

Table of Contents

CHAPTER 10 GEOMETRY: ANGLES, TRIANGLES, AND DISTANCE (3 WEEKS)	2
10.0 ANCHOR PROBLEM: REASONING WITH ANGLES OF A TRIANGLE AND RECTANGLES	6
10.1 ANGLES AND TRIANGLES	8
10.1a Class Activity: <i>Straight and Vertical Angles Review</i>	9
10.1a Homework: <i>Straight and Vertical Angles Review</i>	11
10.1b Class Activity: <i>Special Angles Formed by Transversals</i>	12
10.1b Homework: <i>Special Angles Formed by Transversals</i>	16
10.1c Class Activity: <i>Parallel Lines and Transversals</i>	18
10.1c Homework: <i>Parallel Lines and Transversals</i>	22
10.1d Class Activity: <i>Tessellating Triangles</i>	25
10.1d Homework: <i>Finding Angle Measures in Triangles</i>	29
10.1e Class Activity: <i>Similar Triangles</i>	31
10.1e Homework: <i>Similar Triangles</i>	34
10.1f Self-Assessment: <i>Section 10.1</i>	36
SECTION 10.2 THE PYTHAGOREAN THEOREM	39
10.2a Class Activity: <i>A Proof of the Pythagorean Theorem</i>	40
10.2a Homework: <i>A Proof of the Pythagorean Theorem</i>	46
10.2b Class Activity: <i>The Pythagorean Theorem and Tilted Squares</i>	48
10.2b Homework: <i>The Pythagorean Theorem and Tilted Squares</i>	50
10.2c Class Activity: <i>The Pythagorean Theorem and Unknown Side Lengths</i>	51
10.2c Homework: <i>The Pythagorean Theorem and Unknown Side Lengths</i>	53
10.2d Class Activity: <i>The Converse of the Pythagorean Theorem</i>	56
10.2d Homework: <i>The Converse of the Pythagorean Theorem</i>	60
10.2e Class Activity: <i>Exploration with Pythagorean Triples Extension</i>	61
10.2f Self-Assessment: <i>Section 10.2</i>	62
SECTION 10.3 APPLICATIONS OF THE PYTHAGOREAN THEOREM	68
10.3a Class Activity: <i>Applications of the Pythagorean Theorem</i>	69
10.3a Homework: <i>Applications of the Pythagorean Theorem</i>	73
10.3b Class Activity: <i>Finding Distance Between Two Points</i>	77
10.3b Homework: <i>Finding Distance Between Two Points</i>	82
10.3c Extension: <i>Construction</i>	84
10.3d Self-Assessment: <i>Section 10.3</i>	87

Chapter 10 Geometry: Angles, Triangles, and Distance (3 weeks)

Utah Core Standard(s):

- Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. (8.G.5)
- Explain a proof of the Pythagorean Theorem and its converse. (8.G.6)
- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (8.G.7)
- Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (8.G.8)

Academic Vocabulary:

right triangle, right angle, congruent, leg, hypotenuse, Pythagorean Theorem, converse of Pythagorean Theorem, simplest radical form, Pythagorean triple, rectangular prism, cube, unit cube, distance formula, vertical angles, adjacent angles, straight angles, supplementary, congruent, parallel lines, $\parallel$, transversal, vertex, point of intersection, corresponding angles, alternate interior angles, alternate exterior angles, similar, angle-angle criterion for triangles

Chapter Overview:

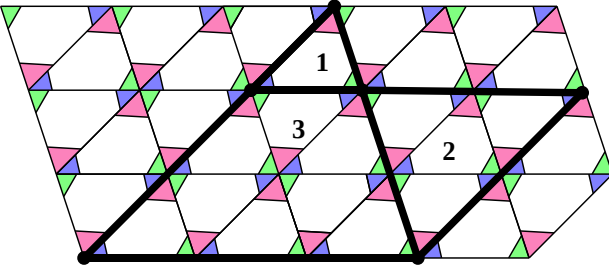
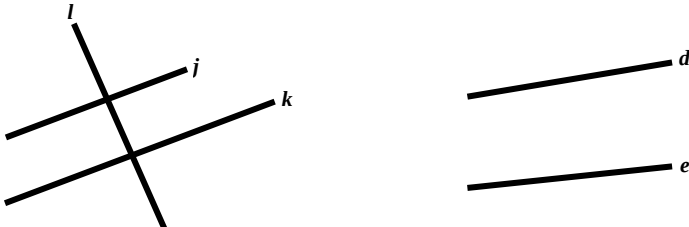
This chapter centers around several concepts and ideas related to angles and triangles. In the first section, students will study theorems about the angles in a triangle, the special angles formed when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. They will apply these theorems to solve problems. In Sections 2 and 3, students will study the Pythagorean Theorem and its converse and realize the usefulness of the Pythagorean Theorem in solving many real-world problems. In this chapter, we are referring to these theorems as a collection of facts. The focus in 8th grade is that students are able to observe these facts through examples, exploration, and concrete models. Students will explain why the theorems are true by constructing mathematical arguments, relying on knowledge acquired throughout the year, particularly the properties of rigid motion and dilations and the understanding of congruence and similarity. The explanations and arguments made by students will come in many different forms, including a bulleted list, a narrative paragraph, a diagram without words, and proof by example. They should give their arguments and explanations within their writing and speaking. The emphasis is on students starting to gain an understanding of what makes a good argument or explanation. Can they explain things in a number of different ways? Can they critique the reasoning of others? They should be asking themselves questions such as: What do I know? What is the question asking? Can I draw a model of the situation? Does my argument/explanation have a claim, evidence, and warrant? What is the connection? These practices engaged in by students set the foundation for a more formal study of proof in Secondary II.

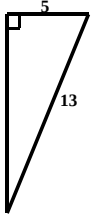
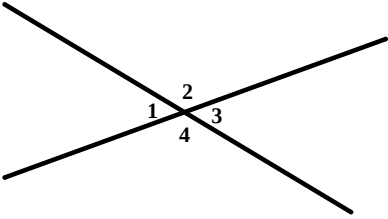
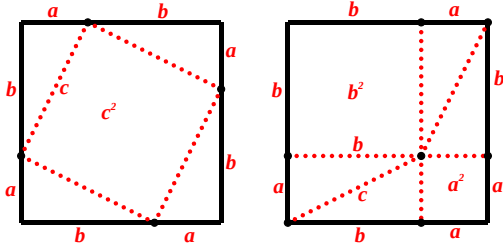
Connections to Content:

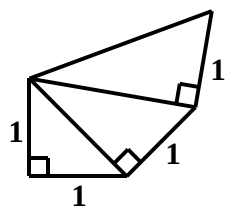
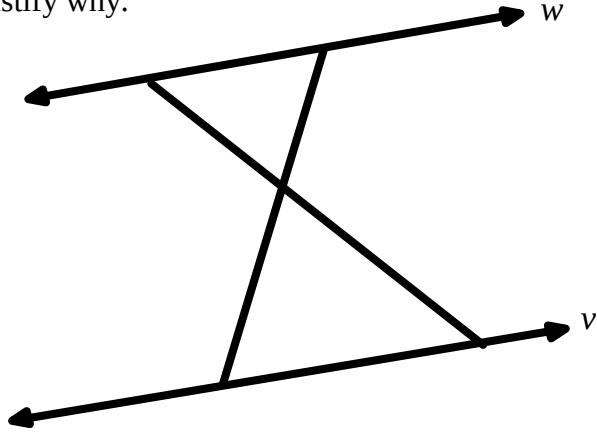
Prior Knowledge: In elementary grades, students have worked with geometric objects such as points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. They have also studied the different types of triangles (right, acute, and obtuse and equilateral, scalene, and isosceles). They have also learned and used facts about supplementary, complementary, vertical, and adjacent angles. In Chapter 9 of this text, students studied rigid motions and dilations and the definition of congruence and similarity.

Future Knowledge: In Secondary II, students will formally prove many of the theorems studied in this chapter about lines, angles, triangles, and similarity. They will also define trigonometric ratios and solve problems involving right triangles.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

	Make sense of problems and persevere in solving them.	 What is the relationship between the triangles formed by the dark lines? Justify your answer. <i>Students will use the concrete model shown above in order to make arguments about several of the theorems studied in this chapter. They will also rely on their knowledge of rigid motions and dilations.</i>
	Model with mathematics.	A new restaurant is putting in a wheelchair ramp. The landing from which people enter the restaurant is 2 feet higher than street level. Safety standards require that for every 1 foot of rise on a wheelchair ramp there must be a run of 12 feet. How long will the ramp have to be to meet safety standards? Round your answer to the nearest tenth. <i>Students will apply the Pythagorean Theorem in order to solve many real-world problems. They will have to analyze the situation to determine if the Pythagorean Theorem can be used to solve the problem, draw a picture of the situation, analyze givens and constraints, and understand what they are solving for.</i>
	Construct viable arguments and critique the reasoning of others.	Suppose you are given two lines j and k in the picture below. You have been asked to determine whether the two lines are parallel. You start by drawing the transversal l through the two lines as shown below. Devise a strategy to determine whether the two lines are parallel using what you know about the properties of rigid motion. Next, use your strategy to determine whether lines d and e are parallel. Just saying they do not look parallel, is not a justification.  <i>Throughout the chapter, students will observe theorems about angles and triangles by example, exploration, and concrete models. Students will construct mathematical arguments as to why the theorems are true, relying on knowledge acquired throughout the year, particularly the properties of rigid motion and dilations and the understanding of congruence and similarity. Students will begin to understand the necessary elements of what makes a good proof as outlined in the chapter overview.</i>

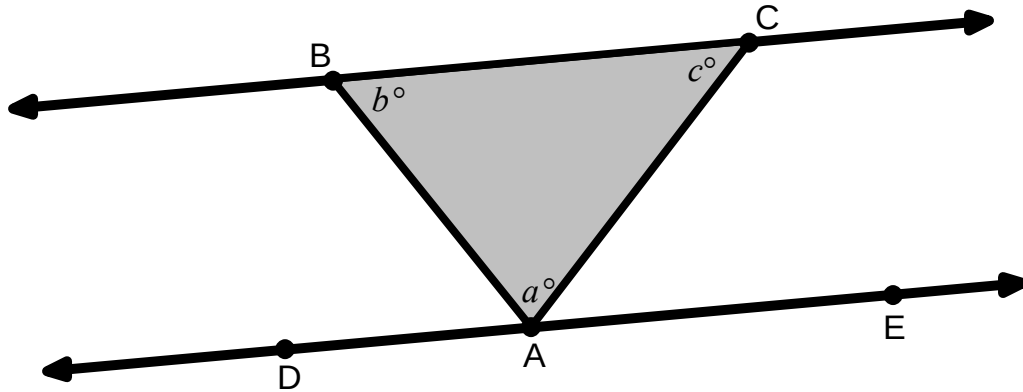
	Attend to Precision	Find, Fix, and Justify: Megan was asked to solve for the unknown side length in the triangle below. Her work is shown below. She made a mistake when solving. Explain the mistake she made and then solve the problem correctly.  <u>Megan's Solution:</u> $a^2 + b^2 = c^2$ $5^2 + 13^2 = c^2$ $25 + 169 = c^2$ $194 = c^2$ $\sqrt{194} = c$ <i>This problem requires that students are clear in their understanding of the Pythagorean Theorem and how to use it to solve for missing side lengths.</i>
	Use appropriate tools strategically.	 Use ideas of rigid motion to prove that $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. Students will rely heavily on the knowledge learned in Chapter 9 about rigid motions and congruence and dilations and similarity. This knowledge will be a tool they apply to understand and informally prove many of the theorems about angles, triangles, and similarity in this chapter.
	Reason abstractly and quantitatively.	 Using the picture above, prove that the sum of the areas of the squares along the two smaller sides of the right triangle equals the area of the square along the larger side of the triangle for <u>any</u> right triangle. <i>Students first begin to study and understand the Pythagorean Theorem using concrete examples. Then, they move to an abstract proof of the Pythagorean Theorem to show that it holds true for any right triangle.</i>

	Look for and express regularity in repeated reasoning.	Use the picture below to answer questions a) and b).  a. Find all the missing side lengths and label the picture with the answers. b. Using the picture above, devise a strategy for constructing a segment with a length of $\sqrt{5}$. Explain your strategy below. <i>In this problem, students should start to notice that the hypotenuse of the new triangle will follow a pattern. This observation gives them a process for constructing any segment of length $\sqrt{n}$ where n is a whole number.</i>
	Look for and make use of structure.	Given that line $w \parallel$ line v, determine if the triangles given below are similar. If they are similar justify why.  <i>In the problem above students must look at the geometric figure above and evaluate the information given to them. They are given that line $w \parallel$ line v. They must recognize that the two intersecting lines that form the triangles are transversals of the parallel lines. Students might do this by extending the transversals beyond the interior of the two parallel lines or by drawing an auxiliary line over these lines that extends beyond the parallel lines. Once they look at these lines as transversals they can use what they know about special angle relationships to determine congruence amongst angles within the triangles. As students view the structure of the intersecting lines their perspective shifts and they are able to derive more information about the figure.</i>

10.0 Anchor Problem: Reasoning with Angles of a Triangle and Rectangles

Part I

Given that $\overrightarrow{BC} \parallel \overrightarrow{DE}$ in the picture below, show that $a + b + c = 180^\circ$.



There are many arguments and explanations that a student can use to show that the interior angles of the triangle sum to 180° . To help students construct their explanations ask them probing questions. What information do I know? What is the question that is being asked? What can I infer from the picture? What other information can I label on the picture? As they make claims ask them to give support and warrants for their claims. For example, if they claim that $\angle DAB = b$ ask them what support they can give for that claim (i.e. how do you know?) The angles are alternate interior angles formed by a transversal that intersects parallel lines.

A sample argument is given below:

In the picture above $\overrightarrow{BC} \parallel \overrightarrow{DE}$ and $\overrightarrow{BA}$ is a transversal of these lines, thus $\angle DAB = b$ because they are alternate interior angles. Similarly, $\angle CAE = c$ by the same argument. Also, $\angle DAB + \angle BAC + \angle CAE = 180^\circ$ since they form a straight angle. By substitution, $180^\circ = \angle DAB + \angle BAC + \angle CAE = b + a + c$. We can conclude that $a + b + c = 180^\circ$.

Part II

Pedro's teacher asks him to classify the quadrilateral below. He claims it is a rectangle. His teacher tells him to give a good argument and explanation. Help Pedro to support his claim using mathematical evidence.

Remember: Opposite sides of a rectangle have the same length and are parallel and the sides of a rectangle meet at right angles.



There are many ways that a student can argue or prove that the given quadrilateral is a rectangle on the coordinate plane. See the illustrative mathematics tasks at the links at the bottom of the page for a more comprehensive list of possible arguments.

Figure 1

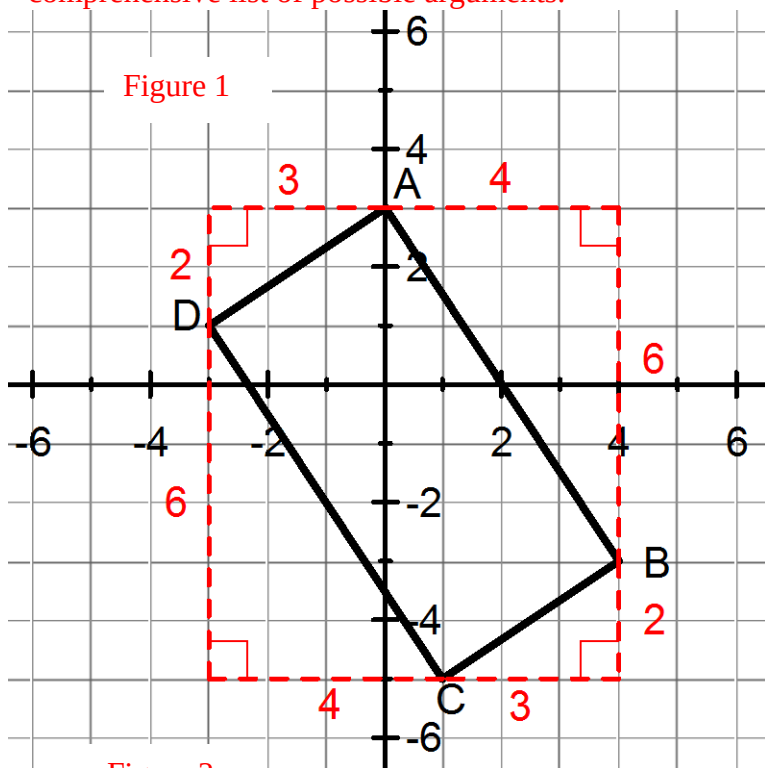
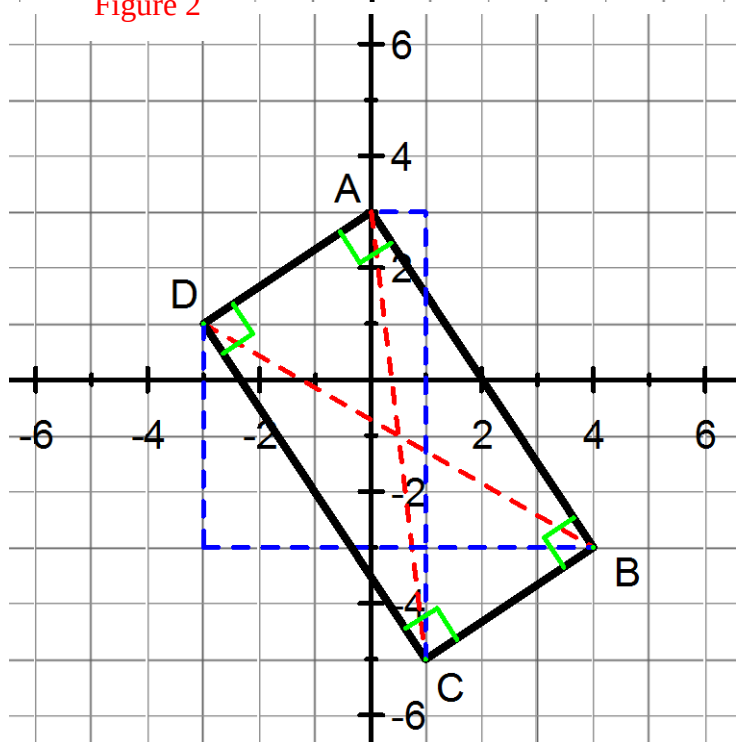


Figure 2



Using the Pythagorean Theorem to find the length of each segment: (Figure 1)

To find the length of each segment of the quadrilateral students can draw a right triangle where each segment is the hypotenuse. They can then use the PT(Pythagorean Theorem) to find the length of each segment. $AB = CD = 2\sqrt{13}$ and $AD = BC = \sqrt{13}$.

Using Slope to show right angles:

The slopes of $\overline{AB}$ and $\overline{CD}$ are both $-\frac{3}{2}$. Since the slopes are the same, the segments are parallel. Similarly, $\overline{AD}$ and $\overline{BC}$ have the same slope which is $\frac{2}{3}$. The product of the slopes of adjacent sides is -1 (slopes are opposite reciprocals) which means the segments are perpendicular or meet at a right angle.

Using the Converse of the PT (Pythagorean Theorem) to show right angles: (Figure 2)

Students might also draw the diagonals of the rectangle (shown in red). They can then draw a right triangle off of each of these diagonals where the diagonals are the hypotenuse of these right triangles (shown in blue). They can use the PT to find the length of these diagonals. Once they have found the length of the diagonals they can use the lengths of the sides of the rectangle, as found above, and the Converse of the PT to show that the diagonals form right triangles. This means that the adjacent sides of the quadrilateral meet at right angles (shown in green).

Since opposite sides are parallel and have the same length and the quadrilateral has 4 congruent angles (all equal to 90°), the quadrilateral is a rectangle.

<https://www.illustrativemathematics.org/illustrations/1302>

<https://www.illustrativemathematics.org/illustrations/1245>

10.1 Angles and Triangles

Section Overview:

The focus of this section is on the development of geometric intuition through exploration with rigid motions and dilations. Through exploration, observation, and the use of concrete models, students will analyze facts about triangles and angles and use these facts to describe relationships in geometric figures. There will also be a focus on making sound mathematical explanations and arguments in order to verify theorems about angles and triangles and when explaining and justifying solutions to problems throughout the section.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Know that straight angles sum to 180° and that vertical angles are congruent.
2. Know that the sum of the angles in a triangle is 180° . Understand that the measure of an exterior angle of a triangle is equal to the sum of the measures of the non-adjacent angles. Use these properties to find missing angle measures related to a triangle.
3. Determine the relationship between angles formed when a transversal intersects parallel lines. Use these relationships to find missing angle measures.
4. Determine whether two lines are parallel based on the angle measures when a transversal intersects the lines.
5. Understand and apply the angle-angle criterion to determine whether two triangles are similar.

Remember to encourage students to ask the questions outlined in the chapter overview as they work through the sections in this chapter. You may even consider posting these questions in your classroom.

What do I know?

What is the question asking?

Can I draw a model of the situation?

Does my argument/explanation have a claim, evidence, and warrant?

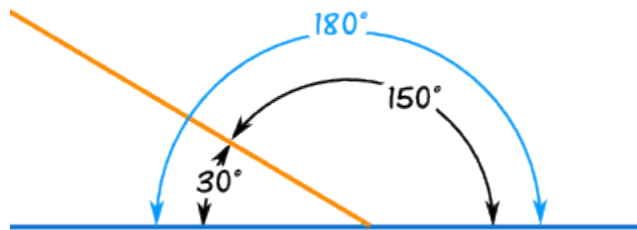
Note: The questions above are related to making a good argument and are in line with the English Language Arts Literacy and Speaking and Listening Standards.

10.1a Class Activity: Straight and Vertical Angles Review

In this section, you will observe and use several different geometric facts learned in previous grades. They will be denoted using bullets.

- **Angles that lie on the same line (straight angles) are supplementary.**

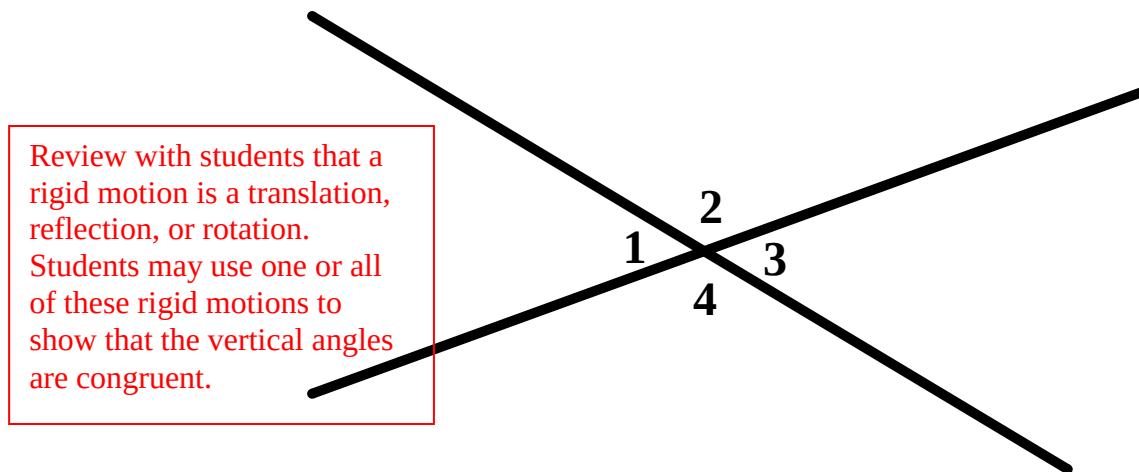
In 7th grade, you learned that a straight angle has a measure of 180° as shown below. Angles that sum to 180° are **supplementary**. In the picture below, 30° and 150° are supplementary and together they form a straight angle.



- **Vertical angles have the same measure.**

Vertical angles are the opposing angles formed by two intersecting lines.

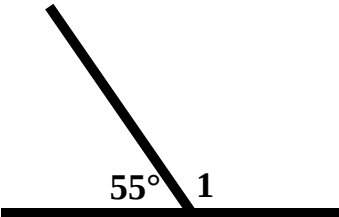
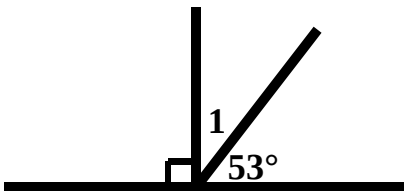
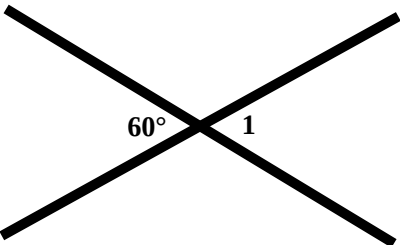
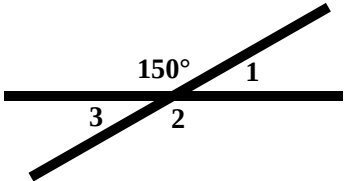
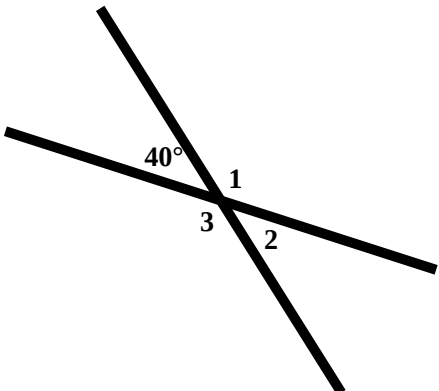
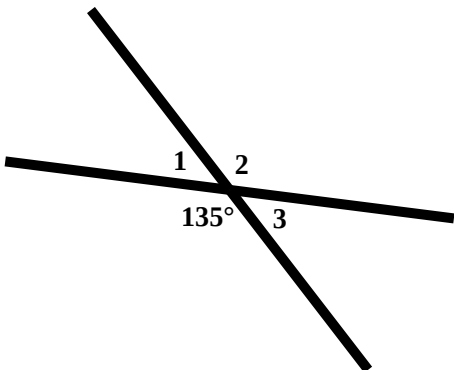
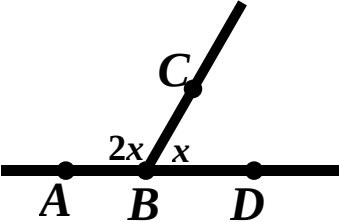
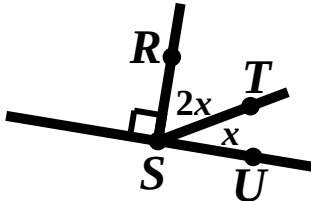
In the picture below, $\angle 1$ and $\angle 3$ are vertical angles and $\angle 2$ and $\angle 4$ are vertical angles.



Review with students that a rigid motion is a translation, reflection, or rotation. Students may use one or all of these rigid motions to show that the vertical angles are congruent.

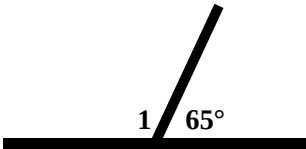
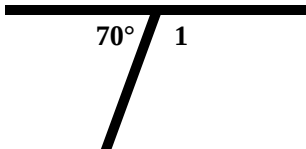
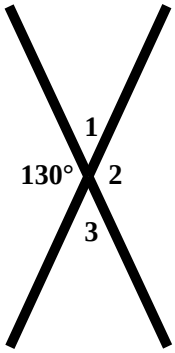
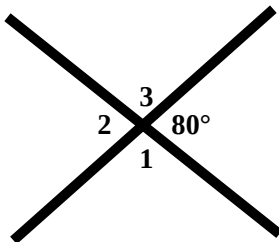
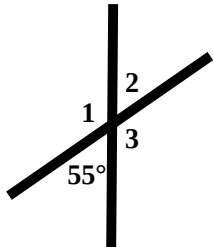
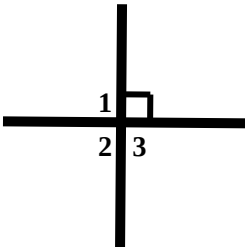
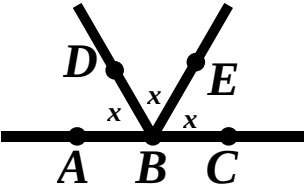
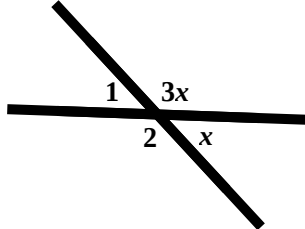
1. Show that $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. (Hint: Think about ideas of rigid motion and straight angles.) 
Students may trace the angles on patty paper and fold, reflecting angle 1 so that it sits on angle 3. They can use a similar method to map angle 2 to angle 4. Alternatively, students can copy angle 1 and rotate it 180° about the vertex to map it to angle 3. The same motion will map angle 2 to angle 4. Since one angle maps to the other using rigid motion, the angles are congruent. Alternatively, they can give an explanation without using rigid motions by saying, that since angles 1 and 3 are both supplementary to angle 2, they must have the same measure. They can set this up as an equation: $\angle 1 + \angle 2 = 180$ and $\angle 2 + \angle 3 = 180$; therefore $\angle 1 + \angle 2 = \angle 2 + \angle 3$. When we solve, we see that $\angle 1 = \angle 3$. They can use a similar process to prove $\angle 2 \cong \angle 4$.
2. Which pairs of angles are supplementary in the picture above?
 $\angle 1$ and $\angle 4$; $\angle 1$ and $\angle 2$; $\angle 2$ and $\angle 3$; $\angle 3$ and $\angle 4$

Review: Find the missing angle measures without the use of a protractor.

3.  $m\angle 1 = \underline{\hspace{1cm}125^\circ\hspace{1cm}}$	4.  $m\angle 1 = \underline{\hspace{1cm}37^\circ\hspace{1cm}}$
5.  $m\angle 1 = \underline{\hspace{1cm}60^\circ\hspace{1cm}}$	6.  $m\angle 1 = \underline{\hspace{1cm}30^\circ\hspace{1cm}}$ $m\angle 2 = \underline{\hspace{1cm}150^\circ\hspace{1cm}}$ $m\angle 3 = \underline{\hspace{1cm}30^\circ\hspace{1cm}}$
7.  $m\angle 1 = \underline{\hspace{1cm}140^\circ\hspace{1cm}}$ $m\angle 2 = \underline{\hspace{1cm}40^\circ\hspace{1cm}}$ $m\angle 3 = \underline{\hspace{1cm}140^\circ\hspace{1cm}}$	8.  $m\angle 1 = \underline{\hspace{1cm}45^\circ\hspace{1cm}}$ $m\angle 2 = \underline{\hspace{1cm}135^\circ\hspace{1cm}}$ $m\angle 3 = \underline{\hspace{1cm}45^\circ\hspace{1cm}}$
9.  $m\angle ABC = \underline{\hspace{1cm}120^\circ\hspace{1cm}}$ $m\angle CBD = \underline{\hspace{1cm}60^\circ\hspace{1cm}}$	10.  $m\angle RST = \underline{\hspace{1cm}60^\circ\hspace{1cm}}$ $m\angle RSU = \underline{\hspace{1cm}90^\circ\hspace{1cm}}$ $m\angle TSU = \underline{\hspace{1cm}30^\circ\hspace{1cm}}$

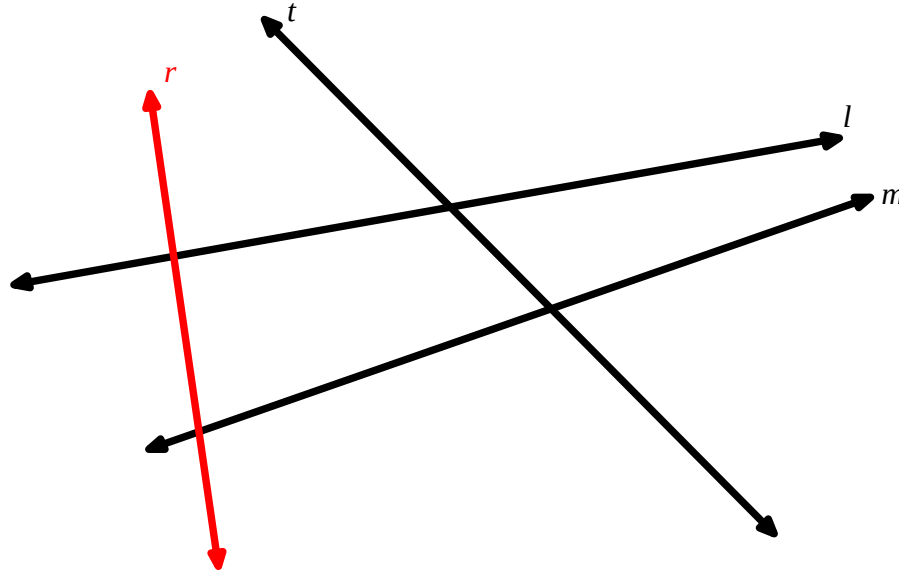
10.1a Homework: Straight and Vertical Angles Review

Review: Find the missing angle measures without the use of a protractor.

1.  $m\angle 1 = \underline{\hspace{1cm}}115^\circ\underline{\hspace{1cm}}$	2.  $m\angle 1 = \underline{\hspace{1cm}}110^\circ\underline{\hspace{1cm}}$
3.  $m\angle 1 = \underline{\hspace{1cm}}50^\circ\underline{\hspace{1cm}}$ $m\angle 2 = \underline{\hspace{1cm}}130^\circ\underline{\hspace{1cm}}$ $m\angle 3 = \underline{\hspace{1cm}}50^\circ\underline{\hspace{1cm}}$	4.  $m\angle 1 = \underline{\hspace{1cm}}100^\circ\underline{\hspace{1cm}}$ $m\angle 2 = \underline{\hspace{1cm}}80^\circ\underline{\hspace{1cm}}$ $m\angle 3 = \underline{\hspace{1cm}}100^\circ\underline{\hspace{1cm}}$
5.  $m\angle 1 = \underline{\hspace{1cm}}125^\circ\underline{\hspace{1cm}}$ $m\angle 2 = \underline{\hspace{1cm}}55^\circ\underline{\hspace{1cm}}$ $m\angle 3 = \underline{\hspace{1cm}}125^\circ\underline{\hspace{1cm}}$	6.  $m\angle 1 = \underline{\hspace{1cm}}90^\circ\underline{\hspace{1cm}}$ $m\angle 2 = \underline{\hspace{1cm}}90^\circ\underline{\hspace{1cm}}$ $m\angle 3 = \underline{\hspace{1cm}}90^\circ\underline{\hspace{1cm}}$
7.  $m\angle ABD = \underline{\hspace{1cm}}60^\circ\underline{\hspace{1cm}}$ $m\angle DBE = \underline{\hspace{1cm}}60^\circ\underline{\hspace{1cm}}$ $m\angle ABE = \underline{\hspace{1cm}}120^\circ\underline{\hspace{1cm}}$ $m\angle ABC = \underline{\hspace{1cm}}180^\circ\underline{\hspace{1cm}}$	8.  $m\angle 1 = \underline{\hspace{1cm}}45^\circ\underline{\hspace{1cm}}$ $m\angle 2 = \underline{\hspace{1cm}}135^\circ\underline{\hspace{1cm}}$

10.1b Class Activity: Special Angles Formed by Transversals

1. In the picture given below line l and line m are cut by a transversal line called t .



2. Define transversal in your own words. Draw another transversal for the two lines above and label it line r . **A transversal is a line that intersects two or more lines at different points.**

Lead a discussion with students to help them develop a definition for transversal. You could ask; what does this word sound like? What does it mean to transverse something?

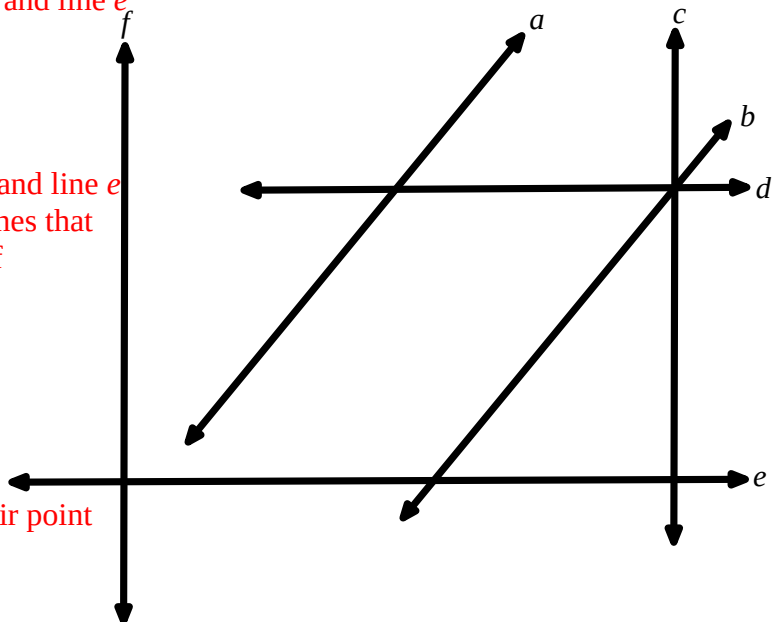
3. Some of the runways at a major airport are shown in the drawing below. Identify at least 2 sets of lines to which each line is a transversal.

- a. line a answers may include;
line d and line e , line f and line c , line f and line e ,
line c and line d , line d and line f , line c and line e

- b. line b answers may include;
line d and line e , line f and line c ,
line f and line e , line d and line f , line c and line e
(line c and line d do not work because lines that intersect two other lines at their point of intersection are not transversals.)

- c. line c answers may include;
line a and line b , line a and line e
line a and line d , line b and line e
line d and line e
(line b and line d do not work because lines that intersect two other lines at their point of intersection are not transversals.)

- d. line e answers may include;
line b and line a , line b and line f
line b and line c , line c and line a
line c and line f , line a and line f



Before you begin the activity below you might find it helpful to review the words in the table below with your class. Ask them; when you hear the following words what do you think of? What key words or ideas will help you remember what these words mean. Write the graphic organizer below together as a class on the board for students to refer to as they complete the activity.

Interior
Inside

Exterior
Outside

Corresponding
The same
location
Matching

Straight Angles
Angles that share a straight line.
A pair of angles that share a line.

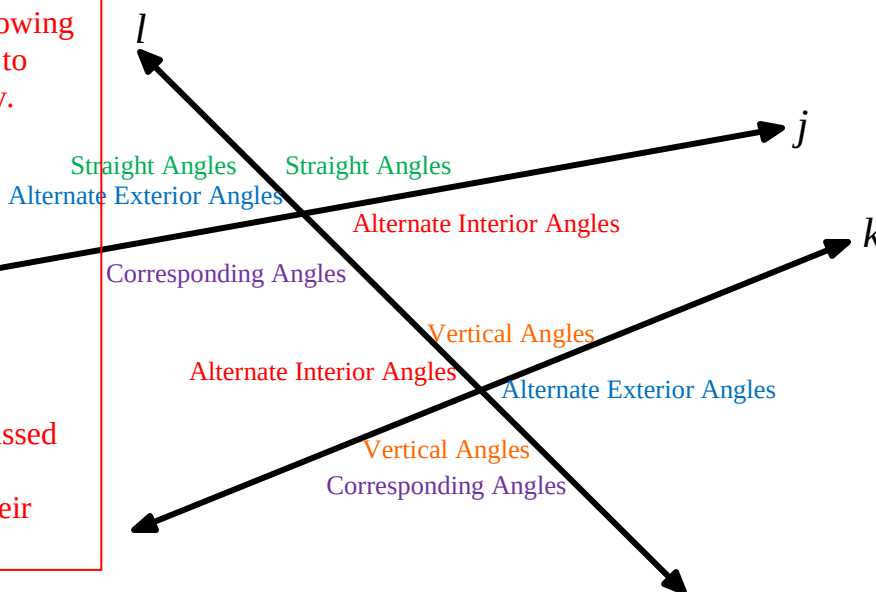
When two lines are intersected by a transversal there are special angle pairs that are formed. Use the angle names provided by your teacher to move the angle names around the picture below until you think you have found its correct location. Be ready to justify your reasoning. There will be several correct locations for each set of angle pairs and more than one term may fit at an angle.

Sample answers are shown.

Make several sets of the following angle pair names to pass out to groups to use for this activity.

Straight Angle/Linear Pair
Alternate Exterior Angles
Corresponding Angles
Alternate Interior Angles
Vertical Angles

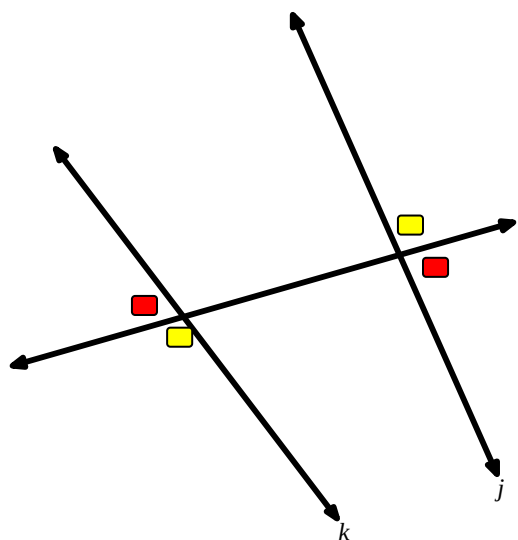
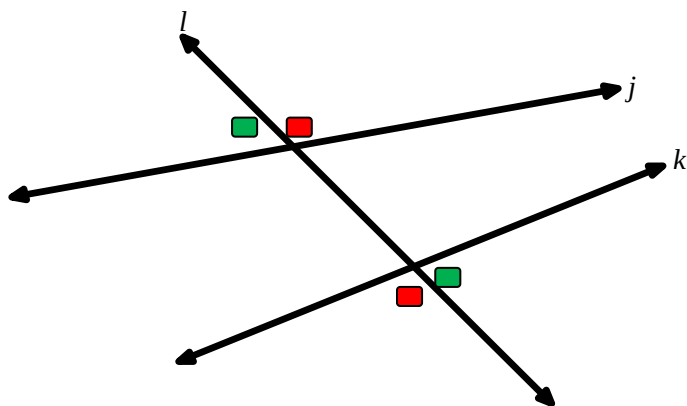
Be sure to make at least two copies of each angle. Once students have finalized the locations and you have discussed them as a class have them write the angles names on their paper in the correct location.



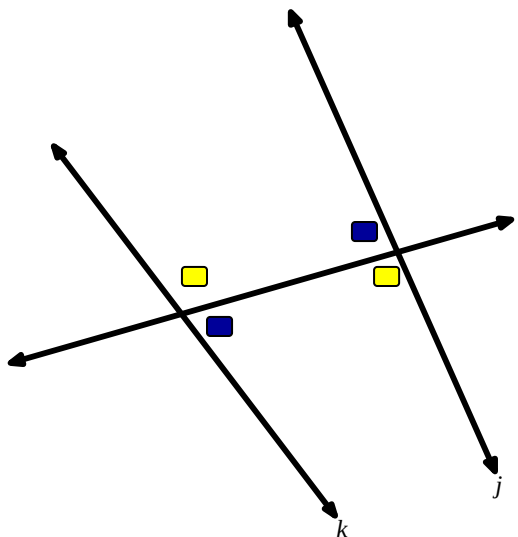
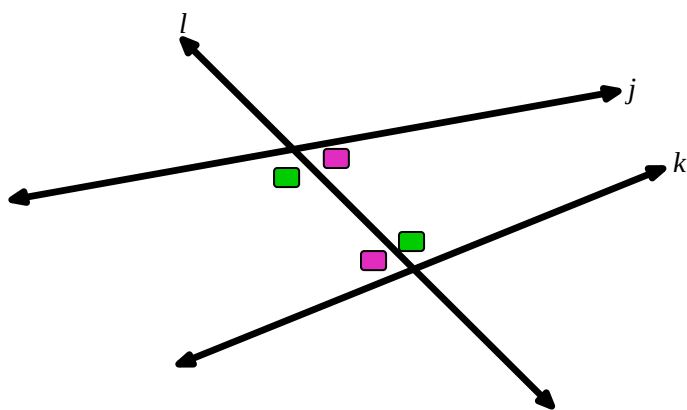
Directions: Color code the following sets of angles by coloring each set of angle pairs the same color. Find at least two sets of the special angles for each drawing.

Sample color coding is given.

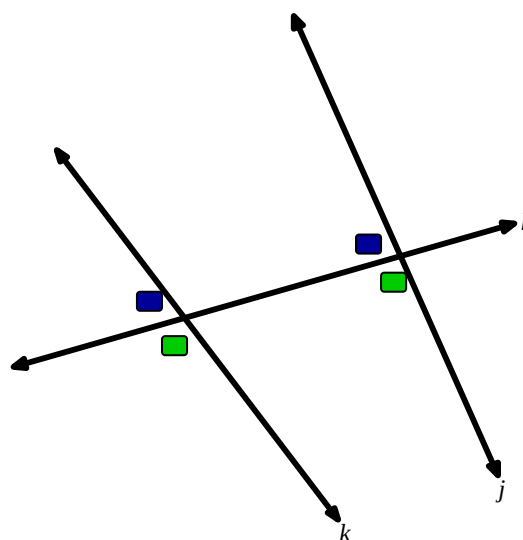
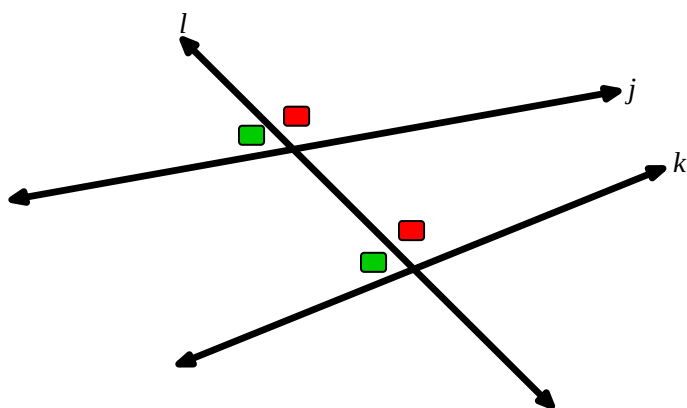
4. Alternate Exterior Angle Pairs



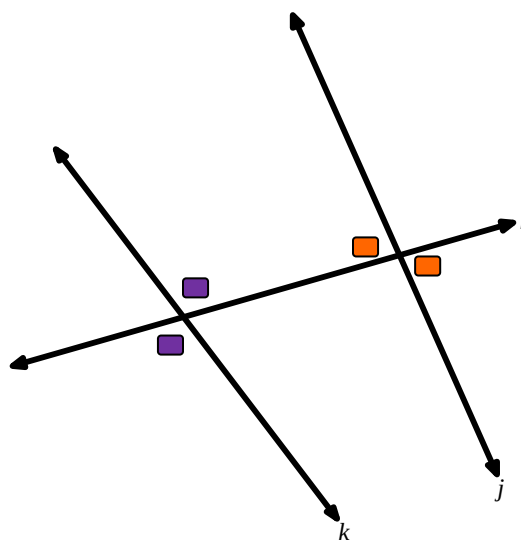
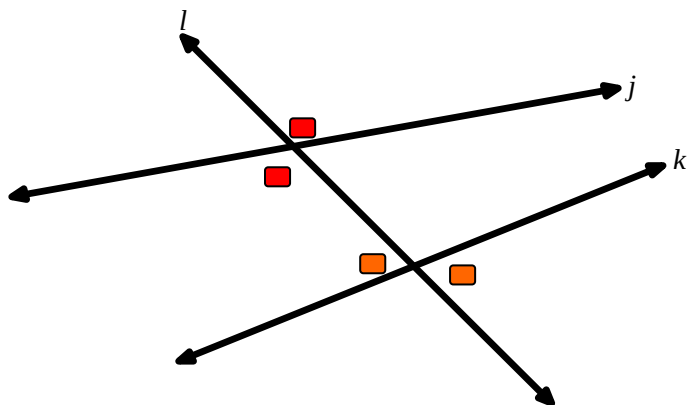
5. Alternate Interior Angle Pairs



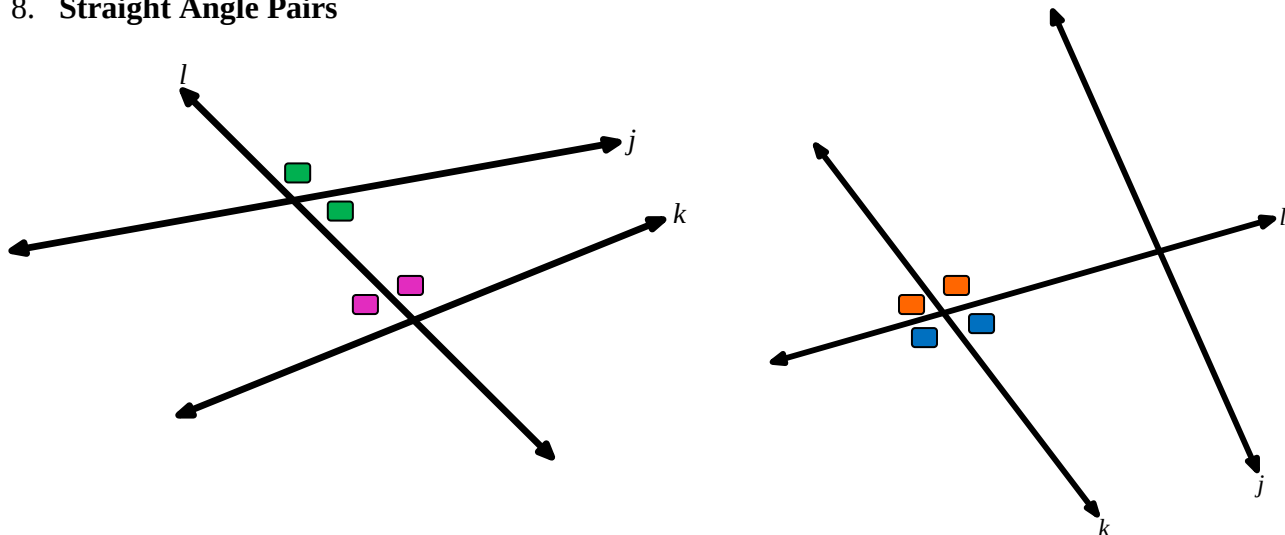
6. Corresponding Angle Pairs



7. Vertical Angle Pairs



8. Straight Angle Pairs



9. Refer to the figure below; identify the following pairs of angles as alternate exterior angles, alternate interior angles, corresponding angles, vertical angles, or straight angles.

a. $\angle 1$ and $\angle 8$ **alternate exterior angles**

b. $\angle 12$ and $\angle 11$ **straight angles**

c. $\angle 13$ and $\angle 21$ **corresponding angles**

d. $\angle 14$ and $\angle 15$ **vertical angles**

e. $\angle 7$ and $\angle 14$ **alternate interior angles**

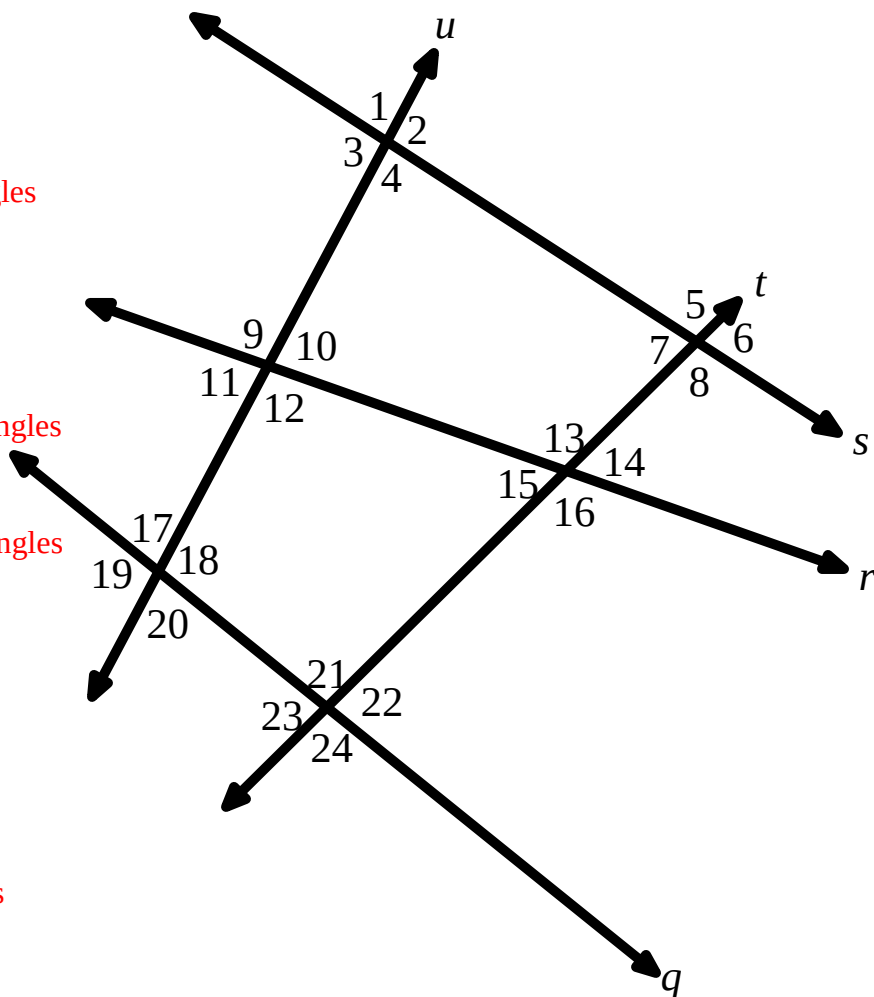
f. $\angle 9$ and $\angle 20$ **alternate exterior angles**

g. $\angle 5$ and $\angle 7$ **straight angles**

h. $\angle 22$ and $\angle 23$ **vertical angles**

i. $\angle 1$ and $\angle 5$ **corresponding angles**

j. $\angle 21$ and $\angle 8$ **alternate interior angles**



10.1b Homework: Special Angles Formed by Transversals

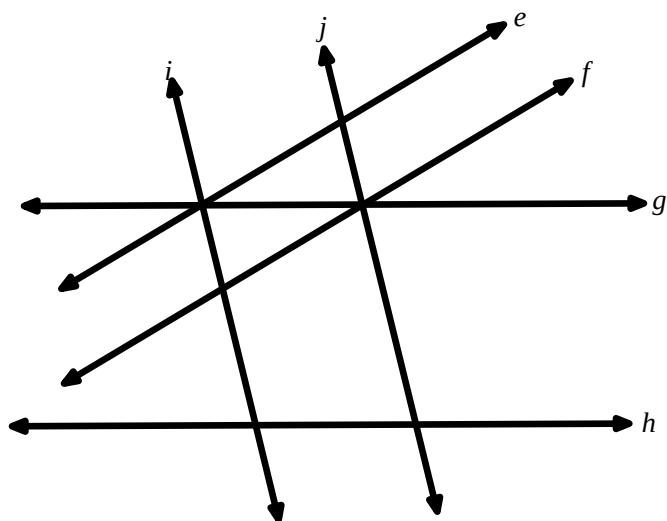
1. Identify the sets of given lines to which each line is a transversal.

a. line e
line i , line j , line g , line h

b. line g
line i , line j , line e , line f

c. line h
line i , line j , line e , line f

d. line j
line e , line f , line g , line h



2. Refer to the figures below. State if $\angle 1$ and $\angle 2$ are alternate exterior angles, alternate interior angles, corresponding angles, vertical angles, or straight angles.

a. Corresponding Angles	b. Vertical Angles
c. Alternate Exterior Angles	d. Alternate Interior Angles

3. Refer to the figure below; state if the following pairs of angles are alternate exterior angles, alternate interior angles, corresponding angles, or straight angles.

a. $\angle 4$ and $\angle 9$ **alternate interior angles**

b. $\angle 12$ and $\angle 11$ **straight angles**

c. $\angle 1$ and $\angle 5$ **corresponding angles**

d. $\angle 1$ and $\angle 8$ **alternate exterior angles**

e. $\angle 6$ and $\angle 7$ **vertical angles**

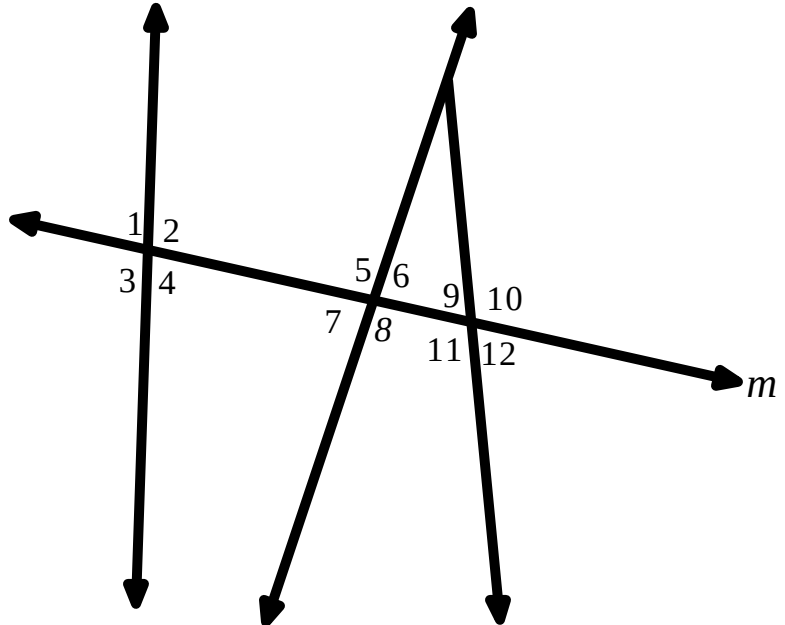
f. $\angle 1$ and $\angle 3$ **straight angles**

g. $\angle 8$ and $\angle 9$ **alternate interior angles**

h. $\angle 7$ and $\angle 11$ **corresponding angles**

i. $\angle 3$ and $\angle 10$ **alternate exterior angles**

j. $\angle 10$ and $\angle 11$ **vertical angles**



Find, Fix, and Justify

4. Patel and Ari are naming alternate interior angles for the figure below. They are listing alternate interior angle pairs for angle 3. Their work is shown below.

Patel

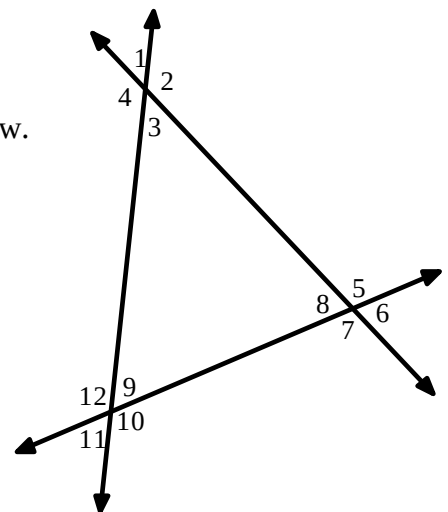
$\angle 3$ and $\angle 12$

$\angle 3$ and $\angle 5$

Ari

$\angle 3$ and $\angle 9$

$\angle 3$ and $\angle 5$



Who is correct? Explain your reasoning.

Patel is correct. Ari listed angle 9 as being an alternate interior angle to angle 3. Angle 9 is an alternate interior angle to angle 4.

10.1c Class Activity: Parallel Lines and Transversals

1. Use the picture given below to describe what parallel lines are. Use the correct notation to denote that line l is parallel to line m .

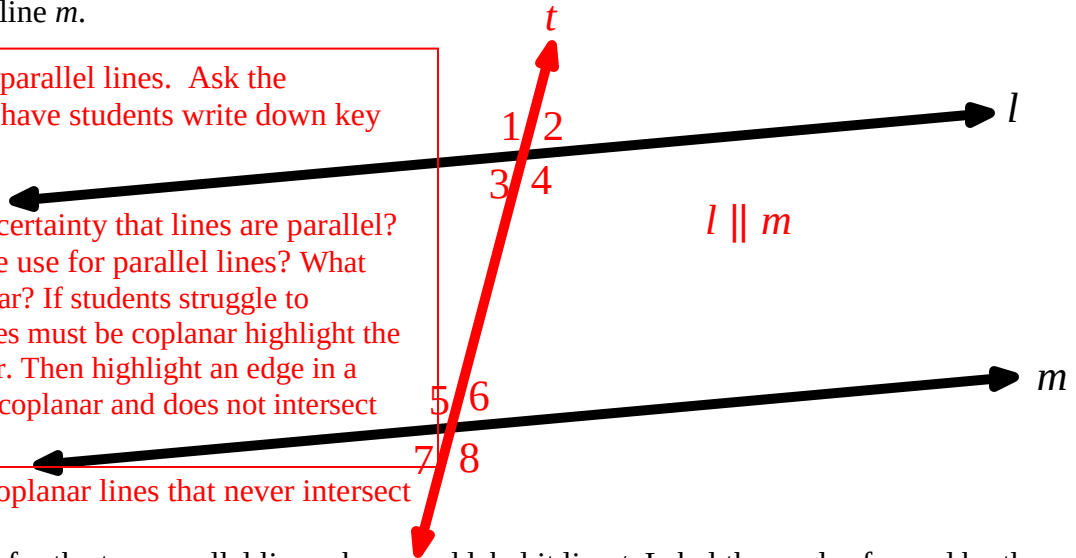
Lead a discussion about parallel lines. Ask the following questions and have students write down key ideas on their paper.

What are parallel lines?

How do you know with certainty that lines are parallel?

What notation do you use for parallel lines? What does it mean to be coplanar? If students struggle to visualize why parallel lines must be coplanar highlight the edge of a box in one color. Then highlight an edge in a different color that is not coplanar and does not intersect with the original edge.

Parallel lines are coplanar lines that never intersect.



2. Draw a transversal for the two parallel lines above and label it line t . Label the angles formed by the transversal and the parallel lines with numbers 1 through 6. *Be sure to number in the same order as your teacher.

Point out to students that a transversal can intersect any two lines, but we are now particularly interested in transversals that intersect parallel lines.

Transversals that intersect two or more parallel lines create angle pairs that have special properties. Use what you know about rigid motions to discover some of these relationships.

3. What type of angle pair is $\angle 2$ and $\angle 6$?



Corresponding angles

4. Copy $\angle 2$ on a piece of tracing paper (or patty paper). Describe the rigid motion that will carry $\angle 2$ to $\angle 6$. Determine the relationship between $\angle 2$ and $\angle 6$.

Students will see that angle 2 can be carried onto angle 6 by a translation. Thus $\angle 2$ and $\angle 6$ are congruent.

5. Use a similar process to see if the same outcome holds true for all of the corresponding angles in the figure. Start by listing the remaining pairs of corresponding angles and then state the relationship.

$\angle 1$ and $\angle 5$ Congruent

$\angle 3$ and $\angle 7$ Congruent

$\angle 4$ and $\angle 8$ Congruent

The other angles can be mapped to their corresponding angles using a similar process. Thus all of the corresponding angles in the figure are congruent to each other. They can also use the fact that vertical angles are congruent and once they know that angle 2 is congruent to angle 6, they also know that angle 3 and angle 7 are also congruent. As students are investigating these angles and making arguments be sure to ask them to give support and warrants for their claims.

6. List the pairs of angles that are vertical angles, what do you know about vertical angles?

$\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 3$, $\angle 5$ and $\angle 8$, $\angle 6$ and $\angle 7$

Vertical angles are congruent.

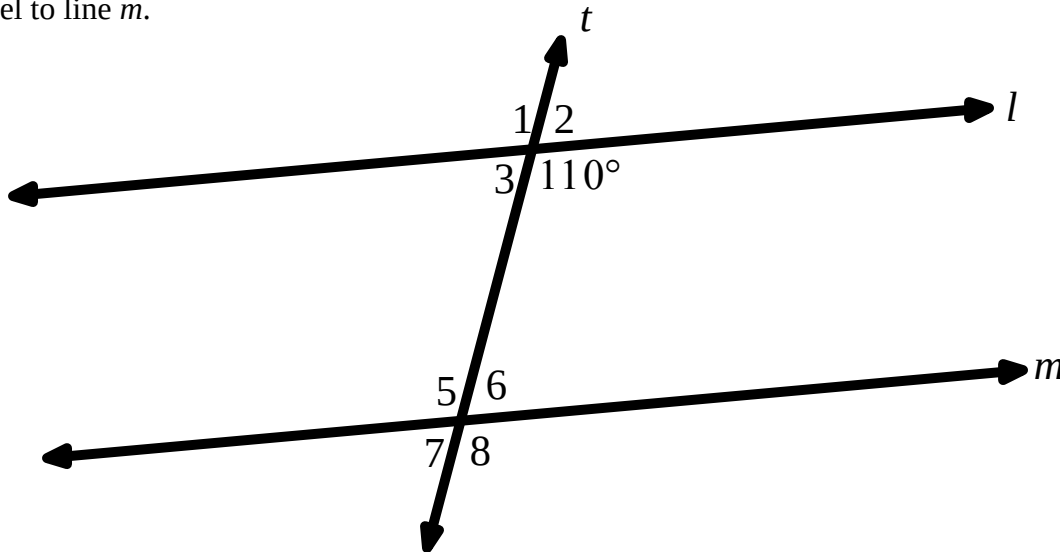
7. Continue to use rigid motions and what you know about vertical angles to discover other relationships that exist between alternate interior angles and alternate exterior angles. Be sure to provide justification for your claims. **Students may argue that $\angle 2$ is congruent to $\angle 6$ because they are corresponding angles, also $\angle 2$ is congruent to $\angle 3$ because they are vertical angles. Therefore $\angle 3$ and $\angle 6$ are congruent. Similar justification can be made to show that every pair of alternate interior angles are congruent and every pair of alternate exterior angles are congruent.**
8. Complete the following statements in the box below.

Properties of Transversals to Parallel Lines

If two parallel lines are intersected by a transversal,

- Corresponding angles are congruent.
- Alternate interior angles are congruent.
- Alternate exterior angles are congruent.

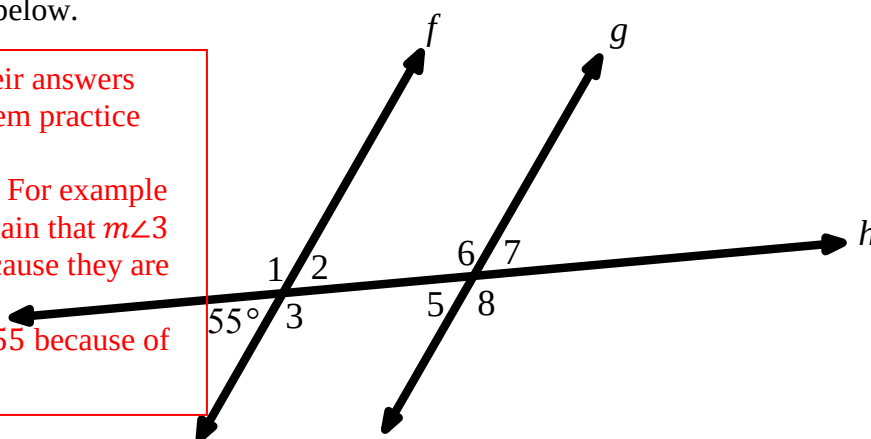
9. In the diagram below one angle measure is given. Find the measure of each remaining angle if line l is parallel to line m .



- a. $m\angle 1 = \underline{110^\circ}$ b. $m\angle 2 = \underline{70^\circ}$ c. $m\angle 3 = \underline{70^\circ}$
- d. $m\angle 5 = \underline{110^\circ}$ e. $m\angle 6 = \underline{70^\circ}$ f. $m\angle 7 = \underline{70^\circ}$
- g. $m\angle 8 = \underline{110^\circ}$

10. Line $f \parallel$ line g and one angle measure is given in the diagram. Determine the measures of the remaining angles in the diagram below.

Have students justify their answers below. This will help them practice making mathematical arguments/explanations. For example in part c they might explain that $m\angle 3$ is congruent to $m\angle 1$ because they are vertical angles or that $m\angle 3$ is equal to $180 - 55$ because of supplementary angles.



- a. $m\angle 1 = \underline{125^\circ}$ b. $m\angle 2 = \underline{55^\circ}$
 c. $m\angle 3 = \underline{125^\circ}$ d. $m\angle 5 = \underline{55^\circ}$
 e. $m\angle 6 = \underline{125^\circ}$ f. $m\angle 7 = \underline{55^\circ}$
 g. $m\angle 8 = \underline{125^\circ}$

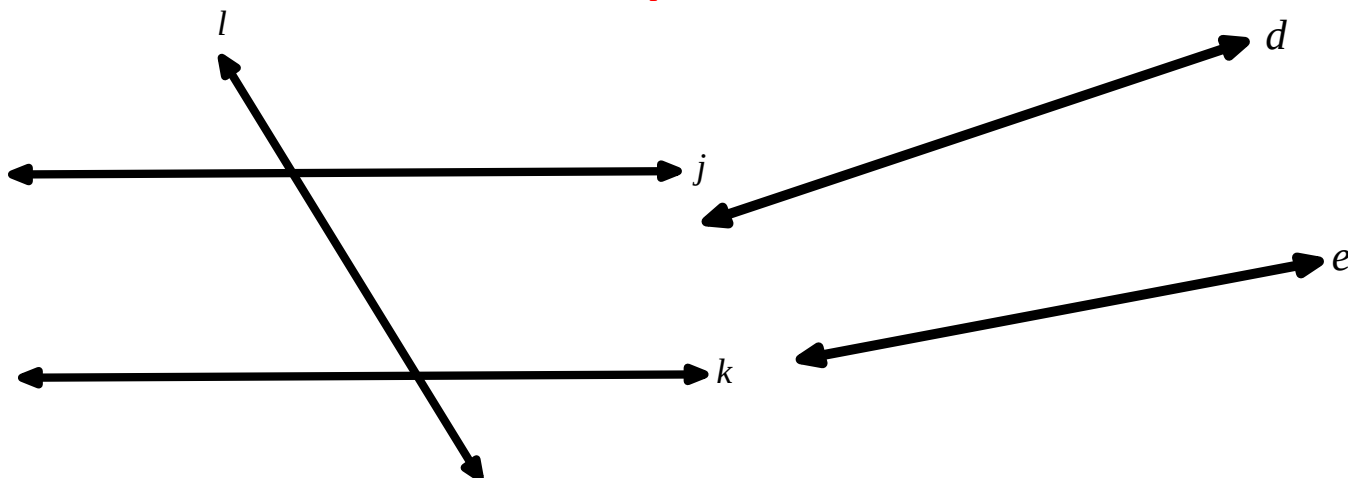
11. Given that line $l \parallel$ line m solve for x and then find the measure of all the remaining angles. Write the angle measures on the picture.

a. $x = 40$	b. $x = 7$
c. $x = 9$	d. $x = 5$

12. Given two lines j and k in a picture below with transversal l devise a strategy to determine whether the two lines are parallel using what you know about the properties of rigid motion. Also use your strategy to determine whether lines d and e are parallel. Stating that the lines do not look parallel, is not a justification.



Copy one angle formed by the transversal and line j . Does it map to its corresponding angle formed by the transversal and line k using rigid motion? Yes, it maps using a translation. Under a translation, corresponding segments are parallel so the segment that is part of the first angle will be parallel to the segment that is part of the second angle. Since the segments sit on lines j and k , lines j and k are also parallel. For lines d and e , we see that if we draw a transversal through the lines and copy one angle formed by the transversal and line d , we cannot map it to its corresponding angle formed by the transversal and line e ; therefore the lines are not parallel.

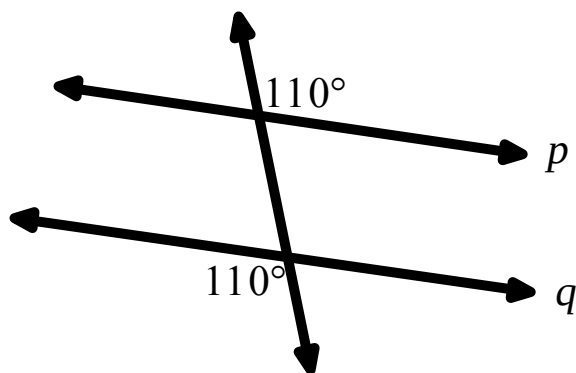


13. Complete the statement below.

Given two lines, if a transversal cuts through both lines so that corresponding angles are congruent, then the two lines are parallel.

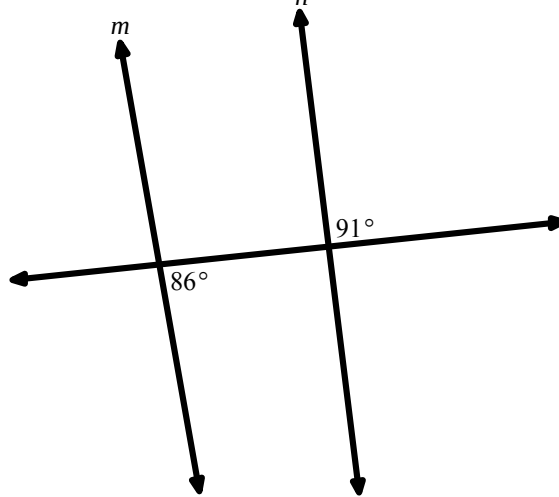
14. Determine whether the following sets of lines are parallel or not. Provide a justification for your response.

a. Is p parallel to q ? Why or why not?



Yes, corresponding angles are congruent.

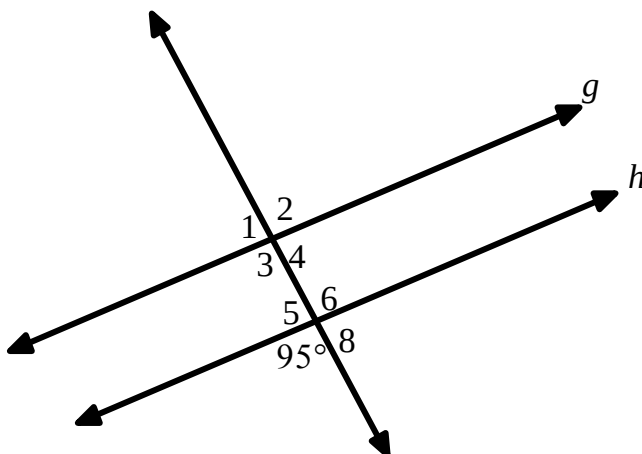
b. Is m parallel to n ? Why or why not?



No, corresponding angles are not congruent.

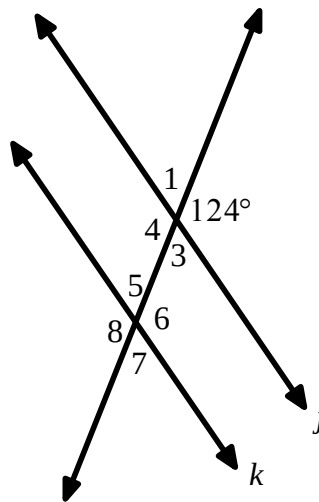
10.1c Homework: Parallel Lines and Transversals

Directions: Use the diagram below to answer questions #1 and 2 given that $g \parallel h$.



1. For each of the following pairs of angles, describe the relationship between the two angles (corresponding angles, alternate interior angles, alternate exterior angles, or vertical angles).
 - a. $\angle 3$ and $\angle 6$ **alt int**
 - b. $\angle 4$ and $\angle 8$ **corresponding**
 - c. $\angle 1$ and $\angle 8$ **alt ext**
 - d. $\angle 1$ and $\angle 5$ **corresponding**
2. One angle measure is given in the diagram. Determine the measures of the remaining angles in the diagram.
 - a. $m\angle 1 = \underline{85^\circ}$
 - b. $m\angle 2 = \underline{95^\circ}$
 - c. $m\angle 3 = \underline{95^\circ}$
 - d. $m\angle 4 = \underline{85^\circ}$
 - e. $m\angle 5 = \underline{85^\circ}$
 - f. $m\angle 6 = \underline{95^\circ}$
 - g. $m\angle 8 = \underline{85^\circ}$

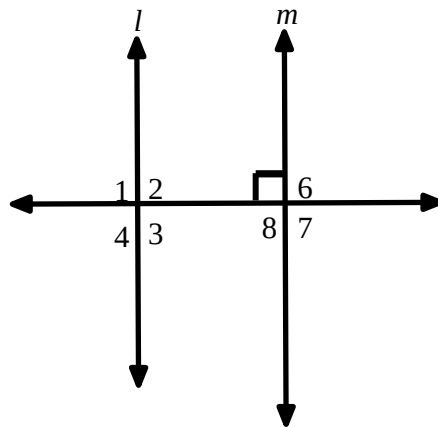
Directions: Use the diagram below to answer question #3 given that line $j \parallel$ line k .



3. One angle measure is given in the diagram. Determine the measures of the remaining angles in the diagram.

- | | |
|--|---------------------------------------|
| a. $m\angle 1 = \underline{56^\circ}$ | b. $m\angle 3 = \underline{56^\circ}$ |
| c. $m\angle 4 = \underline{124^\circ}$ | d. $m\angle 5 = \underline{56^\circ}$ |
| e. $m\angle 6 = \underline{124^\circ}$ | f. $m\angle 7 = \underline{56^\circ}$ |
| g. $m\angle 8 = \underline{124^\circ}$ | |

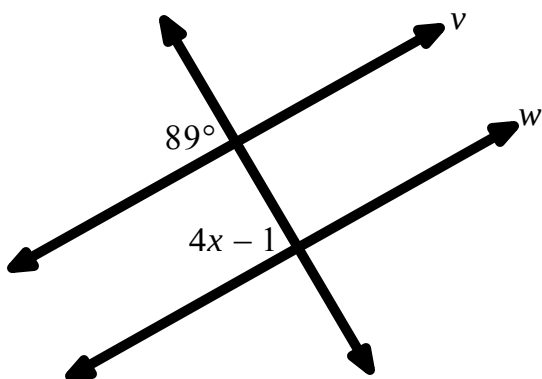
Directions: Use the diagram below to answer question #4 given that line $l \parallel$ line m .



4. One angle measure is given in the diagram. Determine the measures of the remaining angles in the diagram. **All angles measure 90°**

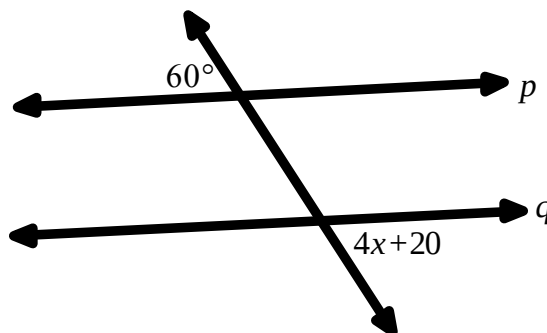
- | | |
|---|---|
| a. $m\angle 1 = \underline{\hspace{2cm}}$ | b. $m\angle 2 = \underline{\hspace{2cm}}$ |
| c. $m\angle 3 = \underline{\hspace{2cm}}$ | d. $m\angle 4 = \underline{\hspace{2cm}}$ |
| e. $m\angle 6 = \underline{\hspace{2cm}}$ | f. $m\angle 8 = \underline{\hspace{2cm}}$ |
| g. $m\angle 7 = \underline{\hspace{2cm}}$ | |

5. Given line $v \parallel$ line w , solve for x .



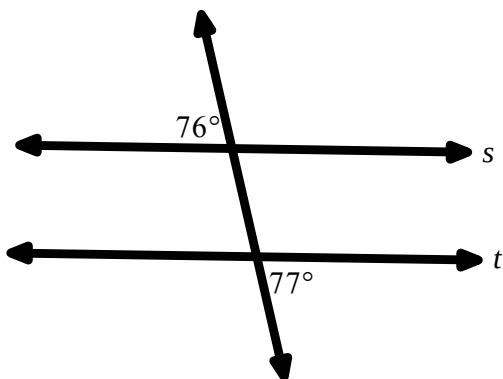
$x = 15$

6. Given line $p \parallel$ line q , solve for x .



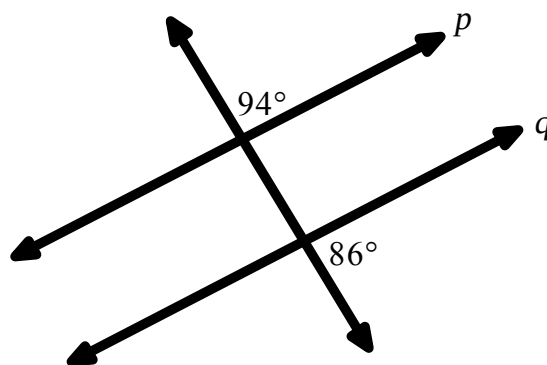
$x = 10$

7. Determine whether lines s and t are parallel. Provide a justification for your response.



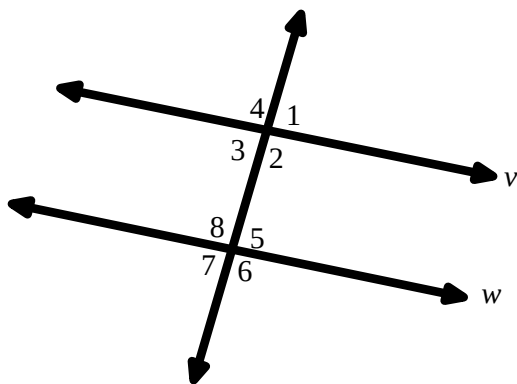
No, alternate exterior angles are not congruent

8. Determine whether lines p and q are parallel. Provide a justification for your response.



Yes, corresponding angles are congruent

9. Given: line $v \parallel$ line w .

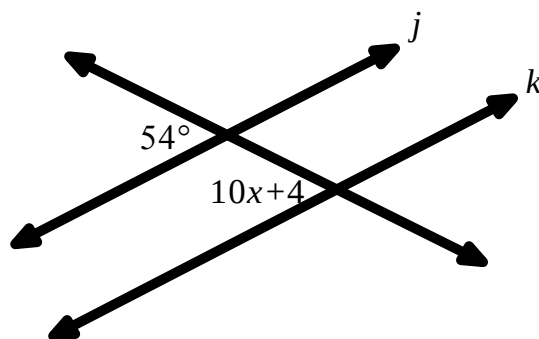


a. Which angles are congruent to $\angle 1$?
 $\angle 3, \angle 5, \angle 7$

b. Which angles are congruent to $\angle 8$?
 $\angle 6, \angle 4, \angle 2$

c. Name **three** pairs of supplementary angles.
 $\angle 2$ and $\angle 5$; $\angle 3$ and $\angle 8$;
 $\angle 1$ and $\angle 4$; $\angle 2$ and $\angle 3$

10. What value of x will make line j parallel to line k ?



$x = 5$

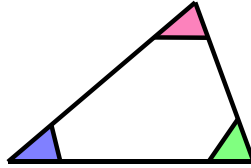
10.1d Class Activity: Tessellating Triangles

Important: Help students to get started on the tessellation. See step-by-step instructions below.

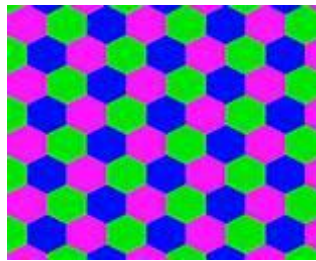
1. Take the index card that has been given to you and using a ruler draw an obtuse scalene triangle or an acute scalene triangle. Remember, in a scalene triangle, the side lengths of the triangle are all different. If the triangle has to be acute or obtuse, that means it can't have a right angle.

As a teacher, you can skip this step and create a variety of triangles in advance using Geogebra or Geometer's Sketchpad and pass one out to each student. The triangles will be more accurate using software to draw them.

2. Cut out the triangle and color the angles each a different color as shown below.



3. Tessellate an 8 1/2" x 11" white piece of paper with copies of your triangle. A **tessellation** is when you cover a surface with one or more geometric shapes, called tiles, with no overlaps or gaps. A tessellation by regular hexagons is shown below.



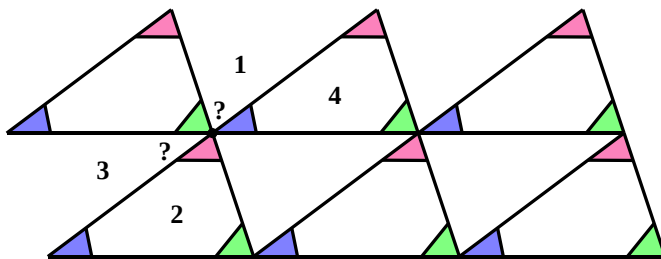
After each tessellation of your triangle, color each angle with its corresponding color.

4. What types of motion did you use to tessellate the plane with your triangle?

Rigid motions (i.e. translations and rotations of 180°)

Teacher Notes: It is very important that you help students to get started on the tessellation; otherwise students may produce a tessellation that will not show many of the theorems we are trying to see. Here is a step-by-step guide of how to get them started on the tessellation:

1. Have students draw, color, and cut out their triangles.
2. Show students the finished product (have one finished ahead of time).
3. Demonstrate the tessellation (you can do this on an overhead transparency so that you can color the angles or on a whiteboard).
4. Start by translating the triangle horizontally and then vertically as shown below.



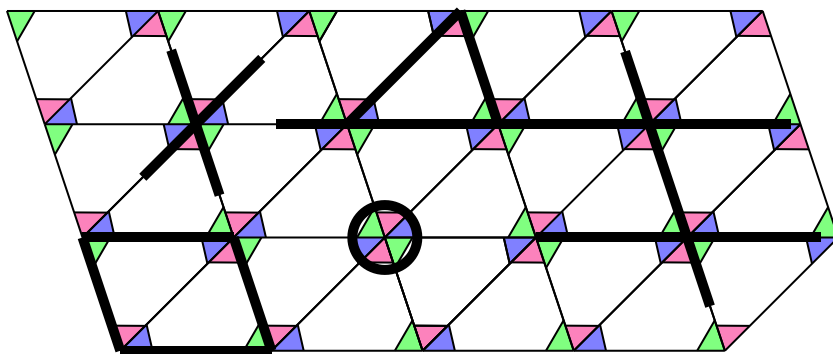
5. Then, ask them, "If we want every straight angle to contain the three different angles (colors) of our triangle, what motion will we use to fill in the remaining spaces? For example, in order to fill in the space at 1, you want the pink angle to be where the question mark is. A rotation of 180° of triangle 2, with the center of rotation at the top vertex of triangle 2 will accomplish this. Students might argue that reflecting triangle 2 will also work. Discuss with them how this will not work because a reflection causes a change in orientation. This change will not allow the angles to match up correctly. A similar process can be used to fill in the question mark at space 3 (a 180° rotation of triangle 4 with center at its lower left vertex will put a blue angle where the question mark is in space 3).
6. From here, students should be able to finish the tessellation.

5. Look back at some of the facts we have studied so far in this section. How does your tessellation support these facts? 

You can have students overlay their tessellation with a transparency or slide it into a plastic sheet protector and use a black overhead marker to observe some facts we have learned about so far – see below or you can have them darken the lines on their tessellation. The color-coding of the angles makes it easy for students to see why the facts we have studied so far are true.

- **Angles that lie on the same line are supplementary and have a common vertex.**
- **Vertical angles have the same measure.**
- **If two lines are parallel and they are intersected by a transversal, then corresponding angles at the points of intersection have the same measure.**
- **Given two lines, if a third line cuts through both lines so that corresponding angles are congruent, then the two lines are parallel.**

The following tessellation is similar to the tessellation that will be created by students. In this tessellation, they can observe many of the theorems we have studied so far.

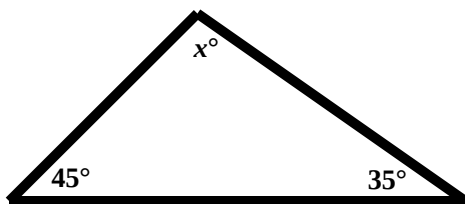


6. The following bolded bullets are additional facts we can observe in our tessellation. Use your tessellation to observe each fact and then provide a mathematical explanation as to why each fact is true. Again, the color coding will help students to observe the bulleted facts below.
- **The sum of the interior angles of a triangle is a straight angle (180°).**
 - **The sum of the interior angles of a quadrilateral is 360° .**
 - **The measure of an exterior angle of a triangle is equal to the sum of the measures of the non-adjacent angles.**

As an extension project, students can create additional tessellations or bring in pictures of tessellations in the real world.

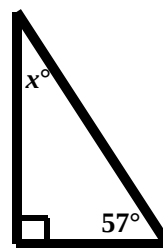
Directions: In the following problems, solve for the missing angle(s).

7.



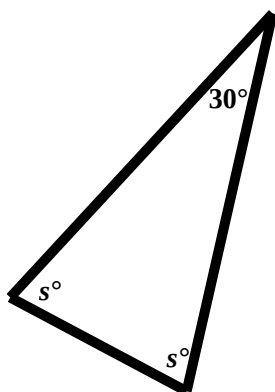
$$x = \underline{\quad 100 \quad}$$

8.



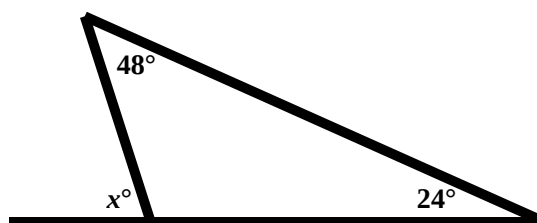
$$x = \underline{\quad 33 \quad}$$

9.



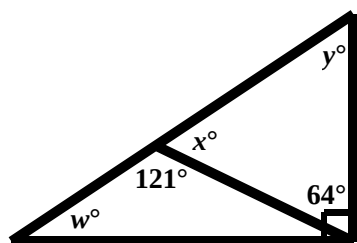
$$s = \underline{\quad 75 \quad}$$

10.



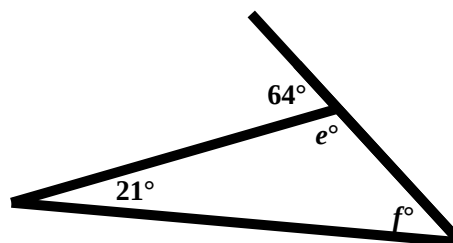
$$x = \underline{\quad 72 \quad}$$

11.



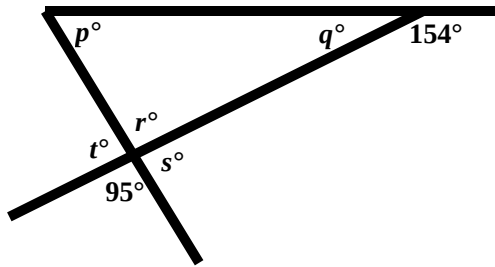
$$w = \underline{\quad 33 \quad} \quad x = \underline{\quad 59 \quad} \quad y = \underline{\quad 57 \quad}$$

12.



$$e = \underline{\quad 116 \quad} \quad f = \underline{\quad 43 \quad}$$

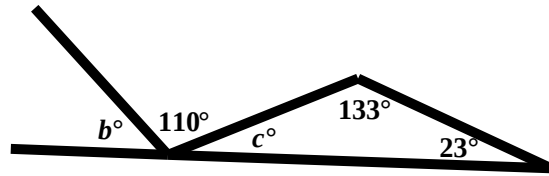
13.



$$p = \underline{59} \quad q = \underline{26} \quad r = \underline{95}$$

$$s = \underline{85} \quad t = \underline{85}$$

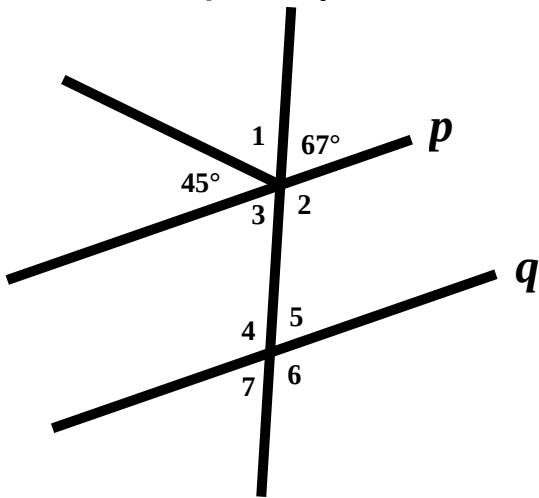
14.



$$b = \underline{46}$$

$$c = \underline{24}$$

15. Given: line $p \parallel$ line q

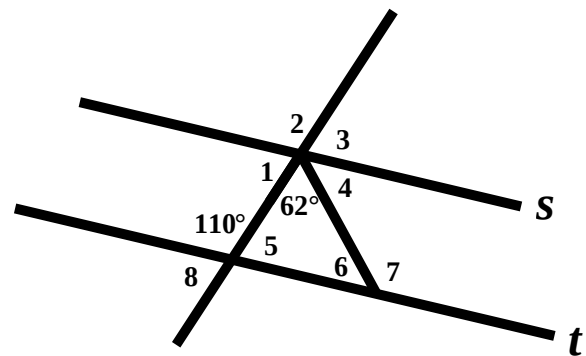


$$\angle 1 = \underline{68} \quad \angle 2 = \underline{113} \quad \angle 3 = \underline{67}$$

$$\angle 4 = \underline{113} \quad \angle 5 = \underline{67} \quad \angle 6 = \underline{113}$$

$$\angle 7 = \underline{67}$$

16. Given line $s \parallel$ line t



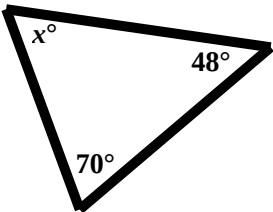
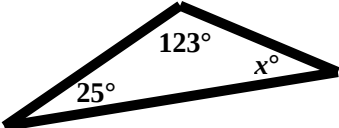
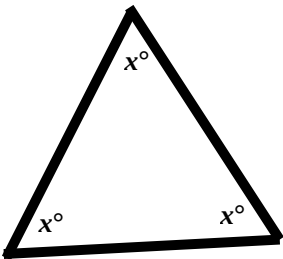
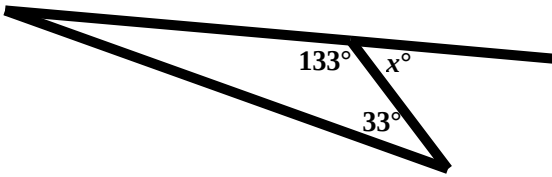
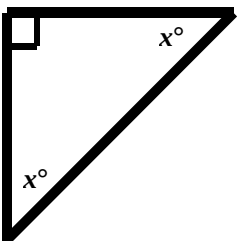
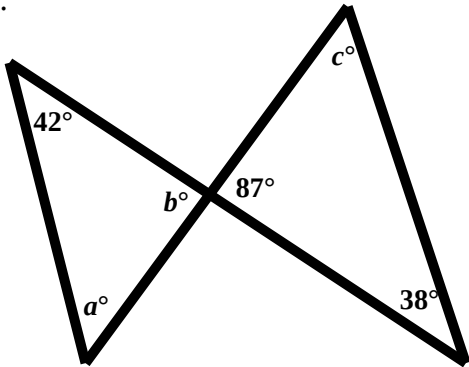
$$\angle 1 = \underline{70} \quad \angle 2 = \underline{110} \quad \angle 3 = \underline{70}$$

$$\angle 4 = \underline{48} \quad \angle 5 = \underline{70} \quad \angle 6 = \underline{48}$$

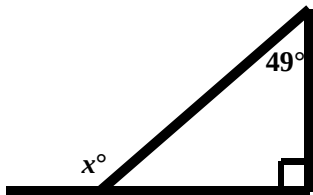
$$\angle 7 = \underline{132} \quad \angle 8 = \underline{70}$$

10.1d Homework: Finding Angle Measures in Triangles

Directions: In the following problems, solve for the missing angle(s).

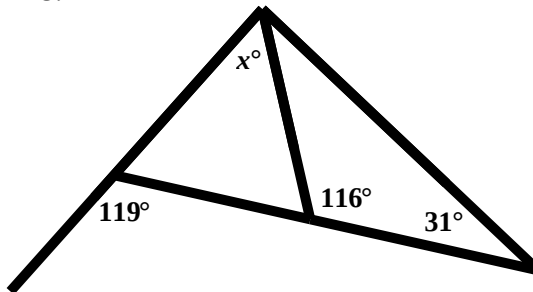
1.  $x = \underline{\quad 62 \quad}$	2.  $x = \underline{\quad 32 \quad}$
3.  $x = \underline{\quad 60 \quad}$	4.  $x = \underline{\quad 47 \quad}$
5.  $x = \underline{\quad 45 \quad}$	6.  $a = \underline{\quad 51 \quad} \quad b = \underline{\quad 87 \quad} \quad c = \underline{\quad 55 \quad}$

7.

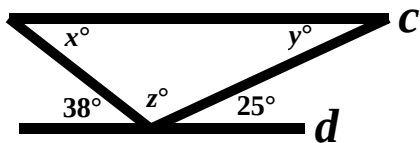


$$x = \underline{139}$$

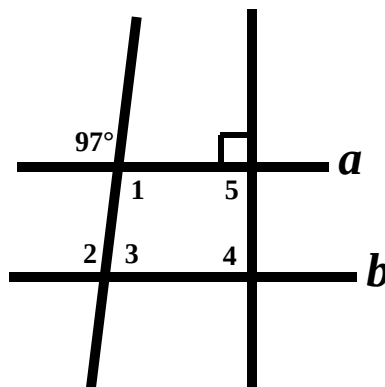
8.



$$x = \underline{55}$$

9. Given: line $c \parallel$ line d 

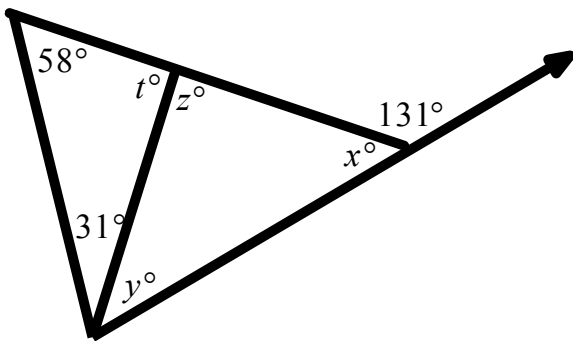
$$x = \underline{38} \quad y = \underline{25} \quad z = \underline{117}$$

10. Given: line $a \parallel$ line b 

$$m\angle 1 = \underline{97} \quad m\angle 2 = \underline{97} \quad m\angle 3 = \underline{83}$$

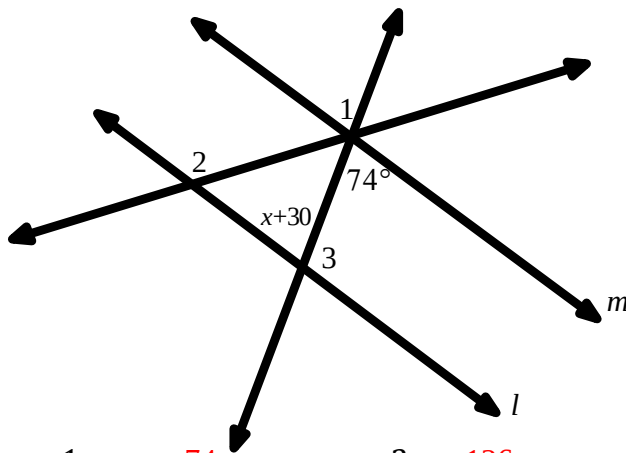
$$m\angle 4 = \underline{90} \quad m\angle 5 = \underline{90}$$

11.



$$t = \underline{91} \quad y = \underline{42}$$

$$z = \underline{89} \quad x = \underline{49}$$

12. Given: line $l \parallel$ line m 

$$m\angle 1 = \underline{74} \quad m\angle 2 = \underline{126}$$

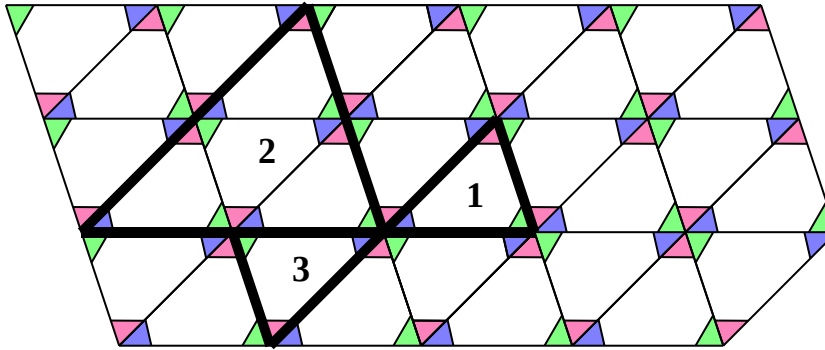
$$m\angle 3 = \underline{106} \quad x = \underline{44}$$

10.1e Class Activity: Similar Triangles

Revisit some of the following facts about similar triangles from Chapter 9.

- If two triangles are similar, then the ratios of the lengths of corresponding sides are the same.
- If two triangles are similar, then corresponding angles have the same measure.

Use the tessellation you made to continue your study of triangles.

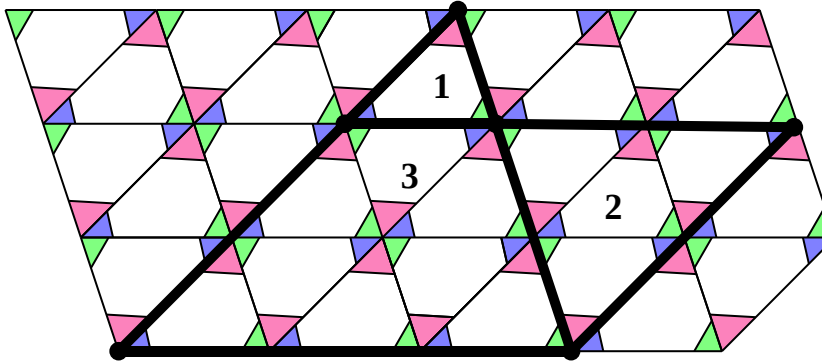


1. In Chapter 9 we learned that if one figure can be carried onto another by a series of rigid motions and dilations, then the two figures are similar.
 - a. In the picture above triangle 1 is similar to triangle 2. Describe the sequence of transformations that will carry triangle 1 onto triangle 2. What is the scale factor?
Answers may vary: A translation of two to the left followed by a dilation with a scale factor of 2 and center at the lower left vertex of triangle 1 or a dilation with factor 2 with center at the vertex just one to the right of the rightmost vertex of triangle 1.
 - b. In the picture above triangle 2 is similar to triangle 3. Describe the sequence of transformations that will carry triangle 2 onto triangle 3. What is the scale factor?
Answers may vary but will likely be some combination of a rotation of 180° and a translation and a dilation with a scale factor of one-half.
 - c. What do you notice about the corresponding angles of similar triangles?
They are congruent
2. Can you find a triangle that is a dilation of triangle 1 with a scale factor of 3? Trace the triangle. What do you notice about the angle measures in the new triangle you created?
Triangles will vary. The angle measures are congruent.

Below is another fact about similar triangles.

- **Given two triangles, if the corresponding angles have the same measure, then the triangles are similar.**

3. We will be using the tessellation you made to explore the proposition above. Find and highlight in black two triangles that have the same angle measures but are a different size.



It is important to note that triangle 1 can be mapped to triangle 2 with a rotation of 180° and a dilation with a scale factor of 2. A reflection will not work because orientation is not maintained.

4. What is the relationship between the triangles formed by the dark lines? Justify your answer. 
- The triangles are similar. One can be mapped to the other through a series of rigid motions and dilations.

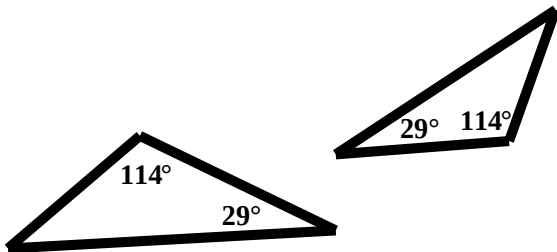
5. Find a third triangle that is a different size than the other two you highlighted. Highlight the third triangle. What is the relationship of this triangle to the other triangles? Justify your answer.
- The triangles are similar. One can be mapped to the other through a series of rigid motions and dilations.

6. Complete the following statement. If two triangles have corresponding angles that are the same measure, then one triangle can be mapped to the other using rigid motions and dilations; therefore the triangles are similar.

7. Do all 3 pairs of corresponding angles have to be congruent in order to say that the two triangles are similar? What if only 2 pairs of corresponding angles are congruent? Would the triangles still be similar? Why or why not?
- If will suffice to know that two angles have the same measure. Due to the triangle sum theorem, we know that the angles sum to 180° so the remaining angle in both triangles will have to be the same in order to sum to 180° .

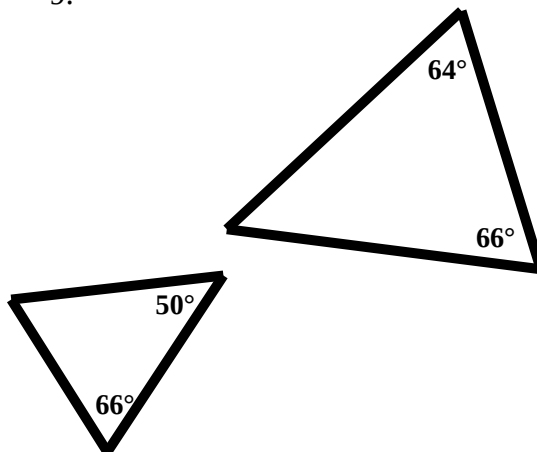
Directions: Are the triangles similar? If they are similar justify why.

8.



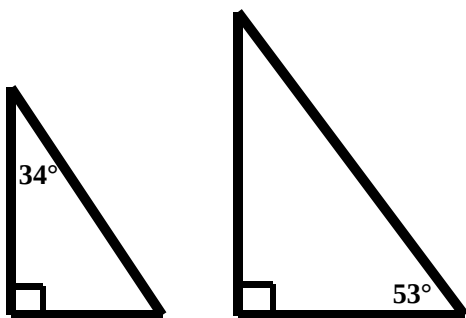
Yes, AA similarity

9.



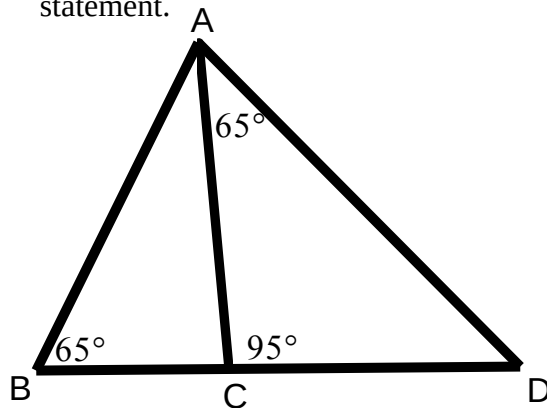
Yes, AA similarity

10.



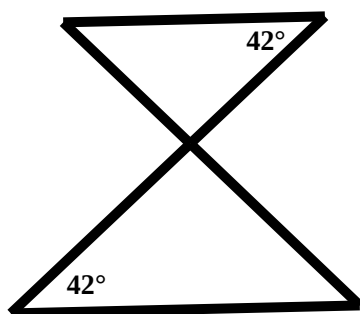
No

11. In the picture below be sure to consider all three triangles shown. If any of the triangles are similar write a similarity statement.



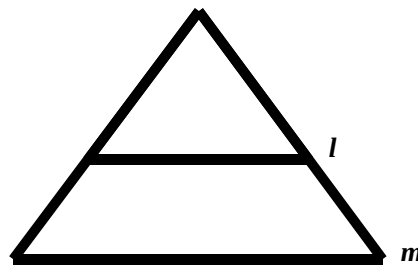
Yes, by AA similarity $\triangle ABD \sim \triangle CAD$.

12.



Yes, AA similarity. We know that the vertical angles are also congruent.

13. Given line $l \parallel$ line m

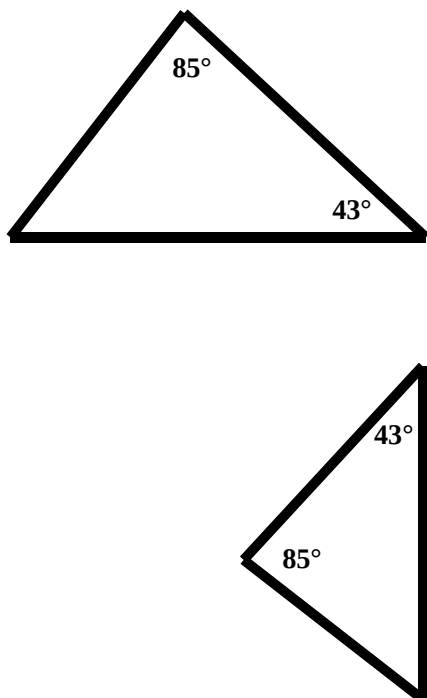


Yes, we know the corresponding angles at the bases of both triangles are congruent and they share the third angle.

10.1e Homework: Similar Triangles

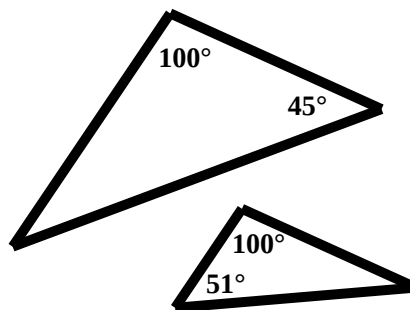
Directions: Are the triangles similar? If they are similar justify why.

1.



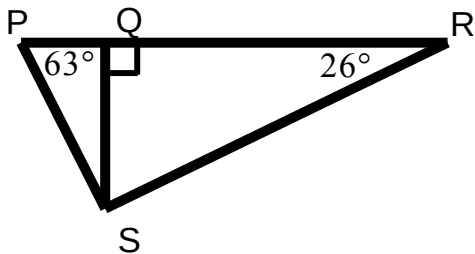
Yes, AA similarity

2.



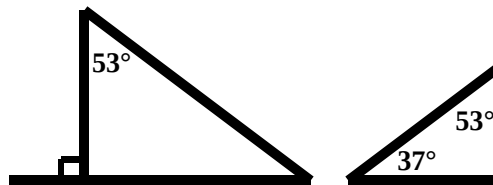
No

3. In the picture below be sure to consider all three triangles shown. If any of the triangles are similar write a similarity statement.



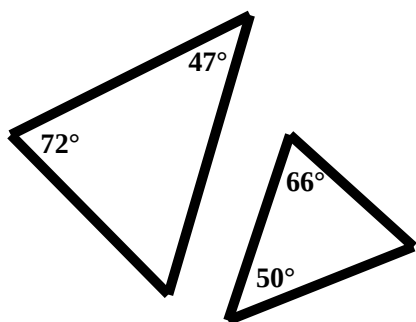
None of the triangles are similar.

4.



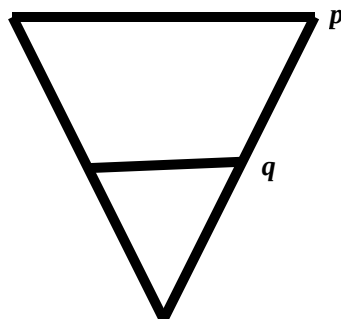
Yes, AA similarity

5.



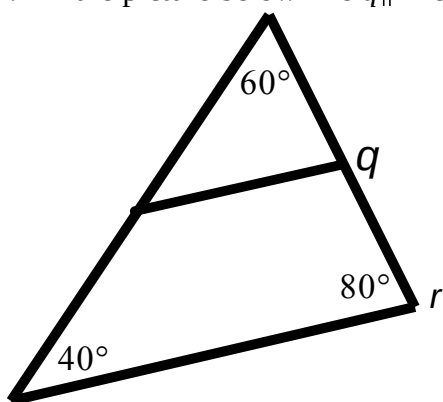
No

6. In the picture below p is not parallel to q .



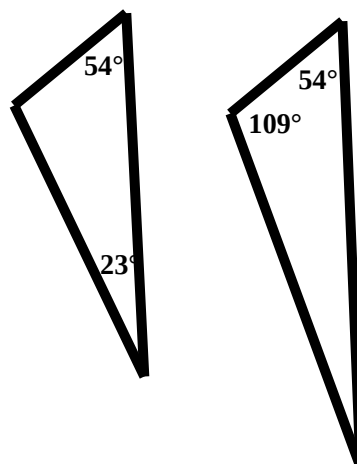
No

7. In the picture below line $q \parallel$ line r .



Yes, AA similarity. The corresponding angles at the base of both triangles are congruent and they share the third angle.

8.



No

10.1f Self-Assessment: Section 10.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provide on the next page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Know that straight angles sum to 180° and that vertical angles are congruent. <i>See sample problem #1</i>				
2. Know that the sum of the angles in a triangle is 180° . Understand that the measure of an exterior angle of a triangle is equal to the sum of the measures of the non-adjacent angles. Use these properties to find missing angle measures related to a triangle. <i>See sample problem #2</i>				
3. Determine the relationship between angles formed when a transversal intersects parallel lines. Use these relationships to find missing angle measures. <i>See sample problem #3</i>				
4. Determine whether two lines are parallel based on the angle measures when a transversal intersects the lines. <i>See sample problem #4</i>				
5. Understand and apply the angle-angle criterion to determine whether two triangles are similar. <i>See sample problem #5</i>				

10.1f Sample Problems: Section 10.1

Sample Problem #1

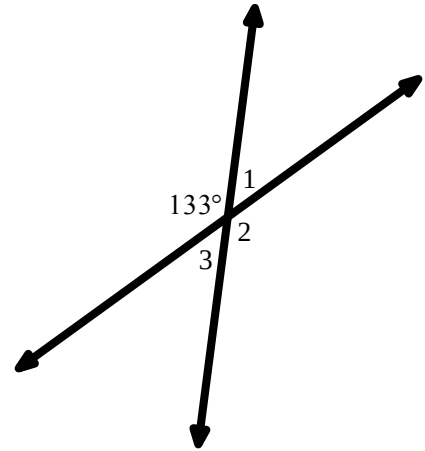
Determine the measures of angles 1, 2, and 3. Justify your answers.

$$m\angle 1 = 47^\circ$$

$$m\angle 2 = 133^\circ$$

$$m\angle 3 = 47^\circ$$

Angles 1 and 3 are straight angles with the given angle that measures 133° , thus $180 - 133 = 47$. Angle 2 is a vertical angle to the given angle so it has the same measure.



Sample Problem #2

In the figure to the right find the value for x , y , and z . Justify your answers.

$$x = 83^\circ$$

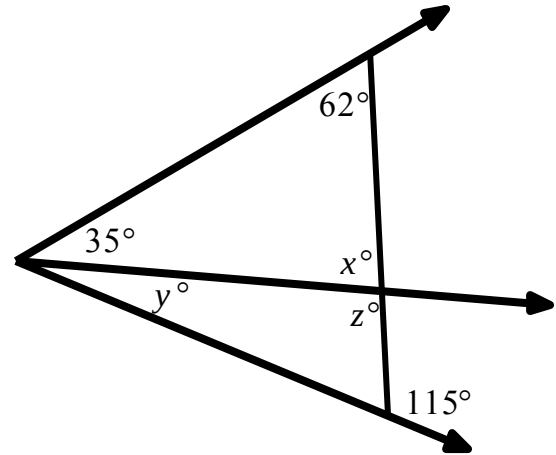
$$y = 18^\circ$$

$$z = 97^\circ$$

$x + 35 + 62 = 180$ because the interior angles of a triangle sum to 180° . Upon solve for x you get $x = 83^\circ$.

$x + z = 180$ because they are straight angles. If you substitute 83 in for x and solve for z you get $z = 97^\circ$.

Finally $z + y = 115$ because the exterior angle of a triangle is the sum of the nonadjacent interior angles of a triangle. If you substitute 97 in for z and solve you get $y = 18^\circ$.



Sample Problem #3

Use the figure to the right to answer each question given that line g is parallel to line h .

- a. State the relationship between the following pairs of angles.

$\angle 1$ and $\angle 8$ **Alternate Exterior Angles**

$\angle 4$ and $\angle 8$ **Corresponding Angles**

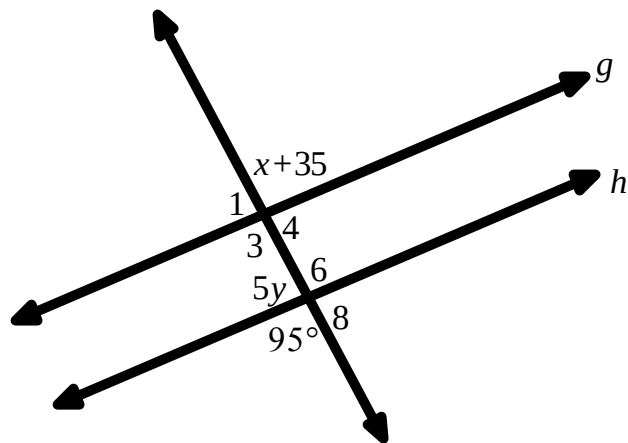
$\angle 3$ and $\angle 6$ **Alternate Interior Angles**

- b. Find the measure of the angles given below.

$$m\angle 1 = \underline{85^\circ} \quad m\angle 3 = \underline{95^\circ}$$

$$m\angle 4 = \underline{85^\circ} \quad m\angle 6 = \underline{95^\circ}$$

$$m\angle 8 = \underline{85^\circ}$$

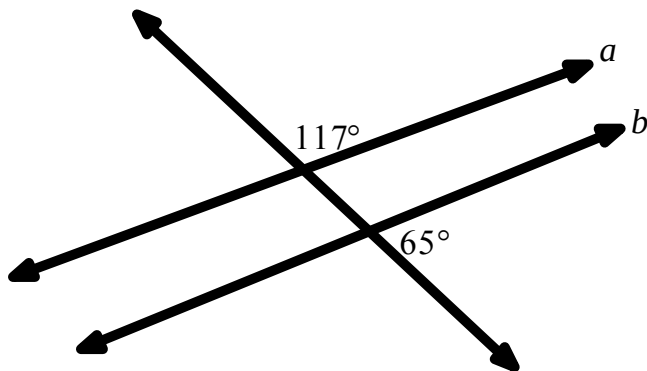


- c. Find the value of x and y .

$$x = \underline{60} \quad y = \underline{17}$$

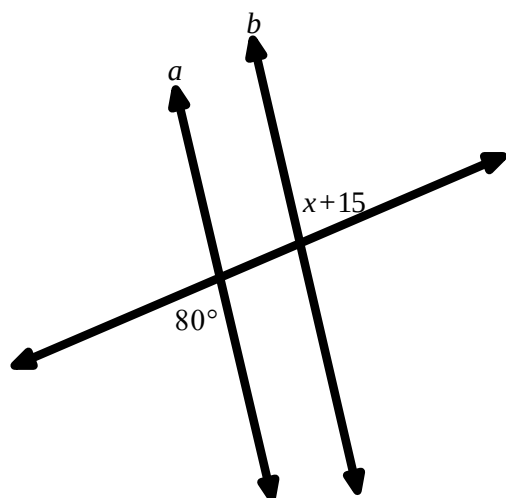
Sample Problem #4

- a. Determine if line $a \parallel$ line b . Justify your answer.



The lines are not parallel. Corresponding angles are not congruent.

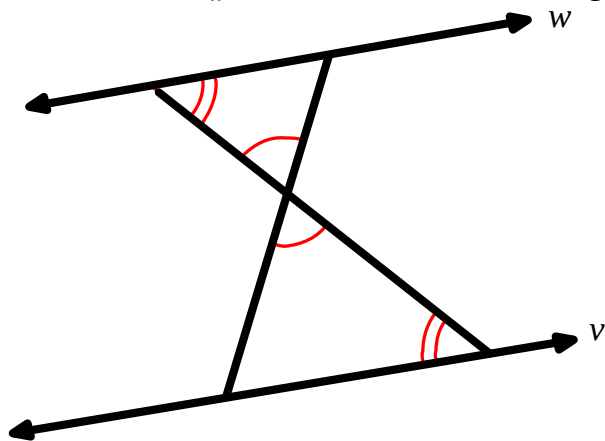
- b. Find the value of x that will make line $a \parallel$ line b .



$$x = \underline{\hspace{1cm}65\hspace{1cm}}$$

Sample Problem #5

Given that line $w \parallel$ line v , determine if the triangles formed below are similar. If they are similar justify why.



The triangles are similar by AA similarity. The vertical angles are congruent. Since line w is parallel to line v the alternate interior angles formed by the transversals are congruent.

Section 10.2 The Pythagorean Theorem

Section Overview:

In this section students begin to formalize many of the ideas learned in Chapter 7. They will transition from using the area of a square to find the length of a segment to generalizing the relationship between the side lengths of a right triangle, i.e. the Pythagorean Theorem, to find the length of a segment. They begin this transition by finding the areas of the squares adjacent to a given right triangle. Using these concrete examples, students describe the relationship between the sides of a right triangle. From here, students work to explain a proof by picture and subsequently a paragraph proof of the Pythagorean Theorem, starting first with a right triangle of side lengths 3, 4, and 5. Students then use a similar process to explain a proof of the Pythagorean Theorem for any right triangle with side lengths a , b , and c where a and b are the legs of the right triangle and c is the hypotenuse. Students arrive at the Pythagorean Theorem: $a^2 + b^2 = c^2$ where a and b are the legs of the right triangle and c is the hypotenuse. Throughout the section, students are connecting the Pythagorean Theorem to work done in Chapter 7. Next, students use the Pythagorean Theorem to solve for the missing side length of a right triangle given the measurements of the other two sides, relying on skills learned in Chapters 7 and 8. This is followed by explaining a proof of the converse of the Pythagorean Theorem: For a triangle with side lengths a , b , and c if $a^2 + b^2 = c^2$, then the triangle is a right triangle. Using this theorem, students determine whether three given side lengths form a right triangle. Throughout this section emphasis is placed on creating good arguments and explanation. Students are not formally proving the Pythagorean Theorem and its converse but explaining why the theorems are true by learning how to provide sufficient explanations and arguments. In addition students are providing evidence and warrants for claims that they make. At the end of the section is an optional exploration on Pythagorean triples.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Know that in a right triangle $a^2 + b^2 = c^2$, where a and b are the legs of the right triangle and c is the hypotenuse.
2. Understand and explain a proof of the Pythagorean Theorem.
3. Use the Pythagorean Theorem to solve for the missing side length of a right triangle given the measurements of the other two sides.
4. Understand and explain a proof of the converse of the Pythagorean Theorem. That is, for a triangle with side lengths a , b , and c if $a^2 + b^2 = c^2$, then the triangle is a right triangle.
5. Use the converse of the Pythagorean Theorem to determine whether three given measurements are possible side lengths of a right triangle.

This section is a great opportunity to review several of the concepts from Chapter 7. Considering spiraling in these concepts as you progress through this section:

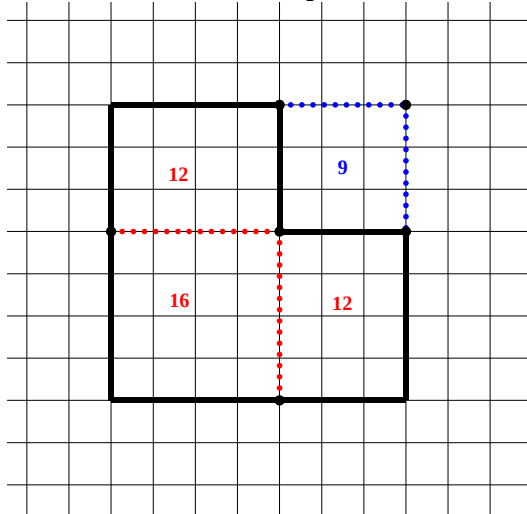
- Classifying numbers as rational and irrational
- Constructing lengths of irrational numbers
- Estimating the value of irrational numbers
- Comparing with rational and irrational numbers

It is also worth pointing out that students studied the triangle inequality theorem in 7th grade, that is, that the sum of the lengths of any two sides of a triangle must be greater than the length of the remaining side. They tried constructing triangles from three measures of angles or sides, noticing when conditions determine a unique triangle, more than one triangle, or no triangle. When a , b , and c are all positive and satisfy the Pythagorean Theorem, there is only one triangle with those given sides and it is a right triangle.

10.2a Class Activity: A Proof of the Pythagorean Theorem

In this lesson students are asked to cut out shapes in order to explain a proof of the Pythagorean Theorem. If you are pressed for time, you may consider creating a class set of the cut-outs on cardstock prior to the lesson. Try to be as precise as possible in your cuts as the students will be fitting the pieces into a puzzle.

- Find the area of the shape below. Each square on the grid has a side length of 1 unit.

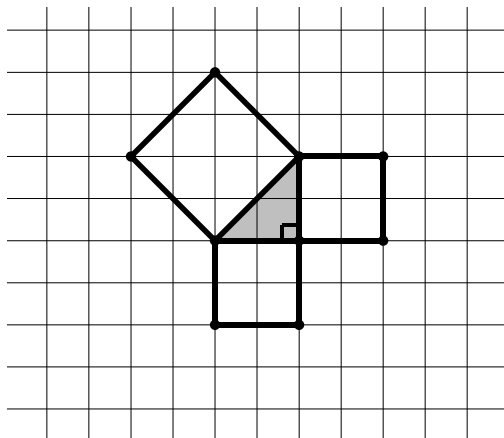


The area of the shape is 40 square units. Students may find the area by breaking the shape into smaller shapes and adding up the areas (shown in red) or finding the area of the large square (49) and subtracting out the area of the small square (9) (shown in blue). This problem is a warm-up for skills needed in the lesson.

In numbers 2 and 3, a right triangle is shown in gray. The shorter sides of a right triangle are referred to as **legs**. The longer side of the right triangle (the side opposite of the right angle) is called the **hypotenuse**.

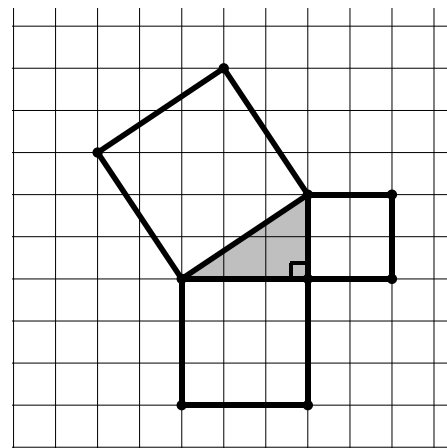
Directions: Squares have been drawn adjacent to the sides of the right triangle. Find the area of each of the squares. Assuming each square on the grid has a side length of 1 unit. Write the areas inside each of the squares. Students can count squares, find the area using the subtraction method for the tilted squares, or use the area formula learned in Chapter 7 ($A = s^2$).

2.



Small squares both have an area of 4. Large square has an area of 8.

3.



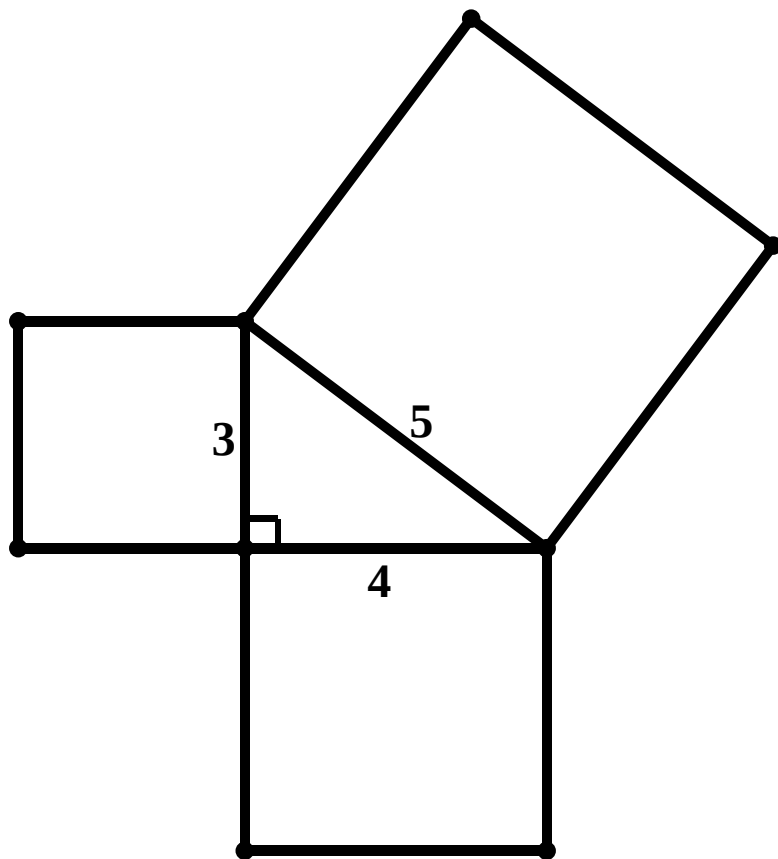
Small squares have areas of 9 and 4. Large square has an area of 13.

You may also have students who recognize that in problem #2, the tilted square consists of 4 copies of the gray triangle and in problem #3, the tilted square consists of 4 copies of the gray triangle + a unit square

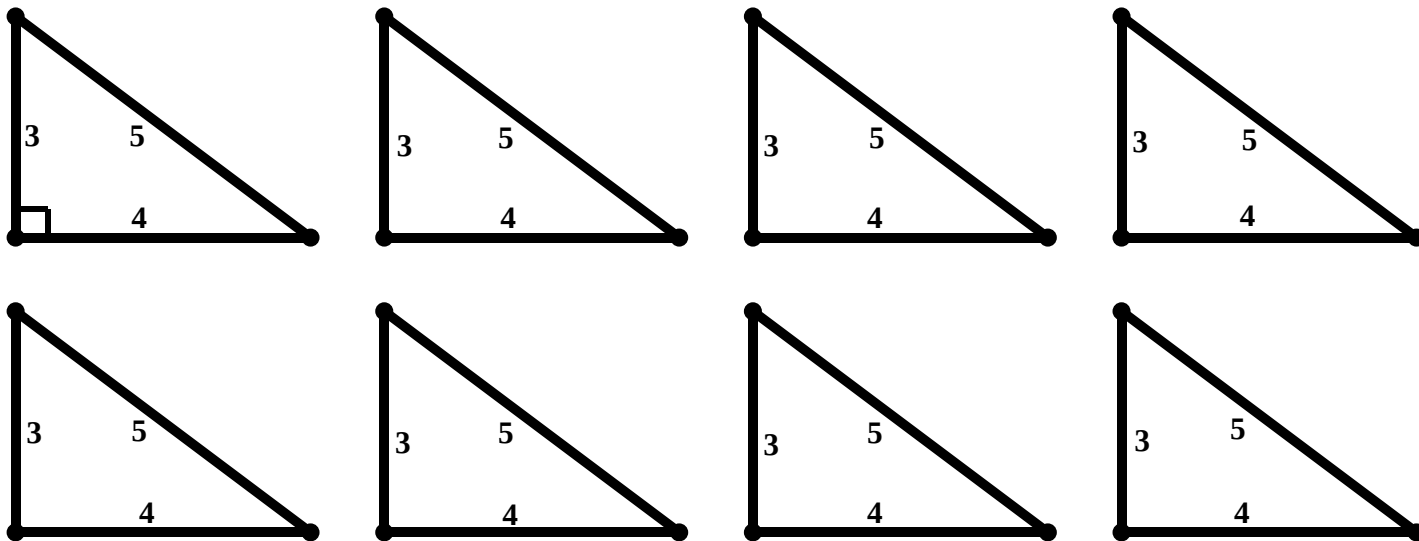
- What do you notice about the relationship between the areas of the squares formed adjacent to the legs of a right triangle?

At this point, students may notice that the sum of the areas of the squares adjacent to the legs of the right triangle equals the area of the square adjacent to the hypotenuse of the triangle.

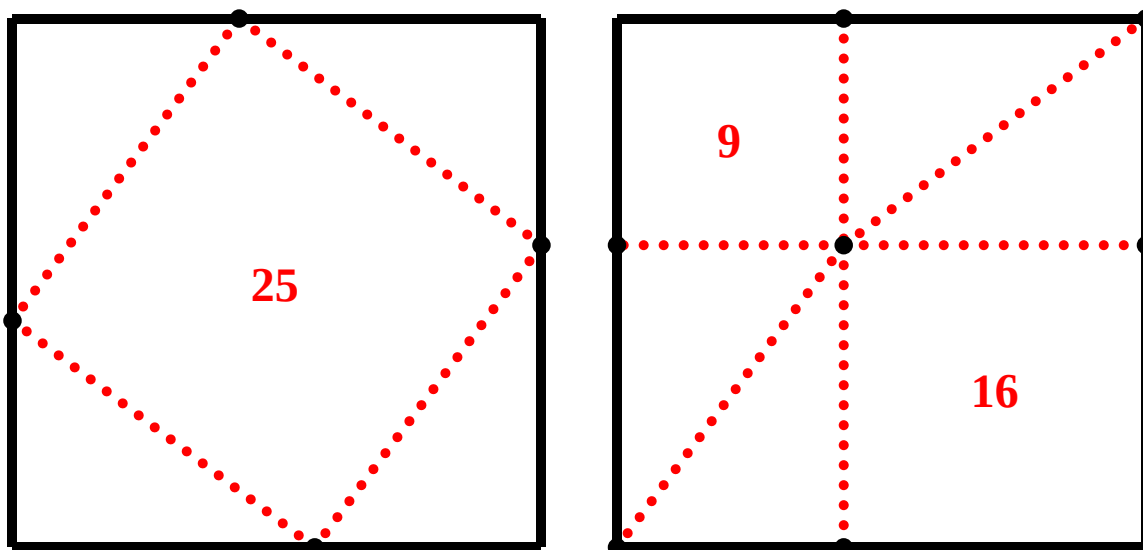
5. Below is a right triangle with side lengths 3, 4, and 5. Squares have been drawn adjacent to the sides of the right triangle.



- a. Find the area of each of the squares. Write the area inside each of the squares. Then, cut out the three squares very carefully. **Students write areas of 9, 16, and 25 inside each square above.**
- b. Below are 8 copies of the original right triangle. Cut out the 8 triangles very carefully.



- c. Below are two **congruent squares**. Since the squares are congruent, we know that their sides have the same length and subsequently they have the same area. Use your square with an area of 25 and four of the triangles from the previous page to cover one of the squares. Use your squares with areas 9 and 16 and four of the triangles from the previous page to cover the other square. Tape the pieces into place. **Possible configurations shown.**



Make sure that students are clear that the squares shown above (25, 9, and 16) are squares formed adjacent to the sides of the right triangle. They can see the original 3, 4, 5 right triangle in the puzzle above as well as the squares adjacent to the sides of the right triangle. In addition, once they have the puzzle pieces in place, have them find the area of the large square to explain that the areas of the squares shown above are congruent.

- d. Use the large squares in part c) to explain the relationship you discovered in #2 – 4 between the squares formed adjacent to the sides of a right triangle.

Students are learning to craft a good explanation and argument here. Help students to focus on providing evidence and warrants for their claims as they explain the relationship between the square and triangles. This is getting them ready to deal with more formal proofs in later grades. They may approach their explanation in several different ways:



Logic:

If the areas of the large squares are the same, I don't need to worry about the area covered by the triangles because the triangles are covering the same amount of area on both large squares. That leaves me with the large square adjacent to the right triangle which is equal in area to the squares adjacent to the two smaller sides of the right triangle.

4 triangles and the large square formed adjacent to the right triangle cover the same area as 4 triangles and the small squares adjacent to the right triangle.

Each triangle has an area of 6 square units. That means, on each large square, the triangles have an area of 24. I know that the large square has an area of 49. If I take the difference between 49 and 24, I am left with 25. On the square on the left, our large square formed adjacent to the right triangle has an area of 25 and on the square on the right, our two smaller squares formed adjacent to the right triangle have areas that sum to 25.

Equation:

Area of Square 1 = Area of Square 2

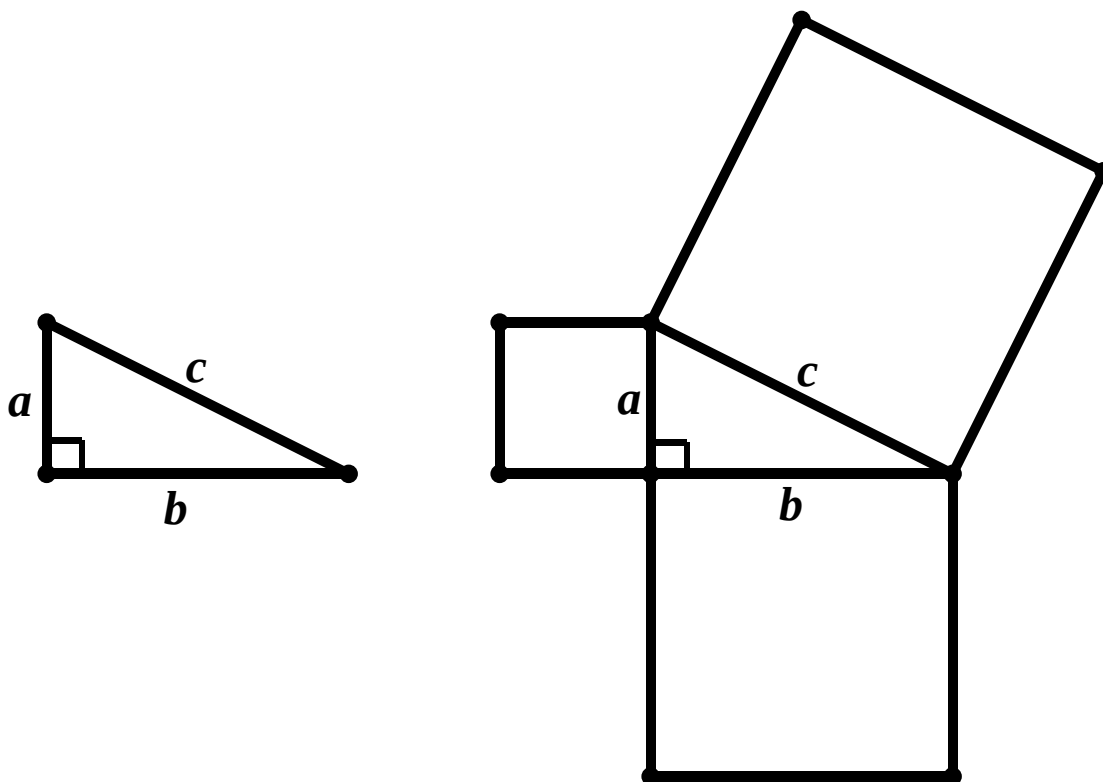
5 squared (25) + area of 4 triangles = 3 squared (9) + 4 squared (16) + area of 4 triangles

$25 = 9 + 16$

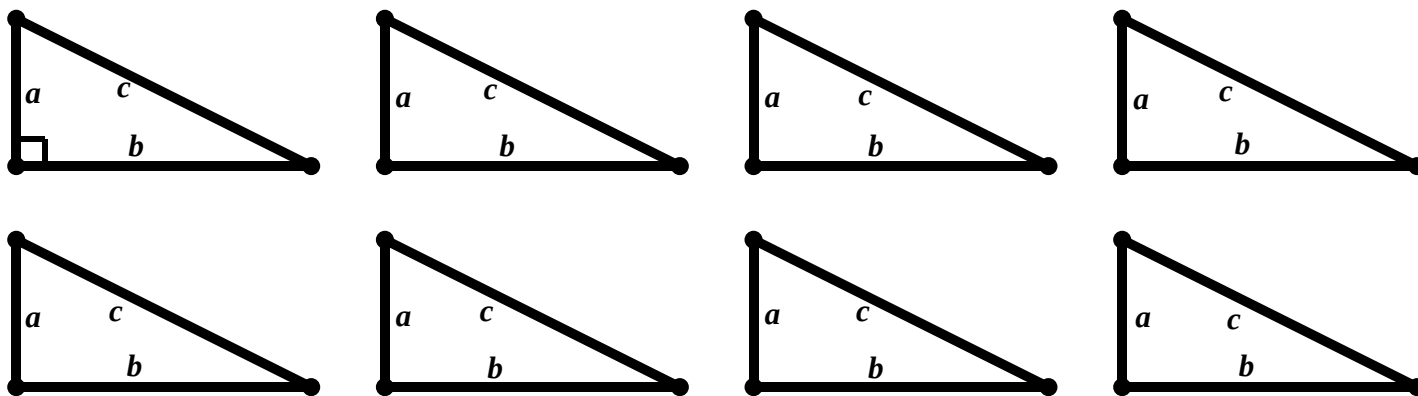
$25 = 25$

6. In the previous problems, we saw that for specific triangles **the sum of the areas of the squares along the legs of the right triangle equals the area of the square along the hypotenuse of the triangle** by looking at several examples. Now, we want to show that this relationship holds true for *any* right triangle.

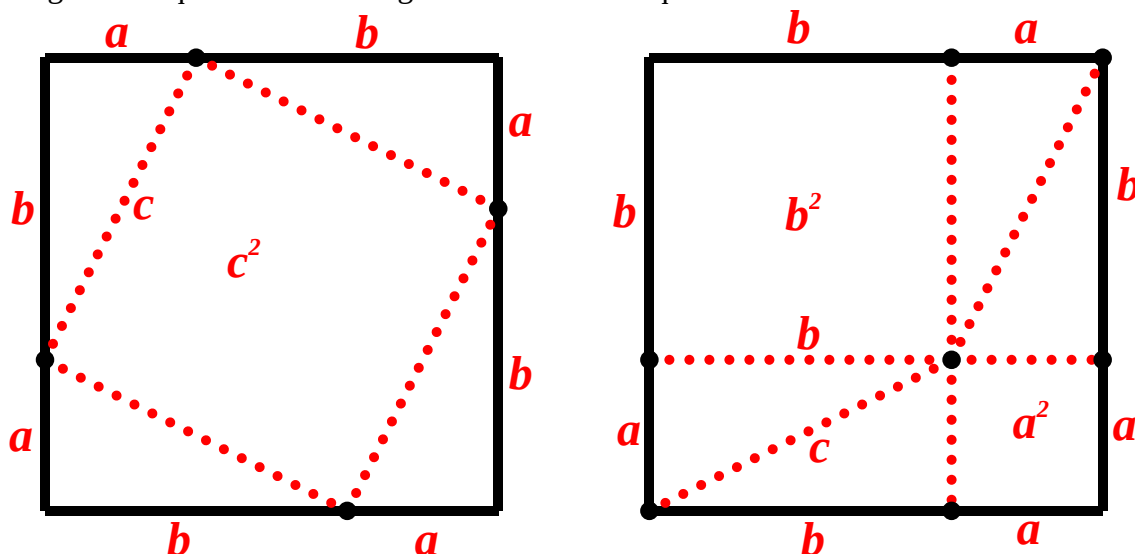
Suppose you have a right triangle with any side lengths a , b , and c where a and b are the legs of the triangle and c is the hypotenuse of the right triangle as shown below. The squares have been drawn along the sides of the right triangle. Our goal is to show that $a^2 + b^2 = c^2$ is always true.



- Find the area of each of the squares adjacent to the sides of the right triangle. Write the areas inside each square.
- Cut out the squares formed on the sides of the triangle above as well as the 8 copies of the triangle with side lengths a , b , and c below.



- c. Arrange the 3 squares and 8 triangles to cover the 2 squares shown below.



You will notice that we do not tell the students that the squares above are congruent. Once they have their puzzles in place, have them show that the two squares are congruent. They can use ideas of rigid motion and patty paper (one square is a translation of the other) or notice that both squares have sides lengths of $(a + b)$. There are several visual demonstrations of this proof on the Geogebra Website.

- d. Using the picture above, show that **the sum of the areas of the squares adjacent to the legs of the right triangle equals the area of the square adjacent to the hypotenuse of the triangle for any right triangle.**



Students can use arguments similar to those in 5d and their puzzle picture above to show that the sum of the areas of the squares adjacent to the small sides of a right triangle equals the area of the square adjacent to the large side of a right triangle.

Area of Square 1 = Area of Square 2 (congruent by rigid motion)
 area of 4 triangles + c^2 = area of 4 triangles + $a^2 + b^2$
 $c^2 = a^2 + b^2$

Area of Square 1 = Area of Square 2 (both squares have side lengths of $(a + b)$)
 area of 4 triangles + c^2 = area of 4 triangles + $a^2 + b^2$
 $4\left(\frac{1}{2}ab\right) + c^2 = 4\left(\frac{1}{2}ab\right) + a^2 + b^2$
 $2ab + c^2 = 2ab + a^2 + b^2$
 $c^2 = a^2 + b^2$

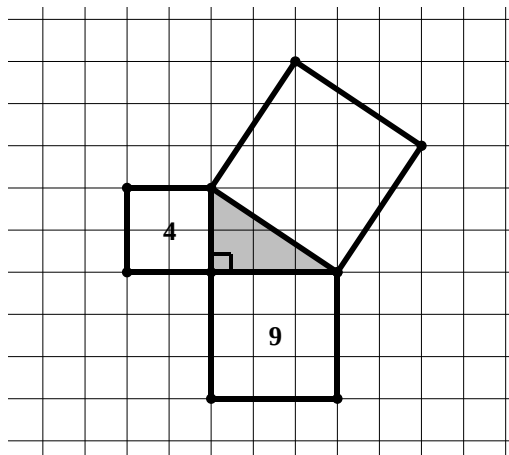
If the algebra shown here enhances your student's understanding then discuss it with them. Otherwise, it is not necessary. The rearrangement of the triangles shows directly that the square adjacent to the hypotenuse is equal to the sum of the squares adjacent to the sides.

- e. Conventionally, the leg lengths of a right triangle are denoted using the variables a and b and the hypotenuse of a right triangle is denoted using the variable c . State the relationship between the side lengths of a right triangle using the words **legs** and **hypotenuse**.
 The sum of the areas of the squares adjacent to the legs of a right triangle equals the area of the square adjacent to the hypotenuse of a right triangle.
- f. Write an **equation** that shows the relationship between the side lengths of a right triangle using a and b for the lengths of the legs and c for the length of the hypotenuse.
The Pythagorean Theorem: $a^2 + b^2 = c^2$ for a right triangle whose leg lengths are a and b and whose hypotenuse is of length c .

Be sure to tell students that this equation that shows the relationship between the side lengths of a right triangle is called the Pythagorean Theorem.

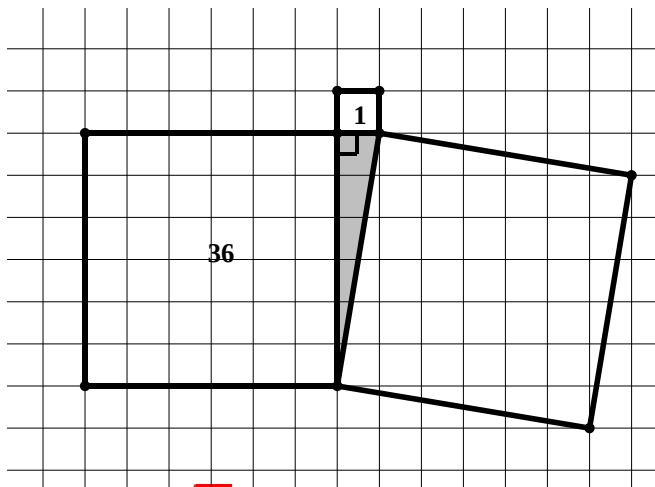
Directions: In each of the problems below, a right triangle is shown in gray. The squares along each of the three sides of the triangles have been drawn. The area of two of the squares is given. Determine the area of the third square. Write your answer in the square. Also find the side length of each square, write the sides lengths below each picture.

1. 13



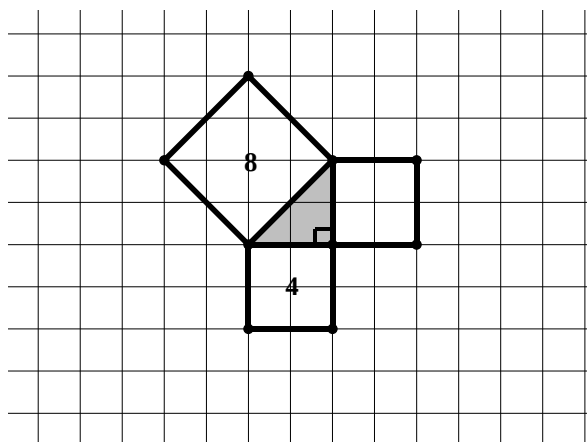
2, 3, $\sqrt{13}$

2. 37



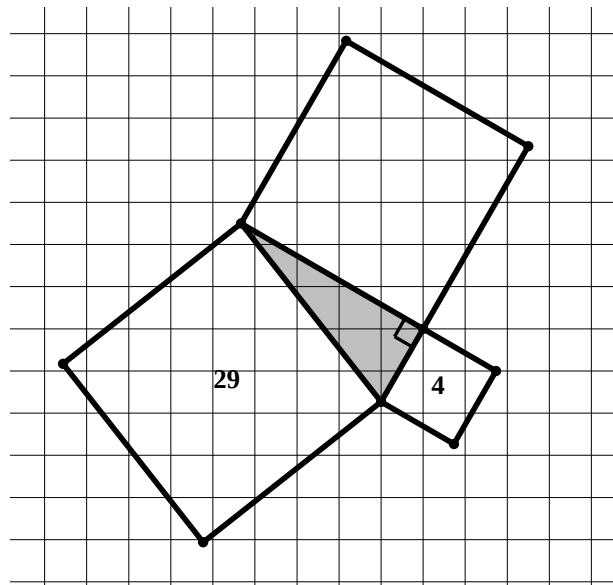
1, 36, $\sqrt{37}$

3. 4



2, 2, $\sqrt{8} = 2\sqrt{2}$

4. 25

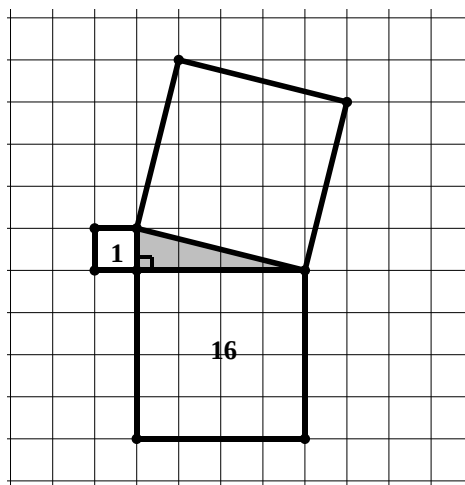


2, 5, $\sqrt{29}$

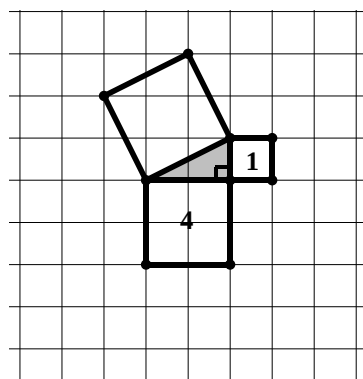
10.2a Homework: A Proof of the Pythagorean Theorem

Directions: In each of the problems below, a right triangle is shown in gray. The squares along each of the three sides of the triangles have been drawn. The area of two of the squares is given. Determine the area of the third square. Write your answer in the square.

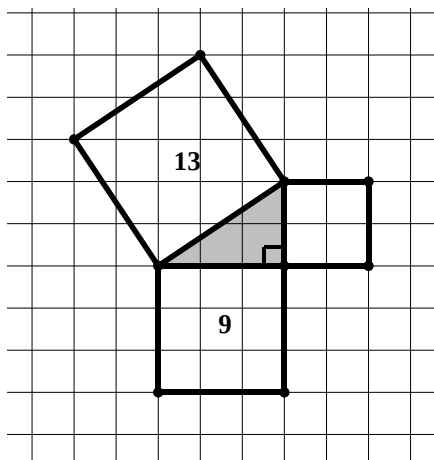
1. 17



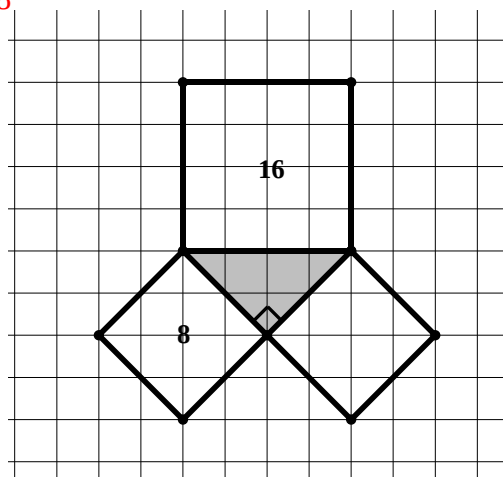
2. 5



3. 4

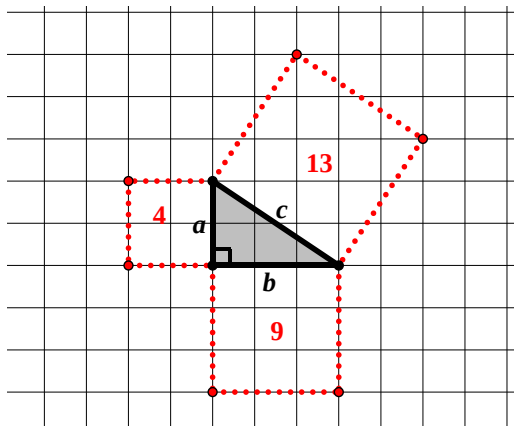


4. 8



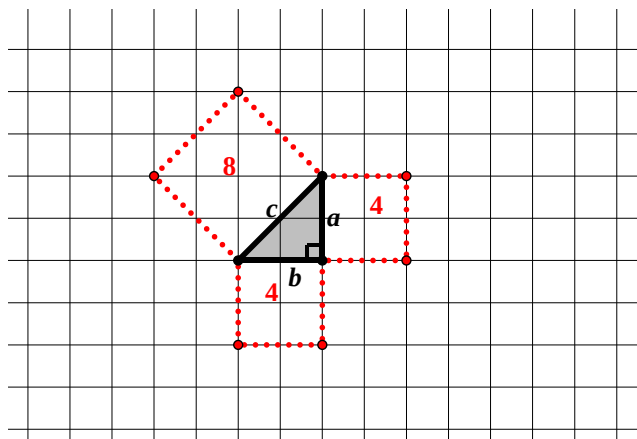
Directions: For each of the following problems, the gray triangle is a right triangle. Draw the squares adjacent to each of the three sides of the triangles. Find the area of each square and write the area in each square. Then, find the side lengths a , b , c of each triangle. **These problems reinforce many ideas from chapter 7 including the relationship between the side length of a square and its area and ideas about irrational numbers.**

5.



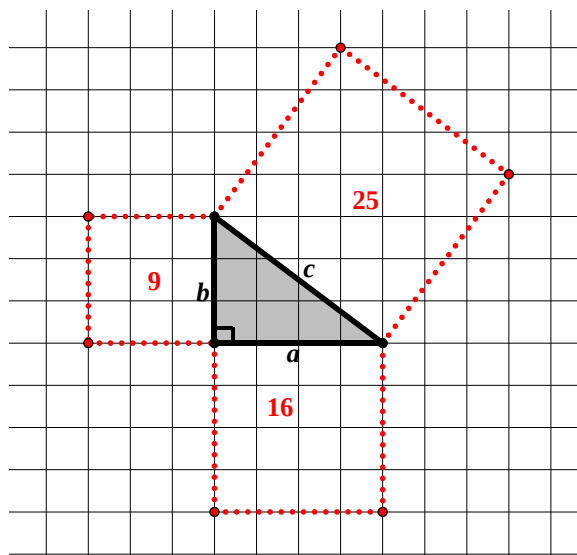
$a = \underline{2}$ $b = \underline{3}$ $c = \underline{\sqrt{13}}$

6.



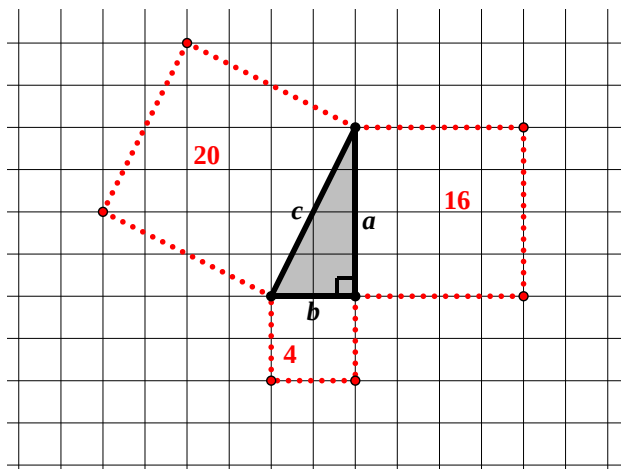
$a = \underline{2}$ $b = \underline{2}$ $c = \underline{2\sqrt{2}}$

7.



$a = \underline{4}$ $b = \underline{3}$ $c = \underline{5}$

8.



$a = \underline{4}$ $b = \underline{2}$ $c = \underline{2\sqrt{5}}$

Honor's Extension: Have students research and present an explanation of alternative proofs of the Pythagorean Theorem.

10.2b Class Activity: The Pythagorean Theorem and Tilted Squares

In this lesson, students will make many connections to content studied in chapter 7. Methods for construction will vary. It is encouraged that students work in pairs or groups on this assignment.

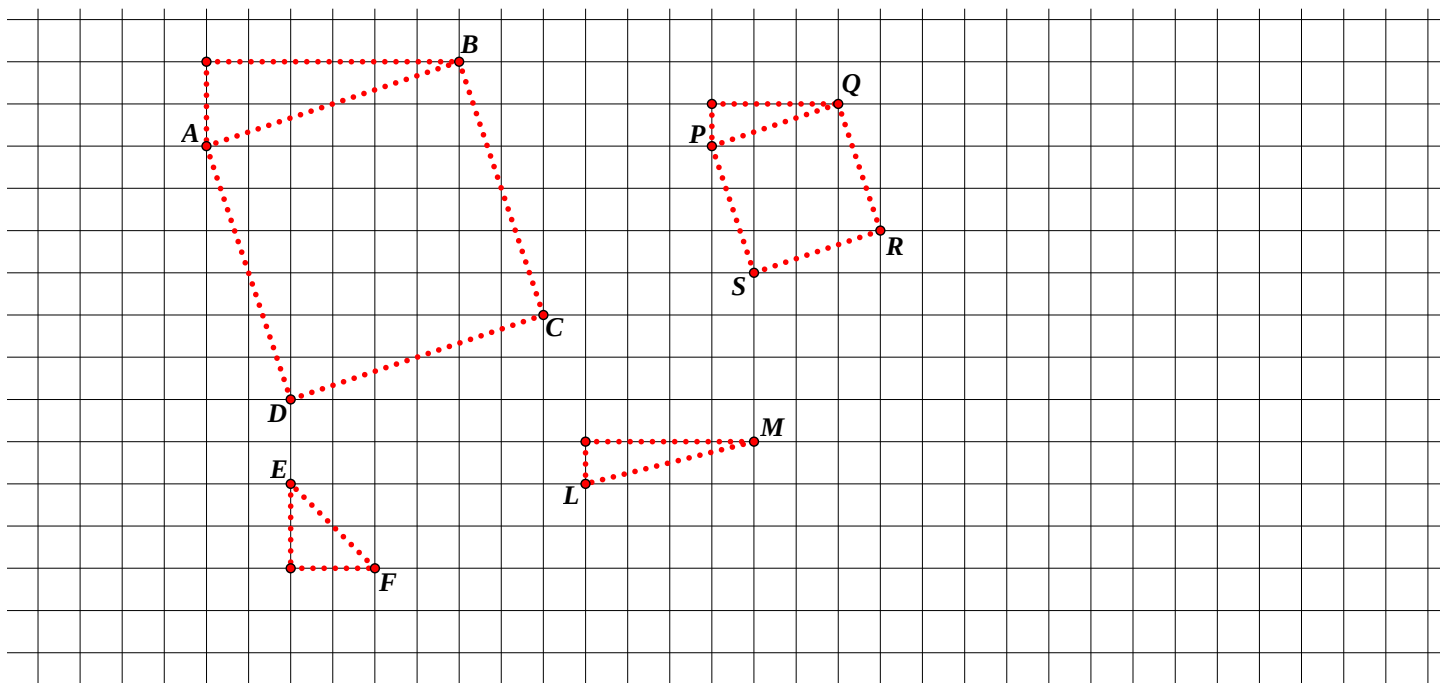
There are many questions you can ask to get them thinking about how to construct the objects below if they are struggling. For example, on question a) you may ask, if I need a square with an area of 40, can I make two smaller squares that add to 40? It will obviously be easier for students to choose perfect squares. In this case, students may choose 36 and 4. Elicit ideas about the Pythagorean Theorem – if the 36 and 4 were squares adjacent to the sides of a right triangle, what would be the measures of the sides of the triangle? Would these be the legs of the right triangle or the hypotenuse?

Alternatively, you may ask, if I need a square with an area of 40 square units, what does the side length of the square need to be. $\sqrt{40}$. Can you construct a right triangle that has a side length of $\sqrt{40}$ or $2\sqrt{10}$? What about using ideas about scaling, can you create a right triangle with a side length of $\sqrt{10}$ and then double all of the lengths of the sides? You can ask similar probing questions for b – c.

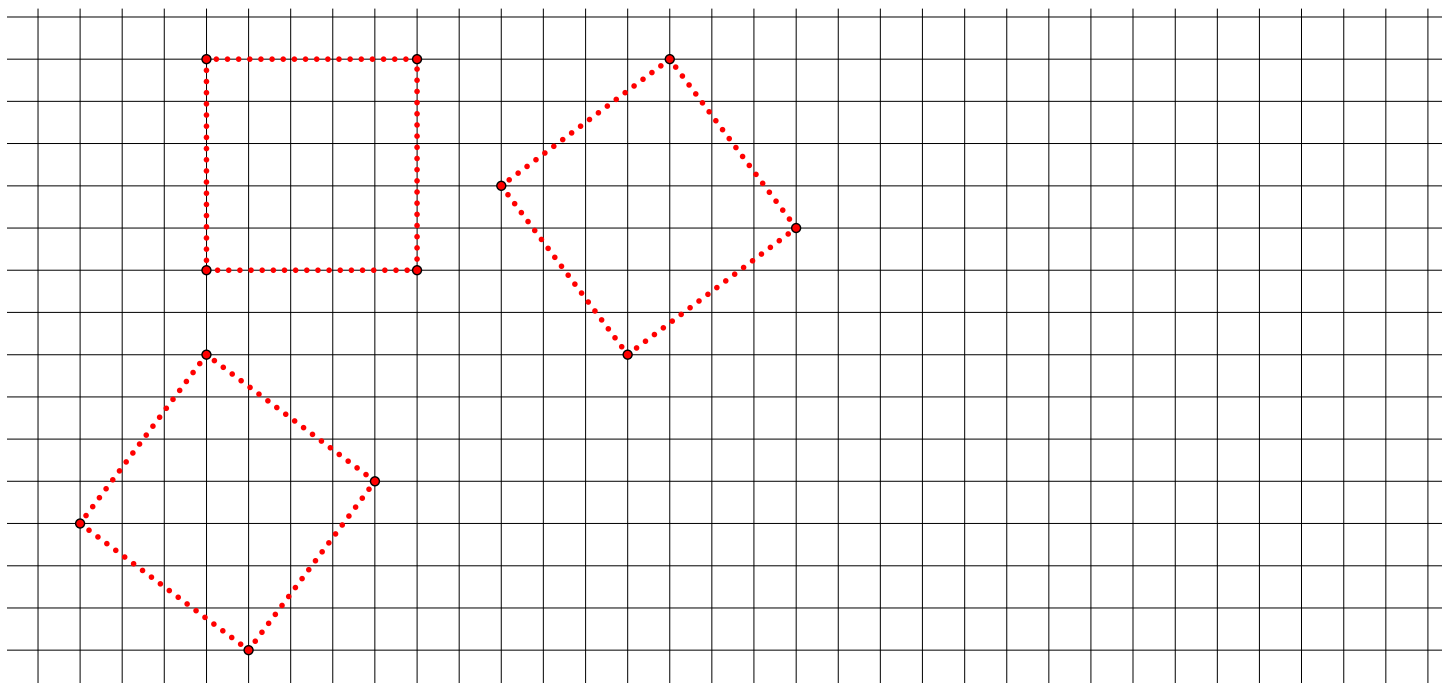


1. On the grids below, construct the following and clearly label each object:

- Square $ABCD$ that has an area of 40 square units
- Square $PQRS$ that has an area of 10 square units
- $\overline{EF}$ that has a length of $\sqrt{8}$ units
- $\overline{LM}$ that has a length of $\sqrt{17}$ units

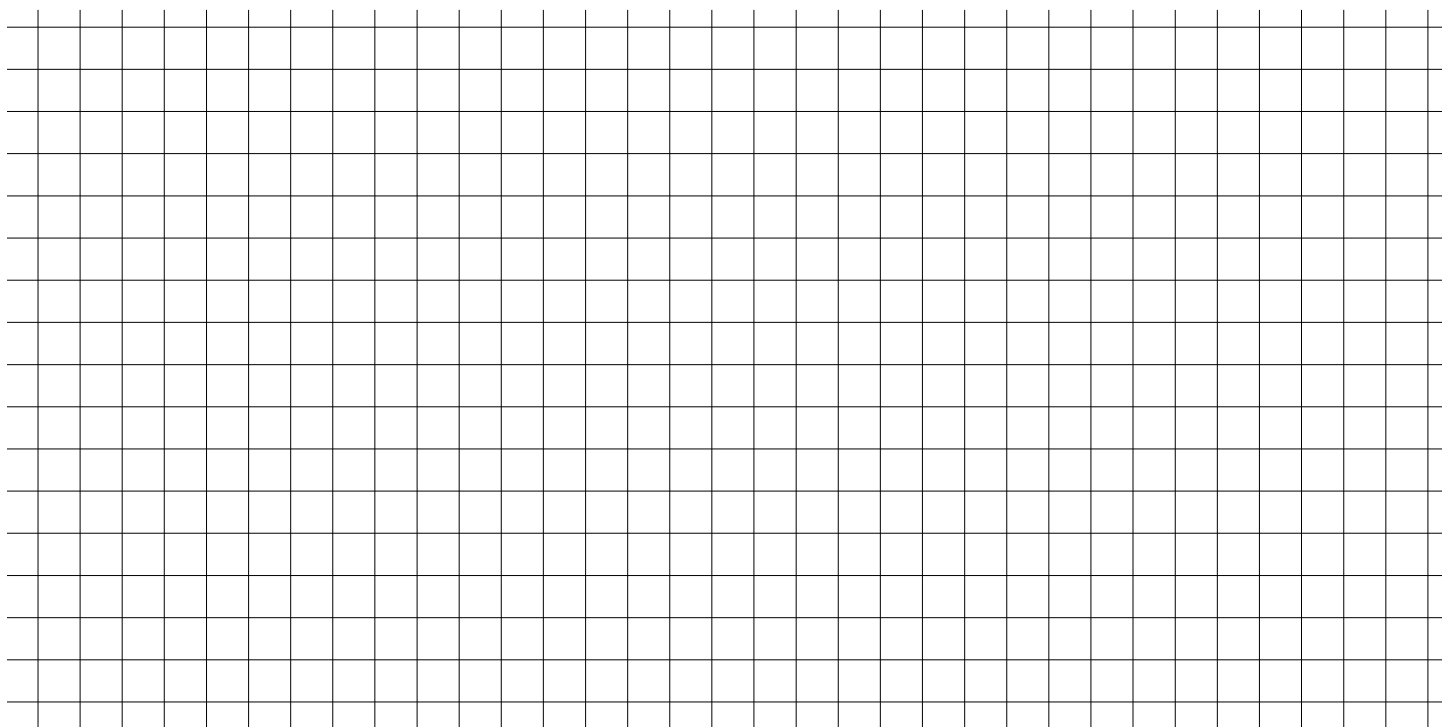
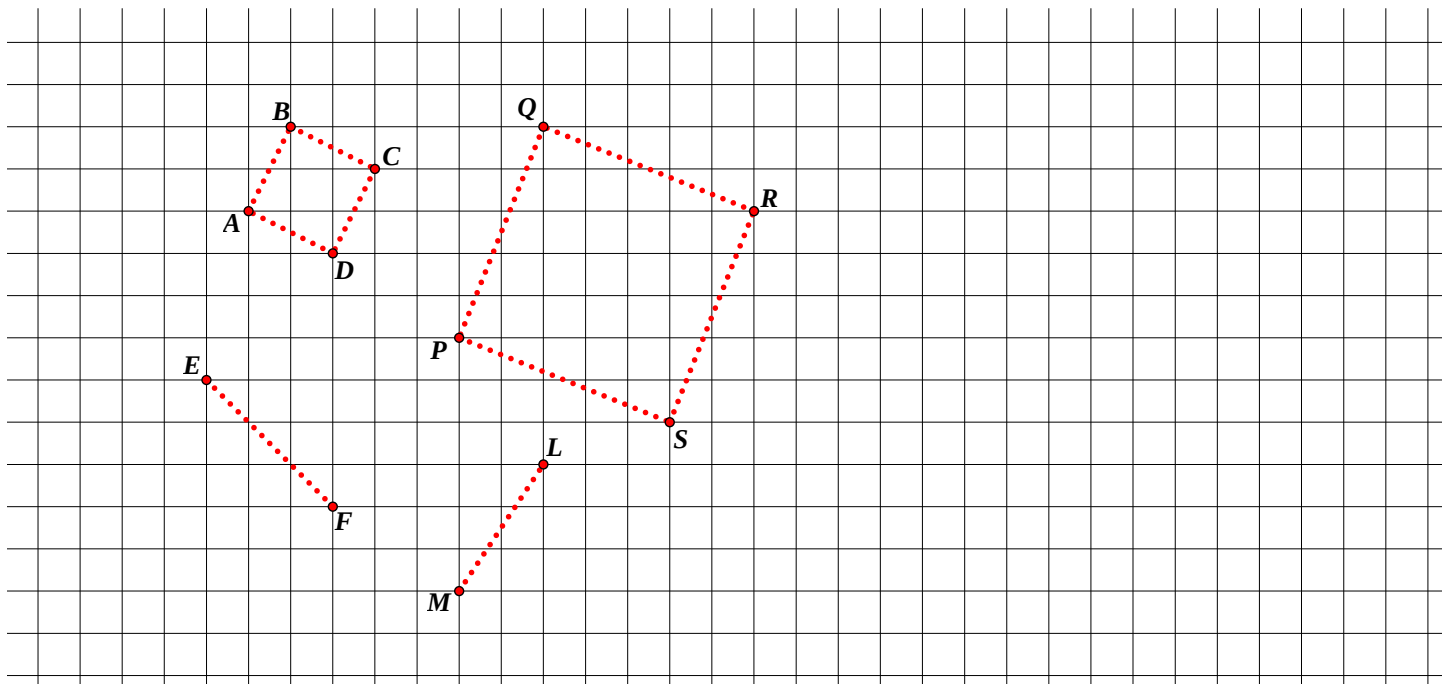


2. Draw as many different squares as you can with an area of 25 square units on the grids below. In this problem, different means that the squares are not tilted the same way. Students should think about their 3, 4, 5 right triangles. If they need a square with an area of 25, they need a side length of 5. This can be constructed with 3, 4, 5 triangles. Changing the rise and run of the right triangles will change the tilt of the square. Again, you can ask probing questions like, “If I want a square with an area of 25, what does the side length need to be? How can I create different segments with lengths of 5?”



10.2b Homework: The Pythagorean Theorem and Tilted Squares

1. On the grids below, construct the following and clearly label each object:
 - a. Square $ABCD$ that has an area of 5 square units
 - b. Square $PQRS$ that has an area of 29 square units
 - c. $\overline{EF}$ that has a length of $\sqrt{18}$ units
 - d. $\overline{LM}$ that has a length of $\sqrt{13}$ units

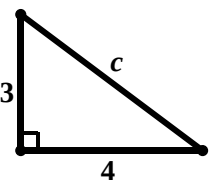
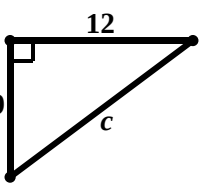
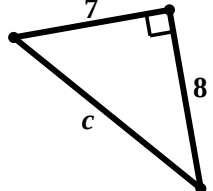
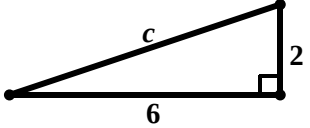
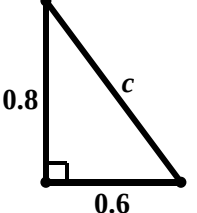
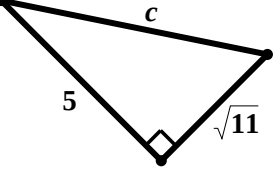


10.2c Class Activity: The Pythagorean Theorem and Unknown Side Lengths

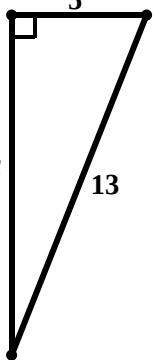
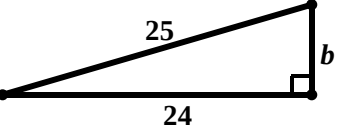
It is good practice to have students start by writing down the PT(Pythagorean Theorem) for each problem and then substituting in the values for a , b , and c .

Directions: Find the length of the hypotenuse of each right triangle shown below using the Pythagorean Theorem. Leave your answer in simplest radical form.

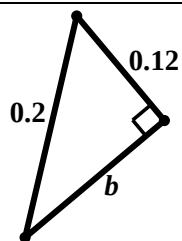


1.  $c = \underline{\text{5}}$	2.  $c = \underline{\text{15}}$
3.  $c = \underline{\sqrt{113}}$	4.  $c = \underline{2\sqrt{10}}$
5.  $c = \underline{\text{1}}$	6.  $c = \underline{\text{6}}$

Directions: Find the length of the leg of each right triangle shown below using the Pythagorean Theorem. Leave your answer in simplest radical form.

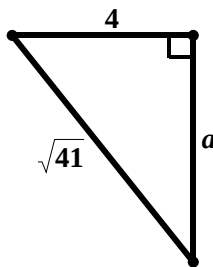
7.  $a = \underline{\text{12}}$	8.  $b = \underline{\text{7}}$
---	---

9.



$$b = \underline{0.16}$$

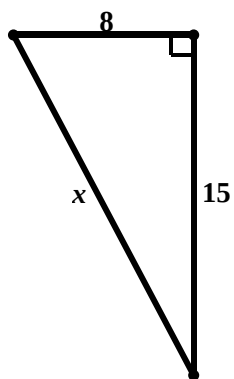
10.



$$a = \underline{5}$$

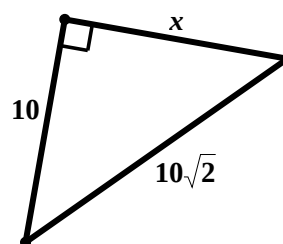
Directions: Find the value of x using the Pythagorean Theorem. Leave your answer in simplest radical form.

11.



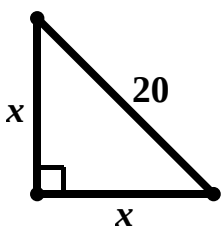
$$x = \underline{17}$$

12.



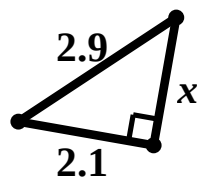
$$x = \underline{10}$$

13.



$$x = \underline{10\sqrt{2}}$$

14.



$$x = \underline{2}$$

10.2c Homework: The Pythagorean Theorem and Unknown Side Lengths

Directions: Two side lengths of a right triangle have been given. Solve for the missing side length if a and b are leg lengths and c is the length of the hypotenuse. Leave your answer in simplest radical form.

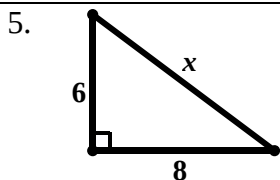
1. $a = 16, b = 30, c = ?$
 34

3. $a = 40, b = ?, c = 50$
 30

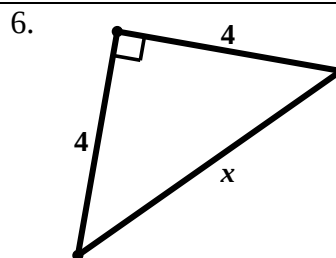
2. $a = 2, b = 2, c = ?$
 $2\sqrt{2}$

4. $a = ?, b = 4\sqrt{3}, c = 8$
 4

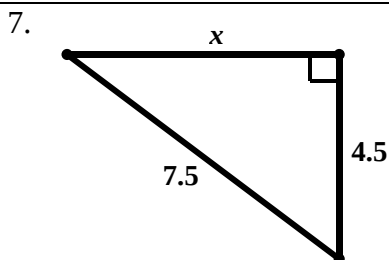
Directions: Find the value of x using the Pythagorean Theorem. Leave your answer in simplest radical form.



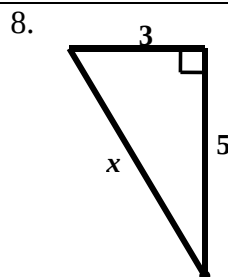
$x = \underline{10}$



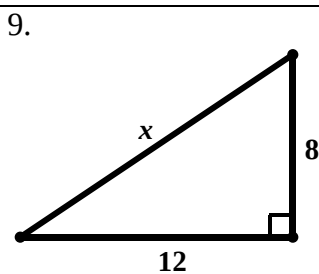
$x = \underline{4\sqrt{2}}$



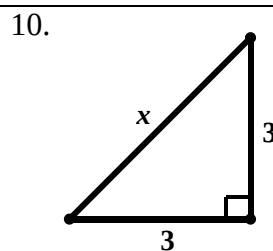
$x = \underline{6}$



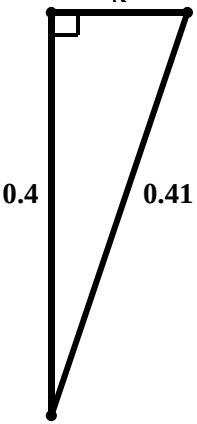
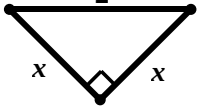
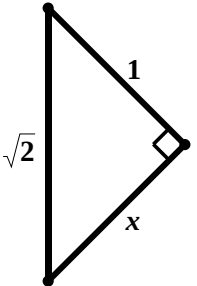
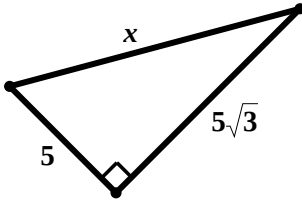
$x = \underline{\sqrt{34}}$



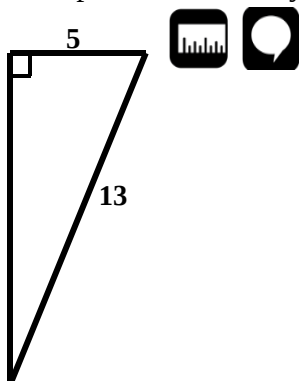
$x = \underline{4\sqrt{13}}$



$x = \underline{3\sqrt{2}}$

11.  $x = \underline{.09}$	12.  $x = \underline{\sqrt{2}}$
13.  $x = \underline{1}$	14.  $x = \underline{10}$

15. **Find, Fix, and Justify:** Megan was asked to solve for the unknown side length in the triangle below. Her work is shown below. She made a mistake when solving. Explain the mistake she made and then solve the problem correctly.



Megan's Solution:

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 5^2 + 13^2 &= c^2 \\
 25 + 169 &= c^2 \\
 194 &= c^2 \\
 \sqrt{194} &= c
 \end{aligned}$$

Correct Solution:

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 5^2 + b^2 &= 13^2 \\
 25 + b^2 &= 169 \\
 b^2 &= 144 \\
 b &= 12
 \end{aligned}$$

Explain Mistake:

a and b are legs of a right triangle. Megan substituted in the length of the hypotenuse for one of the legs.

16. **Find, Fix, and Justify:** Raphael was asked to solve for the length of the hypotenuse in a right triangle with legs that have side lengths of 4 and 5. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.

Raphael's Solution:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 4^2 + 5^2 &= c^2 \\ 16 + 25 &= c^2 \\ 41 &= c \end{aligned}$$

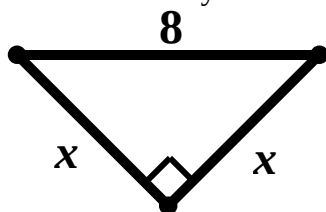
Correct Solution:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 4^2 + 5^2 &= c^2 \\ 16 + 25 &= c^2 \\ 41 &= c^2 \\ \sqrt{41} &= c \end{aligned}$$

Explain Mistake:

Raphael did not take the square root of both sides of the equation in the last step of solving.

17. **Find, Fix, and Justify:** Nataani was asked to solve for the unknown side length in the triangle below. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.



Nataani's Solution:

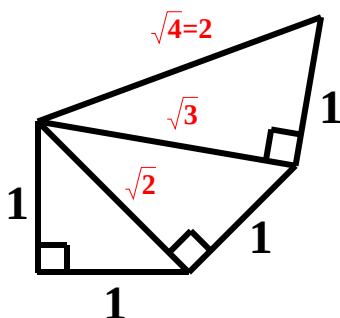
$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 + x^2 &= 8 \\ 2x^2 &= 8 \\ x^2 &= 4 \\ x &= 2 \end{aligned}$$

Correct Solution:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 + x^2 &= 8^2 \\ 2x^2 &= 64 \\ x^2 &= 32 \\ 4\sqrt{2} & \end{aligned}$$

Explain Mistake: Nataani substituted in 8 for the hypotenuse but did not square it.

Extra for Experts: Use the picture below to answer questions a) and b).



- Find all the missing side lengths and label the picture with the answers.
- Using the picture above, devise a strategy for constructing a segment with a length of $\sqrt{5}$. Explain your strategy below. Construct another right triangle with leg lengths 2 and 1 adjacent to the triangle with a hypotenuse of $\sqrt{4}$ (or 2). This would be a good place to talk again about perfect squares and irrational numbers.

10.2d Class Activity: The Converse of the Pythagorean Theorem

Students will need centimeter rulers and something to measure a right angle (a protractor, an index card, etc.) An alternative way of implementing this lesson is to have students first verify that the side lengths satisfy the Pythagorean Theorem, then give them something like pretzel sticks, straws, strips of cardstock, etc. to represent these lengths and ask them to create a triangle with each of the sets. They will see that the only triangle they can create has a right angle. You can also use string that has been knotted (or marked in some way) to show equal “unit” segments. Once students identify a Pythagorean triple that works, they can use the marked string to try to create the triangles and will see that the given lengths determine a right triangle.

1. Mr. Riley’s 8th grade class has been studying the Pythagorean Theorem. One day, he asked his class to find numbers a , b , and c where $a^2 + b^2 = c^2$, and draw triangles with those side lengths. 

Oscar determined that the numbers 5, 12, and 13 satisfy the Pythagorean Theorem as shown below:

$$a^2 + b^2 = c^2$$

$$5^2 + 12^2 = 13^2$$

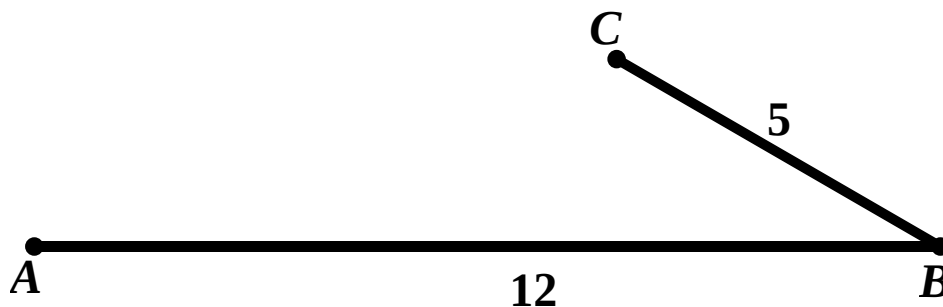
$$25 + 144 = 169$$

$$169 = 169$$

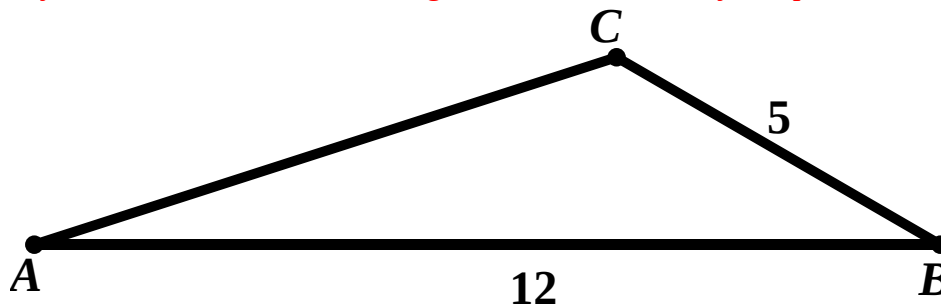
Walk students through this – do 5, 12, and 13 satisfy the PT?

Mr. Riley then said, “OK, so you have found three numbers that satisfy the Pythagorean Theorem. Now, show me that the triangle formed with these side lengths is a right triangle.”

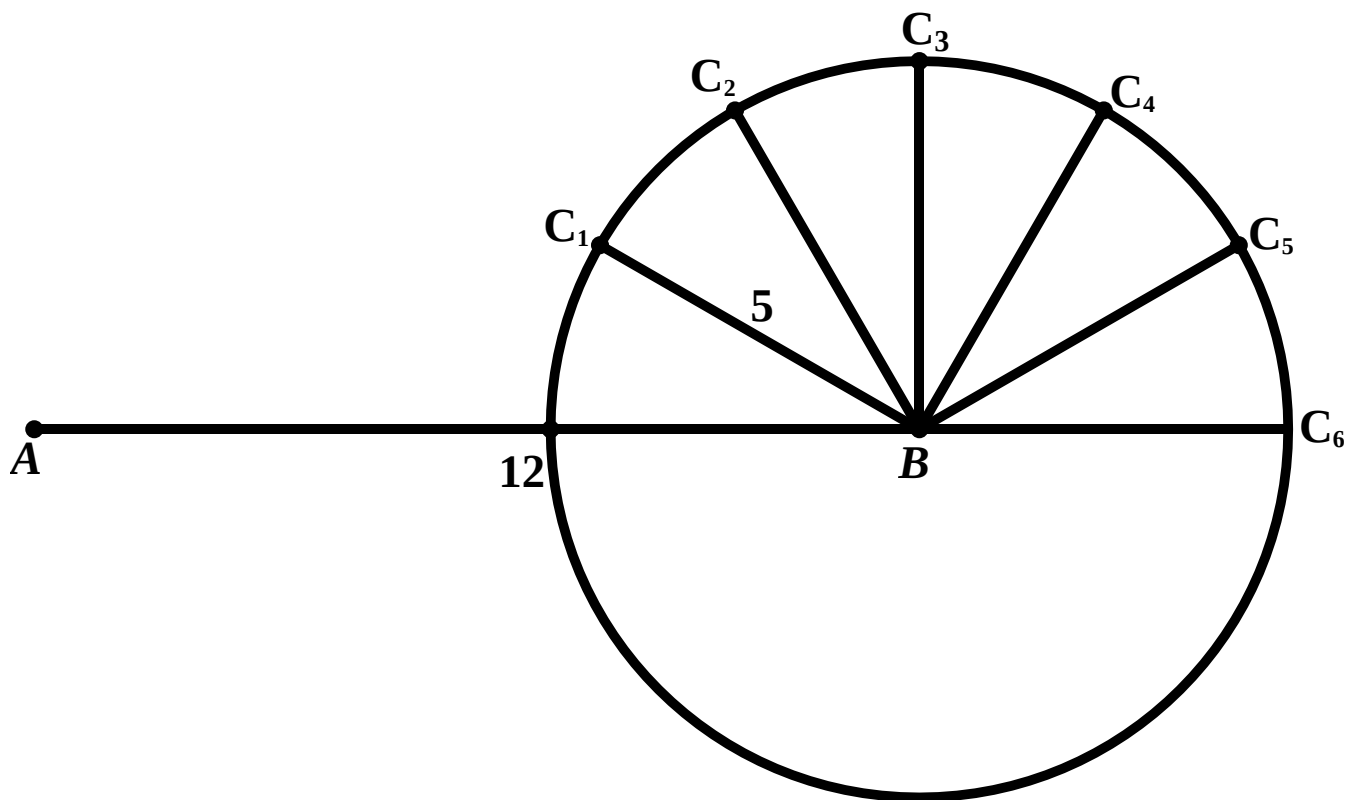
- a. Oscar continued working on the problem. He constructed a segment with a length of 12 cm and labeled the segment AB . From the endpoint B , he constructed a segment with a length of 5 cm and labeled the segment BC as shown in the picture below. Using a ruler, verify the lengths of the segments below. Have students verify the lengths of the segments below



- b. Then, he thought to himself, “I need to make the third side length AC equal to 13 because I know the triple 5, 12, 13 satisfies the Pythagorean Theorem.” He connected A and C as shown below. He measured the length of AC and determined it did not measure 13 cm. Using a ruler, verify that AC does not measure 13 cm. From here, students can play around with the lengths of the segments and see if they can create the 5, 12, 13 triangle on their own or they can proceed to the next page.



- c. Then, he thought to himself, “What if I rotate $\overline{BC}$ around point B until AC measures 13 cm?” He began to rotate $\overline{BC}$ clockwise about B in increments as shown below. Help Oscar to find the location of C on the circle below that will give him a triangle with side lengths 5, 12, and 13.
 Make sure that students understand $\overline{BC}$ all have a length of 5 cm. Why is this true?



- d. What type of triangle is formed when AC equals 13 cm?
 Once students find the location of C where AC measures 13 cm, have them highlight that triangle. Then have them measure $\angle ABC$ to see that it is a 90° angle.

Honor's Extension: You can have students determine when the resulting triangle will be acute or obtuse. In the example above, a triangle with side lengths 5 and 12 will be acute when AC is less than 13 and obtuse when AC is greater than 13.

2. Lucy also found a set of numbers that satisfy the Pythagorean Theorem: 3, 4 and 5. Verify in the space below that Lucy's numbers satisfy the Pythagorean Theorem.

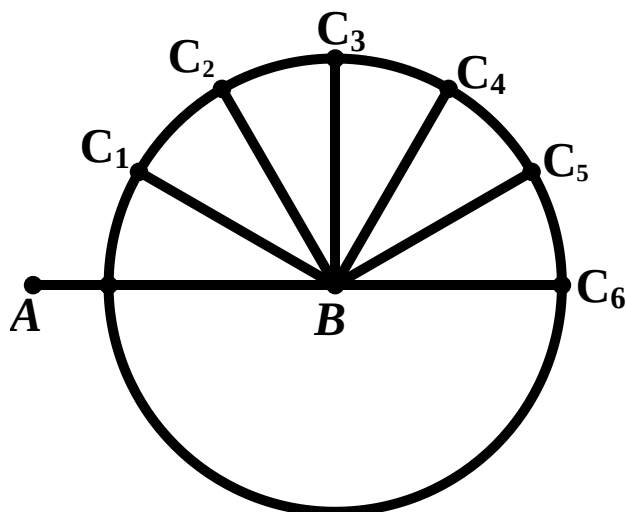
$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

$$25 = 25$$

3. Using a process similar to Oscar's, Lucy set out to prove that a triangle with side lengths 3, 4 and 5 is in fact a right triangle. In the picture below $AB = 4$ cm and $CB = 3$ cm. Help Lucy determine the location of C that will create a triangle with side lengths 3 cm, 4 cm, and 5 cm.



4. What type of triangle is formed when AC equals 5 cm?
 A right triangle
5. Based on the problems above, what type of triangle is formed with side lengths that satisfy the Pythagorean Theorem? Write down the Converse of the Pythagorean Theorem. 

A right triangle will be formed.

Converse of Pythagorean Theorem: For a triangle with side lengths a , b , and c if $a^2 + b^2 = c^2$, then the triangle is a right triangle. It is also worth mentioning that given side lengths a , b , and c , there is at most one triangle that can be created with side lengths a , b , and c (assuming that the sides can be put together to form a triangle). Students learned this in 7th grade when they discussed the uniqueness of triangles.

Be sure to have a discussion with students about being careful to identify the legs (the two shorter sides) and the hypotenuse (the longer side) before plugging the values into the Pythagorean Theorem in the problems below. Have honor's students determine what kind of triangle will be formed (acute or obtuse) if the side lengths do not satisfy the PT.

6. Do the side lengths given below satisfy the Pythagorean Theorem? Remember to distinguish between legs (shorter sides) and the hypotenuse (longest side) and enter them into the equation correctly.

a. 11, 60, 61 Yes	b. 2, 4, 6 No
c. 14, 50, 48 Yes	d. 1, 3, $\sqrt{10}$; Yes
e. 2, 4, and $2\sqrt{5}$. Yes,	f. 5, 6, 8 No

7. Mr. Garcia then asks the class, “What if the tick marks in Lucy’s picture are each 2 cm instead of 1 cm? What are the measures of the side lengths that form the right triangle? Do they satisfy the Pythagorean Theorem?”



6, 8, and 10 – yes they do satisfy the Pythagorean Theorem. This can lead to a great discussion that if you scale any set of triples that satisfy the Pythagorean Theorem, the resulting triples will also satisfy the Pythagorean Theorem. If we think back to the properties of dilations, under a dilation, corresponding angles are congruent. If the 3, 4, 5 triangle has a right angle, then a 6, 8, 10 triangle will also have a right angle.

8. What if the tick marks in Lucy’s picture are each 3 cm? 0.1 cm? 10 cm? What are the measures of the side lengths that form the right triangles given these different scales and do they satisfy the Pythagorean Theorem?

3 cm: 9, 12, 15

0.1 cm: 0.3, 0.4, 0.5

10 cm: 30, 40, 50

10.2d Homework: The Converse of the Pythagorean Theorem

Be sure to have a discussion with students about being careful to identify the legs (the two shorter sides) and the hypotenuse (the longer side) before plugging the values into the Pythagorean Theorem. You can have honors' students classify the triangles that are not right as acute or obtuse.

Directions: Determine whether the three side lengths form a right triangle. Write yes or no on the line provided.

1. 9, 12, 15 __ **Y** __

2. 18, 36, 45 __ **N** __

3. 12, 37, 35 __ **Y** __

4. 8, 15, 16 __ **N** __

5. $\sqrt{6}$, $\sqrt{10}$, 4 __ **Y** __

6. 6.4, 12, 12.2 __ **N** __

7. 8.6, 14.7, 11.9 __ **N** __

8. 8, $8\sqrt{3}$, 16 __ **Y** __

9. 8, 8, $8\sqrt{2}$ __ **Y** __

10. 7, 9, 11.4 __ **N** __

It is good to point out in #7 and #10, that rounding does not make it work (will not form a right triangle).

10.2e Class Activity: Exploration with Pythagorean Triples Extension

EXTENSION exploration – you can assign as a HW project

– it is not intended that you spend class time on this unless time permits



While we have seen several different sets of numbers that form a right triangle, there are special sets of numbers that form right triangles called Pythagorean triples. A **Pythagorean triple** is a set of nonzero **whole numbers** a , b , and c that can be put together to form the side lengths of a right triangle. 3, 4, 5 and 5, 12, 13 are examples of Pythagorean triples. We have seen many other sets of numbers that form a right triangle such as 0.09, 0.4, 0.41 that are not Pythagorean triples because their side lengths are not whole numbers.

- The chart below shows some sets of numbers a , b , and c that are Pythagorean triples. Verify that the sets satisfy the equation $a^2 + b^2 = c^2$.
- Can you find additional Pythagorean triples? Explain the method you used.

Students might decide to scale the triples given or they may start to see recursive patterns emerge. The b column has a difference column that is that of a quadratic (+8, +12, +16, +20, etc.). Similarly, the c column also has a difference column that is that of a quadratic (+8, +12, +16, etc.). There are also explicit equations that show the relationship between a , b , and c as shown in the table below.

a	b	c	a^2	b^2	c^2
3	4	5	Students can use these columns to verify that the triples they find satisfy the Pythagorean Theorem. They may also find relationships between the squares of the side lengths and the lengths of the sides that will help them to find additional triples.		
5	12	13			
7	24	25			
9	40	41			
11	60	61			
13	84	85			
a	$\frac{(a^2 - 1)}{2}$	$\frac{(a^2 + 1)}{2}$ or $(b + 1)$	The explicit equations shown to the left will work for some Pythagorean Triples but not all of them. For example, the triples, 20, 21, 29 and 28, 45, 53 do not work for either of these explicit equations. However the formula below can be used to find most of them. Take any m and n , such that $m > n$ and then find: $2mn, m^2 - n^2, m^2 + n^2$.		
6	8	10			
8	15	17			
a	$\left(\frac{a^2}{4}\right) - 1$	$\left(\frac{a^2}{4}\right) + 1$			

- The chart above starts with values for a that are odd numbers. Why didn't the chart start with a value of 1 for a . The other leg and hypotenuse cannot both be whole numbers when $a = 1$ so the measures would not be a Pythagorean Triple.
- Can you find Pythagorean triples where a is even? What is the smallest Pythagorean triple you can find with a being an even number? See table above for a few PT where a is even. Students can use the fact that the smallest triangle that can be created with a being odd is a 3, 4, 5. If we scale this triangle so that 3 is even and all the numbers remain whole numbers, we would scale all lengths by a factor of 2 leaving us with 6, 8, 10 as the smallest triangle.
- Design a method to confirm that these numbers actually form right triangles. Write a short paragraph describing the method you used, and the results you obtained. Students can construct the triangles with given measurements using toothpicks, hot tamales, pretzel rods, string, GSP, Geogebra, graph paper, etc. to prove that the triples they find form right triangles.

10.2f Self-Assessment: Section 10.2

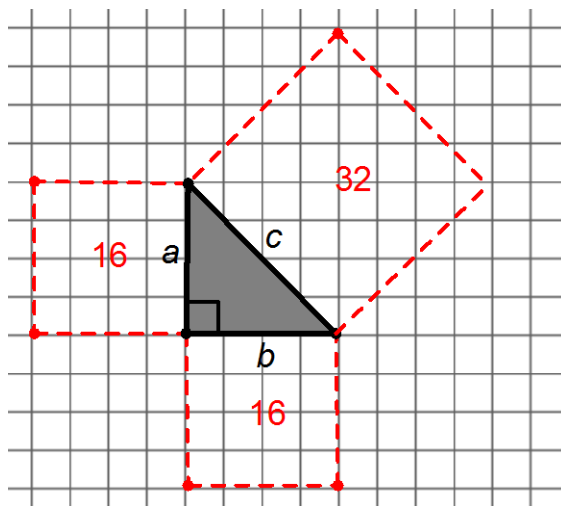
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provided on the next page that match each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Know that in a right triangle $a^2 + b^2 = c^2$, where a and b are the legs of the right triangle and c is the hypotenuse. <i>See sample problem #1</i>				
2. Understand and explain a proof of the Pythagorean Theorem. <i>See sample problem #2</i>				
3. Use the Pythagorean Theorem to solve for the missing side length of a right triangle given the measurements of the other two sides. <i>See sample problem #3</i>				
4. Understand and explain a proof of the converse of the Pythagorean Theorem. That is, for a triangle with side lengths a , b , and c if $a^2 + b^2 = c^2$, then the triangle is a right triangle. <i>See sample problem #4</i>				
5. Use the converse of the Pythagorean Theorem to determine whether three given measurements are possible side lengths of a right triangle. <i>See sample problem #5</i>				

10.2f Sample Problems: Section 10.2

Sample Problem #1

In the picture below the gray triangle is a right triangle. Draw the squares along each of the three sides of the triangles. Find the area of each square and write the area in each square. Then, find the side lengths a , b , c of the triangle.



$$a = \underline{4}, \quad b = \underline{4}, \quad c = \underline{\sqrt{32}} = 4\sqrt{2}$$

Sample Problem #2

Below is a geometric explanation for a proof of the Pythagorean Theorem: Given a right triangle with side lengths a and b and a hypotenuse of c , then $a^2 + b^2 = c^2$. The figures for the proof are given in order. Choose the explanation that provides a sound argument accompanied with reasoning and warrants to support the claims given for each figure. Write the letter that matches each explanation in the space provided.

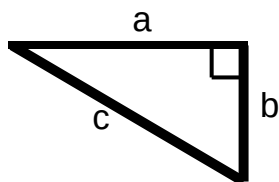


Figure 1 F

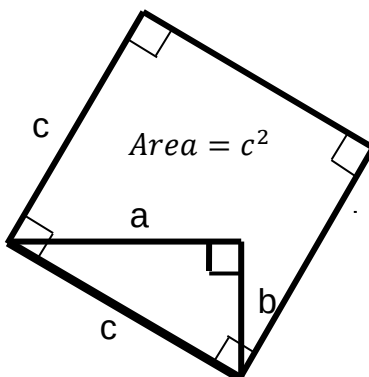


Figure 2 B

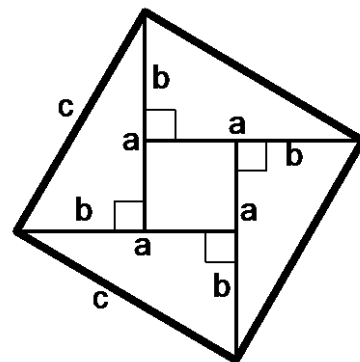


Figure 3 A

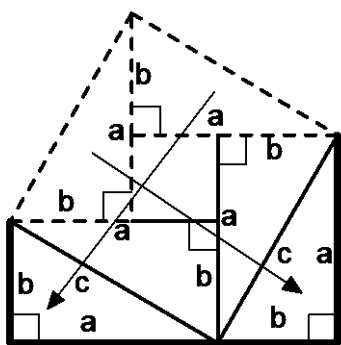


Figure 4 D

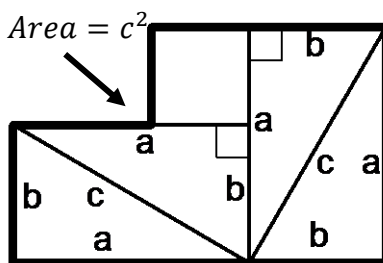


Figure 5 E

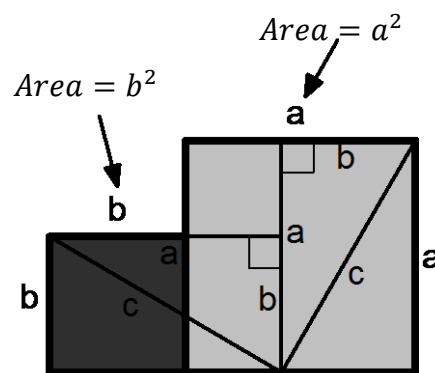


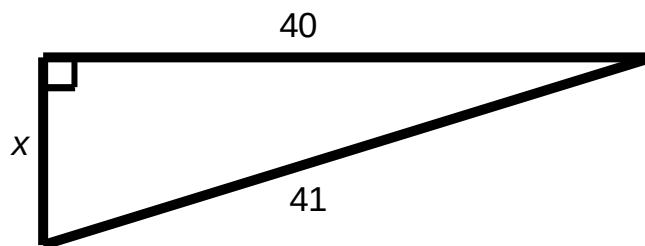
Figure 6 C

- A. Inside of the square draw 4 congruent right triangles with side length a and b and a hypotenuse of c .
- B. Draw a square with off of this triangle with a side length of c . The area of square this square is c^2 . This is because the area of a square is the side length squared
- C. You can view the area of this figure as the composition of two squares with sides length a and b . The area of the darker square is b^2 and the area of the lighter shaded square is a^2 . Thus the area of the whole figure is $a^2 + b^2$. As stated above this is the same as the area of the original square with side length c . Thus $a^2 + b^2 = c^2$.
- D. Rearrange the square by translating the top two triangles to the bottom of the figure.
- E. The area of this figure is the same as the area of the original square because we have not added or removed any of the pieces.
- F. Begin with right triangle with a horizontal side length of a and a vertical side length of b and a hypotenuse of c .

For figure 3 note that, for any side of the given square, there are two right triangles with that side as hypotenuse: once we have done this for one side, this determines the other 3 triangles.
Also if students asks how we know that it is a square and not a rhombus that is formed off of our triangle we can argue that the two angles in our original triangle that are formed off of side c are complementary.

Sample Problem #3

Find the value of x using the Pythagorean Theorem. Leave your answer in simplest radical form.

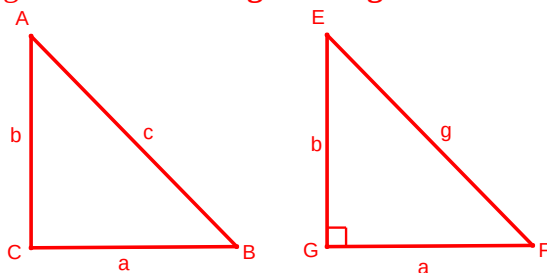


$$x = 9$$

Sample Problem #4

The Converse of the Pythagorean Theorem states that given a triangle with side lengths a , b , and c , if $a^2 + b^2 = c^2$, then the triangle is a right triangle. Explain the proof of the Converse of the Pythagorean Theorem that your teacher provides for you.

We are given $\triangle ABC$ where $a^2 + b^2 = c^2$ as shown in the picture below. We want to show that $\triangle ABC$ is a right triangle. To show this we are going to consider the **right triangle** $\triangle EFG$ that has the same leg lengths, a and b , as $\triangle ABC$.



In $\triangle ABC$, $a^2 + b^2 = c^2$.

This information is given to us.

In right $\triangle EFG$
 $a^2 + b^2 = g^2$.

$\triangle EFG$ is a right triangle so the Pythagorean Theorem is true.

Below is possible explanation of the Converse of the Pythagorean Theorem, given as a flow diagram. You could have students explain this proof in a variety of ways. Some options could be to mix up the explanations given in the ovals and have them match them with their corresponding arguments given in the rectangles. You might ask students to point out what they know given the two triangles. For example, they might say that if you are given side lengths a , b , and c there is one unique triangle that can be formed, or that they know that $c^2 = g^2$ because they are both equal to $a^2 + b^2$. You could state an argument and ask the students to back up the argument with a reason or warrant.

We can set $c^2 = g^2$.

$a^2 + b^2 = c^2$ and $a^2 + b^2 = g^2$.
Since c^2 and g^2 equal the same thing they are equal to each other.

Thus $c = g$.

If you take the square root of both sides of the equation $c^2 = g^2$.

Now we know that
 $\triangle ABC \cong \triangle EFG$.

If three sides of one triangle are equal to three sides of another triangle, then the triangles are congruent.

This means that
 $\angle C = \angle G$.

If two triangles are congruent to each other, then their corresponding parts are congruent to each other.

Also, $\angle C$ is a right angle.

$\angle G$ is a right angle and $\angle C = \angle G$.

Therefore $\triangle ABC$ is a right triangle.

$\angle C$ is a right angle.

Sample Problem #5

Determine whether the three side lengths form a right triangle. Show your work to verify your answer.

5.5, 12.5, 13.5

No, the side lengths do not form a right triangle.

$$5.5^2 + 12.5^2 \neq 13.5^2$$

$$30.25 + 156.25 \neq 182.25$$

$$186.5 \neq 182.25$$

In addition to the sample problems given consider showing several geometric proofs of the Pythagorean Theorem. Discuss these proofs with your class or divide your class into groups and have each group study a different proof. Then have them show the rest of the class how the proof works. There are many interactive versions of the geometric proofs on the Geogebra website.

Section 10.3 Applications of the Pythagorean Theorem

Section Overview:

In this section, students apply the Pythagorean Theorem to solve real-world problems in two- and three-dimensions. Then, students use the Pythagorean Theorem to find the distance between two points. After the students gain an understanding of the process being used to find the distance between two points in a coordinate system, students have the opportunity to derive the distance formula from the Pythagorean Theorem and the process being used. Rather than memorizing the distance formula, the emphasis is placed on the process used to find the distance between two points in a coordinate system and the connection between the Pythagorean Theorem and the distance formula.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Use the Pythagorean Theorem to solve problems in real-world contexts, including three-dimensional contexts.
2. Find the distance between two points in a coordinate system.

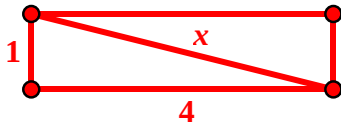
There are many examples of real world applications of the Pythagorean Theorem. You are encouraged to pull from the resources you have and the problems in the mathematical foundation if students need additional practice with applying the Pythagorean Theorem in real-world situations.

10.3a Class Activity: Applications of the Pythagorean Theorem

Directions: For each problem, first draw a picture if one is not provided and then solve the problem.

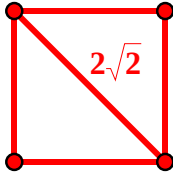


1. What is the length of the diagonal of a rectangle of side lengths 1 inch and 4 inches?



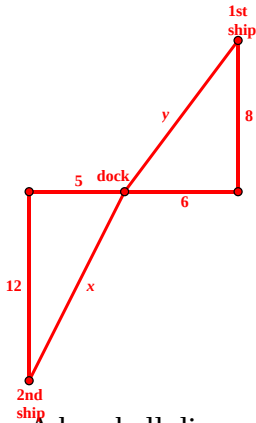
$$x = \sqrt{17}$$

2. A square has a diagonal with a length of $2\sqrt{2}$ inches. What is the side length of the square?



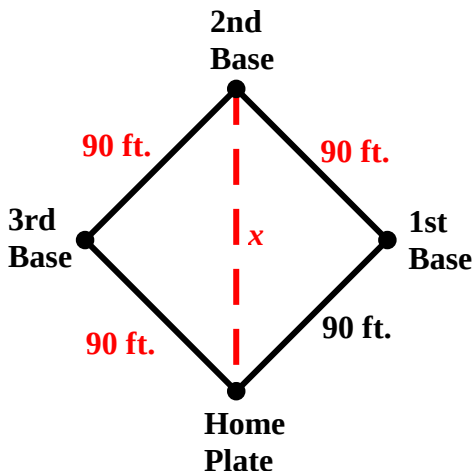
$$x = 2$$

3. Two ships leave a dock. The first ship travels 6 miles east and then 8 miles north and anchors for the night. The second ship travels 5 miles west and then 12 miles south and anchors for the night. How far are each of the ships from the dock when they anchor for the night?



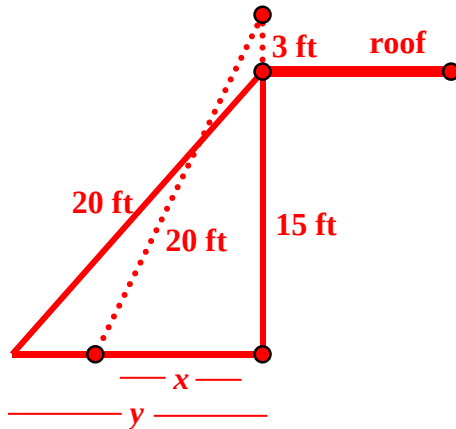
Ship 1 is 10 miles from the dock and ship 2 is 13 miles from the dock.

4. A baseball diamond is in the shape of a square. The distance between each of the consecutive bases is 90 feet. What is the distance from Home Plate to 2nd Base?

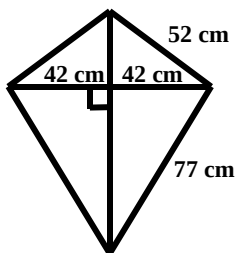


The distance from Home Plate to 2nd Base is $90\sqrt{2}$ feet or about 127 feet.

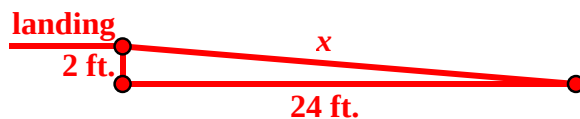
5. Ray is a contractor that needs to access his client's roof in order to assess whether the roof needs to be replaced. He sees that he can access a portion of the roof that is 15 feet from the ground. He has a ladder that is 20 feet long.



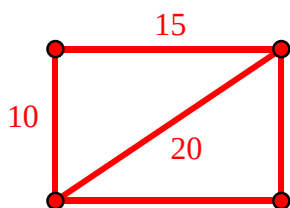
- How far from the base of the house should Ray place the ladder so that it just hits the top of the roof? Round your answer to the nearest tenth of a foot.
 $y \approx 13.2$ feet
 - How far should he place the ladder from the base of the house if he wants it to sit 3 feet higher than the top of the roof? Round your answer to the nearest tenth of a foot.
 $x \approx 8.7$ feet
6. The dimensions of a kite sail are shown below. The support rod that runs from the top of the kite to the bottom of the kite has been broken and needs to be replaced. What length of rod is needed to replace the broken piece? Round your answer to the nearest tenth. ≈ 95.2 cm



7. A new restaurant is putting in a wheelchair ramp. The landing that people enter the restaurant from is 2 feet higher than street level. Safety standards require that for every 1 foot of rise on a wheelchair ramp there must be a run of 12 feet. How long will the ramp have to be to meet safety standards? Round your answer to the nearest tenth. $x \approx 24.1$ feet; Possible extension question: Will a 25-foot long ramp work? If yes, what how far would the ramp be from the base of the landing?



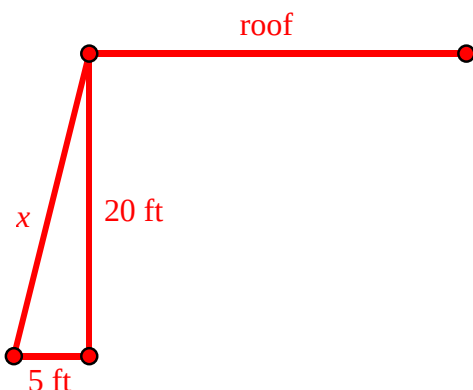
8. Melanie is having a rectangular-shaped patio built in her backyard. It is very important to Melanie that the corners of the patio are right angles. The contractor built a patio with a width of 10 feet and a length of 15 feet. The diagonal measures 20 feet. Does the patio have the right angles that Melanie requested?



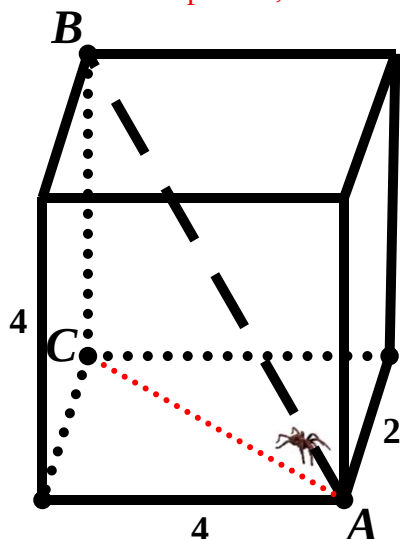
No, the patio does not have the right angles Melanie requested.

$10^2 + 15^2 \neq 20^2$ so by the converse of the Pythagorean Theorem, we know that the patio does not have right angles

9. Fred is safety conscious. He knows that to be safe, the distance between the foot of the ladder and the wall should be $\frac{1}{4}$ the height of the wall. Fred needs to get on the roof of the school building which is 20 ft. tall. How long should the ladder be if he wants it to rest on the edge of the roof and meet safety standards? Round your answer to the nearest tenth. $x \approx 20.6$ or students may also state that as long as the ladder is longer than this, they will meet safety standards, the ladder will just have to be placed further from the base of the building in order to rest on the edge of the roof



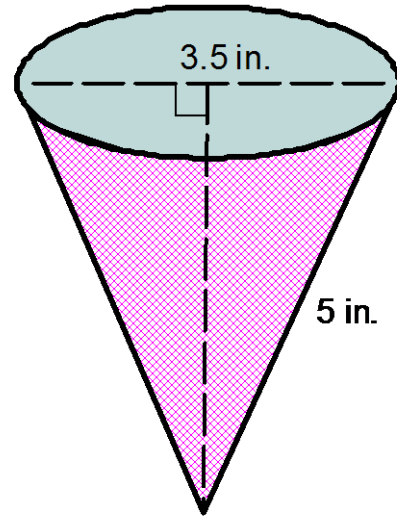
10. A spider has taken up residence in a small cardboard box which measures 2 inches by 4 inches by 4 inches. What is the length, in inches, of a straight spider web that will carry the spider from the lower right front corner of the box to the upper left back corner of the box? In order for students to better visualize this picture, show them a concrete model of a rectangular prism (i.e. a tissue box).



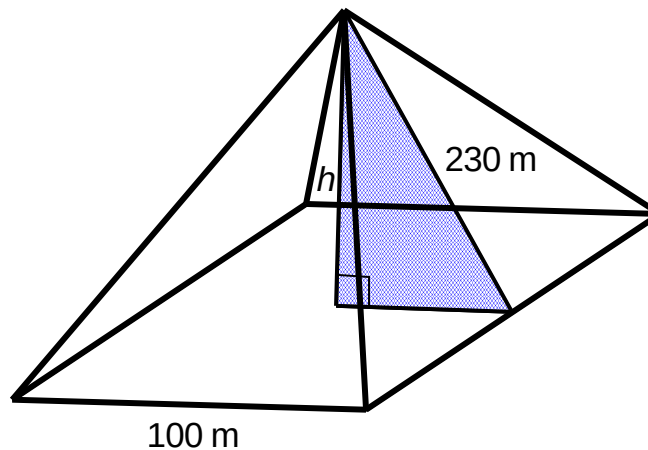
First we find the length of the diagonal of the base AC shown in red in the picture which is the hypotenuse of a right triangle with legs 2 and 4. $AC = 2\sqrt{5}$. Now we consider triangle ABC where $\angle ACB$ is a right angle. Triangle ABC is a right triangle with legs that measure $2\sqrt{5}$ and 4. The length of the hypotenuse is 6 in.

11. Sunny made a paper cone to hold candy for favors for a baby shower. After making the cones she measures the slant height of the cone and the diameter of the base of the cone. Her measurements are shown in the picture below. Find the volume of the cone.

The volume of a cone is approximately 15 in^3 .



12. In the movie Despicable Me, an inflatable model of The Great Pyramid of Giza in Egypt was created by Vector to trick people into thinking that the actual pyramid had not been stolen. When inflated, the false Great Pyramid had a square base of side length 100 m. and the height of one of the side triangles was 230 meters. This is also called the slant height of the pyramid. What is the volume of gas that was used to fully inflate the fake Pyramid? (Hint: Recall the formula for the volume of a pyramid is $\frac{1}{3}Bh$ where B is the area of the base and h is the height of the pyramid (the distance from the base to the apex)).



The volume of the pyramid is approximately $748,333 \text{ m}^3$.

10.3a Homework: Applications of the Pythagorean Theorem

1. What is the length of the diagonal of a square with a side length of 4 cm?

$$x = 4\sqrt{2}$$

2. One side length of a rectangle is 2 inches. The diagonal of the rectangle has a length of $2\sqrt{5}$ inches. What is the length of the other side of the rectangle?

$$x = 4$$

3. A football field is 360 feet long and 160 feet wide. What is the length of the diagonal of a football field assuming the field is in the shape of a rectangle?

$$x \approx 394 \text{ ft}$$

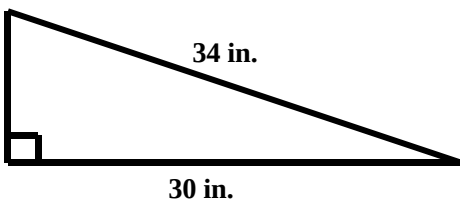
4. The length of an Olympic-size swimming pool is 55 meters. The width of the pool is 25 meters. What is the length of the diagonal of the pool assuming the pool is in the shape of a rectangle?

$$x \approx 60.4 \text{ m}$$

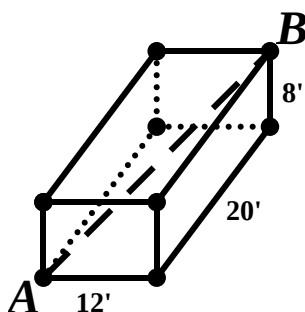
5. You are locked out of your house. You can see that there is a window on the second floor that is open so you plan to go and ask your neighbor for a ladder long enough to reach the window. The window is 20 feet off the ground. There is a vegetable garden on the ground below the window that extends 10 ft. from the side of the house that you can't put the ladder in. What size ladder should you ask your neighbor for?

A ladder that is approximately 22 – 23 feet long.

6. Kanye just purchased a skateboarding ramp. The ramp is 34 inches long and the length of the base of the ramp is 30 inches as shown below. What is the height of the ramp? **16 inches**



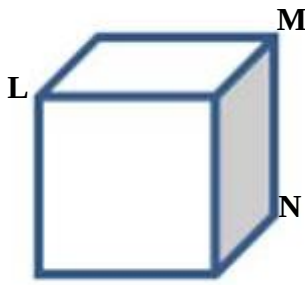
7. A rectangular-shaped room has a width of 12 feet, a length of 20 feet, and a height of 8 feet. What is the approximate distance from one corner on the floor (Point A in the figure) to the opposite corner on the ceiling (Point B in the figure)? **$AB \approx 24.6$ feet or 25 feet**



8. A large pile of sand has been dumped into a conical pile in a warehouse. The slant height of the pile is 20 feet. The diameter of the base of the sand pile is 32 ft. Find the volume of the pile of sand.

The pile of sand has a volume of about 3216 ft^3 .

9. The cube below is a unit cube. A unit cube is a cube of side length 1.



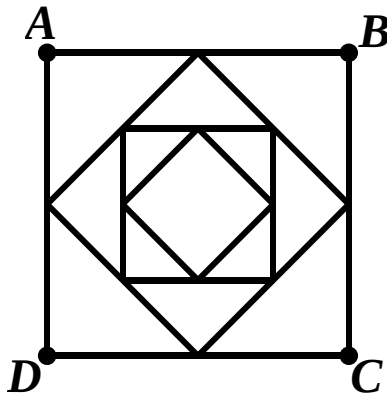
- a. What is the length of $\overline{LM}$? Leave your answer in simplest radical form.

$\sqrt{2}$

- b. What is the length of $\overline{LN}$? Leave your answer in simplest radical form.

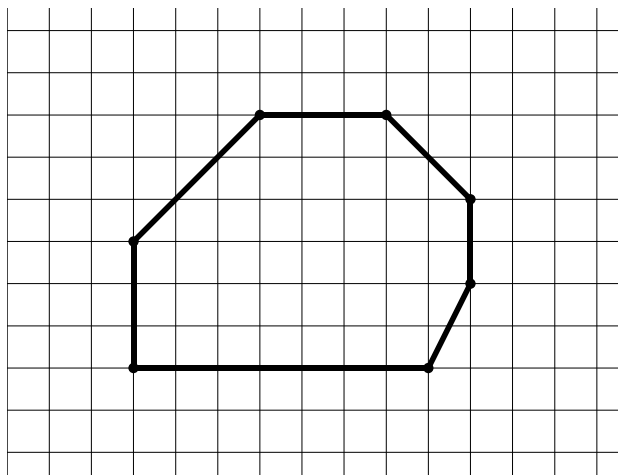
$\sqrt{3}$

Extra for Experts: Square $ABCD$ has side lengths equal to 4 inches. Connecting the midpoints of each side forms the next square inside $ABCD$. This pattern of connecting the midpoints to form a new square is repeated.



- a. What is the side length of the inner-most square? $\sqrt{2}$
- b. What is the area of the inner-most square? 2
- c. What is the ratio of the area of each square to the area of the next square created? 2

Extra for Experts: The following is a scale drawing of a patio that Mr. Davis plans to build in his backyard. Each box in the scale drawing represents 1 unit.



- a. Find the exact value of the perimeter of the scale drawing of the patio. Show all work and thinking.

$$15 + 5\sqrt{2} + \sqrt{5} \text{ units}$$

Students will likely count the squares on the horizontal and vertical segments. In order to find the lengths of the slanted lines, students will likely use the Pythagorean Theorem.

- b. Find the area of the scale drawing of the patio. Show all work and thinking.

$$40\frac{1}{2} \text{ square units}$$

Students will likely use one of three methods: cut the patio into common geometric shapes and find the area of each and add them together, count the squares (combining partial squares) or find the area of the large rectangle that surrounds the patio and subtract out the pieces that are not part of the patio.

- c. If the scale on the drawing above is 1 unit = 3 feet, what is the actual measure of the perimeter of the patio? The area? Show all work.

$$\text{Perimeter} = 45 + 15\sqrt{2} + 3\sqrt{5} \text{ feet (the perimeter is multiplied by 3)}$$

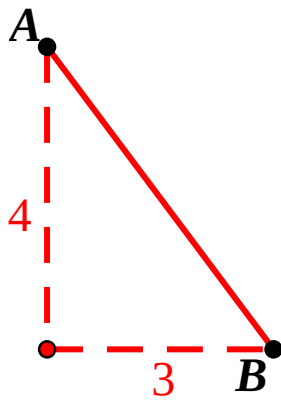
$$\text{Area} = 364.5 \text{ square units (the area is multiplied by } 3^2 \text{ or 9)}$$

10.3b Class Activity: Finding Distance Between Two Points

- Using a centimeter ruler, find the distance between the following sets of points shown below. Then draw the slope triangle of each segment, measure the lengths of the rise and run, and verify that the Pythagorean Theorem holds true. 

- A to B 5 cm
- B to C 13 cm
- C to D 10 cm

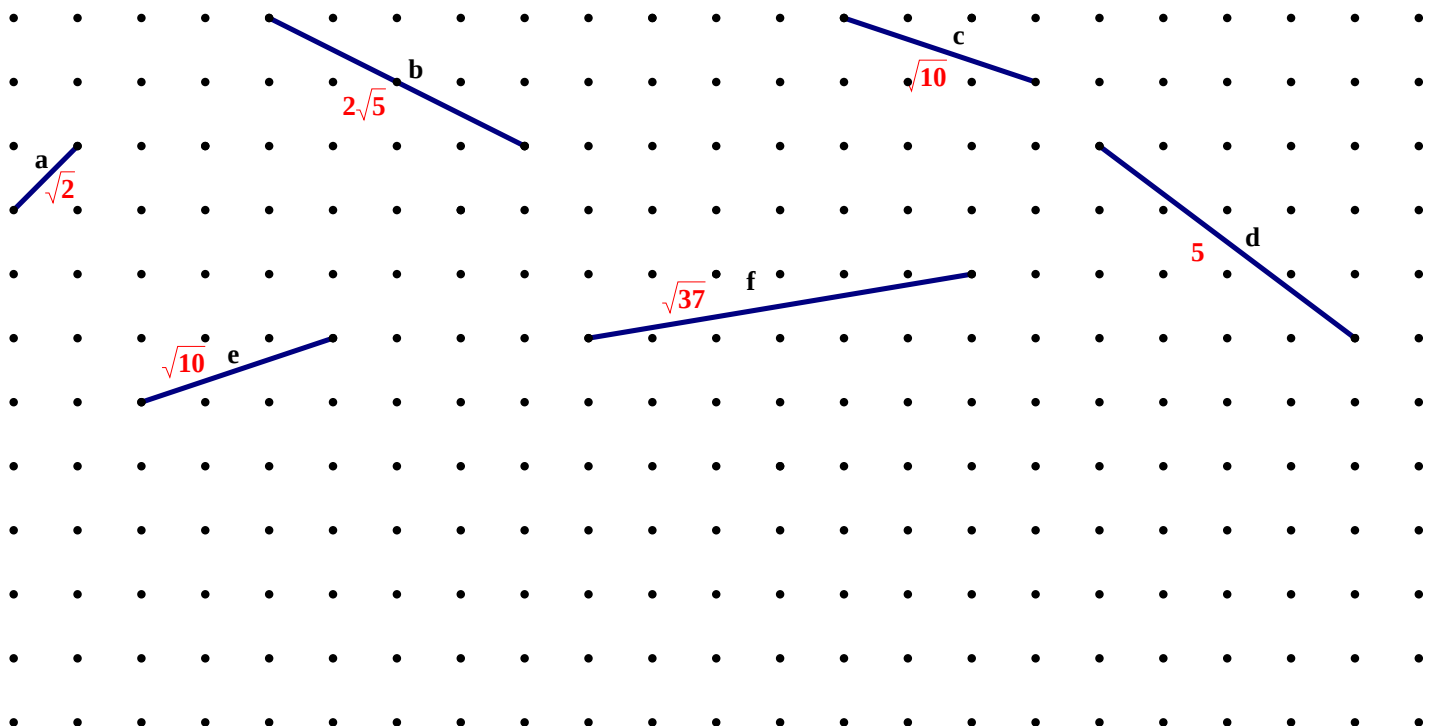
Once students find the lengths of the segments, they should draw the slope triangles using an index card or piece of cardstock as their right angle (as shown below for AB). Then they can measure the lengths of the rise and run and verify that their measurements of the right triangle satisfy the Pythagorean Theorem.



C

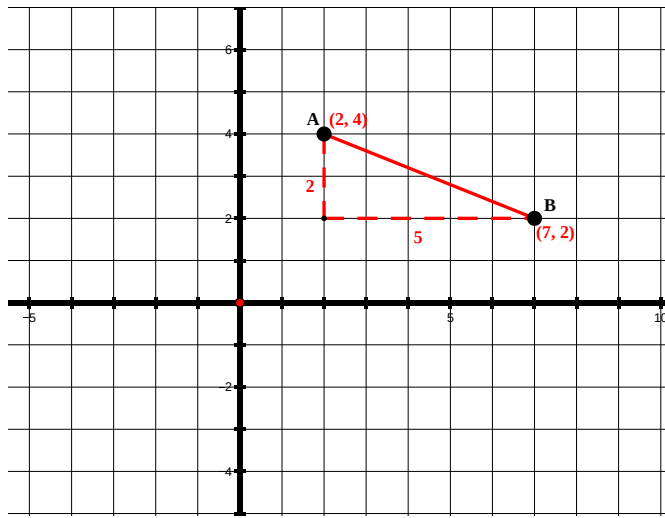
D

- Find the lengths of the segments below. Assume that each horizontal and vertical segment connecting the dots has a length of 1 unit. Students may either use ideas from Chapter 7 (draw squares with the lengths given, find the area of the squares, and then find the side length of the square) or they may use the Pythagorean Theorem.



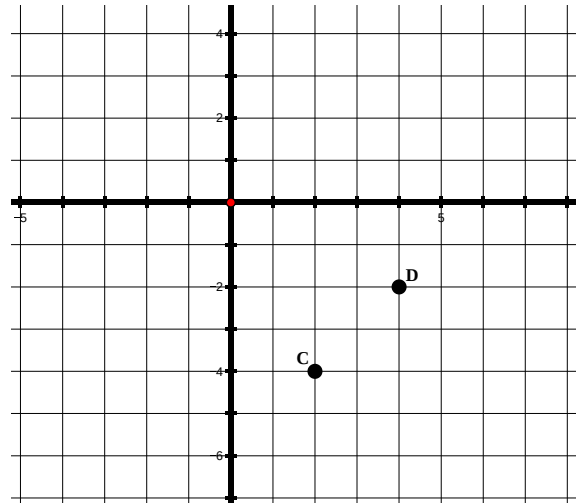
Directions: Label the coordinates of each point. Then, find the distance between the two points shown on each grid below.

3.



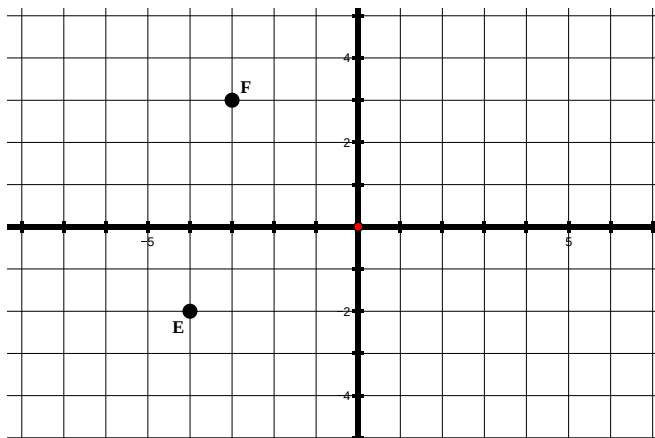
If we draw the slope triangle of $\overline{AB}$, as shown above, we can use the PT to find the distance from point A to B which is $\sqrt{29}$.

4.



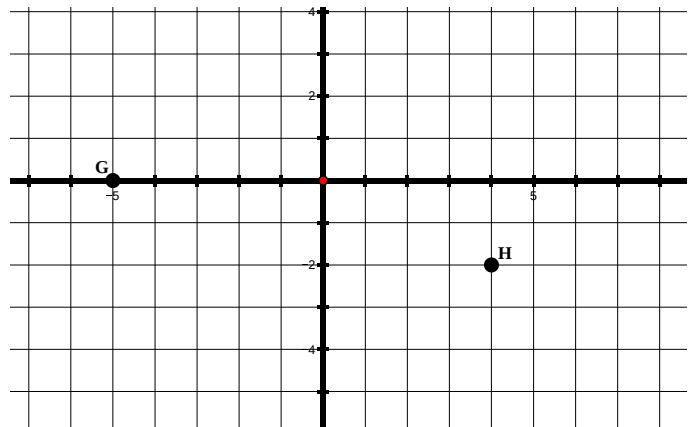
$$CD = 2\sqrt{2}$$

5.



$$EF = \sqrt{26}$$

6.

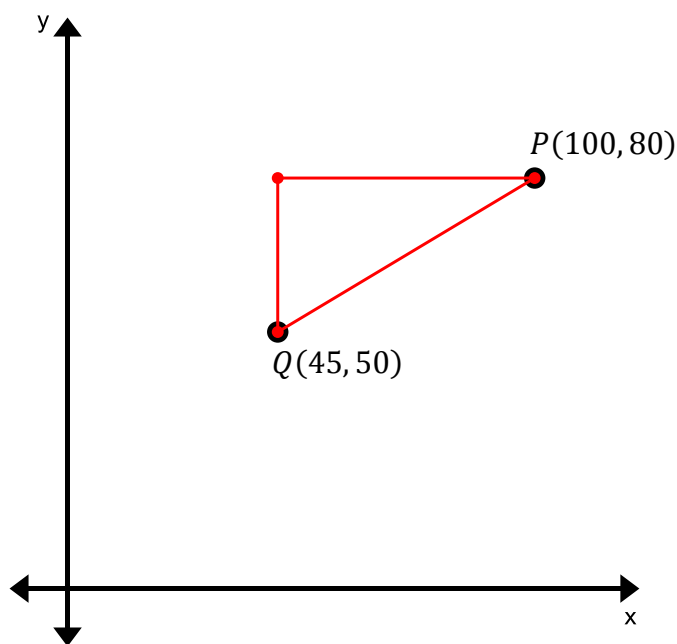


$$GH = \sqrt{85}$$

The Coordinate Distance Formula



7. Find the distance between the two points given on the graph below.



Give students the opportunity to grapple with this problem. Likely, students will begin by drawing a right triangle that connects the two points. When they try to find the length of the legs of the right triangle they will see the issue of not having grid lines to count the distance. Ask them if there is a way to calculate the lengths of these line segments using the ordered pairs. As they explain their reasoning ask them to provide evidence and warrants for their claims. For example, to find the length of the leg between the right angle and point P they might explain that they know that the x coordinate for the point that makes the right angle is 45 (because it is the same horizontal distance away from the origin as point Q that has an x -coordinate of 45). The difference between the x coordinate for P , which is 100, and 45 is 55. Thus the length of this segment is 55. They can use similar reasoning to find the length of the other leg. At this point they are beginning to describe the distance formula which we will derive below.

Once students have articulated the process in their own words, help them to derive the distance formula from the Pythagorean Theorem. (Example 1 is used in the explanation below).

1. Start with the PT: $a^2 + b^2 = c^2$
2. Assume you are using the length of the run of the slope triangle for a (although we could just as easily use the rise). To find the length of a we can count the segments on the grid which equals 5. Alternatively, you can take the difference between the x -coordinates of the two points: (2, 4) and (7, 2) which leaves us with:
 $(7 - 2)^2 + b^2 = c^2$
3. You can use similar thinking to replace b in our equation with the length of the rise:
 $(7 - 2)^2 + (2 - 4)^2 = c^2$
4. Simplify:
 $(5)^2 + (-2)^2 = c^2$
 $25 + 4 = c^2$
 $29 = c^2$
 $\sqrt{29} = c$

You may choose to walk students through this process on the previous page using additional examples from problems 1-4. Once they are comfortable with the process, you may choose to walk them through an abstract derivation of the distance formula using the Pythagorean Theorem as a starting point:

Find the distance between two points $A: (x_1, y_1)$ and $B: (x_2, y_2)$

1. $a^2 + b^2 = c^2$ where c is the distance between our two points and a and b are the lengths of the slope triangle
2. $(x_2 - x_1)^2 + (y_2 - y_1)^2 = c^2$
3. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = c$

Note: It is more important that students understand the process being used to find the distance between two points than it is for them to memorize the formula. If they choose to use the formula, they should understand how it can be derived from the Pythagorean Theorem.

Allow students to use the blank space on the previous page and this page to write down their ideas and notes about finding the distance between two points and to work through the explanations and arguments above.

8. Find the distance between the two points given below. Leave your answers in simplest radical form. Students may either use the distance formula or graph the points, create the slope triangle, and use the PT.

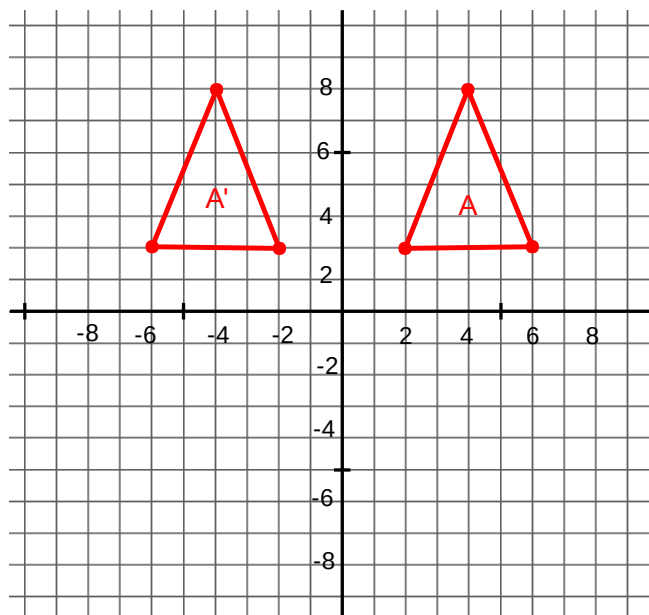
a. $A: (3, 5)$ $B: (6, 9)$
 5

b. $R: (-1, 4)$ $S: (3, 8)$
 $4\sqrt{2}$

c. $C: (0, 5)$ $D: (2, -3)$
 $2\sqrt{17}$

d. $S: (-3, -5)$ $T: (2, -7)$
 $\sqrt{29}$

9. A triangle has vertices at the points (2,3) and (4,8), and (6,3) on the coordinate plane.
- Find the perimeter of the triangle. Use the grid below if needed.
The perimeter of the triangle is approximately 14.8 units. Noticing that the triangle is isosceles with make this computation easier.
 - Find the area of the triangle.
10 units²
 - If the triangle is dilated by a scale factor of 3 what will the new perimeter be?
The new perimeter will be approximately 44.3 units. This is 3 times the old perimeter.
 - If the triangle is dilated by a scale factor of 3 what will the new area be?
The new area will be 90 units². This is 3² times the old area.
 - Plot the original triangle, label it triangle A. Then reflect the triangle over the y-axis, label the new triangle A'. Does this transformation change the perimeter of the triangle? Explain your answer.

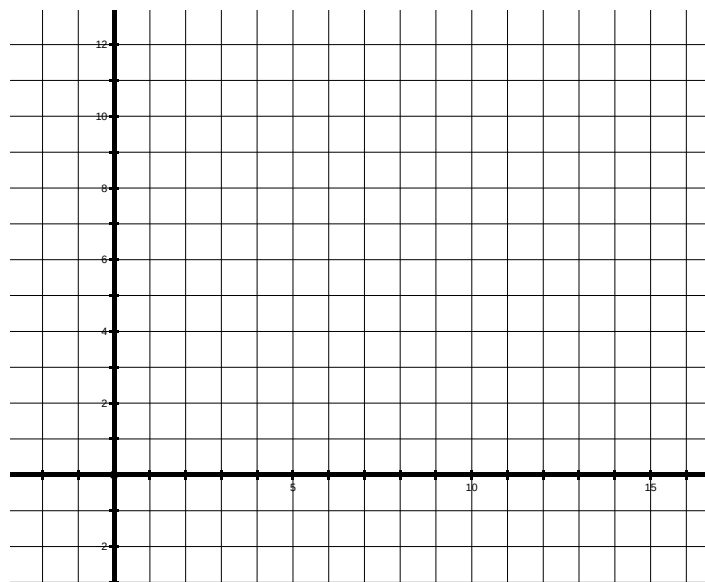


Reflecting the triangle will not change its perimeter because reflections are a rigid motion. Congruence is maintained under a rigid motion so the triangle side lengths remain the same.

10. List **three** coordinate pairs that are 5 units away from the origin in the first quadrant. Describe how to find the points and justify your reasoning. The grid has been provided to help you.
(Note: Points on the axes are not in the first quadrant).



There are many possible answers: (3, 4); (4, 3); ($\sqrt{5}$, $2\sqrt{5}$); ($2\sqrt{21}$, 2)

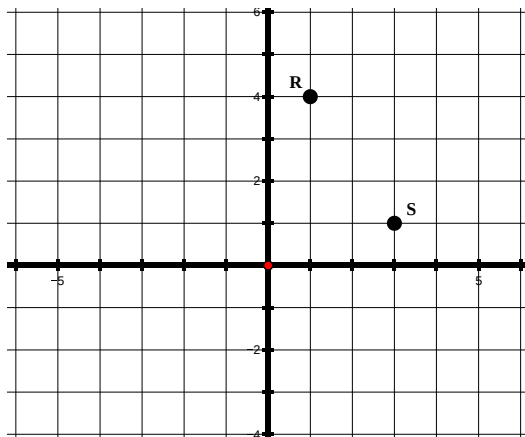


10.3b Homework: Finding Distance Between Two Points

You may wish to provide students with graph paper for #5.

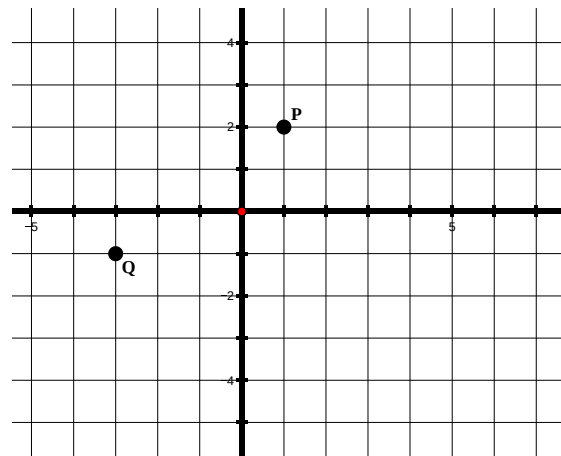
Directions: Find the distance between the two points shown on each grid below. Leave your answers in simplest radical form.

1.



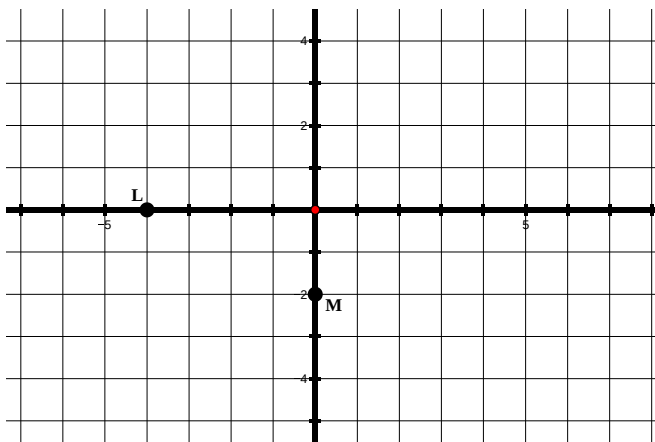
$$\sqrt{13}$$

2.



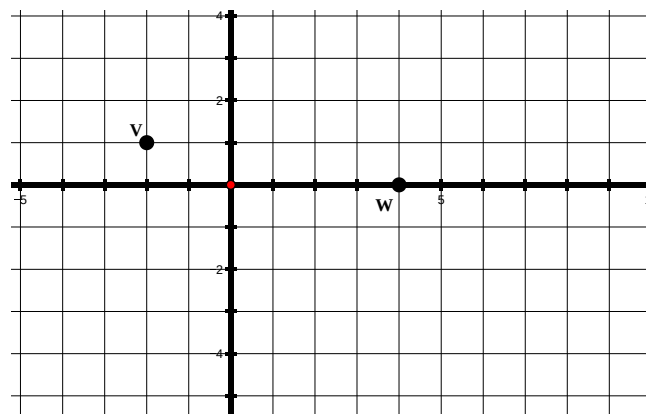
$$5$$

3.



$$2\sqrt{5}$$

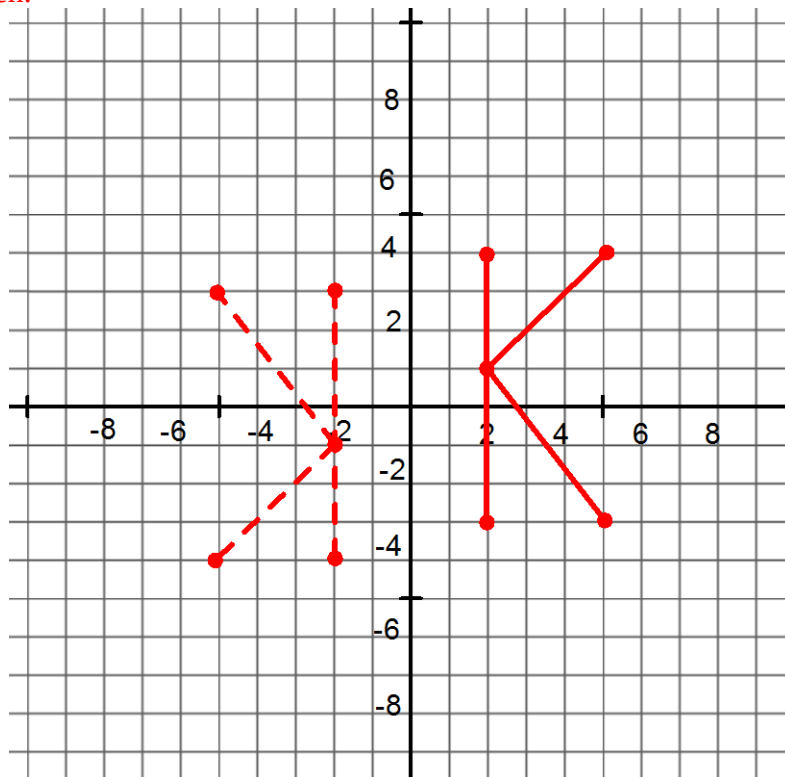
4.



$$\sqrt{37}$$

5. Find the distance between the two points given below. Leave your answers in simplest radical form.
- $A: (2, 1)$ $B: (4, 7)$ $2\sqrt{10}$
 - $R: (2, -1)$ $S: (8, -7)$ $6\sqrt{2}$
 - $C: (1, 0)$ $D: (2, -3)$ $\sqrt{10}$
 - $S: (-2, -4)$ $T: (2, -5)$ $\sqrt{17}$
6. Plot any letter of the alphabet that is made up of segments that are straight lines on the coordinate plane given below. For example you can plot the letter A, E, F, etc. but not the letter B, C, D, etc.

Example letter is given.



- Find the total distance for the segments that make up this letter.
Answers will vary. In the example above the total distance is approximately 16.2 units.
- If you dilated this letter by a scale factor of 4 what is the total distance of the segments that make up your letter?
The total distance will be 4 times the original distance.
- If you dilated this letter by a scale factor of $\frac{1}{5}$ what is the total distance of the segments that make up your letter?
The total distance will be $\frac{1}{5}$ of the original distance.
- Rotate your letter 180 degrees about the origin. Does this transformation change the size or shape of the letter? Explain your answer.
Rotating your letter will not change its size or shape because rotations are a rigid motion. Congruence is maintained under a rigid motion so the segments that make up the letter remain the same length.

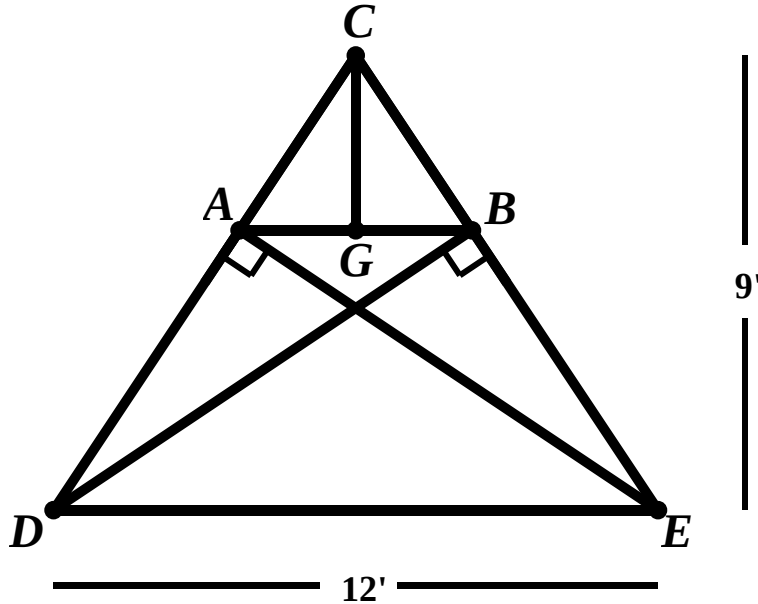
10.3c Extension: Construction

Mario is designing an A-frame for the lodge of a ski resort. Below is a scale drawing of his design.

Given: C lies over the center of the building

$$\overline{AB} \parallel \overline{DE}$$

$\angle DAE$ and $\angle EBD$ are right angles.



What are the lengths of all segments in the diagram?

Key Question #1: “What am I given and what else do I know?” These ideas should be developed intuitively and from the context.

- $\overline{AB} \parallel \overline{DE}$
- $DE = 12'$
- Since C lies over the center of the building, $\overline{CG}$ bisects $\angle ACB$. $\overline{CG}$ is also perpendicular to $\overline{AB}$

Key Question #2: “What am I trying to find specifically?”

The lengths of the all of the segments.

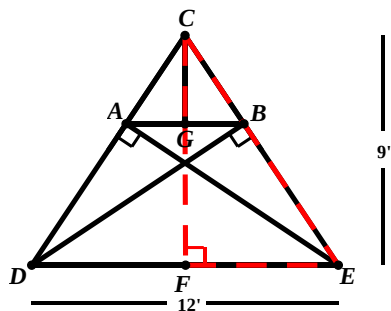
Key Question #3: “What do I know that I can apply? What tools would be useful?”

Students may conclude that given that there are right angles, the Pythagorean Theorem may be a useful tool.

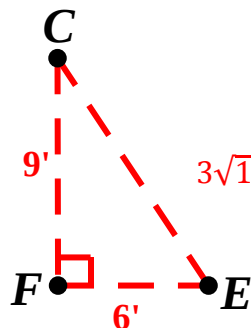
They may also realize that given that $\overline{AB} \parallel \overline{DE}$, ideas of similar triangles will come into play. They may find it useful to have patty paper available to investigate the relationships between the different triangles.

Possible Solution Path with Explanation:

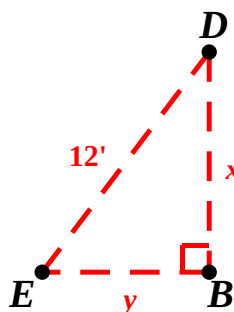
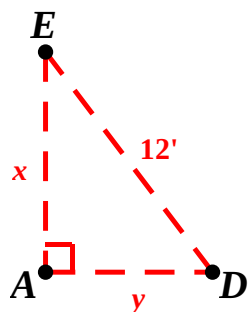
If I extend $\overline{CG}$ down to $\overline{DE}$ and label the point F , F will hit $\overline{DE}$ in the middle (it will bisect it) and it will also form right angles. I now have two congruent right triangles $\triangle DCF$ and $\triangle ECF$. I know that the triangles are congruent because a reflection over $\overline{CF}$ will map $\triangle DCF$ to $\triangle ECF$.



Let's focus on $\triangle ECF$. What do I know? I know $\triangle ECF$ is a right triangle with leg lengths of 9' and 6'.



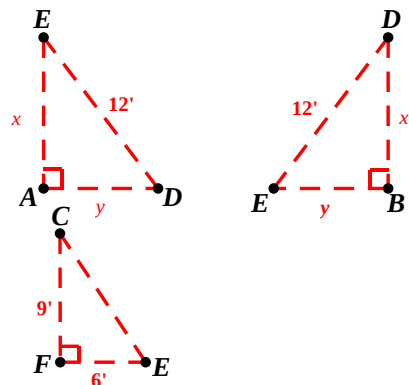
Using the Pythagorean Theorem, I can find the length of $\overline{CE}$ which is $3\sqrt{13}$ or approx. 10.8'.



I also know that $\triangle DBE \cong \triangle EAD$ (one can be mapped to the other using rigid motion). Since this is a right triangle, I know that the following is true:

$x^2 + y^2 = 144$. But I have two variables so I need more information.

What else do I know? I know that $\triangle DBE \sim \triangle CFE$ because they both have a 90 degree angle and they share $\angle CED$ so they are similar by AA similarity for triangles. I can use a similar argument to show that $\triangle EAD \sim \triangle CFE$.



Since the triangles are similar, I know that the lengths of corresponding sides are proportional, therefore

$\frac{x}{y} = \frac{9}{6}$ Solving for x , I get $x = \frac{3}{2}y$. Now I have two equations in two variables: $x^2 + y^2 = 144$ and $x = \frac{3}{2}y$. Using substitution, I have the following:

$$\left(\frac{3}{2}y\right)^2 + y^2 = 144$$

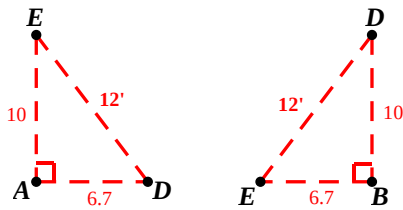
$$\frac{9}{4}y^2 + y^2 = 144$$

$$\frac{13}{4}y^2 = 144$$

$$y^2 = \frac{576}{13}$$

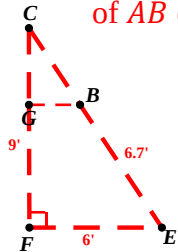
$$y \approx 6.7'$$

Since I know that $x = \frac{3}{2}y$ that means that $x = \frac{3}{2}(6.7)$ so $x \approx 10.1'$. Estimate on the high side – carpenters can always shave off extra material but they can't add more.



So now I have found the lengths of the following segments:
 DB and $EA \approx 10.1'$
 EB and $DA \approx 6.7'$

OK, now which lengths do I have left to find? $\overline{CB}$, $\overline{AC}$, $\overline{CG}$, $\overline{AG}$, and $\overline{GB}$ (Technically we want the length of $\overline{AB}$ due to the fact that it is likely one solid beam).



Let's focus on $\triangle CFE$.

I can find the length of $CB = CE - BE = 10.8 - 6.7 = 4.1'$. This would also be the length of $\overline{CA}$ since $\triangle CGB \cong \triangle CGA$ (one can be mapped to the other through a reflection over $\overline{CG}$).

I also know that $\triangle CGB \sim \triangle CFE$ due to the fact that $\overline{GB} \parallel \overline{FE}$, corresponding angles are congruent, so by AA similarity $\triangle CGB \sim \triangle CFE$.

I can set up the following proportions:

$$\frac{CG}{4.1} = \frac{9}{10.8}$$

$$CG \approx 3.4'$$

$$\frac{BG}{4.1} = \frac{6}{10.8}$$

$$BG \approx 2.3'$$

Since $\overline{BG} \cong \overline{AG}$, this tells us that the length of $\overline{AB} \approx 4.6'$

Students should then go back through and ask the following questions: Have I found the lengths of all segments in the diagram? Am I supporting my arguments with mathematical evidence? Am I communicating clearly? Can someone else follow my logic?

10.3d Self-Assessment: Section 10.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provided on the next page that match each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Use the Pythagorean Theorem to solve problems in real-world contexts, including three-dimensional contexts. <i>See sample problem #1</i>				
2. Find the distance between two points in a coordinate system. <i>See sample problem #2</i>				

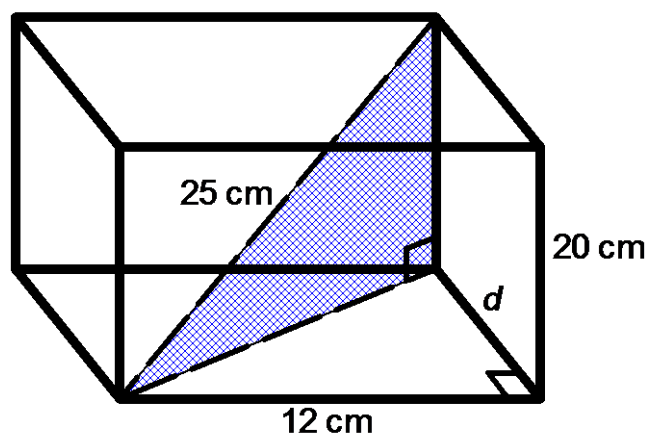
10.3d Sample Problems: Section 10.3

Sample Problem #1

- a. A park is 6 miles east of your home. The bakery is 4 miles north of the park. How far is your home from the bakery as the crow flies?

The distance from the home to the bakery is $\sqrt{52} = 2\sqrt{13}$ miles.

- b. Find the volume of the rectangular prism given below.



The volume of the rectangular prism is 2160 cm^3 .

Sample Problem #2

Find the distance between each set of points.

- a. $A(-10,2)$ and $B(-7,6)$

5

- b. $C(-2,-6)$ and $D(6,9)$

17

- c. $E(3,5)$ and $F(7,9)$

$\sqrt{32} = 4\sqrt{2}$

- d. $G(3,4)$ and $H(-2,-2)$

$\sqrt{61}$