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Chapter 1: Linear Equations in One Variable (4 weeks)

Utah Core Standard(s):

- Solve linear equations in one variable. (8.EE.7)
 - a) Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
 - b) Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Academic Vocabulary: linear expression, simplify, evaluate, linear equation, equivalent expression, solve, solution, inverse operations, like terms, distributive property, ratio, no solution, infinitely many solutions

Chapter Overview:

By the end of eighth grade, students should be able to solve any type of linear equation in one variable. This includes equations with rational number coefficients that require expanding expressions using the distributive property, collecting like terms, and equations with variables on both sides of the equal sign. The chapter utilizes algebra tiles as a tool students can use to create visual, concrete representations of equations. Students manipulate the tiles while simultaneously manipulating the abstract representation of the equation, thus gaining a better understanding of the algebraic processes involved in solving equations. The goal is that students transition from the concrete process of solving equations to the abstract process of solving equations. Students should understand and know the laws of algebra that allow them to simplify expressions and the properties of equality that allow them to transform a linear equation into its simplest form, thus revealing the solution if there is one. Applications are interwoven throughout the chapter in order that students realize the power of being able to create and solve linear equations to help them solve real-world problems. The ability to solve real-world problems by writing and solving linear equations gives purpose to the skills students are learning in this chapter.

Connections to Content:



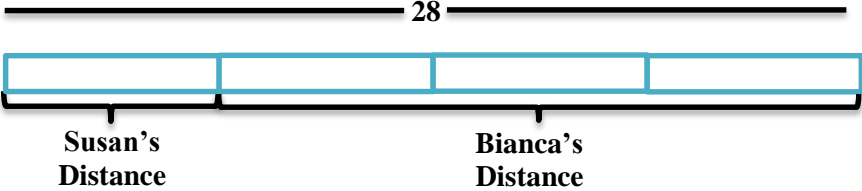
Prior Knowledge



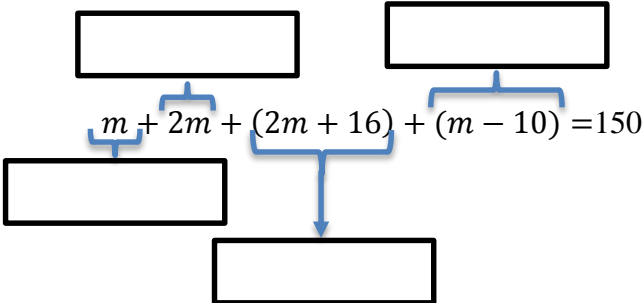
In previous coursework, students used properties of arithmetic to generate equivalent expressions, including those that required expansion using the distributive property. Students solved one- and two-step equations. Students have also solved real-life mathematical problems by creating and solving numerical and algebraic expressions and equations.


Future Knowledge




Later in this book, students will analyze and solve pairs of simultaneous linear equations. They will also create and solve linear equations in two-variables to solve real-world problems. A student's understanding of how to solve a linear equation using inverse operations sets the foundation for understanding how to solve simple quadratic equations later in this course and additional types of equations in subsequent coursework. Additionally, in subsequent coursework, students will be creating equations that describe numbers or relationships for additional types of equations (exponential, quadratic, rational, etc.)

MATHEMATICAL PRACTICE STANDARDS

	<p>Make sense of problems and persevere in solving them.</p>	<p>At Discovery Preschool, parents who have two students enrolled get a discount on the second child. The second child's tuition is 10 dollars less per day than the first child's. If Tess has her two children enrolled for 5 days and her total bill for both children is \$200, how much does she pay each day for her second child to attend daycare?</p> <p>Uncle Hank has another riddle for his nephews. He tells them, "I have the same number of nickels and pennies. I have 4 times as many quarters as nickels. I have 3 more dimes than quarters. I have a total of \$6.14. Whoever can solve my riddle will get my coins."</p> <p>Ben has started the equation for solving the riddle. Finish writing the equation that represents the riddle.</p> $0.01p +$ <p style="text-align: center;">value of pennies</p> <p>How many of each type of coin does Uncle Hank have?</p> <p><i>Students may use a variety of strategies (diagrams, equations, tables, graphs) to solve these and other real-world problems presented throughout the chapter. Regardless of the strategy used, students must analyze givens, constraints, relationships, and goals and identify correspondences between the different approaches and representations used to solve the problems. Ultimately, students will see that these problems can be solved by creating linear equations that describe relationships between quantities. The ability to create equations to model real-world situations is a valuable tool for students going forward.</i></p>
	<p>Reason abstractly and quantitatively.</p>	<p>Bianca ran three times farther than Susan. Together they ran 28 miles. The following is a model that represents this situation.</p>  <p>a. Write an equation that represents this situation. b. How far did each girl run?</p> <p><i>A variety of models are used throughout this chapter to help students make sense of quantities and their relationship in a problem. The concrete bar model shown above helps students to abstract the situation and represent it symbolically. Once the equation is solved, students must interpret the solution in the context and attend to the meaning of the quantities.</i></p>

	<p>Construct viable arguments and critique the reasoning of others.</p>	<p>Two students, Arthur and Oliver, each have some chocolates. They know that they have the same number of chocolates. Arthur has two tubs of chocolates, one bag of chocolates, and twenty-five loose chocolates. Oliver has two tubs of chocolates, two bags of chocolates, and seven loose chocolates. Determine the number of chocolates in a tub. Determine the number of chocolates in a bag. Include any pictures, models, or equations you used to solve the problem and clearly explain the strategy you used.</p> <p>Sam was asked to evaluate the expression $5x + 3x + 20$ for $x = 100$. Sam's work is shown below. What mistake did Sam make? Help Sam to answer the question correctly.</p> <p>Sam's Work:</p> $5x + 3x + 20 = 100$ $8x + 20 = 100$ $8x = 80$ $x = 10$ <p><i>Students may use a variety of strategies to solve the chocolate problem above. They must be able to justify their conclusions, communicate them to others, and respond to the arguments made by others. In the second example, students are asked to examine a problem that has been solved incorrectly, explain what the error is, and solve the problem correctly. This type of error analysis requires students to possess a clear understanding of the mathematical concepts and skills being studied.</i></p>
	<p>Model with mathematics.</p>	<p>A marble jar has twice as many blue marbles as red marbles, 16 more green marbles than blue marbles, and 10 fewer white marbles than red marbles. The jar has a total of 150 marbles. Use this information to answer the questions that follow.</p> <p>The following equation represents this situation. Match each piece of the equation to the appropriate marble color. Write your answer in the boxes provided.</p> <div style="text-align: center;">  </div> <p>Determine how many marbles of each color are in the jar.</p>

		<p>Josh works 40 hours a week as a nurse practitioner. He makes time and a half for every hour he works over 40 hours. Josh works 60 hours one week and earns \$2100. Part of an equation that represents this situation is shown below.</p> <div style="text-align: center;"> <div style="border: 1px solid black; padding: 5px; display: inline-block;">over-time pay rate</div> \downarrow $\text{---}(40) + 1.5p(\text{---}) = 2100$ \uparrow <div style="border: 1px solid black; padding: 5px; display: inline-block;">hours worked at regular rate</div> </div> <p>Fill in the blanks in the equation above so that it matches the story. What is Josh's regular hourly rate? What is Josh's overtime hourly rate?</p> <p><i>Throughout eighth grade, students will build linear models to represent real-world situations, moving fluently between the verbal representation, concrete models, and abstract or symbolic representation. These models map the relationships between quantities in a given situation, allow students to solve many real world problems, and help students to draw conclusions and make decisions in a given situation.</i></p>
	<p>Use appropriate tools strategically.</p>	<p>The following is a model of the equation $7x + 9 - 4x = 2(x + 5)$. Create this model with your tiles and solve the equation, showing your solving actions below.</p> <div style="border: 1px solid black; padding: 10px;"> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <div style="border: 1px solid black; width: 40px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; margin-bottom: 2px;"></div> </div> <div style="text-align: center;"> <div style="border: 1px solid black; width: 20px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-bottom: 2px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px; margin-bottom: 2px;"></div> </div> <div style="text-align: center;"> <div style="background-color: #cccccc; border: 1px solid black; width: 60px; height: 15px; margin-bottom: 2px;"></div> <div style="background-color: #cccccc; border: 1px solid black; width: 60px; height: 15px; margin-bottom: 2px;"></div> <div style="background-color: #cccccc; border: 1px solid black; width: 60px; height: 15px; margin-bottom: 2px;"></div> <div style="background-color: #cccccc; border: 1px solid black; width: 60px; height: 15px; margin-bottom: 2px;"></div> </div> </div> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <div style="border: 2px solid blue; padding: 2px; display: flex; justify-content: space-between; align-items: center;"> <div style="border: 1px solid black; width: 60px; height: 15px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> </div> <div style="border: 2px solid blue; padding: 2px; display: flex; justify-content: space-between; align-items: center; margin-top: 5px;"> <div style="border: 1px solid black; width: 60px; height: 15px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> <div style="border: 1px solid black; width: 20px; height: 15px;"></div> </div> </div> <p><i>Concrete models are used throughout this chapter as a tool to assist students in becoming proficient in the abstract process of solving any type of linear equation. Once students have mastered how to solve a linear equation, this skill becomes a tool for accessing more advanced mathematical content.</i></p> </div>

	<p>Attend to precision.</p>	<p>Find and Fix the Mistake: Ricardo solved the following equation incorrectly. Circle the mistake and describe the mistake in words. Then, solve the equation correctly.</p> $2x + x + 5x = 56$ $7x = 56$ $x = 8$ <p>Combine like terms. Divide both sides by 7.</p> <p><i>When students analyze errors made by others, they must be clear in their understanding of the content and skills being learned. Students identify, explain, and correct common errors that are made when solving equations.</i></p>
	<p>Look for and make use of structure.</p>	<p>Use the following equations to answer the questions that follow.</p> $\frac{x+3}{2} = 5 \qquad \frac{x}{2} + 3 = 5 \qquad \frac{1}{2}(x+3) = 5 \qquad \frac{x}{2} + \frac{3}{2} = 5$ <p>Examine each of the equations above. Circle the equations that are equivalent. Think about the structure of the expressions on the left side of the equation. It may help to use your tiles and draw a model of each equation.</p> <p>Consider the expression $4a - 12$. Write 3 different expressions that if set equal to $4a - 12$ would result in the equation having infinite solutions.</p> <p><i>In order to solve equations, students must make sense of the structure of the expressions in an equation. They must be able to view expressions as a single object or as composed of several objects in order to determine the solving actions and in order to operate on the expression correctly. In order to determine whether two expressions are equivalent and generate equivalent expressions, students must compose and decompose expressions.</i></p>
	<p>Look for and express regularity in repeated reasoning.</p>	<p>Directions: Write the story that goes with the expressions and equation in each problem. Solve for the unknown information and interpret your answer.</p> <p>Number of weeks: w Sophie's Money: $300 - 40w$ Raphael's Money: $180 + 20w$ $300 - 40w = 180 + 20w$</p> <p><i>This chapter deals with linear expressions and equations. Linear functions grow at a constant rate. In the problem above, Sophie's money is decreasing at a rate of \$40 per week while Raphael's is increasing at a rate of \$20 per week. Students may realize that Raphael is closing the gap in the amount of money each child has by \$60 a week and use this reasoning to help determine how long it will take for both children to have the same amount of money.</i></p>

1.0 Anchor Problem: Chocolate



Directions: Consider the following situations. Then answer the questions below. Include any pictures, models, or equations you used to solve the problem and clearly explain the strategy you used.

Students may approach these problems using a variety of methods (diagram, table, equation, logic, etc.). The goal is to get the students thinking about 1) how we can use equations to model real world situations and 2) the different solving outcomes that can occur when solving a linear equation.

Situation 1: Two students, Theo and Lance, each have some chocolates. They know that they have the same number of chocolates. Theo has four full bags of chocolates and five loose chocolates. Lance has two full bags of chocolates and twenty-nine loose chocolates.

Determine the number of chocolates in a bag. Determine the number of chocolates each child has.

Situation 2: Two students, Arthur and Oliver, each have some chocolates. They know that they have the same number of chocolates. Arthur has two tubs of chocolates, one bag of chocolates, and twenty-five loose chocolates. Oliver has two tubs of chocolates, two bags of chocolates, and seven loose chocolates.

Determine the number of chocolates in a tub. Determine the number of chocolates in a bag.

Situation 3: Two students, Abby and Amy have the same number of chocolates. Abby has one full tub of chocolates and 21 remaining chocolates. Amy has one full tub of chocolates and 17 remaining chocolates.

Determine the number of chocolates in a tub.

Section 1.1: Creating and Solving Multi-Step Linear Equations

Section Overview:

This section begins with a review of writing and simplifying algebraic expressions. Students review what an algebraic expression is and what it means to simplify an algebraic expression. They also evaluate algebraic expressions and create expressions to represent real-world situations. This work with expressions sets the foundation for the study of linear equations. Students learn what a linear equation is, what it means to solve a linear equation, and the different outcomes that may occur when solving an equation (one solution, no solution, and infinitely many solutions). Section one focuses on equations with one solution. Students solve linear equations whose solutions require collecting like terms. They then move to equations whose solution requires the use of the distributive property and collecting like terms. Applications that can be solved using the types of equations being studied in a lesson are interwoven throughout. Scaffolding has been provided in order to aide students in the process of creating equations to represent and solve real-world problems.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Understand the meaning of linear expression and linear equation.
2. Solve multi-step linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
3. Write and simplify linear expressions and equations that model real-world problems.

The algebra tiles should be a tool that assist students in understanding how to solve an equation abstractly. Gauge student understanding and allow students to move away from using the tiles as *they* are ready not as the book dictates. Also, the real power in the tiles comes in being able to use them to *physically* represent the equation and to manipulate them in order to solve the equation; therefore it is encouraged that students have a physical set (or virtual set) of tiles while working on this chapter.

1.1a Class Activity: Simplifying Linear Expressions

1. Emma is playing a popular video game and is determined to beat the high score. The game saves her place so that each time she plays it again, she picks up in the same place with the same number of points. Emma downloads the video game on Monday night and starts playing, scoring a bunch of points. On Tuesday, she scores an additional 500 points. On Wednesday she doubles her score from the previous day. On Thursday, she scores the same number of points that she scored on Monday.

- a. Miguel's teacher asks him to write an expression that represents Emma's total score after she is done playing on Thursday. Miguel writes the following expression:

$$2(p + 500) + p$$

Miguel's teacher lets him know that his expression is correct. Write in words what each piece of

Miguel's expression represents in the story problem. 

p : Emma's score on Mon. night

$p + 500$: Emma's score on Tues. night


$2(p + 500)$: Emma's score on Wed. night

$2(p + 500) + p$: Emma's score on Thurs. night

- b. Nevaeh writes the following expression to represent Emma's score on Thursday.

$$2p + 1000 + p$$

The teacher lets her know that she is also correct. How did Nevaeh represent the problem

differently than Miguel? 

Nevaeh doubled the points scored on Mon. night and Tues. night separately as opposed to doubling Emma's total score at the end of Tues. night

- c. Can you think of another expression to represent Emma's score on Thursday?

$$3p + 1000$$

- d. If Emma scored 700 points on Monday, evaluate each of the three expressions above to determine how many points Emma has on Thursday.

$$3,100$$

In this section, continue to emphasize to students what it means to evaluate an expression (substitute in a value for the unknown). Also, since these two expressions are equivalent, we will get the same result when we evaluate the expressions for a specified value of the unknown.

In this lesson, we will study linear expressions. Miguel, Nevaeh, and you all wrote linear expressions to represent Emma's total number of points on Thursday. A **linear expression** is a mathematical phrase consisting of numbers, unknowns (symbols that represent numbers), and arithmetic operations. Linear expressions describe mathematical or real-world situations.

The following are all examples of linear expressions:

- $3x - 5$
- $2x - x - 17$
- $3x$
- $6(2x - 5) + 11$
- $3x + 7x - 3 + 2x$
- 25

Below are **equivalent linear expressions** that represent Emma's score from the video game example. Equivalent expressions have the same meaning. Two expressions are considered **equivalent** when a substitution of any number for the unknown p in each of the expressions produces the same numerical result. Substituting in a specific number for the unknown in an expression and calculating the resulting value is called **evaluating** the expression. **We can say that 2 expressions are equivalent if we can move from one expression to the other using the laws of arithmetic. Have students evaluate the expressions below for different values of p . Continue to emphasize what it means to evaluate an expression and make sure students understand what p represents in the context.**

$$2(p + 500) + p$$

$$2p + 1000 + p$$

$$3p + 1000$$

For ease of communicating mathematical ideas, we will consider a linear expression in the form $Ax + B$ where A and B are numbers and x represents an unknown, the **simplified form of a linear expression**. In the example above, the simplified form of the expression is $3p + 1000$. **While the simplified form of an expression can be useful for the purposes of consistency and ease of communicating mathematical ideas, different forms of the expression are valuable in that they reveal different things about the context that are not easily seen in the simplified form of the expression.**

In this lesson, we will be using tiles to model and simplify linear expressions.

Key for Tiles:

$$\square = 1$$

$$\text{rectangle} = x$$

$$\blacksquare = -1$$

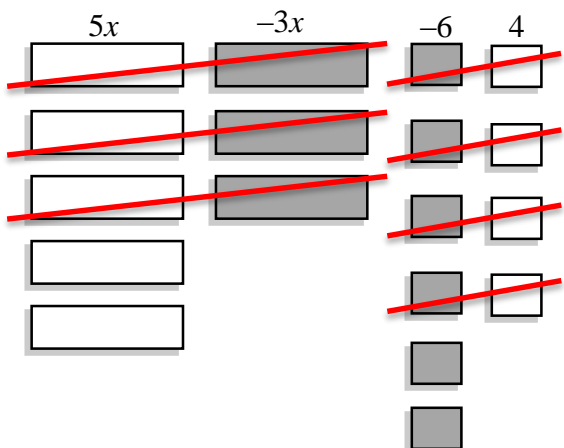
$$\text{shaded rectangle} = -x$$

Remember that a positive tile and a negative tile can be combined to create a **zero pair** or add to zero.

$$\square + \blacksquare = 0$$

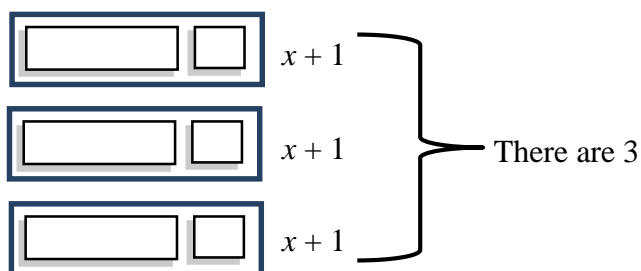
$$\text{rectangle} + \text{shaded rectangle} = 0$$

2. The following is a model of the expression $5x + (-3x) - 6 + 4$



- Find zero pairs, and write the simplified form of this expression. $2x + (-2)$ or $2x - 2$
The red lines represent the zero pairs which cancel out to 0.
- Evaluate this expression when $x = 8$. **To evaluate this expression, substitute 8 in for x and simplify: $2x - 2 \rightarrow 2(8) - 2 \rightarrow 16 - 2 \rightarrow 14$**

3. Using your tiles, model the expression $-4x + 3 + 5x + (-1)$.
 To create this model, lay out or draw 4 negative x tiles, 3 unit tiles, 5 x tiles, and a negative 1 tile. Once they are laid out, you can move tiles around to group the x tiles and group the unit tiles. Draw a model of your tiles in the space below.
- Find zero pairs and write the simplified form of this expression.
 $x + 2$
 - Evaluate this expression when $x = -5$. Substitute in -5 for x and simplify: $x + 2 \rightarrow -5 + 2 \rightarrow -3$
4. The following is a model of the expression $3(x + 1)$. We can think of this as 3 groups or 3 copies of $(x + 1)$

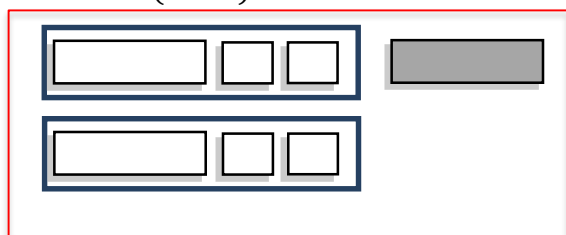


- Write the simplified form of this expression. $3x + 3$
 - Evaluate this expression when $x = -4$. Substitute in -4 for x : $3x + 3 \rightarrow 3(-4) + 3 \rightarrow -12 + 3 \rightarrow -9$
5. Using your tiles, model the expression $2(2x - 1)$.
- Write the simplified form of this expression. $4x - 2$
 - Evaluate this expression when $x = 0$. -2

Directions: Model and simplify each expression.



6. $2(x + 2) - x$



$x + 4$

7. $3 - 3x + 4(x - 3)$
 $x + (-9)$ or $x - 9$

Directions: Simplify each expression. Use the tiles if you need to.

8. $-4x + 3 + 5(2x - 1)$
 $6x + (-2)$

9. $12 - (x - 2) + 4x$
 $3x + 14$

10. Write three expressions that are equivalent to $6x + 12$. Use the tiles if you need to.



Possible expressions: $6(x + 2)$, $2(3x + 6)$, $5x + x + 12$

11. Write three expressions that are equivalent to $4x - 2$. Use the tiles if you need to.

Possible expressions: $4(x - \frac{1}{2})$, $2(2x - 1)$, $5x - x - 2$

Challenge students to use fractions in problems like these.

12. A group of friends goes to a movie on Friday night. Each friend purchases a movie ticket that costs \$8, a small popcorn that costs \$3.50, and a medium drink that costs \$2.25.

- a. Circle the expression(s) below that represent the **total amount of money spent** by the group if f represents the number of friends that went to the movie. There may be more than one answer.



Be sure that students are clear about what the variable represents in the context. Have them highlight the meaning in the problem above or re-write it to the side.

$8 + 3.50 + 2.25$ What does this expression represent?

$f(8 + 3.50 + 2.25)$

$f + 8 + 3.50 + 2.25$

$f + 13.75$

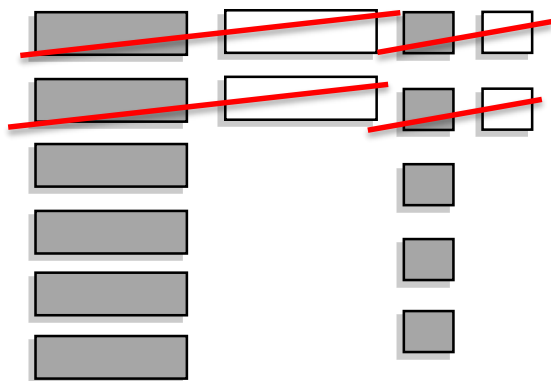
$8f + 3.5f + 2.25f$

$13.75f$

- b. If 5 friends go to the movie, how much money will **each person** spend? How much money will the **entire group** spend? Each person will spend \$13.75. The entire group will spend \$68.75. Help students to see that no matter which expression they use above (as long as it is a correct one) will yield the same result when evaluated for $f = 5$

1.1a Homework: Simplifying Linear Expressions

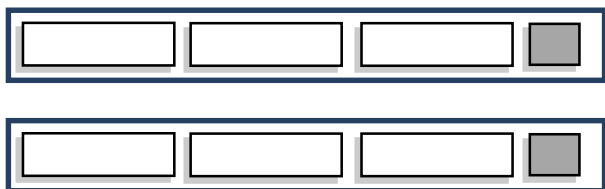
1. The following is a model of the expression $-6x + 2x - 5 + 2$



- Find zero pairs, and write the simplified form of this expression. $-4x - 3$
 - Evaluate this expression for $x = 2$. -11 Substitute 2 in for x and simplify.
2. Model the expression $5x + 2 - x - 4$.

- Simplify the expression modeled above.
- Evaluate this expression for $x = -3$.

3. The following is a model of the expression $2(3x - 1)$.



- Write the simplified form of this expression.
- Evaluate this expression for $x = 4$.

Directions: Simplify each expression. Use the tiles if needed.

4. $2(x + 3) + 4x$ $6x + 6$	5. $4 + 2(x - 2) - 4$
6. $6(2x - 4) - 3x$	7. $4 - (2x + 3) + 5x$ $3x + 1$
8. $-6x - 4 + 5x + 7$	9. $\frac{1}{2}(4x + 12) + 2x$ $4x + 6$

10. Write three expressions that are equivalent to $3x + 15$.

See class activity #10 and 11 for help.

11. Write three expressions that are equivalent to $2x + 4x - 15$.

12. Dan's basketball coaches have set up the following schedule for practice: 10 minutes warm-up and stretching; 15 minutes of defensive drills; 10 minutes of passing drills; 20 minutes of shooting practice, and x minutes of time to scrimmage.

- a. Circle the expression(s) below that represent the amount of time Dan will practice **each week** if they practice **3 days a week**. There may be more than one answer.

$$10 + 15 + 10 + 20 + x$$

$$3(10 + 15 + 10 + 20 + x)$$

$$3(55 + x)$$

$$165 + x$$

$$165 + 3x$$

$$168x$$

- b. If Dan's team scrimmages for 35 minutes each practice, how long is **each** practice? How long do they practice **each week** (again assuming they practice 3 days a week)? **Each practice is 90 minutes (1.5 hours) and they practice 270 minutes (4.5 hours) each week.**

Find and Fix the Mistake: In each of the following problems, a common error has been made when

simplifying the expressions. Identify the mistake, explain it, and simplify the expression correctly.



You may also wish to have students prove that the expressions are not equivalent by finding one number that gives a different result when substituted in for the unknown in the original expression and simplified expression.

<p>13. $5x + 4x - x$ $9x$</p> <p>Explain mistake: Student did not realize there is a 1 in front of the last x in the expression.</p> <p>Correct answer: $8x$</p>	<p>14. $2(x + 5)$ $2x + 5$</p> <p>Explain mistake:</p> <p>Correct answer:</p>
<p>15. $5 - (x - 2)$ $5 - x - 2$ $3 - x$</p> <p>Explain mistake:</p> <p>Correct answer:</p>	<p>16. $-3(2x - 5)$ $-6x - 15$</p> <p>Explain mistake:</p> <p>Correct answer:</p>
<p>17. $2x + 3 + 4x$ $9x$</p> <p>Explain mistake: Student added the 3 to the $6x$ to get $9x$. 3 and $6x$ cannot be combined because they are not like terms.</p> <p>Correct answer: $6x + 3$</p>	<p>18. $x + 3x + 6x$ $10x^2$</p> <p>Explain mistake:</p> <p>Correct answer:</p>

19. Evaluate the expression $2x + 4$ for the following values of x . This skill will help students with graphing later in the book.



<p>a. $x = 1$ Substitute 1 in for x and simplify. 6</p>	<p>b. $x = -1$</p>
<p>c. $x = 0$</p>	<p>d. $x = \frac{1}{2}$</p>

20. Evaluate the expression $-3x + 2$ for the following values of x .

<p>a. $x = 1$</p>	<p>b. $x = -1$ 5</p>
<p>c. $x = 0$</p>	<p>d. $x = \frac{1}{3}$</p>

1.1b Class Activity: Writing Linear Expressions to Model Real World Situations

1. Aria and her friends are playing a game. The expressions below represent the amount of money each player has at the end of the game where m is the amount of money a player started with.

- a. Match each player to the correct expression.  

Expressions		
A. $2(3m - 100)$	B. $\frac{m}{2} + 100 - 25$	C. $2m - 100 - 25$
Player: Hadley	Player: Lea	Player: Peta
D. $\frac{(m+100)}{2} - 25$	E. $2(m - 100) - 25$	F. $2(\frac{m}{3} - 100)$
Player: Miya	Player: Aria	Player: Sierra

Player Scenarios	
<p>Peta collects the following cards:</p> <ul style="list-style-type: none"> You are doing a great job at work and just received a bonus. Double the amount of money you currently have. It's time to pay property taxes. Pay the bank \$100. It's the first day of school. Pay the store \$25 for school supplies. 	<p>Miya collects the following cards:</p> <ul style="list-style-type: none"> Congratulations! Your art submission won first place. Collect \$100. Your house was damaged in a flood. Pay half of your remaining money for repairs. You got a parking ticket. Pay \$25.
<p>Aria collects the following cards:</p> <ul style="list-style-type: none"> It's time to pay property taxes. Pay the bank \$100. You are doing a great job at work and just received a bonus. Double the amount of money you currently have. It's the first day of school. Pay the school \$25 for supplies. 	<p>Hadley collects the following cards:</p> <ul style="list-style-type: none"> You won the lottery! Triple the amount of money you currently have. It's time to pay property taxes. Pay the bank \$100. You are doing a great job at work and just received a bonus. Double the amount of money you currently have.
<p>Lea collects the following cards:</p> <ul style="list-style-type: none"> Your house was damaged in a flood. Pay half of your remaining money for repairs. Congratulations! Your art submission won first place. Collect \$100. You got a parking ticket. Pay \$25. 	<p>Sierra collects the following cards:</p> <ul style="list-style-type: none"> Your two sisters want to join the game. Divide the money you currently have between the three of you. It's time to pay property taxes. Pay the bank \$100. You are doing a great job at work and just received a bonus. Double the amount of money you currently have.

- b. If each player started the game with \$1,000, who won the game?

Hadley

Directions: For #2 – 4, circle the expression(s) that correctly model each situation. There may be more than one

answer.  

You may have students simplify the expressions to verify that the expressions are equivalent.

2. Tim took his friends to the movies. He started with \$40 and bought 3 movie tickets that each cost x dollars. He also bought one tub of popcorn that cost \$5.75.

a. Which of the following expression(s) represent the amount of money Tim has left?

$$40 - x - x - x - 5.75$$

$$40 - 3 - 5.75$$

$$34.25 - 3x$$

$$40 - 3x - 5.75$$

$$-3x + 34.25$$

$$31.25x$$

b. If each movie ticket costs \$6, how much money does Tim have left?

\$16.25

3. Master Tickets charges \$35 for each concert ticket, plus an additional \$2 service fee for each ticket purchased. Kanye purchased x concert tickets.

a. Which of the following expression(s) represent the amount of money Kanye spent?

$$35x + 2$$

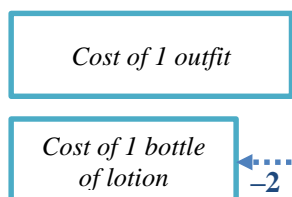
$$35x + 2x$$

$$37x$$

$$x(35 + 2)$$

b. If Kanye purchased 4 concert tickets, how much did he spend? \$148

4. Sara bought 3 baby outfits that cost p dollars each and one bottle of baby lotion. The baby lotion costs 2 dollars less than an outfit. The following is a model of this situation. Students should understand that this model shows that an outfit is \$2 more than a bottle of lotion or a bottle of lotion is \$2 less than an outfit



a. Which of the following expression(s) represent the amount of money Sara spent? Be sure to discuss with students what p represents in the context. Label the model to show that if an outfit is p then a bottle of lotion is $p - 2$

$$p + p + p - 2$$

$$p + p + p + (p - 2)$$

$$3p - 2$$

$$3p + (p - 2)$$

$$4p - 2$$

$$2p$$

b. If each baby outfit costs \$5, how much did Sara spend? \$18

5. Antony and his friends went to a fast food restaurant for lunch. They ordered and ate 3 Big Micks, 2 Mick-Chicken Sandwiches, and 4 large fries. A Mick-Chicken Sandwich has 200 fewer calories than a Big Mick. A large fry has 50 fewer calories than a Big Mick.
- a. Part of an expression that represents the total number of calories consumed by Antony and his friends is shown below. Fill in the remaining pieces of the expression on the lines provided and simplify the expression.

$$\begin{array}{c}
 \text{\# of chicken} \\
 \text{sandwiches} \\
 \downarrow \\
 3(c) + 2(c - 200) + 4(c - 50) \\
 \uparrow \qquad \qquad \qquad \uparrow \\
 \text{calories in a} \qquad \qquad \text{calories in a} \\
 \text{Big Mick} \qquad \qquad \text{large fry}
 \end{array}$$

- b. Write the simplified form of the expression.

$$9c - 600$$

- c. If a Big Mick has 550 calories in it, how many calories did Antony and his friends consume?

$$4,350 \text{ calories}$$

1.1b Homework: Writing Linear Expressions to Model Real World Situations

1. Mateo sends approximately twice as many text messages as his mom each month. His dad sends approximately 500 fewer text messages than Mateo each month. The following expression represents the total number of text messages sent by the family each month:

$$t + \frac{t}{2} + (t - 500)$$

- a. Write in words what each piece of the expression represents in the story.

t : The number of texts sent by Mateo

$\frac{t}{2}$: _____

$(t - 500)$: _____

- b. If Mateo sends approximately 3,000 texts per month, approximately how many texts does his entire family send each month (assuming he has no other family members)? **7,000 texts**

Directions: For #2 – 3, circle the expression(s) that correctly model the situation. There may be more than one answer.

2. Christina is purchasing one of each of the following for her nieces for Easter: a pack of sidewalk chalk that costs \$2.25, a bottle of bubbles that costs \$1, and a chocolate bunny that costs \$3.50.

- a. Which of the following expression(s) represent the amount of money Christina will spend if she has n nieces?

$$6.75 + n$$

$$n(2.25 + 1 + 3.5)$$

$$6.75n$$

$$2.25n + n + 3.5n$$

$$6.75$$

$$n(2.25n + 1n + 3.5n)$$

- b. If Christina has 5 nieces, how much money will she spend on gifts for Easter?

3. At Six East Shoe Store, a pair of boots is \$30 more than a pair of sandals. A pair of sandals costs s dollars each. Emily purchased 2 pairs of boots and 3 pairs of sandals.

- a. Which of the following expression(s) represent the amount of money Emily spent?

$$2(s + 30) + 3s$$

$$(s + 30) + (s + 30) + (s + 30) + s + s$$

$$2s + 3(s + 30)$$

$$(s + 30) + (s + 30) + s + s + s$$

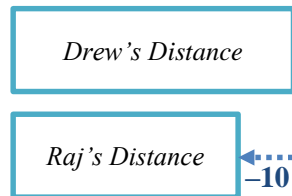
$$5s + 90$$

$$5s + 60$$

- b. If a pair of sandals is \$15, how much did Emily spend in all?

Directions: Write a linear expression **in simplified form** that represents each of the following situations. For these problems, simplified form is $Ax + B$ where A and B are numbers and x represents an unknown.

4. Drew and Raj are both training for a bike race. Raj bikes 10 miles less than Drew each day that they train. The following is a model of this situation.



- Write an expression **in simplified form** that represents the number of miles Drew bikes if Raj bikes m miles each day. $m + 10$
 - Write an expression **in simplified form** that represents the total number of miles Raj and Drew bike if they each train 5 days a week and m represents the number of miles Raj bikes each day. $10m + 50$
 - If Raj bikes 15 miles each day, how many miles do Raj and Drew bike together each week (again assuming they train 5 days a week)? 200 miles
5. Naja is paid p dollars per hour she works. For every hour she works over 40 hours, she is paid time and a half which means she is paid 1.5 times her normal hourly rate. She worked 50 hours last week. The following is a model of this situation.
- Irene tried to write an expression that represents the amount Naja earned last week but needs your help. Help Irene finish the expression by filling in the blanks.

$$40(\underline{\hspace{1cm}}) + \underline{\hspace{1cm}}(1.5p)$$

↑
hours worked
at regular rate

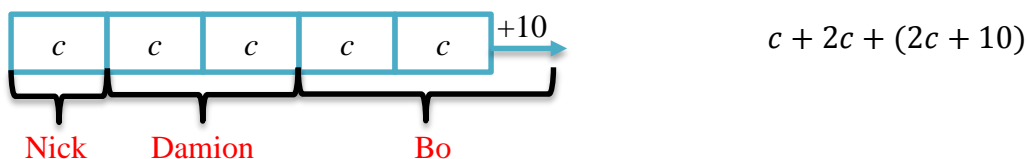
over-time
pay rate
↓

- Simplify the completed expression above. $55p$
- If Naja's regular hourly rate is \$30 per hour, how much did she earn last week?

1.1c Class Activity: Solving Multi-Step Linear Equations (combine like terms)

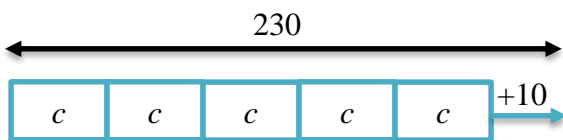
In the lessons up to this point, we have been working with linear expressions. We reviewed how to **simplify** a linear expression and how to **evaluate** a linear expression for a given value of x . We will now begin our work with linear equations. When we **solve** a linear equation, our task is to find the values of the unknown that make the equation true.

1. Damion and his friends went trick-or-treating. The next day, they got together and counted their candy. Damion had twice as much candy as Nick. Bo had 10 more pieces than Damion. The following model and expression represent the amount of candy the boys have together:



- a. Show on the model and the expression which pieces represent the amount of candy each of the boys has. **As we transition to equations, you may want to have students review what they did with expressions (simplify and evaluate). Now, we will be solving for the unknown, that is, determine the values for the unknown that make the equation true.**

What if we also knew that together the boys have 230 pieces of candy? Let's look at a model for this:





$c + 2c + (2c + 10)$ and 230 are both linear expressions that represent the amount of candy the boys have together. When we set two linear expressions equal to each other, we create a linear equation. A **linear equation** is an assertion or statement that two linear expressions are equal to each other. Using the candy example, we can create the following equation:

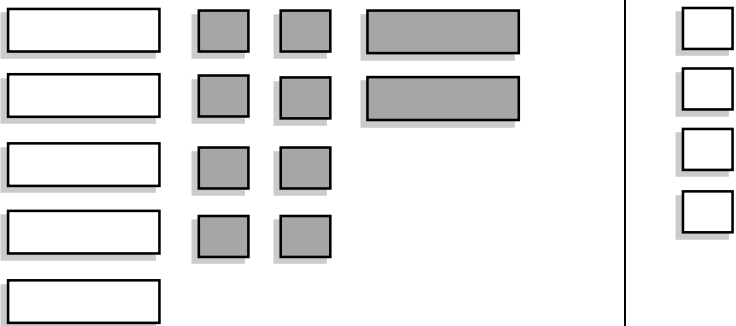
$$\begin{array}{ccc} & \text{"is equal to"} & \\ & \downarrow & \\ \underbrace{c + 2c + (2c + 10)}_{\text{Expression 1}} & = & \underbrace{230}_{\text{Expression 2}} \end{array}$$

- b. Model this equation with your tiles and solve for c . **44**
It is not going to be practical to model 230 unit tiles. Students can use a piece of paper that says 230 or if you are using numbered tiles, create 230.
- c. What does c represent in the context? **The amount of candy Nick has**
- d. How many pieces of candy do each of the boys have? **Damion = 88; Bo = 98**

A **solution** to an equation is a number that makes the equation true when substituted for the unknown. In the example above, the solution is 44. Verify that when you substitute 44 in for c the equation is true.

It is important to note that when we create an equation, the two expressions on either side of the equal sign might be true for 1) **one value of x** (as we saw in the candy example above), 2) **no values of x** (there is not a number that can be substituted for the variable to make the equation true), or 3) **all values of x** (every number we substitute in for the variable will make the equation true). In the first section, we will study equations that have one solution.

2. The following is a model of the equation $5x - 8 - 2x = 4$. Create this model with your tiles and solve the equation, showing the solving actions (steps) below.  

	<div style="border: 1px solid black; height: 100px; width: 100%;"></div>
---	--

- a. Solving Actions (show each step below):

$$5x - 8 - 2x = 4$$

Students should state their solving actions in this space as they manipulate the tiles, recognizing and articulating the inverse operations being used to solve the equations.

$$5x - 8 - 2x = 4$$

$$3x - 8 = 4$$

Combine like terms.

$$3x = 12$$

Add 8 to both sides.

$$x = 4$$

Divide both sides by 3.

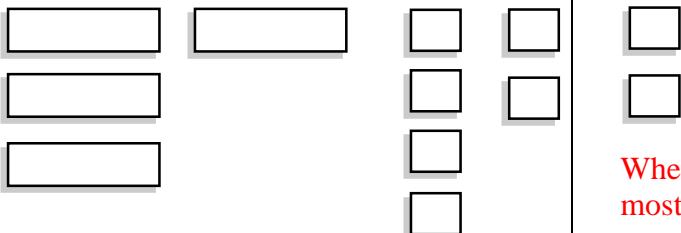
- b. Verify the solution in the space below. Students can substitute 4 into the original equation and verify that the statement is true. Alternatively, they can write a 4 on the tiles above (−4 on the −x tiles) and verify that the two sides of the equation are true.

Directions: Model and solve the following equations. Show the solving actions and verify your solution.

Students can move away from using the tiles as they are ready.

<p>3. $4x + 3x - 1 = 6$ $7x - 1 = 6$ Combine like terms. $7x = 7$ Add 1 to both sides. $x = 1$ Divide both sides by 7.</p>	<p>4. $10 = -x + 3x + 4$ $x = 3$</p>
<p>5. $2x - x + 4 = -8$</p>	<p>6. $10 = -2x - 3 + 4x + 5$</p>

7. The following is a model of an equation.

	<p>When students solve this equation using the tiles, they will most likely start by combining the $3x$ and x. The next logical step is to subtract 6 from both sides; however when they try to subtract 6 from the right side, there are not enough tiles. Students can create zero pairs on to the right side without changing the equation. By creating 4 zero pairs, students can now remove 6 tiles from the right side. Alternatively, students can choose to add 6 negative tiles to both sides. It should be made transparent to students that both methods are valid and that you are essentially doing the same thing.</p>
---	--

a. Write the symbolic representation (equation) for this model.

$$3x + x + 6 = 2$$

b. Solve the equation.

$$3x + x + 6 = 2$$

$$4x + 6 = 2 \quad \text{Combine like terms.}$$

$$4x = -4 \quad \text{Subtract 6 from both sides.}$$

$$x = -1 \quad \text{Divide both sides by 4.}$$

Directions: Solve the following equations.

<p>8. $-7x + 5x + 3 = -9$ $x = 6$</p>	<p>9. $17 = m + 5 - 3m$ $-6 = m$</p>
<p>10. $0.5b + 2b = -50$</p>	<p>11. $-\frac{2}{3} = -\frac{4}{3} + 6r$ $r = \frac{1}{9}$</p>

12. **Find and Fix the Mistake:** Ricardo solved the following equation incorrectly. Circle the mistake and

describe the mistake in words. Then, solve the equation correctly.



$$2x + x + 5x = 56$$

$$7x = 56 \quad \text{Combine like terms.}$$

$$x = 8 \quad \text{Divide both sides by 7.}$$

Ricardo did not realize that the coefficient in front of x is 1. When combined, the result is $8x$. The correct solution is $x = 7$.

13. Carson and his family drove to Disneyland. They started driving on Thursday and then stopped for the night. On Friday, they drove twice as many miles as they had on Thursday. On Saturday, they drove fifty miles more than they had on Friday. Carson's mom asked him to write an equation to determine how many miles they drove each day.



Carson wrote the following equation: $m + 2m + (2m + 50) = 650$

- a. Match each expression with what it represents in the story.

m		The number of miles driven on Friday
$2m$		The number of miles driven on Saturday
$(2m + 50)$		The total number of miles driven.
$m + 2m + (2m + 50)$		The number of miles driven on Thursday.

- b. If Carson and his family live 650 miles from Disneyland, how many miles did Carson's family drive each day?

Thursday: 120 **Friday:** 240 **Saturday:** 290

Encourage students to read back through the story and make sure the answers fit with the story.

14. George started writing a story that matches the expressions and equation shown on the left. Pieces of the story are missing. Help him finish the story, solve the equation, and determine each person's age.



Ages

Talen's age: t

Peter's age: $8t + 3$

$$t + (8t + 3) = 39$$

Story

I am trying to figure out Peter and Talen's ages. Peter tells me that he is three more than... *eight times Talen's age.*

Together, Talen and Peter's ages... *sum to 39.*

How old are Talen and Peter?

Talen's age: 4

Peter's age: 35

1.1c Homework: Solving Multi-Step Linear Equations (combine like terms)

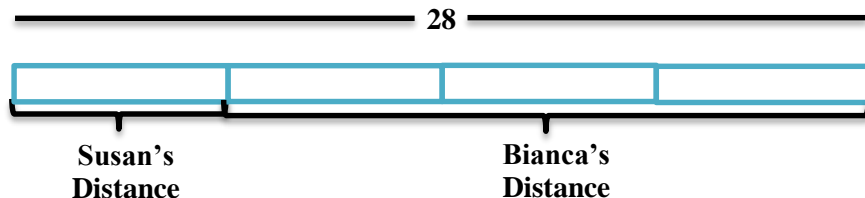
Directions: Solve the following equations. Verify your solutions.

1. $7x - 3x = 24$ $x = 6$	2. $6a + 5a = -11$	3. $-4n - 2n = 6$
4. $5a + 8 + (-2a) = -7$ $a = -5$	5. $18 = 3x + 4 + 2$	6. $-2x - 7 - 4x = 17$
7. $s + 3s = -24$	8. $-5n + 4n - 7 = 8$	9. $x - 4 + 3 = -6$ $x = -5$
10. $6 = 4t - 3t - 2$ $t = 8$	11. $2x - 9x + 17 = -4$	12. $x + 3x + 4x = 56$
13. $8x + 25 - 6x = 35$	14. $-32 = -4x - 2x + 4$	15. $1.3b - 0.7b = 12$ $b = 20$
16. $0.4y + 0.1y = -2.5$	17. $\frac{2}{5}x - \frac{1}{5}x = 9$	18. $r - \frac{5}{4} + \frac{1}{2}r = \frac{13}{4}$ $r = 3$
19. $3x + 5 + 6x - 7 = 25$	20. $7d - 12 + 3d = 15$ $d = 2.7$	21. $22 = 5c + 3c - c + 8$

Find and Fix the Mistake: In #22 – 23, a common mistake has been made. Circle the mistake and describe the mistake in words. Then, solve the equation correctly.

<p>22. $2x + 4x - 2 = 20$</p> <p>$6x - 2 = 20$ Combine like terms ($2x$ and $4x$)</p> <p><u>$4x = 20$</u> Combine like terms ($6x$ and -2)</p> <p>$x = 5$ Divide by 4.</p> <p>Explanation of mistake: $6x$ and -2 are not like terms.</p> <p>Solve correctly: $x = \frac{11}{3}$</p>	<p>23. $3x - 8x - 5 = 10$</p> <p>$5x - 5 = 10$ Combine like terms ($3x$ and $-8x$).</p> <p>$5x = 15$ Add 5 to both sides.</p> <p>$x = 3$ Divide both sides by 5.</p> <p>Explanation of mistake:</p> <p>Solve correctly.</p>
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24. Bianca ran three times farther than Susan. Together they ran 28 miles. The following is a model that represents this situation.



- Write an equation that represents this situation. $4x = 28$ where x represents Susan's distance
- How far did each girl run?

Susan: 7 miles Bianca: 21 miles

25. Write a story that matches the expressions and equation shown on the left. Then solve the equation and find each person's age.

Ages

Felipe's age: f

Felipe's sister's age: $f - 6$

Felipe's mom's age: $3f - 9$

$$f + (f - 6) + (3f - 9) = 60$$

Story

Felipe's age: _____

Felipe's sister's age: _____

Felipe's mom's age: _____

Hint: When writing stories like the one above, be sure to relate the unknown to its meaning in the story. Don't use the actual unknown in the story. For example, in the situation above, do not write "Felipe's sister is f minus 6 years old." Think about how Felipe's sister's age is related to Felipe's age which is represented by f in the expression. Also, remember that your story needs a question.

1.1d Class Activity: Equations with Fractions

This section was included to help students who struggle with fractions. The idea is that students would clear the fractions prior to solving the equation. That being said, some students may opt to solve some of the equations without clearing the fractions first.


1. Use the following equations to answer the questions that follow.

$$\frac{x+3}{2} = 5$$

$$\frac{x}{2} + 3 = 5$$

$$\frac{1}{2}(x+3) = 5$$

$$\frac{x}{2} + \frac{3}{2} = 5$$

- a. Examine each of the equations above. Circle the equations that are equivalent. Think about the structure of the expressions on the left side of the equation. It may help to use your tiles and draw a model of each equation. 

- b. Solve each of the equations in the space above. Did you find that some equations were easier to solve than others? Why or why not?

Equations 1, 3, and 4: $x = 7$

Equation 2: $x = 4$

When faced with an equation with fractions, we can transform it into an equation that does not contain fractions. This is called **clearing of fractions**. In the problems above, in order to clear the fractions, we need to get rid of the 2 in the denominator of each equation.

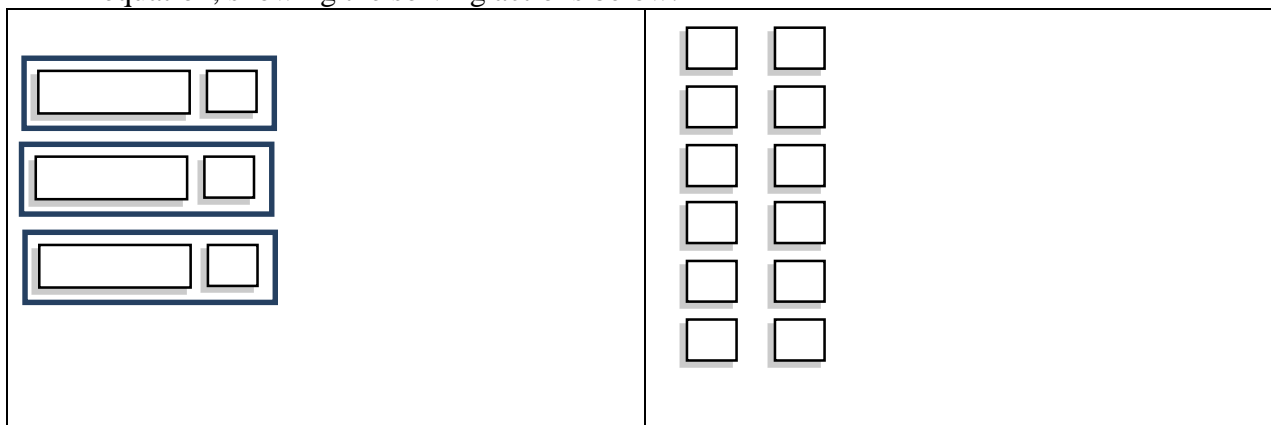
- c. Can you think of a way to eliminate the 2 in each equation before you start to solve the equation? Test your method and re-solve each of the equations above. **Work through each equation with the students, showing them how to clear the fractions before starting to solve. Compare this method with other methods of solving and allow students to choose the method that they like the best for a given problem.**

Directions: Solve each equation by first clearing the fractions.

<p>2. $\frac{x-1}{4} = 6$ $x = 25$</p> <p>In this problem, we can clear the 4 in the denominator by multiplying both sides of the equation by 4. When we do that, we are left with: $x - 1 = 24$ Then continue solving.</p>	<p>3. $\frac{x}{2} + \frac{1}{4} = \frac{7}{4}$ $x = 3$</p> <p>To clear the fractions in this problem, we can multiply both sides of the equation by 4. Remember, on the left side, you must multiply each term by 4. When we do this, we are left with: $2x + 1 = 7$ Then continue solving.</p>	<p>4. $\frac{1}{3}(2x - 4) = -6$ $x = -7$</p>
<p>5. $\frac{2x}{3} + 4 = \frac{14}{3}$</p>	<p>6. $\frac{4}{5} = 3x + \frac{1}{5}$ $x = \frac{1}{5}$</p>	<p>7. $\frac{-2x-5}{3} = 3$</p>
<p>8. $\frac{x}{12} + \frac{1}{3} = \frac{1}{4}$</p>	<p>9. $-6 = \frac{(3x-12)}{3}$ $x = -2$</p>	<p>10. $\frac{1}{2}(4x + 12) = 2$ $x = -2$</p>

1.1e Class Activity: Solving Multi-Step Linear Equations (distribute and combine like terms)

- The following is a model of the equation $3(x + 1) = 12$. Create this model with your tiles and solve the equation, showing the solving actions below.



Solving Actions:

$$3(x + 1) = 12$$

$$x = 3$$

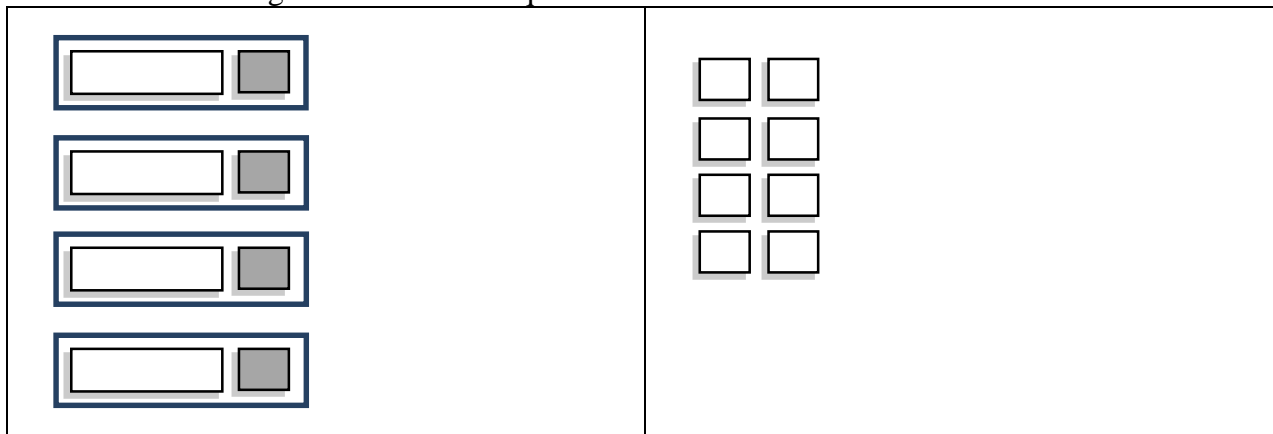
There are two valid ways to solve this problem and students should be exposed to both. The first way is to distribute the 3 to $(x + 1)$ and continue solving. Alternatively, one can start by dividing both sides by 3. Show students on the model what happens when you divide both sides by 3. Talk about when it makes sense to start with distributing and when it makes sense to start by dividing.

Directions: Model and solve the following equations.

<p>2. $2(x + 5) = 14$ $x = 2$</p>	<p>3. $2(3x + 1) - 2x = 10$ $2(3x + 1) - 2x = 10$ $6x + 2 - 2x = 10$ Distribute. $4x + 2 = 10$ Combine like terms. $4x = 8$ Subtract 2 from both sides. $x = 2$ Divide both sides by 4.</p>	<p>4. $-12 = 3(x - 2)$ $x = -2$</p>
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There are many different ways to solve these equations and the others in this chapter. Encourage students to share different solving approaches with their classmates. With practice, students will begin to develop a sense of the sequence that best leads to the solution.

5. The following is a model of an equation.



a. Write the symbolic representation for this model.

$$4(x - 1) = 8$$

b. Solve the equation.

$$x = 3$$

Directions: Solve the following equations without the use of the tiles.

<p>6. $-2(x + 1) = 8$ $x = -5$</p>	<p>7. $13 = -3(x - 4) - 8$ $13 = -3(x - 4) - 8$ $13 = -3x + 12 - 8$ Distribute the -3. $13 = -3x + 4$ Combine like terms 12 and -8. $9 = -3x$ Subtract 4 from both sides. $-3 = x$ Divide both sides by -3.</p>
<p>8. $5 + 2(3a - 1) = 15$</p>	<p>9. $\frac{1}{2}(2t + 4) = -8$ $t = -10$</p>

$$10. \frac{x}{3} + \frac{x-2}{5} = 6$$

$$x = 12$$

$$11. 14 = 5 - 3(x - 2)$$

$$14 = 5 - 3x + 6 \quad \text{Distribute the } -3.$$

$$14 = 11 - 3x \quad \text{Combine like terms 5 and 6.}$$

$$3 = -3x \quad \text{Subtract 11 from both sides.}$$

$$-1 = x \quad \text{Divide both sides by } -3.$$

12. Part of a story that matches the expressions and equation shown on the left has been written for you. Finish the story, solve the equation, and determine how much time Theo spends training in each sport.

Triathlon Training Schedule

Minutes spent swimming: x

Minutes spent running: $2x$

Minutes spent biking: $2x + 30$

$$3x + 4(2x) + 2(2x + 30) = 510 \text{ min.}$$

Story

Theo is training for a triathlon. He runs twice as long as he swims. He bikes...*thirty minutes more than he runs.*

He swims three times a week, runs four times a week, and bikes...*twice a week.*

If he spends a total of 510 minutes per week training, how many minutes does he spend on each exercise at a time?

Minutes spent swimming: 30 Minutes spent running: 60 Minutes spent biking: 90

13. Write a story that matches the expressions and equation shown on the left. Then, solve the equation and determine how much each ride at the fair costs.

A Trip to the Fair

Cost of a pony ride: b

Cost to ride the Ferris wheel: $\frac{1}{2}b$

Cost to bungee jump: $2b + 5$

$$3b + 4\left(\frac{1}{2}b\right) + (2b + 5) = \$33$$

Story

The cost to ride the Ferris wheel is half as much as the cost of a pony ride. The cost to bungee jump is 5 dollars more than twice the cost of a pony ride. Lucas went on three pony rides, rode the Ferris wheel four times, and bungee jumped once. He spent a total of \$33. How much does each ride cost?

Cost of a pony ride: \$4

Cost to ride the Ferris wheel: \$2 Cost to bungee jump: \$13

14. Solve this riddle. “Consider the numbers 3, 8, and 7. Find a fourth number so that the average of the numbers is 7.” The following equation represents this situation.

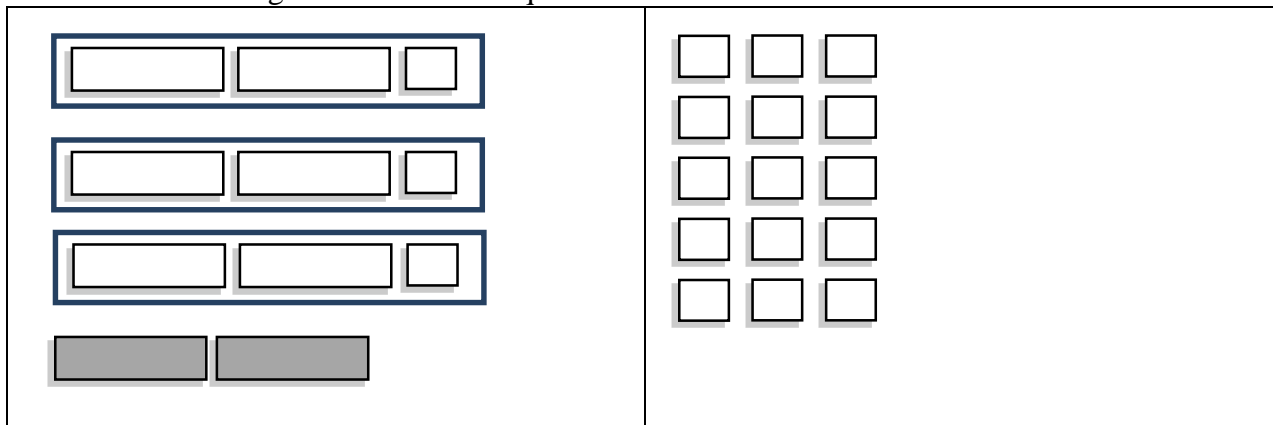
The diagram shows the equation $\frac{3 + 8 + 7 + x}{4} = 7$. Three boxes with red text are connected to parts of the equation by blue arrows:

- A box labeled "sum of the data points" has an arrow pointing to the numerator $3 + 8 + 7 + x$.
- A box labeled "average or mean" has an arrow pointing to the right side of the equation, $= 7$.
- A box labeled "number of data points" has an arrow pointing to the denominator 4 .

- Fill in the boxes above telling what each piece of the equation represents.
- Solve the equation and find the fourth number. **10**

1.1e Homework: Solving Multi-Step Linear Equations (distribute and combine like terms)

1. The following is a model of an equation.



a. Write the symbolic representation (equation) for this model.

$$3(2x + 1) - 2x = 15$$

b. Solve the equation.

$$3(2x + 1) - 2x = 15$$

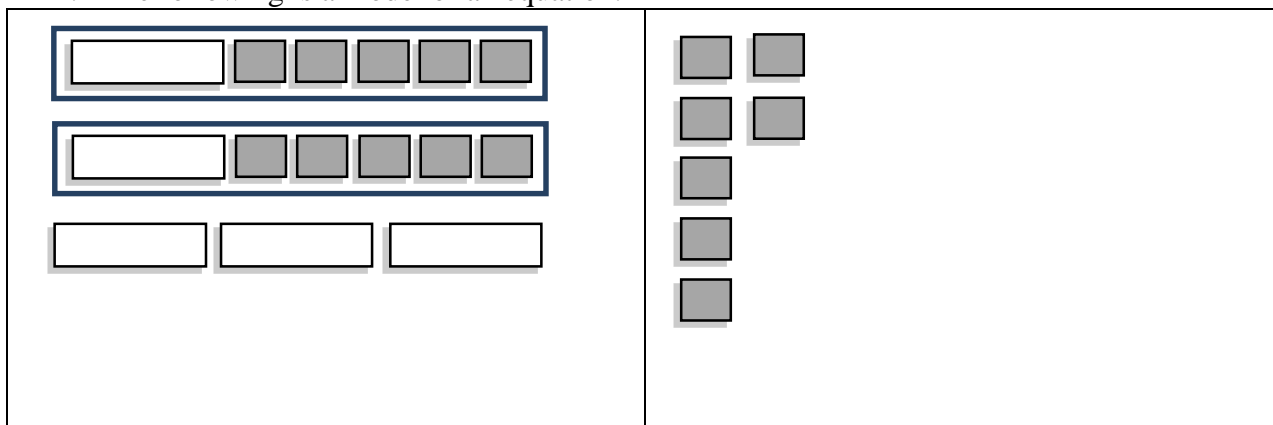
$$6x + 3 - 2x = 15 \text{ Distribute the 3.}$$

$$4x + 3 = 15 \text{ Combine like terms } 6x \text{ and } -2x.$$

$$4x = 12 \text{ Subtract 3 from both sides.}$$

$$x = 3 \text{ Divide both sides by 4.}$$

2. The following is a model of an equation.




a. Write the symbolic representation for this model.

b. Solve the equation.

$$x = \frac{3}{5}$$

Directions: Solve the following equations. Verify your solutions.

3. $3(4x - 2) = 30$	4. $-24 = 4(2 + 2x)$ $x = -4$	5. $-16 = 2(4x + 8)$
6. $3(x + 10) + 5 = 11$	7. $3t - 2 + t - 5t = -1$ $t = -1$	8. $-24 = 2(1 - 5x) + 4$
9. $-2(a + 3) + 4a = 18$	10. $28 = 5x + 3(x + 4)$	11. $4x - 3(x - 2) = 21$ $x = 15$
12. $0 = -2(x + 5) + 3x$	13. $\frac{1}{3}(x + 6) = 1$	14. $5 - 4(2b - 5) + 3b = 15$
15. $10 = 3(x - 2) - 2(5x - 1)$ $x = -2$	16. $0.2(10t - 4) - t = 1.2$	17. $-\frac{1}{7}(x - 7) + 22 = 26$ $x = -21$
18. $-10 = \frac{3x}{4} + \frac{x}{2}$ $-8 = x$	19. $\frac{x-2}{3} = \frac{1}{2}$ $x = \frac{7}{2}$	20. $-(x - 5) + 2 - x = 3$ $x = 2$

Find and Fix the Mistake: In the following problems, a common mistake has been made. Circle the mistake and describe the mistake in words. Then, solve the equation correctly. 

<p>21. $7 + 2(3x + 4) = 3$</p> <p>$7 + 6x + 4 = 3$ Distribute the 2.</p> <p>$11 + 6x = 3$ Combine like terms (7 and 4).</p> <p>$6x = -8$ Subtract 11 from both sides.</p> <p>$x = -\frac{8}{6}$ Divide both sides by 6.</p> <p>$x = -\frac{4}{3}$ Simplify the fraction.</p> <p>Explanation of Mistake: The 2 was only distributed to the $3x$ and not to the entire quantity in the parentheses.</p> <p>Solve Correctly: $x = -2$</p>	<p>22. $-2(y - 3) + 5 = 3$</p> <p>$-2y - 6 + 5 = 3$ Distribute the -2.</p> <p>$-2y - 1 = 3$ Combine like terms (-6 and 5)</p> <p>$-2y = 4$ Add 1 to both sides.</p> <p>$y = -2$ Divide both sides by -2.</p> <p>Explanation of Mistake:</p> <p>Solve Correctly:</p>
<p>23. $5 - (2x - 7) = 14$</p> <p>$5 - 2x - 7 = 14$ Distribute the negative sign.</p> <p>$-2 - 2x = 14$ Combine like terms (5 and -7).</p> <p>$-2x = 16$ Add 2 to both sides.</p> <p>$x = -8$ Divide both sides by -2.</p> <p>Explanation of Mistake: The negative sign was only distributed to the $2x$ and not the -7.</p> <p>Solve Correctly: $x = -1$</p>	<p>24. $\frac{1}{3}(x + 15) = 11$</p> <p>$\frac{1}{3}x + 5 = 11$ Distribute the $\frac{1}{3}$.</p> <p>$\frac{1}{3}x = 6$ Subtract 5 from both sides.</p> <p>$x = 2$ Divide both sides by 3.</p> <p>Explanation of Mistake:</p> <p>Solve Correctly:</p>

25. The expressions below show the grams of fat in sandwiches at a popular fast food restaurant. Use these expressions and the equation to write a story and determine the number of grams of fat in each sandwich.

Fast Food Calories

Crispy Chicken: f

Single burger with cheese: $f + 11$

Double burger with cheese: $2f$

$3f + 2(f + 11) + 4(2f) = 204$ g.

Story

Fat grams in a Crispy Chicken: _____

Fat grams in a single burger with cheese: _____

Fat grams in a double burger with cheese: _____

26. Mia has taken two quizzes in math so far this quarter and scored a 75% on the first and an 82% on the second. What must she score on the third quiz in order to have an average of 80% on her three quizzes?

See #14 from Class Activity for a similar problem

1.1f Class Activity: Creating and Solving Linear Equations to Model Real World Problems Part I



Select problems. Access to a calculator is highly recommended. As students work through these problems, make sure they are clear on what the unknown stands for in the context. Ask questions such as: What quantities are involved? What is the relationship between the quantities? What units are involved? What question(s) are we trying to answer? What does the solution to the equation tell us? What does the solution represent in the context? Do our answers make sense and fit the parameters defined in the word problem?

1. Use the story below about Chloe and her friends to answer the questions that follow.

Going on a Picnic

Cost of a sandwich: 6x

Cost of a bag of chips: 1.5x

Cost of a cookie: x

Cost of a soda: 2x

Story

Chloe and her friends are going on a picnic.
A sandwich is 6 times the cost of a cookie. A
bag of chips is one and a half times the cost
of a cookie. A soda is twice the cost of a
cookie.

- a. Write expressions for the cost of each item on the lines provided above if the cost of a cookie is x .
- b. Chloe and her friends buy 2 sandwiches, 3 bags of chips, 4 cookies, and 2 sodas. They spend a total of \$12.25. Use this information and the expressions you wrote above to write an equation representing this situation.

$$2(6x) + 3(1.5x) + 4x + 2(2x) = 12.25$$

- c. Solve your equation to determine the cost of each item.

Sandwich: \$3 Bag of chips: \$0.75 Cookie: \$0.50 Soda: \$1

2. Uncle Hank loves riddles. Uncle Hank tells his nephews, "I have twice as many dimes as quarters. I have 12 more nickels than quarters. I have \$4.60 total. Whoever can solve my riddle will get my coins."
 - a. Owen has a good start on an equation for solving this riddle. Help Owen fill in the missing pieces of the equation on the lines below.

$$0.25q + \underline{0.1}(2q) + 0.05(\underline{q + 12}) = 4.60$$

- b. How many of each type of coin does Uncle Hank have?

of quarters: 8

of dimes: 16

of nickels: 20

3. Use the story below about Farmer Ted and his animals to answer the questions that follow.

Farmer Ted's Animals

Weight of a cow: c

Weight of a horse: $2c$

Weight of a sheep: $\frac{1}{4}c - 100$

Weight of a pig: $\frac{1}{4}c$

Story

Farmer Ted is weighing his animals. He knows that a pig weighs approximately $\frac{1}{4}$ as much as a cow. He also knows that a Clydesdale horse weighs about twice what a cow weighs. A sheep weighs approximately 100 pounds less than a pig.

- On the lines above to the left, write the expressions that match the weight of each of the animals if a cow weighs c pounds.
- Write an expression for the weight of one cow, one horse, one sheep, and one pig.
 $c + 2c + \left(\frac{1}{4}c - 100\right) + \frac{1}{4}c$
- If Farmer Ted puts 3 cows, 2 Clydesdale horses, 4 sheep and 1 pig on a giant scale used for weighing semi-trucks, the scale reads 7,850 pounds. Approximately how much does each animal weigh?

$$3c + 2(2c) + 4\left(\frac{1}{4}c - 100\right) + \frac{1}{4}c = 7,850$$

* Units are pounds.

Cow: 1,000 Clydesdale horse: 2,000 Sheep: 150 Pig: 250

4. Miley is trying to solve the following riddle: "The sum of three consecutive integers is 84. What are the integers?" She writes part of an equation that can be used to solve this riddle.

second
integer
↓

$$n + (n + 1) + (\text{ }n + 2\text{ }) = 84$$

↑
first integer

- Help Miley complete the equation above by filling in the blank.
- Find the three integers. **27, 28, 29**

5. Eli is making lemonade for a party. Expressions showing the ratio of water to sugar to lemon juice used to make lemonade are shown on the left.

Making Lemonade

Cups of water: c

Cups of sugar: $\frac{1}{4}c$

Cups of lemon juice: $\frac{1}{2}c$

$$c + \frac{1}{4}c + \frac{1}{2}c = 14 \text{ cups}$$

Story

The ratio of water to sugar to lemon juice used to make lemonade is $1:\frac{1}{4}:\frac{1}{2}$. Eli used a total of 14 cups of ingredients to make a batch of lemonade. How many cups of each ingredient did Eli use?

- Write a story that matches the expressions and equation shown on the left.
- Solve the equation above. How many cups of each ingredient is Eli planning to use?

Cups of water: 8

Cups of sugar: 2

Cups of lemon juice: 4

6. Use the incomplete story and the expressions and equation below to answer the questions that follow.

Triangles

$m\angle A: x$

$m\angle B: 3x$

$m\angle C: x - 20$

$$x + 3x + (x - 20) = 180^\circ$$

Story

In $\triangle ABC$, the measure of $\angle B$ is three times larger than...the measure of $\angle A$.

The measure of $\angle C$ is 20° less than...the measure of $\angle A$.

The sum of the angles in a triangle is... 180° .

What is the measure of each angle in the triangle?

- Finish the story above so that it matches the expressions and equation shown on the left.
- What is the measure of each angle in the triangle?

$m\angle A =$ 40°

$m\angle B =$ 120°

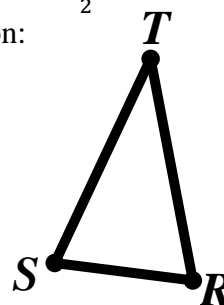
$m\angle C =$ 20°

7. In $\triangle RST$, $\angle R$ and $\angle S$ have the same measure. The measure of $\angle T$ is $\frac{1}{2}$ the measure of $\angle R$ and $\angle S$. Marie drew the following model and picture to represent this situation:

$\angle R$

$\angle S$

$\angle T$



- a. Help Marie write an equation that represents the sum of the angles in $\triangle RST$. Remember the sum of the angles in a triangle is 180° .

$x + x + \frac{1}{2}x = 180$ or $x + 2x + 2x = 180$ depending on how you define x . This is a good place to use suggestive variables resulting in the following possible equations $T + 2T + 2T = 180$ or $R + R + \frac{R}{2} = 180$

- b. Solve the equation and find the measure of each angle.

$m\angle R$: 72° $m\angle S$: 72° $m\angle T$: 36°

8. Use the expressions and equation below to answer the questions that follow.

Rectangles

Width of a rectangle: w

Length of a rectangle: $2w$

$2w + w + 2w + w = 42$ ft.

Story

The length of a rectangle is twice its width.

The perimeter of the rectangle is 42 feet.

What is the measure of the length and width of the rectangle?

- a. Draw a picture of the rectangle that accurately depicts the ratio of the length of the rectangle to its width. *Pictures will vary but the length should be approximately double the width.*
- b. Write a story in the space provided that matches the expressions and equation.
- c. What is a different but equivalent way of writing the equation above?
 $2(2w) + 2(w) = 42$
- d. Solve the equation and find the length and width of the rectangle.

Length: 14 ft.

Width: 7 ft.

9. Josh works 40 hours a week as a nurse practitioner. He makes time and a half for every hour he works over 40 hours. Josh works 60 hours one week and earns \$2100. Part of an equation that represents this situation is shown below.

over-time
pay rate

↓

$$\underline{\textcolor{red}{p}}(40) + 1.5p(\underline{\textcolor{red}{20}}) = 2100$$

↑

hours worked
at regular
rate

- a. Fill in the blanks in the equation above so that it matches the story.
 - b. What is Josh's regular hourly rate? $\textcolor{red}{\$30/\text{hour}}$
 - c. What is Josh's overtime hourly rate? $\textcolor{red}{\$45/\text{hour}}$
10. The ratio of girls to boys at The Gymnastics Preparation Center is 3:2. If there are 180 kids that train at The Gymnastics Preparation Center, how many of them are girls? How many of them are boys?
 $\textcolor{red}{\text{Consider using a bar model as an interim representation. Students can use the bar model to create the equation. There are 108 girls and 72 boys.}}$
11. The average of three numbers is 14. The largest number is two more than twice the smallest. The second largest number is twice the smallest number. Find the three numbers.
 $\textcolor{red}{\text{Encourage students to write the expressions first. The three numbers are 8, 16, and 18}}$

See Class Activity for problems that are similar to the ones in the homework.

- ### Points Scored

Sanjeet's points: _____

Cole's points: _____

Equation: _____

Sanjeet and his team members were looking at the total points scored by each player during the season. Sanjeet scored twice as many points as Terrence. Cole scored 12 more points than Sanjeet. Together the boys scored 992 points during the season. How many points did each boy score?

- Cole:** _____

Terrence: _____

Sanjeet: _____

- $0.01p +$
value of pennies

- c. How many of each type of coin does Uncle Hank have?

Quarters: _____

Dimes: _____

Nickels: _____

Pennies: _____

3. During the summer, Victoria plays soccer and takes swim and piano lessons. Each swim lesson is 15 minutes shorter than a soccer practice. Each piano lesson is twice as long as a soccer practice. Use this information to answer the questions that follow.

- a. The following expressions represent how long an activity is each time she goes. Write the name of the activity that matches each expression on the lines provided.

t : _____

$t - 15$: _____

$2t$: _____

- b. Victoria has soccer three times a week, swimming four times a week, and piano twice a week. She spends a total of 435 minutes each week doing these three activities a week. Write an equation that represents this situation.

$$3t + 4(t - 15) + 2(2t) = 435$$

- c. How long is one session of each activity?

Soccer: _____

Swimming: _____

Piano: _____

4. The ratio of freshmen to sophomores to juniors to seniors in band is 1:2:3:2. If there are a total of 240 students in the band, how many are in each grade level?

Freshmen: _____

Sophomores: _____

Juniors: _____

Seniors: _____

5. The art teacher is making salt dough for an upcoming project. The ratio of flour to salt to water used to make salt dough is shown below.

Making Salt Dough

Cups of flour: $2c$

Cups of salt: c

Cups of water: $\frac{3}{4}c$

$$2c + c + \frac{3}{4}c = 60 \text{ cups}$$

Story

The ratio of flour to salt to water used to make salt dough is $2:1:\frac{3}{4}$. The art teacher used 60 cups of ingredients to make a batch of salt dough. How many cups of each ingredient did he use?

- Write a story that matches the expressions and equation shown on the left.
- Solve the equation. How many cups of each ingredient is the art teacher planning to use?

Cups of flour: 32 cups

Cups of salt: 16 cups

Cups of water: 12 cups

6. Use the story below about a triangle to answer the questions that follow.

Angles in a Triangle

$m\angle A$: _____

$m\angle B$: _____

$m\angle C$: x

Equation: _____

Story

In $\triangle ABC$, the measure of $\angle B$ is three times larger than the measure of $\angle C$. The measure of $\angle A$ is twice as large as the measure of $\angle C$. The sum of the angles in a triangle is 180° .

- Write the expressions and equation matching the story on the lines provided above.
- What is the measure of each angle in the triangle?

$m\angle A =$ _____

$m\angle B =$ _____

$m\angle C =$ _____

7. The width of a rectangle is six more than four times its length. A model of this situation has been drawn below. Use this information to answer the questions that follow.

Length:

Width:

--	--	--	--

→ +6

- a. Write an expression that represents the perimeter of the rectangle.
 $2(x) + 2(4x + 6)$
- b. If the perimeter of the rectangle is 112 feet, write an equation to represent this situation and find the length and width of the rectangle.

Length: _____

Width: _____

8. A marble jar has twice as many blue marbles as red marbles, 16 more green marbles than blue marbles, and 10 fewer white marbles than red marbles. The jar has a total of 150 marbles. Use this information to answer the questions that follow.
- a. The following equation represents this situation. Match each piece of the equation to the appropriate marble color. Write your answer in the boxes provided.

Diagram illustrating the distribution of 150 items into four categories: red, blue, green, and yellow. The equation $m + 2m + (2m + 16) + (m - 10) = 150$ is shown with brackets linking terms to categories:

- m is linked to red.
- $2m$ is linked to blue.
- $(2m + 16)$ is linked to green.
- $(m - 10)$ is linked to yellow.

- b. Determine how many marbles of each color are in the jar.

Blue: _____ **Red:** _____ **Green:** _____ **White:** _____

9. Use the expressions and equation below about the cost of clothes to answer the questions that follow.

The Cost of Clothes

Cost of a shirt: c

Cost of a pair of jeans: $c + 12$

$$3c + 2(c + 12) = \$164$$

Story

- Write a story that matches the expressions and equation in the space provided.
- Solve the equation to determine the cost of a shirt and the cost of a pair of jeans.

Cost of a shirt: _____ **Cost of a pair of jeans:** _____

Directions: Write and solve an equation to answer each of the following problems. Use pictures and models to help you. Refer back to similar problems you have already seen in the chapter to help you if you get stuck. Make sure your answers are displayed clearly with the appropriate units.

10. The width of a rectangle is five less than three times the length of the rectangle. If the perimeter of the rectangle is 70 inches what are the dimensions of the rectangle?

11. At Shoes for Less, a pair of shoes is \$15 less than a pair of boots. Cho purchased three pairs of shoes and two pairs of boots for \$120. How much does a pair of boots cost?

Cost of a pair of boots: b

Cost of a pair of shoes: $b - 15$

$$3(b - 15) + 2b = 120$$

A pair of shoes costs \$18 and a pair of boots costs \$33.

12. Central Lewis High School has five times as many desktop computers as laptops. The school has a total of 360 computers. How many laptops does Central Lewis High School have?

13. In $\triangle LMN$, the measure of $\angle L$ is equal to the measure of $\angle M$. The measure of $\angle N$ is twice the measure of $\angle M$. Find the measure of each angle in $\triangle LMN$.

$$m\angle L = \underline{\hspace{2cm}}$$

$$m\angle M = \underline{\hspace{2cm}}$$

$$m\angle N = \underline{\hspace{2cm}}$$

14. Adam is trying to solve the following riddle: “The sum of three consecutive integers is -36 . What are the integers?” Solve Adam’s riddle.

15. Afua got a 90% on her first math exam, a 76% on her second math exam, and a 92% on her third math exam. What must she score on her fourth exam to have an average of 88% in the class?

16. At Discovery Preschool, parents who have two students enrolled get a discount on the second child. The second child’s tuition is 10 dollars less per day than the first child’s. If Tess has her two children enrolled for 5 days and her total bill for both children is \$200, how much does she pay each day for her second child to attend daycare?

1st child daily tuition: t

2nd child daily tuition: $t - 10$

$$5t + 5(t - 10) = 200$$

Tess pays \$15 a day for her second child to attend daycare.

1.1g Self-Assessment: Section 1.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Understand the meaning of linear expression and linear equation.	I am struggling with the difference between a linear equation and a linear expression.	If given a list of expressions and equations, I can tell which ones are expressions and which are equations.	I can define linear expression and equation in my own words and provide examples of each.	I can define linear expression and equation in my own words and provide examples of each. I know what it means to simplify an expression, to evaluate an expression, and to solve an equation.
2. Solve multi-step linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	I am struggling to solve most of the equations on the following page.	I can solve the equations in Set A on the following page.	I can solve all of the equations in Set A and all but one of the equations in Set B.	I can solve all of the equations in Sets A and B on the following page.
3. Write and solve linear expressions and equations that model real world problems.	When faced with a word problem similar to those in this chapter, I am having a difficult time seeing how the pieces of an equation relate to the story and I usually skip these problems.	When faced with a word problem similar to those in this chapter, I can match the different pieces of an equation that has been given to me to the story and solve the equation.	When faced with a word problem similar to those in this chapter, I can identify the important quantities in a practical situation, complete partial expressions and equations that have been given to me, solve the equation, and interpret the solution in the context.	When faced with a word problem similar to those in this chapter, I can identify the important quantities in a practical situation, write expressions and equations showing the relationship between the quantities, solve the equation, and interpret the solution in the context.

1. Sam was asked to evaluate the expression $5x + 3x + 20$ for $x = 100$. Sam's work is shown below.

Sam's Work:

$$5x + 3x + 20 = 100$$

$$8x + 20 = 100$$

$$8x = 80$$

$$x = 10$$

- a. What mistake did Sam make? Help Sam to answer the question correctly.

2. Solve the following equations.

Set A

1. $8x - 6x + 1 = 11$	2. $-4x - 6 + x = -12$	3. $2(x + 3) + 4x = 40$

Set B

4. $-2(2x + 3) + 3x = -2$	5. $7k - (k + 6) + 2k = 22$	6. $\frac{1}{2}(4x - 16) + 8x = 12$

3. Jesse and her brothers Nick and Owen are saving money over the summer. Each week, Jesse saves twice as much as Owen. Owen saves \$5 more than Nick. At the end of four weeks, the three of them have saved a total of \$220. How much money does each person save per week?

Section 1.2: Creating and Solving Multi-Step Linear Equations with Variables on Both Sides

Section Overview:


In this section students will solve equations with unknowns on both sides of the equal sign. From here, students will apply the skills learned so far in the chapter and solve a variety of linear equations with rational number coefficients. Up to this point, students have only encountered linear equations with a unique solution (one solution). In the latter part of this section, students will be introduced to linear equations in one variable with no solution or infinitely many solutions. Students will analyze what it is about the structure of an equation and the solving outcome ($x = a$, $a = b$, or $a = a$ where a and b are different numbers) that results in one solution, infinitely many solutions, or no solution (is true for a unique value, no value, or all values of the unknown).

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Solve multi-step linear equations that have one solution, infinitely many solutions, or no solution.
2. Understand what it is about the structure of a linear equation that results in equations with one solution, infinitely many solutions, or no solutions.
3. Identify and provide examples of equations that have one solution, infinitely many solutions, or no solution.

1.2a Class Activity: Solving Multi-Step Linear Equations (variables on both sides)

1. The following is a model of the equation $5x + 2 = 3x + 12$. Create this model with your tiles and solve the equation, showing your solving actions below. 

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- a. Solving Actions:

$$5x + 2 = 3x + 12$$

$$2x + 2 = 12 \text{ Subtract } 3x \text{ from both sides.}$$

$$2x = 10 \text{ Subtract } 2 \text{ from both sides.}$$

$$x = 5 \text{ Divide both sides by } 2.$$

This problem is different than the ones we have studied so far because the variable is on both sides of the equation.

- b. How can you verify your solution?

Students can verify the solution by substituting into the equation. Alternatively, they can label the pieces of the model above by writing the values on each of the tiles and verifying that the two sides are equal.

Directions: Model and solve the following equations.

<p>2. $2x = x + 4$ $x = 4$ Subtract x from both sides.</p>	<p>3. $3x + 3 = 2x + 7$ $x = 4$</p>	<p>4. $x + 10 = 2x + 5$ $x = 5$</p>
<p>5. $x + 5 = -x - 3$ $2x + 5 = -3$ Add x to both sides. $2x = -8$ Subtract 5 from both sides. $x = -4$ Divide both sides by 2.</p>	<p>6. $4x = -2x + 12$</p>	<p>7. $2 - 5x = -6x + 5$</p>

Directions: Solve the following equations. You may use the tiles to help you.

<p>8. $4x = 2x + 12$</p>	<p>9. $5x + 3 = x + 27$</p>
<p>10. $-4x + 6 = 3x - 36$ $-7x + 6 = -36$ Subtract $3x$ from both sides. $-7x = -42$ Subtract 6 from both sides. $x = 6$ Divide both sides by -7.</p>	<p>11. $0.7x - 0.6 = -0.2x - 0.42$ $x = 0.2$</p>
<p>12. $8 - 4x = 4x$</p>	<p>13. $\frac{1}{3}x - 8 = 12 + \frac{4}{3}x$ $x = -20$</p>
<p>14. $\frac{x+3}{2} = \frac{x-1}{4}$ One way to solve this problem is to multiply both sides of the equation by 4 (clearing the denominators) and leaving: $2(x + 3) = x - 1$ Multiply both sides by 4. $2x + 6 = x - 1$ Distribute. $x + 6 = -1$ Subtract x from both sides. $x = -7$ Subtract 6 from both sides. Alternatively, you can cross multiply – see #15.</p>	<p>15. $\frac{(x-3)}{3} = \frac{(2x+4)}{5}$ $5(x - 3) = 3(2x + 4)$ Cross multiply. $5x - 15 = 6x + 12$ Distribute. $-15 = x + 12$ Subtract $5x$ from both sides. $-27 = x$ Subtract 12 from both sides.</p>

1.2a Homework: Solving Multi-Step Linear Equations (variables on both sides)

See class activity for worked-out examples.

Directions: Solve the following equations. Verify your solutions.

1. $3x = 2x + 2$	2. $3x + 5 = 4x + 1$ $4 = x$	3. $6x + 3 = 3x + 12$
4. $3x + 8 = 2x + 10$	5. $5x + 3 = 3x + 7$	6. $5a - 5 = 7 + 2a$
7. $3b - 6 = 8 - 4b$ $b = 2$	8. $-3y + 12 = 3y - 12$	9. $3x = -3x + 1$ $x = \frac{1}{6}$
10. $-2x + 8 = -3x - 1$	11. $3p - 4 = 5p + 4$	12. $3 - 0.25x = -\frac{1}{2}x + 9$ $x = 24$

Directions: In the following problems, a common mistake has been made. Circle the mistake and describe the mistake in words. Then, solve the equation correctly.

13. $4x - 8 = -2x + 20$

$2x - 8 = 20$

Combine like terms ($4x$ and $-2x$)

$2x = 28$

Add 8 to both sides.

$x = 14$

Divide both sides by 2.

Explanation of Mistake:

The $4x$ and $-2x$ are on different sides of the equation so cannot be combined. If students think about what really happened in this step, they subtracted $2x$ from the left side and added $2x$ to the right side so they added different quantities to both sides.

Solve Correctly: $x = \frac{14}{3}$

14. $6x + 4 = -2x$

$8x = 4$

Add $2x$ to both sides.

$x = \frac{4}{8}$

Divide both sides by 8.

$x = \frac{1}{2}$

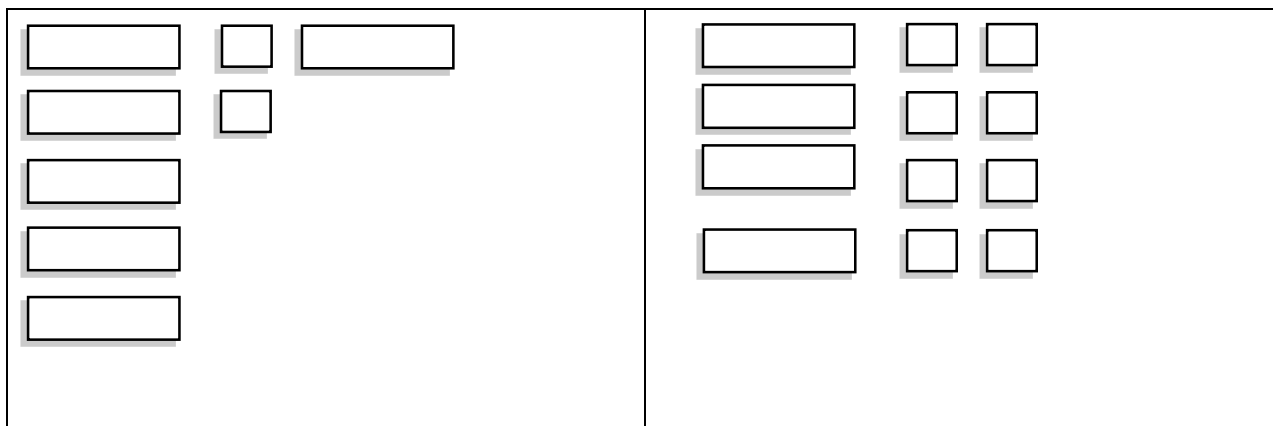
Simplify the fraction.

Explanation of Mistake:

Solve Correctly:

1.2b Class Activity: Solving Multi-Step Linear Equations (putting it all together)

- The following is a model of the equation $5x + 2 + x = 4x + 8$. Create this model with your tiles and solve the equation, showing your solving actions below.



- Solving Actions:

$$5x + 2 + x = 4x + 8$$

$$6x + 2 = 4x + 8 \text{ Combine like terms.}$$

$$2x + 2 = 8 \text{ Subtract } 4x \text{ from both sides.}$$

$$2x = 6 \text{ Subtract 2 from both sides.}$$

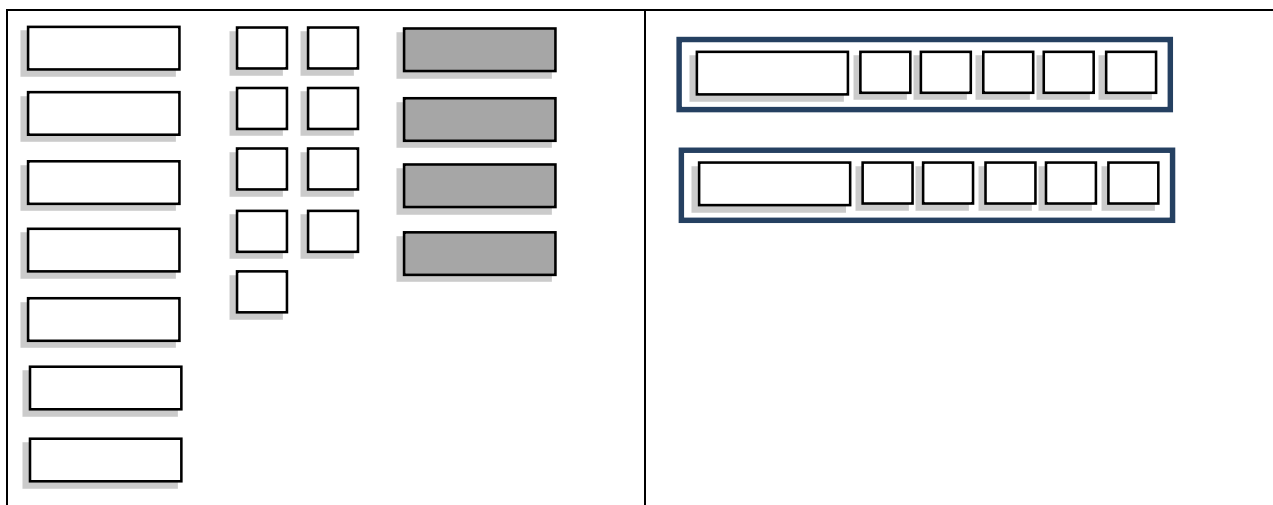
$$x = 3 \text{ Divide both sides by 2.}$$

Again, there are many different ways to solve the given equation and arrive at the correct solution; however some solution pathways are easier than others. Practice develops a sense of the solving sequence that is the easiest and most direct path to the solution.

- Verify your solution.

Substitute 3 in for x into the original equation and verify that both sides equal each other when simplified.

- The following is a model of the equation $7x + 9 - 4x = 2(x + 5)$. Create this model with your tiles and solve the equation, showing your solving actions below.



- Solving Actions:

$$7x + 9 - 4x = 2(x + 5)$$

$$x = 1$$

Directions: Model and solve the following equations.

<p>3. $2(x + 3) = 5x - 3$ $2x + 6 = 5x - 3$ Distribute. $6 = 3x - 3$ Subtract $2x$ from both sides. $9 = 3x$ Add 3 to both sides. $3 = x$ Divide both sides by 3.</p>	<p>4. $8x + 3 - 2x = 3x + 12$ $x = 3$</p>	<p>5. $10x + 2 - 3x = 3(2x + 2)$ $x = 4$</p>
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6. The following is a model of an equation.

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a. Write the symbolic representation for this model.

$$3(x - 2) + (-x) = 2(2x + 3)$$

b. Solve the equation.

$$3(x - 2) + (-x) = 2(2x + 3)$$

$$3x - 6 + (-x) = 4x + 6 \text{ Distribute.}$$

$$2x - 6 = 4x + 6 \text{ Combine like terms } 3x \text{ and } -x.$$

$$-2x - 6 = 6 \text{ Subtract } 4x \text{ from both sides.}$$

$$-2x = 12 \text{ Add 6 to both sides.}$$

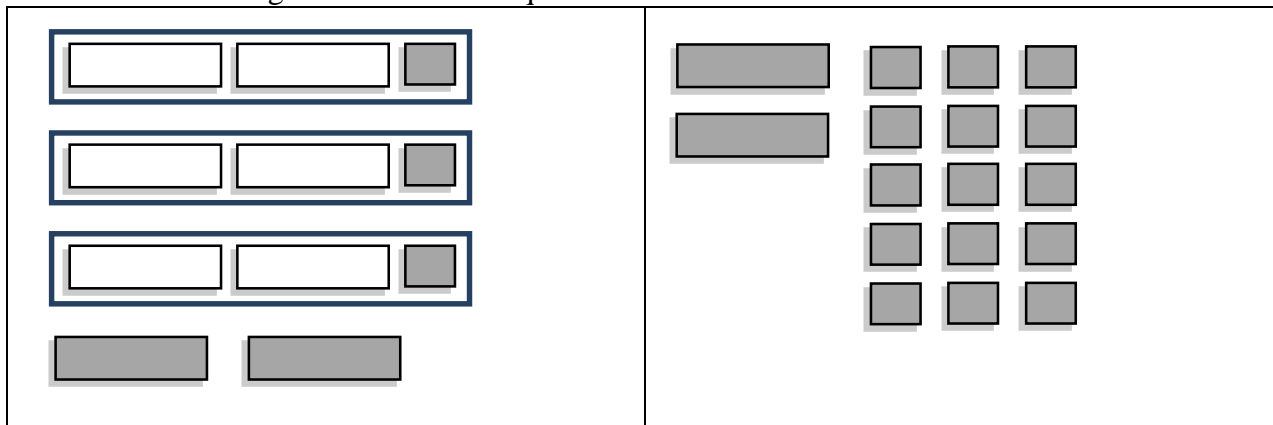
$$x = -6 \text{ Divide both sides by } -2.$$

Directions: Solve the following equations without the use of the tiles.

<p>7. $9x - 4 = 7 - 2x$ $x = 1$</p>	<p>8. $2(a + 2) = 11 + a$</p>
<p>9. $5x - 4x + 18 = 3x + 2$</p>	<p>10. $2t + 21 = 3(t + 5)$</p>
<p>11. $7x + 2 - 4x = 7 + 2x + 4$ $x = 9$</p>	<p>12. $4(1 - x) + 3x = -2(x + 1)$</p>
<p>13. $\frac{1}{4}(12x + 16) = 10 - 3(x - 2)$ $3x + 4 = 10 - 3x + 6$ Distribute the $\frac{1}{4}$ and -3. $3x + 4 = 16 - 3x$ Combine like terms 10 and 6. $6x + 4 = 16$ Add $3x$ to both sides. $6x = 12$ Subtract 4 from both sides. $x = 2$ Divide both sides by 6.</p>	<p>14. $\frac{2x-9}{3} = 8 - 3x$ $3 = x$</p>
<p>15. $\frac{y}{3} + 5 = \frac{y}{2} + 3$ $2y + 30 = 3y + 18$ Multiply both sides by 6 to clear the fractions. $30 = y + 18$ Subtract $2y$ from both sides. $12 = y$ Subtract 18 from both sides.</p>	<p>16. $\frac{1}{2}(2n + 6) = 5n - 12 - n$ $5 = n$</p>

1.2b Homework: Solving Multi-Step Linear Equations (putting it all together)

1. The following is a model of an equation.



a. Write the symbolic representation of the equation for this model.

$$3(2x - 1) + (-2x) = -2x - 15$$

b. Solve the equation.

$$6x - 3 + (-2x) = -2x - 15 \text{ Distribute the 3.}$$

$$4x - 3 = -2x - 15 \text{ Combine like terms.}$$

$$6x - 3 = -15 \text{ Add } 2x \text{ to both sides.}$$

$$6x = -12 \text{ Add 3 to both sides.}$$

$$x = -2 \text{ Divide both sides by 6.}$$

Directions: Solve the following equations. Verify your solutions.

2. $x + 3x = 9 + x$	3. $4c + 4 = c + 10$	4. $3(4x - 1) = 2(5x - 7)$ $x = -\frac{11}{2}$
5. $3x + 10 + 2x = 2(x + 8)$ $x = 2$	6. $2(x + 8) = 2(2x + 1)$	7. $4(x + 3) = x + 26 + x$

8. $3a + 5(a - 2) = 6(a + 4)$	9. $13 - (2c + 2) = 2(c + 2) + 3c$ $c = 1$	10. $2(4x + 1) - 2x = 9x - 1$
11. $2 - (2x + 2) = 2(x + 3) + x$	12. $3(x - 6) = 4(x + 2) - 21$	13. $3(y + 7) = 2(y + 9) - y$ $y = \frac{-3}{2}$
14. $-4(x - 3) = 6(x + 5)$	15. $\frac{1}{2}(12 - 2x) - 4 = 5x + 2(x - 7)$ $x = 2$	16. $\frac{1}{2}(12n - 4) = 14 - 10n$

1.2c Class Activity: Creating and Solving Linear Equations to Model Real World Problems Part II

Directions: Write a story that goes with the expressions and equation in each problem. Solve for the unknown information and interpret your answer.



1. Birthday Parties

Number of people at a birthday party: p

Cost of party at Boondocks: $8p + 60$

Cost of party at Raging Waters: $20p$

$$8p + 60 = 20p$$

Story

The cost of a birthday party at Boondocks is \$8 per person plus \$60 to rent the party room. The cost of a party at Raging Waters is \$20 per person. How many kids would attend for the cost of the parties to be the same?

- Write a story that matches the expressions and equations.
- Solve the equation in the space above. $p = 5$
- Interpret your answer. If there are 5 people, the cost of the parties is the same.

2. A Number Trick

Starting number: n

Lily's number: $3(n + 5)$

Kali's number: $(n - 5)$

$$3(n + 5) = (n - 5)$$

Story

Lily starts with a number, adds 5 to it, and multiplies the result by 3. Kali starts with the same number and subtracts 5 from it. When they are finished, they realize they have the same result. What number did the girls start with?

- Write a story that matches the expressions and equations.
- Solve the equation in the space above. $n = -10$
- Interpret your answer. Both girls started with the number -10.

3. Savings

Number of weeks: w

Sophie's Money: $300 - 40w$

Raphael's Money: $180 + 20w$

$$300 - 40w = 180 + 20w$$

Story

Sophie currently has \$300 and is spending money at a rate of \$40 per week. Raphael currently has \$180 and is saving money at a rate of \$20 per week. After how many weeks will Sophie and Raphael have the same amount of money?

- Write a story that matches the expressions and equations.
- Solve the equation in the space above. $w = 2$
- Interpret your answer.

After 2 weeks, Sophie and Raphael will have the same amount of money.

Directions: Write an expression for each unknown quantity in the word problem. Then write an equation for each problem. Solve your equation and interpret your answer.

4. Horizon Phone Company charges \$15 a month plus 10 cents per text. G-Mobile charges a flat rate of \$55 per month with unlimited texting. At how many texts would the two plans cost the same? Which plan is the better deal if you send 200 texts per month?

Number of texts sent per month: t

Horizon monthly charge: $15 + 0.1t$

G-Mobile monthly charge: 55

$$15 + 0.1t = 55$$

At 400 texts, the two plans cost the same. If you send 200 texts per month, Horizon is the better deal.

5. The enrollment in dance class is currently 80 students and is increasing at a rate of 4 students per term. The enrollment in choir is 120 students and is decreasing at a rate of 6 students per term. After how many terms will the number of students in dance equal the number of students in choir? How many students will be in each class?

Number of terms: t

Number of students in dance: $80 + 4t$

Number of students in choir: $120 - 6t$

$$80 + 4t = 120 - 6t$$

After 4 terms, the number of students in dance will equal the number of students in choir. The number of students in each class will be 96.

6. You burn approximately 230 calories less per hour if you ride your bike versus go on a run. Lien went on a two-hour run plus burned an additional 150 calories in his warm-up and cool down. Theo went on a 4 hour bike ride. If Lien and Theo burned the same amount of calories on their workouts, approximately how many calories do you burn an hour for each type of exercise?

Calories: c

Calories burned biking: $c - 230$

Calories burned running: c

Calories burned by Lien: $2c + 150$

Calories burned by Theo: $4(c - 230)$

$$2c + 150 = 4(c - 230)$$

You burn approximately 305 calories per hour riding a bike and approximately 535 calories per hour running.

1.2c Homework: Creating and Solving Linear Equations to Model Real World Problems Part II

Directions: Write the story that goes with the expressions and equation in each problem. Solve for the unknown information and interpret your answer.

See class activity for similar problems.

1. Fixing Your Car

Time (hours): h

Cost of Mike's Mechanics: $15h + 75$

Cost of Bubba's Body Shop: $25h$

$$15h + 75 = 25h$$

Story

- Write a story that matches the expressions and equations.
- Solve the equation in the space above.
- Interpret your answer.

2. World Languages

Number of years: t

of students in French: $160 - 9t$

of students in Spanish: $85 + 6t$

$$160 - 9t = 85 + 6t$$

Story

- Write a story that matches the expressions and equations.
- Solve the equation in the space above.
- Interpret your answer.

3. Downloading Music

of songs downloaded: s

Monthly cost at bTunes: $0.99s$

Monthly cost at iMusic: $10 + 0.79s$

$$0.99s = 10 + 0.79s$$

Story

- Write a story that matches the expressions and equations.
- Solve the equation in the space above.
- Interpret your answer.

Directions: Write an expression for each unknown quantity in the word problem. Then write an equation for each problem. Solve your equation and interpret your answer.

4. Underground Floors charges \$8 per square foot of wood flooring plus \$150 for installation. Woody's Hardwood Flooring charges \$6 per square foot plus \$200 for installation. At how many square feet of flooring would the two companies charge the same amount for flooring? If you were going to put flooring on your kitchen floor that had an area of 120 square feet, which company would you choose?

5. Owen and Charlotte's mom give them the same amount of money to spend at the fair. They both spent all of their money. Owen goes on 8 rides and spends \$5 on pizza while Charlotte goes on 5 rides and spends \$6.50 on pizza and ice cream. How much does each ride cost?

Cost per ride: r

Amount Owen spends: $8r + 5$

Amount Charlotte spends: $5r + 6.50$

$$8r + 5 = 5r + 6.50$$

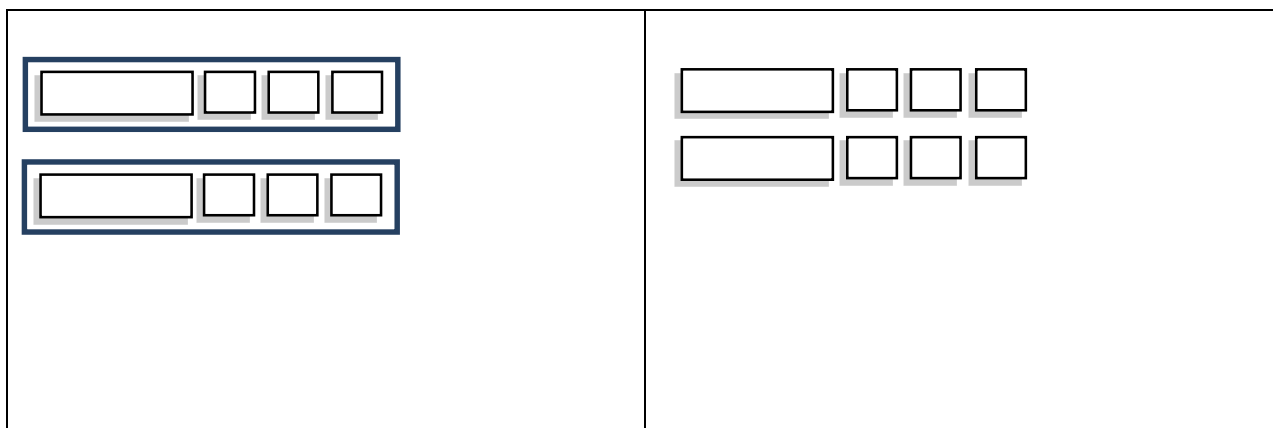
The cost of each ride is \$0.50.

6. Ashton and Kamir are arguing about how a number trick they heard goes. Ashton tells Andrew to think of a number, multiply it by five and subtract three from the result. Kamir tells Andrew to think of a number add five and multiply the result by three. Andrew says that whichever way he does the trick he gets the same answer. What was the number?

1.2d Class Activity: Solving Multi-Step Linear Equations (the different solving outcomes)

Up to this point, we have solved linear equations with a unique solution (**one solution**). In this lesson, we encounter equations that when solved have **infinitely many solutions** and **no solution**.

1. Consider the following model:




- a. Make some observations about the model above.

Students may make observations that the two sides contain the same number of x tiles and the same number of unit tiles but that they are grouped differently.

- b. Write the symbolic representation (equation) for this model and then solve the equation you wrote.

$$2(x + 3) = 2x + 6$$

$$6 = 6$$

- c. What happened when you solved the equation? What is it about the structure of the equation that led to the solution? 

When this equation was transformed into its simplest form, the result was $a = a$.

Since both sides of the equation are equivalent expressions, this equation would be true for all values of x .

To help to solidify what is happening here, select multiple values for x and have students substitute them into the symbolic or concrete model of the representation and observe what happens. This will guide them toward the conclusion that this equation has infinitely many solutions.

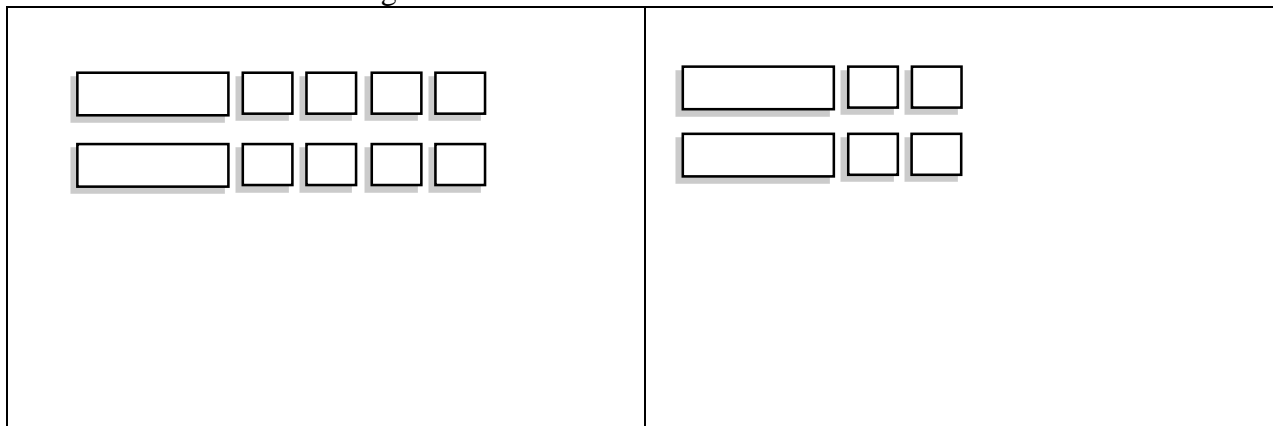
- d. Build or draw your own equation using your tiles that would result in the same solution as the one above.

Answers will vary but the expressions on both sides of the equal sign should be equivalent.

- e. Solve the equation you built. What do you notice?

When the equation is transformed into its simplest form, the result should be $a = a$.

2. Consider the following model:



a. Make some observations about the model above.



Students may observe that the left and right side both have 2 x tiles and the right side 8 unit tiles while the right side has 4 unit tiles.

b. Write the symbolic representation for this model and then solve the equation you wrote.

$$2x + 8 = 2x + 4$$

$$8 = 4$$

The claim that the original equation becomes true for some value of x is false, since the original equation is equivalent to the equation $8 = 4$.

c. What happened when you solved the equation? What is it about the structure of the equation that led to the solution?  

When this equation was transformed into its simplest form, the result was $a = b$ where a and b are different numbers.

To help to solidify what is happening here, select multiple values for x and have students substitute them into the symbolic or concrete model of the representation and observe what happens. This will guide them toward the conclusion that this equation has no solution.

d. Build or draw your own equation using your tiles that would result in the same solution as the one above.


Answers will vary

e. Solve the equation you built. What do you notice?

When this equation is transformed into its simplest form, the result should be $a = b$ where a and b are different numbers.

Directions: Solve the following equations. If there is one solution, state what the solution is. Otherwise, state if there are infinitely many solutions or no solution.


3. $x - 1 = x + 1$ $-1 = 1$ Subtract x from both sides. no solution	4. $5x - 10 = 10 - 5x$ $x = 2$
5. $4(m - 3) = 10m - 6(m + 2)$ $4m - 12 = 10m - 6m - 12$ Distribute. $4m - 12 = 4m - 12$ Combine like terms. $-12 = -12$ Subtract $4m$ from both sides. infinitely many solutions	6. $4(x - 4) = 4x - 16$
7. $2x - 5 = 2(x - 5)$	8. $3x = 3x - 4$ $0 = -4$ Subtract $3x$ from both sides. no solution
9. $3v + 5 + 2v = 5(2 + v)$	10. $5 - (4a + 8) = 5 - 4a - 8$ infinitely many solutions
11. $\frac{2x+8}{2} = x + 4$	12. $\frac{1}{3}(x - 2) = \frac{x}{3} - \frac{2}{3}$ infinitely many solutions

13. What is it about the structure of an expression that leads to one solution, infinitely many solutions, or no solution? Provide examples to support your claim. 

When an equation is transformed into its simplest form and the result is $x = a$, then the equation is true for one value of x . This value is a solution of the equation. Examples may vary

When an equation is transformed into its simplest form and the result is $a = a$, then the equation is true for all values of x . In the original equation, both sides of the equation are the same expression. Examples may vary

When an equation is transformed into its simplest form and the result is $a = b$, then the equation is true for no values of x . The original claim that these two expressions are equal to each other is false; therefore there are no values of x that make this equation true. Examples may vary

Directions: Without solving completely, determine the number of solutions by examining the structure of the equation. 

14. $6a - 3 = 3(2a - 1)$ infinitely many solutions	15. $5x - 2 = 5x$ no solution	16. $8x - 2x + 4 = 6x - 1$ no solution
17. $5m + 2 = 3m - 8$ one solution	18. $2(3a - 12) = 3(2a - 8)$ infinitely many solutions	19. $\frac{3x-12}{3} = x + 4$ no solution
20. $\frac{2x+2}{4} = \frac{x+1}{2}$ infinitely many solutions	21. $x + \frac{1}{5} = \frac{x+1}{5}$ one solution	22. $\frac{x}{2} - 4 = \frac{1}{2}(x - 8)$ infinitely many solutions

23. Consider the expression $4a - 12$. Write 3 different expressions that if set equal to $4a - 12$ would result in the equation having infinite solutions.

Possible answers:

$$4(a - 3)$$

$$2(2a - 6)$$

$$2a + 2a - 12$$

24. Consider the expression $x + 1$. Write 3 different expressions that if set equal to $x + 1$ would result in the equation having no solution.

Possible answers:

$$x + 2$$

$$x$$

$$x - 7$$

25. Consider the expression $2x + 6$. Write 3 different expressions that if set equal to $2x + 6$ would result in the equation having one solution.

Possible answers:

$$5x$$

$$15$$

$$3x - 7$$

26. Determine whether the equation $7x = 5x$ has one solution, infinitely many solutions, or no solution. If it has one solution, determine what the solution is.

This equation has one solution $x = 0$. This may confuse some students as at first glance it may appear as though the equation has no solution. Talk them through how this equation is different than the ones that they have seen that have no solution.

1.2d Homework: Solving Multi-Step Linear Equations (the different solving outcomes)



Directions: Without solving completely, determine the number of solutions of each of the equations.

1. $x - 211 = x$	2. $3(m - 3) = 3m - 9$	3. $5 - x = -x + 5$ infinitely many solutions
4. $-4m + 12 = 4m + 12$	5. $-3(x + 2) = -3x + 6$ no solution	6. $\frac{x-3}{5} = \frac{x}{5} - \frac{3}{5}$

Directions: Solve the following equations. If there is one solution, state what the solution is. Otherwise, state if there are infinitely many solutions or no solution. Show all your work.

7. $3x + 1 - 3(x - 1) = 4$	8. $3(a + 6) - 2(a - 6) = 6$ $a = -24$
9. $3(r - 4) = 3r - 4$	10. $2(x + 1) = 3x + 4$
11. $3 - (4b - 2) = 3 - 4b + 2$ infinitely many solutions	12. $3 - (4b - 2) = 3 - 4b - 2$ no solution

13. $2y - 5y + 6 = -(3y - 6)$	14. $f + 1 = 7f + 12 - 11 - 6f$
15. $12 + 8a = 6a - 6$	16. $\frac{1}{2}(6m - 10) = 3m - 5$

Directions: Fill in the blanks of the following equations to meet the criteria given. In some cases, there may be more than one correct answer.  

17. An equation that yields one solution: $8x + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}x + 10$

18. An equation that yields no solution: $8x + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}x + 10$

19. An equation that yields infinitely many solutions: $8x + 24 = \underline{\hspace{1cm}}(\underline{\hspace{1cm}}x + \underline{\hspace{1cm}})$

Directions: Create your own equations to meet the following criteria.

20. An equation that yields one solution of $x = 5$.

21. An equation that yields no solution.

22. An equation that yields infinitely many solutions.

23. **Challenge:** Can you think of an equation with two solutions?

1.2e Class Activity: Abstracting the Solving Process



In 8th grade, students are pushed to understanding concepts on an algebraic, abstract level. This lesson challenges students to abstract the solving process. This lesson requires that students are comfortable manipulating linear expressions and equations.

1. Solve the following equations for x . State the solving actions.

a. $x + 4 = 10$

$x = 6$ Subtract 4 from both sides.

b. $x + b = 10$ where b represents any number

$x = 10 - b$ Subtract b from both sides.

c. $x + b = c$ where b and c represent any number

$x = c - b$ Subtract b from both sides.

2. Solve the following equations for x . State the solving actions.

a. $2x = -16$

$x = -8$ Divide both sides by 2.

b. $ax = -16$ where a represents any number not equal to zero

$x = \frac{-16}{a}$ Divide both sides by a .

c. $ax = c$ where a and c represent any number not equal to zero

$x = \frac{c}{a}$ Divide both sides by a .

3. Solve each of the following equations for x . State the solving actions.

a. $3x + 4 = 19$

$3x = 15$ Subtract 4 from both sides.

$x = 5$ Divide both sides by 3.

- b. Rewrite the equation in part a. by replacing the 4 in the equation with b which represents any number, and the 3 in the equation with a which represents any number, and the 19 with c which represents any number. Solve your equation, stating the solving actions.

$ax + b = c$

$ax = c - b$ Subtract b from both sides.

$x = \frac{c-b}{a}$ Divide both sides by a .

4. Hugo and Maggie both solved the following equation for x .

$$3(x + 2) = 12$$

Hugo's Method: $3(x + 2) = 12$ $3x + 6 = 12$ Distribute the 3 $3x = 6$ Subtract 6 from both sides $x = 2$ Divide both sides by 3	Maggie's Method: $3(x + 2) = 12$ $x + 2 = 4$ Divide both sides by 3 $x = 2$ Subtract 2 from both sides
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- a. Examine the solutions. Did both people solve the equation correctly?



Yes

- b. The equations below are a mirror of the equations above; however the numbers 2, 3, and 12 in the original equation have been replaced with p , q , and r which represent any number. Solve the equations below for x , using both Hugo and Maggie's methods.

Hugo's Method: $p(x + q) = r$ $px + pq = r$ Distribute the p $px = r - pq$ Subtract pq from both sides $x = \frac{r-pq}{p}$ Divide both sides by p	Maggie's Method: $p(x + q) = r$ $x + q = \frac{r}{p}$ Divide both sides by p $x = \frac{r}{p} - q$ Subtract q from both sides
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As an extension have students show that the two resulting expressions in Hugo and Maggie's methods above are equivalent.

5. Solve the following equations for x .

a. $2x + 3x = 15$

$5x = 15$ Combine like terms.

$x = 3$ Divide both sides by 5.

Alternative method of solving.

$x(2 + 3) = 15$ Factor out an x .

$x(5) = 15$ Simplify.

$x = 3$ Divide both sides by 5.

b. $ax + bx = 15$

$(a + b)x = 15$ Combine like terms (add coefficients in front of x)

$x = \frac{15}{(a+b)}$ Divide both sides by $(a + b)$

c. Can you think of a different way of solving $ax + bx = 15$ for x ?

$x(a + b) = 15$ Factor out an x .

$x = \frac{15}{(a+b)}$ Divide both sides by $(a + b)$

6. Solve the following equations for x if a , b , and c represent real numbers not equal to 0. If you get stuck, put actual numbers in for a , b , and c and think about how you would solve these equations and then apply that thinking to the equations below.

a. $ax = bx + c$
 $x = \frac{c}{(a - b)}$

b. $\frac{ax+b}{c} = 5$
 $x = \frac{5c - b}{a}$

1.2e Homework: Abstracting the Solving Process

1. Solve the following equations for x . State the solving actions.

a. $x - 6 = 8$

$x = 14$ Add 6 to both sides.

b. $x - b = 8$ where b represents any number

$x = 8 + b$ Add b to both sides.

c. $x - b = c$ where b and c represent any number

$x = c + b$ Add b to both sides.

2. Solve the following equations for x . State the solving actions.

a. $\frac{x}{4} = 20$

$x = 80$ Multiply both sides by 4.

b. $\frac{x}{d} = 20$ where d represents any number not equal to zero

$x = 20d$ Multiply both sides by d .

c. $\frac{x}{d} = c$ where c and d represent any number not equal to zero

$x = cd$ Multiply both sides by d .

3. Solve each of the following equations for x . State the solving actions.

a. $\frac{x}{3} + 5 = -1$

$\frac{x}{3} = -6$ Subtract 5 from both sides.

$x = -18$ Multiply both sides by 3.

b. $\frac{x}{a} + b = c$

$\frac{x}{a} = c - b$ Subtract b from both sides.

$x = \frac{c-b}{a}$ Divide both sides by a .

4. Solve the following equations for x where a, b, c, d, e , and f are real numbers not equal to 0. If you get stuck, put actual numbers in for a, b , and c and think about how you would solve these equations and then apply that thinking to the equations below.

<p>a. $bx + d = a$ $x = \frac{a - d}{b}$</p>	<p>b. $\frac{ax - c}{d} = b$ $x = \frac{bd + c}{a}$</p>
<p>c. $\frac{x}{ab} = c$ $x = abc$</p>	<p>d. $abx = c$ $x = \frac{c}{ab}$</p>
<p>e. $ex + fx - c = d$ $x = \frac{d + c}{e + f}$</p>	<p>f. $ax - c = -bx + d$ $x = \frac{d + c}{a + b}$</p>

1.2f Self-Assessment: Section 1.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Solve multi-step linear equations that have one solution, infinitely many solutions, or no solution.	I am struggling to solve most of the equations on the following page.	I can solve the equations in Set A on the following page.	I can solve all of the equations in Sets A and B on the following page.	I can solve all of the equations in Sets A, B, and C on the following page.
2. Understand what it is about the structure of a linear equation that results in equations with one solution, infinitely many solutions, or no solutions.	I don't understand what the different solving outcomes of a linear equation are.	I can tell whether an equation has one, no, or infinite solutions only after I have solved it but I sometimes get confused about the difference between no solution and infinite solutions.	I can tell whether an equation has one, no, or infinite solutions only after I have solved it.	I can look at an equation and without solving the equation entirely determine whether it will have one solution, no solution, or infinite solutions just by examining the structure of the equation.
3. Identify and provide examples of equations that have one solution, infinitely many solutions, or no solutions.	I don't know what it means for an equation to have one, no, or infinite solutions.	When given a list of equations, I can identify equations that have one, no, or infinite solutions but I have to draw a model or solve the equation in order to tell.	When given a list of equations, I can identify equations that have one, no, or infinite solutions without solving the equations.	When given a list of equations, I can determine whether the equations will have one, no, or infinite solutions. I can also generate my own equations that will result in one, no, or infinite solutions.

1. Solve the following equations.
2. Before solving each equation, examine the structure of the equation and determine whether it will have one, no, or infinite solutions.

Set A

1. $8g + 9 = 4g + 1$	2. $x + 1 = x - 2$	3. $-2x + 4x - 10 = -3x$
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Set B

4. $2a - 12 = 3(a - 6)$	5. $-5x + 2(x - 1) = -12 + 2x$	6. $5 - (2a - 3) = 5 - 2a + 3$
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Set C

7. $-2(x - 1) + 4x = 3(2x - 2)$	8. $\frac{1}{2}(6x - 8) + 2x = -x - 16$	9. $\frac{1}{4}(16x - 1) = 4x - \frac{1}{4}$
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