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# Chapter 3: Representations of a Line (4 weeks)

## Utah Core Standard(s):

- Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line. (8.F.3)
- Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (8.F.4)
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. (8.F.2)

**Vocabulary:** constant difference, context, difference table, equation, geometric model, graph, horizontal, initial value, linear, parallel, perpendicular, rate of change, reflection, rotation, slope, slope-intercept form, table, transformation, translation, unit rate, vertical, y-intercept.

## Chapter Overview:


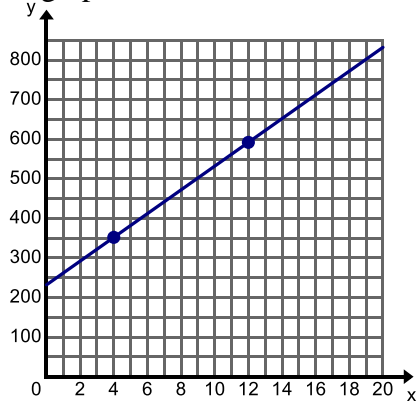


In this chapter, students solidify their understanding of the slope-intercept form of a linear equation. They write the equation for a linear relationship in slope-intercept form given a slope and y-intercept, two points, or a graph. They also write equations in slope-intercept form from a given context. In conjunction with writing equations students will graph equations given a variety of conditions. They may be given an equation in a variety of forms to graph, a slope and point, or a context. This chapter mainly focuses on the procedural process of graphing and writing equations for linear relationships. The transition from equation to relation to function is an important and difficult one. Chapter 5 will specifically address this transition and help students make the change in thinking.


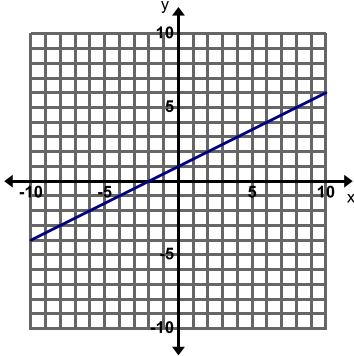


## Connections to Content:


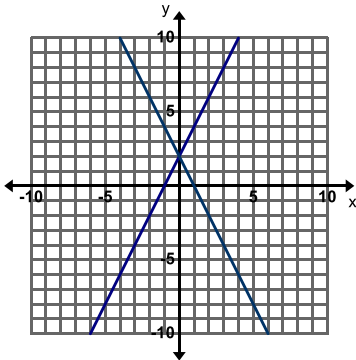

Prior Knowledge: Up to this point, students have been studying what makes a linear relationship and how it is composed. They have graphed linear equations by plotting ordered pairs generated in a table as solutions to an equation. They have written linear relationships by focusing on how the relationship grows by a constant rate of change and looking at an initial value. For the most part the slope-intercept form of a linear equation has been addressed mostly on a conceptual level. This chapter allows students to further their study of linear relationships by focusing on the procedural methods for graphing and writing linear equations.

Future Knowledge: In this chapter students will gain the skills and knowledge to work with simultaneous linear relationships in Chapter 4. This chapter sets them up to be able to write and graph a system of linear equations in order to examine its solution. In addition this chapter is the building block for a student's understanding of the idea of function. In Chapter 5, students will solidify the concept of function, construct functions to model linear relationships between two quantities, and interpret key features of a linear function. This work will provide students with the foundational understanding and skills needed to work with other types of functions in future courses.

# MATHEMATICAL PRACTICE STANDARDS (emphasized):

	<p>Make sense of problems and persevere in solving them.</p>	<p>The graph below shows the weight of a baby elephant where <math>x</math> is the time (in weeks) since the elephant's birth and <math>y</math> is the weight (in pounds). At 4 weeks, the elephant weighed 352 lbs. and at 12 weeks, the elephant weighed 592 lbs. Label the axes of the graph.</p>  <p>Use your graph and equation to tell the story of this elephant.  <i>Students are using the skills they learned for writing equations of lines to solve a real world problem. They are translating between the different representations of a line and recognizing important features of the representations. Following this work, the teacher is prompted to ask the students if it really makes sense that this elephant gains exactly 30 pounds each week, leading to a conversation about real world data and statistics.</i></p>
	<p>Reason abstractly and quantitatively.</p>	<p>The cost to rent a jet ski is \$80 per hour. The cost also includes a flat fee for a lesson on how to use the jet ski. Steve rented a jet ski for 3 hours and the total bill was \$265. Write the equation for the total cost of renting a jet ski <math>y</math> for <math>x</math> hours.</p> <p>Use your equation to add more details to the story about renting a jet ski.</p> <p><i>Throughout this chapter students must write equations for many linear relationship contexts. In some cases it might be abstracting an initial value as the y-intercept or by looking at the constant rate of change as the slope. In other cases students must comprehend the intended meaning of given quantities not just how to compute them. For example, in the problem above a student must discern that they can represent renting a jet ski for 3 hours at a costs of \$265 as the point (3, 265). They must then recognize how to use the point to write an equation.</i></p>
	<p>Look for and make use of structure</p>	<p>Brainstorm ideas on how you could write the equation of the line without graphing when you are given a point and the slope. Consider how you could use the equation <math>y = mx + b</math> to find the y-intercept if you know the slope and a point on the line. Using an example from the previous page may help you work through the process.</p> <p><i>In the problem above, students make use of the structure of the slope-intercept form of an equation. They recognize that by substituting the ordered pair values of <math>x</math> and <math>y</math> and the slope into the equation they can obtain the value for <math>b</math>, the y-intercept. To do so they must recognize that they can change the structure of the equation (solve for <math>b</math>) and obtain their intended value. This process allows them to then write an equation in slope-intercept form.</i></p>

	<p>Construct viable arguments and critique the reasoning of others.</p>	<p><b>Find, Fix, and Justify:</b> Kevin was asked to graph the line <math>y = -\frac{1}{2}x + 1</math>. Kevin graphed the line below and made a common error. Describe Kevin's error and then graph the line correctly on the grid.</p>  <p><i>There are many Find, Fix, and Justify problems in this chapter where a student must find a common mistake and fix it. As student's critique another student's work they must analyze the key components of a linear relationship. This reinforces a student's understanding and often clears up common misconceptions.</i></p>
	<p>Model with Mathematics.</p>	<p>A Health Teacher is writing a test with two sections. The entire test is worth 40 points. He wants the questions in Section A to be worth 2 points each and the questions in Section B to be worth 4 points each. Let <math>x</math> represent the number of questions in Section A and <math>y</math> represent the number of questions in Section B.</p> <ol style="list-style-type: none"> <li>Write an equation that describes all the different combinations of number of questions in Section A and B.</li> <li>Graph this equation to show all possible numbering outcomes for this test.</li> </ol> <p><i>This question is asking students to model a situation with an equation and graph. The equation allows the student to see how the unknown values are related to each other. The graph provides a pictorial representation of all of the possible solutions. Through modeling this situation with mathematics, students better understand how to represent more than one solution algebraically and graphically.</i></p>
	<p>Use appropriate tools strategically</p>	<p>Use a graphing calculator to graph the following equation.  <math>x - 3y = -9</math></p> <p><i>A graphing calculator is not only a useful tool in graphing an equation but also helpful when used to check your work. A teacher may allow students to only use the graphing calculator when checking their work. Another strategic way to use the graphing calculator is to examine how a line can change if the quantities in the equation are changed. In the example above, a student must employ the strategy of changing the equation into slope-intercept form before entering it into the graphing calculator.</i></p>

	<p>Attend to Precision</p>	<p>Determine if the following sets of lines in each graph are parallel or perpendicular. Justify your answer. If they are not parallel or perpendicular describe the transformation that carries one line to the other.</p>  <p><i>Upon first glance these lines appear to be perpendicular. But by calculating the slopes of the lines it is determined that they are not. This is a good example of attending to precision. Students must rely upon accurately and efficiently calculating the slope of each line to see that the slopes are not opposite reciprocals of each other.</i></p>
	<p>Look for and express regularity in repeated reasoning</p>	<p>Graph the equation <math>y = x + 3</math> and label the line with the equation.</p> <ol style="list-style-type: none"> <li>Predict how the graph of <math>y = x + 1</math> will compare to the graph of <math>y = x + 3</math>.</li> <li>Predict how the graph of <math>y = x - 3</math> will compare to the graph of <math>y = x + 3</math>.</li> <li>Graph the following equations on the same grid and label each line with its equation.  <math>y = x + 1</math>  <math>y = x - 3</math> </li> <li>Were your predictions correct? Why or why not?</li> <li>What is the relationship between the lines <math>y = x + 3</math>, <math>y = x + 1</math>, and <math>y = x - 3</math>?</li> <li>Write a different equation that would be parallel to the equations in this problem.</li> <li>Describe the movement of a line when <math>b</math> is increased or decreased while <math>m</math> is held constant.</li> </ol> <p><i>In section 3.2 students investigate how changing different parts of the equation result in different transformations of a line. They do this through repeatedly examining how these changes affect the line. For example a student might state, "Every time that you change the y-intercept in the equation the line keeps the same slope but moves up or down the y-axis."</i></p>

## Section 3.0 Anchor Problem: Solutions to a Linear Equation

Recall from Chapter 1 that you wrote and solved equations with one variable. Find the solution to each equation below.

1.  $x + 7 = 10$

2.  $5y = 15$

3.  $4x - 6 = 10$

4.  $3x - 11 = 2x + 9$

5. In your own words describe what a solution is.

Talk with your neighbor about what they think a solution is.

As we continue the study of linear relations it is important for students to understand that when we graph a linear equation we are showing all of the solutions to the equation. Since it is impossible to list all of the solutions to a linear equation the graph provides a “picture” of all of the solutions.

6. Refine your definition of a solution now that we have discussed it as a class.

**A solution is:** Any value that when substituted in for the variable makes the equation true.

7. Can there be more than one solution to an equation.

Yes, if there is more than one value that will make the equation true then they are all solutions. There can be one, none, or infinitely many solutions to a linear equation.

Now find the solutions to each equation below (it is okay to guess).

8.  $x + y = 12$

9.  $m - n = 12$

10.  $xy = 24$

Answers will vary in this section.

Any values that make the equation true are possible solutions.

11.  $y = 5x$

12.  $y = x^2$

At this point students begin to see that there are infinitely many solutions to these equations. This prompts the need for a way to list or show all of the solutions; thus the need for a graph.

Compare your solutions with your neighbor.

13. Is the definition for a solution the same if you have two different variables in your equation as opposed to above where we have only one variable? Yes, the definition is the same. A solution is any value(s) that makes the equation true.

14. How many total solutions are there for an equation with more than one variable?

Find at least four solutions to each equation. Write the solutions as ordered pairs.

15.  $y = 2x$  Sample answers are given below.

$x$	$y$	$(x,y)$
-1	-2	$(-1, -2)$
0	0	$(0,0)$
1	2	$(1,2)$
2	4	$(2,4)$

16.  $x + y = 5$

$x$	$y$	$(x,y)$

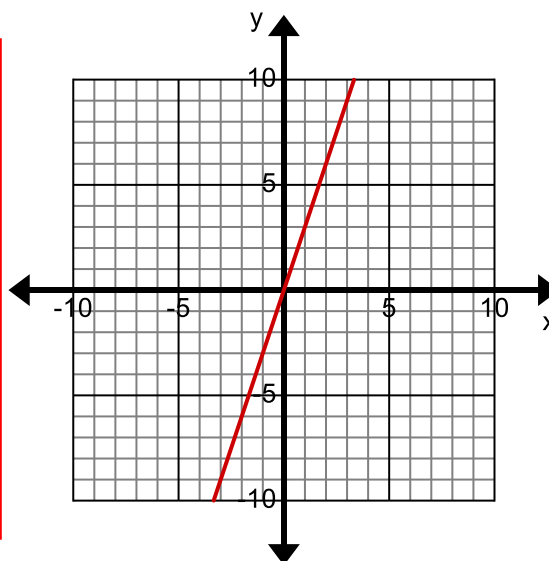
17. Is it possible to list every possible solution to these equations?

18. Why do you think the instructions prompted you to write your answers as ordered pairs?

19. To show every solution to an equation with two different variables you graph the equation.

20. Show the solutions to the equation  $y=3x$  on the graph below and describe in detail how the graph shows all the solutions to this equation.

It might be helpful to demonstrate how the line is formed by the solutions to the equation above. Project the grid on the board and ask students to come up and plot an ordered pair that is a solution to the equation. They will soon see that the ordered pairs follow a pattern. Some students may even come up with solutions that include fractions. If not, ask them if there are solutions that fall between integer ordered pairs. Begin filling in all of these solutions as well. Soon a line will start to appear because all of the fractional solutions will start to “merge” together. You could have this discussion either before or after students do problems # 21-23.



21. Why do you draw arrows on your graph?

The arrows indicate that the line goes on infinitely in both directions. This means that there are infinitely many solutions.

22. Put a star on the graph where a solution is a fraction.

23. Put a smiley on the graph where there is a negative solution.

## Section 3.1: Graph and Write Equations of Lines

### Section Overview:

Now that students have an understanding of the parameters  $m$  and  $b$  in the slope-intercept form of a linear equation, this section will transition students into the procedural work of being able to write and graph the equation of a line from any set of givens. Students apply the skills they have learned to write linear equations that model real world situations.

### Concepts and Skills to Master:

*By the end of this section, students should be able to:*

1. Write a linear equation in the form  $y = mx + b$  given any of the following:
  - slope and y-intercept
  - slope and a point
  - two points
  - a table
  - a graph of a linear relationship
  - a context of a real world situation
2. Graph linear relationships given any of the following:
  - an equation
  - slope and a point

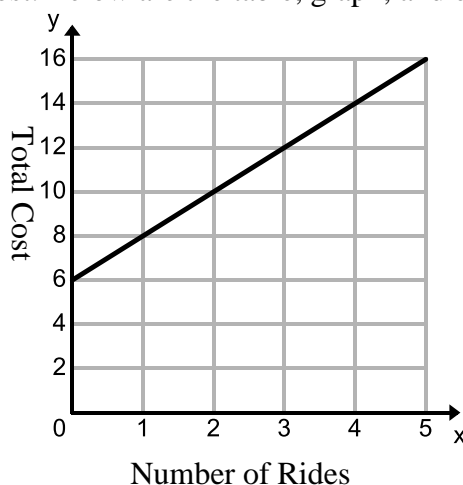


### 3.1a Class Activity: Write Equations in Slope-Intercept Form **n#**

Revisit a situation from the previous chapter:

You and your friends go to the state fair. It costs \$6 to get into the fair and \$2 each time you go on a ride. Consider the relationship between number of rides and total cost. Below are the table, graph, and equation that model this linear relationship.

Number of Rides ( $x$ )	Total Cost ( $y$ )
0	6
1	8
2	10
3	12
4	14
5	16



You modeled this situation with the equation  $y = 2x + 6$

Discuss the following questions with a partner. Highlight your answers on the table, graph and equation above.

What is the slope of the graph? Where do you see the slope in the equation? What does the slope represent in the context?

After students have discussed the questions above, review the following ways to demonstrate that the slope of the graph is 2: Have students show the slope of 2 on the table with a difference column, on the graph by showing the rise and run between two points, in the equation by underlining it (noting that it comes before  $x$ ), and in the context by underlining it, discussing that it is the cost of each additional ride (rate of change).

What is the y-intercept of the graph? Where do you see the y-intercept in the equation? What does the y-intercept represent in the context?

Review with students the following ways to demonstrate that the y-intercept is 6: Have students show the y-intercept on the graph by circling it, on the table, probing students to think about the value of  $x$  at the y-intercept, circling it in the equation and noting that it is the constant in the equation, and circling it in the context, discussing that it is the initial fee to get into the park (what you have to pay when you enter the park but don't go on any rides).

By looking at the problems done in the previous chapter, you can see that one way to represent a linear equation is in slope-intercept form. In the previous chapter you also derived the equation  $y = mx + b$ .

**Slope-intercept form** of a linear equation is

$$y = mx + b$$

where  $m$  represents the slope (rate of change)

and  $b$  represents the y-intercept (initial value or starting point)

**If you are given a representation of a linear relationship, you can write the equation for the relationship in slope-intercept form by finding the slope ( $m$ ) and y-intercept ( $b$ ) and substituting them into the slope-intercept form of a linear equation shown above.**

**Directions:** Write the equation of each line in slope-intercept form.



1. The slope of the line is 3. The y-intercept is (0, 4).

$$y = 3x + 4$$

2. The slope of the line is  $-2$ . The y-intercept is (0, 0).

3. The slope of the line is  $\frac{1}{2}$ . The y-intercept is (0,  $-2$ ).

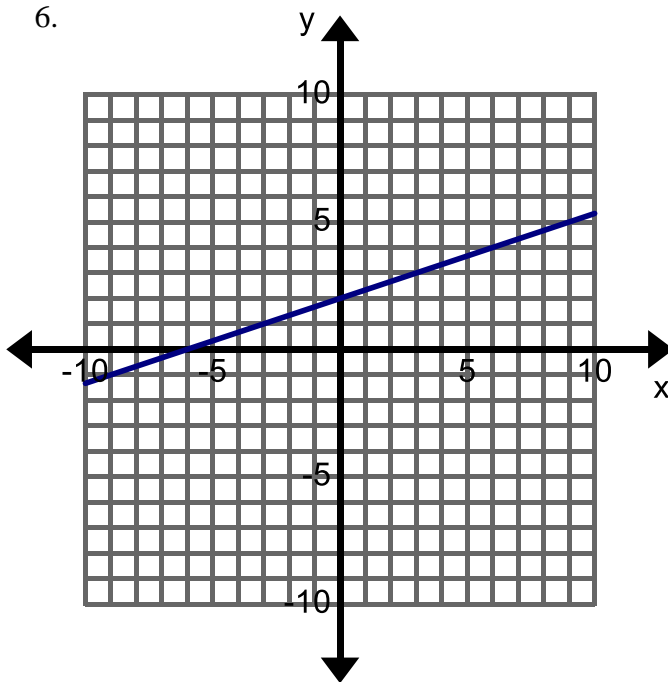
4. The slope of the line is  $-\frac{4}{3}$ . The y-intercept is (0,  $-1$ ).

$$y = -\frac{4}{3}x - 1$$

5. The slope of the line is 0. The y-intercept is (0, 2).

**Directions:** Find the slope and y-intercept from the graph, table, or story below. Then write the equation of each line in slope-intercept form. If you have a hard time determining where the line intersects a point be sure to check at least three points.

6.

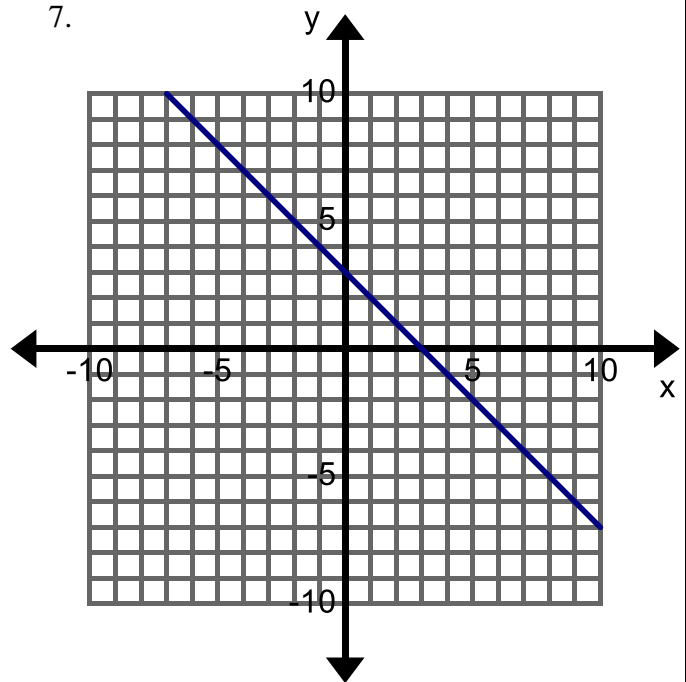


$m$ :  $\frac{1}{3}$   $b$ : 2

Equation:  $y = \frac{1}{3}x + 2$

If students need help finding the slope of a line from a graph, refer to 2.3d.

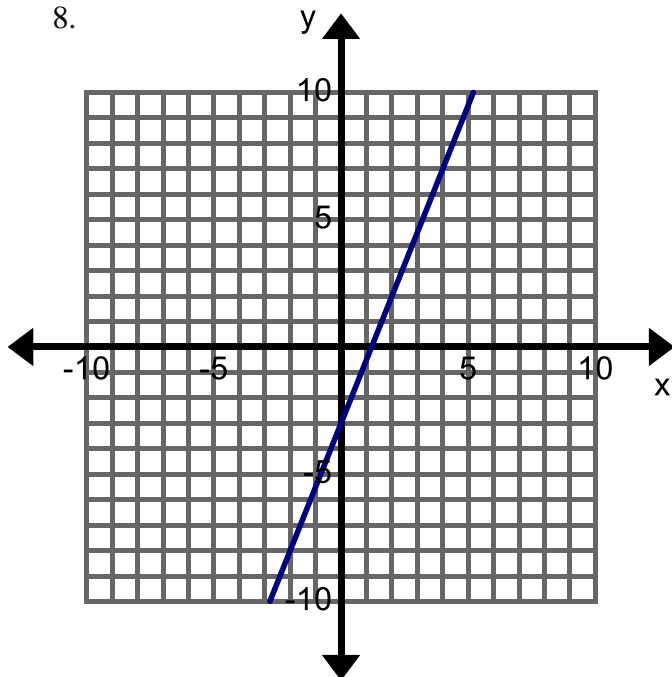
7.



$m$ :  $-1$   $b$ : 3

Equation:  $y = -x + 3$

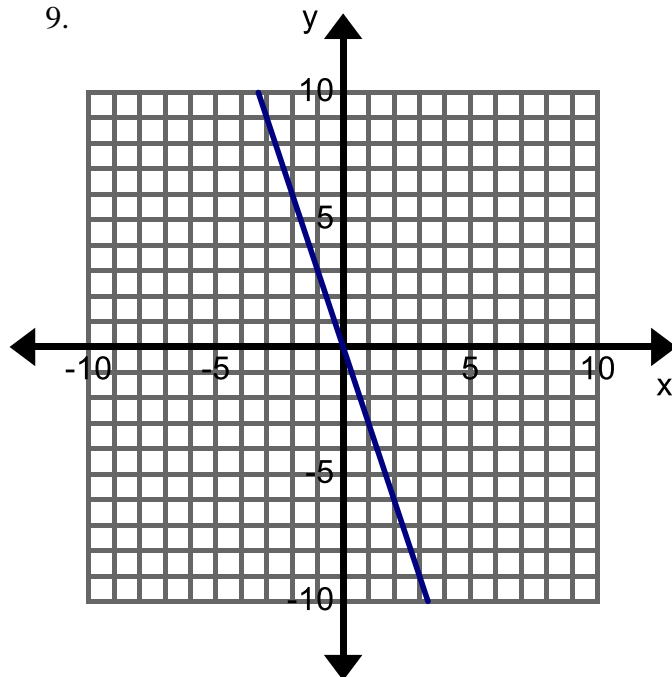
8.



$m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

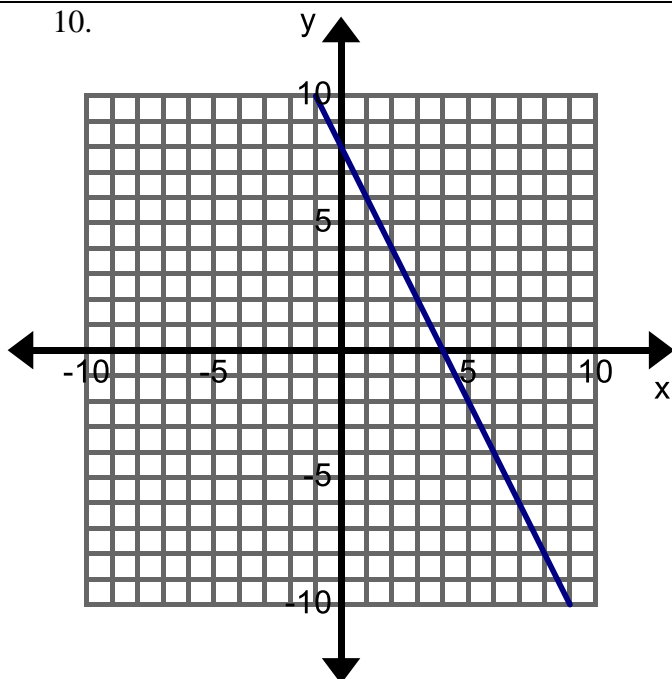
9.



$m$ : -3  $b$ : 0

Equation:  $y = -3x$

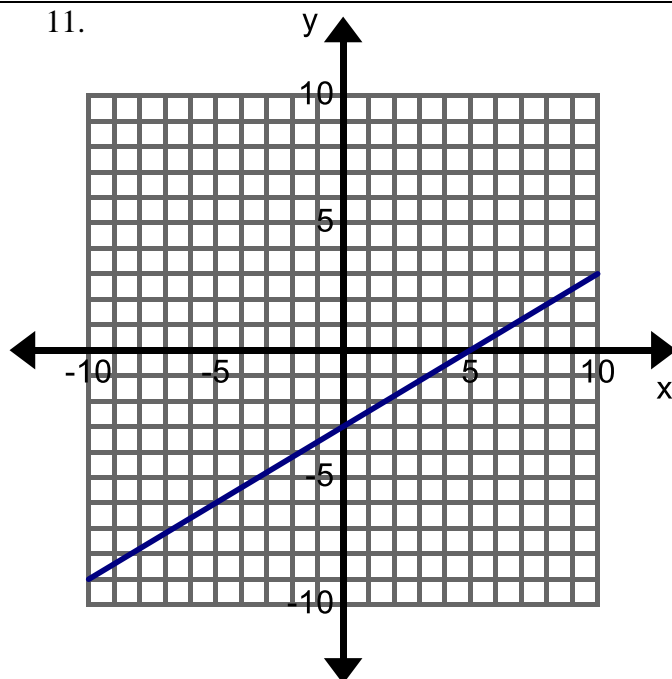
10.



$m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

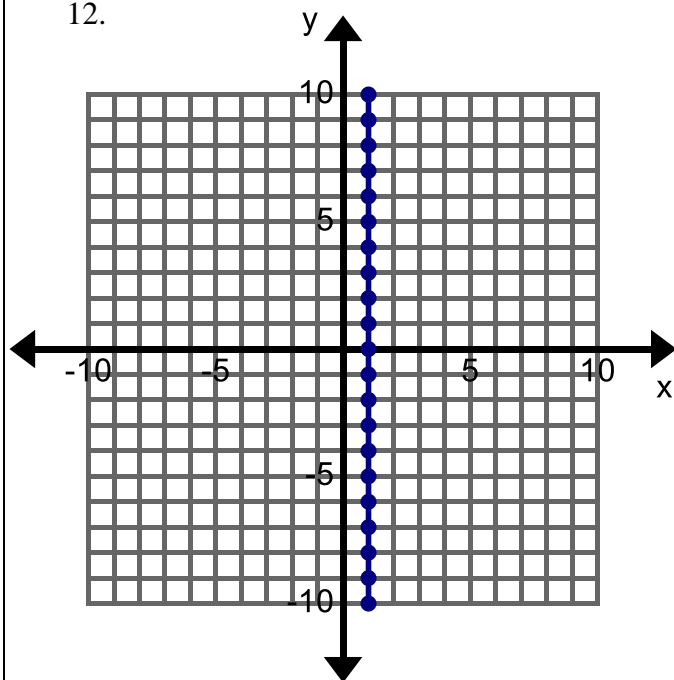
11.



$m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

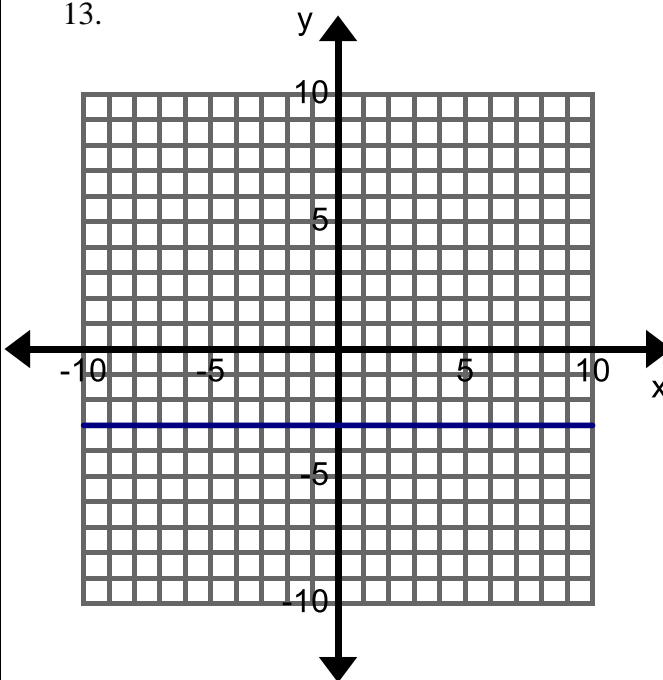
12.



$m$ : undefined  $b$ : does not cross

Equation:  $x = 1$

13.



$m$ : 0  $b$ : -3

Equation:  $y = -3$

14.

$x$	$y$
0	4
1	6
2	8
3	10

$m$ : 2  $b$ : 4

Equation:  $y = 2x + 4$

If students need help finding slope from 2 points, refer to 2.3e.

15.

$x$	$y$
-1	-1
0	-2
1	-3
2	-4

$m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

16.

$x$	$y$
-2	-1
0	0
2	1
4	2

$m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

17. You are going on a road trip with your family. You are already 30 miles into your trip and the speed limit is 75 miles per hour on the freeway. Let  $x$  be the number of hours from now and  $y$  be the total distance traveled.

$m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

18. A Basset Hound weighs 100 pounds and is on a special diet to lose 4 pounds per month. Let  $x$  represent the number of months passed and  $y$  the weight of the dog.

$m$ : -4  $b$ : 100

Equation:  $y = -4x + 100$

19. Your cell phone plan has a flat rate of \$30 each month. For each text you send it costs \$0.20. Let  $x$  represent the number of texts that you send and  $y$  your total monthly bill.

$m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

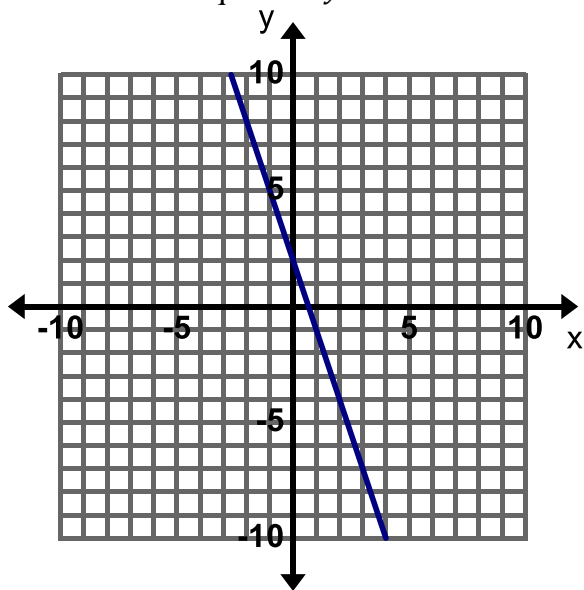
Equation:

**Find, Fix, and Justify:** In each of the following problems, a common mistake of writing the equation of a line



has been made. Describe the error and write the correct equation.

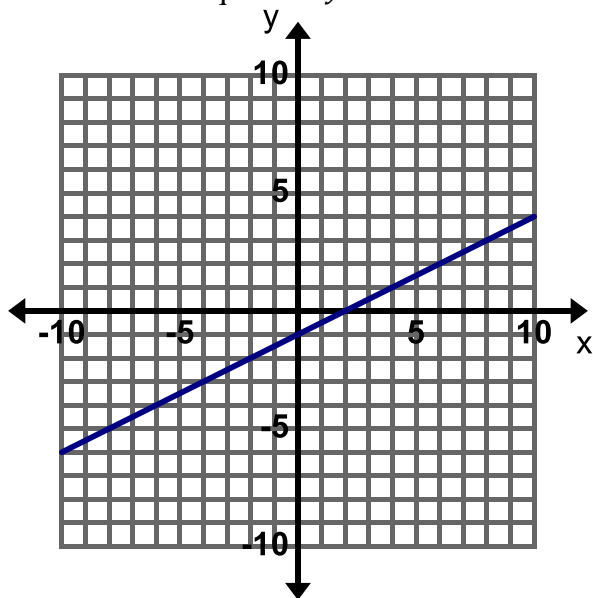
20. Incorrect Equation:  $y = 3x + 2$



Mistake: the slope should be negative 3

Correct Equation:  $y = -3x + 2$

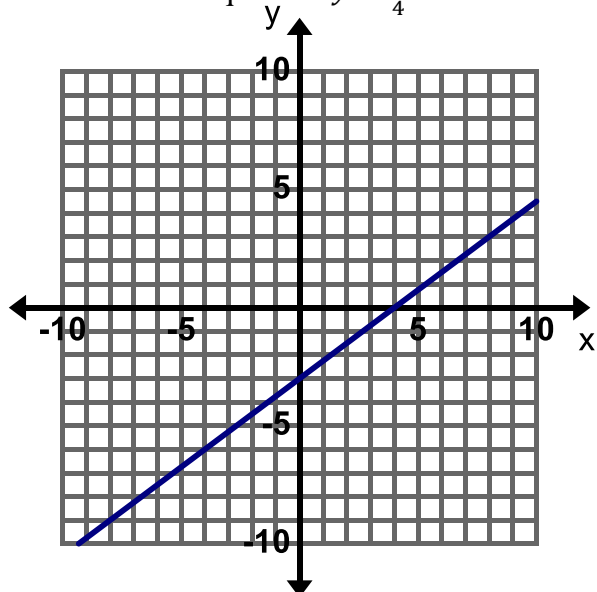
21. Incorrect Equation:  $y = 2x - 1$



Mistake:

Correct Equation:

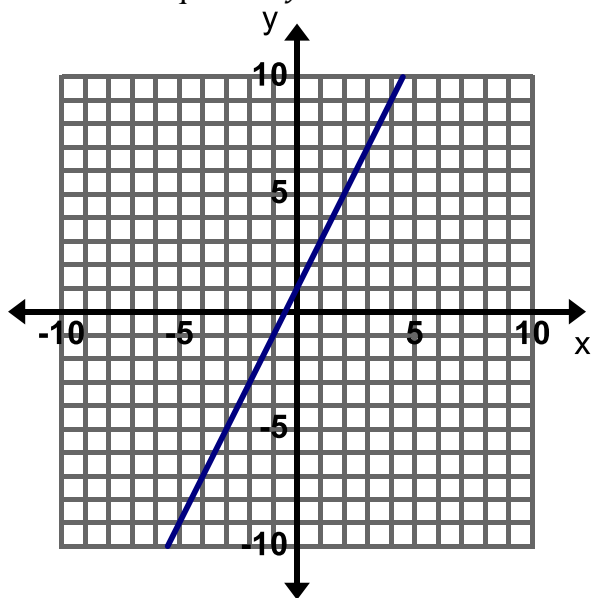
22. Incorrect Equation:  $y = \frac{3}{4}x + 4$



Mistake:

Correct Equation:

23. Incorrect Equation:  $y = x + 2$



Mistake:

Correct Equation:

### 3.1a Homework: Write Equations in Slope-Intercept Form

**Directions:** Write the equation of each line in slope-intercept form.

1. The slope of the line is 5. The y-intercept is  $(0, -1)$ .

$$y = 5x - 1 \text{ or } y = 5x + (-1)$$

↑ slope      ↓ y-intercept

2. The slope of the line is  $-1$ . The y-intercept is  $(0, -6)$ .

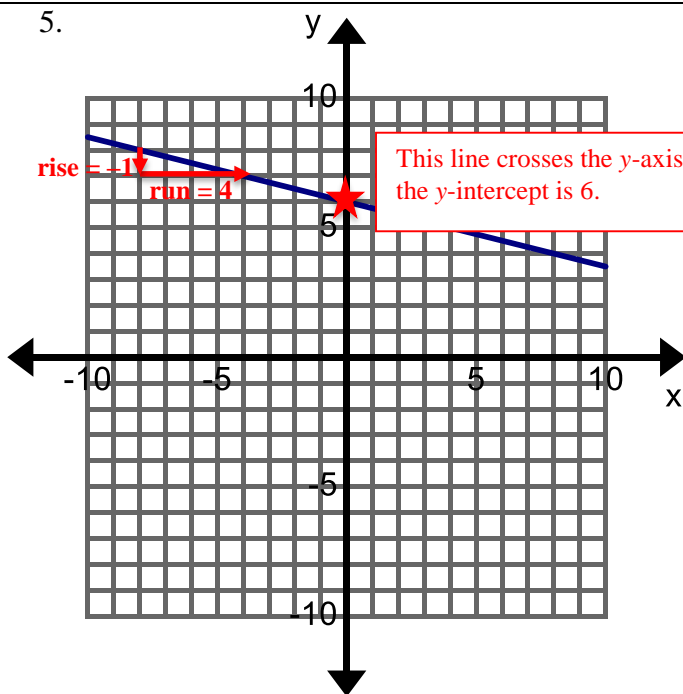
3. The slope of the line is  $\frac{1}{4}$ . The y-intercept is  $(0, 0)$ .

4. The slope of the line is  $-\frac{3}{5}$ . The y-intercept is  $(0, 10)$ .

$$y = -\frac{3}{5}x + 10$$

**Hint:** Slope-intercept form of the equation of a line is  $y = mx + b$  where  $m$  represents the slope of the line and  $b$  represents the y-intercept. In order to write the equation of a line, students must find the slope and y-intercept of the line. The slope of a line can be determined by calculating the  $\frac{\text{rise}}{\text{run}}$  (the vertical change over the horizontal change). The y-intercept is the point where the line crosses the y-axis ( $x = 0$ ).

5.

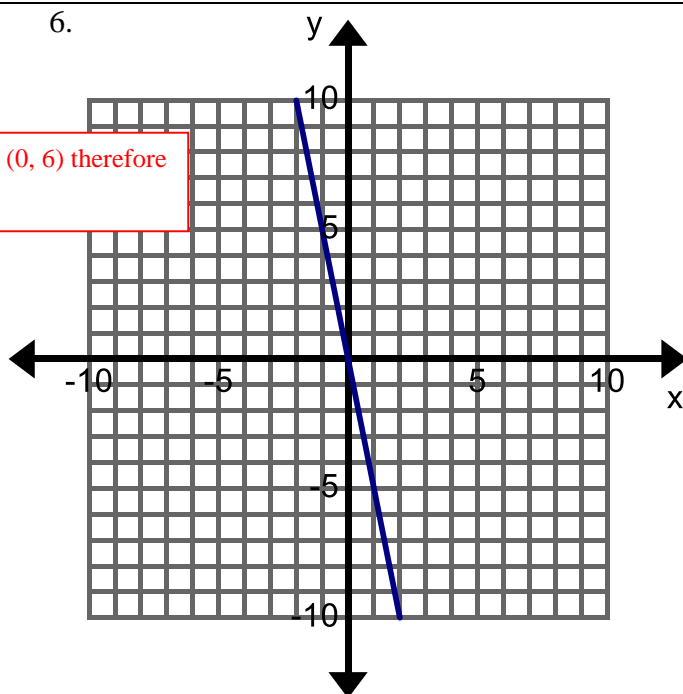


This line crosses the y-axis at  $(0, 6)$  therefore the y-intercept is 6.

$m$ :  $-\frac{1}{4}$      $b$ :  $6$

Equation:  $y = -\frac{1}{4}x + 6$

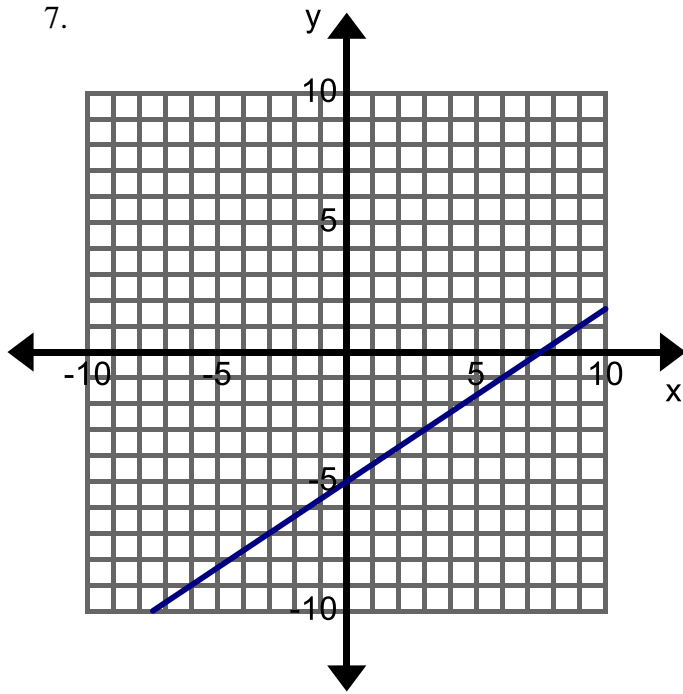
6.



$m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

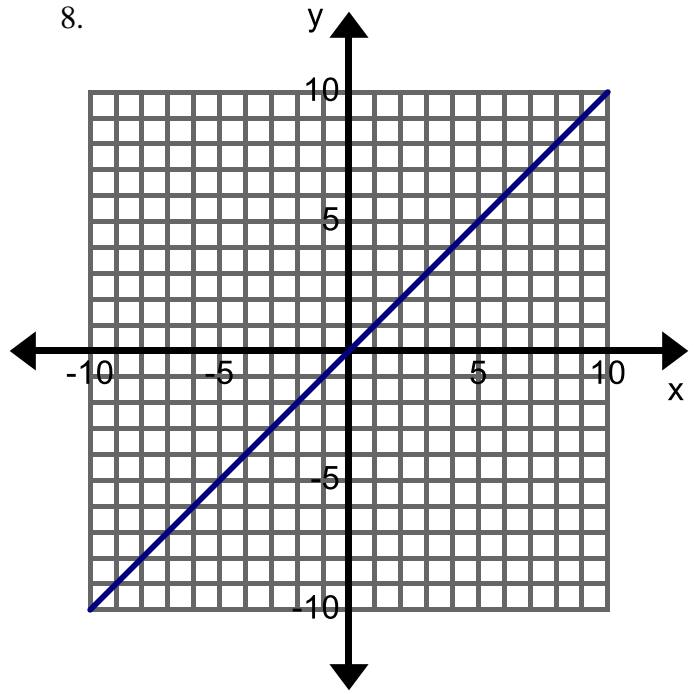
7.



$m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

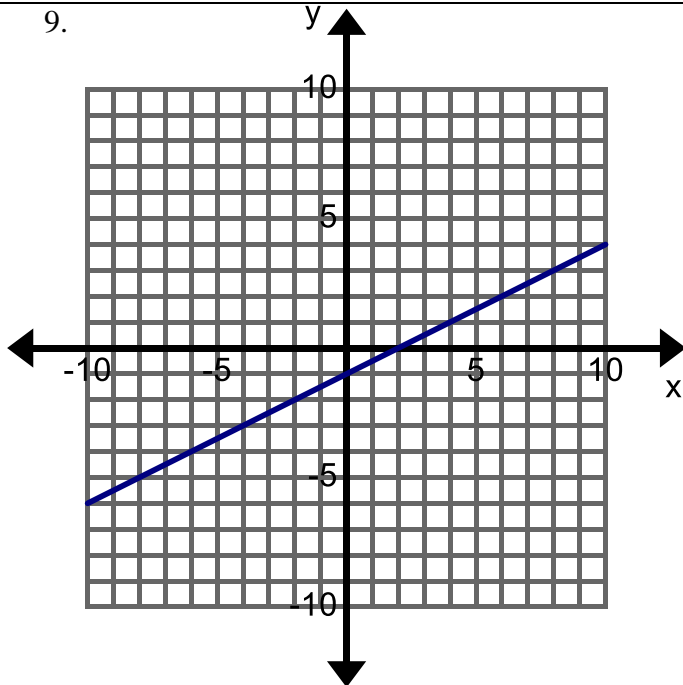
8.



$m$ : 1  $b$ : 0

Equation:  $y = x$

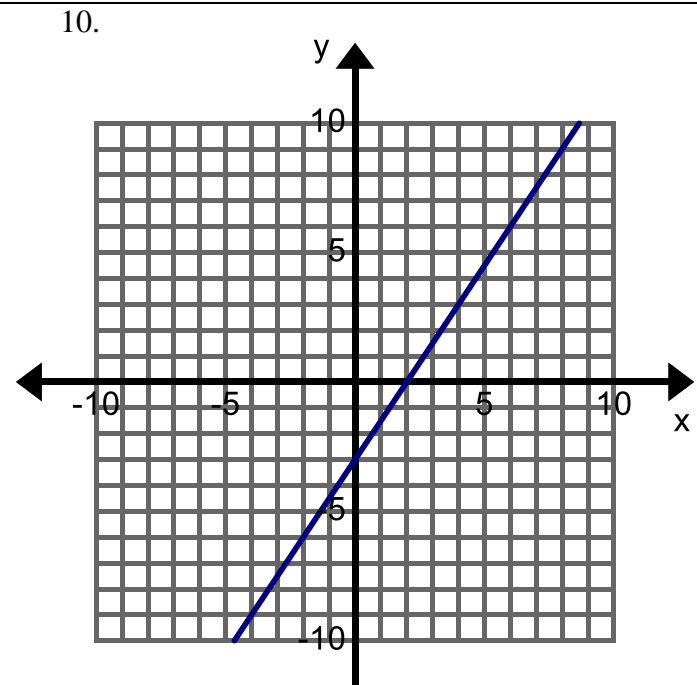
9.



$m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

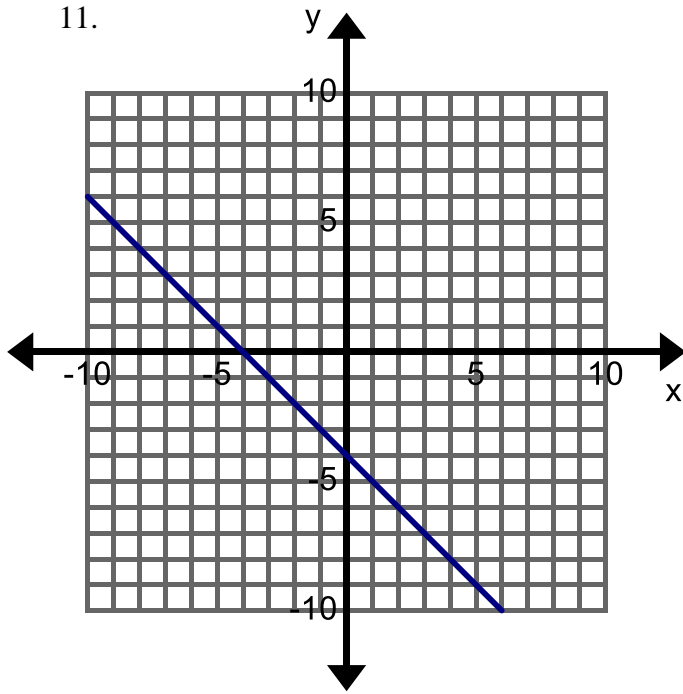
10.



$m$ :  $\frac{3}{2}$   $b$ : -3

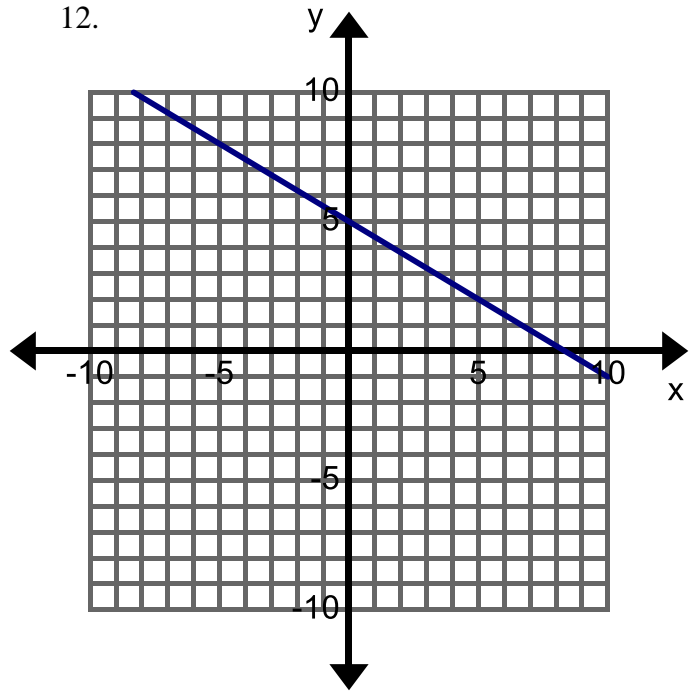
Equation:  $y = \frac{3}{2}x - 3$

11.


 $m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

12.


 $m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

13.

$x$	$y$
0	2
1	0
2	-2
3	-4

 $m$ : -2  $b$ : 2
Equation:  $y = -2x + 2$ 

**Hint:** To find the slope of a line from a table, choose any 2 points from the table. Either graph the points and determine the rise and run of the line or use the slope formula  $\frac{y_2 - y_1}{x_2 - x_1}$ . In #13, if we use the points

$(0, 2)$  and  $(1, 0)$ , we would have:  
 $\frac{0-2}{1-0} = \frac{-2}{1} = -2$ . The y-intercept of the graph is where  $x = 0$ .

$x$	$y$
-1	5
0	5
1	5
2	5

 $m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

16. You want to ship Science Textbooks from Florida. The textbooks cost \$60 each plus \$150 for shipping costs. Let  $x$  represent the number of textbooks shipped and  $y$  the total cost.

 $m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

17. Velma has \$450 in her checking account and withdraws \$25 each week. Let  $x$  represent the number of weeks that have past and  $y$  the total amount of money in her account.



 $m$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Equation:

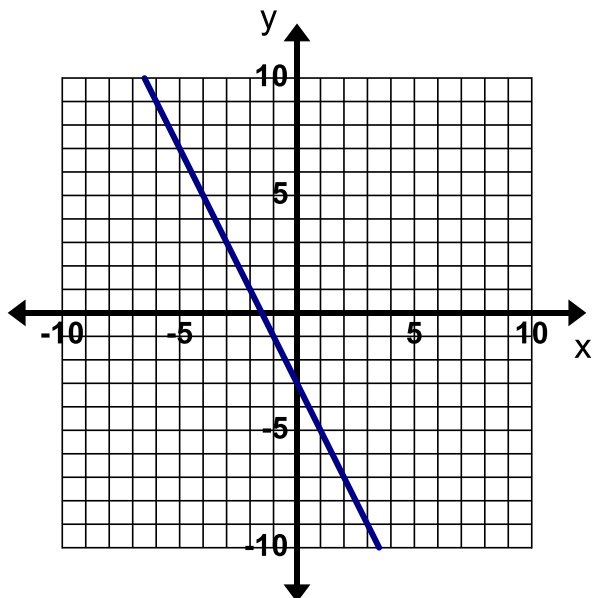
16. Jarius is 15 feet away from his car and walks toward it at a rate of 2 feet per second. Let  $x$  represent the number of seconds that have passed and  $y$  the distance away from the car.

 $m$ : -2  $b$ : 15
Equation:  $y = -2x + 15$



**Find, Fix, and Justify:** In each of the following problems, a common mistake of writing the equation of a line has been made. Describe the error and write the correct equation.  

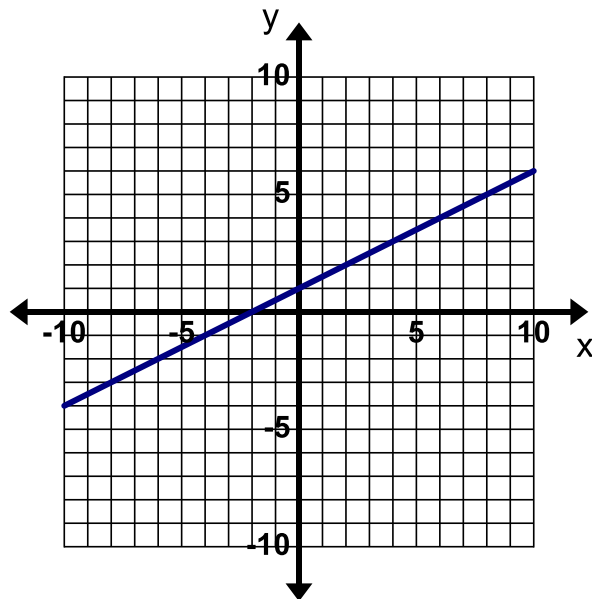
20. Incorrect Equation:  $y = -2x + 3$



Mistake:

Correct Equation:

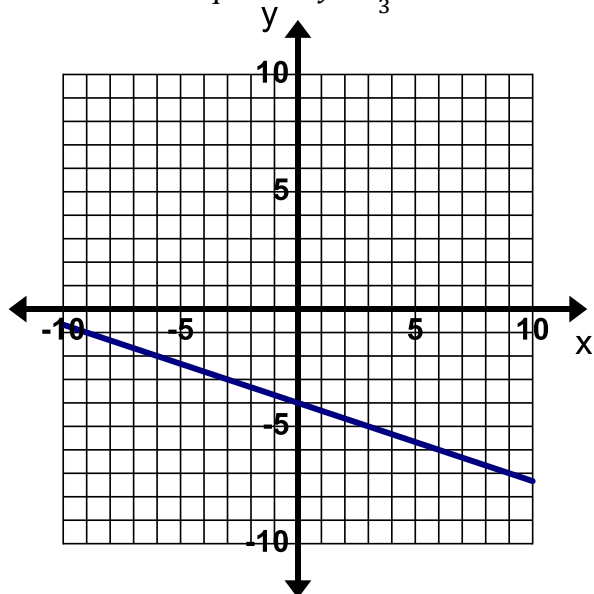
21. Incorrect Equation:  $y = x + \frac{1}{2}$



Mistake: **interchanged  $m$  and  $b$  in the equation**

Correct Equation:  **$y = \frac{1}{2}x + 1$**

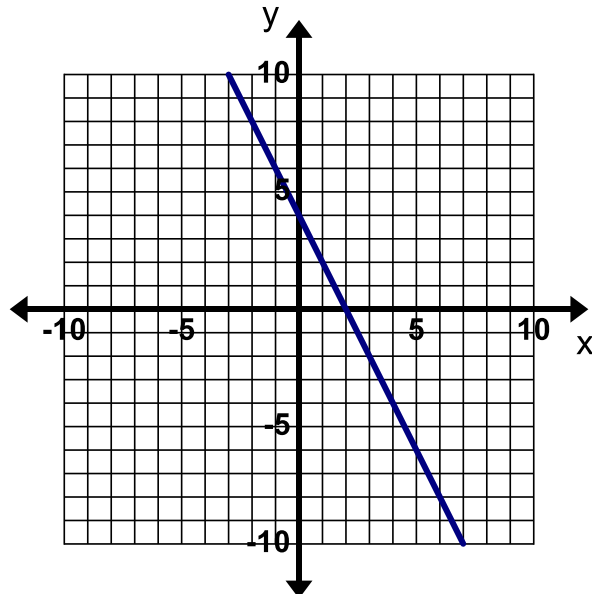
22. Incorrect Equation:  $y = \frac{1}{3}x - 4$



Mistake:

Correct Equation:

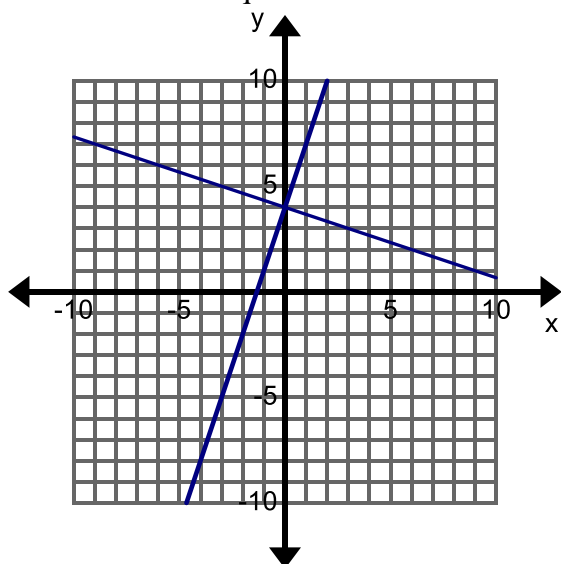
23. Incorrect Equation:  $y = -2x + 2$



Mistake:

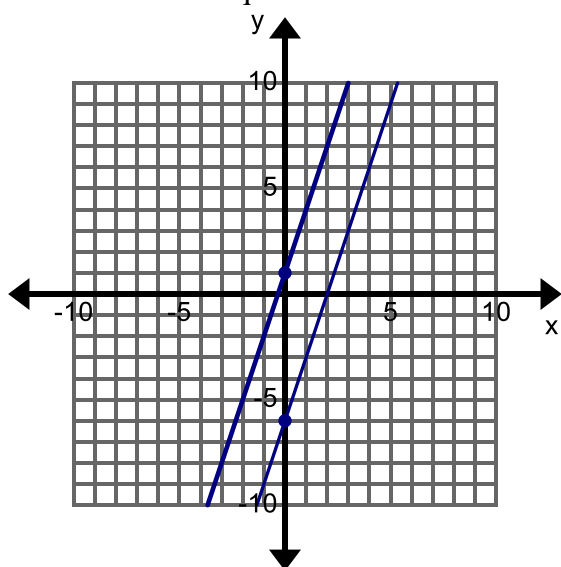
Correct Equation:

24. Write the equation of each line in the graph below. Label each line with its equation.



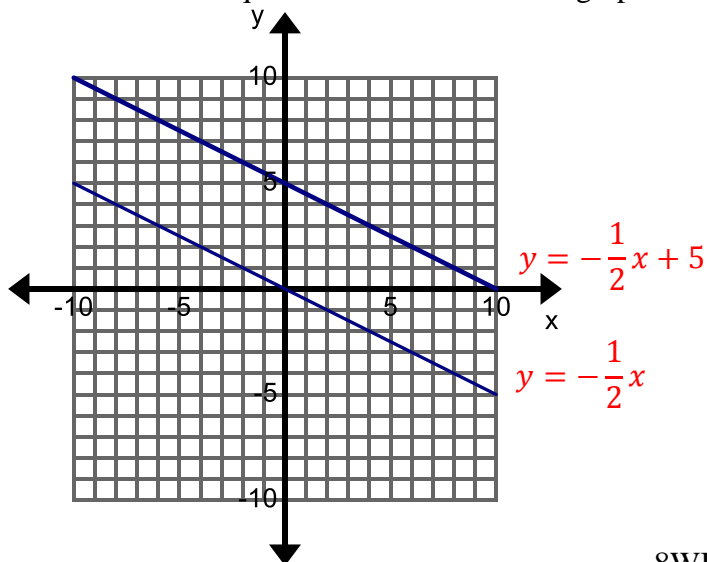
Compare the two lines. What is the same?  
What is different?

25. Write the equation of each line in the graph below. Label each line with its equation.



Compare the two lines. What is the same?  
What is different?

26. Write the equation of each line in the graph below. Label each line with its equation.



Compare the two lines. What is the same?  
What is different?

The lines have the same slope but different  $y$ -intercepts. Some students may point out that the lines are parallel – this will be studied in more detail in the next section.

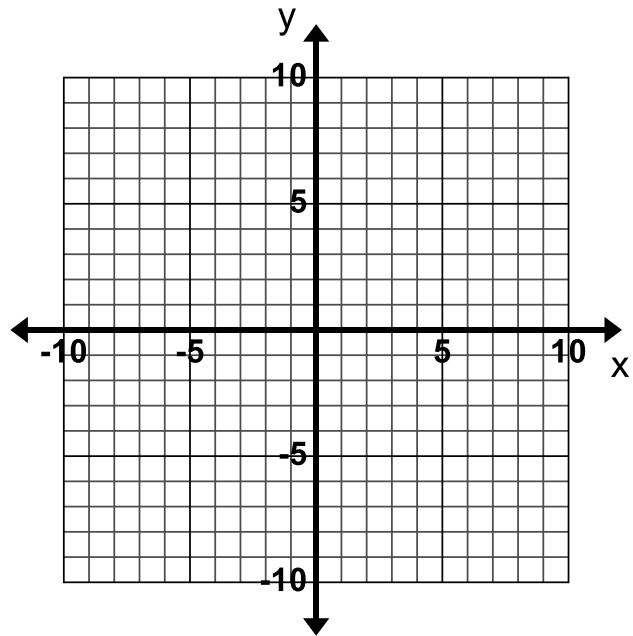
### 3.1b Classwork: Graph from Slope-Intercept Form

1. On the following coordinate plane, draw a line with a slope of  $\frac{1}{3}$ .

a. How do you know that your line has a slope

of  $\frac{1}{3}$ ?

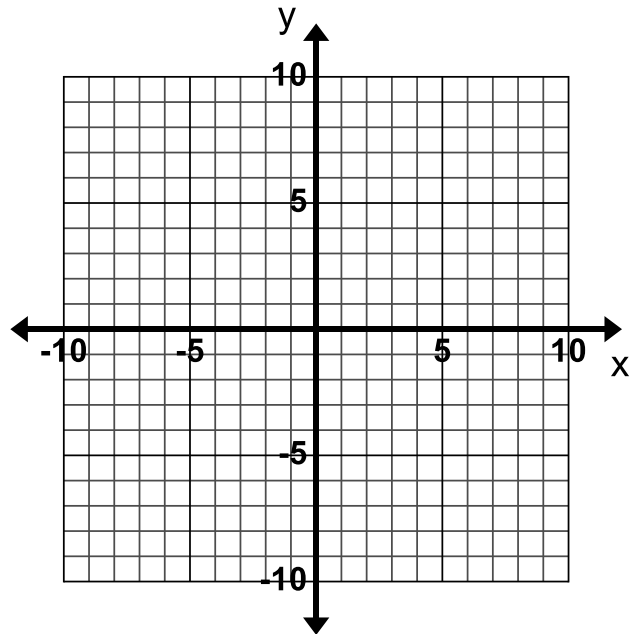
b. What did you do to draw your line to ensure that you ended with a slope of  $\frac{1}{3}$ ?



2. Draw a line with a slope of -2.

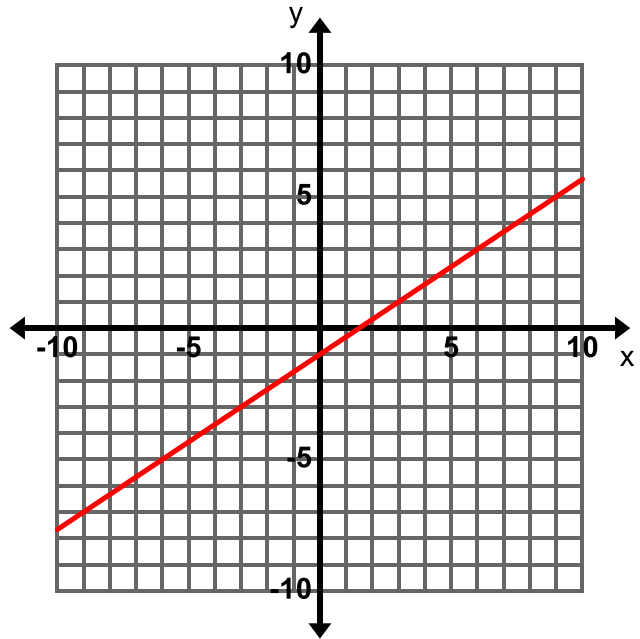
a. How do you know that your line has a slope of -2?

b. What did you do to draw your line to ensure that you ended with a slope of -2?



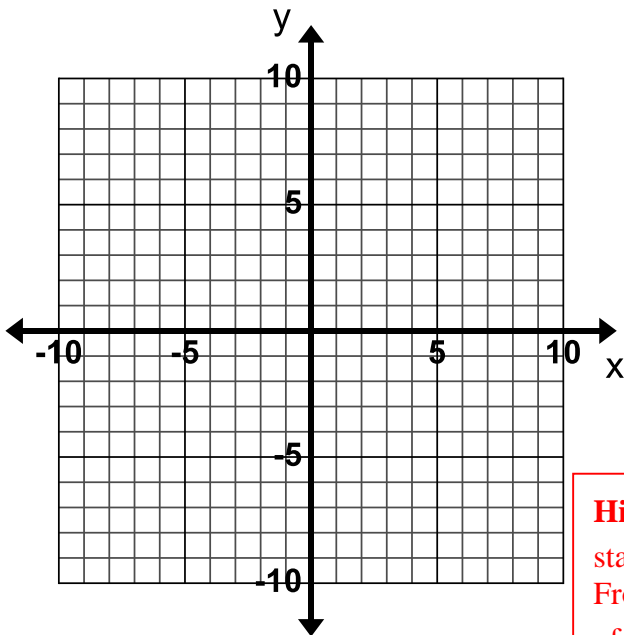
3. Consider the following equation  $y = \frac{2}{3}x - 1$ .

- What is the y-intercept?  $(0, -1)$
- What is the slope?  $\frac{2}{3}$
- Graph the line on the grid to the right by first plotting the y-intercept and then drawing a line with the slope that goes through the y-intercept. Use what you wrote in the first two questions to help you.



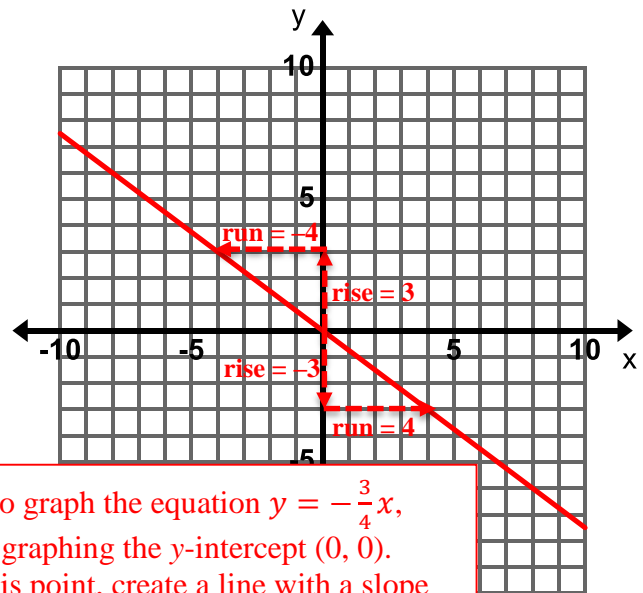
4. Graph  $y = 3x + 1$

- What is the y-intercept?
- What is the slope?



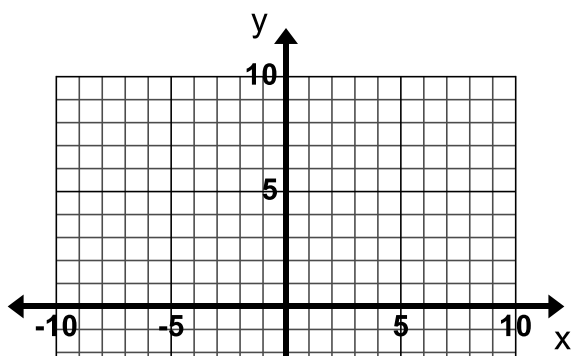
5. Graph  $y = -\frac{3}{4}x$

- What is the y-intercept?  $(0, 0)$
- What is the slope?  $-\frac{3}{4}$



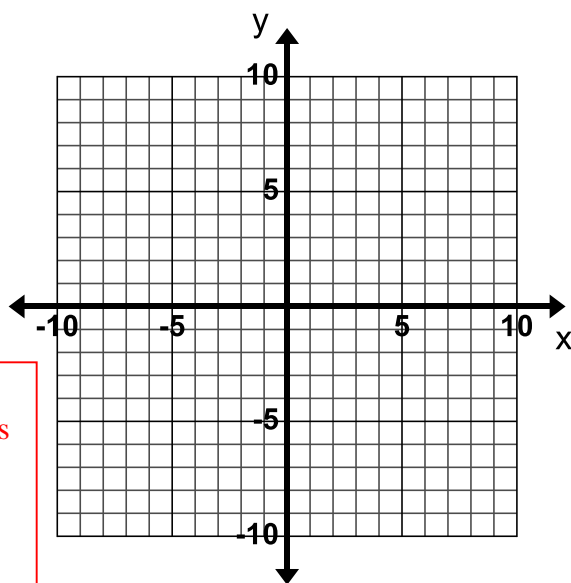
**Hint:** To graph the equation  $y = -\frac{3}{4}x$ , start by graphing the y-intercept  $(0, 0)$ . From this point, create a line with a slope of  $-\frac{3}{4}$ .

6. Graph  $y = -2 + x$

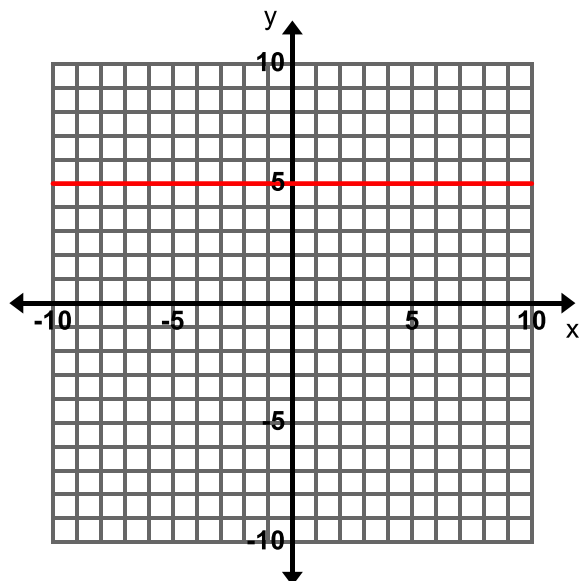


**Hint:** Remember when an equation is written in slope-intercept form, the slope is the number in front of the  $x$ . In #6, what number is in front of the  $x$ ?

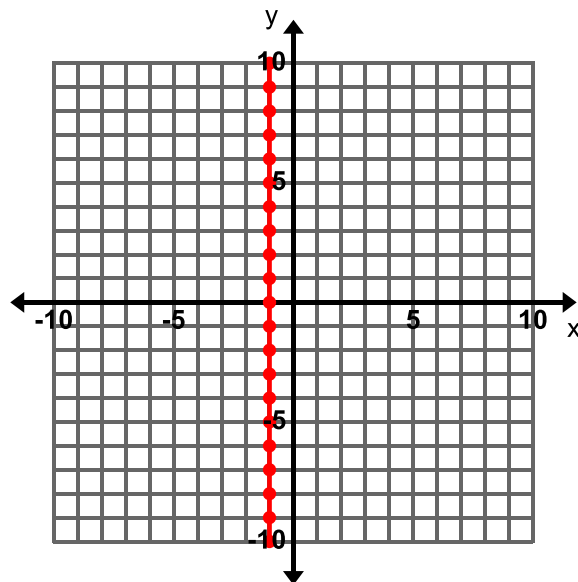
7. Graph  $y = -2x + 3$



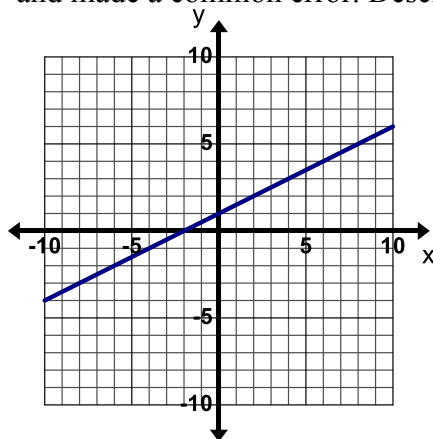
8. Graph  $y = 5$



9. Graph  $x = -1$

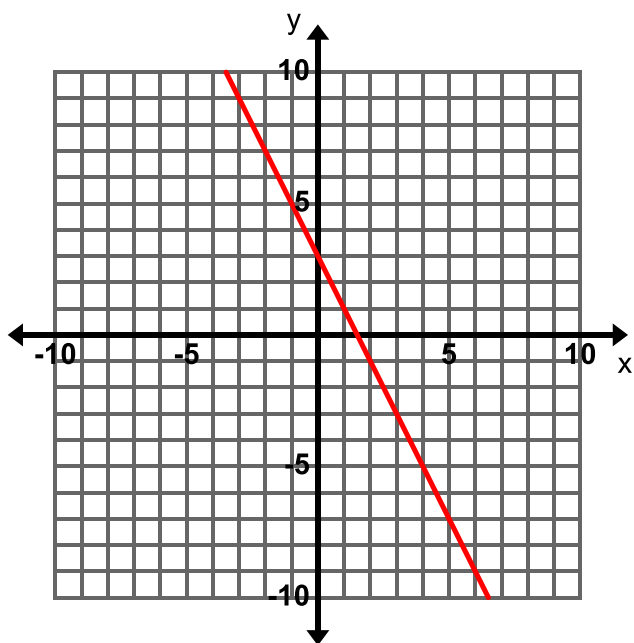


10. **Find, Fix, and Justify:** Kevin was asked to graph the line  $y = -\frac{1}{2}x + 1$ . Kevin graphed the line below and made a common error. Describe Kevin's error and then graph the line correctly on the grid.

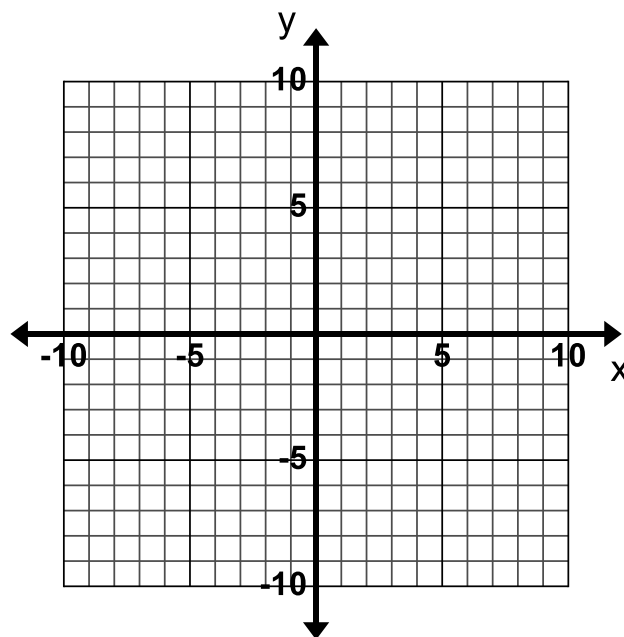


### 3.1b Homework: Graph from Slope-Intercept Form

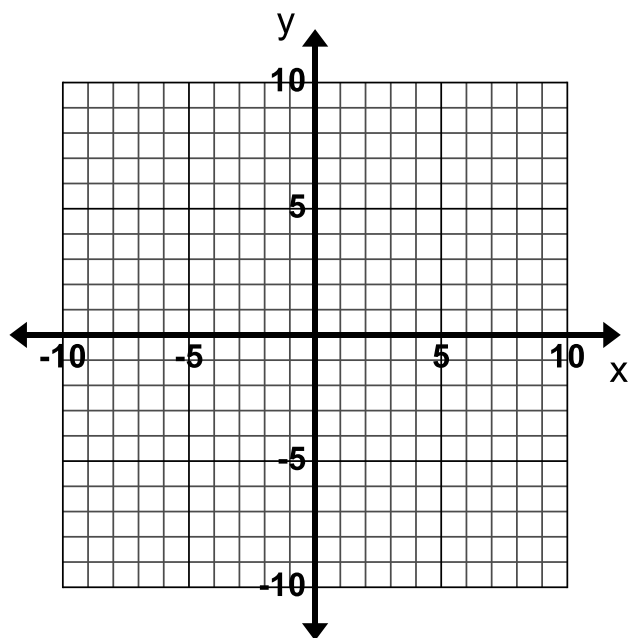
1. Graph  $y = -2x + 3$



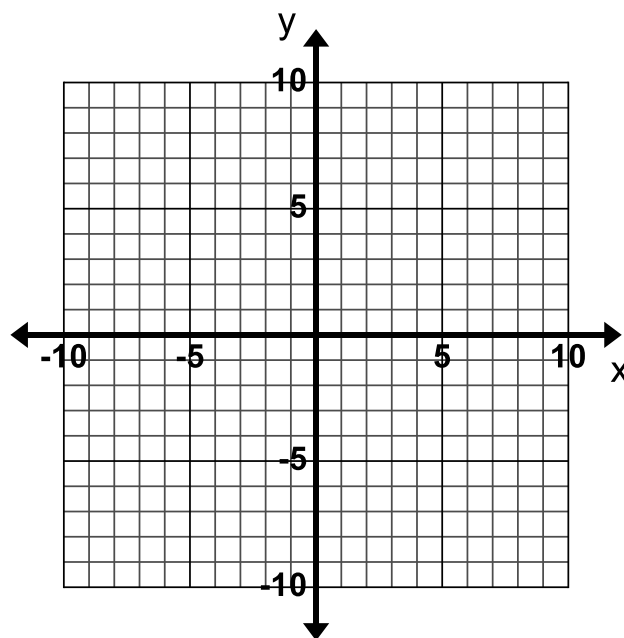
2. Graph  $y = 4x - 3$



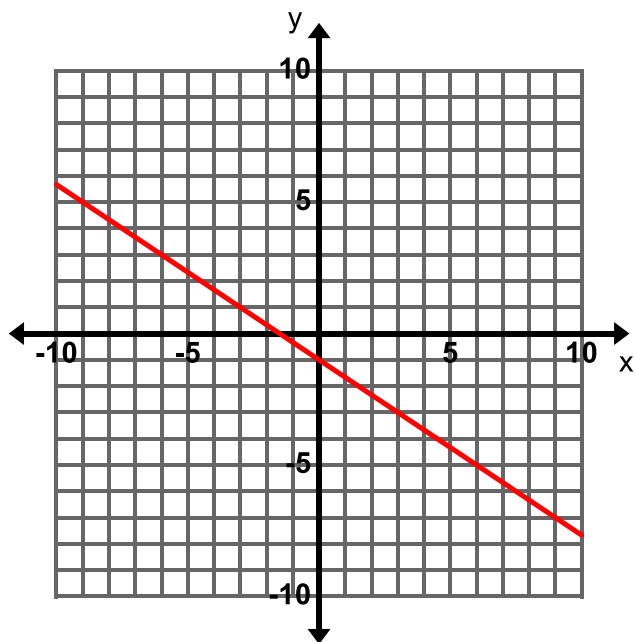
3. Graph  $y = -4x$



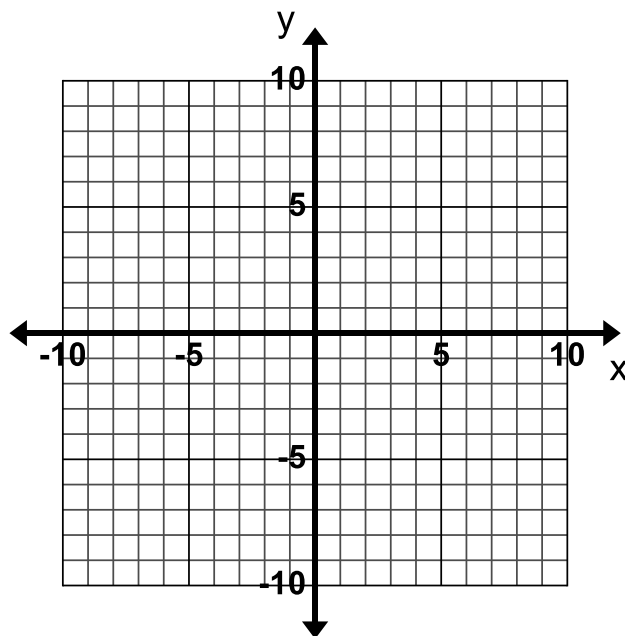
4. Graph  $y = 5 + \frac{2}{3}x$



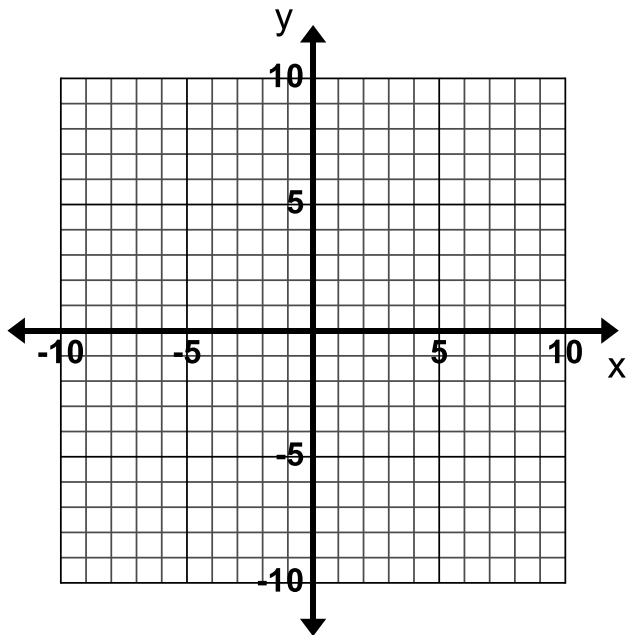
5. Graph  $y = -\frac{2}{3}x - 1$



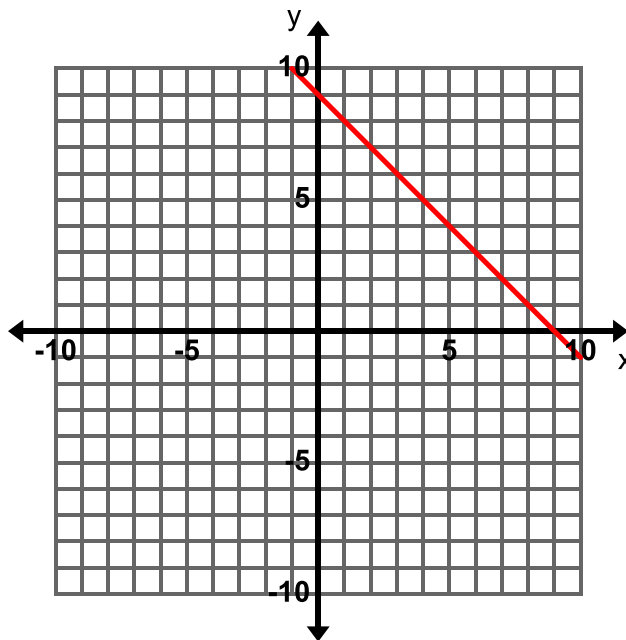
6. Graph  $y = 3 + x$



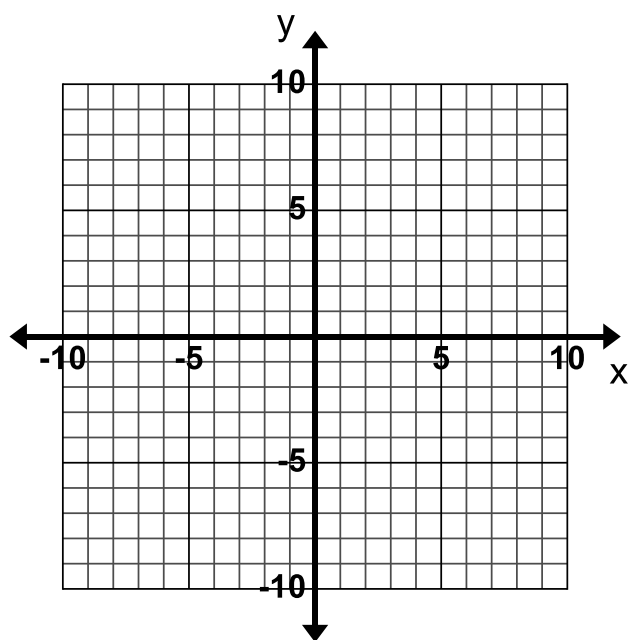
7. Graph  $y = 2x - 9$



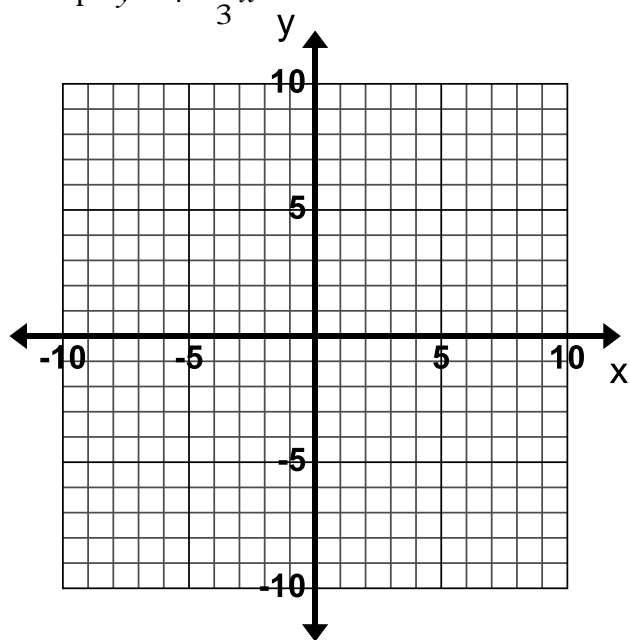
8. Graph  $y = 9 - x$



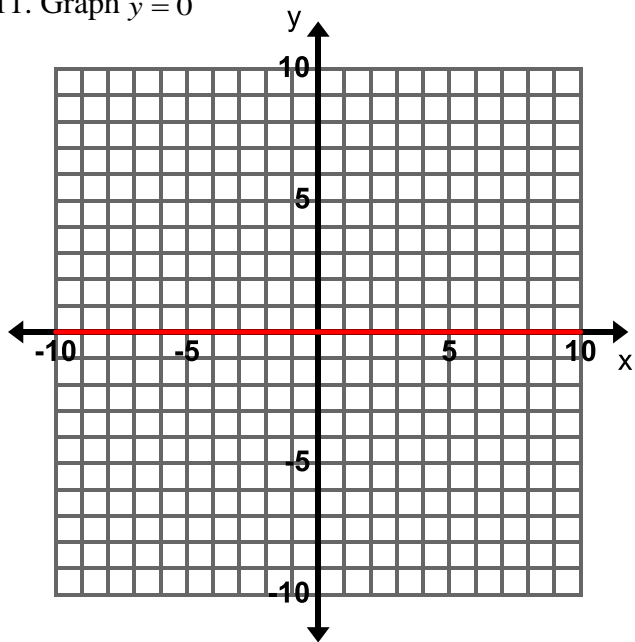
9. Graph  $y = 3x - 2$



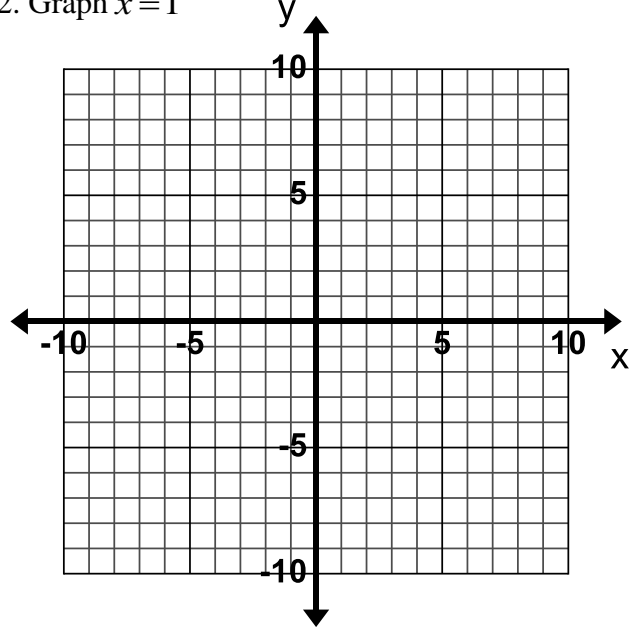
10. Graph  $y = 4 - \frac{1}{3}x$



11. Graph  $y = 0$

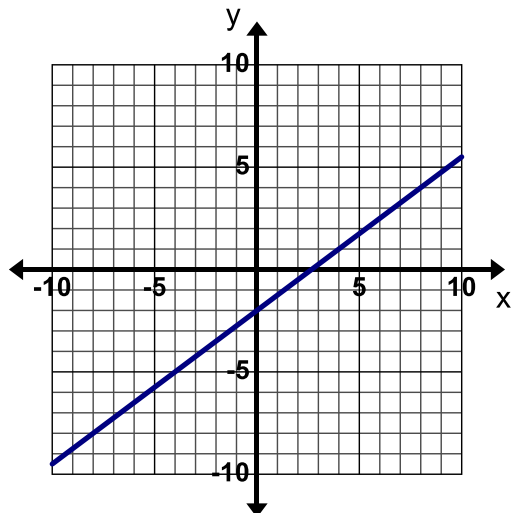


12. Graph  $x = 1$

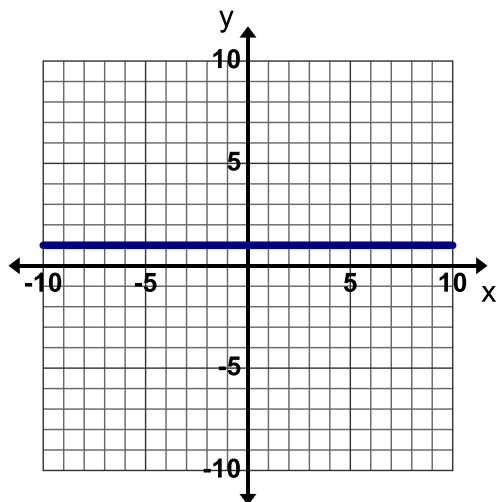




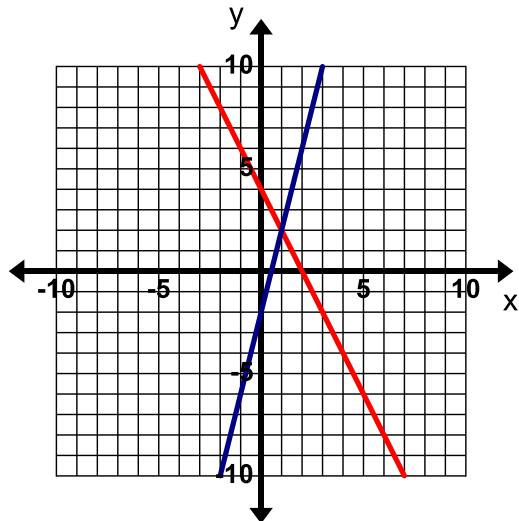
13. **Find, Fix, and Justify:** Lani was asked to graph the line  $y = \frac{4}{3}x - 2$ . Lani graphed the line below and made a common error. Describe Lani's error and then graph the line correctly on the grid.



14. **Find, Fix, and Justify:** Janeen was asked to graph the line  $x = 1$ . Janeen graphed the line below and made a common error. Describe Janeen's error and then graph the line correctly on the grid.



15. **Find, Fix, and Justify:** Zach was asked to graph the line  $y = 4 - 2x$ . Zach graphed the line below and made a common error. Describe Zach's error and then graph the line correctly on the grid.



Zach mixed up the slope and y-intercept in the equation. He graphed the line with a slope of 4 and a y-intercept of -2.

### 3.1c Class Activity: Write and Graph in Slope-Intercept Form



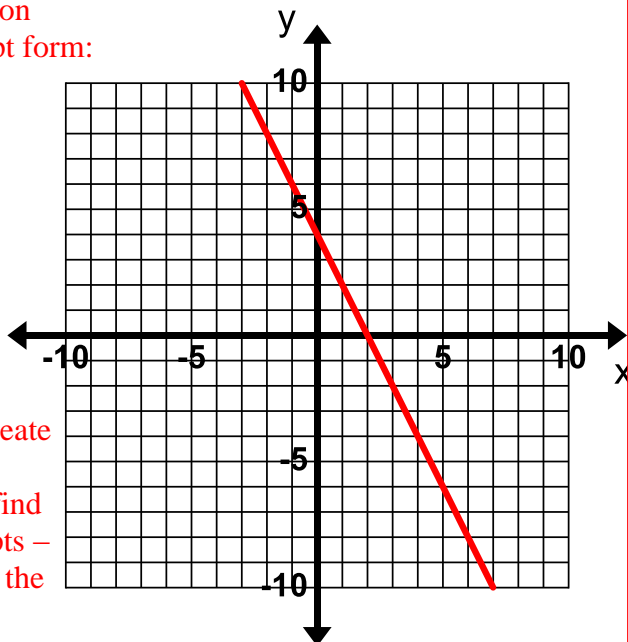
Graph the equation given below. Be ready to discuss your ideas with the class.

One way to graph this equation is to put it into slope-intercept form:

$$\begin{aligned} 4x + 2y &= 8 \\ 4x + 2y &= 8 \\ -4x &= -4x \\ \frac{2y}{2} &= \frac{8-4x}{2} \\ y &= 4-2x \\ y &= -2x+4 \end{aligned}$$

Graph the point (0, 4) and create a line with a slope of -2.

Alternatively, students may find and plot the x- and y-intercepts – see explanation in the box to the right.



Graphing with x- and y-intercepts:

The x-intercept is the point where the line crosses the x-axis. At this point,  $y = 0$ . To find the x-intercept, substitute in 0 for y in the equation:

$$\begin{aligned} 4x + 2y &= 8 \\ 4x + 2(0) &= 8 \\ 4x + 0 &= 8 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

The x-intercept is (2, 0).

The y-intercept is the point where the line crosses the y-axis. At this point,  $x = 0$ .

Substitute in 0 for x in the original equation and solve. The y-intercept is (0, 4).

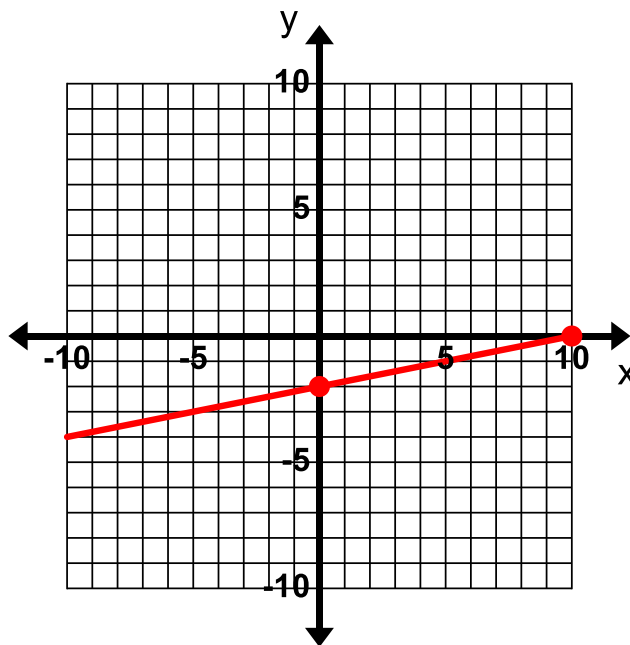
Write down how to graph an equation that is not written in Slope-Intercept Form in the space below.

To graph an equation that is not in slope-intercept form; change it to slope-intercept form by solving the equation for y. You can also find the x- and y- intercepts and connect them with a line – see explanation above.

1. Graph  $x - 5y = 10$

$$\begin{aligned} x - 5y &= 10 \\ -x &= -x \\ \frac{-5y}{-5} &= \frac{10-x}{-5} \\ y &= -2 + \frac{1}{5}x \\ y &= \frac{1}{5}x - 2 \end{aligned}$$

The x-intercept is (10, 0) and the y-intercept is (0, -2).



2. Graph  $3x + 2y = -6$

$$3x + 2y = -6$$

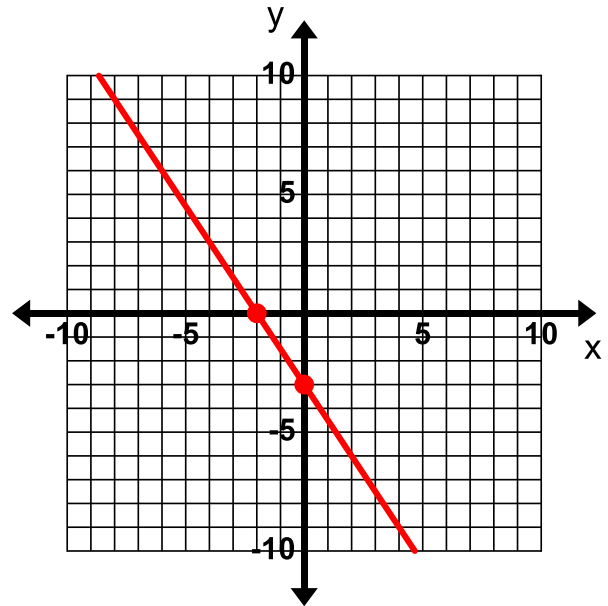
$$\underline{-3x = -3x} \quad \text{Subtract } 3x \text{ from both sides.}$$

$$\frac{2y}{2} = \frac{-6-3x}{2} \quad \text{Divide both sides by 2.}$$

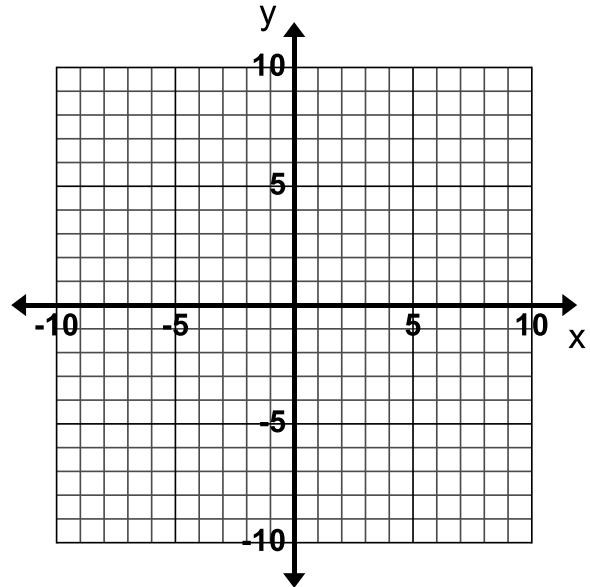
$$y = -3 - \frac{3}{2}x \quad \text{Simplify.}$$

$$y = -\frac{3}{2}x - 3 \quad \text{Write in slope-intercept form.}$$

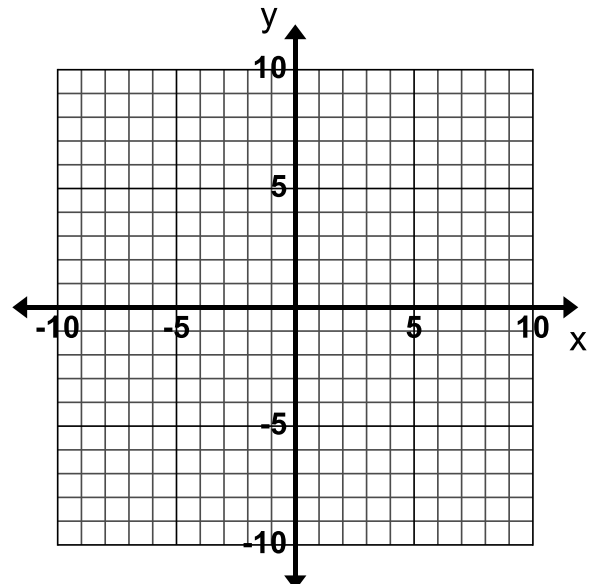
Plot the y-intercept  $(0, -3)$ . From this point, create a line with a slope of  $-\frac{3}{2}$  (go up 3 and to the left 2 or go down 3 and to the right 2).



3. Graph  $4x + 8y = -24$



4. Graph  $3x + y = 9$



5. Graph  $5x - y = -5$

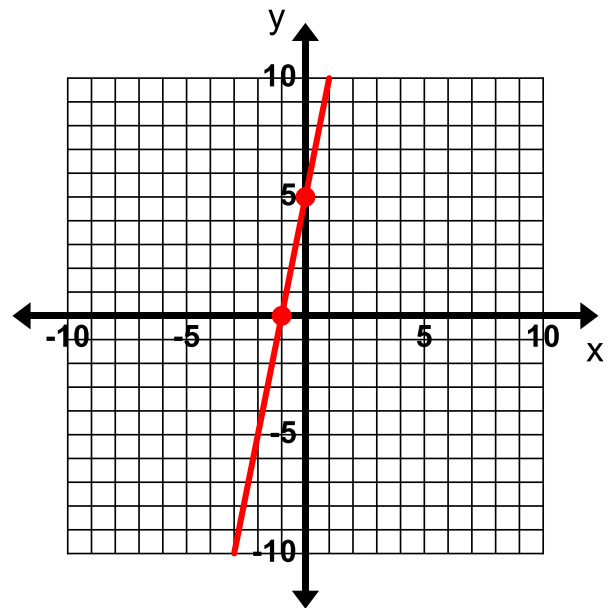
$5x - y = -5$  Subtract  $5x$  from both sides.

$-5x = -5x$

$\frac{-y}{-1} = \frac{-5-5x}{-1}$  Divide both sides by  $-1$ .

$y = 5 + 5x$

$y = 5x + 5$  Write in slope-intercept form.



6. Graph  $x + 2y = -8 + x$

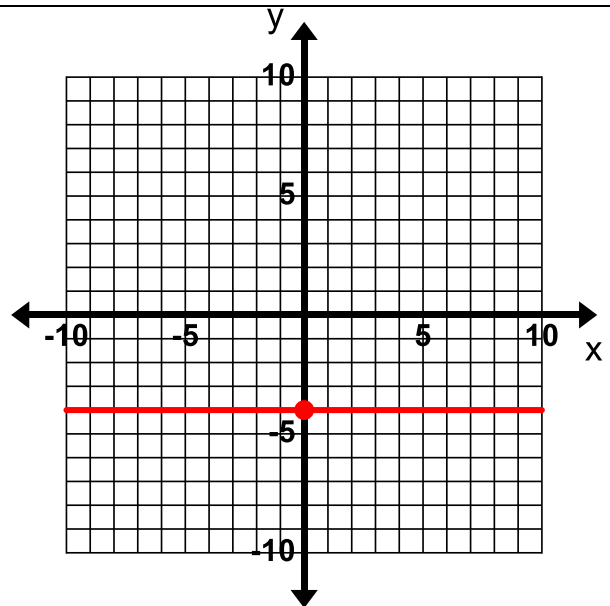
$x + 2y = -8 + x$

$-x = -x$

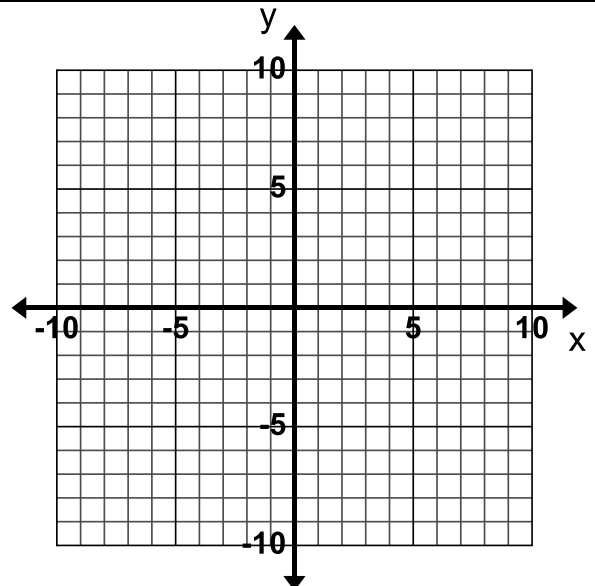
$\frac{2y}{2} = \frac{-8}{2}$

$y = -4$

There is not an  $x$ -intercept and the  $y$ -intercept is  $(0, -4)$ .



7. Graph  $2(y - x) = 2y - 14$



8. A Health Teacher is writing a test with two sections. The entire test is worth 40 points. He wants the questions in Section A to be worth 2 points each and the questions in Section B to be worth 4 points each. Let  $x$  represent the number of questions in Section A and  $y$  represent the number of questions in Section B.

- a. Write an equation that describes all the different combinations of number of questions in Section A and B.

$$2x + 4y = 40$$

- b. Graph this equation to show all possible numbering outcomes for this test.

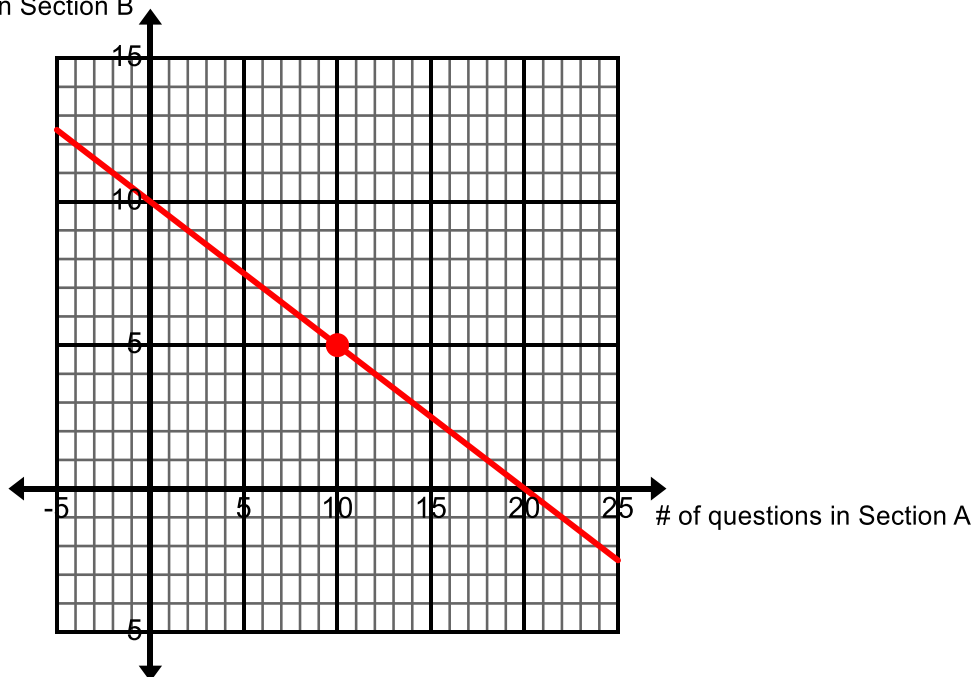
$$2x + 4y = 40 \text{ \# questions in Section B}$$

$$\frac{-2x}{4} = \frac{-2x}{4}$$

$$\frac{4y}{4} = \frac{40 - 2x}{4}$$

$$y = 10 - \frac{1}{2}x$$

$$y = -\frac{1}{2}x + 10$$



- c. Highlight an ordered pair that falls on the line and explain what it represents.

Each ordered pair represents the possible number of questions in section A and section B of the test. The ordered pair (10,5) on the graph means that if there are 10 questions in Section A then there will be 5 questions in section B.

Use the graph to answer the following questions.

- d. If you have 16 questions in Section A of the test how many questions will be in Section B of the test?

If there are 16 questions in Section A then there will be 2 questions in section B. This is because for an  $x$  value of 16 the corresponding  $y$ -value is 2.

- e. If you have 8 questions worth 4 points each, how many questions will be worth 2 points each?

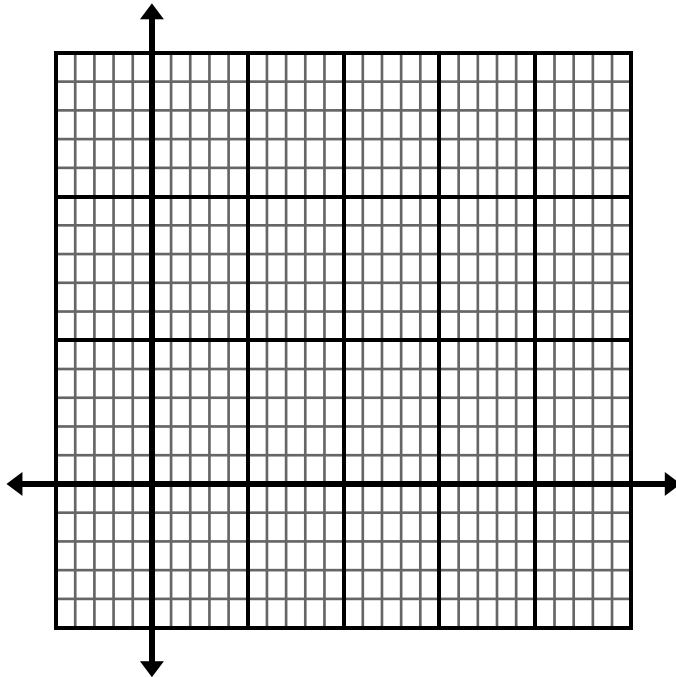
If there are 8 questions worth 4 points each then there will be 4 questions worth 2 points each. This is because for the  $y$ -value of 8 the corresponding  $x$ -value is 4.

- f. Is it realistic for there to be 9 questions in Section A on the test? Explain your answer.

It is not realistic for there to be 9 questions in Section A because the corresponding  $y$ -value is 5.5. This means that there would need to be  $5\frac{1}{2}$  questions in Section B. You can't have half of a question.

9. The Hernandez family wants to eat out on Monday night. Salads cost \$8.00 each and sandwiches cost \$6.00 each. They have a gift card for \$42 and want to spend all of it. Let  $x$  represent the number of salads that the family can buy and  $y$  represent the number of sandwiches that they can buy.

- Write an equation that represents all the possible combinations of salads and sandwiches that they can buy with \$42.
- Graph this equation to show all the different salad and sandwich combinations.



- What do the ordered pairs on the graph represent?
- List the realistic combinations for the order. Mark the ordered pairs on the graph that represent these combinations. Explain why these are the only solutions that would work in the real world.



10. The difference between Eugene's age and Wyatt's age's is 5 years. Eugene is older than Wyatt. Let  $x$  represent Eugene's age and  $y$  represent Wyatt's age.

- a. Write an equation that represents all the possible different ages that Eugene and Wyatt can be.

$$x - y = 5$$

- b. Graph this equation to show all the age combinations.

$$x - y = 5$$

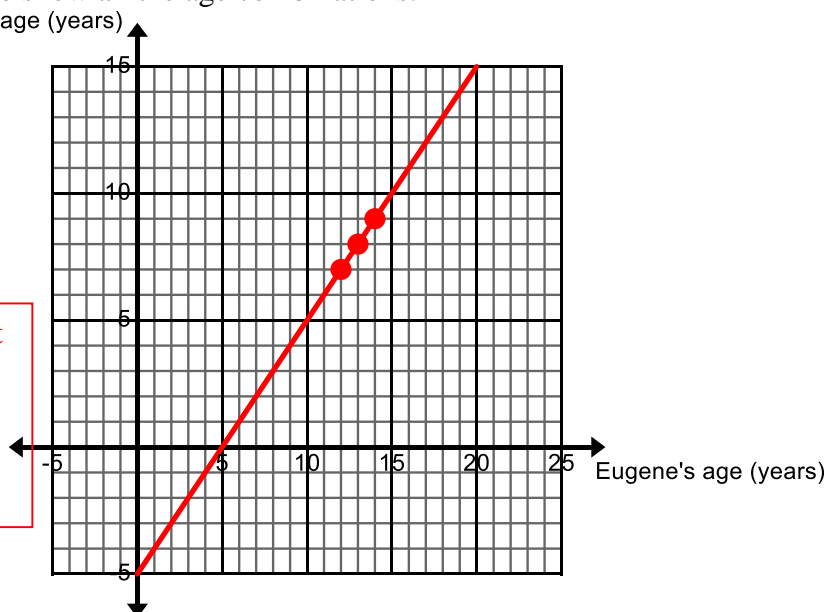
$$-x = -x$$

$$\frac{-y}{-1} = \frac{5-x}{-1}$$

$$y = -5 + x$$

$$y = x - 5$$

It is worthwhile to point out that in part *b* you can also solve the equation by adding  $y$  to both sides and subtracting 5 from both sides.



- c. Mark an ordered pair on the graph that represents the ages of Eugene and Wyatt if Eugene is the same age as you.

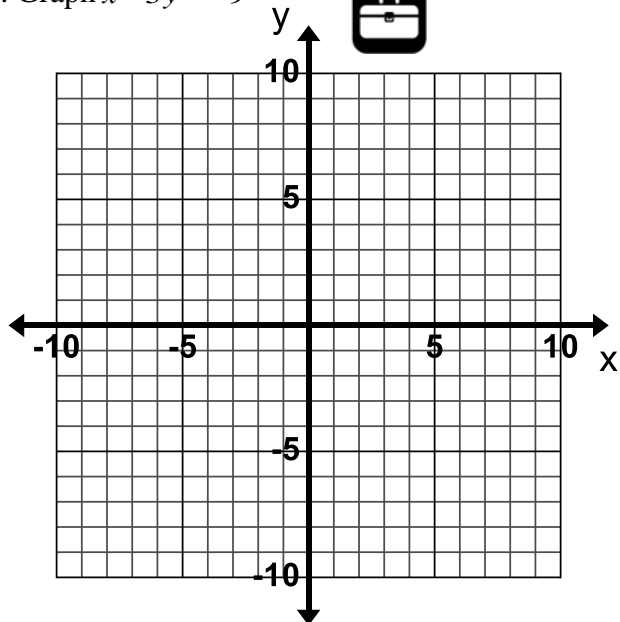
Possible answers are shown.

- d. Why does the graph only include mainly the first quadrant?

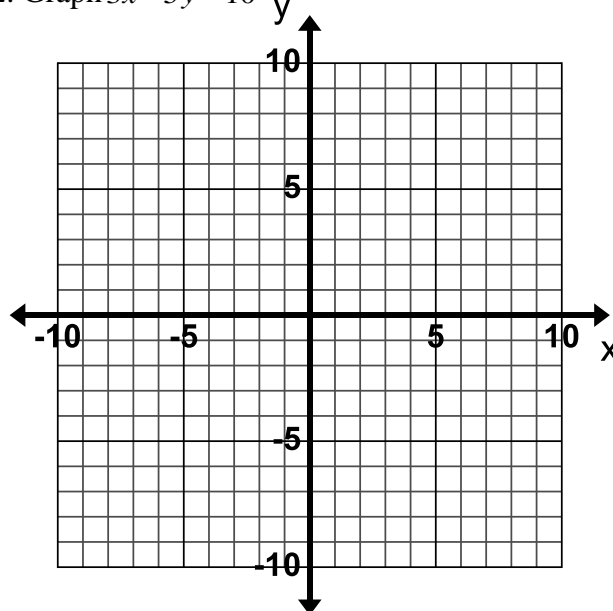
The other quadrants would include negative numbers, it is impossible to have a negative age.

Graph each equation first by hand and then use a graphing calculator to check your line.

11. Graph  $x - 3y = -9$

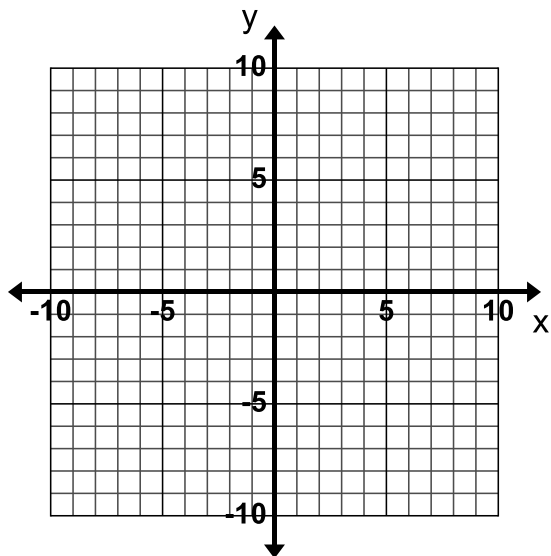


12. Graph  $3x - 5y = 10$



### 3.1c Homework: Write and Graph in Slope-Intercept Form

1. Graph  $x + y = 4$



2. Graph  $2x + 3y = -12$

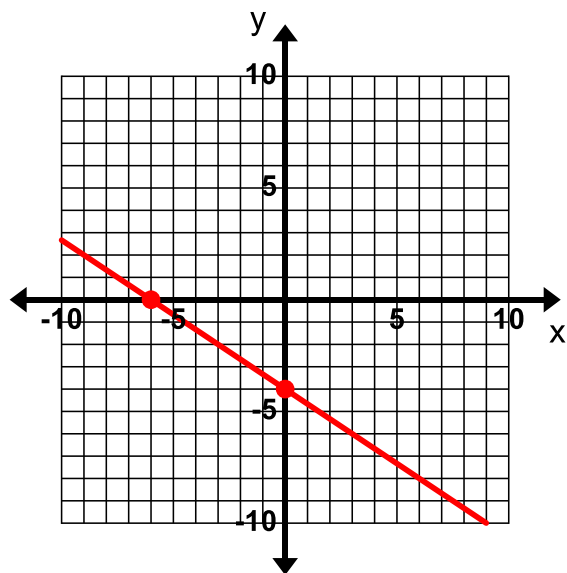
$$2x + 3y = -12$$

$$\underline{-2x = -2x}$$

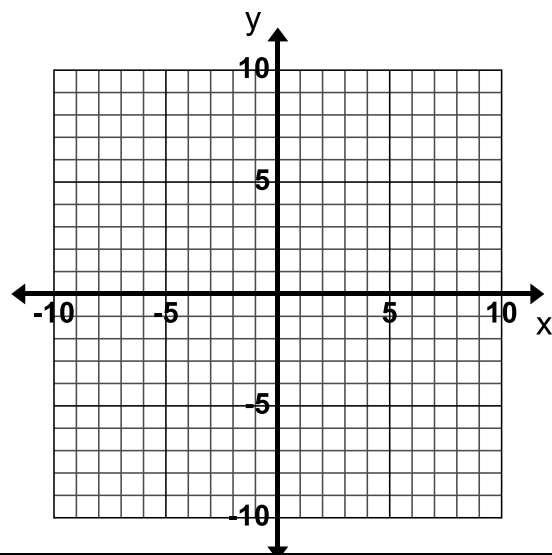
$$\frac{3y}{3} = \frac{-12 - 2x}{3}$$

$$y = -4 - \frac{2}{3}x$$

$$y = -\frac{2}{3}x - 4$$

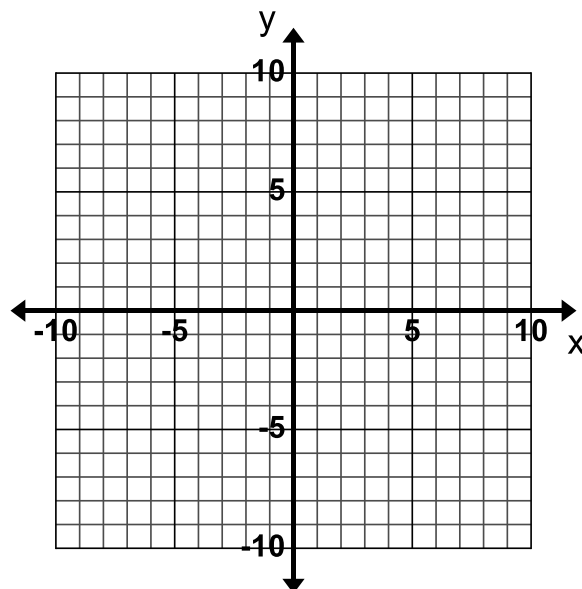


3. Graph  $-2x + y = 3$

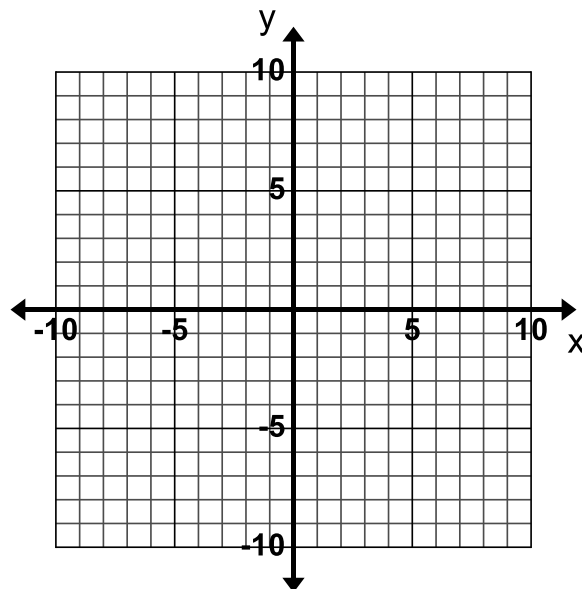




4. Graph  $x - y = 10$



5. Graph  $x - 4y = -8$



6. Graph  $-\frac{2}{3}x + 3 = 3 + y$

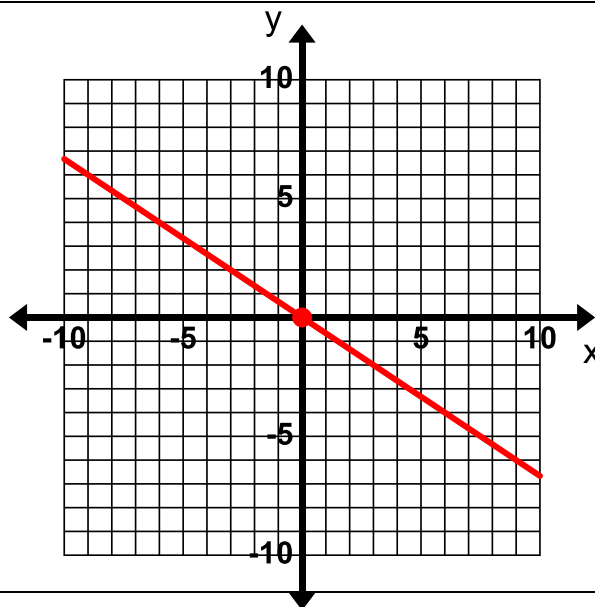
$$-\frac{2}{3}x + 3 = 3 + y$$

$$\underline{-3 = -3}$$

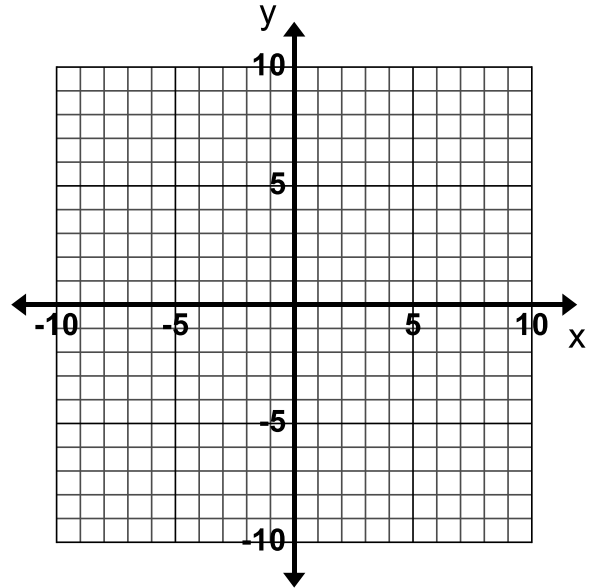
$$-\frac{2x}{3} = y$$

$$y = -\frac{2x}{3}$$

The  $x$ -intercept is  $(0,0)$  and the  $y$ -intercept is  $(0,0)$ .



7. Graph  $-10 + 5y = 5(y + x)$

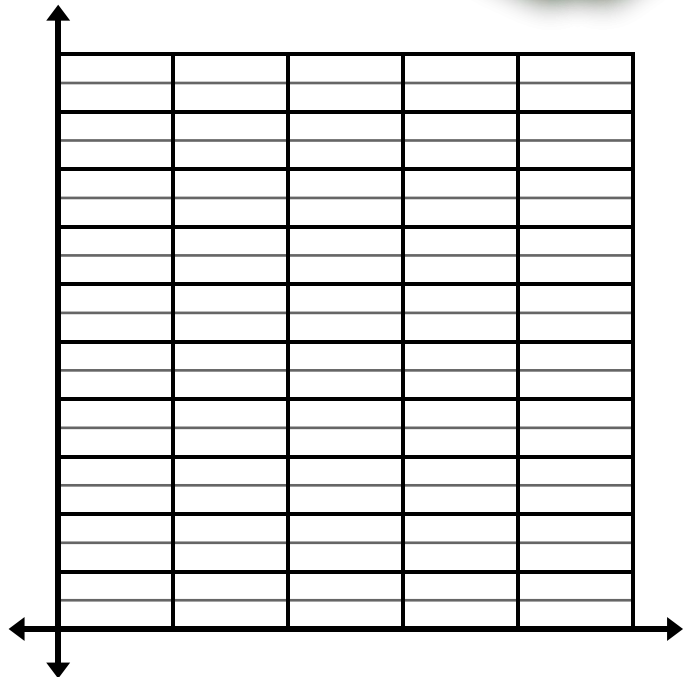


8. You have \$15 in five-dollar bills and one-dollar bills. Let  $x$  represent the number of five-dollar bills you have and  $y$  represent the number of one-dollar bills you have.

- a. Write an equation that represents all the possible combinations of five-dollar bills and one-dollar bills you could have with \$15.

$$5x + y = 15$$

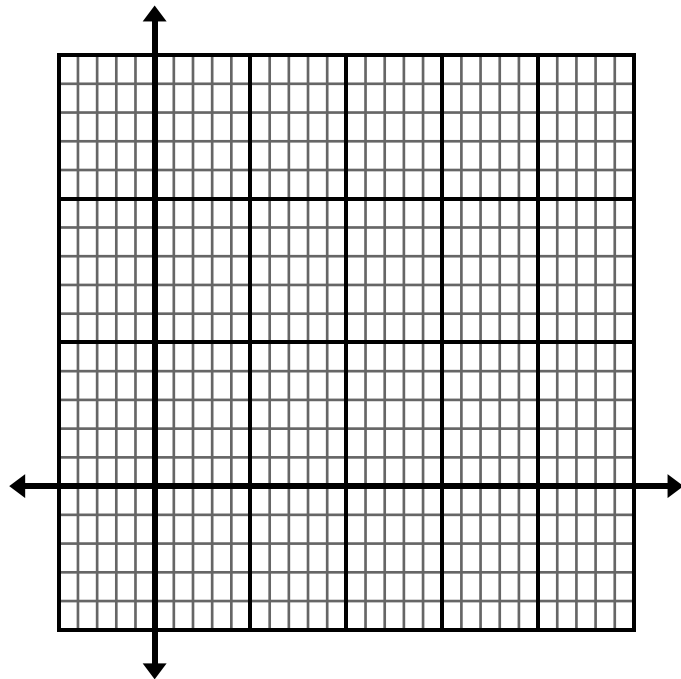
- b. Graph this equation to show all the different dollar bill combinations.



- c. What do the ordered pairs on the graph represent?
- d. How many one-dollar bills would you have if you have 2 five-dollar bills?
- e. How many five-dollar bills would you have if you have 10 one-dollar bills?
- f. Find and describe the  $x$  and  $y$ -intercepts for this context.

9. The difference between Lily's age and twice Kenny's age is 6 years. Lily is older than Kenny. Let  $x$  represent Lily's age and  $y$  represent Kenny's age.

- a. Write an equation that represents all the possible different ages that Lily and Kenny can be.
- b. Graph this equation to show all the age combinations.

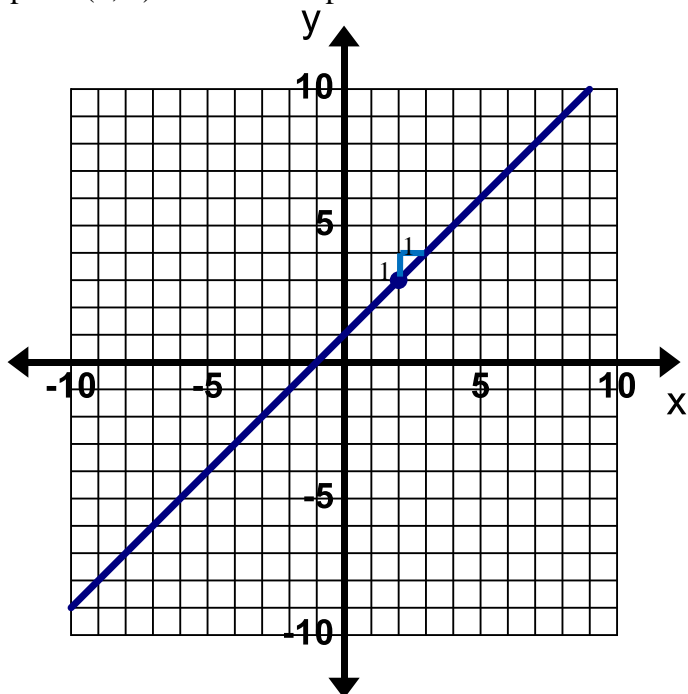


- c. Mark an ordered pair on the graph that represents the age of Lily and Kenny if Lily is 10.
- d. List at least 6 possible age combinations for Lily and Kenny?

### 3.1d Class Activity: Graph and Write Equations for Lines Given the Slope and a Point



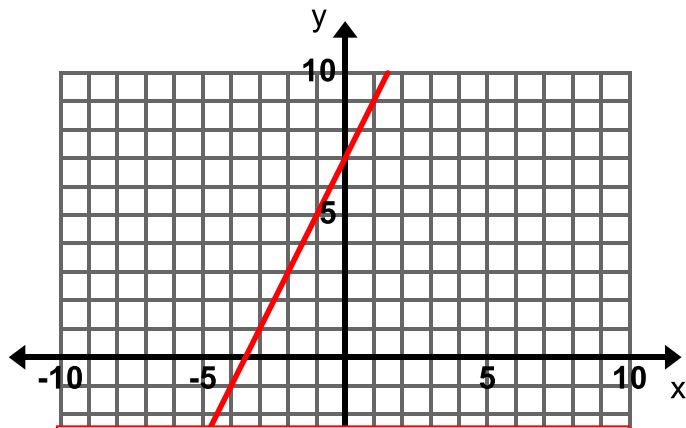
**Example:** Graph the line that passes through the point (2, 3) and has a slope of 1.



Write the equation of the line that you drew.

$$y = x + 1$$

- Graph the line that passes through the point (-1, 5) and has a slope of 2. **Start by plotting the point (-1, 5). From this point, create a line with a slope of 2. Refer to 2.3 for help with slope.**

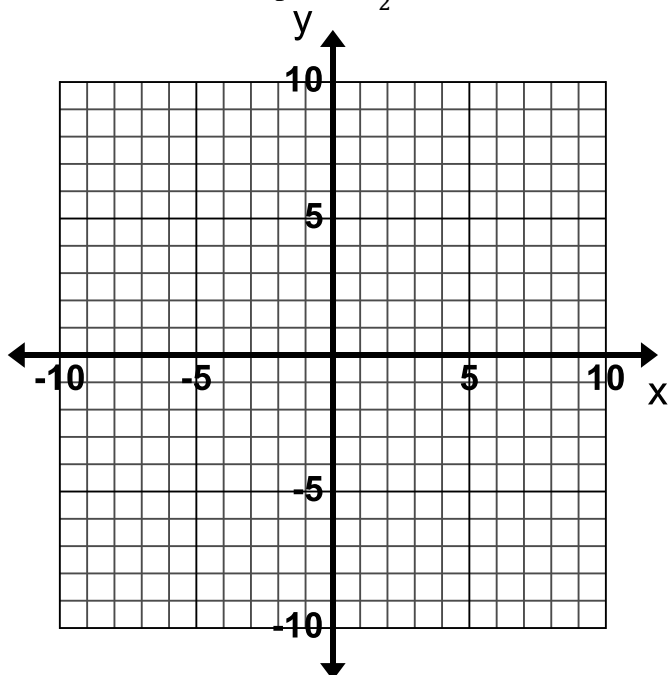


**Hint:** Remember that the slope-intercept form of the equation of a line is  $y = mx + b$  where  $m$  represents the slope of the line and  $b$  represents the  $y$ -intercept. The equation of this line is  $y = 2x + 7$ .

Write the equation of the line that you drew.

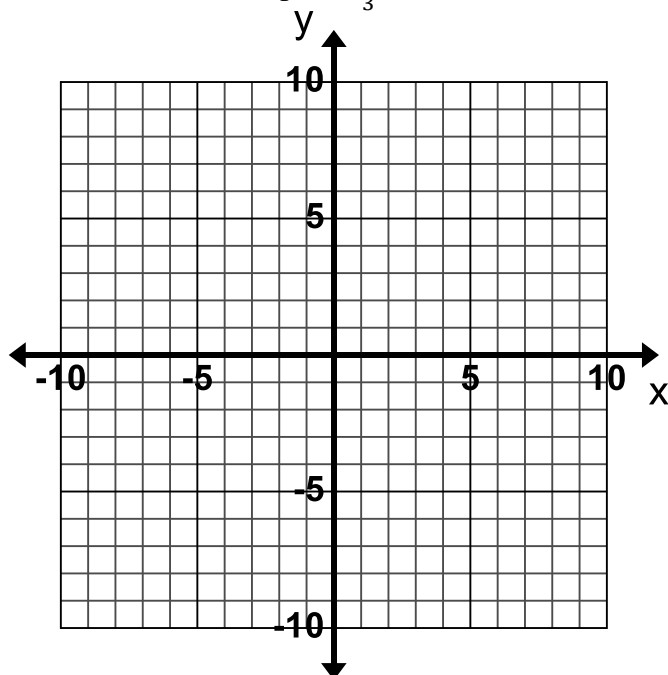
$$y = 2x + 7$$

- Graph the line that passes through the point (4, 1) and has a slope of  $-\frac{1}{2}$ .



Write the equation of the line that you drew.

- Graph the line that passes through the point (-6, 2) and has a slope of  $\frac{1}{3}$ .



Write the equation of the line that you drew.

4. How did you use the graph to write the equation of the lines above?

The graph can be used to find the y-intercept.

5. Would it be practical to always graph to find the equation? Why or why not?

One possible response would be that it is not always practical because if the y-intercept is not an integer value, you won't know what the exact value is.

6. Brainstorm ideas on how you could write the equation of the line without graphing when you are given a point and the slope. Consider how you could use the equation  $y = mx + b$  to find the y-intercept if you know the slope and a point on the line. Using an example from the previous page may help you work through the process.

Substitute in the slope and the  $x$  and  $y$  coordinates of the point and then solve for the y-intercept,  $b$ . Then, write the equation of the line.

For example, given a slope of 4 and the point  $(-1, -6)$ .

$$-6 = 4(-1) + b$$

$$-6 = -4 + b$$

$$\begin{array}{r} +4 \quad +4 \\ \hline \end{array}$$

$$-2 = b$$

$$\Rightarrow y = 4x - 2$$

**Directions:** Find the equation of the line that passes through the given point with the given slope.

7. Through  $(-1, -6)$ ;  $m = 4$   
 $y = 4x - 2$

8. Through  $(-3, 4)$ ;  $m = -\frac{2}{3}$

9. Through  $(4, -1)$ ;  $m = \frac{3}{2}$

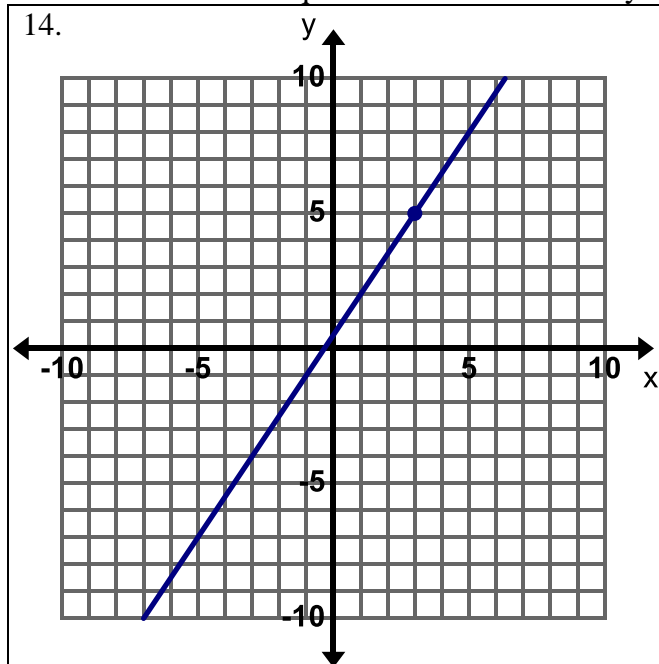
10. Through  $(3, 2)$ ;  $m = 1$

11. Through  $(3, 5)$ ;  $m = \text{undefined}$   
 $x = 3$

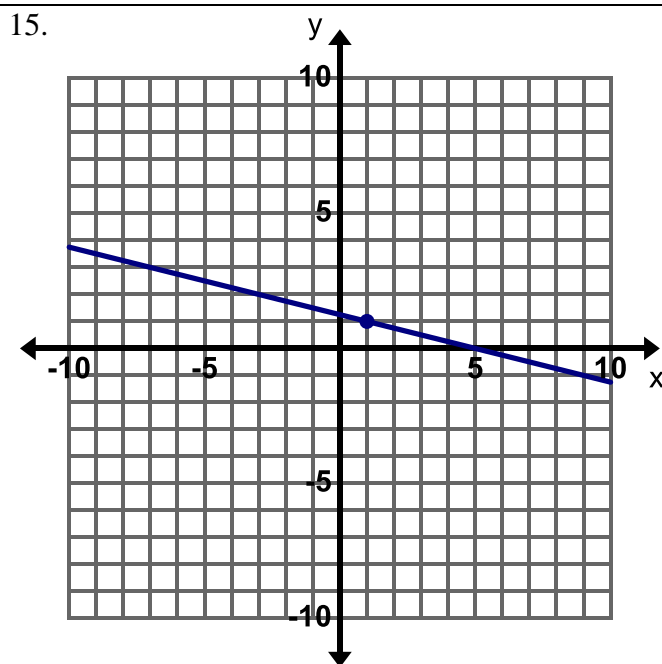
12. Through  $(3, -4)$ ;  $m = 0$

13. **Find, Fix, and Justify:** Felipe was asked to write the equation of the line that has a slope of  $\frac{1}{3}$  and passes through the point  $(6, 4)$ . Felipe made a common error and wrote the equation  $y = \frac{1}{3}x + 4$ . Describe Felipe's error and write the correct equation in the space below.

**Directions:** Write the equation of the line. Show your work.



Equation:



Equation:  $y = -\frac{1}{4}x + \frac{5}{4}$

16. Harper is at the bowling alley. She has spent \$13 so far renting bowling shoes and playing two rounds of bowling. The cost for each round is \$5 per person. Let  $x$  represent the number of rounds she has played and  $y$  represent the total cost.

- a. What is the rate of change for the situation above?

It costs \$5 per round of bowling.

- b. What point is addressed in the situation above?

The point (2,13) represents 2 rounds of bowling at a cost of \$13.

- c. Write an equation in Slope-Intercept form to represent the relationship between the number of rounds of bowling played and the total cost.

$$m = 5 \quad (2, 13)$$

$$y = mx + b$$

$$13 = 5(2) + b$$

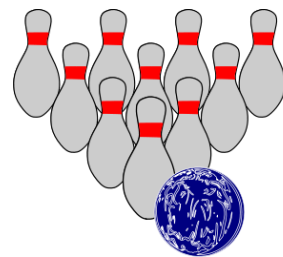
$$13 = 10 + b$$

$$b = 3$$

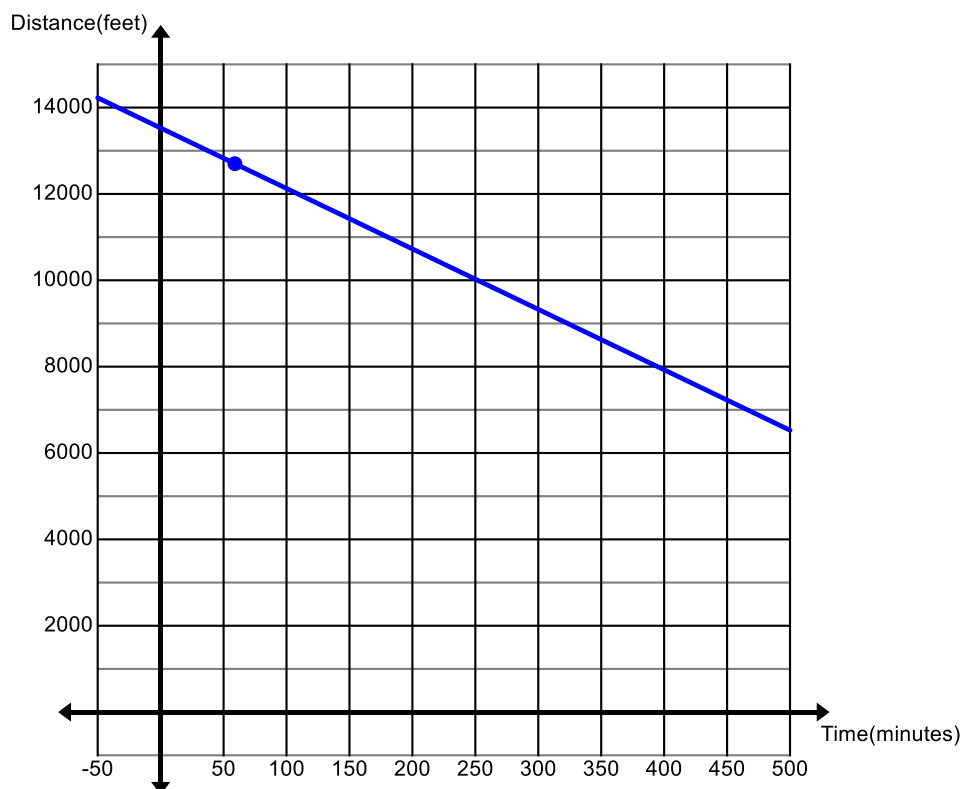
$$\Rightarrow y = 5x + 3$$

- d. What does the  $y$ -intercept in this relationship represent?

The  $y$ -intercept means that it costs \$3 to rent bowling shoes.



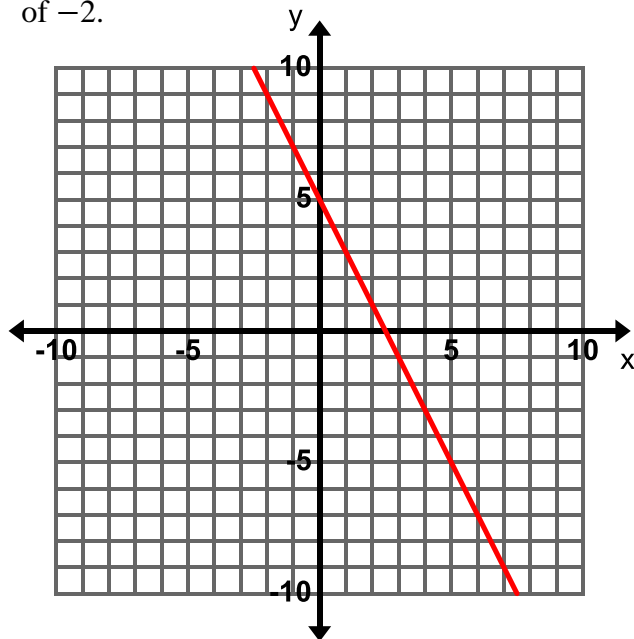
17. Art and Sierra are descending King's Peak, the highest peak in Utah. They have been climbing down the mountain losing 14 feet of elevation every minute. They reach Anderson Pass 59 minutes after leaving the summit (the top of the peak). The graph represents the relationship between the time that has passed since leaving the summit on the  $x$ -axis and the elevation represented on the  $y$ -axis.



- What is the rate of change for the situation above?
- What is the elevation of Anderson Pass?
- Write an equation in Slope-Intercept form to represent the relationship between the number of minutes climbing down the peak and the current elevation.
- How high is King's peak?
- Use your equation to predict how long it will take Art and Sierra to get to Gunsight Pass which has an elevation of 11,888 feet if they continue to descend at the same rate.
- Label the summit for King's Peak, Anderson Pass, and Gunsight Pass. Also use the graph or equation to predict the elevation of Dollar Lake if Art and Sierra reach it after 3 hours and 16 min. Once you have determined the elevation for Dollar Lake label it on the graph as well.

### 3.1d Homework: Graph and Write Equations for Lines Given the Slope and a Point

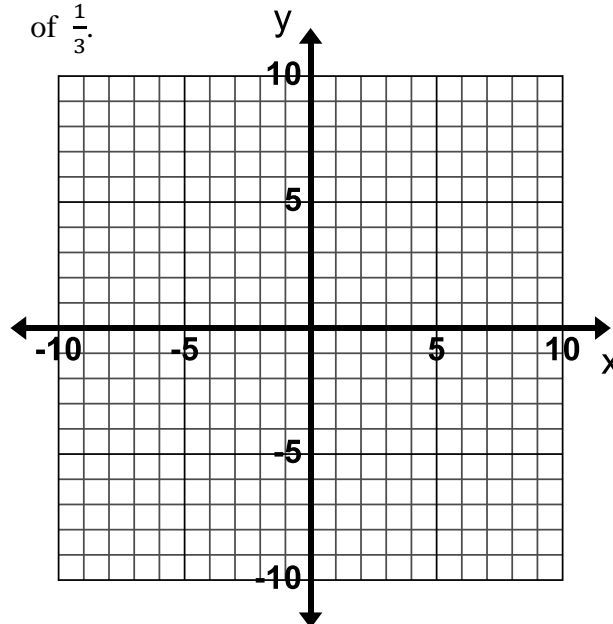
1. Graph a line that does the following:  
Passes through the point  $(4, -3)$  and has a slope of  $-2$ .



Write the equation of the line that you drew.

$$y = -2x + 5$$

2. Graph a line that does the following:  
Passes through the point  $(-6, 3)$  and has a slope of  $\frac{1}{3}$ .



Write the equation of the line that you drew.

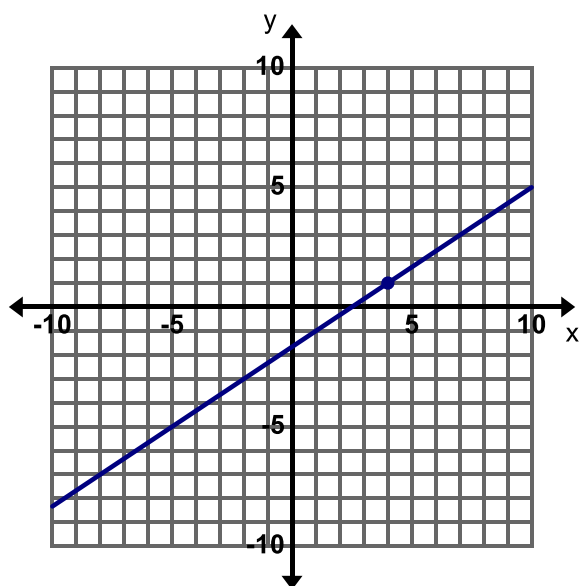
**Directions:** Write the equation for the line that has the given slope and contains the given point.

<p>3. slope = 1 passes through <math>(3, 7)</math></p> $y = x + 4$	<p>4. slope = <math>\frac{2}{3}</math> passes through <math>(3, 4)</math></p> $y = \frac{2}{3}x + 2$	<p>5. slope = 5 passes through <math>(6, -10)</math></p>
<p>6. slope = <math>-2</math> passes through <math>(3, 1)</math></p>	<p>7. slope = 5 passes through <math>(-2, 8)</math></p>	<p>8. slope = <math>\frac{1}{3}</math> passes through <math>(0, 2)</math></p>



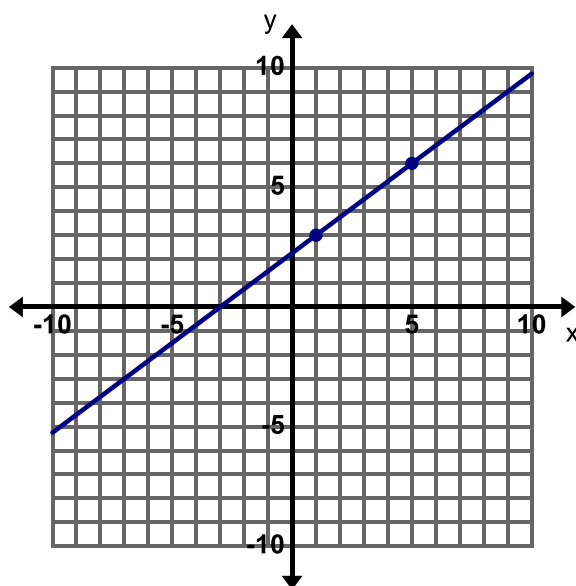
**Directions:** Write the equation of the line.

9.



Equation:

10.



You can use either point to find  $b$ . Try it to verify.

Equation:  $y = \frac{3}{4}x + \frac{9}{4}$

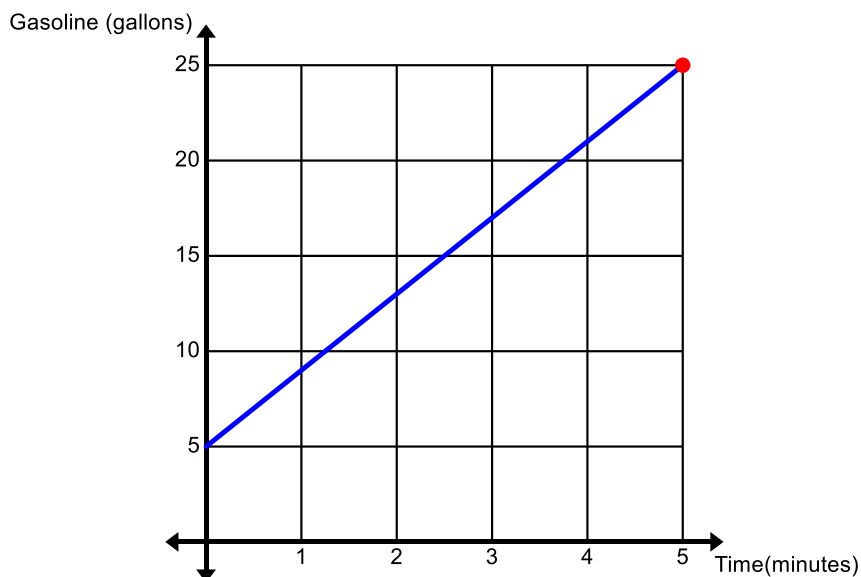
11. In your own words, explain how to write the equation of a line in slope-intercept form when you are given the slope and a point.

You have to substitute in the slope and point in order to find the y-intercept. Once you have solved for the y-intercept, you now have the pieces you need to plug into the slope-intercept form of a linear equation.

12. At the beginning of the year Monica puts a set amount of money into her health benefit account. Every month she withdraws \$15 from this account for her contact lenses. After 3 months she has \$255 left in her account.
- What is the rate of change for this situation?
  - What point on the line is described in the story above?
  - Write an equation in slope-intercept form to represent the relationship between the time that has passed and the amount of money left in Monica's account. Let  $x$  represent the time in months and  $y$  represent the amount of money remaining in the account.

- d. If Monica does not use all of the money in her account by the end of the year she loses it. Monica only uses the money in the account for contact lenses; will she lose money at the end of the year?

13. The graph shown describes the amount of gasoline being put into a truck that has a 25 gallon tank. The gasoline is pumped at a rate of 4 gallons per minute.



- a. Label the point on the graph where you can determine how long it takes to fill the 25 gallon tank up with gas. Then state how the point helps you to determine the time.

- b. What is the rate of change for this story?
- c. Write an equation in slope-intercept form that describes the relationship between the time that has passed and the amount of gasoline in the tank.
- d. How much gasoline was in the tank before the tank was filled?
- e. Is there more than one method for finding the equation of this line?  
To find the equation of the line you can use a point and the slope, the slope and the y-intercept (as found on the graph), or you could use two points.

**14. Think about this...**

In this lesson, you were given the slope and a point on the line and used this information to write the equation of the line in slope-intercept form. In the next lesson, you will be given 2 points and asked to write the equation in slope-intercept form. Write down your thoughts on how you might do this.

Now try it...

Write the equation of the line that passes through the points (1, 4) and (3, 10).

$y = 3x + 1$

### 3.1e Class Activity: Write Equations for Lines Given Two Points

- Describe how to write the equation of a line in slope-intercept form when you are given two points on the line.

**Directions:** Write the equation of the line that passes through the points given.

<p>2. (0, 4), (-1, 3)</p> <p>In order to solve this problem, students must first find the slope of the line by either graphing or using the slope formula:</p> $\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 4}{-1 - 0} = \frac{-1}{-1} = 1$ <p>Once students have found the slope, they can plug the slope and one of the points (either one) into the equation <math>y = mx + b</math> to find the y-intercept:</p> $y = mx + b$ $3 = (1)(-1) + b$ $3 = -1 + b$ $4 = b$ <p>It so happens in this problem that the y-intercept is one of the points given in the problem: (0, 4).</p> <p>Equation: <math>y = x + 4</math></p>	<p>3. (-5, 9), (-2, 0)</p> $y = -3x - 6$	<p>4. (0, 0), (3, -6)</p>
<p>5. (2, 2), (4, 3)</p>	<p>6. (1, 2), (1, -6)</p> <p>To find the slope,</p> $\frac{-6 - 2}{1 - 1} = \frac{-8}{0} = \text{undefined}$ <p>A vertical line has a slope that is undefined. The equation for the vertical line that passes through the points given is <math>x = 1</math>.</p>	<p>7. (-2, 4), (0, 4)</p> <p>To find the slope,</p> $\frac{4 - 4}{0 - -2} = \frac{0}{2} = 0$ <p>A horizontal line has a slope of 0. The equation for the horizontal line that passes through the points given is <math>y = 4</math>.</p>

- A box containing 20 cans of SPAM weighs 250 ounces. A box containing 28 cans of SPAM weighs 346 ounces.
  - Write an equation that relates the number of cans of SPAM to the weight of the box. Let  $x$  represent the number of cans of SPAM and  $y$  represent the weight of the box in ounces.  
 $y = 12x + 10$
  - What does the y-intercept in the equation represent?  
The empty box weighs 10 ounces.
  - Use your equation to predict the weight of a box that contains 40 cans of SPAM.  
A box with 40 cans of SPAM will weigh 490 ounces.

**Directions:** Write an equation for a line from the information given in each table.

9.

$x$	$y$
3	8
4	9
5	10
6	11

Equation:  $y = x + 5$

10.

$x$	$y$
2	3
4	6
6	9
8	12

Equation:

11.

$x$	$y$
-12	-1
-10	-1
-8	-1
-6	-1

Equation:

12. Toa takes the freeway home from work so he can use his cruise control. The table below shows the time  $x$  in minutes since he entered the freeway related to the distance  $y$  in miles he is from his exit at several points on his journey.

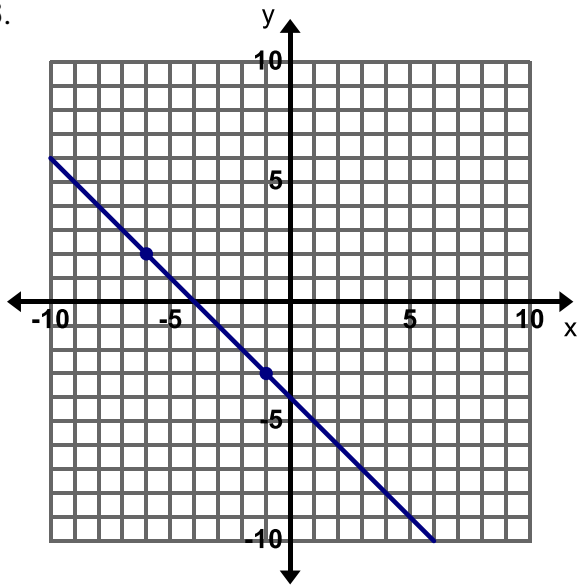
$Time(x)$	$Distance(y)$
8	34
20	25
32	16
40	10



- a. Write an equation that relates the time Toa has been on the freeway to the distance he is from his exit. Choose any 2 points from the table to find the equation. Using the points (8, 34) and (20, 25), find the slope:  $\frac{25-34}{20-8} = \frac{-9}{12} = \frac{-3}{4}$ . The slope tells us that each minute Toa is decreasing his distance from his exit by  $\frac{3}{4}$  of a mile.  
Then use the slope and a point in the table to find the y-intercept. Using the point (8, 34):  
 $y = mx + b$   
 $34 = -\frac{3}{4}(8) + b$   
 $34 = -6 + b$   
 $40 = b$   
**Equation:**  $y = -\frac{3}{4}x + 40$  where  $y$  = distance from exit (in miles) and  $x$  = time (in minutes)
- b. What does the y-intercept represent in this equation?  
The y-intercept means that Toa's total distance on the freeway is 40 miles (i.e. when he enters the freeway, he is 40 miles from his exit).
- c. Use your equation to predict how much time will pass before Toa reaches his exit.  
When Toa is at his exit  $y = 0$ . Using the equation from above:  
 $y = -\frac{3}{4}x + 40$   
 $0 = -\frac{3}{4}x + 40$   
 $\frac{3}{4}x = 40$   
 $x = \frac{160}{3} = 53\frac{1}{3}$  minutes = 53 minutes and 20 seconds

**Directions:** Write an equation for the line given.

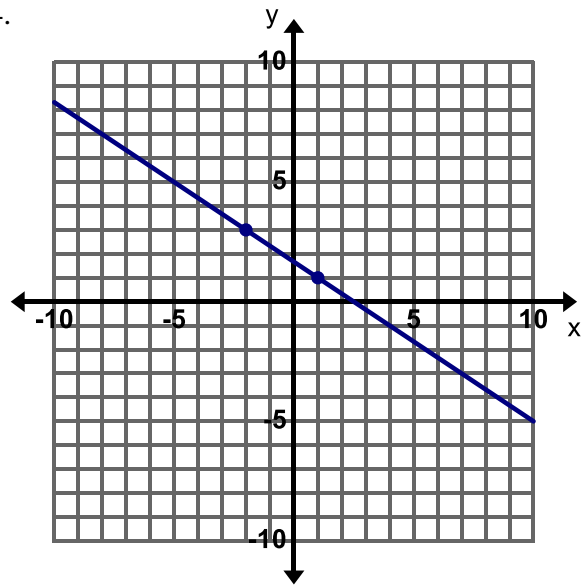
13.



This is a good problem to review all the different methods we have used to write the equation of the line, we can pull the slope and y-intercept right from the graph or use the points given to find the slope and y-intercept.

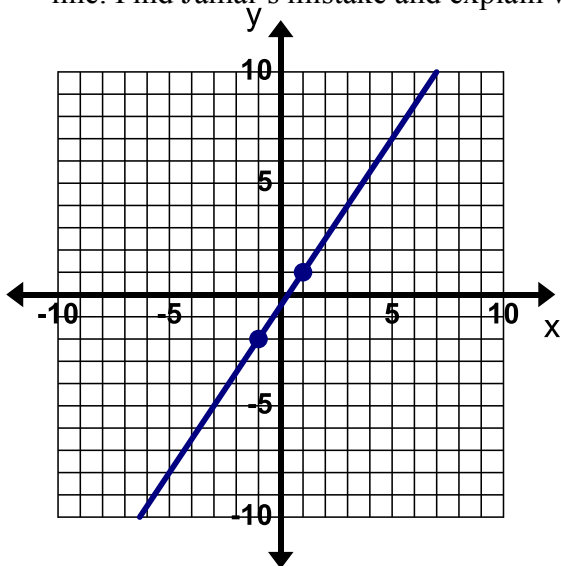
Equation:  $y = -x - 4$

14.



Equation:  $y = -\frac{2}{3}x + \frac{5}{3}$

**15. Find, Fix, and Justify:** Jamal was asked to write an equation for the line on the graph below. Jamal's work is shown to the right of the graph, he has made a common mistake in writing the equation for the line. Find Jamal's mistake and explain what he did wrong. Then write the correct equation for the line.



$(-1, -2) \quad (1, 1)$

$$m = \frac{-2 - 1}{-1 - 1} = \frac{-3}{-2} = \frac{3}{2}$$

$$1 = \frac{3}{2}(1) + b$$

$$1 = \frac{3}{2} + b \Rightarrow b = \frac{3}{2}$$

$$y = \frac{3}{2}x + \frac{3}{2}$$

Jamal did not solve the equation for  $b$ . The correct equation is  $y = \frac{3}{2}x - \frac{1}{2}$

### 3.1e Homework: Write Equations for Lines Given Two Points

**Directions:** Write the equation of the line that passes through the points given.

1. (0, 2) and (-2, 0)	2. (5, 0) and (-10, -5)  In order to solve these problems, students must first find the slope of the line by either graphing or using the slope formula: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 0}{-10 - 5} = \frac{-5}{-15} = \frac{1}{3}$ Once students have found the slope, they can plug the slope and one of the points (either one) into the equation $y = mx + b$ to find the y-intercept: $y = mx + b$ $0 = \frac{1}{3}(5) + b$ $0 = \frac{5}{3} + b$ $-\frac{5}{3} = b$ Equation: $y = \frac{1}{3}x - \frac{5}{3}$	3. (1, 1) and (3, 3)
4. (4, 2) and (0, -2)  $y = x - 2$	5. (2, 3) and (-2, 3)	6. (0, -1) and (3, -2)

7. Clarissa is saving money at a constant rate. After 2 months she has \$84 in her savings account. After 5 months she has \$210 in her account.
- Write an equation that relates the amount of money she has in her savings account to the number of months that have passed. Let  $x$  represent the number of months and  $y$  represent the total amount of money in the account.
  - Interpret the y-intercept and slope of the equation for this context.
  - Clarissa would like to purchase a plane ticket to visit her sister exactly one year after she began saving money. The plane ticket costs \$450. Will she have enough money in the account to pay for the ticket.

**Directions:** Write an equation for the line from the information given in each table.

8.

$x$	$y$
-1	5
0	2
1	-1
2	-4

Equation:  $y = -3x + 2$

Again, students need to find the slope and y-intercept in order to write the equation for the line that passes through these points.

9.

$x$	$y$
-2	2
2	3
6	4
10	5

Equation:

10.

$x$	$y$
0	-1
0	0
0	1
0	2

Equation:

11. Create your own real-world story that matches the table below. Write an equation to represent the relationship between your variables.



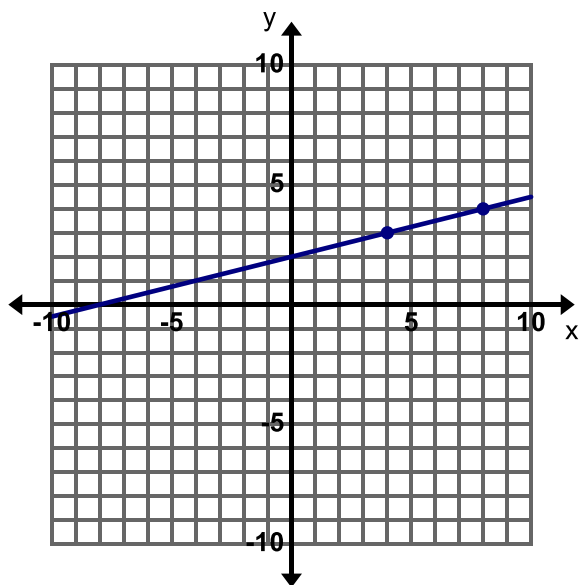
$x$	$y$
5	20
12	41
20	65
22	71

Possible Story: Owen has \$5 in his piggy bank. His parents start giving him allowance and he adds \$3 to his piggy bank each week. Let  $y$  = the amount of money Owen has in his piggy bank and  $x$  = the number of weeks that have passed since his parents started giving him allowance.

Equation:  $y = 3x + 5$

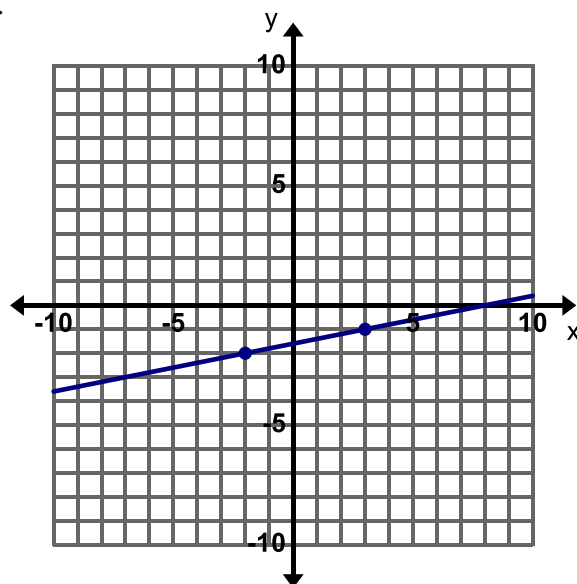
**Directions:** Write an equation for the line given.

12.



Equation:

13.



Equation:  $y = \frac{1}{5}x - \frac{8}{5}$

**Extra for Experts:** Consider the three points  $(-2, 4)$ ,  $(1, 2)$  and  $(4, r)$  on the same line. Find the value of  $r$  and explain your steps.

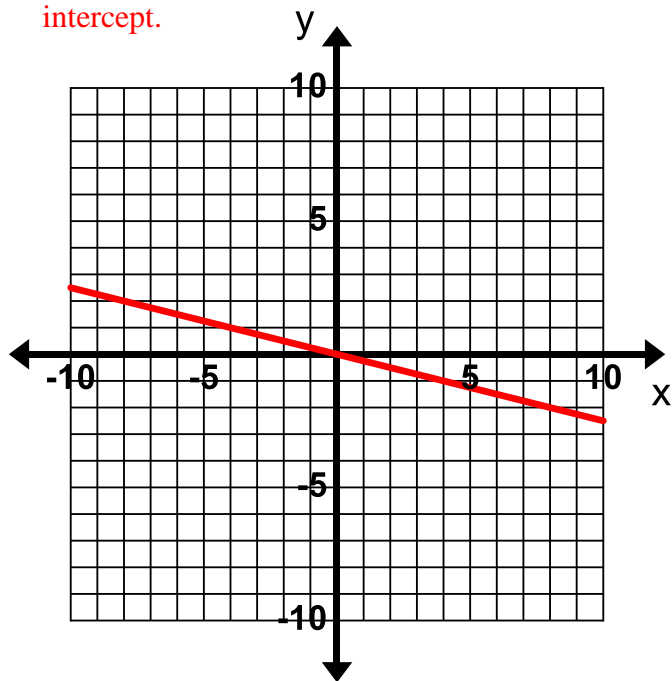


### 3.1f Class Activity: Graphing and Writing Equations for Lines, Mixed Review

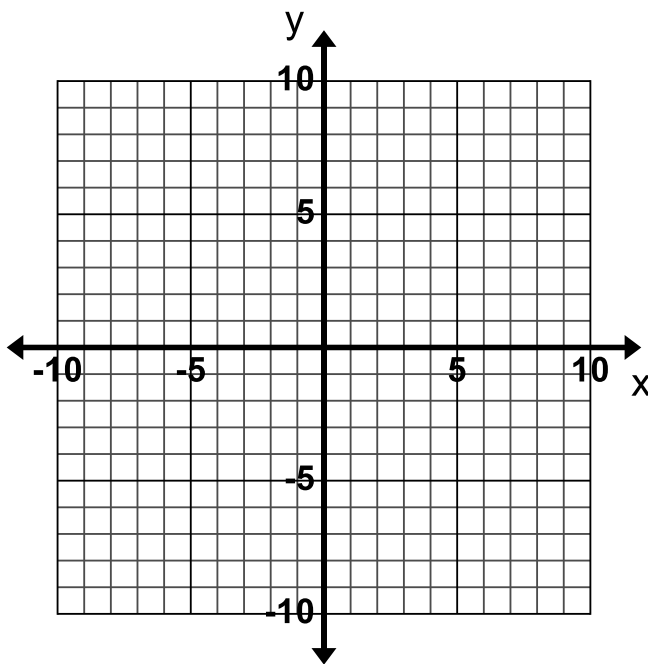
**Directions:** Graph the lines for the following given information.

1. The equation of the line is  $y = -\frac{1}{4}x$ .

When an equation is written in slope-intercept form,  $y = mx + b$ , the number in front of the  $x$  is the slope and the constant or  $b$  is the  $y$ -intercept.

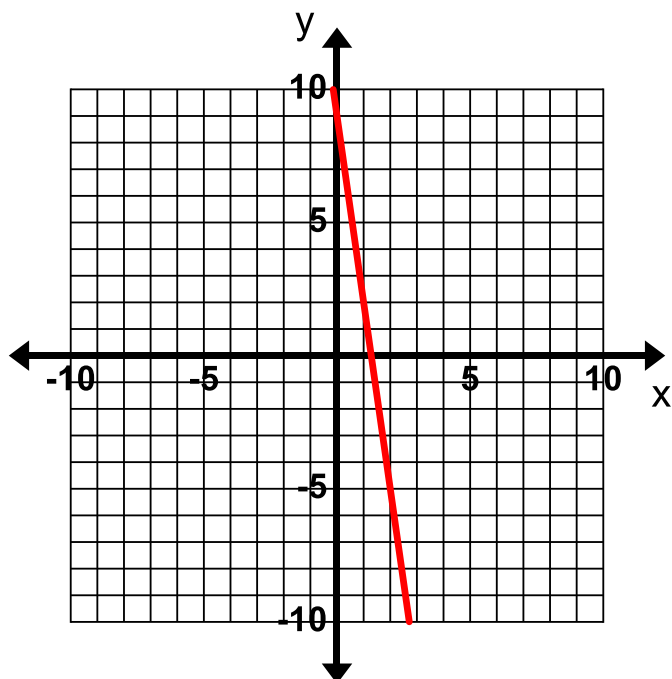


2. The equation of the line is  $y = x - 8$ .

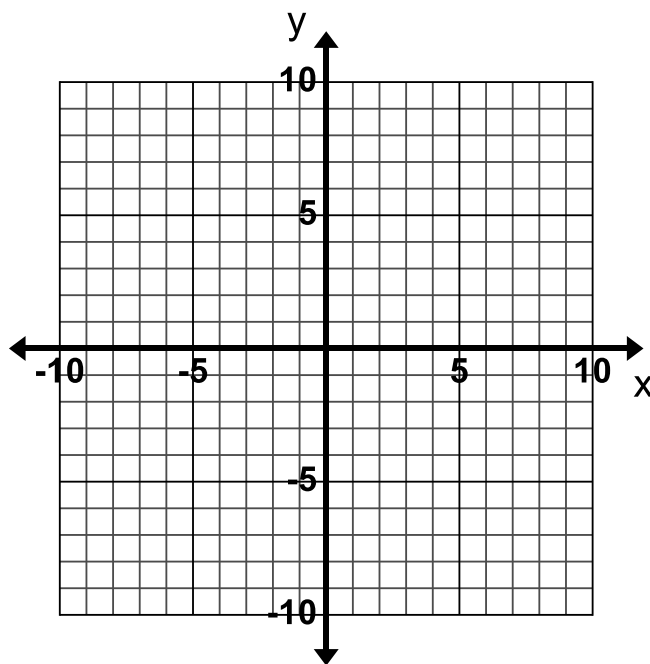


3. The equation of the line is  $7x + y = 9$ .

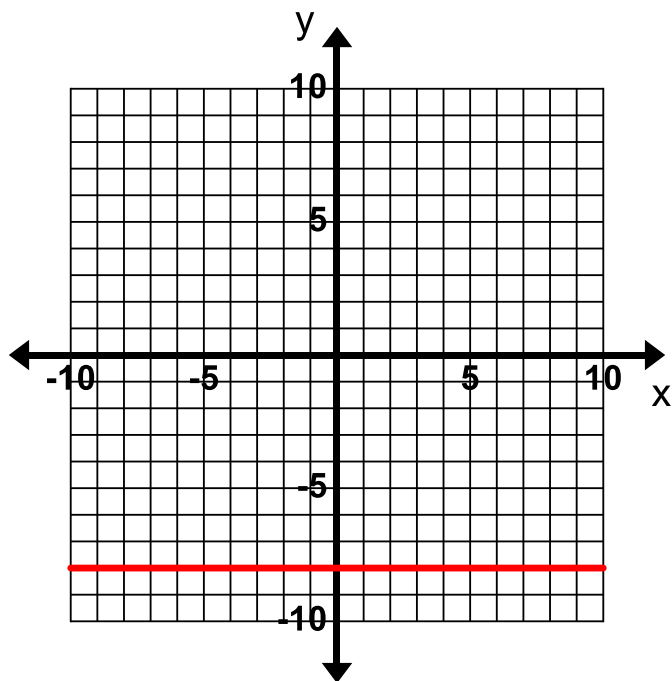
Remember to write this equation in slope-intercept form first (solve for  $y$ ).



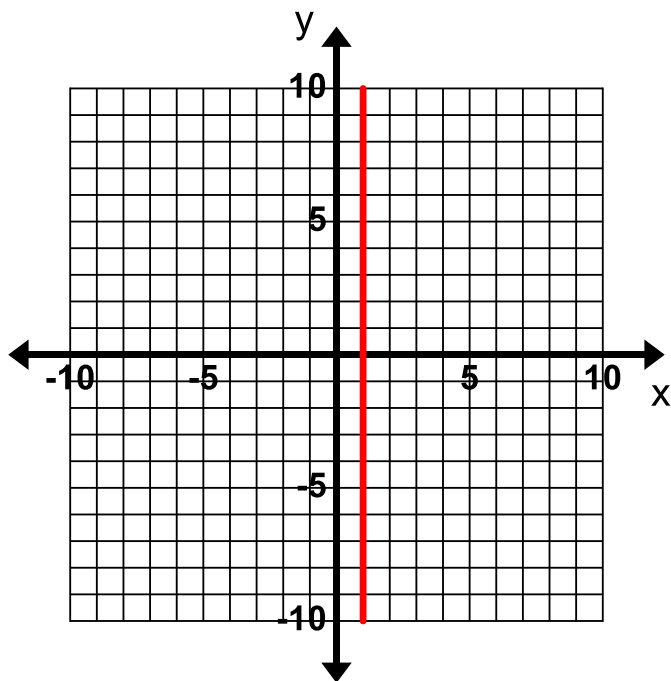
4. The equation of the line is .



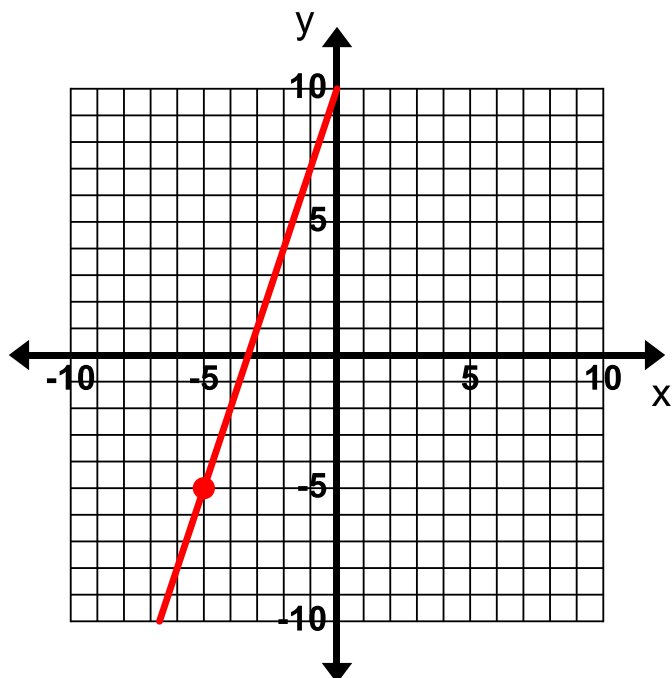
5. The equation of the line is  $x + 2(y + 1) = x - 14$ .



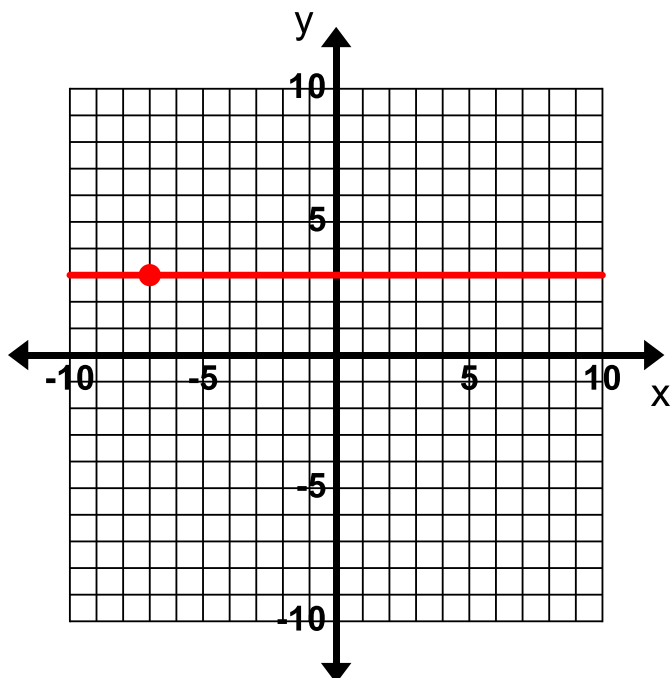
6. The equation of the line is  $x = 1$ .



7. The line contains the point  $(-5, -5)$  and has a slope of 3.



8. The line contains the point  $(-7, 3)$  and has a slope of 0.



**Directions:** Write the equation in slope-intercept form for each line based on the information given.

9. The slope of the line is  $-\frac{1}{2}$  and the y-intercept is  $-5$ .

$$y = -\frac{1}{2}x - 5$$

10. The line has a slope of 4 and goes through the point  $(6, -1)$ .

$$y = 4x - 25$$

11. The line contains the points  $(-2, 7)$  and  $(3, -3)$ .

$$y = -2x + 3$$

12. The line contains the points in the table.

x	y
0	-6
1	-2
2	2
3	6

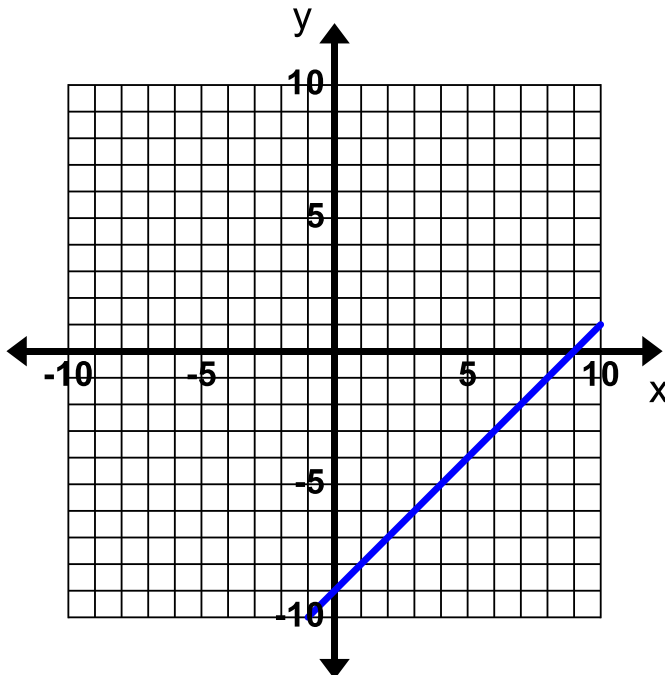
$$y = 4x - 6$$

13. The line contains the points in the table.

x	y
2	2
4	1
8	-1
14	-4

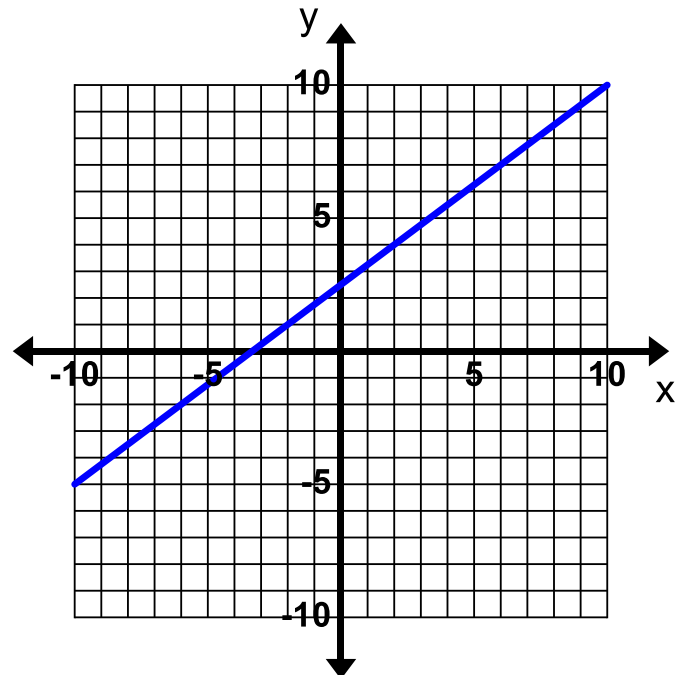
$$y = -\frac{1}{2}x + 3$$

14. The line graphed below.



$$y = x - 9$$

15. The line graphed below.

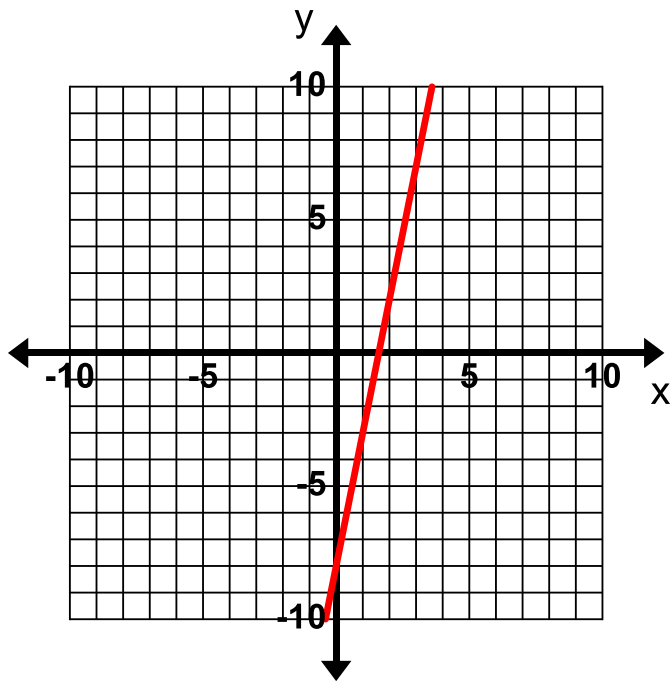


$$y = \frac{3}{4}x + 2.5$$

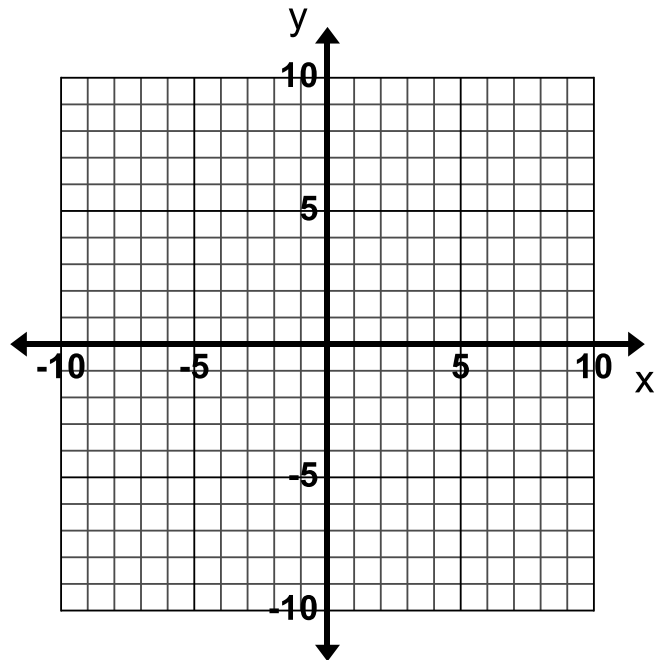
### 3.1f Homework: Graphing and Writing Equations for Lines, Mixed Review

**Directions:** Graph the lines for the following given information.

1. The equation of the line is  $y = 5x - 8$ .

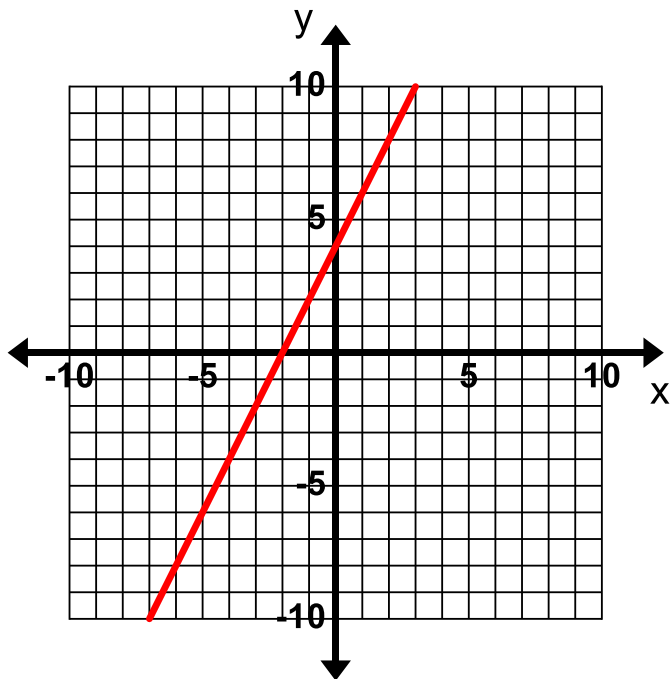


2. The equation of the line is  $y = -\frac{1}{3}x + 7$ .

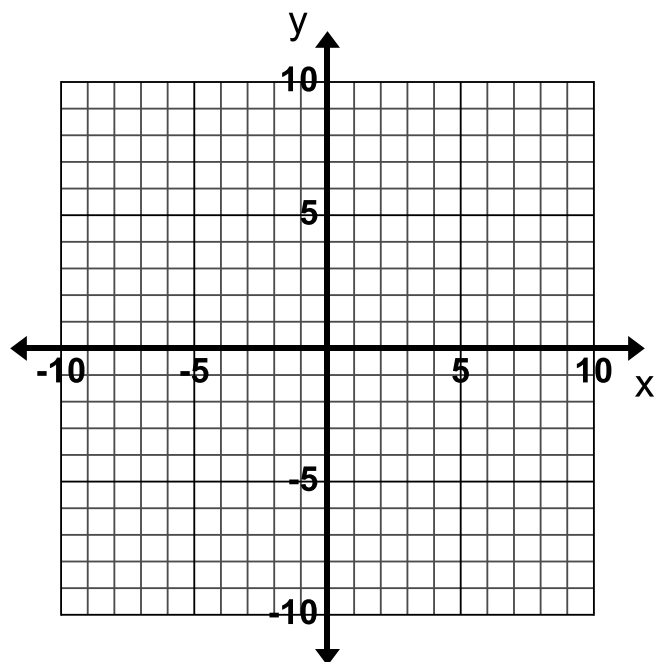


3. The equation of the line is  $-4x + 2y = 8$ .

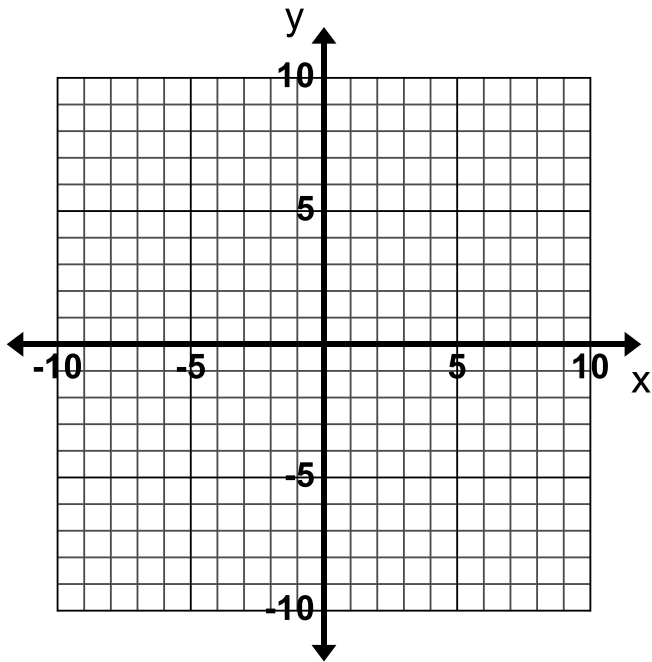
Remember to write this equation in slope-intercept form first (solve for  $y$ ).



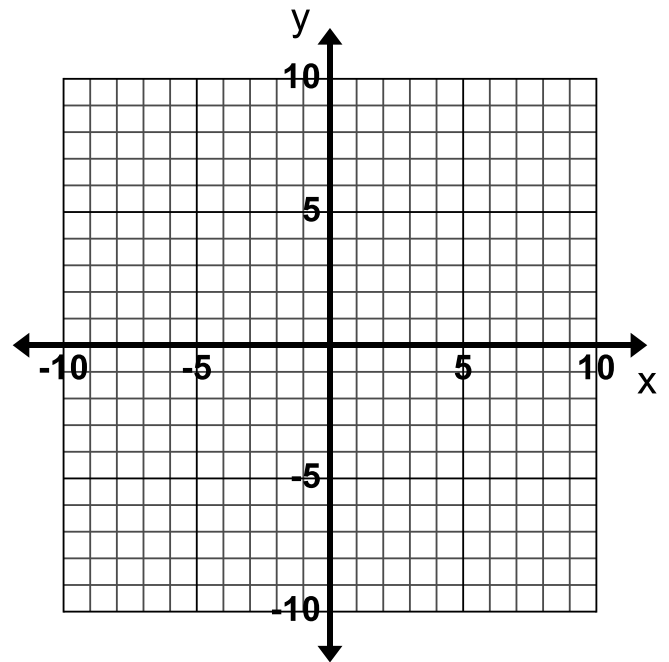
4. The equation of the line is  $x - 3y = 9$ .



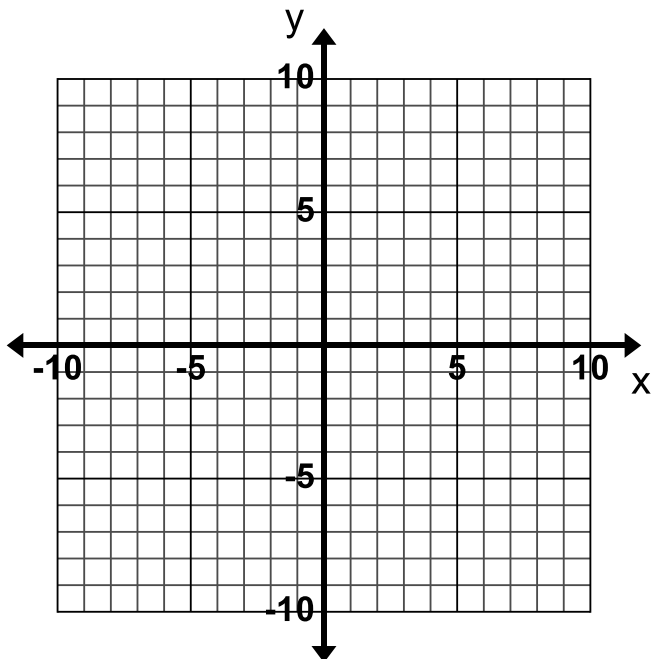
5. The equation of the line is  $y + 2x = y - 2$ .



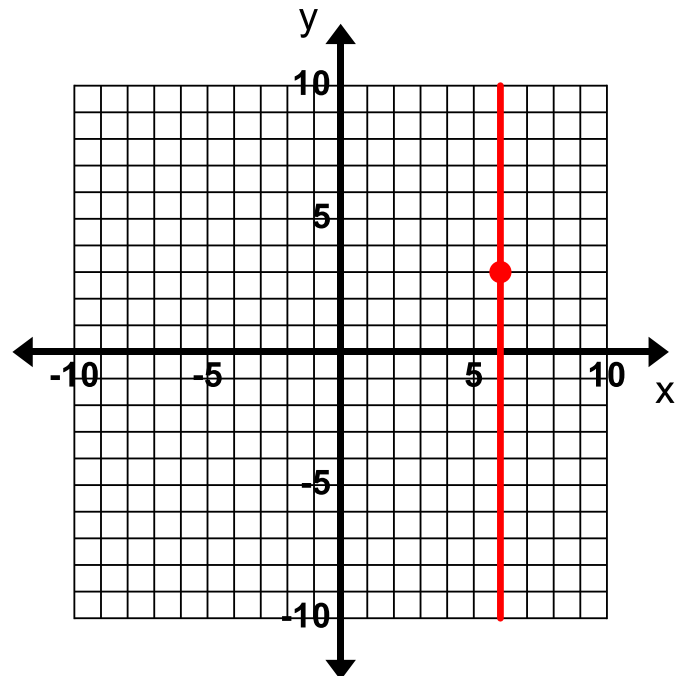
6. The equation of the line is  $y = 6$ .



7. The line contains the point  $(1, 2)$  and has a slope of  $-\frac{5}{2}$ .



8. The line contains the point  $(6, 3)$  and the slope is undefined.



**Directions:** Write the equation in slope-intercept form for each line based on the information given.

9. The slope of the line is 1 and the y-intercept is  $-4$ .

10. The line has a slope of  $-\frac{1}{4}$  and goes through the point  $(-2, 4)$ .

$$y = -\frac{1}{4}x + 3.5$$

11. The line contains the points  $(1, -2)$  and  $(2, 4)$ .

12. The line contains the points in the table.

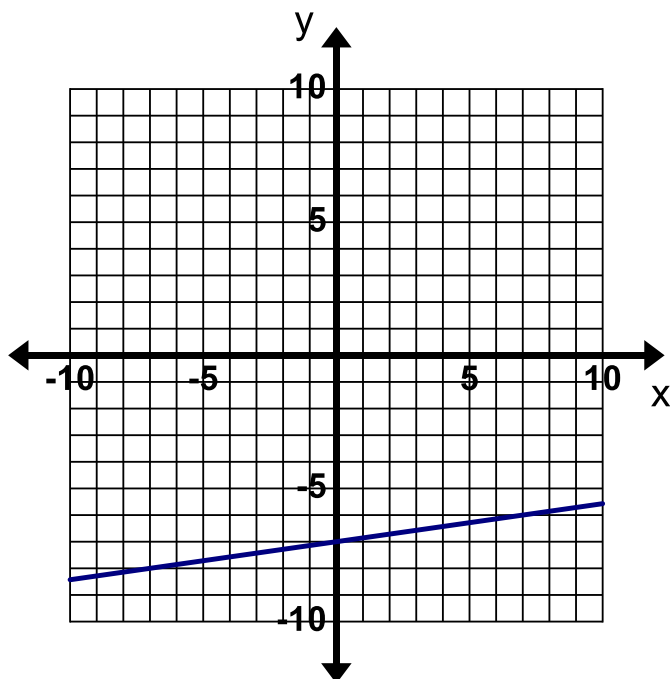
$x$	$y$
0	0
1	$-2$
2	$-4$
3	$-6$

13. The line contains the points in the table.

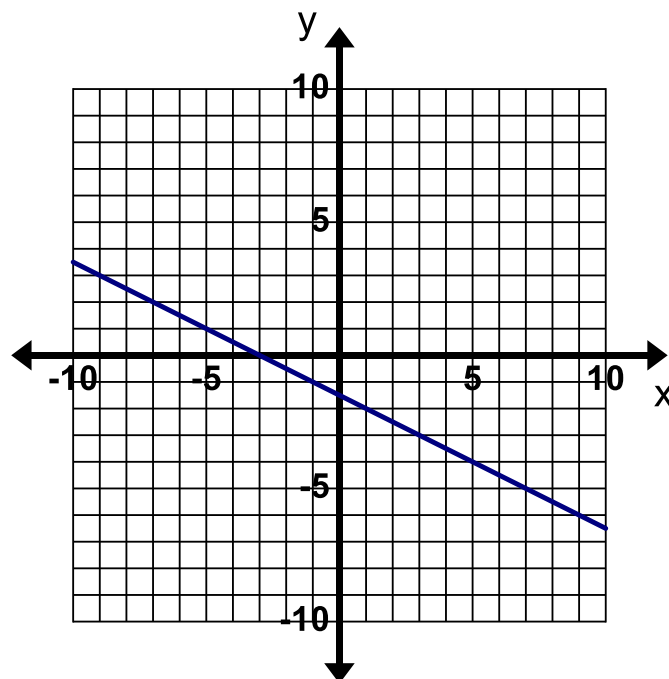
$x$	$y$
1	5
3	$-1$
4	$-4$
6	$-10$

$$y = -3x + 8$$

14. The line graphed below.



15. The line graphed below.

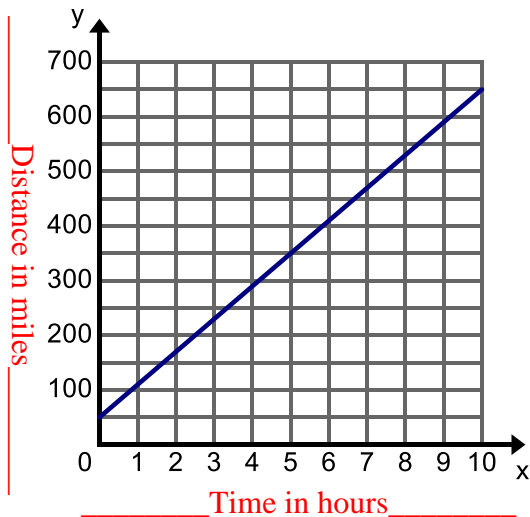


$$y = -\frac{1}{2}x - 1.5$$

### 3.1g Classwork: Write Equations to Solve Real-world Problems

**Directions:** Write the equation for each of the following real-world problems.

1. The graph below shows a trip taken by a car where  $x$  is time (in hours) the car has driven and  $y$  is the distance (in miles) from Salt Lake City. Label the axes of the graph.



Equation:  $y = 60x + 50$

Use your graph and equation to tell the story of this trip taken by the car.

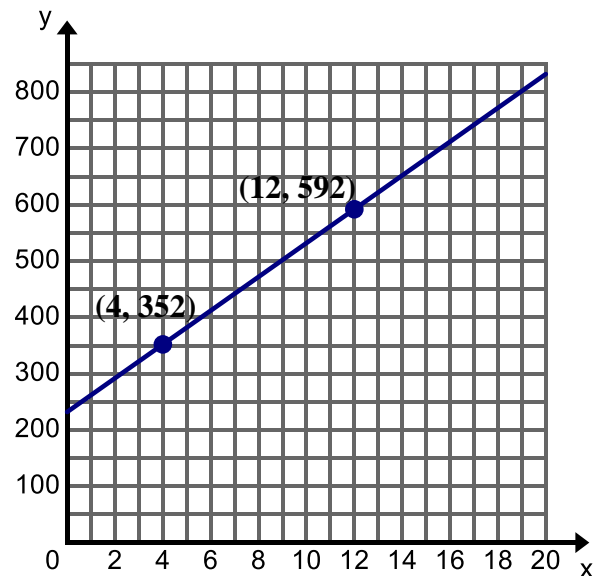
Answers will vary:

A car started a trip 50 miles from Salt Lake City.  
The car drove away from Salt Lake City at a constant rate of 60 mph for 10 hours.

You can ask students additional questions about this situation.

How far was the car from SLC after 6 hours? After 8 hours?

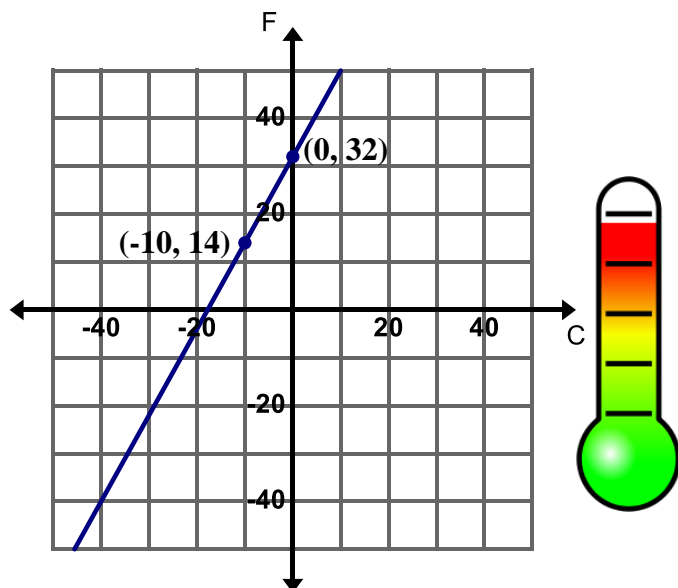
2. The graph below shows the weight of a baby elephant where  $x$  is the time (in weeks) since the elephant's birth and  $y$  is the weight (in pounds). At 4 weeks, the elephant weighed 352 lbs. and at 12 weeks, the elephant weighed 592 lbs. Label the axes of the graph.



Equation:

Use your graph and equation to tell the story of this elephant.

3. The graph below shows the relationship between temperature in degrees Celsius and temperature in degrees Fahrenheit.



Equation:  $y = \frac{9}{5}x + 32$

5. A handyman charges \$40 an hour plus the cost of materials. Rosanne received a bill from the handyman for \$477 for 8 hours of work.

Equation:  $y = 40x + 157$

Use your equation to add more details to the story about the work the handyman did for Roseanne.

The cost of materials was \$157.

4. Peter is draining his hot tub so that he can clean it. He puts a hose in the hot tub to drain the water at a constant rate. After 5 minutes there are 430 gallons of water left in the hot tub. After 20 minutes there are 370 gallons of water left in the hot tub. Let  $x$  be time (in minutes) and  $y$  be water remaining (in gallons).

Equation:

Use your equation to add more details to the story of Peter draining the hot tub.

6. The table below shows the height  $h$  (in feet) of a hot air balloon  $t$  minutes after it takes off from the ground. It rises at a constant rate.

$t$ (minutes)	$h$ (feet)
2	300
5	750
9	1,350



Equation:

Use the table and equation to tell the story of the hot air balloon.

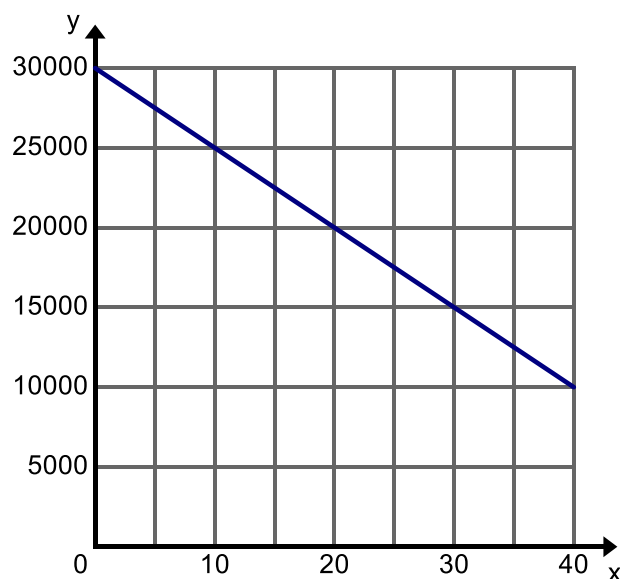


### 3.1g Homework: Write Equations to Solve Real-world Problems

**Directions:** Write the equation for each of the following real-world problems.



1. The graph below shows the descent of an airplane where  $x$  is time (in minutes) since the plane started its descent and  $y$  is the altitude (in feet) of the plane. Label the axes of the graph.



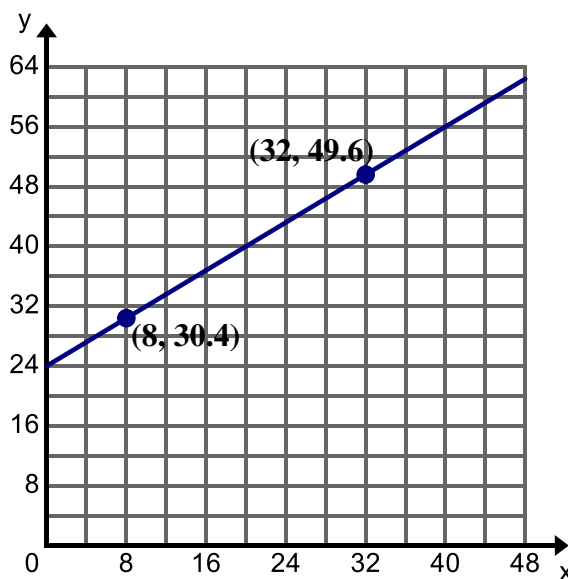
Equation:  $y = -500x + 30,000$

Use the graph and equation to tell the story of this airplane.

An airplane starts its descent 30,000 feet above the ground. It descends at a constant rate of 500 feet per minute.

Ask students how long it will take for the airplane to reach the ground at this rate.

2. The graph below shows the length of a boa constrictor where  $x$  is time (in weeks) since the boa constrictor's birth and  $y$  is length (in inches). The boa constrictor was 30.4 in. at 8 weeks and 49.6 in. at 32 weeks. Label the axes of the graph.



Equation:

Use the graph and equation to tell the story of this boa constrictor.

3. The table below shows the amount of money Lance has in his savings account where  $x$  is time (in months) and  $y$  is the account balance (in dollars).

$x$ (time)	$y$ (account balance)
0	160
3	385
6	610
9	835

Equation:

Use the table and equation to tell the story of Lance's savings.

4. The cost to rent a jet ski is \$80 per hour. In addition there also includes fee for a lesson on how to use the jet ski. Steve rented a jet ski for 3 hours and the total bill was \$265. Write the equation for the total cost of renting a jet ski  $y$  for  $x$  hours.

Equation:  $y = 80x + 25$

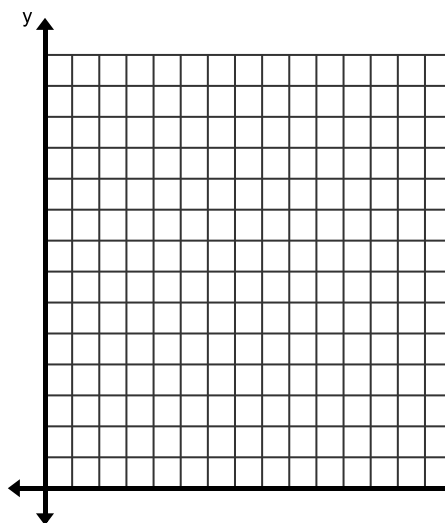
Use your equation to add more details to the story about renting a jet ski.

The initial fee of renting a jet ski is \$25 (cost of the lesson).

5. In order to make the playoff a soccer team must get 20 points during the regular season. The team gets 2 points for a win and 1 point for a tie. A team earns just enough points to make the playoffs. Let  $x$  represent the number of wins and  $y$  represent the number of ties.

- Write an equation to relating the all the possible values of  $x$  and  $y$  that will let the team make the playoffs.
- Write the equation in Slope-Intercept Form.
- If the team wins 8 games, how many tie games will need to occur?

6. The cost of a party at The Little Gym is \$250 which includes cake, pizza, and admission for any number of children. Create the graph and equation of this situation where  $x$  is the number of children and  $y$  is the total cost.



Equation:

### 3.1h Self-Assessment: Section 3.1

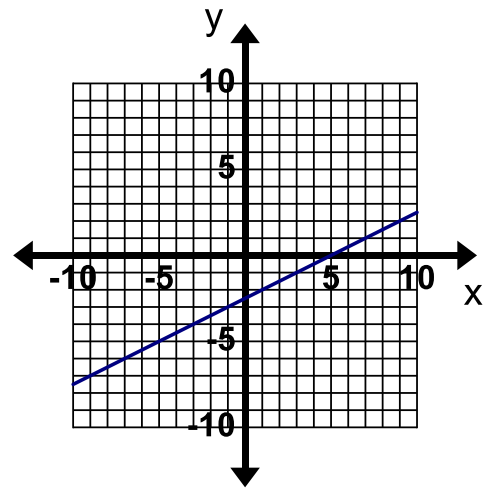
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
<p>1. Write a linear equation in the form <math>y = mx + b</math> given any of the following:</p> <ul style="list-style-type: none"> <li>• slope and y-intercept</li> <li>• slope and a point</li> <li>• two points</li> <li>• a table</li> <li>• a graph of a linear relationship</li> <li>• a context of a real world situation</li> </ul> <p><i>See sample problem #1</i></p>	I can write an equation for 1 or 2 of the given conditions.	I can write an equation for 3 or 4 of the given conditions.	I can write an equation for 5 or 6 of the given conditions.	I can write an equation for all six of the given conditions. In addition I can explain my steps in my own words.
<p>2. Graph linear relationships given any of the following:</p> <ul style="list-style-type: none"> <li>• an equation</li> <li>• slope and a point</li> </ul> <p><i>See sample problem #2</i></p>	I can graph the linear relationship given an equation in slope-intercept form.	I can graph the linear relationship given a slope and point.	I can graph the linear relationship from an equation in slope-intercept form. I can graph a linear relationship given a slope and point.	I can graph the linear relationship from an equation given in slope-intercept form and an equation that is not in slope intercept form. I can graph a linear relationship given a slope and point.

### Sample Problem #1

For each problem below write a linear equation in the form  $y = mx + b$  for the given conditions.

- a. The line has a slope of  $-\frac{4}{3}$ , and a y-intercept of  $(0, -1)$ .
- b. The line has a slope of  $-2$  and passes through the point  $(4, -3)$ .
- c. The line contains the points  $(0, -1)$  and  $(3, -2)$ .
- d.

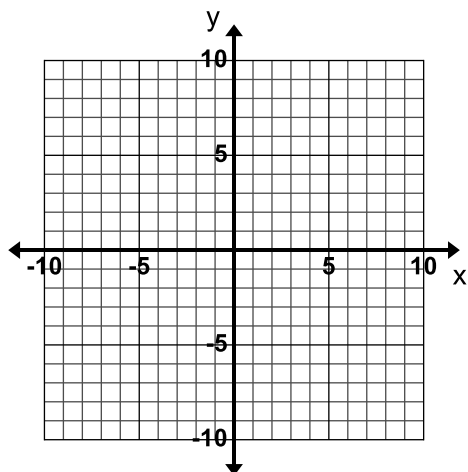


- e. A cat is running away from a dog. After 5 seconds it is 16 feet away from the dog and after 11 seconds it is 28 feet away from the dog. Let  $x$  represent the time in seconds that have passed and  $y$  represent the distance in feet that the cat is away from the dog.

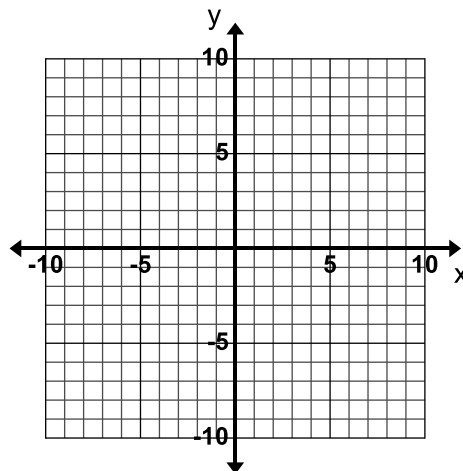
### Sample Problem #2

Graph the linear relationships given the following conditions.

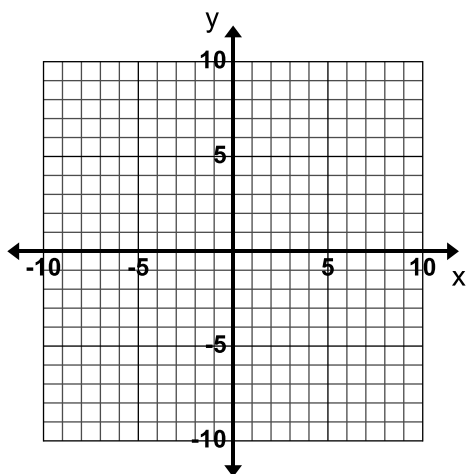
a. The equation of the line is  $y = -\frac{4}{3}x - 1$ .



b. The equation of the line is  $2x + 3y = -6$ .

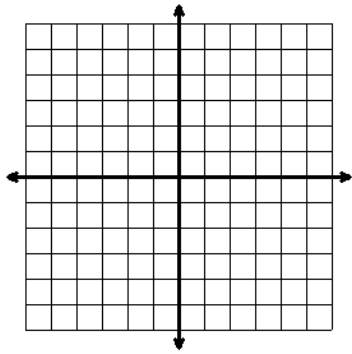


c. The line has a slope of  $-2$  and passes through the point  $(-4, -4)$ .

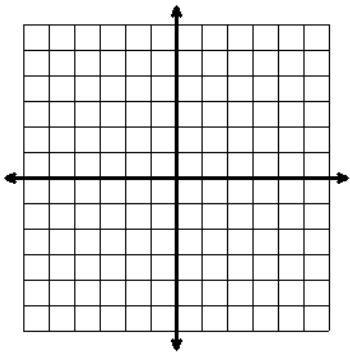


Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

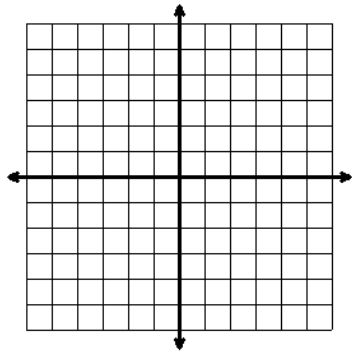
**MONDAY**



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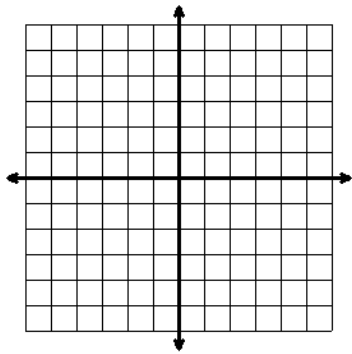


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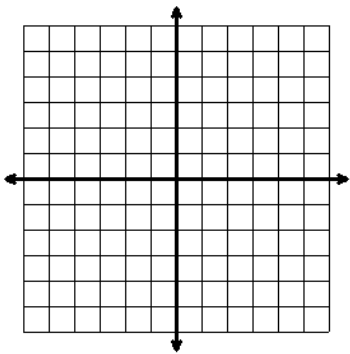


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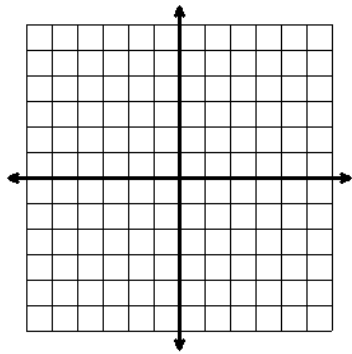
**TUESDAY**



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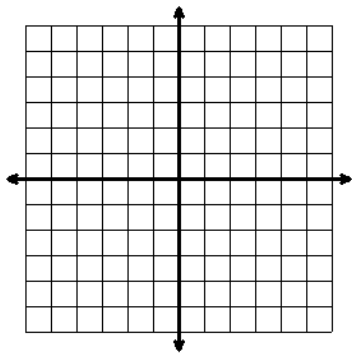


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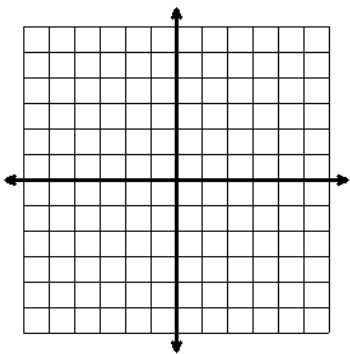


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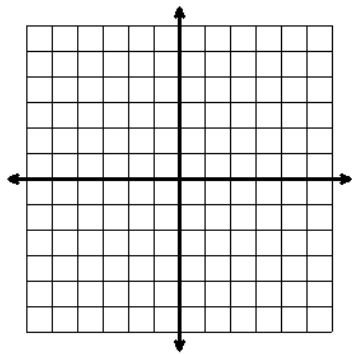
**WEDNESDAY**



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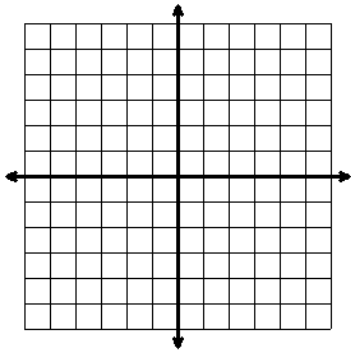


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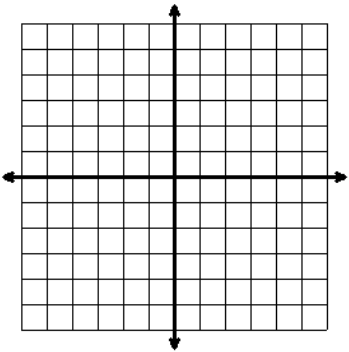


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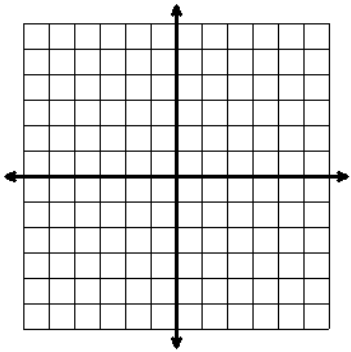
THURSDAY



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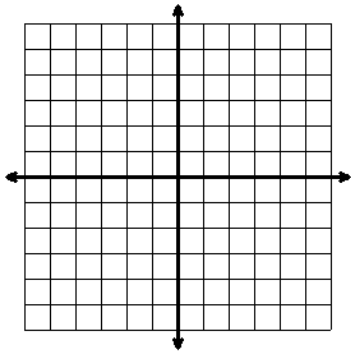


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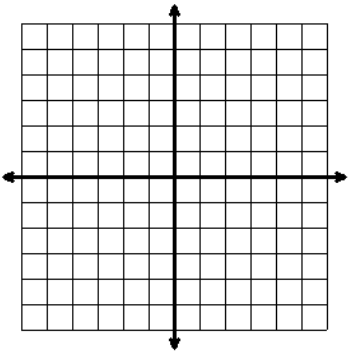


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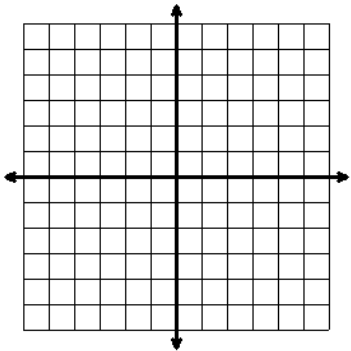
FRIDAY



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## Section 3.2: Relate Slopes and Write Equations for Parallel and Perpendicular Lines

### Section Overview:

Students begin this section by investigating the effects of changes in the slope and y-intercept of a line, describing the transformation (translations, rotations, and reflections) that has taken place and write new equations that reflect the changes in  $m$  and  $b$ . In the next lesson students use transformations to discover how the slopes of parallel and perpendicular lines are related. Once students have an understanding of the relationship between the slopes of parallel and perpendicular lines, students write equations of lines that are parallel or perpendicular to a given line.

### Concepts and Skills to Master:

*By the end of this section, students should be able to:*

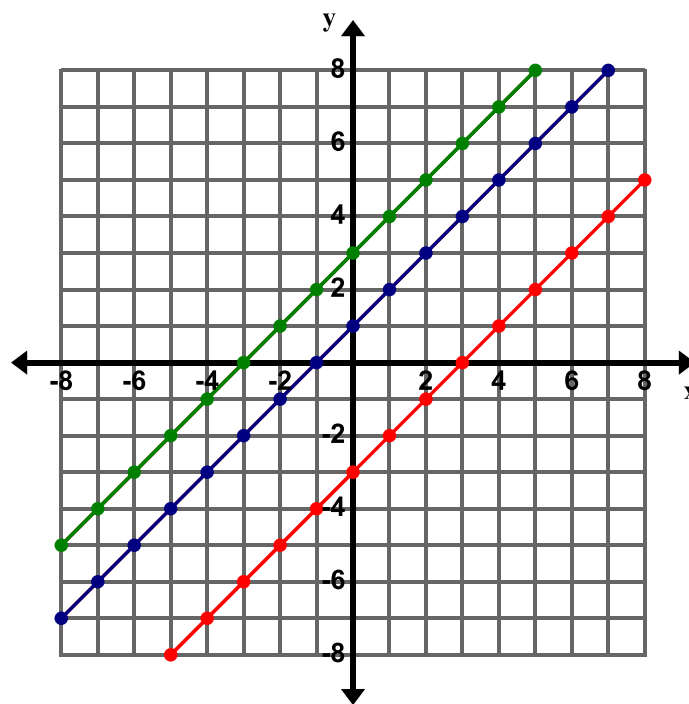
- Write a new equation for a line given a described transformation.
- Describe a transformation of a graph given a change in its equation (change in the slope or y-intercept).
- Compare the slopes of parallel lines and explain the transformation that creates parallel lines.
- Compare the slopes of perpendicular lines and explain the transformation that creates perpendicular lines.
- Write the equation of a line parallel to a given line that passes through a given point.
- Write the equation of a line perpendicular to a given line that passes through a given point.



### 3.2a Class Activity: Equations for Graph Shifts



1. Graph the equation  $y = x + 3$  and label the line with the equation.
  - a. Predict how the graph of  $y = x + 1$  will compare to the graph of  $y = x + 3$ .
  - b. Predict how the graph of  $y = x - 3$  will compare to the graph of  $y = x + 3$ .
  - c. Graph the following equations on the same grid and label each line with its equation.  
 $y = x + 1$   
 $y = x - 3$
  - d. Were your predictions correct? Why or why not?
  - e. What is the relationship between the lines  $y = x + 3$ ,  $y = x + 1$ , and  $y = x - 3$ ?  
 The lines have the same slope but different y-intercepts. The lines are a vertical translation of each other (they are also a horizontal translation of each other). Students may also describe it as a shift up or down.  
 The lines are parallel.



It may help students to color code their lines in this section using colored pencils. It would also be great to have patty paper so that students can investigate the transformations taking place. Remind students that in Chapter 2 they looked at a transformation of a line called a translation. This line has been translated.

- f. Write a different equation that would be parallel to the equations in this problem.  
 Answers will vary but the equation has to have a slope of 1.
- g. Describe the movement of a line when  $b$  is increased or decreased while  $m$  is held constant.  
 The line is shifted up or down (vertical translation).

2. Graph the equation  $y = 2x - 4$  and label the line with the equation.

a. Predict how the graph of  $y = x - 4$  will compare to the graph of  $y = 2x - 4$ .

b. Predict how the graph of  $y = \frac{1}{2}x - 4$  will compare to the graph of  $y = 2x - 4$ .

c. Predict how the graph of  $y = -2x - 4$  will compare to the graph of  $y = 2x - 4$ .

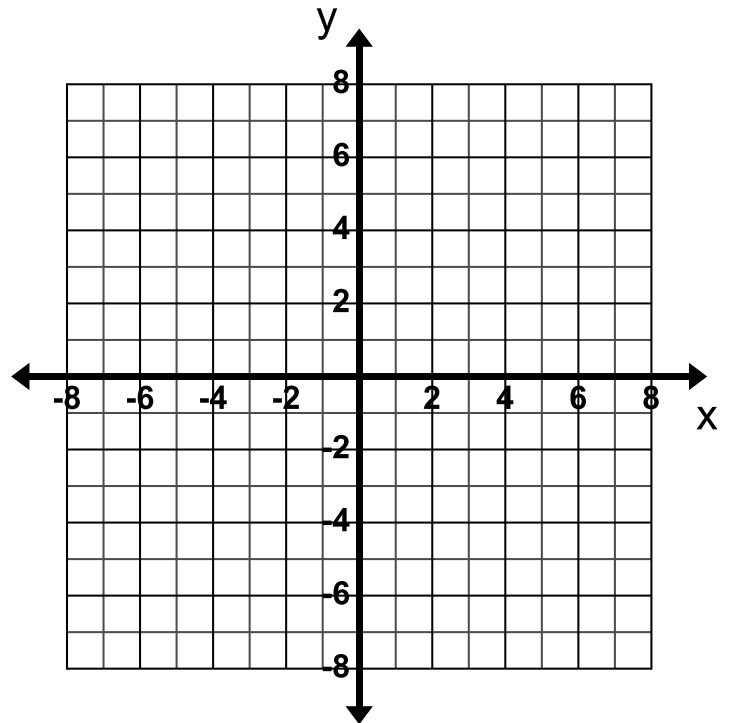
d. Graph the following equations and label each line with its equation.

$$y = x - 4$$

$$y = \frac{1}{2}x - 4$$

$$y = -2x - 4$$

e. Were your predictions correct? Why or why not?



f. Describe the movement of a line when the slope is increased or decreased while the y-intercept is held constant.

g. Describe the movement of a line when  $m$  is changed to  $-m$ .

h. Write the equation of a line that would be steeper than all of the equations in this problem.

3. Consider the equation  $y = 2x + 4$ . Write a new equation that would transform the graph of  $y = 2x + 4$  in the ways described below.
  - a. I want the slope to stay the same but I want the line to be shifted up 2 units.  
 $y = 2x + 6$
  - b. I want the y-intercept to stay the same but I want the line to be less steep.  
 $y = \frac{1}{2}x + 4$  (answers will vary)
  - c. I want a line that is parallel to  $y = 2x + 4$  but I want the line to be translated down 7 units.  
 $y = 2x - 3$
4. Describe the relationship and transformation of the graphs of the following equations compared to the graph of the equation  $y = 4x - 7$ .
  - a.  $y = 2x - 7$
  - b.  $y = 4x + 9$
  - c.  $y = -4x - 7$
  - d.  $y = 4x - 5$
5. Describe the relationship and transformation of the graphs of the following equations compared to the graph of the equation  $y = -\frac{1}{2}x - 3$ .
  - a.  $y = -\frac{1}{2}x$  This line is a vertical translation 3 units up from  $y = -\frac{1}{2}x - 3$ . The lines are parallel.
  - b.  $y = -2x - 3$  This line is a rotation of  $y = -\frac{1}{2}x - 3$  about the point  $(0, -3)$ . This line is steeper.
  - c.  $y = -\frac{1}{4}x - 3$  This line is a rotation of  $y = -\frac{1}{2}x - 3$  about the point  $(0, -3)$ . This line is less steep.
  - d.  $y = \frac{1}{2}x - 3$  This line is a reflection of  $y = -\frac{1}{2}x - 3$  across the y-axis (can also be described as a rotation about the point  $(0, -3)$ ). It is also a reflection over the line  $y=b$ .
  - e.  $y = -\frac{1}{2}x + 5$  This line is a vertical translation 8 units up from  $y = -\frac{1}{2}x - 3$ . The lines are parallel.

6. Consider the equation  $y = 3x + 2$ . Complete the chart below if the equation  $y = 3x + 2$  is changed in the in the ways described below.

Change the equation $y = 3x + 2 \dots$	New Equation	Describe the Graph Shift (from the original Equation) (use the words rotation, reflection and/or translation)
Change the y-intercept to 5 while keeping the slope constant	$y = 3x + 5$	The new line is translated 3 units up from the original line $y = 3x + 2$ .
Change the y-intercept to -3 while keeping the slope constant	$y = 3x - 3$	The new line is translated 5 units down from the original line $y =$ $3x + 2$ .
Change the slope to 1 while keeping the y-intercept constant		
Change the slope to 4 while keeping the y-intercept constant	$y = 4x + 2$	The new line is a rotation of the original line $y = 3x + 2$ about the point (0, 2) and the new line is steeper.
		The new line is translated 2 units down from the original line $y = 3x + 2$ .
Change the slope to $-3$ while keeping the y-intercept constant		
		The new line is a rotation of the original line $y = 3x + 2$ about the point (0, 2) and the new line is steeper.

### 3.2a Homework: Equations for Graph Shifts

1. Consider the equation  $y = x - 4$ . Write a new equation that would transform the graph of  $y = x - 4$  as described below. **See classwork #3 for a similar problem.**
  - a. I want the slope to stay the same but I want the line to be shifted up 3 units.
  - b. I want the y-intercept to stay the same but I want the line to be less steep.
  - c. I want a line that is parallel to  $y = x - 4$  but I want the line to be translated down 6 units.
2. Describe the relationship and transformation of the graphs of the following equations compared to the graph of the equation  $y = -3x$ .
  - a.  $y = 3x$
  - b.  $y = -3x - 4$   
**Vertical translation 4 units down; lines are parallel**
  - c.  $y = -2x$
  - d.  $y = -3x + 4$   
**Vertical translation 4 units up; lines are parallel**
3. Describe the relationship and the transformation of the graphs of the following equations compared to the graph of the equation  $y = \frac{4}{3}x + 4$ .
  - a.  $y = \frac{4}{3}x - 1$   
**Vertical translation 5 units down; lines are parallel**
  - b.  $y = \frac{4}{3}x$
  - c.  $y = 2x + 4$   
**y-intercepts are the same; this line is steeper**
  - d.  $y = -\frac{4}{3}x + 4$
  - e.  $y = \frac{1}{3}x + 4$

4. Consider the equation  $y = \frac{1}{2}x + 3$ . Complete the chart below if the equation  $y = \frac{1}{2}x + 3$  is changed in the ways described.

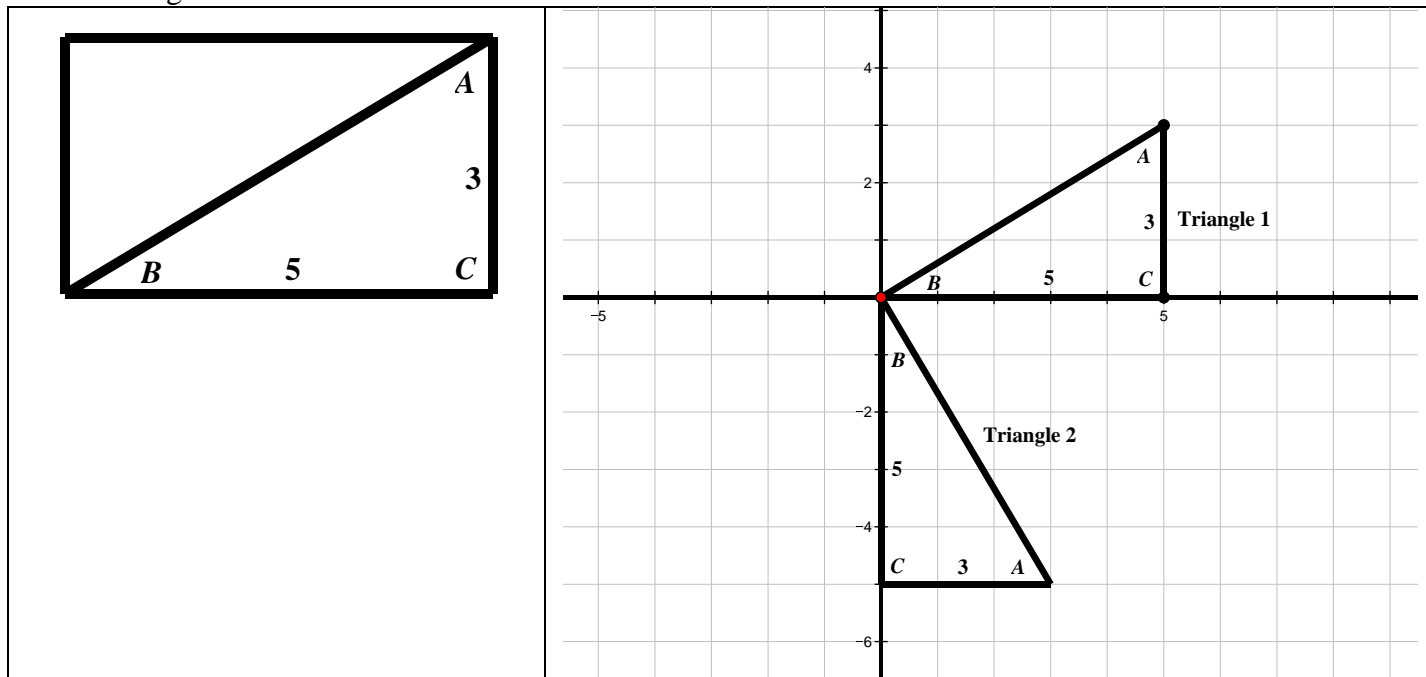
Change the equation $y = \frac{1}{2}x + 3 \dots$	New Equation	Describe the Graph Shift (from the original Equation) (use the words rotation, reflection and/or translation)
Change the y-intercept to 6 while keeping the slope constant	$y = \frac{1}{2}x + 6$	The new line is translated 3 units up from the line $y = \frac{1}{2}x + 3$ .
Change the slope to $-\frac{1}{2}$ while keeping the y-intercept constant		
	$y = \frac{1}{2}x - 2$	
Change the y-intercept to 0 while keeping the slope constant		
		The new line is a rotation of the equation $y = \frac{1}{2}x + 3$ about the point (0, 3) and the new line is less steep.
Change the slope to 2 while keeping the y-intercept constant		
Change the slope to $-2$ while keeping the y-intercept constant	$y = -2x + 3$	The new line is a rotation of the equation $y = \frac{1}{2}x + 3$ about the point (0, 3) and the new line is steeper. This is actually a $90^\circ$ rotation so these lines are perpendicular – students will explore this in the next section.

5. Describe the transformation (graph shift) that occurs in each of the following situations. Use words like translation, reflection, and rotation.
- The slope is increased or decreased while the y-intercept is held constant
  - The y-intercept is decreased while the slope is held constant
  - The slope  $m$  is changed to  $-m$
  - The y-intercept is increased while the slope is held constant

### 3.2b Class Activity: Slopes of Perpendicular Lines

Materials: Graph paper (one inch grid), 3 by 5 card, straight edge, scissors.

1. On your 3 x 5 card, draw the diagonal (as shown in the 1<sup>st</sup> box below). Label as shown below. Then cut the card into two triangles.
2. On your graph paper, draw the  $x$  and  $y$  axis as shown in the 2<sup>nd</sup> box below. Trace your triangle to create Triangles 1 and 2 as shown below.



3. Highlight the hypotenuse  $\overline{AB}$  of each triangle. Find the slope and equation of each hypotenuse:

a. Triangle 1	b. Triangle 2
Hypotenuse Slope: $\frac{3}{5}$	Hypotenuse Slope: $-\frac{5}{3}$
Equation of the Hypotenuse Line: $y = \frac{3}{5}x$	Equation of the Hypotenuse Line: $y = -\frac{5}{3}x$

**Important NOTE:** For purposes of the questions below, it is given that the 3 x 5 card is a rectangle and therefore has 90 degree angles. Use the card to help view perpendicular lines or a 90 degree rotation.

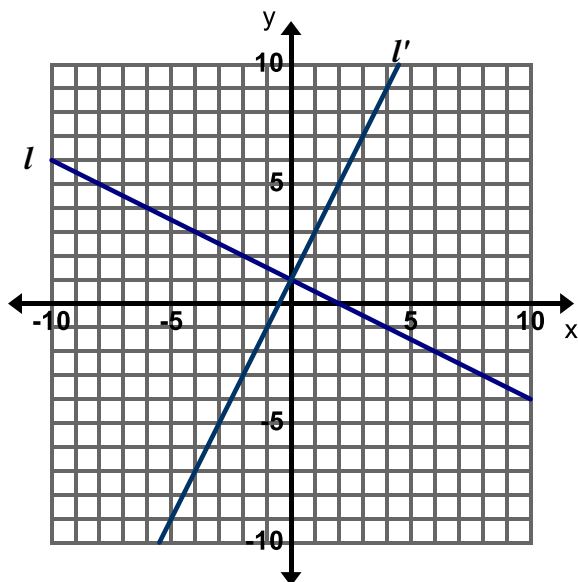
4. Describe the transformation(s) needed to carry Triangle 1 onto Triangle 2.  
**Rotation of 90° clockwise about the origin.**
5. What is the angle formed by the two hypotenuses at their  $y$ -intercept intersection? How do you know? How can you prove the measure of that angle?  
**90°; one way to informally prove it is to use the corner of the index card to measure the angle**
6. Consider the transformation that carries Triangle 1 to Triangle 2. What happens to the rise and run of the slope of the hypotenuse when you rotate the triangle 90°? Relate this to the slopes in your equations above.  
**The rise and run interchange and there is a sign change in just one of them.**

7. Is there another way you can rotate Triangle 1 so that the hypotenuses of Triangle 1 and Triangle 2 are perpendicular? Observe what happens to the rise and run that form the slope of  $\overline{AB}$ .  
 Rotate it  $90^\circ$  counter-clockwise about the origin. The rise and run interchange and there is a sign change in just one of them.

8. What does this activity tell us about the slopes of perpendicular lines?  
 The slopes of perpendicular lines are opposite reciprocals (the product of the slopes is  $-1$ )

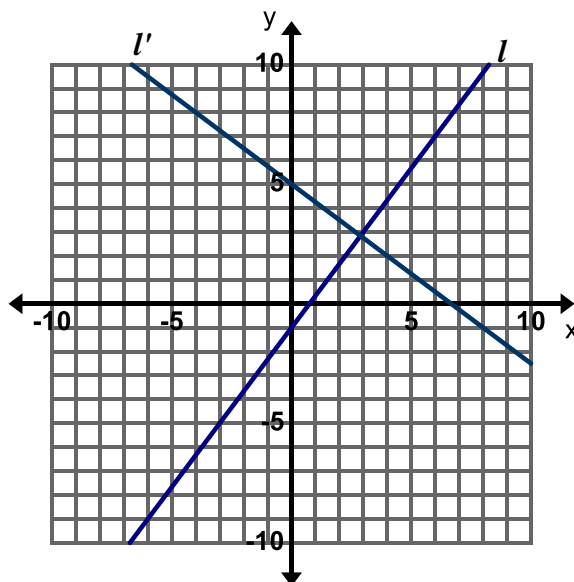
**Directions:** Use the pairs of lines in the graphs below to answer the questions that follow. Remember a transformation is a translation (slide), reflection (flip), or rotation (turn).

9. Pair 1



- Describe the transformation that carries line  $l$  to line  $l'$ .  
 Rotation of  $90^\circ$  clockwise about the  $(0, 1)$
- Find the slope of each of the lines.  
 $l = -\frac{1}{2}$ ;  $l' = 2$
- Describe how the lines and slopes are related.  
 The lines are perpendicular and the slopes are opposite reciprocals (the product of the slopes is  $-1$ )

10. Pair 2



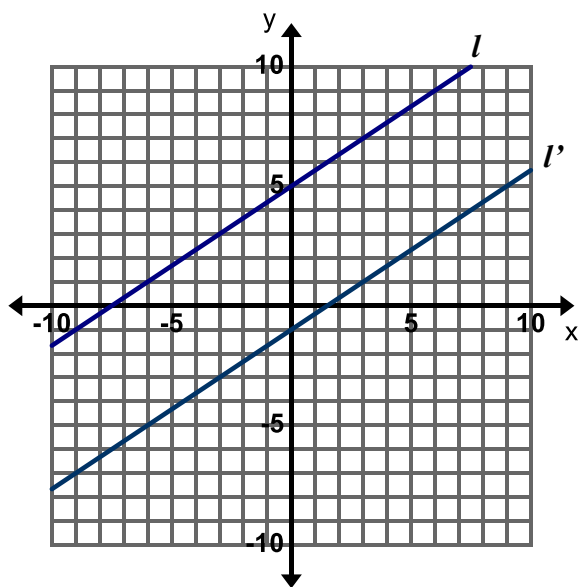
- Describe the transformation that carries line  $l$  to line  $l'$ .
- Find the slope of each of the lines.
- Describe how the lines and slopes are related.



### 3.2b Homework: Slopes of Parallel Lines

**Directions:** Use the pairs of lines in the graphs below to answer the questions that follow. Remember a transformation is a translation (slide), reflection (flip), or rotation (turn).

1. Pair 1



- a. Describe the transformation that carries line  $l$  to line  $l'$ .

vertical translation 6 units down

- b. Find the slope of each line. What do you observe about the slopes?

$l$ :  $\frac{2}{3}$

$l'$ :  $\frac{2}{3}$

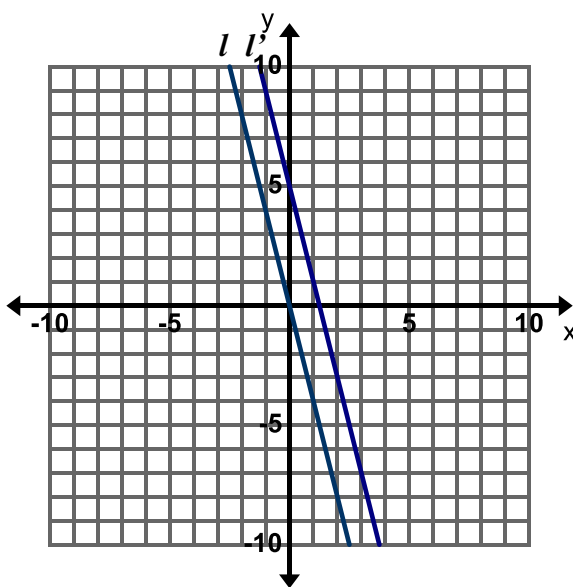
The slopes are the same.

- c. Write an equation for each line. How are the equations the same and how are they different?

$l$ :  $y = \frac{2}{3}x + 5$

$l'$ :  $y = \frac{2}{3}x - 1$

2. Pair 2



- a. Describe the transformation that carries line  $l$  to line  $l'$ .

- b. Find the slope of each line. What do you observe about the slopes?

$l$ :

$l'$ :

- c. Write an equation for each line. How are the equations the same and how are they different?

$l$ :

$l'$ :

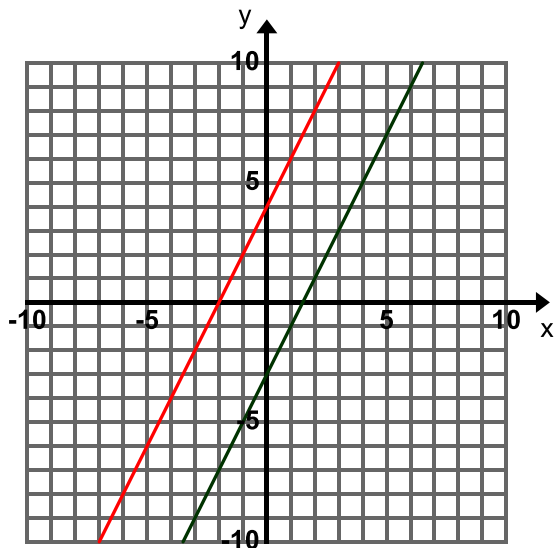
3. Given the graphs of two or more lines how can you determine if they are parallel?

### 3.2c Class Activity: Equations of Parallel and Perpendicular Lines

**Directions:** In the following problems, lines A and B are **parallel**. Graph and label both lines. Then write the equation of line B.

1. Line A:  $y = 2x - 3$

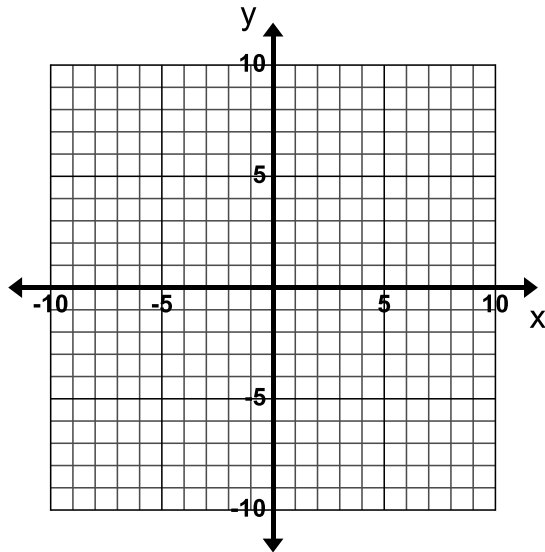
Line B: passes through  $(0, 4)$



Equation of Line B:  $y = 2x + 4$

2. Line A:  $y = -4x + 1$

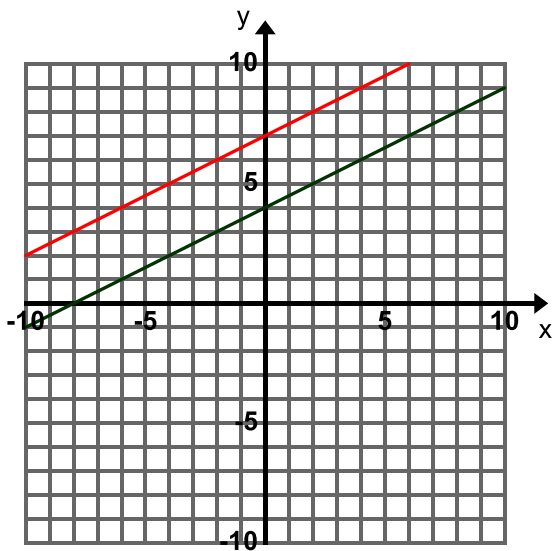
Line B: passes through  $(0, -5)$



Equation of Line B:

3. Line A:  $y = \frac{1}{2}x + 4$

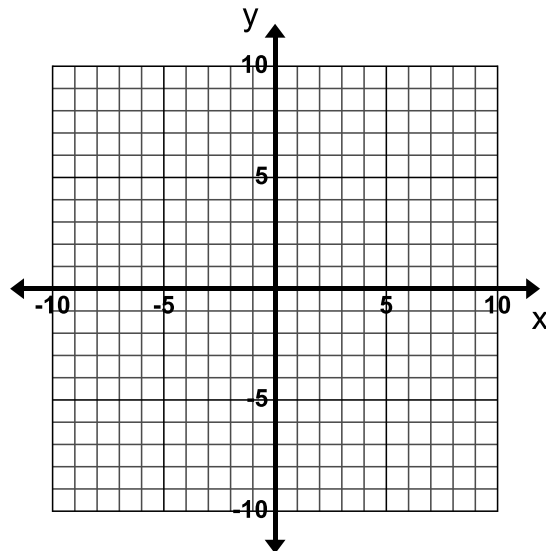
Line B: passes through  $(0, 7)$



Equation of Line B:  $y = \frac{1}{2}x + 7$

4. Line A:  $y = 4x$

Line B: passes through  $(3, -3)$



Equation of Line B:

**Directions:** In the following problems, lines A and B are parallel. Find the equation for line B without graphing.

5. Find the equation of line B which is **parallel** to line A and passes through (2, 3).

Line A:  $y = -3x + 7$

Line B:  $y = -3x + 9$

Hint: Remember that parallel lines have the same slope. In this problem, line A has a slope of  $-3$ . If Line B is parallel to line A, it will also have a slope of  $-3$ . In order to determine the  $y$ -intercept of line B, we use the slope and a point it passes through (2, 3).

$$y = mx + b$$

$$3 = (-3)(2) + b$$

$$3 = -6 + b$$

$$9 = b$$

6. Find the equation of line B which is **parallel** to line A and passes through  $(-6, 2)$ .

Line A:  $y = \frac{1}{3}x + 2$

Line B:

7. Given the slope of a line, how do you figure out the slope of a line **perpendicular** to it?

The slope of the perpendicular line is the opposite reciprocal of the given slope.

8. Give the slope of a line that is **perpendicular** to the following lines:

a.  $y = 3x - 2$ ;  $m$  of perpendicular line:  $-\frac{1}{3}$

b.  $y = -\frac{2}{3}x$ ;  $m$  of perpendicular line:

c.  $y = -x + 2$ ;  $m$  of perpendicular line:  $1$

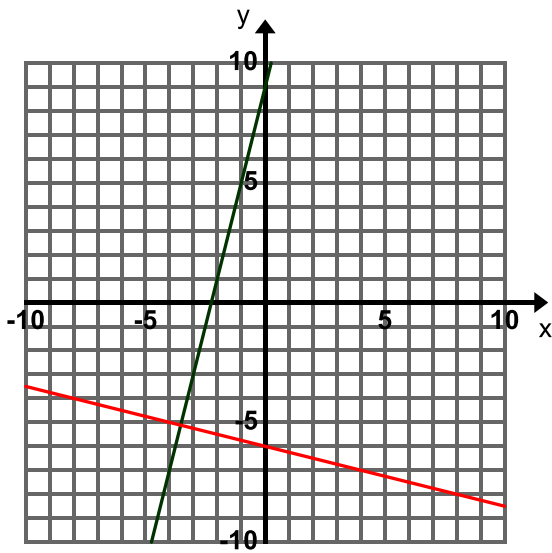
d.  $y = -2x + 6$ ;  $m$  of perpendicular line:

**Directions:** In the following problems, lines A and B are **perpendicular**. Graph and label both lines. Then write the equation of line B.

9. Line A:  $y = 4x + 9$

What is the slope of line B?  $-\frac{1}{4}$

Line B: passes through  $(4, -7)$



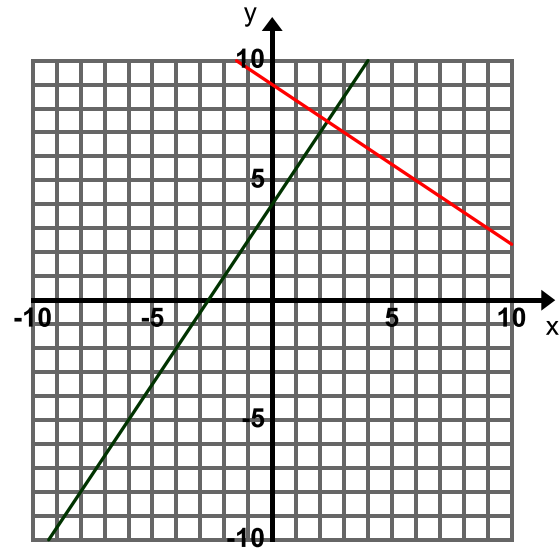
Equation of Line B:  $y = -\frac{1}{4}x - 6$

10. Line A:  $2y = 3x + 8$

Rewrite as  $y = \frac{3}{2}x + 4$

What is the slope of line B?  $-\frac{2}{3}$

Line B: passes through  $(3, 7)$



Equation of Line B:  $y = -\frac{2}{3}x + 9$

**Directions:** In the following problems, lines A and B are **perpendicular**. Find the equation for line B.

<p>11. Find the equation of the line B which is <b>perpendicular</b> to line A and passes through (3, 7).</p> <p>Line A: <math>y = -3x + 7</math></p> <p>Line B:</p> <p>Remember slopes of perpendicular lines are opposite reciprocals so the slope of line B will be <math>\frac{1}{3}</math>. Use this information and the point (3, 7) to find the y-intercept for line B. Then, you will have the pieces you need to write the equation of Line B.</p>	<p>12. Find the equation of Line B which is <b>perpendicular</b> to line A and passes through (2, 4).</p> <p>Line A: <math>y = -\frac{1}{2}x - 2</math></p> <p>Line B:</p>
-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	----------------------------------------------------------------------------------------------------------------------------------------------------------------------------

**Directions:** Determine if the following sets of lines are parallel, perpendicular, or neither. Justify your answer.

<p>13. Line A: <math>y = \frac{3}{4}x + 1</math></p> <p>Line B: <math>y = \frac{3}{4}x - 5</math></p> <p>Parallel; same slope</p>	<p>14. Line A: <math>y = \frac{3}{4}x + 1</math></p> <p>Line B: <math>y = -\frac{3}{4}x + 1</math></p> <p>Neither; the slopes are not the same nor negative reciprocals of each other</p>
<p>15. Line A: <math>y = \frac{3}{4}x + 1</math></p> <p>Line B: <math>y = \frac{4}{3}x + 1</math></p>	<p>16. Line A: <math>y = \frac{3}{4}x + 1</math></p> <p>Line B: <math>y = -\frac{4}{3}x + 1</math></p>
<p>17. Line A: <math>y = 3x + 2</math></p> <p>Line B: <math>y = -3x + 2</math></p>	<p>18. Line A: <math>y = 3x + 2</math></p> <p>Line B: <math>y = 3x + 5</math></p>

19. Line A: $y = 3x + 2$ Line B: $y = -\frac{1}{3}x$ Perpendicular; slopes are opposite reciprocals	20. Line A: $y = 3x + 2$ Line B: $y = \frac{1}{3}x + 2$
21. Line A: $y = \frac{1}{2}x + 1$ Line B: $6x + 3y = 18$	22. Line A: $4x - 2y = -6$ Line B: $-6x + y = -4(x - 2)$

**Directions:** Determine whether the lines through the pairs of points are parallel, perpendicular, or neither.

23. $(-3, 1)$ and $(2, 3)$ $(-3, 5)$ and $(-1, 0)$ perpendicular	24. $(-3, -1)$ and $(-1, -3)$ $(-1, 2)$ and $(-4, -1)$	25. $(1, 8)$ and $(-1, 1)$ $(0, 7)$ and $(2, 4)$ neither
26. $(2, 0)$ and $(1, 6)$ $(1, 3)$ and $(7, 4)$	27. $(-3, 0)$ and $(-2, 4)$ $(2, -1)$ and $(1, -5)$ parallel	28. $(-3, 4)$ and $(3, 7)$ $(4, 2)$ and $(-2, 6)$

For #23 – 28, use the slope formula to find the slope of the line that passes through each set of points.

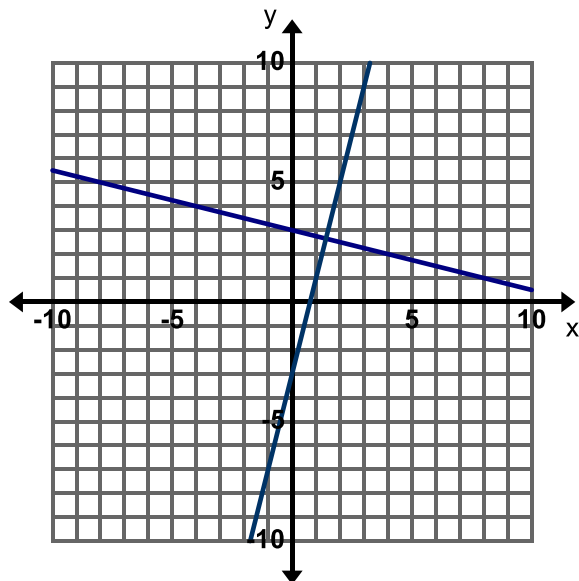
Example:  $(-3, 1)$  and  $(2, 3)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{2 - -3} = \frac{2}{5}$$

### 3.2c Homework: Equations of Parallel and Perpendicular Lines

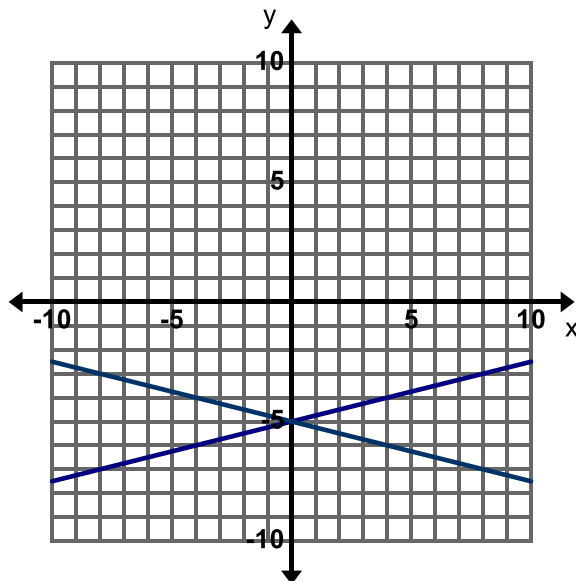
**Directions:** Determine if the following sets of lines in each graph are parallel or perpendicular. Justify your answer. If they are not parallel or perpendicular describe the transformation that carries one line to the other.

1. Parallel, Perpendicular, or Neither?



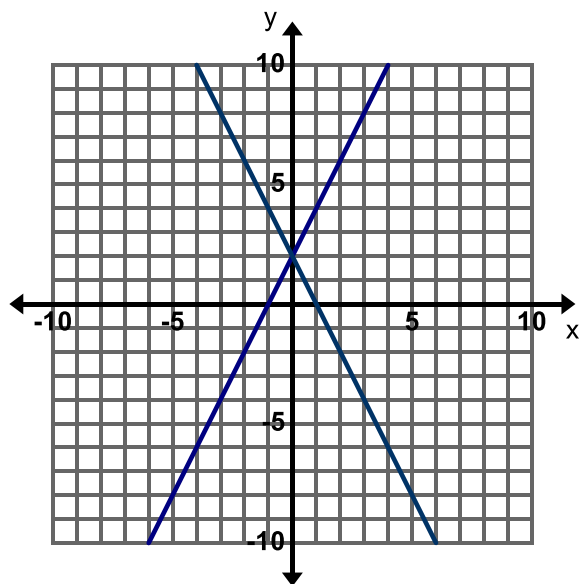
Justification: **Perpendicular**; slope of one of the lines is 4 and the slope of the other line is  $-\frac{1}{4}$

2. Parallel, Perpendicular, or Neither?



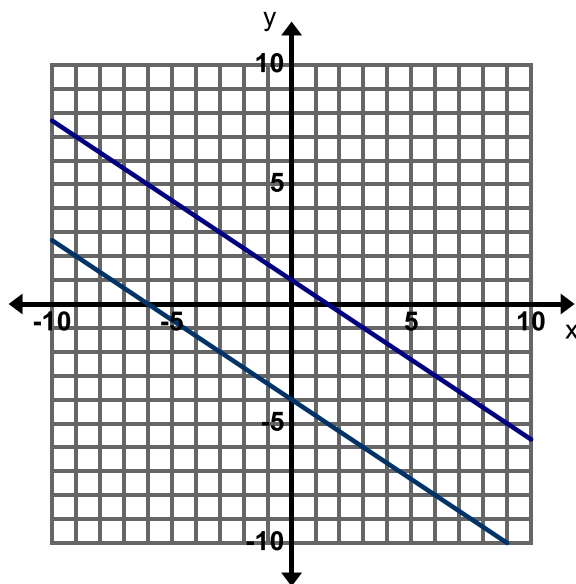
Justification:

3. Parallel, Perpendicular, or Neither?



Justification:

4. Parallel, Perpendicular, or Neither?



Justification:

**Directions:** Determine if the following sets of lines are parallel, perpendicular, or neither. Justify your answer.

<p>5. Line A: <math>y = \frac{1}{4}x - 3</math></p> <p>Line B: <math>y = -4x + 3</math></p> <p><b>Perpendicular, slopes are opposite reciprocals</b></p>	<p>6. Line A: <math>y = \frac{1}{4}x - 3</math></p> <p>Line B: <math>y = \frac{1}{4}x + 3</math></p> <p><b>Parallel, slopes are the same</b></p>
<p>7. Line A: <math>y = \frac{1}{4}x + 2</math></p> <p>Line B: <math>y = -\frac{1}{4}x + 2</math></p>	<p>8. Line A: <math>y = \frac{1}{4}x - 5</math></p> <p>Line B: <math>y = 4x + 5</math></p>
<p>9. Line A: <math>y = \frac{2}{3}x + 1</math></p> <p>Line B: <math>2x - 3y = 6</math></p>	<p>10. Line A: <math>2x - 3y = -9</math></p> <p>Line B: <math>3x + 2y = -8</math></p>

**Directions:** Determine whether the lines through the pairs of points are parallel, perpendicular, or neither.

**Hint:** Use the slope formula to determine the slope of the line that passes through each set of points. See class work #23 – 28 for an example.

<p>11. <math>(-2, 0)</math> and <math>(4, 3)</math> <math>(0, 0)</math> and <math>(1, -2)</math></p>	<p>12. <math>(-2, -11)</math> and <math>(-1, -7)</math> <math>(2, -11)</math> and <math>(-1, 1)</math></p> <p><b>neither</b></p>	<p>13. <math>(0, 0)</math> and <math>(3, 4)</math> <math>(-1, -1)</math> and <math>(2, 3)</math></p>
<p>14. <math>(4, 12)</math> and <math>(2, 6)</math> <math>(4, -12)</math> and <math>(2, -6)</math></p>	<p>15. <math>(-1, -5)</math> and <math>(0, -4)</math> <math>(-1, -3)</math> and <math>(0, -4)</math></p> <p><b>perpendicular</b></p>	<p>16. <math>(-2, 9)</math> and <math>(0, 1)</math> <math>(3, 13)</math> and <math>(-1, -3)</math></p>

**Directions:** Write the equation of the line using algebra. (Do not graph the equations to write the equation.)

17. Write the equation of the line that is **perpendicular** to  $y = \frac{2}{3}x - 5$  and passes through the point  $(2, 5)$ .

$$y = -\frac{3}{2}x + 8$$

18. Write the equation of the line that is **perpendicular** to  $y = -5x + 2$  and passes through the point  $(10, -4)$ .

19. Write the equation of the line that is **parallel** to  $y = -3x + 2$  and passes through the point  $(-3, -2)$ .

20. Find the equation of the line that is **parallel** to  $y = \frac{3}{5}x - 4$  and passes through the point  $(5, 4)$ .



### 3.2d Self-Assessment: Section 3.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
<p>1. Write a new equation for a line given a described transformation.</p> <p><i>See sample problem #1</i></p>	I can identify the transformation but I do not know how it relates to the equation.	I can write a new equation for a line for one described transformation.	I can write a new equation for a line from both described transformations.	I can write a new equation for a line for both described transformations. I can also explain in my own words why the transformation changed the equation.
<p>2. Describe a transformation of a graph given a change in its equation(change in the slope or y-intercept).</p> <p><i>See sample problem #2</i></p>	I don't know how to see a transformation given a change in an equation.	I can describe one transformation.	I can accurately describe both transformations.	I can accurately describe both transformations. I can explain in my own words how the parts of the equation are related to the transformation.
<p>3. Compare the slopes of parallel lines and explain the transformation that creates parallel lines.</p> <p><i>See sample problem #3</i></p>	I can find the slopes of the lines but cannot describe the transformation that takes line $l$ to line $l'$ .	I can describe the transformation that takes line $l$ to line $l'$ but do not know how to find the slopes of the lines.	I can describe the transformation that takes line $l$ to line $l'$ and state the slope of each line.	I can describe the transformation that takes line $l$ to line $l'$ and state the slope of each line. I can also explain how the equations will be similar and different.

<p>4. Write the equation of a line parallel to a given line that passes through a given point.</p> <p><i>See sample problem #4</i></p>	<p>I know that parallel lines have the same slope but I do not know how to write the equation of the line that passes through the point (2,3).</p>	<p>I can graph the line given and the line that is parallel to it and goes through the point (2,3).</p>	<p>I can write the equation of the line that is parallel to <math>y = -3x + 7</math> and passes through the point (2, 3).</p>	<p>I can write the equation of the line that is parallel to <math>y = -3x + 7</math> and passes through the point (2, 3). I can also determine the transformation that moves the original line to the given point.</p>
<p>5. Compare the slopes of perpendicular lines and explain the transformation that creates perpendicular lines.</p> <p><i>See sample problem #5</i></p>	<p>I can find the slope of one of the lines.</p>	<p>I know how to find the slope of the lines but cannot describe the transformation that creates these lines.</p>	<p>I know how to find the slopes of perpendicular lines and can describe the transformation that creates these lines.</p>	<p>I know how to find the slopes of perpendicular lines and can describe the transformation that creates these lines. I can explain how the transformation will affect the equation of the line.</p>
<p>6. Write the equation of a line perpendicular to a given line that passes through a given point.</p> <p><i>See sample problem #6</i></p>	<p>I can find the slopes of the lines but cannot describe the transformation that takes line <math>l</math> to line <math>l'</math>.</p>	<p>I can describe the transformation that takes line <math>l</math> to line <math>l'</math> but do not know how to find the slopes of the lines.</p>	<p>I can describe the transformation that takes line <math>l</math> to line <math>l'</math> and state the slope of each line.</p>	<p>I can describe the transformation that takes line <math>l</math> to line <math>l'</math> and state the slope of each line. I can also create my own examples of lines that are perpendicular through a given point.</p>

**Sample Problem #1**

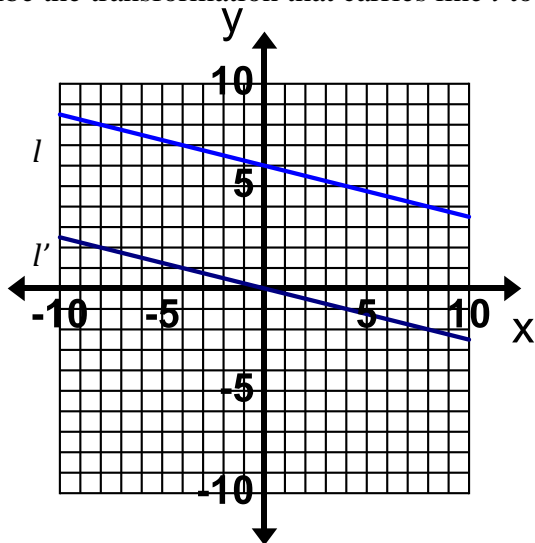
Consider the equation  $y = 3x + 2$ . Write a new equation that represents a line that is parallel to the original line and shifted down 3 units.

**Sample Problem #2**

What is the relationship and transformation of the graph of the equation  $y = \frac{4}{3}x + 4$  compared to the graph of the equation  $y = -\frac{4}{3}x + 5$ .

**Sample Problem #3**

Describe the transformation that carries line  $l$  to line  $l'$ .

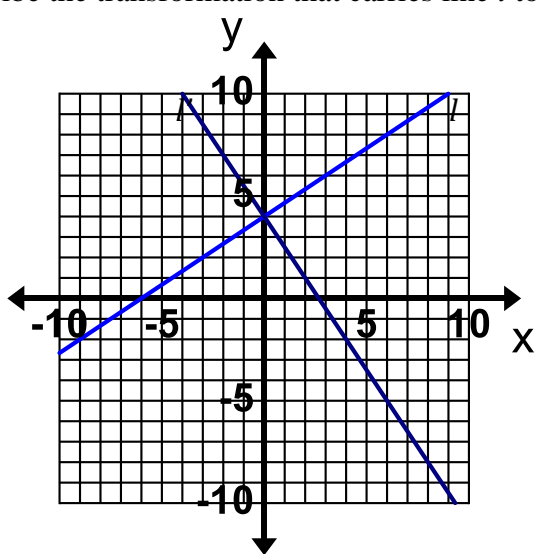


What is the slope of line  $l$ ?

What is the slope of line  $l'$ ?

**Sample Problem #5**

Describe the transformation that carries line  $l$  to line  $l'$ .



What is the slope of line  $l$ ?

What is the slope of line  $l'$ ?

**Sample Problem #4**

Write the equation of a line that is parallel to  $y = -3x + 7$  and passes through  $(2, 3)$ .

**Sample Problem #6**

Write the equation of the line that is perpendicular to  $y = -5x + 2$  and passes through the point  $(10, -4)$ .