First published in 2013 by the University of Utah in association with the Utah State Office of Education.

Copyright © 2013, Utah State Office of Education. Some rights reserved.

This work is published under the Creative Commons Attribution License ("CC BY") which is available online at http://creativecommons.org/licenses/by/3.0/ and provides permissions for the reusing, revising, remixing, and redistributing of this work. This work is an *open educational resource* (OER).

Table of Contents

CHAPTER 1: LINEAR EQUATIONS IN ONE VARIABLE (4 WEEKS)	2
1.0 ANCHOR PROBLEM: CHOCOLATE	7
SECTION 1.1: CREATING AND SOLVING MULTI-STEP LINEAR EQUATIONS	8
1.1a Class Activity: Simplifying Linear Expressions	9
1.1a Homework: Simplifying Linear Expressions	13
1.1b Class Activity: Writing Linear Expressions to Model Real World Situations	16
1.1b Homework: Writing Linear Expressions to Model Real World Situations	19
1.1c Class Activity: Solving Multi-Step Linear Equations (combine like terms)	21
1.1c Homework: Solving Multi-Step Linear Equations (combine like terms)	25
1.1d Class Activity: Equations with Fractions	
1.1e Class Activity: Solving Multi-Step Linear Equations (distribute and combine like terms)	
1.1e Homework: Solving Multi-Step Linear Equations (distribute and combine like terms)	
1.1f Class Activity: Creating and Solving Linear Equations to Model Real World Problems Part I	
1.1f Homework: Creating and Solving Linear Equations to Model Real World Problems Part I	
1.1g Self-Assessment: Section 1.1	
SECTION 1.2: CREATING AND SOLVING MULTI-STEP LINEAR EQUATIONS WITH VARIABLES ON BOTH SIDES	51
1.2a Class Activity: Solving Multi-Step Linear Equations (variables on both sides)	
1.2a Homework: Solving Multi-Step Linear Equations (variables on both sides)	54
1.2b Class Activity: Solving Multi-Step Linear Equations (putting it all together)	55
1.2b Homework: Solving Multi-Step Linear Equations (putting it all together)	
1.2c Class Activity: Creating and Solving Linear Equations to Model Real World Problems Part II	60
1.2c Homework: Creating and Solving Linear Equations to Model Real World Problems Part II	63
1.2d Class Activity: Solving Multi-Step Linear Equations (the different solving outcomes)	66
1.2d Homework: Solving Multi-Step Linear Equations (the different solving outcomes)	
1.2e Class Activity: Abstracting the Solving Process	
1.2e Homework: Abstracting the Solving Process	75
1.2f Self-Assessment: Section 1.2	77

Chapter 1: Linear Equations in One Variable (4 weeks)

Utah Core Standard(s):

- Solve linear equations in one variable. (8.EE.7)
 - a) Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where *a* and *b* are different numbers).
 - b) Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Academic Vocabulary: linear expression, simplify, evaluate, linear equation, equivalent expression, solve, solution, inverse operations, like terms, distributive property, ratio, no solution, infinitely many solutions

Chapter Overview:

By the end of eighth grade, students should be able to solve any type of linear equation in one variable. This includes equations with rational number coefficients that require expanding expressions using the distributive property, collecting like terms, and equations with variables on both sides of the equal sign. The chapter utilizes algebra tiles as a tool students can use to create visual, concrete representations of equations. Students manipulate the tiles while simultaneously manipulating the abstract representation of the equation, thus gaining a better understanding of the algebraic processes involved in solving equations. The goal is that students transition from the concrete process of solving equations to the abstract process of solving equations. Students should understand and know the laws of algebra that allow them to simplify expressions and the properties of equality that allow them to transform a linear equation into its simplest form, thus revealing the solution if there is one. Applications are interwoven throughout the chapter in order that students realize the power of being able to create and solve linear equations to help them solve real-world problems. The ability to solve real-world problems by writing and solving linear equations gives purpose to the skills students are learning in this chapter.

Connections to Content:

Prior Knowledge

In previous coursework, students used properties of arithmetic to generate equivalent expressions, including those that required expansion using the distributive property. Students solved one- and two-step equations. Students have also solved real-life mathematical problems by creating and solving numerical and algebraic expressions and equations.

Future Knowledge

Later in this book, students will analyze and solve pairs of simultaneous linear equations. They will also create and solve linear equations in two-variables to solve real-world problems. A student's understanding of how to solve a linear equation using inverse operations sets the foundation for understanding how to solve simple quadratic equations later in this course and additional types of equations in subsequent coursework. Additionally, in subsequent coursework, students will be creating equations that describe numbers or relationships for additional types of equations (exponential, quadratic, rational, etc.)

MATHEMATICAL PRACTICE STANDARDS

	Make sense of problems and	for 5 days and her tot, she pay each day for 1 Uncle Hank has anoth the same number of n quarters as nickels. I 1 \$6.14. Whoever can s Ben has started the ec- equation that represent 0.01p	al bill for both children is her second child to attend her riddle for his nephewn nickels and pennies. I hav have 3 more dimes than a solve my riddle will get n quation for solving the ric nts the riddle.	s \$200, how much does 1 daycare? s. He tells them, "I have e 4 times as many quarters. I have a total of ny coins." Idle. Finish writing the
x - x	solving them.	pennie	S	
	-	How many of each type of coin does Uncle Hank have?		
		Students may use a va graphs) to solve these throughout the chapte analyze givens, const correspondences betv used to solve the prob problems can be solve relationships between model real-world situ	ariety of strategies (diagr e and other real-world pr er. Regardless of the stra raints, relationships, and ween the different approa blems. Ultimately, studen ed by creating linear equ n quantities. The ability to ations is a valuable tool	rams, equations, tables, roblems presented tegy used, students must goals and identify thes and representations ts will see that these fations that describe o create equations to
		forward.		jor sindenis going
		<i>forward.</i> Bianca ran three time The following is a mo	s farther than Susan. Tog	gether they ran 28 miles.
		<i>forward.</i> Bianca ran three time The following is a mo	es farther than Susan. Tog odel that represents this s 28	gether they ran 28 miles.
		<i>forward.</i> Bianca ran three time The following is a mo	es farther than Susan. Tog odel that represents this s 28	gether they ran 28 miles.
		forward. Bianca ran three time The following is a mo Susan's	es farther than Susan. Tog odel that represents this s 28 Bianca	gether they ran 28 miles. ituation.
D #	Reason abstractly and	forward. Bianca ran three time The following is a mo Susan's Distance	es farther than Susan. Tog odel that represents this s 28 Bianca Distan	gether they ran 28 miles. ituation.

Construct viable arguments and critique the reasoning of others.	Two students, Arthur and Oliver, each have some chocolates. They know that they have the same number of chocolates. Arthur has two tubs of chocolates, one bag of chocolates, and twenty-five loose chocolates. Oliver has two tubs of chocolates, two bags of chocolates, and seven loose chocolates. Determine the number of chocolates in a tub. Determine the number of chocolates in a bag. Include any pictures, models, or equations you used to solve the problem and clearly explain the strategy you used. Sam was asked to evaluate the expression $5x + 3x + 20$ for $x = 100$. Sam's work is shown below. What mistake did Sam make? Help Sam to answer the question correctly. Sam's Work: 5x + 3x + 20 = 100 8x = 80 x = 10 <i>Students may use a variety of strategies to solve the chocolate problem</i> <i>above. They must be able to justify their conclusions, communicate them</i> <i>to others, and respond to the arguments made by others. In the second</i> <i>example, students are asked to examine a problem that has been solved</i> <i>incorrectly, explain what the error is, and solve the problem correctly.</i> <i>This type of error analysis requires students to possess a clear</i> <i>understanding of the mathematical concepts and skills being studied.</i>
Model with mathematics.	A marble jar has twice as many blue marbles as red marbles, 16 more green marbles than blue marbles, and 10 fewer white marbles than red marbles. The jar has a total of 150 marbles. Use this information to answer the questions that follow. The following equation represents this situation. Match each piece of the equation to the appropriate marble color. Write your answer in the boxes provided. $\underbrace{m+2m+(2m+16)+(m-10)=150}$ Determine how many marbles of each color are in the jar.

		Josh works 40 hours a week as a nurse practitioner. He makes time and a half for every hour he works over 40 hours. Josh works 60 hours one
		is shown below.
		over-time pay rate
		(40) + 1.5p() = 2100
		hours worked at regular rate
		Fill in the blanks in the equation above so that it matches the story. What is Josh's regular hourly rate? What is Josh's overtime hourly rate? Throughout eighth grade, students will build linear models to represent real-world situations, moving fluently between the verbal representation, concrete models, and abstract or symbolic representation. These models map the relationships between quantities in a given situation, allow students to solve many real world problems, and help students to draw conclusions and make decisions in a given situation.
	Use appropriate	The following is a model of the equation $7x + 9 - 4x = 2(x + 5)$. Create this model with your tiles and solve the equation, showing your solving actions below.
Ē	strategically.	Concrete models are used throughout this chapter as a tool to assist students in becoming proficient in the abstract process of solving any type of linear equation. Once students have mastered how to solve a linear equation, this skill becomes a tool for accessing more advanced mathematical content.

		Find and Fix the Mistake: Ricardo solved the following equation		
		incorrectly. Circle the mistake and describe the mistake in words. Then,		
		solve the equation correctly.		
	A thornal to	2x + x + 5x = 56		
	Attend to	7x = 56 Combine like terms.		
ենքեր	precision.	x = 8 Divide both sides by 7.		
		When students analyze errors made by others, they must be clear in		
		their understanding of the content and skills being learned. Students		
		identify, explain, and correct common errors that are made when		
		solving equations.		
		Use the following equations to answer the questions that follow.		
		x+3 $x + 3 - 5$ $x + 3 - 5$ $(x+3) - 5$ $x + 3 - 5$		
		$\frac{1}{2} = 5$ $\frac{1}{2} + 3 = 5$ $\frac{1}{2}(x+3) = 5$ $\frac{1}{2} + \frac{1}{2} = 5$		
		Examine each of the equations above. Circle the equations that are		
		equivalent. Think about the structure of the expressions on the left side		
		of the equation. It may help to use your tiles and draw a model of each		
		equation.		
	Look for and			
	make use of	Consider the expression $4a - 12$. Write 3 different expressions that if		
Ľ	structure.	set equal to $4a - 12$ would result in the equation having infinite		
		solutions.		
		In order to solve equations, students must make sense of the structure of		
		the expressions in an equation. They must be able to view expressions as		
		a single object or as composed of several objects in order to determine		
		the solving actions and in order to operate on the expression correctly.		
		In order to determine whether two expressions are equivalent and		
		generate equivalent expressions, students must compose and decompose		
		expressions.		
		Directions: Write the story that goes with the expressions and equation		
		in each problem. Solve for the unknown information and interpret your		
		answer.		
		Number of weeks: w		
	Look for and	Sophie's Money: $300 - 40w$		
	express	Raphael's Money: $180 + 20w$		
_+'	regularity in	300 - 40w = 180 + 20w		
L	repeated	This chapter deals with linear expressions and equations. Linear		
	reasoning.	<i>functions grow at a constant rate. In the problem above, Sophie's money</i>		
	O [*]	is decreasing at a rate of \$40 per week while Raphael's is increasing at		
		a rate of \$20 per week. Students may realize that Raphael is closing the		
		gap in the amount of money each child has by \$60 a week and use this		
		reasoning to help determine how long it will take for both children to		
		have the same amount of money.		

1.0 Anchor Problem: Chocolate

💢 📲 💽 🏠

Directions: Consider the following situations. Then answer the questions below. Include any pictures, models, or equations you used to solve the problem and clearly explain the strategy you used.

Situation 1: Two students, Theo and Lance, each have some chocolates. They know that they have the same number of chocolates. Theo has four full bags of chocolates and five loose chocolates. Lance has two full bags of chocolates and twenty-nine loose chocolates.

Determine the number of chocolates in a bag. Determine the number of chocolates each child has.

Situation 2: Two students, Arthur and Oliver, each have some chocolates. They know that they have the same number of chocolates. Arthur has two tubs of chocolates, one bag of chocolates, and twenty-five loose chocolates. Oliver has two tubs of chocolates, two bags of chocolates, and seven loose chocolates.

Determine the number of chocolates in a tub. Determine the number of chocolates in a bag.

Situation 3: Two students, Abby and Amy have the same number of chocolates. Abby has one full tub of chocolates and 21 remaining chocolates. Amy has one full tub of chocolates and 17 remaining chocolates.

Determine the number of chocolates in a tub.

Section 1.1: Creating and Solving Multi-Step Linear Equations

Section Overview:

This section begins with a review of writing and simplifying algebraic expressions. Students review what an algebraic expression is and what it means to simplify an algebraic expression. They also evaluate algebraic expressions and create expressions to represent real-world situations. This work with expressions sets the foundation for the study of linear equations. Students learn what a linear equation is, what it means to solve a linear equation, and the different outcomes that may occur when solving an equation (one solution, no solution, and infinitely many solutions). Section one focuses on equations with one solution. Students solve linear equations whose solutions require collecting like terms. They then move to equations whose solution requires the use of the distributive property and collecting like terms. Applications that can be solved using the types of equations being studied in a lesson are interwoven throughout. Scaffolding has been provided in order to aide students in the process of creating equations to represent and solve real-world problems.

Concepts and Skills to Master:

By the end of this section, students should be able to:

- 1. Understand the meaning of linear expression and linear equation.
- 2. Solve multi-step linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
- 3. Write and simplify linear expressions and equations that model real-world problems.

1.1a Class Activity: Simplifying Linear Expressions

- 1. Emma is playing a popular video game and is determined to beat the high score. The game saves her place so that each time she plays it again, she picks up in the same place with the same number of points. Emma downloads the video game on Monday night and starts playing, scoring a bunch of points. On Tuesday, she scores an additional 500 points. On Wednesday she doubles her score from the previous day. On Thursday, she scores the same number of points that she scored on Monday.
 - a. Miguel's teacher asks him to write an expression that represents Emma's total score after she is done playing on Thursday. Miguel writes the following expression:

2(p + 500) + p

Miguel's teacher lets him know that his expression is correct. Write in words what each piece of Miguel's expression represents in the story problem.

b. Nevaeh writes the following expression to represent Emma's score on Thursday.

2p + 1000 + p

The teacher lets her know	that she is also	correct. How	v did Nevaeh re	present the problem
differently than Miguel?	•			

- c. Can you think of another expression to represent Emma's score on Thursday?
- d. If Emma scored 700 points on Monday, evaluate each of the three expressions above to determine how many points Emma has on Thursday.

In this lesson, we will study linear expressions. Miguel, Nevaeh, and you all wrote linear expressions to represent Emma's total number of points on Thursday. A **linear expression** is a mathematical phrase consisting of numbers, unknowns (symbols that represent numbers), and arithmetic operations. Linear expressions describe mathematical or real-world situations.

8WB1 - 9

The following are all examples of linear expressions:

- 3*x* 5
- 2x x 17
- 3*x*
- 6(2x-5)+11
- $\bullet \quad 3x + 7x 3 + 2x$
- 25

Below are **equivalent linear expressions** that represent Emma's score from the video game example. Equivalent expressions have the same meaning. Two expressions are considered **equivalent** when a substitution of any number for the unknown p in each of the expressions produces the same numerical result. Substituting in a specific number for the unknown in an expression and calculating the resulting value is called **evaluating** the expression.

2(p + 500) + p2p + 1000 + p3p + 1000

For ease of communicating mathematical ideas, we will consider a linear expression in the form Ax + B where A and B are numbers and x represents an unknown, the **simplified form of a linear expression**. In the example above, the simplified form of the expression is 3p + 1000.

In this lesson, we will be using tiles to model and simplify linear expressions.





Remember that a positive tile and a negative tile can be combined to create a zero pair or add to zero.



2. The following is a model of the expression 5x + (-3x) - 6 + 4



- a. Find zero pairs, and write the simplified form of this expression.
- b. Evaluate this expression when x = 8.

- 3. Using your tiles, model the expression -4x + 3 + 5x + (-1).
 - a. Find zero pairs and write the simplified form of this expression.
 - b. Evaluate this expression when x = -5.
- 4. The following is a model of the expression 3(x + 1).



- a. Write the simplified form of this expression.
- b. Evaluate this expression when x = -4.
- 5. Using your tiles, model the expression 2(2x 1).
 - a. Write the simplified form of this expression.
 - b. Evaluate this expression when x = 0.

Directions: Model and simplify each expression.

6.
$$2(x+2) - x$$

7. $3 - 3x + 4(x - 3)$

Directions: Simplify each expression. Use the tiles if you need to.

8. $-4x + 3 + 5(2x - 1)$	9. $12 - (x - 2) + 4x$

10. Write three expressions that are equivalent to 6x + 12. Use the tiles if you need to.

11. Write three expressions that are equivalent to 4x - 2. Use the tiles if you need to.

- 12. A group of friends goes to a movie on Friday night. Each friend purchases a movie ticket that costs \$8, a small popcorn that costs \$3.50, and a medium drink that costs \$2.25.
 - a. Circle the expression(s) below that represent the **total amount of money spent** by the group if f represents the number of friends that went to the movie. There may be more than one answer.

8 + 3.50 + 2.25 f(8 + 3.50 + 2.25) f + 8 + 3.50 + 225 f + 13.75 8f + 3.5f + 2.25f13.75f

b. If 5 friends go to the movie, how much money will **each person** spend? How much money will the **entire group** spend?

1.1a Homework: Simplifying Linear Expressions

1. The following is a model of the expression -6x + 2x - 5 + 2



- a. Find zero pairs, and write the simplified form of this expression.
- b. Evaluate this expression for x = 2.
- 2. Model the expression 5x + 2 x 4.
 - a. Simplify the expression modeled above.
 - b. Evaluate this expression for x = -3.
- 3. The following is a model of the expression 2(3x 1).



- a. Write the simplified form of this expression.
- b. Evaluate this expression for x = 4.

Directions: Simplify each expression. Use the tiles if needed.

4. $2(x+3) + 4x$	5. $4 + 2(x - 2) - 4$
6. $6(2x-4) - 3x$	7. $4 - (2x + 3) + 5x$
8. $-6x - 4 + 5x + 7$	9. $\frac{1}{2}(4x + 12) + 2x$

10. Write three expressions that are equivalent to 3x + 15.

- 11. Write three expressions that are equivalent to 2x + 4x 15.
- 12. Dan's basketball coaches have set up the following schedule for practice: 10 minutes warm-up and stretching; 15 minutes of defensive drills; 10 minutes of passing drills; 20 minutes of shooting practice, and *x* minutes of time to scrimmage.
 - a. Circle the expression(s) below that represent the amount of time Dan will practice **each week** if they practice **3 days a week**. There may be more than one answer.

10 + 15 + 10 + 20 + x 3(10 + 15 + 10 + 20 + x) 3(55 + x) 165 + x 165 + 3x168x

b. If Dan's team scrimmages for 35 minutes each practice, how long is **each** practice? How long do they practice **each week** (again assuming they practice 3 days a week)?

Find and Fix the Mistake: In each of the following problems, a common error has been made when

simplifying the expressions. Identify the mistake, explain it, and simplify the expression correctly.

13. $5x + 4x - x$	14. $2(x+5)$
9 <i>x</i>	2x + 5
Explain mistake:	Explain mistake:
Correct answer:	Correct answer:
15.5 - (x - 2)	163(2x - 5)
5 - x - 2	-6x - 15
3-x	Explain mistake:
Explain mistake:	
Correct answer:	Correct answer:
17.2x + 3 + 4x	18. x + 3x + 6x
9 <i>x</i>	$10x^{2}$
Explain mistake:	Explain mistake:
Correct answer:	Correct answer:

19. Evaluate the expression 2x + 4 for the following values of *x*.

a. <i>x</i> = 1	b. $x = -1$
	1
c. $x = 0$	d. $x = \frac{1}{2}$

20. Evaluate the expression -3x + 2 for the following values of *x*.

A	5
a. $x = 1$	b. $x = -1$
c. $x = 0$	d. $x = \frac{1}{3}$

1.1b Class Activity: Writing Linear Expressions to Model Real World Situations

- 1. Aria and her friends are playing a game. The expressions below represent the amount of money each player has at the end of the game where *m* is the amount of money a player started with.
 - a. Match each player to the correct expression.

Expressions			
A. $2(3m - 100)$	B. $\frac{m}{2} + 100 - 25$	C. 2 <i>m</i> – 100 – 25	
	2		
Player:	Player:	Player:	
D. $\frac{(m+100)}{2} - 25$	E. $2(m - 100) - 25$	F. $2(\frac{m}{2}-100)$	
2		3	
Player:	Player:	Player:	

Player Scenarios				
Peta collects the following cards:	Miya collects the following cards:			
 You are doing a great job at work and just received a bonus. Double the amount of money you currently have. It's time to pay property taxes. Pay the bank \$100. It's the first day of school. Pay the store \$25 for school supplies. 	 Congratulations! Your art submission won first place. Collect \$100. Your house was damaged in a flood. Pay half of your remaining money for repairs. You got a parking ticket. Pay \$25. 			
Aria collects the following cards:	Hadley collects the following cards:			
 It's time to pay property taxes. Pay the bank \$100. You are doing a great job at work and just received a bonus. Double the amount of money you currently have. It's the first day of school. Pay the school \$25 for supplies 	 You won the lottery! Triple the amount of money your currently have. It's time to pay property taxes. Pay the bank \$100. You are doing a great job at work and just received a bonus. Double the amount of money you currently have. 			
Lea collects the following cards:	Sierra collects the following cards:			
 Your house was damaged in a flood. Pay half of your remaining money for repairs. Congratulations! Your art submission won first place. Collect \$100. You got a parking ticket. Pay \$25. 	 Your two sisters want to join the game. Divide the money you currently have between the three of you. It's time to pay property taxes. Pay the bank \$100. You are doing a great job at work and just received a bonus. Double the amount of money you currently have. 			

b. If each player started the game with \$1,000, who won the game?

Directions: For #2 - 4, circle the expression(s) that correctly model each situation. There may be more than one answer.

- 2. Tim took his friends to the movies. He started with \$40 and bought 3 movie tickets that each cost x dollars. He also bought one tub of popcorn that cost \$5.75.
 - a. Which of the following expression(s) represent the amount of money Tim has left?

40 - x - x - x - 5.75	40 - 3x - 5.75
40 - 3 - 5.75	-3x + 34.25
34.25 - 3x	31.25 <i>x</i>

- b. If each movie ticket costs \$6, how much money does Tim have left?
- 3. Master Tickets charges \$35 for each concert ticket, plus an additional \$2 service fee for each ticket purchased. Kanye purchased *x* concert tickets.
 - a. Which of the following expression(s) represent the amount of money Kanye spent?

$$35x + 2$$
 $37x$
 $35x + 2x$ $x(35 + 2)$

- b. If Kanye purchased 4 concert tickets, how much did he spend?
- 4. Sara bought 3 baby outfits that $\cot p$ dollars each and one bottle of baby lotion. The baby lotion $\cot 2$ dollars less than an outfit. The following is a model of this situation.



a. Which of the following expression(s) represent the amount of money Sara spent?

$$p + p + p - 2 \qquad \qquad 3p + (p - 2)$$

$$p + p + p + (p - 2)$$
 $4p - 2$
 $3p - 2$ $2p$

b. If each baby outfit costs \$5, how much did Sara spend?

- 5. Antony and his friends went to a fast food restaurant for lunch. They ordered and ate 3 Big Micks, 2 Mick-Chicken Sandwiches, and 4 large fries. A Mick-Chicken Sandwich has 200 fewer calories than a Big Mick. A large fry has 50 fewer calories than a Big Mick.
 - a. Part of an expression that represents the total number of calories consumed by Antony and his friends is shown below. Fill in the remaining pieces of the expression on the lines provided and simplify the expression.



b. Write the simplified form of the expression.

c. If a Big Mick has 550 calories in it, how many calories did Antony and his friends consume?

1.1b Homework: Writing Linear Expressions to Model Real World Situations

1. Mateo sends approximately twice as many text messages as his mom each month. His dad sends approximately 500 fewer text messages than Mateo each month. The following expression represents the total number of text messages sent by the family each month:

$$t + \frac{t}{2} + (t - 500)$$

a. Write in words what each piece of the expression represents in the story.

b. If Mateo sends approximately 3,000 texts per month, approximately how many texts does his entire family send each month (assuming he has no other family members)?

Directions: For #2 - 3, circle the expression(s) that correctly model the situation. There may be more than one answer.

- 2. Christina is purchasing one of each of the following for her nieces for Easter: a pack of sidewalk chalk that costs \$2.25, a bottle of bubbles that costs \$1, and a chocolate bunny that costs \$3.50.
 - a. Which of the following expression(s) represent the amount of money Christina will spend if she has *n* nieces?

6.75 + n	n(2.25 + 1 + 3.5)
6.75 <i>n</i>	2.25n + n + 3.5n
6.75	n(2.25n + 1n + 3.5n)

- b. If Christina has 5 nieces, how much money will she spend on gifts for Easter?
- 3. At Six East Shoe Store, a pair of boots is \$30 more than a pair of sandals. A pair of sandals costs *s* dollars each. Emily purchased 2 pairs of boots and 3 pairs of sandals.
 - a. Which of the following expression(s) represent the amount of money Emily spent?

$$2(s + 30) + 3s \qquad (s + 30) + (s + 30) + (s + 30) + s + s$$
$$2s + 3(s + 30) \qquad (s + 30) + (s + 30) + s + s + s$$
$$5s + 90 \qquad 5s + 60$$

b. If a pair of sandals is \$15, how much did Emily spend in all?

Directions: Write a linear expression in simplified form that represents each of the following situations. For these problems, simplified form is Ax + B where A and B are numbers and x represents an unknown.

4. Drew and Raj are both training for a bike race. Raj bikes 10 miles less than Drew each day that they train. The following is a model of this situation.



- a. Write an expression **in simplified form** that represents the number of miles Drew bikes if Raj bikes *m* miles each day.
- b. Write an expression **in simplified form** that represents the total number of miles Raj and Drew bike if they each train 5 days a week and *m* represents the number of miles Raj bikes each day.
- c. If Raj bikes 15 miles each day, how many miles do Raj and Drew bike together each week (again assuming they train 5 days a week)?
- 5. Naja is paid p dollars per hour she works. For every hour she works over 40 hours, she is paid time and a half which means she is paid 1.5 times her normal hourly rate. She worked 50 hours last week. The following is a model of this situation.
 - a. Irene tried to write an expression that represents the amount Naja earned last week but needs your help. Help Irene finish the expression by filling in the blanks.

over-time
pay rate
$$40(__) + __(1.5p)$$

hours worked
at regular rate

- b. Simplify the completed expression above.
- c. If Naja's regular hourly rate is \$30 per hour, how much did she earn last week?

1.1c Class Activity: Solving Multi-Step Linear Equations (combine like terms)

In the lessons up to this point, we have been working with linear expressions. We reviewed how to **simplify** a linear expression and how to **evaluate** a linear expression for a given value of x. We will now begin our work with linear equations. When we **solve** a linear equation, our task is to find the values of the unknown that make the equation true.

1. Damion and his friends went trick-or-treating. The next day, they got together and counted their candy. Damion had twice as much candy as Nick. Bo had 10 more pieces than Damion. The following model and expression represent the amount of candy the boys have together:



a. Show on the model and the expression which pieces represent the amount of candy each of the boys has.

What if we also knew that together the boys have 230 pieces of candy? Let's look at a model for this:



c + 2c + (2c + 10) and 230 are both linear expressions that represent the amount of candy the boys have together. When we set two linear expressions equal to each other, we create a linear equation. A **linear** equation is an assertion or statement that two linear expressions are equal to each other. Using the candy example, we can create the following equation:



- b. Model this equation with your tiles and solve for c.
- c. What does *c* represent in the context?
- d. How many pieces of candy do each of the boys have?

A solution to an equation is a number that <u>makes the equation true</u> when substituted for the unknown. In the example above, the solution is 44. Verify that when you substitute 44 in for c the equation is true.

It is important to note that when we create an equation, the two expressions on either side of the equal sign might be true for 1) **one value of** x (as we saw in the candy example above), 2) **no values of** x (there is not a number that can be substituted for the variable to make the equation true), or 3) **all values of** x (every number we substitute in for the variable will make the equation true). In the first section, we will study equations that have one solution.

2. The following is a model of the equation 5x - 8 - 2x = 4. Create this model with your tiles and solve

the equation, showing the solving actions (steps) below.



a. Solving Actions (show each step below): 5x - 8 - 2x = 4

b. Verify the solution in the space below.

Directions: Model and solve the following equations. Show the solving actions and verify your solution.

3. $4x + 3x - 1 = 6$	4. $10 = -x + 3x + 4$
5. $2x - x + 4 = -8$	6. $10 = -2x - 3 + 4x + 5$

7. The following is a model of an equation.



- a. Write the symbolic representation (equation) for this model.
- b. Solve the equation.

Directions: Solve the following equations.

8. $-7x + 5x + 3 = -9$	9. $17 = m + 5 - 3m$
$10.\ 0.5b + 2b = -50$	11. $-\frac{2}{3} = -\frac{4}{3} + 6r$
	5 5

12. Find and Fix the Mistake: Ricardo solved the following equation incorrectly. Circle the mistake and

describe the mistake in words. Then, solve the equation correctly.

2x + x + 5x = 56

- 7x = 56 Combine like terms.
- x = 8 Divide both sides by 7.

13. Carson and his family drove to Disneyland. They started driving on Thursday and then stopped for the night. On Friday, they drove twice as many miles as they had on Thursday. On Saturday, they drove fifty miles more than they had on Friday. Carson's mom asked him to write an equation to determine

how many miles they drove each day.

Carson wrote the following equation: m + 2m + (2m + 50) = 650

a. Match each expression with what it represents in the story.

m	The number of miles driven on Friday
2m	The number of miles driven on Saturday
(2m + 50)	The total number of miles driven.
m + 2m + (2m + 50)	The number of miles driven on Thursday.

b. If Carson and his family live 650 miles from Disneyland, how many miles did Carson's family drive each day?

Thursday: _____

Friday: _____ Saturday: _____

14. George started writing a story that matches the expressions and equation shown on the left. Pieces of the story are missing. Help him finish the story, solve the equation, and determine each person's age.



<u>Ages</u> Talen's age: tPeter's age: 8t + 3t + (8t + 3) = 39

Story

I am trying to figure out Peter and Talen's

ages. Peter tells me that he is three more

than...

Together, Talen and Peter's ages ...

How old are Talen and Peter?

Talen's age: _____

Peter's age: _____

1.1c Homework: Solving Multi-Step Linear Equations (combine like terms)

Directions:	Solve	the foll	owing e	equations.	Verify	your solutions
-------------	-------	----------	---------	------------	--------	----------------

1. $7x - 3x = 24$ 2. $6a + 5a = -11$ 3. $-4n - 2n = 6$ 4. $5a + 8 + (-2a) = -7$ 5. $18 = 3x + 4 + 2$ 6. $-2x - 7 - 4x = 17$ 7. $s + 3s = -24$ 8. $-5n + 4n - 7 = 8$ 9. $x - 4 + 3 = -6$ 10. $6 = 4t - 3t - 2$ 11. $2x - 9x + 17 = -4$ 12. $x + 3x + 4x = 56$ 13. $8x + 25 - 6x = 35$ 14. $-32 = -4x - 2x + 4$ 15. $1.3b - 0.7b = 12$ 16. $0.4y + 0.1y = -2.5$ $17.\frac{2}{5}x - \frac{1}{5}x = 9$ $18.r - \frac{5}{4} + \frac{1}{2}r = \frac{13}{4}$ 19. $3x + 5 + 6x - 7 = 25$ 20. $7d - 12 + 3d = 15$ 21. $22 = 5c + 3c - c + 8$	-	in course source the romo wing equa	tions. Venify your solutions.	
4. $5a + 8 + (-2a) = -7$ 5. $18 = 3x + 4 + 2$ 6. $-2x - 7 - 4x = 17$ 7. $s + 3s = -24$ 8. $-5n + 4n - 7 = 8$ 9. $x - 4 + 3 = -6$ 10. $6 = 4t - 3t - 2$ 11. $2x - 9x + 17 = -4$ 12. $x + 3x + 4x = 56$ 13. $8x + 25 - 6x = 35$ 14. $-32 = -4x - 2x + 4$ 15. $1.3b - 0.7b = 12$ 16. $0.4y + 0.1y = -2.5$ 17. $\frac{2}{5}x - \frac{1}{5}x = 9$ 18. $r - \frac{5}{4} + \frac{1}{2}r = \frac{13}{4}$ 19. $3x + 5 + 6x - 7 = 25$ 20. $7d - 12 + 3d = 15$ 21. $22 = 5c + 3c - c + 8$		1. $7x - 3x = 24$	2. $6a + 5a = -11$	3. $-4n - 2n = 6$
7. $s + 3s = -24$ 8. $-5n + 4n - 7 = 8$ 9. $x - 4 + 3 = -6$ 10. $6 = 4t - 3t - 2$ 11. $2x - 9x + 17 = -4$ 12. $x + 3x + 4x = 56$ 13. $8x + 25 - 6x = 35$ 14. $-32 = -4x - 2x + 4$ 15. $1.3b - 0.7b = 12$ 16. $0.4y + 0.1y = -2.5$ $17. \frac{2}{5}x - \frac{1}{5}x = 9$ $18. r - \frac{5}{4} + \frac{1}{2}r = \frac{13}{4}$ 19. $3x + 5 + 6x - 7 = 25$ $20. 7d - 12 + 3d = 15$ $21. 22 = 5c + 3c - c + 8$		4. $5a + 8 + (-2a) = -7$	5. $18 = 3x + 4 + 2$	6. $-2x - 7 - 4x = 17$
10. $6 = 4t - 3t - 2$ 11. $2x - 9x + 17 = -4$ 12. $x + 3x + 4x = 56$ 13. $8x + 25 - 6x = 35$ 14. $-32 = -4x - 2x + 4$ 15. $1.3b - 0.7b = 12$ 16. $0.4y + 0.1y = -2.5$ $17. \frac{2}{5}x - \frac{1}{5}x = 9$ $18. r - \frac{5}{4} + \frac{1}{2}r = \frac{13}{4}$ 19. $3x + 5 + 6x - 7 = 25$ $20. 7d - 12 + 3d = 15$ $21. 22 = 5c + 3c - c + 8$		7. $s + 3s = -24$	8. $-5n + 4n - 7 = 8$	9. $x - 4 + 3 = -6$
13. $8x + 25 - 6x = 35$ 14. $-32 = -4x - 2x + 4$ 15. $1.3b - 0.7b = 12$ 16. $0.4y + 0.1y = -2.5$ $17. \frac{2}{5}x - \frac{1}{5}x = 9$ $18. r - \frac{5}{4} + \frac{1}{2}r = \frac{13}{4}$ 19. $3x + 5 + 6x - 7 = 25$ 20. $7d - 12 + 3d = 15$ 21. $22 = 5c + 3c - c + 8$		10.6 = 4t - 3t - 2	$11.\ 2x - 9x + 17 = -4$	12. $x + 3x + 4x = 56$
16. $0.4y + 0.1y = -2.5$ $17.\frac{2}{5}x - \frac{1}{5}x = 9$ $18. r - \frac{5}{4} + \frac{1}{2}r = \frac{13}{4}$ 19. $3x + 5 + 6x - 7 = 25$ 20. $7d - 12 + 3d = 15$ $21. 22 = 5c + 3c - c + 8$		$13.\ 8x + 25 - 6x = 35$	1432 = -4x - 2x + 4	$15.\ 1.3b - 0.7b = 12$
19. $3x + 5 + 6x - 7 = 25$ 20. $7d - 12 + 3d = 15$ 21. $22 = 5c + 3c - c + 8$		$16.\ 0.4y + 0.1y = -2.5$	$17.\frac{2}{5}x - \frac{1}{5}x = 9$	$18. r - \frac{5}{4} + \frac{1}{2}r = \frac{13}{4}$
		$1\overline{9}.\ 3x + 5 + 6x - 7 = 25$	20.7d - 12 + 3d = 15	$2\overline{1.22} = 5c + 3c - c + 8$

Find and Fix the Mistake: In #22 - 23, a common mistake has been made. Circle the mistake and describe the mistake in words. Then, solve the equation correctly.

$22.\ 2x + 4x - 2 = 20$		23.	3x - 8x - 5 =	10
6x - 2 = 20	Combine like terms $(2x \text{ and } 4x)$		5x - 5 = 10	Combine like terms $(3x \text{ and } -8x)$.
4x = 20	Combine like terms ($6x$ and -2)		5r - 15	Add 5 to both sides
x = 5	Divide by 4.		5x - 15	Add 5 to both sides.
Explanation of mis	take:		x = 3	Divide both sides by 5.
•		Expla	nation of mistal	ke:
Solve correctly:		Solve	correctly.	

24. Bianca ran three times farther than Susan. Together they ran 28 miles. The following is a model that represents this situation.



b. How far did each girl run?

Susan: _____ Bianca: _____

25. Write a story that matches the expressions and equation shown on the left. Then solve the equation and find each person's age.

Ages Felipe's age: f Felipe's sister's age: $f - 6$	Sta	ry		
Felipe's mom's age: $3f - 9$	9			
f + (f - 6) + (3f - 9) =	60			
Felipe's age:	Felipe's sister's a	ge:	Felipe's mom's age:	

Hint: When writing stories like the on above, be sure to relate the unknown to its meaning in the story. Don't use the actual unknown in the story. For example, in the situation above, do not write "Felipe's sister is f minus 6 years old." Think about how Felipe's sister's age is related to Felipe's age which is represented by f in the expression. Also, remember that your story needs a question.

1.1d Class Activity: Equations with Fractions

1. Use the following equations to answer the questions that follow.

$$\frac{x+3}{2} = 5 \qquad \qquad \frac{x}{2} + 3 = 5 \qquad \qquad \frac{1}{2}(x+3) = 5 \qquad \qquad \frac{x}{2} + \frac{3}{2} = 5$$

- a. Examine each of the equations above. Circle the equations that are equivalent. Think about the structure of the expressions on the left side of the equation. It may help to use your tiles and draw a model of each equation.
- b. Solve each of the equations in the space above. Did you find that some equations were easier to solve than others? Why or why not?

When faced with an equation with fractions, we can transform it into an equation that does not contain fractions. This is called **clearing of fractions**. In the problems above, in order to clear the fractions, we need to get rid of the 2 in the denominator of each equation.

c. Can you think of a way to eliminate the 2 in each equation before you start to solve the equation? Test your method and re-solve each of the equations above.

Directions: Solve each equation by first clearing the fractions.

2. $\frac{x-1}{4} = 6$	3. $\frac{x}{2} + \frac{1}{4} = \frac{7}{4}$	$4. \ \frac{1}{3}(2x-4) = -6$
5. $\frac{2x}{3} + 4 = \frac{14}{3}$	6. $\frac{4}{5} = 3x + \frac{1}{5}$	7. $\frac{-2x-5}{3} = 3$
$8. \ \frac{x}{12} + \frac{1}{3} = \frac{1}{4}$	9. $-6 = \frac{(3x-12)}{3}$	$10.\frac{1}{2}(4x+12) = 2$

1.1e Class Activity: Solving Multi-Step Linear Equations (distribute and combine like terms)

1. The following is a model of the equation 3(x + 1) = 12. Create this model with your tiles and solve the equation, showing the solving actions below.



Solving Actions: 3(x + 1) = 12

Directions: Model and solve the following equations.

2. $2(x+5) = 14$	$3. \ 2(3x+1) - 2x = 10$	4. $-12 = 3(x - 2)$

5. The following is a model of an equation.



- a. Write the symbolic representation for this model.
- b. Solve the equation.

Directions:	Solve	the fo	llowing	equations	without	the use	of the tiles.
0 0 0 00 0							

8 1	
6. $-2(x+1) = 8$	7. $13 = -3(x - 4) - 8$
8 5 + 2(3a - 1) = 15	$0^{-1}(2t+4) = 0$
0.012(5u - 1) = 15	9. $\frac{1}{2}(2l+4) = -8$

$10.\frac{x}{3} + \frac{x-2}{5} = 6$	11. 14 = 5 - 3(x - 2)

12. Part of a story that matches the expressions and equation shown on the left has been written for you. Finish the story, solve the equation, and determine how much time Theo spends training in each sport.

<u>Triatholon Training Schedule</u> Minutes spent swimming: <i>x</i>	Story
Minutes spent running: $2x$ Minutes spent biking: $2x + 30$	Theo is training for a triathlon. He runs
3x + 4(2x) + 2(2x + 30) = 510 min.	twice as long as he swims. He bikes
	He swims three times a week, runs four times
	a week, and bikes
	If he spends a total of 510 minutes per week
	training, how many minutes does he spend
	on each exercise at a time?

 Minutes spent swimming:
 Minutes spent running:
 Minutes spent biking:

13. Write a story that matches the expressions and equation shown on the left. Then, solve the equation and determine how much each ride at the fair costs.

A Trip to the Fair	Story
Cost of a pony ride: <i>b</i>	
Cost to ride the Ferris wheel: $\frac{1}{2}b$	
Cost to bungee jump: $2b + 5^{2}$	
$3b + 4\left(\frac{1}{2}b\right) + (2b + 5) = \33	
(2)	
Cost of a pony ride:	Cost to ride the Ferris wheel: Cost to bungee jump:
	8WB1 - 32 ©2014 University of Utah Middle School Math Project in partnership with the Utah State Office of Education. Licensed under Creative Commons, cc-by.

14. Solve this riddle. "Consider the numbers 3, 8, and 7. Find a fourth number so that the average of the numbers is 7." The following equation represents this situation.



- a. Fill in the boxes above telling what each piece of the equation represents.
- b. Solve the equation and find the fourth number.

1.1e Homework: Solving Multi-Step Linear Equations (distribute and combine like terms)

1. The following is a model of an equation.



- a. Write the symbolic representation (equation) for this model.
- b. Solve the equation.

2. The following is a model of an equation.



- a. Write the symbolic representation for this model.
- b. Solve the equation.
Directions: Solve the following equations. Verify your solutions.

Silve the following equa	dons. venny your solutions.	
3. $3(4x - 2) = 30$	4. $-24 = 4(2 + 2x)$	5. $-16 = 2(4x + 8)$
6. $3(x + 10) + 5 = 11$	7. $3t - 2 + t - 5t = -1$	824 = 2(1 - 5x) + 4
9. $-2(a+3) + 4a = 18$	$10.\ 28 = 5x + 3(x + 4)$	11.4x - 3(x - 2) = 21
$12.\ 0 = -2(x+5) + 3x$	$13.\frac{1}{3}(x+6) = 1$	14.5 - 4(2b - 5) + 3b = 15
15. $10 = 3(x-2) - 2(5x-1)$	$16.\ 0.2(\overline{10t-4}) - t = 1.2$	$17\frac{1}{7}(x-7) + 22 = 26$
$1810 = \frac{3x}{4} + \frac{x}{2}$	$19. \frac{x-2}{3} = \frac{1}{2}$	20(x-5) + 2 - x = 3

Find and Fix the Mistake: In the following problems, a common mistake has been made. Circle the mistake and describe the mistake in words. Then, solve the equation correctly.

21.7 + 2(3x + 4) = 3	222(y-3) + 5 = 3		
7 + 6x + 4 = 3 Distribute the 2.	-2y - 6 + 5 = 3 Distribute the -2 .		
11 + 6x = 3 Combine like terms (7 and 4).	-2y - 1 = 3 Combine like terms (-6 and 5)		
6x = -8 Subtract 11 from both sides.	-2y = 4 Add 1 to both sides.		
$x = -\frac{8}{6}$ Divide both sides by 6.	y = -2 Divide both sides by -2 .		
$x = -\frac{4}{3}$ Simplify the fraction.	Explanation of Mistake:		
Explanation of Mistake			
	Solve Correctly:		
Solve Correctly:			
22 = (2x - 7) = 14	24 1 (47) 44		
25.5 - (2x - 7) = 14	$24. \frac{1}{3}(x+15) = 11$		
5 - 2x - 7 = 14 Distribute the negative sign.	$\frac{1}{3}x + 5 = 11$ Distribute the $\frac{1}{3}$.		
-2 - 2x = 14 Combine like terms (5 and -7).	5 5		
	$\frac{1}{3}x = 6$ Subtract 5 from both sides.		
-2x = 16 Add 2 to both sides.			
x = -8 Divide both sides by -2.	x = 2 Divide both sides by 3.		
Explanation of Mistake:	Explanation of Mistake:		
Solve Correctly:	Solve Correctly:		

25. The expressions below show the grams of fat in sandwiches at a popular fast food restaurant. Use these expressions and the equation to write a story and determine the number of grams of fat in each sandwich.

Fast Food Calories	Story
Crispy Chicken: f Single burger with cheese: $f + 11$	
Double burger with cheese: $2f$ 2f + 2(f + 11) + 4(2f) = 204 α	
3j + 2(j + 11) + 4(2j) = 204 g.	
Fat grams in a Crispy Chicken:	Fat grams in a single burger with cheese:
Fat grams in a double burger with chee	ese:

26. Mia has taken two quizzes in math so far this quarter and scored a 75% on the first and an 82% on the second. What must she score on the third quiz in order to have an average of 80% on her three quizzes?

1.1f Class Activity: Creating and Solving Linear Equations to Model Real World Problems Part I

1. Use the story below about Chloe and her friends to answer the questions that follow.

Going on a Picnic	Story
Cost of a sandwich:	Chilos, and have briandy and asing on a mining
Cost of a bag of chips:	A can durish is 6 times the south of a southing A
Cost of a cookie: r	A surface is b times the cost of a cooke. A
$cost of a cookie. \{x}_{cookie}$	of a cookie. A code is twice the cost
Cost of a soda:	of a cookie. A sour is twice the cost of a
	<u>cookce</u>

- a. Write expressions for the cost of each item on the lines provided above if the cost of a cookie is x.
- b. Chloe and her friends buy 2 sandwiches, 3 bags of chips, 4 cookies, and 2 sodas. They spend a total of \$12.25. Use this information and the expressions you wrote above to write an equation representing this situation.
- c. Solve your equation to determine the cost of each item.

Sandwich: _____ Bag of chips: _____ Cookie: _____ Soda: _____

- 2. Uncle Hank loves riddles. Uncle Hank tells his nephews, "I have twice as many dimes as quarters. I have 12 more nickels than quarters. I have \$4.60 total. Whoever can solve my riddle will get my coins."
 - a. Owen has a good start on an equation for solving this riddle. Help Owen fill in the missing pieces of the equation on the lines below.

 $0.25q + _(2q) + 0.05 _ = 4.60$

b. How many of each type of coin does Uncle Hank have?

of quarters: _____

of dimes: _____

of nickels: _____

8WB1 - 38

@2014 University of Utah Middle School Math Project in partnership with the Utah State Office of Education. Licensed under Creative Commons, cc-by.

3. Use the story below about Farmer Ted and his animals to answer the questions that follow.

Farmer Ted's Animals	Story		
Weight of a cow:c	Farmer Ted is weighing his animals. He		
Weight of a horse:	knows that a pig weighs approximately $\frac{1}{4}$ as		
Weight of a sheep:	much as a cow. He also knows that a		
Waight of a right	Clydesdale horse weighs about twice what a		
Weight of a pig:	cow weighs. A sheep weighs approximately		
	100 pounds less than a pig.		

- a. On the lines above to the left, write the expressions that match the weight of each of the animals if a cow weighs *c* pounds.
- b. Write an expression for the weight of one cow, one horse, one sheep, and one pig.
- c. If Farmer Ted puts 3 cows, 2 Clydesdale horses, 4 sheep and 1 pig on a giant scale used for weighing semi-trucks, the scale reads 7,850 pounds. Approximately how much does each animal weigh?

Cow: Clydesdale horse: Sheep: Pig:	Pig:
------------------------------------	------

4. Miley is trying to solve the following riddle: "The sum of three consecutive integers is 84. What are the integers?" She writes part of an equation that can be used to solve this riddle.

second
integer
$$n + (n + 1) + (___) = 84$$

first integer

- a. Help Miley complete the equation above by filling in the blank.
- b. Find the three integers.

5. Eli is making lemonade for a party. Expressions showing the ratio of water to sugar to lemon juice used to make lemonade are shown on the left.

Making Lemonade	Story			
Cups of sugar: $\frac{1}{4}c$				
Cups of lemon juice: $\frac{1}{2}c$				
$c + \frac{1}{4}c + \frac{1}{2}c = 14$ cups				
Ŧ Z				
a. Write a story tha	t matches the expressions and e	quation shown on the left.		
b. Solve the equation	on above. How many cups of ea	ch ingredient is Eli planning to use?		
Cups of water:	Cups of sugar:	Cups of lemon juice:		
6. Use the incomplete story	v and the expressions and equati	on below to answer the questions that follow.		
$\frac{\text{Triangles}}{m \angle A: x}$	Story	-		
$m \angle B: 3x$ $m \angle C: x - 20$	In $\triangle ABC$, the mea	sure of $\angle B$ is three times		
$x + 3x + (x - 20) = 180^{\circ}$	larger than	Larger than		
	The measure of 2	The measure of $\angle C$ is 20° less than		
	The sum of the a	ngles in a triangle is		
	What is the mea	wre of each angle in the		
	triangle?			

- a. Finish the story above so that it matches the expressions and equation shown on the left.
- b. What is the measure of each angle in the triangle?

 $m \angle B =$ _____

 $m \angle A =$ _____

m∠C = _____

8WB1 - 40

7. In $\triangle RST$, $\angle R$ and $\angle S$ have the same measure. The measure of $\angle T$ is $\frac{1}{2}$ the measure of $\angle R$ and $\angle S$. Marie drew the following model and picture to represent this situation: T



- a. Help Marie write an equation that represents the sum of the angles in ΔRST . Remember the sum of the angles in a triangle is 180°.
- b. Solve the equation and find the measure of each angle.

<i>m∠R</i> :	<i>m∠S</i> :	<i>m∠T</i> :

8. Use the expressions and equation below to answer the questions that follow.

<u>Rectangles</u> Width of a rectangle: w	Story
Length of a rectangle: 2w	
2w + w + 2w + w = 42 ft.	

- a. Draw a picture of the rectangle that accurately depicts the ratio of the length of the rectangle to its width.
- b. Write a story in the space provided that matches the expressions and equation.
- c. What is a different but equivalent way of writing the equation above?
- d. Solve the equation and find the length and width of the rectangle.

Length: _____

Width: _____

9. Josh works 40 hours a week as a nurse practitioner. He makes time and a half for every hour he works over 40 hours. Josh works 60 hours one week and earns \$2100. Part of an equation that represents this situation is shown below.



- a. Fill in the blanks in the equation above so that it matches the story.
- b. What is Josh's regular hourly rate?
- c. What is Josh's overtime hourly rate?
- 10. The ratio of girls to boys at The Gymnastics Preparation Center is 3:2. If there are 180 kids that train at The Gymnastics Preparation Center, how many of them are girls? How many of them are boys?

11. The average of three numbers is 14. The largest number is two more than twice the smallest. The second largest number is twice the smallest number. Find the three numbers.

1.1f Homework: Creating and Solving Linear Equations to Model Real World Problems Part I

1. Use the story below about Sanjeet and his friends' end-of-season basketball statistics to answer the questions that follow.

Points Scored	Story	
Terrence's points:x	Sanjeet and his team members were looking	
Sanjeet's points: Cole's points:	at the total points scored by each player	
	during the season. Sanjeet scored twice as	
Equation:	many points as Terrence. Cole scored 12	
	more points than Sanjeet. Together the boys	
	scored 992 points during the season. How	
	many points did each boy score?	

- a. Write expressions in the spaces provided above for the total points scored by each player during the season if Terrence scored *x* points.
- b. Write an equation that represents this situation in the space above.
- c. Solve your equation to determine the number of points scored by each boy during the season.

Cole: _____

Terrence: _____

Sanjeet: _____

Utah State Office of Education. Licensed under Creative Commons, cc-by.

- d. Double check your answers. Do your answers show that Sanjeet scored twice as many points as Terrence? That Cole scored 12 more points that Sanjeet? Do the scores sum to 992?
- 2. Uncle Hank has another riddle for his nephews. He tells them, "I have the same number of nickels and pennies. I have 4 times as many quarters as nickels. I have 3 more dimes than quarters. I have a total of \$6.14. Whoever can solve my riddle will get my coins."
 - a. Ben has started the equation for solving the riddle.
 - b. Finish writing the equation that represents the riddle.

0.01*p* + value of pennies

c. How many of each type of coin does Uncle Hank have?

Quarters:	Dimes:	Nickels:	Pennies:
	8	3WB1 - 43 ©20	14 University of Utah Middle School Math Project in partnership with the

- 3. During the summer, Victoria plays soccer and takes swim and piano lessons. Each swim lesson is 15 minutes shorter than a soccer practice. Each piano lesson is twice as long as a soccer practice. Use this information to answer the questions that follow.
 - a. The following expressions represent how long an activity is each time she goes. Write the name of the activity that matches each expression on the lines provided.

t:	 	
<i>t</i> – 15:	 	
2t:		

- b. Victoria has soccer three times a week, swimming four times a week, and piano twice a week. She spends a total of 435 minutes each week doing these three activities a week. Write an equation that represents this situation.
- c. How long is one session of each activity?

Soccer:	Swimming:	Piano:
---------	-----------	--------

4. The ratio of freshmen to sophomores to juniors to seniors in band is 1:2:3:2. If there are a total of 240 students in the band, how many are in each grade level?

Freshmen:	Sophomores:	Juniors:	Seniors:
	8	WB1 - 44	

5. The art teacher is making salt dough for an upcoming project. The ratio of flour to salt to water used to make salt dough is shown below.

Making Salt Dough	Story	
Cups of salt: <i>c</i>		
Cups of water: $\frac{3}{c}c$		
$2c + c + \frac{3}{2}c = 60$ cups		
4 4		
a. Write a story that mb. Solve the equation.	atches the expressions and eq How many cups of each ingre	uation shown on the left. edient is the art teacher planning to use?
Cups of flour:	Cups of salt:	Cups of water:
6. Use the story below about a	a triangle to answer the questi	ons that follow.
Angles in a Triangle	Story	
<i>m∠A</i> :	51019	
<i>m∠B</i> :	In $\triangle ABC$, the m	easure of $\angle B$ is three times
<i>m</i> ∠ <i>C</i> : <i>x</i>	larger than the	r measure of $\angle C$. The measure
Equation:	$-$ of $\angle A$ is twice a	is large as the measure of
	∠C.The sum of	the angles in a triangle is
	180°.	

a. Write the expressions and equation matching the story on the lines provided above.

b. What is the measure of each angle in the triangle?

 $m \angle A =$ _____

 $m \angle B =$

7. The width of a rectangle is six more than four times its length. A model of this situation has been drawn below. Use this information to answer the questions that follow.

Length:			
Width:			+6

- a. Write an expression that represents the perimeter of the rectangle.
- b. If the perimeter of the rectangle is 112 feet, write an equation to represent this situation and find the length and width of the rectangle.

Length: _____

Width: _____

- 8. A marble jar has twice as many blue marbles as red marbles, 16 more green marbles than blue marbles, and 10 fewer white marbles than red marbles. The jar has a total of 150 marbles. Use this information to answer the questions that follow.
 - a. The following equation represents this situation. Match each piece of the equation to the appropriate marble color. Write your answer in the boxes provided.



b. Determine how many marbles of each color are in the jar.

Blue:	Red:	Green:	White:
		0 WD 1 AC	

8WB1 - 46

9. Use the expressions and equation below about the cost of clothes to answer the questions that follow.

<u>The Cost of Clothes</u> Cost of a shirt: <i>c</i>	Story
Cost of a pair of jeans: $c + 12$ 3c + 2(c + 12) = \$164	

a. Write a story that matches the expressions and equation in the space provided.

b. Solve the equation to determine the cost of a shirt and the cost of a pair of jeans.

Cost of a shirt: _____ Cost of a pair of jeans: _____

Directions: Write and solve an equation to answer each of the following problems. Use pictures and models to help you. Refer back to similar problems you have already seen in the chapter to help you if you get stuck. Make sure your answers are displayed clearly with the appropriate units.

10. The width of a rectangle is five less than three times the length of the rectangle. If the perimeter of the rectangle is 70 inches what are the dimensions of the rectangle?

11. At Shoes for Less, a pair of shoes is \$15 less than a pair of boots. Cho purchased three pairs of shoes and two pairs of boots for \$120. How much does a pair of boots cost?

12. Central Lewis High School has five times as many desktop computers as laptops. The school has a total of 360 computers. How many laptops does Central Lewis High School have?

13. In ΔLMN , the measure of $\angle L$ is equal to the measure of $\angle M$. The measure of $\angle N$ is twice the measure of $\angle M$. Find the measure of each angle in ΔLMN .

14. Adam is trying to solve the following riddle: "The sum of three consecutive integers is -36. What are the integers?" Solve Adam's riddle.

15. Afua got a 90% on her first math exam, a 76% on her second math exam, and a 92% on her third math exam. What must she score on her fourth exam to have an average of 88% in the class?

16. At Discovery Preschool, parents who have two students enrolled get a discount on the second child. The second child's tuition is 10 dollars less per day than the first child's. If Tess has her two children enrolled for 5 days and her total bill for both children is \$200, how much does she pay each day for her second child to attend daycare?

1.1g Self-Assessment: Section 1.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Understand the meaning of linear expression and linear equation.	I am struggling with the difference between a linear equation and a linear expression.	If given a list of expressions and equations, I can tell which ones are expressions and which are equations.	I can define linear expression and equation in my own words and provide examples of each.	I can define linear expression and equation in my own words and provide examples of each. I know what it means to simplify an expression, to evaluate an expression, and to solve an equation.
2. Solve multi-step linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	I am struggling to solve most of the equations on the following page.	I can solve the equations in Set A on the following page.	I can solve all of the equations in Set A and all but one of the equations in Set B.	I can solve all of the equations in Sets A and B on the following page.
3. Write and solve linear expressions and equations that model real world problems.	When faced with a word problem similar to those in this chapter, I am having a difficult time seeing how the pieces of an equation relate to the story and I usually skip these problems.	When faced with a word problem similar to those in this chapter, I can match the different pieces of an equation that has been given to me to the story and solve the equation.	When faced with a word problem similar to those in this chapter, I can identify the important quantities in a practical situation, complete partial expressions and equations that have been given to me, solve the equation, and interpret the solution in the context.	When faced with a word problem similar to those in this chapter, I can identify the important quantities in a practical situation, write expressions and equations showing the relationship between the quantities, solve the equation, and interpret the solution in the context.

8WB1 - 49

1. Sam was asked to evaluate the expression 5x + 3x + 20 for x = 100. Sam's work is shown below.

Sam's Work: 5x + 3x + 20 = 100 8x + 20 = 100 8x = 80x = 10

- a. What mistake did Sam make? Help Sam to answer the question correctly.
- 2. Solve the following equations.

1. $8x - 6x + 1 = 11$	2. $-4x - 6 + x = -12$	3. $2(x+3) + 4x = 40$

Set B

4. $-2(2x+3) + 3x = -2$	5. $7k - (k+6) + 2k = 22$	6. $\frac{1}{2}(4x - 16) + 8x = 12$

3. Jesse and her brothers Nick and Owen are saving money over the summer. Each week, Jesse saves twice as much as Owen. Owen saves \$5 more than Nick. At the end of four weeks, the three of them have saved a total of \$220. How much money does each person save per week?

Section 1.2: Creating and Solving Multi-Step Linear Equations with Variables on Both Sides

Section Overview:

In this section students will solve equations with unknowns on both sides of the equal sign. From here, students will apply the skills learned so far in the chapter and solve a variety of linear equations with rational number coefficients. Up to this point, students have only encountered linear equations with a unique solution (one solution). In the latter part of this section, students will be introduced to linear equations in one variable with no solution or infinitely many solutions. Students will analyze what it is about the structure of an equation and the solving outcome (x = a, a = b, or a = a where a and b are different numbers) that results in one solution, infinitely many solutions, or no solution (is true for a unique value, no value, or all values of the unknown).

Concepts and Skills to Master:

By the end of this section, students should be able to:

- 1. Solve multi-step linear equations that have one solution, infinitely many solutions, or no solution.
- 2. Understand what it is about the structure of a linear equation that results in equations with one solution, infinitely many solutions, or no solutions.
- 3. Identify and provide examples of equations that have one solution, infinitely many solutions, or no solution.

1.2a Class Activity: Solving Multi-Step Linear Equations (variables on both sides)

1. The following is a model of the equation 5x + 2 = 3x + 12. Create this model with your tiles and solve



- a. Solving Actions: 5x + 2 = 3x + 12
- b. How can you verify your solution?

Directions: Model and solve the following equations.

2 $2r = r + 4$	3 3r + 3 = 2r + 7	4 r + 10 = 2r + 5
2. 2x = x + 1	$5.5 \times 10 = 2 \times 17$	$+.$ $\lambda + 10 - 2\lambda + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2-5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2-5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2-5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2-5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2-5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2-5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2 - 5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2 - 5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2 - 5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2 - 5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2 - 5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2 - 5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2 - 5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2 - 5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2 - 5x = -6x + 5$
5. $x + 5 = -x - 3$	6. $4x = -2x + 12$	7. $2 - 5x = -6x + 5$

Directions: Solve the following equations. You may use the tiles to help you.

8. $4x = 2x + 12$	9. $5x + 3 = x + 27$
104x + 6 = 3x - 36	$11.\ 0.7x - 0.6 = -0.2x - 0.42$
	1 4
12. $8 - 4x = 4x$	$13.\frac{1}{3}x - 8 = 12 + \frac{4}{3}x$
$14.\frac{x+3}{2} = \frac{x-1}{4}$	$15. \frac{(x-3)}{2} = \frac{(2x+4)}{5}$
	3 5

1.2a Homework: Solving Multi-Step Linear Equations (variables on both sides)

In centing. Solve the following equa	atons. Venny your solutions.	
1. $3x = 2x + 2$	2. $3x + 5 = 4x + 1$	3. $6x + 3 = 3x + 12$
4. $3x + 8 = 2x + 10$	5. $5x + 3 = 3x + 7$	6. $5a - 5 = 7 + 2a$
7. $3b - 6 = 8 - 4b$	8. $-3y + 12 = 3y - 12$	9. $3x = -3x + 1$
102x + 8 = -3x - 1	11. $3p - 4 = 5p + 4$	$12.\ 3 - 0.25x = -\frac{1}{2}x + 9$

Directions: Solve the following equations. Verify your solutions

Directions: In the following problems, a common mistake has been made. Circle the mistake and describe the mistake in words. Then, solve the equation correctly. 13. 4x - 8 = -2x + 2014. 6x + 4 = -2x

2x - 8 = 20	Combine like terms $(4x \text{ and } -2x)$	8x = 4	Add $2x$ to both sides.
2x = 28	Add 8 to both sides.	$x = \frac{4}{8}$	Divide both sides by 8.
x = 14	Divide both sides by 2.	$x = \frac{1}{2}$	Simplify the fraction.
Explanation of M	listake:	Explanation of	of Mistake:
Solve Correctly:		Solve Correc	tly:

1.2b Class Activity: Solving Multi-Step Linear Equations (putting it all together)

1. The following is a model of the equation 5x + 2 + x = 4x + 8. Create this model with your tiles and solve the equation, showing your solving actions below.



- a. Solving Actions: 5x + 2 + x = 4x + 8
- b. Verify your solution.
- 2. The following is a model of the equation 7x + 9 4x = 2(x + 5). Create this model with your tiles and solve the equation, showing your solving actions below.



a. Solving Actions: 7x + 9 - 4x = 2(x + 5)

Directions: Model and solve the following equations.

3. $2(x+3) = 5x - 3$	4. $8x + 3 - 2x = 3x + 12$	5. $10x + 2 - 3x = 3(2x + 2)$

6. The following is a model of an equation.



- a. Write the symbolic representation for this model.
- b. Solve the equation.

Directions: Solve the following equations without the use of the tiles.

$\begin{bmatrix} 7 & 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0$	9 - 2(a + 2) = 11 + a
1. 7x - 4 - 1 - 2x	0. $2(u+2) = 11 + u$
9. $5x - 4x + 18 = 3x + 2$	10.2t + 21 = 3(t + 5)
	12 4(1 - w) + 2w = 2(w + 1)
11. $/x + 2 - 4x = / + 2x + 4$	12. 4(1-x) + 3x = -2(x+1)
$13\frac{1}{2}(12r+16) = 10 - 3(r-2)$	$14 \frac{2x-9}{2} = 8 - 3r$
$\frac{15.4}{4}$ (12x + 10) = 10 5(x 2)	3 3 3
$15.\frac{1}{3}+5=\frac{1}{2}+3$	16(2n+6) = 5n - 12 - n
	<u> </u>

1.2b Homework: Solving Multi-Step Linear Equations (putting it all together)

1. The following is a model of an equation.



- a. Write the symbolic representation of the equation for this model.
- b. Solve the equation.

Directions: Solve the following equations. Verify your solutions.

2 x + 3x = 9 + x	3 4c + 4 = c + 10	4. $3(4x-1) = 2(5x-7)$
2		
5. $3x + 10 + 2x = 2(x + 8)$	6. $2(x+8) = 2(2x+1)$	7. $4(x+3) = x + 26 + x$
5. $3x + 10 + 2x = 2(x + 8)$	6. $2(x+8) = 2(2x+1)$	7. $4(x+3) = x + 26 + x$
5. $3x + 10 + 2x = 2(x + 8)$	6. $2(x+8) = 2(2x+1)$	7. $4(x+3) = x + 26 + x$
5. $3x + 10 + 2x = 2(x + 8)$	6. $2(x+8) = 2(2x+1)$	7. $4(x+3) = x + 26 + x$
5. $3x + 10 + 2x = 2(x + 8)$	6. $2(x+8) = 2(2x+1)$	7. $4(x+3) = x + 26 + x$
5. $3x + 10 + 2x = 2(x + 8)$	6. $2(x+8) = 2(2x+1)$	7. $4(x+3) = x + 26 + x$
5. $3x + 10 + 2x = 2(x + 8)$	6. $2(x+8) = 2(2x+1)$	7. $4(x+3) = x + 26 + x$
5. $3x + 10 + 2x = 2(x + 8)$	6. $2(x+8) = 2(2x+1)$	7. $4(x+3) = x + 26 + x$
5. $3x + 10 + 2x = 2(x + 8)$	6. $2(x+8) = 2(2x+1)$	7. $4(x+3) = x + 26 + x$
5. $3x + 10 + 2x = 2(x + 8)$	6. $2(x+8) = 2(2x+1)$	7. $4(x+3) = x + 26 + x$
5. $3x + 10 + 2x = 2(x + 8)$	6. $2(x+8) = 2(2x+1)$	7. $4(x+3) = x + 26 + x$

8. $3a + 5(a - 2) = 6(a + 4)$	9. $13 - (2c + 2) = 2(c + 2) + 3c$	10. $2(4x+1) - 2x = 9x - 1$
11. $2 - (2x + 2) = 2(x + 3) + x$	12.3(x-6) = 4(x+2) - 21	13. $3(y+7) = 2(y+9) - y$
14 - 4(r - 3) = 6(r + 5)	$15 \frac{1}{2}(12-2r) - 4 - 5r + 2(r-7)$	$16^{-1}(12n-4) = 14 - 10n$
11.1(x - 3) = 0(x + 3)	13. 2(12 - 2x) + - 3x + 2(x - 7)	$10. \frac{1}{2}(12n-4) = 14 - 10n$

1.2c Class Activity: Creating and Solving Linear Equations to Model Real World Problems Part II

Directions: Write a story that goes with the expressions and equation in each problem. Solve for the unknown

information and interpret your answer.

1. <u>Birthday Parties</u> Number of people at a birthday party: p Cost of party at Boondocks: $8p + 60$ Cost of party at Raging Waters: $20p$ 8p + 60 = 20p	Story

- a. Write a story that matches the expressions and equations.
- b. Solve the equation in the space above.
- c. Interpret your answer.

2. <u>A Number Trick</u>	Story
Starting number: <i>n</i>	
Lily's number: $3(n+5)$	
Kall's number: $(n-5)$	
3(n+5) = (n-5)	

- a. Write a story that matches the expressions and equations.
- b. Solve the equation in the space above.
- c. Interpret your answer.

3. Savings

Number of weeks: wSophie's Money: 300 - 40wRaphael's Money: 180 + 20w300 - 40w = 180 + 20w

	Story	
) — 40w 30 + 20w		
- 20 <i>w</i>		

- a. Write a story that matches the expressions and equations.
- b. Solve the equation in the space above.
- c. Interpret your answer.

Directions: Write an expression for each unknown quantity in the word problem. Then write an equation for each problem. Solve your equation and interpret your answer.

4. Horizon Phone Company charges \$15 a month plus 10 cents per text. G-Mobile charges a flat rate of \$55 per month with unlimited texting. At how many texts would the two plans cost the same? Which plan is the better deal if you send 200 texts per month?

5. The enrollment in dance class is currently 80 students and is increasing at a rate of 4 students per term. The enrollment in choir is 120 students and is decreasing at a rate of 6 students per term. After how many terms will the number of students in dance equal the number of students in choir? How many students will be in each class?

6. You burn approximately 230 calories less per hour if you ride your bike versus go on a run. Lien went on a two-hour run plus burned an additional 150 calories in his warm-up and cool down. Theo went on a 4 hour bike ride. If Lien and Theo burned the same amount of calories on their workouts, approximately how many calories do you burn an hour for each type of exercise?

1.2c Homework: Creating and Solving Linear Equations to Model Real World Problems Part II

Directions: Write the story that goes with the expressions and equation in each problem. Solve for the unknown information and interpret your answer.

1. Fixing Your Car	Story
Time (hours): <i>h</i>	
Cost of Mike's Mechanics: $15h + 75$	
Cost of Bubba's Body Shop: 25 <i>h</i>	
15h + 75 = 25h	

- a. Write a story that matches the expressions and equations.
- b. Solve the equation in the space above.
- c. Interpret your answer.

2. <u>World Languages</u> Number of years: <i>t</i>	Story
# of students in French: $160 - 9t$	
# of students in Spanish: $85 + 6t$	
160 - 9t = 85 + 6t	

- a. Write a story that matches the expressions and equations.
- b. Solve the equation in the space above.
- c. Interpret your answer.

 3. <u>Downloading Music</u> # of songs downloaded: s Monthly cost at bTunes: 0.99s Monthly cost at iMusic: 10 ± 0.79s 	Story
0.99s = 10 + 0.79s	

- a. Write a story that matches the expressions and equations.
- b. Solve the equation in the space above.
- c. Interpret your answer.

Directions: Write an expression for each unknown quantity in the word problem. Then write an equation for each problem. Solve your equation and interpret your answer.

4. Underground Floors charges \$8 per square foot of wood flooring plus \$150 for installation. Woody's Hardwood Flooring charges \$6 per square foot plus \$200 for installation. At how many square feet of flooring would the two companies charge the same amount for flooring? If you were going to put flooring on your kitchen floor that had an area of 120 square feet, which company would you choose?

5. Owen and Charlotte's mom give them the same amount of money to spend at the fair. They both spent all of their money. Owen goes on 8 rides and spends \$5 on pizza while Charlotte goes on 5 rides and spends \$6.50 on pizza and ice cream. How much does each ride cost?

6. Ashton and Kamir are arguing about how a number trick they heard goes. Ashton tells Andrew to think of a number, multiply it by five and subtract three from the result. Kamir tells Andrew to think of a number add five and multiply the result by three. Andrew says that whichever way he does the trick he gets the same answer. What was the number?

1.2d Class Activity: Solving Multi-Step Linear Equations (the different solving outcomes)

Up to this point, we have solved linear equations with a unique solution (**one solution**). In this lesson, we encounter equations that when solved have **infinitely many solutions** and **no solution**.

1. Consider the following model:



- a. Make some observations about the model above.
- b. Write the symbolic representation (equation) for this model and then solve the equation you wrote.
- c. What happened when you solved the equation? What is it about the structure of the equation that led to the solution?
- d. Build or draw your own equation using your tiles that would result in the same solution as the one above.

e. Solve the equation you built. What do you notice?

2. Consider the following model:



- a. Make some observations about the model above.
- b. Write the symbolic representation for this model and then solve the equation you wrote.

- c. What happened when you solved the equation? What is it about the structure of the equation that led to the solution?
- d. Build or draw your own equation using your tiles that would result in the same solution as the one above.

e. Solve the equation you built. What do you notice?

Directions: Solve the following equations. If there is one solution, state what the solution is. Otherwise, state if there are infinitely many solutions or no solution.

3. $x - 1 = x + 1$	4. $5x - 10 = 10 - 5x$
5. $4(m-3) = 10m - 6(m+2)$	6. $4(x-4) = 4x - 16$
7 2x - 5 = 2(x - 5)	8 3r = 3r - 4
	0 , 0 , 0 , 0 , 0 , 0 , 0
9. $3v + 5 + 2v = 5(2 + v)$	10.5 - (4a + 8) = 5 - 4a - 8
2x+8	x = 1 ($x = 2$
$11.\frac{11.\frac{11}{2}}{2} = x + 4$	$12.\frac{-}{3}(x-2) = \frac{-}{3} - \frac{-}{3}$

13. What is it about the structure of an expression that leads to one solution, infinitely many solutions, or no solution? Provide examples to support your claim.

Directions: Without solving completely, determine the number of solutions by examining the structure of the equation.

14.6a - 3 = 3(2a - 1)	15.5x - 2 = 5x	$16.\ 8x - 2x + 4 = 6x - 1$
17.5m + 2 = 3m - 8	$18.\ 2(3a - 12) = 3(2a - 8)$	$19.\frac{3x-12}{2} = x + 4$
		3
$20.\frac{2x+2}{2} = \frac{x+1}{2}$	21. $x + \frac{1}{x} = \frac{x+1}{x}$	$22, \frac{x}{2} - 4 = \frac{1}{2}(x - 8)$
4 2	5 5	2 2 2

- 23. Consider the expression 4a 12. Write 3 different expressions that if set equal to 4a 12 would result in the equation having infinite solutions.
- 24. Consider the expression x + 1. Write 3 different expressions that if set equal to x + 1 would result in the equation having no solution.
- 25. Consider the expression 2x + 6. Write 3 different expressions that if set equal to 2x + 6 would result in the equation having one solution.
- 26. Determine whether the equation 7x = 5x has one solution, infinitely many solutions, or no solution. If it has one solution, determine what the solution is.

1.2d Homework: Solving Multi-Step Linear Equations (the different solving outcomes)

Directions: Without solving completely, determine the number of solutions of each of the equations.

1. $x - 211 = x$	2. $3(m-3) = 3m - 9$	3. $5 - x = -x + 5$
4. $-4m + 12 = 4m + 12$	5. $-3(x+2) = -3x + 6$	6. $\frac{x-3}{5} = \frac{x}{5} - \frac{3}{5}$

Directions: Solve the following equations. If there is one solution, state what the solution is. Otherwise, state if there are infinitely many solutions or no solution. Show all your work.

7. $3x + 1 - 3(x - 1) = 4$	8. $3(a+6) - 2(a-6) = 6$
9. $3(r-4) = 3r-4$	$10.\ 2(x+1) = 3x + 4$
11.3 - (4b - 2) = 3 - 4b + 2	12.3 - (4b - 2) = 3 - 4b - 2
$13.\ 2y - 5y + 6 = -(3y - 6)$	14. f + 1 = 7f + 12 - 11 - 6f
--------------------------------	------------------------------------
$15.\ 12 + 8a = 6a - 6$	$16.\frac{1}{2}(6m - 10) = 3m - 5$
$15.\ 12 + 8a = 6a - 6$	$16.\frac{1}{2}(6m - 10) = 3m - 5$
$15.\ 12 + 8a = 6a - 6$	$16.\frac{1}{2}(6m - 10) = 3m - 5$
$15.\ 12 + 8a = 6a - 6$	$16.\frac{1}{2}(6m - 10) = 3m - 5$
$15.\ 12 + 8a = 6a - 6$	$16.\frac{1}{2}(6m - 10) = 3m - 5$
$15.\ 12 + 8a = 6a - 6$	$16.\frac{1}{2}(6m-10) = 3m-5$
$15.\ 12 + 8a = 6a - 6$	$16.\frac{1}{2}(6m - 10) = 3m - 5$

Directions: Fill in the blanks of the following equations to meet the criteria given. In some cases, there may be more than one correct answer.

17. An equation that yields one solution: $8x + __ = __x + 10$

18. An equation that yields no solution: $8x + __ = __x + 10$

19. An equation that yields infinitely many solutions: 8x + 24 = (x + y)

Directions: Create your own equations to meet the following criteria. 20. An equation that yields one solution of x = 5.

21. An equation that yields no solution.

22. An equation that yields infinitely many solutions.

23. Challenge: Can you think of an equation with two solutions?

1.2e Class Activity: Abstracting the Solving Process

- 1. Solve the following equations for *x*. State the solving actions.
 - a. x + 4 = 10
 - b. x + b = 10 where *b* represents any number
 - c. x + b = c where *b* and *c* represent any number
- 2. Solve the following equations for x. State the solving actions. a. 2x = -16
 - b. ax = -16 where *a* represents any number not equal to zero
 - c. ax = c where *a* and *c* represent any number not equal to zero
- 3. Solve each of the following equations for x. State the solving actions. a. 3x + 4 = 19
 - b. Rewrite the equation in part a. by replacing the 4 in the equation with b which represents any number, and the 3 in the equation with a which represents any number, and the 19 with c which represents any number. Solve your equation, stating the solving actions.

4. Hugo and Maggie both solved the following equation for *x*.

3(x+2) = 12

Hugo's Method: Ma $3(x+2) = 12$ $3(x)$		Maggie's Method: 3(x + 2) = 12	
3x + 6 = 12	Distribute the 3	x + 2 = 4	Divide both sides by 3
3x = 6	Subtract 6 from both sides	x = 2	Subtract 2 from both sides
x = 2	Divide both sides by 3		

a. Examine the solutions. Did both people solve the equation correctly?



b. The equations below are a mirror of the equations above; however the numbers 2, 3, and 12 in the original equation have been replaced with p, q, and r which represent any number. Solve the equations below for *x*, using both Hugo and Maggie's methods.

Hugo's Method:Mag $p(x+q) = r$ $p(x)$	ggie's Method: (r + q) = r

- 5. Solve the following equations for *x*.
 - a. 2x + 3x = 15
 - b. ax + bx = 15
 - c. Can you think of a different way of solving ax + bx = 15 for x?
- 6. Solve the following equations for *x* if *a*, *b*, and *c* represent real numbers not equal to 0. If you get stuck, put actual numbers in for *a*, *b*, and *c* and think about how you would solve these equations and then apply that thinking to the equations below.

a. $ax = bx + c$	b. $\frac{ax+b}{c} = 5$

1.2e Homework: Abstracting the Solving Process

- 1. Solve the following equations for *x*. State the solving actions. a. x - 6 = 8
 - b. x b = 8 where *b* represents any number
 - c. x b = c where *b* and *c* represent any number
- 2. Solve the following equations for x. State the solving actions. a. $\frac{x}{4} = 20$
 - b. $\frac{x}{d} = 20$ where *d* represents any number not equal to zero
 - c. $\frac{x}{d} = c$ where c and d represent any number not equal to zero
- 3. Solve each of the following equations for x. State the solving actions. a. $\frac{x}{3} + 5 = -1$

b.
$$\frac{x}{a} + b = c$$

4. Solve the following equations for *x* where *a*, *b*, *c*, *d*, *e*, and *f* are real numbers not equal to 0. If you get stuck, put actual numbers in for *a*, *b*, and *c* and think about how you would solve these equations and then apply that thinking to the equations below.

a. $bx + d = a$	b. $\frac{ax-c}{d} = b$
c. $\frac{x}{ab} = c$	d. $abx = c$
e. $ex + fx - c = d$	f. $ax - c = -bx + d$

1.2f Self-Assessment: Section 1.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Solve multi-step linear equations that have one solution, infinitely many solutions, or no solution.	I am struggling to solve most of the equations on the following page.	I can solve the equations in Set A on the following page.	I can solve all of the equations in Sets A and B on the following page.	I can solve all of the equations in Sets A, B, and C on the following page.
2. Understand what it is about the structure of a linear equation that results in equations with one solution, infinitely many solutions, or no solutions.	I don't understand what the different solving outcomes of a linear equation are.	I can tell whether an equation has one, no, or infinite solutions only after I have solved it but I sometimes get confused about the difference between no solution and infinite solutions.	I can tell whether an equation has one, no, or infinite solutions only after I have solved it.	I can look at an equation and without solving the equation entirely determine whether it will have one solution, no solution, or infinite solutions just by examining the structure of the equation.
3. Identify and provide examples of equations that have one solution, infinitely many solutions, or no solutions.	I don't know what it means for an equation to have one, no, or infinite solutions.	When given a list of equations, I can identify equations that have one, no, or infinite solutions but I have to draw a model or solve the equation in order to tell.	When given a list of equations, I can identify equations that have one, no, or infinite solutions without solving the equations.	When given a list of equations, I can determine whether the equations will have one, no, or infinite solutions. I can also generate my own equations that will result in one, no, or infinite solutions.

- 1. Solve the following equations.
- 2. Before solving each equation, examine the structure of the equation and determine whether it will have one, no, or infinite solutions.

Set A

1. $8g + 9 = 4g + 1$	2. $x + 1 = x - 2$	3. $-2x + 4x - 10 = -3x$

Set B

4. $2a - 12 = 3(a - 6)$	5. $-5x + 2(x - 1) = -12 + 2x$	6. $5 - (2a - 3) = 5 - 2a + 3$

Set C

7. $-2(x-1) + 4x = 3(2x-2)$	8. $\frac{1}{2}(6x-8) + 2x = -x - 16$	9. $\frac{1}{4}(16x - 1) = 4x - \frac{1}{4}$