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Chapter 2: Exploring Linear Relations (4 weeks)

Utah Core Standard(s):

- Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has a greater speed.* (8.EE.5)
- Use similar triangles to explain why the slope *m* is the same between any two distinct points on a nonvertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at *b*. (8.EE.6)
- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (8.F.4)

Academic Vocabulary: proportional relationship, proportional constant, unit rate, rate of change, linear relationship, slope(*m*), translation, dilation, *y*-intercept(*b*), linear, right triangle, origin, rise, run, graph, table, context, geometric model, constant difference, difference table, initial value, slope-intercept form.

Chapter Overview:

Students begin this chapter by reviewing proportional relationships from 6th and 7th grade, recognizing, representing, and comparing proportional relationships. In eighth grade, a shift takes place as students move from proportional linear relationships, a special case of linear relationships, to the study of linear relationships in general. Students explore the growth rate of a linear relationship using patterns and contexts that exhibit linear growth. During this work with linear patterns and contexts, students begin to surface ideas about the two parameters of a linear relationship: constant rate of change (slope) and initial value (*y*-intercept) and gain a conceptual understanding of the slope-intercept form of a linear equation. This work requires students to move fluently between the representations of a linear relationship, students are introduced to the concept of slope and use the properties of dilations to show that the slope is the same between any two distinct points on a non-vertical line. Finally, students synthesize concepts learned and derive the equation of a linea.

Connections to Content:

<u>Prior Knowledge</u>: This chapter relies heavily on a student's knowledge about ratios and proportional relationships from 6th and 7th grade. Students should come with an understanding of what a unit rate is and how to compute it. In addition they need to be able to recognize and represent proportional relationships from a story, graph, table, or equation. In addition they must identify the constant of proportionality or unit rate given different representations.

<u>Future Knowledge:</u> After this chapter students continue to work with linear relationships and begin work with functions. They will work more formally with slope-intercept form as they write and graph equations for lines. This will set the stage for students to be able to graph and write the equation of a line given any set of conditions. Students use their knowledge of slope and proportionality to represent and construct linear functions in a variety of ways. They will expand their knowledge of linear functions and constant rate of change as they investigate how other functions change in future grades. The work done in this chapter is the foundation of the study of how different types of functions grow and change.

MATHEMATICAL PRACTICE STANDARDS:

		Gourmet jellybeans cost \$9 for 2 pounds.									
		a. Complete the table.									
		Pounds .5			3	4		8	10		20
		Total Cost		\$9			\$27				
				Ψ			Ψ21				
		h Label the even Cr	onh th			• W	hot is t	hanni	tmotol		
		b. Label the axes. Graph the c. what is the unit rate?									
	Make sense of	^y ↑		d. Write a sentence with correct units to describe the rate of change.				t units			
	problems and										
7	in solving										
_	them.					\mathbf{w}	rita an	aquati	on to f	ind the	oost
						c. w	r anv a	mount	of iell	vbeans.	COSt
							j		•- J•) = = =====	
						f. W	hy is th	ie data	graph	ed only	in in
						un	e first c	luadra	nt?		
		0			×						
		<i>x</i>									
		As students approach thi	is prob	lem the	ey are	given	some r	eal wo	rld dat	a and a	asked to
		graph and analyze it. They must make conjectures about the unit rate of the line an							line and		
		understand the correspondence of the corresp	ndence	es betw	een th	e table	e, graph m by h	i, and wing t	equation and the second s	on. The rolain y	e final why only
		the first quadrant was us	ed.	piuuiz	e me	proble	m by ne	iving i	пет ел	piun v	vny oniy
		Graphing points can be t	ime-co	onsumi	ng. De	evelop	a proce	edure f	for calc	ulating	; the
		slope without graphing e	each po	oint. Ex	plain	your p	rocedu	re belo	w. Sho	w that	it
		works for problems 1-4 above.									
		Write what you think ab	out the	e metho	ope w ds.	mout	using I	igin u	langles	on a g	jiapii.
				$y_2 - y_2$	y_1 y_1	. 1	•,	0.11	1	•,	1.0
		Now discuss this formula	a:. <i>m</i> =	$\frac{x_2}{x_2-x_1}$	$\frac{1}{x_1}$ WI	hat doe	es it me	an? Ho	ow doe	s it wo	rk?
		By examining how the ri	se and	run is	1 found	amon	gst a va	iriety d	of point	s stude	ents
	Reason	begin understand that the rise is the difference of the y values and the run is the							the		
n #	abstractly and	difference of the x values	s. The	y must	abstra	ict the	given i	nforma	ation a	nd repr	esent it
"	quantitatively.	symbolically as they dev	elop a	nd ana	lyze th	e slope	e formu	ıla.			

	On the line to the right choose any two points that fall on the line. (To make your examination easier choose two points that fall on an intersection of the gridlines). From the two points create a right triangle, the line itself will be the hypotenuse and the legs will extend from the two points and meet at a right angle.
viable arguments and critique the reasoning of others.	Compare the points that you chose and your triangle with someone in your class. Discuss the following: Did you both choose the same points? How are your triangles the same? How are your triangles different? What relationship exists between your triangles? Upon comparing their triangle with a class member students begin to discover that any right triangle constructed on the line is related through a dilation. By talking with one another they can analyze different triangles and discuss the proportionality that exists between them. This also gives students the opportunity to help one another learn how to accurately construct the right triangle on the graph used to find slope. In addition, they begin to make conjectures about how slope can be found
Model with mathematics.	 from any two points on the line. a. Create your own story that shows a proportional relationship. b. Complete a table and graph to represent this relationship. Be sure to label the axes of your graph. c. Write an equation that represents your proportional relationship. This question asks students to not only create their own proportional relationship but to model it with a table, graph, and equation. Students show their understanding of how a momentum proportional relationship.
Attend to precision.	<i>how a proportional relationship is shown in several representations.</i> The graph below shows the distance a cat is from his bowl of milk over time. Which sentence is a good match for the graph? A. The cat was 12 feet away from the milk and ran toward it reaching it after 4 seconds. B. The cat was 4 feet away from the milk and ran toward it reaching it after 12 seconds. C. The cat ran away from the milk at a rate of 3 feet per second. D. The cat ran away from the milk at a rate of 4 feet per second. E. The cat was 12 feet away from the milk at a rate of 4 feet per second. E. The cat was 12 feet away from the milk at a rate of 4 feet per second. Upon examining the graph students must attend to precision as they discuss the ordered pair (4,12) and analyze exactly what it is telling us about the cat. Students often confuse the given information with rate of change and fail to recognize what quantity each number in the ordered pair represents. Also they must communicate what the direction of the line is telling us. Later on they are asked to make their own graphs and must use the correct units and labels to communicate their thinking.

		 Your group task is to build a set of stairs and a handicap ramp at the side. Both the stairs and the ramp will begin at the same place (at ground level) and end at the height of three feet. Answer the following questions as you develop your design. How many steps do you want or need? 						
		• How deep should each step be (we will call this the run)? Why do you want						
	Use	this run depth?						
	appropriate	• How tall will each step be (we will call this the rise)? Why do you want this						
	tools stratogically	rise height?						
	strategicany.	 What is the total distance (total depth for all steps) you will need (at the base) for all of the stairs? This would be a measurement at ground level from stair/ramp start point to stair/ramp end point. Sketch the ramp as viewed from the side on graph paper below. Label and sketch the base and height, for example: stair-base (in inches or feet) and height (in inches or feet). As students design a set of stairs and ramps they must use decide how they can use the tools (graph paper, ruler, pencil) provided them most efficiently. They will need to generate a graph that displays their designs and use the graph as a tool to analyze the slope of the stairs and ramp. 						
	Look for and make use of structure.	Examine the graphs and equations given above. Describe the general form of a linear equation. In other words, in general, how is a linear equation written? What are its different parts? As students examine many equations in slope-intercept form and interpret the slope and y-intercept they see the structure of the general form for a linear equation begin to emerge. Even if they write the initial value or y-intercept first they can step back and look at an overview of the general form of the equation and shift their perspective to see that the order in which you write your slope and initial value does not matter						
╡ ┺	Look for and express regularity in repeated reasoning.	In each graph below, how many right triangles do you see? • Trace the triangles by color. • For each triangle write a ratio comparing the lengths of its legs or $\frac{height}{base}$. Then simplify the ratio $\frac{height}{base} = -$. In this series of problems students repeatedly find the height/base ratio of triangles that are dilations of one another and infer that slope can be calculated with the						
		rise/run ratio by choosing any two points. A general method for finding slope as rise/run is discovered.						

2.0 Anchor Problem: Proportionality and Unit Rate

Toby the snail is crossing a four foot wide sidewalk at a constant rate. It takes him 1 minute and 36 seconds to scoot across **half** the width of the sidewalk, as pictured below.



- 1. Find the unit rate for this proportional relationship. Be sure to explain what this unit rate means.
- 2. Write an equation that describes this proportional relationship if x is the amount of time it takes Toby to cross the sidewalk and y is the distance he has traveled. Use this equation to make a table of values to graph the first 5 seconds of Toby's journey.





3. How long will it take Toby to cross the sidewalk?

A large group of cyclists are on the sidewalk heading in Toby's direction. The graph below shows the rate at which they travel.

- Find and describe the unit rate for the group of cyclists. Highlight this unit rate on the graph.
- 5. Describe how you can find the unit rate at a different location on the graph.
- 6. The cyclists are 4300 feet away from Toby on the sidewalk. Will Toby cross the side walk before the cyclists reach him? Justify your answer.



Section 2.1: Analyze Proportional Relationships

Section Overview:

The section begins by reviewing proportional relationships that were studied in 6^{th} and 7^{th} grade. By investigating several contexts, students study the proportional constant or unit rate in tables, graphs, and equations. They recognize that a proportional relationship can be represented with a straight line that goes through the origin and compare proportional relationships represented in many ways. In the last lesson of the section a bridge from proportional relationships to linear relationships is achieved as students translate the graph of a proportional relation away from the origin and analyze that there is no effect on steepness of the line or rate at which it changes but that the relation is no longer proportional. They begin to see a proportional relationship as a special subset of a linear relationship where the rate of change is a proportional constant or unit rate and the graph of the relationship is a line that goes through the origin. Students also investigate the transition of the proportional constant or unit rate to rate of change, that is, if the input or *x*-coordinate changes by an amount *A*, the output or *y*-coordinate changes by the amount *m* times *A*.

Concepts and Skills to Master:

By the end of this section, students should be able to:

- 1. Graph and write equations for a proportional relationship and identify the proportional constant or unit rate given a table, graph, equation, or context.
- 2. Compare proportional relationships represented in different ways.
- 3. Know that the graph of a proportional relationship goes through the origin.

2.1a Class Activity: Proportional Relationships

In the previous chapter, you wrote equations with one variable to describe many situations mathematically. In this chapter you will learn how to write an equation that has two variables to represent a situation. In addition to writing equations you can represent real-life relationships in others ways. In 6^{th} and 7^{th} grade you studied **proportional relationships** and represented these relationships in various ways. The problems given below will help you to review how ratio and proportion can help relate and represent mathematical quantities from a given situation.

- 1. Julie is picking teammates for her flag football team. She picks three girls for every boy.
 - a. Complete the table below to show the relationship of boys to girls on Julie's team.

Boys	Girls
1	
2	
	9
4	

b. Graph the girl to boy relationship for Julie's team with boys on the *x*-axis and girls on the *y*-axis.



c. Find the ratio of girls to boys for several different ordered pairs in the table. $\frac{\text{number of girls}}{\text{number of boys}} =$

d. Fill in the boxes to show the relationship between girls and boys on Julie's team.



- e. Use the equation and graph to determine how many girls would be on the team if Julie chose 10 boys to be on the team.
- f. Use the equation and graph to determine how many boys are on the team if Julie chose 18 girls.

In the previous example the two quantities of interest are in a **proportional relation**.

Recall that when two quantities are proportionally related, the ratio of each *y* value to its corresponding *x* value is constant. This constant is called the **constant of proportionality** or **proportional constant**.

The ratio that related the number of boys to girls was 3. This is the proportional constant for this relationship.

- 2. Carmen is making homemade root beer for an upcoming charity fundraiser. The number of pounds of dry ice to the ounces of root beer extract (flavoring) is proportionally related. If Carmen uses 12 pounds of dry ice she will need to use 8 ounces of root beer extract.
 - a. Write a ratio/proportional constant that relates the number of pounds of dry ice to the number of ounces of root beer extract.
 - b. Write a ratio/proportional constant that relates the number of ounces of root beer extract to the number pounds of dry ice.

Notice that the proportional constant depends on how you define your items.

c. Complete the table below to show the relationship between number ounces of root beer extract *x* and number of pounds of dry ice *y* needed to make homemade root beer.

Ounces of	Pounds of
Root Beer	Dry Ice
Extract	(y)
(<i>x</i>)	
0	
	$\frac{3}{2} = 1.5$
2	
3	
	6
8	

d. Graph and label this relationship below.



- d. What is the proportional constant for this relationship?
- e. Write an equation that shows the relationship between the number of ounces of root beer extract (x) and the number of pounds of dry ice (y) needed to make homemade root beer.

Every ratio has an associated rate. **Unit rate** is another way of interpreting the ratio's proportional constant. The statement below describes how unit rate defines *y* and *x* in a proportional relationship.

If quantities y and x are in proportion then the **unit rate** of y with respect to x is the amount of y that corresponds to one unit of x. If we interchange the roles of y and x, we would speak of the unit rate of x with respect to y.

- 5. In the previous problem Carmen was making homemade root beer. Express the proportional constant as a unit rate.
- 6. What would the unit rate be if we interchanged the roles of *x* and *y*?

In the problem below use the properties of a proportional relationship to help you answer the question.

- 7. Doug is pouring cement for his backyard patio that is 100 square feet. The cement comes out of the truck at a constant rate. It is very important that he gets all the cement poured before 12:00 noon when it gets too hot for the cement to be mixed properly. It is currently 11:00 AM and he has poured 75 square feet of concrete in the last 3 hours. At this rate will he finish before noon?
 - a. Fill in the missing items in the table if *x* represents the number of hours that have passed since Doug began pouring concrete and *y* represents the amount of concrete poured

Time elapsed	Amount of
(hours)	concrete poured
x	(square feet)
	у
0	
1	
2	
3 (11:00AM)	75
4	

b. Graph the relationship below.



- c. What is the unit rate for this relationship? In other words, how many square feet of concrete can Doug pour in 1 hour.
- d. Which equation given below best describes this relationship?

a)	y=25x	b)	y=75x
c)	<i>x</i> =25 <i>y</i>	d)	y=11x

- e. Will Doug finish the job in time? Justify your answer.
- 10. Vanessa is mixing formula for her baby. The graph given to the right describes the relationship between the ounces of water to the scoops of formula to make a properly mixed bottle.



- d. Write an equation to relate the ounces of water to the scoops of formula.
- e. How many scoops of formula must Vanessa use to make 9 ounce bottle for her baby?

11. Ben, Boston, and Bryton have each designed a remote control monster truck. They lined them up to crush some mini cars in the driveway. The lines on the graph below show the distance in inches that each monster truck travels over time in seconds.



- a. Bryton states that each truck is traveling at a constant rate. Is his statement correct? Why or why not.
- b. What do the values in the ordered pairs given on the graph represent?
- c. Find the unit rate for each boy's truck. Ben:

Bryton:

Boston:

- d. Which boy's truck is moving the fastest?
- 12. Explain why the graph of a proportional relationship makes a straight line.
- 13. Summarize what you know about proportional relationships using bulleted list in the space below.

2.1a Homework: Proportional Relationships.

- 1. A florist is arranging flowers for a wedding. For every 2 pink flowers in a vase, he also includes 8 white flowers.
 - a. Complete the table below to show the relationship of white to pink flowers in each vase.



b. Graph the white flower to pink flower relationship for each vase with pink flowers on the *x*-axis and white flowers on the *y*-axis.

c. Find the ratio of white to pink for several different ordered pairs in the table.

 $\frac{\text{number of white flowers}}{\text{number of pink flowers}} =$

d. Fill in the boxes to show the relationship between white flowers and pink flowers in a vase.



e. Use the equation and graph to determine how many white flowers there would be if the florist included 20 pink flowers.

- 2. You are going to Europe for vacation and must exchange your money. The exchange rate of Euros to Dollars is a proportional relationship. The table below shows the exchange rate for Euros *y* to Dollars *x*.
 - a. Complete the table.

Dollars(x)	Euros(y)
2	
4	3
6	
	6

c. What is the unit rate for this relationship?

b. Graph the Euro to Dollar relationship.



- d. Write an equation that represents this relationship.
- e. If you exchanged \$20, how many Euro would you get?
- f. If you received 36 Euro, how many dollars did you exchange?
- 3. In the problem given above we found that the unit rate described the number of Euros in a Dollar. Now interchange the roles of *y* and *x* for this relationship. Find the exchange rate for Dollars (*y*) to Euros (*x*). Use the questions below to help you do this.
 - a. Make a table of values to represent the Dollar to Euro relationship.

$\operatorname{Euros}(x)$	Dollars(y)

- b. What is the unit rate for this Dollar to Euro relationship?
- c. Write an equation that represents this relationship.

Use the equation to answer the following questions.

- d. While in Europe you find a shirt that you want to buy that is marked at 25 Euros. You only have \$32 to exchange for the Euros. Do you have enough money? Explain.
- e. Upon returning home from Europe you have 100 Euros left. How many Dollars can you get for 100 Euros?
- 4. The graph given below shows the gas mileage that Penny gets in her car. The ratio 192:6 describes the miles to gallons fuel rate for her car.
 - a. What is the unit rate for this relationship?
 - b. Use the graph to approximate how many miles Penny can go if she has a 15 gallon tank in her car.



- 5. A proportional constant of $\frac{1}{3}$ relates the number of inches a flower grows to the number of weeks since being planted.
 - a. Fill in the missing items in the table if *x* represents the number of weeks that have past and *y* represents the height of the flower.

x (weeks)	1	3		9	30
y (height)			2		

- b. Write an equation that represents this relationship and use the equation to predict how tall the flower will be after 8 weeks.
- c. Is it probable for the flower to continue to grow in this manner forever?
- 6. Padma needs to buy 5 pounds of candy to throw at her city's annual 4th of July Parade; the picture below shows how much it costs to buy Salt Water Taffy at her local grocery store.



- a. Find the unit rate for this proportional relationship. Be sure to describe what this unit rate means.
- b. Write an equation that describes this proportional relationship if x is the number of pounds and y is the cost. Use this equation to make a table of values to graph how much up to five pounds of taffy will cost.



c. How much will it cost Padma to buy the 5 pounds of candy that she needs?

- 7. Padma also sees that she can buy Tootsie Rolls at the grocery store. The graph below shows cost of tootsies rolls per pound purchased. (Note: When cost is involved unit rate is often referred to as *unit price*.)
 - Money(dollars) a. Find and describe the unit rate for 30 the Tootsie Rolls. Highlight this 28 unit rate on the graph. 26 24 22 20 b. Describe how you can find the unit 18 rate at a different location on the graph. 16 14 12 10 8 c. Graph the line for the Salt Water 6 Taffy on the grid with the Tootsie Roll. Label each line with the candy 4 it represents. 2 2 3 4 5 1 Weight(pounds)
 - d. What is the better deal for Padma, should she buy the Salt Water Taffy or the Tootsie Rolls? Justify your answer.

- 8. Create your own relationship.
 - a. Create your own story that shows a proportional relationship.
 - b. Complete a table and graph to represent this situation. Be sure to label your graph and table.



c. Write an equation that represents your proportional relationship.

2.1b Class Activity: Comparing Proportional Relationships



Proportional relationships can help us to compare and analyze quantities and to make useful decisions. Complete the tasks given below that compare proportional relationships.

Emma is putting together an order for sugar, flour, and salt for her restaurant pantry. The graph below shows the cost y to buy x pounds of sugar and flour. One line shows the cost of buying x pounds of flour and the other line shows the cost of buying x pounds of sugar.



- 1. From the graph which ingredient costs more to buy per pound? Justify your answer.
- 2. The cost to buy salt by the pound is less than sugar and flour. Draw a possible line that could represent the cost to buy *x* pounds of salt.

Don and Betsy are making super smoothies to re-energize them after a long workout. Betsy follows the recipe which calls for 2 cups of strawberries for every 3 bananas. Don wants twice as much as Betsy so he makes a smoothie with 4 cups of strawberries and 5 bananas.

Don tastes his smoothie and says, "This tastes too tart, there are too many strawberries!"

3. Explain why Don's smoothie is too tart.



4. Find and describe the unit rate for Besty's smoothie.

- 5. Find and describe the unit rate for Don's smoothie.
- 6. Write an equation that relates the number of strawberries(*x*) to the number of bananas(*y*) for Besty's smoothie.
- 7. Write an equation that relates the number of strawberries(*x*) to the number of bananas(*y*) for Don's smoothie.
- 8. Use your equations to make tables to graph both of these lines on the same grid. Be sure to label which line belongs to which person.



9. Explain how the steepness of the lines relates to the unit rate.

10. For the recreational activities below, compare the cost *y* per hour *x* by looking at graphs and equations.

- Fill in the missing representations. If the information is given in a table, fill in the story and equation. If the information is given in an equation, fill in the story and table, etc.
- Find the unit rate or slope for each situation.
- Graph all situations on the given graph on the next page. Remember to <u>Label the axes</u>. <u>Label the lines</u> with the situation names.

a. Long Distance Phone Call: It costs \$10 per hour to talk on	b. Roller skating	c. Music Lessons The bill for private guitar	
the phone long distance.		lessons was \$75. The lesson	
		lasted 3 hours.	
Hours(x) Cost(y)	Hours(x) Cost(y)	Hours(x) = Cost(y)	
	2 5		
Equation: $y = 10x$	Equation:	Equation:	
Unit Rate:	Unit Rate:	Unit Rate:	
d. Parks	e. Bungee Jumping	f.	
	It costs \$20 to bungee jump		
	for 15 minutes.		
Hours(x) Cost(y)	Hours(x) Cost(y)	Hours(x) Cost(y)	
24 6			
Equation:	Equation:	Equation:	
Unit Rate:	Unit Rate:	Unit Rate:	

Graph



2.1b Homework: Comparing Proportional Relationships

Use the graph completed during the class activity to answer questions 1-10 below.

- 1. Order the five activities from highest cost to lowest cost per hour.
- 2. How do you compare the cost per hour by looking at the graph?
- 3. How do you compare the cost per hour by looking at the equations?
- 4. Create a sixth activity in column f on page 23. Think of a situation which would be less expensive than Bungee Jumping, but more expensive than the others. Fill in the table and make the graph.

Answer the questions below:

5. As the rate gets_____, the line gets _____.

6. Renting the pavilion at the park for 3 hours costs _____.

7. Talking on the phone for 2.5 hours costs _____.

- 8. Bungee jumping for_____ hours costs \$40.
- 9. For \$10 you can do each activity for approximately how much time?

a.	Long Distance	
b.	Roller Skating	
с.	Music Lessons	
d.	Park	
e.	Bungee	
f.		

10. Did you use the tables, equations or graphs to answer questions 5-10? Why?

11. The graph below shows the distance two snowboarders have traveled down a hill for several seconds. Hannah is traveling 18 meters per second.



- a. Which equation below is the best choice to describe the distance Torah travels after x seconds. a) y = 29x b) y = 17x
- c) y = 10x d) y = -18x
- b. Explain your reasoning for your choice above.
- c. The unit rate of 10 meters per seconds describes Christina's speed going down the same hill. Draw a line that could possibly represent her speed.
- 12. At Sweet Chicks Bakery the equation y = 3.25x represents the total cost to purchases cupcakes; where x represents the number of cupcakes and y represents the total cost. The graph given below shows the cost for buying cupcakes at Butter Cream Fairy Bakery.



- a. Which bakery offers the better deal? Use the equation and graph to justify your answer.
- b. Use the information given above to determine how much it will cost to buy 10 cupcakes at the bakery with the better deal.

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13. The table given below shows how much money Charlie earned every day that he worked last week. He gets paid the same rate every hour.

	Tuesday	Wednesday	Friday
Hours Worked	4	5	3.5
Money Earned	\$38.00	\$47.50	\$33.25

Sophia earns \$10.50 per hour at her job.

a. Using the same coordinate plane, draw a line that represents Charlie's earnings if *x* represents the number of hours worked and *y* represents the amount of money earned. Also draw a line that represents how much Sophia earns. Label each line with the person's name.



b. How can you use this graph to determine who makes more money?

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2.1c Class Activity: Proportional Relationships as Linear Relationships

1. Two cousins, Grace and Kelly, are both headed to the same summer camp. They both leave from their own house for camp at the same time. The graph below represents the girls' trips to camp.



a. Analyze the graph to determine which girl is traveling faster.

Grace					
Time(x)	Time(x) Distance(y)				
0					
1	60				
	120				
3					
	240				

b. Complete the table below for Grace and Kelly.

Kelly				
Time(x)	Distance(y)	$\frac{y}{x}$		
0				
	120			
2				
	200			
4				

c. What do you notice about the ratio $\frac{y}{x}$ for Grace? What do you notice about the ratio $\frac{y}{x}$ for Kelly? What is this ratio describing?

- d. Describe why Kelly's driving relationship is not proportional?
- e. Is it possible to still describe the rate at which Kelly drives? If so, what is it?

Often the rate at which a relationship changes is shown by seeing that the changes from one measurement to another are proportional; that is, the quotient of the change in y values with respect to the x values is constant. This is called the **Rate of Change**.

Both of the relationships described above have a constant rate of change of 60 mph. This constancy defines them as **linear relationships.** (Their graphs produce straight lines).

f. Use what you learned above to see if you can write an equation that represents each girl's distance *y* from Grace's house after *x* hours.

Grace: _____

Kelly: _____

2. Agatha makes \$26 for selling 13 bags of popcorn at the Juab County Fair.

- a. Find and describe the rate of change for this relationship.
- b. Complete the table that shows the amount of money Agatha makes for selling up to three bags of popcorn.



c. Graph the dollars to bags of popcorn relationship.



- d. Highlight on the graph where you can see the rate of change.
- e. Write an equation that represents the relationship between the number of bags of popcorn that Agatha sells(*x*) and the amount of money she makes (*y*). Equation:
- 3. At the Sanpete County Fair Fitz gets paid \$8 a day plus \$2 for every bag of popcorn that he sells.
 - a. Find and describe the rate of change for this relationship.
 - b. Complete the table that shows the amount of money that Fitz makes for selling up to three bags of popcorn.

x	У

- c. Graph this relationship on the same coordinate plane as Agatha's line on the previous page.
- d. Highlight on the graph where you can see the rate of change.
- f. Write at least two sentences that explain the similarities and differences between Agatha's and Fitz's relationship.

2.1c Homework: Proportional Relationships as Linear Relationships

1. Nate and Landon are competing in a 5 minute long Hot Dog eating contest. Nate has a special strategy to eat 4 hot dogs before the competition even begins to stretch out his stomach. The graph below represents what happened during the competition.



a. Compete the tables where t = time in minutes and h = number of total hotdogs consumed.

Landon		Nate			
t	h	$\frac{h}{t}$	t	h	$\frac{h}{t}$
0	0		0		
1	2.5		1	5.5	
			2		
3	7.5			8.5	
	12.5		5		

- b. Determine the rate of change (the number of hot dogs consumed per minute for each boy).
- c. Write an equation that represents the number of hotdogs *h* for each boy after *t* minutes. Landon: ______

Nate:

d. For which person, Landon or Nate, is the relationship between time and number of hot dogs eaten proportional?? Justify your answer.

- 2. During her Tuesday shift at Sweater Barn, Fiona sells the same amount of sweaters per hour. Two hours into her shift Fiona has sold 8 sweaters.
 - a. Find and describe the rate of change for this relationship.
 - b. Complete the table given below where x is the number of hours worked and y is the total number of sweaters sold.



c. Graph the relationship on the grid below.



d. Write an equation that represents the relationship between the number of hours Fiona works(*x*) and the amount of sweaters she sells(*y*).

Equation:_____

e. Does this represent a proportional relationship? Explain how you know.

- 3. On Saturday Fiona gets to work 15 minutes early and sells three sweaters before her shift even begins. She then sells 4 sweaters every hour for the rest of her shift.
 - a. Find and describe the rate of change for this relationship.
 - b. Complete the table that represents this relationship.
 - c. Graph this relationship on the same coordinate plane as Tuesday's information on the previous page.
 - d. Write an equation that represents the relationship between the number of hours Fiona works(*x*) and the amount of sweaters she sells(*y*).

Equation:_____

- e. Does this represent a proportional relationship? Explain how you know.
- f. Compare the rate of change of both of the lines on the previous page by highlighting the change on the graph. What do you notice?
- 4. Nayala bought 5 pounds of mangos for \$6.25.
 - a. What is the price per pound for the mangos that she bought?
 - b. Which line below, A, B, or C, represents the cost in dollars(y) to weight in pounds(x) relationship? Cost (dollars)



x	у
(hours)	(sweaters sold)
0	3
1	
	11
3	

2.1d Self Assessment: Section 2.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample criteria are provided along with sample problems for each skill/concept on the following page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
 Graph and write equations for a proportional relationship and identify the proportional constant or unit rate given a table, equation, or contextual situation. 	I can correctly answer only 1 of the three parts of the question.	I can correctly answer 2 of the three parts of the question.	I can correctly answer all three parts of the question but cannot explain my answers.	I know how to find the unit rate for both Callie and Jeff and state what the unit rate is describing. I can accurately label the graphs and write an equation that represents the amount of money earned in relationship to the number of papers each
 Compare proportional relationships represented in different ways. 	I do not know how to compare these proportional relationships.	I can find the unit rate for only one relationship.	I can find the unit rate for each relationship represented and then compare the unit rates to determine who makes more money. I do not know how to make a third representation for someone who makes more money than Addy and Rachel.	I can find the unit rate for each relationship represented and then compare the unit rates to determine who makes more money. I can also create a third representation for someone who makes more money than Addy and Rachel.
See sample problem #2				
Skill/Concept	Minimal	Partial	Sufficient	Substantial
--	---	--	--	---
	Understanding	Understanding	Understanding	Understanding
	1	2	3	4
3. Know that the graph of a proportional relationship goes through the origin.See sample problem #3	I might be able to make a graph of Jeff's savings but I do not know how it relates to the other graph.	I can make a graph that show's Jeff's savings but I don't know how it relates to the other graph.	I can make a graph that shows Jeff's savings. I can discuss some of the similarities and differences between the two graphs.	I can make (and clearly label) a graph that shows Jeff's savings. I can discuss the similarities and differences between the two graphs including a discussion about proportionality and slope.

Sample Problem #1

Callie and Jeff each have a job delivering newspapers. Jeff gets paid \$140 dollars for delivering 350 papers. Callie gets paid \$100 for delivering 200 papers.

a. Find the unit rate for each person. Be sure to state what the unit rate is describing. Callie:

Jeff:

b. The graph given below shows the money earned to papers delivered relationship. Label which line represents each person.



c. Write an equation that represents how much each person earns if x is the number of papers they deliver and y represents how much money they make.

Callie:

Jeff:

Sample Problem #2

Below is a table of how much money Rachel earns on her paper route. She gets paid the same amount of money per paper delivered.

Number of Papers	Money
Delivered	Earned
75	\$36
150	\$72
225	\$108

The equation below represents how much money Addy makes delivering papers. In the equation p represents the number of papers delivered and d represents the money earned.

$$d = .45p$$

- a. Who makes more money? How do you know?
- b. Create a representation for someone who delivers papers and makes more than both Addy and Rachel.

Sample Problem #3

The graph provided below shows the amount of money that Jeff earns delivering papers. Suppose that Jeff had \$75 dollars in savings before he started his job of delivering newspapers. Jeff saves all of his money earned from delivering newspapers. Graph this relationship below where *x* is the number of papers he delivers and *y* is the amount of money he has in savings. Correctly label each line has Total Savings and Money Earned.



How does the savings line compare to his money earned line? Be sure to discuss the proportionality of each graph.

Section 2.2: Linear Relations in Pattern and Context

Section Overview:

In this section, students start by writing rules for linear patterns. They use the skills and tools learned in Chapter 1 to write these equations. Students connect their rules to the geometric model and begin to surface ideas about the rate of change and initial value (starting point) in a linear relationship. Students continue to use linear patterns and identify the rate of change and initial value in the different representations of a linear pattern (table, graph, equation, and geometric model). They also begin to understand how linear functions change. Rate of change is investigated as students continue to interpret the parameters m and b in context and advance their understanding of a linear relationship. Students move fluently between the representations of a linear relationship and make connections between the representations. This conceptual foundation will set the stage for students to be able to derive the equation of a line using dilations in section 2.3.

Concepts and Skills to Master:

By the end of this section, students should be able to:

- 1. Write rules for linear patterns and connect the rule to the pattern (geometric model).
- 2. Understand how a linear relationship grows as related to rate of change and show how that growth can be seen in each of the representations.
- 3. Create the additional representations (table, graph, equation, context, geometric model) of a linear relationship when given one representation and make connections between them.
- 4. Identify the rate of change and initial value of a linear relationship in the table, graph, equation, context, and geometric model of a linear pattern.

2.2a Class Activity: Connect the Rule to the Pattern

Linear relationships can be used to illustrate many patterns. The patterns in the problems below exhibit linear relationships.

1. Use the pattern below to answer the questions that follow.



a. Draw the figure at stage 4 in the space above. How did you draw your figure for stage 4 (explain or show on the picture how you see the pattern growing from one stage to the next)?

b. How many blocks are in stage 4? Stage 10? Stage 100?

c. Write a rule that gives the total number of blocks *t* for any stage *s*. Show how your rule relates to the pattern (geometric model).





- e. Use your rule to determine the number of blocks in stage 100.
- f. Use your rule to determine which stage has 25 blocks.
- g. Draw or describe stage 0 of the pattern. How does the number of blocks *n* in stage 0 relate to the simplified form of your rule?

2. Use the pattern below to answer the questions that follow.



a. Draw the figure at stage 5 in the space above. How did you draw your figure in stage 5 (explain or show on the picture how you see the pattern growing from one stage to the next)?

b. How many blocks are in stage 5? Stage 10? Stage 100?





- e. Use your rule to determine the number of blocks in Stage 100.
- f. Use your rule to determine which stage has 58 blocks.
- g. Draw or describe stage 0 of the pattern. How does the number of blocks in stage 0 relate to the simplified form of your rule?

2.2a Homework: Connect the Rule to the Pattern

1. Use the pattern below to answer the questions that follow.



- a. Draw the figure at stage 4 in the space above. How did you draw your figure in stage 4 (explain or show on the picture how you see the pattern growing from one step to the next)?
- b. How many blocks are in stage 4? Stage 10? Stage 100?
- c. Write a rule that gives the total number of blocks *t* for any stage *s*. Show how your rule relates to the pattern (geometric model).





- e. Use your rule to determine the number of blocks in stage 100.
- f. Use your rule to determine which stage has 28 blocks.
- g. Draw or describe stage 0 of the pattern. How does the number of blocks in stage 0 relate to the simplified form of your rule?

2. Use the pattern below to answer the questions that follow.



- a. Draw the figure at stage 4 in the space above. How did you draw your figure in stage 4 (explain or show on the picture how you see the pattern growing from one stage to the next)?
- b. How many blocks are in stage 4? Stage 10? Stage 100?
- c. Write a rule that gives the total number of blocks *t* for any stage, *s*. Show how your rule relates to the pattern (geometric model).





- e. Use your rule to determine the number of blocks in stage 100.
- f. Use your rule to determine which stage has 37 blocks.
- g. Draw or describe Stage 0 of the pattern. How does the number of blocks in stage 0 relate to the simplified form of your rule?

2.2b Class Activity: Representations of a Linear Pattern



1. You studied this pattern in the previous lesson. Use your work from the previous lessons to answer the questions that follow.



- a. How many new blocks are added to the pattern from one stage to the next?
- b. Complete the table.
- c. Show where you see the rate of change in your table.

Stage (s)	# of Blocks (t)
1	
2	
3	
4	
5	

d. Create a graph of this data. Where do you see the rate of change on your graph?



e. What is the simplified form of the equation that gives the number of blocks *t* for any stage *s* (see previous lesson)? Where do you see the different pieces of the equation in the geometric model, table, and graph?

f. The pattern above is a **linear** pattern. Describe how a linear pattern grows. Describe what the graph of a linear pattern looks like.

2. You studied this pattern in the previous lesson. Use your work from the previous lesson to answer the questions that follow.



- a. How many new blocks are added to the pattern from one stage to the next?
- b. Complete the table.
- c. Show where you see the rate of change in your table.

Stage (s)	# of Blocks (t)
1	
2	
3	
4	
5	

d. Create a graph of these data. Where do you see the rate of change on your graph?



- e. What is the equation that gives the number of blocks *t* for any stage *s* (see previous lesson)? Where do you see the different pieces of the equation in the geometric model, table, and graph?
- f. Is this pattern a linear pattern? Use supporting evidence from each of the representations to justify your answer.

3. Describe what you see in each of the representations (geometric model, table, graph, and equation) of a linear pattern. Make connections between the different representations.

Geometric Model:

Table:

<u>Graph</u>:

Equation:

2.2b Homework: Representations of a Linear Pattern

1. You studied this pattern in the previous lesson. Use your work from the previous lesson to answer the questions that follow.



- a. How many new blocks are added to the pattern from one stage to the next?
- b. Complete the table.
- c. Show where you see the rate of change in your table.

Stage (s)	# of Blocks (t)
1	
2	
3	
4	
5	

d. Create a graph of this data. Where do you see the rate of change on your graph?



- e. What is the equation that gives the total number of blocks *t* for any stage *s* (see previous lesson)? Where do you see the different pieces of the equation in the geometric model, table, and graph?
- f. Is this pattern a linear pattern? Use supporting evidence from each of the representations to justify your answer.

2. You studied this pattern in the previous lesson. Use your work from the previous lesson to answer the questions that follow.



- a. How many new blocks are added to the pattern from one stage to the next?
- b. Complete the table.
- c. Show where you see the rate of change in your table.

Stage (s)	# of Blocks (t)
1	
2	
3	
4	
5	

d. Create a graph of this data. Where do you see the rate of change on your graph?



- e. What is the equation that gives the total number of blocks *t* for any stage *s* (see previous lesson)? Where do you see the different pieces of the equation in the geometric model, table, and graph?
- f. What do you notice about this pattern? Use supporting evidence from each of the representations to justify your answer.

3. Create your own geometric model of a linear pattern in the space below. Then complete the table, graph, and equation for your pattern. Use these representations to prove that your pattern is linear.



Stage (s)	# of Blocks (t)
1	
2	
3	
4	
5	



Equation: _____

Prove that your pattern is linear using the representations (geometric model, table, graph, and equation) as evidence.

4. Draw or describe a pattern that can be represented by the equation t = 1 + 6s where *t* is the total number of blocks and *s* is the stage.

2.2c Class Activity: Representations of a Linear Context



- 1. Courtney is collecting coins. She has 2 coins in her collection to start with and plans to add 4 coins each week.
 - a. Complete the table and graph to show how many coins Courtney will have after 6 weeks.



Time	# of
(weeks)	Coins
0	
1	
2	
3	
4	
5	
6	

- b. Write an equation for the number of coins *c* Courtney will have after *w* weeks.
- c. Is this context linear? Use evidence from the context, graph, table, and equation to support your answer.

- 2. Jack is filling his empty swimming pool with water. The pool is being filled at a constant rate of four gallons per minute.
 - a. Complete the table and graph below to show how much water will be in the pool after 6 minutes.





- b. Write an equation for the number of gallons g that will be in the pool after m minutes.
- c. Is this context linear? Use evidence from the context, graph, table, and equation to support your answer.
- d. Compare this swimming pool problem to the previous problem about coins. How are the problems similar? How are they different?
- e. How would you change the coin context so that it could be modeled by the same equation as the swimming pool context?
- f. How would you change the swimming pool context so that it could be modeled by the same equation as the coin context?

3. An airplane is at an elevation of 3000 ft. The table below shows its elevation(y) for every 2 miles(x) it travels.

Miles	Elevation
0	3000
2	2000
4	1000

a. Complete the graph to show how many miles it will take for the airplane to reach the ground.



- b. Use the table and the graph to find the rate of change.
- c. Write an equation that represents this relationship
- d. Explain the how equation can be used to determine how many miles it will take for the plane to reach the ground.

2.2c Homework: Representations of a Linear Context

- 1. Hillary is saving money for college expenses. She is saving \$200 per week from her summer job. Currently, she does not have any money saved.
 - a. Complete the table and graph to show how much money Hillary will have 6 weeks from now.

Time (weeks)	Amount Saved (dollars)
0	
1	
2	
3	
4	
5	
6	





- b. Write an equation for the amount of money m Hillary will have saved after w weeks if she continues saving at the same rate.
- c. Is this context linear? Use evidence from the context, graph, table, and equation to support your answer.

- 2. The cost for a crew to come and landscape your yard is \$200 per hour. The crew charges an initial fee of \$100 for equipment.
 - a. Complete the table and graph below to show how much it will cost for the crew to work on your yard for 6 hours.

Time (hours)	Cost (dollars)
0	
2	
4	
6	



- b. Write an equation for the cost c of landscaping for h hours.
- c. Is this context linear? Use evidence from the context, graph, table, and equation to support your answer.
- d. Compare this landscaping problem to the problem with Hillary's savings. How are the problems similar? How are they different?
- e. How would you change the savings context so that it could be modeled by the same equation as the landscaping context?
- f. How would you change the landscaping context so that it could be modeled by the same equation as the savings context?

3. Linda is always losing her tennis balls. At the beginning of tennis season she has 20 tennis balls. The table below represents how many balls she has as the season progresses; where *x* represents the number of weeks and *y* represents the number of tennis balls.

Weeks	Number of tennis balls	
0	20	
2	16	
4	12	
6	8	

a. Complete the graph to show how many weeks will pass until Linda runs out of balls.



- b. Use the table and the graph to find the rate of change.
- c. Write an equation that represents this relationship.
- d. Explain how to use the equation to determine how many weeks will pass until Linda runs out of balls.

2.2d Class Activity: Rate of Change in a Linear Relationship

Explore and investigate the rate of change in linear relationships below.

1. The graph below shows the distance a cat is from his bowl of milk over time. Which sentence is a good match for the graph?



- 3. Create a table at the right which also tells the story of the graph.
- 4. Is this a proportional relationship? Justify your answer.

5. Find the unit rate in this story.

6. Sketch a graph for each of the four stories from number 1 on the previous page which you didn't choose. Label the graphs by letter to match the story. Find the rate of change for each story as well.



7. A baby was 9 feet from the edge of the porch. He crawled toward the edge for 6 seconds. Then his mother picked him up a few feet before he reached the edge. Circle the graph below that matches this



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©2014 University of Utah Middle School Math Project in partnership with the Utah State Office of Education. Licensed under Creative Commons, cc-by. 8. The graph and table below describe a runner's distance from the finish line in the last seconds of the race. Which equation tells the same story as the table and graph? Use the ordered pairs given in the table to test your chosen equation and explain your choice.



9. Create a table for this graph,



10. Write a story for this graph.

11. Which equation matches your story, the graph and the table? Explain your choice.

- a) y = 5 + 6x
- b) y = 5x + 23
- c) y = 3x + 5
- d) y = 5x + 3

2.2d Homework: Rate of Change in a Linear Relationship

Explore and investigate the linear relationships below.

1. The graph below shows the distance a mouse is from her cheese over time. Which sentence is a good match for the graph?



- 2. Write everything you can say about the mouse and the distance she is from the cheese during this time.
- 3. Create a table at the right which also tells the story of the graph.
- 4. Is this a proportional relationship? If so what is the proportional constant?

5. Sketch a graph for the four stories from number 1 above which you didn't choose in the space provided. Label the graphs by letter to match the story. Find the rate of change for each story as well.



6. Compare the rate of change with the steepness of each line above, how does the rate of change relate to the steepness of the line?

7. Triss opens a bank account and adds \$25 to the account every week. Circle the graph below that matches this story.



8. Write a story for each of the remaining graphs above.

Use the graph at the right to complete the following.

9. Create a table for this graph,

Time	Distance
0	
1	
2	
3	
4	

10. Write a story for this graph.

- Distance (4, 26)
- 11. Which equation matches your story, the graph and the table?
 - a) y = 5 + 6x
 - b) y = 4x + 26
 - c) y = 5x + 6
 - d) y = 6x + 4

2.2e Class Activity: More Representations of a Linear Context

Directions: In each of the following problems, you are given one of the representations of a linear relationship. Complete the remaining 3 representations. Be sure to label the columns in your table and the axes on your graph.



- a. What is the rate of change in this problem? What does the rate of change represent in the context?
- b. What is the *y*-intercept of your graph? Where do you see the *y*-intercept in the table and in the equation? What does the *y*-intercept represent in the context?
- c. How would you change the context so that the relationship between total cost and number of rides can be modeled by the equation y = 2x?
- d. How would you change the context so that the relationship between total cost and number of rides can be modeled by the equation y = 6?



- a. What is the rate of change in this problem? What does the rate of change represent in the context?
- b. What is the *y*-intercept of your graph? Where do you see the *y*-intercept in the table and in the equation? What does the *y*-intercept represent in the context?
- c. How would your equation change if your gas tank held 18 gallons of gas and used 2.5 gallons per hour of driving? What would these changes do to your graph?

3.				
Context	<u>Table</u>			
		Time (hours)	Cost (dollars)	
		0	7	
		1	9	
		2	11	
		3	13	
		4	15	
		5	17	
Graph	Equation			
↑				

- a. What is the rate of change in this problem? What does the rate of change represent in your context?
- b. What is the y-intercept of your graph? Where do you see the y-intercept in the table and in the equation? What does the y-intercept represent in your context?
- c. How would your context change if the rate of change was 3?



- a. What is the rate of change in this problem? What does the rate of change represent in your context?
- b. What is the *y*-intercept of the graph? Where do you see the *y*-intercept in the table and in the equation? What does the *y*-intercept represent in your context?
- c. How would your context and equation change if the *y*-intercept of the graph was changed to 75? How would this change affect the graph?
- d. How would your context and equation change if the rate of change in this problem was changed to -2? Would the graph of the new line be steeper or less steep than the original?

Table Context Graph **Equation** y = 4x + 10

- a. How would your context change if the equation above was changed to y = 2x + 10? How would this change affect the graph?
- b. How would your context change if the equation above was changed to y = 4x + 8? How would this change affect the graph?
- 6. Describe in your own words what a linear relationship is composed of. Think about all of the equations that you have written to represent a linear relationship, what do they have in common, what do the different parts of the equations represent?

5.

2.2e Homework: More Representations of a Linear Context

Directions: In each of the following problems, you are given one of the representations of a linear relationship. Complete the remaining 3 representations. Be sure to label the columns in your table and the axes on your graph.

1. A Community Garden



- a. What is the rate of change in this problem? What does the rate of change represent in the context?
- b. What is the y-intercept of your graph? What does the y-intercept represent in the context?
- c. How would you change the context so that the relationship between total cost *c* and number of flats *f* purchased was c = 12f?

2. Enrollment



- a. What is the rate of change in this problem? What does the rate of change represent in the context?
- b. What is the *y*-intercept of your graph? What does the *y*-intercept represent in the context?
- c. How would you change the context so that the relationship between number of years and number of students enrolled was y = 40 + 6x?

3. Write your own context

Context	Table	
	Time# of Fis(hours)	h
	0 300	
	2 270	
	4 240	
	6 210	
	8 180	
	10 150	
	Equation	

- a. What is the rate of change in this problem? What does the rate of change represent in the context?
- b. What is the *y*-intercept of your graph? What does the *y*-intercept represent in the context?
- c. How would you change your context so that the amount of fish remaining y after x hours could be represented by the equation y = 300 30x?
4. Write your own context

Context	Table
<u>Graph</u>	<u>Equation</u>
	y = 30 + 5x

a. What is the rate of change in this problem? What does the rate of change represent in the context?

b. What is the *y*-intercept of your graph? What does the *y*-intercept represent in the context?

5. Write your own context



- a. What is the rate of change in this problem? What does the rate of change represent in the context?
- b. What is the *y*-intercept of your graph? What does the *y*-intercept represent in your context?
- c. How would your context and equation change if the rate of change in this problem was changed to 10? Would the graph of the new line be steeper or less steep?

2.2f Self-Assessment: Section 2.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Some sample criteria are provided as well as sample problems on the following page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding	Substantial Understanding 4
	-	-	0	-
1. Write rules for linear patterns and connect the rule to the pattern (geometric model).	I don't know how to write a rule for the pattern and I don't know how to find the number of blocks in the 50 th stage.	I don't know how to write a rule for the pattern. The only way I know how to find the number of blocks in the 50 th stage is by adding the rate of change 50	I can write a rule that describes the pattern but don't know how my rule connects to the pattern.	I can write a rule that describes the pattern and explain how my rule connects to the pattern. I can also use the rule to predict the number of
Saa sampla problem #1		times.		blocks in stage
 2. Understand how a linear relationship grows as related to rate of change and show how that growth can be seen in each of the representations. 3. Create the additional representations (table, graph, equation, context, geometric model) of a linear relationship when given one representation and make connections 	I can find the rate of change in only one of the representations. I can only make one linear representation.	I can find the rate of change in some of the representations. I can make some of the linear representations.	I know how to find the rate of change in all the representations but have a hard time explaining how you can see the growth. I can make all the different representations of a linear pattern but I don't know how they are	I know how to find the rate of change in all the representations. I can also show how the rate of change can be seen. I can fluently move between the different representations of a linear relationship and make
between them. See sample problem #3			connected.	connections between them.
4. Identify the rate of change and initial value of a linear relationship in the table, graph, equation, context, and geometric model of a linear pattern. See sample problem #4	I can identify the rate of change and initial value in 2 or less of the representations of a linear model.	I can identify the rate of change and initial value in 3 of the representations of a linear model.	I can identify the rate of change and initial value in 4 of the representations of a linear model.	I can identify the rate of change and initial value in all of the representations of a linear model.

Sample Problem #1

Write a rule that describes the pattern given below where *s* is the stage number and *t* is the total number of blocks. Be sure to explain how your rule connects to the pattern. Then use the rule to predict the number of blocks in the 50^{th} stage.



Sample Problem #2

3 4 5

For the pattern given below find the number of blocks in the next stage by determining the rate of change. Fill in the table and graph and explain how you can see the pattern grow in each of these representations.





Sample Problem #3

Given the following equation create a table, graph, and a context or geometrical model that the equation could possibly describe. Write a sentence that describes how the different representations are related.

y = -3x + 12

Table:

x	у

Graph:



Context/Model:

Sample Problem #4

For each of the representations given below identify the rate of change and initial value.

1.
$$y = \frac{1}{2}x - 3$$

Rate of change:

Initial value:

3.



Rate of change:

Initial Value:



Rate of Change:

Initial Value:

2. The local community center charges a monthly fee of \$15 to use their facilities plus \$2 per visit.

Rate of change:

Initial Value:



Rate of Change:

Initial Value:

Section 2.3: Investigate The Slope of a Line

Section Overview:

This section uses proportionality to launch an investigation of slope. Transformations are integrated into the study of slope by looking at the proportionality exhibited by dilations. Students dilate a triangle on lines to show that a dilation produces triangles that have proportional parts and thus the slope is the same between any two distinct points on a non-vertical line. They further their investigation of slope and proportional relationships and derive the slope formula. Adequate time and practice is given for students to solidify their understanding of this crucial aspect of linear relationships. The last few lessons provide an opportunity for students to use all of the knowledge and tools acquired throughout the chapter to formally derive the equations y=mx and y=mx+b. They will use proportionality produced by a dilation to do this derivation.

Concepts and Skills to Master:

By the end of this section, students should be able to:

- 1. Show that the slope of a line can be calculated as rise/run. Also explain why the slope is the same between any two distinct points on the line.
- 2. Find the slope of a line from a graph, set of points, and table. Recognize when there is a slope of zero or when the line is undefined.
- 3. Given a context, find slope from various starting points (2 points, table, line, equation).
- 4. Recognize that m in y=mx and y=mx+b represents the rate of change or slope of a line. Understand that b is where the line crosses the y-axis or is the y-intercept.
- 5. Derive the equation y=mx and y=mx+b using dilations and proportionality.

2.3a Class Activity: Building Stairs and Ramps.

In the previous section you saw that a constant rate of change is an attribute of a linear relationship. When a linear relationship is graphed on a line you call the constant rate of change of the line the **slope** of the line.

The **slope** of a line describes how steep it is. It describes the change in *y* values compared to the change in the *x* values.

The following investigation will examine how slope is measured.

On properly built staircases all of the stairs have the same measurements. The important measurements on a stair are what we call the *rise* and the *run*. When building a staircase these measurements are chosen carefully to prevent the stairs from being too steep, and to get you to where you need to go.

One step from three different staircases has been given below.



The vertical measurement is the "rise".

The horizontal measurement is the "run".

1. State the rise and run for each staircase.

a. Staircase #1	b. Staircase #2	c. Staircase #3
rise =run =	rise =run =	rise =run =

 Using the run and rise for each step, graph the height a person will be at after each step for the first 5 steps. Do this for each staircase. Height (Rise) ▲



3. Which staircase is the steepest? _____

Just like staircases, the measurement of the steepness of a line is also very important information. On the graph on the previous page draw a connecting line from the origin(0,0) through the tip of each stair step.

Find the slope of each line representing a staircase using the ratio: $\frac{rise}{run}$ or $\frac{y}{x}$, and by simplifying this fraction.

- 4. Calculate the slope ratio for each staircase.
 - a. Staircase #1: _____
 b. Staircase #2: _____
 c. Staircase #3: _____
- 5. If you didn't have the graph to look at, only the ratios you just calculated, how would you know which staircase would be the steepest?
- 6. Calculate the slope for climbing 1, 2, & 3 steps on each of the staircases.

	Staircase #1				Staircase #2	2	Staircase #3			
	Total Rise	Total Run	Slope (rise/run)	Total Rise	Total Run	Slope (rise/run)	Total Rise	Total Run	Slope (rise/run)	
1 step										
2 steps										
3 steps										

- 7. Does the slope of the staircase change as you climb each step?
- 8. Using your knowledge of how slope is calculated, see if you can figure out the slope of the ramp found at your school. Take measurements at two locations on the ramp. Use the table below to help you.

	Rise	Run	Slope
1 st			
measurement			
2^{nd}			
measurement			

2.3a Homework: Measuring the Slope of Stairs and Ramps

- 1. Your task is to design a set of stairs and a wheelchair ramp at the side. Both the stairs and the ramp will begin at the same place (at ground level) and end at the height of 3 feet. Answer the following questions as you develop your design.
- How many steps do you want or need?
- How deep should each step be? Why do you want this run depth?
- How tall will each step be? Why do you want this rise height?
- What is the total distance (total depth for all steps) you will need (at the base) for all of the stairs—this would be a measurement at ground level from stair/ramp start point to stair/ramp end point?
- 2. Sketch the ramp (as viewed from the side) on graph paper below. Label and sketch the base and height, for example: Ramp-base (in inches or feet) and Height (in inches or feet).



3. From the sketch of the ramp, find and record the following measurements.

	Rise height (total height you've climbed on the ramp at this point)	Run depth (total distance – at ground level – covered from stair-base beginning)	Ratio	rise run
12 inches in from				
the start of the ramp				
24 inches in from				
the start of the ramp				
Where the ramp				
meets the top				

4. Sketch the stairs (as viewed from the side) on graph paper below. Label the sketch base and height, for example: Stair-base (in inches) and Height (in inches).



From the sketch above, find and record the following measurements.

	Rise height (total height you've climbed at this point)	Run depth (total distance covered from stair-base beginning)	Ratio	rise run	Reduced ratio
At the first					
step					
At the third					
step					
At the last					
step					

- 5. What do you notice about the Ratio column for both the ramp and the stairs?
- 6. On your stair drawing, draw a line from the origin (0,0) through the very tip of each stair. Now look at your ramp drawing. What do you observe?
- 7. Explain what would happen to the slope of the line for your stairs if the rise of your stairs was higher or lower?
- 8. Fill in the blanks
 - a. Slope = —
 - b. In the picture below the rise is ______ units, and the run is ______ units. Thus, the slope of





9. Suppose you want to make a skateboard ramp that is **not** as steep as the one show below. Write down two different slopes that you could use.



10. Find the slope of the waterslide. Calculate it in two different ways.



Extra Practice: Measure the Slope of a Hill

Instructions for measuring the grade of a hill or a road:

Rest the level (or clear bottle filled with water) on the ground. Lift up the lower end of the level/bottle (i.e, the end nearest the bottom of the hill) until the level measures level. (If using a bottle, the water level in the bottle should be parallel to the side of the bottle.)



While holding the level/bottle still in this position, measure the distance between the end of the level and the ground using the ruler, as shown in the image below.



- 1. Calculate the grade by dividing the distance measured with the ruler (the "rise") by the length of the level or straight edge (the "run") and multiplying by 100:
- 2. Roadway signs such as the one to the right are used to warn drivers of an upcoming steep down grade that could lead to a dangerous situation. What is the grade, or slope, of the hill described on the sign? (Hint: Change the percent to a decimal and then change the decimal to a fraction)



2.3b Class work: Dilations and Proportionality

When an object, such as a line, is moved in space it is called a transformation. A special type of transformation is called a dilation. A dilation transforms an object in space from the center of dilation, usually the origin, by a scale factor called r. The dilation moves every point on the object so that the point is r times away from the center of dilation as it was originally. This means that the object is enlarged or reduced in size.

For example, if you dilate the set of points (0,0), (0,4),(3, 0), and (3,4) with a scale factor of 2 and the center of dilation is at the origin (0,0) the distance of each point from the center will be 2 times as long as it was originally. Algebraically this means that you multiply each point by 2.

$$(x, y) \rightarrow (2x, 2y)$$

To confirm this, investigate this transformation below.



A special notation is used to differentiate between the pre-image and the image. If the pre-image is called A then the image is called A, pronounced "A prime".

2. Try another shape to see what kind of relationship exists between the pre-image and the image.



Now that you know how a dilation works you will investigate what kind of relationship is formed between the pre-image and the image after the dilation.



- 4. What do you notice about $\overline{B'C'}$ in relationship to \overline{BC} ?
- 5. What do you notice about the size of the two triangles?
- 6. Write a ratio that compares the corresponding parts of the pre-image with the image.
- 7. What kind of relationship exists between the pre-image and image?

2.3b Homework: Dilations and Proportionality

Continue your investigation of dilations and the relationships between the pre-image and image below.

- 1. Using the figure below:
 - a. Connect point B to C
 - b. Double the length of \overline{AB} on the line \overline{AE} . Label the new segment $\overline{AB'}$
 - c. Double the length of \overline{AC} on the line \overline{AD} . Label the new segment $\overline{AC'}$.
 - d. Connect B' to C'.
- 2. What do you notice about $\overline{B'C'}$ in relationship to \overline{BC} ?





- 4. Using the figure below:
 - a. Connect point B to C.
 - b. HALF the length of \overline{AB} on the line \overline{AE} . Label the new segment $\overline{AB'}$.
 - c. HALF the length of \overline{AC} on the line \overline{AD} . Label the new segment $\overline{AC'}$.
 - d. Connect B' to C'.
- 5. What do you notice about $\overline{B'C'}$ in relationship to \overline{BC} ?
- 6. What kind of relationship exists between the pre-image and image?



- 7. Using the figure below:
 - a. Connect point B to C.
 - b. HALF the length of \overline{AB} on the line \overline{AE} . Label the new segment $\overline{AB'}$.
 - c. HALF the length of \overline{AC} on the line \overline{AD} . Label the new segment $\overline{AC'}$.
 - d. Connect B' to C'.
- 8. What do you notice about $\overline{B'C'}$ in relationship to \overline{BC} ?
- 9. What kind of relationship exists between the pre-image and image?



2.3c Class Activity: Proportional Triangles and Slope

1. On the line to the right choose any two points that lie on the line. (To make your examination easier choose two points that lie on an intersection of the gridlines).

From the two points create a right triangle, the line itself will be the hypotenuse and the legs will extend from the two points and meet at a right angle. An example is shown below.





2. Compare the points that you choose and your triangle with someone in your class. Discuss the following;

Did you both chose the same points? How are your triangles the same? How are your triangles different? What relationship exists between your triangles?

Given any two triangles with hypotenuse on the given line and legs horizontal and vertical, then there is a dilation that takes one on the other. In particular, the lengths of corresponding sides are all multiplied by the factor of the dilation, and so the ratio of the length of the vertical leg to the horizontal leg is the same for both triangles.

The graphs given below have several different triangles formed from two points that lie on the line. These triangles are dilations of one another and their corresponding parts are proportional. Answer the questions below about these lines to observe what this tells us about slope.

- In each graph below, how many right triangles do you see?
- Trace each triangle you see with a different color.
- For each triangle write a ratio comparing the lengths of its legs or $\frac{height}{base}$. Then simplify the ratio

 $\frac{height}{base} = ----.$



7. Do the ratios (rise to run) always simplify to the same fraction (even quadrants with negative ordered pairs)? Why or why not?

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8. How does the "rise over run ratio" describe the steepness of the line?



11. Why do graphs 9 and 10 have the same ratio but they are different lines?

- 12. How could you differentiate between the slopes of these lines?
- 13. How does the rise related to the run of a negative slope affect the steepness of the line?



16. Are the ratios for graphs 14 and 15 positive or negative? How do you know?

17. Why are the slopes for graphs 14 and 15 the same if they are different lines?

2.3c Homework: Similar Triangles and Slope

For each line graphed below,

• Draw a Right Triangle to calculate the slope of the line. The slope of a line is denoted by the letter *m*. Thus $slope = m = \frac{rise}{m}$

• Label each triangle with a ratio and simplify the ratio $m = \frac{rise}{run} = ----$



2. What does the sign of the slope tell us about the line?

For each line graphed below, calculate the slope of the line.



6. Briefly, explain how to calculate slope when looking at a graph.

2.3d Class Activity: Finding Slope from Graphs

1. Do the graphs below have positive or negative slopes? How do you know?



- Draw a right triangle to calculate the slope of the line.
- Label each triangle with a ratio and simplify the ratio $m = \frac{rise}{m} = \frac{rise}{m}$





For each line graphed below, calculate the slope of the line.





2.3d Homework: Finding Slope from Graphs

1. Do the graphs below have positive or negative slopes? How do you know?



- For each line graphed below,
 - Draw a right triangle to calculate the slope of the line.
 - Label each triangle with a ratio and simplify the ratio $m = \frac{rise}{run} = ----$



For each line graphed below, calculate the slope of the line.





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2.3e Class Activity: Finding Slope from Two Points

Calculate the slope of the following graphs:



Graph the following pairs of points. Use the graph to determine the slope.



Find the rise and run and slope of each line shown below. You will have to think of a way to use the coordinate points to find the rise and run.



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14. Graphing points can be time-consuming. Develop a procedure for calculating the slope without graphing each point. Explain your procedure below. Show that it works for problems 1-4 above.

Discuss and compare your method for calculating slope without using right triangles on a graph with someone else.

15. Now discuss this formula: $slope = m = \frac{y_2 - y_1}{x_2 - x_1}$ What does it mean? How does it work?

Fill in the missing information in the problems below. Use the empty box to calculate slope using the formula,







29.	30.	31.	32.
(6.1)	(1.6)(1.6)		(0, 2)
(-0, 1)	(-1, 0) (-4, 0)		(0,-2)
$\Delta y =$	$\Delta y =$	$\frac{\Delta y}{\Delta y} = \frac{3}{2}$	$\Delta y = -3$
Δx	Δx	$\Delta x = 2$	$\Delta x = 4$

2.3e Homework: Finding Slope from Two Points



Graph the following pairs of points. Use the graph to determine the slope.

Calculate the slope of the line connecting each pair of points.

	1	0	-	-			
3.	(1,42) and (4,40)				4.	(-21, -2) and (-20, -	5)

5. (3, -10) and (-6, -10) 6. (10, -11) and (11, -12)

7. (5, 1) and (-7, 13) 8. (14, -3) and (14, -7)

- 9. (8, 41) and (15, 27) 10. (17, 31) and (-1, -5)
- 11. (-5, 36) and (-4, 3) 12. (32, -23) and (-6, -2)

2.3f Class Activity: Practice Finding the Slope of a Line

Calculate the slope of the line on each graph.



For each pair of points,

a.

- Calculate the slope of the line passing through each pair.
- Find one other point that lies on the line containing the given points.
- 3. (10, -6) and (-5, 4) 4. (7, 3) and (-3, 0)
- 5. (0, 4) and (1, 0) 6. (-5, 1) and (-5, -2)

Calculate the slope of the line that contains the points given in each table. Calculate the slope twice, one time by using the Slope Formula with two points and the other time by finding the rate of change or unit rate in the table.



11. Why are the slopes the same no matter what two points you use to find the slope?

10 x

5

2.3f Homework: Practice Finding the Slope of a Line

Calculate the slope of the line on each graph.



Calculate the slope of the line passing through each pair of points.

7. (3, 9) and (4, 12)

8. (5, 15) and (6, 5)

9. (6, 9) and (18, 7) 10. (-8, -8) and (-1, -3)

For numbers 11 and 12;

- Calculate the slope of the line passing through each pair
- Find one other point that lies on the line containing the given points

11. (-6,-5) and (4,0) 12. (4,1) and (0,7)

Calculate the slope of the line that contains the points given in each table.

13.	x y	14.	x	У
	3 -9		8	1
	5 -15		6	3
	9 -27		2	7
			-4	13
15.	x y	16.	X	У
	0 4	_	10	1
	1 -5		8	1
	2 -14		-12	1
	3 -23		-14	1
1.5	5 25	10		_
17.	x y	18.	X	У
	-5 2	_	-3	5
	5 6		-3	10
	10 8		-3	15
			-3	20

19. Why doesn't it matter which two points you use to find the slope?

2.3g Class Activity: Finding Slope from a Context

- 1. Gourmet jellybeans cost \$9 for 2 pounds.
 - a. Complete the table.

Pounds	.5		3	4		8	10		20
Total Cost		\$9			\$27				
b. Graph and label	the relat	ionship.		c. V	Vhat is t	he slope	of the li	ne?	

d. Write the slope of the line as a rate of change that describes what the slope means in this context.
e. Write an equation to find the cost for any amount of jellybeans.
f. Why is the data graphed only in the first quadrant?

- 2. Kaelynn takes the same amount of time to solve each of the equations on her math homework. She can solve 10 equations in 8 minutes.
 - a. Complete the table.

Minutes	2		8		20	
Equations Solved		5		15		30

b. Graph and label the relationship.	c. What is the slope of the line?
	 d. Write the slope of the line as a rate of change that describes what the slope means in this context. e. Write an equation to find the number of equations solved for any number of minutes.
3. Mr. Irving and Mrs. Hendrickson pay babysitters differently.

a. Ex	amine the	table. De	escribe the different	nce. e.	Write each slope as a rate of change to
	TT	т ·	TT 1 1		interpret its meaning?
	Hours	Irving	Hendrickson		T .
	0	\$0	\$4		Irving:
	1	\$9	\$12		
	3	\$27	\$28		Hendrickson:
	5	\$45	\$44		
b. Ar	e both rela	tionships	s proportional?	f.	Explain the different <i>y</i> -intercepts.
c. Gr dif	aph and lal ferent lines	oel the tw s).	vo pay rates (two	g.	Is one babysitting job better than the other? Why or why not?
d. WI	hat is the sl	lope of e	ach line?	→	Write an equation for each situation.Irving:Hendrickson:
	0				

2.3g Homework: Finding Slope from a Context

1. The soccer team is going out for hot dogs. Greg's Grill is having a special on hot dogs: four hot dogs for three dollars. Each hot dot costs the same amount of money.

Hot Dogs	1			16		28	40
Total Cost		\$3	\$9		\$18		

b.	b. Label the graph axes and then graph the relationship.									n tl	c.	What is the slope of the line?									
	↑																				
								+			-									d.	Write a sentence with correct units that
								t		t		L									describes what the slope means.
				\neg	+	+	+	╀	╋	┝	┢	┝	Н		Н	Н					
					1	4	#	t		t	E										
		┢	\vdash	+	┥	+	+	╀	╈	┢	┢	┝	Н		Н	Н	\vdash				
		F			\neg	\neg	\mp	Ŧ		F	F	F	\square								
								\pm												e.	Write an equation to find the cost for any
	_	┝		\dashv	+	+	+	╀		┝	┢	┝	Н		Н	Н					amount of not dogs.
					1	1	1	‡	t	t	t										
																		↦			

2. The state fair costs \$2 to get in plus \$.50 per ticket to go on rides. Complete the following table, showing the cost for getting into the fair with additional tickets for rides.

a. Complete the tabl	e.						
Tickets	0	1		10		30	
Total Expense	2		\$4.50		\$12		\$20

b.	La	ıbe	el	th	e g	gra	ap	h	ax	es	s a	nc	1 t	he	en	gı	ap	ph	tł	ne	c.	Wl
	re]	lat	io	ns	hi	p.																
		Γ																			d.	W1
																						of
			_				_		_												е	WI
		┝	-	-		-	-	-	-	-		-			\square			-			С.	
		┢																				
																					f.	W1
																						the

- hat is the slope of the line?
- rite a sentence that describes what the slope the line means.
- hy does the line not pass through 0?
- rite an equation to find the total expense at fair with any amount of tickets purchased.

3. Excellent Bakers and Delicious Delights bakeries charge differently for sandwiches for business lunches.

a. Examin	ne the table.	Describe the d	lifference.	d.	Gr La	aph the bel.	two pay	y rates (t	wo diffe	erent lin	es).
Sandwich Meals 0 1 3 5 b. Are box c. Find the relation Excellent: Delicious :	Excellent Bakers \$0 \$20 \$60 \$100 th relationsh e rate of chanship.	Delicious Delights \$15 \$30 \$60 \$90 ange for each	al?	e. f.	W Ex W sit	hat is the cellent hat doe uations	ne slope s the slope he differ	of each Delic	line? cious: rou abou	t these t	two
h. Is one situa	ation better	than the other?	Why or wh	ny ne	ot?						
i. Write an eaExcellaDelicion	quation for o ent Bakery _ ous Delights	each situation.									

2.3h Class Activity: The Equation of a Linear Relationship

Throughout this chapter the components of a linear relationship have be investigated. A linear relationship is defined by the constant rate of change that it possesses and it can be represented in many ways. In this lesson the focus will be more on the equation that represents a linear equation.

The equation given below represents a linear relationship.

y = 2x + 5

1. Graph this equation of this line by making a table of values that represent solutions to this equation. (This if often called a T-chart).



- 2. Find the slope of this line?
- 3. Where do you see the slope in the equation?

Often the letter *b* is used to denote the *y*-intercept.

- 4. What is the *y*-intercept of this line? b=
- 5. Where do you see the *y*-intercept in the equation?



For each of the equations given below, make a table of values to help you graph the line. Then identify the slope and *y*-intercept. Circle the slope in your equation and put a star next to the *y*-intercept.





- 12. Examine the graphs and equations given above. Describe the general form of a linear equation. In other words, in general, how is a linear equation written? What are its different parts?
- 13. Write down he general form of a linear equation in the box below based off of your class discussion.

Slope-intercept form of a linear equation is

where *m* represents the slope (rate of change) and *b* represents the *y*-intercept (initial value or starting point)

14. What if the *y*-intercept is zero, how do you write the general form of the equation?

2.3h Homework: The Equation of a Linear Relationship

For each of the equations given below, make a table of values to help you graph the line. Then identify the slope and y-intercept. Circle the slope in your equation and put a star next to the y-intercept.







2.3i Class Activity: Use Dilations and Proportionality to Derive the Equation y = mx.



Up to this point you have been investigating how to describe many patterns and stories with a linear relationship. You have begun to get a sense of how linear relationships are formed and described.

Write down what you know about linear relationships below.

Now write down what you know about linear proportional relationships below.

You are going to use the facts listed above to derive the equation y=mx using dilations. Begin by looking at the example below.

- 1. Graph a line on the coordinate plane to the right that goes through the origin and has a slope of $\frac{2}{3}$. Label the rise and run on your graph with a right triangle. Redraw and label this triangle in the space provided below the graph.
- 2. Does this line describe a proportional relationship?
- Choose any point (*x*, *y*) on your line and draw a slope triangle that describes the rise and run. Redraw and label this triangle in the space provided below the graph.



- 4. Write a proportional statement with your ratios.
- 5. Solve the equation that you wrote above for *y*.

Now look at the general case for the form y=mx.

- 6. Graph a line on the coordinate plane to the right that goes through the origin and has a slope of *m*; remember that slope is the same as a unit rate which compares your *y*-value to an *x* value of 1. Label the rise and run on your graph with a right triangle. Redraw and label this triangle in the space provided below the graph.
- 7. Does this line describe a proportional relationship?
- Choose any point (*x*, *y*) on your graph and draw a slope triangle the describes that rise and run. Redraw and label this triangle in the space provided below the graph.
- 9. Write a proportional statement with your ratios.
- 10. Solve the equation that you wrote above for y.

Notice that this is the equation y=mx that describes a proportional relationship.



11. Show that the equation for a line that goes through the origin and has a slope of $\frac{2}{5}$ is $y = \frac{2}{5}x$ using dilations and proportionality.



2.3i Homework: Use Dilations and Proportionality to Derive the Equation y = mx.

1. Show that the equation for a line that goes through the origin and has a slope of $\frac{1}{3}$ is $y = \frac{1}{3}x$ using dilations and proportionality.



2. Show that the equation for a line that goes through the origin and has a slope of $-\frac{3}{5}$ is $y = -\frac{3}{5}x$ using dilations and proportionality.



2.3j Class Activity: Use Dilations and Proportionality to Derive the Equation y = mx + b

What about linear relationships that are not proportional? You are going to further investigate the general form of a linear equation with a transformation. A geometric transformation can relate a linear proportional relationship to a linear non-proportional relationship.

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In previous sections we used the geometric transformation called a dilation. Another type of transformation is called a translation. Shifting a line or moving all the points on the line the same distance and direction is a transformation is a **translation**.

For example, if you transform the line $y = \frac{1}{2}x$ upwards by 3 units, every ordered pair that lies on that line gets moved up 3 units. Algebraically that means that you add 3 to every *y* value since this is a vertical shift.

$$(x, y) \rightarrow (x, y+3)$$

To confirm this, investigate this transformation below.



- 5. Graph your new ordered pairs.
- 6. Write an equation for your image and compare it with your equation for the pre-image.

In section 2.3h you investigated the general form for a linear equation given below.

The general form of a linear equation is y=mx+b where *m* is the slope or rate of change and *b* is the initial value or *y*-intercept.

It is also seen in the example above. The equation $y = \frac{1}{2}x + 3$ represents the linear relationship where the slope is $\frac{1}{2}$ and 3 is the y-intercept or initial value.

Use the information on the previous page to compare and contrast the two relations using the Venn Diagram below. Be sure to list similarities and differences about their graphs and equations.



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So how is this general form, y=mx+b, for a linear equation derived? Start with an example.

- 17. Graph a line on the coordinate plane to the right that goes through the point (0,4) and has a slope of $\frac{2}{3}$. Label the rise and run on your graph with a right triangle. Redraw and label this triangle in the space provided below the graph.
- 18. Does this line describe a proportional relationship? Explain.



19. Choose any point (*x*, *y*) on your graph and draw a slope triangle the describes the rise and run. Redraw and label this triangle in the space provided below the graph.

- 20. Write a proportional statement with your ratios.
- 21. Solve the equation that you wrote above for *y*.

Now look at the general case for the form y = mx + b.

- 22. Graph a line on the coordinate plane to the right that goes through any point on the *y*-axis(call the *y*-intercept *b*) and has a slope of *m*. Label the rise and run on your graph with a right triangle. Redraw and label this triangle in the space provided below the graph.
- 23. Does this line describe a proportional relationship? Explain.



- 24. Choose any point (x, y) on your graph and draw a slope triangle that describes the rise and run. Redraw and label this triangle in the space provided below the graph.
- 25. Write a proportional statement with your slope ratios.

26. Solve the equation that you wrote above for *y*.

Notice that this is the general form of a linear equation. This form is called Slope-Intercept form.

Slope-Intercept form of a linear equation is y = mx + bwhere *m* represents the slope (rate of change) and *b* represents the *y*-intercept (initial value or starting point)

27. Show that the equation of a line that goes through the point (0,3) and has a slope of $\frac{1}{6}$ is $y = \frac{1}{6}x + 3$.



2.3j Homework: Use Similar Triangles to Derive the Equation y = mx + b

1. Show that the equation of a line that goes through the point (0, -2) and has a slope of $\frac{3}{2}$ is $y = \frac{3}{2}x - 2$.



2. Show that the equation of a line that goes through the point (0,4) and has a slope of -2 is y = -2x + 4.



2.3k Self-Assessment: Section 2.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provided on the next page that match each skill/concept

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
 Show that the slope of a line can be calculated as rise/run. Also explain why the slope is the same between any two distinct points on the line. See sample problem #1 	I can find the slope of the line but do not know how I did it.	I can find the slope of the line by finding the rise and run but do not know how to show how you can find the slope between any two points on the line.	I can find the slope of the line by finding the rise and run. I can explain how to find the slope using any two points that lie on gridlines.	I can find the slope of the line by finding the rise and run and show or explain why the slope can be found from any two points that fall on the line.
 2. Find the slope of a line from a graph, set of points, and table. Recognize when there is a slope of zero or when the slope of the line is undefined. See sample problem #2 	I can find the slope of the line from one of the three representations.	I can find slope of the line from two of the three representations.	I can find the slope of the line from the graph, points, and table but I did not simplify all of my answers.	I can accurately find the slope of the line from the graph, points, and table. I can also express my answers in simplest form.
3. Given a context, find slope from various starting points (2 points, table, line, equation).	I can only find the slope of the line from one of the starting points. I don't know what the slope means given then context.	I can find the slope of the line from some of the starting points but I don't know what the slope means given the context.	I can find the slope of a line from any starting point but I don't know what the slope means in the given context.	I can find the slope of the linear relationship from any starting point and interpret what it means given the
5.86 sample problem #3 4. Recognize that m in y = mx and y = mx + b represents the rate of change or slope of a line. Understand that b is where the line crosses the y-axis or is the y- intercept.	I do not know how to find the slope and y- intercept given in the equation.	I can identify the y-intercept but not the slope in this equation.	I can identify the slope but not the y-intercept in the equation.	I can identify the slope and y- intercept in the equation.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
5. Derive the equation y = mx and $y = mx + busing dilations andproportionality.$	I can show the y- intercept and slope for both equations but I don't know how they relate to dilations of triangles or proportions.	I can derive only one of the equations using similar triangles.	I can accurately go through the steps of deriving both of the equations of the line using similar triangles and proportionality.	I can accurately go through the steps of deriving both of the equations of the line using dilations of triangles and proportionality. I can explain the reasoning at each step in my own
See sample problem #5				words.

Find the slope of the line given below by finding the rise and run. Also show or explain why the slope can be found from any two points that fall on the line.



Find the slope of each line given in the graph, set of points and table below.



b. Find the slope of the line that goes through the points (-2,2), (6,-10).

c. Find the slope of the line that goes through the points in the table given below.

х	У
4	-8
6	-8
8	-8

Find the slope of the line from each context given below.

a. The cost to fix a car at Bubba's Body Shop is shown in the table below. What is the slope of this linear relationship, be sure to explain what is means given the context.

Total Cost								
t (time in	C (cost in							
(0)	0							
1	25							
2	50							
3	75							

b. The graph given below describes the height over time of a Norfolk Pine that is planted in Rudy's backyard. Find and describe the slope of this relationship as related to the context.



- c. The equation y=2x+10 describes the monthly cost to rent a movie at the local video kiosk, where *x* represents the number of movies rented and *y* represents the total cost. Find and describe the slope of this relationship.
- d. The points (1, 16) (4, 15) describe the height of a lit candle that is burning at two different times of day. Here *x* represents the number of hours it has been burning and *y*-represents the candle's height in inches. Find the slope of this relationship and describe what it means given the context.

Sample Problem #4

For the equation given below identify and the rate of change or slope and the y-intercept.

$$y = -\frac{1}{3}x + 5$$

y-intercept(*b*):

Show that the equation for a line that goes through the origin and has a slope of $\frac{5}{4}$ is $y = \frac{5}{4}x$ using dilations and proportionality.



Show that the equation for a line that goes through (0,3) and has a slope of 2 is y = 2x + 3.

