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Chapter 2: Exploring Linear Relations (4 weeks)

Utah Core Standard(s):

- Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has a greater speed.* (8.EE.5)
- Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. (8.EE.6)
- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (8.F.4)

Academic Vocabulary: proportional relationship, proportional constant, unit rate, rate of change, linear relationship, slope(m), translation, dilation, y-intercept(b), linear, right triangle, origin, rise, run, graph, table, context, geometric model, constant difference, difference table, initial value, slope-intercept form.

Chapter Overview:
Students begin this chapter by reviewing proportional relationships from 6th and 7th grade, recognizing, representing, and comparing proportional relationships. In eighth grade, a shift takes place as students move from proportional linear relationships, a special case of linear relationships, to the study of linear relationships in general. Students explore the growth rate of a linear relationship using patterns and contexts that exhibit linear growth. During this work with linear patterns and contexts, students begin to surface ideas about the two parameters of a linear relationship: constant rate of change (slope) and initial value (y-intercept) and gain a conceptual understanding of the slope-intercept form of a linear equation. This work requires students to move fluently between the representations of a linear relationship and make connections between the representations. After exploring the rate of change of a linear relationship, students are introduced to the concept of slope and use the properties of dilations to show that the slope is the same between any two distinct points on a non-vertical line. Finally, students synthesize concepts learned and derive the equation of a line.

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Connections to Content:

Prior Knowledge: This chapter relies heavily on a student's knowledge about ratios and proportional relationships from 6th and 7th grade. Students should come with an understanding of what a unit rate is and how to compute it. In addition they need to be able to recognize and represent proportional relationships from a story, graph, table, or equation. In addition they must identify the constant of proportionality or unit rate given different representations.

Future Knowledge: After this chapter students continue to work with linear relationships and begin work with functions. They will work more formally with slope-intercept form as they write and graph equations for lines. This will set the stage for students to be able to graph and write the equation of a line given any set of conditions. Students use their knowledge of slope and proportionality to represent and construct linear functions in a variety of ways. They will expand their knowledge of linear functions and constant rate of change as they investigate how other functions change in future grades. The work done in this chapter is the foundation of the study of how different types of functions grow and change.

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MATHEMATICAL PRACTICE STANDARDS:

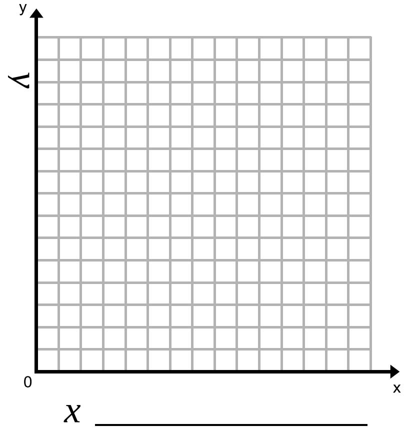
Make sense of problems and persevere in solving them.

Gourmet jellybeans cost $9 for 2 pounds.

a. Complete the table.

Pounds	.5		3	4		8	10		20
Total Cost		$9			$27				

b. Label the axes. Graph the relationship.



c. What is the unit rate?

d. Write a sentence with correct units to describe the rate of change.

e. Write an equation to find the cost for any amount of jellybeans.

f. Why is the data graphed only in the first quadrant?

As students approach this problem they are given some real world data and asked to graph and analyze it. They must make conjectures about the unit rate of the line and understand the correspondences between the table, graph, and equation. The final question asks students to conceptualize the problem by having them explain why only the first quadrant was used.

Reason abstractly and quantitatively.

Graphing points can be time-consuming. Develop a procedure for calculating the slope without graphing each point. Explain your procedure below. Show that it works for problems 1-4 above.

Discuss the methods for calculating slope without using right triangles on a graph. Write what you think about the methods.

Now discuss this formula:. $m = \frac{y_2 - y_1}{x_2 - x_1}$ What does it mean? How does it work?

By examining how the rise and run is found amongst a variety of points students begin understand that the rise is the difference of the y values and the run is the difference of the x values. They must abstract the given information and represent it symbolically as they develop and analyze the slope formula.

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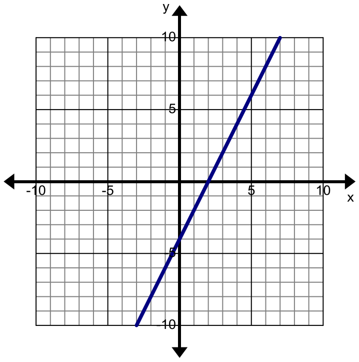
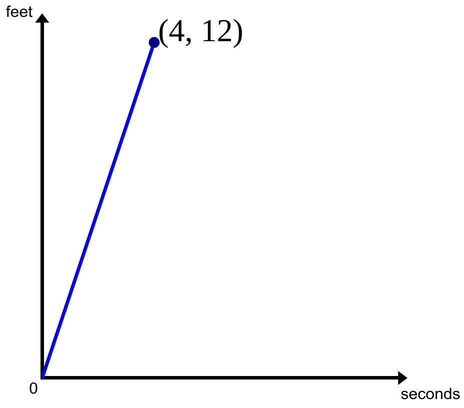
💬	**Construct viable arguments and critique the reasoning of others.**	On the line to the right choose any two points that fall on the line. (To make your examination easier choose two points that fall on an intersection of the gridlines). From the two points create a right triangle, the line itself will be the hypotenuse and the legs will extend from the two points and meet at a right angle.  Compare the points that you chose and your triangle with someone in your class. Discuss the following: Did you both choose the same points? How are your triangles the same? How are your triangles different? What relationship exists between your triangles? *Upon comparing their triangle with a class member students begin to discover that any right triangle constructed on the line is related through a dilation. By talking with one another they can analyze different triangles and discuss the proportionality that exists between them. This also gives students the opportunity to help one another learn how to accurately construct the right triangle on the graph used to find slope. In addition, they begin to make conjectures about how slope can be found from any two points on the line.*
📈	**Model with mathematics.**	a. Create your own story that shows a proportional relationship. b. Complete a table and graph to represent this relationship. Be sure to label the axes of your graph. c. Write an equation that represents your proportional relationship. *This question asks students to not only create their own proportional relationship but to model it with a table, graph, and equation. Students show their understanding of how a proportional relationship is shown in several representations.*
📏	**Attend to precision.**	The graph below shows the distance a cat is from his bowl of milk over time. Which sentence is a good match for the graph? A. The cat was 12 feet away from the milk and ran toward it reaching it after 4 seconds. B. The cat was 4 feet away from the milk and ran toward it reaching it after 12 seconds. C. The cat ran away from the milk at a rate of 3 feet per second. D. The cat ran away from the milk at a rate of 4 feet per second. E. The cat was 12 feet away from the milk and ran away from it at a rate of 4 feet per second.  *Upon examining the graph students must attend to precision as they discuss the ordered pair (4,12) and analyze exactly what it is telling us about the cat. Students often confuse the given information with rate of change and fail to recognize what quantity each number in the ordered pair represents. Also they must communicate what the direction of the line is telling us. Later on they are asked to make their own graphs and must use the correct units and labels to communicate their thinking.*

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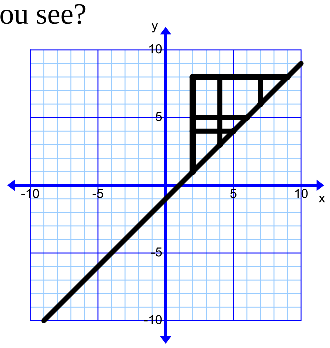
	Use appropriate tools strategically.	Your group task is to build a set of stairs and a handicap ramp at the side. Both the stairs and the ramp will begin at the same place (at ground level) and end at the height of three feet. Answer the following questions as you develop your design. • How many steps do you want or need? • How deep should each step be (we will call this the run)? Why do you want this run depth? • How tall will each step be (we will call this the rise)? Why do you want this rise height? • What is the total distance (total depth for all steps) you will need (at the base) for all of the stairs? This would be a measurement at ground level from stair/ramp start point to stair/ramp end point. Sketch the ramp as viewed from the side on graph paper below. Label and sketch the base and height, for example: stair-base (in inches or feet) and height (in inches or feet). *As students design a set of stairs and ramps they must use decide how they can use the tools (graph paper, ruler, pencil) provided them most efficiently. They will need to generate a graph that displays their designs and use the graph as a tool to analyze the slope of the stairs and ramp.*
	Look for and make use of structure.	Examine the graphs and equations given above. Describe the general form of a linear equation. In other words, in general, how is a linear equation written? What are its different parts? *As students examine many equations in slope-intercept form and interpret the slope and y-intercept they see the structure of the general form for a linear equation begin to emerge. Even if they write the initial value or y-intercept first they can step back and look at an overview of the general form of the equation and shift their perspective to see that the order in which you write your slope and initial value does not matter.*
	Look for and express regularity in repeated reasoning.	In each graph below, how many right triangles do you see? • Trace the triangles by color. • For each triangle write a ratio comparing the lengths of its legs or $\frac{height}{base}$. Then simplify the ratio $\frac{height}{base} = \underline{\qquad}$.  *In this series of problems students repeatedly find the height/base ratio of triangles that are dilations of one another and infer that slope can be calculated with the rise/run ratio by choosing any two points. A general method for finding slope as rise/run is discovered.*

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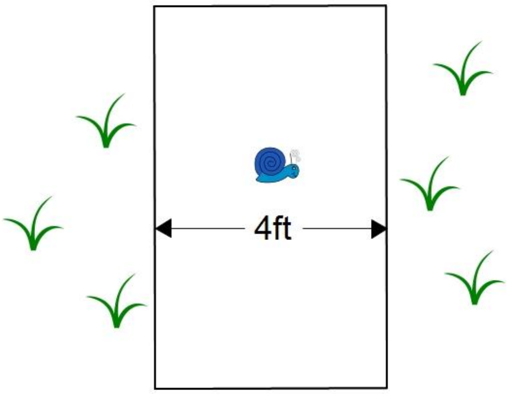
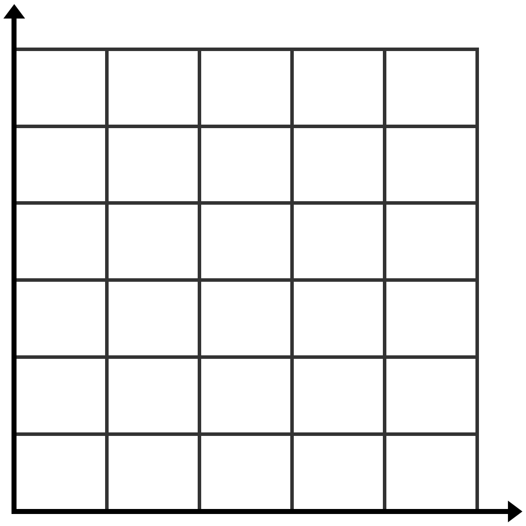
2.0 Anchor Problem: Proportionality and Unit Rate

Toby the snail is crossing a four foot wide sidewalk at a constant rate. It takes him 1 minute and 36 seconds to scoot across **half** the width of the sidewalk, as pictured below.



1. Find the unit rate for this proportional relationship. Be sure to explain what this unit rate means.

2. Write an equation that describes this proportional relationship if *x* is the amount of time it takes Toby to cross the sidewalk and *y* is the distance he has traveled. Use this equation to make a table of values to graph the first 5 seconds of Toby's journey.



3. How long will it take Toby to cross the sidewalk?

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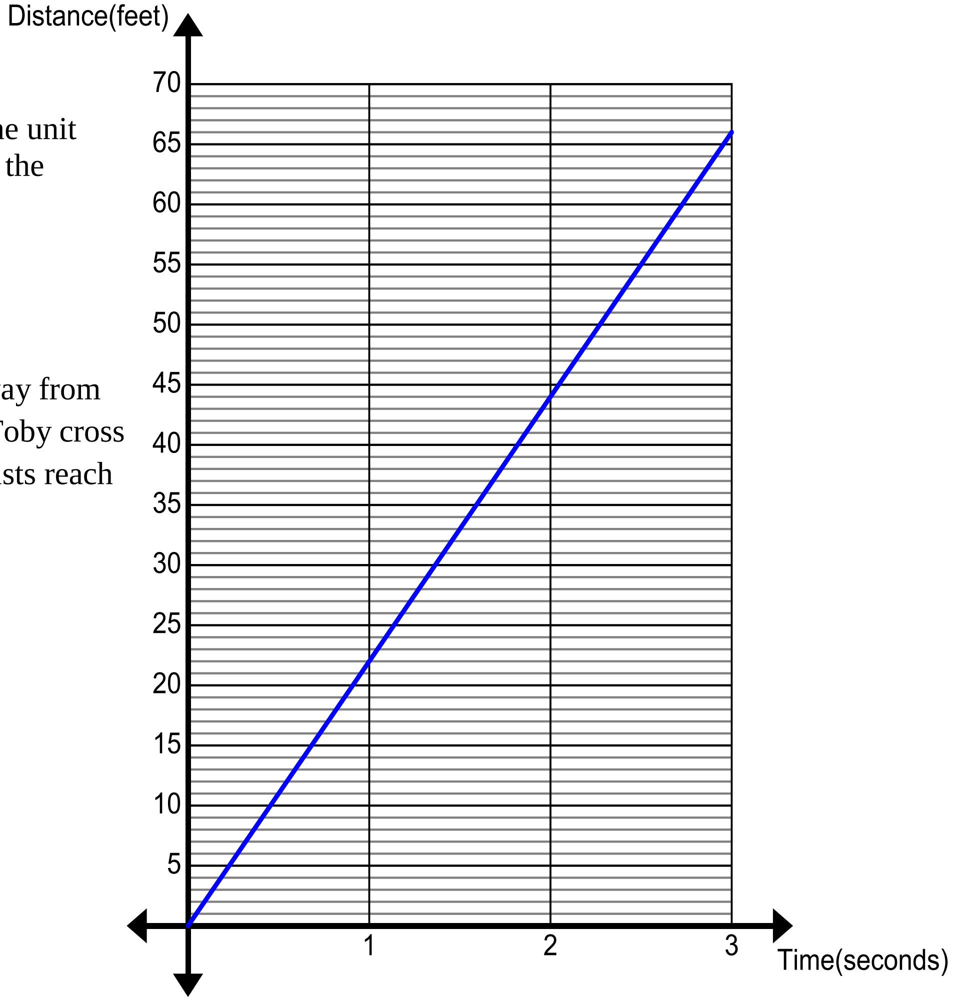
A large group of cyclists are on the sidewalk heading in Toby's direction. The graph below shows the rate at which they travel.

4. Find and describe the unit rate for the group of cyclists. Highlight this unit rate on the graph.

5. Describe how you can find the unit rate at a different location on the graph.

6. The cyclists are 4300 feet away from Toby on the sidewalk. Will Toby cross the side walk before the cyclists reach him? Justify your answer.



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Section 2.1: Analyze Proportional Relationships

Section Overview:

The section begins by reviewing proportional relationships that were studied in 6th and 7th grade. By investigating several contexts, students study the proportional constant or unit rate in tables, graphs, and equations. They recognize that a proportional relationship can be represented with a straight line that goes through the origin and compare proportional relationships represented in many ways. In the last lesson of the section a bridge from proportional relationships to linear relationships is achieved as students translate the graph of a proportional relation away from the origin and analyze that there is no effect on steepness of the line or rate at which it changes but that the relation is no longer proportional. They begin to see a proportional relationship as a special subset of a linear relationship where the rate of change is a proportional constant or unit rate and the graph of the relationship is a line that goes through the origin. Students also investigate the transition of the proportional constant or unit rate to rate of change, that is, if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount m times A.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Graph and write equations for a proportional relationship and identify the proportional constant or unit rate given a table, graph, equation, or context.
2. Compare proportional relationships represented in different ways.
3. Know that the graph of a proportional relationship goes through the origin.

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2.1a Class Activity: Proportional Relationships

In the previous chapter, you wrote equations with one variable to describe many situations mathematically. In this chapter you will learn how to write an equation that has two variables to represent a situation. In addition to writing equations you can represent real-life relationships in others ways. In 6th and 7th grade you studied **proportional relationships** and represented these relationships in various ways. The problems given below will help you to review how ratio and proportion can help relate and represent mathematical quantities from a given situation.

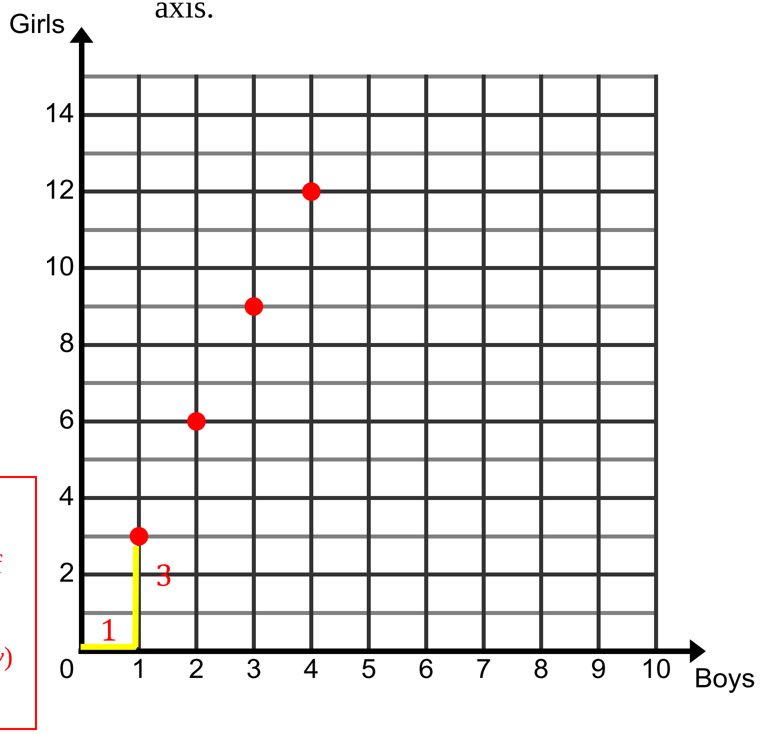
1. Julie is picking teammates for her flag football team. She picks three girls for every boy.

a. Complete the table below to show the relationship of boys to girls on Julie's team.

Boys	Girls
$1 \xrightarrow{\times 3}$	3
$2 \xrightarrow{\times 3}$	6
$3 \xrightarrow{\times 3}$	9
$4 \xrightarrow{\times 3}$	12

You can observe the proportional constant in the table, graph, and equation. In the table you can see that the number of boys is related to the corresponding number of girls by multiplying by three. In the graph the corresponding y-value for an x value of one is three. Finally, in the equation three relates the number of girls(y) to the number of boy(x).

b. Graph the girl to boy relationship for Julie's team with boys on the x-axis and girls on the y-axis.



c. Find the ratio of girls to boys for several different ordered pairs in the table.

$$\frac{\text{number of girls}}{\text{number of boys}} = \frac{12}{4} = \frac{6}{2} = \frac{3}{1}$$

Sample answers are given. All ratios will reduce to $\frac{3}{1}$.

d. Fill in the boxes to show the relationship between girls and boys on Julie's team.

$$\boxed{\text{Number of girls } y} = 3 \bullet \boxed{\text{Number of boys } x}$$

Sample answers are given above.

e. Use the equation and graph to determine how many girls would be on the team if Julie chose 10 boys to be on the team.

If Julie chose 10 boys to be on the team there would be 30 girls on the team.

f. Use the equation and graph to determine how many boys are on the team if Julie chose 18 girls.

If there are 18 girls on the team then there are 6 boys on the team as well.

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In the previous example the two quantities of interest are in a **proportional relation**.

Recall that when two quantities are proportionally related, the ratio of each *y* value to its corresponding *x* value is constant. This constant is called the **constant of proportionality** or **proportional constant**.

The ratio that related the number of boys to girls was 3. This is the proportional constant for this relationship.

2. Carmen is making homemade root beer for an upcoming charity fundraiser. The number of pounds of dry ice to the ounces of root beer extract (flavoring) is proportionally related. If Carmen uses 12 pounds of dry ice she will need to use 8 ounces of root beer extract.

   a. Write a ratio/proportional constant that relates the number of pounds of dry ice to the number of ounces of root beer extract.

$$\frac{\text{pounds of dry ice}}{\text{ounces of root beer}} = \frac{12}{8} = \frac{3}{2}$$

   b. Write a ratio/proportional constant that relates the number of ounces of root beer extract to the number pounds of dry ice.

$$\frac{\text{ounces of root beer}}{\text{pounds of dry ice}} = \frac{8}{12} = \frac{2}{3}$$

Notice that the proportional constant depends on how you define your items.

   c. Complete the table below to show the relationship between number ounces of root beer extract *x* and number of pounds of dry ice *y* needed to make homemade root beer.

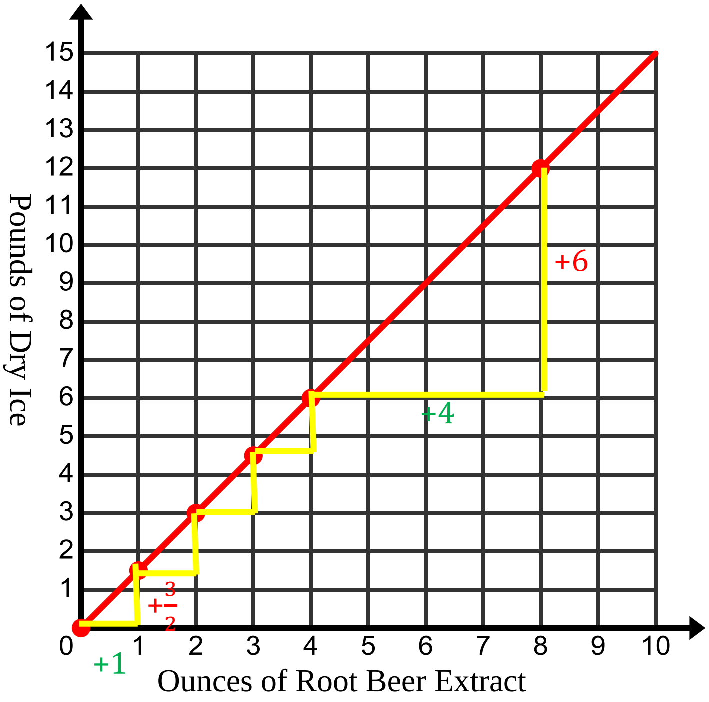
Ounces of Root Beer Extract (x)	Pounds of Dry Ice (y)
0	0
1	$\frac{3}{2} = 1.5$
2	3
3	$\frac{9}{2} = 4.5$
4	6
8	12

$\times \frac{3}{2}$ $\times \frac{3}{2}$ $\times \frac{3}{2}$ $\times \frac{3}{2}$ $\times \frac{3}{2}$ $\times \frac{3}{2}$

+1 +1 +1 +1 +4

$+\frac{3}{2}$ $+\frac{3}{2}$ $+\frac{3}{2}$ $+\frac{3}{2}$ +6

   d. Graph and label this relationship below.



After looking at the unit rate in number 5 below you can see the unit rate in the table (note the jump of the *x* value from 4 to 8) and graph. Looking at the unit rate on the graph is essentially the same as finding the slope. At this point in the chapter view this as unit rate. Found by looking at the vertical increase (the number of *y* units) that relate to a horizontal increase of 1(1 unit of *x*.). The connection to slope will come later. Proportional constant and unit rate have the same value they are just different ways of interpreting the relationship between your quantities.

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d. What is the proportional constant for this relationship?

The proportional constant for this relationship is $\frac{3}{2}$.

Students may try to argue that the proportional constant is 2/3. In this case the number of ounces of extract is defined as x and the pounds of dry ice is defined as y. If you were to switch the x and y variables than the proportional constant would be 2/3.

e. Write an equation that shows the relationship between the number of ounces of root beer extract (x) and the number of pounds of dry ice (y) needed to make homemade root beer.

$$y = \frac{3}{2}x$$

Every ratio has an associated rate. **Unit rate** is another way of interpreting the ratio's proportional constant. The statement below describes how unit rate defines y and x in a proportional relationship.

If quantities y and x are in proportion then the **unit rate** of y with respect to x is the amount of y that corresponds to one unit of x. If we interchange the roles of y and x, we would speak of the unit rate of x with respect to y.

5. In the previous problem Carmen was making homemade root beer. Express the proportional constant as a unit rate.

   The proportional constant is $\frac{3}{2}$. This means that she will need $\frac{3}{2}$ or 1.5 pounds of dry ice for every one ounce of root beer extract.

6. What would the unit rate be if we interchanged the roles of x and y?

   If you were to switch the x and y variables then the unit rate would be $\frac{2}{3}$. You will need two thirds of an ounce of root beer extract for every one pound of dry ice.

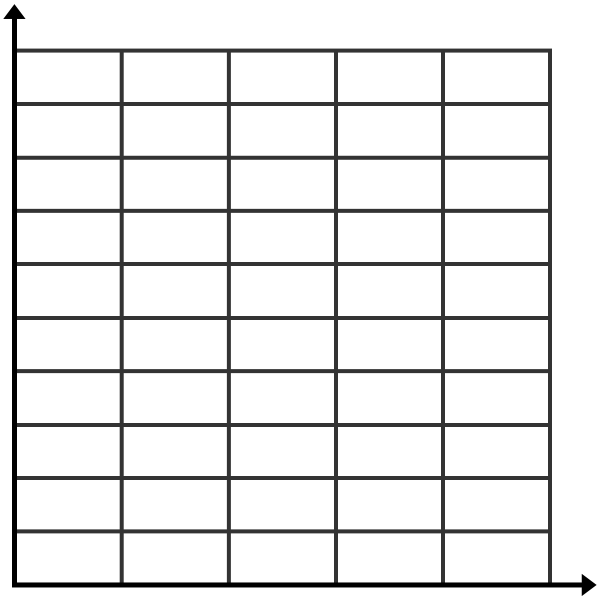
In the problem below use the properties of a proportional relationship to help you answer the question.

7. Doug is pouring cement for his backyard patio that is 100 square feet. The cement comes out of the truck at a constant rate. It is very important that he gets all the cement poured before 12:00 noon when it gets too hot for the cement to be mixed properly. It is currently 11:00 AM and he has poured 75 square feet of concrete in the last 3 hours. At this rate will he finish before noon?

   a. Fill in the missing items in the table if x represents the number of hours that have passed since Doug began pouring concrete and y represents the amount of concrete poured

Time elapsed (hours) x	Amount of concrete poured (square feet) y
0	
1	
2	
3 (11:00AM)	75
4	

   b. Graph the relationship below.



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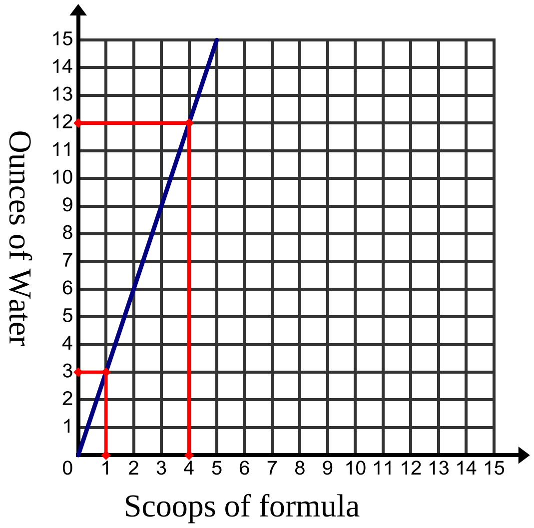
c. What is the unit rate for this relationship? In other words, how many square feet of concrete can Doug pour in 1 hour.

d. Which equation given below best describes this relationship?

a) $y=25x$	b) $y=75x$
c) $x=25y$	d) $y=11x$

e. Will Doug finish the job in time? Justify your answer.

10. Vanessa is mixing formula for her baby. The graph given to the right describes the relationship between the ounces of water to the scoops of formula to make a properly mixed bottle.



a. Does the graph describe a proportional relationship? Justify your answer.
This is a proportional relationship because the graph is a straight line going through the origin.

b. What is the unit rate for this relationship? Show on the graph how you can see the unit rate.
The unit rate for this relationship is 3 ounces of water per one scoop of formula.

c. At a different location on the graph show and explain how you can find the unit rate. Since this is a proportional relationship the unit rate can be found by dividing any two points that fall on the line. For example $\frac{12}{4} = 3$.

d. Write an equation to relate the ounces of water to the scoops of formula.
$y=3x$

e. How many scoops of formula must Vanessa use to make 9 ounce bottle for her baby?
Vanessa will need to use 3 scoops of formula to make a 9 ounce bottle.

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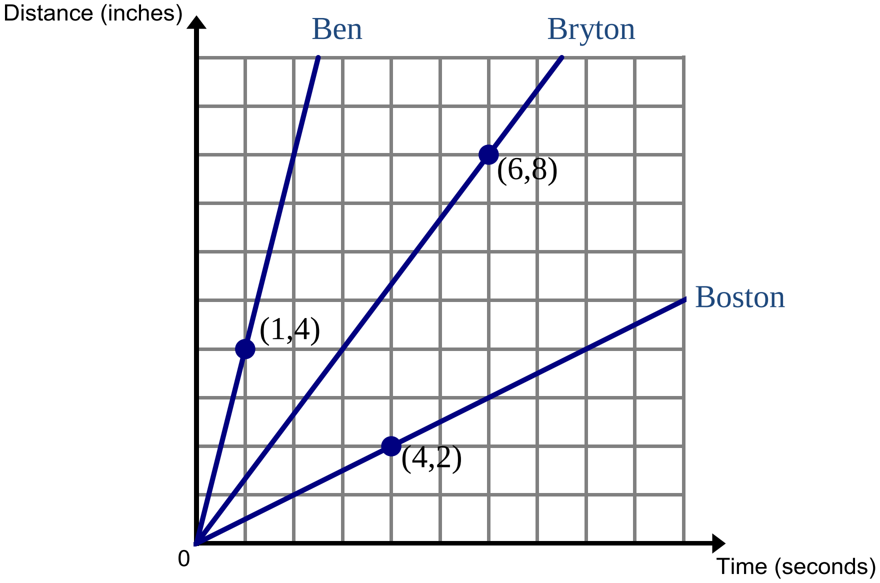
11. Ben, Boston, and Bryton have each designed a remote control monster truck. They lined them up to crush some mini cars in the driveway. The lines on the graph below show the distance in inches that each monster truck travels over time in seconds.



a. Bryton states that each truck is traveling at a constant rate. Is his statement correct? Why or why not.
Bryton is correct. The graph for each truck is a straight line going through the origin. This means that they are representing a proportional relationship.

b. What do the values in the ordered pairs given on the graph represent?
At each particular point the *x* values represent the time that has passed and the *y* values represent the distance the truck has traveled.

c. Find the unit rate for each boy's truck.
Ben: 4 inches per second

Bryton: $\frac{4}{3}$ *or* $1\frac{1}{3}$ inches per second

Boston: $\frac{1}{2}$ of an inch per second

d. Which boy's truck is moving the fastest?
Ben's truck is the fastest.

12. Explain why the graph of a proportional relationship makes a straight line.

13. Summarize what you know about proportional relationships using bulleted list in the space below.

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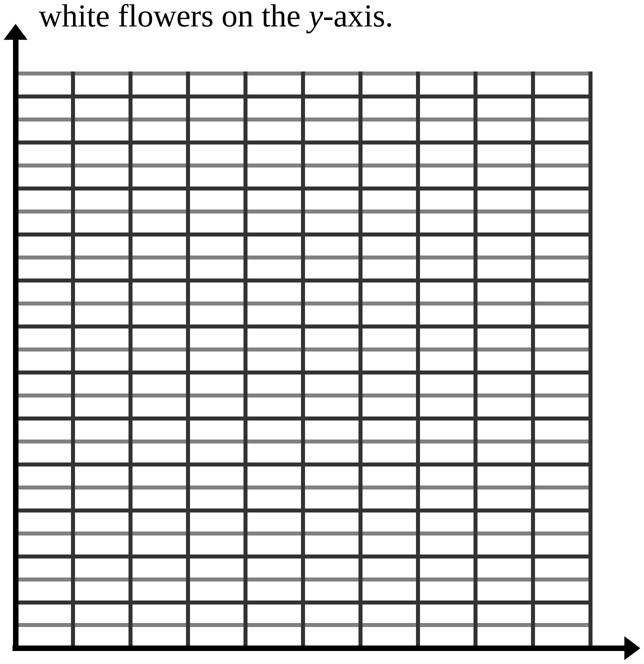
2.1a Homework: Proportional Relationships.

1. A florist is arranging flowers for a wedding. For every 2 pink flowers in a vase, he also includes 8 white flowers.

   a. Complete the table below to show the relationship of white to pink flowers in each vase.

Pink	White
1	
	8
	12
6	

   b. Graph the white flower to pink flower relationship for each vase with pink flowers on the *x*-axis and white flowers on the *y*-axis.



   c. Find the ratio of white to pink for several different ordered pairs in the table.

$$\frac{\text{number of white flowers}}{\text{number of pink flowers}} =$$

   d. Fill in the boxes to show the relationship between white flowers and pink flowers in a vase.

$$\boxed{\qquad\qquad\qquad} = 4\bullet\boxed{\qquad\qquad\qquad}$$

   e. Use the equation and graph to determine how many white flowers there would be if the florist included 20 pink flowers.

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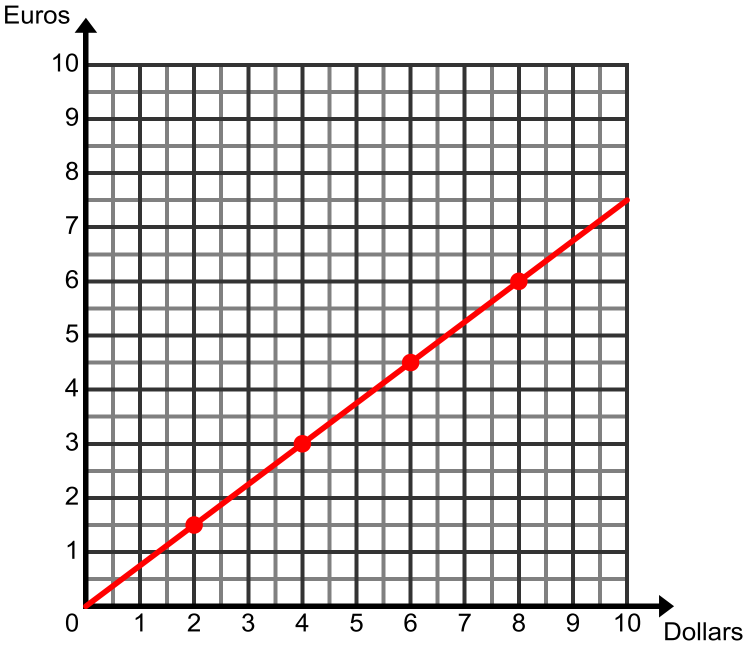
2. You are going to Europe for vacation and must exchange your money. The exchange rate of Euros to Dollars is a proportional relationship. The table below shows the exchange rate for Euros y to Dollars x.

a. Complete the table.

Dollars(x)	Euros(y)
2	1.5
4	3
6	4.5
8	6

b. Graph the Euro to Dollar relationship.



c. What is the unit rate for this relationship?

The unit rate is $\frac{3}{4}$ of a Euro per dollar. Meaning that one dollar equals three fourths of a Euro.

d. Write an equation that represents this relationship.

$y = \frac{3}{4}x$ or $y = .75x$

e. If you exchanged \$20, how many Euro would you get?

You will get 15 Euro for \$20.

f. If you recieved 36 Euro, how many dollars did you exchange?

If you get 36 Euro you exchange \$48.

3. In the problem given above we found that the unit rate described the number of Euros in a Dollar. Now interchange the roles of y and x for this relationship. Find the exchange rate for Dollars (y) to Euros (x). Use the questions below to help you do this.

a. Make a table of values to represent the Dollar to Euro relationship.

Euros(x)	Dollars(y)

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b. What is the unit rate for this Dollar to Euro relationship?

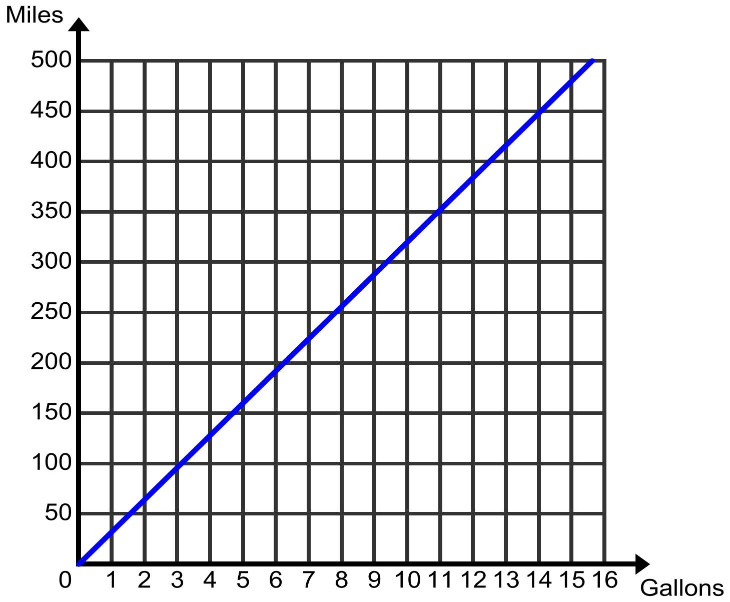
c. Write an equation that represents this relationship.

Use the equation to answer the following questions.

d. While in Europe you find a shirt that you want to buy that is marked at 25 Euros. You only have $32 to exchange for the Euros. Do you have enough money? Explain.

e. Upon returning home from Europe you have 100 Euros left. How many Dollars can you get for 100 Euros?

4. The graph given below shows the gas mileage that Penny gets in her car. The ratio 192:6 describes the miles to gallons fuel rate for her car.



a. What is the unit rate for this relationship?
32 mpg or Penny can go 32 miles on one gallon of gas.

b. Use the graph to approximate how many miles Penny can go if she has a 15 gallon tank in her car.
Penny can go 480 miles on one tank of gas.

5. A proportional constant of $\frac{1}{3}$ relates the number of inches a flower grows to the number of weeks since being planted.

a. Fill in the missing items in the table if *x* represents the number of weeks that have past and *y* represents the height of the flower.

x *(weeks)*	1	3		9	30
y *(height)*			2		

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b. Write an equation that represents this relationship and use the equation to predict how tall the flower will be after 8 weeks.

c. Is it probable for the flower to continue to grow in this manner forever?

6. Padma needs to buy 5 pounds of candy to throw at her city's annual 4th of July Parade; the picture below shows how much it costs to buy Salt Water Taffy at her local grocery store.

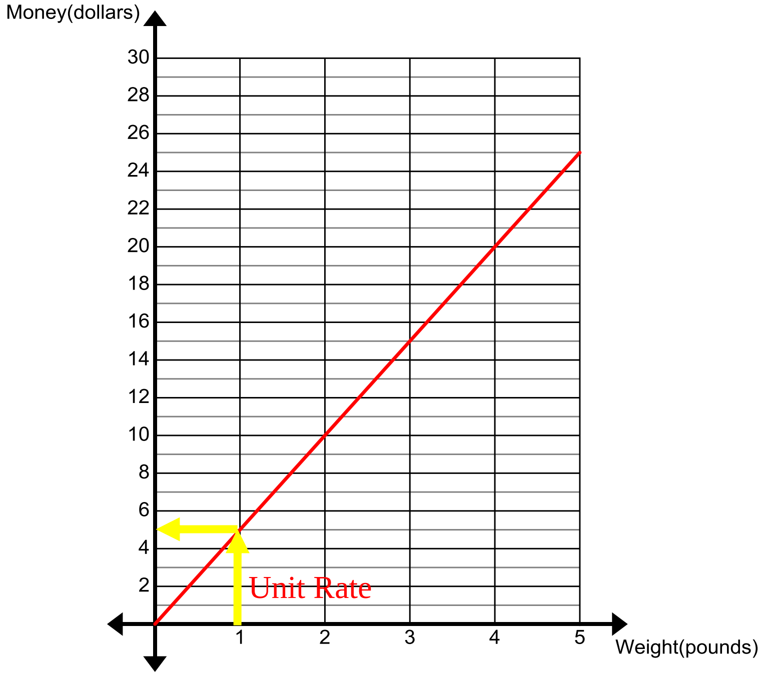
Salt Water Taffy
3 pounds for
$15.00

a. Find the unit rate for this proportional relationship. Be sure to describe what this unit rate means.
The unit rate is 5. This means that it costs $5.00 per pound of taffy.

b. Write an equation that describes this proportional relationship if *x* is the number of pounds and *y* is the cost. Use this equation to make a table of values to graph how much up to five pounds of taffy will cost.

$y = 5x$

x	y
0	0
1	5
2.	10
3	15
4	20
5	25



c. How much will it cost Padma to buy the 5 pounds of candy that she needs?
It will cost Anna $25 to buy 5 pounds of candy.

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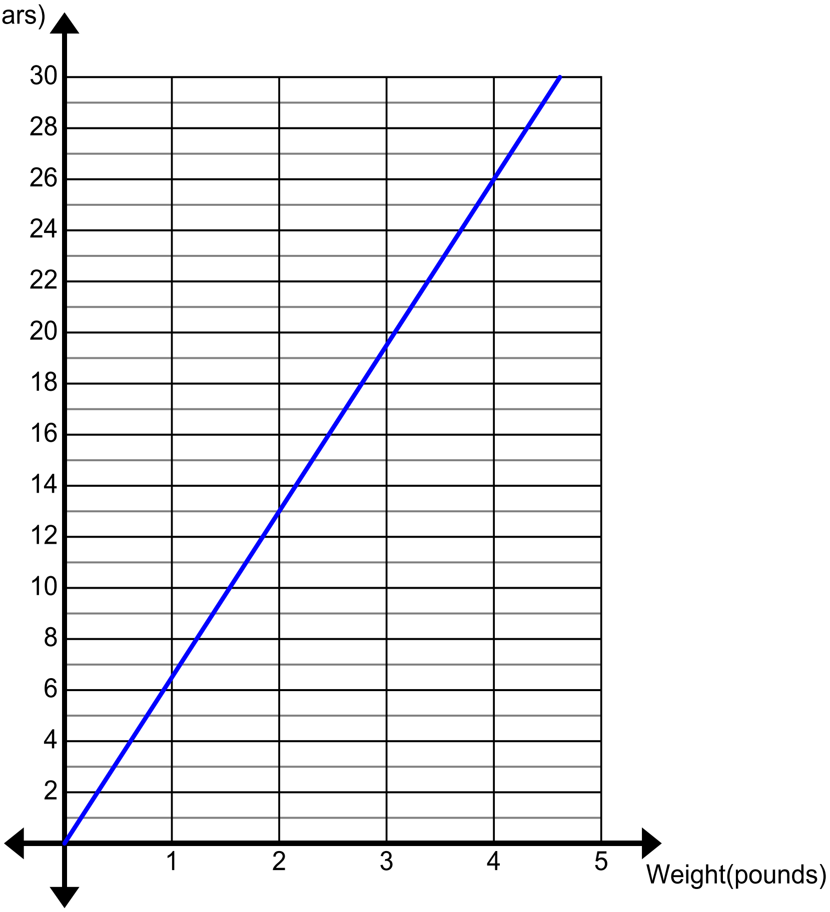
7. Padma also sees that she can buy Tootsie Rolls at the grocery store. The graph below shows cost of tootsies rolls per pound purchased. (Note: When cost is involved unit rate is often referred to as *unit price*.)



a. Find and describe the unit rate for the Tootsie Rolls. Highlight this unit rate on the graph.

b. Describe how you can find the unit rate at a different location on the graph.

c. Graph the line for the Salt Water Taffy on the grid with the Tootsie Roll. Label each line with the candy it represents.

d. What is the better deal for Padma, should she buy the Salt Water Taffy or the Tootsie Rolls? Justify your answer.

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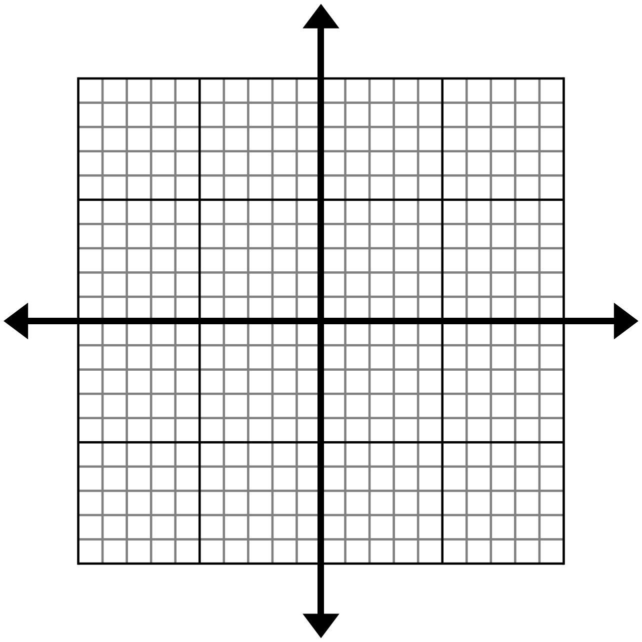
8. Create your own relationship.

   a. Create your own story that shows a proportional relationship.

   b. Complete a table and graph to represent this situation. Be sure to label your graph and table.



   c. Write an equation that represents your proportional relationship.

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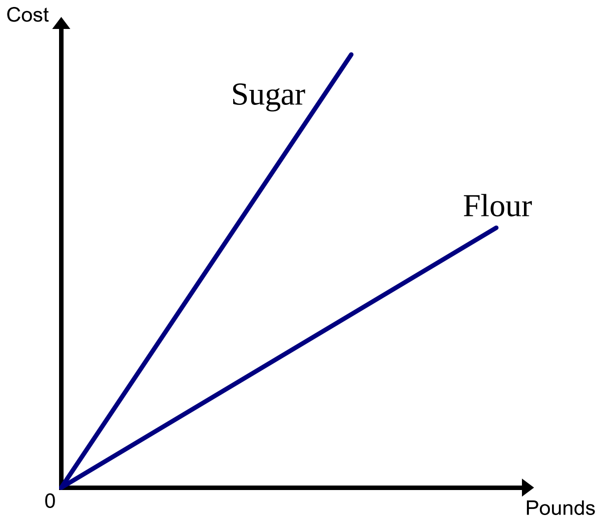
2.1b Class Activity: Comparing Proportional Relationships

Proportional relationships can help us to compare and analyze quantities and to make useful decisions. Complete the tasks given below that compare proportional relationships.

Emma is putting together an order for sugar, flour, and salt for her restaurant pantry. The graph below shows the cost *y* to buy *x* pounds of sugar and flour. One line shows the cost of buying *x* pounds of flour and the other line shows the cost of buying *x* pounds of sugar.



1. From the graph which ingredient costs more to buy per pound? Justify your answer.
   The sugar costs more to buy per pound. Since the line for the sugar is steeper that means that it has a higher unit rate.

2. The cost to buy salt by the pound is less than sugar and flour. Draw a possible line that could represent the cost to buy *x* pounds of salt.

Don and Betsy are making super smoothies to re-energize them after a long workout. Betsy follows the recipe which calls for 2 cups of strawberries for every 3 bananas. Don wants twice as much as Betsy so he makes a smoothie with 4 cups of strawberries and 5 bananas.

Don tastes his smoothie and says, “This tastes too tart, there are too many strawberries!”

3. Explain why Don’s smoothie is too tart.
   Don did not add the right amount of bananas he doubled the amount of strawberries by adding 2 cups, but he also added 2 bananas. He should have doubled the bananas from 3 to 6. The smoothie was too tart because he did not add enough bananas to balance out the tartness of the strawberries.

4. Find and describe the unit rate for Besty’s smoothie.

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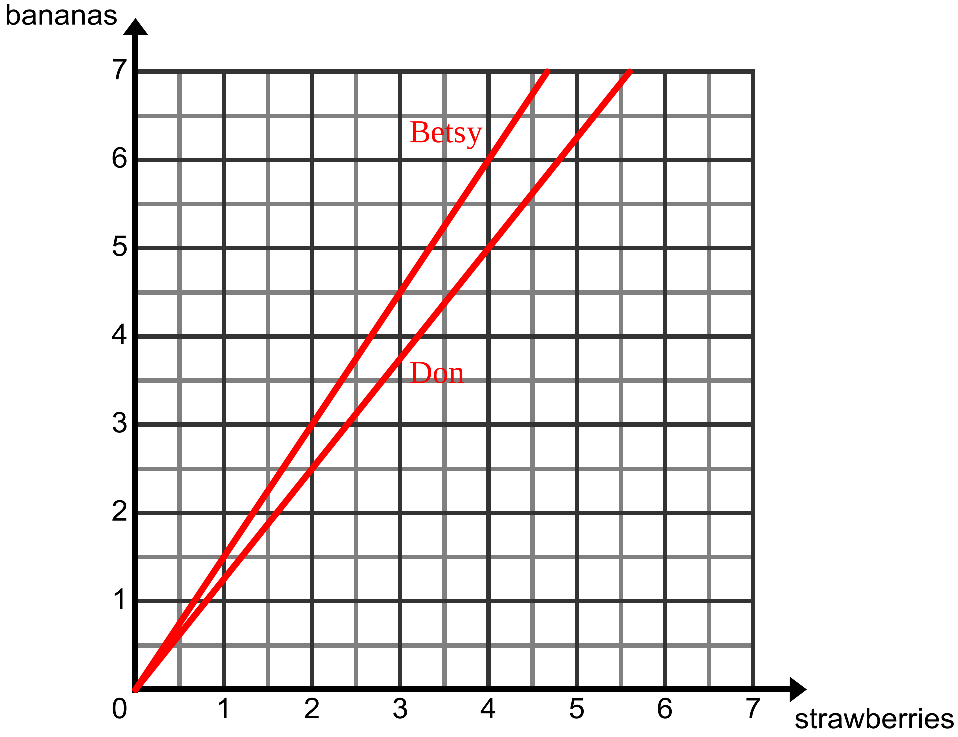
5. Find and describe the unit rate for Don's smoothie.
   The unit rate for Don's line is 1.25 or 6/5. This means that for every 1 cup of strawberries there are 1.25 bananas.

6. Write an equation that relates the number of strawberries(x) to the number of bananas(y) for Besty's smoothie.
   Besty $y=1.5x$

7. Write an equation that relates the number of strawberries(x) to the number of bananas(y) for Don's smoothie.

8. Use your equations to make tables to graph both of these lines on the same grid. Be sure to label which line belongs to which person.



9. Explain how the steepness of the lines relates to the unit rate.

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10. For the recreational activities below, compare the cost y per hour x by looking at graphs and equations.

- Fill in the missing representations. If the information is given in a table, fill in the story and equation. If the information is given in an equation, fill in the story and table, etc.
- Find the unit rate or slope for each situation.
- Graph all situations on the given graph on the next page. Remember to Label the axes. Label the lines with the situation names.
  Answers given in the tables may vary.

a. **Long Distance Phone Call:**
It costs $10 per hour to talk on the phone long distance.

Hours(x)	Cost(y)
0	0
1	10
2	20
3	30
4	40

Equation: $y = 10x$

Unit Rate: $10 per hour.

b. **Roller skating**
Roller skating for two hours costs $5.

Hours(x)	Cost(y)
0	0
2	5
4	10
6	15
8	20

Equation: $y = \frac{5}{2}x$

Unit Rate: $2.50 per hour

c. **Music Lessons**
The bill for private guitar lessons was $75. The lesson lasted 3 hours.

Hours(x)	Cost(y)

Equation:

Unit Rate:

d. **Parks**

Hours(x)	Cost(y)
24	6

Equation:

Unit Rate:

e. **Bungee Jumping**
It costs $20 to bungee jump for 15 minutes.

Hours(x)	Cost(y)
0	0
1	80
2	160
3	240
4	320

Equation: $y = 80x$

Unit Rate: $80 per hour.

f.

Hours(x)	Cost(y)

Equation:

Unit Rate:

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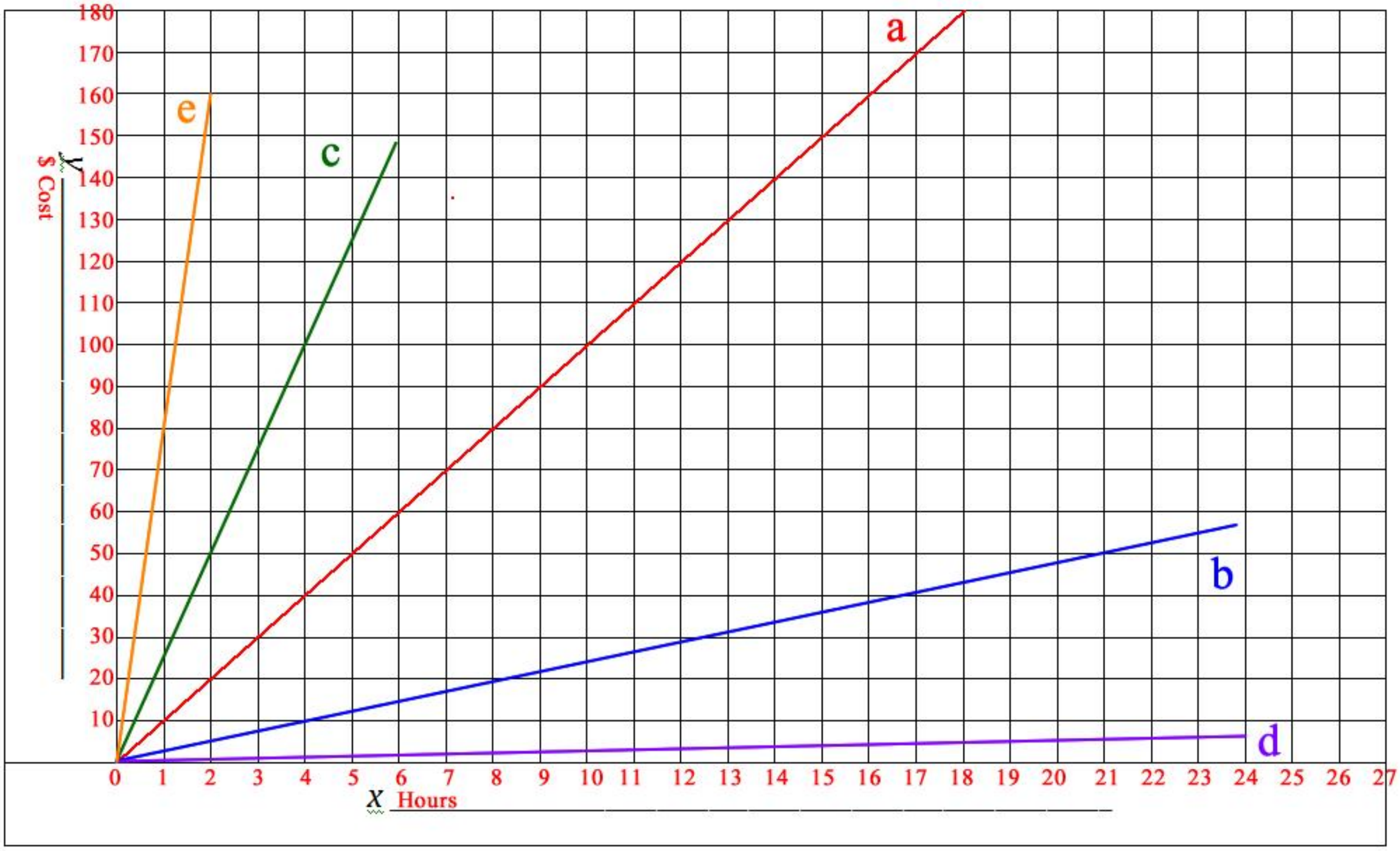
Graph



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2.1b Homework: Comparing Proportional Relationships

Use the graph completed during the class activity to answer questions 1-10 below.

1. Order the five activities from highest cost to lowest cost per hour.
   Bungee Jumping, Music Lessons, Long Distance, Roller Skating, Parks

2. How do you compare the cost per hour by looking at the graph?

3. How do you compare the cost per hour by looking at the equations?
   The greater the unit rate(the number in front of x) the more expensive it is per hour.

4. Create a sixth activity in column f on page 23. Think of a situation which would be less expensive than Bungee Jumping, but more expensive than the others. Fill in the table and make the graph.

Answer the questions below:

5. As the rate gets higher, the line gets steeper.
   lower less steep

6. Renting the pavilion at the park for 3 hours costs________.

7. Talking on the phone for 2.5 hours costs $25.

8. Bungee jumping for______hours costs $40.

9. For $10 you can do each activity for approximately how much time?

a. Long Distance	1 hour
b. Roller Skating	
c. Music Lessons	24 minutes
d. Park	
e. Bungee	7½ minutes
f.	

10. Did you use the tables, equations or graphs to answer questions 5-10? Why?

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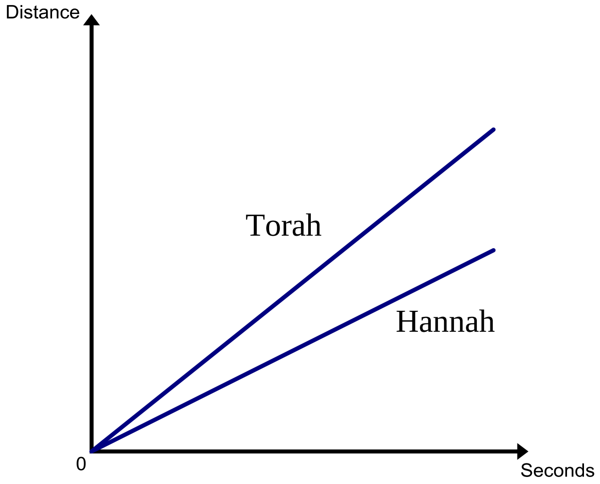
11. The graph below shows the distance two snowboarders have traveled down a hill for several seconds. Hannah is traveling 18 meters per second.



a. Which equation below is the best choice to describe the distance Torah travels after x seconds.

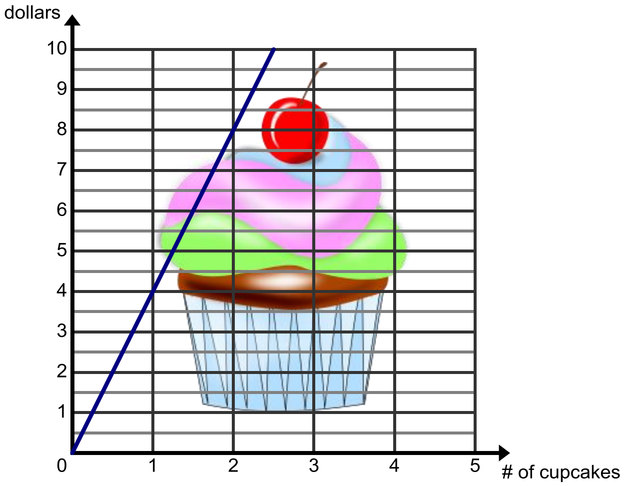
a) $y = 29x$ b) $y = 17x$

c) $y = 10x$ d) $y = -18x$

b. Explain your reasoning for your choice above.

c. The unit rate of 10 meters per seconds describes Christina's speed going down the same hill. Draw a line that could possibly represent her speed.

12. At Sweet Chicks Bakery the equation $y = 3.25x$ represents the total cost to purchases cupcakes; where x represents the number of cupcakes and y represents the total cost. The graph given below shows the cost for buying cupcakes at Butter Cream Fairy Bakery.



a. Which bakery offers the better deal? Use the equation and graph to justify your answer. Sweet Chicks Bakery offers the better deal because it costs $3.25 per cupcake at this bakery. At Butter Cream Fairy Bakery it costs $4.00 per cupcake.

b. Use the information given above to determine how much it will cost to buy 10 cupcakes at the bakery with the better deal. It will cost $32.50 to buy ten cupcakes at Sweet Chicks Bakery.

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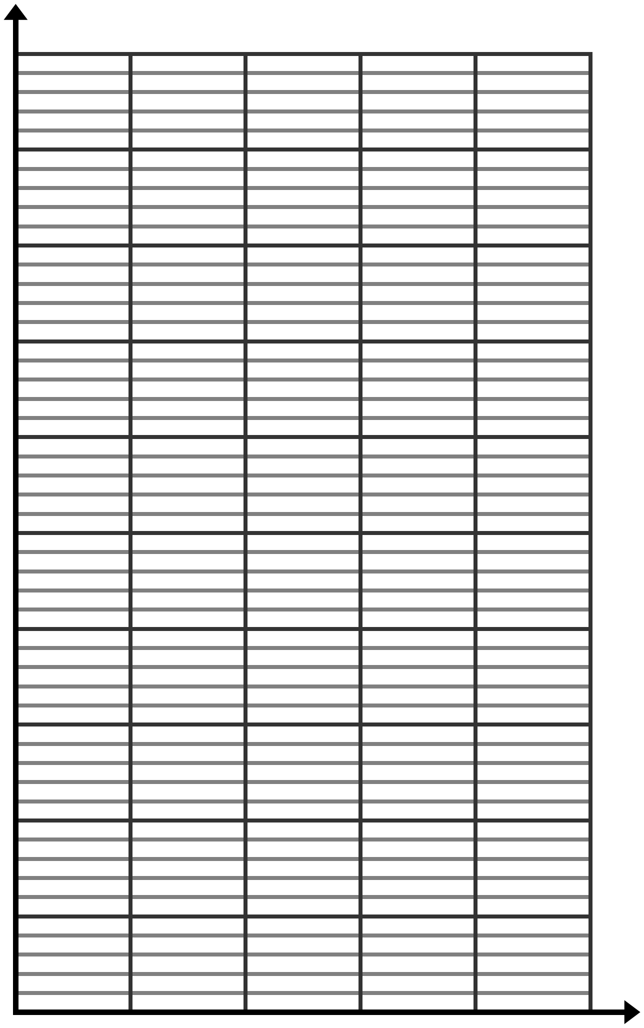
13. The table given below shows how much money Charlie earned every day that he worked last week. He gets paid the same rate every hour.

	Tuesday	Wednesday	Friday
Hours Worked	4	5	3.5
Money Earned	$38.00	$47.50	$33.25

Sophia earns $10.50 per hour at her job.

a. Using the same coordinate plane, draw a line that represents Charlie's earnings if *x* represents the number of hours worked and *y* represents the amount of money earned. Also draw a line that represents how much Sophia earns. Label each line with the person's name.



b. How can you use this graph to determine who makes more money?

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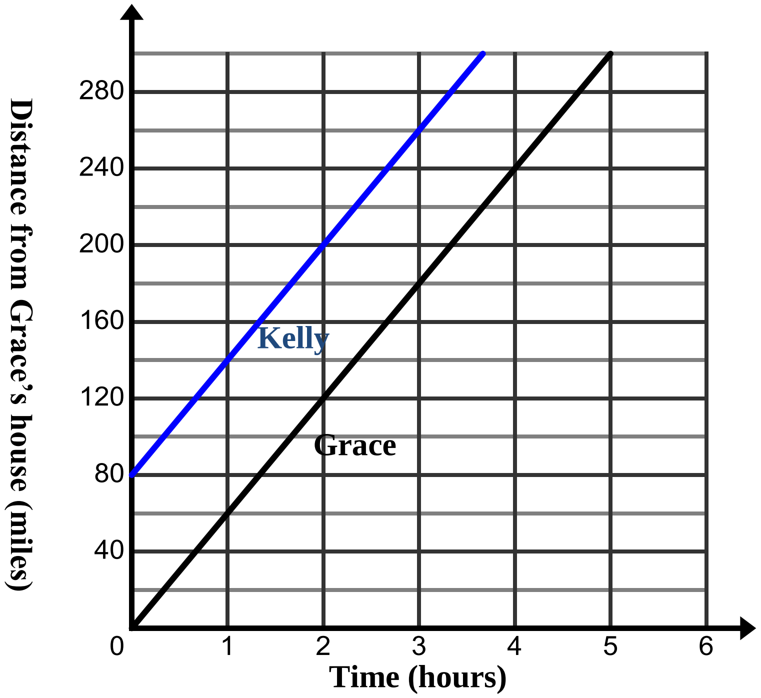
2.1c Class Activity: Proportional Relationships as Linear Relationships

1. Two cousins, Grace and Kelly, are both headed to the same summer camp. They both leave from their own house for camp at the same time. The graph below represents the girls' trips to camp.



a. Analyze the graph to determine which girl is traveling faster.
Both girls are driving 60 mph.

b. Complete the table below for Grace and Kelly.

Grace		
Time(x)	***Distance(y)***	$\frac{y}{x}$
0	0	Ø
1	60	60
2	120	60
3	180	60
4	240	60

Kelly		
Time(x)	***Distance(y)***	$\frac{y}{x}$
0	80	Ø
1	120	120
2	160	80
3	200	$66\frac{2}{3}$
4	240	60

c. What do you notice about the ratio $\frac{y}{x}$ for Grace? What do you notice about the ratio $\frac{y}{x}$ for Kelly? What is this ratio describing?
The ratio is the same for Grace. The ratio is not the same for Kelly. The ratio describes the proportional constant or unit rate that relates the time to the distance. The unit rate for grace is 60 mph. There is not a unit rate for Kelly.

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d. Describe why Kelly's driving relationship is not proportional?
There is not a proportional constant that relates the time to the distance. Her line is a straight line but does not go through the origin.

e. Is it possible to still describe the rate at which Kelly drives? If so, what is it?
It is possible to describe the rate that Kelly drives, you just have to find the change in distance over time. Kelly also travels 60 mph, she just lives 80 miles closer to camp.

> Even though Kelly's driving relationship is not proportional, it still exhibits a constant rate of change, as seen in the graph by a straight line. You can see this in the table as well by looking at the change that occurs between variables. In Kelly's case the distance increases by 60 miles as the time increases by 1 hour.

> Often the rate at which a relationship changes is shown by seeing that the changes from one measurement to another are proportional; that is, the quotient of the change in *y* values with respect to the *x* values is constant. This is called the **Rate of Change**.

Both of the relationships described above have a constant rate of change of 60 mph. This constancy defines them as **linear relationships.** (Their graphs produce straight lines).

f. Use what you learned above to see if you can write an equation that represents each girl's distance *y* from Grace's house after *x* hours.

Grace: ______$y=60x$________________________

Kelly: ______$y=60x+80$________________________

> You can see the constant rate of change in the tables. 60 is added to each row in the last column.

2. Agatha makes $26 for selling 13 bags of popcorn at the Juab County Fair.

a. Find and describe the rate of change for this relationship.
The rate of change is 2 dollars for every bag of popcorn sold.

b. Complete the table that shows the amount of money Agatha makes for selling up to three bags of popcorn.

x	*y*
0	0
1 $\xrightarrow{\times 2}$	2
2 $\xrightarrow{\times 2}$	4
3 $\xrightarrow{\times 2}$	6

+2
+2
+2
Rate of change pattern is seen vertically

Proportional constant pattern is seen horizontally. Constant ratio is $\frac{y}{x}$ or $\frac{2}{1}$

c. Graph the dollars to bags of popcorn relationship.

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d. Highlight on the graph where you can see the rate of change.
See graph.

e. Write an equation that represents the relationship between the number of bags of popcorn that Agatha sells(x) and the amount of money she makes (y).
Equation:_______ $y = 2x$ ______

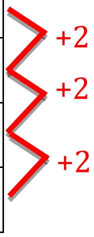
3. At the Sanpete County Fair Fitz gets paid $8 a day plus $2 for every bag of popcorn that he sells.

a. Find and describe the rate of change for this relationship. The unit rate for this relationship is two dollars for every bag of popcorn sold.

b. Complete the table that shows the amount of money that Fitz makes for selling up to three bags of popcorn.

> This is where students begin to see that a proportional relationship is a subset of a linear relationship. That it is in fact a special linear relationship. Be sure to talk about how the two relationships exhibited on this page are both linear relationships because they both have constant rate of change. Agatha's relationship shows a special case where the linear relationship is also proportional.

x	y
0	8
1	10
2	12
3	14



c. Graph this relationship on the same coordinate plane as Agatha's line on the previous page.
See graph on previous page.

d. Highlight on the graph where you can see the rate of change.
See graph.

e. Write an equation that represents the relationship between the number of bags of popcorn that Fitz sells(x) and the amount of money he makes(y).
Equation:______ $y = 8 + 2x$ ________

f. Write at least two sentences that explain the similarities and differences between Agatha's and Fitz's relationship.
Agatha and Fitz's relationships are both linear because they exhibit a constant rate of change. They both get paid the same amount of money per bag of popcorn because their rates of change are the same. Agatha's relationship is proportional because a proportional constant of $\frac{2}{1}$ relates the number of bags of popcorn she sells to the amount of money she makes. However, Fitz will make more money, if they sell the same amount of bags, because he starts with $8.

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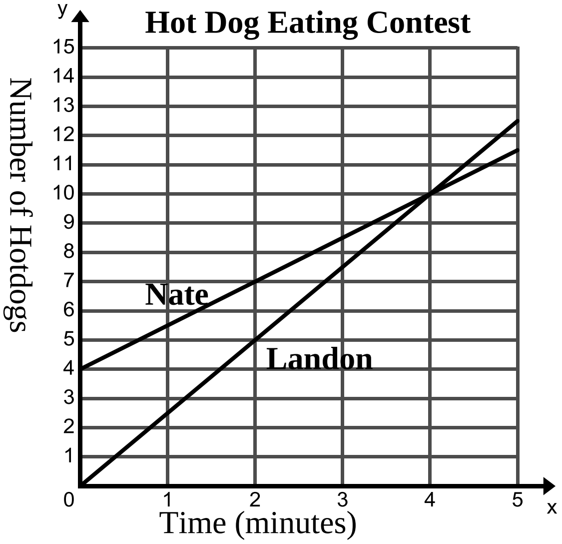
2.1c Homework: Proportional Relationships as Linear Relationships

1. Nate and Landon are competing in a 5 minute long Hot Dog eating contest. Nate has a special strategy to eat 4 hot dogs before the competition even begins to stretch out his stomach. The graph below represents what happened during the competition.



a. Compete the tables where t = time in minutes and h= number of total hotdogs consumed.

Landon		
t	h	$\frac{h}{t}$
0	0	
1	2.5	
3	7.5	
	12.5	

Nate		
t	h	$\frac{h}{t}$
0		
1	5.5	
2		
	8.5	
5		

b. Determine the rate of change (the number of hot dogs consumed per minute for each boy).

c. Write an equation that represents the number of hotdogs h for each boy after t minutes.
Landon: ____________________________

Nate: ____________________________

d. For which person, Landon or Nate, is the relationship between time and number of hot dogs eaten proportional?? Justify your answer.

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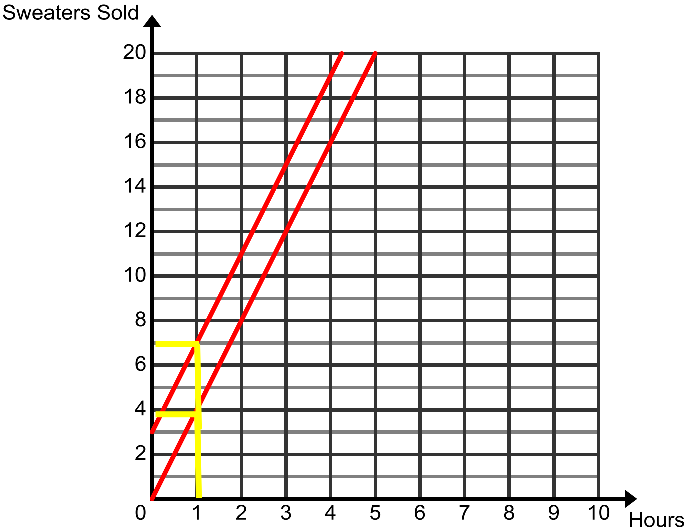
2. During her Tuesday shift at Sweater Barn, Fiona sells the same amount of sweaters per hour. Two hours into her shift Fiona has sold 8 sweaters.

   a. Find and describe the rate of change for this relationship.
   The rate of change for this relationship is 4 sweaters sold every hour.

   b. Complete the table given below where x is the number of hours worked and y is the total number of sweaters sold.

x	y
0	0
1	4
2	8
3	12

   c. Graph the relationship on the grid below.



   d. Write an equation that represents the relationship between the number of hours Fiona works(x) and the amount of sweaters she sells(y).

   Equation:______ $y = 4x$ _________

   e. Does this represent a proportional relationship? Explain how you know.
   This is a proportional relationship because the proportional constant is 4 and when the relationship is graphed, it is a straight line going through the origin.

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3. On Saturday Fiona gets to work 15 minutes early and sells three sweaters before her shift even begins. She then sells 4 sweaters every hour for the rest of her shift.

   a. Find and describe the rate of change for this relationship.

   b. Complete the table that represents this relationship.

   c. Graph this relationship on the same coordinate plane as Tuesday's information on the previous page. See graph on previous page.

   d. Write an equation that represents the relationship between the number of hours Fiona works(x) and the amount of sweaters she sells(y).
   Equation:____________________

x (hours)	y (sweaters sold)
0	3
1	
	11
3	

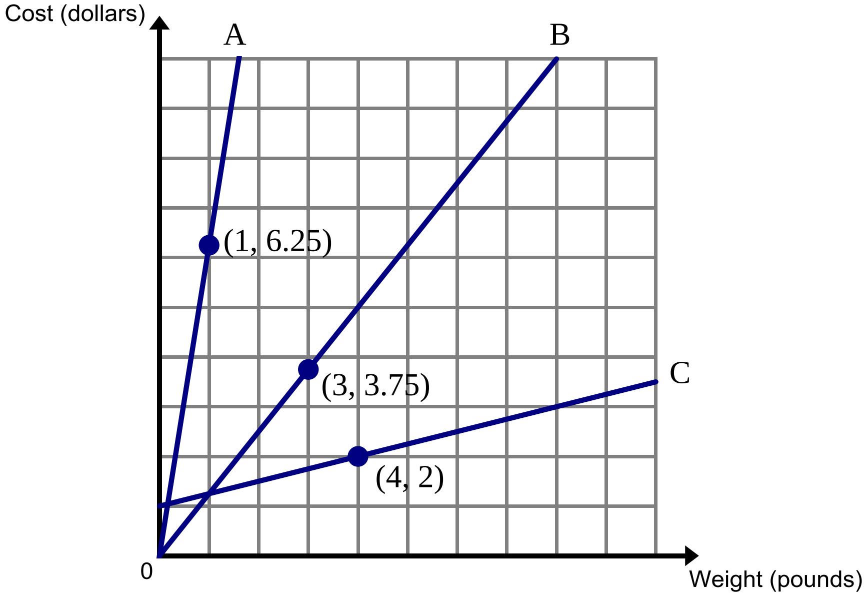
   e. Does this represent a proportional relationship? Explain how you know.

   f. Compare the rate of change of both of the lines on the previous page by highlighting the change on the graph. What do you notice? The rate of change for both of the lines is 4.

4. Nayala bought 5 pounds of mangos for $6.25.

   a. What is the price per pound for the mangos that she bought?
   The mangos cost $1.25 per pound.

   b. Which line below, A, B, or C, represents the cost in dollars(y) to weight in pounds(x) relationship?



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2.1d Self Assessment: Section 2.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample criteria are provided along with sample problems for each skill/concept on the following page.

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Understanding 3**	**Substantial Understanding 4**
1. Graph and write equations for a proportional relationship and identify the proportional constant or unit rate given a table, equation, or contextual situation. *See sample problem #1*	I can correctly answer only 1 of the three parts of the question.	I can correctly answer 2 of the three parts of the question.	I can correctly answer all three parts of the question but cannot explain my answers.	I know how to find the unit rate for both Callie and Jeff and state what the unit rate is describing. I can accurately label the graphs and write an equation that represents the amount of money earned in relationship to the number of papers each person delivered.
2. Compare proportional relationships represented in different ways. *See sample problem #2*	I do not know how to compare these proportional relationships.	I can find the unit rate for only one relationship.	I can find the unit rate for each relationship represented and then compare the unit rates to determine who makes more money. I do not know how to make a third representation for someone who makes more money than Addy and Rachel.	I can find the unit rate for each relationship represented and then compare the unit rates to determine who makes more money. I can also create a third representation for someone who makes more money than Addy and Rachel.

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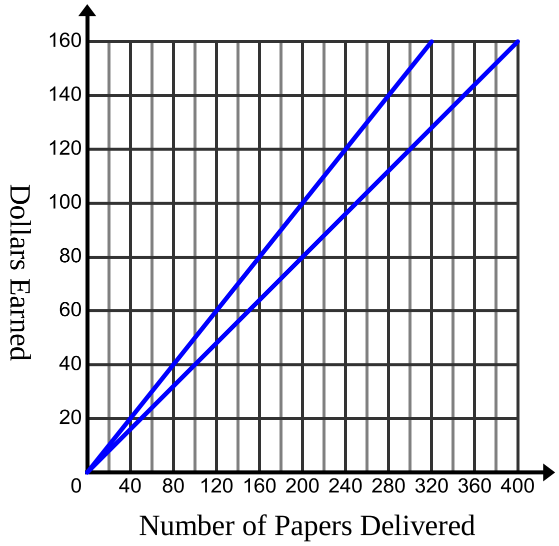
Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Understanding 3**	**Substantial Understanding 4**
3. Know that the graph of a proportional relationship goes through the origin. *See sample problem #3*	I might be able to make a graph of Jeff's savings but I do not know how it relates to the other graph.	I can make a graph that show's Jeff's savings but I don't know how it relates to the other graph.	I can make a graph that shows Jeff's savings. I can discuss some of the similarities and differences between the two graphs.	I can make (and clearly label) a graph that shows Jeff's savings. I can discuss the similarities and differences between the two graphs including a discussion about proportionality and slope.

Sample Problem #1
Callie and Jeff each have a job delivering newspapers. Jeff gets paid $140 dollars for delivering 350 papers. Callie gets paid $100 for delivering 200 papers.

a. Find the unit rate for each person. Be sure to state what the unit rate is describing.
Callie:

Jeff:

b. The graph given below shows the money earned to papers delivered relationship. Label which line represents each person.



c. Write an equation that represents how much each person earns if *x* is the number of papers they deliver and *y* represents how much money they make.

Callie: Jeff:

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Sample Problem #2

Below is a table of how much money Rachel earns on her paper route. She gets paid the same amount of money per paper delivered.

Number of Papers Delivered	Money Earned
75	$36
150	$72
225	$108

The equation below represents how much money Addy makes delivering papers. In the equation p represents the number of papers delivered and d represents the money earned.

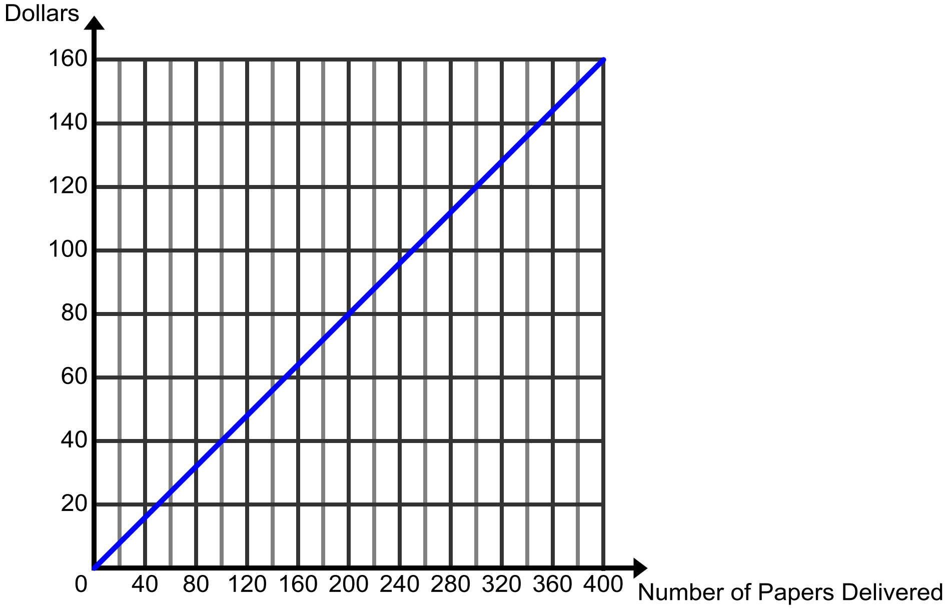
$$d = .45p$$

a. Who makes more money? How do you know?

b. Create a representation for someone who delivers papers and makes more than both Addy and Rachel.

Sample Problem #3

The graph provided below shows the amount of money that Jeff earns delivering papers. Suppose that Jeff had $75 dollars in savings before he started his job of delivering newspapers. Jeff saves all of his money earned from delivering newspapers. Graph this relationship below where x is the number of papers he delivers and y is the amount of money he has in savings. Correctly label each line has Total Savings and Money Earned.



How does the savings line compare to his money earned line? Be sure to discuss the proportionality of each graph.

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Section 2.2: Linear Relations in Pattern and Context

Section Overview:

In this section, students start by writing rules for linear patterns. They use the skills and tools learned in Chapter 1 to write these equations. Students connect their rules to the geometric model and begin to surface ideas about the rate of change and initial value (starting point) in a linear relationship. Students continue to use linear patterns and identify the rate of change and initial value in the different representations of a linear pattern (table, graph, equation, and geometric model). They also begin to understand how linear functions change. Rate of change is investigated as students continue to interpret the parameters m and b in context and advance their understanding of a linear relationship. Students move fluently between the representations of a linear relationship and make connections between the representations. This conceptual foundation will set the stage for students to be able to derive the equation of a line using dilations in section 2.3.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Write rules for linear patterns and connect the rule to the pattern (geometric model).
2. Understand how a linear relationship grows as related to rate of change and show how that growth can be seen in each of the representations.
3. Create the additional representations (table, graph, equation, context, geometric model) of a linear relationship when given one representation and make connections between them.
4. Identify the rate of change and initial value of a linear relationship in the table, graph, equation, context, and geometric model of a linear pattern.

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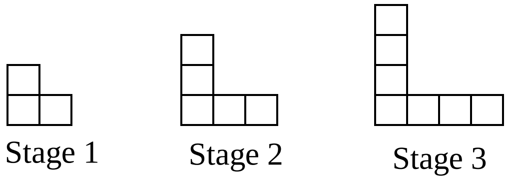
2.2a Class Activity: Connect the Rule to the Pattern

Linear relationships can be used to illustrate many patterns. The patterns in the problems below exhibit linear relationships.

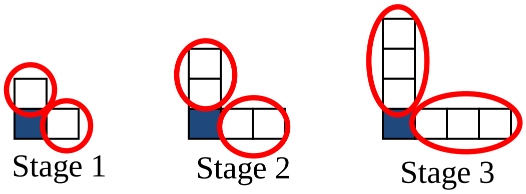
1. Use the pattern below to answer the questions that follow.



a. Draw the figure at stage 4 in the space above. How did you draw your figure for stage 4 (explain or show on the picture how you see the pattern growing from one stage to the next)?
Some responses may be: "I drew the previous stage and added 1 block to each end." "I drew the corner square and realized that stage 4 would have four blocks on each arm attached to the corner block."

b. How many blocks are in stage 4? Stage 10? Stage 100? Stage 4 has 9 blocks. Students will most likely find the number of blocks in stage 10 (21) by adding two until they reach stage 10 or creating a sequence or table. Some may draw out all the stages and you may want to ask if that is an efficient way of finding the total number of blocks for stage 10 (201). If students are using one of the methods mentioned they will run into difficulty finding the number of blocks in stage 100 - this gives purpose to finding an equation that relates the number of blocks to the stage number.

c. Write a rule that gives the total number of blocks *t* for any stage *s*. Show how your rule relates to the pattern (geometric model).
The answer to question a. (how students drew the figure in stage 4) will help students to come up with a rule. Here is one way students might see the pattern.



The rule for this way of seeing the pattern is

$$t = 1 + 2s$$

The student is seeing the corner block plus 2 groups of *s*.
If students have a difficult time coming up with the rule, have them write it out numerically:
1+2(1) for stage 1
1+2(2) for stage 2
1+2(3) for stage 3
1+2(4) for stage 4
1+2(100) for stage 100
$t = 1 + 2s$ for stage *s*

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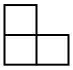
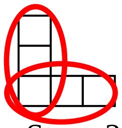
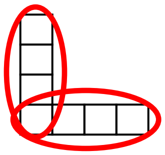
d. Try to think of a different rule that gives the total number of blocks t for any stage, s. Show how your rule relates to the pattern (geometric model).


Stage 1


Stage 2


Stage 3

The rule for this way of seeing the pattern is

$$t = 2(s + 1) - 1$$

The student is seeing the 2 groups of $(s + 1)$ and subtracting out one block because they counted the corner block twice. If students have a difficult time coming up with the rule, have them write it out numerically:

2(2)-1 for stage 1
2(3)-1 for stage 2
2(4)-1 for stage 3
2(5)-1 for stage 4
$2(s + 1) - 1$ for stage s

e. Use your rule to determine the number of blocks in stage 100.
201

f. Use your rule to determine which stage has 25 blocks.
12

g. Draw or describe stage 0 of the pattern. How does the number of blocks n in stage 0 relate to the simplified form of your rule?


Stage 0

It is the number of blocks in stage 0 (the number of blocks the pattern starts with). In the simplified form of the rule, it is the constant.

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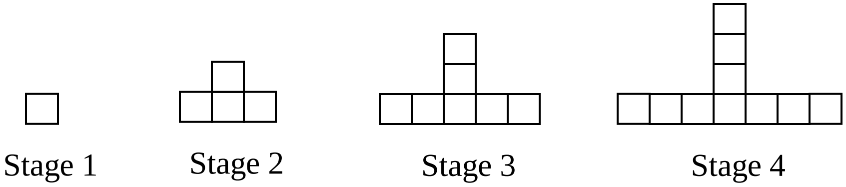
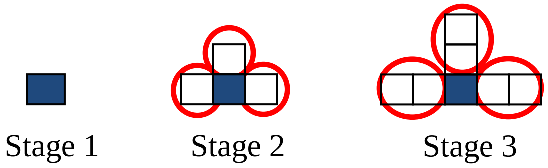
2. Use the pattern below to answer the questions that follow.



a. Draw the figure at stage 5 in the space above. How did you draw your figure in stage 5 (explain or show on the picture how you see the pattern growing from one stage to the next)?
Some responses may be: “I added a block to the end of each leg.” “I saw the middle square and drew three legs each with 4 blocks coming off the middle square.”

b. How many blocks are in stage 5? Stage 10? Stage 100?
Step 5 has 13 blocks. Students will most likely find the number of blocks in stage 10 (28) by adding 3 until they reach stage 10 or creating a sequence or table. Some may draw out all the stages and you may want to ask if that is an efficient way of finding the total number of blocks for stage 10. If students are using one of the methods mentioned they will run into difficulty finding the number of blocks in stage 100 - this gives purpose to finding an equation that relates the number of blocks to the stage number.

c. Write a rule that gives the total number of blocks *t* for any stage, *s*. Show how your rule relates to the pattern (geometric model).



The rule for this way of seeing the pattern is

$$t = 1 + 3(s - 1)$$

The student is seeing the middle block plus 3 groups of -1 . If students have a difficult time coming up with the rule, have them write it out numerically:
1+3(0) for stage 1
1+3(1) for stage 2
1+3(2) for stage 3
1+3(3) for stage 4
1+3(99) for stage 100
$1 + 3(s - 1)$ for stage *s*

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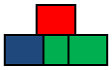
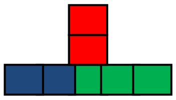
d. Try to think of a different rule that gives the total number of blocks *t* for any stage, *s*. Show how your rule relates to the pattern (geometric model).

Stage 1



Stage 2



Stage 3

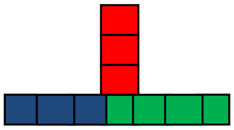
The rule for this way of seeing the pattern is

$$t = (s-1) + s + (s-1)$$

The student is seeing the base as comprised of the the previous stage number $(s-1)$ + current stage number s.

The vertical column is comprised of the previous stage number $(s-1)$.

If students have a difficult time coming up with the rule, have them write it out numerically:



Stage 4

0+1+0 for stage 1
1+2+1 for stage 2
2+3+2 for stage 3
3+4+3 for stage 4
99+100+99 for stage 100
$(s-1) + s + (s-1)$ for stage *s*

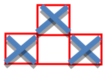
Then probe them to think about finding the number of blocks in the 10th stage? 100th stage? any stage *s*?

e. Use your rule to determine the number of blocks in Stage 100.
298

f. Use your rule to determine which stage has 58 blocks.
20

g. Draw or describe stage 0 of the pattern. How does the number of blocks in stage 0 relate to the simplified form of your rule?
If we think about going back from step 1 to step 0, we see that we have to take away three blocks, thus leaving us with -2 blocks.

Encourage students to articulate and show how they see these “negative” blocks. One way to think about this in the geometric model is to think of the three blocks that make up the arms as negative blocks (or blocks that take away from the total number of blocks in the model).



Stage 0

It is the number of blocks in step 0 (the number of blocks the pattern starts with). In the simplified form of the rule, it is the constant.

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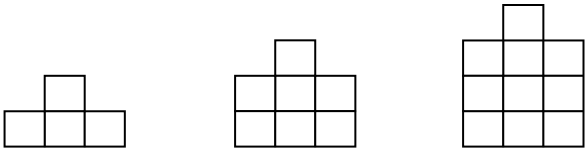
2.2a Homework: Connect the Rule to the Pattern

1. Use the pattern below to answer the questions that follow.

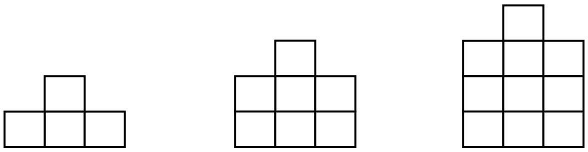


Stage 1 Stage 2 Stage 3

   a. Draw the figure at stage 4 in the space above. How did you draw your figure in stage 4 (explain or show on the picture how you see the pattern growing from one step to the next)?

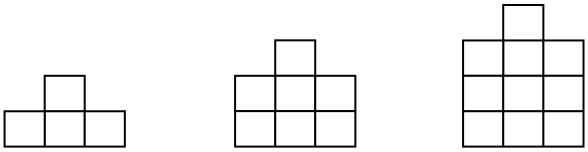
   b. How many blocks are in stage 4? Stage 10? Stage 100?

   c. Write a rule that gives the total number of blocks *t* for any stage *s*. Show how your rule relates to the pattern (geometric model).



Stage 1 Stage 2 Stage 3

   d. Try to think of a different rule that gives the total number of blocks *t* for any stage, *s*. Show how your rule relates to the pattern (geometric model).



Stage 1 Stage 2 Stage 3

   e. Use your rule to determine the number of blocks in stage 100.

   f. Use your rule to determine which stage has 28 blocks.

   g. Draw or describe stage 0 of the pattern. How does the number of blocks in stage 0 relate to the simplified form of your rule?

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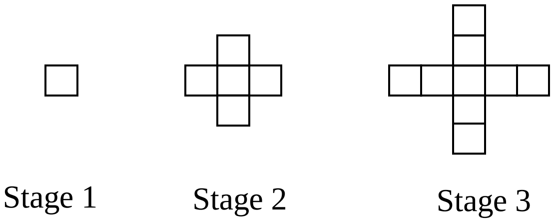
2. Use the pattern below to answer the questions that follow.

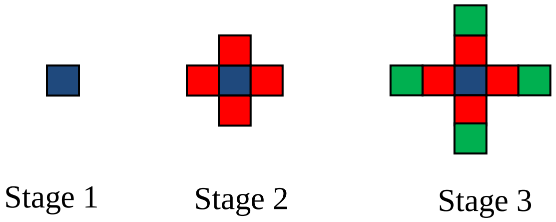


Some students may see this pattern as being very similar to #2 in class. They may realize that they can use their rule from #2 in class and determine an expression to add on that represents the number of blocks in the bottom leg.

a. Draw the figure at stage 4 in the space above. How did you draw your figure in stage 4 (explain or show on the picture how you see the pattern growing from one stage to the next)?

b. How many blocks are in stage 4? Stage 10? Stage 100? stage 4 = 13, stage 10 = 37

c. Write a rule that gives the total number of blocks *t* for any stage, *s*. Show how your rule relates to the pattern (geometric model).

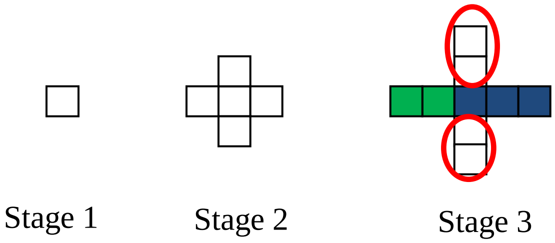


The rule for this way of seeing the pattern is

$$t = 1 + (s - 1)4$$

The block in the middle + $(s - 1)$ copies of 4

d. Try to think of a different rule that gives the total number of blocks *t* for any stage, *s*. Show how your rule relates to the pattern (geometric model).



The rule for this way of seeing the pattern is

$$t = 2(s - 1) + s + (s - 1)$$

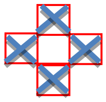
e. Use your rule to determine the number of blocks in stage 100.
397

f. Use your rule to determine which stage has 37 blocks.
Stage 10

g. Draw or describe Stage 0 of the pattern. How does the number of blocks in stage 0 relate to the simplified form of your rule?

Similar to #2 from the classwork, if we think about going back from stage 1 to stage 0, we see that we have to take away four blocks, thus leaving us with -3 blocks.

One way to think about this in the geometric model is to think of the three blocks that make up the arms as negative blocks (or blocks that take away from the total number of blocks in the model).



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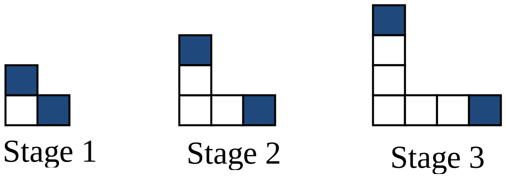
2.2b Class Activity: Representations of a Linear Pattern

1. You studied this pattern in the previous lesson. Use your work from the previous lessons to answer the questions that follow.



a. How many new blocks are added to the pattern from one stage to the next? 2

b. Complete the table.

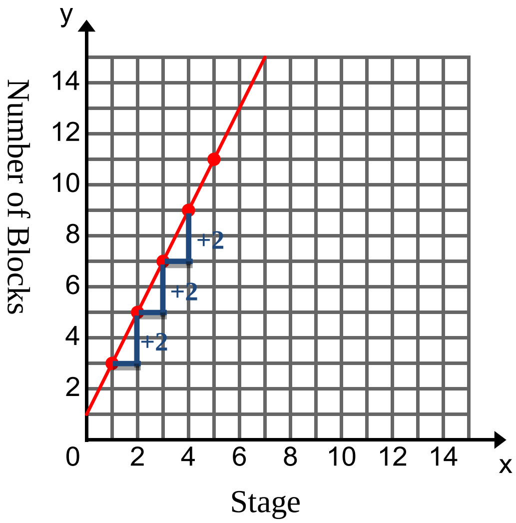
c. Show where you see the rate of change in your table.

Stage (s)	***# of Blocks (t)***
1	3
2	5
3	7
4	9
5	11

+1
+1
+1
+1

+2
+2
+2
+2

d. Create a graph of this data. Where do you see the rate of change on your graph?



e. What is the simplified form of the equation that gives the number of blocks t for any stage s (see previous lesson)? Where do you see the different pieces of the equation in the geometric model, table, and graph? $t = 2s + 1$
The 2 in the equation connects the slope on the graph, the difference column on the table, and the new blocks being added to the pattern from one stage to the next. The 1 in the equation connects to the y-intercept on the graph.

f. The pattern above is a **linear** pattern. Describe how a linear pattern grows. Describe what the graph of a linear pattern looks like.
There are many ways to describe how a linear pattern grows: constant rate of change, equal differences over equal intervals, first difference in the table is constant. The graph is a line.

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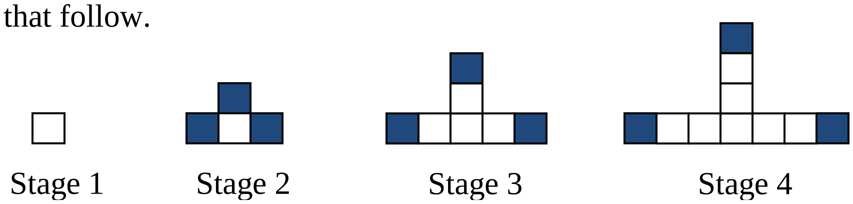
2. You studied this pattern in the previous lesson. Use your work from the previous lesson to answer the questions that follow.



a. How many new blocks are added to the pattern from one stage to the next? 3

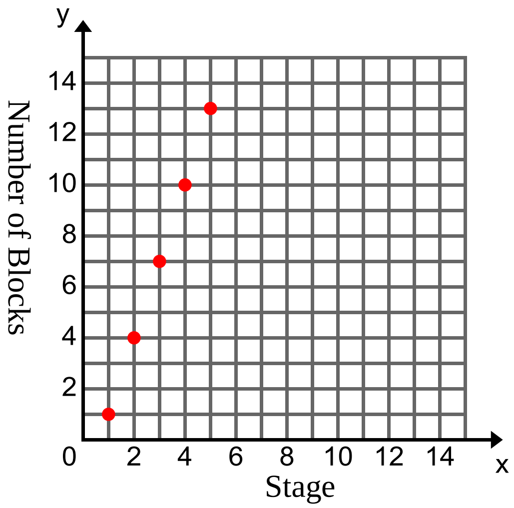
b. Complete the table.

c. Show where you see the rate of change in your table.

***Stage* (*s*)**	***# of Blocks* (*t*)**
1	1
2	4
3	7
4	10
5	13

+3
+3
+3
+3

d. Create a graph of these data. Where do you see the rate of change on your graph?



e. What is the equation that gives the number of blocks *t* for any stage *s* (see previous lesson)? Where do you see the different pieces of the equation in the geometric model, table, and graph?

$t = 3s - 2$

f. Is this pattern a linear pattern? Use supporting evidence from each of the representations to justify your answer.

Yes, all models show a constant rate of change. Geometric Model: same number of blocks added each time; Table: the first difference is constant; Graph is a straight line.

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3. Describe what you see in each of the representations (geometric model, table, graph, and equation) of a linear pattern. Make connections between the different representations.

Geometric Model:

The same number of blocks is added to the figure each time to get from one stage to the next. This shows a constant rate of change. Students can think about this recursive process as now = previous + 3. Students may also just show arrows above the pattern, annotating the +3 every time.

Table:

In the table, the first difference is constant.

Graph:

The constant rate of change makes a straight line when graphed.

Equation:

$y = mx + b$ (While students may not make the connection to the general form of a linear equation they may be able to communicate the different pieces as a constant/y-intercept/initial value + rate of change(x).)

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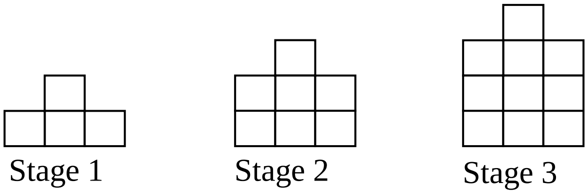
2.2b Homework: Representations of a Linear Pattern

1. You studied this pattern in the previous lesson. Use your work from the previous lesson to answer the questions that follow.



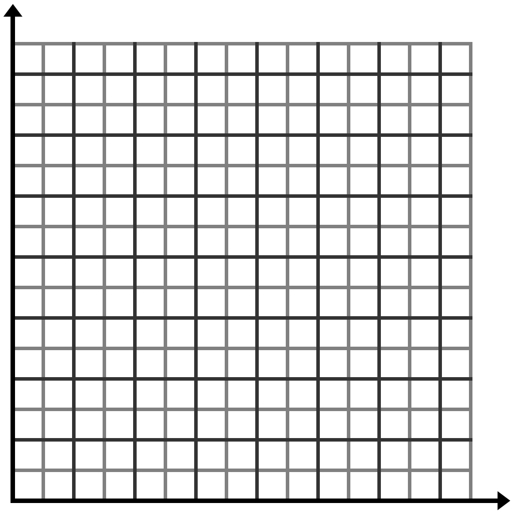
   a. How many new blocks are added to the pattern from one stage to the next?

   b. Complete the table.

   c. Show where you see the rate of change in your table.

Stage (s)	***# of Blocks (t)***
1	
2	
3	
4	
5	

   d. Create a graph of this data. Where do you see the rate of change on your graph?



   e. What is the equation that gives the total number of blocks *t* for any stage *s* (see previous lesson)? Where do you see the different pieces of the equation in the geometric model, table, and graph?

   f. Is this pattern a linear pattern? Use supporting evidence from each of the representations to justify your answer.

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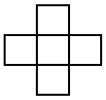
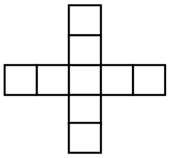
2. You studied this pattern in the previous lesson. Use your work from the previous lesson to answer the questions that follow.


Stage 1


Stage 2


Stage 3

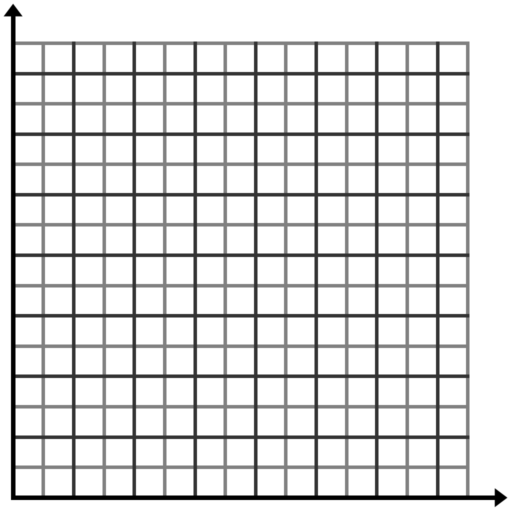
a. How many new blocks are added to the pattern from one stage to the next?

b. Complete the table.

c. Show where you see the rate of change in your table.

Stage (s)	***# of Blocks (t)***
1	1
2	5
3	9
4	13
5	17

d. Create a graph of this data. Where do you see the rate of change on your graph?



e. What is the equation that gives the total number of blocks *t* for any stage *s* (see previous lesson)? Where do you see the different pieces of the equation in the geometric model, table, and graph?
$t = 4s - 3$

f. What do you notice about this pattern? Use supporting evidence from each of the representations to justify your answer.
The pattern is linear. There is a constant rate of change found in the table and the graph is a straight line.

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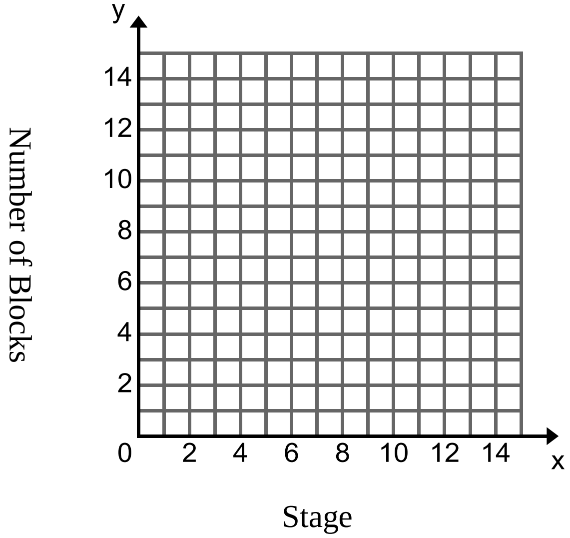
3. Create your own geometric model of a linear pattern in the space below. Then complete the table, graph, and equation for your pattern. Use these representations to prove that your pattern is linear.

Stage (s)	***# of Blocks (t)***
1	
2	
3	
4	
5	



Equation: ______________________________

Prove that your pattern is linear using the representations (geometric model, table, graph, and equation) as evidence.

4. Draw or describe a pattern that can be represented by the equation $t = 1 + 6s$ where t is the total number of blocks and s is the stage

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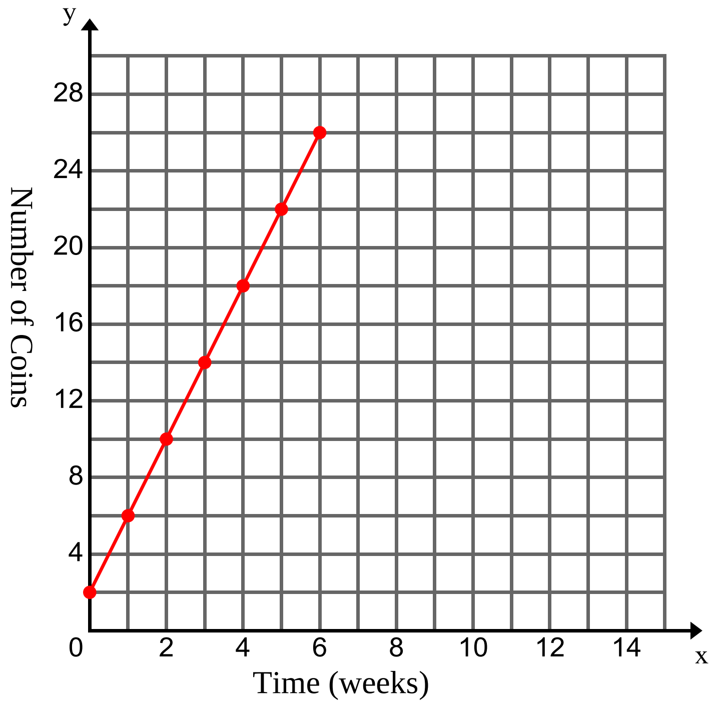
2.2c Class Activity: Representations of a Linear Context

1. Courtney is collecting coins. She has 2 coins in her collection to start with and plans to add 4 coins each week.

   a. Complete the table and graph to show how many coins Courtney will have after 6 weeks.



Time (weeks)	***# of Coins***
0	2
1	6
2	10
3	14
4	18
5	22
6	26

   b. Write an equation for the number of coins c Courtney will have after w weeks.

   $c = 2 + 4w$

   c. Is this context linear? Use evidence from the context, graph, table, and equation to support your answer.

   Yes. Context – add four coins **each week**(constant rate of change); Graph is a line; Table – constant difference of 4 in the difference column; Equation: $y = mx + b$ (while students may not make the connection to the general form of a linear equation they may be able to communicate the different pieces as a constant/y-intercept/initial value + slope/(x)

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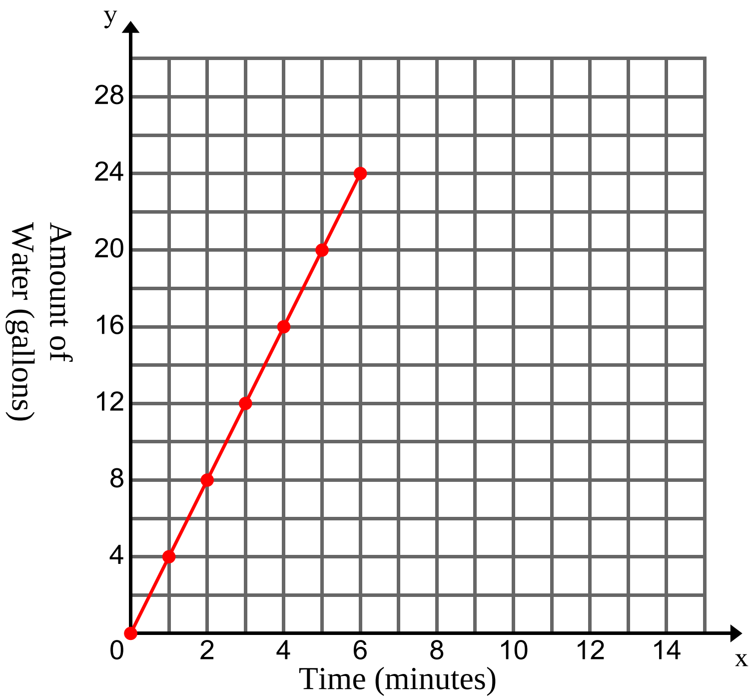
2. Jack is filling his empty swimming pool with water. The pool is being filled at a constant rate of four gallons per minute.
   a. Complete the table and graph below to show how much water will be in the pool after 6 minutes.



Time (minutes)	***Amount of Water (gallons)***
0	0
1	4
2	8
3	12
4	16
5	20
6	24

   b. Write an equation for the number of gallons *g* that will be in the pool after *m* minutes.

   $g = 4m$

   c. Is this context linear? Use evidence from the context, graph, table, and equation to support your answer.

   Yes. Context – constant rate of 4 gallons **per minute**; Graph is a line; Table – constant difference of 4 in the difference column; Equation: The equation is of the form $y = mx + b$.

   d. Compare this swimming pool problem to the previous problem about coins. How are the problems similar? How are they different?

   The rate of change is 4 in both problems; however in the coin problem the initial value is 2 while in the swimming pool problem the initial value is 0.

   e. How would you change the coin context so that it could be modeled by the same equation as the swimming pool context?

   Start with 0 coins in the collection.

   f. How would you change the swimming pool context so that it could be modeled by the same equation as the coin context?

   Start with 2 gallons of water in the pool

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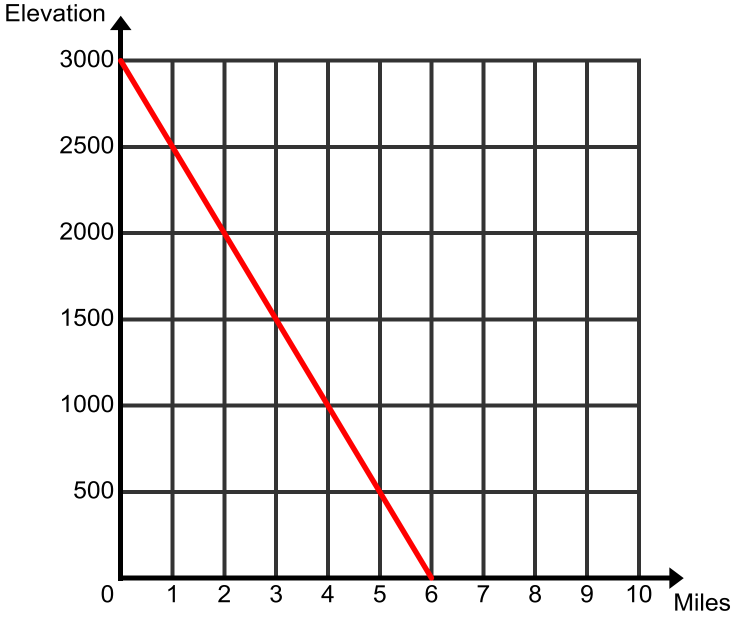
3. An airplane is at an elevation of 3000 ft. The table below shows its elevation(y) for every 2 miles(x) it travels.

	Miles	Elevation	
	0	3000	
+2			-1000
	2	2000	
+2			-1000
	4	1000	

a. Complete the graph to show how many miles it will take for the airplane to reach the ground.



According to the graph it will take 6 miles to reach the ground.

b. Use the table and the graph to find the rate of change.
The rate of change is -500 ft per mile.

c. Write an equation that represents this relationship
$y = -500m + 3000$

d. Explain the how equation can be used to determine how many miles it will take for the plane to reach the ground.
The ground would be an elevation of 0 so if you plug 0 into your equation for y and solve for x you will get 6 miles.

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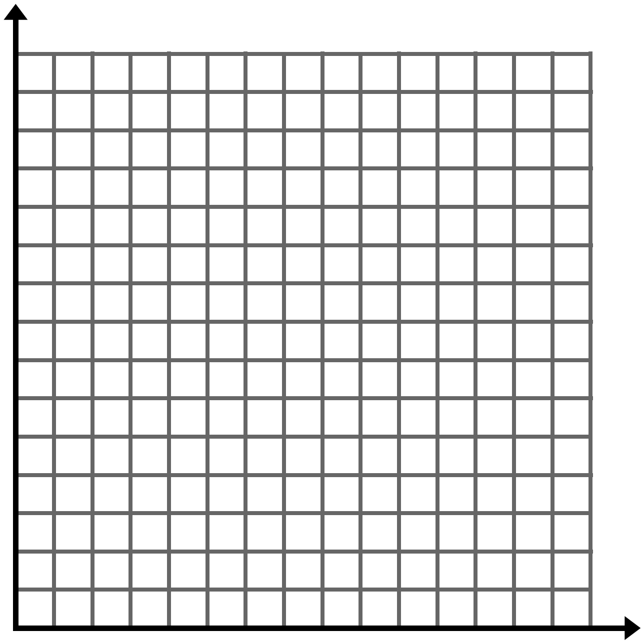
2.2c Homework: Representations of a Linear Context

1. Hillary is saving money for college expenses. She is saving \$200 per week from her summer job. Currently, she does not have any money saved.

   a. Complete the table and graph to show how much money Hillary will have 6 weeks from now.

Time (weeks)	***Amount Saved (dollars)***
0	
1	
2	
3	
4	
5	
6	



   b. Write an equation for the amount of money *m* Hillary will have saved after *w* weeks if she continues saving at the same rate.

   c. Is this context linear? Use evidence from the context, graph, table, and equation to support your answer.

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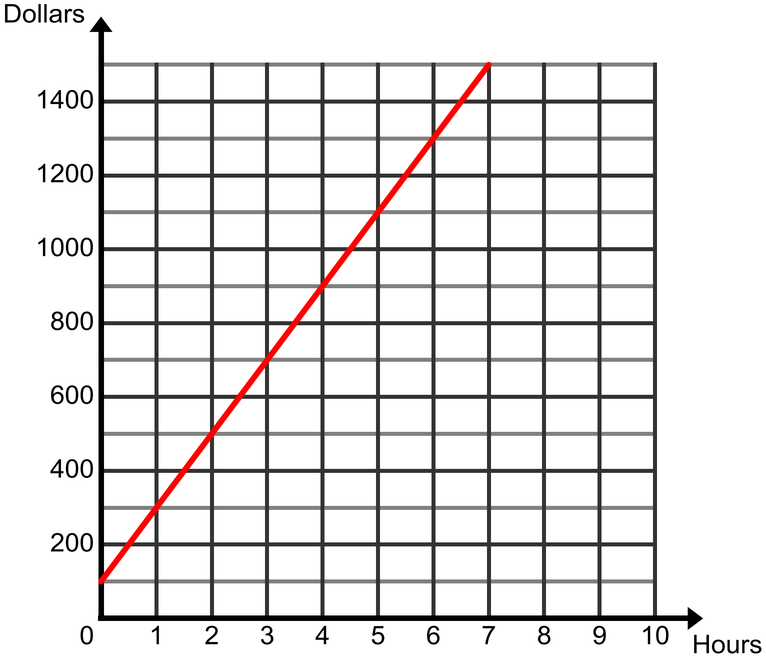
2. The cost for a crew to come and landscape your yard is \$200 per hour. The crew charges an initial fee of \$100 for equipment.

   a. Complete the table and graph below to show how much it will cost for the crew to work on your yard for 6 hours.

Time (hours)	***Cost (dollars)***
0	100
2	500
4	900
6	1300



   b. Write an equation for the cost *c* of landscaping for *h* hours.
   $c = 100 + 200h$

   c. Is this context linear? Use evidence from the context, graph, table, and equation to support your answer.
   Yes. Context – 200 dollars **per hour**; Graph is a line; Table – constant difference of 200 in the difference column; Equation: $y = mx + b$

   d. Compare this landscaping problem to the problem with Hillary's savings. How are the problems similar? How are they different?
   The rate of change is 200 in both problems; however in Hillary's savings the initial value is 0 and in the landscaping problem, the initial value is 100.

   e. How would you change the savings context so that it could be modeled by the same equation as the landscaping context?
   Hillary would have \$100 in her bank account to start.

   f. How would you change the landscaping context so that it could be modeled by the same equation as the savings context?
   The crew would not charge an initial fee of \$100 for equipment – they would only charge an hourly fee of \$200.

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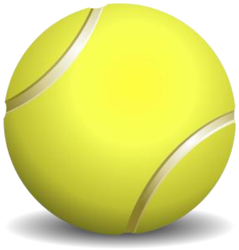
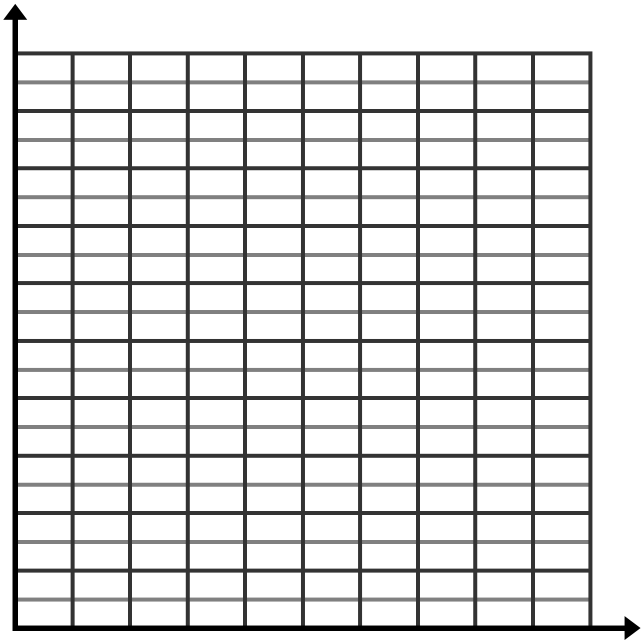
3. Linda is always losing her tennis balls. At the beginning of tennis season she has 20 tennis balls. The table below represents how many balls she has as the season progresses; where x represents the number of weeks and y represents the number of tennis balls.

Weeks	***Number of tennis balls***
0	20
2	16
4	12
6	8



a. Complete the graph to show how many weeks will pass until Linda runs out of balls.



b. Use the table and the graph to find the rate of change.

c. Write an equation that represents this relationship.

d. Explain how to use the equation to determine how many weeks will pass until Linda runs out of balls.

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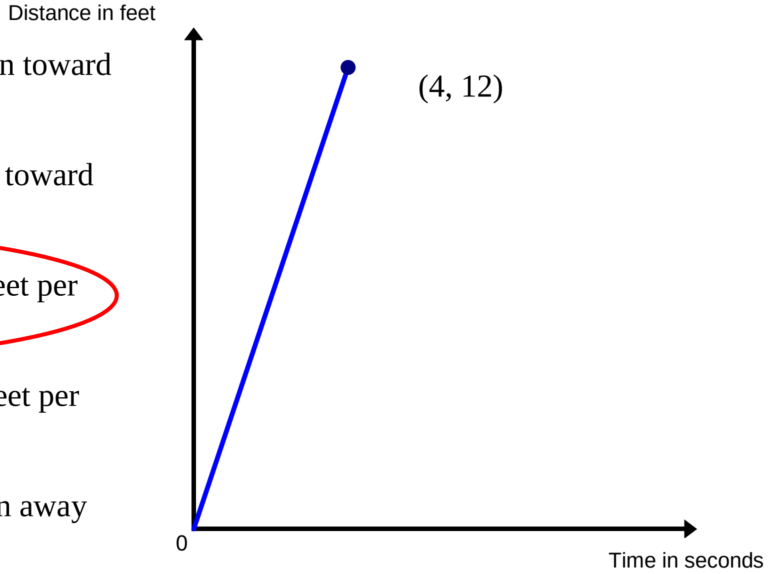
2.2d Class Activity: Rate of Change in a Linear Relationship

Explore and investigate the rate of change in linear relationships below.

1. The graph below shows the distance a cat is from his bowl of milk over time. Which sentence is a good match for the graph?



A. The cat was 12 feet away from the milk and ran toward it reaching it after 4 seconds.

B. The cat was 4 feet away from the milk and ran toward it reaching it after 12 seconds.

[✓ C. The cat ran away from the milk at a rate of 3 feet per second.]

D. The cat ran away from the milk at a rate of 4 feet per second.

E. The cat was 12 feet away from the milk and ran away from it at a rate of 4 feet per second.

2. Write everything you can say about the cat and the distance he is from the milk during this time.
   The cat starts at the bowl of milk. After 4 seconds he is 12 feet away from the milk. He travels at a rate of 3 ft per second.

3. Create a table at the right which also tells the story of the graph.
   See table

Time (seconds)	Distance (feet)
0	0
1	3
2	6
3	9
4	12

4. Is this a proportional relationship? Justify your answer.
   Yes, the proportional constant is 3 feet per second.

5. Find the unit rate in this story.
   The cat travels 3 feet per second.

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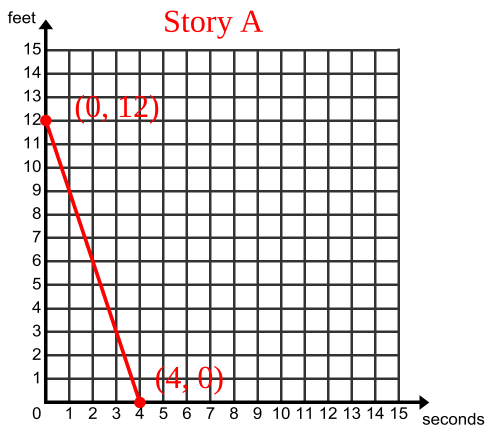
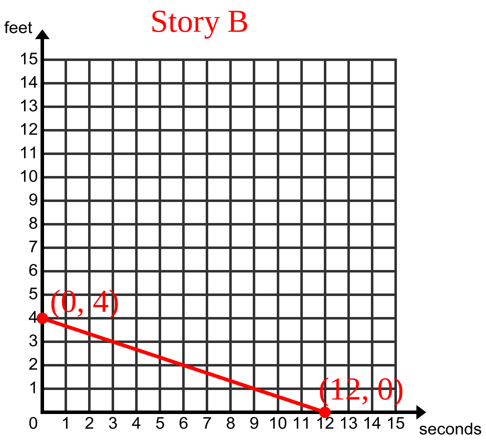
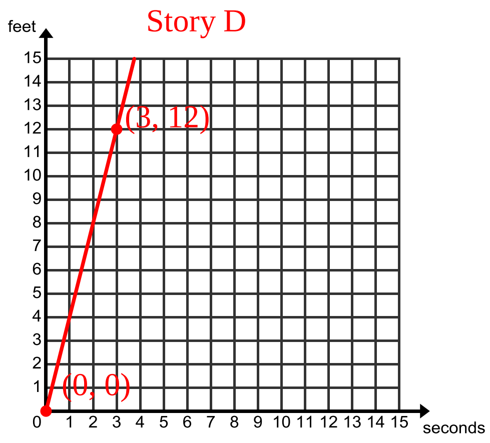
6. Sketch a graph for each of the four stories from number 1 on the previous page which you didn't choose. Label the graphs by letter to match the story. Find the rate of change for each story as well.



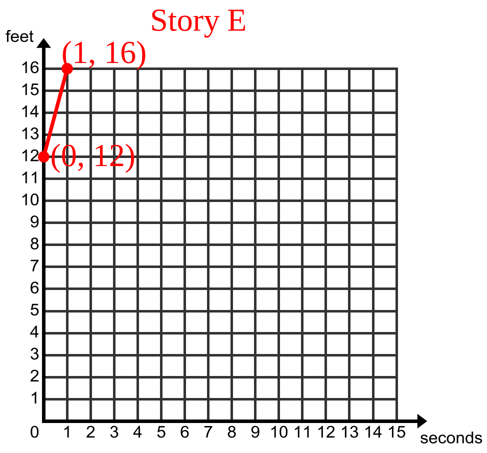
Rate of change: -3 feet per second



Rate of change: -1/3 feet per second



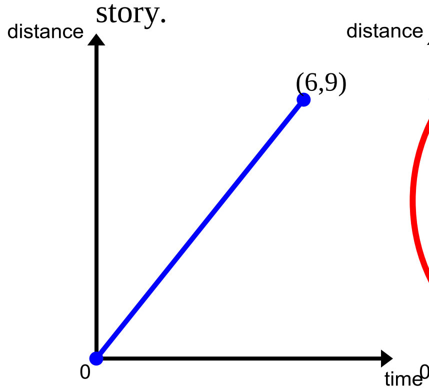
Rate of change: 4 feet per second

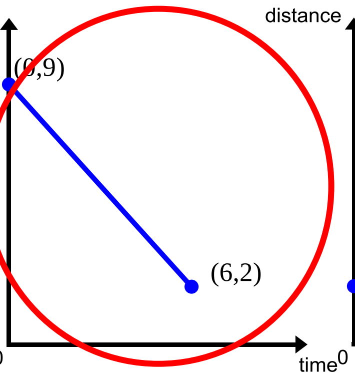


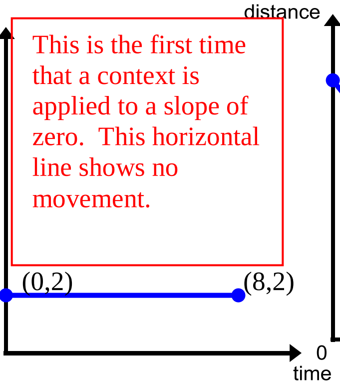
Rate of change: 4 feet per second

All of the graphs above are linear because they are a straight line and have a constant rate of change. Story D is a special linear relationship because it is also proportional. Tnegative unit rates denote that the cat is moving toward the milk.

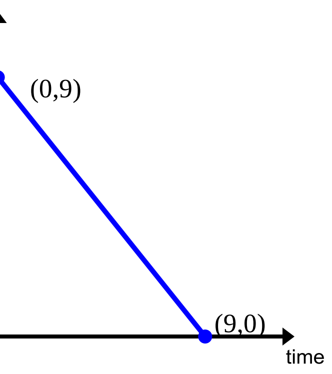
7. A baby was 9 feet from the edge of the porch. He crawled toward the edge for 6 seconds. Then his mother picked him up a few feet before he reached the edge. Circle the graph below that matches this story.







This is the first time that a context is applied to a slope of zero. This horizontal line shows no movement.



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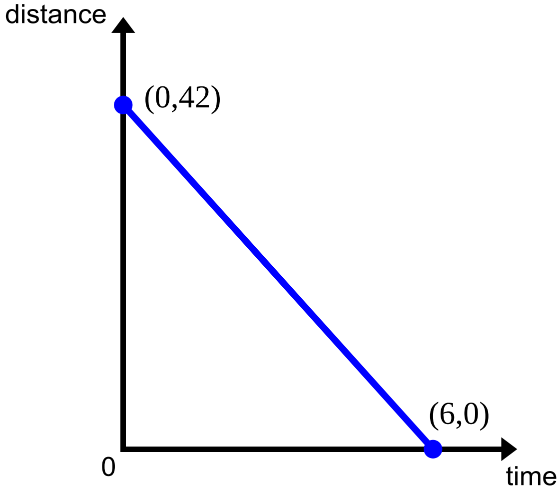
8. The graph and table below describe a runner's distance from the finish line in the last seconds of the race. Which equation tells the same story as the table and graph? Use the ordered pairs given in the table to test your chosen equation and explain your choice.

Time (seconds)	**Distance (meters)**
0	42
1	35
2	28
3	21
4	14
5	7
6	0



a) $y = 7x + 42$

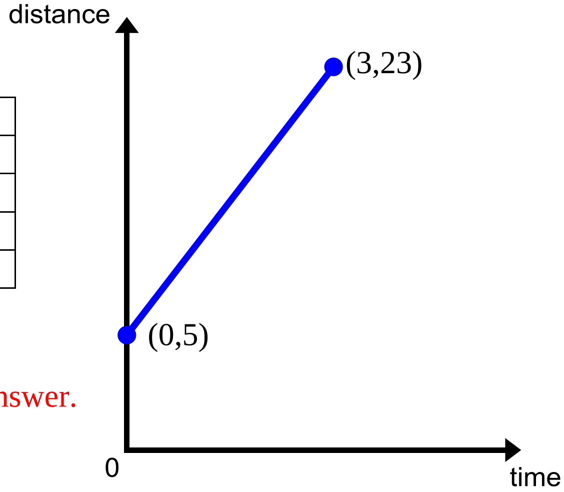
b) $y = 42 - 6x$

[✓ c) $y = 42 - 7x$]

d) $y = 42x + 6$

9. Create a table for this graph,

Time	**Distance**
0	5
1	11
2	17
3	23



10. Write a story for this graph. See student answer.

11. Which equation matches your story, the graph and the table? Explain your choice.

[✓ a) $y = 5 + 6x$]

b) $y = 5x + 23$

c) $y = 3x + 5$

d) $y = 5x + 3$

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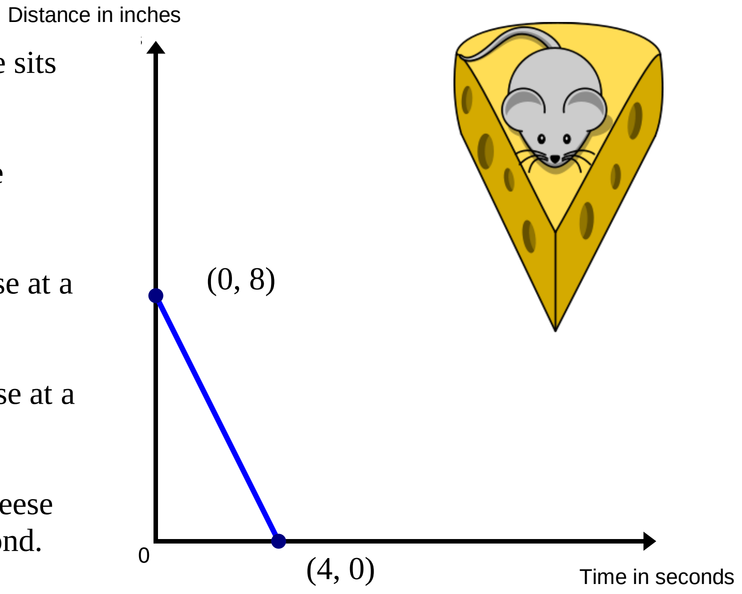
2.2d Homework: Rate of Change in a Linear Relationship

Explore and investigate the linear relationships below.

1. The graph below shows the distance a mouse is from her cheese over time. Which sentence is a good match for the graph?



A. The mouse is 8 inches away from the cheese, she sits there and does not move.

B. The mouse is 8 inches away from the cheese, she scurries towards it and reaches it after 4 seconds.

C. The mouse scurries away from the piece of cheese at a rate of 2 inches per second.

D. The mouse scurries away from the piece of cheese at a rate of 4 inches per second.

E. The mouse is 8 inches away from the piece of cheese and scurries away from it a rate of 2 inches per second.

2. Write everything you can say about the mouse and the distance she is from the cheese during this time.
The mouse is 8 inches away from her cheese. She scurries at a rate of -2 inches per second and reaches it after 4 seconds.

3. Create a table at the right which also tells the story of the graph and your writing.

4. Is this a proportional relationship? If so what is the proportional constant?

No, there is not a constant ratio and the line does not go through the origin.

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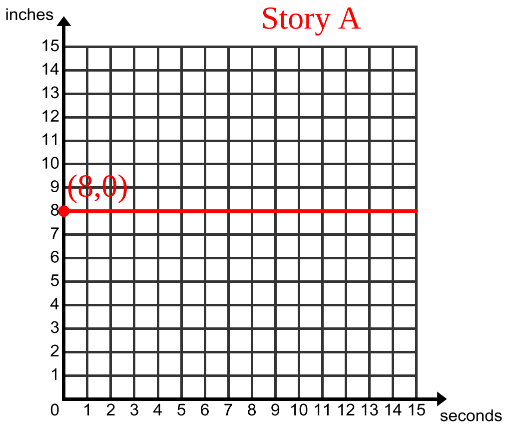
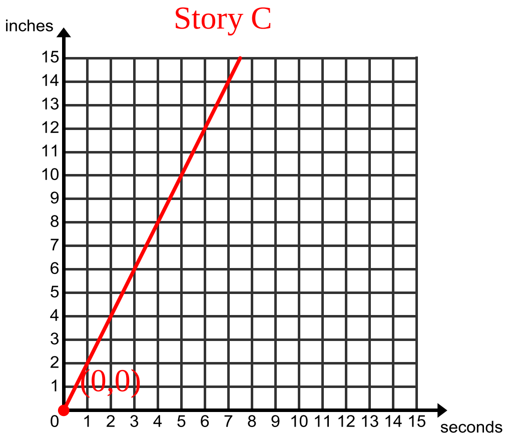
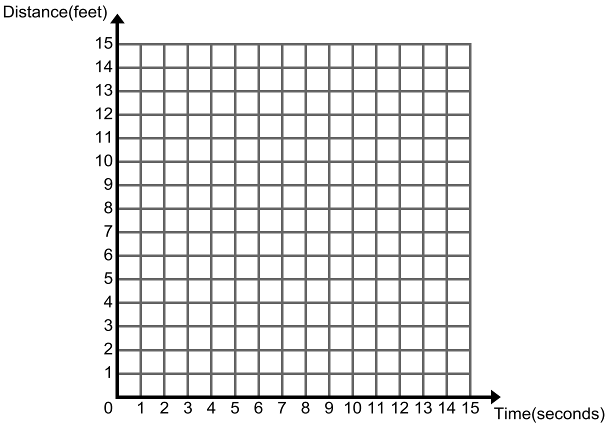
5. Sketch a graph for the four stories from number 1 above which you didn't choose in the space provided. Label the graphs by letter to match the story. Find the rate of change for each story as well.



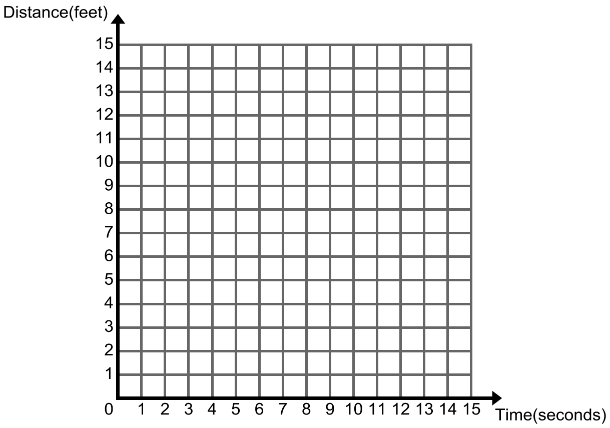
Rate of change: 0 inches per second



Rate of change: 2 inches per second



Rate of change:



Rate of change:

6. Compare the rate of change with the steepness of each line above, how does the rate of change relate to the steepness of the line?
The higher the rate of change is the steeper the line is. The smaller the rate of change the less steep the line is.

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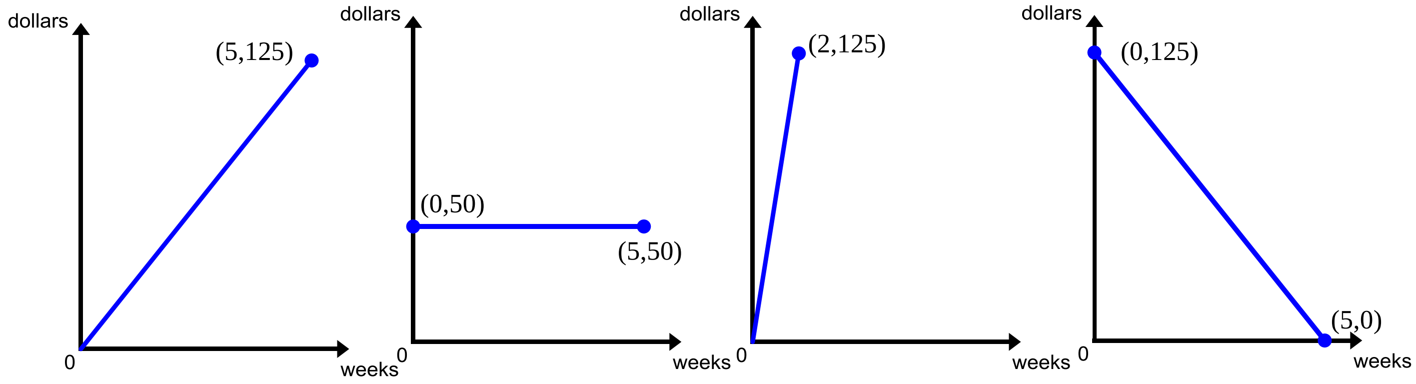
7. Triss opens a bank account and adds $25 to the account every week. Circle the graph below that matches this story.



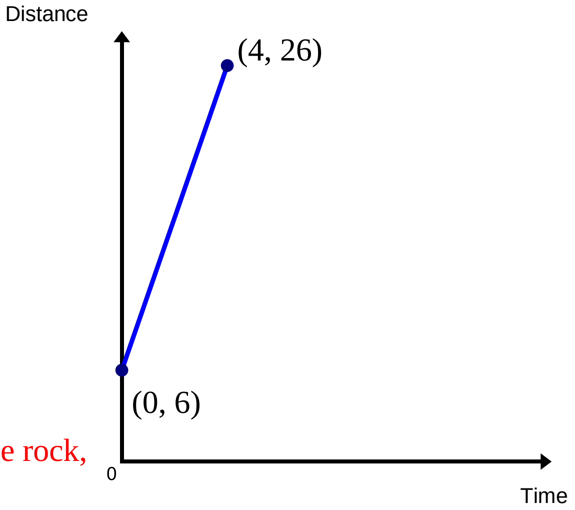
8. Write a story for each of the remaining graphs above.

Triss adds $75 to her account every week.

Use the graph at the right to complete the following.

9. Create a table for this graph,

Time	Distance
0	
1	
2	
3	
4	



10. Write a story for this graph.
Sample Answer- The lizard was 6 feet away from the rock, after 4 second he was 26 feet away from the rock.

11. Which equation matches your story, the graph and the table?
a) $y=5+6x$
b) $y=4x+26$
c) $y=5x+6$
d) $y=6x+4$

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2.2e Class Activity: More Representations of a Linear Context

Directions: In each of the following problems, you are given one of the representations of a linear relationship. Complete the remaining 3 representations. Be sure to label the columns in your table and the axes on your graph.

1. The State Fair

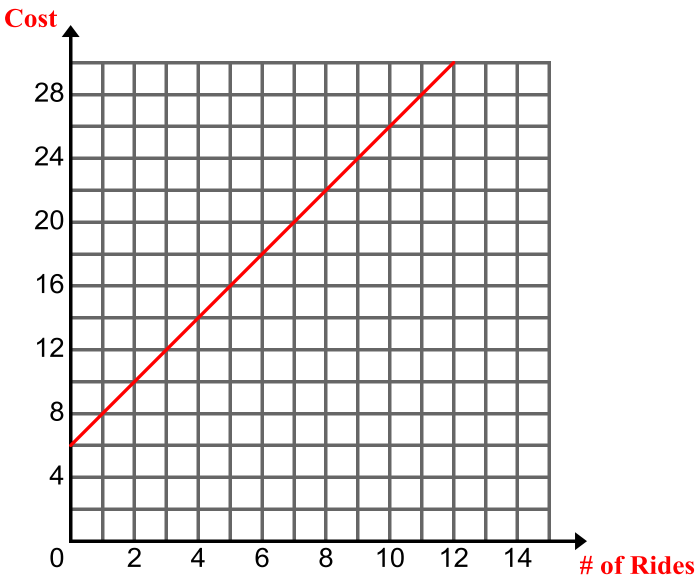
Context

You and your friends go to the state fair. It costs \$6 to get into the fair and \$2 each time you go on a ride. Consider the relationship between number of rides and total cost.

Table

# of rides	Cost
0	6
1	8
2	10
3	12
4	14
5	16

Graph



Equation

$$y = 6 + 2x$$

a. What is the rate of change in this problem? What does the rate of change represent in the context?
+2; the cost per ride

b. What is the y-intercept of your graph? Where do you see the y-intercept in the table and in the equation? What does the y-intercept represent in the context?
(0, 6); in table where $x = 0$ (have students star or circle this), on the graph at $x = 0$ rides (have students star or circle this; in the context the y-intercept is the initial fee to get into the park (what you will pay if you don't ride any rides)

c. How would you change the context so that the relationship between total cost and number of rides can be modeled by the equation $y = 2x$?
It is free to get into the park – you only have to pay 2 dollars for each ride you take.

d. How would you change the context so that the relationship between total cost and number of rides can be modeled by the equation $y = 6$? It costs \$6 to get into the park and the rides are free.

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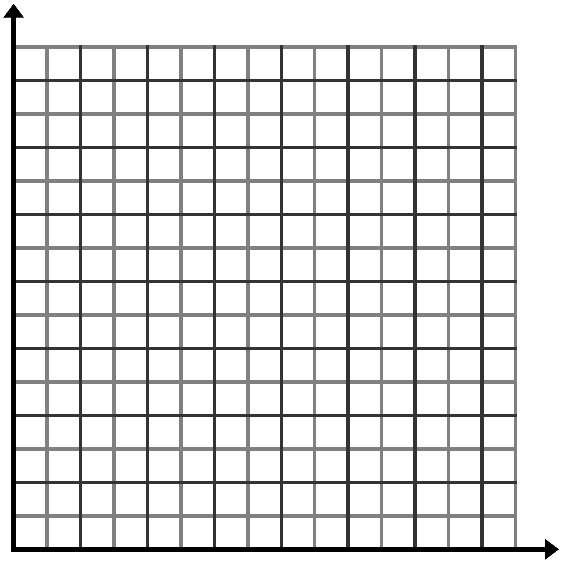
2. Road Trip

Context

You are taking a road trip. You start the day with a full tank of gas. Your tank holds 16 gallons of gas. On your trip, you use 2 gallons per hour. Consider the relationship between time in hours and amount of gas remaining in the tank.

Table

Graph



Equation

a. What is the rate of change in this problem? What does the rate of change represent in the context?
-

b. What is the y-intercept of your graph? Where do you see the y-intercept in the table and in the equation? What does the y-intercept represent in the context?

c. How would your equation change if your gas tank held 18 gallons of gas and used 2.5 gallons per hour of driving? What would these changes do to your graph?

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3. Students can title their context.

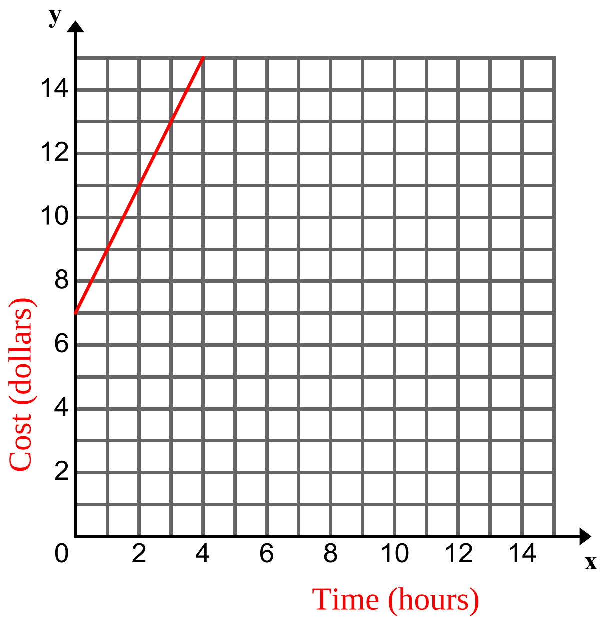
Context

Answers will vary.

Table

Time (hours)	Cost (dollars)
0	7
1	9
2	11
3	13
4	15
5	17

Graph



Equation

$$y = 7 + 2x$$

a. What is the rate of change in this problem? What does the rate of change represent in your context?
+2; answers will vary depending on the context

b. What is the y-intercept of your graph? Where do you see the y-intercept in the table and in the equation? What does the y-intercept represent in your context?
7; answers will vary depending on the context

c. How would your context change if the rate of change was 3?
Answers will vary depending on the initial context

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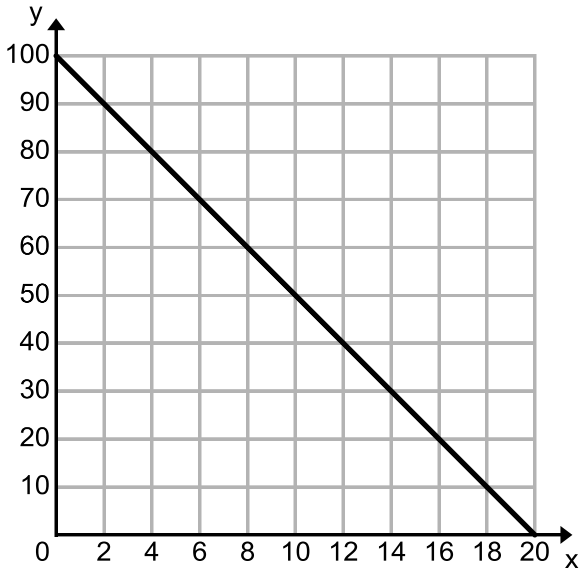
4.

Context

Table

Graph



Equation

a. What is the rate of change in this problem? What does the rate of change represent in your context?

b. What is the y-intercept of the graph? Where do you see the y-intercept in the table and in the equation? What does the y-intercept represent in your context?

c. How would your context and equation change if the y-intercept of the graph was changed to 75? How would this change affect the graph?

d. How would your context and equation change if the rate of change in this problem was changed to -2? Would the graph of the new line be steeper or less steep than the original?

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5. Students can title their context.

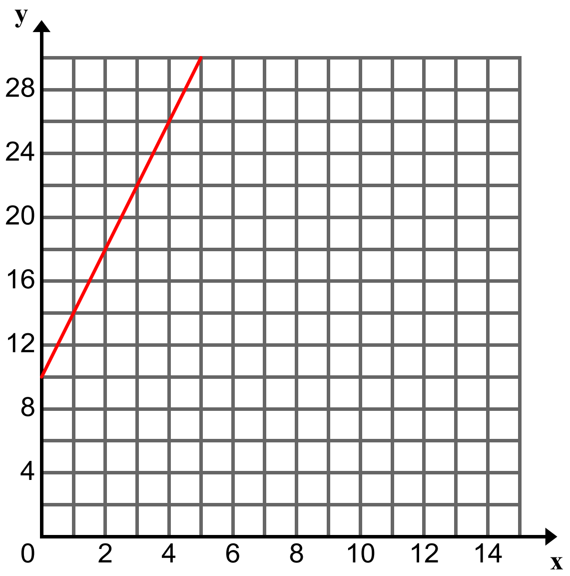
Context

Answers will vary.

Table

x	y
0	10
1	14
2	18
3	22
4	26
5	30

Graph



Equation

$$y = 4x + 10$$

Graph and table labels will depend on context.

a. How would your context change if the equation above was changed to $y = 2x + 10$? How would this change affect the graph?
Answers will vary. The slope of the line will be less steep.

b. How would your context change if the equation above was changed to $y = 4x + 8$? How would this change affect the graph?
Answers will vary. The line would be shifted down two units.

6. Describe in your own words what a linear relationship is composed of. Think about all of the equations that you have written to represent a linear relationship, what do they have in common, what do the different parts of the equations represent?

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2.2e Homework: More Representations of a Linear Context

Directions: In each of the following problems, you are given one of the representations of a linear relationship. Complete the remaining 3 representations. Be sure to label the columns in your table and the axes on your graph.

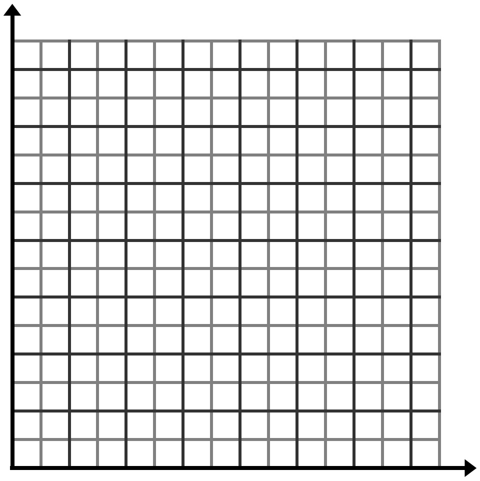
1. A Community Garden

Context

Gavin is buying tomato plants to plant in his local community garden. Tomato plants are $9 per flat (a flat contains 36 plants). Consider the relationship between total cost and number of flats purchased.

Table

Graph



Equation

a. What is the rate of change in this problem? What does the rate of change represent in the context?

b. What is the y-intercept of your graph? What does the y-intercept represent in the context?

c. How would you change the context so that the relationship between total cost c and number of flats f purchased was $c = 12f$?

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2. Enrollment

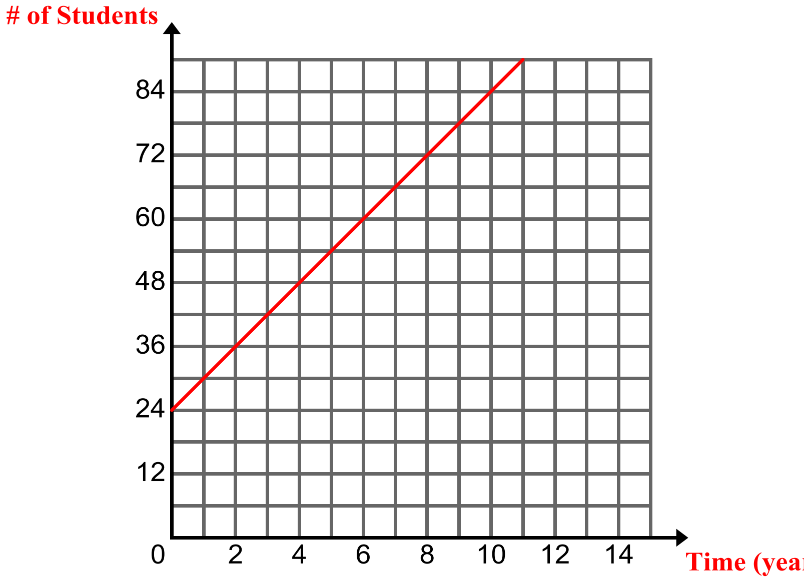
Context

The number of students currently enrolled at Discovery Place Preschool is 24. Enrollment is going up by 6 students each year. Consider the relationship between the number of years from now and the number of students enrolled.

Table

Time (years)	# of Students
0	24
1	30
2	36
3	42
4	48
5	54

Graph



Equation

$y = 24 + 6x$

a. What is the rate of change in this problem? What does the rate of change represent in the context?

+6; the increase in enrollment each year

b. What is the y-intercept of your graph? What does the y-intercept represent in the context?

(0, 24); the number of students currently enrolled

c. How would you change the context so that the relationship between number of years and number of students enrolled was $y = 40 + 6x$?

The number of students currently enrolled is 40

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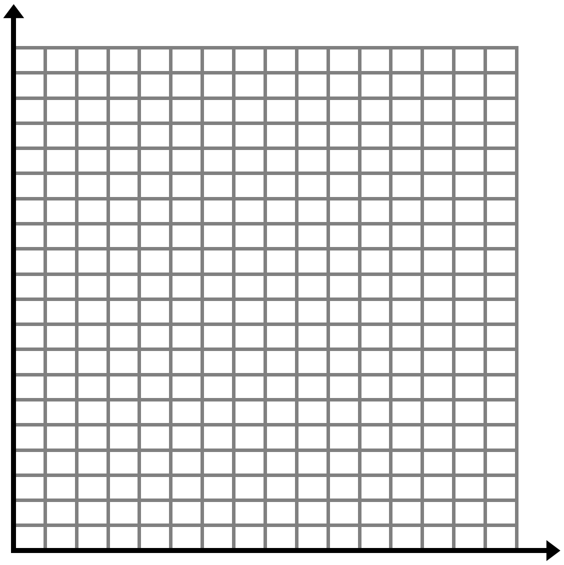
3. Write your own context

Context

Table

Time (hours)	# of Fish
0	300
2	270
4	240
6	210
8	180
10	150

Graph



Equation

a. What is the rate of change in this problem? What does the rate of change represent in the context?

b. What is the *y*-intercept of your graph? What does the *y*-intercept represent in the context?

c. How would you change your context so that the amount of fish remaining *y* after *x* hours could be represented by the equation $y = 300 - 30x$?

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4. Write your own context

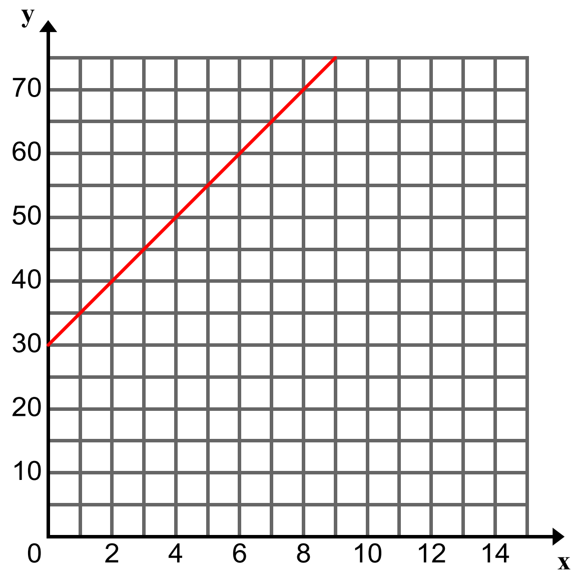
Context

Answers will vary

Table

x	y
0	30
1	35
2	40
3	45
4	50
5	55

Graph



Equation

$$y = 30 + 5x$$

a. What is the rate of change in this problem? What does the rate of change represent in the context?
+5; answers will vary

b. What is the *y*-intercept of your graph? What does the *y*-intercept represent in the context?
(0, 30); answers will vary

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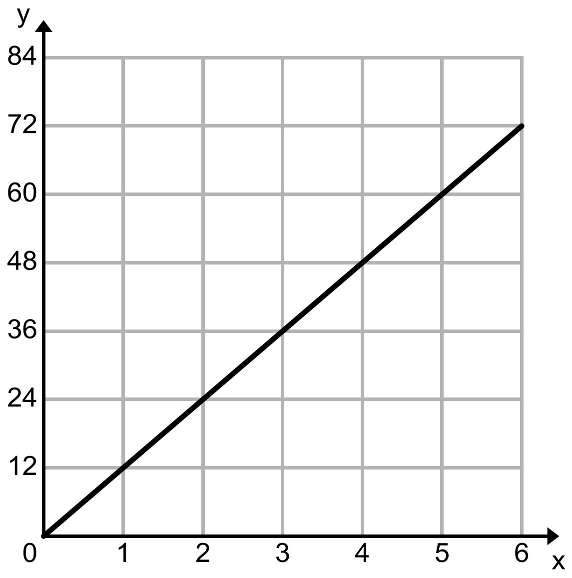
5. Write your own context

Context

Table

Graph



Equation

a. What is the rate of change in this problem? What does the rate of change represent in the context?

b. What is the *y*-intercept of your graph? What does the *y*-intercept represent in your context?

c. How would your context and equation change if the rate of change in this problem was changed to 10? Would the graph of the new line be steeper or less steep?

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2.2f Self-Assessment: Section 2.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Some sample criteria are provided as well as sample problems on the following page.

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Understanding 3**	**Substantial Understanding 4**
1. Write rules for linear patterns and connect the rule to the pattern (geometric model). *See sample problem #1*	I don't know how to write a rule for the pattern and I don't know how to find the number of blocks in the 50th stage.	I don't know how to write a rule for the pattern. The only way I know how to find the number of blocks in the 50th stage is by adding the rate of change 50 times.	I can write a rule that describes the pattern but don't know how my rule connects to the pattern.	I can write a rule that describes the pattern and explain how my rule connects to the pattern. I can also use the rule to predict the number of blocks in stage 50.
2. Understand how a linear relationship grows as related to rate of change and show how that growth can be seen in each of the representations. *See sample problem #2*	I can find the rate of change in only one of the representations.	I can find the rate of change in some of the representations.	I know how to find the rate of change in all the representations but have a hard time explaining how you can see the growth.	I know how to find the rate of change in all the representations. I can also show how the rate of change can be seen.
3. Create the additional representations (table, graph, equation, context, geometric model) of a linear relationship when given one representation and make connections between them. *See sample problem #3*	I can only make one linear representation.	I can make some of the linear representations.	I can make all the different representations of a linear pattern but I don't know how they are connected.	I can fluently move between the different representations of a linear relationship and make connections between them.
4. Identify the rate of change and initial value of a linear relationship in the table, graph, equation, context, and geometric model of a linear pattern. *See sample problem #4*	I can identify the rate of change and initial value in 2 or less of the representations of a linear model.	I can identify the rate of change and initial value in 3 of the representations of a linear model.	I can identify the rate of change and initial value in 4 of the representations of a linear model.	I can identify the rate of change and initial value in all of the representations of a linear model.

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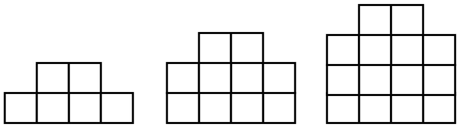
Sample Problem #1

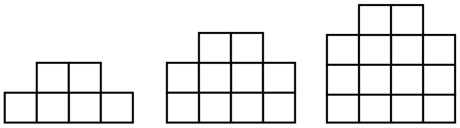
Write a rule that describes the pattern given below where *s* is the stage number and *t* is the total number of blocks. Be sure to explain how your rule connects to the pattern. Then use the rule to predict the number of blocks in the 50th stage.



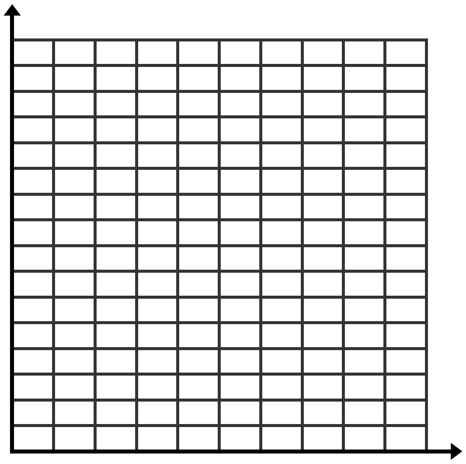
Stage 1 Stage 2 Stage 3

Sample Problem #2

For the pattern given below find the number of blocks in the next stage by determining the rate of change. Fill in the table and graph and explain how you can see the pattern grow in each of these representations.



Stage (s)	***# of Blocks (b)***
1	
2	
3	
4	
5	



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Sample Problem #3

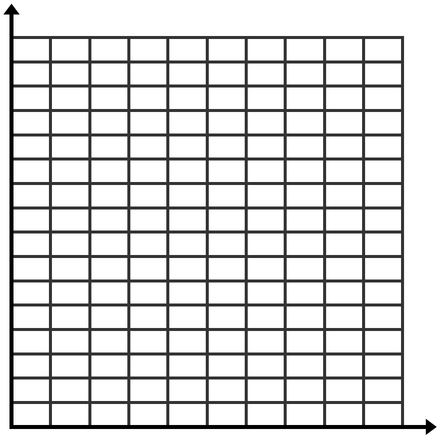
Given the following equation create a table, graph, and a context or geometrical model that the equation could possibly describe. Write a sentence that describes how the different representations are related.

$$y = -3x + 12$$

Table:

x	y

Graph:



Context/Model:

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Sample Problem #4

For each of the representations given below identify the rate of change and initial value.

a. $y = \frac{1}{2}x - 3$

Rate of change:

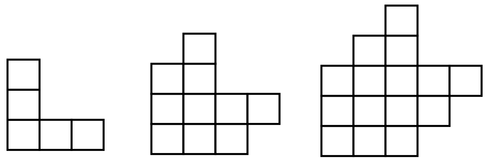
Initial value:

b. The local community center charges a monthly fee of $15 to use their facilities plus $2 per visit.

Rate of change:

Initial Value:

c.



Stage 1 Stage 2 Stage 3

Rate of change:Type equation here.

Initial Value:

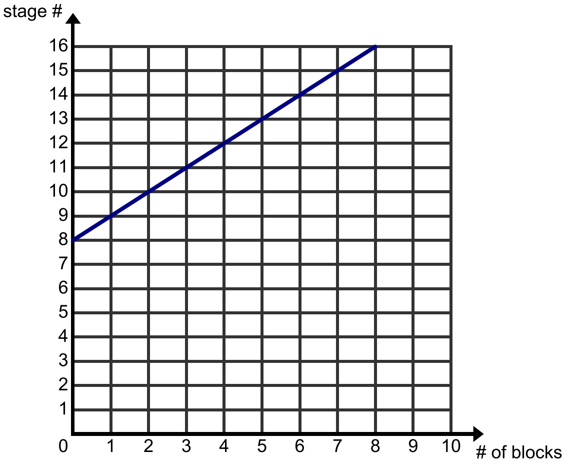
d.

x	y

Rate of Change:

Initial Value:

e.



Rate of Change:

Initial Value:

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Section 2.3: Investigate The Slope of a Line

Section Overview:

This section uses proportionality to launch an investigation of slope. Transformations are integrated into the study of slope by looking at the proportionality exhibited by dilations. Students dilate a slope triangle on lines to show that a dilation produces triangles that have proportional parts and thus the slope is the same between any two distinct points on a non-vertical line. They further their investigation of slope and proportional relationships and derive the slope formula. Adequate time and practice is given for students to solidify their understanding of this crucial aspect of linear relationships. The last few lessons provide an opportunity for students to use all of the knowledge and tools acquired throughout the chapter to formally derive the equations $y=mx$ and $y=mx+b$. They will use proportionality produced by a dilation to do this derivation.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Show that the slope of a line can be calculated as rise/run. Also explain why the slope is the same between any two distinct points on the line.
2. Find the slope of a line from a graph, set of points, and table. Recognize when there is a slope of zero or when the slope of the line is undefined.
3. Given a context, find slope from various starting points (2 points, table, line, equation).
4. Recognize that m in $y=mx$ and $y=mx+b$ represents the rate of change or slope of a line. Understand that b is where the line crosses the y-axis or is the y-intercept.
5. Derive the equation $y=mx$ and $y=mx+b$ using dilations and proportionality.

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2.3a Class Activity: Building Stairs and Ramps.

In the previous section you saw that a constant rate of change is an attribute of a linear relationship. When a linear relationship is graphed on a line you call the constant rate of change of the line the **slope** of the line.

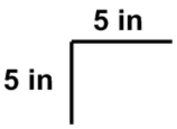
> The **slope** of a line describes how steep it is. It describes the change in *y* values compared to the change in the *x* values.

The following investigation will examine how slope is measured.

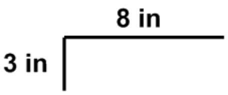
On properly built staircases all of the stairs have the same measurements. The important measurements on a stair are what we call the *rise* and the *run*. When building a staircase these measurements are chosen carefully to prevent the stairs from being too steep, and to get you to where you need to go.

One step from three different staircases has been given below.

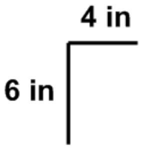
Staircase #1



Staircase #2



Staircase #3

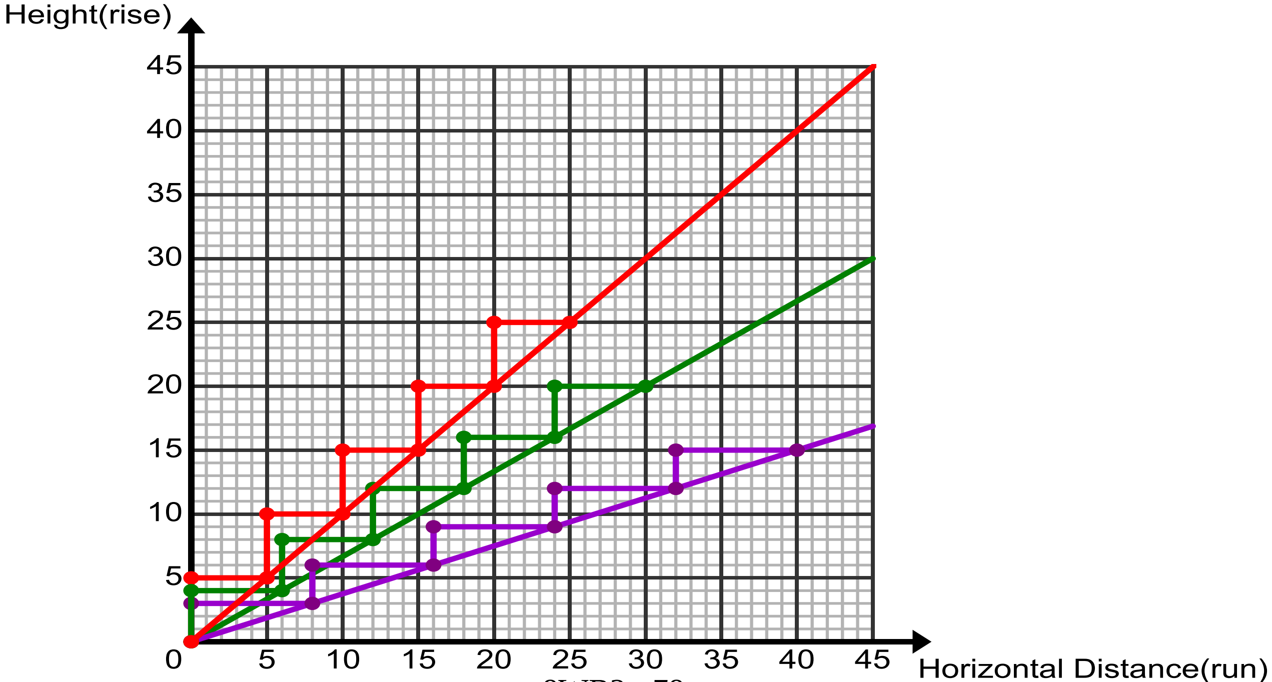


The **vertical** measurement is the "**<u>rise</u>**". The **horizontal** measurement is the "**<u>run</u>**".

1. State the rise and run for each staircase.

a. ***Staircase #1***	b. ***Staircase #2***	c. ***Staircase #3***
rise = __5__run = __5__	**rise = ___3___run = __8__**	**rise = _____run = ____**

2. Using the run and rise for each step, graph the height a person will be at after each step for the first 5 steps. Do this for each staircase.



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3. Which staircase is the steepest? __Staircase #3____

Just like staircases, the measurement of the steepness of a line is also very important information. On the graph on the previous page draw a connecting line from the origin (0,0) and through the tip of each stair step.

This line shows the slope of your stairs. For each step you climb, you move up $\boldsymbol{y}$ inches and forward $\boldsymbol{x}$ inches.

Find the slope of each line representing a staircase using the ratio: $\boldsymbol{\frac{rise}{run}}$ $\boldsymbol{or}$ $\boldsymbol{\frac{y}{x}}$**,** and by simplifying this fraction.

4. Calculate the slope ratio for each staircase.

   a. Staircase #1: _$\frac{5}{5} = 1$____

   b. Staircase #2: _______

   c. Staircase #3: __$\frac{6}{4} = \frac{3}{2}$____

5. If you didn't have the graph to look at, only the ratios you just calculated, how would you know which staircase would be the steepest? The greater the ratio the steeper the line.

6. Calculate the slope for climbing 1, 2, & 3 steps on each of the staircases.

	Staircase #1			**Staircase #2**			**Staircase #3**		
	Total Rise	Total Run	Slope (rise/run)	Total Rise	Total Run	Slope (rise/run)	Total Rise	Total Run	Slope (rise/run)
1 step	5	5	$\frac{5}{5}$	3		$\frac{3}{8}$		4	$\frac{6}{4}$
2 steps	10	10		6	16		12		$\frac{12}{8}$
3 steps		15	$\frac{15}{15}$		24	$\frac{9}{24}$	18	12	

7. Does the slope of the staircase change as you climb each step?
   The slope of the staircase is the same as you climb each step. The ratios all simplify to the same number.

8. Using your knowledge of how slope is calculated, see if you can figure out the slope of the ramp found at your school. Take measurements at two locations on the ramp. Use the table below to help you.

	Rise	Run	Slope
1st measurement			
2nd measurement			

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2.3a Homework: Measuring the Slope of Stairs and Ramps

1. Your task is to design a set of stairs and a wheelchair ramp at the side. Both the stairs and the ramp will begin at the same place (at ground level) and end at the height of 3 feet. Answer the following questions as you develop your design.

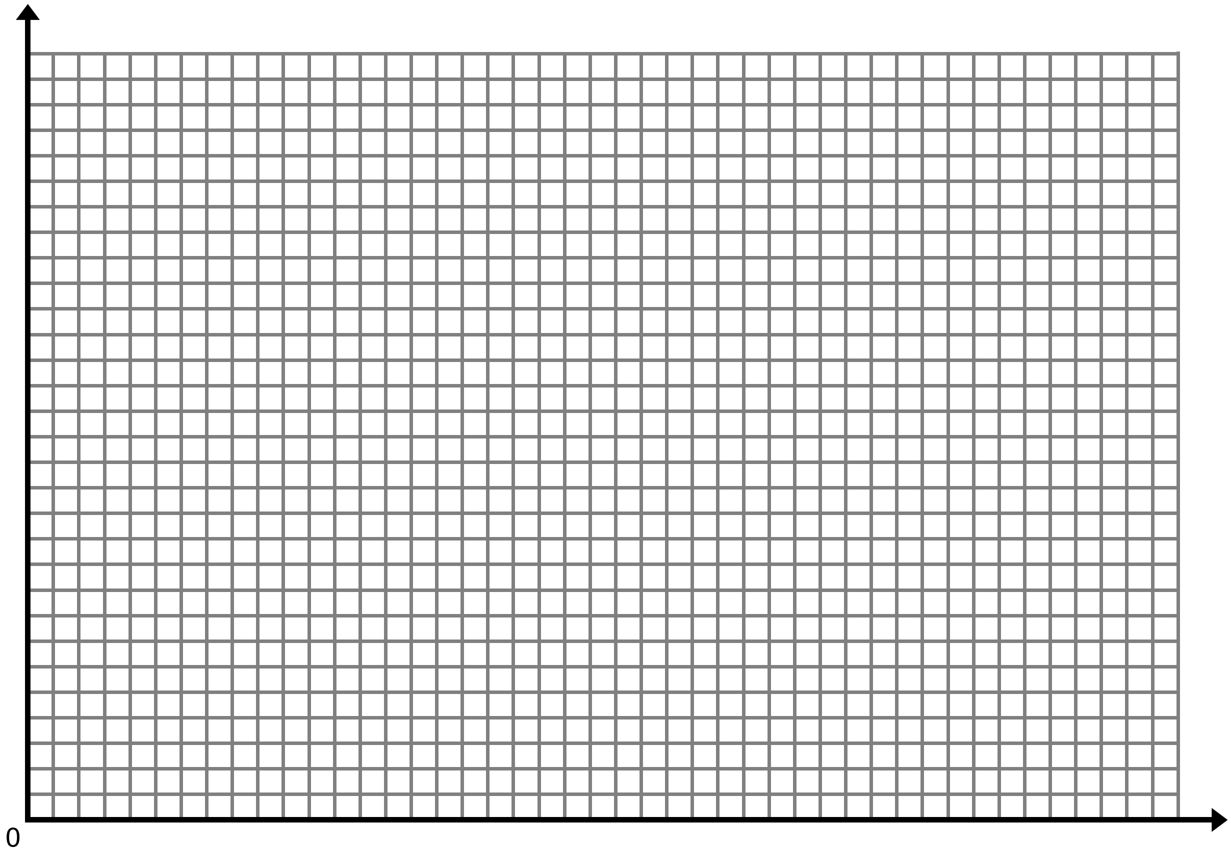
- How many steps do you want or need?
  Answers will vary.

- How deep should each step be? Why do you want this run depth?
  Answers will vary.

- How tall will each step be? Why do you want this rise height?
  Answers will vary.

- What is the total distance (total depth for all steps) you will need (at the base) for all of the stairs—this would be a measurement at ground level from stair/ramp start point to stair/ramp end point?
  Answers will vary.

2. Sketch the ramp (as viewed from the side) on graph paper below. Label and sketch the base and height, for example: Ramp-base (in inches or feet) and Height (in inches or feet). Answers will vary.



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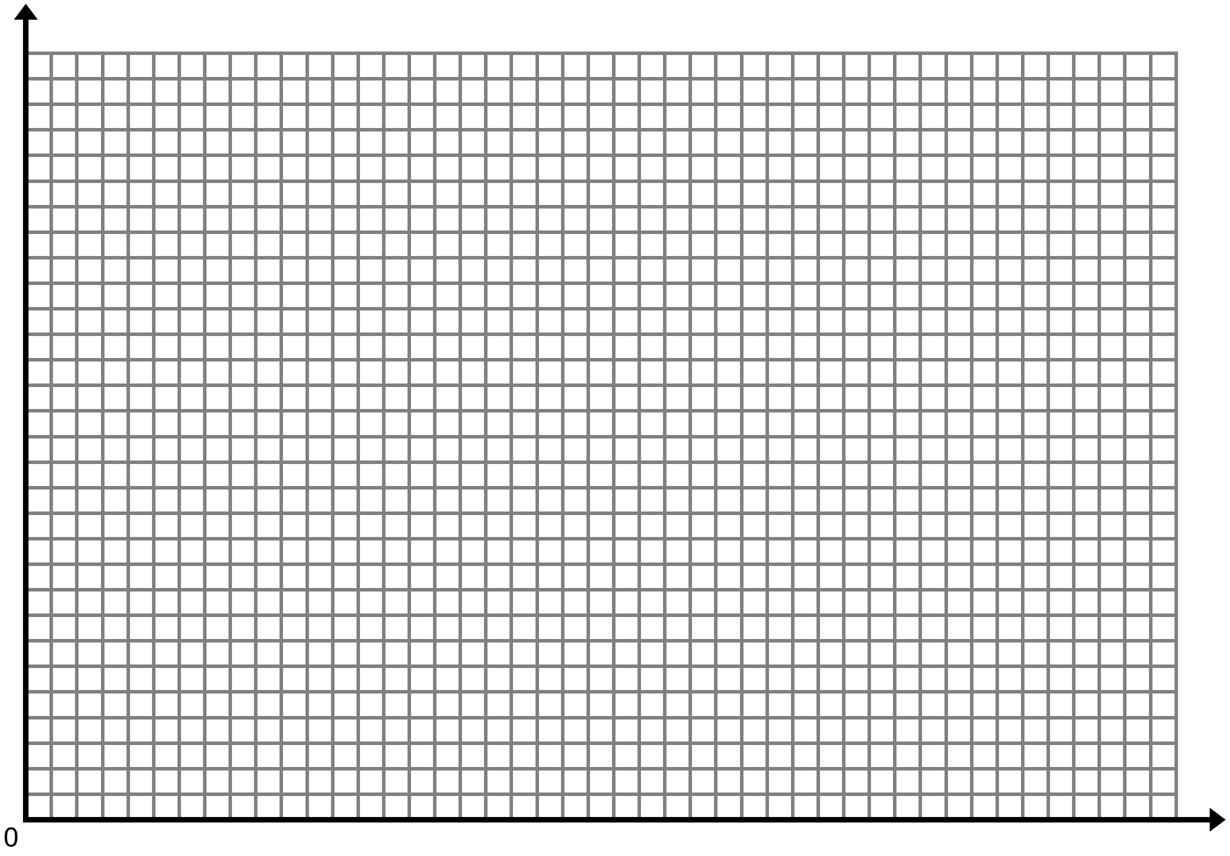
3. From the sketch of the ramp, find and record the following measurements. Answers will vary.

	Rise height (total height you've climbed on the ramp at this point)	Run depth (total distance – at ground level – covered from stair-base beginning)	Ratio $\frac{rise}{run}$
12 inches in from the start of the ramp			
24 inches in from the start of the ramp			
Where the ramp meets the top			

4. Sketch the stairs (as viewed from the side) on graph paper below. Label the sketch base and height, for example: Stair-base (in inches) and Height (in inches). Answers will vary.



From the sketch above, find and record the following measurements. Answers will vary.

	Rise height (total height you've climbed at this point)	Run depth (total distance covered from stair-base beginning)	Ratio $\frac{rise}{run}$	Reduced ratio
At the first step				
At the third step				
At the last step				

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5. What do you notice about the Ratio column for both the ramp and the stairs?

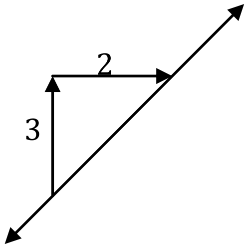
   They have the same ratios.

6. On your stair drawing, draw a line from the origin (0,0) through the very tip of each stair. Now look at your ramp drawing. What do you observe?

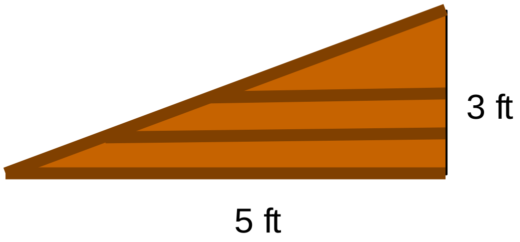
7. Explain what would happen to the slope of the line for your stairs if the rise of your stairs was higher or lower? If the rise is higher then the slope is bigger meaning that the stairs would be steeper. If the rise is lower than the slope is smaller meaning that the stairs would be less steep.

8. Fill in the blanks
   a. Slope =

   b. In the picture below the rise is________ units, and the run is________ units. Thus, the slope of this line is___.

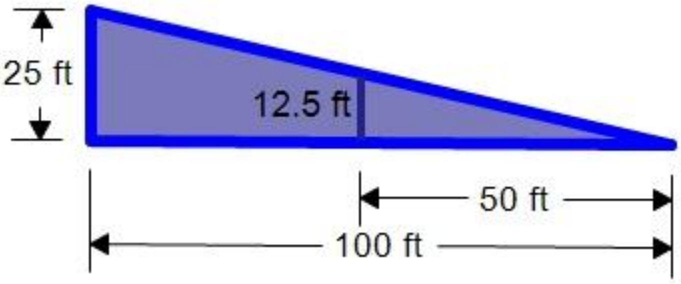


9. Suppose you want to make a skateboard ramp that is **not** as steep as the one show below. Write down two different slopes that you could use.



Answers will vary.

10. Find the slope of the waterslide. Calculate it in two different ways.



The slope of the water slide is $-\frac{1}{4}$.

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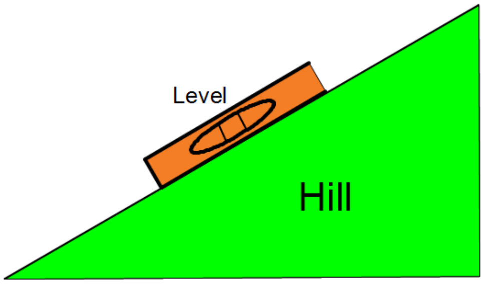
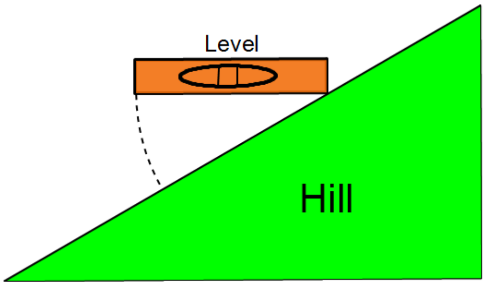
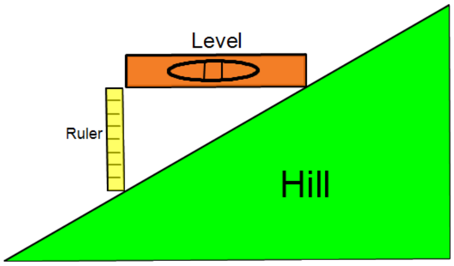
Extra Practice: Measure the Slope of a Hill

Instructions for measuring the grade of a hill or a road:
Rest the level (or clear bottle filled with water) on the ground. Lift up the lower end of the level/bottle (i.e, the end nearest the bottom of the hill) until the level measures level. (If using a bottle, the water level in the bottle should be parallel to the side of the bottle.)



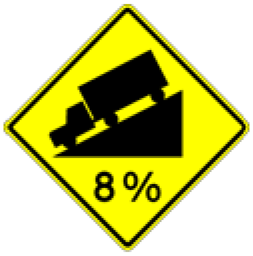


While holding the level/bottle still in this position, measure the distance between the end of the level and the ground using the ruler, as shown in the image below.



1. Calculate the grade by dividing the distance measured with the ruler (the "rise") by the length of the level or straight edge (the "run") and multiplying by 100:

2. Roadway signs such as the one to the right are used to warn drivers of an upcoming steep down grade that could lead to a dangerous situation. What is the grade, or slope, of the hill described on the sign? (Hint: Change the percent to a decimal and then change the decimal to a fraction)



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2.3b Class work: Dilations and Proportionality

When an object, such as a line, is moved in space it is called a transformation. A special type of transformation is called a dilation. A dilation transforms an object in space from the center of dilation, usually the origin, by a scale factor called *r*. The dilation moves every point on the object so that the point is *r* times away from the center of dilation as it was originally. This means that the object is enlarged or reduced in size.

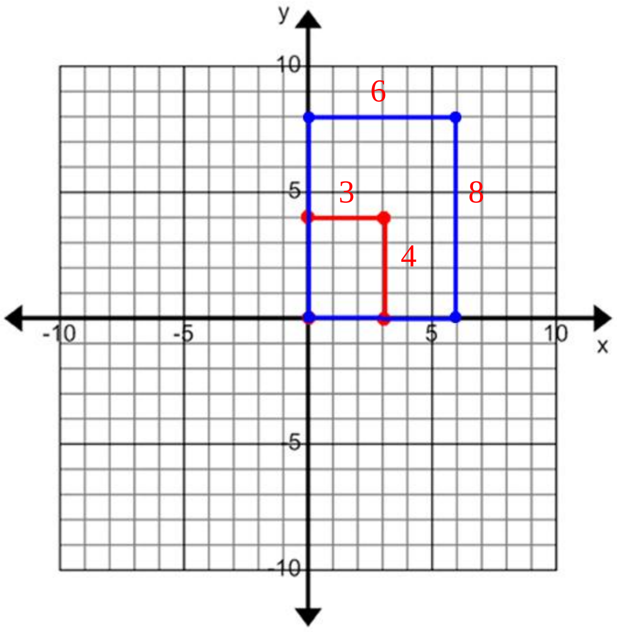
For example, if you dilate the set of points (0,0), (0,4),(3, 0), and (3,4) with a scale factor of 2 and the center of dilation is at the origin (0,0) the distance of each point from the center will be 2 times as long as it was originally. Algebraically this means that you multiply each point by 2.

$$(x, y) \rightarrow (2x, 2y)$$

To confirm this, investigate this transformation below.

1.

a. Graph and connect the ordered pairs (0,0), (0,4), (3, 0), and (3,4).



b. Find the length of each segment and dilate it by a scale factor of two. Draw and label the new lengths from the center of dilation (the origin) in a different color on the coordinate plane above.

The original object is called the ***pre-image*** and the transformed object is called the ***image***.

c. Compare the size of the pre-image with the image.
Each side is twice the length and the area is 4 times the size.

d. Dilate by a scale factor of 2 algebraically using the ordered pairs. Algebraically this is written as $(x, y) \rightarrow (2x, 2y)$. Write the ordered pairs below.
(0,0), (0,8), (6,0), (6,8)

e. Graph and connect your new ordered pairs for the image on the coordinate plane in part a.
See Graph

f. What do you observe about the transformation when you do it graphically and algebraically?
Both times you end up with the same new image.

g. How do the lines that correspond to one another in the image and pre-image compare?
They have the same slope and their lengths are proportional.

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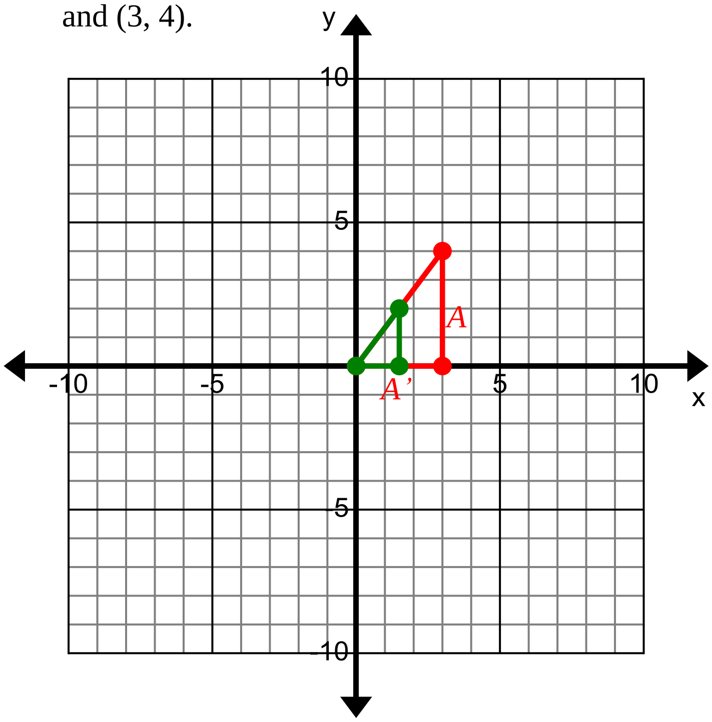
A special notation is used to differentiate between the pre-image and the image. If the pre-image is called ***A*** then the image is called ***A'***, pronounced "A prime".

2. Try another shape to see what kind of relationship exists between the pre-image and the image.

a. Graph and connect the ordered pairs (0,0), (3,0), and (3, 4).



b. Find the length of each segment and dilate it by a scale factor of $\frac{1}{2}$. (The length of the hypotenuse is 5) Draw the new lengths from the center of dilation (the origin) in a different color.

c. Labe the pre-image A and the image A'. Compare the size of the pre-image with the image.
Each side measures half the length of the pre-image. The image has ¼ of the area of the pre-image.

d. Dilate by a scale factor of $\frac{1}{2}$ algebraically using the ordered pairs. That it is $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$.
Write the ordered pairs below.

(0,0), (0.5,0), and (1.5, 2)

e. Graph your new ordered pairs for the image.
See Graph

f. What do you observe about the transformation when you do it graphically and algebraically?
Both times you end up with the same new image.

g. How do the lines that correspond to one another in the image and pre-image compare?
They have the same slope and their lengths are proportional.

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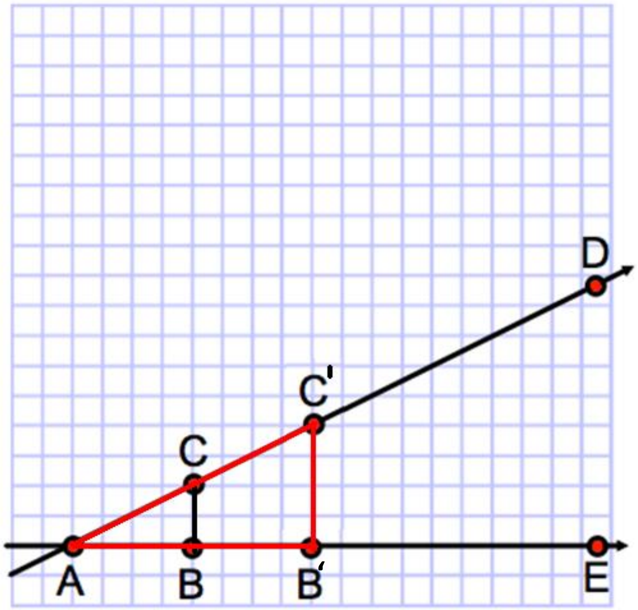
Now that you know how a dilation works you will investigate what kind of relationship is formed between the pre-image and the image after the dilation.

3. Using the figure below.	
a. Connect point B to C b. Double the length of $\overline{AB}$ on the line $\overline{AE}$. Label the new segment $\overline{AB'}$ c. Double the length of $\overline{AC}$ on the line $\overline{AD}$. Label the new segment $\overline{AC'}$. d. Connect B' to C'.	

4. What do you notice about $\overline{B'C'}$ in relationship to $\overline{BC}$?

   The length of $\overline{B'C'}$ is double the length of BC.

5. What do you notice about the size of the two triangles?

   The image has an area that is 4 times the area of the pre-image.

6. Write a ratio that compares the corresponding parts of the pre-image with the image.

   $$\frac{\overline{AB}}{\overline{A'B'}} = \frac{4}{8} = \frac{1}{2} = \frac{\overline{BC}}{\overline{B'C'}} = \frac{2}{4} = \frac{1}{2}$$

7. What kind of relationship exists between the pre-image and image?

   The corresponding parts of the image and pre-image are proportional to one another.

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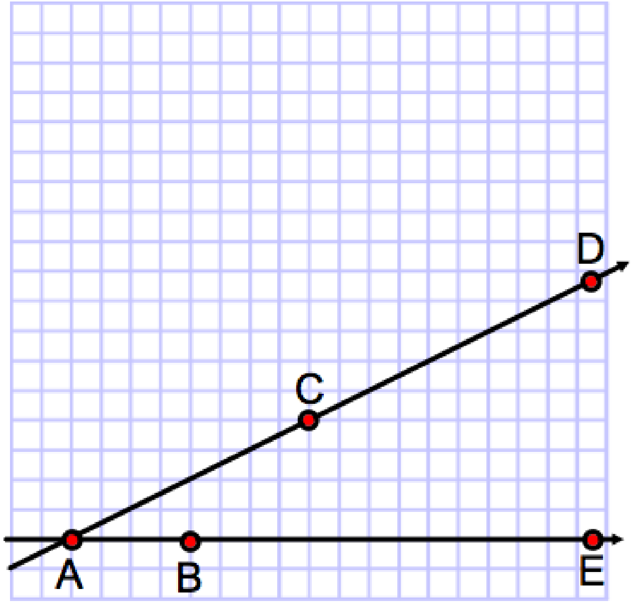
2.3b Homework: Dilations and Proportionality

Continue your investigation of dilations and the relationships between the pre-image and image below.

1. Using the figure below:
   a. Connect point B to C
   b. Double the length of $\overline{AB}$ on the line $\overline{AE}$. Label the new segment $\overline{AB'}$
   c. Double the length of $\overline{AC}$ on the line $\overline{AD}$. Label the new segment $\overline{AC'}$.
   d. Connect B' to C'.



2. What do you notice about $\overline{B'C'}$ in relationship to $\overline{BC}$?

3. What kind of relationship exists between the pre-image and image?

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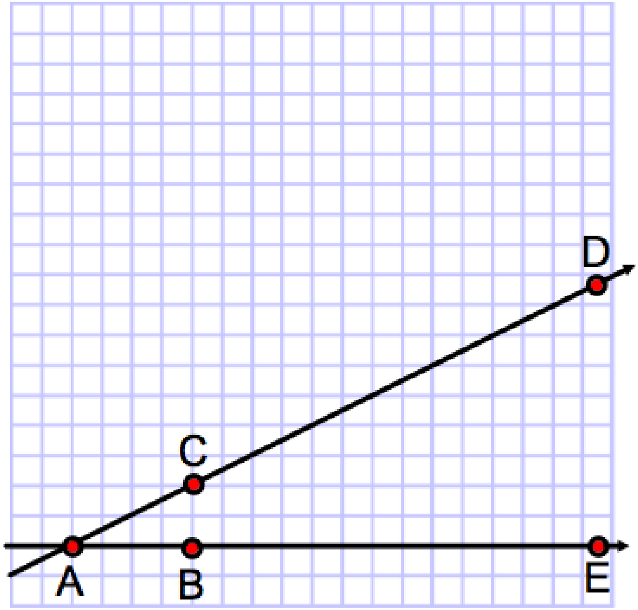
4. Using the figure below:
    a. Connect point B to C.
    b. HALF the length of $\overline{AB}$ on the line $\overline{AE}$. Label the new segment $\overline{AB'}$.
    c. HALF the length of $\overline{AC}$ on the line $\overline{AD}$. Label the new segment $\overline{AC'}$.
    d. Connect B' to C'.



5. What do you notice about $\overline{B'C'}$ in relationship to $\overline{BC}$?

6. What kind of relationship exists between the pre-image and image?

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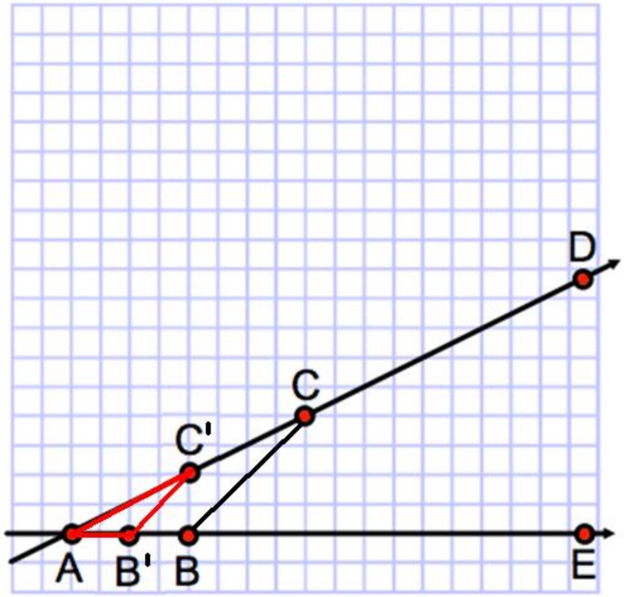
7. Using the figure below:

   a. Connect point B to C.

   b. HALF the length of $\overline{AB}$ on the line $\overline{AE}$. Label the new segment $\overline{AB'}$.

   c. HALF the length of $\overline{AC}$ on the line $\overline{AD}$. Label the new segment $\overline{AC'}$.

   d. Connect B' to C'.



8. What do you notice about $\overline{B'C'}$ in relationship to $\overline{BC}$?

   It is half the length.

9. What kind of relationship exists between the pre-image and image?
   The corresponding parts of the image and pre-image are proportional to one another.

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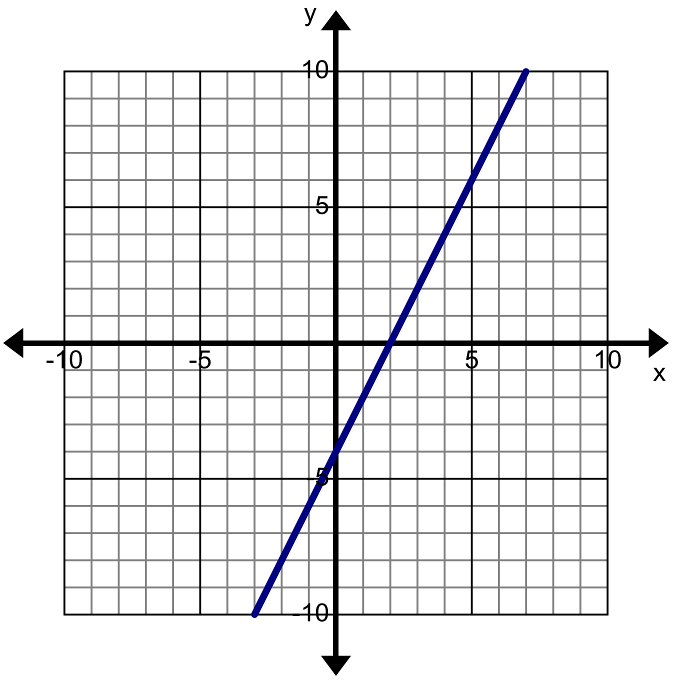
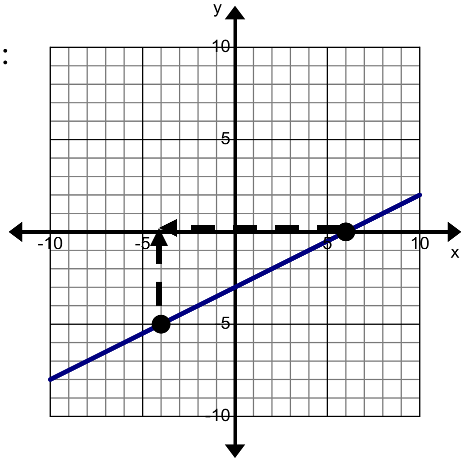
2.3c Class Activity: Proportional Triangles and Slope

1. On the line to the right choose any two points that lie on the line. (To make your examination easier choose two points that lie on an intersection of the gridlines).



From the two points create a right triangle, the line itself will be the hypotenuse and the legs will extend from the two points and meet at a right angle. An example is shown below.

Example:



2. Compare the points that you choose and your triangle with someone in your class. Discuss the following;

   Did you both chose the same points?
   How are your triangles the same?
   How are your triangles different?
   What relationship exists between your triangles?

All of the triangles will be dilations of one another no matter what two points you choose. Since they are dilations of one another their corresponding parts are proportional.

Given any two triangles with hypotenuse on the given line and legs horizontal and vertical, then there is a dilation that takes one on the other. In particular, the lengths of corresponding sides are all multiplied by the factor of the dilation, and so the ratio of the length of the vertical leg to the horizontal leg is the same for both triangles.

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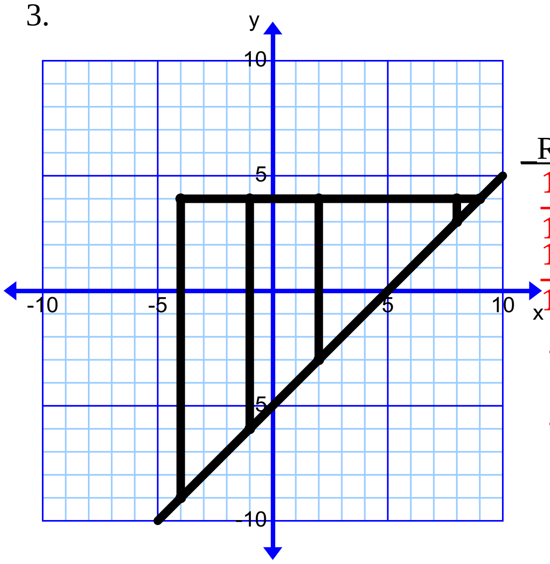
The graphs given below have several different triangles formed from two points that lie on the line. These triangles are dilations of one another and their corresponding parts are proportional. Answer the questions below about these lines to observe what this tells us about slope.

- In each graph below, how many right triangles do you see?
- Trace each triangle you see with a different color.
- For each triangle write a ratio comparing the lengths of its legs or $\frac{height}{base}$. Then simplify the ratio

$\frac{height}{base} = \_\_\_\_\_\_$.

In the future we will refer to the ratio as $\frac{rise}{run}$, instead of $\frac{height}{base}$.

3.



Ratios

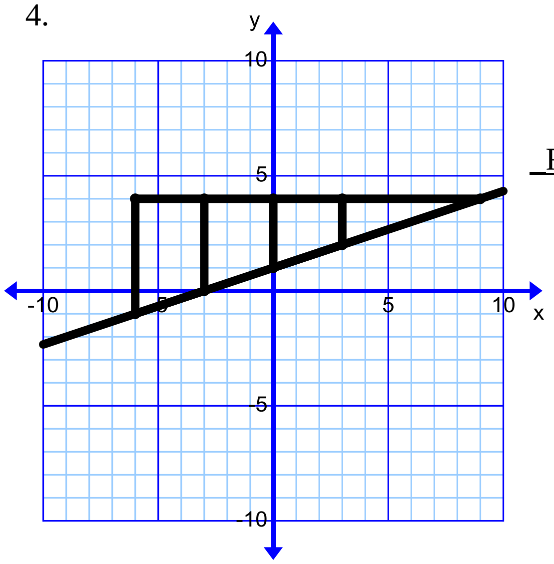
$\frac{13}{13} = 1$

$\frac{10}{10} = 1$

$\frac{7}{7} = 1$

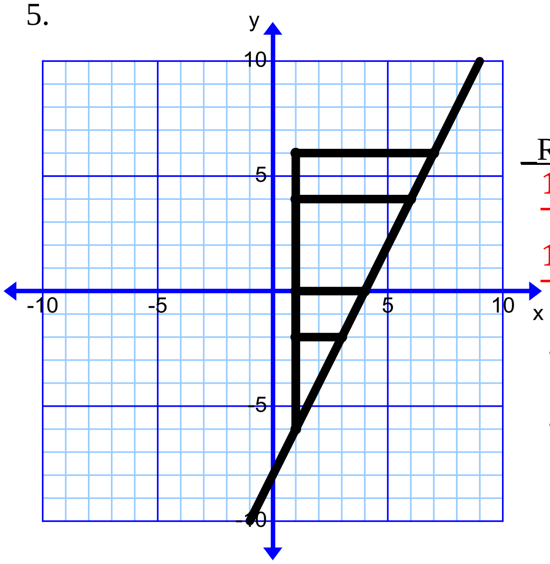
$\frac{1}{1} = 1$

4.



Ratios

5.



Ratios

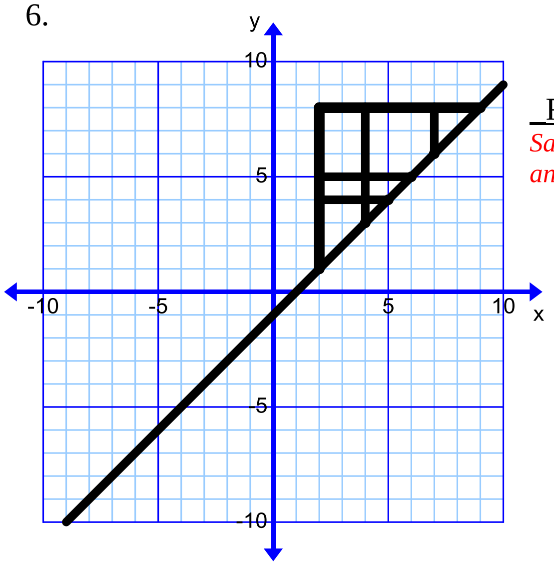
$\frac{12}{6} = 2$

$\frac{10}{5} = 2$

$\frac{6}{3} = 2$

$\frac{4}{2} = 2$

6.



Ratios

Sample answers

$\frac{7}{7} = 1$

$\frac{3}{3} = 1$

$\frac{2}{2} = 1$

$\frac{5}{5} = 1$

7. Do the ratios (rise to run) always simplify to the same fraction (even quadrants with negative ordered pairs)? Why or why not? Yes, the lines that create the triangles are dilations of one another. These dilations make these triangles similar to one another. Similar triangles have parts that are proportional to one another, that is why the ratios are equal.
8. How does the "*rise over run ratio*" describe the steepness of the line?

The higher the number the more steep the line is, the smaller the number the less steep the line is.

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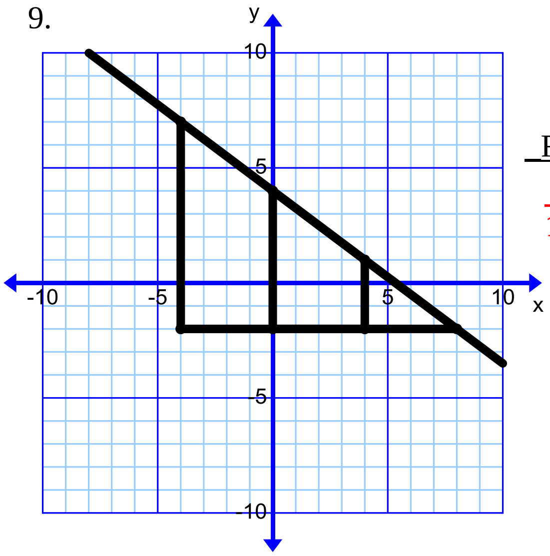
9.



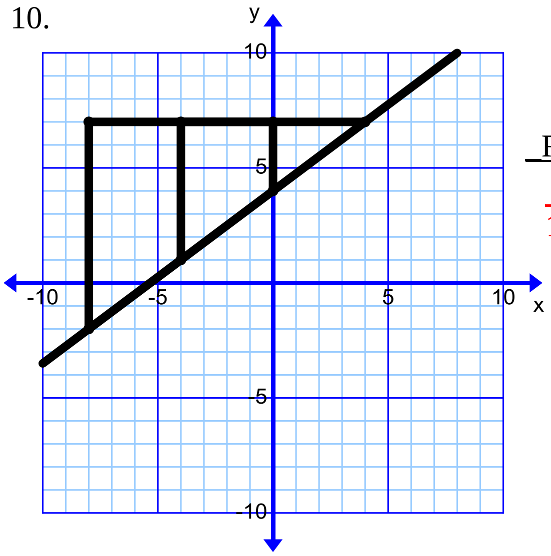
Ratios

$\frac{9}{12} = \frac{3}{4}$

$\frac{6}{9} = \frac{3}{4}$

$\frac{3}{4} = \frac{3}{4}$

10.



Ratios

$\frac{9}{12} = \frac{3}{4}$

$\frac{6}{9} = \frac{3}{4}$

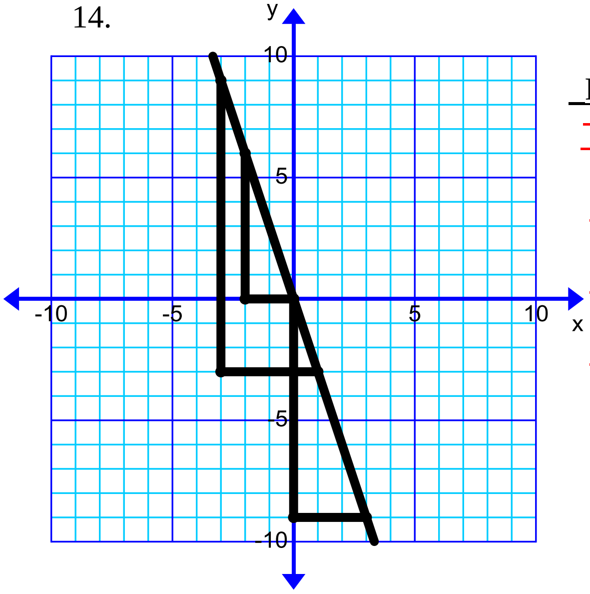
$\frac{3}{4} = \frac{3}{4}$

11. Why do graphs *9* and *10* have the same ratio but they are different lines?
One of the lines is going down from left to right and the other is going up from left to right. They have the same ratio because they are the same steepness.

12. How could you differentiate between the slopes of these lines?
Make the ratio for the line that is going down negative. The rise is negative because you are going down as you move from left to right.

13. How does the rise related to the run of a negative slope affect the steepness of the line?
The steepness of the line does not change, the negative slope means it is going down.

14.



Ratios

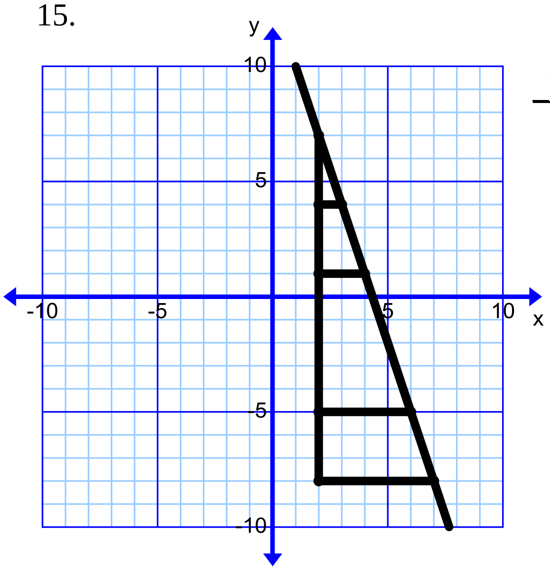
$\frac{-12}{4} = -3$

$\frac{-6}{2} = -3$

$\frac{-3}{1} = -3$

$\frac{-9}{3} = -3$

15.



Ratios

16. Are the ratios for graphs *14* and *15* positive or negative? How do you know?
Negative, as you move from left to right the rise is going down so it is negative. This makes the entire ratio negative.

17. Why are the slopes for graphs *14* and *15* the same if they are different lines?
The lines have the same steepness even though they are different lines.

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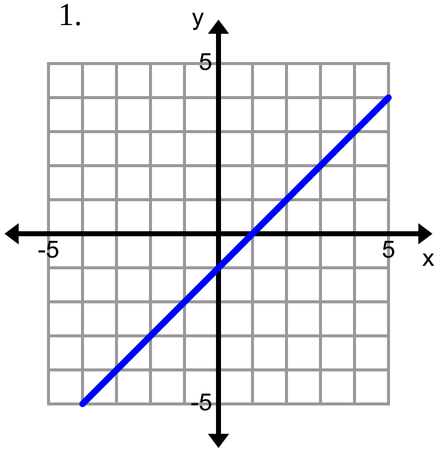
2.3c Homework: Similar Triangles and Slope

For each line graphed below,

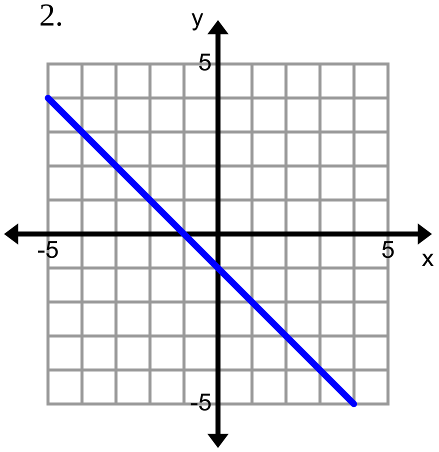
- Draw a Right Triangle to calculate the slope of the line. The slope of a line is denoted by the letter *m*.
  Thus $slope = m = \frac{rise}{run}$
- Label each triangle with a ratio and simplify the ratio $m = \frac{rise}{run} = \underline{\qquad}$

1.



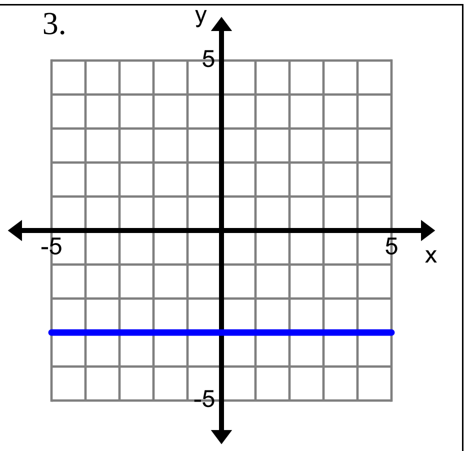
$m = \frac{rise}{run} = \frac{1}{1} = 1$

2.



$m = \frac{rise}{run} =$

3.

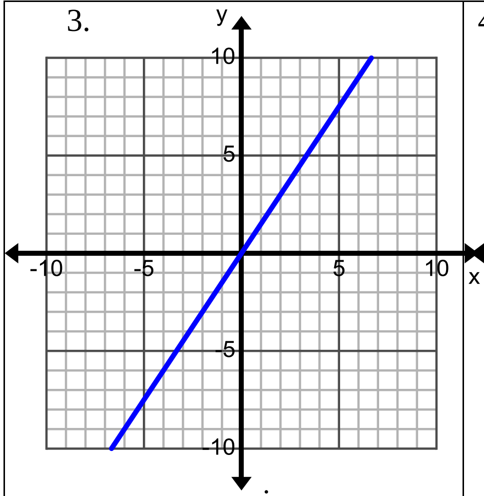


$m = \frac{rise}{run} = \frac{0}{1} = 0$

2. What does the sign of the slope tell us about the line?
   The sign of the slope determines if the line is going up or down from left to right.

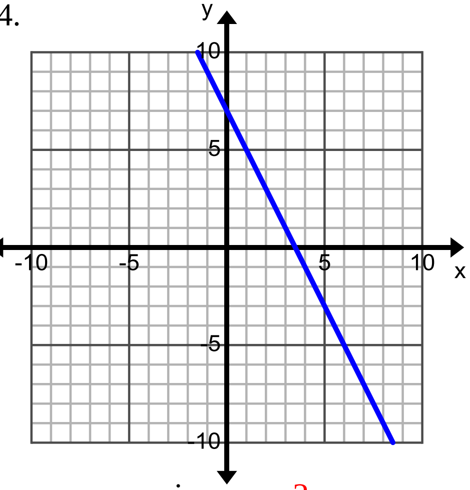
For each line graphed below, calculate the slope of the line.

3.



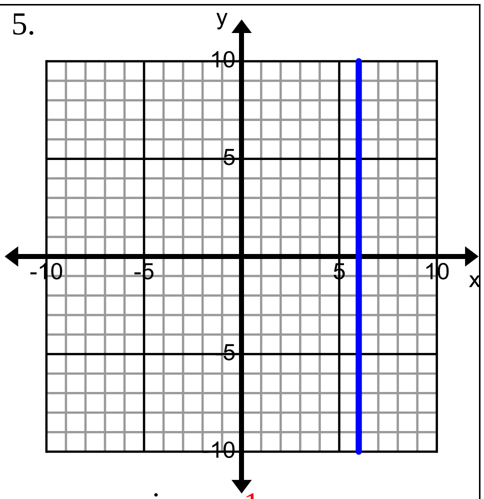
$m = \frac{rise}{run} =$

4.



$m = \frac{rise}{run} = \frac{-2}{1} = -2$

5.



$m = \frac{rise}{run} = \frac{1}{0} = undefined$

6. Briefly, explain how to calculate slope when looking at a graph.

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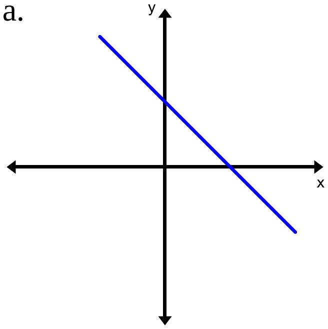
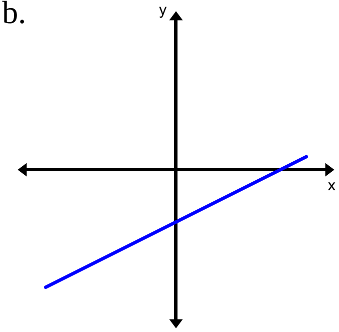
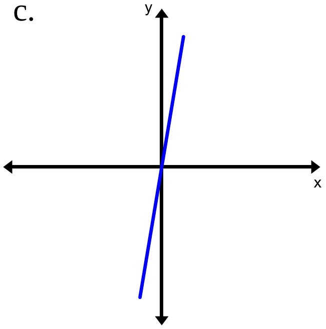
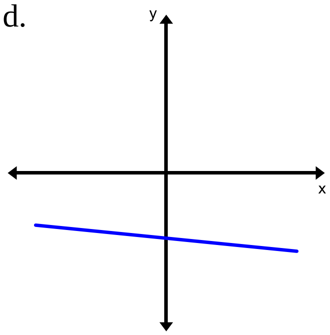
2.3d Class Activity: Finding Slope from Graphs

1. Do the graphs below have positive or negative slopes? How do you know?

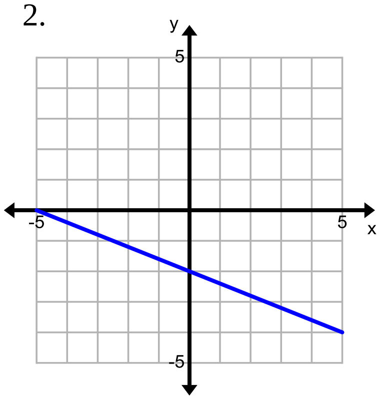
a. 	b. 	c. 	d. 
Positive or negative? Negative	Positive		
e. Explain how you know whether a line of a graph has a positive or negative slope. If the line is going down from left to right it has a negative slope. If the line is going up from left to right it has a positive slope.			

For each line graphed below,

- Draw a right triangle to calculate the slope of the line. Answers will vary
- Label each triangle with a ratio and simplify the ratio $m = \frac{rise}{run} = \frac{\quad}{\quad}$

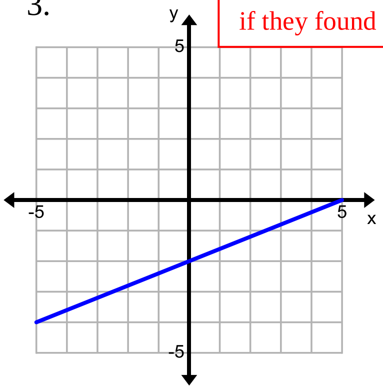
If you have a hard time seeing where the line intersects a point on the graph find at least 3 points and they can verify if they found the correct intersections.

2.



$\frac{rise}{run} = \frac{-2}{5}$

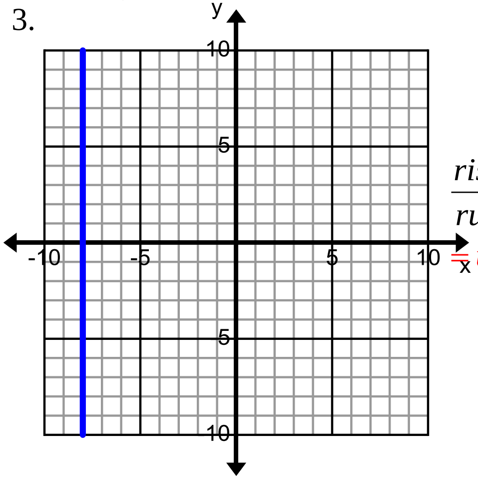
3.



$\frac{rise}{run} = \frac{2}{5}$

For each line graphed below, calculate the slope of the line.

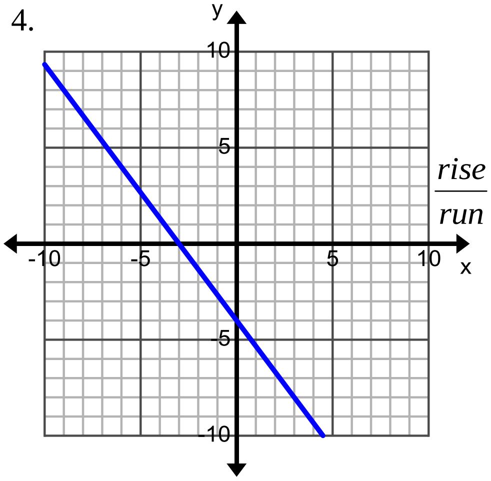
3.



$\frac{rise}{run} = \frac{2}{0}$

= *undefined*

4.



$\frac{rise}{run} = \frac{-4}{3}$

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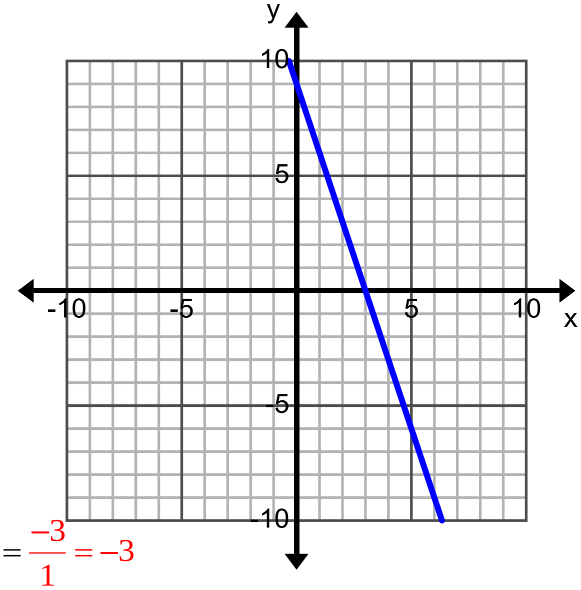
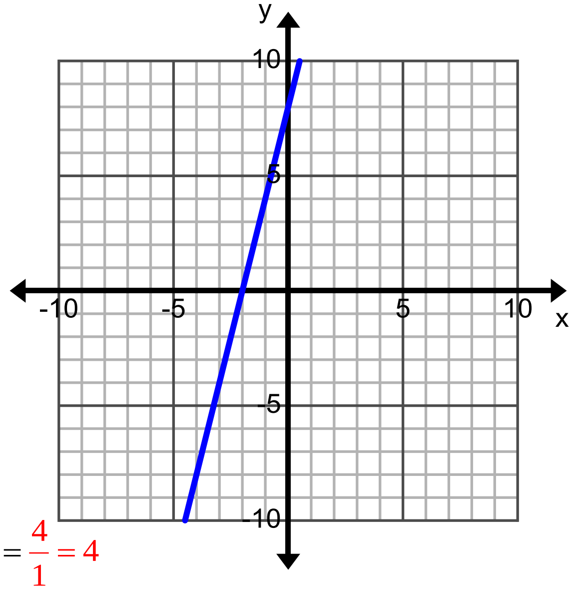
5.



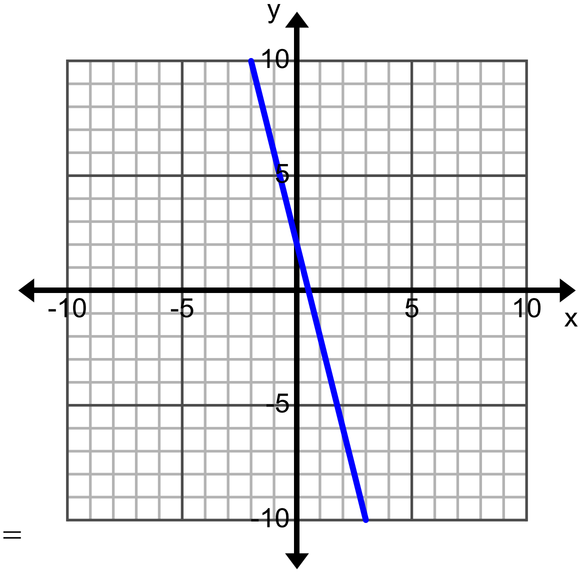
$m = \frac{-3}{1} = -3$

6.



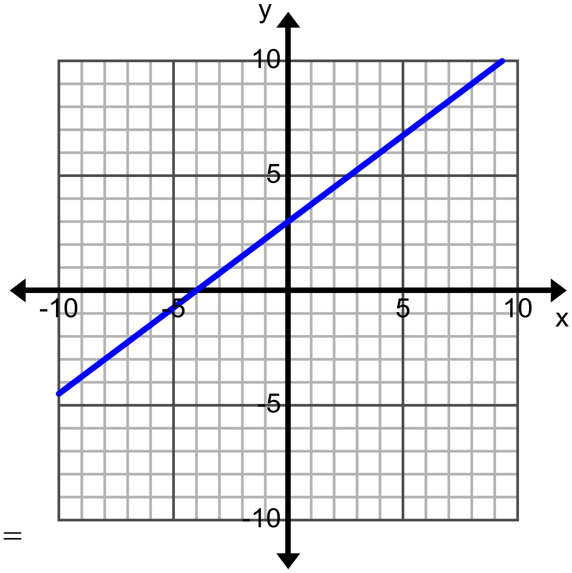
$m = \frac{4}{1} = 4$

7.



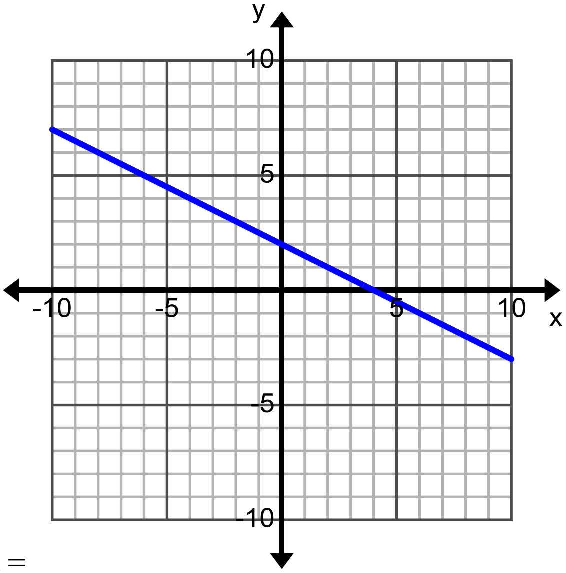
$m =$

8.



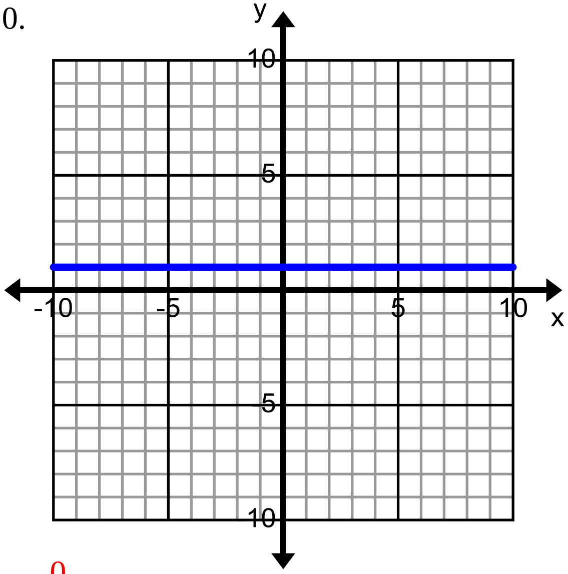
$m =$

9.



$m =$

10.



$m = \frac{0}{1} = 0$

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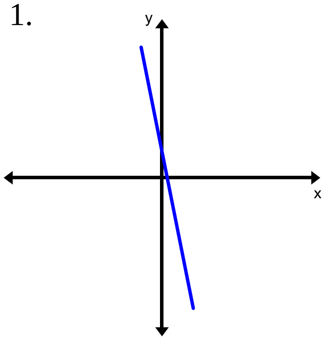
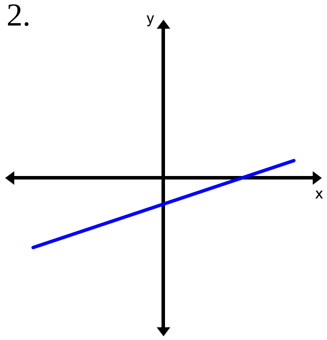
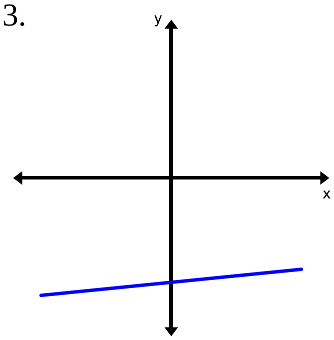
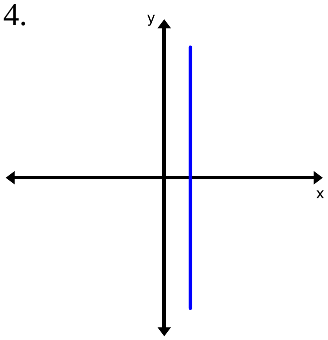
2.3d Homework: Finding Slope from Graphs

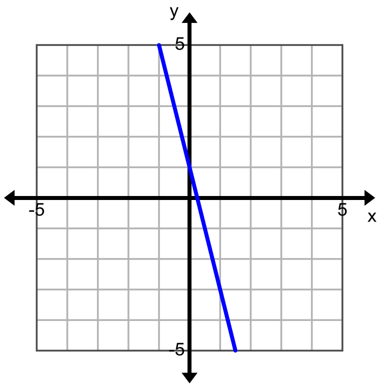
1. Do the graphs below have positive or negative slopes? How do you know?

1. 	2. 	3. 	4. 
Positive or negative?	Positive		Neither, the line is undefined.
5. Explain how you know whether a line of a graph has a positive or negative slope.			

For each line graphed below,

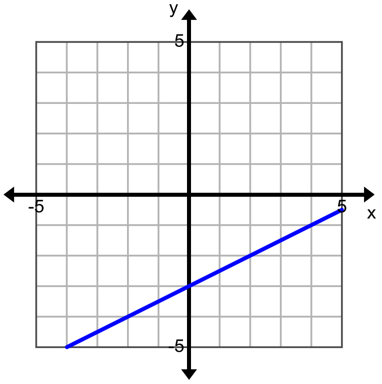
- Draw a right triangle to calculate the slope of the line. Answers will vary
- Label each triangle with a ratio and simplify the ratio $m = \frac{rise}{run} = \underline{\qquad}$

2.



$\frac{rise}{run} = \frac{-4}{1} = -4$

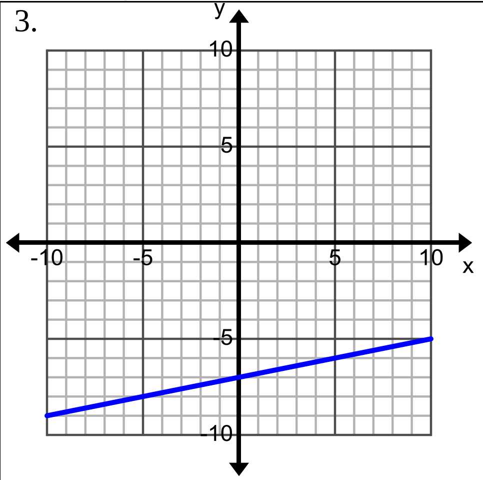
3.



$\frac{rise}{run} =$

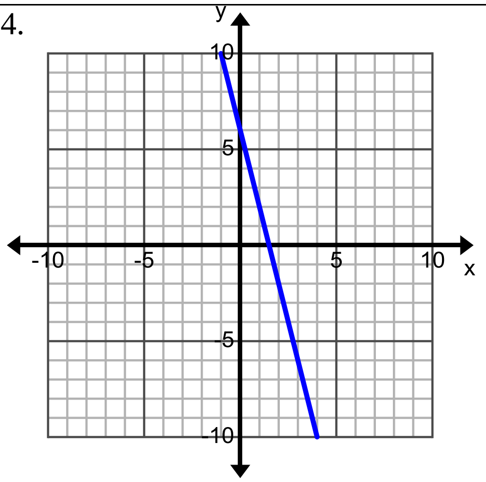
For each line graphed below, calculate the slope of the line.

3.



$\frac{rise}{run} =$

4.



$\frac{rise}{run} =$

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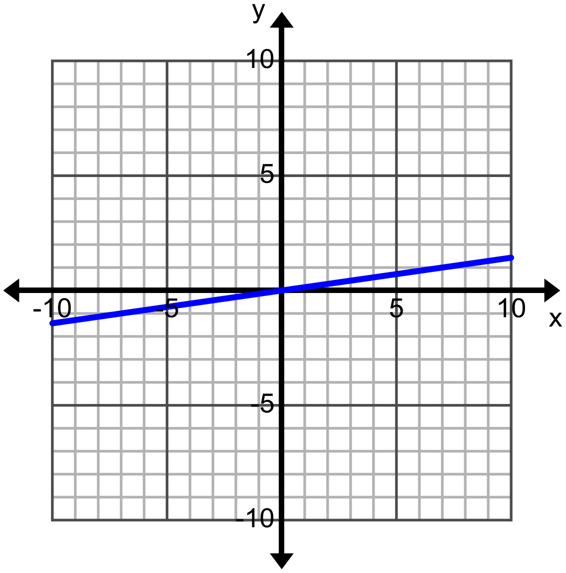
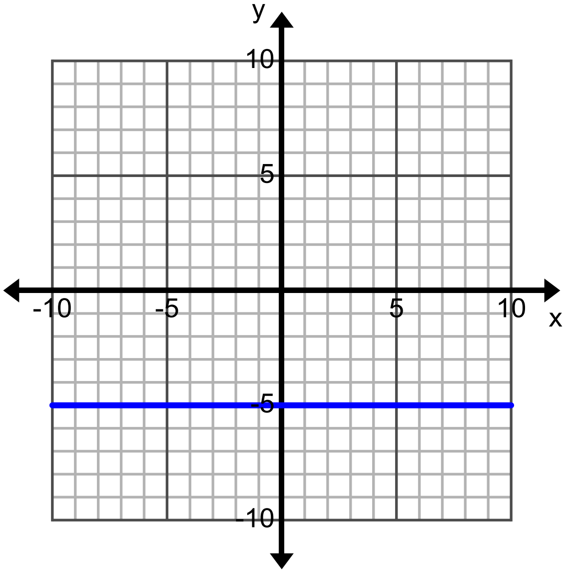
5.



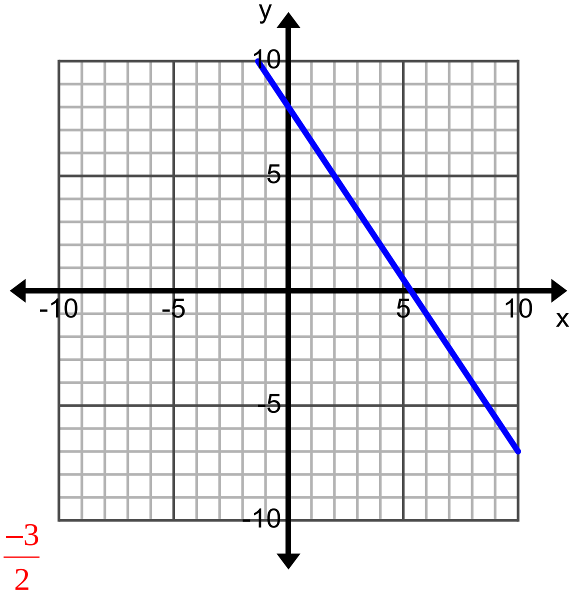
$m =$

6.



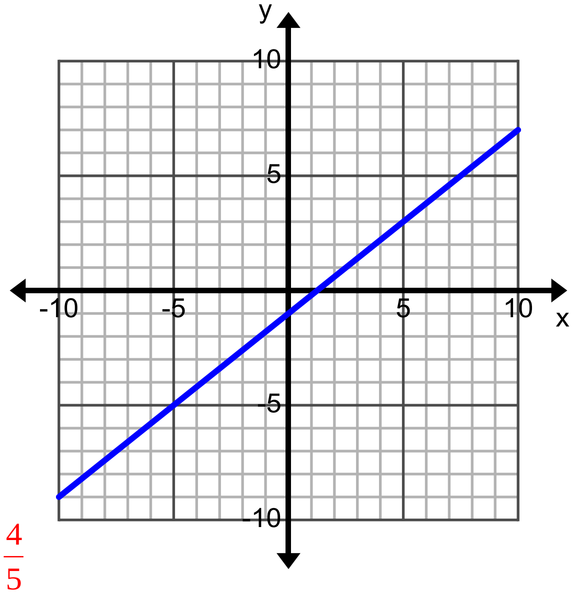
$m =$

7.



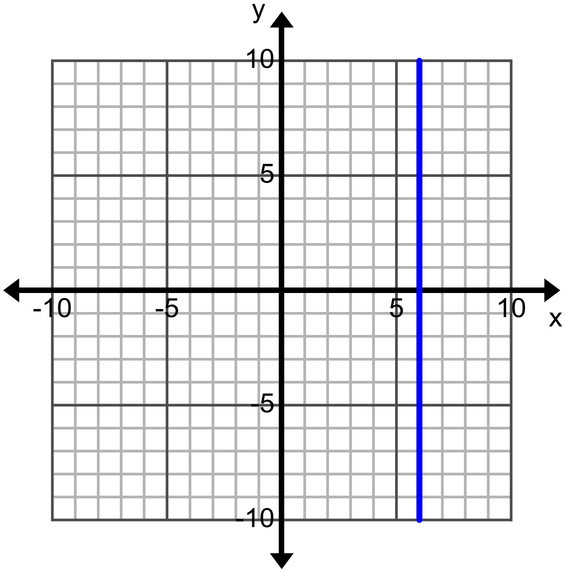
$m = \frac{-3}{2}$

8.



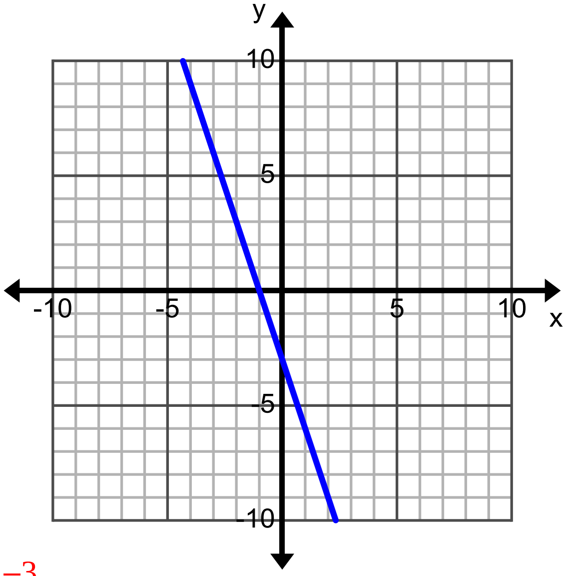
$m = \frac{4}{5}$

9.



$m =$

10.



$m = \frac{-3}{1} = -3$

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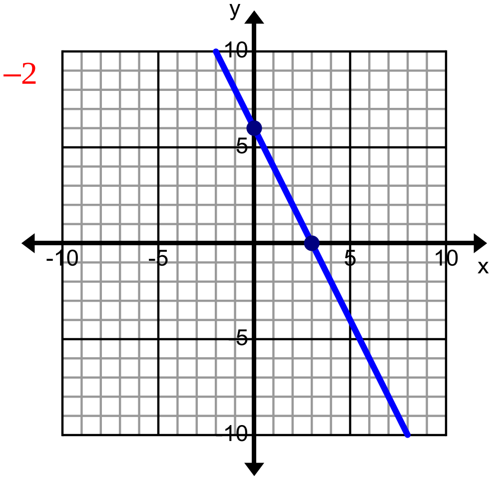
2.3e Class Activity: Finding Slope from Two Points

Calculate the slope of the following graphs:

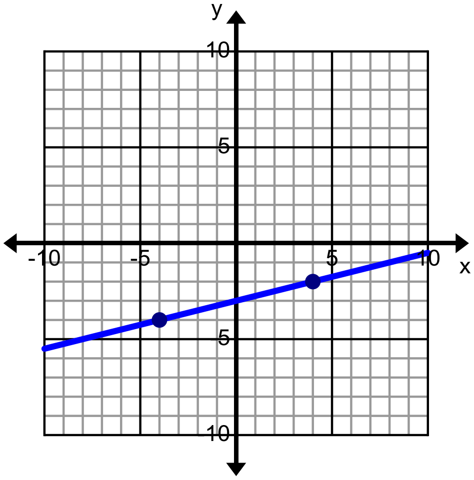
1.

$m = \frac{-6}{3} = -2$



2.

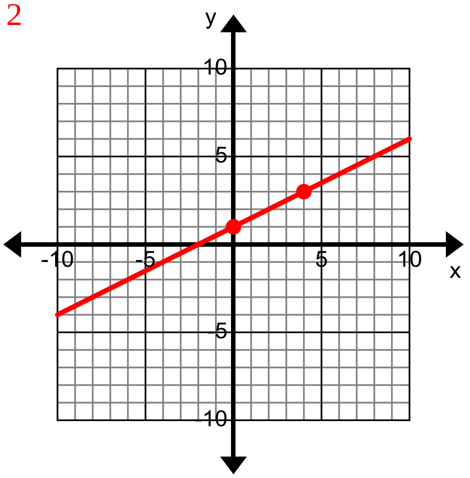
$m =$



Graph the following pairs of points. Use the graph to determine the slope.

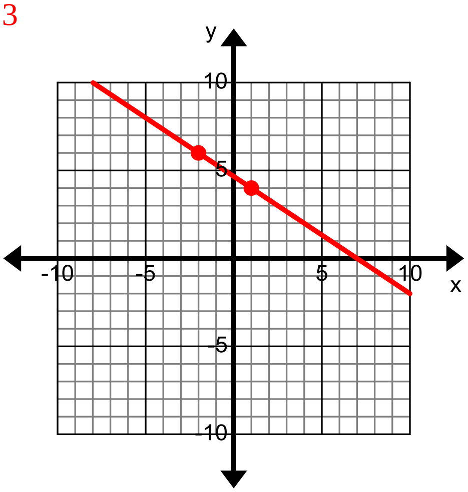
3. points: (4, 3) and (0, 1)

$m = \frac{1}{2}$



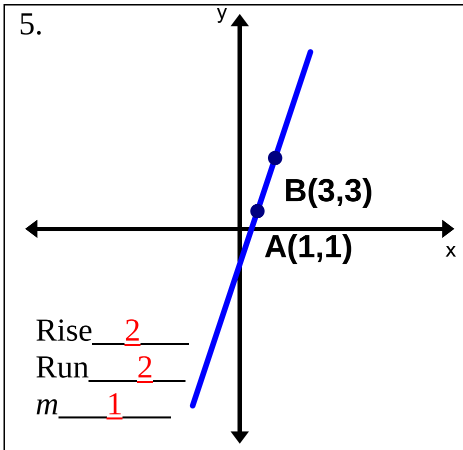
4. points: (1, 4) and (-2, 6)

$m = \frac{-2}{3}$



Find the rise and run and slope of each line shown below. You will have to think of a way to use the coordinate points to find the rise and run.

5.

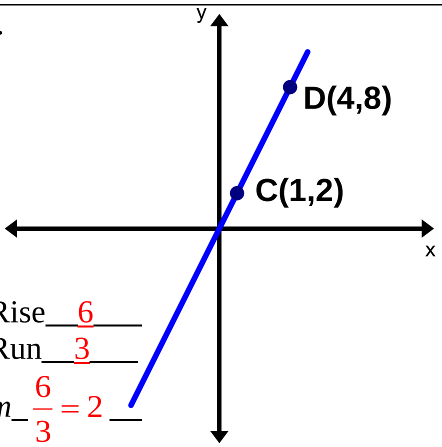


Rise__2___

Run__2___

m___1___

6.

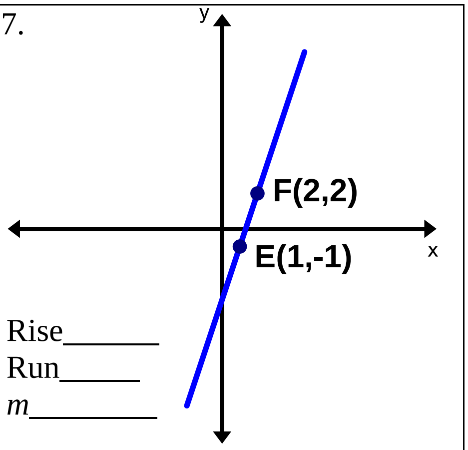


Rise__6___

Run__3___

m_ $\frac{6}{3} = 2$ __

7.



Rise______

Run_____

m_________

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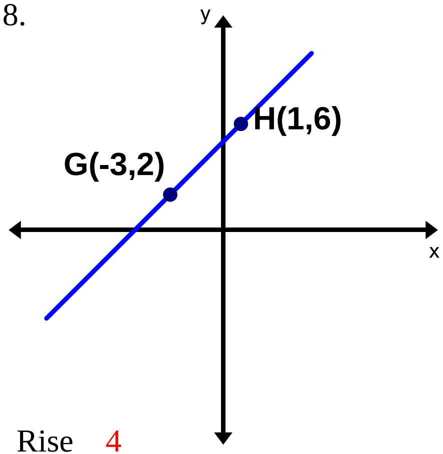
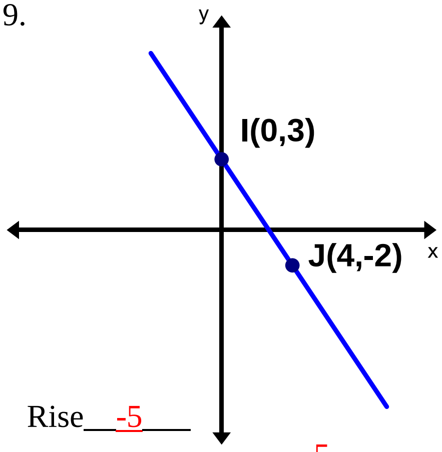
8.



Rise 4

Run 4 m 1

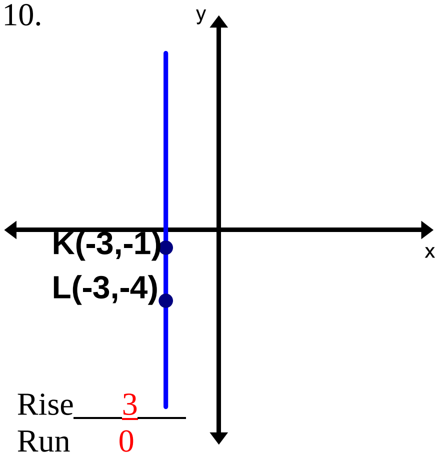
9.



Rise -5

Run 4 m $\frac{-5}{4}$

10.

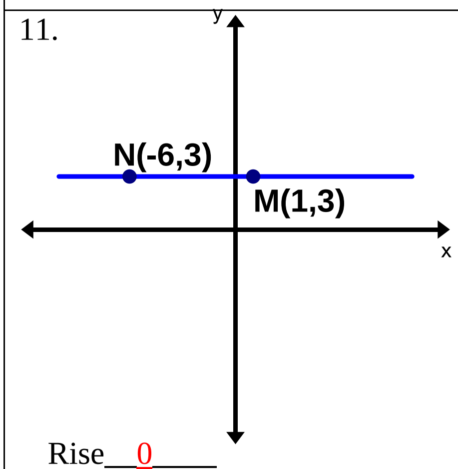


Rise 3

Run 0

m *undefined*

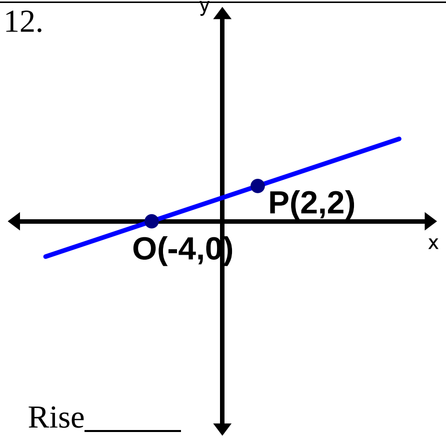
11.



Rise 0

Run 7 m 0

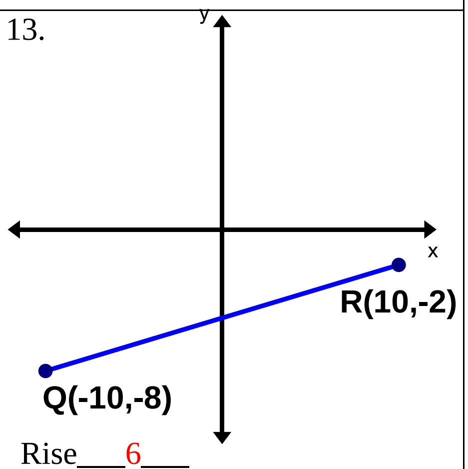
12.



Rise

Run m

13.



Rise 6

Run 20 m $\frac{6}{20}=\frac{3}{10}$

14. Graphing points can be time-consuming. Develop a procedure for calculating the slope without graphing each point. Explain your procedure below. Show that it works for problems 1-4 above.
See student answer

Discuss and compare your method for calculating slope without using right triangles on a graph with someone else.

15. Now discuss this formula: $slope = m = \frac{y_2 - y_1}{x_2 - x_1}$ What does it mean? How does it work? See student answer

The order that you choose to define your first and second point does not matter.

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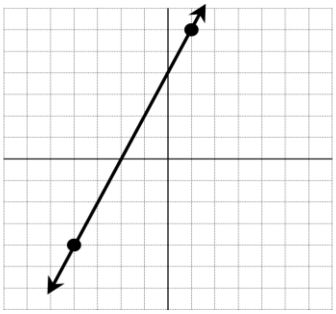
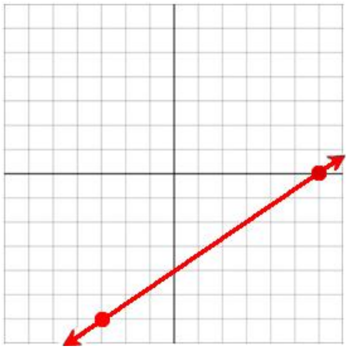
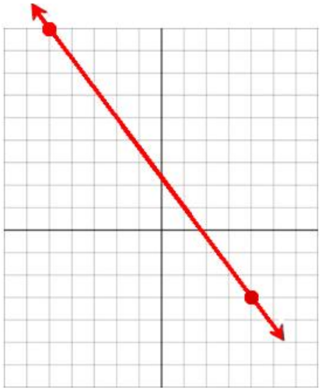
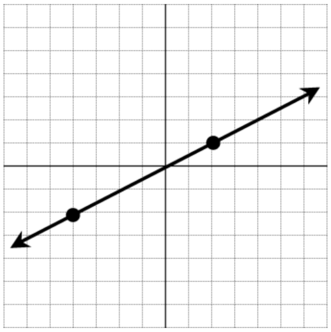
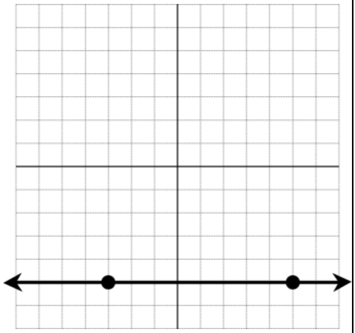
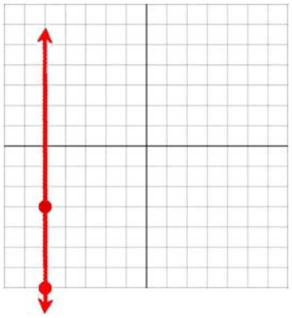
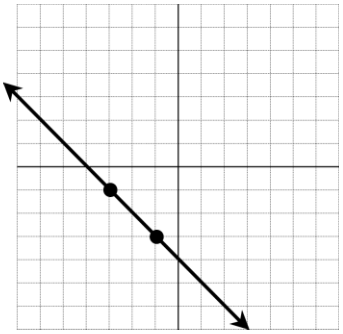
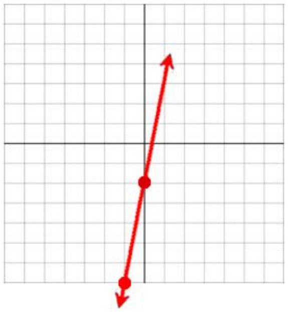
Fill in the missing information in the problems below. Use the empty box to calculate slope using the formula, $slope = m = \frac{y_2 - y_1}{x_2 - x_1}$. The first one has been done for you.

17. 	18. 	19. 	20. 
(-4, -4) (1, 6)	(-3, -6) (6, 0)	(4, -3) (-5, 9)	(-4, -2) (2, 1)
$m = \frac{6 - -4}{1 - -4}$	$m = \frac{0 - -6}{6 - -3}$	$m = \frac{9 - -3}{-5 - 4}$	
$\frac{\Delta y}{\Delta x} = \frac{10}{5} = 2$	$\frac{\Delta y}{\Delta x} = \frac{6}{9} = \frac{2}{3}$	$\frac{\Delta y}{\Delta x} = \frac{12}{-9} = -\frac{4}{3}$	$\frac{\Delta y}{\Delta x} = \frac{3}{6} = \frac{1}{2}$

21. 	22. 	23. 	24. 
(-3, -5) (5, -5)	(-5, -7) (-5, -3)		(-1 , -7) (0, -2)
$m = \frac{-5 - -5}{5 - -3}$	$m = \frac{-3 - -7}{-5 - -5}$		$m = \frac{-2 - -7}{0 - -1}$
$\frac{\Delta y}{\Delta x} =$	$\frac{\Delta y}{\Delta x} = \frac{4}{0} = \mathit{undefined}$	$\frac{\Delta y}{\Delta x} =$	$\frac{\Delta y}{\Delta x} = 5$

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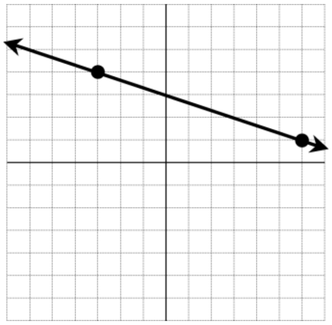
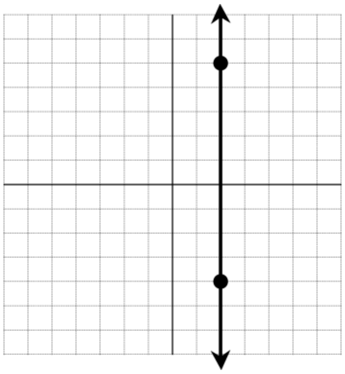
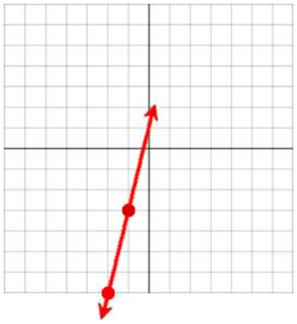
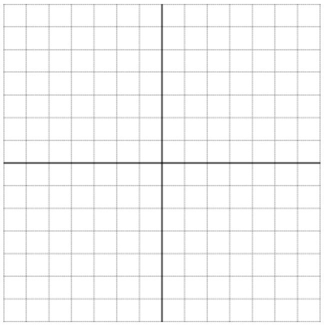
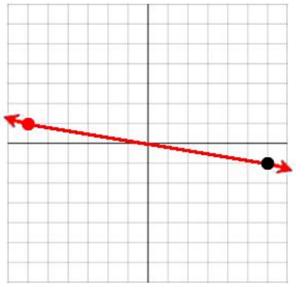
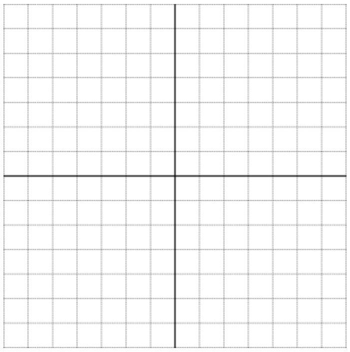
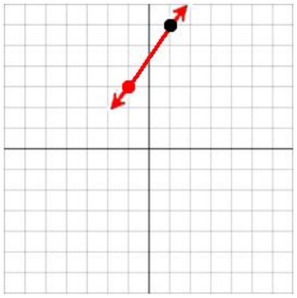
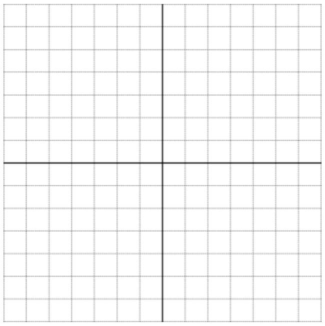
25. 	26. 	27. 	28. 
	(2, 5) (2, -4)	(-2, -7) (-1, -3)	(-7, 6) (6, -7)
$m = \frac{1-4}{6--3}$	$m = \frac{-4-5}{2-2}$	$m = \frac{-3--7}{-1--2}$	$m = \frac{-7-6}{6--7}$
$\frac{\Delta y}{\Delta x} = \frac{-3}{9} = \frac{-1}{3}$	$\frac{\Delta y}{\Delta x} = \frac{-9}{0} = \mathit{undefined}$	$\frac{\Delta y}{\Delta x} =$	$\frac{\Delta y}{\Delta x} = \frac{-13}{13} = -1$

29. 	30. 	31. 	32. 
(-6, 1) (6, -1)	(-1, 6) (-4, 6)	(	(0, -2) (4, -5)
	$m = \frac{6-6}{-4--1}$	$m = \frac{6-3}{1--1}$	$m = \frac{-5--2}{4-0}$
$\frac{\Delta y}{\Delta x} = \frac{-2}{12} = \frac{-1}{6}$	$\frac{\Delta y}{\Delta x} = \frac{0}{-3} = 0$	$\frac{\Delta y}{\Delta x} = \frac{3}{2}$	$\frac{\Delta y}{\Delta x} = \frac{-3}{4}$

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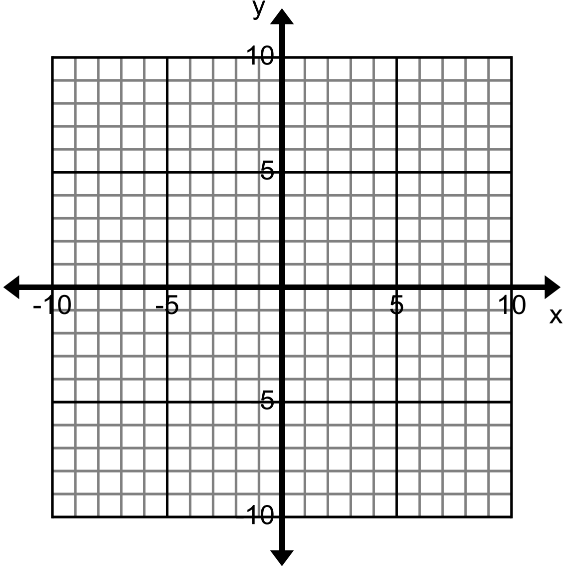
2.3e Homework: Finding Slope from Two Points

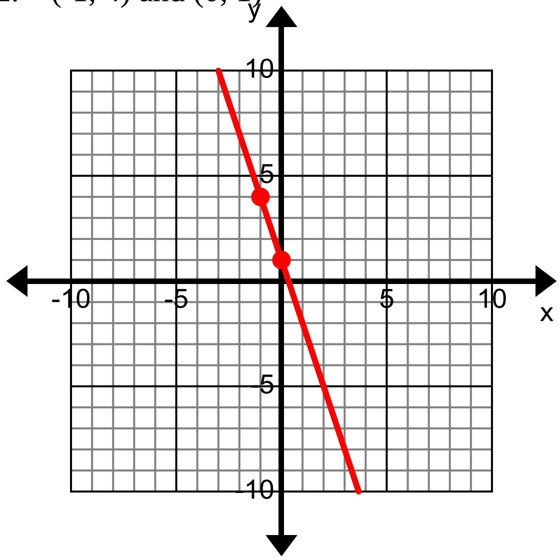
Graph the following pairs of points. Use the graph to determine the slope.

1. (-4, 3) and (2, 6)



$m=$

2. (-1, 4) and (0, 1)



$m = -3$

Calculate the slope of the line connecting each pair of points.

3. (1, 42) and (4, 40) $\frac{-2}{3}$

4. (-21, -2) and (-20, -5)

5. (3, -10) and (-6, -10) 0

6. (10, -11) and (11, -12) -1

7. (5, 1) and (-7, 13)

8. (14, -3) and (14, -7) $undefined$

9. (8, 41) and (15, 27) -2

10. (17, 31) and (-1, -5)

11. (-5, 36) and (-4, 3)

12. (32, -23) and (-6, -2) $-\frac{21}{38}$

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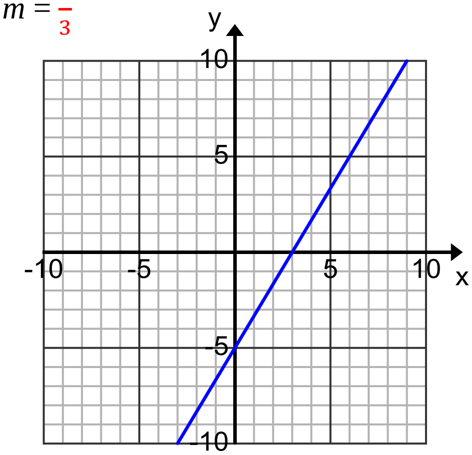
2.3f Class Activity: Practice Finding the Slope of a Line

Calculate the slope of the line on each graph.

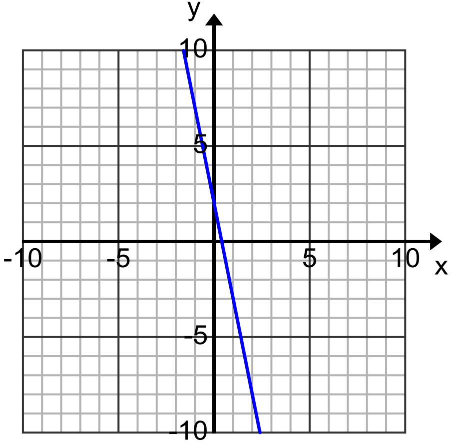
a.

$m = \frac{5}{3}$



b.

m



For each pair of points,

- Calculate the slope of the line passing through each pair.
- Find one other point that lies on the line containing the given points.

3. (10, -6) and (-5, 4)

4. (7, 3) and (-3, 0) $m = \frac{3}{10}$
Answers will vary

5. (0, 4) and (1, 0)

6. (-5, 1) and (-5, -2) *m=undefined*
Answers will vary

Calculate the slope of the line that contains the points given in each table. Calculate the slope twice, one time by using the Slope Formula with two points and the other time by finding the rate of change or unit rate in the table.

7.

x	y
3	4
4	5
5	6
6	7

m=__________

m=__________

8.

x	y
0	4
1	9
2	14
3	19

m=___5_____

m=___5_____

9.

x	y
0	9
3	12
6	15
9	18

m=___1_____

m=___1_____

10.

x	y
2	4
4	12
6	20
8	28

m=________

m=________

11. Why are the slopes the same no matter what two points you use to find the slope? Regardless of what points are chosen a triangle can be drawn that is similar to all other triangles that can be created from any other set of points. Since the triangles are similar their corresponding parts are proportional resulting in ratios that are the same. These ratios represent the rise and run of the line.

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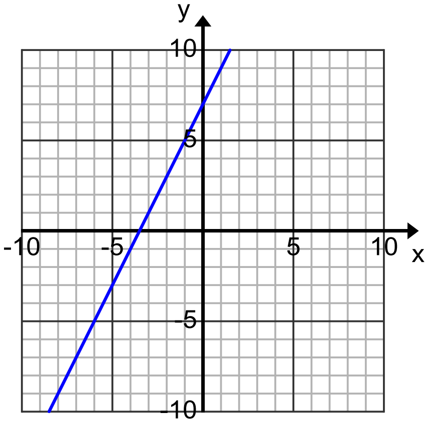
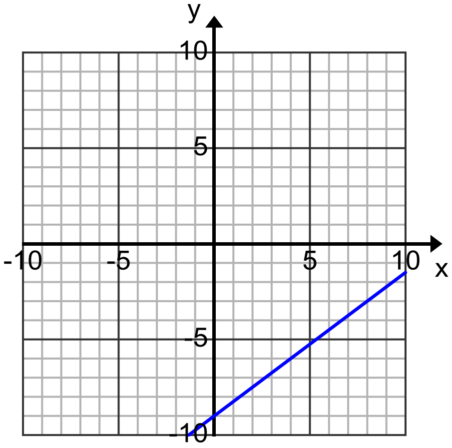
2.3f Homework: Practice Finding the Slope of a Line

Calculate the slope of the line on each graph.

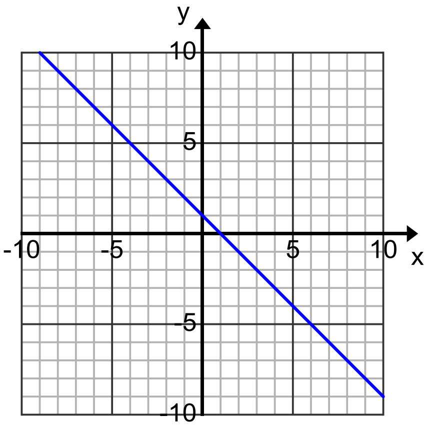
1. $m = 2$



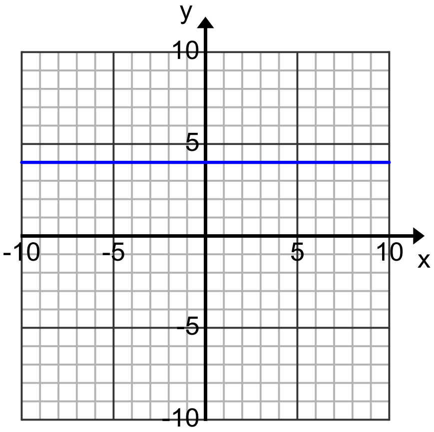
2. $m =$



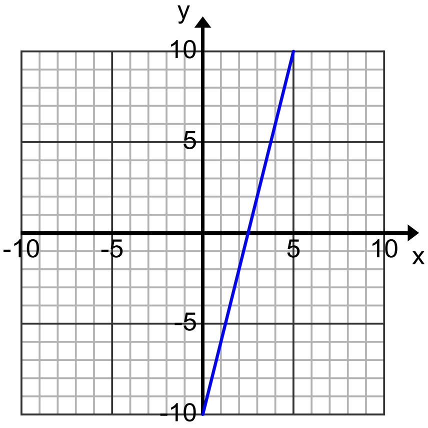
3. $m = -1$



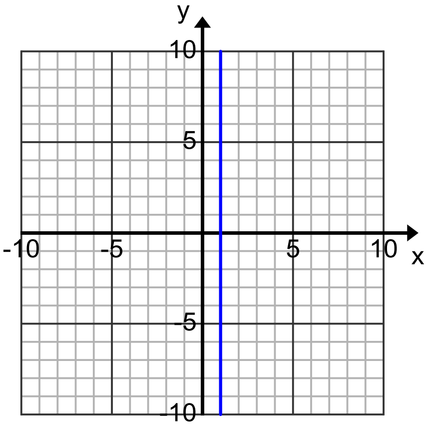
4. $m =$



5. $m = 4$



6. $m =$



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Calculate the slope of the line passing through each pair of points.

7. (3, 9) and (4, 12)

$m = 3$

8. (5, 15) and (6, 5)

9. (6, 9) and (18, 7)

$m = \frac{-1}{6}$

10. (-8, -8) and (-1, -3)

For numbers 11 and 12;

- Calculate the slope of the line passing through each pair
- Find one other point that lies on the line containing the given points

11. (-6,-5) and (4,0)

$m = \frac{1}{2}$

Answer will vary

12. (4,1) and (0,7)

Calculate the slope of the line that contains the points given in each table.

13. m = -3

x	y
3	-9
5	-15
9	-27

14.

x	y
8	1
6	3
2	7
-4	13

15. m = -9

x	y
0	4
1	-5
2	-14
3	-23

16.

x	y
10	1
8	1
-12	1
-14	1

17. m = 2/5

x	y
-5	2
5	6
10	8

18. m

x	y
-3	5
-3	10
-3	15
-3	20

19. Why doesn’t it matter which two points you use to find the slope?

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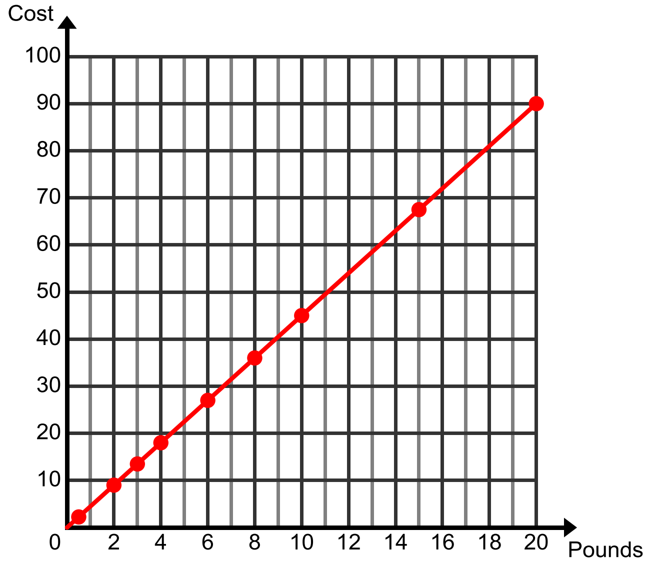
2.3g Class Activity: Finding Slope from a Context

1. Gourmet jellybeans cost $9 for 2 pounds.

   a. Complete the table.

Pounds	.5	2	3	4	$6	8	10	15	20
Total Cost	$2.25	$9	$13.5	$18	$27	$36	$45	$67.5	$90

   b. Graph and label the relationship.



   c. What is the slope of the line?

$$\frac{9}{2}$$

   d. Write the slope of the line as a rate of change that describes what the slope means in this context. $\frac{9}{2} = \frac{4.5}{1}$

For every pound of jelly beans that are bought it cost $4.50.

   e. Write an equation to find the cost for any amount of jellybeans.

$$y = 4.50x \ or\ y = \frac{9}{2}x$$

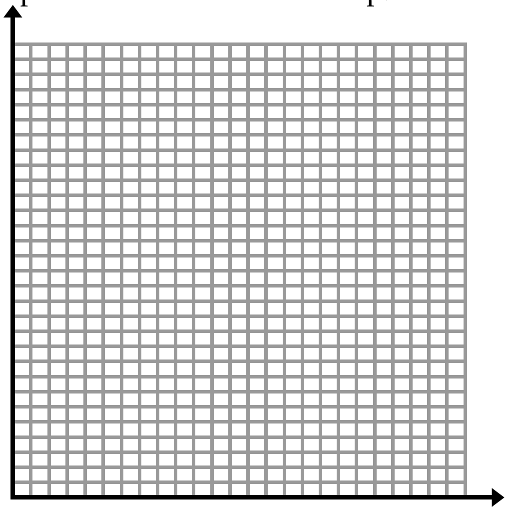
   f. Why is the data graphed only in the first quadrant? It is impossible to buy a negative amount of jelly beans.

2. Kaelynn takes the same amount of time to solve each of the equations on her math homework. She can solve 10 equations in 8 minutes.

   a. Complete the table.

Minutes	2		8			20	
Equations Solved		5		15			30

   b. Graph and label the relationship.



   c. What is the slope of the line?

   d. Write the slope of the line as a rate of change that describes what the slope means in this context.

   e. Write an equation to find the number of equations solved for any number of minutes.

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3. Mr. Irving and Mrs. Hendrickson pay babysitters differently.

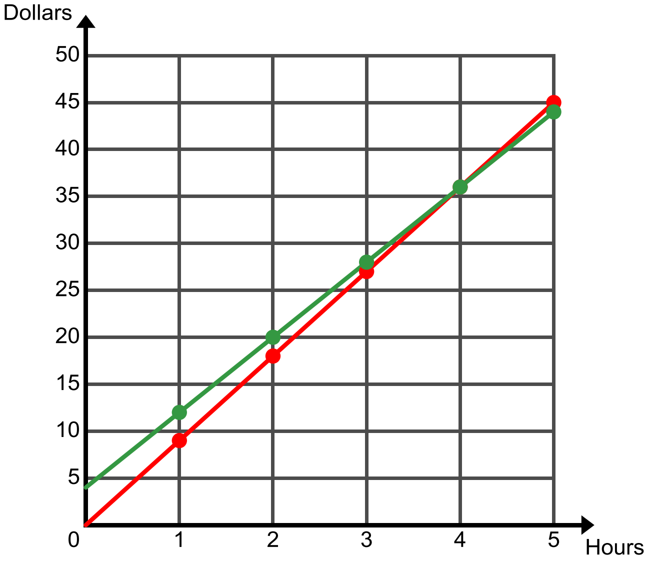
a. Examine the table. Describe the difference.

Hours	Irving	Hendrickson
0	\$0	\$4
1	\$9	\$12
3	\$27	\$28
5	\$45	\$44

Mr. Irving pays \$9 for every hour of babysitting. It appears that Mrs. Hendrickson pays \$8 an hour except for the first hour.

b. Are both relationships proportional?
No, Mrs. Hendrickson's relationship is not proportional.

c. Graph and label the two pay rates (two different lines).



d. What is the slope of each line?

Irving: $\frac{9}{1} = 9$ Hendrickson: $\frac{8}{1} = 8$

e. Write each slope as a rate of change to interpret its meaning?

Irving: _\$9 per hour________

Hendrickson: __\$8 per hour__

f. Explain the different *y* -intercepts.
The *y*-intercept for Mr. Irving is 0. The *y*-intercept for Mrs. Hendrickson is 4, she pays \$4 dollars to her babysitter up front, then pays \$8 per hour.

g. Is one babysitting job better than the other? Why or why not?
It depends on how long the babysitter is babysitting. Mrs. Hendrickson is better if the babysitting is less than 4 hours. Mr. Irving is better if the babysitting is more than 4 hours.

h. Write an equation for each situation.

- Irving: __$y = 9x$________
- Hendrickson: __$y = 8x + 4$______

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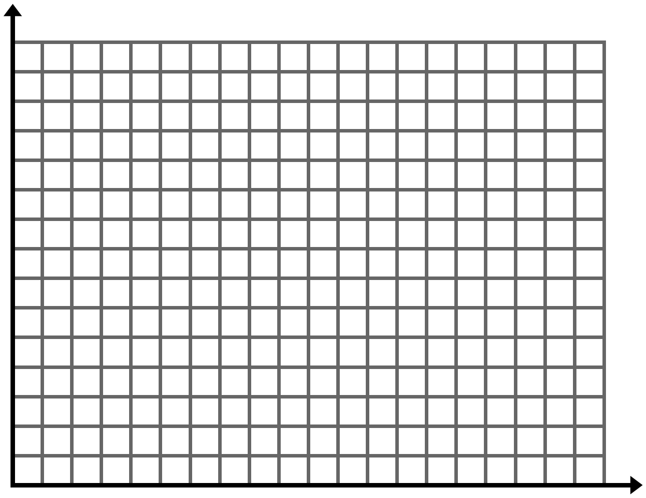
2.3g Homework: Finding Slope from a Context

1. The soccer team is going out for hot dogs. Greg's Grill is having a special on hot dogs: four hot dogs for three dollars. Each hot dot costs the same amount of money.
   a. Complete the table.

Hot Dogs	1			16		28	40
Total Cost		$3	$9		$18		

b. Label the graph axes and then graph the relationship.



c. What is the slope of the line?

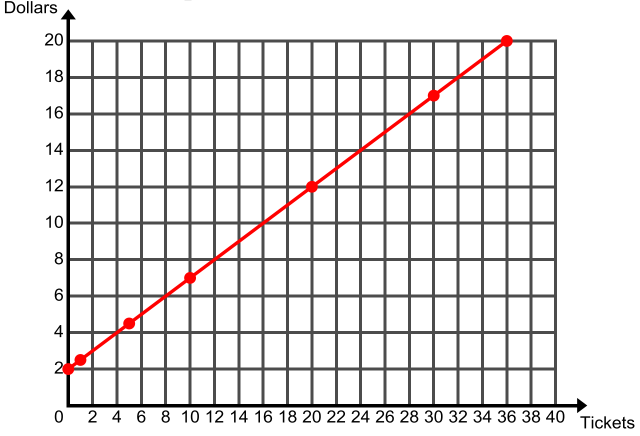
d. Write a sentence with correct units that describes what the slope means.

e. Write an equation to find the cost for any amount of hot dogs.

2. The state fair costs $2 to get in plus $.50 per ticket to go on rides. Complete the following table, showing the cost for getting into the fair with additional tickets for rides.
   a. Complete the table.

Tickets	0	1	5	10	20	30	36
Total Expense	2	$2.50	$4.50	7	$12	$17	$20

b. Label the graph axes and then graph the relationship.



c. What is the slope of the line?

$\frac{1}{2}$

d. Write a sentence that describes what the slope of the line means.
It costs $0.50 per ticket.

e. Why does the line not pass through 0?
The fair costs $2 to get into before you buy any tickets

f. Write an equation to find the total expense at the fair with any amount of tickets purchased.

$y = .5x + 2 \text{ or } y = \frac{1}{2}x + 2$

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3. Excellent Bakers and Delicious Delights bakeries charge differently for sandwiches for business lunches.

a. Examine the table. Describe the difference.

Sandwich Meals	Excellent Bakers	Delicious Delights
0	$0	$15
1	$20	$30
3	$60	$60
5	$100	$90

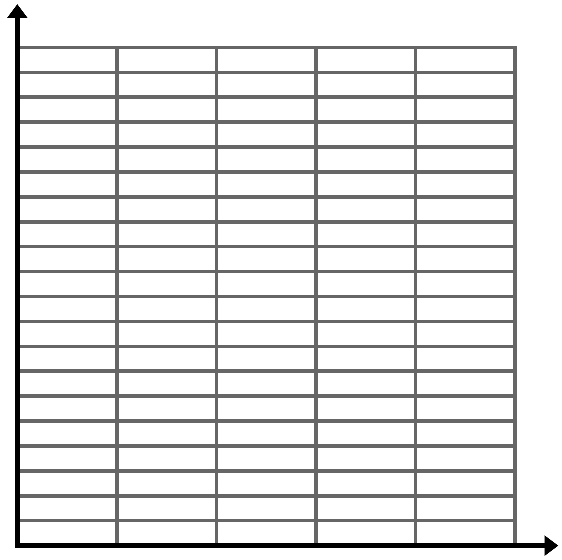
b. Are both relationships proportional?

c. Find the rate of change for each relationship.

Excellent :

Delicious:

d. Graph the two pay rates (two different lines). Label.



e. What is the slope of each line?

Excellent: Delicious:

f. What does the slope tell you about these two situations?

g. Explain the different *y* intercepts.

h. Is one situation better than the other? Why or why not?

i. Write an equation for each situation.

- Excellent Bakery _________
- Delicious Delights _______

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2.3h Class Activity: The Equation of a Linear Relationship

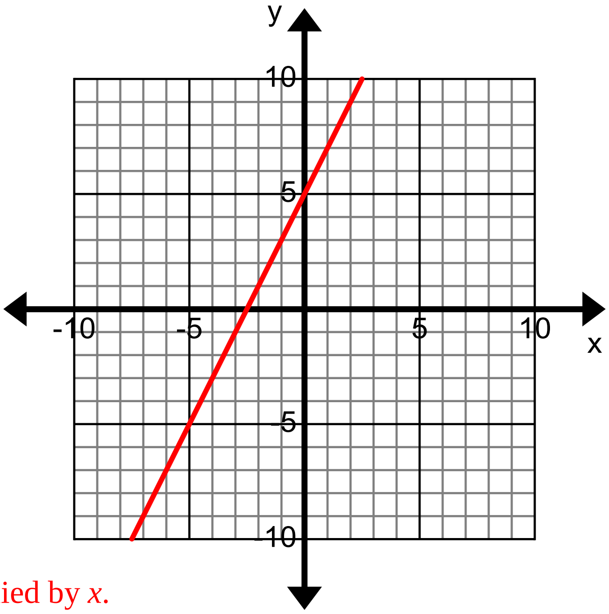
Throughout this chapter the components of a linear relationship have be investigated. A linear relationship is defined by the constant rate of change that it possesses and it can be represented in many ways. In this lesson the focus will be more on the equation that represents a linear equation.

The equation given below represents a linear relationship.

$$y = 2x + 5$$

1. Graph this equation of this line by making a table of values that represent solutions to this equation. (This if often called a T-chart).

x	*y*
-1	3
0	5
1	7
2	9



2. Find the slope of this line?

   $m=2$

3. Where do you see the slope in the equation?

   The slope is in front of the *x*. It is being multiplied by *x*.

Often the letter *b* is used to denote the *y*-intercept.

4. What is the *y*-intercept of this line?

   $b=5$ or (0, 5)

5. Where do you see the *y*-intercept in the equation?

   The *y*-intercept is the constant that is being added or subtracted.

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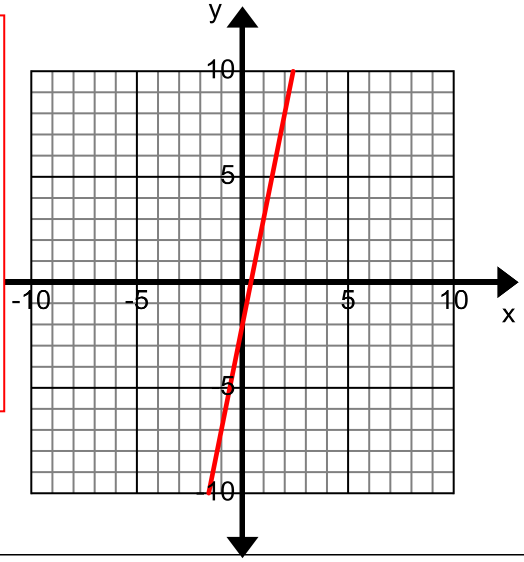
For each of the equations given below, make a table of values to help you graph the line. Then identify the slope and *y*-intercept. Circle the slope in your equation and put a star next to the *y*-intercept.

6. $y = \textcircled{5}x - 2*$

x	y

Table will vary.

The *y*-intercept in this problem is -2. This can sometimes be an issue for students. Remind them that they are adding negative 2 which is same thing as subtracting 2. Also talk about adding zero for a *y*-intercept of zero.



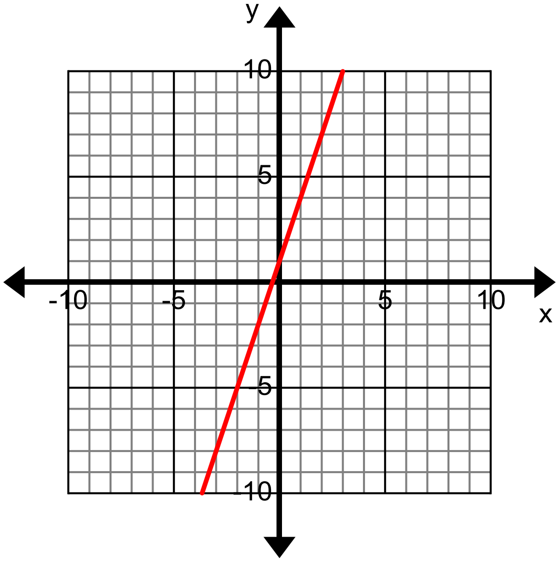
Slope(m): 5

y-intercept(b): -2

7. $y = \textcircled{3}x + 1*$

x	y

Table will vary.



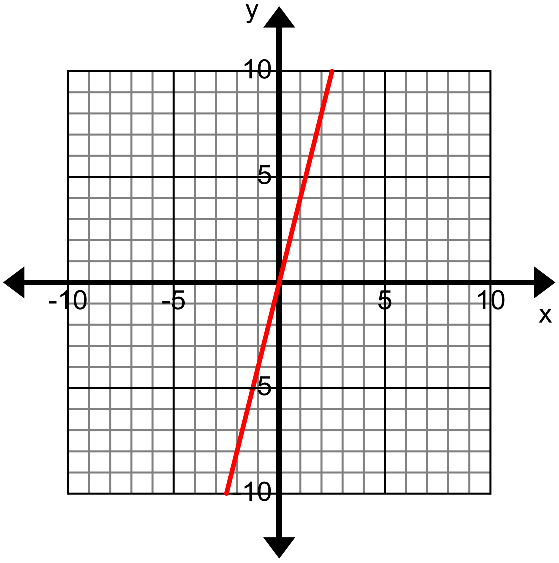
Slope(m): 3

y-intercept(b): 1

8. $y = \textcircled{4}x + 0*$

x	y

Table will vary.



Slope(m): 4

y-intercept(b): 0

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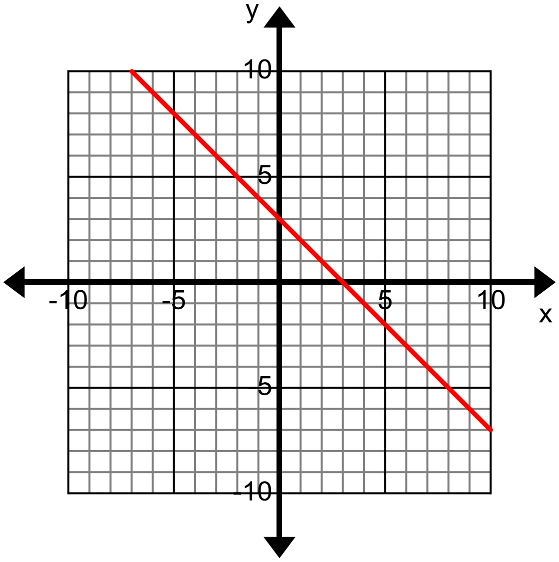
9. $x + y = 3$

$y = -1x + 3$*

x	y

Table will vary.

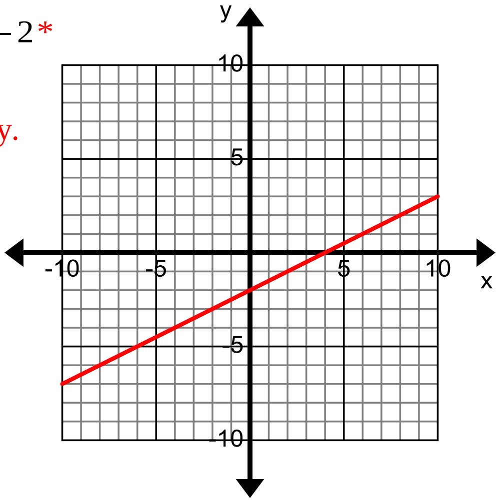


Slope(m): -1

y-intercept(b): 3

10. $y = \frac{1}{2}x - 2$*

Table will vary.

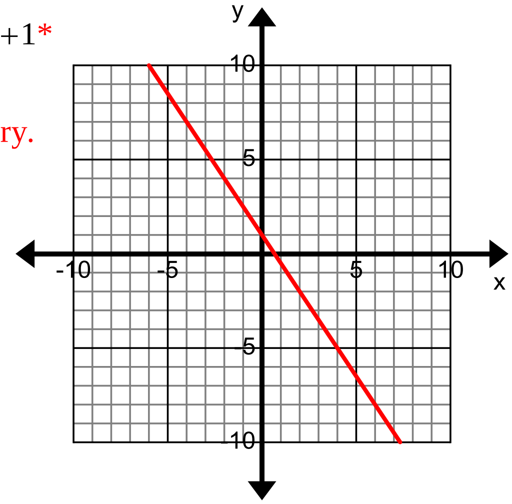


Slope(m): $\frac{1}{2}$

y-intercept(b): -2

11. $y = -\frac{4}{3}x + 1$*

Table will vary.



Slope(m): $-\frac{4}{3}$

y-intercept(b): 1

12. Examine the graphs and equations given above. Describe the general form of a linear equation. In other words, in general, how is a linear equation written? What are its different parts?

Answers may include; $y=mx+b$ or $y=b+mx$, where m is the slope and b is the y-intercept. The desired outcome/y is equal to the slope/rate of change times the input/x plus your initial value/y-intercept/starting point.

13. Write down he general form of a linear equation in the box below based off of your class discussion.

Slope-intercept form of a linear equation is

__________***y = mx + b***________________

where m represents the slope (rate of change)

and b represents the y-intercept (initial value or starting point)

14. What if the y-intercept is zero, how do you write the general form of the equation?

If the y-intercept is zero then the general form of the equation is $y=mx+0$ or $y=mx$

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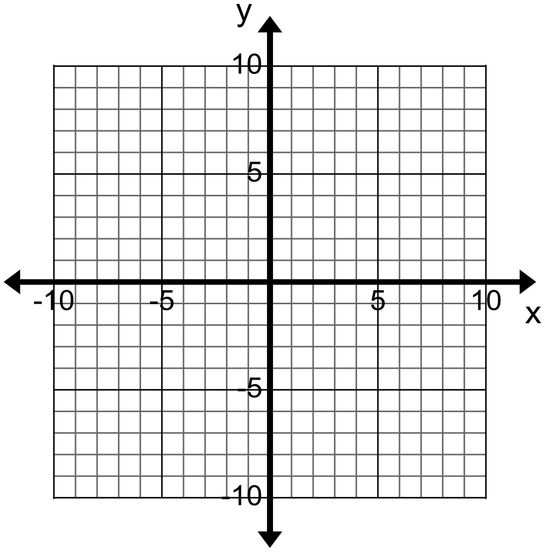
2.3h Homework: The Equation of a Linear Relationship

For each of the equations given below, make a table of values to help you graph the line. Then identify the slope and y-intercept. Circle the slope in your equation and put a star next to the y-intercept.

1. $y = 3x - 3$

x	y

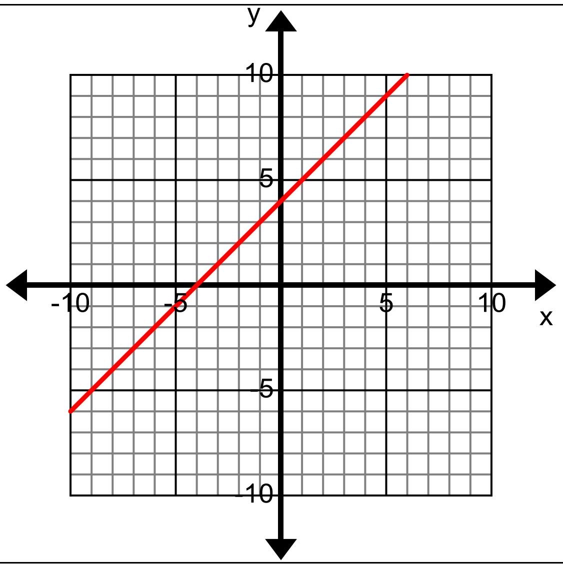


Slope(m): y-intercept(b):

2. $y = \textcircled{1}x + 4$*

x	y

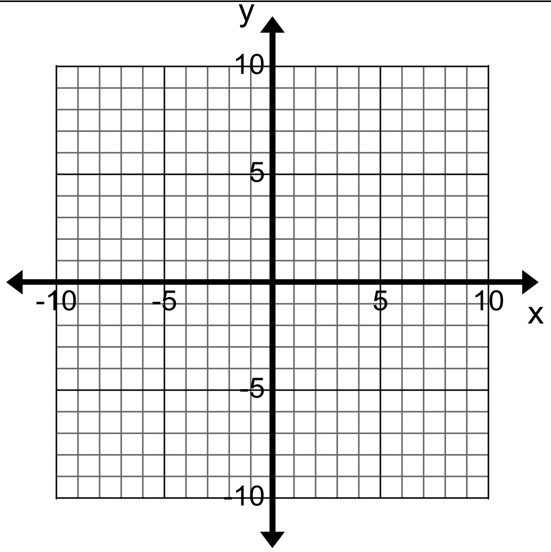
Table will vary.



Slope(m): 1 y-intercept(b): 4

3. $y = -5x$

x	y



Slope(m): y-intercept(b):

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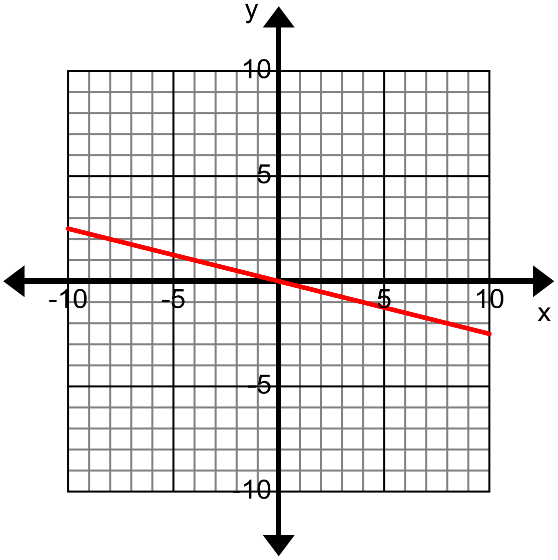
4. $y = -\frac{1}{4}x$ *

x	y

Table will vary.

Slope(m): $-\frac{1}{4}$

y-intercept(b): 0



5. $x + y = 2$

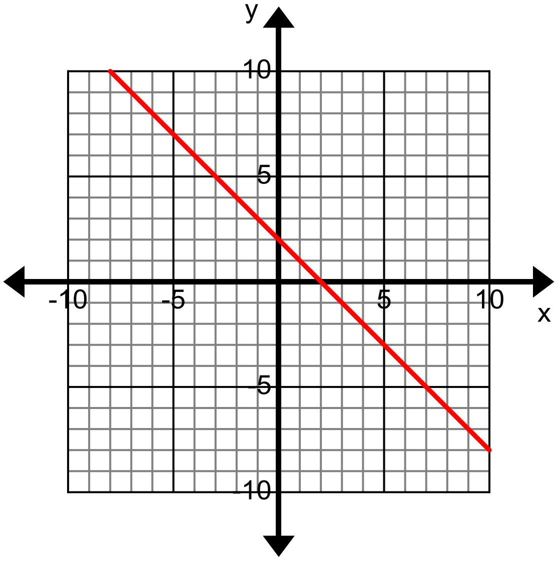
$y = -1x + 2$ *

x	y

Table will vary.

Slope(m): -1

y-intercept(b): 2

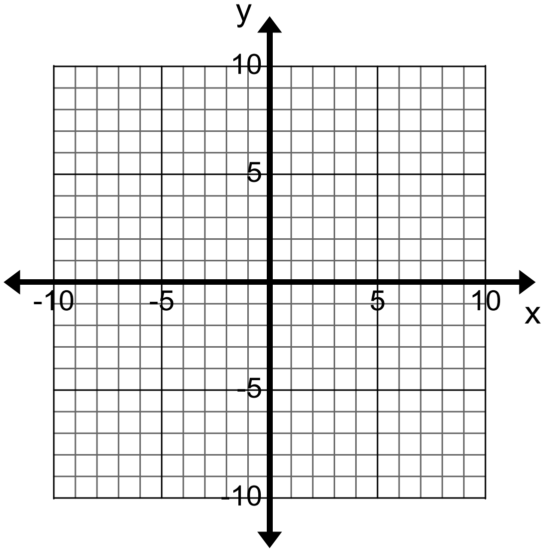


6. $-x + y = -1$

x	y

Slope(m):

y-intercept(b):



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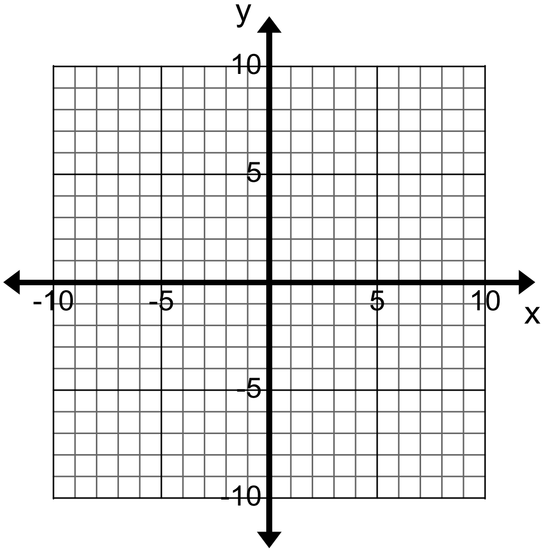
7. $y = -2x + 6$

x	y

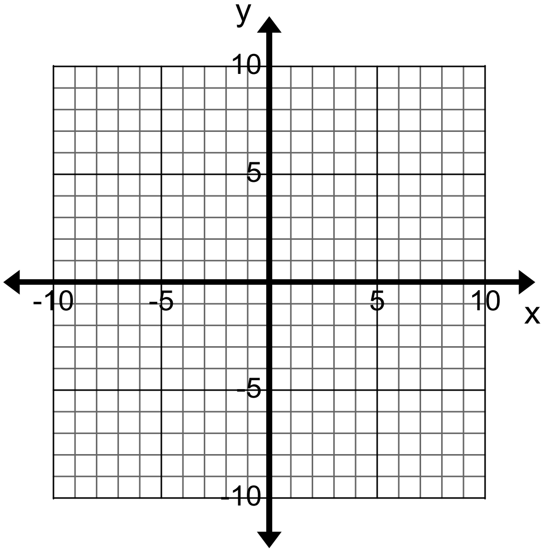
Slope(m): y-intercept(b):



8. $y = -\frac{4}{3}x - 2$

x	y

Slope(m): y-intercept(b):

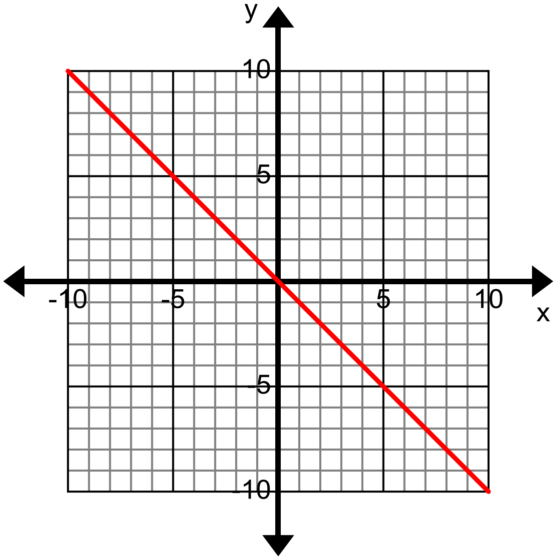


9. $y = -1x + 0*$

x	y

Table will vary.

Slope(m): -5 y-intercept(b): 0



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2.3i Class Activity: Use Dilations and Proportionality to Derive the Equation $y = mx$.

Up to this point you have been investigating how to describe many patterns and stories with a linear relationship. You have begun to get a sense of how linear relationships are formed and described.

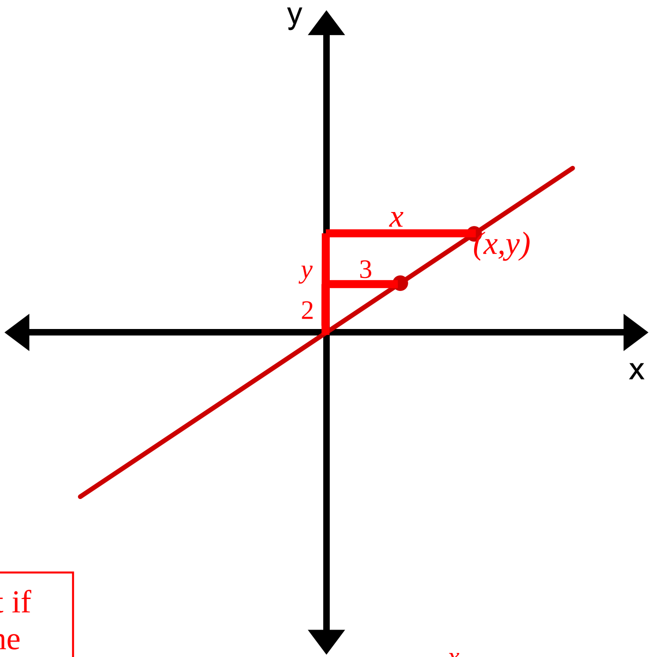
Write down what you know about linear relationships below.

- A linear relationship is defined by its constant rate of change and its initial value.
- The constant rate of change in the graph of a linear relationship is called slope and the initial value is the *y*-intercept.
- The equation $y=mx+b$ relates the x to y where m is the slope and b it the *y*-intercept.

Now write down what you know about linear proportional relationships below.

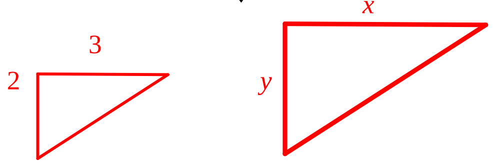
- A proportional relationship is a special linear relationship.
- A proportional relationship is defined by a proportional constant or unit rate that relates x to y.
- The proportional constant is the same as a unit rate in a proportional relationship. These can be interpreted as rate of change
- When a proportional relationship is graphed it is a straight line going through the origin.
- A proportional relation can also be represented by an equation where the proportional constant relates your x value to your y value.

You are going to use the facts listed above to derive the equation $y=mx$ using dilations. Begin by looking at the example below.



1. Graph a line on the coordinate plane to the right that goes through the origin and has a slope of $\frac{2}{3}$. Label the rise and run on your graph with a right triangle. Redraw and label this triangle in the space provided below the graph. See graph.
2. Does this line describe a proportional relationship? Yes, the graph is linear and goes through the origin.
3. Choose any point (x,y) on your line and draw a slope triangle that describes the rise and run. Redraw and label this triangle in the space provided below the graph. See graph.

Recall that the slope ratio is constant, meaning that if you draw a right triangle from any two points on the line the ratio of its side lengths will be proportional to the side lengths of any and all other right triangles drawn from the line. This is true because the triangles are dilations of each other. This means that you can set your slope ratios for each triangle that you drew equal to each other*(#4).*



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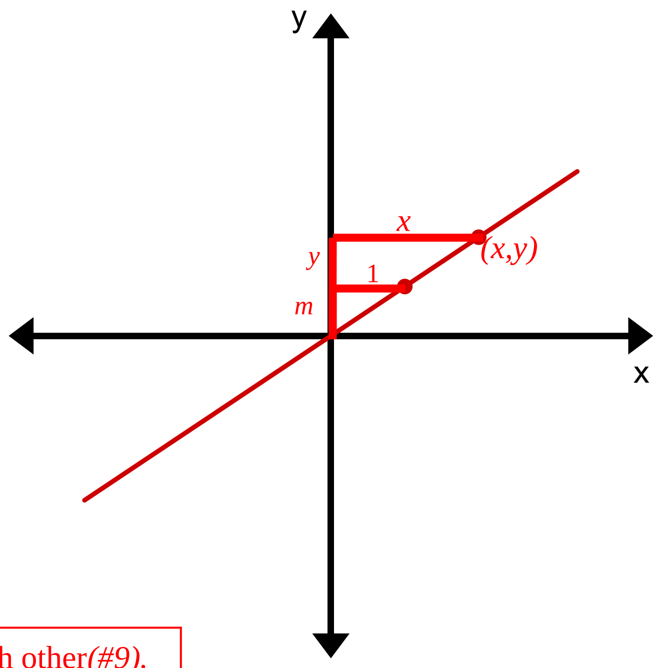
4. Write a proportional statement with your ratios. $\frac{2}{3}=\frac{y}{x}$

5. Solve the equation that you wrote above for *y*. $y=\frac{2}{3}x$

Notice that this equation is of the form *y*=*mx* where *m* is the slope of the line*(#5).*

Now look at the general case for the form *y*=*mx*.

6. Graph a line on the coordinate plane to the right that goes through the origin and has a slope of *m*; **remember that slope is the same as a unit rate which compares your *y*-value to an *x* value of 1.** Label the rise and run on your graph with a right triangle. Redraw and label this triangle in the space provided below the graph. See graph.

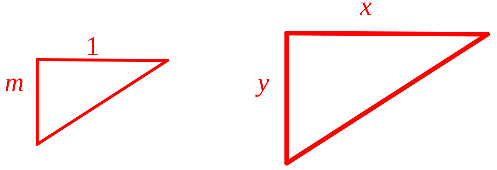


7. Does this line describe a proportional relationship? Yes, it is a straight line going through the origin.

8. Choose any point (*x*,*y*) on your graph and draw a slope triangle the describes that rise and run. Redraw and label this triangle in the space provided below the graph. See graph.

Similarly you can set your slope ratios equal to each other*(#9).*

9. Write a proportional statement with your ratios. $\frac{m}{1}=\frac{y}{x}$



10. Solve the equation that you wrote above for *y*.
$y=mx$

Notice that this is the equation *y*=*mx* that describes a proportional relationship.

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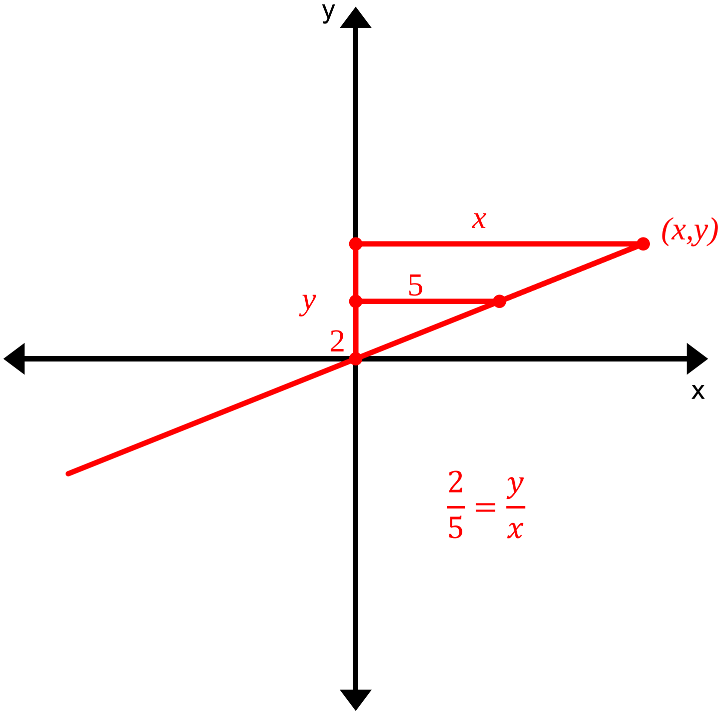
11. Show that the equation for a line that goes through the origin and has a slope of $\frac{2}{5}$ is $y = \frac{2}{5}x$ using dilations and proportionality.



$$\frac{2}{5} = \frac{y}{x}$$

The rise for the right triangle formed from the point (5,2) and the origin is 2 and the run is 5. Choose any point (*x*,*y*) on the line and form a right triangle where the rise is *x* and the run is *y*. Since the right triangles formed are dilations of one another their corresponding parts are proportional; thus$\frac{2}{5} = \frac{y}{x}$. Upon solving for *y* you get $y = \frac{2}{5}x$.

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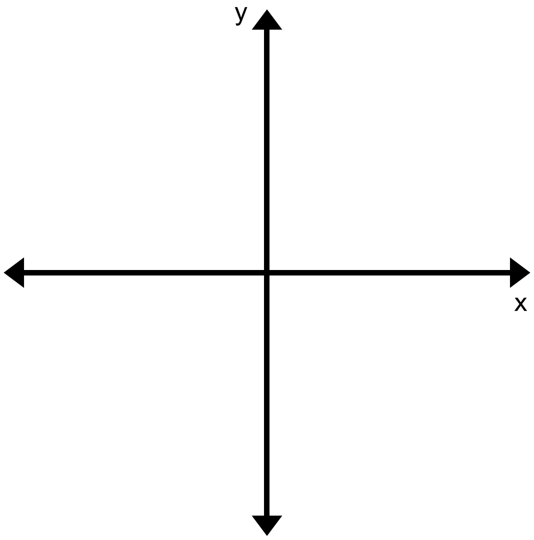
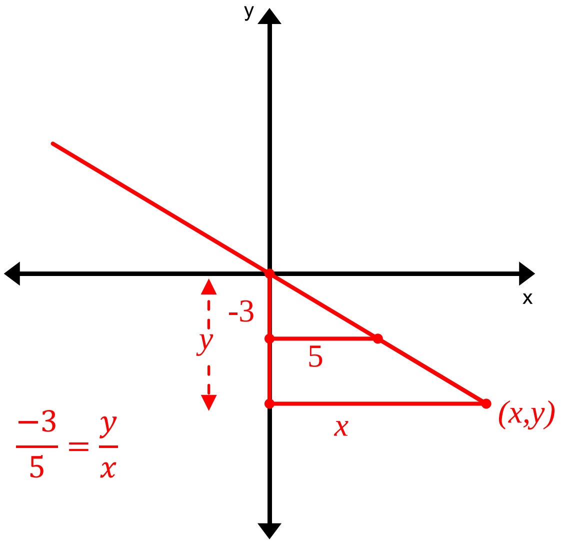
2.3i Homework: Use Dilations and Proportionality to Derive the Equation $y = mx$.

1. Show that the equation for a line that goes through the origin and has a slope of $\frac{1}{3}$ is $y = \frac{1}{3}x$ using dilations and proportionality.



See number 11 on page 118 for an example similar to this problem.

2. Show that the equation for a line that goes through the origin and has a slope of $-\frac{3}{5}$ is $y = -\frac{3}{5}x$ using dilations and proportionality.



$$\frac{-3}{5} = \frac{y}{x}$$

The rise for the right triangle formed from the point (5,-3) and the origin is -3 and the run is 5. Choose any point (x,y) on the line and form a right triangle where the rise is x and the run is y. Since the right triangles formed are dilations of one another their corresponding parts are proportional; thus $\frac{-3}{5} = \frac{y}{x}$. Upon solving for y you get $y = \frac{-3}{5}x$.

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2.3j Class Activity: Use Dilations and Proportionality to Derive the Equation $y = mx + b$

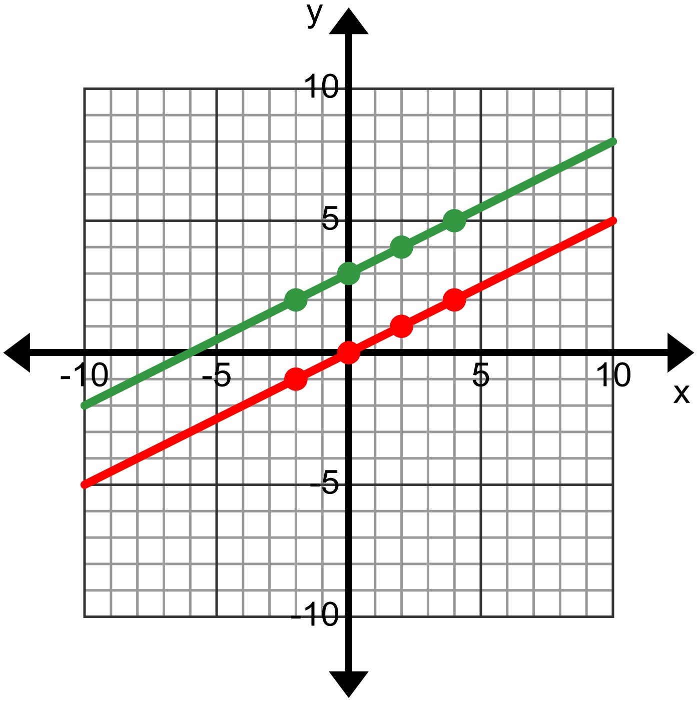
What about linear relationships that are not proportional? You are going to further investigate the general form of a linear equation with a transformation. A geometric transformation can relate a linear proportional relationship to a linear non-proportional relationship.

In previous sections we used the geometric transformation called a dilation. Another type of transformation is called a translation. Shifting a line or moving all the points on the line the same distance and direction is a transformation is a **translation**.

For example, if you transform the line $y = \frac{1}{2}x$ upwards by 3 units, every ordered pair that lies on that line gets moved up 3 units. Algebraically that means that you add 3 to every *y* value since this is a vertical shift.

$$(x, y) \rightarrow (x, y+3)$$

To confirm this, investigate this transformation below.



Consider the relation $y = \frac{1}{2}x$

1. Make a table of values for this relation.

x	y
-2	-1
0	0
2	1
4	2

2. Graph the relation on the coordinate plane to the right.
   See graph.
3. On the same coordinate plane translate every point 3 units up and draw the new graph or the image in a different color.
   See graph
4. Using your table of values add 3 to every *y* value. $(x, y) \rightarrow (x, y+3)$

x	y	$y+3$
-2	-1	2
0	0	3
2	1	4
4	2	5

5. Graph your new ordered pairs.
   See graph.

6. Write an equation for your image and compare it with your equation for the pre-image.

$y = \frac{1}{2}x + 3$

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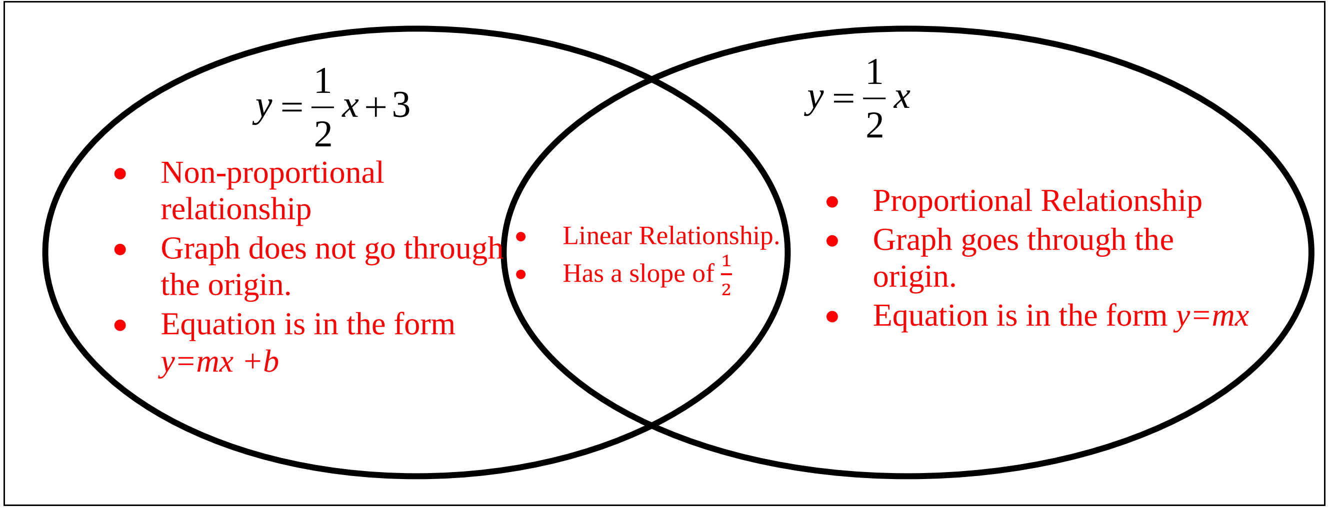
In section 2.3h you investigated the general form for a linear equation given below.

> The general form of a linear equation is $y=mx+b$ where m is the slope or rate of change and b is the initial value or y-intercept.

It is also seen in the example above. The equation $y=\frac{1}{2}x+3$ represents the linear relationship where the slope is $\frac{1}{2}$ and 3 is the y-intercept or initial value.

Use the information on the previous page to compare and contrast the two relations using the Venn Diagram below. Be sure to list similarities and differences about their graphs and equations.



$y=\frac{1}{2}x+3$

- Non-proportional relationship
- Graph does not go through the origin.
- Equation is in the form $y=mx+b$

Both:

- Linear Relationship.
- Has a slope of $\frac{1}{2}$

$y=\frac{1}{2}x$

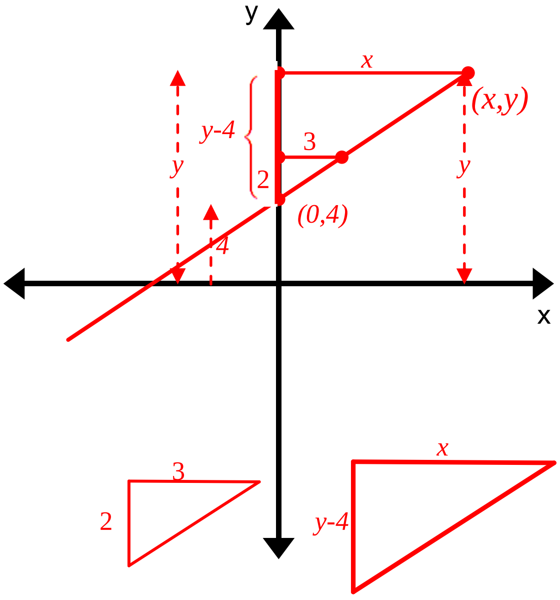
- Proportional Relationship
- Graph goes through the origin.
- Equation is in the form $y=mx$

So how is this general form, $y=mx+b$, for a linear equation derived? Start with an example.

17. Graph a line on the coordinate plane to the right that goes through the point (0,4) and has a slope of $\frac{2}{3}$. Label the rise and run on your graph with a right triangle. Redraw and label this triangle in the space provided below the graph.

18. Does this line describe a proportional relationship? Explain. This is not a proportional relationship because the line does not go through the origin.

19. Choose any point (x,y) on your graph and draw a slope triangle the describes the rise and run. Redraw and label this triangle in the space provided below the graph. See graph.



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20. Write a proportional statement with your ratios. $\frac{2}{3}=\frac{y-4}{x}$

21. Solve the equation that you wrote above for *y*. $y=\frac{2}{3}x+4$

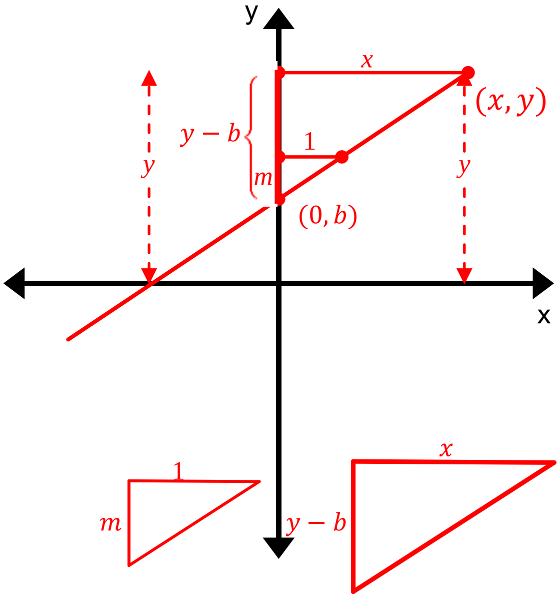
Once again, recall that all right triangle formed from any two points on the line are dilations of one another; so the ratios of their side lengths are proportional. This means that you can set your slope ratios for each triangle that you drew equal to each other*(#20.)* Notice that this equation is of the form *y=mx+b* where *m* is the slope of the line and *b* is the *y*-intercept*(#21).*

Now look at the general case for the form *y=mx+b*.

22. Graph a line on the coordinate plane to the right that goes through any point on the *y*-axis(call the *y*-intercept *b*) and has a slope of *m*. Label the rise and run on your graph with a right triangle. Redraw and label this triangle in the space provided below the graph.

23. Does this line describe a proportional relationship? Explain. This is not a proportional relationship because the line does not go through the origin.

24. Choose any point (x, y) on your graph and draw a slope triangle that describes the rise and run. Redraw and label this triangle in the space provided below the graph. See graph.



Once again, recall that all right triangle formed from any two points on the line are dilations of one another; so the ratios of their side lengths are proportional*(#25).*

25. Write a proportional statement with your slope ratios. $\frac{m}{1}=\frac{y-b}{x}$

26. Solve the equation that you wrote above for *y*. $y=mx+b$

Notice that this is the general form of a linear equation. This form is called Slope-Intercept form.

Slope-Intercept form of a linear equation is

$$\boldsymbol{y=mx+b}$$

where *m* represents the slope (rate of change)
and *b* represents the *y*-intercept (initial value or starting point)

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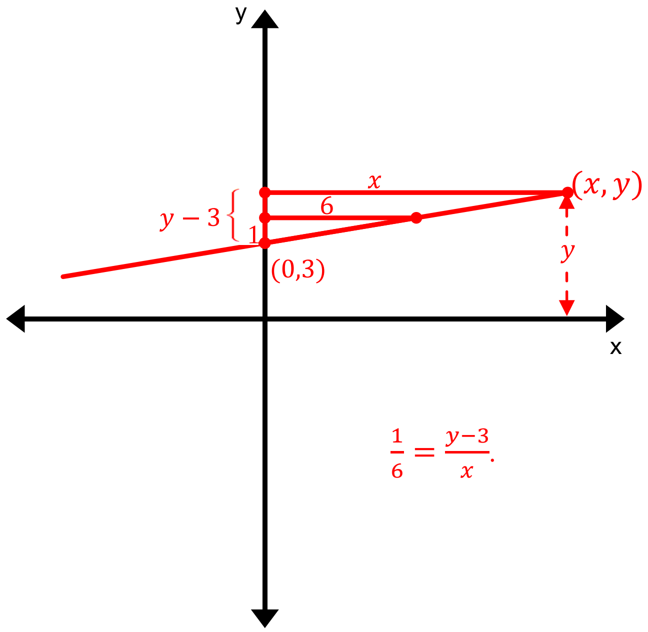
27. Show that the equation of a line that goes through the point (0,3) and has a slope of $\frac{1}{6}$ is $y=\frac{1}{6}x+3$.



The rise for the right triangle formed from the point (0,3) with a slope of $\frac{1}{6}$ is 1 and the run is 6. Choose any point (x, y) on the line and form a right triangle where the rise is x and the run is $y-3$. Since the right triangles formed are dilations of one another their corresponding parts are proportional; thus $\frac{1}{6}=\frac{y-3}{x}$. Upon solving for y you get $y=\frac{1}{6}x+3$.

Interactive lesson similar to the ones in derivations in this activity can be found at:
http://learnzillion.com/lessons/1472-derive-ymx-using-similar-triangles
http://learnzillion.com/lessons/1473-derive-ymxb-using-similar-triangles

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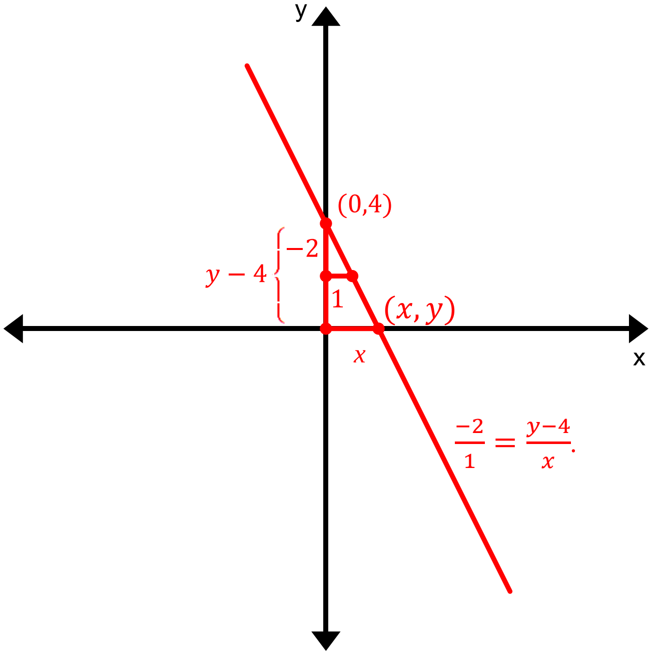
2.3j Homework: Use Similar Triangles to Derive the Equation $y = mx + b$

1. Show that the equation of a line that goes through the point $(0, -2)$ and has a slope of $\frac{3}{2}$ is $y = \frac{3}{2}x - 2$.

See number 127 on page 123 for an example that is similar to this problem.

2. Show that the equation of a line that goes through the point $(0,4)$ and has a slope of -2 is $y = -2x + 4$.



The rise for the right triangle formed from the point $(0,4)$ with a slope of -2 is -2 and the run is 1. Choose any point (x, y) on the line and form a right triangle where the rise is x and the run is $-y + 4$. Since the right triangles formed are dilations of one another their corresponding parts are proportional; thus $\frac{-2}{1} = \frac{y-4}{x}$. Upon solving for y you get $y = -2x + 4$.

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2.3k Self-Assessment: Section 2.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provided on the next page that match each skill/concept

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Understanding 3**	**Substantial Understanding 4**
1. Show that the slope of a line can be calculated as rise/run. Also explain why the slope is the same between any two distinct points on the line. *See sample problem #1*	I can find the slope of the line but do not know how I did it.	I can find the slope of the line by finding the rise and run but do not know how to show how you can find the slope between any two points on the line.	I can find the slope of the line by finding the rise and run. I can explain how to find the slope using any two points that lie on gridlines.	I can find the slope of the line by finding the rise and run and show or explain why the slope can be found from any two points that fall on the line.
2. Find the slope of a line from a graph, set of points, and table. Recognize when there is a slope of zero or when the slope of the line is undefined. *See sample problem #2*	I can find the slope of the line from one of the three representations.	I can find slope of the line from two of the three representations.	I can find the slope of the line from the graph, points, and table but I did not simplify all of my answers.	I can accurately find the slope of the line from the graph, points, and table. I can also express my answers in simplest form.
3. Given a context, find slope from various starting points (2 points, table, line, equation). *See sample problem #3*	I can only find the slope of the line from one of the starting points. I don't know what the slope means given then context.	I can find the slope of the line from some of the starting points but I don't know what the slope means given the context.	I can find the slope of a line from any starting point but I don't know what the slope means in the given context.	I can find the slope of the linear relationship from any starting point and interpret what it means given the context.
4. Recognize that *m* in $y = mx$ and $y = mx + b$ represents the rate of change or slope of a line. Understand that b is where the line crosses the *y*-axis or is the *y*-intercept. *See sample problem #4*	I do not know how to find the slope and *y*-intercept given in the equation.	I can identify the *y*-intercept but not the slope in this equation.	I can identify the slope but not the *y*-intercept in the equation.	I can identify the slope and *y*-intercept in the equation.

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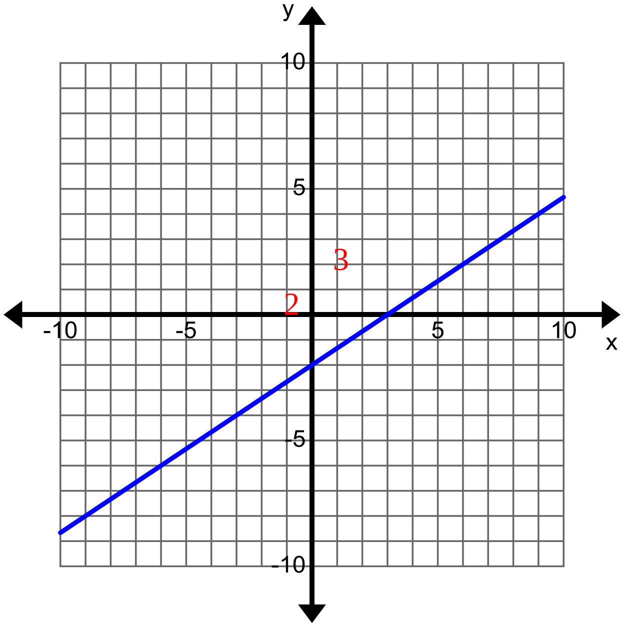
Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
5. Derive the equation $y = mx$ and $y = mx + b$ using dilations and proportionality. *See sample problem #5*	I can show the y-intercept and slope for both equations but I don't know how they relate to dilations of triangles or proportions.	I can derive only one of the equations using similar triangles.	I can accurately go through the steps of deriving both of the equations of the line using similar triangles and proportionality.	I can accurately go through the steps of deriving both of the equations of the line using dilations of triangles and proportionality. I can explain the reasoning at each step in my own words.

Sample Problem #1

Find the slope of the line given below by finding the rise and run. Also show or explain why the slope can be found from any two points that fall on the line.



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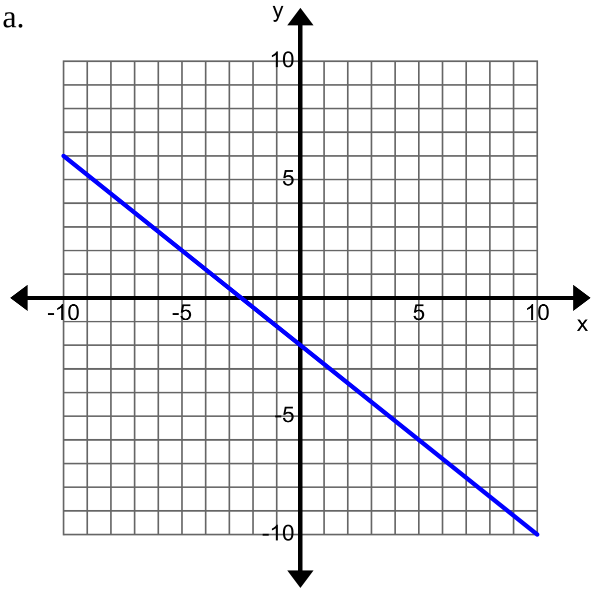
Sample Problem #2

Find the slope of each line given in the graph, set of points and table below.

a.



b. Find the slope of the line that goes through the points (-2,2), (6,-10).

c. Find the slope of the line that goes through the points in the table given below.

x	*y*
4	-8
6	-8
8	-8

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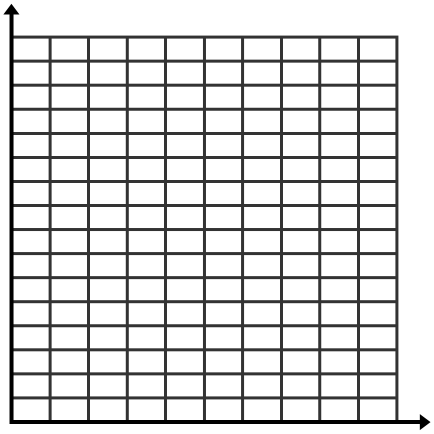
Sample Problem #3
Find the slope of the line from each context given below.

a. The cost to fix a car at Bubba's Body Shop is shown in the table below. What is the slope of this linear relationship, be sure to explain what is means given the context.

Total Cost	
t (time in hours)	***C (cost in dollars)***
0	0
1	25
2	50
3	75

b. The graph given below describes the height over time of a Norfolk Pine that is planted in Rudy's backyard. Find and describe the slope of this relationship as related to the context.



c. The equation *y=2x+10* describes the monthly cost to rent a movie at the local video kiosk, where *x* represents the number of movies rented and *y* represents the total cost. Find and describe the slope of this relationship.

d. The points (1, 16) (4, 15) describe the height of a lit candle that is burning at two different times of day. Here *x* represents the number of hours it has been burning and *y*-represents the candle's height in inches. Find the slope of this relationship and describe what it means given the context.

Sample Problem #4
For the equation given below identify and the rate of change or slope and the *y*-intercept.

$$y = -\frac{1}{3}x + 5$$

Slope(*m*): *y*-intercept(*b*):

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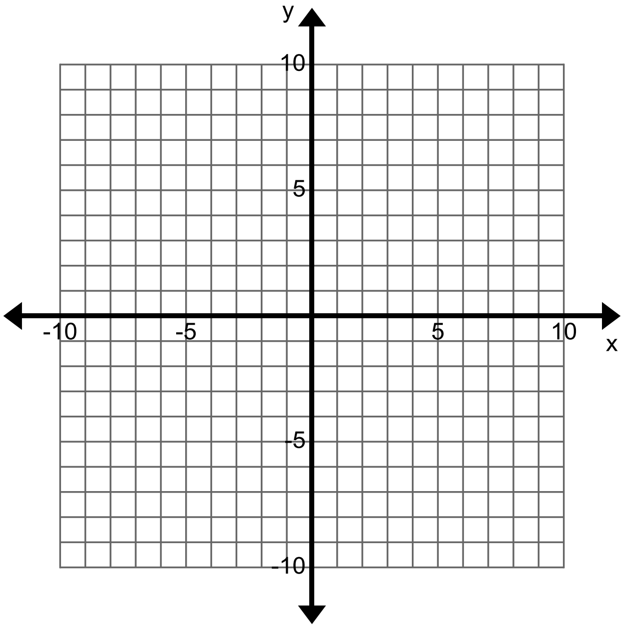
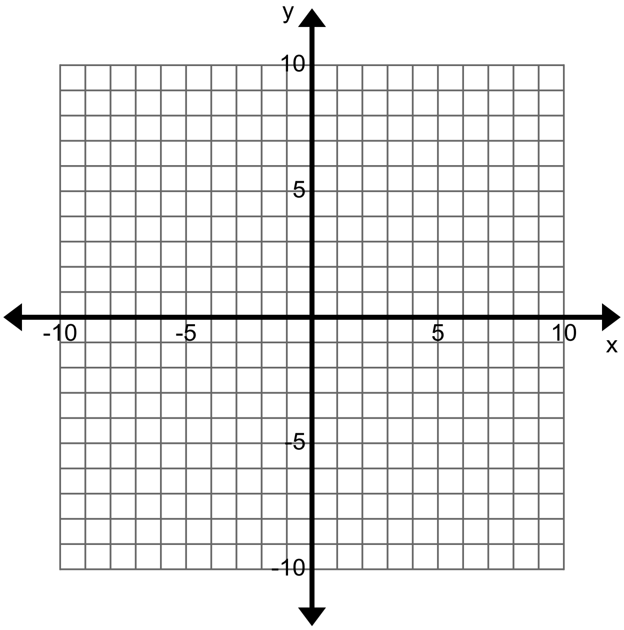
Sample Problem #5

Show that the equation for a line that goes through the origin and has a slope of $\frac{5}{4}$ is $y = \frac{5}{4}x$ using dilations and proportionality.



Show that the equation for a line that goes through (0,3) and has a slope of 2 is $y = 2x + 3$.



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