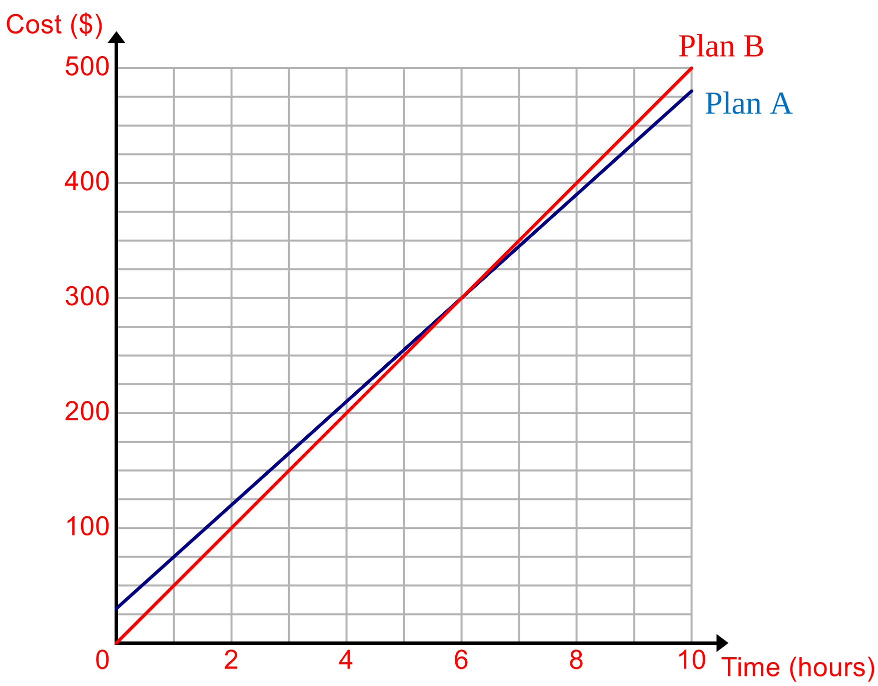
4.2g Class Activity: Proportional and Nonproportional Relationships

1. Your dad has researched renting jet skis for your family vacation. He found the following two cost fee plans:
   Rental plan A: $30.00 initial one time rental fee plus $45.00 per hour.
   Rental plan B: $50.00 per hour with no initial one time rental fee.

   a. Create data tables to show how much it will cost to rent jet skis from each company.

Rental Plan A			Rental Plan B		
Hours	Expression for Total Cost	Total Cost (in dollars), simplified	Hours	Expression for Total Cost	Total Cost (in dollars), simplified
1	$30(1) + $45(1)	$75	1	$50 (1)	$50
2	$30(1) + $45(2)	$120	2	$50(2)	$100
3	$30(1) + $45(3)	$165	3	$50(3)	$150
4	$30(1) + $45(4)	$210	4	$50(4)	$200
5	$30(1) + $45(5)	$255	5	$50(5)	$250
x	$30(1) + $45(*x*)	$30 + $45*x*	*x*	$50(*x*)	$50*x*

   b. Create graphs for each situation on the same grid. Label the axes. Put a title on the graph. Label graphed lines by plan A or B.



Directions: Answer the questions related to the two plans shown on the tables and graphs above.

c. How long can you rent a jet ski if your family has budgeted $350 for the rental fee?

Plan A: _just over 7 hours__________________ Plan B: ____7 hours___________________

d. Look at the tables. Compare the total cost to the number of hours for each row of the table. Is there a set unit rate? Does the table show a proportional relationship for this plan?

Plan A: ____No_____________________ Plan B: ______Yes___________________

e. Write an equation that relates the total cost, *y*, and the number of hours, *x*.

Plan A: ___$y = 30 + 45x$__________________ Plan B: ___$y = 50x$____________________

It is not expected that students in Grade 7 are able to write equations in the form $y = mx + b$ as in Plan A. Students should be table to talk through how they calculated the cost for Plan A and you can help them to write the equation.

f. Use your graph to find the cost to rent one jet ski for 10 hours.

Plan A: ___$480________________________ Plan B: ____$500_______________________

g. Explain why and when you would you choose to use each plan.

Plan A is the better option if you are going to rent the jet ski for more than 6 hours. Plan B is the better plan if you are going to rent the jet ski for less than 6 hours. At 6 hours, the plans cost the same.

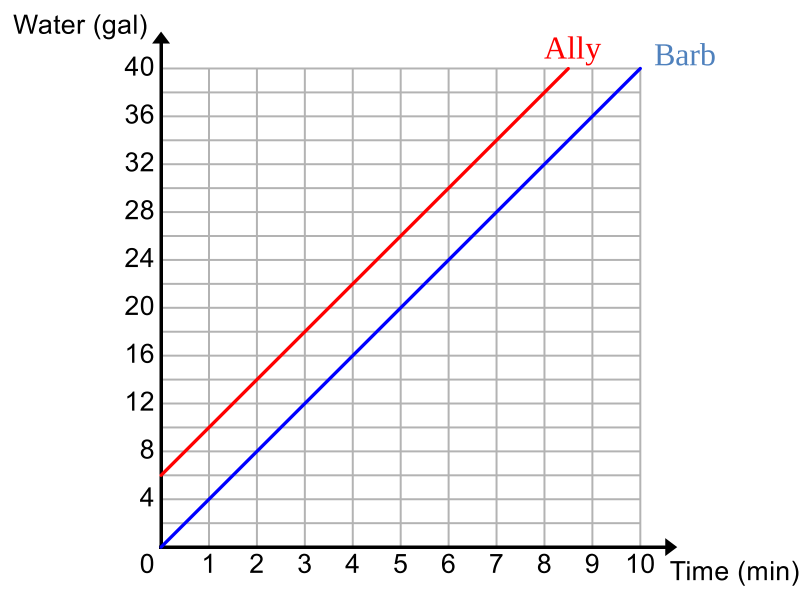
2. Aly and Barb are both filling swimming pools at a constant rate of 2 gallons every $\frac{1}{2}$ minute. Aly's pool started with 6 gallons of water in it while Barb's pool was empty.

a. Complete the tables below. Graph each relationship on the same grid using a different color. Label the lines.

Barb's Pool	
Time (min)	Water (gal)
0	0
1	4
2	8
3	12
4	16
5	20
8	32

Aly's Pool	
Time (min)	Water (gal)
0	6
1	10
2	14
3	18
4	22
5	26
8	38

+1 +1 +1 / +4 +4 +4



b. Write an equation that shows the amount of water y, in gallons, after x minutes.

Barb's Pool: $y = 4x$ Aly's Pool: $y = 6 + 4x$

c. Are the relationships proportional? Explain how you know using each of the representations.

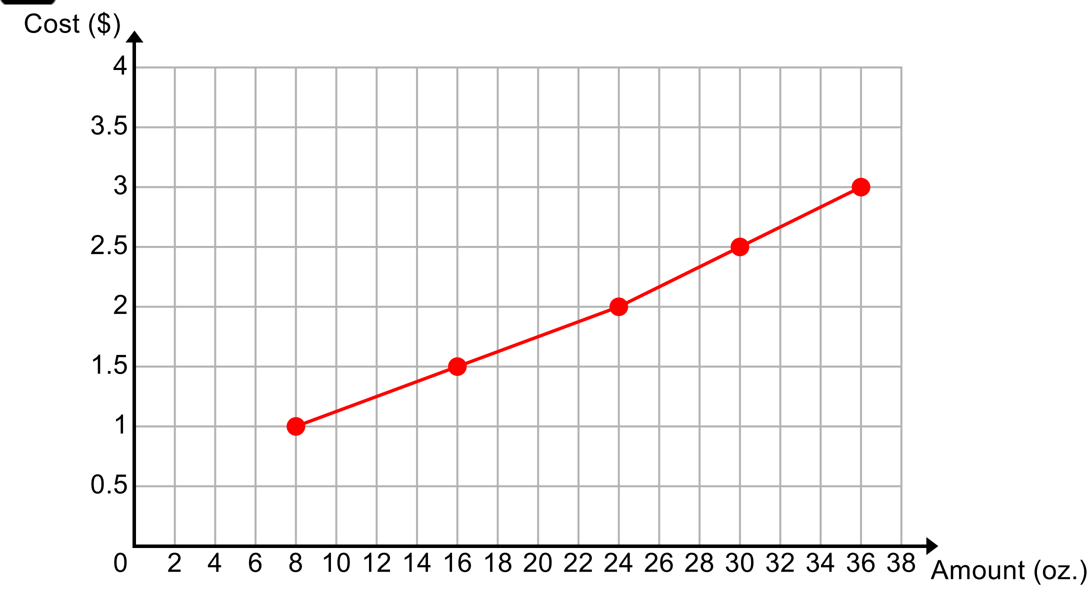
Barb's Pool:	Aly's Pool:
At this point, students should know that Barb's is in a proportional relationship because ratio of y to x is constant. In this lesson, we are solidifying for students that the graph of a proportional relationship is a straight line that passes through the origin and that the equation takes the form $y = kx$.	At this point, students should realize that this is not a proportional relationship because the ratio of y to x is not constant. The graph shows that this line does not pass through the origin. Additionally the equation is not in the form $y = kx$. Students will likely notice that the additive structure still holds, for each additional unit of x, y increases by the same amount as shown on the table. Work with students to find the equation. **You are not teaching $y = mx + b$.** Rather you're helping students gain an understanding that while the relationship is not proportional it is predictable. Students should be encouraged to arrive at the equation using numerical reasoning. Ask students how the contexts are different. Aly's pool starts with water in it.

3. The following table shows the relationship between the number of ounces in soup cans and cost.

Amount (oz.)	8	16	24	30	36
Cost ($)	1.00	1.50	2.00	2.50	3.00

a. Is this a proportional relationship? Explain.
No, the unit rate is not constant

b. Graph the information on the table below. How does the graph support your claim made above?

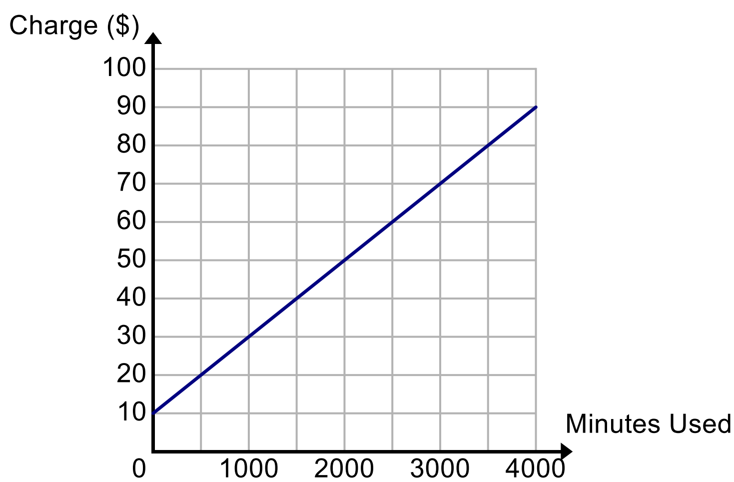


The graph supports this conclusion because the points do not lie on a straight line.

4. The graph shows the monthly charge for a cell phone based on the number of minutes used.

a. Complete the table.

Minutes Used	**Charge ($)**
0	10
500	20
1000	30
1500	40
2000	50
5000	110



b. Are cost and time proportionally related in this situation? Justify your answer using the representations shown above. No, the unit rate is not constant. The line does not pass through the origin. In the table, $t = 0$ corresponds to 10. A proportional relationship will contain the point (0, 0).

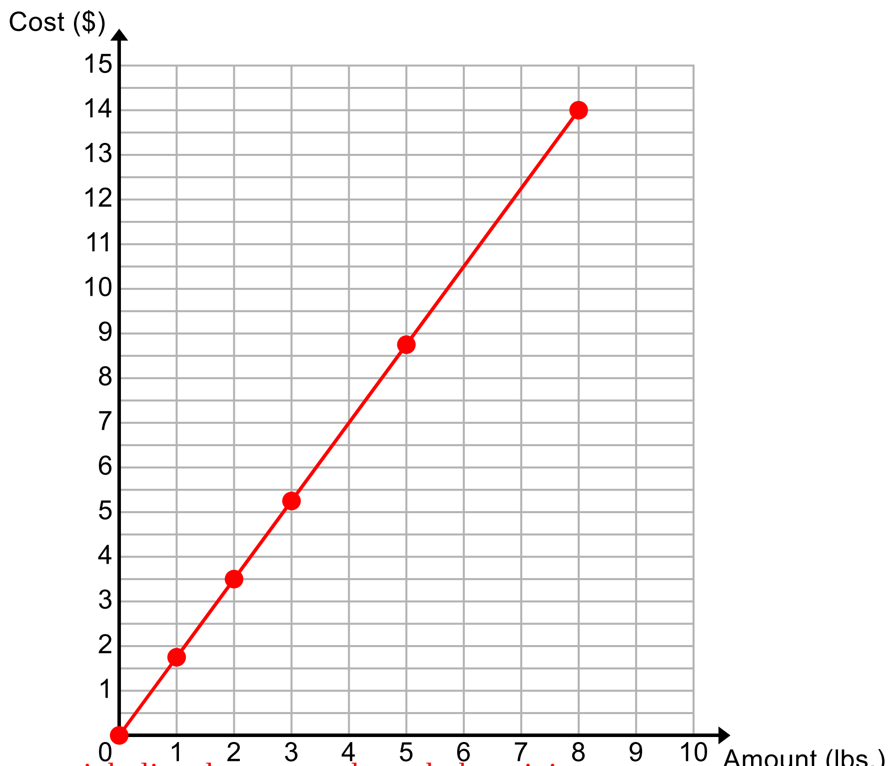
5. The following table shows the relationship between the amount of apples, in pounds, and cost, in dollars.

Amount (lbs.)	1	2	3	5	8
Cost ($)	1.75	3.50	5.25	8.75	14

a. Is this a proportional relationship? Explain.
Yes, the ratio of y to x is constant (i.e. the unit rate is constant)

b. If the relationship is proportional, write an equation that represents the cost y, in dollars, of x pounds of apples. $y = 1.75x$

c. Graph the information on the table below. How does the graph support your claim made above?



The graph is a straight line that passes through the origin.
Talk about a quick and efficient way to graph. Consider using the points (2, 3.50) and (8, 14).

6. Genevieve is filling a glass beaker with water. The weight of the beaker y, in grams, can be modeled by the equation $y = x + 60$ where x represents the amount of water, in millimeters. Complete the table and graph to represent this situation.

Amount of Water (ml)	0	1	5	10	100
Weight (g)	60	61	65	70	160

a. Complete the following ordered pair: (0, _60__). What does this point represent in the situation?
The beaker weighs 60 grams when it is empty.

b. Is the relationship between weight and amount of water proportional in this situation? Explain.
No, the unit rate is not constant. A graph would not pass through the point (0, 0).

7. A flower that is 3 inches tall when planted grows $\frac{1}{2}$ inch every 4 weeks. Complete the table to show this relationship.

Time (weeks)	0	4	8	12	16
Height (inches)	3			$4\frac{1}{2}$	

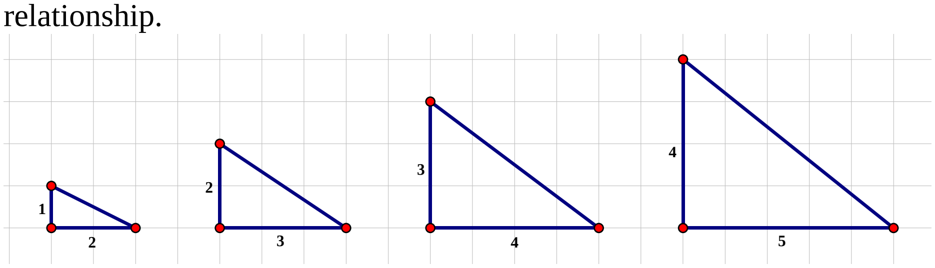
a. Is the relationship between time and height in this situation proportional? Explain.

8. The sum of two numbers, x and y, is 0. Complete the table to show this relationship.

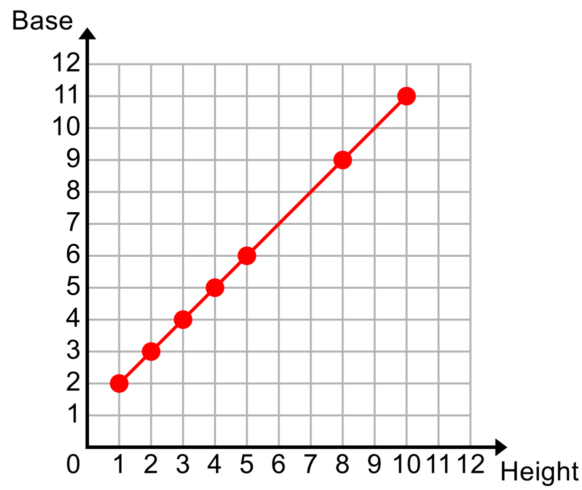
x	y
−2	2
−1	1
0	0
1	−1
2	−2

a. Is this relationship proportional? Explain.
Yes

9. Max is making triangles following the pattern below. Complete the table and graph to show this relationship.



Height	Base
1	2
2	3
3	4
4	5
5	6
8	9
10	11



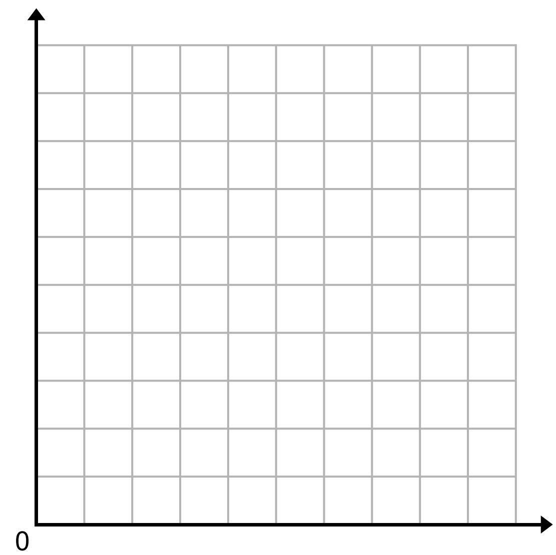
a. Is the relationship between the height and base of these triangles proportional? Explain.

No, the unit rate is not constant. The graph does not pass through the origin.

10. Create your own set of triangles or rectangles that show a proportional relationship between the height and the base. Answers will vary

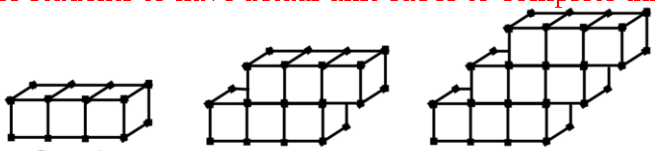
a. Complete the table and graph for your figures.

Height	**Base**



b. How do the table and graph show that the relationship is proportional?

11. Penny is using blocks to build stairs.
It would be helpful for students to have actual unit cubes to complete this problem.



a. Complete the table for this pattern.

Stage #	Volume	Surface Area
1	3	14
2	6	24
3	9	34
4	12	44
5	15	54
10	30	104
25	75	254
x	$3x$	$10x + 4$

b. Describe the growth of the volume as related to the stage.
Volume is increased by 3 units with each stage

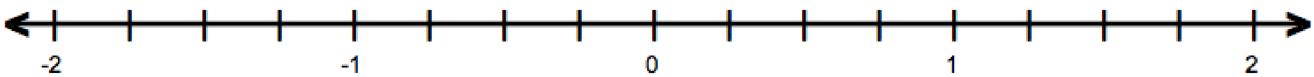
c. Describe the growth of the surface area as related to the stage.
Surface area is increased by 10 units with each stage.

d. Which of the relationships is proportional? Which of the relationships is not proportional? Explain. The relationship between stage # and volume is proportional. The relationship between stage # and surface area is not proportional.

Spiral Review

1. Place the fractions on the number line below.

$$-\frac{3}{4}, \quad \frac{5}{12}, \quad -1\frac{1}{4}, \quad -\frac{4}{3}$$



2. Write $\frac{1}{5}$ as a percent and decimal.

3. Solve $-3x + 5.5x + 4 = 7.2$

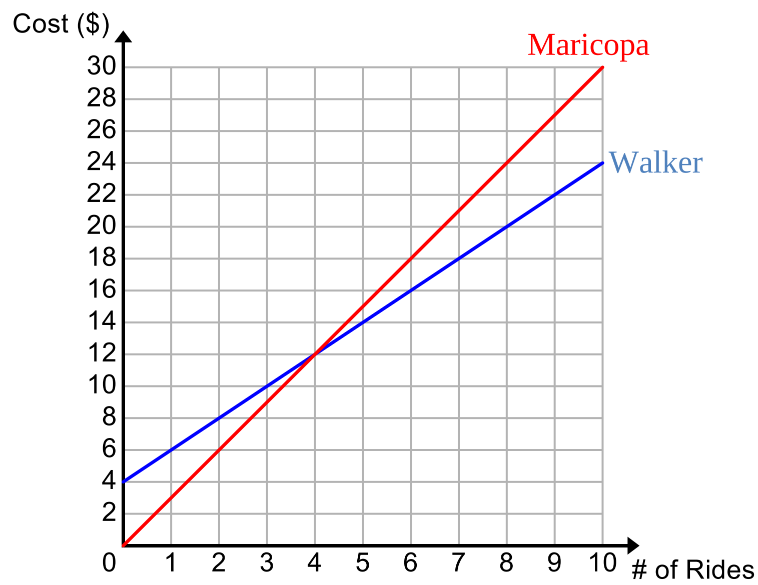
4. Simplify the following expression. Use a model if needed.
   $2(m - 1) + 3m - 4 + m$

4.2g Homework: Proportional and Nonproportional Relationships

1. At the Walker County Fair, it costs $4 to get into the fair and $2 for each ride. At the Maricopa County Fair, it is free to get into the fair and each ride costs $3.
   a. Complete the tables below. Graph each relationship on the same grid using a different color. Label the lines.

Walker County Fair	
# of Rides (x)	Cost (y)
0	4
1	6
2	8
3	10
4	12
5	14
8	20

Maricopa County Fair	
# of Rides (x)	Cost (y)
0	0
1	3
2	6
3	9
4	12
5	15
8	24



   b. Write an equation for both fairs that shows the cost y for x rides. *The equation for the Walker County Fair is a bonus problem.

   *Walker County Fair: $y = 2x + 4$ Maricopa County Fair: $y = 3x$

   c. Are the relationships proportional? Explain how you know using each of the representations.

Walker County Fair:	Maricopa County Fair:
Not proportional; Have students share their explanations.	Proportional; Have students share their explanations.

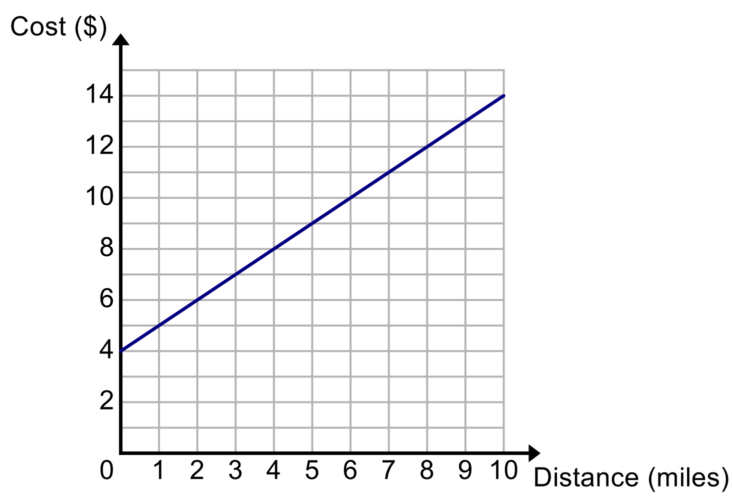
2. Use the graph below to fill in the table relating cost to cab-ride distance.

Miles	Cost	Ordered Pair	Write a complete sentence describing the meaning of this point on the graph.

a. Is this a proportional graph? Why or why not?

b. Looking at the graph, describe what's happening to the cost as miles increase.

c. Is the unit rate constant?



d. Challenge: Write the equation relating cost to cab-ride distance. Explain the equation in words.

3. The following table shows the relationship between eggs and cups of milk used in a recipe.

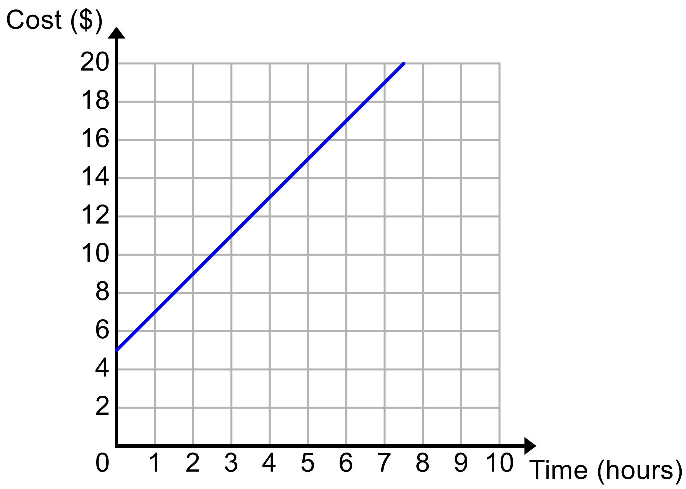
Eggs	1	2	3	4	5
Cups of Milk	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$

a. Is this a proportional relationship? Explain.
Yes, the unit rate is constant, one uses $\frac{1}{2}$ cup milk for each egg

b. Describe what a graph of this situation would look like.
It would be a straight line through the origin.

c. If the relationship is proportional, write an equation that represents this situation.
$m = \frac{e}{2}$

4. The graph below shows the amount Sunset Splash charges based on the amount of time you spend at the park.
   a. Complete the table to represent this situation.



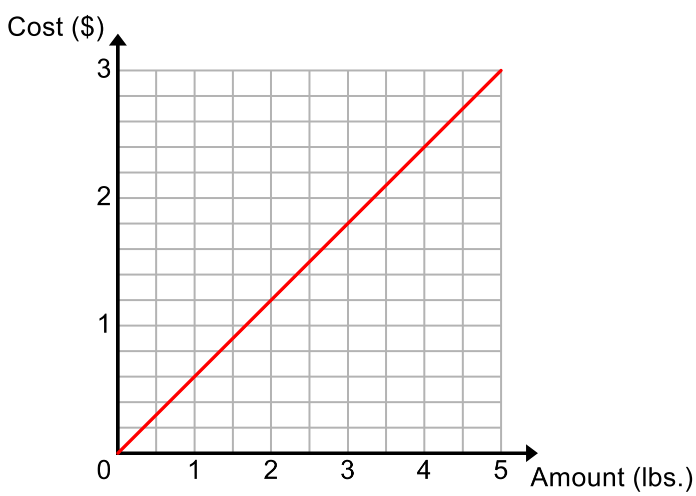
Time (hours)	Cost ($)
0	
1	
2	
3	
4	

   b. Are cost and time proportionally related in this situation? Justify your answer using the representations shown above.

5. The following table shows the relationship between the amount of bananas, in pounds, and cost, in dollars.

Amount (lbs.)	0	1	1.5	5	8
Cost ($)	0	0.6	0.9	3	4.80

   a. Graph the information.



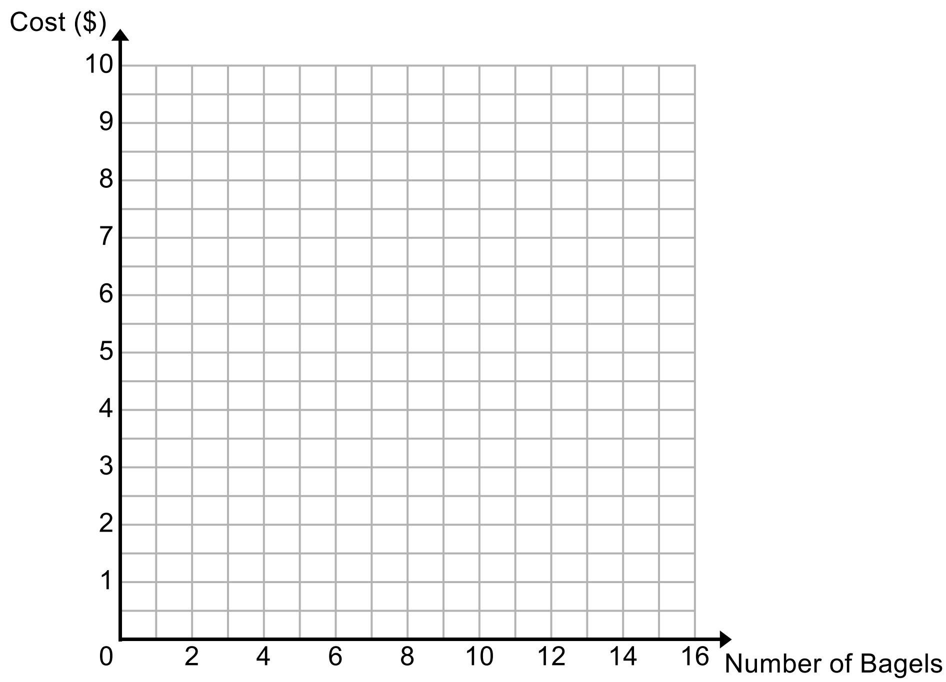
   b. Is this a proportional relationship? Explain.
   Yes

   c. If it is a proportional relationship, write an equation for the cost y, in dollars, of x pounds of bananas. $y = 0.6x$

6. The following table shows the relationship between number of bagels purchased and cost.

Number of Bagels	1	2	6	13	26
Price ($)	0.75	1.50	4.50	9.00	18.00

a. Graph the information.



b. Is this a proportional relationship? Explain.

c. If it is a proportional relationship, write an equation for the cost *y*, in dollars, of *x* bagels.

7. The total weight *y*, in pounds, of *x* math textbooks can be modeled by the equation $y = 5x$. Complete the table and graph to represent this situation.

Number of Books	0	1	5	10	100
Weight (lbs.)					

a. Complete the following ordered pair: $(0, \_\_\_\_)$. What does this point represent in the situation?

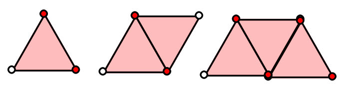
b. Is the relationship between number of textbooks and weight proportional in this situation? Explain.

8. The sum of two numbers, x and y, is 1. Complete the table to show this relationship.

x	y
−1	2
0	1
2	−1
5	−4
3	−2

a. Is this relationship proportional? Explain. No

9. Penny is making triangle trains with her blocks as shown.



a. Complete the table to show the pattern. Recall that perimeter is the distance around a shape.

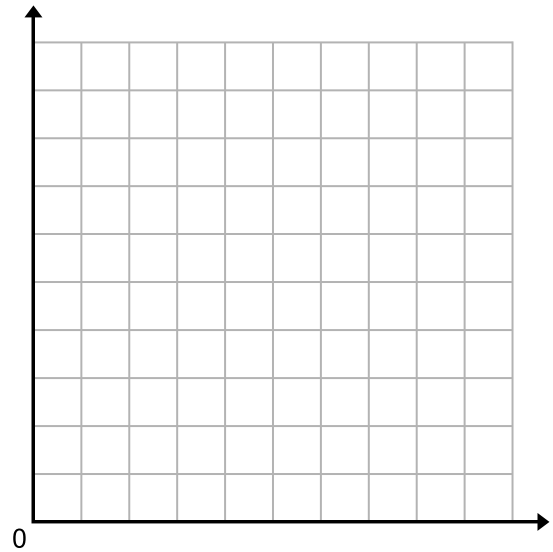
Stage # or area	Perimeter
1	3
2	
3	
4	
5	
10	
100	
x	

b. Describe how the perimeter changes from one stage to the next.

c. Is this relationship proportional? Why or why not?

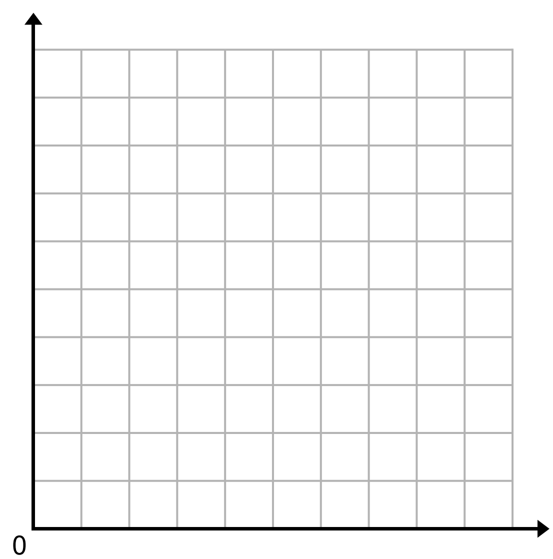
10. Make up your own situation that would show a relationship that is proportional. Create the table, graph, and equation for your situation and use the representations to show that the relationship is proportional.

Situation:



11. Make up your own situation that would show a relationship that is **<u>not</u>** proportional. Create the table, graph, and equation for your situation and use the representations to show that the relationship is proportional.

Situation:



4.2h Class Activity: More Proportional and Nonproportional Relationships

1. Explain how you can determine whether a relationship is proportional or non-proportional. Provide examples to help support your explanations.

    a. …looking at tables.

    If there is a constant ratio for each of the rows then the relationship is proportional.

    b.…looking at graphs.

    If the graph is a straight line through the origin (0,0), then the relationship is proportional.

    c.…looking at equations.

    If the equation is in the form $y = kx$ then it is a proportional relationship.

2. Determine whether the tables show a proportional relationship between *x* and *y*. For each table that is proportional, write an equation that shows the relationship between *x* and *y*.

a.

x	**y**
$\frac{1}{2}$	1
3	6
5	10
10	20

Yes
$y = 2x$

b.

x	**y**
−2	−1
0	0
2	1
5	2.5

Yes
$y = \frac{x}{2}$

c.

x	**y**
0	2
1	3
2	4
3	5

No

d.

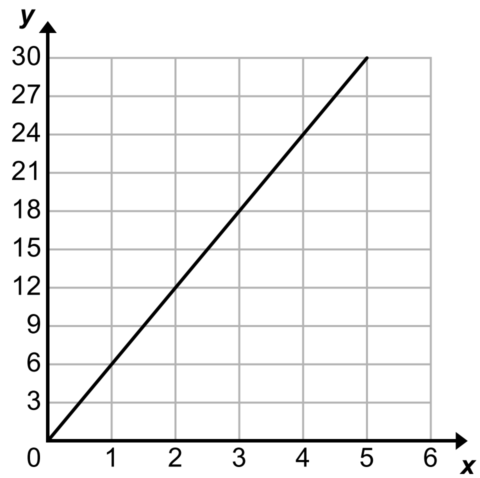
x	**y**
0	0
1	3
2	4
3	5

3. Determine whether the equations represent a proportional relationship.

a. $y = 2x$ Proportional: Yes or No	b. $y = \frac{x}{3}$ Proportional: Yes or No	c. $y = 2x + 1$ Proportional: Yes or No
d. $\frac{1}{2}y = x$ Proportional: Yes or No	e. $y = \frac{1}{2}x + \frac{1}{2}$ Proportional: Yes or No	f. $y = 0.75x$ Proportional: Yes or No

4. Determine whether the graphs show a proportional relationship between x and y. For each graph that is proportional, write an equation that shows the relationship between x and y.

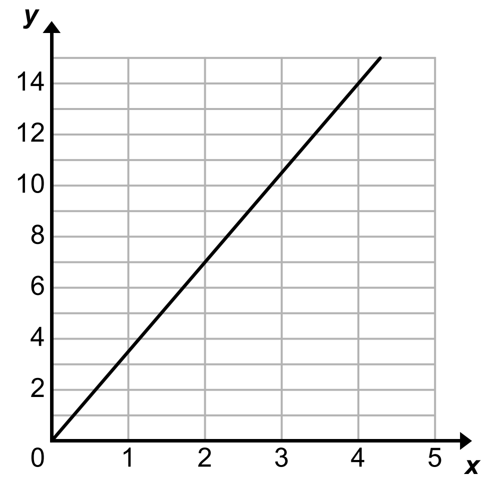
a.



Proportional: Yes or No

Equation: ___$y = 6x$___________________

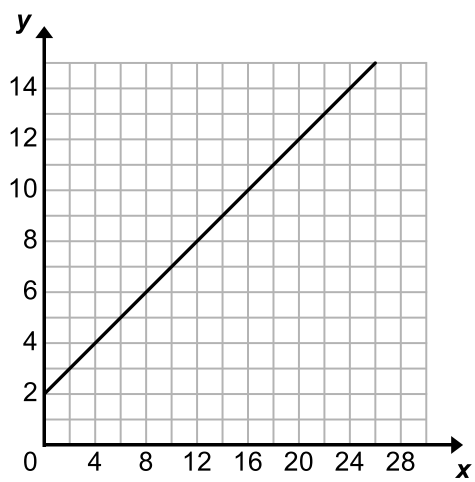
b.



Proportional: Yes or No

Equation: ___$y = \frac{7}{2}x$______________________

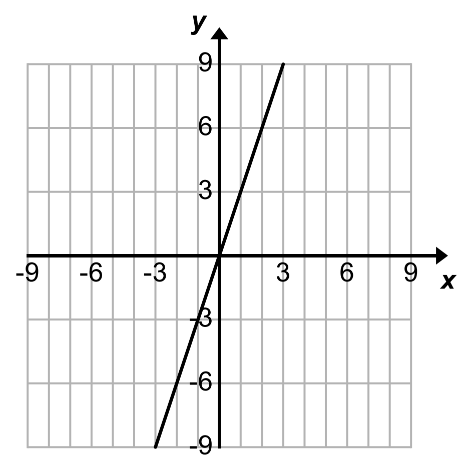
c.



Proportional: Yes or No

Equation: ________________________

d.



Proportional: Yes or No

Equation: ____________________

Spiral Review

1. Add $\frac{2}{7} + \frac{2}{3}$

2. Simplify the following expression. Use a model if needed.
   $71b - 4a + 4b - 4a$

3. Find the sum or product of each:
   a. $-3 + -7$
   b. $(-3)(-7)$
   c. $-3 - (-7)$
   d. $3(-7)$
   e. $-3 + (7)$
   f. $-3 - 7$

4.2h Homework: More Proportional and Nonproportional Relationships

1. Determine whether the tables show a proportional relationship between x and y. For each table that is proportional, write an equation that shows the relationship between x and y.

a.

x	y
0	0
1	3
2	6
4	12

b.

x	y
0	10
1	20
2	30
3	40

c.

x	y
3	0
5	2
7	4
9	6

No

d.

x	y
5	2
10	4
12	4.8
15	6

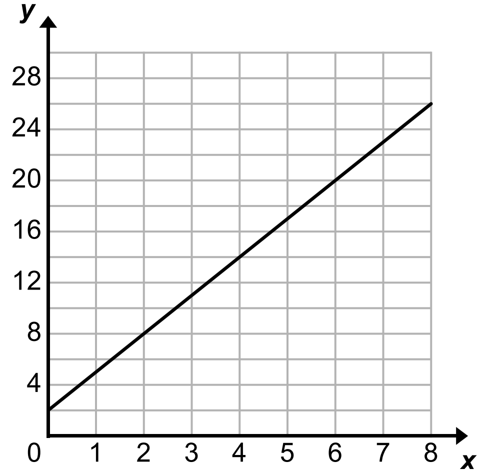
Yes
$y = 0.4x$

2. Determine whether the equations represent a proportional relationship.

a. $y = \frac{5}{2}x$ Proportional: Yes or No	b. $y = x$ Proportional: Yes or No	c. $y = x + 1$ Proportional: Yes or No
d. $y = 3x + 2$ Proportional: Yes or No	e. $y = 2.5x$ Proportional: Yes or No	f. $2y = 3x$ Proportional: Yes or No

3. Determine whether the graphs show a proportional relationship between *x* and *y*. For each graph that is proportional, write an equation that shows the relationship between *x* and *y*.

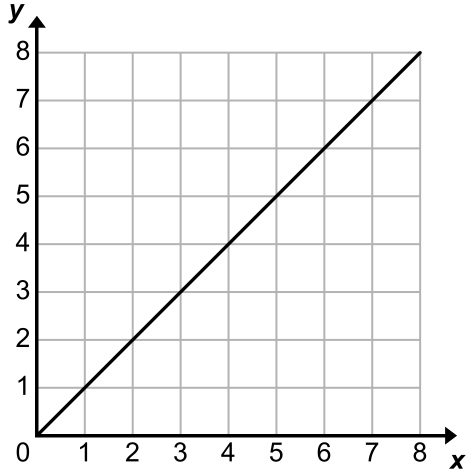
a.



Proportional: Yes or No

Equation: ____________________________

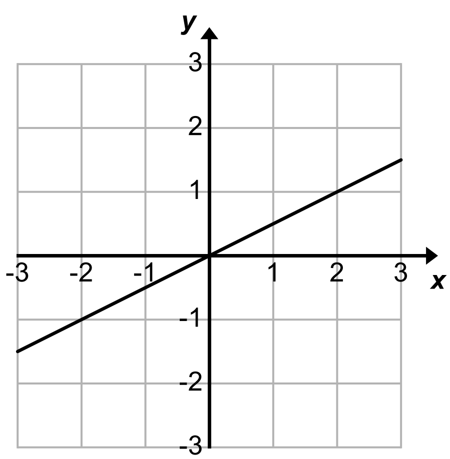
b.



Proportional: Yes or No

Equation: _______________________

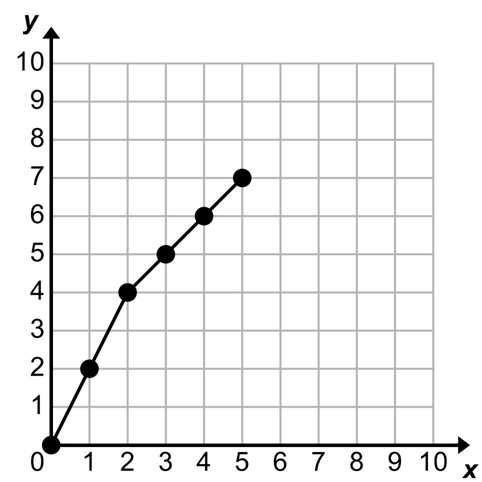
c.



Proportional: Yes or No

Equation: ____$y = \frac{1}{2}x$____________________

d.



Proportional: Yes or No

Equation: ______________________

4.2i Self-Assessment: Section 4.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Create a table of values to represent a relationship that is proportional.	I struggle to create a table of values for a proportional relationship.	I can complete a partially filled in table of values for a proportional relationship but struggle when the table involves fractions.	I can complete a table of values for any proportional relationship, including situations that involve fractions or when the tables skip around.	I can complete a table of values for any proportional relationship, including situations that involve fractions or when the tables skip around. I can construct my own table to represent a proportional relationship, accurately labeling the columns and understanding what values make sense to choose for the inputs.
2. Create a graph to represent a relationship that is proportional.	I struggle to graph a relationship from a table of values.	I can graph a relationship from a table of values, when the labels and scale are given.	I can create my own table of values in order graph a relationship. I can appropriately and accurately label and scale both axes.	I can create my own table of values in order to graph a relationship. I can appropriately and accurately label and scale both axes. I understand what the graph of a relationship should look like and I use this to help me scale the graph and create the graph quickly and efficiently. I can make connections between the graph and the other representations of a proportional relationship.
3. Explain the meaning of the point (x, y) in the context of a relationship that is proportional.	I am not sure about the meaning of the point (x, y) in a proportional relationship.	When given a context, I can write the ordered pair that corresponds to that context.	When given a context, I can write the ordered pair that corresponds to that context. I can also write the context that corresponds with a given ordered pair.	When given a context, I can write the ordered pair that corresponds to that context. I can also write the context that corresponds with a given ordered pair. I am aware of the different quantities involved and their units and use these appropriately in my explanation.
4. Explain the significance of the points $(0, 0)$ and $(1, r)$ in the graph of a proportional relationship, where r is the unit rate.	I struggle to understand the meaning of these points in a proportional relationship.	I can explain the meaning of the point (0, 0) but I am not sure about the point (1, *r*).	I can explain the contextual meaning of both of these points in a proportional relationship.	I can explain the contextual meaning of both of these points in a proportional relationship. I can generate the ordered pair (1, *r*) given any representation of a proportional relationship.

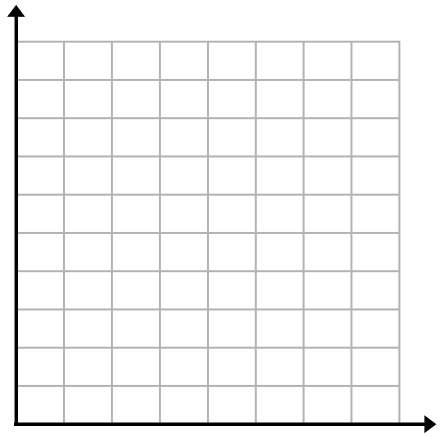
5. Write an equation to represent a relationship that is proportional.	I struggle to begin writing an equation for a proportional relationship.	I know what an equation for a proportional relationship should look like, but I often mix up the parts. I struggle to write an equation when given some of the representations.	I can write an equation for a proportional relationship given any representation.	I can write an equation for a proportional relationship given any representation. I can make connections between the equation and the other representations.
6. Identify the unit rate from a table, graph, equation, diagram, or verbal description of a relationship that is proportional.	I am not sure what a unit rate is.	I can determine a unit rate from some of the representations of a proportional relationship but not all of them. I also struggle to find the unit rate when the problem involves fractions.	I can find a unit rate given any representation of a proportional relationship and I label the unit rate with the correct units. I can determine the unit rate even when fractions are involved.	I can find a unit rate given any representation of a proportional relationship and I label the unit rate with the correct units. I can show the unit rate on a graph, table, and equation of a proportional relationship. I can also write the ordered pair that represents the unit rate in a proportional relationship.
7. Analyze the representations of a proportional relationship to solve real-world problems.	When asked questions that require me to analyze the representations of a proportional relationship, I am not sure what to do.	I can solve real-world and mathematical problems using one or two of the representations of a proportional relationship but there are a few that I am not comfortable with.	I can solve real-world and mathematical problems using several of the representations of proportional relationships.	I can solve real-world and mathematical problems using all of the representations of proportional relationships. I understand when one representation might be a more efficient way to solve a problem.
8. Determine whether two quantities are in a proportional relationship given a verbal description (context), table, graph, or equation. Explain why a relationship is or is not a proportional relationship.	I don't understand how to determine if a relationship is proportional.	I can usually determine if a relationship is proportional from a graph but I struggle when given one of the other representations.	I can determine if a relationship is proportional given any representation. I can explain why or why not in my own words.	I can determine if a relationship is proportional given any representation. I can explain why or why not in my own words. I can create my own examples of the representations of relationships that are proportional and those that are not.

Sample Problems for Section 4.2

Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. Roxana bikes the same distance to and from work every day. By the end of the third day, she had biked 42 km. Complete the table [1], graph [2], and equation [5] to show this relationship.

Time (days)	**Total Distance (km)**
3	42



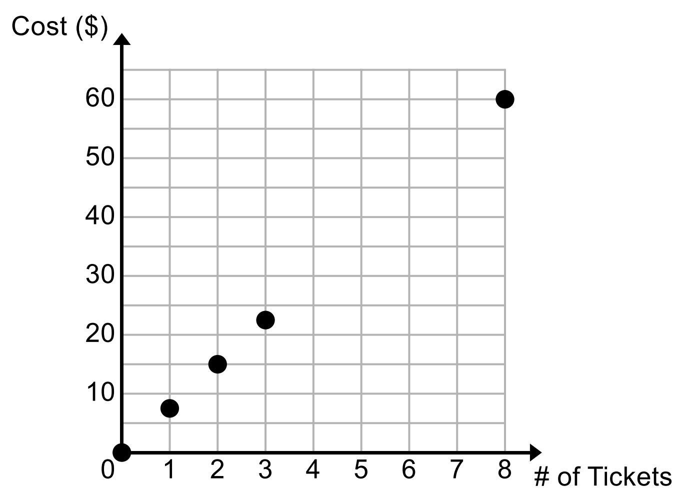
Equation: ____________

   a. Where do you see the unit rate in each of the representations? [4][6]

   b. What does the point $(4, 56)$ represent in this situation? [3]

   c. How far does Roxana bike in 5 days? [7]

2. The graph below shows the cost of movie tickets. Complete the table [1] and equation [5] to represent this relationship.



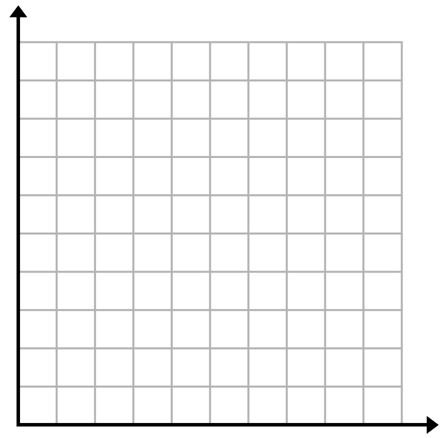
Equation: ________________

   a. What does the point $(10, 75)$ represent in the situation? [3]

   b. What is the unit rate? [6] Write the unit rate as an ordered pair. [4]

3. Wendy is making her famous chocolate milkshakes. Complete the table [1], graph [2], and equation [5] to show the relationship between cups of vanilla ice cream and tablespoons of chocolate syrup. You will complete the third column in part d. below.

Vanilla Ice Cream (c.)	**Chocolate Syrup (T.)**	
$\frac{1}{2}$		
1		
	10	
8	16	
10		
	25	



Equation: ____________

a. How many tablespoons of chocolate syrup does Wendy need for 5 cups of ice cream? [7] Write your answer as an ordered pair. [3]

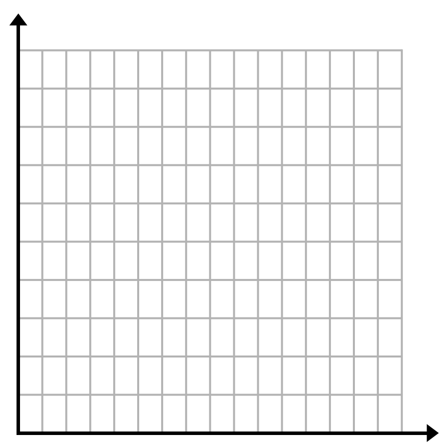
b. What does the point $(1, 2)$ represent in the situation? [4]

c. What does the point $(3.5, 7)$ represent in the situation? [3]

d. Add your favorite ingredient to the chocolate milkshakes. Put your ingredient in the third column, decide on the ratio of your ingredient to the vanilla ice cream and chocolate syrup, and complete the table [1] accordingly.

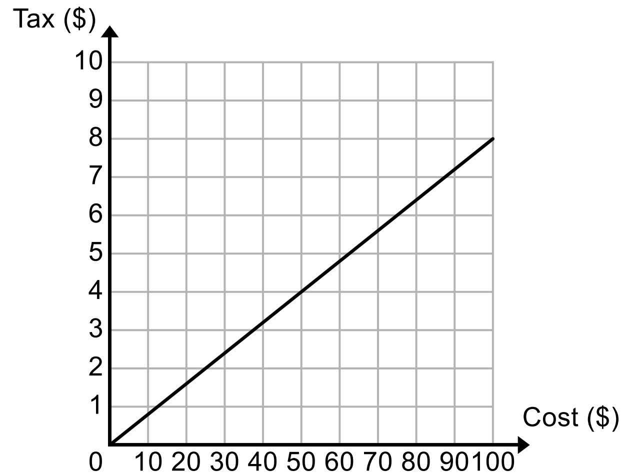
4. A recipe for salad dressing calls for 2 teaspoons of mustard for every $\frac{1}{2}$ cup of oil. Complete the table [1], graph [2], and equation [5] to show this relationship.

Oil (c.)	Mustard (tsp.)
$\frac{1}{4}$	
$\frac{1}{2}$	
1	
2	
3	
8	



Equation: ____________

5. The graph below shows the tax charged on an item based on the cost of an item. Create the table [1] and equation [5] that match the graph.



Equation: _____________

a. What is the unit rate in this situation? Where do you see the unit rate in each of the representations? [6]

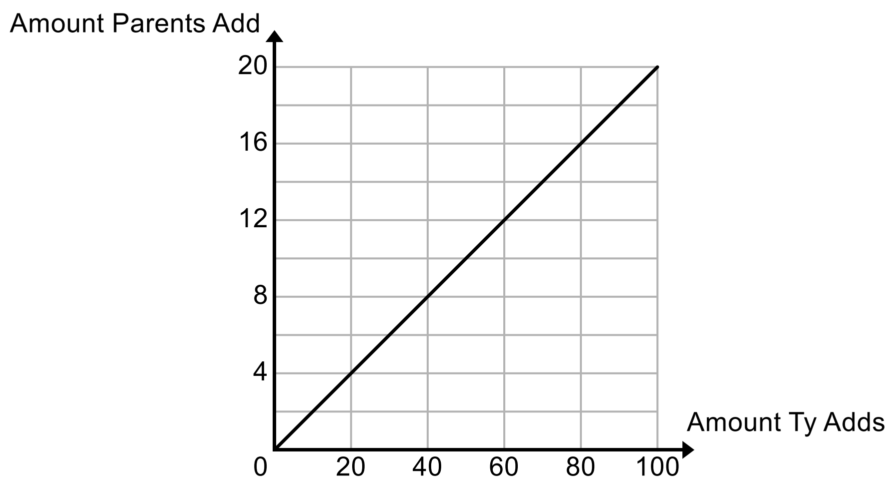
b. What is the tax rate as a percentage?

c. How much tax would you pay on a pair of jeans that costs $75? [7]

d. Norah paid $7.20 in tax. How much was her bill before tax? [7]

6. Ty is saving money. His parents told him they would also contribute to his savings. The graph shows the amount his parents add to his savings based on the amount he saves. Complete the table [1] and equation [5] to show this relationship.

1	
5	
10	
	8
100	
	30



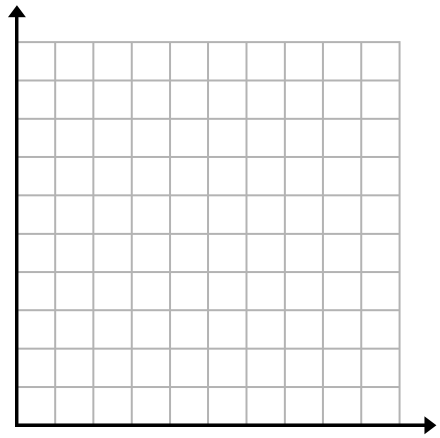
Equation: ____________________

a. What does the point $(75, 15)$ represent in the situation? [3]

b. Which point represents a total of $144 (Ty's savings plus his parents' contribution)? [3]

c. What percentage do Ty's parents match?

7. On a map, 1 centimeter represents 25 miles. Complete the table [1], graph [2], and equation [5] to represent this situation.

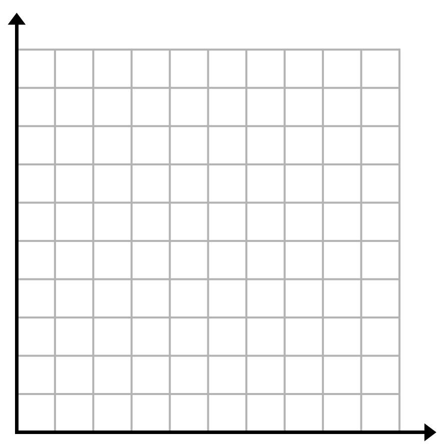


Equation: ______________

a. What is the actual distance between two cities if they are 3.6 cm apart on the map? [7]

b. If the actual distance between two cities is 360 miles, how far apart are the cities on the map? [7]

8. The equation $y = 5x$ shows the relationship between a student's score y and the number of correct answers x on a test. The maximum score a student can earn on the test is 100. Use the equation to complete the table [1] and graph [2].



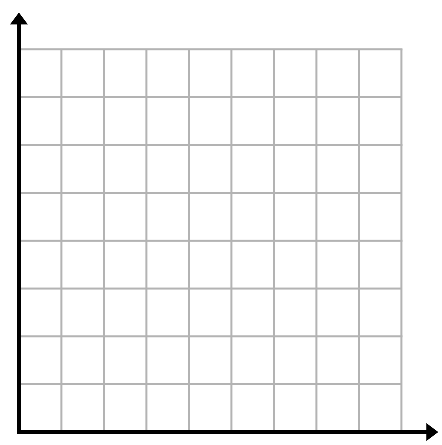
a. What does the point $(1, 5)$ represent in this situation? [4]

9. The equation $y = 0.1x$ gives the amount y, in dollars, that Susan makes in commission based on her sales x, in dollars. Complete the table [1] to show the relationship between Susan's sales and her commission.

Sales ($)	Commission ($)
0	
100	
500	
	100
	200
	1,000

a. What percentage does Susan make in commission?
b. If Susan made $150 in commission, how much did she sell? [7]
c. If Susan has a $625 sale, how much will her commission be? [7]
d. Describe what a graph of this situation would look like.

10. Complete the table [1], graph [2], and equation [5] to show the relationship between minutes and hours.

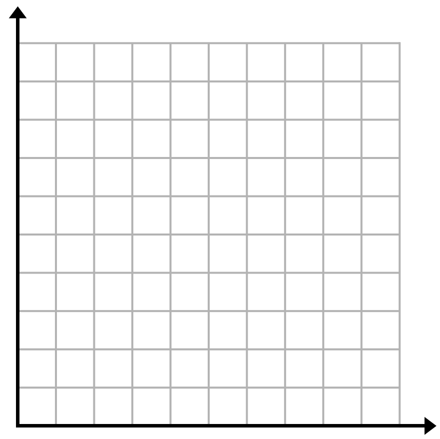


Equation: _______________

a. Use your equation to determine the number of minutes in 4.5 hours. [7]

b. Use your equation to determine the number of hours in 400 minutes. [7]

11. Complete the table [1], graph [2], and equation [5] to show the relationship between feet and inches.



Equation: ______________

a. How many inches are in 7.5 feet? [7]

b. How many feet are in 75 inches? [7]

12. Last year, Sheryl got a Labrador puppy. The table shows the age and weight of the puppy over several months. Is the relationship between weight and age proportional? Explain. [8]

Weight of dog (in pounds)	Age of dog (in months)
23	3
30	4
45	6
55	9
60	12

13. Are the relationships shown in each of the tables proportional? Explain. [8]

a.

Kelsey's Run	
Miles	Hours
$\frac{1}{2}$	$\frac{1}{8}$
2	$\frac{1}{2}$
3.5	$\frac{7}{8}$
12	3

b.

Charlie's Pay	
Sales ($)	Pay ($)
500	1,200
1,000	1,400
1,500	1,600
2,000	1,800

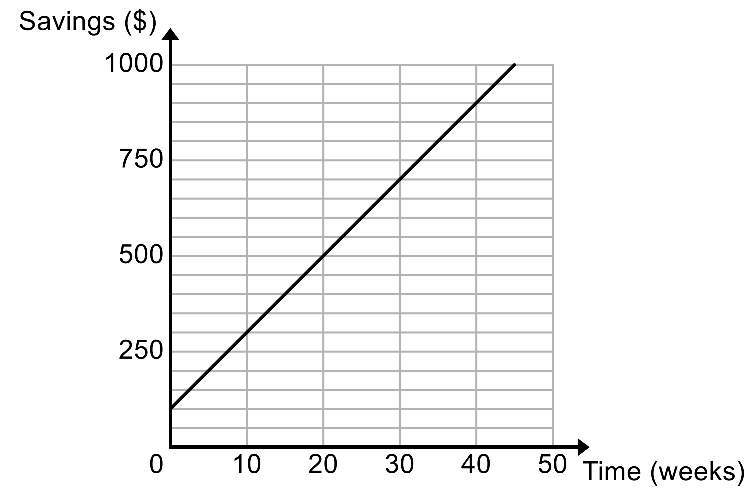
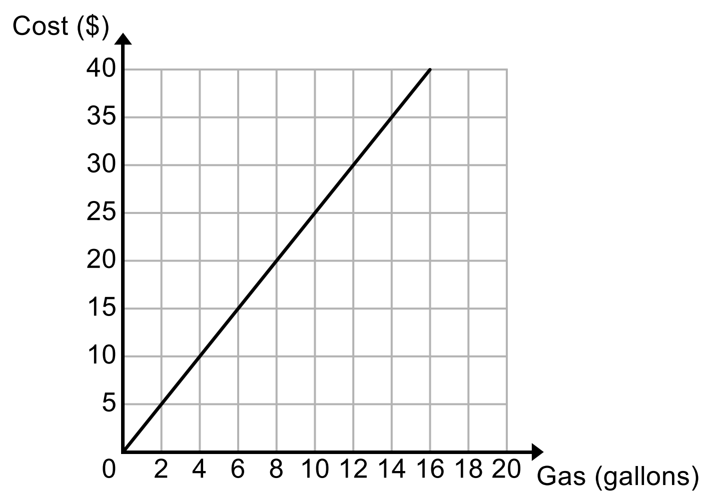
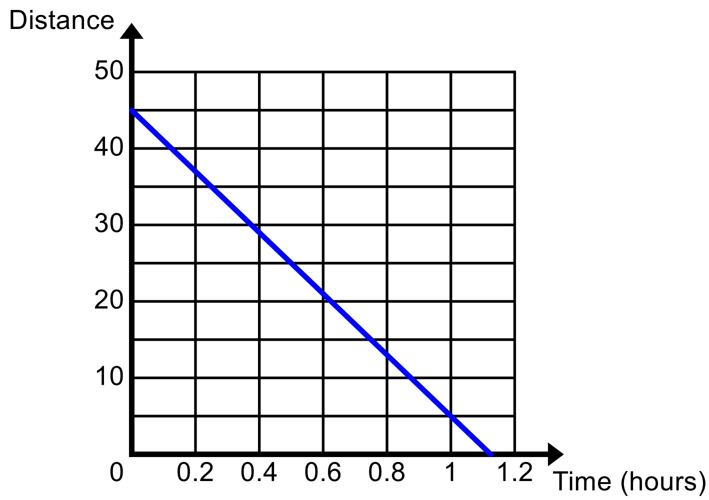
c.

Oscar's Pay	
Time (hours)	Pay ($)
1	13.95
3	41.85
8	111.60
40	558

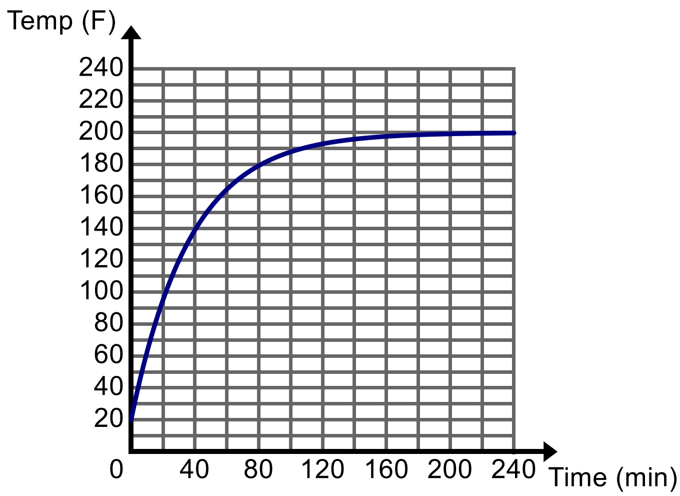
d.

Cost of Peanut Butter	
Ounces	Cost ($)
0	0
15	2.34
40	4.99
64	7.25

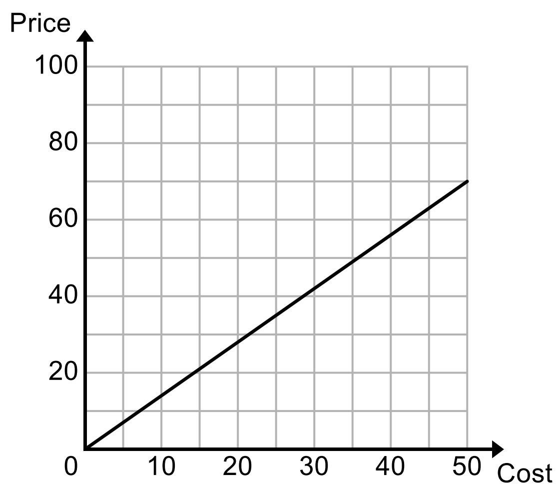
14. Are the relationships shown in each of the graphs proportional? Explain. [8]

a. 	
b. 	
c. 	

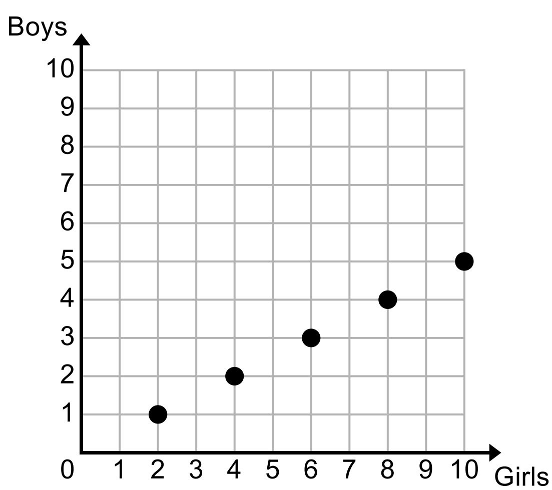
d.


Temp (F)
240
220
200
180
160
140
120
100
80
60
40
20
0
40
80
120
160
200
240
Time (min)

e.


Price
100
80
60
40
20
0
10
20
30
40
50
Cost

f.


Boys
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
Girls

15. Determine whether the equations represent a proportional relationship. Explain. [8]

a. $y = x + 12$ where *y* represents number of water bottles and *x* represents number of people Proportional: Yes or No	b. $y = 8x + 50$ where *y* represents cost of a birthday party, in dollars, and *x* represents number of guests Proportional: Yes or No	c. $y = 16 - 2x$ where *y* represents water remaining, in gallons, and *x* represents time, in minutes Proportional: Yes or No
d. $y = \frac{1}{2}x$ where *y* represents number of boys and *x* represents number of girls Proportional: Yes or No	e. $y = 8.25x$ where *y* represents cost and *x* represents # of tickets Proportional: Yes or No	f. $5y = 40x$ where *y* represents number of pictures and *x* represents number of pages Proportional: Yes or No

Section 4.3: Analyze and Use Proportional Relationships and Models to Solve Real-World Problems

Section Overview:

The concepts studied in sections 4.1 and 4.2 will be applied throughout this section. The formal definition of a proportion will be introduced, and students will set up and solve proportions for real-world problems, including problems with percentages of increase and decrease. It is likely, especially for advanced classes, that before the topic is formally introduced students will set up proportions and solve them (a) using properties of equality, (b) by finding a common denominator on both sides, or even (c) by cross multiplying, if they have seen this in previous years or at home. Any method that can be justified using proportional reasoning can lead to a meaningful discussion. Students should be encouraged to justify their answer with other representations (graph, bar model, table, or unit rates) to construct viable arguments for their understanding or bring out the possible misconception that proportions can be solved with an additive rather than a multiplicative strategy (e.g. $\frac{1}{2} = \frac{3}{6}$ so $(1 \cdot 3)/2$ $(1 \cdot 3)/2$ $(1 \cdot 3)/2$ $\frac{1 \cdot 3}{2} = \frac{3 \cdot 3}{6}$ but $\frac{1+3}{2} \neq \frac{3+3}{6}$).

In addition to solving proportions algebraically, students will examine part-to-whole ratios and compare them to the part-to-part ratios they have been using in the first two sections of the chapter. Odds and probability will be examined together to give students practice in fluently switching between part-to-part and part-to-whole relationships and solving for missing quantities using either type of ratio.

Domain(s) and Standard(s) from CCSS-M

1. Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. 7.RP.3

Primary Concepts and Skills to Master in this Section:

By the end of this section, students should be able to:

1. Fluidly transition between part:part and part:whole relationship statements.
2. Write and solve a proportion to find missing values.
3. Explain the difference between probability and odds.
4. Solve one- and multi-step problems involving percents using proportional reasoning.

4.3a Class Activity: Writing Proportions

In the previous sections, students used models and representations to find missing quantities in proportional relationships. This lesson will introduce a proportion as another tool students can use to model relationships between quantities and solve real-world problems.

1. Review: Jennifer received \$9.00 for 2 hours of babysitting. At this rate, if she baby-sat for 27 hours last month, how much did she make?

Method 1: We have solved similar problems using bar models. Solve this problem using the bar model below, and **explain your reasoning in the space next to the bar model**.

\$9.00	
1 hour	1 hour

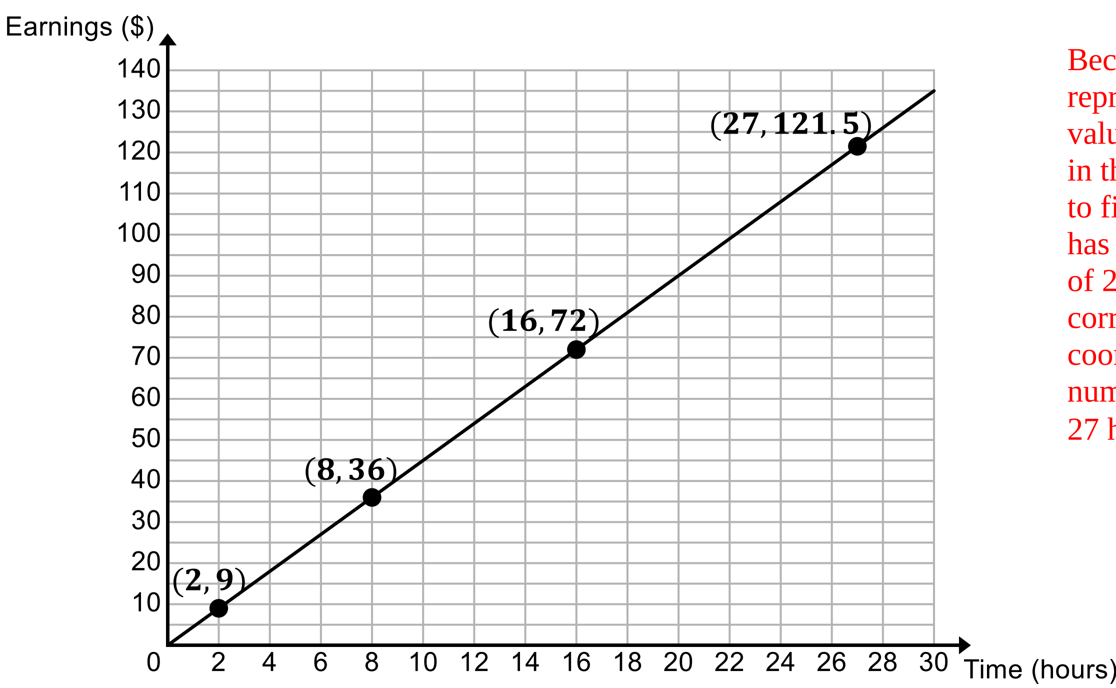
2 hours and \$9 measure the same event in different ways. Each hour is \$4.50. So, the unit rate is 4.5 and we can use that measure to answer questions relating to this relationship. To get 27 hours, we multiply 27 by \$4.50.

Method 2: Solve this problem using the table, and **explain your reasoning in the space next to the table.**

Hours (x)	2	1	27
Dollars (y)	9		

If I divide the pay by hours, I can see that Jennifer makes \$4.50 per hour. The ratio of $y : x$ is always 4.5 : 1. We see the unit rate at the point (1, r). The proportional constant 4.5/1, so $y/x = 4.5$ for any pair of values; e.g. the relationship between hours (x) and dollars (y) is $4.5x = y$. If $x = 27$ then $y = 4.5 \times 27 = 121.50$.

Method 3: We have also solved similar problems using graphs. Solve this problem using the graph below, and **explain the reasoning next to the graph**.



Because each point represents a pair of values that could be in the table, we need to find which point has an x coordinate of 27. The corresponding y coordinate is the number of dollars for 27 hours.

Method 4: We have used equations to solve problems like this also. Write an equation for this situation and **explain how the equation can be used to solve the problem.**

The proportional constant is $\frac{y}{x} = 4.5$ where x is the number of hours and y is the pay. Thus,

$y = 4.5x$
$y = 4.5(27)$
$y = 121.50$

a. State the proportional constant in this situation and describe how you found it. 4.5; $\frac{y}{x} = 4.5$; Students should be able to explain the relationship between money earned and hours in all four situations. Push students to attend to how the different representations show the same idea in different ways.

b. How many hours did Jennifer work if she earned $150 dollars? $33\frac{1}{3}$ or 33 hours and 20 minutes

c. Explain how you can use our method for finding the proportional constant to answer d.

$\frac{150}{x} = 4.5$ or $\frac{150}{4.5} = x$

The equation $\frac{y}{x} = 4.5$ is a proportional relationship. Each side of the equation is a ratio: $\frac{y}{x}$ is the ratio of y to x or money earned to hours worked, and 4.5 is the ratio of 4.5 to 1 or 4.5 dollars to 1 hour**.** ***A proportion is an equation stating that two ratios are equivalent.***

In general:

$\frac{\boldsymbol{a}}{\boldsymbol{b}} = \frac{\boldsymbol{c}}{\boldsymbol{d}}$ This proportion means "*a* is to *b* as *c* is to *d*"

Be sure you are consistent in the units of the quantities you are comparing. In the proportion below, both quantities are comparing dollars to hours: Students need to attend to precision and think about the structure of the proportion to make sure they are setting it up correctly. Additionally, they need to make sense of the quantities involved and their relationships.

We can write the following proportion for the situation above:

$\frac{\text{dollars}}{\text{hours}}: \frac{9}{2} = \frac{D}{27}$

"9 dollars is to 2 hours as how many (D) dollars is to 27 hours?"

d. Is this the only possible proportion for the question? Explain. There are a number of ways this proportion can be written. $\frac{2}{9} = \frac{27}{D}$; focus on consistent ratio across proportion. Additionally, students may use an equivalent ratio in the proportion: $\frac{1}{4.5} = \frac{27}{D}$

e. What strategies can you use to solve the proportion $\frac{9}{2} = \frac{D}{27}$?

Connect strategies to previous ideas with tables, graph and rates. Be sure to specifically refer to equivalent ratios in 4.1. Recall: a proportion can be two ratios comparing different quantities of similar units (like apples and oranges or girls and boys) or ratios comparing two different units for the same event/situation (like miles and hours or dollars and servings).

Directions: Set up *two* proportions for *each* problem. Be sure to write the units of your comparisons. You do NOT need to answer the question yet. You will come back and solve for the missing quantities for these problems as part of 4.3b homework. The first one has been done for you.

2. A 5-inch frog can leap 50 inches in one jump. If jumping distance is proportional to body length, how far should a 60-inch human be able to jump? Set up two possible proportions.

Solution:

We will first compare body length to jumping distance:

$$\frac{\text{body}}{\text{jumping}}: \frac{5}{50} = \frac{60}{J}$$

Then we will compare jumping distance to body length:

$$\frac{\text{jumping}}{\text{body}}: \frac{50}{5} = \frac{J}{60}$$

Be sure to emphasize a) units in the proportion (body to jumping) and b) our answer will create equivalent ratios. Also, students may use an equivalent ratio such as the unit rate in the proportion, replacing $\frac{50}{5}$ with $\frac{10}{1}$.

Students do NOT need to solve these problems now. They will return to solve them in 4.3b Homework.

3. Eli won 2 out of 3 chess games one day. At this rate, if he played 33 games, how many did he win?

$$\frac{\text{won}}{\text{played}}: \frac{2}{3} = \frac{x}{33} \quad \text{or} \quad \frac{\text{played}}{\text{won}}: \frac{3}{2} = \frac{33}{x}$$

4. Eli won first place in a spelling contest. Millie won second place in the contest. They had to share a cash prize in the ratio 8:5. Eli received 72 dollars. How much did Millie receive?

$$\frac{\text{first}}{\text{second}}: \frac{8}{5} = \frac{72}{x} \quad \text{or} \quad \frac{\text{second}}{\text{first}}: \frac{5}{8} = \frac{x}{72}$$

5. Bargain Betty's sells sneakers and scooters in the ratio 3:8. The store sold 21 sneakers yesterday. How many scooters were sold?

$$\frac{\text{sneakers}}{\text{scooters}}: \frac{3}{8} = \frac{21}{x} \quad \text{or} \quad \frac{\text{scooters}}{\text{sneakers}}: \frac{8}{3} = \frac{x}{21}$$

6. In 1.5 hours, a sprint-race sled dog can run 30 miles. How long would it take the dog to run 12 miles?

$$\frac{\text{hours}}{\text{miles}}: \frac{1.5}{30} = \frac{x}{12} \quad \text{or} \quad \frac{\text{miles}}{\text{hours}}: \frac{30}{1.5} = \frac{12}{x}$$

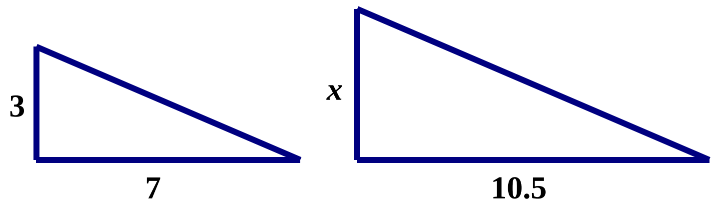
7. Bliss chocolates have 220 calories for 6 pieces. How many pieces should you eat if you want 150 calories of chocolate?

$\frac{\text{calories}}{\text{pieces}}: \frac{220}{6} = \frac{150}{x}$ or $\frac{\text{pieces}}{\text{calories}}: \frac{6}{220} = \frac{x}{150}$

8. An ant can crawl $\frac{3}{4}$ of a yard in $\frac{1}{2}$ minute. How long will it take for it to crawl 12 yards?

$\frac{\text{yards}}{\text{minute}}: \frac{3/4}{1/2} = \frac{\mathbf{12}}{\boldsymbol{x}}$ or $\frac{\text{minutes}}{\text{yards}}: \frac{\mathbf{1/2}}{\mathbf{3/4}} = \frac{\boldsymbol{x}}{\mathbf{12}}$

9. The side lengths of the two triangles below are proportional. What is the measure of the side labeled *x*?



$\frac{\text{little}}{\text{big}}: \frac{3}{7} = \frac{x}{10.5}$ or $\frac{\text{big}}{\text{little}}: \frac{7}{3} = \frac{10.5}{x}$

10. Tim is on the basketball team. His ratio of shots made to shots attempted is 1:4. If Tim attempts 40 shots, how many would you expect him to make?

$\frac{\text{attempts}}{\text{makes}}: \frac{4}{1} = \frac{40}{x}$ or $\frac{\text{makes}}{\text{attempts}}: \frac{1}{4} = \frac{x}{40}$

If students need additional practice setting up proportions, go back to the problems in 4.2 and choose additional problems.

Spiral Review

1. Write a ratio for the following situations.

   a. Flora knits 4.5 scarves in 8 days.

   b. It takes Henry 24 days to knit 13.5 scarves.

   c. Who knits faster? Justify your answer.

2. Name the Property of Arithmetic demonstrated by each statement.

   a. $\left(\frac{3}{5}\right)\left(\frac{5}{3}\right)=1$

   b. $1 \cdot mn = mn$

   c. $(2)\frac{x}{2} = 8(2)$

3. Solve each equation.

   a. $\frac{x}{1.7} = 3.5$

   b. $\frac{1}{2} = \frac{x}{4}$

4.3a Homework: Writing Proportions

Directions: Set up *two* proportions for *each* problem. Be sure to write the units of your comparisons. You do NOT need to answer the question yet.

1. 12-packs of soda are on sale for 5 for $12. At this rate, what is the cost for three 12-packs?

2. You are making a cinnamon sugar topping for your Snicker Doodles.
   a. The ratio of cinnamon to sugar is 1:4. You use 3 tablespoons of cinnamon. How much sugar?

   $\frac{\text{cinnamon}}{\text{sugar}}: \frac{1}{4} = \frac{3}{x}$ or $\frac{\text{sugar}}{\text{cinnamon}}: \frac{4}{1} = \frac{x}{3}$

   b. A different recipe has a ratio of cinnamon to sugar that is 2:3. If you need a total of 20 tablespoons of topping, how much cinnamon and sugar will you need?

   $\frac{\text{cinnamon}}{\text{total}}: \frac{2}{5} = \frac{x}{20}$ or $\frac{\text{sugar}}{\text{total}}: \frac{3}{5} = \frac{y}{20}$

   Or you could do total as numerator for either proportion.

***notice that you needed to switch to a part-to-whole ratio here to solve. There will be more of this later in section 4.4d. You can show where the 5 comes from using a model.

3. For your batch of cookies, you'll need 2 cups of chocolate chips for every 5 cups of cookie dough mix.
   a. You have 30 cups of cookie dough mix, how many cups of chocolate chips?

   b. You have 1 cup of chocolate chips, how much cookie dough mix do you need?

4. Students are collecting cans of food for the needy. They count cans every 3 days and get an average of 145 cans when they count. At this rate, how many cans will they have at the end of the month (20 school days)?

   $\frac{\text{cans}}{\text{days}}: \frac{145}{3} = \frac{x}{20}$ or $\frac{\text{days}}{\text{cans}}: \frac{3}{145} = \frac{20}{x}$

4.3b Class Activity: Solving Proportions

This lesson deliberately avoids "cross multiply." Rather, it is designed to help students understand how to solve proportions with properties of arithmetic.

The following proportion compares two quantities:

$\frac{3}{x} = \frac{5}{9}$ means "3 is to what number, as 5 is to 9"

What is another comparison that could be made between the two quantities?

Some number is to 3, as 9 is to 5; 3 is to 5 as some number is to 9; etc.: $\frac{x}{3} = \frac{9}{5}$

Now consider the equation $\frac{x}{3} = \frac{9}{5}$. We can solve this equation for *x* using the *multiplication property of equality*. Examples 1 and 2 show a path for the solution. Write a justification of why each step shows an equation with the same solution as the previous step.

Students must examine the structure of the proportion equation in order to solve for *x*. Additionally, students are required to provide a justification for each step in the solving process.

Example #1: Write a justification for each step, using algebraic properties. Refer to section 3.1a for a review of algebraic properties.

Equation	Justification
$\frac{x}{3} = \frac{9}{5}$	*Given*
$(3)\frac{x}{3} = \frac{9}{5}(3)$	*Multiplication property of equality; multiply both sides of the equation by 3*
$x = \frac{27}{5}$	*Simplifying each expression. 3/3 is 1.*

Example #2: Write a justification for each step, using algebraic properties.

Equation	Justification
$\frac{3}{x} = \frac{5}{9}$	*Given*
$(x)\frac{3}{x} = \frac{5}{9}(x)$	*Multiplication property of equality; multiply both sides of the equation by x.*
$3 = \frac{5}{9}x$	*Simplifying each expression. x/x is 1.*
$\left(\frac{9}{5}\right)3 = \frac{5}{9}x\left(\frac{9}{5}\right)$	*Multiplication property of equality; multiply both sides of the equation by 9/5.*
$\frac{27}{5} = x$	*Simplifying each expression. 9/5 3 is 27/5; 5/9*9/5 is 1*

Example 3 and 4 show another way to think about solving a proportion.

Example #3: Write a justification for each step, using algebraic properties.

Equation	Justification
$\frac{x}{3} = \frac{9}{5}$	*Given*
$(15)\frac{x}{3} = \frac{9}{5}(15)$	*Multiplication property of equality; multiply both sides of the equation by the LCM, 15.*
$5x = 27$	*Simplify each expression*
$\left(\frac{1}{5}\right)5x = 27\left(\frac{1}{5}\right)$	*Multiplication property of equality; multiply both sides of the equation by 1/5*
$x = \frac{27}{5}$	*Simplify each expression*

Example #4: Write a justification for each step, using algebraic properties.

Equation	Justification
$\frac{3}{x} = \frac{5}{9}$	*Given*
$(9x)\frac{3}{x} = \frac{5}{9}(9x)$	*Multiplication property of equality; multiply both sides of the equation by the LCM, 9x.*
$27 = 5x$	*Simplifying each expression. x/x and 9/9 is 1.*
$\left(\frac{1}{5}\right)27 = 5x\left(\frac{1}{5}\right)$	*Multiplication property of equality; multiply both sides of the equation by 1/5.*
$\frac{27}{5} = x$	*Simplifying each expression.*

Directions: For the proportions below, **solve using properties of equality,** use any method from above you choose. You may also reverse the order of the ratio comparison. Make sure that students clearly show what they are multiplying both sides by so that they can use their work to understand why cross multiplication works as explained on the next page.

1. $\frac{3}{2} = \frac{12}{x}$

8

2. $\frac{3}{2} = \frac{y}{1}$

1.5

3. $\frac{5}{2} = \frac{8}{x}$

3.2

4. $\frac{x}{27} = \frac{4}{6}$

18

5. $\frac{5}{x} = \frac{2}{3}$

7.5

6. $\frac{9}{x} = \frac{6}{5}$

7.5

There is a short-cut for solving the problems from the previous page called cross-multiplication.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } ab = cd$$

7. Go back through the problems on the previous page and examine why this short-cut works.

It would now be appropriate to show students why short-cut of cross multiplying works to solve proportions.

Abstract Method:

$\frac{a}{b} = \frac{c}{d}$

$(b)\frac{a}{b} = \frac{c}{d}(b)$

$a = \frac{cb}{d}$

$(d)a = \frac{cb}{d}(d)$

$ad = cb$

Numerical Example (#1 from previous page)

$\frac{3}{2} = \frac{12}{x}$

$(2)\frac{3}{2} = \frac{12}{x}(2)$

$3 = \frac{12 \bullet 2}{x}$

$(x)3 = \frac{12 \bullet 2}{x}(x)$

$3x = 12 \bullet 2$

Continue to solve.

8. Solve the following two equations and describe how it is different than solving for a missing value in a proportion:

$\frac{3}{2} + x = \frac{7}{2}$ $\quad\quad$ $5 + \frac{x}{2} = \frac{3}{2}$

9. Look at #1 above. Show that the missing value you found for the proportion creates equivalent ratios.

$$\frac{3}{2}\left(\frac{4}{4}\right) = \frac{12}{8}$$

10. Create a table with the values from the proportion in #1 above to find the constant of proportionality.

x	***y***
2	3
8	12

This idea is reinforcing concepts taught in 4.1 and 4.2. From the table we can see that the constant of proportionality is 1.5. The table also reminds students that in this relationship, the 2 and the 3 are a pair of values that correspond to each other and the 8 and 12 are a pair of values that correspond to each other.

11. Look at #3 above. Show that the missing value you found for the proportion creates equivalent ratios.

$$\frac{5}{2}\left(\frac{1.6}{1.6}\right) = \frac{8}{3.2}$$

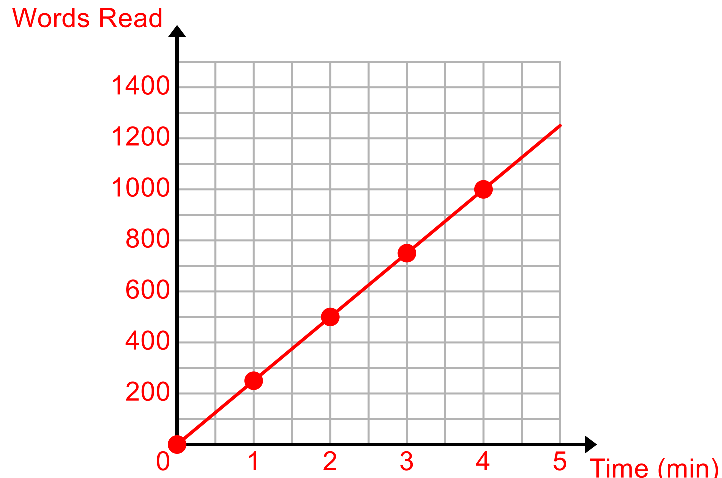
12. Use a table with the values from the proportion in #3 above to find the constant of proportionality.

x	***y***
2	5
3.2	8

From the table we can see that the constant of proportionality is 2.5.

13. In 4.2, you created a table, graph and equation for the proportional relationship described as: Megan can read 125 words in 30 minutes. Recreate them here:

Time (min.)	Words Read
0	0
1	250
2	500
3	750
4	1000



Equation: _______$y = 250x$_________

a. Use a proportion to find out how many words she can read in 11 minutes and then explain how your answer is related to the three other ways you represented the relationship above.

All of the above are ways to model this relationship, as well as the proportion below. Students should attend to precision when setting up the proportion and when labeling their answer with the appropriate units.

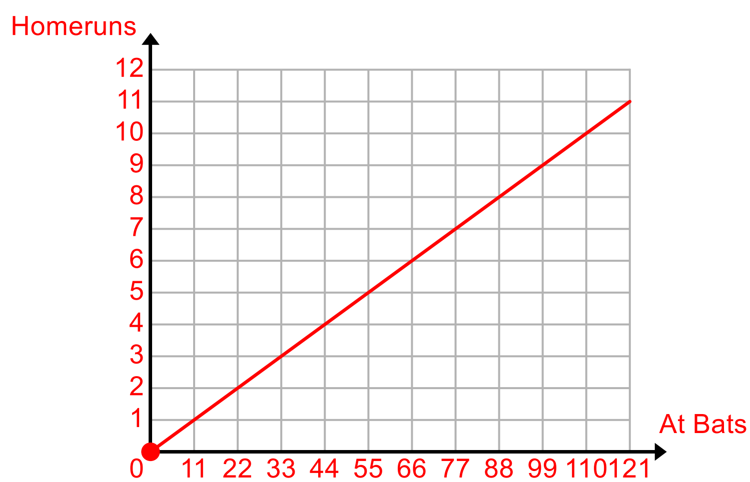
$\frac{\text{words read}}{\text{time}}$: $\frac{250}{1} = \frac{x}{11}$ or notice that students can use any pair of values in the proportion:

$\frac{\text{words read}}{\text{time}}$: $\frac{750}{3} = \frac{x}{11}$

$x = 2{,}750$ words; She can read 2,750 words in 11 minutes. Students should understand that this corresponds to a point on the graph $(11, 2{,}750)$ and that this ordered pair makes the equation true.

14. In 4.2e you used a table to create a graph and equation for the proportional relationship describing Mark's hits based on the number of at-bats. Recreate it here:

At Bats	Homeruns
0	0
11	1
22	2
33	3
44	4



Equation: _____$y = \frac{1}{11}x$_____________

a. Use a proportion to find out approximately how many at-bats you expect Mark had if he had 20 homeruns and then explain how your answer is related to the graph and equation you created to represent the relationship.

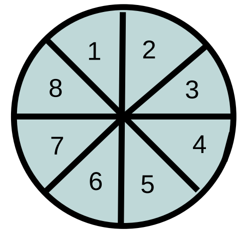
$\frac{\text{at bats}}{\text{homeruns}}$: $\frac{22}{2} = \frac{x}{20}$

220 at bats; We would expect Mark to have 220 at bats if he had 20 homeruns.

For additional practice setting up and solving proportions, we encourage you to revisit additional problems in 4.2. There are also several additional practice problems at the end of this section.

Spiral Review

1. A spinner like the one shown is used in a game. Determine the probability of each outcome if the spinner is equally likely to land on each section. Express each probability as a fraction and percent.



   a. *P*(5)

   b. *P*(greater than 2)

   c. *P*(not 6)

2. Sal is 37 years old. This 4 more than 3 times the age of Shae. Write and solve an equation to determine the age of Shae.

3. Determine which fraction is larger by drawing a model.

$$\frac{3}{4} \qquad \frac{3}{5}$$

4.3b Homework: Solving Proportions

Directions: For the proportions below, **solve using properties of equality**. You may also reverse the order of the ratio comparison.

1. $\frac{5}{3} = \frac{15}{x}$

2. $\frac{2}{4} = \frac{x}{15}$
7.5

3. $\frac{7}{3.5} = \frac{9}{x}$

4. $\frac{5}{3} = \frac{y}{1}$
$1\frac{2}{3}$

5. $\frac{5}{x} = \frac{9}{6}$

6. $\frac{9}{x} = \frac{12}{8}$
6

7. Now go back to #1 – 6 and solve using cross multiplication.

8. Go back to Lesson 4.3a Class Activity and solve problems #2 – 10 (where you wrote proportion equations but did not yet solve). You can use any method you wish to solve the proportions.
Odd answers are given below.

   3) He won 22 games.
   5) 56 scooters were sold.
   7) You would eat approximately 4 pieces.
   9) The measure of *x* is 4.5 inches.

4.3c Class Activity: Odds and Probability: Chance Proportions (part-to-part and part-to-whole) Problems

Review:
Use proportions, models, tables, or equations to solve the problems below.
Students will need to distinguish between part-to-part and part-to-whole ratios in this activity. Drawing models and/or clearly labeling ratios will be helpful for solidifying the two representations of the ratio. For example, if you know 1/3 of the students in a class are boys, then the ratio of boys to girls is 1:2. You may find in helpful to review the fundamentals of probability covered in Chapter 1.

1. Ginger and her brother Cal have red and green planting buckets in the ratio of 3:1.
   a. If there are 5 green buckets, how many red buckets are there?

   $$\frac{\text{red}}{\text{green}}:\ \frac{3}{1}=\frac{x}{5};x=15$$

   b. Ginger and Cal bought more buckets because they have more to plant. They purchased the buckets in the same red:green ratio of 3:1. If they now have 28 buckets total, how many red and green buckets do they have?

   $$\frac{\text{red}}{\text{total}}:\ \frac{3}{4}=\frac{x}{28};x=21$$

   There are 21 red buckets and 7 green buckets. Notice that the ratio of red:green is still 3:1.

   c. How are the problems different? You may also want to prompt: Which one is part-to-part? part-to-whole?

   Problem a. is part-to-part and problem b. is part-to-whole.

2. Write proportion equations for finding the missing number. Use a model, table or equation if necessary.
   a. In a group of 72 people, 2 out of every 9 people are left-handed. How many people are likely right-handed?

   $$\frac{\text{left-handed}}{\text{total}}:\ \frac{2}{9}=\frac{x}{72};x=16$$

   16 is the likely number of left handed people. 72 – 16 = 56, the number of likely right handed people.

   b. Five out of 8 students voted for Jose. The other students voted for Carlos. If 39 students voted for Carlos, how many students voted in all?

   104 students voted in all

3. Rewrite the problems a. and b. from #2 above to be part-to-part. Use a model, table, graph or equation to help you solve. Write proportion equations to find the missing number.
    a. In a group of 72 people, the ratio of left-handed to right-handed people is 2:7. How many people are right handed?

    b. The ratio of students voting for Jose and Carlos was 5:3. If 39 students voted for Carlos, how many students voted in all?

Throughout this lesson, it is important students remember what each quantity represents. Set a norm of labeling quantities.

In Chapter 1 we talked about probability. Recall that probability is a part-to-whole relationship. For example, the probability of rolling a 5 on a six sided dice is $\frac{1}{6}$ OR the probability of flipping a heads on a coin is $\frac{1}{2}$.

4. Now consider the following: A school basketball team wins 3 games for every 1 game it loses. The odds that the school basketball team will lose a game is 1 to 3. Is this a part-to-part or part-to-whole ratio? Explain any parts or wholes.
   Part to part: 1 loss (part) to 3 wins (part)

5. Rewrite the statement above to state the probability of the basketball team winning. Explain how this new statement is related to the previous statement. Explain any parts or wholes.
   The probability of winning is $\frac{3}{4}$; 3 wins (part) to 4 games played (whole)

6. What is the difference between odds and probability?
   Odds are part-to-part & probability is part-whole.
   Odds represent a part-to-part ratio, where each possible outcome is one of the parts. Probability (sometimes referred to as "chance") represents a part-to-whole relationship, where the whole is all the possible outcomes, including the outcome being measured (the part).

Directions: Use the information in 4 – 6 to answer the next two questions. Set up a proportion for each.

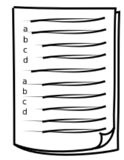
7. If the basketball team has won 63 games, how many games do you suspect they have lost?
   21 games

8. If the basketball team has played a total of 96 games, how many games do you suspect they won?
   72 games

Directions: Set up proportion equations to find needed information to answer the questions. Draw models or make tables if desired.

9. The probability of drawing a red marble from a bag of red and blue marbles is $\frac{2}{5}$. If there are 30 red marbles in the bag, how many blue marbles are there?
Odds are 2:3 red:blue, $\frac{\text{red}}{\text{blue}} = \frac{2}{3} = \frac{30}{x}$; x = 45 = blue; or $\frac{\text{red}}{\text{total}} = \frac{2}{5} = \frac{30}{x}$; x = 75 = total; 75 – 30 = 45 blue

10. The odds of randomly guessing the answer right on a multiple choice test are 1:4. What is the probability of guessing the correct answer? If the test has 20 questions, how many of them would I expect to guess correctly?
Probability is $\frac{1}{5}$, expect 4 correct answers on 20 questions; $\frac{\text{correct guesses}}{\text{total guesses}} = \frac{1}{5} = \frac{x}{20}$



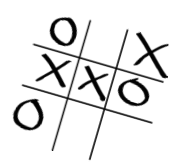
11. The chance of rain during April is $\frac{5}{6}$. How many days can you expect rain during the month of April?
25 days of rain. (Note, there are 30 days in April.) $\frac{\text{rain}}{\text{total days}} = \frac{5}{6} = \frac{x}{30}$

12. What is the probability of rolling a number less than 3 on a six-sided number cube? What are the odds? If you roll the cube 27 times, how many rolls (theoretically) would have a number less than 3?
Probability is $\frac{\text{less than 6}}{\text{total possibilities}} = \frac{2}{6} = \frac{1}{3}$; $\frac{\text{less than 6}}{\text{total possibilities}} = \frac{2}{6} = \frac{x}{27}$, 9 of the rolls should come up less than 3; odds are 1:2

13. The odds of winning a carnival game is $\frac{2}{9}$. If 80 people won, how many people do you expect lost? How many people played?
$\frac{\text{won}}{\text{lost}} = \frac{2}{9} = \frac{80}{x}$ 360 lost, 440 played

14. A certain student is late for school 3 out of 8 times. What are the odds for him/her to be late? If there are 180 days of school, how many days would he/she likely be late?
The odds are 3:5; the student can be expected to be late 67 or 68 days

15. Eli has a record for winning Tic-Tac-Toe 3 out of 7 times. What are the odds that Eli wins? What is the probability of him winning? How many games would he need to play in order to be likely to win 12 games?



Spiral Review

1. Fill in the blank with $<$, $>$, or $=$.
   a. $1\frac{5}{25}$ $\qquad$ 1.525

   b. $-1\frac{5}{25}$ $\qquad$ -1.2

   c. -2.6 $\qquad$ $-3\frac{3}{5}$

2. Use a model to answer each percent problem
   a. What number is 40% of 40?

   b. What percent of 45 is 15?

3. Bryton spent half of the money that he earned working at the car wash on motorcycle parts and 1/3 of the remaining money on a new T-shirt. He has $40 left. How much money did he make at the car wash? Use a model to solve.

4.3c Homework: Odds and Probability: Chance Proportions

Directions: Set up proportion equations to find needed information to answer the questions. Draw models or make tables if desired.

1. The chance of snowfall on any day in February is $\frac{6}{7}$. What are the odds of snow on a day in February? If there are 28 days in February, how many of those days would you expect there to be snow fall?

2. Approximately 12 out of every 100 males are left-handed. What are the odds that a randomly chosen boy is left-handed? Out of 75 boys how many of them would you expect to be left-handed?

   Odds are 12:88 or 3:22; 9 boys

3. Over time, you have found that your probability of bowling at least one strike in a game is $\frac{3}{10}$. What are the odds of getting at least one strike in a game? If you bowled 24 games, about how many games would you expect to bowl at least one strike?

4. The odds of finding a four-leaf clover in a clover patch are 1:34. What is the probability that a certain clover has four leaves? If I pick 245 clovers, how many of them would I expect to have four leaves?

   Probability is $\frac{1}{35}$; 7 of them

5. The odds of getting an even number on a given spinner are 4:3. What is the probability of getting an even number on this spinner? How many times would you expect to spin the spinner if you wanted to land on an even number 32 times?

6. The probability of randomly selecting a pair of blue socks from a given sock drawer is $\frac{5}{9}$. If I randomly draw a pair of socks 45 times, how many of those draws (theoretically) will result in a pair of blue socks?

   25 times

7. The odds of selecting a green marble from a given bag are 5 : 18. How many times (theoretically) would I need to randomly select marbles in order to pull out a green marble 75 times?

4.3d Class Activity: Percent Proportions

Review: Percentages involve part-to-whole relationships. If we use a percent in a proportion equation, we write the percent as a fraction out of 100, for example 72% is $\frac{72}{100}$. In a percent proportion equation, we will always be looking to find one of three possible missing pieces of information, the "percent" out of 100, the "part", or the "whole":

$$\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}}$$

Directions: For #1 – 3, write a percent proportion equation to solve this question.

Problems #1-3 are to practice putting the variable in the appropriate position.

1. A survey reveals 252 out of 350 students in 7th grade like to read. What percent like to read?
$\frac{x}{100} = \frac{252}{350}$

2. 72% of the students in 8th grade like to read and there are 350 students in 8th grade. How many students in 8th grade like to read?
$\frac{72}{100} = \frac{x}{350}$

3. Let's say 72% of the students in 6th grade like to read and we know that 252 students like to read, but we don't know how many students are in 6th grade.
$\frac{72}{100} = \frac{252}{x}$

Directions: Write proportion equations to solve the problems below. Then, solve the problem.

4. You are taking a math test and you get 32 problems correct on the test. Your grade is 80%. How many problems were on the test?
$\frac{80}{100} = \frac{32}{x}$
40 problems

5. Your best friend takes a math test with 40 problems on it. She solves 75% of the problems correctly. How many problems did your friend get correct?
$\frac{75}{100} = \frac{x}{40}$
30 problems

6. Another friend gets 36 out of 40 problems correct. What percent did this friend solve correctly?
$\frac{x}{100} = \frac{36}{40}$
90%

7. Tony bought a CD on sale for \$3 off the original price of \$15. What percent was the CD marked down?
$\frac{x}{100} = \frac{3}{15}$
20%

8. Serena got the same CD as Tony for \$10.50 at another store. What percent did she get off?
$\frac{x}{100} = \frac{4.5}{15}$
30%

9. Eddie bought a book of sports stories. It was 25% off the original price of \$15. How much did he pay?
$\frac{25}{100} = \frac{x}{15}$; \$15 – x = \$11.25 OR
25% off means pay 75% of price
$\frac{75}{100} = \frac{y}{15}$ and y = \$11.25

10. There are 75 students in the eighth grade. Forty-five of those go to music class. The rest of the students have art. What percent have art? What percent have music?
$\frac{x}{100} = \frac{45}{75}$; Music is 60%, Art is 40%

11. Of all the 8th grade students, 60 walk to and from school. If 80% of the 8th graders walk to and from school, how many students are in the 8th grade?
$\frac{80}{100} = \frac{60}{x}$; 75 students

Consider the following situations:

12. Elias has 28 marbles, 7 are red the rest are blue. Calvin has 64 marbles with 16 red and the rest blue.

a. What percent of Elias marbles are red? What percent of Calvin's are red?

25% of Elias's marbles are red; 25% of Calvin's marbles are red.

$\frac{red}{total} = \frac{x}{100} = \frac{7}{28}$ $\frac{red}{total} = \frac{x}{100} = \frac{16}{64}$

b. If they pool their marbles, what percentage will be red?

$\frac{red}{total} = \frac{x}{100} = \frac{7+16}{28+64} = \frac{23}{92}$; x = 25% Ask students why the answer is not 50%

c. What is the ratio of red to blue marbles for Elias? What is the ratio of red to blue for Calvin?

Elias, red:blue = 7:21 = 1:3. Calvin, red:blue = 16:48 = 1:3

d. If they pool their marbles, what is the ratio of red to blue?

Red:blue = (7 + 16):(21+48) = 23:69 = 1:3

13. Sara has 50 marbles, 5 are red the rest are blue. Ginger has 100 marbles with 20 red and the rest blue.

a. What percent of Sara's are red? What percent of Ginger's are red?

10% of Sara's marbles are red; 20% of Ginger's marbles are red.

$\frac{red}{total} = \frac{x}{100} = \frac{5}{50}$ =10% $\frac{red}{total} = \frac{x}{100} = \frac{20}{100}$ =20%

b. If the pool their marbles, what percentage will be red?

$\frac{red}{total} = \frac{x}{100} = \frac{5+20}{50+100} = \frac{25}{150}$; $x = 16\frac{2}{3}$% Ask students why the answer is not 30%. You may also consider asking students why the answer is answer is not an average

c. What is the ratio of red to blue marbles for Sara? What is the ratio of red to blue for Ginger?

Sara, red:blue = 5:45 = 1:9; Ginger, red:blue = 20:80 = 1:4

d. If they pool their marbles, what is the ratio of red to blue?

Red:blue = (5 + 20):(45 + 80) = 25:125 = 1:5

Spiral Review

1. Anita has a piece of licorice that is 12 inches long. She bites off 25% of the piece. How long is the licorice now? Use a model to solve.

2. Rachel’s piece of licorice is 25% longer than Anita’s 12-inch piece of licorice. How long is Rachel’s licorice? Use a model to solve.

3. A vine in a garden has grown 32% in the last 2 weeks. It was originally 2 feet long, how tall is it now? Use a numerical expression to solve.

4. The new model of a pick-up truck is 230 inches long. The old model’s length is 13% shorter. What was the length of the old model?

4.3d Homework: Write and Solve Three Percent Problems:

- Brainstorm as a class to make a list of situations in which you might find percent problems.
- In your groups of four, decide on four different contexts for writing percentage problems.
- Assign one to each member of the group.
- Each member of the group writes three word problems.
  - One in which the percent is not known but the “whole” and “part” are known.
  - One in which the percent and the “whole” are known, but the “part” is not known.
  - One in which the percent and the “part” are known, but the “whole” is not known.
- Each member of the group solves the three word problems he/she wrote.
- Group members present and critique problems and solutions.
- Twelve completed problems are turned in as homework for the group.

See student responses.

4.3e Class Activity: Proportional Constants in Markups and Markdowns

Review:

1. A sales manager at a clothing store is decreasing prices on six items by 20%.
   a. Draw a bar model to show a decrease of 20% off the original cost of an item.

				20% decrease

We find 20% of the original (by cutting it up into 5 parts) and then remove that part. We are left with 80% of the original.

   b. Suppose one of the items is a scarf that originally cost \$14.50. What would the sale price for the scarf be? \$11.60

   c. Explain in your own words this relationship: $\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}}$

   d. Use the relationship in c. to write a proportional relationship between percentage and the cost of the scarf.

$$\frac{80}{100} = \frac{\text{sale price}}{\text{original price}}$$

The original price is \$14.50, we want 80% of the amount (since there was a 20% discount and 100% – 20% = 80%). Note: percent is part of a whole that is 100; we want part of a whole that is \$14.50

   e. Using your model, proportions, or another method, find the sale price for each item (sale price of each item is 20% off original price):

Original price (x)	\$14.50	\$18.00	\$25.25	\$40.10	\$39.95	\$5.00
Sale price (y)	\$11.60	\$14.40	\$20.20	\$32.08	\$31.96	\$4.00

   f. What is the constant of proportionality? $\frac{y}{x} = \frac{\text{sale price}}{\text{original price}} = \frac{0.8}{1}$

   g. Convert your constant of proportionality into a percentage and use your model to explain why this makes sense.

   80% because $\frac{0.8}{1} = \frac{80}{100}$ and a 20% discount means paying 80% of original price.

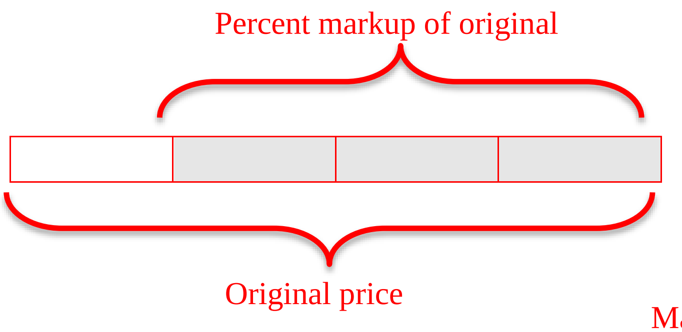
   h. Write an equation for y, the sale price. Verify that your equation is true for two columns in the table.
   $y = 0.8x$

   i. If the sale price is \$29.80 for a pair of shoes at this store, how much was the original price?
   \$37.25

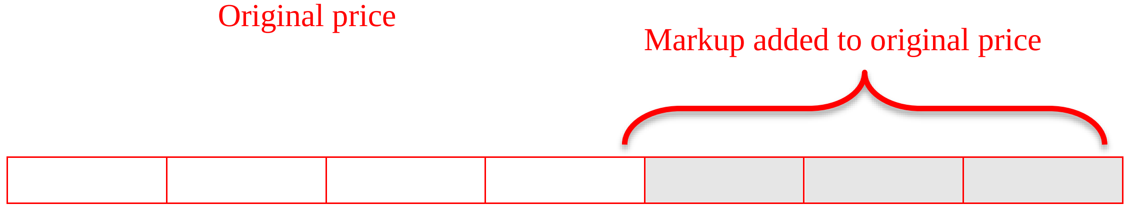
   j. Use a calculator or graph paper to graph the ordered pairs for (original price, sale price). What do you notice? The ordered pairs line up on a straight line through the origin, so they represent a proportional relationship.

2. A used car dealership marks up the cost of the cars they buy at auctions by 75%.

a. Draw a bar model for increasing by 75%, where the original bar represents the original cost of the car at auction.



We find 75% of the original (first model) and then add it to the original amount. Thus, we have 1 "original" plus 0.75 of the original; or 1.75 of the original.



b. If 100% is the original whole, what part of the original is the new amount? Justify your answer.

175%. It is 100% of the original plus 75% of the original.

c. Use a proportion in the form $\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}}$ to find the selling price (part) for a car whose original auction price (whole) was $1500.

$$\frac{175}{100} = \frac{x}{1500} \qquad x = \$2,625$$

d. Using your model, proportions, or another method, find the selling price if there is always a 75% mark-up of the auction price:

Auction price (x)	$1500	$1900	$1750	$6400	$9350	$6044
Selling price (y)	$2,625	$3,325	$3,062.50	$11,200	$16,362.50	$10,577

e. What is the proportional constant for $\frac{\text{selling price}}{\text{auction price}}$? $\frac{1.75}{1}$

f. Write an equation for y, the selling price. Verify that your equation is true for two columns in the table.

$y = 1.75x$

g. If you go to the car dealership and see a selling price of $7,700, how much did the car cost at auction? Justify your answer.

$4,400

h. Using a graphing calculator or on graph paper, graph the ordered pairs (auction price, selling price). What do you notice? See student responses

3. Find the missing price for each. Use the proportional constant, proportions, equations or models to solve, showing your work.
Remember to first identify the **whole**. e.g. to what does the 100% refer? This is key in setting up proportions or creating models. You may need to remind students that the proportional constant is output/input (y/x) thus in this case it is: $\frac{\text{final price}}{\text{original price}}$ which is the same as the percentage. Percentage of increase is $\frac{100 + \text{ \% increase}}{100}$; percentage of decrease is $\frac{100 - \text{ \% decrease}}{100}$.

a. Original Price: $16.20
Increase by 40%
Final Price: $22.68
Proportional Constant: 1.4

b. Original Price: $1,050.00
Increase by 0.5%
Final Price: $1055.25
Proportional Constant: 1.005

c. Original Price: $19.80
Decrease by 5%
Final Price: $18.81
Proportional Constant: 0.95

d. Original Price: $86.60
Decrease by 85%
Final Price: $12.99
Proportional Constant: 0.15

e. Original Price: $29.00
Increase by 180%
Final Price: $81.20
Proportional Constant: 2.8

f. Original Price: $55.00
Increase by 150%
Final Price: $137.50
Proportional Constant: 2.5

g. Original Price: $100.00
Decrease by 20%
THEN increase by 20%
Final Price: 1.20 × (0.80 × $100) = $96.00
How does this compare with the original price?

h. Original Price: $80.00
Increase by 15%
THEN decrease by 15%
Final Price: 0.85 × (1.15 × $80) = $78.20
How does this compare with the original price?

i. Original Price: $40.00
Decrease by 25% (product is on sale)
THEN increase by 6% (pay sales tax)
Final Price: 1.06 × (0.75 × $40) = $31.80

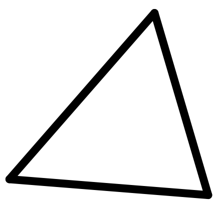
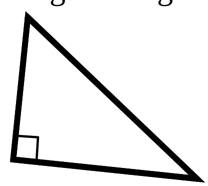
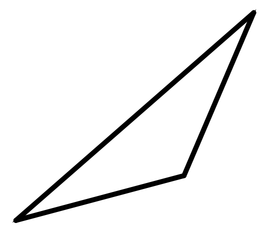
j. Original Price: $150.00
Decrease by 30% (item is on sale)
THEN decrease by 10% (apply store coupon)
Final Price: 0.90 × (0.70 × $150) = $94.50

On problems g. - j., help students understand that the second percent increase/decrease applies to the "whole" generated after the first step has been applied. In problem g. the 20% increase applies to a smaller whole ($80) than the original price of $100. Therefore, the 20% increase doesn't make up for the 20% decrease.
***For additional practice, use percent markup and markdown problems from Chapter 3 where students used models and equations to solve these types of problems.**

Spiral Review

1. Solve the following equations.
   a. $-3 = m - 2$
   b. $4k = -50$
   c. $\frac{x}{3} = -4$
   d. $-2 + y = -10$

2. Solve the following equations.
   a. $6(r - 2) = 12$
   b. $25 = 5(2d + 4)$

3. Classify each triangle as an acute, obtuse, or right triangle.
   a.
   
   b.
   
   c.
   

4.3e Homework: Proportional Constants in Markups and Markdowns

1. All the dimensions of a drawing must be decreased by 15%.
   a. Draw a bar model for decreasing by 15%, where the original bar represents the original length of an object in the drawing.

   b. Using your model or another method, find the length if it is decreased by 15%:

Original length (x)	18 cm	9.6 cm	12 cm	1 cm	22.8 cm	32 cm
Final length (y)	15.3 cm	8.16 cm	10.2 cm	0.85 cm	19.38 cm	27.2 cm

   c. What is the unit rate of $\frac{\text{length in final picture}}{\text{length in original picture}}$? $\frac{0.85}{1}$

   d. Convert your unit rate into a percentage and use your model to explain why this makes sense.
   85% because $\frac{0.85}{1} = \frac{85}{100}$

   e. Write an equation for y, the final length. Verify that your equation is true for two columns in the table.
   $y = 0.85x$

   f. If the roof of a barn in the final picture is 20.4 cm long, how long was the roof of the barn in the original picture?
   24 cm

2. At the bake sale fundraiser, George marks up all items 40% to make a profit for the math club.
   a. Draw a bar model for increasing by 40%.

   b. Using your model or another method, find the new price if the original is increased by 40%:

Original price (x)	\$0.50	\$1.00	\$0.75	\$1.05	\$3.00	\$2.50
Sale price (y)						

   c. What is the unit rate of $\frac{\text{sale price}}{\text{original price}}$?

   d. Convert your unit rate into a percentage and use your model to explain why this makes sense.

   e. Write an equation for y, the sale price. Verify that your equation is true for two columns in the table.

   f. If you go to the bake sale and purchase a brownie for $3.15, how much did the brownie originally cost? Justify your answer.

3. Find the missing price for each. Use the proportional constant, proportions, equations or models to solve, showing your work.

a. Original Price: $22.00
Increase by 40%
Final Price: $30.80
Proportional Constant: 1.4

b. Original Price: $9,000.00
Decrease by 0.5%
Final Price:
Proportional Constant:

c. Original Price: $39.80
Decrease by 10%
Final Price: $35.82
Proportional Constant: 0.9

d. Original Price:
Decrease by 85%
Final Price: $25.98
Proportional Constant:

e. Original Price: $55.00
Increase by 120%
Final Price: $121.00
Proportional Constant: 1.2

f. Original Price: $35.00
Increase by 250%
Final Price:
Proportional Constant:

g. Original Price: $60.00
Decrease by 25%
THEN increase by 25%
Final Price: $1.25 \times (0.75 \times \$60) = \$56.25$
How does this compare with the original price?

h. Original Price: $100.00
Decrease by 30%
THEN increase by 10%
Final Price:
How does this compare with the original price?

i. Original Price: $72.00
Increase by 12.5%
THEN increase by 10%
Final Price: $89.10

j. Original Price: $900.00
Decrease by 40% (item is on sale)
THEN decrease by 5% (apply store coupon)
Final Price:

4.3f Self-Assessment: Section 4.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Fluidly transition between part:part and part:whole relationship statements.	I struggle to transition between part:part and part:whole relationship statements.	I can rewrite relationship statements in the other form with a model.	I can transition between relationship statements without drawing a model.	I can transition between relationships without a model. I can justify how I arrived at the relationship statement and explain the relationships relative to tables, graphs, and equations.
2. Write and solve a proportion to find missing values.	I struggle to know how to use a proportion to find a missing value.	I can find a missing value when given a proportion, but I struggle to write my own proportions.	I can write and solve proportions to solve real-world problems.	I can write and solve proportions to solve real-world problems. I can explain how the proportion and answer relate to the context, graph, table of values, and equation.
3. Explain the difference between probability and odds.	I struggle to know the difference between odds and probability.	I understand what probability is but I am not sure what odds are or how they are related.	I know the difference between probability and odds. I can solve problems with either probability or odds.	I can explain the difference between probability and odds and solve problems with either. I can transform a probability situation into an odds situation and vice-versa.
4. Solve one- and multi-step problems involving percents using proportional reasoning.	I struggle to solve problems involving percents using proportional reasoning.	I can solve percent problems using proportional reasoning if they only have one step.	I can solve multi-step percent problems using proportional reasoning.	I can solve multi-step percent problems using proportional reasoning. I can explain the reasoning behind each step and connect my method to other methods and the different representations.

Sample Problems for Section 4.3

1. The ratio of boys to girls Ms. Garcia's class is 4:5.
   a. If there are 12 boys in the class, how many girls are there? [2]

   b. If she randomly selects a student from the class to present their solution to a math problem, what is the probability the student will be a girl? [1] [2]

2. Twenty percent of the students in Mr. Manycattle's class earned a reward for perfect attendance. What is the ratio of students with perfect attendance to those without perfect attendance? [1]

3. The odds of winning a carnival game is $\frac{2}{11}$. If 160 people played the game one evening, how many people do you expect lost? [1] [2] [3]

4. A slushy recipe calls for a ratio of 3.5 cups of water to 1.5 cups of juice. [1] [2]
   a. Use two different methods to determine how much water you will need if you have one cup of juice.

   b. Use two different methods to determine how much juice and water you would need if you wanted to end up with 25 cups of liquid.

5. Solve the following proportion equations: [2]

   a. $\frac{3}{4} = \frac{15}{x}$ b. $\frac{9}{x} = \frac{15}{25}$ c. $\frac{x}{6.2} = \frac{3}{4.3}$ d. $\frac{\frac{2}{3}}{\frac{3}{5}} = \frac{x}{3}$

6. Use proportional reasoning (unit rate, proportional constant, and/or proportion equations) to find the following missing prices: [4]
   a. Original Price:
      Increase by 30%
      Final Price: $84.50
   b. Original Price: $36.50
      Increase by 45%
      Final Price:

c. Original Price:
Decrease by 20%
Final Price: $67.20

d. Original Price: $185.00
Decrease by 45%
Final Price:

e. Original Price: $125.00
Decrease by 20% (product is on sale)
THEN decrease by 15% (apply store coupon)
Final Price:

f. Original Price: $59.00 (wholesale price)
Increase by 50% (store mark-up)
THEN decrease by 25% (product is on sale)
Final Price:

7. Carter's Sporting Goods is selling everything at a 25% discount. [4]
   a. Below is a list of items with either the sale or original price. Fill in any missing values.

Item	Original Price	Sale Price
Sweat	64.00	
Parka		273.75
Yoga Pants	95.00	
Light Jacket		127.50

   b. Create a graph to show the relationship between the original price and the sale price. Be sure to label each axis.

   c. Write a proportion for this context and then use it to find the proportional constant. [2] [4]

4.3g Extra Practice with part-to-part and part-to-whole relationships

Directions: Label the ratio in the problem as part-to-part (pp) or part-to-whole (pw). Draw models if desired. Write proportion equations to solve. (Be careful! To answer the question, you might have to do two steps.)

1. Two numbers are in the ratio 9:5. If the smaller number is 20, what is the <u>sum</u> of the two numbers?

   The sum is 56, (pp)

2. Clarence the Clown has 20 purple and pink balloons to give out to children at the circus. 1 out of 4 balloons is pink. How many balloons are purple?

3. One out of every 4 children attending the art show is a girl. There are 21 boys. What is the total number of children at the art show?

   There are 28 children at the art show, (pw)

4. Tina and Alma shared a cash prize in the ratio 6:7. If Alma received 35 dollars, how much money did Tina receive?

5. Maria downloaded pop tunes and hip hop tunes to her computer in the ratio 3:2. She has 10 hip-hop tunes. How many <u>more</u> pop tunes than hip-hop tunes does Maria have?

   She has 5 more pop tunes than hip-hop tunes, (pp)

6. Jorge built a gaming website. The website had 40 visitors on Monday. Three out of every 8 visitors played Planet Zak. The other visitors played Cosmic Blobs. How many visitors played Cosmic Blobs?

7. For every 4 candy bars that Ella sells, Ben sells three. Ella sold 24 candy bars last month. How many candy bars did Ben sell?
   Ben sold 18 candy bars, (pp)

8. The ratio of the weight of Christian's hamster to the weight of Javier's hamster is 1:3. Christian's hamster weighs 7 ounces. How much do the two hamsters weigh together?

9. Gustavo spends 4 out of every 5 dollars he earns on software. He uses the rest of the money to buy snacks. Last month, Gustavo spent 20 dollars on software. How much less money did he spend on snacks?

He spent $5 on snacks and $20 on software so he spent $15 less on snacks than software. (pw)

10. Kevin spends 3 out of every 4 dollars he earns on games. He uses the rest of the money to buy comic books. Last month, Kevin spent 18 dollars on games. How much less money did he spend on comic books?

11. The ratio of the weight of Rachel's radio to the weight of Belle's radio is 5:7. Rachel's radio weighs 35 ounces. How much less does Rachel's radio weigh than Belle's radio?

Rachel's radio weighs 14 ounces less than Bell's radio, (pp)

12. Mrs. Ruiz has 20 students in her class. One out of 4 students stayed after school yesterday for soccer practice. The other students stayed for band. How many students stayed for band practice?

13. For every 7 push-ups Dulce can do, Sara can do 6. If Dulce did 28 push-ups during gym class. How many push-ups did they do altogether?

They did 52 push-ups together, (pp)

14. The ratio of the number of comic books in Yazmin's collection to the number of comic books in Itzel's collection is 4:1. Itzel has 6 comic books. How many comic books do they have altogether?

15. A group of fourth grade students voted for their favorite sport. One out of 9 students voted for baseball. The other students voted for basketball. There were a total of 36 votes. How many students voted for basketball?

32 students voted for basketball, (pw)

16. Angela and Diana share some lollipops in the ratio 4:7. Diana has 28 lollipops. How many lollipops do they have altogether?

17. For every 3 hats that Lexi sells, Maddie sells two. Lexi sold 12 hats last month. How many hats did they sell in all?

    20 hats all together, (pp)

18. For every 7 games the Cherico Cheetahs played, the team won 6. If the Cherico Cheetahs won 30 more games than they lost, how many games did they win?

19. A website conducted a survey about musical tastes. According to the survey, 1 out of 8 people prefers hip hop music. The others prefer rock music. If 56 people took part in the survey, how many more people prefer rock music?

    42 more people prefer rock, (pw)

20. On Friday morning, Harmon's had 35 boxes of Captain Crush on the shelf. 1 out of every 5 boxes contained a prize. How many boxes did not contain a prize?

21. Marco and Justin shared a cash prize in the ratio 3:4. If Justin received 16 dollars, how much money did Marco receive?

    Marco received $12, (pp)

22. The ratio of girls to boys at the track meet was 4:1. Eric counted 9 boys. How many children were at the track meet altogether?

23. One out of every 4 children attending the soccer game is a girl. There are 27 boys. What is the total number of children at the soccer game?

    There were 36 children at the game, (pw)

24. Clarence the Clown has 36 pink and white balloons to give out to children at the circus. 1 out of 4 balloons is white. How many more balloons are pink?

25. The ratio of the length of Jake's wire to the length of Maddie's wire is 1:4. Maddie's wire measures 28 inches. How long is Jake's wire?

    Jake's wire is 7 inches long, (pp)

26. For every 7 sit-ups Naomi can do, Morgan can do 6. If Naomi did 49 sit-ups, how many more sit-ups did Naomi do than Morgan?

27. On Wednesday morning, Smith's had 99 boxes of Sweet Smacks on the shelf. 4 out of every 11 boxes contained a prize. How many boxes did not contain a prize?

63 boxes had no prize, (pw)

28. Kyle and Megan participated in a team bike-athon. Kyle rode 5 out of every 8 miles in the course. Megan covered the rest of the mileage. If Kyle rode 45 miles, how many fewer miles did Megan ride than Kyle?

29. For every 3 books that Corey sells, Lexi sells one. Corey sold 21 books last week. How many fewer books did Lexi sell than Corey?

Lexi sold 14 fewer books than Corey, (pp)

30. Will and Abby shared some marshmallows in the ratio 7:3. Abby had 27 marshmallows. How many fewer marshmallows did Abby have than Will?