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# Chapter 8: Geometry Part 2: Measurement in 2- and 3-Dimensions, Plane Sections of Solids (2 weeks)

## UTAH CORE Standard(s)

**Draw, construct, and describe geometrical figures and describe the relationships between them.**

1. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. 7.G. 3

**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**

2. Know the formulas for the area and circumference of a circle and use them to solve problems. 7.G.4
3. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. 7.G.6

## Chapter Summary:

Throughout this chapter students develop and explore ideas in geometry around measures in one-, two-, and three-dimensions. Additionally, students will tie concepts learned previously in 6<sup>th</sup> and 7<sup>th</sup> grade to content in this chapter. Section 1 begins with a brief review and practice with perimeter and area of polygonal figures. Students then use their knowledge of area and perimeter to solve real-world problems, find the area of non-standard shapes and review ideas around percent increase/decrease and scale factor, and connect their understanding of one- and two-dimensional measures to adding and multiplying algebraic expressions by finding the perimeter and area of figures with variable expression side lengths.

Section 2 starts with students exploring plane sections of 3-D solids. Exercises in this section are designed to help students develop an intuitive understanding of dimension (building up and cutting down). Students will notice that plane sections of 3-D objects are affected by: a) the type of solid with which one starts, e.g. right prism versus a pyramid and b) the angle of the cut to the base and/or other faces. This observation should help students understand when and why parallel (and/or perpendicular) cuts to a specific face are needed to create uniform cross sections, and then by extension, uniform cubed units for finding volume. In other words, the study of plane sections here is to help students develop an understanding of the structure of a solid and procedures for finding volumes. Throughout the exercises, students should develop a stronger understanding of units of measure for one-, two-, and three-dimensions. Towards the end of this section students review the use of nets (a concept from 6<sup>th</sup> grade) to find surface area of prisms and cylinders and then to differentiate this measure from volume, which they will also find.



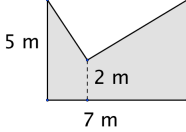




**VOCABULARY:** area, axis of symmetry, circle, circumference, cross-section, cube, cylinder, edges, equidistant, face, plane section, perimeter, polygon, polyhedron, prism, pyramid, quadrilateral, rectangle, square, surface area, symmetry, trapezoid, triangle, volume, vertices.



## **CONNECTIONS TO CONTENT:**

Prior Knowledge: In 6<sup>th</sup> grade students found areas of special quadrilaterals and triangles. They also found areas of other objects by decomposing them into rectangles and triangles. In Chapter 5 of this text, students extended that understanding to finding the area of a circle. Also in 6<sup>th</sup> grade, students found volume of rectangular prisms and their surface areas by using nets. During their study of geometry in 6<sup>th</sup> grade, students should have learned that the height and base of an object are always perpendicular to each other. They will build on this understanding as they apply their knowledge of area and volume to real life contexts and as they explore cross sections and plane sections.

Future Knowledge: In 8<sup>th</sup> grade, students will continue working with volume, formalizing algorithms for volume of cylinders and adding methods for finding the volume of cones and spheres. Students explore cross sections of objects in 7<sup>th</sup> grade to understand how dimensions are related to each other and the algorithms for surface area and volume. Further, ideas developed through cross section activities are foundational in the study of calculus. Lastly, ideas about planar sections will be extended in secondary mathematics when students explore Cavalieri's Principle.

## MATHEMATICAL PRACTICE STANDARDS (emphasized):

|   |   |   |    |    |                |  |    |    |                     |   |
|---|---|---|----|----|----------------|--|----|----|---------------------|---|
|    | <p>Make sense of problems and persevere in solving them.</p>            | <p>Students will be given a variety of contextual problems with which they will need to make sense and persevere. For example, 8.0 Anchor Problem: The city is looking to build a new swimming pool in city park. In their city council meeting, they have determined that they want the pool to hold no more than <math>2500 \text{ m}^3</math> of water, or it will cost too much to keep it filled.</p> <p>Help the city council to choose a design for the swimming pool. Design 3 different swimming pools that will each hold at least <math>2000 \text{ m}^3</math> but no more than <math>2500 \text{ m}^3</math> of water.</p>   |    |    |                |  |    |    |                     |   |
|    | <p>Reason abstractly and quantitatively.</p>                            | <p>Students will use their understanding of area and volume to reason in a variety of contexts such as the 8.1e Class Activity: Write an expression to find the area. If possible, find the exact area.</p> <div style="text-align: center;">  </div>   |    |    |                |  |    |    |                     |   |
|    | <p>Construct viable arguments and critique the reasoning of others.</p> | <p>Students will apply their understanding of perimeter and area to construct and critique arguments. 8.1g Class Activity: <i>Mike, Juliana and Joe were working together to make a garden larger. Mike said, "We have to buy more fencing because if we increase the area of the garden we will need more fencing to go around."</i></p> <p><i>Juliana had a different opinion. "That's not true," she said. "We can use the same amount of fencing and move it to make the area of the garden larger."</i></p> <p><i>Joe disagreed with both Mike and Juliana. He said, "I know a way that we can make the garden larger and use less fencing."</i></p> <p><i>Who is right?</i></p> |    |    |                |  |    |    |                     |   |
|  | <p>Model with Mathematics.</p>  | <p>Students will use models to explore concepts in geometry such as using play-dough and string to create cross sections of prisms. Throughout the chapter, they will connect models to algorithms.</p>   |    |    |                |  |    |    |                     |   |
|  | <p>Attend to Precision.</p>   | <p>Careful attention should be paid to explanations and units throughout this chapter. Students will be expected to attend to several ideas at the same time. Students should attend to precision as they explain ideas throughout this chapter. For example, when discussing cross sections, students should use precise language in describing the angle of cuts to the base, faces and/or edges.</p>   |    |    |                |  |    |    |                     |   |
|  | <p>Look for and make use of structure.</p>                              | <p>Students will connect ideas of one-, two-, and three-dimensional measures to simplifying numeric and algebraic expressions, including this example from the 8.2 d Homework: Determine which expression(s) will give the surface area or volume for a 3-D object.</p> <table style="width: 100%; border: none;"> <tr> <td style="width: 50%;">a.</td> <td style="width: 50%;">c.</td> </tr> <tr> <td><math>2(6 + 2 + 3)</math></td> <td><math>(2 \cdot 6 + 2 \cdot 3 + 3 \cdot 6)2</math></td> </tr> <tr> <td>b.</td> <td>d.</td> </tr> <tr> <td><math>3 \cdot 2 \cdot 6</math></td> <td><math>2 \cdot 3 \cdot 2 + 2 \cdot 3 \cdot 6 + 2 \cdot</math></td> </tr> </table>           | a. | c. | $2(6 + 2 + 3)$ | $(2 \cdot 6 + 2 \cdot 3 + 3 \cdot 6)2$ | b. | d. | $3 \cdot 2 \cdot 6$ | $2 \cdot 3 \cdot 2 + 2 \cdot 3 \cdot 6 + 2 \cdot$ |
| a.  | c.  |   |    |    |                |  |    |    |                     |   |
| $2(6 + 2 + 3)$  | $(2 \cdot 6 + 2 \cdot 3 + 3 \cdot 6)2$                                  |   |    |    |                |  |    |    |                     |   |
| b.  | d.  |   |    |    |                |  |    |    |                     |   |
| $3 \cdot 2 \cdot 6$   | $2 \cdot 3 \cdot 2 + 2 \cdot 3 \cdot 6 + 2 \cdot$                       |   |    |    |                |  |    |    |                     |   |

|   |   |  |
|---|---|--|
|   |   |  |
|  | <p>Use appropriate tools strategically.</p>                   | <p>Students will use a variety of tools in this chapter including play dough, rulers, graph paper, and calculators. Encourage students to make sense of ideas with tools.</p>  |
|  | <p>Look for and express regularity in repeated reasoning.</p> | <p>Students will note in this chapter that length is one-dimensional, area is two-dimensional and requires 2 length (2 one-dimensional measures) that are perpendicular, and volume is three dimensional and requires 3 lengths that are each perpendicular to each other.</p> |

## 8.0 Anchor Problem; Designing a Swimming Pool:

The city is looking to build a new swimming pool at the city park. In their city council meeting, they determined that they want the pool to hold no more than  $2500 \text{ m}^3$  of water in order to conserve water.

- A. Help the city council to choose a design for the swimming pool. Design three different swimming pools that each hold at least  $2000 \text{ m}^3$  but no more than  $2500 \text{ m}^3$  of water.
- B. Choose your favorite of the three designs from part A. Find new measures for the pool so that: a) it holds twice as much water as your original design and b) it remains the same overall shape (but not size) as the original.



## Section 8.1: Measurement in Two Dimensions

### Section Overview:

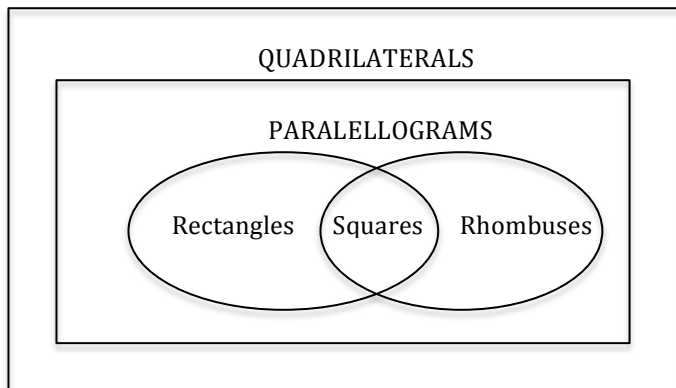
This section involves a review and extension of previously learned skills (from 6<sup>th</sup> grade) involving perimeter and area of plane figures and volumes of right solids. The section begins with a review of perimeter and area and then moves to finding area of irregular figures, first with numeric side lengths and then with variable expression side length. Additionally, students continue work on skills developed earlier in the year including percent increase/decrease, scale factor for perimeter and area, and simplifying algebraic expressions. The end of the section focuses on all these skill in various real-world and mathematical contexts.

### Concepts and Skills to be Mastered (from standards)

1. Know the formula for the area and circumference of a circle and use them to solve problems.
2. Give an informal derivation of the relationship between area and circumference of a circle.
3. Solve real-world and mathematical problems involving area, volume, and surface are of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

## 8.1a Class Activity: Differentiate Area and Perimeter

REVIEW:



1. Rectangle Riddle #1: Can you figure out the dimensions for each rectangle? Use the clues below to draw the four different rectangles described.
- Rectangle A has a perimeter equal to that of Rectangle B.
  - Rectangle A is a square.
  - Rectangle B has an area of 24 square units. It is as close to a square as possible for that area if the side lengths are *whole numbers*.
  - Rectangle C has a perimeter equal to the measure of the area of Rectangle B. Rectangle C's length is 3 times the width.
  - Rectangle D has two different odd integers as length and width; each is 1 unit greater than the length and width of Rectangle B.

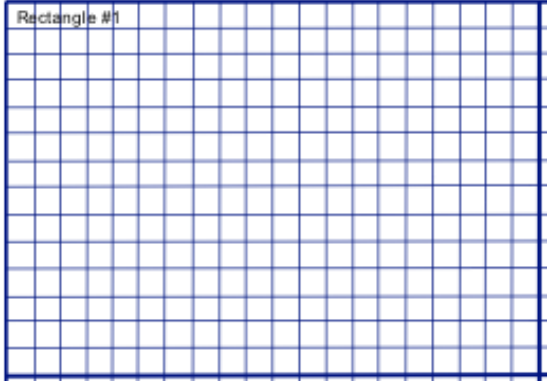
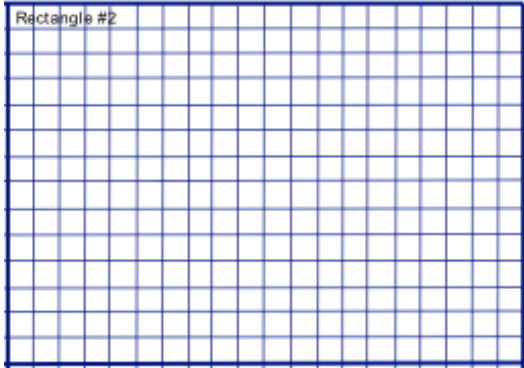
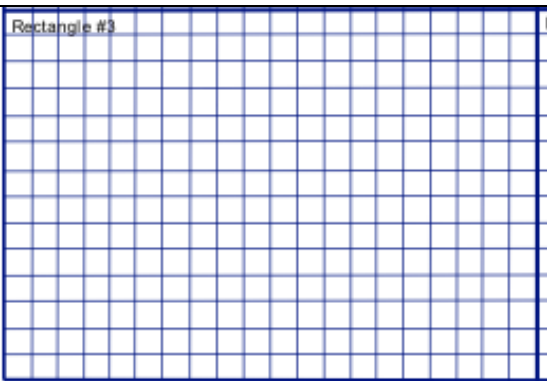
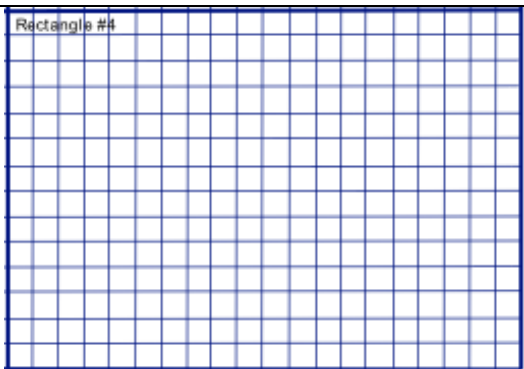
|   |  |   |  |
|---|--|---|--|
| A |  | B |  |
| C |  | D |  |



Use words and models to explain the difference between perimeter and area.

2. Rectangle Riddle #2: Can you figure out the dimensions for each rectangle? Use the clues below to draw the four different rectangles described.

- The numeric measure of the area of Rectangle #1 is twice the number representing the perimeter. Rectangle #1 is 8 units wide.
- Rectangle #2 has the same number representing the area and the perimeter; the perimeter is  $\frac{1}{2}$  the perimeter of Rectangle #1.
- Rectangle #3 has the same length as Rectangle #1. The area of Rectangle #3 is equal to the difference between the areas of Rectangles #1 and #2.
- Rectangle #4 has an area 6 square units less than the area of Rectangle #3 and a width 2 units more than the width of Rectangle #2.

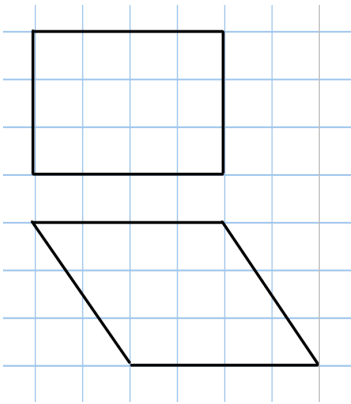
|  |   |
|--|---|
| A<br>  | B<br>  |
| C<br> | D<br> |

3. What helped you most to accomplish “Rectangle Riddle #2”?

For questions #4-6 use a piece of graph paper to answer each:

4. List as many rectangles as you can that have an area of 36?

5. DRAW two different parallelograms that have the same area. State their base and height.



6. DRAW two different triangles that have an area of 12. State their base and height.

7. Suppose you start with a rectangle that has a base length of 6 and height of 4. If you triple the length of the base, what do you have to do to the height to have a new rectangle of the same area?



### Activity:

*Mike, Juliana and Joe were working together on a garden area and perimeter problem their teacher gave them. The problem is to make the area of a garden larger without having to buy more fencing (increasing the perimeter.) Their task is to figure out if that's possible.*

*Mike said, "We have to buy more fencing because if we increase the area of the garden we will need more fencing to go around."*

*Juliana had a different opinion. "That's not true," she said, "We can use the same amount of fencing and move it to make the area of the garden larger."*

*Joe disagreed with both Mike and Juliana. He said, "I know a way that we can make the garden larger and use less fencing."*

### Who is right?

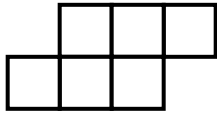
- A. Group response: Use graph paper and string to help you think about the problem.
  - a. Decide who you think is right (Mike, Juliana or Joe.)
  - b. Come up with a possible answer with your group.
  - c. Prepare your group explanation and presentation.
  - d. Be certain to justify your conclusion with reasoning and calculation.
  
- B. Challenge: Examine the Garden Problem using a spreadsheet. Consider all possible rectangles with perimeters of 36 units. How does the area change as you change the configuration of the perimeter?



## 8.1a Homework: Perimeter and Area Problem Solving

Answer each of the questions below.

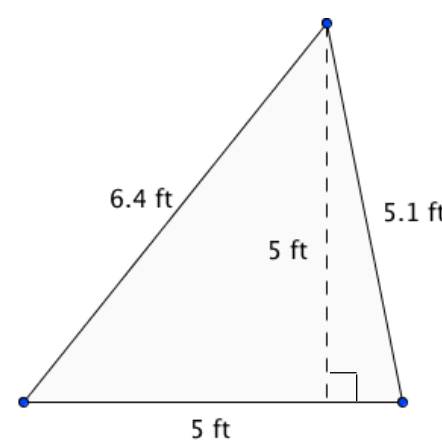
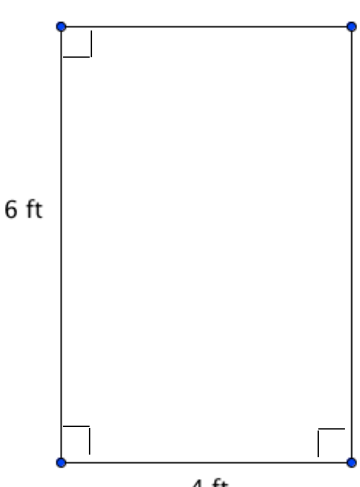
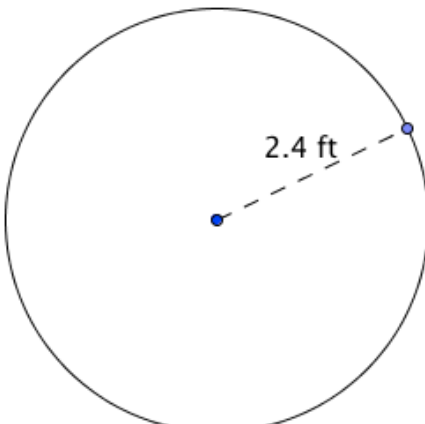
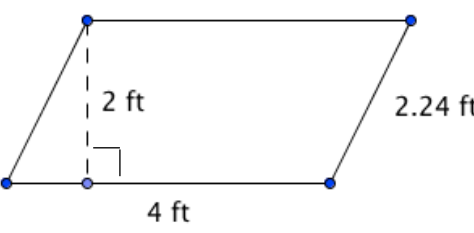
1. What is the least number of tiles you can add to the figure below to create a shape with a perimeter of 16?  
Note: When adding a tile, the new tile must share at least one side with the original shape; each tile is 1 unit by 1 unit.



2. Use the original figure given above to answer a-d. Draw your answers. Note, an answer of just “yes” or “no” is not sufficient. Use pictures or words to justify your answer.
  - a. Can you add a tile to this figure to increase the perimeter by 1? If so, how?
  - b. Can you add a tile to this figure to increase the perimeter by 2? If so, how?
  - c. Can you add a tile to this figure to increase the perimeter by 3? If so, how?
  - d. Can you add a tile to this figure so that the perimeter doesn't change? If so, how?
3. Can you make more than one shape with the same perimeter, but different areas? Show your ideas with grid paper.
4. Can you make more than one shape with the same area, but different perimeters? Show your ideas with grid paper.
5. If you pick any whole number between 12 and 24, use that many unit tiles, can you make a shape where the area and perimeter are equal? Show your ideas.

# Spiral Review

1. Find the area of each of the following shapes:

|  |   |
|--|---|
| <p>a.</p>   | <p>b.</p>   |
| <p>c.</p>  | <p>d.</p>  |

2. Solve:  $-8 < -3m + 10$

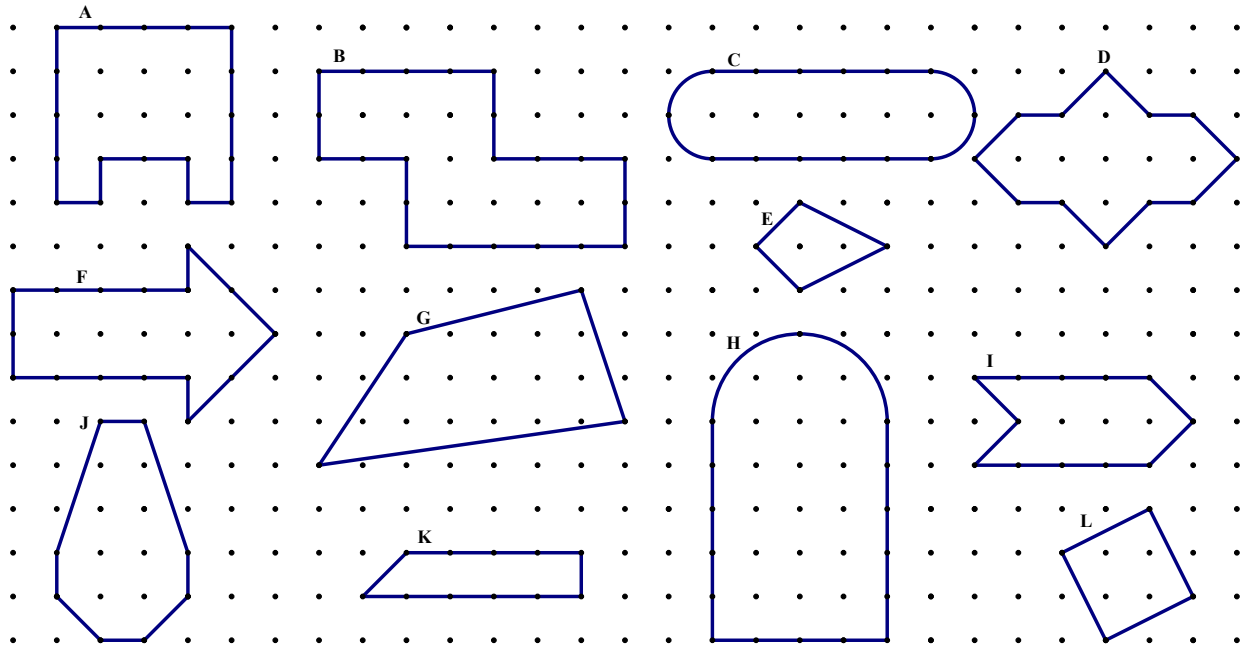
3. Solve:  $-12 > 3x$

4. A website says that the odds of Mr. and Mrs. Durrand having a baby with blue or hazel eyes is 50:50. Describe a simulation that models the color of eyes their baby will have.

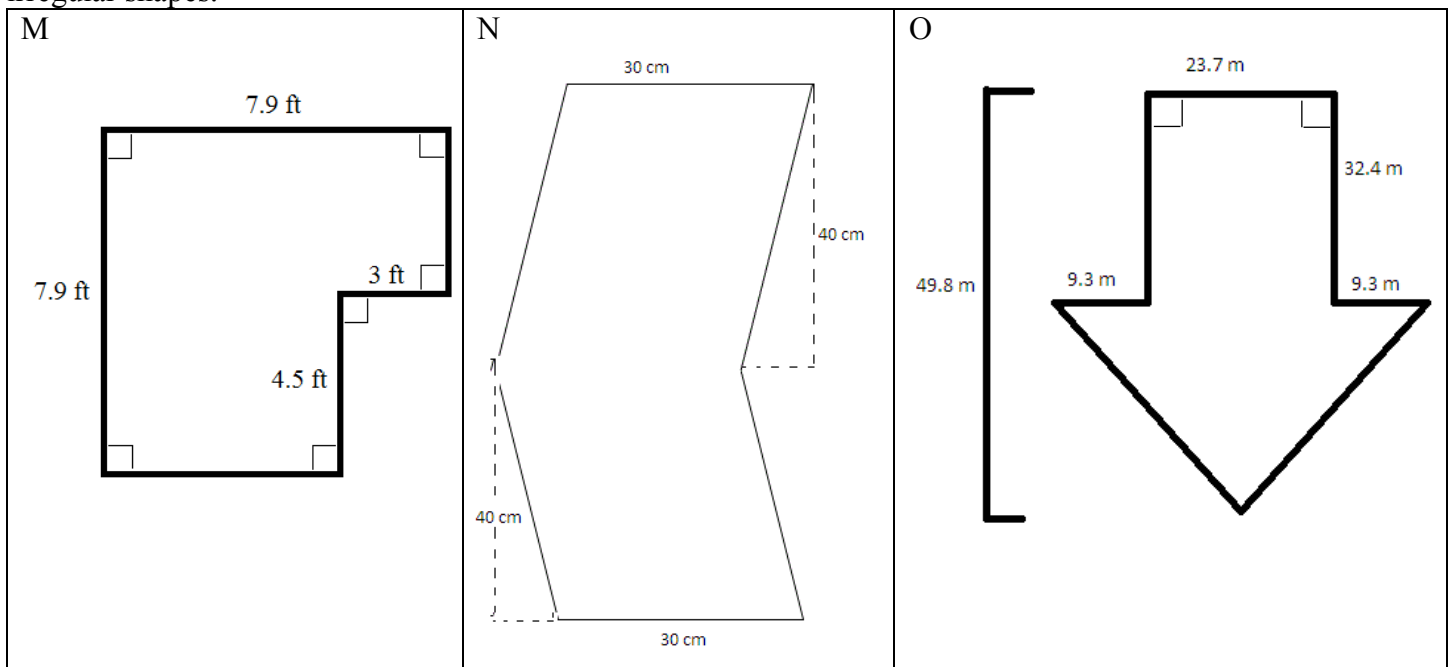
5. Write  $\frac{3}{5}$  as a percent and decimal.

### 8.1b Class Activity: Areas of Irregular Shapes

The following shapes have been drawn on square dot paper. The distance between each dot represents one unit. Use what you have learned about area to find the area of each shape (A-L).



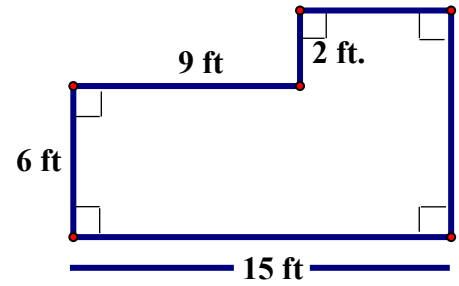
Use your knowledge of area and the problems that you completed above to find the area of the following irregular shapes.



1 a. Johona is building a deck off the back of her house. To the right is a sketch of it. She will need to have a full concrete foundation below the deck. Find the surface area of the concrete foundation.

b. Suppose Johona wants to build her deck onto a concrete foundation that is 1.5 ft. thick and has the same surface area as the deck. How many cubic feet of concrete will she need?

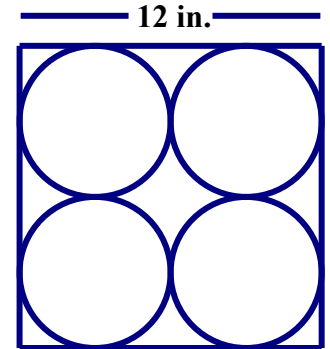
c. How many cubic yards will she need?



2. a. Hugo is making the tile pattern shown. The tile is a square with four circles of the same size inside. He will paint the circles blue and the remaining part of the tile yellow. Find the area of the portion of the tile that will be yellow.

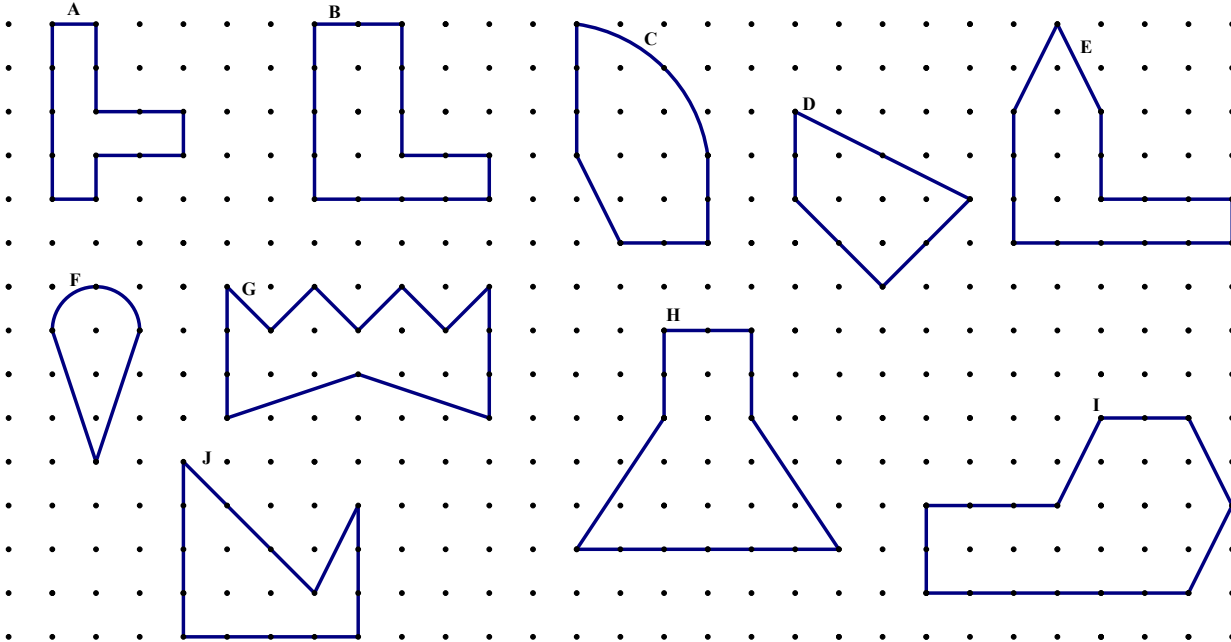
b. Find the area of the diamond-shaped piece in the middle of the tile.

c. What portion (percent) of the tile will be yellow?



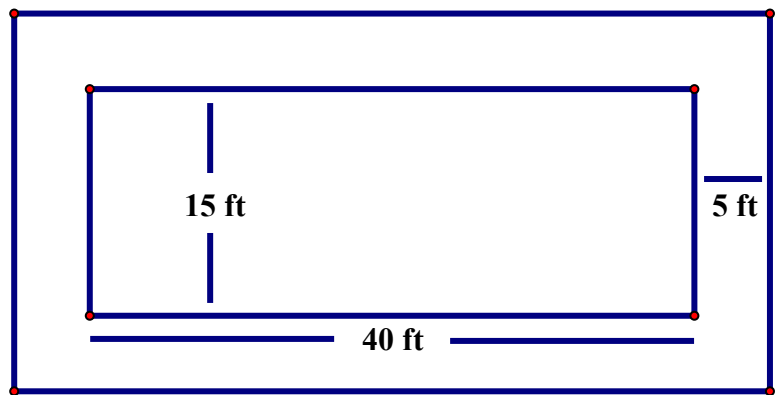
### 8.1b Homework: Areas of Irregular Shapes

1. The following shapes have been drawn on square dot paper. The distance between each dot represents one unit. Use what you have learned about area to find the area of each shape (A-J).



Solve the following area problems.

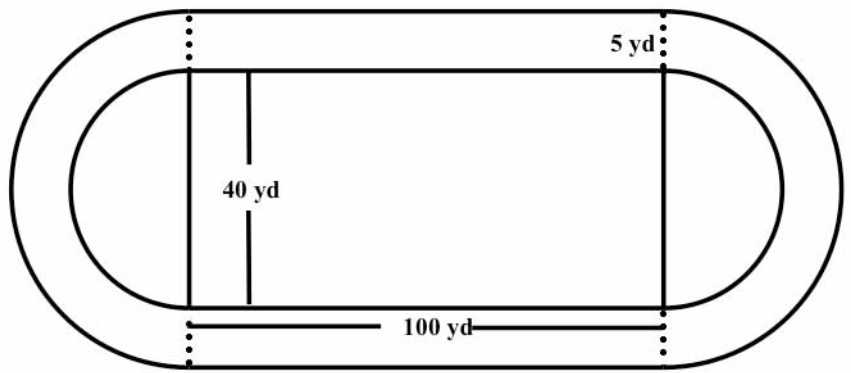
2 a. A rectangular lap pool with a length of 40 ft. and a width of 15 ft. is surrounded by a 5-ft. wide deck. Find the area of the deck.



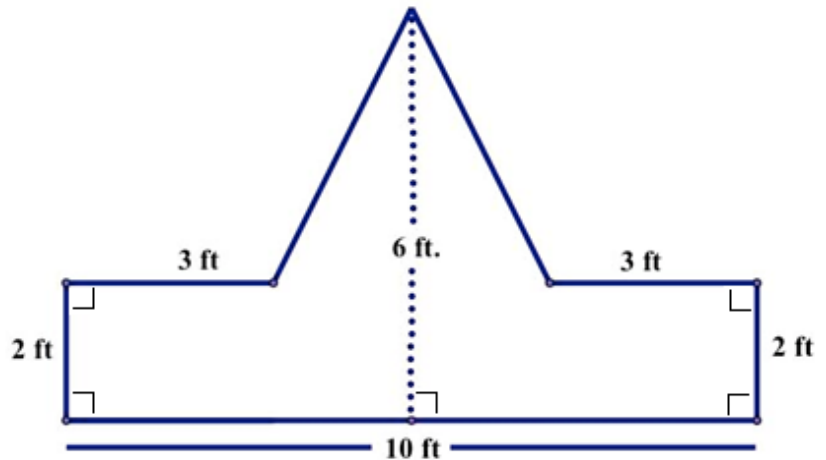
2 b. Draw a picture of the deck if the deck is extended 3 feet in every direction, then find the area of the new deck.



3. A rectangular field with two semi-circles at each of the shorter ends of the field measures 100 yards long and 40 yards wide. It is surrounded by a track that is 5 yards wide. Find the area of the field which includes the two semi-circles on each of the shorter ends of the rectangle. Find the area of the track.



4. Laura is painting a sign for the new Post Office. She will paint the triangular portion blue and the lower rectangular portion red. Find the area of sign that she will paint blue. What percent of the sign will be in blue?



### Spiral Review

1. Simplify the following:

$$2.6 + (-2.6) =$$

$$\frac{1}{6} + \frac{3}{7} =$$

2. Simplify the following expressions:

$$-60 + 5x - 1 - 17x$$

$$9(5 - x) + 3x - 8$$

3. Anya and Bartholomew clean windows. Anya charges 50 cents per window plus \$10 per job. Bartholomew charges 90 cents per window plus \$6 per job. If on one job, they make the same amount of money, how many windows did they each clean?

4. Alfonso is tossing his baby brother's cylinder toy. The following table shows the results of the tosses. Based on the observations, what is the probability the cylinder will land on its side?

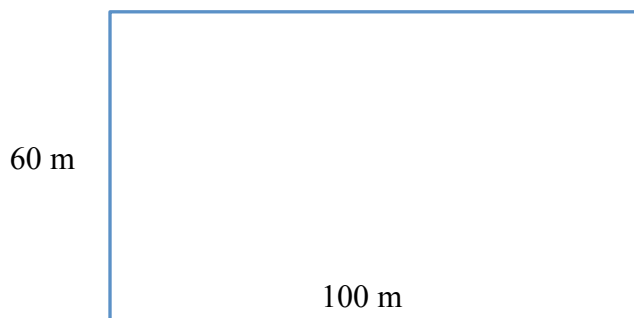
| Land on side | Land on top or bottom |
|--------------|-----------------------|
| 9            | 11                    |

## 8.1c Class Activity: Areas of Irregular Shapes and Expressions



Use your knowledge of area to find the area of the following shapes, if possible. Write an expression to show how you arrived at your answer.

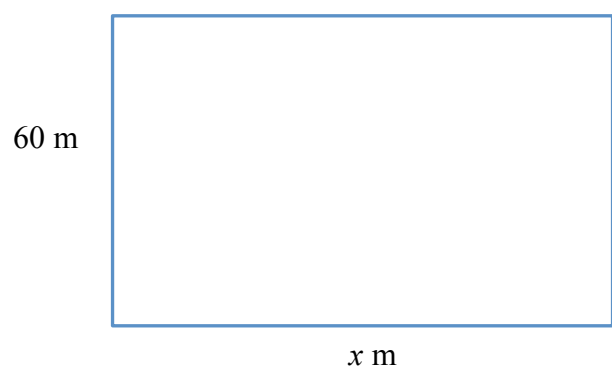
1 a.



1 b.

Suppose you had a  $60 \times 100$  meter plot of ground on which you were going to plant a garden. In the corner of your plot you want to build  $5 \times 8$  storage shed. How much area will be left for your garden? Does it matter which way the storage shed is oriented?

2 a



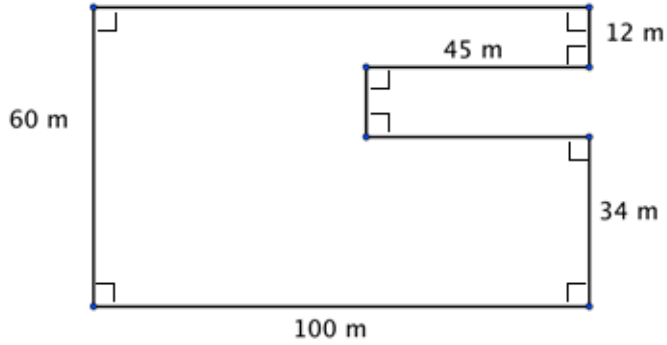
2 b.

Suppose you have a 60 by 200 meter plot of ground on which you are going to plant a garden. You don't want to plant the whole thing, just an area of  $8000 \text{ m}^2$ . If one side is 60 meters, how long will the other side (the 200 meter side) have to be to have an area of  $8000 \text{ m}^2$ ? What percent of the other side would you be using?

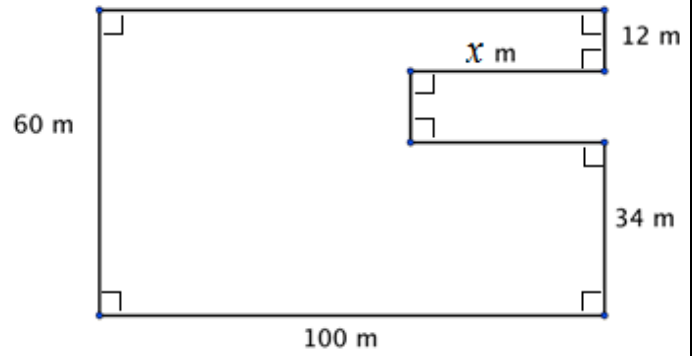
2 c.

What percent of your  $60 \times 200$  meter plot of ground will be planted?

3. a

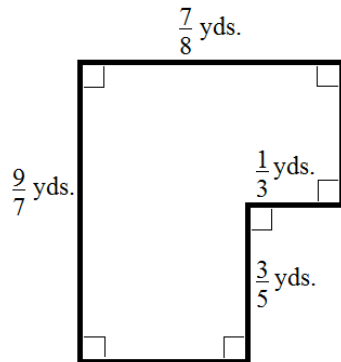


3 b.

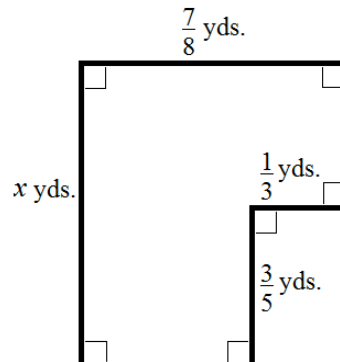


3 c. Suppose you're building a rectangular storage enclosure. The base of the enclosure is to be 60 by 100 meters. You need to construct a ramp into the enclosure as illustrated in 3 b. How long should the ramp be if you want the remaining area to be  $5250 \text{ m}^2$ ?

4a.

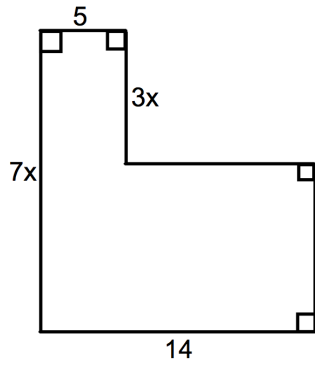


4b.

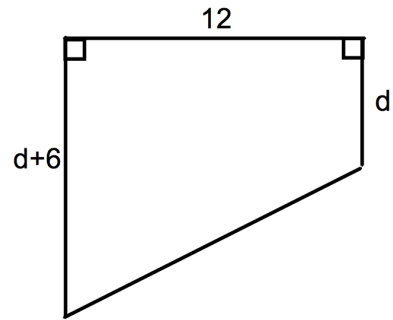


Write an expression to represent the indicated measure for each of the following irregular shapes.

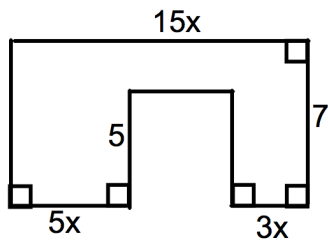
5. Find the area and perimeter for the figure below.



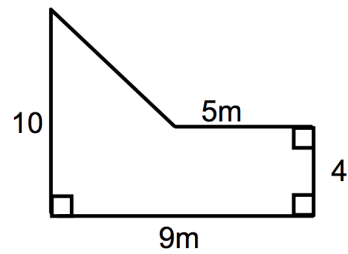
6. Find the area of the figure below.



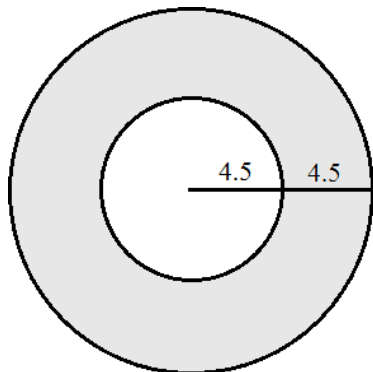
7. Find the area and perimeter of the figure below:



8. Find the area of the figure below:



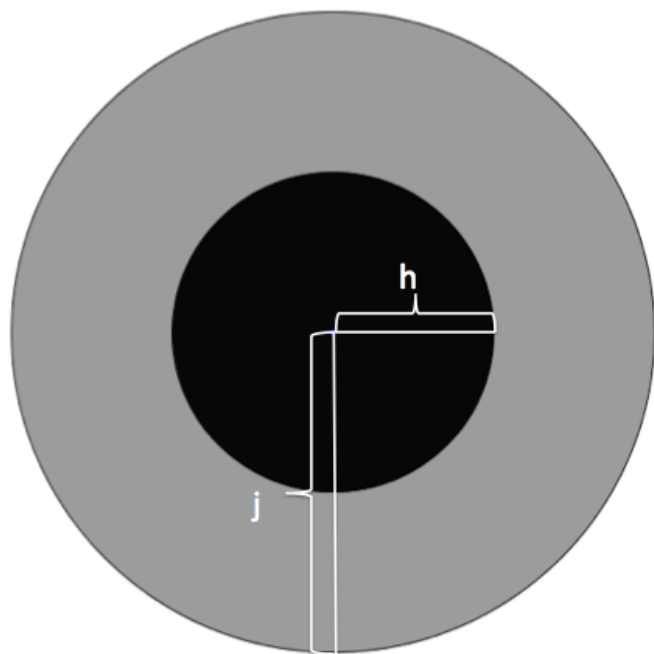
9. A bull's eye is made of two concentric circles as shown below. The radius of the smaller circle is 4.5 inches. The larger circle has a radius of 9 inches.
- Use 3.14 to approximate  $\pi$  in calculating the area of:
- The smaller circle
  - The larger circle
  - The space between the smaller circle and larger circle (the outer ring)



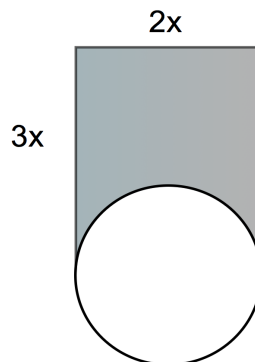
Assuming the dart hits the target somewhere, what is the probability of hitting the smaller circle's area?

- What is the probability of hitting the outer ring?

10. Find the area of the grey part of the figure below.

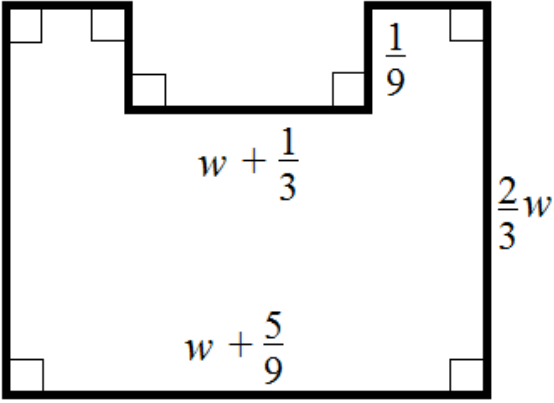
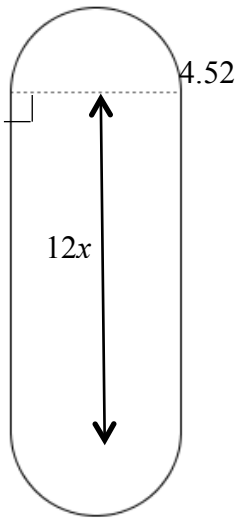


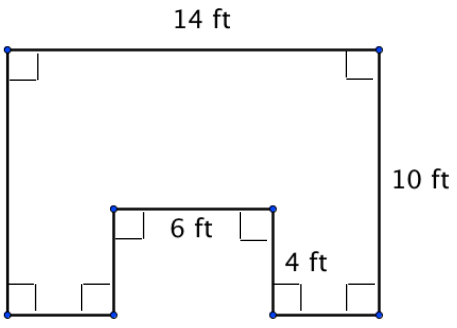
11. The figure below shows a circle and a rectangle. The circle's diameter is equal to the rectangle's base. Find area of the shaded region and its perimeter. (use 3.14 for  $\pi$ )



### 8.1c Homework: Areas of Irregular Shapes and Expressions

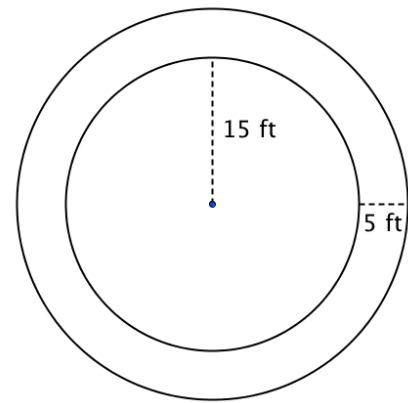
Solve the following area problems. Write an expression showing how you got the area.

|  |   |
|--|---|
| <p>1. Find the area of the region.</p>  | <p>2. Find the perimeter and area of the region. (use 3.14 for pi)</p>  |
|--|---|

|  |   |
|--|---|
| <p>3. Jeremy needs to buy soil for the garden spot in his backyard. A sketch of the plot is to the right.</p> <ol style="list-style-type: none"> <li>Find the area of the garden.</li> <li>How many cubic feet of soil will he need to buy if he covers the area in 6 inches of soil?</li> <li>How many cubic feet of soil will he need to buy if he covers the area in one and a half feet of soil?</li> <li>How many feet of fencing will he need to enclose the garden if he fences the exact shape of the garden?</li> </ol> |  |
|--|---|

4. Nico is building a deck around the circular pool in his back yard. The pool has a radius of 15 ft. The deck will be 5 ft wide.

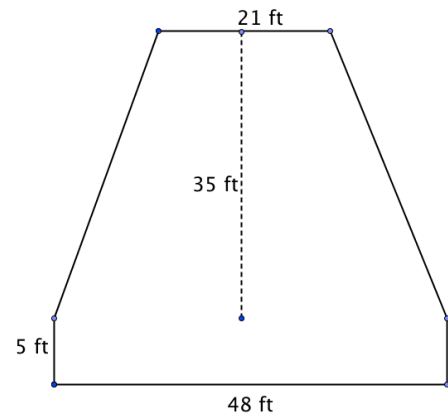
a. Find the area of the deck.



5. a. A stage with a trapezoidal area upstage and a rectangular area downstage is illustrated in the figure to the right. Find the area of the stage.

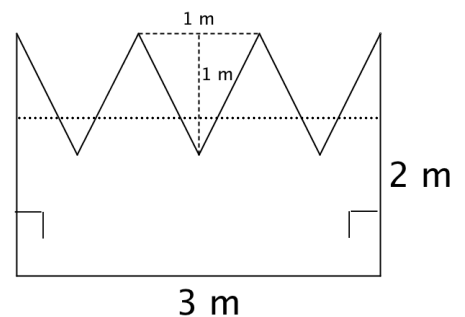
b. What is the area of just the rectangular portion of the stage?

c. What portion of the stage is the rectangular portion?



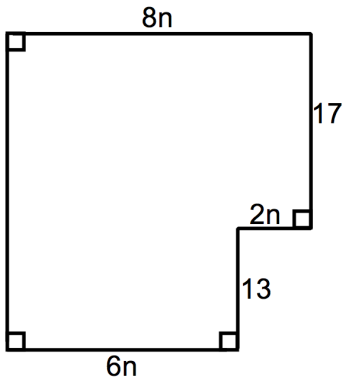
6. Rachel is painting a sign for the new Health Center. Find the area of sign that she will need to paint red if she paints the entire area red.

Suppose Rachel decides to paint the lower portion (the  $3 \times 1$  bottom portion, shown below the dotted line) of the sign blue. What percent of the sign would be blue?



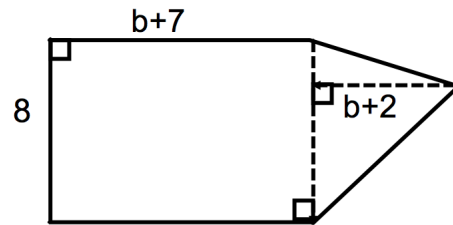
Write an expression to represent the **white area** of the following irregular shapes.

7.

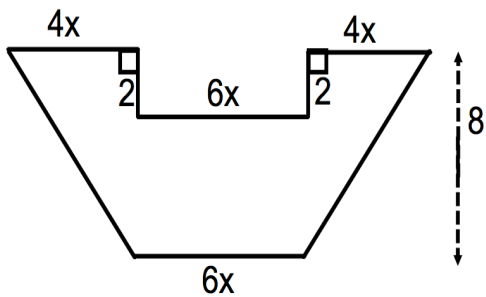


Find the perimeter of this region:

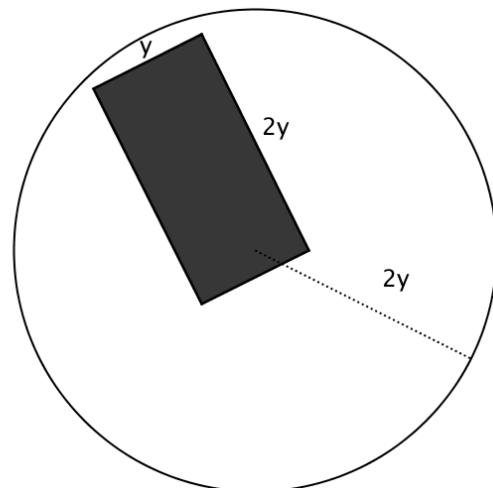
8.



9.

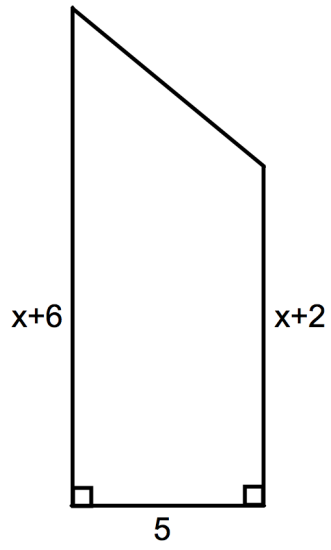


10. The circle has a radius of  $2y$ .

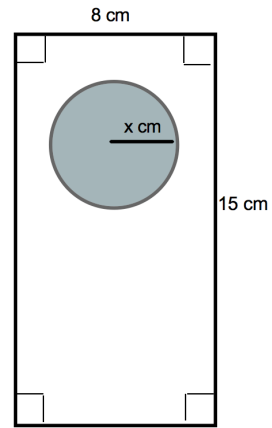




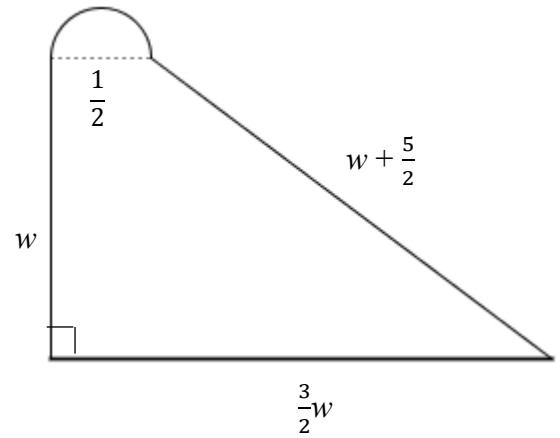
11.



12.



13.



## Spiral Review

1. Write and solve a proportion to find what percent of 27 is 3?
2. A local college football team is known for its awesome offense. The table below shows a season's rushing yards for 9 players. Find the mean absolute deviation of their rushing yards.

| Rushing Yards |
|---------------|
| 460           |
| 399           |
| 180           |
| 158           |
| 110           |
| 95            |
| 55            |
| 15            |
| 1             |

3. Two angles are supplementary. One angle is 49 degrees less than the other. What are the measures of each angle?
4. Write 0.672 as a percent.
5. Sam has lots of bracelets. She gets 3 more bracelets. Then she sells  $\frac{1}{3}$  of the bracelets. Write two different equivalent expressions showing how many bracelets Sam has left.

## 8.1d Class Activity: Review Areas of Triangles, Parallelograms, Trapezoids; Circle Area and Circumference

Activity: In pairs, answer each of the following. Use a model to justify your answer.

1. Look back at question #3 from your homework (8.1c). How much more soil will Jeremy need to buy if he decided to cover his garden in 1.5 feet of soil rather than six inches of soil? Give your answer as a scale factor (twice as much? Three times as much? 1.5 times as much?) Explain:
2. Kara and Sharice are in a quilting competition. Both are stitching rectangular-shaped quilts. So far Kara's has an area of 2,278 square inches with a height of 44 inches. Sharice's quilt has an area of 2,276 square inches with a height of 47 inches. Whose quilt is wider? By how many inches is it wider?
3. Rufina bought two 12-foot pieces of lumber and two 8-foot pieces of lumber to create a border for her garden in her yard. She wants to use all the wood to enclose her garden. How should she use the four pieces to create a rectangular garden of largest possible area? What is the largest area she can get with her four pieces of wood?
4. A triangle has an area of 90 square centimeters.
  - a. If its height is 15 cm, what is the length of the base?
  - b. Draw a triangle with an area of 90 square centimeters, height of 15 cm and base you found in part a. Is there more than one triangle you can draw with those dimensions?
  - c. Draw a triangle that has an area that is 150% of the original 90 cm<sup>2</sup> triangle. What are the dimensions of your larger triangle?

5. Julius drew a trapezoid that had bases of 15 and 11 inches and a height of 4 inches.
  - a. What is the area of the trapezoid Julius drew?
  - b. Can you draw a trapezoid that has the same area, but different dimensions?
  - c. Draw a trapezoid with the same height but with bases that are a  $\frac{5}{2}$  scale factor of the original trapezoid.
  - d. What is the area of the new trapezoid?

Review:

Draw a model and explain how to find the circumference of a circle:

Draw a model and explain how to find the area of a circle:

6. You're making a 12 inch diameter pizza. You want the sauce to cover the pizza with a 1.5 inch ring left around the outside without sauce. (Use 3.14 as an approximation for pi.)
  - a. What is the area that the sauce will cover?
  
  - b. What percent of the dough will be covered by sauce?
  
  - c. If one 8 oz can of tomato sauce covers about 125 sq. inches of pizza dough, how many cans of sauce will you need to buy?

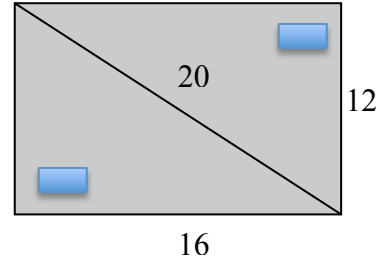
7. A circular swimming pool with a diameter of 32 feet is located exactly in the middle of a  $40 \text{ ft} \times 40 \text{ ft}$  square lot. For safety reasons the lot needs to have an 8 ft fence on the perimeter of the entire lot.
- How long will the fence need to be?
  - If the fence was around only the circular pool, how long would the fence be?
  - Explain how much longer a fence around the whole yard is than a fence only around the pool using percent increase.
  - What percent of the yard area does the pool take up?
  - What is the area of the yard NOT taken up by the pool?
8. Look back at question #4 from the homework (8.1c). Nico's pool currently takes up approximately  $706.5 \text{ ft}^2$  of space in his yard (using 3.14 as an approximation for pi). If Nico adds the five foot deck around the pool, what percent increase of space will this be?

### 8.1d Homework: Review Areas of Triangles, Parallelograms, Trapezoids; Circle Area and Circumference

1. Wallpaper comes in rolls that are 60 feet long and 2 feet wide. How many rolls of wallpaper will it take to cover 700 square feet?
  
2. A rectangular garden has an area of 45 square feet. One of the sides is 6 feet.
  - a. What is the other side?
  - b. You want to put a fence around it. How long will the fence need to be?
  - c. You decide you want to increase the length of each side by a scale factor of 3.2. What are the new dimensions of your garden?
  - d. What is the area of your new garden?
  
3. Mrs. Garcia has a table shaped like an isosceles trapezoid in her third grade classroom. The two parallel sides have lengths of 6 feet and 8 feet. The distance between them is 4 feet.
  - a. What is the area of the top of Mrs. Garcia's table?
  - b. Suppose Mrs. Garcia has a 1.75 ft by 0.8 ft puzzle on the table. How much surface area is now available on her table?
  
4. The diameter of the earth is about 7926 miles.
  - a. Find the distance around the earth at the equator.
  
  - b. If there are 5280 feet in every mile, what is the distance around the Earth in feet?
  
  - c. Suppose you can jog at a rate of 2 miles every 15 minutes. At this rate, how long would it take you to walk around the Earth?

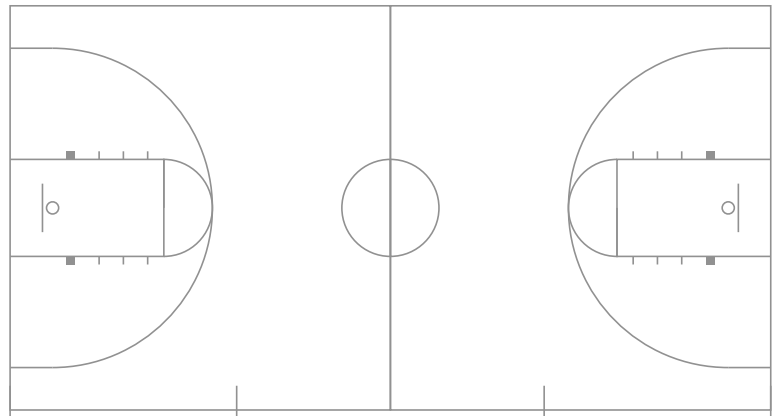
5. A 12 foot by 16 foot rectangular office is being sectioned off into two triangular areas so that desks can be placed in opposite corners. The diagonal of the office (from corner to corner) is 20 feet. The manager needs a dividing curtain to hang from the ceiling around one of the triangles.

a. How long does the curtain need to be?



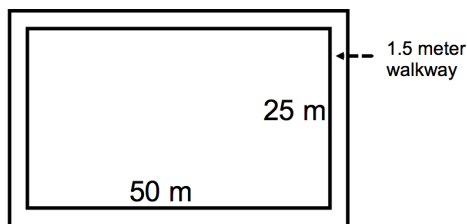
b. The carpet in each section will be a different color. How many square feet of carpet will be needed to cover each triangular section?

6. The three-point line in basketball is approximately a semi-circle with a radius of 19 feet and 9 inches. The entire court is a rectangle 50 feet wide by 94 feet long. What is the approximate area of the court that results in 3 points for a team?



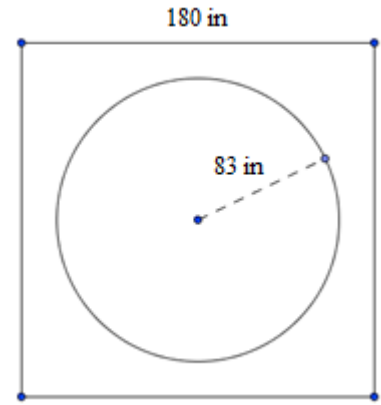
7. Your neighbor's backyard lawn is shaped like a rectangle. The back fence is 38.2 feet long and the side fence is 32.6 feet long. He will pay you \$0.04 per square foot for mowing and \$0.11 per foot for trimming all the edges. How much will you get paid total for mowing and trimming? Remember to show all your work.

8. You're the manager of a county recreation center that has a 50 by 25 meter rectangular pool. Currently, there is an 1.5 meter cement walkway around the pool (see diagram). The community is concerned about the safety of the walkway and would like to cover it with a non-slip rubber substance that costs \$78 a square meter to be installed. The county has budgeted \$15,000 for the project. Is that enough money to cover the walkway? Explain your answer.



## Spiral Review

1. A hot tub is surrounded by a square deck as pictured to the right. What is the area of the deck?



2. Cristian is building a rectangular garden in his backyard. The width of the garden is set at 29 inches. He wants the fence to be 5 inches longer than the garden on each side. If he wants the area enclosed by the fence to be 2028 square inches, how long should the garden be?
3. Eugene's math class has 20 boys and 10 girls. If the teacher draws a student's name at random for a candy bar, what is the probability Eugene will be chosen? What is the probability that a girl will be chosen?
4. There are a total of 214 cars and trucks on a lot. If the number of cars is four more than twice the number of trucks, how many cars and trucks are on the lot?
5.  $-1(-4)(-7)$



## 8.1f Self-Assessment: Section 8.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

| Skill/Concept   | Beginning Understanding  | Developing Skill and Understanding  | Practical Skill and Understanding   | Deep Understanding, Skill Mastery  |
|---|--|---|---|--|
| 1. Solve problems involving area and circumference of a circle with numeric measures.   | I can solve problems finding circumference or area of any circle as long as I'm given the radius, formulas and a picture of the situation. | I can solve problems finding circumference or area of any circle as long as I'm given a picture of the situation.         | I can solve problems finding circumference or area of any circle when information is given pictorially or in context.                     | I can solve problems finding circumference or area of any circle regardless of how the information is given. I can also apply information to new contexts.                             |
| 2. Solve real world and mathematical problems involving area and perimeter of two-dimensional objects composed of triangles and quadrilaterals. | I struggle to solve problems involving area and perimeter of triangles and quadrilaterals.   | I can solve problems involving area and perimeter of triangles and quadrilaterals if the needed information is given.     | I can solve problems involving area and perimeter of triangles and quadrilaterals even if I have to find missing information to solve it. | I can solve problems involving area and perimeter of triangles and quadrilaterals even if I have to find missing information to solve it. I can also explain why my answer is correct. |
| 3. Understand and explain the difference between perimeter and area.  | I struggle to understand the difference between perimeter and area.  | I can calculate perimeter and area of various two-dimensional objects.  | I understand and can show visually the difference between perimeter and area.   | I understand and can explain in my own words the difference between perimeter and area.  |
| 4. Find area or perimeter of an object with algebraically measured lengths.   | I can find the area or perimeter of an object with numeric measured lengths, but struggle if there are variables involved.                 | I can usually find perimeter or area of an object with algebraic measured lengths.  | I can always find perimeter or area of an object with algebraically measured lengths.   | I can solve area and perimeter problems involving missing sides with algebraically measured lengths.   |
| 5. Solve problems involving scale or percent increase/decrease and area and/or perimeter.   | I can find the perimeter or area of objects, but I struggle to solve problems involving scale or percent increase/decrease.                | I can find the perimeter and areas of two scaled objects. I can usually find the scale factor between those measurements. | I can solve problems involving scale or percent increase/decrease and area and/or perimeter.  | I can solve problems involving scale or percent increase/decrease and perimeter or area. I can also explain the relationship between perimeters or areas of those objects.             |

### Sample Problems for Section 8.1

1. Use the given information to find the missing information. Round each answer to the nearest hundredth unit.

a. Diameter: 18 km  
Circumference: \_\_\_\_\_

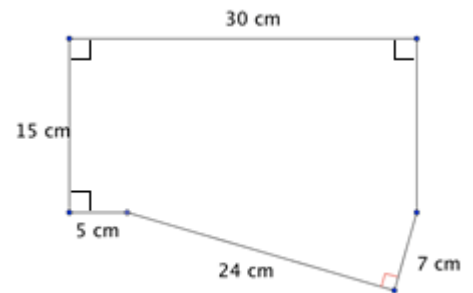
c. Circumference: 50.24 in  
Radius: \_\_\_\_\_

b. Radius: 0.1 m  
Area: \_\_\_\_\_

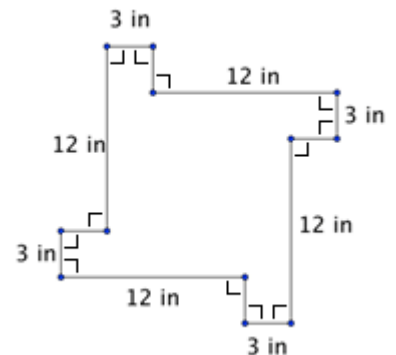
d. Area:  $105.63 \text{ in}^2$   
Radius: \_\_\_\_\_

2. Answer the following questions using the object pictured.

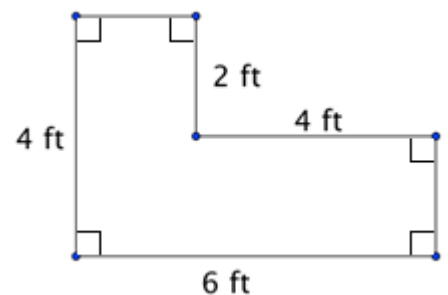
a. Find the perimeter and area of the object to the left.



b. What percent of the total area is the area of one of the outer squares in the object to the left?



c. Phoebe is buying an L-shaped desk. How much area will the desk pictured take up in her apartment?



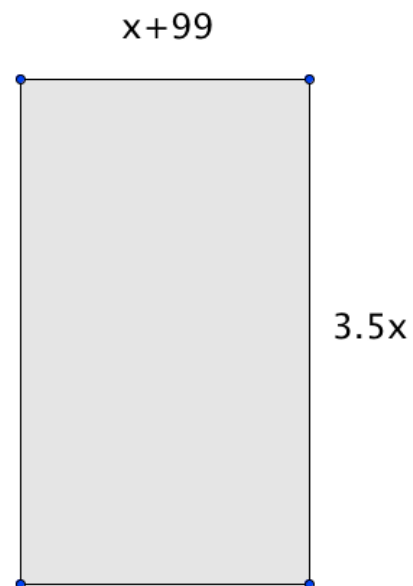
3. Explain the difference between perimeter and area in words and/or pictures. Use the figure in 2c to help you in your explanation.

4. Use the rectangle to the right to answer the following questions:

a. What is the perimeter?

b. What is the area?

c. If the perimeter is 1098 units, what is the value of  $x$ ?



5. The dimensions of Gabrielle's rectangular garden are 12 feet by 16 feet. Daniel is building a garden. He is going to decrease the dimensions by 25%.

a. Find the perimeter and area of each garden.

b. Find the percent of decrease between the perimeters of each garden.

c. Find the percent of decrease the areas of each garden.

d. In general, how are the percents of change of perimeters and areas of scaled objects related?

## Section 8.2: 2D Plane Sections from 3D Figures and 3D Measurement

### Section Overview:

The goal of this section is to help students better understand: a) attributes of various prisms, b) how those attributes affect finding the surface area and volume of different prisms, c) the relationship between measurements in 1-, 2-, and 3-dimensional figures and d) connect ideas of scale factor/percent change to ideas of surface area and volume. Students begin this section by examining three-dimensional figures. They should observe which faces for an object are parallel and/or perpendicular and which are the same size and shape. Students quickly move to taking cross-sections of various figures to notice what two dimensional shapes are generated by different types of cuts. Attention will be paid to when parallel plane sections generate surfaces that are the same and when they are different. The exercises in this section are designed to solidify students' understanding of the algorithms for finding perimeter, area, and volume. Additionally, exercises should help students better understand units of measure for perimeter, area, and volume. Next, students examine the nets of 3D figures. Nets were introduced in 6<sup>th</sup> grade; in 7<sup>th</sup> grade, students extend their understanding by differentiating surface area from volume. Students will use their understanding of surface area and volume to solve various problems. Specifically, attention will be paid to scale factor and percent change in problems involving volume and surface area.

### Concepts and Skills to be Mastered (from standards)

1. Describe the two-dimensional figures that result from slicing three dimensional figures.
2. Solve real-world and mathematical problems involving volume and surface area of three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

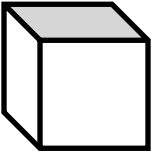
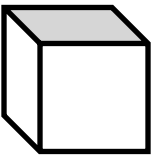
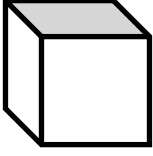
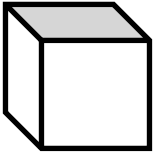
Background knowledge for making and describing 2D cross-sections:

- Right regular polyhedron prisms are three dimensional figures with at minimum two parallel polygonal faces of the same size and shape (called bases); remaining lateral faces are rectangles not necessarily the same as the bases (note that the base may be any polygon but the lateral faces are always rectangles.)
- Right cylinders have parallel circular bases of the same size (radii are equal) whose centers are aligned directly above each other (centers lie on a line that is perpendicular to both bases).
- There are several ways to take plane sections of a prism or cylinder (for the descriptions below, we assume the prism or cylinder is standing on one of its bases):
  - Parallel to the base
  - Perpendicular to the base
  - Slice at an angle (in a tilted direction) to the base
- A cross-section is a special type of plane section cut perpendicular to a face or base. Most cuts in this section will be cross-sections.
- “Equidistant” means an equal distance. “Vertices” are the corners of the figure.

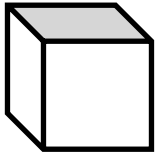
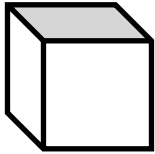
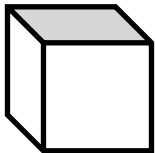
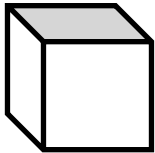
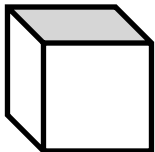
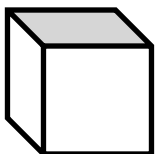
This section is primarily exploratory in nature. Attention to precision will be very important as the teacher and students explain their thinking.

## 8.2a Class Activity: 2D Plane Sections of Cubes and Prisms (play dough & dental floss)

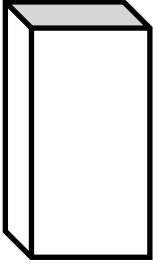
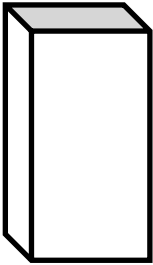
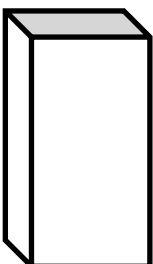
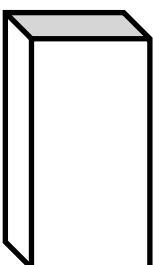
1. Mold a **CUBE** from play-dough.

| Perform the following cuts.   | Sketch where you cut  | Sketch the exposed surface(s). | NOTES   |
|---|---|--------------------------------|---|
| a. Cut <i>parallel</i> to the base.   |    |                                |   |
| b. Cut parallel to the base again, but at a different distance from the base.   |    |                                | How does the new exposed surface compare to the previous surface? |
| <p>What is true about all cuts of a cube parallel to the base?</p> <p>What is true about other cuts of a cube NOT parallel to the base but through at least one lateral face?</p>             |   |                                |   |
| c. Cut perpendicular to the base and parallel to a face.  |  |                                |   |
| d. Cut perpendicular to the base and parallel to a face again, but at a different location on the cube.   |  |                                | How does the new exposed surface compare to the previous surface? |
| <p>What is true about all cuts of a cube perpendicular to the base and parallel to a face?</p> <p>What is true about cuts of a cube perpendicular to the base but not parallel to a face?</p> |   |                                |   |

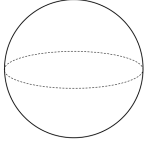
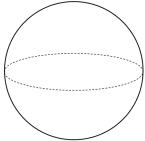
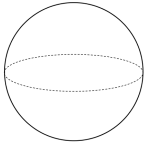
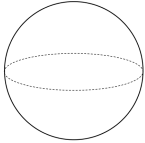
2. Mold a **CUBE** again. Perform a single cut to create the following:

| Create...              | Sketch where you cut  | NOTES |
|------------------------|---|-------|
| a. Triangle            |    |       |
| b. Square              |    |       |
| c. Rectangle           |   |       |
| d. Pentagon            |  |       |
| e. Hexagon             |  |       |
| f. Circle or Trapezoid |  |       |

3. Mold a Right **RECTANGULAR PRISM** (that is not a cube.)

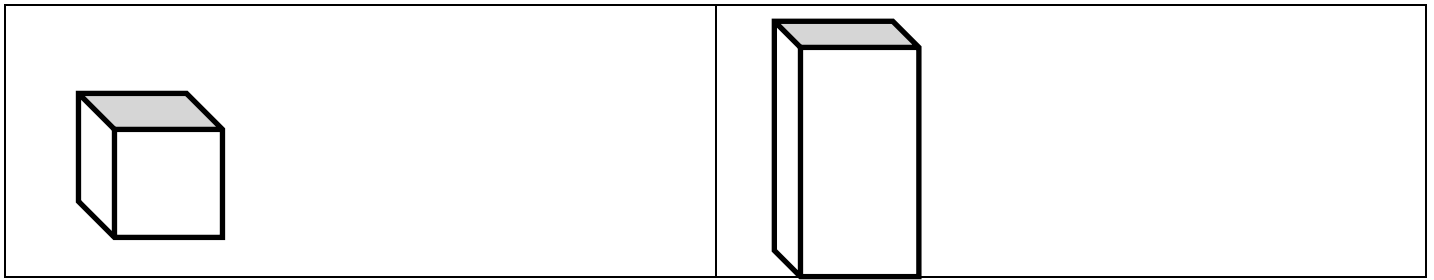
| Perform the following cuts.  | Sketch where you cut  | <b>NOTES:</b> describe the plane section and what might happen with other similar cuts. |
|--|---|---|
| a. Cut parallel to the base.   |    |   |
| b. Cut parallel to the base again, but at a different distance from the base.  |    |   |
| <p>What is true about all cuts of a rectangular prism parallel to the base?</p> <p>What is true about cuts of a rectangular prism NOT parallel to the base?</p>  |   |   |
| c. Cut perpendicular to the base and parallel to a face.   |  |   |
| d. Cut perpendicular to the base but not parallel to a face.   |  |   |
| <p>What is true about all plane sections of a rectangular prism perpendicular to the base and parallel to a face?</p> <p>What is true about any plane section of a rectangular prism perpendicular to the base but not parallel to a face?</p> |   |   |

4. Mold a **SPHERE**.

| Perform the following cuts.   | Sketch where you cut  | NOTES: describe the plane section and what might happen with other similar cuts. |
|---|---|--|
| a. Cut parallel to the table at different distances from the table.   |    |  |
| b. Make cuts that are not parallel to the table.  |    |  |
| <p>What is true about all cuts of a sphere parallel to the table?</p> <p>What is true about all cuts of a sphere NOT parallel to the table?</p> |   |  |
| c. Cut perpendicular to the table.  |  |  |
| d. Cut perpendicular to the table again, but at a different location on the sphere.   |  |  |
| <p>What is true about all cuts of a sphere?</p>   |   |  |



5. Make a cube and a rectangular prism like in exercises #1 and #3 again. Orient them as in the figure below:

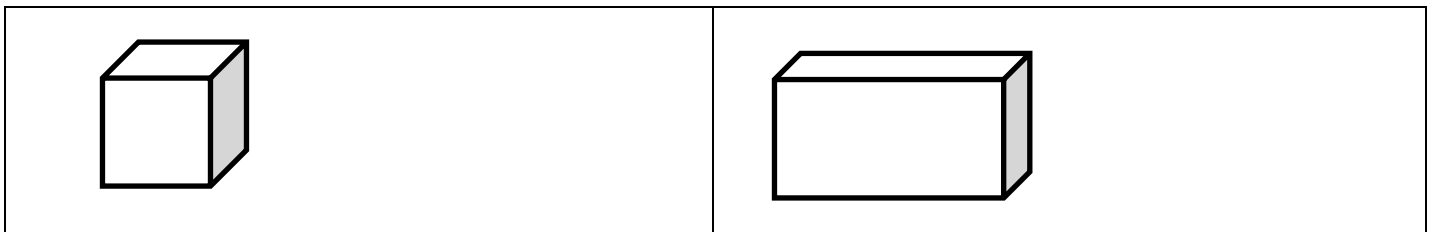


What is the shape of the base of the cube and the rectangular prism?

Will all cuts parallel to the base result in the same planar figure for the cube?

Will all cuts parallel to the base result in the same planar figure for the rectangular prism?

Now rotate each  $90^\circ$  as shown in the figure below:



What is the shape of the base of the cube and the rectangular prism now?

Will all cuts parallel to the base result in the same planar figure for the cube?

Will all cuts parallel to the base result in the same planar figure for the rectangular prism?

6. Compare and contrast plane sections of rectangular prisms, cubes and spheres.

## 8.2a Homework: 3D Objects

Use your knowledge of each three-dimensional object to answer the following questions.

### Cubes

1. How many faces does a cube have?
2. What do you know about each face of a cube?
3. How many edges does a cube have?
4. How many vertices does a cube have?

### Rectangular Prisms


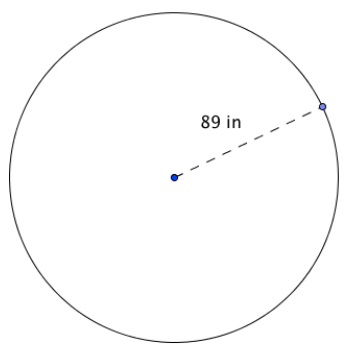
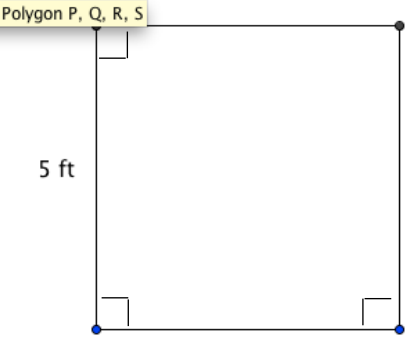
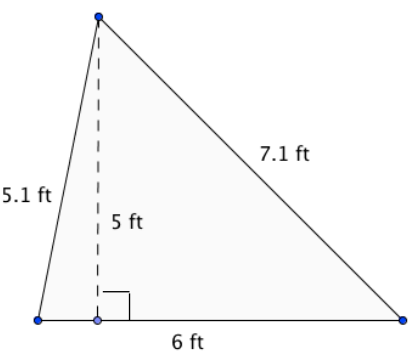
5. How many faces does a rectangular prism have?
6. How many edges and vertices does a rectangular prism have?
7. How are a rectangular prism and a cube similar and different?

### Sphere

8. Does a sphere have any edges or vertices?
9. What makes a sphere different from all the other 3D objects named above?

## Spiral Review

1. Find the area of each figure below:

|  |   |
|--|---|
| <p>a.</p>   | <p>b.</p>   |
| <p>c. The figure below is a square.</p> <p>Polygon P, Q, R, S</p>  | <p>d.</p>  |

2. Simone rolls a die 64 times. Approximately, how many times will she roll a 6?

3. Find the unit rate for BOTH units.  
Izzy drove 357 miles on 10 gallons of gasoline.

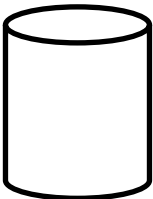
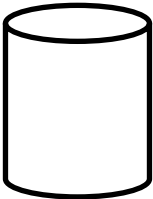
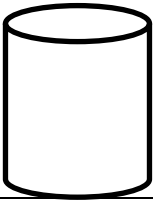
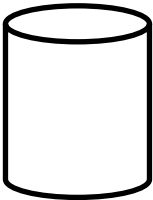
4. Convert the following units using the ratios given:

\_\_\_\_\_ feet = 37 inches (1 foot = 12 inches)

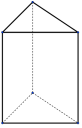
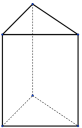
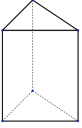
5. The temperature at midnight was  $8^{\circ}\text{C}$ . By 8 am, it had risen  $1.5^{\circ}$ . By noon, it had risen another  $2.7^{\circ}$ . Then a storm blew in, causing it to drop  $2.7^{\circ}$  by 6 pm. What was the temperature at 6 pm?

## 8.2b Class Activity: 2D Plane Sections on Cylinders and More

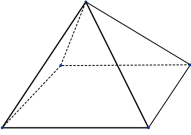
1. Mold a right **CYLINDER**, put the circular base on the table.

| Perform the following cuts.  | Sketch where you cut  | NOTES: describe the plane section and what might happen with other similar cuts. |
|--|---|--|
| a. Cut parallel to the base.   |    |  |
| b. Make cuts not parallel to the base but through the lateral face of the cylinder.  |    |  |
| <p>What is true about all cuts of a cylinder parallel to the base?</p> <p>What is true about all cuts of a cylinder NOT parallel to the base?</p>  |   |  |
| c. Cut perpendicular to the base.  |  |  |
| d. Make cuts that go through at least one base of the cylinder but are not perpendicular to the base.  |  |  |
| <p>What is true about all cuts of a cylinder parallel to the to the base?</p> <p>What is true about cuts of a cylinder perpendicular to the base?</p> <p>What is true about any other cut (NOT parallel to the base or perpendicular to the base)?</p> |   |  |

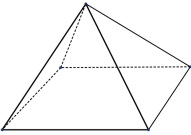
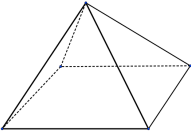
2. Make a **TRIANGULAR-BASED RIGHT PRISM**. Put the triangle base on the table.

| Perform the following cuts.   | Sketch where you cut   | NOTES: describe the plane section and what might happen with other similar cuts. |
|---|--|--|
| a. Cut parallel to the base.  |   |  |
| b. Make cuts that are not parallel to the base but go through at least one face of the prism. |   |  |
| What is true about all cuts of a triangular prism parallel to the base?                       |  |  |
| c. Cut perpendicular to the base.   |  |  |
| What is true about all cuts of a triangular prism perpendicular to the base?                  |  |  |

3. Make a **SQUARE-BASED RIGHT PYRAMID**. Put the square base on the table.

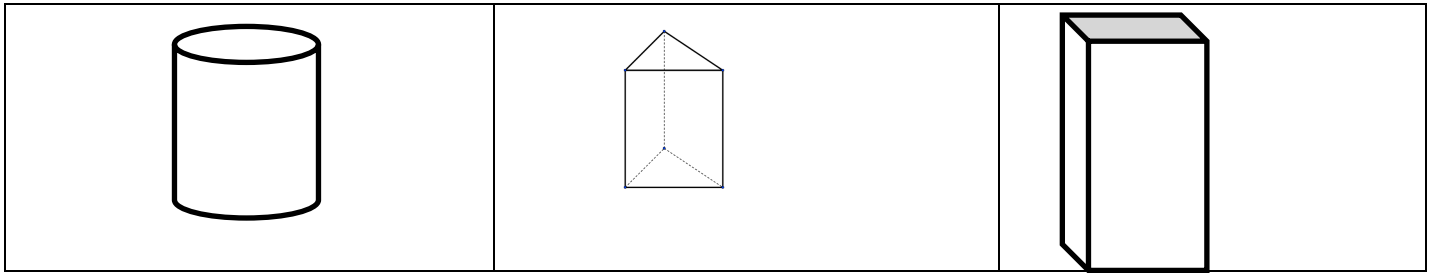
| Perform the following cuts.        | Sketch where you cut  | NOTES: describe the plane section and what might happen with other similar cuts. |
|------------------------------------|---|--|
| a. Make cuts parallel to the base. |  |  |

What do you notice about cuts of a square-based pyramid parallel to the base?

|   |   |  |
|---|---|--|
| b. Cut perpendicular to the base and parallel to one of the base's edges.           |  |  |
| c. Cut perpendicular to the base again, but at a different location on the pyramid. |  |  |

What do you notice about cuts of a triangular pyramid perpendicular to the table?

4. Make a cylinder and a triangular prism like in exercises #1 and #2 again. Also make a rectangular prism. Orient them as in the figure below:



What is the shape of the base of the cylinder?

What is the shape of the base of the triangular prism?

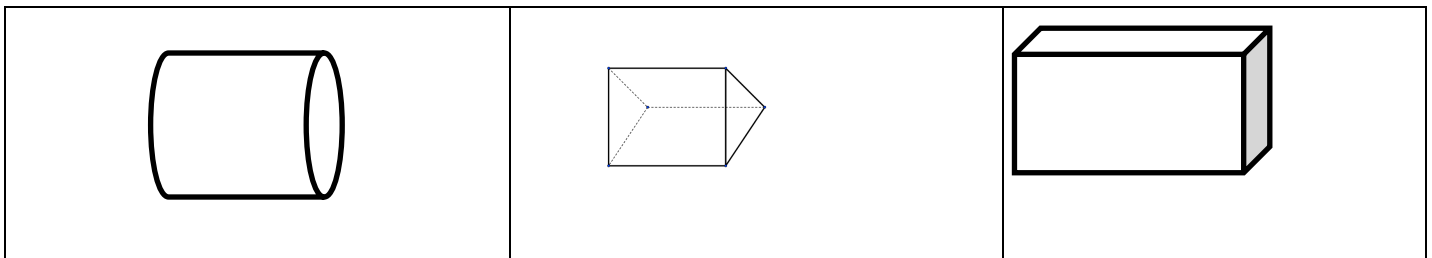
What is the shape of the base of the rectangular prism?

Will all cuts parallel to the base result in the same planar figure for the cylinder?

Will all cuts parallel to the base result in the same planar figure for the triangular prism?

Will all cuts parallel to the base result in the same planar figure for the rectangular prism?

5. Now rotate each  $90^\circ$  as shown in the figure below:



What is the shape of the part of the cylinder resting on the table?

What is the shape of the part of the triangular prism resting on the table?

What is the shape of the part of the rectangular prism resting on the table?

Will all cuts parallel to the table result in the same planar figure for the cylinder?

Will all cuts parallel to the table result in the same planar figure for the triangular prism?

Will all cuts parallel to the table result in the same planar figure for the rectangular prism?

## 8.2b Homework: Area of Plane Sections

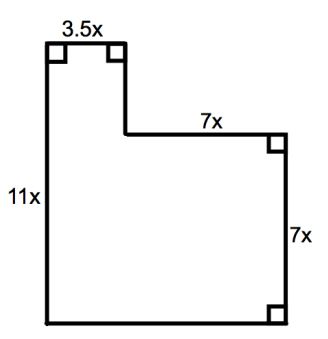
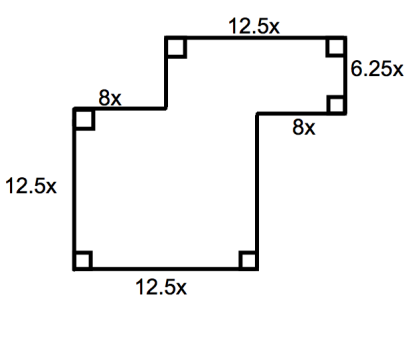
For each plane-section described below, state the shape of the plane-section and its area. Refer to previous class activities, as needed.

1. Imagine cutting a cube, parallel to the base.
  - a. What shape is the plane section?
  - b. If you cut a  $5 \times 5 \times 5$  inch cube parallel to any face, what will the area of the plane section be?
2. Imagine a right square-based prism with edge lengths  $\frac{3}{4} \times \frac{3}{4} \times 4\frac{1}{2}$  inches.
  - a. If you make a cut parallel to the square base, what will the plane section be?
  - b. What will the area of the plane section described in “a” be?
  - c. What will the plane section be if the cut is made parallel to the lateral face?
  - d. What will the area of the plane section in “c” be?
3. Imagine cutting a sphere with diameter 10 cm parallel to the table through the center.
  - a. What shape will any plane section be?
  - b. What will the area of the plane section be?
4. Imagine cutting a cylinder of diameter 12.62 cm and height 8 cm, parallel to the base.
  - a. What shape is the plane section and what is its area?
  - b. What shape is the plane section if the cut is perpendicular to the base? What do you know about the figure?
5. Imagine a triangular prism:
  - a. What shape is the plane section parallel to the base?
  - b. What shape is a plane section perpendicular to the base?
  - c. If the area of the plane section in “b” is  $6.25 \text{ cm}^2$  and the height of the prism 5 cm. What was the length of the cut?
6. Imagine cutting a square based right pyramid parallel to the base.
  - a. What shape is the plane section?
  - b. If the dimensions of the length and the width of the plane section are  $\frac{3}{2}$  in. and  $\frac{3}{2}$  in., what is the area of the plane section?



## Spiral Review

1. Find the perimeter and area of each figure below:

|   |  |
|---|--|
| <p>a.</p>  | <p>b.</p>  |
|---|--|

2. Ten percent of the population is left-handed. Otto believes left-handed people have an advantage in boxing. If he observes 3 people boxing, describe a simulation that would show the probability one is left-handed.

3. Solve and graph the following inequality:  $-97 \geq 4(19 - 2x) + 3$



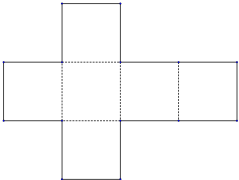
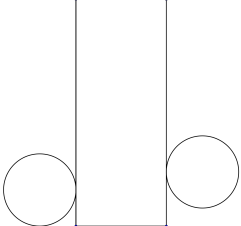
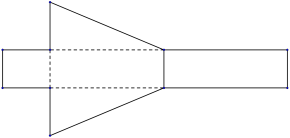
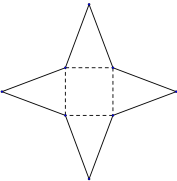
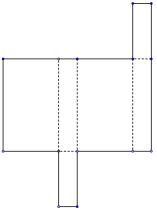
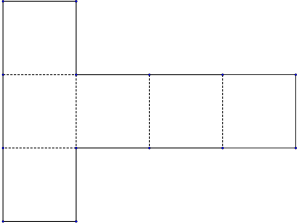
4. Solve  $-17 + 5$

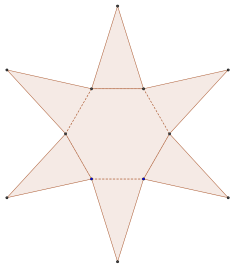
5. Angel owes his mom \$124. Angel made two payments of \$41 to his mom. How much does Angel now owe his mother?

## 8.2c Class Activity: Nets of 3D Objects

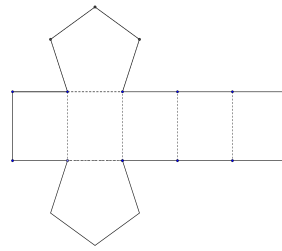
*Review from 6<sup>th</sup> grade:* A net is a two dimensional figure that can be folded to make a three dimensional object. You can learn about a 3D object by examining its net.

Look at each net below. Determine if the net can be folded into a three-dimensional object for which you could make equal parallel planar cuts (e.g. opposite sides are the same). Remember, you can orient objects any way you want. If it does not allow for equal parallel planar cuts, explain why.

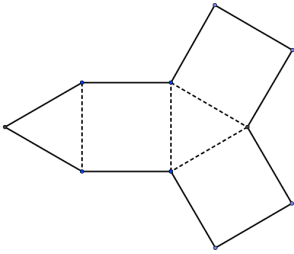
| NET   | NET  |
|---|--|
|  <p>1. Can you create equal parallel plane sections?</p>   |  <p>2. Can you create equal parallel plane sections?</p>   |
|  <p>3. Can you create equal parallel plane sections?</p>  |  <p>4. Can you create equal parallel plane sections?</p>   |
|  <p>5. Can you create equal parallel plane sections?</p> |  <p>6. Can you create equal parallel plane sections?</p> |



7. Can you create equal parallel plane sections?



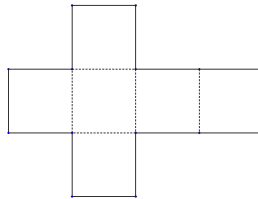
8. Can you create equal parallel plane sections?



9. Can you create equal parallel plane sections?

Notes:

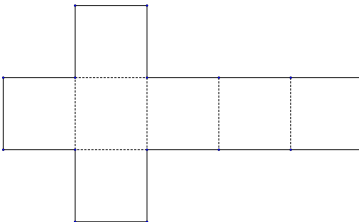
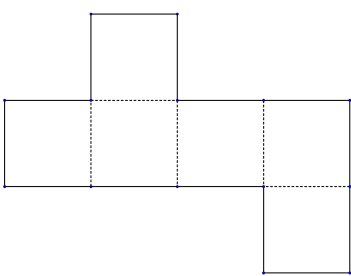
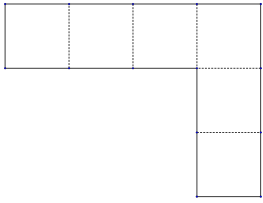

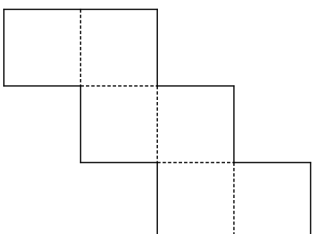
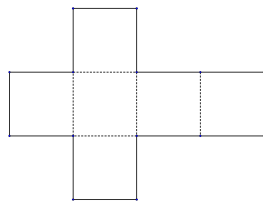
10. The net to the right forms a cube.



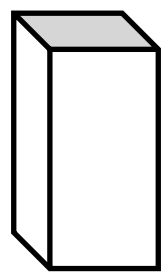
- a. If each square face of the cube has a side length of 4.2 cm, what is the surface area of the cube?
- b. If each square face of the cube has a length of 4.2 cm, what is the volume of the cube?
- c. If you created a new cube with edge lengths that are 150% of the original cube (edge lengths of 4.2 cm), what would the new surface area and volume be?
- d. If you created a new cube with edge lengths that are 70% of the original cube (edge lengths of 4.2 cm), what would the new surface area and volume be?

## 8.2c Homework: Nets of 3D Objects

1. Which of the following nets make a cube?

|   |  |   |
|---|--|---|
| <p>a.</p>  | <p>b.</p>   | <p>c.</p>  |
| <p>d.</p>  | <p>e.</p>  | <p>f.</p>  |

2. Draw a net for the square-based rectangular prism to the right:

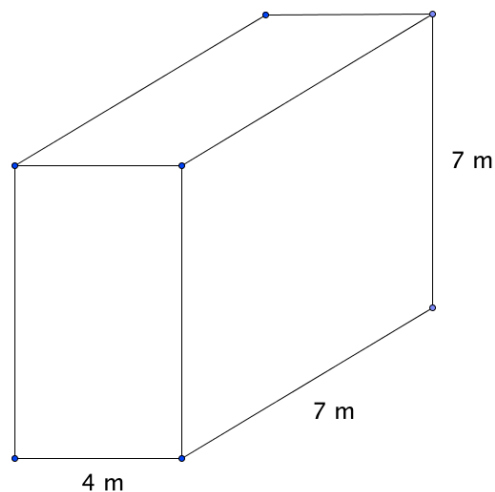
|  |  |
|--|--|
|  |  |
|--|--|

3. Suppose the prism in # 2 above has base edges of  $8 \times 8 \times 12$  inches. Find the surface area and volume of the prism. Show all your work.
  
4. Still using the prism above, find the new dimensions of the prism if you scaled it by  $\frac{3}{2}$ . Then find the new surface area and volume.
  
5. Still using the prism above, find the new dimensions of the prism if you scale it by  $\frac{2}{3}$ . Then find the new surface area and volume.

## Spiral Review

1. Solve:
  - a.  $3 - 9$
  - b.  $47 + (-65)$
  
2. On a map of New York City, with a scale of 1 inch =  $\frac{1}{2}$  mile, Central Park is 1 inch wide and 5 inches long. What is the area of the park?

3. Find the surface area and volume of the following rectangular prism:



4. Find the additive inverse and multiplicative inverse of each of the following numbers:

| Number         | Additive Inverse | Multiplicative Inverse |
|----------------|------------------|------------------------|
| 2              |                  |                        |
| $\frac{17}{7}$ |                  |                        |
| -2.15          |                  |                        |

5. Given the following table, find the indicated unit rate:

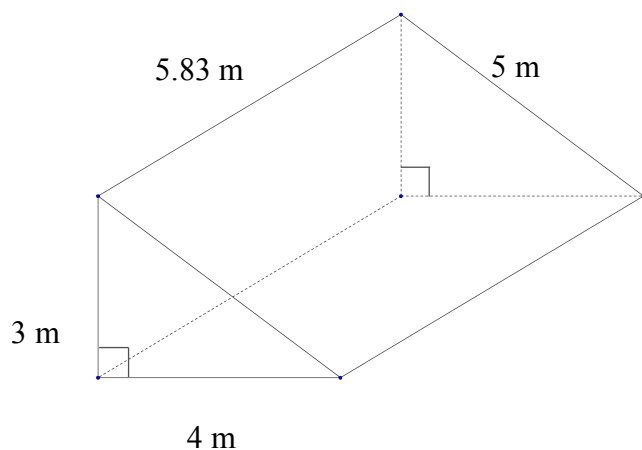
| Days | Total Push-ups |
|------|----------------|
| 2    | 30             |
| 4    | 60             |
| 29   | 435            |

\_\_\_\_\_ push-ups per day

## 8.2c Additional Practice: Surface Area and Volume



1. Explain your procedure for finding surface area.
2. Explain your procedure for finding volume.
3. Find the surface area and volume of the right prism below. Note that the triangular bases are right triangles with the right angle occurring where the 4m and 3m sides meet.



Find the surface areas and volume for each. You may wish to sketch the figure. Show work. Include units.

| Shape and Dimensions   | Surface Area | Volume |
|--|--------------|--------|
| 4. Rectangular prism: <ul style="list-style-type: none"> <li>• length 10 in</li> <li>• width 8 in</li> <li>• height 5 in.</li> </ul>       |              |        |
| 5. Rectangular prism: <ul style="list-style-type: none"> <li>• length 4 feet</li> <li>• width 2 feet</li> <li>• height 6.2 feet</li> </ul> |              |        |

6. If the base area of a right rectangular prism is  $28 \text{ cm}^2$  and the volume is  $177.8 \text{ cm}^3$ , what is the height of the prism?
7. If the height of a square based prism is 13 in and its volume is  $637 \text{ in}^3$ , what is the length of each side of the base?
8. Give three possible edge lengths for a prism of volume  $96 \text{ m}^3$ .

## 8.2d Class Activity: Growing and Shrinking Stuff

For each of the following problems, find the necessary measurements to answer all the questions.

1. A cube with side lengths of 8 centimeters is enlarged so its side lengths are now 24 centimeters.
  - a. By what scale factor did the side lengths increase? Show your work or justify your answer.
  
  - b. By what scale factor will the surface area increase? Show your work or justify your answer.
  
  - c. By what scale factor did the volume increase? Show your work or justify your answer.
  
2. A rectangular prism with side lengths of 5 yards, 10 yards, and 15 yards is reduced by a scale factor resulting in new sides of 1 yd, 2 yd, and 3 yd.
  - a. What was the scale factor for the reduction?
  
  - b. What would be the new surface area and volume for the prism?
  
  - c. By what scale factor did the surface area and volume each change?
  
3. A rectangular prism with side lengths 2 mm, 3 mm, and 5 mm is enlarged by some scale factor. The new volume of the prism is  $240 \text{ mm}^3$ .
  - a. By what scale factor were the side lengths increased?
  
  
  
  
  
  
  
  
  
  
  - b. What is the new length of each of the sides?
  
  
  
  
  
  
  
  
  
  
  - c. What is the new surface area of the prism?

## 8.2d Homework: Growing and Shrinking Stuff

For each of the following problems, find the necessary measurements to answer all the questions.

1. A mini cereal box has the following dimensions: 4.5 in by 6 in by 2 in.
  - a. If all the dimensions are doubled, will it require double the amount of cardboard to make the box? Why or why not?
  - b. If all the dimensions are doubled, will it hold double the amount of cereal? Why or why not?
  - c. If *one* of the dimensions is doubled, will it require double the amount of cardboard to make the box? Why or why not?
  - d. If one of the dimensions is doubled, will it hold double the amount of cereal? Why or why not?
  
2. A container of chocolate milk mix has the following dimensions: a square base with sides of 6 in. and height 9 in.
  - a. If all the dimensions are reduced by a scale factor of  $\frac{1}{3}$ , will it require a third the amount of materials to make the container? Why or why not?
  - b. If all the dimensions are reduced by a scale factor of  $\frac{1}{3}$ , will it hold a third the amount of chocolate milk mix? Why or why not?
  - c. If the length of each side of the base is reduced by a scale factor of  $\frac{1}{3}$ , how much surface area and volume will it now have?
  - d. What is the percent change of surface area and volume for “c”?



3. Refer back to the mini cereal box with dimensions, 4.5'' by 6'' by 2''.
  - a. If all the dimensions are increased by 4 inches to: 8.5'' by 10'' by 6'' how does this change the amount of cardboard needed to make the box?
  
  
  
  
  
  
  
  
  
  
  - b. If all the dimensions are increased by a value of 4 inches, as described above, how will this change the amount of cereal the box can hold?

### Spiral Review

1. Mrs. Zamora will not tell you how the class in general did on the last test. You really want to know how you compare, so you survey 10 random students out of 35. Estimate the average score for your class and describe how far off the estimate might be.

|     |    |    |     |    |
|-----|----|----|-----|----|
| 77  | 89 | 79 | 100 | 94 |
| 100 | 80 | 81 | 88  | 87 |

2. Draw and describe the plane section that results from the following cut: a cylinder cut parallel to the base.
  
  
  
  
  
  
  
  
  
  
3. Katherine is visiting patients in a hospital. She visits 9 patients in 3 hours. She visits 18 patients in 6 hours. Is this situation proportional? If so, what is the unit rate?
  
  
  
  
  
  
  
  
  
  
4. A candidate for Congress is trying to decide what to focus her campaign on in order to win the election. Describe how she might obtain a random sample to know what issues are important to her constituents.
  
  
  
  
  
  
  
  
  
  
5. Kurt puts 70% of his earnings into his savings. Write and solve an equation to find how much money he earned if he had \$165 to spend.

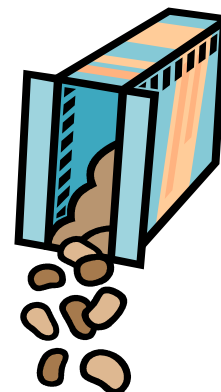
## 8.2e Project: Packing Packages

Have you ever notice that cereal generally comes in tall thin boxes and that laundry soap generally comes in short wide boxes? Why do you think they come as they do?

Think about your experiences with perimeter, surface area, and with volume in this chapter. What kind of box do you predict might hold the most and take the least amount of cardboard? Explain your thinking.

Suppose your favorite cereal comes in a box that is 24 cm. high, 20 cm. long, and 6 cm. wide. This box of cereal costs \$4.35.

1. Draw a model of the box. Find the surface area and volume for the box. Show work and label answers



The  $24\text{ cm} \times 20\text{ cm} \times 6\text{ cm}$  dimensions of the box can also be thought of as a height of 24 cm and a girth (distance around the box) of 52 cm. If we add these two measures (height and girth), we get 76 cm.

For this project you will:

- a. Draw a model of a box with a total girth plus height of no more than 76 cm that holds the most cereal possible with the least surface area possible.
- b. Build the box as described in “a”.
- c. Provide a table, graph or spreadsheet to show how you arrived at your dimensions (remember: use only height and girth for your table, graph, or spreadsheet.)
- d. Find the percent decrease in surface area for your new box from the original box.
- e. Find the percent increase in volume for your new box from the original box.
- f. Write two paragraphs about your project. In the first paragraph, state and justify how much you would charge for cereal in this box. In the second paragraph, explain why you think cereal does not come in the type of box you designed.

## 8.2f Self-Assessment: Section 8.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

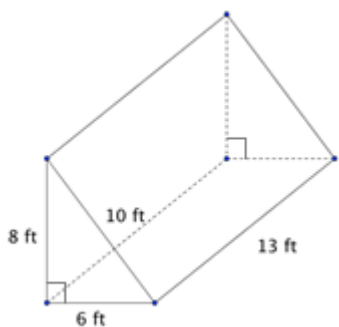
| Skill/Concept  | Beginning Understanding  | Developing Skill and Understanding   | Practical Skill and Understanding  | Deep Understanding, Skill Mastery  |
|--|--|--|--|--|
| 1. Describe the type of plane sections of right prisms and pyramids that result from different cuts.   | I struggle to know or describe the type of plane section that will result from different cuts of right prisms and pyramids regardless of how I try to do it. | I know and can describe the type of plane section that will result from different cuts of right prisms and pyramids if I'm able to use objects or apps to make cuts. | I know and can describe the type of plane section that will result from different cuts of right prisms and pyramids with and without manipulatives/apps. | I know and can describe the type of plane section that will result from different cuts of right prisms and pyramids. I can explain how the plane section varies with different cuts. |
| 2. Solve real world and mathematical problems involving volume and surface area of three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. | I struggle to solve real world problems involving volume and surface area.   | I can find volume and surface area of three-dimensional objects in context with some help.   | I can solve real world and mathematical problems involving volume and surface area of three-dimensional objects.   | I can solve real world and mathematical problems involving volume and surface area of three-dimensional objects, explain how I got my answer and why it is correct.                  |
| 3. Find the scale factor and/or percent change in surface area and/or volume when dimensions are changed.  | I struggle to find scale factor and/or percent change between measurements.  | I can find scale factor and/or percent change between measurements with some help.   | I can find scale factor and/or percent change between measurements.  | I can find scale factor and/or percent change between measurements, explain my procedure and why my answer is correct.   |

## Sample Problems for Section 8.2

1. Draw and describe the plane section that results from the following cuts.
  - a. A right rectangular prism is cut parallel to the base.
  - b. A cube is cut perpendicular to the base and parallel to a face.
  - c. A right pyramid is cut parallel to the base.
  - d. Describe how each cut above is similar and different from other similar cuts for the same object.

2. Answer each question about volume and surface area.

- a. Find the volume and surface area of the triangular prism below.



- b. Find the volume and surface area of a right rectangular prism with dimensions 8 cm by 15 cm by 2 cm.
  - c. If a right rectangular prism with dimensions 8 cm by 15 cm by 2 cm is scaled by a factor of 3, what would the new volume and surface area be?
  - d. What is the length of a box with a surface area of  $472 \text{ cm}^2$  and a width of 11 cm and a height of 6 cm?
  - e. Ellie is filling a sandbox for her son. The sandbox dimensions are 4 ft by 4 ft by 1 ft. How many cubic feet of sand does she need to buy to fill the sandbox?
3. The dimensions of Hans' turtle aquarium are 18 in by 18 in by 12 in. The dimensions of Anna's rabbit cage are all doubled.
    - a. Find the surface area and volume of each animal habitat.
    - b. Find the scale factor between the surface areas of each container.
    - c. Find the scale factor between the volumes of each container.
    - d. In general, how are the scale factors of surface areas related? Volumes related?

## **Play Dough Recipe**

### Ingredients:

- 2 C flour
- 2 C warm water (food coloring is optional)
- 1 C salt
- 2 T oil
- 1 T cream of tartar

### Directions:

In a medium pot, add water and oil together and stir. Then add the flour, salt, and cream of tartar. Keep stirring until all the ingredients are blended together and the mixture is not sticking to the sides of the pot. Then knead the mixture. For storage, keep play dough in plastic bags or sealed containers.