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Chapter 8: Geometry Part 2: Measurement in 2- and 3-Dimensions, Plane Sections of Solids (2 weeks)

UTAH CORE Standard(s)

Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. 7.G. 3

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

2. Know the formulas for the area and circumference of a circle and use them to solve problems. 7.G.4
3. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. 7.G.6

Chapter Summary:

Throughout this chapter students develop and explore ideas in geometry around measures in one-, two-, and three-dimensions. Additionally, students will tie concepts learned previously in 6th and 7th grade to content in this chapter. Section 1 begins with a brief review and practice with perimeter and area of polygonal figures. Students then use their knowledge of area and perimeter to solve real-world problems, find the area of non-standard shapes and review ideas around percent increase/decrease and scale factor, and connect their understanding of one- and two-dimensional measures to adding and multiplying algebraic expressions by finding the perimeter and area of figures with variable expression side lengths.

Section 2 starts with students exploring plane sections of 3-D solids. Exercises in this section are designed to help students develop an intuitive understanding of dimension (building up and cutting down). Students will notice that plane sections of 3-D objects are affected by: a) the type of solid with which one starts, e.g. right prism versus a pyramid and b) the angle of the cut to the base and/or other faces. This observation should help students understand when and why parallel (and/or perpendicular) cuts to a specific face are needed to create uniform cross sections, and then by extension, uniform cubed units for finding volume. In other words, the study of plane sections here is to help students develop an understanding of the structure of a solid and procedures for finding volumes. Throughout the exercises, students should develop a stronger understanding of units of measure for one-, two-, and three-dimensions. Towards the end of this section students review the use of nets (a concept from 6th grade) to find surface area of prisms and cylinders and then to differentiate this measure from volume, which they will also find.

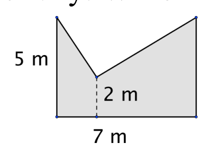
VOCABULARY: area, axis of symmetry, circle, circumference, cross-section, cube, cylinder, edges, equidistant, face, plane section, perimeter, polygon, polyhedron, prism, pyramid, quadrilateral, rectangle, square, surface area, symmetry, trapezoid, triangle, volume, vertices.

CONNECTIONS TO CONTENT:

Prior Knowledge: In 6th grade students found areas of special quadrilaterals and triangles. They also found areas of other objects by decomposing them into rectangles and triangles. In Chapter 5 of this text, students extended that understanding to finding the area of a circle. Also in 6th grade, students found volume of rectangular prisms and their surface areas by using nets. During their study of geometry in 6th grade, students should have learned that the height and base of an object are always perpendicular to each other. They will build on this understanding as they apply their knowledge of area and volume to real life contexts and as they explore cross sections and plane sections.

Future Knowledge: In 8th grade, students will continue working with volume, formalizing algorithms for volume of cylinders and adding methods for finding the volume of cones and spheres. Students explore cross sections of objects in 7th grade to understand how dimensions are related to each other and the algorithms for surface area and volume. Further, ideas developed through cross section activities are foundational in the study of calculus. Lastly, ideas about planar sections will be extended in secondary mathematics when students explore Cavalieri's Principle.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

	Make sense of problems and persevere in solving them.	Students will be given a variety of contextual problems with which they will need to make sense and persevere. For example, 8.0 Anchor Problem: The city is looking to build a new swimming pool in city park. In their city council meeting, they have determined that they want the pool to hold no more than 2500 m^3 of water, or it will cost too much to keep it filled. Help the city council to choose a design for the swimming pool. Design 3 different swimming pools that will each hold at least 2000 m^3 but no more than 2500 m^3 of water.
	Reason abstractly and quantitatively.	Students will use their understanding of area and volume to reason in a variety of contexts such as the 8.1e Class Activity: Write an expression to find the  area. If possible, find the exact area.
	Construct viable arguments and critique the reasoning of others.	Students will apply their understanding of perimeter and area to construct and critique arguments. 8.1g Class Activity: *Mike, Juliana and Joe were working together to make a garden larger. Mike said, "We have to buy more fencing because if we increase the area of the garden we will need more fencing to go around."* *Juliana had a different opinion. "That's not true," she said. "We can use the same amount of fencing and move it to make the area of the garden larger."* *Joe disagreed with both Mike and Juliana. He said, "I know a way that we can make the garden larger and use less fencing.* *Who is right?*
	Model with Mathematics.	Students will use models to explore concepts in geometry such as using play-dough and string to create cross sections of prisms. Throughout the chapter, they will connect models to algorithms.
	Attend to Precision.	Careful attention should be paid to explanations and units throughout this chapter. Students will be expected to attend to several ideas at the same time. Students should attend to precision as they explain ideas throughout this chapter. For example, when discussing cross sections, students should use precise language in describing the angle of cuts to the base, faces and/or edges.
	Look for and make use of structure.	Students will connect ideas of one-, two-, and three-dimensional measures to simplifying numeric and algebraic expressions, including this example from the 8.2 d Homework: Determine which expression(s) will give the surface area or volume for a 3-D object. a. $2(6+2+3)$ b. $3\cdot2\cdot6$ c. $(2\cdot6+2\cdot3+3\cdot6)2$ d. $2\cdot3\cdot2+2\cdot3\cdot6+2\cdot($

	Use appropriate tools strategically.	Students will use a variety of tools in this chapter including play dough, rulers, graph paper, and calculators. Encourage students to make sense of ideas with tools.
	Look for and express regularity in repeated reasoning.	Students will note in this chapter that length is one-dimensional, area is two-dimensional and requires 2 length (2 one-dimensional measures) that are perpendicular, and volume is three dimensional and requires 3 lengths that are each perpendicular to each other.

8.0 Anchor Problem; Designing a Swimming Pool:

The city is looking to build a new swimming pool at the city park. In their city council meeting, they determined that they want the pool to hold no more than 2500 m^3 of water in order to conserve water.

A. Help the city council to choose a design for the swimming pool. Design three different swimming pools that each hold at least 2000 m^3 but no more than 2500 m^3 of water.
B. Choose your favorite of the three designs from part A. Find new measures for the pool so that: a) it holds twice as much water as your original design and b) it remains the same overall shape (but not size) as the original.

A) Allow students to experiment with this problem. You may need to briefly review how to find perimeter, area and volume. Recall that perimeter and area were discussed in Chapter 5. This chapter will connect ideas there to volume and continue to explore scale factor.

Note that students are not limited to a rectangular prism design. They will need to think about how to compute volumes for irregular shapes. Help them extend their understanding of how to find the area for different figures to finding volume for different figures.

Have students draw their ideas for pools and label dimensions. It will be useful to hang these in your room and refer back to them as you explore ideas of dimensions and scale factor throughout the chapter.

Answers will vary, but must fit the following inequality: $2000 \le xyz \le 2500$.

Some examples of possible sizes are:

- 20 m by 10 m by 10 m
- 25 m by 10 m by 10 m
- 20 m by 25 m by 5 m
- 30 m by 10 m by 8 m
- 8 m by 18 m by 15 m
- 2 m by 12 m by 100 m
- 3 m by 8 m by 100 m

For students that struggle to understand concepts in volume, use unit cubes to model ideas. For example, you might lay out a rectangle of 20×10 cubes. Ask students what the volume is (20×10×1). Students should see that the volume is 200. Then lay another 20×10 layer so that students can see that an addition of 1 unit height results in a volume increase of 200 (20×10×2).

Students should have done similar activities in 6th grade. Help students connect those ideas with targeting a desired volume.

B) Again, let students experiment with this question. They will likely try to first double the lengths of all the sides of their favorite pool configuration. However, this will result in a volume 800% (8 fold) of the original. Ask students why that happened. They should recall from chapter 5 that doubling a side measure will double the perimeter (one dimensional measure) but quadruple the area (two dimensional measure; two factors of 2 e.g. 2×2). So for volume, doubling the length of each side results in an 8-fold increase in volume because the effect is 2×2×2.

Another error students are likely to make is wanting to add a specific length to each side. For example, if one starts with a 20×10×10 meter pool of volume 2000 m^3 and adds 3.146 to each side, 23.146 × 13.146 × 13.146, the new volume will be about 4000.03 m^3. While volume certainly doubled, the new pool is not the same shape as the original (the sides are not proportional to the original). You may need to review concepts from chapter 5 with students.

In order to double the volume while maintaining the same size and shape of the pool, the scale factor will need to be the cube root of 2 (1.259921…). Students will study irrational numbers in 8th grade. That is not the point of this exercise. For now, they might use a calculator to estimate the cubed root of 2. Have students use a trial and error approach to find the number. Once they discover an approximate value, they can multiply each side of their favorite pool by 1.25992. That will scale up their pool and result in a volume twice their original. Note: the perimeter's length is ≈126% the original (a scale factor of 1.25991), the new area is ≈159% the original (a scale factor of 1.5874) and the new volume is ≈200% the original (a scale factor of 2.)

Note: it may not occur to your class to look for the cube root of 2 for the scale factor. Rather, they may just try to find the scale factor by trial and error. If they attack it that way, let them. When they come up with 1.2599, discuss with them how this is related to what they learned in chapter 5 and then show them that cubing that number results in 2 and how this is related to ideas in chapter 5.

Section 8.1: Measurement in Two Dimensions

Section Overview:

This section involves a review and extension of previously learned skills (from 6th grade) involving perimeter and area of plane figures and volumes of right solids. The section begins with a review of perimeter and area and then moves to finding area of irregular figures, first with numeric side lengths and then with variable expression side length. Additionally, students continue work on skills developed earlier in the year including percent increase/decrease, scale factor for perimeter and area, and simplifying algebraic expressions. The end of the section focuses on all these skill in various real-world and mathematical contexts.

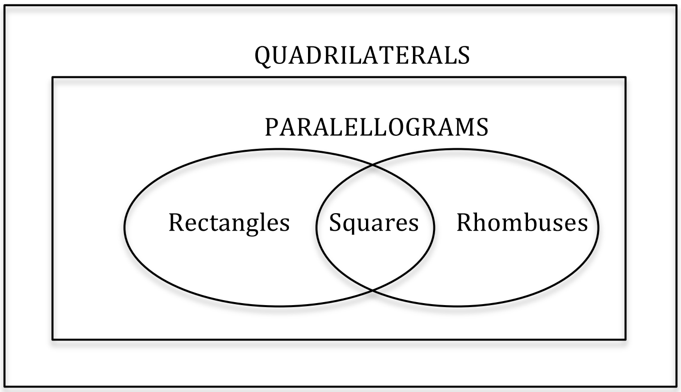
Concepts and Skills to be Mastered (from standards)

1. Know the formula for the area and circumference of a circle and use them to solve problems.
2. Give an informal derivation of the relationship between area and circumference of a circle.
3. Solve real-world and mathematical problems involving area, volume, and surface are of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

   Area of rectangles, squares, and triangles should be a review for students. Be careful not to spend too much time re-teaching, just use the opening sections to resurface their knowledge of area.

8.1a Class Activity: Differentiate Area and Perimeter

REVIEW:

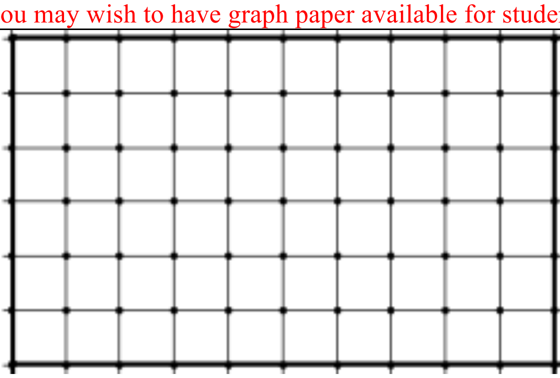
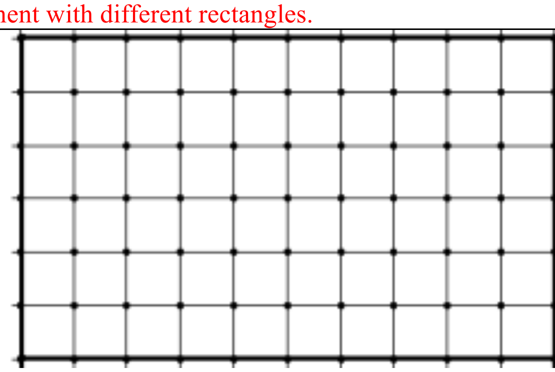
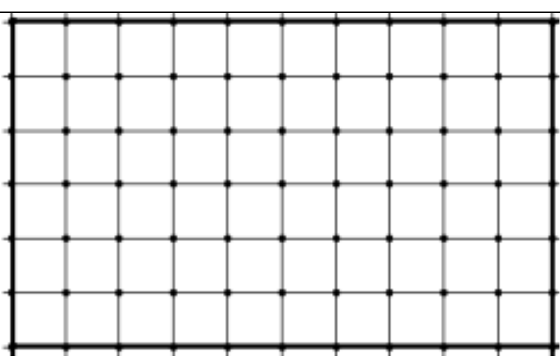
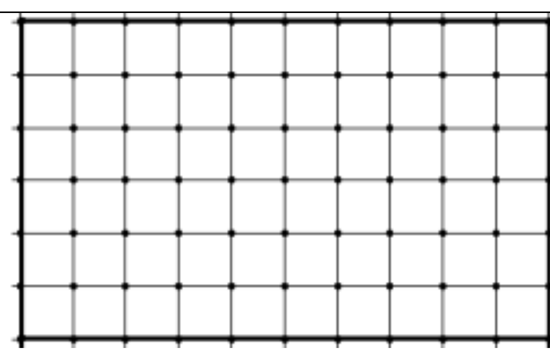


Several ideas should be discussed as student engage in the "riddles":

- Quadrilaterals can be defined by sides, angles or a combination.
- Logic: Are all squares rectangles? Yes—why? Are all rectangles squares? No—why? etc.
- Relationship between perimeter and area:
  - Two rectangles with the same perimeter can have different areas.
  - Discuss similarities and differences in how each is found ("counting" linear units v counting square units).
- What happens when side lengths are not whole units? Connect ideas here to fraction arithmetic.

1. Rectangle Riddle #1: Can you figure out the dimensions for each rectangle? Use the clues below to draw the four different rectangles described.
   - Rectangle A has a perimeter equal to that of Rectangle B.
   - Rectangle A is a square.
   - Rectangle B has an area of 24 square units. It is as close to a square as possible for that area if the side lengths are ***whole numbers***.
   - Rectangle C has a perimeter equal to the measure of the area of Rectangle B. Rectangles C's length is 3 times the width.
   - Rectangle D has two different odd integers as length and width; each is 1 unit greater than the length and width of Rectangle B.

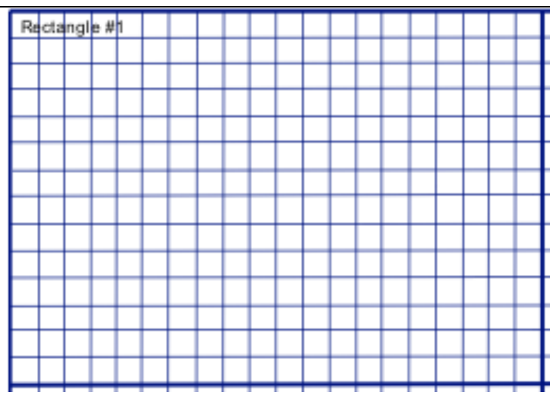
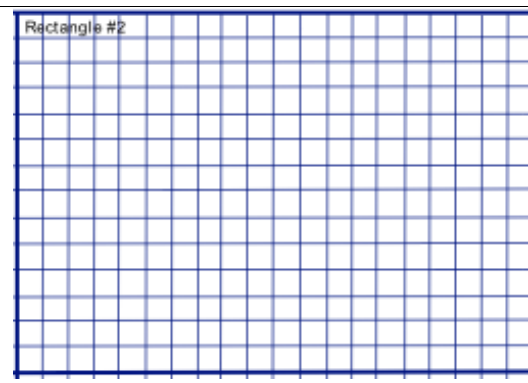
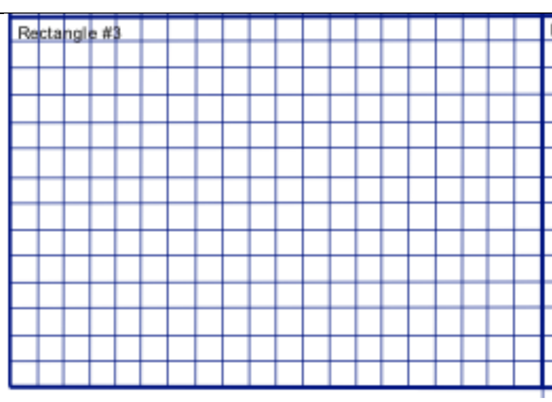
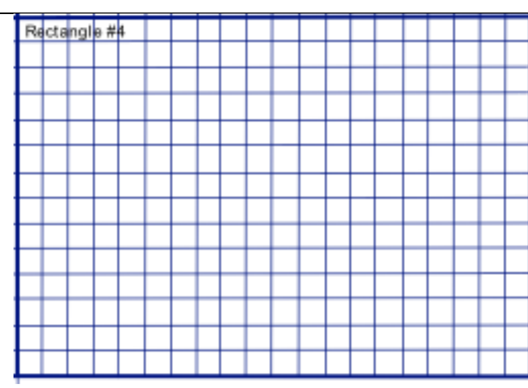
Note: You may wish to have graph paper available for students to experiment with different rectangles.

A	 5 by 5	B	 6 by 4
C	 9 by 3	D	 7 by 5

Use words and models to explain the difference between perimeter and area. Answers may vary. Perimeter is a one-dimensional (length) unit of measure and area is a two-dimensional (square) unit of measure. Stress to students that for area, length and width are perpendicular to each other.

2. Rectangle Riddle #2: Can you figure out the dimensions for each rectangle? Use the clues below to draw the four different rectangles described.

- The numeric measure of the area of Rectangle #1 is twice the number representing the perimeter. Rectangle #1 is 8 units wide.
- Rectangle #2 has the same number representing the area and the perimeter; the perimeter is 1/2 the perimeter of Rectangle #1.
- Rectangle #3 has the same length as Rectangle #1. The area of Rectangle #3 is equal to the difference between the areas of Rectangles #1 and #2.
- Rectangle #4 has an area 6 square units less than the area of Rectangle #3 and a width 2 units more than the width of Rectangle #2.

A	 8 by 8	B	 4 by 4
C	 6 by 8	D	 6 by 7

3. What helped you most to accomplish "Rectangle Riddle #2"?

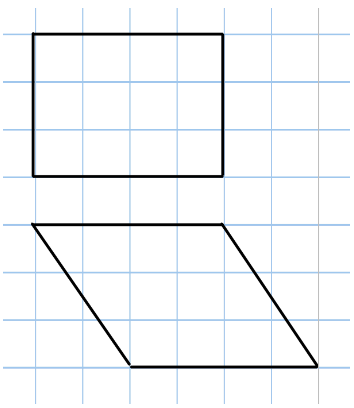
For questions #4-6 use a piece of graph paper to answer each:

4. List as many rectangles as you can that have an area of 36?

Answers will vary, but the product of the length and width need to result in 36 square units:
Students will likely quickly note: 1 unit by 36 units, 2 units by 18 units, 3 units by 12 units, 4 units by 9 units, 6 units by 6 units. Encourage them to find dimensions that are not whole units: (1/2) unit by 72 units, 1.5 units by 24 units, 2.25 units by 16, etc. This is a good opportunity to review fraction arithmetic. You may also choose to connect ideas here to solving equations, e.g. $(5/3)x = 36$, find x. For honors students, you might explore $xy=36$, noting neither x nor y can ever equal 0 and that as $x \rightarrow \infty$, $y \rightarrow 0$ and as $y \rightarrow \infty$, $x \rightarrow 0$. Further, you might discuss domain here—the lengths must be positive values.

5. DRAW two different parallelograms that have the same area. State their base and height.

Answers will vary, but must satisfy the statement above. For each pair, the base and height must be the same.



An example is to the left (not shown in the student workbook.) This would be a good time to review again how area and perimeter are different types of measures and that holding one constant does not force the other to remain constant. Also review vocabulary with students: "height" and "base." Point out that these measure must be perpendicular to each other and why. Further, discuss orientation with students, e.g. translate figures and discuss choices for height and base. Lastly, discuss with student that area is a measure of two dimensions, while perimeter is a measure of one dimension. These ideas will resurface when students explore cross sections in section 2.

6. DRAW two different triangles that have an area of 12. State their base and height.

Answers will vary. Discuss with students that there are many ways this might be achieved. All answers will satisfy the equation $0.5(b)(h) = 12$. Again, encourage students to find non-whole number lengths for the base and/or height. See #5 for extension ideas.

Ask students to write in one or two sentences how they were able to answer #6 and #7.

Vocabulary note: "side" refers to a two-dimensional figure, while "face" and "edge" refer to a three-dimensional figure.

7. Suppose you start with a rectangle that has a base length of 6 and height of 4. If you triple the length of the base, what do you have to do to the height to have a new rectangle of the same area?

Divide the height by 3. Explore this idea with students on a grid as well as arithmetically.
On the following page is an activity you might use as a take home group problem or as a class activity or class discussion.
The homework for this section may be challenging. You might want to help students get started.

Activity:

Mike, Juliana and Joe were working together on a garden area and perimeter problem their teacher gave them. The problem is to make the area of a garden larger without having to buy more fencing (increasing the perimeter.) Their task is to figure out if that's possible.

Mike said, "We have to buy more fencing because if we increase the area of the garden we will need more fencing to go around."
Mike's statement is not true. For example, a fence of length 100 feet can enclose a garden that is 1 foot by 49 feet; the area would then be only 49 feet squared. However, the same amount of fencing could be altered to contain a 10 foot by 40 foot garden; now the area is 400 feet squared. Same perimeter, different area.
Juliana had a different opinion. "That's not true," she said, "We can use the same amount of fencing and move it to make the area of the garden larger."

See above; this statement is entirely true.
Joe disagreed with both Mike and Juliana. He said, "I know a way that we can make the garden larger and use less fencing.
There is truth in this statement. For example if they had 100 feet of fencing and their original garden was 1 foot by 49 feet the garden would have an area of 49 sq. ft., then using 90 feet of fencing to enclose a 5 foot by 40 foot garden would result in an area of 200 sq. ft. which is larger than the original 49 square foot area. The maximum area 100 feet of fencing might enclose is 625 square feet (a 25 foot by 25 foot garden) IF we limit the shape of the garden to a rectangle. We can get even more area from regular polygons with more sides and the most with a circular garden.
Who is right?
Note: students will have to choose length of fencing with which to experiment.

A. Group response: Use graph paper and string to help you think about the problem.
   a. Decide who you think is right (Mike, Juliana or Joe.)
   b. Come up with a possible answer with your group.
   c. Prepare your group explanation and presentation.
   d. Be certain to justify your conclusion with reasoning and calculation.

Some students may ask if they are limited to rectangles, see above teacher comments. You might start limiting to rectangles and then open it up to other plane figures.

B. Challenge: Examine the Garden Problem using a spreadsheet. Consider all possible rectangles with perimeters of 36 units. How does the area change as you change the configuration of the perimeter?

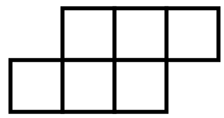
   Suggest that students:
   a. Name the variables in the columns (perimeter, width, length, area).
   b. Look for a pattern when they fill in their spreadsheet.
   c. Create graphs (charts) to get a visual picture of what's happening to area as width and length vary.
   d. Ask them: What do you know about the garden problem from looking at the table and the graphs?

8.1a Homework: Perimeter and Area Problem Solving

Answer each of the questions below.

1. What is the least number of tiles you can add to the figure below to create a shape with a perimeter of 16? Note: When adding a tile, the new tile must share at least one side with the original shape; each tile is 1 unit by 1 unit.
   Least number of tiles is 2

You might suggest to students to draw the figure on a separate sheet of paper and then cut out "tiles" to manipulate.



2. Use the original figure given above to answer a-d. Draw your answers. Note, an answer of just "yes" or "no" is not sufficient. Use pictures or words to justify your answer.

   a. Can you add a tile to this figure to increase the perimeter by 1? If so, how?

No, no matter how you add a tile, the perimeter either does not change or increases by two units.

   b. Can you add a tile to this figure to increase the perimeter by 2? If so, how?

Yes. Notice though that adding a tile to the "missing" corners does not result in an increase in the perimeter. Ask students to explain why this is true. Also ask them why adding a tile to the left most or right most side did not increase the perimeter by 3.

   c. Can you add a tile to this figure to increase the perimeter by 3? If so, how?

No, this is not possible. Ask them to explain in their own words why this is true.

   d. Can you add a tile to this figure so that the perimeter doesn't change? If so, how?

Yes, if we add a tile to either corner.

3. Can you make more than one shape with the same perimeter, but different areas? Show your ideas with grid paper.

Yes, answers may vary; i.e.: add 2 tiles to the corners.

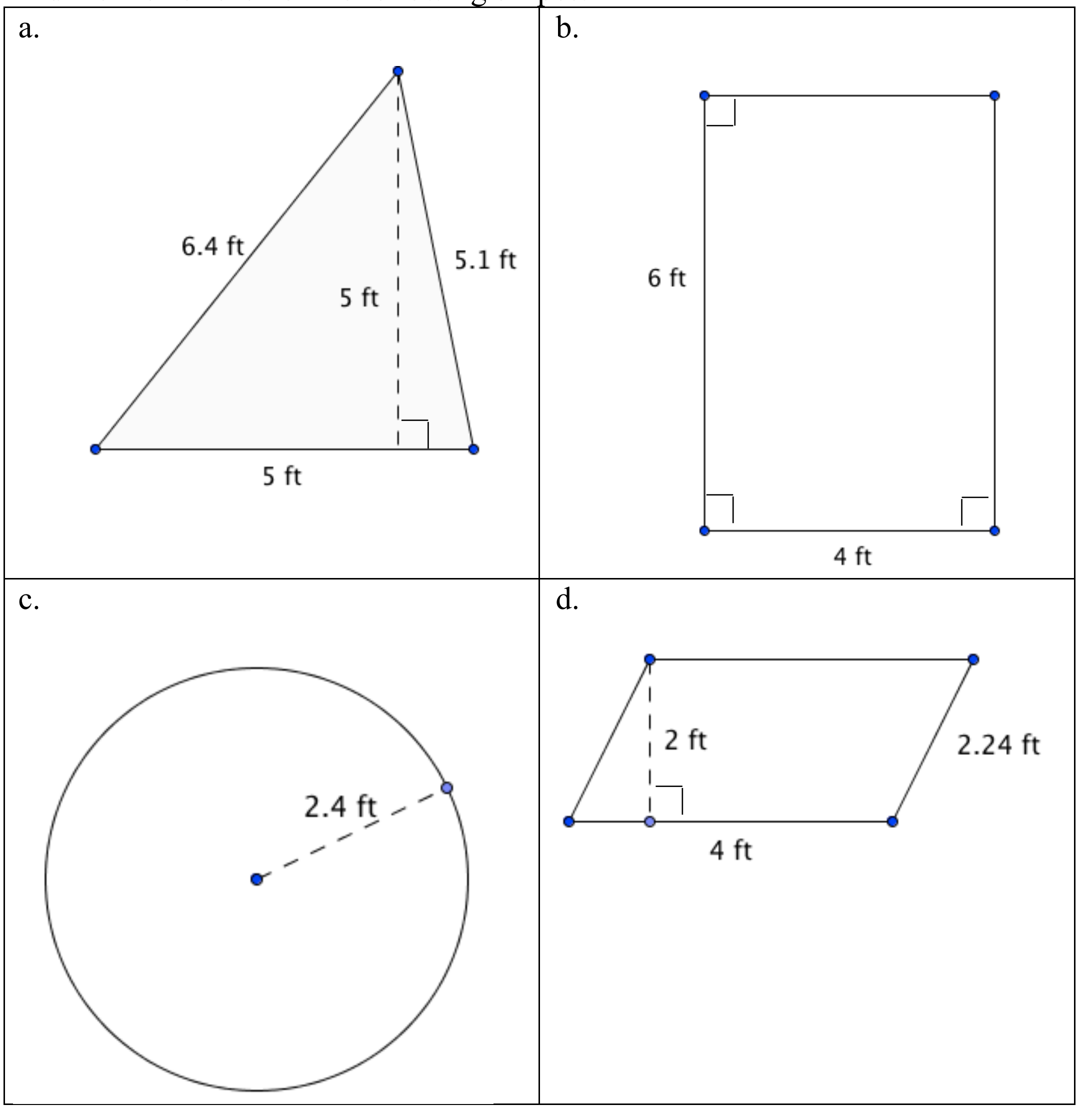
4. Can you make more than one shape with the same area, but different perimeters? Show your ideas with grid paper.

Yes, answers may vary; i.e.: shift the top 3 tiles so that they can line up with the bottom 3 tiles.

5. If you pick any whole number between 12 and 24, use that many unit tiles, can you make a shape where the area and perimeter are equal? Show your ideas.

Spiral Review

1. Find the area of each of the following shapes:



2. Solve: $-8 < -3m + 10$ $m < 6$

3. Solve: $-12 > 3x$ $x < -4$

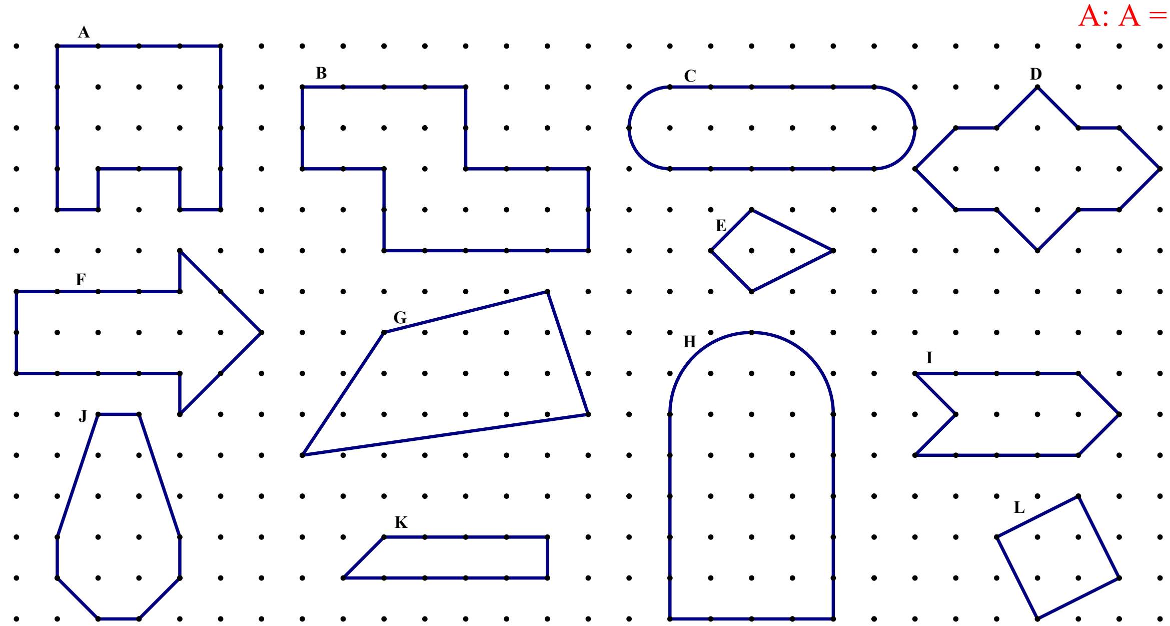
4. A website says that the odds of Mr. and Mrs. Durrand having a baby with blue or hazel eyes is 50:50. Describe a simulation that models the color of eyes their baby will have.
Answers may vary. Possible simulations include tossing a coin or rolling a fair die (assigning 1-3 to one color 4-6 to the other; or odds to one color even to the other color).

5. Write $\frac{3}{5}$ as a percent and decimal. 60% 0.6

8.1b Class Activity: Areas of Irregular Shapes

The following shapes have been drawn on square dot paper. The distance between each dot represents one unit. Use what you have learned about area to find the area of each shape (A-L).

A: A =

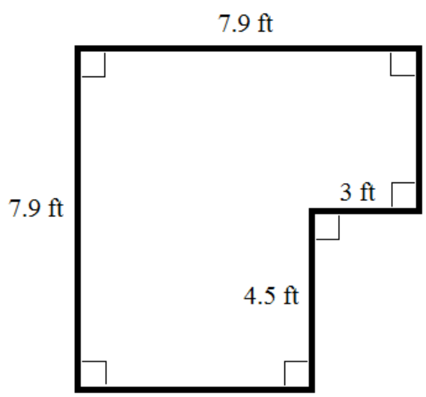


14 sq. units   B: A = 18 sq. units   C: A = 13.14 sq. units   D: A = 12 sq. units
E: A = 3 sq. units   F: A = 12 sq. units   G: A = 16 sq. units   H: A = 26.28 sq. units
I: A = 8 sq. units   J: A = 11 sq. units   K: A = 4.5 sq. units   L: A = 5 sq. units

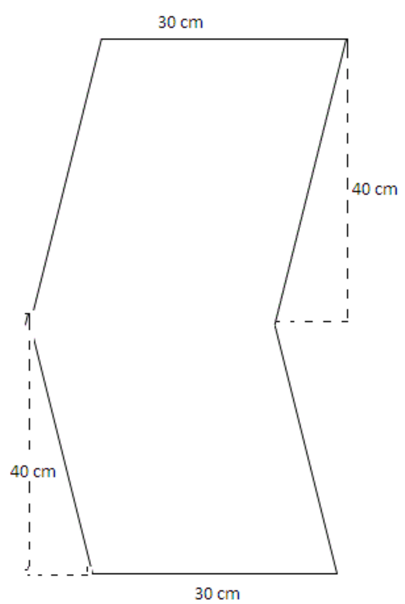
Allow students ample time to find the area of each. You might want to review with students that they can "count" squares, "cut" an object into smaller parts and then add all the parts together, or find the area of a larger shape and then subtract parts to get to the desired shape. Have students present their solutions to the class.

Use your knowledge of area and the problems that you completed above to find the area of the following irregular shapes.

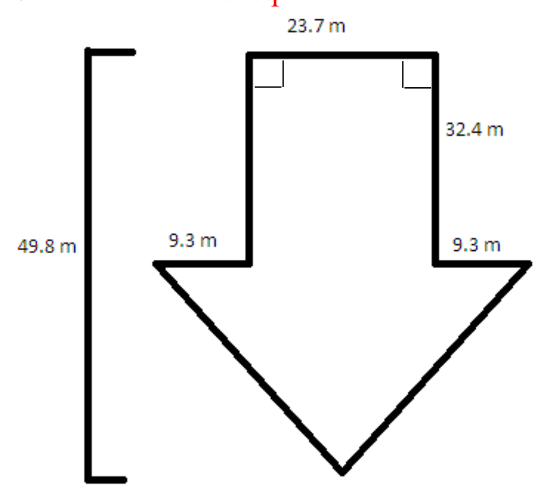
M   A = 48.91 sq. ft



N   A = 2400 sq. cm

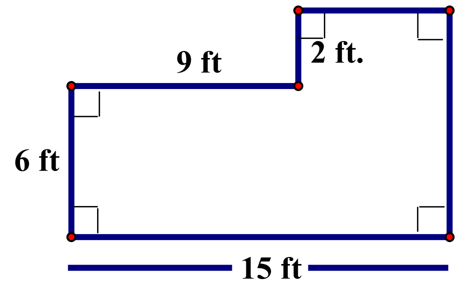


O   A = 1135.89 sq. m



1 a. Johona is building a deck off the back of her house. To the right is a sketch of it. She will need to have a full concrete foundation below the deck. Find the surface area of the concrete foundation.

A = 102 sq. ft



b. Suppose Johona wants to build her deck onto a concrete foundation that is 1.5 ft. thick and has the same surface area as the deck. How many cubic feet of concrete will she need?

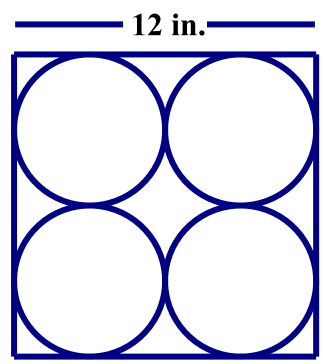
$V = 102 \times 1.5 = 153 \text{ ft}^3$

c. How many cubic yards will she need?

$153/27 = 5\ 2/3 \text{ yd}^3$ To help students understand why they need to divide by 27, you might create a 3×3×3 cube with unit blocks and help students connect the fact that 3 ft = 1 yard to this situation.

2. a. Hugo is making the tile pattern shown. The tile is a square with four circles of the same size inside. He will paint the circles blue and the remaining part of the tile yellow. Find the area of the portion of the tile that will be yellow.

Area of yellow portion is 30.96 sq. in



b. Find the area of the diamond-shaped piece in the middle of the tile.

Area of diamond-shaped piece is 7.74 in^2

c. What portion (percent) of the tile will be yellow?

30.96/144 = 21.5%

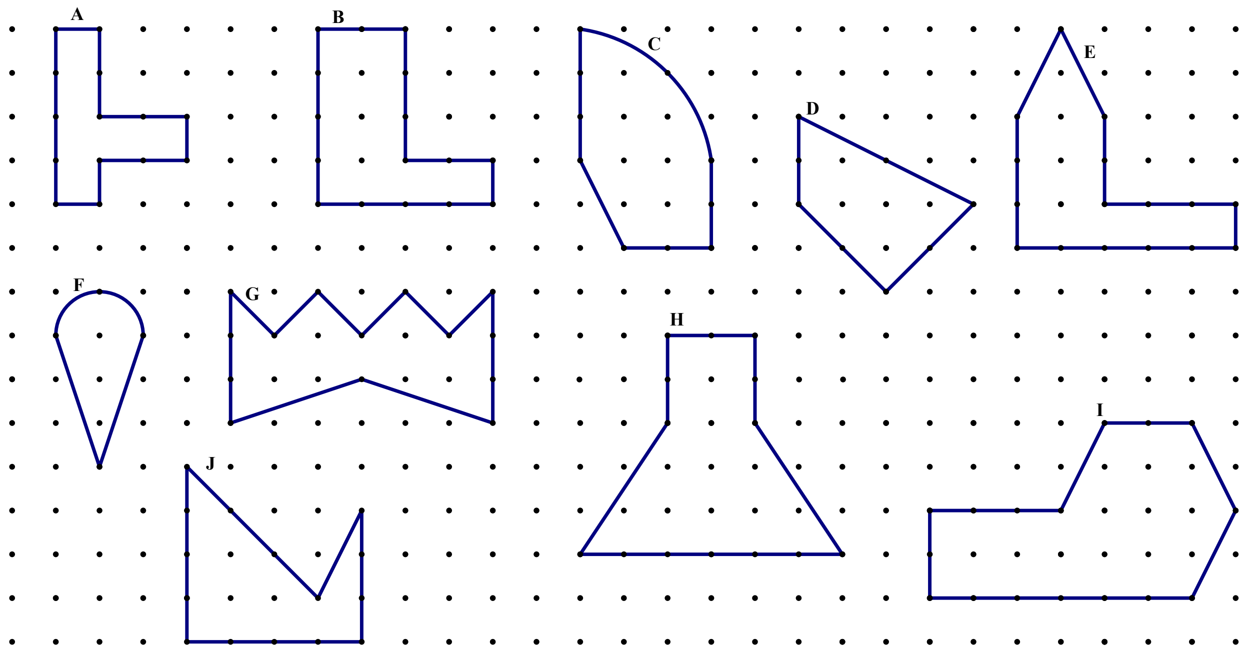
Help students draw their ideas for finding areas of irregular shapes. For example, on #2 a student might draw a 12×12 square then write "–" four circles of radius 3 (see below). Transition student to then writing it as: $12^2 - (4(3^2\pi))]$ inches as they become ready.

$12^2 \quad - \quad 3^2\pi \quad - \quad 3^2\pi \quad - \quad 3^2\pi \quad - \quad 3^2\pi$

8.1b Homework: Areas of Irregular Shapes

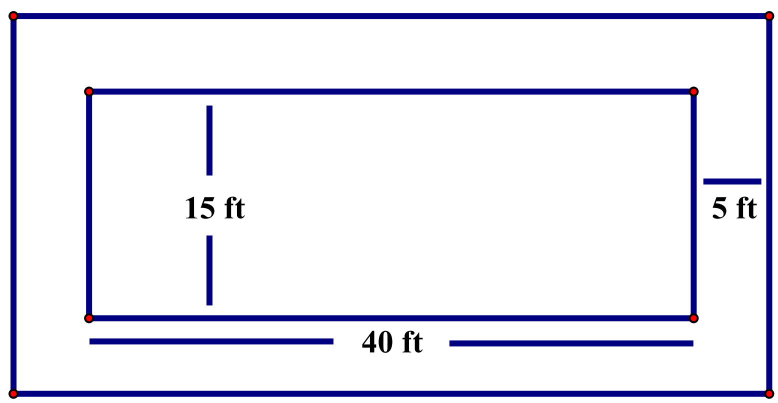
1. The following shapes have been drawn on square dot paper. The distance between each dot represents one unit. Use what you have learned about area to find the area of each shape (A-J).

A: A = 6 sq. units   B: A = 10 sq. units   C: A = 12.065 sq. units   D: A= 8 sq. units   E: A = 11 sq. units
F: A = 4.57 sq. units   G: A = 12 sq. units   H: A = 16 sq. units   I: A = 19 sq. units   J: A = 9.5 sq. units



Solve the following area problems.

2 a. A rectangular lap pool with a length of 40 ft. and a width of 15 ft. is surrounded by a 5-ft. wide deck. Find the area of the deck.
Area of the deck is 650 sq. ft.



2 b. Draw a picture of the deck if the deck is extended 3 feet in every direction, then find the area of the new deck. $31 \times 56 - 15 \times 40 = 1736 - 600 = 1136$ sq. ft.

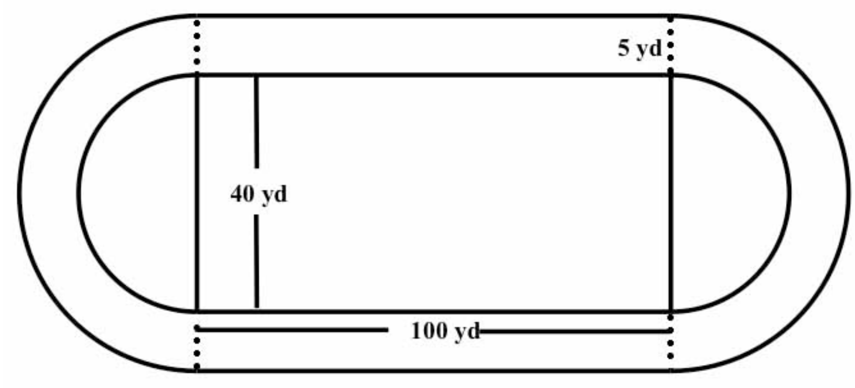
*When reviewing this problem in class, you might ask students if there is a scale factor that takes the smaller deck to the larger one. NO: 25→31 the scale factor is 31/25 but 50→56 the scale factor is 28/25. They are not the same. The two rectangles are not proportional (not similar, the word "similar" is not part of the 7th grade core). Note: this may be very confusing to students. Discuss the ratio of the sides.

3. A rectangular field with two semi-circles at each of the shorter ends of the field measures 100 yards long and 40 yards wide. It is surrounded by a track that is 5 yards wide. Find the area of the field which includes the two semi-circles on each of the shorter ends of the rectangle. Find the area of the track.

Area of the field is approximately 5256 sq. yds

Area of the track is approximately 1706.5 sq. yds

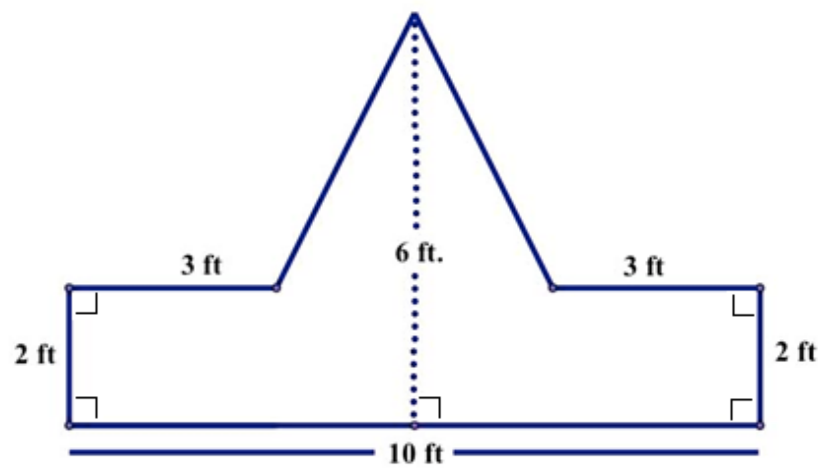
Discuss why these are approximations.



4. Laura is painting a sign for the new Post Office. She will paint the triangular portion blue and the lower rectangular portion red. Find the area of sign that she will paint blue. What percent of the sign will be in blue?

Area in blue is 0.5(4 ft)(4 ft) = 8 ft^2.

Area of the whole sign is 28 ft^2, the portion in blue is 8/28 ≈ 28.57%



Spiral Review

1. Simplify the following:

$2.6 + (-2.6) = 0$ (additive inverse)

$\frac{1}{6} + \frac{3}{7} =$ 7/42 + 18/42 = 25/42

2. Simplify the following expressions:

$-60 + 5x - 1 - 17x$ $-61 - 12x$

$9(5 - x) + 3x - 8$ $37 - 6x$

3. Anya and Bartholomew clean windows. Anya charges 50 cents per window plus $10 per job. Bartholomew charges 90 cents per window plus $6 per job. If on one job, they make the same amount of money, how many windows did they each clean? $0.5w + 10 = 0.9w + 6$, $w = 10$; 10 windows

4. Alfonso is tossing his baby brother's cylinder toy. The following table shows the results of the tosses. Based on the observations, what is the probability the cylinder will land on its side?

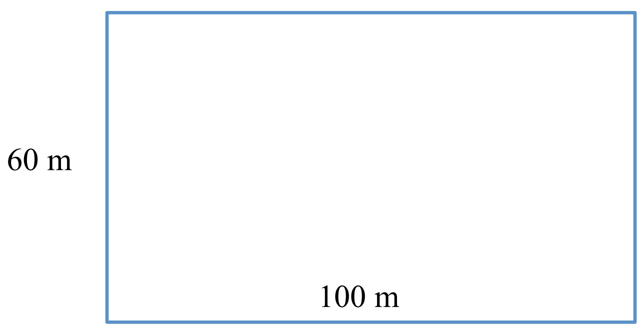
Land on side	Land on top or bottom
9	11

8.1c Class Activity: Areas of Irregular Shapes and Expressions

The goal in this section is to transition students to writing algebraic expressions for finding area of irregular polygons.

Use your knowledge of area to find the area of the following shapes, if possible. Write an expression to show how you arrived at your answer.

1 a.



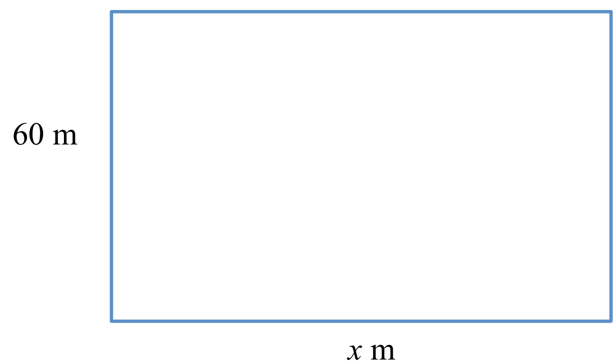
$A = 60 \times 100 = 6000\ m^2$

1 b.

Suppose you had a 60×100 meter plot of ground on which you were going to plant a garden. In the corner of your plot you want to build 5×8 storage shed. How much area will be left for your garden? Does it matter which way the storage shed is oriented?

$6000 - 40 = 5960\ m^2$. It does not matter where you put it, the area will always be the same.

2 a



$A = 60x\ m^2$

2 b.

Suppose you have a 60 by 200 meter plot of ground on which you are going to plant a garden. You don't want to plant the whole thing, just an area of 8000 m^2. If one side is 60 meters, how long will the other side (the 200 meter side) have to be to have an area of 8000m^2? What percent of the other side would you be using?

$60x = 8000$

$x = 400/3$ or $133\ 1/3$ meters

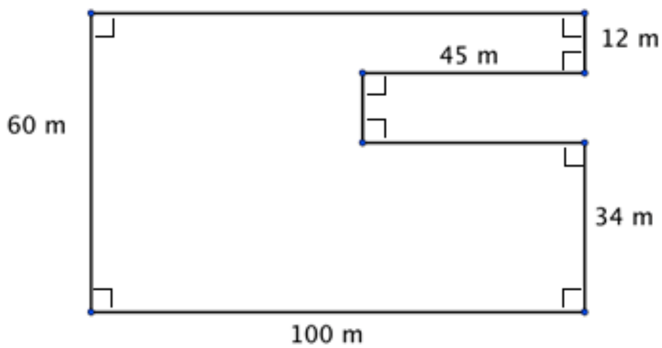
$(133\ 1/3)/200 = 66\ 2/3\ \%$

2 c.

What percent of your 60×200 meter plot of ground will be planted?

$8000/12000 = 66\ 2/3\ \%$

3. a



Area of Big Rectangle + Area of Medium Rectangle + Area of Small Rectangle

(60)(55) + (45)(34) + (12)(45)

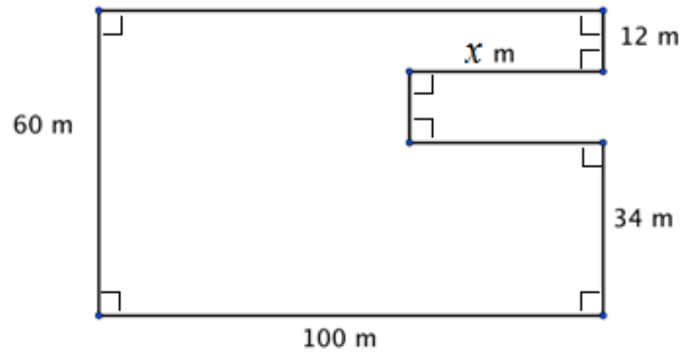
or

Big rectangle – Little rectangle

(60)(100) – (14)(45) =

A = 5370 sq. m

3 b.



Area of Big Rectangle + Area of Medium Rectangle + Area of Small Rectangle

(60)(100 – x) + (34)(x) + (12)(x)

6000 – 60x + 34x +12x

or

Big rectangle – Little rectangle

(60)(100) – (14)(x) =

A = 6000 – 14x sq. m

3 c. Suppose you're building a rectangular storage enclosure. The base of the enclosure is to be 60 by 100 meters. You need to construct a ramp into the enclosure as illustrated in 3 b. How long should the ramp be if you want the remaining area to be 5250 m^2?

6000 – 14x = 5250

x = (approximately) 53.57 meters

For 4 – 8 only one method is shown. Student methods may vary.

4a.

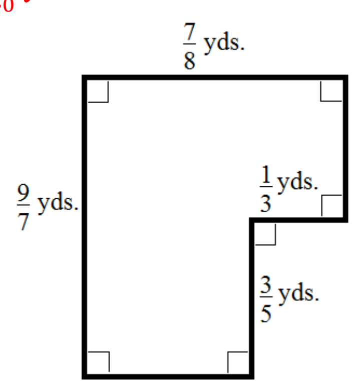
Area of Big Rectangle – Area of Small Rectangle

$\frac{9}{7} \cdot \frac{7}{8} - (\frac{1}{3} \cdot \frac{3}{5})$ or

Area of left rectangle + Area of right rectangle

$$\frac{9}{7} \cdot \left(\frac{7}{8} - \frac{1}{3}\right) + \left(\frac{1}{3} \cdot \left(\frac{9}{7} - \frac{3}{5}\right)\right)$$

A = $\frac{37}{40}$ yds



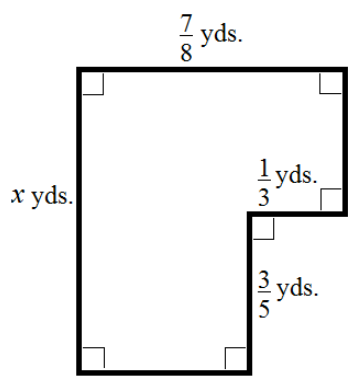
4b.

Area of Big Rectangle – Area of Small Rectangle

$x \cdot \frac{7}{8} - \frac{1}{3} \cdot \frac{3}{5}$

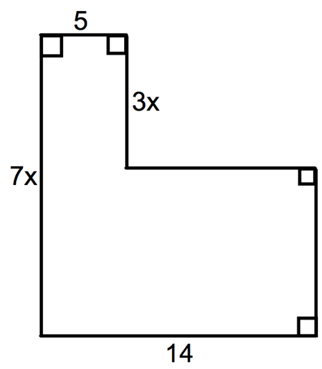
You may also add together the two smaller rectangles

$$\frac{7}{8}x - \frac{1}{5}$$



Write an expression to represent the indicated measure for each of the following irregular shapes.

5. Find the area and perimeter for the figure below.

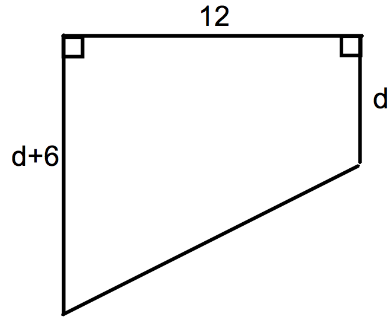


Area of Big Rectangle + Area of Small Rectangle
$5(3x) + 4x(14) = 15x + 56x = 71x$ sq units
Perimeter: $5 + 3x + 9 + 4x + 14 + 7x = 28 + 14x$ units

Students might struggle with finding the length of the right side, help them see that it is $7x – 3x$.

The larger issue is helping students connect ideas about perimeter and area to addition and multiplication.

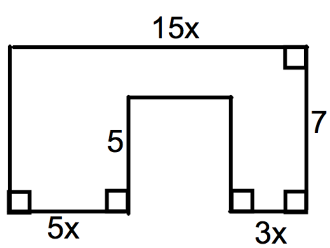
6. Find the area of the figure below.



Area of large Rectangle – Area of Triangle
$12(d + 6) – 0.5(6)(12) = 12d + 72 – 36 = 12d + 36$ sq. units
or
Area of the small rectangle plus the triangle: $12d + 0.5(6)(12) = 12d + 36$ square units
or
Area of a Trapezoid ("on its side")
$0.5(12)(d + 6 + d)$
$6(2d + 6) = 12d + 36$ square units.

There are several ways students might find the area. Encourage students to find more than one way.

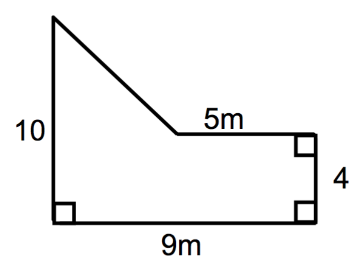
7. Find the area and perimeter of the figure below:



Area: $15x(7) – 5(7x) = 105x – 35x = 70x$ square units
or
$5x(7) + 3x(7) + 2(7x) = 35x + 21x + 14x = 70x$ square units

Perimeter: $15x + 7 + 3x + 5 + 7x + 5 + 5x + 7 = 30x + 24$ units

8. Find the area of the figure below:

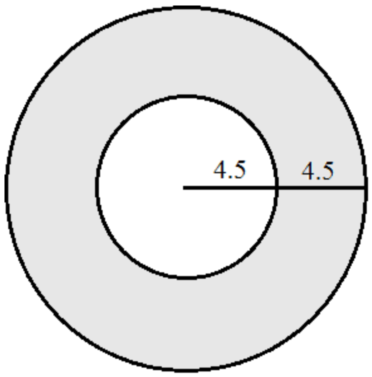


Area: $9m(4m) + 0.5(6m)(4m) = 36m^2 + 12m^2 = 48$ square m

9. A bull's eye is made of two concentric circles as shown below. The radius of the smaller circle is 4.5 inches. The larger circle has a radius of 9 inches.
Use 3.14 to approximate π in calculating the area of:
   a. The smaller circle 63.585 in^2
   b. The larger circle 254.34 in^2
   c. The space between the smaller circle and larger circle (the outer ring) 190.755 in^2

Ask students why the area of the larger circle isn't twice the area of the smaller circle given the radius of the larger circle is twice that of the smaller circle.

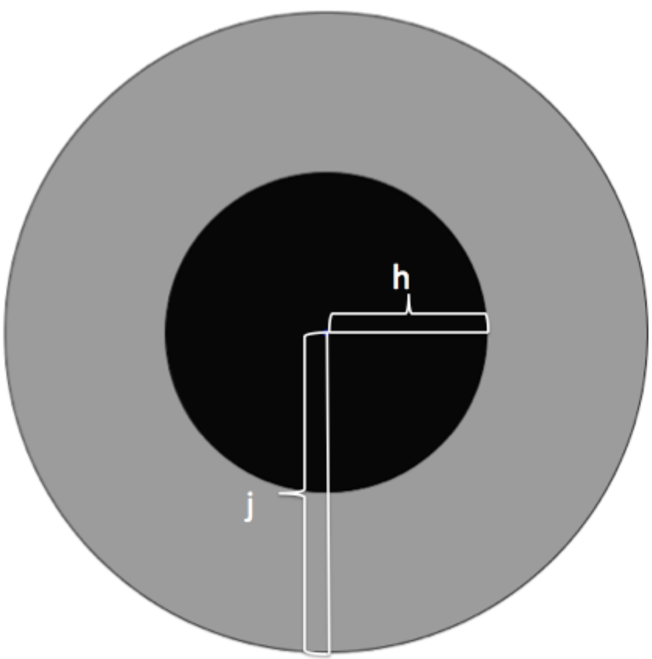
Also ask: what is the scale factor of the area of the smaller circle to the area of the larger circle? (Ans. 4).



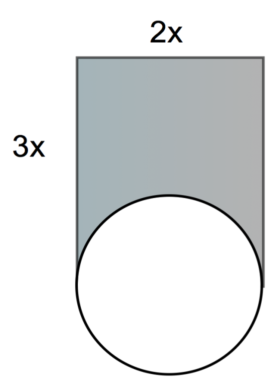
Assuming the dart hits the target somewhere, what is the probability of hitting the smaller circle's area? 63.585/254.24 approximately 25%

   d. What is the probability of hitting the outer ring? 190.755/254.34 or approximately 75%. Point out that there is really no need to compute this, given we know that the total probability must equal 1, hence 1 – (the probability of hitting the smaller target) gives us the probability of hitting the outer ring.

10. Find the area of the grey part of the figure below.
Area of Big Circle – Area of Small Circle
$j^2\pi - h^2\pi$ or $\approx 3.14j^2 - 3.14h^2$



11. The figure below shows a circle and a rectangle. The circle's diameter is equal to the rectangle's base. Find area of the shaded region and its perimeter. (use 3.14 for pi)
Area of rectangle - Area of Semi-Circle
$(3x)(2x) - (0.5)(\pi)(x^2)$
$6x^2 - 1.57x^2 = 4.43x^2$ square units;
Perimeter: $8x + 0.5(2x)\pi = 11.14x$ units



8.1c Homework: Areas of Irregular Shapes and Expressions

Note: this is a very long homework assignment. You may want to make adjustments.

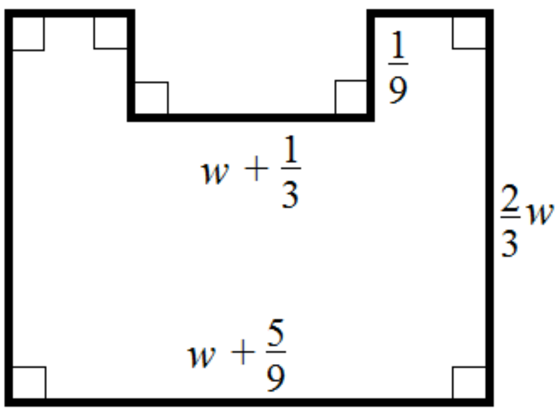
Solve the following area problems. Write an expression showing how you got the area.

1. Find the area of the region.

Area of Big Rectangle – Area of Small Rectangle

$\left(w+\frac{5}{9}\right)\left(\frac{2}{3}w\right)-\left(w+\frac{1}{3}\right)\left(\frac{1}{9}\right)$

$\frac{2}{3}w^2+\frac{10}{27}w-\frac{1}{27}$ square units



2. Find the perimeter and area of the region. (use 3.14 for pi)

Area of Circle + Area of Rectangle

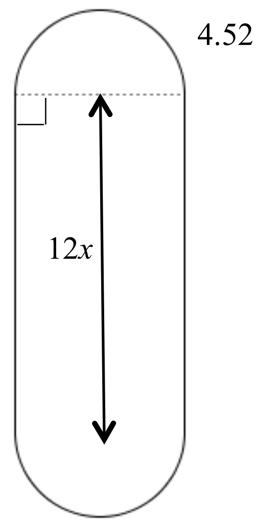
$\pi(2.26)^2+(4.52)(12x)$

$16.038+54.24x$ square units

Perimeter: $2(12x)+\pi(4.52)$

$24x+14.19$ units

Note: this is an approximation



3. Jeremy needs to buy soil for the garden spot in his backyard. A sketch of the plot is to the right.
   a. Find the area of the garden.
   b. How many cubic feet of soil will he need to buy if he covers the area in 6 inches of soil?
   c. How many cubic feet of soil will he need to buy if he covers the area in one and a half feet of soil?
   d. Howmany feet of fencing will he need to enclose the garden if he fences the exact shape of the garden?

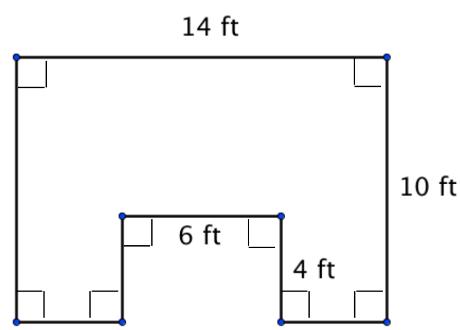
a. Area of Big Rectangle – Area of Small Rectangle

(14 ft)(10 ft) – (4 ft)(6 ft); A = 116 ft^2

b. 116 $ft^2 \times 0.5$ ft = 58 ft^3

c. 116 $ft^2 \times 1.5$ = 174 ft^3

d. 14 + 10 + 4 + 4 + 6 + 4 + 4 + 10 = 56 ft.



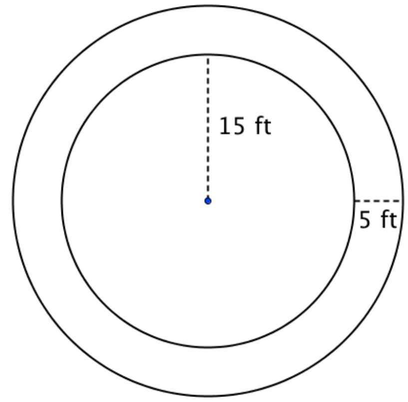
4. Nico is building a deck around the circular pool in his back yard. The pool has a radius of 15 ft. The deck will be 5 ft wide.

a. Find the area of the deck.

Area of the Outer Circle – Area of the Inner Circle

$\pi(20)^2 - \pi(15)^2$

Area of the deck is 549.5 ft^2



5. a. A stage with a trapezoidal area upstage and a rectangular area downstage is illustrated in the figure to the right. Find the area of the stage.

Area of Trapezoid + Area of Rectangle

$(0.5)(35)(48 + 21) + (48)(5)$

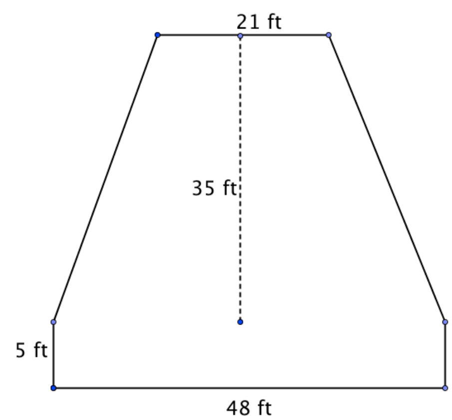
$A = 1447.5\ ft^2$

b. What is the area of just the rectangular portion of the stage?

240 ft^2

c. What portion of the stage is the rectangular portion?

$240/1447.5 \approx 20.92\%$



6. Rachel is painting a sign for the new Health Center. Find the area of sign that she will need to paint red if she paints the entire area red.

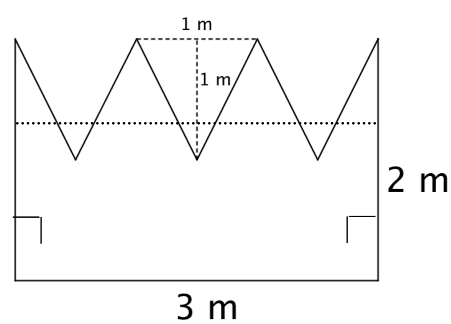
Area of Rectangle – 3 Area of Triangles

$(3)(2) - 3(0.5)(1)(1)$

$A = 4.5\ m^2$

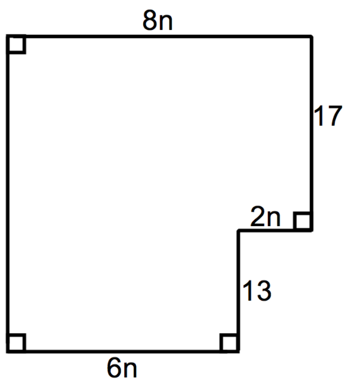
Suppose Rachel decides to paint the lower portion (the 3×1 bottom portion, shown below the dotted line) of the sign blue. What percent of the sign would be blue?

$3/4.5 \approx 66\ 2/3\ \%$



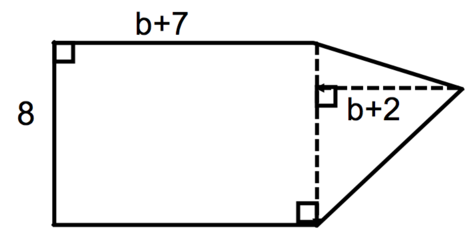
Write an expression to represent the **white area** of the following irregular shapes.

7.
Area of Big Rectangle – Area of Small Rectangle
$8n(30) - 2n(13) = 240n - 26n = 214n$ sq units.

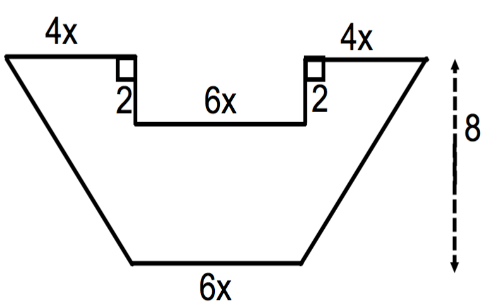


Find the perimeter of this region:
$8n + 17 + 2n + 13 + 6n + 30 = 16n + 60$ units

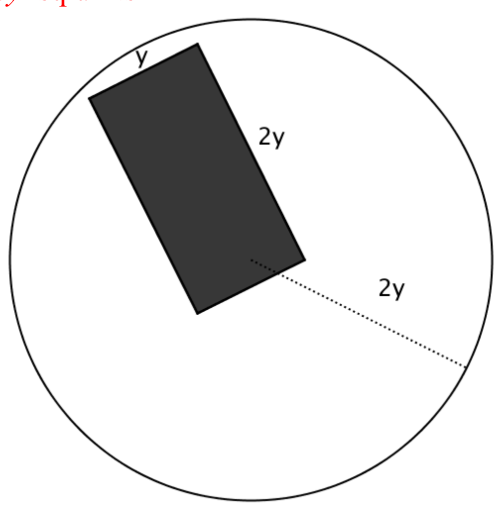
8.
Area of Rectangle + Area of Triangle
$(8)(b + 7) + (0.5)(8)(b + 2)$
$8b + 56 + 4b + 8$
$12b + 64$ sq. units



9.
Area of Trapezoid – Area of Rectangle
$(0.5)(8)(6x + 14x) - 2(6x) = 80x - 12x = 68x$ sq units



10. The circle has a radius of 2y.
Area of Circle – Area of Rectangle
$\pi(2y)^2 - (y)(2y)$
$10.56y^2$ sq units

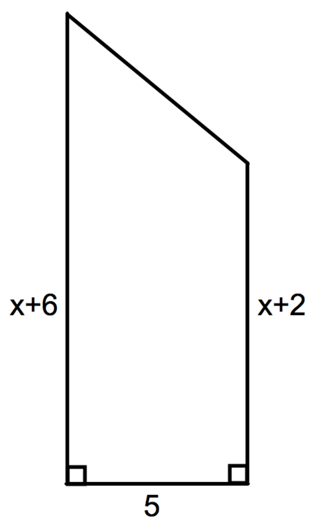


11.

Area of the trapezoid

$0.5(5)(x + 6 + x + 2)$

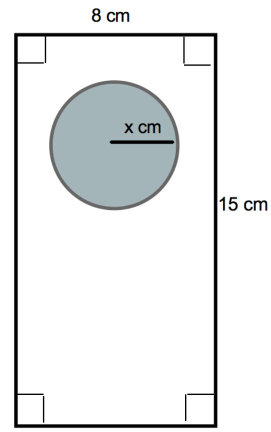
$2.5(2x + 8) = 5x + 20$ sq units



12.

Area of the Rectangle – Area of the circle

$(8)(15) - x^2\pi = 120 - x^2\pi$ cm^2



13.

Area of Semi-Circle + Area of Triangle + Area of Rectangle

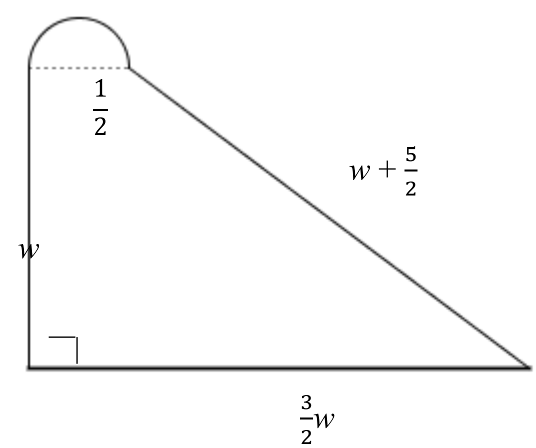
$\frac{1}{2}\pi(\frac{1}{4})^2 + \frac{1}{2}(w)\left(\frac{3}{2}w - \frac{1}{2}\right) + w(\frac{1}{2})$

$\frac{3}{2}w^2 + \frac{1}{4}w + \frac{1}{32}\pi$

or Area of Semi-Circle + Area of Trapezoid

$\frac{1}{2}\pi(\frac{1}{4})^2 + \frac{1}{2}(w)\left(\frac{3}{2}w + \frac{1}{2}\right)$

$\frac{3}{4}w^2 + \frac{1}{4}w + \frac{1}{32}\pi$



Spiral Review

1. Write and solve a proportion to find what percent of 27 is 3? 3 / 27 = x / 100. x = 300/27 = 11.111…%

2. A local college football team is known for its awesome offense. The table below shows a season's rushing yards for 9 players. Find the mean absolute deviation of their rushing yards. 121.777…

Rushing Yards
460
399
180
158
110
95
55
15
1

3. Two angles are supplementary. One angle is 49 degrees less than the other. What are the measures of each angle? 65.5° and 114.5°

4. Write 0.672 as a percent. 67.2 %

5. Sam has lots of bracelets. She gets 3 more bracelets. Then she sells $\frac{1}{3}$ of the bracelets. Write two different equivalent expressions showing how many bracelets Sam has left.
$\frac{2}{3}(x+3)$
$\frac{2}{3}x+1$
$\frac{2}{3}(x)+\frac{2}{3}(3)$

8.1d Class Activity: Review Areas of Triangles, Parallelograms, Trapezoids; Circle Area and Circumference

Questions #1-5 relate to polygonal figures, questions #6-8 relate to circles.

Activity: In pairs, answer each of the following. Use a model to justify your answer.

1. Look back at question #3 from your homework (8.1c). How much more soil will Jeremy need to buy if he decided to cover his garden in 1.5 feet of soil rather than six inches of soil? Give your answer as a scale factor (twice as much? Three times as much? 1.5 times as much?) Explain:

   3 times as much; 3×0.5 feet is 1.5 feet.

2. Kara and Sharice are in a quilting competition. Both are stitching rectangular-shaped quilts. So far Kara's has an area of 2,278 square inches with a height of 44 inches. Sharice's quilt has an area of 2,276 square inches with a height of 47 inches. Whose quilt is wider? By how many inches is it wider?

   The width of Kara's quilt is 51.77 in. The width of Sharice's quilt is 48.43 in.
   Kara's quilt is wider by 3.34 in.

3. Rufina bought two 12-foot pieces of lumber and two 8-foot pieces of lumber to create a border for her garden in her yard. She wants to use all the wood to enclose her garden. How should she use the four pieces to create a rectangular garden of largest possible area? What is the largest area she can get with her four pieces of wood?

   Rufina has 40 linear feet of wood with which to create a garden. Encourage students to try a variety of configurations: a 12 ft $\times$8 ft garden would give an area of 96 ft^2, a 15 ft $\times$ 5 ft would give and area of 75 ft^2, etc. Ask students what they notice happening? Help them discover that a 10 ft $\times$ 10 ft garden would result in the largest area of 100 ft^2.

4. A triangle has an area of 90 square centimeters.
   a. If its height is 15 cm, what is the length of the base?
   b. Draw a triangle with an area of 90 square centimeters, height of 15 cm and base you found in part a. Is there more than one triangle you can draw with those dimensions?
   c. Draw a triangle that has an area that is 150% of the original 90 cm^2 triangle. What are the dimensions of your larger triangle?

   a. Base is 12 cm
   b. Drawings might be vary Yes, infinitely many triangles with those dimensions
   c. Original triangle had b = 12 cm, h = 15 cm, and A = 0.5(12 cm)(15 cm) = 90 cm^2. We want A = 1.5(90) cm^2 or 135 cm^2. One way: 135 = 0.5(15)b, so b = 18 cm. There are other answers.

5. Julius drew a trapezoid that had bases of 15 and 11 inches and a height of 4 inches.
   a. What is the area of the trapezoid Julius drew?
   b. Can you draw a trapezoid that has the same area, but different dimensions?
   c. Draw a trapezoid with the same height but with bases that are a 5/2 scale factor of the original trapezoid.
   d. What is the area of the new trapezoid?

a) A = 52 in^2
b) answers will vary
c) The base of 15 will be 75/2 or 37.5 inches and the 11 base will become 55/2 or 27.5 inches. The height remains the same.
d) A = 0.5(4)(37.5 + 27.5) = 130 in^2

Notice again that the area changed by the same scale factor (this happened with #4 also). We only changed one dimension, not both, so the area changed in the same way. In other words, if we scaled both the height and base by 5/2, our area would scale by $(5/2)^2$, but because we only changed one dimension, our area scaled by one factor of 5/2. Discuss this point with students.

Review: These ideas are review from chapter 5.

Draw a model and explain how to find the circumference of a circle:

Draw a model and explain how to find the area of a circle:

6. You're making a 12 inch diameter pizza. You want the sauce to cover the pizza with a 1.5 inch ring left around the outside without sauce. (Use 3.14 as an approximation for pi.)
   a. What is the area that the sauce will cover?
      If D = 12 in, r = 6 in. We want a 1.5 inch ring, so we use r = 4.5. Area of the sauce is 63.585 in^2
   b. What percent of the dough will be covered be sauce?
      About 56%
   c. If one 8 oz can of tomato sauce covers about 125 sq. inches of pizza dough, how many cans of sauce will you need to buy?
      1 can of tomato sauce

7. A circular swimming pool with a diameter of 32 feet is located exactly in the middle of a 40 ft × 40 ft square lot. For safety reasons the lot needs to have an 8 ft fence on the perimeter of the entire lot.
   a. How long will the fence need to be?
      P = 160 feet

   b. If the fence was around only the circular pool, how long would the fence be?
      C = 100.48 feet

   c. Explain how much longer a fence around the whole yard is than a fence only around the pool using percent increase.
      59% increase

   d. What percent of the yard area does the pool take up?
      About 50.2%

   e. What is the area of the yard NOT taken up by the pool?

      About 49.8%

8. Look back at question #4 from the homework (8.1c). Nico's pool currently takes up approximately 706.5 ft^2 of space in his yard (using 3.14 as an approximation for pi). If Nico adds the five foot deck around the pool, what percent increase of space will this be?

   Old area = 706.5 ft^2; new area is 400π ft^2 or approximately 1,256 ft^2. 1,256/709.5 = 1.778. Thus the pool with the deck is now 177.778% the area of just the pool alone. Notice that the scale factor from the 15 foot radius to the 20 foot radius is 4/3, so the percent increase will be $(4/3)^2$. Clarify with students that for finding area of a circle, the radius is squared (review with them why this is true from the picture they drew before doing #6), thus when this length is increased by a scale factor of 4/3, the area will scale at $(4/3)^2$.

8.1d Homework: Review Areas of Triangles, Parallelograms, Trapezoids; Circle Area and Circumference

1. Wallpaper comes in rolls that are 60 feet long and 2 feet wide. How many rolls of wallpaper will it take to cover 700 square feet?

6 rolls of wallpaper—talk about this with students. One roll covers 120 ft^2 of area. Five rolls cover 600 ft^2, so you'll need a 6^{th} roll. There will be 20 ft^2 left over.

2. A rectangular garden has an area of 45 square feet. One of the sides is 6 feet.
   a. What is the other side? 7.5 ft
   b. You want to put a fence around it. How long will the fence need to be? 27 ft.
   c. You decide you want to increase the length of each side by a scale factor of 3.2. What are the new dimensions of your garden? 7.5(3.2) by 6(3.2) or 24 ft by 19.2 ft
   d. What is the area of your new garden? 460.8 ft^2

3. Mrs. Garcia has a table shaped like an isosceles trapezoid in her third grade classroom. The two parallel sides have lengths of 6 feet and 8 feet. The distance between them is 4 feet.

   a. What is the area of the top of Mrs. Garcia's table? $A = 28\ ft^2$
   b. Suppose Mrs. Garcia has a 1.75 ft by 0.8 ft puzzle on the table. How much surface area is now available on her table? $28 - (1.75)(0.8) = 26.6\ ft^2$

4. The diameter of the earth is about 7926 miles. This is a fun problem to do as a class.
   a. Find the distance around the earth at the equator.

Using a diameter of 7926 miles and 3.14 for pi, the distance around the Earth at the equator is approximately 24,862.52 miles

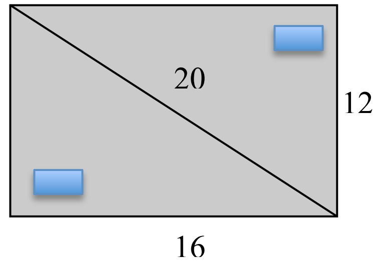
   b. If there are 5280 feet in every mile, what is the distance around the Earth in feet?
      $24862.52 \times 5280 = 131{,}274{,}105.6$ feet.

   c. Suppose you can jog at a rate of 2 miles every 15 minutes. At this rate, how long would it take you to walk around the Earth?
      24,862.52(15/2) = 186,468.9 minute; 186,468.9/60 = 3,107.815 hours; 3,107.815/24 = 129.49 days IF you walked 24 hours a day 7 days a week. If you walked 10 hours a day (3,107.815/10), it would take 310.7815 10 hour days of walking. Of course, we're assuming that one could actually walk around the equator, there's actually a lot of ocean to contend with.

5. A 12 foot by 16 foot rectangular office is being sectioned off into two triangular areas so that desks can be placed in opposite corners. The diagonal of the office (from corner to corner) is 20 feet. The manager needs a dividing curtain to hang from the ceiling around one of the triangles.

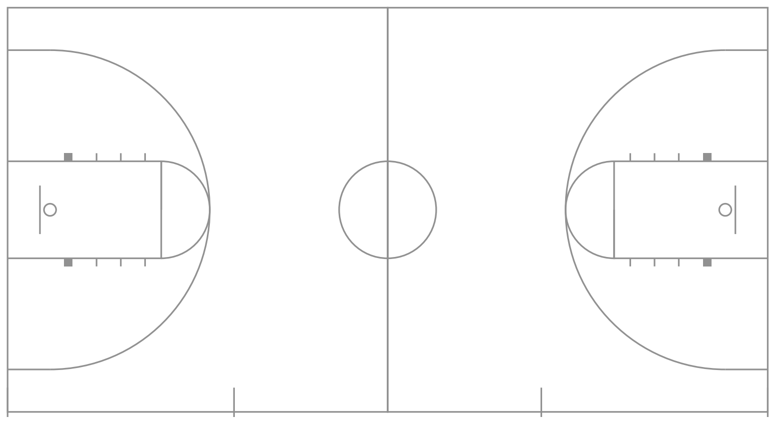
   a. How long does the curtain need to be?

      48 ft long



   b. The carpet in each section will be a different color. How many square feet of carpet will be needed to cover each triangular section?
      96 sq. ft will be needed to cover each triangular section. Point out to students that the area for each part can be found by either finding the area of the triangle (0.5×12 ft $\times$ 16 ft) or finding the area of the rectangle and then dividing it by 2.

6. The three-point line in basketball is approximately a semi-circle with a radius of 19 feet and 9 inches. The entire court is a rectangle 50 feet wide by 94 feet long. What is the approximate area of the court that results in 3 points for a team?

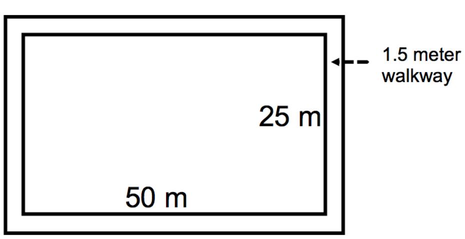


$A = 94\times50 - (.5)19.75^2\pi \approx 2043.8$ ft^2 (rounded the area of the circle to the nearest sq. ft). Teachers should explain the 3-pt rule for students who don't know the rules of basketball.

7. Your neighbor's backyard lawn is shaped like a rectangle. The back fence is 38.2 feet long and the side fence is 32.6 feet long. He will pay you $0.04 per square foot for mowing and $0.11 per foot for trimming all the edges. How much will you get paid total for mowing and trimming? Remember to show all your work.

Mowing: $49.81 ($0.04 × 38.2 × 32.6)
Trimming: $15.58 [$0.11 × (38.2 + 32.6 + 38.2 + 32.6)] Total: $65.39

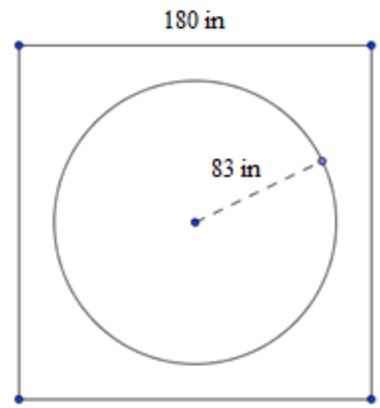
8. You're the manager of a county recreation center that has a 50 by 25 meter rectangular pool. Currently, there is an 1.5 meter cement walkway around the pool (see diagram). The community is concerned about the safety of the walkway and would like to cover it with a non-slip rubber substance that costs $78 a square meter to be installed. The county has budgeted $15,000 for the project. Is that enough money to cover the walkway? Explain you answer.



No it is not enough money. The area to be covered is (53)(28) – (50)(25) or 234 sq ft. That will cost 234(78) or $18, 252. They are short $3,252.

Spiral Review

1. A hot tub is surrounded by a square deck as pictured to the right. What is the area of the deck?
   Area = $(180 \text{ in})^2 - \pi(83 \text{ in})^2 \approx 21{,}631.46 \text{ in}^2$



2. Cristian is building a rectangular garden in his backyard. The width of the garden is set at 29 inches. He wants the fence to be 5 inches longer than the garden on each side. If he wants the area enclosed by the fence to be 2028 square inches, how long should the garden be?
   $(29+5+5)(x+5+5)=2028$
   $39(x+10)=2028$ 42 inches long
   $x=42$

3. Eugene's math class has 20 boys and 10 girls. If the teacher draws a student's name at random for a candy bar, what is the probability Eugene will be chosen? What is the probability that a girl will be chosen? P(Eugene)= 1/30; P(girl) = 10/30 = 1/3

4. There are a total of 214 cars and trucks on a lot. If the number of cars is four more than twice the number of trucks, how many cars and trucks are on the lot?
   $(2t+4)+t=214$ trucks = 70
   $3t=210$ cars = 144
   $t=70$

5. $-1(-4)(-7)$ -28

8.1f Self-Assessment: Section 8.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Solve problems involving area and circumference of a circle with numeric measures.	I can solve problems finding circumference or area of any circle as long as I'm given the radius, formulas and a picture of the situation.	I can solve problems finding circumference or area of any circle as long as I'm given a picture of the situation.	I can solve problems finding circumference or area of any circle when information is given pictorially or in context.	I can solve problems finding circumference or area of any circle regardless of how the information is given. I can also apply information to new contexts.
2. Solve real world and mathematical problems involving area and perimeter of two-dimensional objects composed of triangles and quadrilaterals.	I struggle to solve problems involving area and perimeter of triangles and quadrilaterals.	I can solve problems involving area and perimeter of triangles and quadrilaterals if all the information is given.	I can solve problems involving area and perimeter of triangles and quadrilaterals even if I have to find missing information to solve it.	I can solve problems involving area and perimeter of triangles and quadrilaterals even if I have to find missing information to solve it. I can also explain why my answer is correct.
3. Understand and explain the difference between perimeter and area.	I struggle to understand the difference between perimeter and area.	I can calculate perimeter and area of various two-dimensional objects.	I understand and can show visually the difference between perimeter and area.	I understand and can explain in my own words the difference between perimeter and area.
4. Find area or perimeter of an object with algebraically measured lengths.	I can find the area or perimeter of an object with numeric measured lengths, but struggle if there are variables involved.	I can usually find perimeter or area of an object with algebraic measured lengths.	I can always find perimeter or area of an object with algebraically measured lengths.	I can solve area and perimeter problems involving missing sides with algebraically measured lengths.
5. Solve problems involving scale or percent increase/decrease and area and/or perimeter.	I can find the perimeter or area of objects, but I struggle to solve problems involving scale or percent increase/decrease.	I can find the perimeter and areas of two scaled objects. I can usually find the scale factor between those measurements.	I can solve problems involving scale or percent increase/decrease and area and/or perimeter.	I can solve problems involving scale or percent increase/decrease and perimeter or area. I can also explain the relationship between perimeters or areas of those objects.

Sample Problems for Section 8.1

1. Use the given information to find the missing information. Round each answer to the nearest hundredth unit.

   a. Diameter: 18 km
   Circumference: ____________

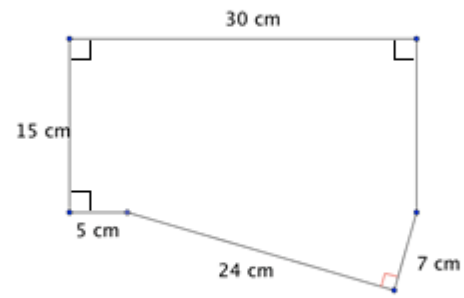
   b. Radius: 0.1 m
   Area: ____________

   c. Circumference: 50.24 in
   Radius: ____________

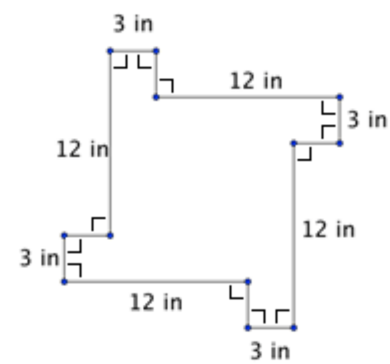
   d. Area: 105.63 in^2
   Radius: ____________

2. Answer the following questions using the object pictured.

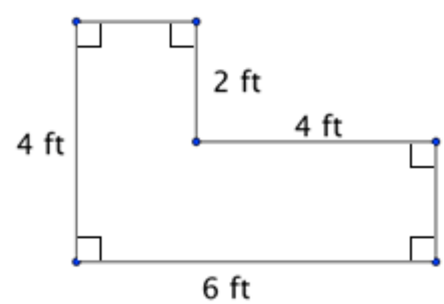
   a. Find the perimeter and area of the object to the left.



   b. What percent of the total area is the area of one of the outer squares in the object to the left?



   c. Phoebe is buying an L-shaped desk. How much area will the desk pictured take up in her apartment?

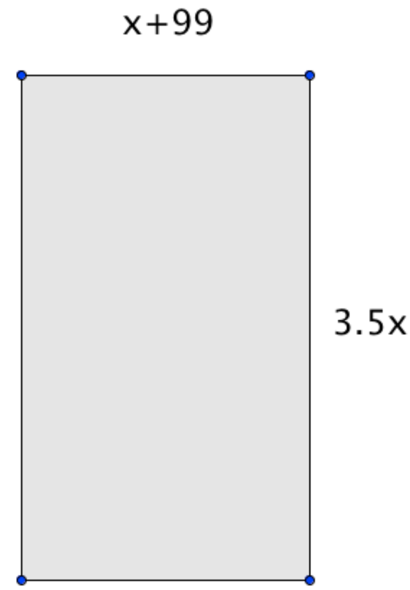


3. Explain the difference between perimeter and area in words and/or pictures. Use the figure in 2c to help you in your explanation.

4. Use the rectangle to the right to answer the following questions:
   a. What is the perimeter?

   b. What is the area?

   c. If the perimeter is 1098 units, what is the value of *x*?



5. The dimensions of Gabrielle's rectangular garden are 12 feet by 16 feet. Daniel is building a garden. He is going to decrease the dimensions by 25%.
   a. Find the perimeter and area of each garden.
   b. Find the percent of decrease between the perimeters of each garden.
   c. Find the percent of decrease the areas of each garden.
   d. In general, how are the percents of change of perimeters and areas of scaled objects related?

Section 8.2: 2D Plane Sections from 3D Figures and 3D Measurement

Section Overview:

The goal of this section is to help students better understand: a) attributes of various prisms, b) how those attributes affect finding the surface area and volume of different prisms, c) the relationship between measurements in 1-, 2-, and 3-dimensional figures and d) connect ideas of scale factor/percent change to ideas of surface area and volume. Students begin this section by examining three-dimensional figures. They should observe which faces for an object are parallel and/or perpendicular and which are the same size and shape. Students quickly move to taking cross-sections of various figures to notice what two dimensional shapes are generated by different types of cuts. Attention will be paid to when parallel plane sections generate surfaces that are the same and when they are different. The exercises in this section are designed to solidify students' understanding of the algorithms for finding perimeter, area, and volume. Additionally, exercises should help students better understand units of measure for perimeter, area, and volume. Next, students examine the nets of 3D figures. Nets were introduced in 6^{th} grade; in 7^{th} grade, students extend their understanding by differentiating surface area from volume. Students will use their understanding of surface area and volume to solve various problems. Specifically, attention will be paid to scale factor and percent change in problems involving volume and surface area.

Concepts and Skills to be Mastered (from standards)

1. Describe the two-dimensional figures that result from slicing three dimensional figures.
2. Solve real-world and mathematical problems involving volume and surface area of three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Background knowledge for making and describing 2D cross-sections:

- Right regular polyhedron prisms are three dimensional figures with at minimum two parallel polygonal faces of the same size and shape (called bases); remaining lateral faces are rectangles not necessarily the same as the bases (note that the base may be any polygon but the lateral faces are always rectangles.)
- Right cylinders have parallel circular bases of the same size (radii are equal) whose centers are aligned directly above each other (centers lie on a line that is perpendicular to both bases).
- There are several ways to take plane sections of a prism or cylinder (for the descriptions below, we assume the prism or cylinder is standing on one of its bases):
    - Parallel to the base
    - Perpendicular to the base
    - Slice at an angle (in a tilted direction) to the base
- A cross-section is a special type of plane section cut perpendicular to a face or base. Most cuts in this section will be cross-sections.
- "Equidistant" means an equal distance. "Vertices" are the corners of the figure.

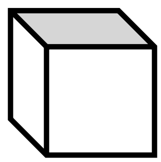
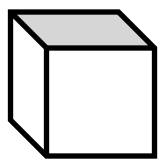
This section is primarily exploratory in nature. Attention to precision will be very important as the teacher and students explain their thinking.

8.2a Class Activity: 2D Plane Sections of Cubes and Prisms (play dough & dental floss)

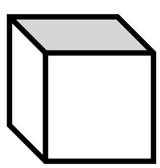
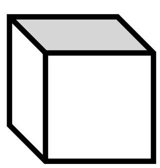
Students can make their own play-dough using the provided recipe at the end of the chapter OR you may use the links below to view demonstrations of plane sections OR you might use a combination of the two.
http://www.learner.org/courses/learningmath/geometry/session9/part_c/index.html
http://www.shodor.org/interactivate/activities/CrossSectionFlyer/
Review vocabulary (side, vertices, face, edge) with students. Many ideas will be explored in this section, take time to discuss concepts thoroughly with students.

OBJECTIVE THROUGHOUT PLANE SECTION EXERCISES: STUDENTS WILL UNDERSTAND WHEN THEY CAN MAKE PARALLEL PLANAR CUTS THAT GENERATE EQUAL SECTIONS. This will lead into ideas about volume.

1. Mold a **CUBE** from play-dough.

Perform the following cuts.	**Sketch where you cut**	**Sketch the exposed surface(s).**	**NOTES**
a. Cut *parallel* to the base.		Square	Examine the prism with students. Note parallel and perpendicular sides. Also note number of faces and that the faces are all the same size and shape.
b. Cut parallel to the base again, but at a different distance from the base.		Discuss what would happen if the cube was placed on a different face.	How does the new exposed surface compare to the previous surface? It's the ***same*** (this will not be true when they do spheres.)

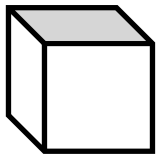
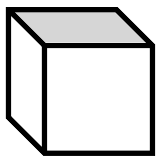
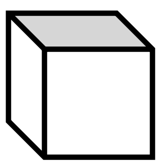
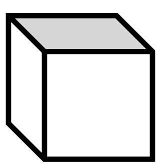
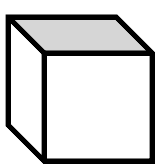
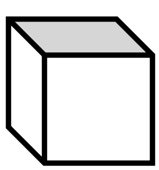
What is true about all cuts of a cube parallel to the base? You always get a cross section that is a square surface that is the same as the face of the cube.
What is true about other cuts of a cube NOT parallel to the base but through at least one lateral face? You get different polygons—have students explore this.

c. Cut perpendicular to the base and parallel to a face.			If the cut is parallel to a face (and thus perpendicular to the base), then all cross sections will be a square.
d. Cut perpendicular to the base and parallel to a face again, but at a different location on the cube.		Ask students to make cuts perpendicular to the base but **not** parallel to any of the faces. Discuss the cross sections they find.	How does the new exposed surface compare to the previous surface? All plane sections parallel to a face will be a square

What is true about all cuts of a cube perpendicular to the base and parallel to a face? You always get a surface that is the same as the face of the cube.

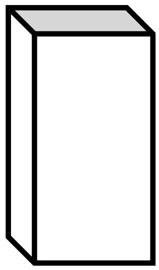
What is true about cuts of a cube perpendicular to the base but not parallel to a face? You get rectangles that are not necessarily the same, neither shape nor size.

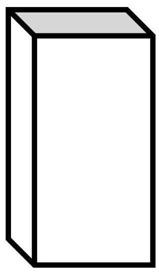
2. Mold a **CUBE** again. Perform a single cut to create the following:

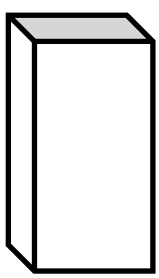
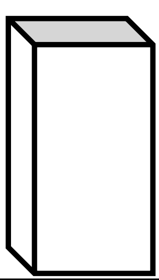
Create…	**Sketch where you cut**	**NOTES**
a. Triangle ask students if they can create an equilateral, isosceles and/or scalene triangle		Discuss the characteristics of cuts needed to make different triangles. e.g. for an equilateral triangle, the cut must intersect three edges the same distance (equidistant) from a vertex. Help students attend to precision and use appropriate language such as face, edge, vertex, side, plane, equidistant, parallel, perpendicular, etc. Throughout these exercises, it will be helpful to have students try to draw where the plane cuts the solid figure.
b. Square		Ask students to describe from where each side of the plane section came. Answer: each side of the plane section is a line from a face of the cube. In other words, to get a triangle, the plane section must pass through three sides, for a quadrilateral it must pass through four sides. Students may believe that a cut diagonally through a face and perpendicular to the opposite face will result in a square. Help them see that the diagonal is not equal to an edge length.
c. Rectangle		A cut perpendicular to the base but not parallel to a face will result in a rectangle. A cut through the diagonal of a face perpendicular to the opposite face results in a rectangle. All cuts perpendicular to the base result in a rectangle. All such rectangles will have two sides equal to the length of a face. The other two sides will vary in length.
d. Pentagon		This will be difficult for students. Help them understand that to get a five sided polygon, the cut must pass through five sides of the cube. Also note that the vertices of the plane section come from the edges of the cube.
e. Hexagon		A hexagon can be created by cutting/passing through all six faces of the cube. Remember, the cut must be straight—students will want to make "curved" cuts. Note that no other polygons can be made by plane sections because there are only 6 sides of a cube.
f. Circle or Trapezoid		Not possible.

3. Mold a Right **RECTANGULAR PRISM** (that is not a cube.)

<table>
<tr><th>Perform the following cuts.</th><th>Sketch where you cut
prism may be turned the other way</th><th>NOTES: describe the plane section and what might happen with other similar cuts.</th></tr>
<tr><td rowspan="2">a. Cut parallel to the base.

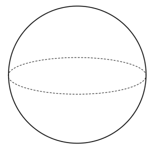
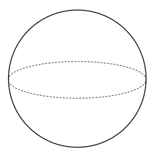
b. Cut parallel to the base again, but at a different distance from the base.</td><td></td><td>Examine the prism with students. Note parallel and perpendicular faces. Also note number of faces and if the faces are of the same size and shape. Discuss what would happen is the prism were placed on a different face.

All cuts parallel to the base result in a plane section the same size and shape as the base.</td></tr>
<tr><td></td><td>All cuts parallel to the base, regardless of its distance from the base, result in plane sections that are the same (this will not be true when students cut pyramids or spheres). Ask students to make a conjecture about why all these plane sections are all the same.
Have students make cuts NOT parallel to the base but through at least two lateral faces and note what happens—this will result in polygons that are not all the same.</td></tr>
<tr><td colspan="3">What is true about all cuts of a rectangular prism parallel to the base? The plane section will always be the same as the base of the prism.
What is true about cuts of a rectangular prism NOT parallel to the base? The plane sections will NOT all be the same as the base.</td></tr>
<tr><td rowspan="2">c. Cut perpendicular to the base and parallel to a face.

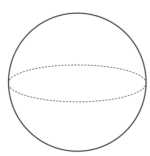
d. Cut perpendicular to the base but not parallel to a face.</td><td></td><td>All plane sections parallel to a face will be the same as the face the cut is parallel to.</td></tr>
<tr><td></td><td>These plane sections are not all the same. All are rectangles with heights equal to the face heights, but bases of the rectangles will vary.</td></tr>
<tr><td colspan="3">What is true about all plane sections of a rectangular prism perpendicular to the base and parallel to a face? The surface will be the same as the face of the prism.
What is true about any plane section of a rectangular prism perpendicular to the base but not parallel to a face? The plane sections will vary.</td></tr>
</table>

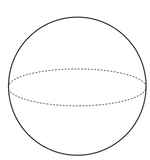
4. Mold a **SPHERE**.

<table>
<tr><th>Perform the following cuts.</th><th>Sketch where you cut</th><th>NOTES: describe the plane section and what might happen with other similar cuts.</th></tr>
<tr><td rowspan="2">a. Cut parallel to the table at different distances from the table.

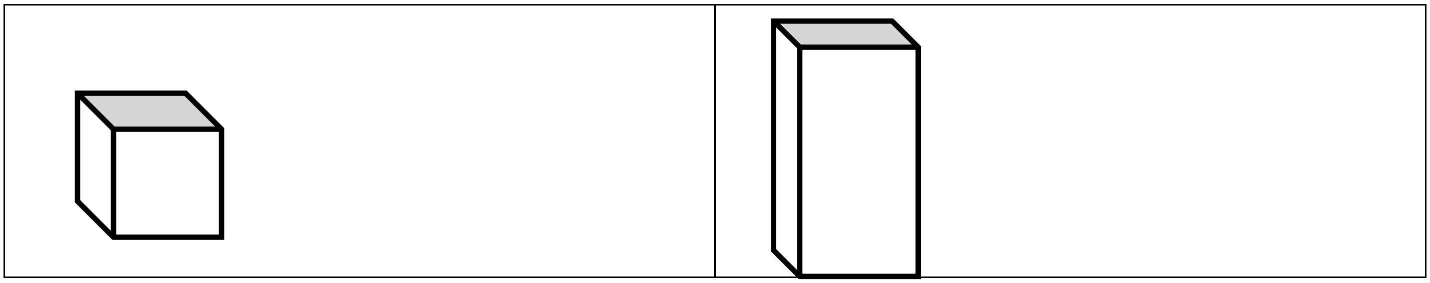
b. Make cuts that are not parallel to the table.</td><td></td><td>Discuss the figure with students. Ask students how the sphere is different than a right prism. Students may note: it doesn’t have “sides”, no “base”, no matter how it’s oriented, it always looks the same. Ask students how a sphere is related to a circle: a circle is the locus of points in a *plane* all equidistant from a fixed point, while a sphere is the locus of points in *space* all equidistant from a fixed point. Students will likely be comfortable with the idea that plane sections are all circles, of different sizes, for cuts ***parallel*** to the table regardless of where the cut is made.</td></tr>
<tr><td></td><td>Students might find it hard to understand why cuts regardless of the angle, of a sphere always results in a circle. Some students may believe that cuts of various angles will result in an ellipse. Help students understand that because a sphere has no “base”, orientation of the cut does not affect the plane section.</td></tr>
<tr><td colspan="3">What is true about all cuts of a sphere parallel to the table? They are all circles of various sizes.

What is true about all cuts of a sphere NOT parallel to the table? They are all circles.</td></tr>
<tr><td rowspan="2">c. Cut perpendicular to the table.

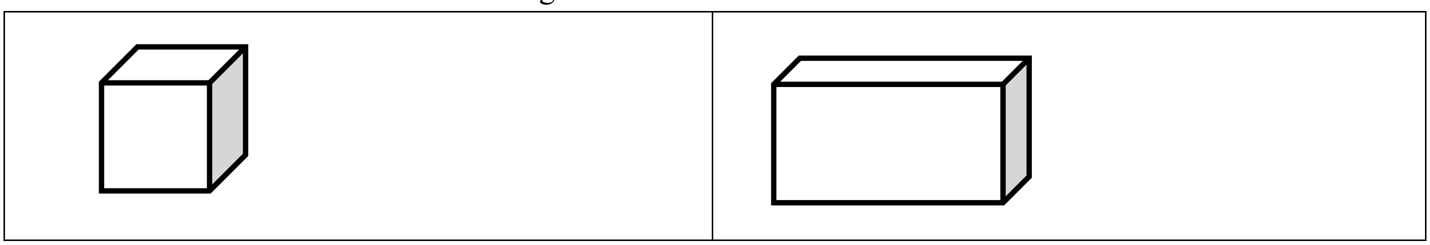
d. Cut perpendicular to the table again, but at a different location on the sphere.</td><td></td><td>Review the mathematical foundations document for ideas on how to extend concepts with perpendicular cuts.

The radius of the circle (size) changes as the distance from the table changes. All circles are scaled versions of each other (similar.)</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="3">What is true about all cuts of a sphere? Regardless of where the cut is made, you always get a circular surface.</td></tr>
</table>

5. Make a cube and a rectangular prism like in exercises #1 and #3 again. Orient them as in the figure below:



What is the shape of the base of the cube and the rectangular prism? Cube→square; prism→square
Will all cuts parallel to the base result in the same planar figure for the cube? Yes, they will all be squares
Will all cuts parallel to the base result in the same planar figure for the rectangle prism? Yes, they will all be squares
Help students understand that the volume of either can be thought of as area of the base time the height.
Now rotate each 90° as shown in the figure below:



What is the shape of the base of the cube and the rectangular prism now? Cube→square; prism→rectangle
Will all cuts parallel to the base result in the same planar figure for the cube? Yes, they will all be squares.
Will all cuts parallel to the base result in the same planar figure for the rectangle prism? Yes, they will all be rectangles the same size as the base (face touching the table).
Again, discuss with students that volume can be represented as area of the base times the height. Help them see how this is connect to $l \times w \times h$. This will be explored further for the cylinder and triangular prism in the next lesson.

6. Compare and contrast plane sections of rectangular prisms, cubes and spheres.

Possible responses from students:

For prisms, students will observe that restricting cuts to being parallel to the base will always result in plane sections of the same size and shape as the base.

Cuts parallel to a face will result in cross-sections of the same size and shape as the face only if the base is rectangular. In the next section students will cut triangular prisms. You may want to discuss this further then.

All cuts parallel to a face will be perpendicular to the base for a right prism.

Cuts perpendicular to the base but not necessarily parallel to a face.

Various cuts perpendicular to the base will not necessarily result in cross-sections that are all the same size and shape, however they will all be quadrilaterals when cutting rectangular prisms.

Cross-sections of spheres are always circles, but the size of the circle varies depending on how far from the center the cross-section was taken.

8.2a Homework: 3D Objects

Use your knowledge of each three-dimensional object to answer the following questions.

Cubes

1. How many faces does a cube have? 6 faces

2. What do you know about each face of a cube? 6 squares that are all the same size

3. How many edges does a cube have? 12 edges

4. How many vertices does a cube have? 8 vertices

Rectangular Prisms

5. How many faces does a rectangular prism have? 6 faces

6. How many edges and vertices does a rectangular prism have? 12 edges; 8 vertices

7. How are a rectangular prism and a cube similar and different? Answers will vary, possible responses: Similarities: same number of faces, edges, vertices; opposite faces are the same size and shape; plane sections parallel to a face result in plane sections that are all the same size and shape as the face. Differences: in a cube, all faces are the same, in a rectangular prism, opposite sides are the same however, all faces are not necessarily the same.

Sphere

8. Does a sphere have any edges or vertices? No edges or vertices

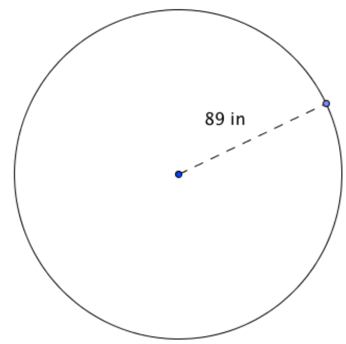
9. What makes a sphere different from all the other 3D objects named above? It has no edges, vertices, or faces; all cuts, no matter the angle, result in circles, orientation of a sphere does not affect what we "see." For example, when we look at a prism, we "see" whichever face we're looking at. The base or other faces may be different, but we can't tell unless we turn it. With a sphere, no matter how it's turned, one always "sees" a circle.

Spiral Review

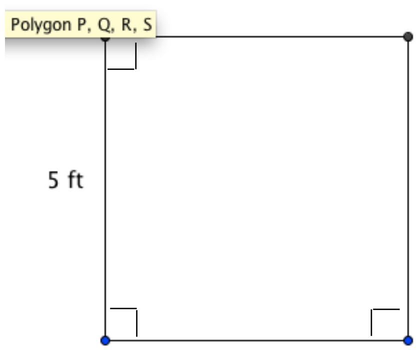
1. Find the area of each figure below:

a.

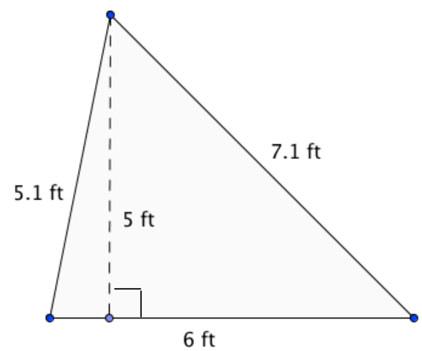
b.



c. The figure below is a square.



d.



2. Simone rolls a die 64 times. Approximately, how many times will she roll a 6? $1/6 \times 64 \approx 10 - 11$

3. Find the unit rate for BOTH units.

Izzy drove 357 miles on 10 gallons of gasoline. $\frac{357}{10} = \frac{35.7 miles}{1 gal}$ *and* $\frac{10}{357} = \frac{0.028 gal}{1 mile}$

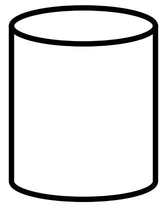
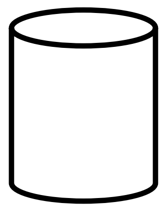
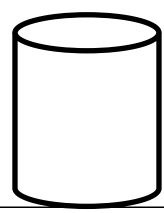
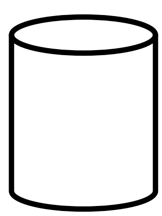
4. Convert the following units using the ratios given:

____$3\frac{1}{12}$___ feet = 37 inches (1 foot = 12 inches)

5. The temperature at midnight was 8° C. By 8 am, it had risen 1.5°. By noon, it had risen another 2.7°. Then a storm blew in, causing it to drop 2.7° by 6 pm. What was the temperature at 6 pm? 9.5°

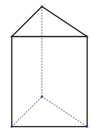
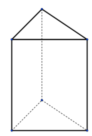
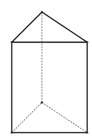
8.2b Class Activity: 2D Plane Sections on Cylinders and More

1. Mold a right **CYLINDER,** put the circular base on the table.

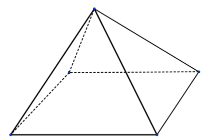
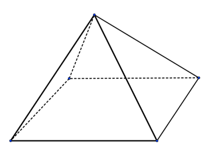
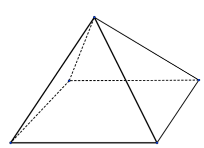
<table>
<tr><th>Perform the following cuts.</th><th>Sketch where you cut</th><th>NOTES: describe the plane section and what might happen with other similar cuts.</th></tr>
<tr><td>a. Cut parallel to the base.</td><td></td><td>Discuss attributes of cylinder with students. Help students recognize that the lateral face of the cylinder is continuous and how it is related to the circumference of the base; e.g. the base length of the lateral face is the circumference of the base of the cylinder. You might help students recognize that the lateral face is a rectangle with base = C and height = height of cylinder.
All cuts parallel to the base result in a circular plane section. All circles are the same size and shape as the base.</td></tr>
<tr><td>b. Make cuts not parallel to the base but through the lateral face of the cylinder.</td><td></td><td>There are several ways one might make a cut that's not parallel to the base: 1) through the lateral face only. These cuts will be elliptical. 2) Through one base and the lateral face. This cut will have a straight side (the one generated by going through the base, and then a curved side—an ellipse with a portion cut out of it. 3) through both the bases. This will look like an ellipse with two ends cut off.</td></tr>
<tr><td colspan="3">What is true about all cuts of a cylinder parallel to the base? You always get circles of the same size as the base (same radius).
What is true about all cuts of a cylinder NOT parallel to the base? Students will not get circles. Discuss why this is true. Discuss how these plane sections differ from plane sections of spheres.</td></tr>
<tr><td>c. Cut perpendicular to the base.</td><td></td><td>These cuts will result in rectangles. Take time to discuss and have students experiment with this.</td></tr>
<tr><td>d. Make cuts that go through at least one base of the cylinder but are not perpendicular to the base.</td><td></td><td>See b. above.</td></tr>
<tr><td colspan="3">What is true about all cuts of a cylinder parallel to the to the base? They are all circles of the same size as the base.
What is true about cuts of a cylinder perpendicular to the base? All will result in different rectangles.

What is true about any other cut (NOT parallel to the base or perpendicular to the base)? These result in various plane sections.</td></tr>
</table>

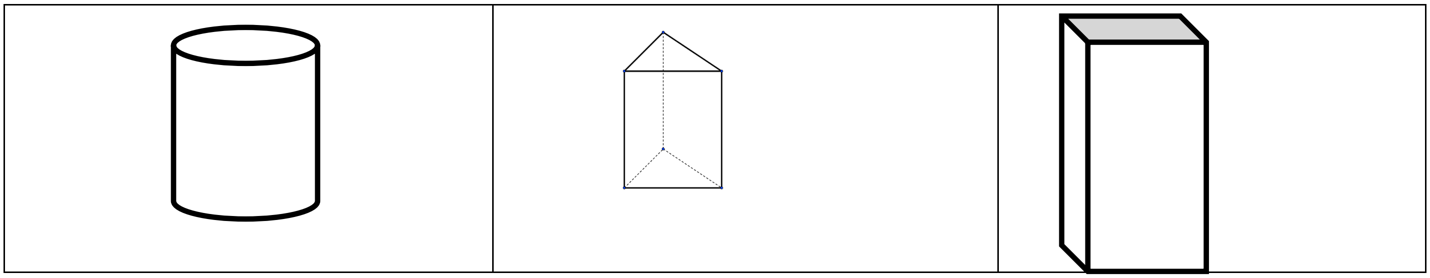
2. Make a **TRIANGULAR-BASED RIGHT PRISM**. Put the triangle base on the table.

Perform the following cuts.	**Sketch where you cut**	**NOTES: describe the plane section and what might happen with other similar cuts.**
a. Cut parallel to the base.		Discuss attributes of the prism with students. Help students recognize that one might see the lateral surface area of this prism as three separate rectangles with base b and height h OR as one large rectangle of base $3b$ and height h. Also discuss with students why we are choosing the triangular faces to be the base.
b. Make cuts that are not parallel to the base but go through at least one face of the prism.		These cuts will result in different polygons.
What is true about all cuts of a triangular prism parallel to the base? They all result in triangular plane sections that are the same size and shape as the base.		
c. Cut perpendicular to the base.		Students might expect that cuts perpendicular to the base and parallel to a face result in plane sections that are the same size and shape as the face. But in this case the plane section will not be the same shape as the face. Indeed, plane sections will be rectangular, with heights equal to the height of the prism but they will have varying bases. You might ask students if this is because there are an odd number of faces (this is only part of the issue, a right prism with a hexagonal base will also not give plane sections of the same size and shape as a face with a cut that is perpendicular to the base and parallel to a face). Only a right prism with four congruent faces results in plane sections perpendicular to the base and parallel to a face that are the same size and shape as the face.
What is true about all cuts of a triangular prism perpendicular to the base? The plane section will be a rectangle of the same height as the prism but with different lengths for the base of the rectangle. The result of 2c should lead to a discussion about volume and orientation (volume for right prisms is area of the base multiplied by the height. This "works" because all plane sections of right prisms parallel to the base are the same size and shape.) Only for right rectangular prisms can we change its orientation and still use the same values to find volume. If a right prism that has a non-parallelogram base, the easiest way to find its volume is to choose for the base a face that has an opposite face that is the same size and shape. In the case of a triangular prism, orienting it as seen in the picture above means we can find the volume simply by finding the area of the base and then multiplying it by the height. If we rotate the prism 90 degrees (lay it on its side), we cannot merely find the area of the rectangular base and multiply it by the height (height of the triangle) because all the plane sections are not the same.		

3. Make a **SQUARE-BASED RIGHT PYRAMID**. Put the square base on the table.

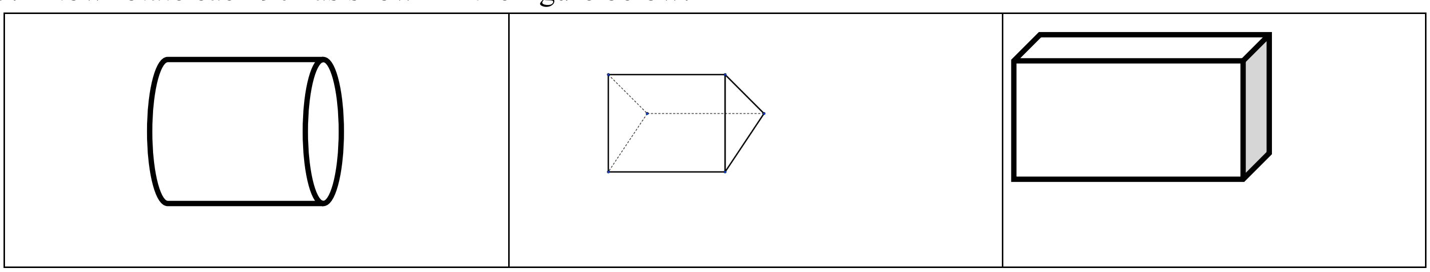
Perform the following cuts.	**Sketch where you cut**	**NOTES: describe the plane section and what might happen with other similar cuts.**
a. Make cuts parallel to the base.		Discuss attributes of pyramid with students. Help students see that there is only one base and why each of the lateral faces are of the same size and shape for this prism—the base is square and the height of the prism is on a line perpendicular to the base at its "center." Review concepts from the mathematical foundations document for how you might discuss the solid. All cuts parallel to the base result in squares, however squares are of different sizes. You might discuss with students that the size of the squares are changing at a constant rate as plane sections move up the height. Thus, the rate of change for the measure of the square plane section is affected by the height of the pyramid. These idea will be explored further in math beyond this course.
What do you notice about cuts of a square-based pyramid parallel to the base? Continue the discussion about volume from #2. Help students see that finding the area of the base and then multiplying it by the height will not result in the volume of a pyramid (plane sections change as we move up the height), but that we can intuitively see a scale factor affecting the base of the square plane sections—all the squares are scaled versions of each other.		
b. Cut perpendicular to the base and parallel to one of the base's edges.		All cuts will form trapezoids with one base the same for all trapezoids. Discuss with students that the cross-section passes through four faces.
c. Cut perpendicular to the base again, but at a different location on the pyramid.		If the cut passes through only three faces, the cross-section will be a triangle. Plane sections will vary.
What do you notice about cuts of a triangular pyramid perpendicular to the table? Students should notice that no matter how one cuts a pyramid, the resulting plane sections vary.		

4. Make a cylinder and a triangular prism like in exercises #1 and #2 again. Also make a rectangular prism. Orient them as in the figure below:



What is the shape of the base of the cylinder? Circle
What is the shape of the base of the triangular prism? Triangle
What is the shape of the base of the rectangular prism? Rectangle
Will all cuts parallel to the base result in the same planar figure for the cylinder? Yes, they will all be circles
Will all cuts parallel to the base result in the same planar figure for the triangular prism? Yes, they will all be triangles
Will all cuts parallel to the base result in the same planar figure for the rectangular prism? Yes, they will all be rectangles
Thus, volume for these figures is area of the base time the height ***if we orient them this way—see below.***

5. Now rotate each 90° as shown in the figure below:



What is the shape of the part of the cylinder resting on the table? This question will throw students. Students might say it's resting on a line or a very narrow rectangle.
What is the shape of the part of the triangular prism resting on the table? If they lay it flat, it is a rectangle.
What is the shape of the part of the rectangular prism resting on the table? If they lay it flat, it is a rectangle.
Will all cuts parallel to the table result in the same planar figure for the cylinder? NO, varying rectangles.
Will all cuts parallel to the table result in the same planar figure for the triangular prism? NO, varying rectangles.
Will all cuts parallel to the table result in the same planar figure for the rectangular prism? Yes, they will all be rectangles
Discuss 1) the volume of the figure remains the same despite orientation. 2) For V = Bh, we want all the plane sections parallel to the base to be the same. 3) Orientation is irrelevant for rectangular prisms since all plane sections parallel to the base are the same no matter the orientation.

Again: remember that you're not just trying to help students recognize the 2 dimensional plane sections; you're also helping them understand that one can often orient a three-dimensional object in such a way that opposite sides will be the same and thus one can create equal sections. If this can be done, finding the volume of the object will be the base (the section that's always the same) times the height.

8.2b Homework: Area of Plane Sections

For each plane-section described below, state the shape of the plane-section and its area. Refer to previous class activities, as needed.

1. Imagine cutting a cube, parallel to the base.

   a. What shape is the plane section?

      Same shape as the base, which is square

   b. If you cut a $5 \times 5 \times 5$ inch cube parallel to any face, what will the area of the plane section be?

      A = 25 sq. in

2. Imagine a right square-based prism with edge lengths $\frac{3}{4} \times \frac{3}{4} \times 4\frac{1}{2}$ inches.

   a. If you make a cut parallel to the square base, what will the plane section be?

      Same shape as the base, a square with side length of ¾ in.

   b. What will the area of the plane section described in "a" be? $9/16\ \text{in}^2$

   c. What will the plane section be if the cut is made parallel to the lateral face? Rectangle

   d. What will the area of the plane section in "c" be?

      $A = \frac{27}{8}\ \text{in}^2$

3. Imagine cutting a sphere with diameter 10 cm parallel to the table through the center.

   a. What shape will any plane section be? Circle

   b. What will the area of the plane section be? $25\pi\ \text{cm}^2$

4. Imagine cutting a cylinder of diameter 12.62 cm and height 8 cm, parallel to the base.

   a. What shape is the plane section and what is its area?

      Circle; area $(6.31)^2\pi\ \text{cm}^2 = 39.8161\pi\ \text{cm}^2$ or approximately $125.02\ \text{cm}^2$

   b. What shape is the plane section if the cut is perpendicular to the base? What do you know about the figure?

      It is a rectangle and it will have a height of 8cm. Students might also say that the biggest rectangle possible would be 12.62 by 8 cm.

5. Imagine a triangular prism:

   a. What shape is the plane section parallel to the base?

      Triangle

   b. What shape is a plane section perpendicular to the base? Various rectangles

   c. If the area of the plane section in "b" is $6.25\ \text{cm}^2$ and the height of the prism 5 cm. What was the length of the cut? 1.25 cm

6. Imagine cutting a square based right pyramid parallel to the base.

   a. What shape is the plane section?

      Square

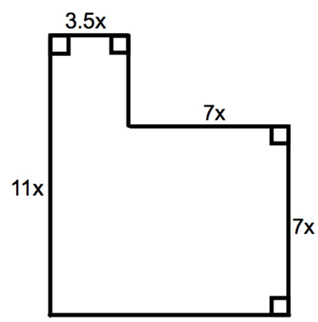
   b. If the dimensions of the length and the width of the plane section are $\frac{3}{2}$ in. and $\frac{3}{2}$ in., what is the area of the plane section?

      $A = \frac{9}{4}\ \text{in}^2$

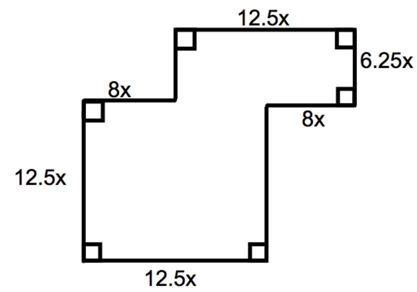
Spiral Review

1. Find the perimeter and area of each figure below:

a.



b.



2. Ten percent of the population is left-handed. Otto believes left-handed people have an advantage in boxing. If he observes 3 people boxing, describe a simulation that would show the probability one is left-handed.

3. Solve and graph the following inequality: $-97 \geq 4(19-2x)+3$ $x \geq 22$

4. Solve $-17+5$ -12

5. Angel owes his mom \$124. Angel made two payments of \$41 to his mom. How much does Angel now owe his mother?

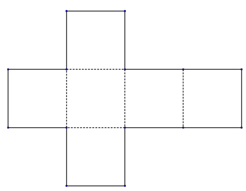
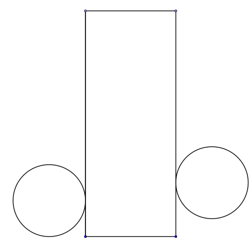
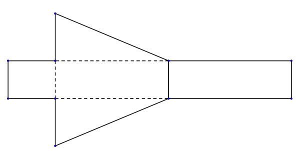
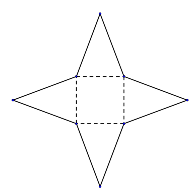
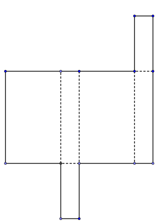
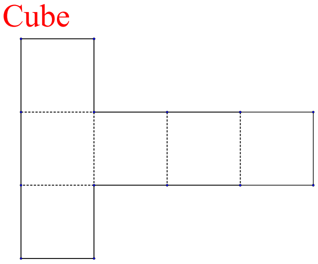
$-124+(41 \cdot 2)=-42$

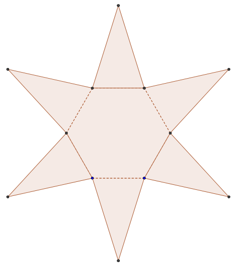
8.2c Class Activity: Nets of 3D Objects

Right rectangular prisms are first introduced/explored in 5th Grade (5.MD.C.5). Students find volume in 6th Grade (6.GA.2).

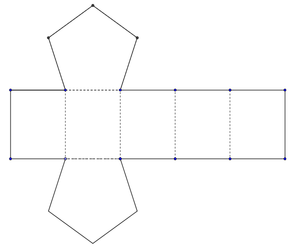
Review from 6th grade: A net is a two dimensional figure that can be folded to make a three dimensional object. You can learn about a 3D object by examining its net.

Look at each net below. Determine if the net can be folded into a three-dimensional object for which you could make equal parallel planar cuts (e.g. opposite sides are the same). Remember, you can orient objects any way you want. If it does not allow for equal parallel planar cuts, explain why.

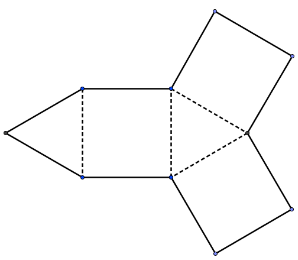
NET	NET
 1. Can you create equal parallel plane sections? Yes. Square base. All faces are squares, so this can be turned any direction.	 Discuss with students that nets show the entire surface area of a 3D figure and how surface area is different than volume. 2. Can you create equal parallel plane sections? Yes. Circular base. This object must be oriented with circles as the base.
 3. Can you create equal parallel plane sections? Yes. This prism must be oriented with the triangle as the base. This one is challenging.	 4. Can you create equal parallel plane sections? No matter how the object is oriented, there is no way to make parallel cuts that result in plane sections that are the same.
 5. Can you create equal parallel plane sections? Yes, any face can serve as the base.	Cube  6. Can you create equal parallel plane sections? Yes, any face can serve as the base.



7. Can you create equal parallel plane sections? No. No matter how the object is oriented, there is no way to make parallel cuts that result in plane sections that are the same.



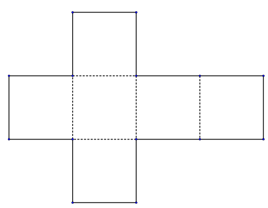
8. Can you create equal parallel plane sections? Yes. The base muse be the pentagon so that all the parallel plane sections are the same size and shape.



9. Can you create equal parallel plane sections? Yes. The base muse be the triangle so that all the parallel plane sections are the same size and shape.

Notes: You may want to enlarge the nets and allow students to put them together as solids.

10. The net to the right forms a cube.



a. If each square face of the cube has a side length of 4.2 cm, what is the surface area of the cube? 105.84 cm^2

b. If each square face of the cube has a length of 4.2 cm, what is the volume of the cube? 74.088 cm^3

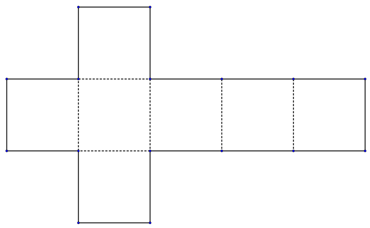
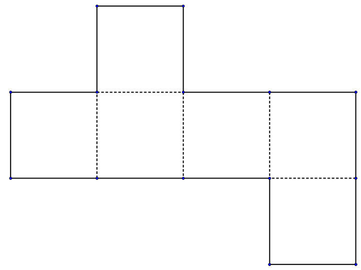
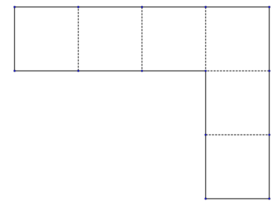
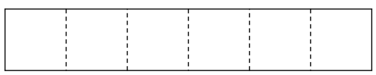
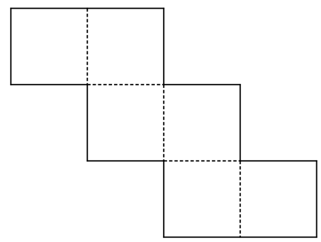
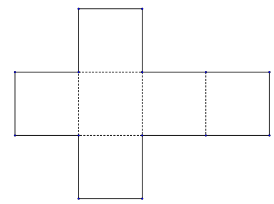
You might talk with students about why the numeric value for surface area is larger than the numeric value for volume. A good way to do this is to build and discuss a 3×3×3 cube.

c. If you created a new cube with edge lengths that are 150% of the original cube (edge lengths of 4.2 cm), what would the new surface area and volume be? SA = 238.14 cm^2 ; V = 250.047 cm^3
There are two ways students might think about this: Method 1: 1.5(4.2 cm) = 6.3 cm; so the new surface area is $6(6.3\text{ cm})^2 = 238.14\text{cm}^2$ and the new volume is $(6.3\text{ cm})^3 = 250.047\text{cm}^3$. Method 2: new area is old area multiplied by scale factor squared: $105.84(1.5)^2 = 238.17$ and new volume is old volume multiplied by scale factor cubed: $74.088(1.5)^3 = 250.047\text{cm}^3$.

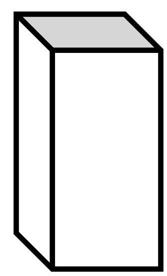
d. If you created a new cube with edge lengths that are 70% of the original cube (edge lengths of 4.2 cm), what would the new surface area and volume be? $SA = 0.7^2(105.84\text{ cm}^2) = 51.861\text{cm}^2$;
$V = 0.7^3(74.088\text{ cm}^3) = 25.4122\text{ cm}^3$

8.2c Homework: Nets of 3D Objects

1. Which of the following nets make a cube? b, e, f

a.	b	c
		
d.	e.	f.
		

2. Draw a net for the square-based rectangular prism to the right:



3. Suppose the prism in # 2 above has base edges of $8 \times 8 \times 12$ inches. Find the surface area and volume of the prism. Show all your work.

   SA = $4(8\times12) + 2(8\times8) = 512$ in^2
   V = $12(8\times8) = 768$ in^3

4. Still using the prism above, find the new dimensions of the prism if you scaled it by 3/2. Then find the new surface area and volume.

   New dimensions: $8(3/2)=12$, $12(3/2)= 18$, $12\times12\times18$ inches
   New SA = $512(3/2)^2$ in^2 = 1,152 in^2
   New V = $768(3/2)^3$ in^3 = 2,592 in^3

5. Still using the prism above, find the new dimensions of the prism if you scale it by 2/3. Then find the new surface area and volume.

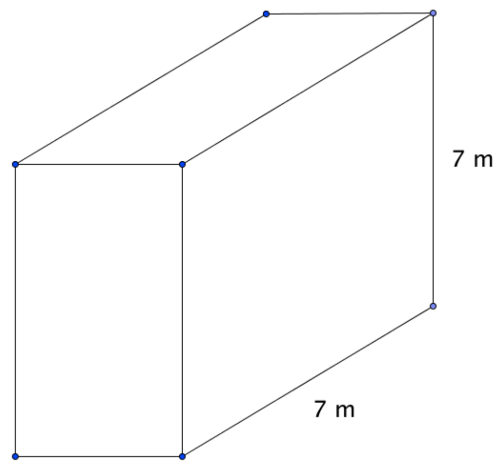
   New dimensions: $8(2/3) = 16/3$, $12(2/3) = 8$, $(16/3) \times (16/3) \times (8)$ inches
   New SA = $512(2/3)^2$ in^2 = 2,048/9 in^2
   New V = $768(2/3)^3$ in^3 = 6144/27 in^3

Spiral Review

1. Solve:
   a. $3-9$ -6
   b. $47+(-65)$ -18

2. On a map of New York City, with a scale of 1 inch= ½ mile, Central Park is 1 inch wide and 5 inches long. What is the area of the park? $0.5 \text{ mi} \times 2.5 \text{ mi} = 1.25 \text{ mi}^2$

3. Find the surface area and volume of the following rectangular prism: SA = 210 m^2; V = 196 m^3



4. Find the additive inverse and multiplicative inverse of each of the following numbers:

Number	Additive Inverse	Multiplicative Inverse
2	–2	½ or 0.5
$\frac{17}{7}$	$\frac{-17}{7}$	$\frac{-7}{17}$
–2.15	2.15	1 / 2.15 (approx. 0.465)

5. Given the following table, find the indicated unit rate:

__15___ push-ups per day

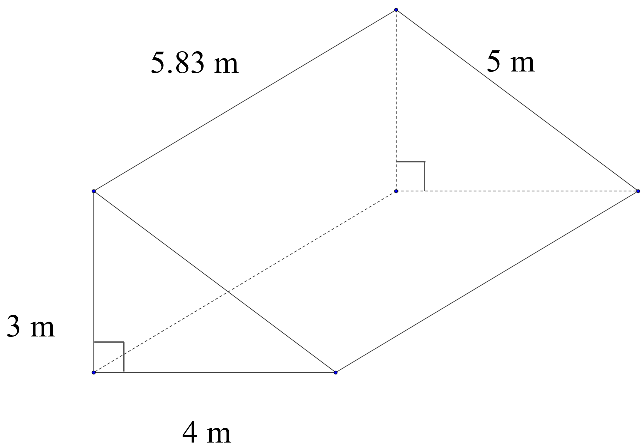
Days	Total Push-ups
2	30
4	60
29	435

8.2c Additional Practice: Surface Area and Volume

1. Explain your procedure for finding surface area.

2. Explain your procedure for finding volume.

3. Find the surface area and volume of the right prism below. Note that the triangular bases are right triangles with the right angle occurring where the 4m and 3m sides meet.



SA = 81.96 m^2

V = 0.5(4)(3)(5.83) m^3 = 34.98 m^3

Find the surface areas and volume for each. You may wish to sketch the figure. Show work. Include units.

Shape and Dimensions	Surface Area	Volume
4. Rectangular prism: • length 10 in • width 8 in • height 5 in.	SA = 340 in^2	V = 400 in^3
5. Rectangular prism: • length 4 feet • width 2 feet • height 6.2 feet	SA = 90.4 ft^2	V = 49.6 ft^3

6. If the base area of a right rectangular prism is 28 cm^2 and the volume is 177.8 cm^3, what is the height of the prism? 6.35cm

7. If the height of a square based prism is 13 in and its volume is 637 in^3, what is the length of each side of the base? The square is 7 in × 7 in

8. Give three possible edge lengths for a prism of volume 96 m^3. Answers will vary. The product of the three lengths must be 96. Encourage students to come up with non-whole number values.

8.2d Class Activity: Growing and Shrinking Stuff

For each of the following problems, find the necessary measurements to answer all the questions.

1. A cube with side lengths of 8 centimeters is enlarged so its side lengths are now 24 centimeters.
   a. By what scale factor did the side lengths increase? Show your work or justify your answer.

      3 times

   b. By what scale factor will the surface area increase? Show your work or justify your answer.

      9 times

   c. By what scale factor did the volume increase? Show your work or justify your answer.

      27 times

2. A rectangular prism with side lengths of 5 yards, 10 yards, and 15 yards is reduced by a scale factor resulting in new sides of 1 yd, 2 yd, and 3 yd.

   a. What was the scale factor for the reduction? 1/5

   b. What would be the new surface area and volume for the prism? SA = 2(1)(2) + 2(2)(3) + 2(1)(3) = 22 yd^2. V = 1(2)(3) = 6 yd^3

   c. By what scale factor did the surface area and volume each change? SA changed by 1/25 and V by 1/125.

3. A rectangular prism with side lengths 2 mm, 3 mm, and 5 mm is enlarged by some scale factor. The new volume of the prism is 240 mm^3.

   a. By what scale factor were the side lengths increased?
      Scale factor of 2. Notice $2 \times 3 \times 5 \times x^3 = 240$; $30x^3 = 240$; $x^3 = 8$. So students are looking for a number that they can cube to get 8. With number sense, they should reason that the scale factor was 2.

   b. What is the new length of each of the sides? 4 mm, 6 mm, 10 mm.

   c. What is the new surface area of the prism? 2(4)(6) + 2(6)(10) + 2(4)(10) = 248 mm^2

8.2d Homework: Growing and Shrinking Stuff

For each of the following problems, find the necessary measurements to answer all the questions.

1. A mini cereal box has the following dimensions: 4.5 in by 6 in by 2 in.
   a. If all the dimensions are doubled, will it require double the amount of cardboard to make the box? Why or why not?

      No, it's surface area so we need 4 times as much cardboard (2×2)

   b. If all the dimensions are doubled, will it hold double the amount of cereal? Why or why not?

      No, it's volume so it will hold 8 times as much $(2 \times 2 \times 2)$

   c. If *one* of the dimensions is doubled, will it require double the amount of cardboard to make the box? Why or why not?

      No, the original surface area is $2(4.5)(6) + 2(6)(2) + 2(4.5)(2) = 96 \text{ in}^2$. If we double just one side we get:
      $2(9)(6) + 2(6)(2) + 2(9)(2) = 170 \text{ in}^2$ or
      $2(4.5)(12) + 2(12)(2) + 2(4.5)(2) = 174 \text{ in}^2$ or
      $2(4.5)(6) + 2(6)(4) + 2(4.5)(4) = 138 \text{ in}^2$
      In other words, the surface area changes depending on which length is doubled. Notice that the new box for any doubled side length is not a scaled version of the original.

   d. If one of the dimensions is doubled, will it hold double the amount of cereal? Why or why not?

      Yes, it will hold double the amount of cereal. Students will likely use a picture to explain. A simple mathematical explanation is that we are using the properties of arithmetic (multiplication): $2 \times (4.5 \times 6 \times 2) = (2 \times 4.5) \times (6 \times 2) = 2 \times (4.5 \times 6) \times 2$. We can also change the order (commute.)

2. A container of chocolate milk mix has the following dimensions: a square base with sides of 6 in. and height 9 in.
   a. If all the dimensions are reduced by a scale factor of $\frac{1}{3}$, will it require a third the amount of materials to make the container? Why or why not?

      No, we need $\frac{1}{9}$ times as much material e.g. (1/3)(1/3)

   b. If all the dimensions are reduced by a scale factor of $\frac{1}{3}$, will it hold a third the amount of chocolate milk mix? Why or why not?

      No, it will only hold $\frac{1}{27}$ of the original volume

   c. If the length of each side of the base is reduced by a scale factor of $\frac{1}{3}$, how much surface area and volume will it now have?

      $SA = 2(2)(2) + 4(2)(9) = 80 \text{ in}^2$
      $V = 2 \times 2 \times 9 = 36 \text{ in}^3$

   d. What is the percent change of surface area and volume for "c"?

      SA→ original SA was $2(6)(6) + 4(6)(9) = 288 \text{ in}^2$. $(288 – 80)/288 \approx 72.22\%$ reduction.
      VA→original V was $6 \times 6 \times 9 = 324 \text{ in}^3$. $(324 – 36)/324 \approx 88.889\%$ reduction.

3. Refer back to the mini cereal box with dimensions, 4.5” by 6” by 2”.
   a. If all the dimensions are increased by 4 inches to: 8.5” by 10” by 6” how does this change the amount of cardboard needed to make the box?
      Originally, the surface area was 2(4.5)(6) + 2(6)(2) + 2(4.5)(2) = 96 in^2. Now the SA is 2(8.5)(10) + 2(10)(6) + 2(8.5)(6) = 392 in^2
      So, (392 – 96)/96 ≈ 3.0833 = 308.33% increase.

   b. If all the dimensions are increased by a value of 4 inches, as described above, how will this change the amount of cereal the box can hold?
      Originally the volume was 4.5×6×2 = 54 in^2
      Now the volume is 8.5×10×6 = 510 in^3
      So, (510 – 54)/54 ≈ 8.4444 = 844.44% increase

Spiral Review

1. Mrs. Zamora will not tell you how the class in general did on the last test. You really want to know how you compare, so you survey 10 random students out of 35. Estimate the average score for your class and describe now far off the estimate might be.

77	89	79	100	94
100	80	81	88	87

2. Draw and describe the plane section that results from the following cut: a cylinder cut parallel to the base.

3. Katherine is visiting patients in a hospital. She visits 9 patients in 3 hours. She visits 18 patients in 6 hours. Is this situation proportional? If so, what is the unit rate? Yes. She visits 3 patients per hour.

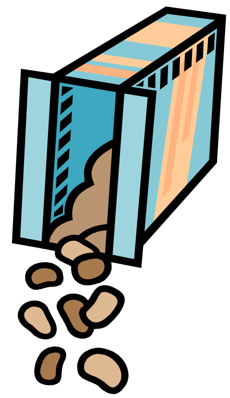
4. A candidate for Congress is trying to decide what to focus her campaign on in order to win the election. Describe how she might obtain a random sample to know what issues are important to her constituents. Answers will vary. She may obtain a list of registered voters and use a computer to select a random sample.

5. Kurt puts 70% of his earnings into his savings. Write and solve an equation to find how much money he earned if he had $165 to spend.
   $x(1-0.7)=165$
   $0.3x=165$ He earned $550.
   $x=550$

8.2e Project: Packing Packages

Have you ever notice that cereal generally comes in tall thin boxes and that laundry soap generally comes in short wide boxes? Why do you think they come as they do?

Think about your experiences with perimeter, surface area, and with volume in this chapter. What kind of box do you predict might hold the most and take the least amount of cardboard? Explain your thinking.



Suppose your favorite cereal comes in a box that is 24 cm. high, 20 cm. long, and 6 cm. wide. This box of cereal costs $4.35.

1. Draw a model of the box. Find the surface area and volume for the box. Show work and label answers

SA = 1488 cm^2
V = 2880 cm^3

The 24 cm × 20 cm × 6 cm dimensions of the box can also be thought of as a height of 24 cm and a girth (distance around the box) of 52 cm. If we add these two measures (height and girth), we get 76 cm.

For this project you will:

a. Draw a model of a box with a total girth plus height of no more than 76 cm that holds the most cereal possible with the least surface area possible.

b. Build the box as described in "a".

c. Provide a table, graph or spreadsheet to show how you arrived at your dimensions (remember: use only height and girth for your table, graph, or spreadsheet.)

d. Find the percent decrease in surface area for your new box from the original box.

e. Find the percent increase in volume for your new box from the original box.

f. Write two paragraphs about your project. In the first paragraph, state and justify how much you would charge for cereal in this box. In the second paragraph, explain why you think cereal does not come in the type of box you designed.

8.2f Self-Assessment: Section 8.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Describe the type of plane sections of right prisms and pyramids that result from different cuts.	I struggle to know or describe the type of plane section that will result from different cuts of right prisms and pyramids regardless of how I try to do it.	I know and can describe the type of plane section that will result from different cuts of right prisms and pyramids if I'm able to use objects or apps to make cuts.	I know and can describe the type of plane section that will result from different cuts of right prisms and pyramids with and without manipulatives/apps.	I know and can describe the type of plane section that will result from different cuts of right prisms and pyramids. I can explain how the plane section varies with different cuts.
2. Solve real world and mathematical problems involving volume and surface area of three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	I struggle to solve real world problems involving volume and surface area.	I can find volume and surface area of three-dimensional objects in context with some help.	I can solve real world and mathematical problems involving volume and surface area of three-dimensional objects.	I can solve real world and mathematical problems involving volume and surface area of three-dimensional objects, explain how I got my answer and why it is correct.
3. Find the scale factor and/or percent change in surface area and/or volume when dimensions are changed.	I struggle to find scale factor and/or percent change between measurements.	I can find scale factor and/or percent change between measurements with some help.	I can find scale factor and/or percent change between measurements.	I can find scale factor and/or percent change between measurements, explain my procedure and why my answer is correct.

Sample Problems for Section 8.2

1. Draw and describe the plane section that results from the following cuts.
   a. A right rectangular prism is cut parallel to the base.

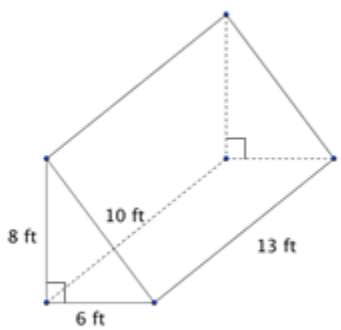
   b. A cube is cut perpendicular to the base and parallel to a face.

   c. A right pyramid is cut parallel to the base.

   d. Describe how each cut above is similar and different from other similar cuts for the same object.

2. Answer each question about volume and surface area.

   a. Find the volume and surface area of the triangular prism below.



   b. Find the volume and surface area of a right rectangular prism with dimensions 8 cm by 15 cm by 2 cm.

   c. If a right rectangular prism with dimensions 8 cm by 15 cm by 2 cm is scaled by a factor of 3, what would the new volume and surface area be?

   d. What is the length of a box with a surface area of 472 cm^2 and a width of 11 cm and a height of 6 cm?

   e. Ellie is filling a sandbox for her son. The sandbox dimensions are 4 ft by 4 ft by 1 ft. How many cubic feet of sand does she need to buy to fill the sandbox?

3. The dimensions of Hans' turtle aquarium are 18 in by 18 in by 12 in. The dimensions of Anna's rabbit cage are all doubled.
   a. Find the surface area and volume of each animal habitat.
   b. Find the scale factor between the surface areas of each container.
   c. Find the scale factor between the volumes of each container.
   d. In general, how are the scale factors of surface areas related? Volumes related?

Play Dough Recipe

Ingredients:

- 2 C flour
- 2 C warm water (food coloring is optional)
- 1 C salt
- 2 T oil
- 1 T cream of tartar

Directions:

In a medium pot, add water and oil together and stir. Then add the flour, salt, and cream of tartar. Keep stirring until all the ingredients are blended together and the mixture is not sticking to the sides of the pot. Then knead the mixture. For storage, keep play dough in plastic bags or sealed containers.