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Chapter 4: Simultaneous Linear Equations (3 weeks)

Utah Core Standard(s):

- Analyze and solve pairs of simultaneous linear equations. (8.EE.8)
 - a) Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
 - b) Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
 - c) Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Academic Vocabulary: system of linear equations in two variables, simultaneous linear equations, solution, intersection, ordered pair, elimination, substitution, parallel, no solution, infinitely many solutions

Chapter Overview:



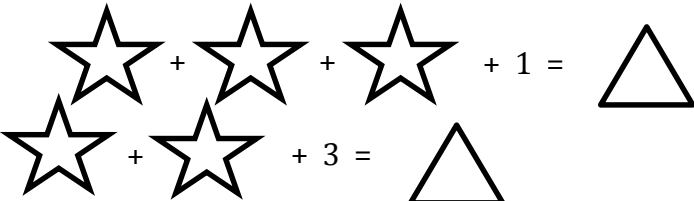
In this chapter we discuss intuitive, graphical, and algebraic methods of solving simultaneous linear equations; that is, finding all pairs (if any) of numbers (x, y) that are solutions of both equations. We will use these understandings and skills to solve real world problems leading to two linear equations in two variables.


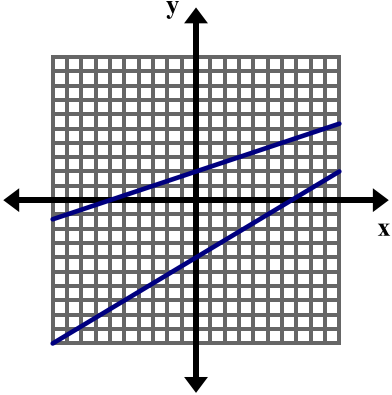


Connections to Content:




Prior Knowledge: In chapter 1, students learned to solve one-variable equations using the laws of algebra to write expressions in equivalent forms and the properties of equality to solve for an unknown. They solved equations with one, no, and infinitely many solutions and studied the structure of an equation that resulted in each of these outcomes. In chapter 3, students learned to graph and write linear equations in two-variables. Throughout, students have been creating equations to model relationships between numbers and quantities.

Future Knowledge: In subsequent coursework, students will gain a conceptual understanding of the process of elimination, examining what is happening graphically when we manipulate the equations of a linear system. They will also solve systems that include additional types of functions.

MATHEMATICAL PRACTICE STANDARDS

	<p>Make sense of problems and persevere in solving them.</p>	<p>Kevin and Nina are competing in a bike race. When Kevin is ninety miles into the race, he is in first place. Nina is in second place and is 15 miles behind Kevin.</p> <ol style="list-style-type: none"> From this point, Kevin continues the race at a constant rate of 25 mph and Nina continues the race at a constant rate of 30 mph. When will Nina catch Kevin? Solve this problem using any method you wish. If the race is 150 miles long, who will win? Assume Nina and Kevin bike at the speeds given in part a). Now suppose the following: Ninety miles into the race, Kevin is still in first place and Nina is still in second place, 15 miles behind Kevin. But now Kevin and Nina both finish out the race at a speed of 30 mph. When will Nina catch Kevin? If the race is 150 miles long, who will win? <p><i>The goal of this problem is that students will have the opportunity to explore a problem that can be solved using simultaneous linear equations from an intuitive standpoint, providing insight into graphical and algebraic methods that will be explored in the chapter. Students also gain insight into the meaning of the solution(s) to a system of linear equations. This problem requires students to analyze givens, constraints, relationships, and goals. Students may approach this problem using several different methods: picture, bar model, guess and check, table, equation, graph, etc.</i></p>
	<p>Reason abstractly and quantitatively.</p>	<p>Write a system of equations for the model below and solve the system using substitution.</p> <div style="text-align: center;">  </div> <p><i>This chapter utilizes a pictorial approach in order to help students grasp the concepts of substitution and elimination. Students work with this concrete model and then transition into an abstract model as they begin to manipulate the equations in order to solve the system.</i></p>

	<p>Construct viable arguments and critique the reasoning of others.</p>	<p>How many solutions does the system of linear equations graphed below have? How do you know?</p>  <p><i>In order to answer this question students must understand that the graph of an equation shows all of the ordered pairs that satisfy the equation and that when we graph the equation of a line we see a limited view of that line. They must also understand what the solution to a system of linear equations is and how the solution is determined graphically. Students will use this information, along with additional supporting statements, in order to make an argument as to the number of solutions to this system of equations.</i></p>
	<p>Model with mathematics.</p>	<p>The student officers are buying packs of streamers and balloons to decorate for a school dance. Packs of balloons cost \$3.50 and packs of streamers cost \$2. If the student officers bought a total of 12 packs of decorations and spent \$31.50, how many packs of balloons did they buy? How many packs of streamers did they buy? Write the solution in a complete sentence.</p> <p><i>The ability to create and solve equations gives students the power to solve many real world problems. They will apply the strategies learned in this chapter to solve problems arising in everyday life that can be modeled and solved using simultaneous linear equations.</i></p>
	<p>Use appropriate tools strategically.</p>	<p>A farmer saw some chickens and pigs in a field. He counted 60 heads and 176 legs. Determine exactly how many chickens and pigs he saw.</p> <ol style="list-style-type: none"> Solve the problem using the methods and strategies studied in this chapter. Which method do you prefer using to solve this problem? Use your preferred method to determine the number of chickens and pigs in a field with 45 heads and 146 legs. <p><i>While solving this problem, students should be familiar with and consider all possible tools available: graphing calculator, graph paper, concrete models, tables, equations, etc. Students may gravitate toward the use of a graphing calculator given the size of the numbers. This technological tool may help them to explore this problem in greater depth.</i></p>

	<p>Attend to precision.</p>	<p>Consider the equations $-2x + y = -1$ and $y = 2x + 4$. Make sure both equations are written in slope-intercept form, then graph both equations on the coordinate plane below and solve the system of linear equations. <i>Solving systems of equations both graphically and algebraically requires students to attend to precision while executing many skills including using the properties of equality and laws of algebra in order to simplify and rearrange equations, producing graphs of equations, and simplifying and evaluating algebraic expressions in order to find and verify the solution to a system of linear equations.</i></p>
	<p>Look for and make use of structure.</p>	<p>One equation in a system of linear equations is $6x + 4y = -12$.</p> <ol style="list-style-type: none"> Write a second equation for the system so that the system has one solution. Write a second equation for the system so that the system has no solution. Write a second equation for the system so that the system has infinitely many solutions. <p><i>In this problem, students must analyze the structure of the first equation in order to discern possible second equations that will result in one, infinitely many, or no solution.</i></p>
	<p>Look for and express regularity in repeated reasoning.</p>	<p>Gabriela and Camila like to race each other. Gabriela can run 10 feet/second while Camila can run 12 feet/second. Being a good sport, Camila gives Gabriela a 20-foot head start.</p> <p>How long will it take Camila to catch Gabriela?</p> <p><i>Students can use repeated reasoning in order to solve this problem. Realizing that each second Camila closes the gap between her and Gabriela by 2 feet, students may determine that it will take 10 seconds in order for Camila to catch Gabriela.</i></p>

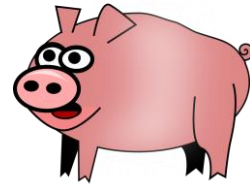
4.0 Anchor Problem: Chickens and Pigs

A farmer saw some chickens and pigs in a field. He counted 30 heads and 84 legs. Determine exactly how many chickens and pigs he saw. There are many different ways to solve this problem, and several strategies have been listed below. Solve the problem in as many different ways as you can and show your strategies below.



Strategies for Problem Solving

- Make a List or Table
- Draw a Picture or Diagram
- Guess, Check, and Revise
- Write an Equation or Number Sentence
- Find a Pattern
- Work Backwards
- Create a Graph
- Use Logic and Reasoning



Section 4.1: Understand Solutions of Simultaneous Linear Equations

Section Overview:

In this section, students are solving simultaneous linear equations that have one, no, or infinitely many solutions using intuitive and graphical methods. In order to access the problems initially students may use logic, and create pictures, bar models, and tables. They will solve simultaneous linear equations using a graphical approach, understanding that the solution is the point of intersection of the two graphs. Students will understand what it means to solve two linear equations, that is, finding all pairs (if any) of numbers (x, y) that are solutions to both equations and they will interpret the solution in a context.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Solve simultaneous linear equations by graphing.
2. Understand what it means to solve a system of equations.
3. Identify and provide examples of systems of equations that have one solution, infinitely many solutions, or no solution.
4. Interpret the solution to a system in a context.

4.1a Class Activity: The Bake Sale

- The student council is planning a bake sale to raise money for a local food pantry. They are going to be making apple and peach pies. They have decided to make 10 pies. Each pie requires 2 pounds of fruit; therefore they need a total of 20 pounds of fruit.



- In the table below, fill out the **first two columns only** with **8** possible combinations that will yield 20 pounds of fruit. **Some possible combinations are shown.**

# of Pounds of Apples	# of Pounds of Peaches	Cost of Apples	Cost of Peaches	Total Cost
0	20	0	20	20
20	0	40	0	40
10	10	20	10	30
These columns represent some of the different combinations of fruit that will yield 20 total pounds of fruit.		These columns represent the corresponding cost of the apples and peaches for each combination listed.		

- One pound of apples costs \$2 and one pound of peaches cost \$1. Fill out the rest of the table above to determine how much the student council will spend for each of the combinations.
- Mrs. Harper, the student council advisor, tells the students they have exactly \$28 to spend on fruit. How many pounds of each type of fruit should they buy so that they have the required 20 pounds of fruit **and** spend exactly \$28?

- d. If p represents the number of pounds of peaches purchased and a represents the number of pounds of apples purchased, the situation above can be modeled by the following equations:

$$p + a = 20$$

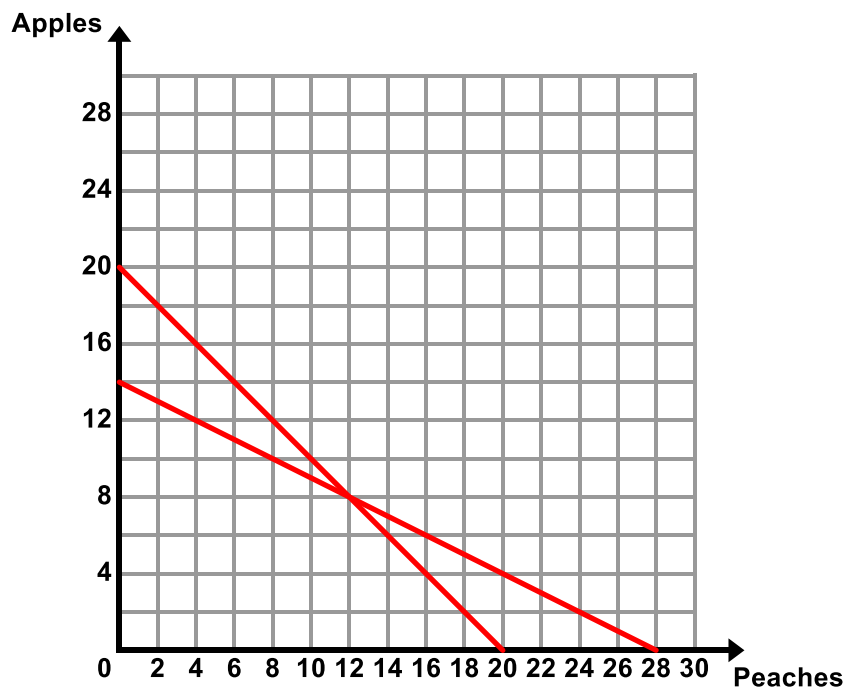
$$2a + p = 28$$

Write in words what each of these equations represents in the context.

$p + a = 20$ _____ **Total Number of Pounds of Fruit** _____

$2a + p = 28$ _____ **Total Cost of the Fruit** _____


- e. Does the solution you found in part c) make both equations true?
- f. Graph the equations from part d) on the coordinate plane below. Label the lines according to what they represent in the context. To graph the first equation $p + a = 20$, you can plot the different ordered pairs from the first two columns of the table. For example, (0, 20) and (20, 0) are both combinations that make this equation true. To graph the second equation, find different ordered pairs that make this equation true. For example, if I bought 4 pounds of apples, I would buy 20 pounds of peaches:
- $$2(4) + p = 28$$
- $$8 + p = 28$$
- $$p = 20$$
- Alternatively, you can put this equation into slope-intercept form and graph using the slope and y-intercept: $a = -\frac{1}{2}p + 14$. The y-intercept is 14 and the slope is $-\frac{1}{2}$.



- g. Find the point of intersection in the graph above. What do you notice?

- h. The Bake Sale problem can be modeled and solved using a **system of linear equations**. Write in your own words what a **system of linear equations** is.

Answers will vary. A situation that has more than one constraint/requirement. Two or more equations working together to explain a situation.

- i. Explain, in your own words, what the **solution** to a system of linear equations is. How can you find the solution in the different representations (table, graph, equation)? 

The solution to a system of linear equations (if there is one) is the ordered pair (or pairs) that satisfies all of the equations of a system (makes them true). In a graph, this is the point or points of intersection of the two lines (if there is one). In this example, the solution is the ordered pair that yields 20 pounds of fruit AND costs \$28.

- j. Josh really likes apple pie so he wants to donate enough money so that there are an equal number of pounds of peaches and apples. How much does he need to donate?

- k. What if the students had to spend exactly \$25? Exactly \$20?
How would the equations change?

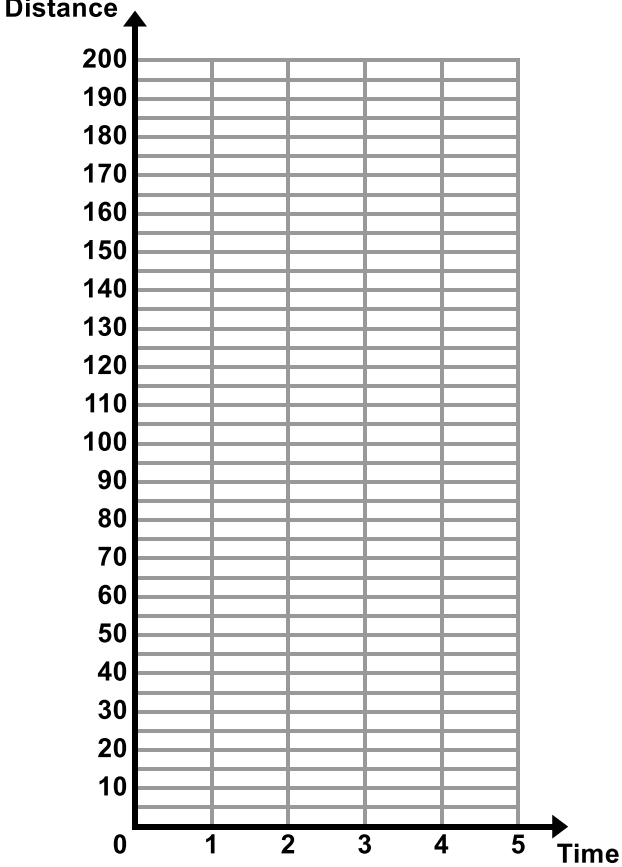
How would the graphs change?

What would the new solutions be?


- l. What if the students wanted to make 20 pies and had exactly \$64 to spend? Write the system of equations that models this problem. Find a combination that works.

4.1b Class Activity: Who Will Win the Race

1. Kevin and Nina are competing in a bike race. When Kevin is ninety miles into the race, he is in first place. Nina is in second place and is 15 miles behind Kevin.
 - a. From this point, Kevin continues the race at a constant rate of 25 mph and Nina continues the race at a constant rate of 30 mph. When will Nina catch Kevin? Solve this problem using any method you wish.

<p><u>Picture:</u></p> <div style="margin-top: 20px;"> <div style="border-bottom: 2px solid black; width: 350px; margin-bottom: 10px;"></div> <div>Kevin</div> </div> <div style="margin-top: 20px;"> <div style="border-bottom: 2px solid black; width: 350px; margin-bottom: 10px;"></div> <div>Nina</div> </div>	<p><u>Table:</u></p> <table border="1" style="width: 100%; border-collapse: collapse; margin-top: 10px;"> <tr> <td style="padding: 5px;">Time (hours)</td> <td style="width: 25px;"></td> <td style="width: 25px;"></td> <td style="width: 25px;"></td> <td style="width: 25px;"></td> <td style="width: 25px;"></td> </tr> <tr> <td style="padding: 5px;">Kevin</td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td style="padding: 5px;">Nina</td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table> <table border="1" style="width: 100%; border-collapse: collapse; margin-top: 10px;"> <tr> <td style="width: 25px;"></td> <td style="width: 25px;"></td> <td style="width: 25px;"></td> <td style="width: 25px;"></td> <td style="width: 25px;"></td> <td style="width: 25px;"></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table>	Time (hours)						Kevin						Nina																							
Time (hours)																																					
Kevin																																					
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<p><u>Graph:</u></p> 	<p><u>Other Methods:</u></p> <p><i>Write an Equation, Use Logic and Reasoning, Guess, Check, and Revise, Find a Pattern, Work Backwards.</i></p>																																				

- b. If the race is 150 miles long, who will win? Assume Nina and Kevin bike at the speeds given in part a).

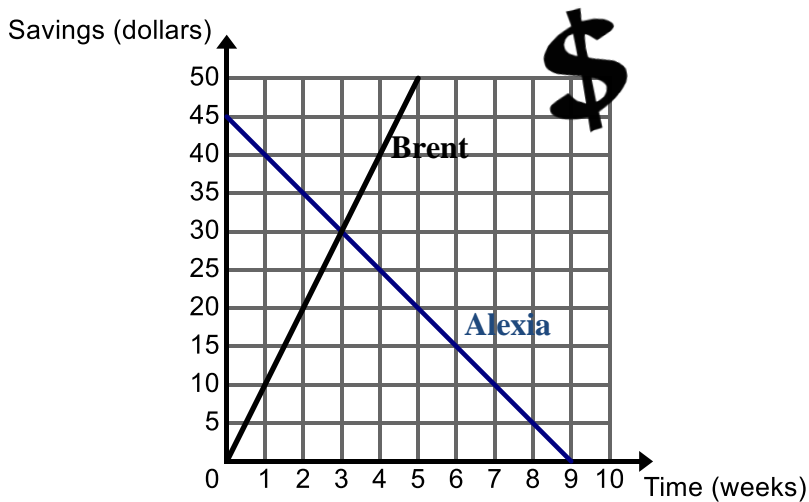
- c. Now suppose the following: Ninety miles into the race, Kevin is still in first place and Nina is still in second place, 15 miles behind Kevin. But now Kevin and Nina both finish out the race at a constant speed of 30 mph. When will Nina catch Kevin? If the race is 150 miles long, who will win? 

2. The graph below shows the amount of money Alexia and Brent have in savings.

- a. Write an equation to represent the amount y that each person has in savings after x weeks:

Alexia: $y = 45 - 5x$

Brent:





A common mistake here is for students to neglect the scales on the axes. You may want to remind students to take note of the scales.

- b. Tell the story of the graph. Be sure to include what the point of intersection means in the context.

Alexia starts with \$45 and spends \$5 each week. Brent starts with \$0 and makes/saves \$10 each week. At 3 weeks, Alexia and Brent will have the same amount of money - \$30. Consider having students verify using the equations that the ordered pair (3, 30) satisfies both equations. When these values are substituted in for x and y , this ordered pair will make both equations true.

If you look at the graph, you can see that each week, Brent closes the gap by \$15 until the point of intersection and then he increases the gap by \$15 each week after the point of intersection.

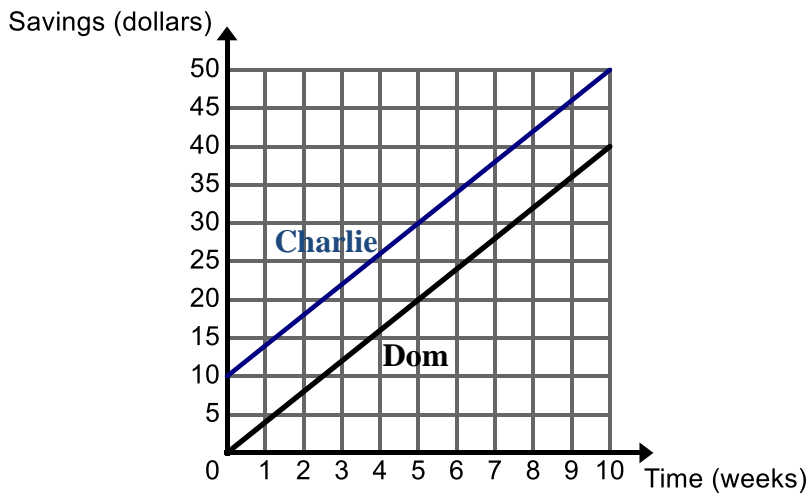
4.1b Homework: Who Will Win the Race

1. Gabriela and Camila like to race each other. Gabriela can run 10 feet/second while Camila can run 12 feet/second. Being a good sport, Camila gives Gabriela a 20-foot head start. 
 - a. How long will it take Camila to catch Gabriela? (For ideas on how to solve this problem, see the strategies used in the classwork) 
 - b. If the girls are racing to a tree that is 30 yards away, who will win the race? (*Remember there are 3 feet in 1 yard*).
2. Darnell and Lance are both saving money. Darnell currently has \$40 and is saving \$5 each week. Lance has \$25 and is saving \$8 each week.
 - a. When will Darnell and Lance have the same amount of money? **There are a few different ways that students can solve this part of the problem. Students may choose to create a table for each boy with one column labeled # of weeks and the other column labeled amount of money saved. Another option is to create an equation for each boy. For example, the equation that represents Darnell's amount of savings is $y = 40 + 5x$ where y = the amount of money Darnell has saved and x = the number of weeks that have passed. Once students have the equations, they must determine the amount of time x it will take for the boys to have the same amount of money. Another approach to solving this problem is a graph, where time is on the x -axis and amount of money saved is on the y -axis. The point of intersection of the two lines, if there is one, represents the amount of time it will take for the boys to have the same amount of money AND how much they will both have.**
 - b. How much will each boy have when they have the same amount of money?
 - c. If both boys continue saving at this rate, who will have \$100 first?

3. The graph below shows the amount of money Charlie and Dom have in savings.

a. Write an equation to represent the amount y that each person has in savings after x weeks:

Charlie: $y = 10 + 4x$ Dom: _____

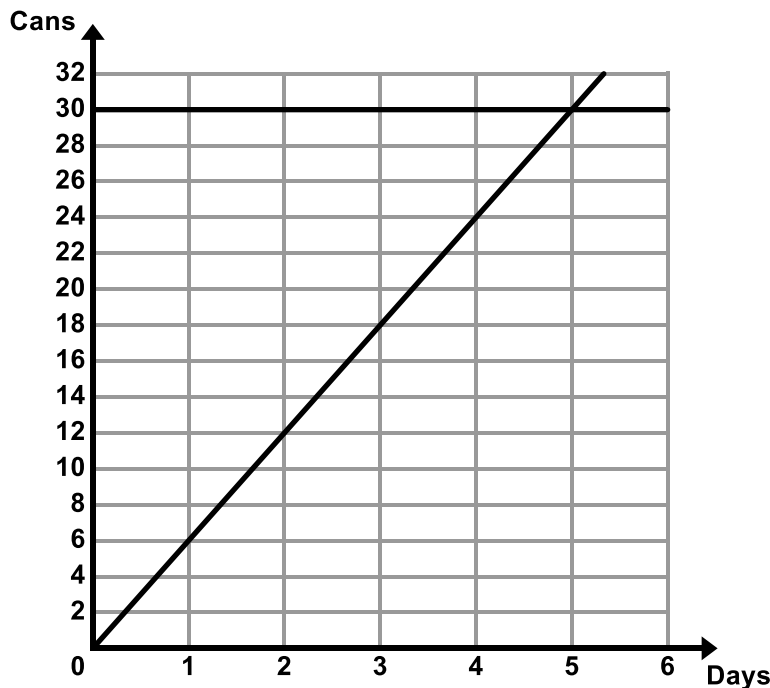


b. Tell the story of the graph.

4. Lakeview Middle School is having a food drive. The graph below shows the number of cans each class has collected for the food drive with time 0 being the start of week 3 of the food drive.

a. Write an equation to represent the number of cans y that each class has collected after x days.

Mrs. Lake's Class: _____ Mr. Luke's Class: _____



b. Tell the story of the graph.

Mrs. Lake's Class begins the week with 30 cans of food and no one brings any more food. Mr. Luke's class starts the week with zero cans of food and they bring 6 cans of food per day. At day 5 both classes have exactly the same number of cans, 30.

4.1c Class Activity: Solving Simultaneous Linear Equations by Graphing

One method for solving simultaneous linear equations is graphing. In this method, both equations are graphed on the same coordinate grid, and the solution is found at the point where the two lines intersect.

Consider the simultaneous linear equations shown below and answer the questions that follow:

$$2x + y = 4$$

$$y = 4x - 2$$

1. What problems might you encounter as you try to graph these two equations?

The first equation is not in a form we are used to graphing.

2. What form of linear equations do we typically use when graphing?

Slope-Intercept form or $y = mx + b$.

As we have seen, it is possible to rearrange an equation that is not in slope-intercept form using the same rules we used when solving equations. We can rearrange this equation to put it in slope-intercept form. Remember, slope-intercept form is the form $y = mx + b$, so our goal here will be to isolate y on the left side of the equation, then arrange the right side so that our slope comes first, followed by the y -intercept.

$$2x + y = 4$$

Subtract $2x$ from both sides to isolate y

$$y = 4 - 2x$$

(Remember that 4 and $-2x$ are not like terms and cannot be combined)

$$y = -2x + 4$$

Rearrange the right side so that the equation is truly in slope-intercept form

3. Let's look at an example that is a little more challenging. With your teacher's help, write in the steps you complete as you go.

$$4x - 8y = 16$$

Subtract $4x$ from both sides to isolate y term. _____

$$-8y = 16 - 4x$$

Remember that 16 and $-4x$ are not like terms and cannot be combined. _____

$$y = -2 + \frac{1}{2}x$$

Divide both sides of the equation by -8 _____

$$y = \frac{1}{2}x - 2$$

Rearrange the right side of the equation so that it is in slope-intercept form. _____
(Make sure to keep the correct signs with each term.)

4. **Skill Review:** Put the following equations into slope-intercept form.



a. $5x + y = 9$

$$y = -5x + 9$$

Steps: Subtract $5x$ from both sides

c. $4y - x = 16$

$$y = \frac{1}{4}x + 4$$

Steps: Add x to both sides. Divide both sides by 4

e. $-y = x - 2$

b. $4x + 2y = -12$

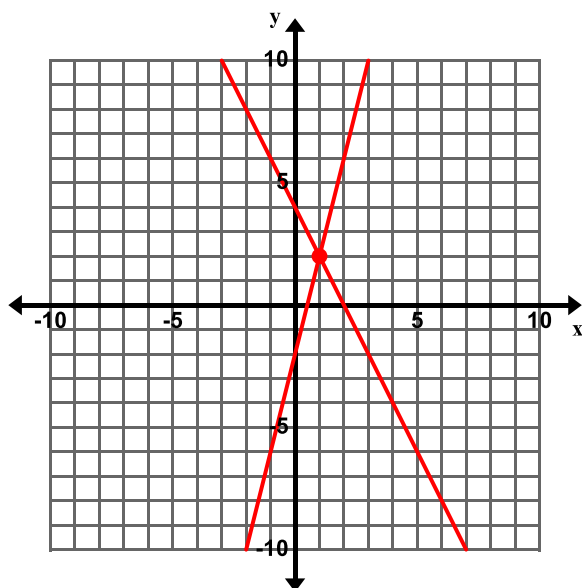
$$y = -2x - 6$$

Steps: Subtract $4x$ from both sides. Divide both sides by 2 .

d. $4x - 2y = -24$

f. $-2x + 5y = 3$

5. Consider the linear equations $2x + y = 4$ and $y = 4x - 2$ from the previous page. Graph both equations on the coordinate plane below. **Remind students to put both equations into slope-intercept form first.**



- a. Find the coordinates (x, y) of the point of intersection.

$(1, 2)$

- b. Verify that the point of intersection you found satisfies both equations. **Remind students to check their solution(s) using the original equations.**

$$2x + y = 4$$

$$2(1) + 2 = 4$$

$$2 + 2 = 4$$

$$4 = 4$$

$$y = 4x - 2$$

$$2 = 4(1) - 2$$

$$2 = 4 - 2$$

$$2 = 2$$

The **solution(s) to a pair of simultaneous linear equations** is all pairs (if any) of numbers (x, y) that are solutions of both equations, that is (x, y) satisfy both equations. When solved graphically, the solution is the point or points of intersection (if there is one).

6. Determine whether $(3, 8)$ is a solution to the following system of linear equations:

$$2x + y = 14$$

$$x + y = 11$$

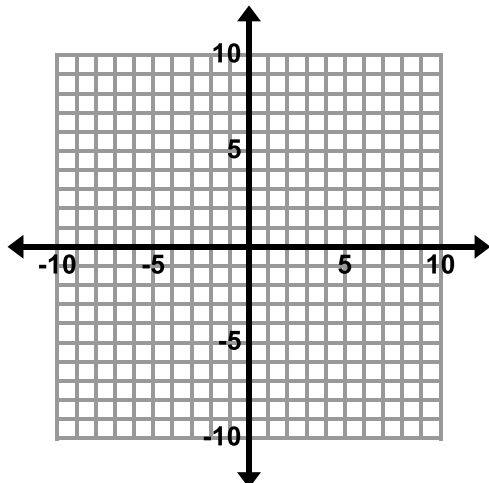
$(3, 8)$ is a solution to this system of linear equations. To determine whether or not $(3, 8)$ is a solution to this system, plug this ordered pair into each equation. In order for this to be a solution of the system, it must make **BOTH** equations true.


7. Determine whether $(0, -5)$ is a solution to the following system of linear equations:

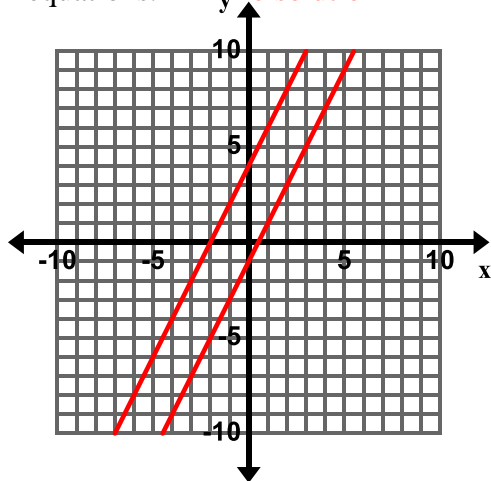
$$y = 2x - 5$$

$$4x + 5y = 25$$

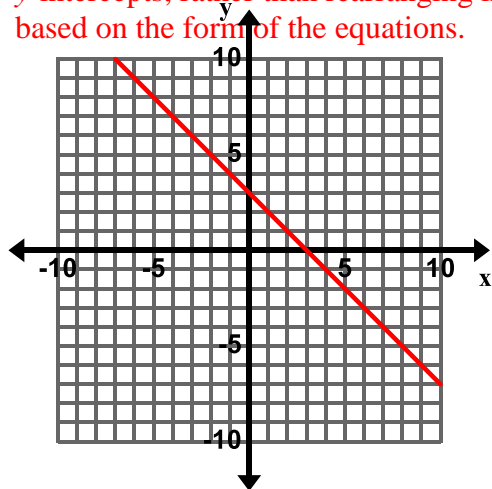
8. Consider the equations $y = -2x$ and $y = -\frac{1}{2}x - 3$. Make sure both equations are written in slope-intercept form, then graph both equations on the coordinate plane below and find the solution. Verify that the solution satisfies both equations. **The solution to this system is $(2, -4)$. Graph both equations and ensure that the point of intersection is $(2, -4)$.**



9. Consider the equations $-2x + y = -1$ and $y = 2x + 4$. Make sure both equations are written in slope-intercept form, then graph both equations on the coordinate plane below and solve the system of linear equations.  **no solution**

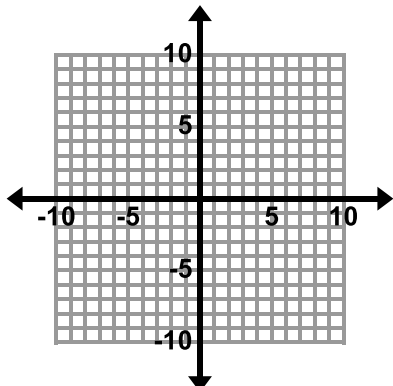
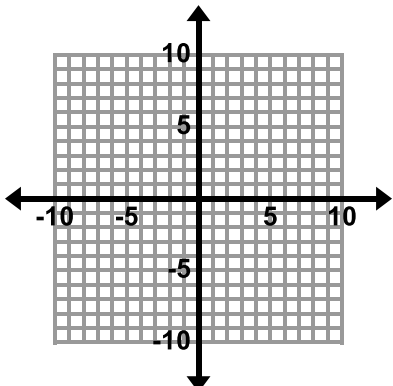
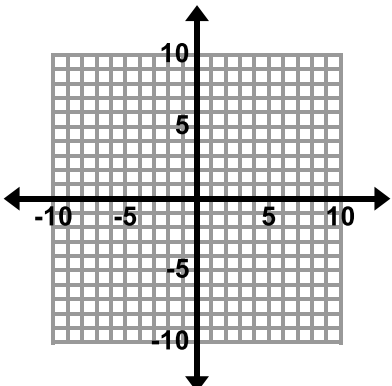



10. Consider the equations $x + y = 3$ and $3x + 3y = 9$. Graph both equations on the coordinate plane below and solve the system of linear equations. **Students may choose to graph this by finding the x - and y -intercepts, rather than rearranging into slope-intercept form. Discuss which method is easier/faster based on the form of the equations.**



Infinitely many solutions, have students identify 2 – 3 solutions and verify the solutions in the equation. It is important that students understand that not just any point on the plane works; the solutions are the infinitely many points on the line. For example, $(0, 3)$ and $(5, -2)$ are both solutions because both points lie on the line. $(5, 0)$ would not be a solution because it does not lie on the line.

11. In the table below, draw an example of a graph that represents the different solving outcomes of a system of linear equations:

One Solution	No Solution	Infinitely Many Solutions
		

12. Without graphing, determine whether the following systems of linear equations will have one solution, no solution, or infinitely many solutions. 

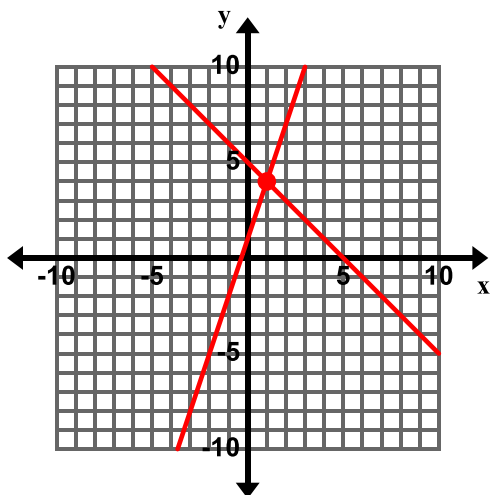
<p>a. $y = 8x + 2$ and $y = -4x$</p> <p>Just by examining the original equations, we can see that the graphs of these equations would not be parallel (they do not have the same slope). We can also see that the equations are not equivalent. Therefore, we know that this system has one solution.</p>	<p>b. $y = -\frac{2}{3}x - 5$ and $y + \frac{2}{3}x = 1$</p> <p>The slope of the first equation is $-\frac{2}{3}$. If we put the second equation into slope-intercept form, we see that its slope is also $-\frac{2}{3}$. Since these lines have the same slope, they are parallel. Parallel lines do not intersect; therefore this system has no solution.</p>
<p>c. $2x + y = 8$ and $y = 2x - 2$</p>	<p>d. $x + y = 5$ and $-2x - 2y = -10$</p> <p>Examining these equations, we see that the second equation is a multiple of the first (to obtain the second equation, the first equation was multiplied by -2). The two equations are equivalent, meaning that when graphed, the equations would produce the same line. Therefore this system has infinitely many solutions – the points that lie on the line.</p>
<p>e. $3x + 2y = 5$ and $3x + 2y = 6$</p>	<p>f. $y = 2x + 5$ and $4x - 2y = -10$</p>

13. One equation in a system of linear equations is $6x + 4y = -12$.
- Write a second equation for the system so that the system has only **one solution**. Answers will vary, possible equation: $x + 2y = -6$
 - Write a second equation for the system so that the system has **no solution**. Answers will vary – line should have the same slope but different y-intercept as the first equation, possible equation: $6x + 4y = 10$
 - Write a second equation for the system so that the system has **infinitely many solutions**. Answers will vary – line should have the same slope and same y-intercept as the first equation, possible equation: $2x + 2y = -6$

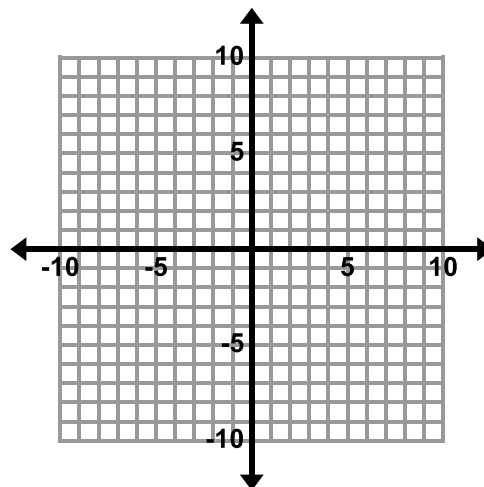
4.1c Homework: Solving Simultaneous Linear Equations by Graphing

1. Solve the system of linear equations graphically. If there is one solution, verify that your solution satisfies both equations. **Hint: The solution to a system of linear equations is the point where the two lines intersect. If the lines do not intersect (are parallel), then there are no solutions. If the lines intersect at all points, there are infinitely many solutions.**

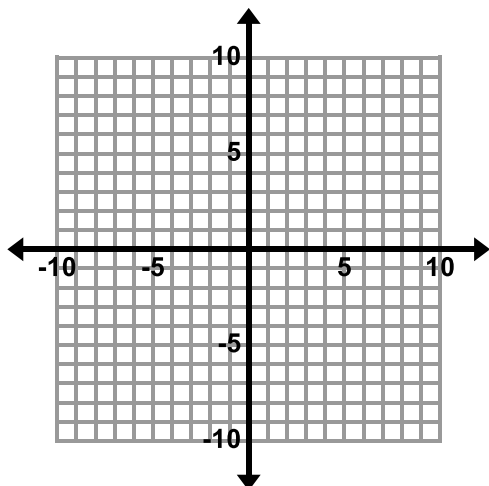
a. $y = 3x + 1$ and $x + y = 5$
(1, 4)



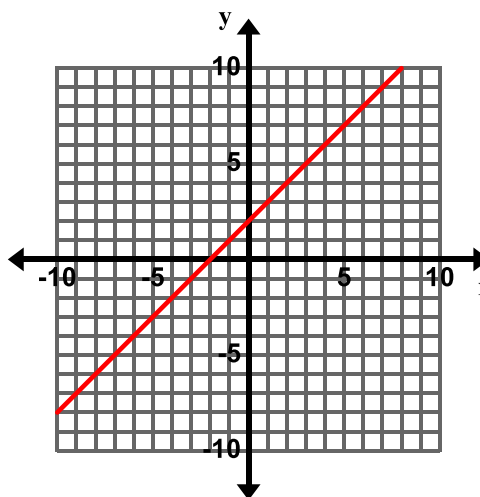
b. $y = -5$ and $2x + y = -3$



c. $y = -3x + 4$ and $y = \frac{1}{2}x - 3$

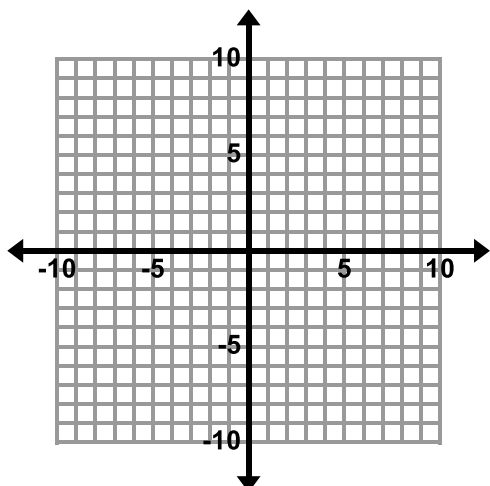


d. $x - y = -2$ and $-x + y = 2$
Infinitely many solutions

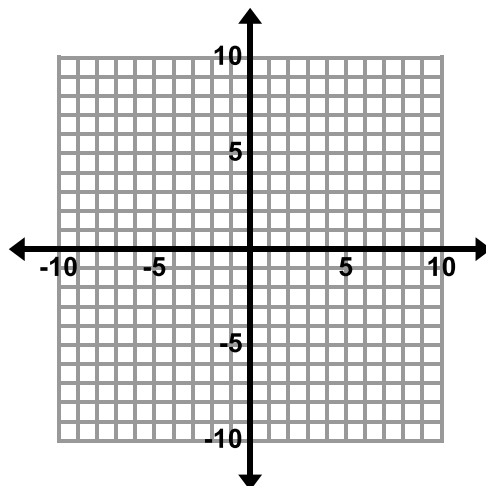


List 2 points that are solutions to this system.
Answers will vary but points should be on the line. Possible answers $(-2, 0)$, $(0, 2)$, $(2, 4)$

e. $y = \frac{1}{2}x - 2$ and $y = \frac{1}{2}x + 4$



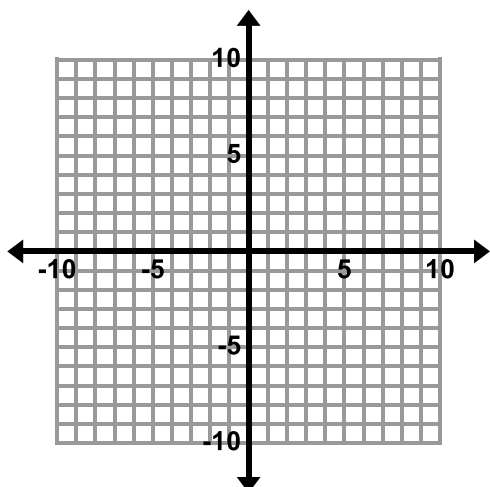
f. $2x - 8y = 6$ and $x - 4y = 3$



Circle the ordered pair(s) that are solutions to this system.

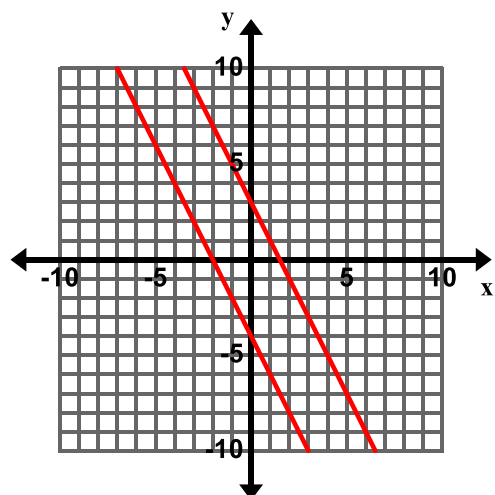
(0, 0) (0, -1) (3, 0) (9, 3)

g. $y = 6x - 6$ and $y = 3x - 6$



h. $2x + y = -4$ and $y + 2x = 3$

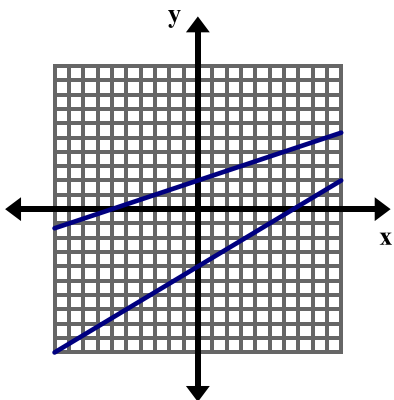
No solution



2. Without graphing, determine whether the following systems of linear equations will have one solution, no solution, or infinitely many solutions. **Hint: Make sure to write the equations in slope-intercept form first. If the lines have the same slope, they are parallel; therefore there is no solution. If they equations are equivalent (one is a multiple of the other) then the lines will be the same and there are infinitely many solutions.**

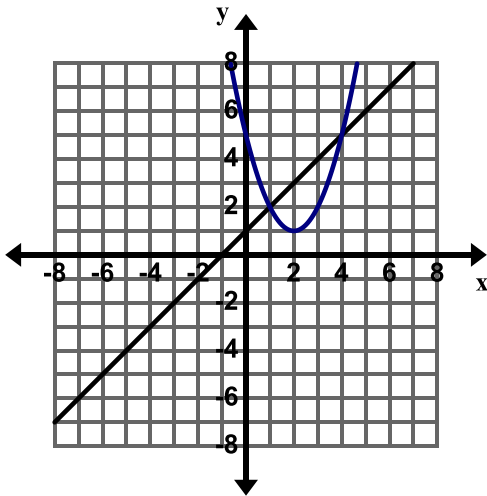
<p>a. $x + y = 5$ and $x + y = 6$ Write both equations in slope-intercept form first: $y = -x + 5$ and $y = -x + 6$</p> <p>The slope of both lines is -1; therefore the lines are parallel. Since the lines do not intersect there is no solution to this system of equations.</p>	<p>b. $-3x + 9y = 15$ and $y = \frac{1}{3}x + \frac{5}{3}$</p>
<p>c. $y = 6$ and $y = 2x + 1$</p>	<p>d. $x - y = 5$ and $x + y = 5$</p> <p>a</p>

3. How many solutions does the system of linear equations graphed below have? How do you know?



4. One equation in a system of linear equations is $y = x - 4$. **Refer to #13 in the classwork for help.**
- Write a second equation for the system so that the system has only **one solution**.
 - Write a second equation for the system so that the system has **no solution**.
 - Write a second equation for the system so that the system has **infinitely many solutions**.

5. The grid below shows the graph of a line and a parabola (the curved graph).



- a. How many solutions do you think there are to this system of equations? Explain your answer.
- b. Estimate the solution(s) to this system of equations.
- c. The following is the system of equations graphed above.
- $$y = x + 1$$
- $$y = (x - 2)^2 + 1$$

How can you verify whether the solution(s) you estimated in part b) are correct?

- d. Verify the solution(s) from part b).

4.1d Self-Assessment: Section 4.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Solve simultaneous linear equations by graphing. Sample Problem #1	I can identify the solution to a system of linear equations when given the graphs of both equations.	I know that I can find a solution to a system of equations by graphing, but I often mess up with the graphing or getting the equations in slope-intercept form.	I can graph to find the solution to a system of equations, but I am not sure how to verify using algebra that the solution is correct.	I can re-write equations in slope-intercept form, graph them to find the solution, and plug the solution back in to verify my answer with very few mistakes.
2. Understand what it means to solve a system of equations. Sample Problem #1	I know that when I graph a system of equations, the answer is where the lines cross.	I know that when I graph a system of equations, the answer is the point where the two lines intersect, and is written as an ordered pair (x, y) .	I know that the solution to a system of equations is the point where the lines intersect, and that if you plug this point into the equations they should both be true.	I understand that the solution to a system of equations is the point on the coordinate plane where two lines intersect and because of this, it is also an ordered pair that satisfies both equations at the same time.
3. Identify and provide examples of systems of equations that have one solution, infinitely many solutions, or no solution. Sample Problems #2, #3	I can look at the graph of a system of equations and tell if it has one, no, or infinitely many solutions, but I sometimes get them mixed up.	I can look at the graph of a system of equations and tell if it has one, no, or infinitely many solutions. I can sometimes tell just by looking at the equations as well.	I know how to tell how many solutions a system of equations has by looking at a graph. I can also tell how many solutions a system of equations has by looking at the equations. When given an equation, I can write another equation that would give the system of equations one, no, or infinitely many solutions.	I know how to tell how many solutions a system of equations has by looking at a graph and by looking at just the equations. I understand what it is about the structure of the equations that makes the graphs look the way they do. I can write a system of equations that would have one solution, no solution, or infinitely many solutions.
4. Interpret the solution to a system in a context. Sample Problem #4	When I solve a story problem involving a system of equations I struggle to explain what the solution represents in the context.	When I solve a story problem involving a system of equations, I understand what the solution means, and I can explain to someone what my answer means most of the time.	When given a story problem involving a system of equations, I can write a sentence explaining what the answer means in the context.	When given a story problem involving a system of equations, I can write a sentence describing what the answer means in the context. I can also answer additional questions about the situation.

Section 4.1 Sample Problems (For use with self-assessment)

- Graph the following systems of equations to find the solution. After you have found your solution, verify that it is correct.

$\begin{cases} y = 3x - 5 \\ y = \frac{1}{2}x \end{cases}$	$\begin{cases} y = -2x + 7 \\ x + 3y = -9 \end{cases}$	$\begin{cases} -4x + 6y = 6 \\ x + y = 6 \end{cases}$
Verify:	Verify:	Verify:

- Tell whether the system of equations has one solution, infinitely many solutions, or no solutions.

		$\begin{cases} y = 3x + 4 \\ 2y = 6x + 8 \end{cases}$
		$\begin{cases} y = -\frac{1}{4}x + 6 \\ y = -\frac{1}{4}x - 4 \end{cases}$

3. One equation in a system of linear equations is $y = -2x + 4$.
- Write a second equation for the system so that the system has only **one solution**.
 - Write a second equation for the system so that the system has **no solution**.
 - Write a second equation for the system so that the system has **infinitely many solutions**.
4. At the county fair, you and your little sister play a game called Honey Money. In this game she covers herself in honey and you dig through some sawdust to find hidden money and stick as much of it to her as you can in 30 seconds. The fair directors have hid only \$1 bills and \$5 bills in the sawdust. During the game your little sister counts as you put the bills on her. She doesn't know the difference between \$1 bills and \$5 bills, but she knows that you put 16 bills on her total. You were busy counting up how much money you were going to make, and you came up with a total of \$40. After the activity you put the all the money into a bag and your little sister takes it to show her friends and loses it. The fair directors find a bag of money, but say they can only give it to you if you can tell them how many \$1 bills you had, and how many \$5 bills you had. What will you tell the fair directors so you can get your money back?
- Solve this problem using any method you wish. Show your work in the space below.
 - Write your response to the fair directors in a complete sentence on the lines provided.

Section 4.2: Solve Simultaneous Linear Equations Algebraically

Section Overview:

In this section, students are solving simultaneous linear equations that have one, no, or infinitely many solutions using algebraic methods. The section utilizes concrete models and real world problems in order to help students grasp the concepts of substitution and elimination. Students then solve systems of linear equations abstractly by manipulating the equations. Students then apply the skills they have learned in order to solve real world problems that can be modeled and solved using simultaneous linear equations.

Concepts and Skills to Master:

By the end of this section, students should be able to:

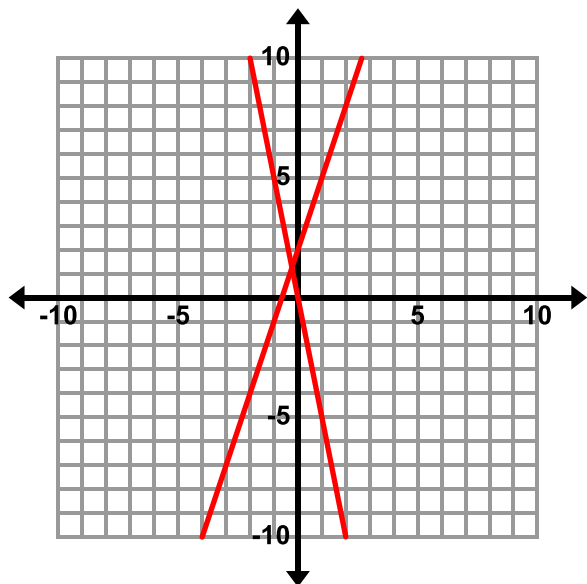
1. Determine which method of solving a system of linear equations may be easier depending on the problem.
2. Solve simultaneous linear equations algebraically.
3. Create a system of linear equations to model a real world problem, solve the system, and interpret the solution in the context.

4.2a Class Activity: Introduction to Substitution

In the previous section, you learned how to solve a system of linear equations by graphing. In this section, we will learn another way to solve a system of linear equations. Solve the following system by graphing.

$$y = 3x + 2$$

$$y = -5x$$



Give some reasons as to why graphing is not always the best method for solving a system of linear equations.

Answers will vary but a few reasons, the graph does not cross at an integer point, the numbers are large and difficult to put onto a graph.

In this section, we will learn about algebraic methods for solving systems of linear equations. These methods are called substitution and elimination.

Directions: Find the value of each shape. Verify your answers.

The idea with these models is that students will “replace equal parts” or substitute the symbols from one equation with an expression from the other equation so that one of the equations contains only one symbol (or variable). Once students find the value of one symbol, they can find the value of the other. **Encourage students to check the solution in each of the original models by writing the value of each shape inside the shape and verifying that both equations are true – see #2 for sample check.**

1.

$$\begin{array}{c} \bigcirc + \bigcirc + \square = 25 \\ \square = 5 \end{array}$$

$$\bigcirc = \underline{10}$$

How did you determine the circle's value?

Students should see that we can replace the square in the first equation with a 5 leaving us with:
 $2c + 5 = 25$ where c = circle. Solving this equation leaves us with $c = 10$.

2.

After students have found the solution, have them substitute in the values and verify their answers.

$$\begin{array}{c} \textcircled{8} + \textcircled{8} + \boxed{2} = 18 \\ \textcircled{8} + \boxed{2} = 10 \end{array}$$

$$\begin{array}{c} \boxed{} = \underline{2} \\ \textcircled{} = \underline{8} \end{array}$$

How did you determine the value of each shape?

Students should see that we can replace a square and circle in the first equation with 10, leaving us with: $c + 10 = 18$, solving this equation leaves us with $c = 8$. Now that we know that $c = 8$, we can substitute in 8 for the circle in the second equation, leaving us with $8 + s = 10$. Solving this equation gives us $s = 2$. It is important to note that the value of the shapes may change from problem to problem. For example, the circle in example #1 has a value of 10 while the circle in this problem has a value of 8. The shapes represent variables so they may change from problem to problem.

3.

$$\begin{array}{c} \textcircled{} + \textcircled{} + \boxed{} + \boxed{} = 20 \\ \textcircled{} + \textcircled{} + \boxed{} = 17 \end{array}$$

$$\begin{array}{c} \boxed{} = \underline{} \\ \textcircled{} = \underline{} \end{array}$$

How did you determine the value of each shape?

4.

$$\begin{array}{c} \triangle + \triangle + \triangle = 27 \\ \triangle + \boxed{} = 8 \end{array}$$

$$\begin{array}{c} \triangle = \underline{} \\ \boxed{} = \underline{} \end{array}$$

How did you determine the value of each shape?

5.

$$\begin{array}{c} \star + \star = 16 \\ \star + \star + \triangle + \triangle = 26 \end{array}$$

$$\begin{array}{c} \star = \underline{} \\ \triangle = \underline{} \end{array}$$

How did you determine the value of each shape?

6.

$$\begin{array}{ccccccc} \star & + & \star & + & \bigcirc & + & \bigcirc & + & \bigcirc & = & 19 \\ \star & + & \star & + & \bigcirc & = & 13 \end{array}$$

$$\begin{array}{l} \star = \underline{\hspace{2cm}} \\ \bigcirc = \underline{\hspace{2cm}} \end{array}$$

How did you determine the value of each shape?

7.

$$\begin{array}{ccccccc} \bigcirc & + & \square & + & \square & = & 30 \\ \square & = & \bigcirc & + & \bigcirc \end{array}$$

$$\begin{array}{l} \square = \underline{12} \\ \bigcirc = \underline{6} \end{array}$$

How did you determine the value of each shape?

The structure changes here a bit. One way for students to do this problem is to replace each square in the first equation with 2 circles, giving the equation $5c = 30$. Solving for c , $c = 6$. Replace each circle in the second equation with 6, giving the equation $s = 6 + 6$. Solving for s , $s = 12$.

8.

$$\begin{array}{ccccccc} \bigcirc & + & \bigcirc & + & \triangle & + & \triangle & = & 16 \\ \bigcirc & + & \bigcirc & = & \triangle & + & \triangle & + & 4 \end{array}$$

$$\begin{array}{l} \bigcirc = \underline{\hspace{2cm}} \\ \triangle = \underline{\hspace{2cm}} \end{array}$$

How did you determine the value of each shape?

9.

$$\begin{array}{ccccccc} \square & + & \square & + & \star & = & 21 \\ \square & + & \square & + & \square & + & \square & + & \star & + & \star & = & 42 \end{array}$$

$$\begin{array}{l} \square = \underline{\hspace{2cm}} \\ \star = \underline{\hspace{2cm}} \end{array}$$

Infinitely many solutions – Examine with students the structure of these two equations that leads to infinitely many solutions. We have doubled both sides of the first equation so the two equations are equivalent. Tie back to ideas in chapter 1. Ask students how this is different from the other problems. In the other problems, we were left with a shape and it was equal to a value. Here all the shapes are gone. Have students try some different values for the stars and circles. What if I put in a 4 for the squares, what are the stars equal to? (13) Be sure to show that this solution works in both equations. What if I put a 5 in for the squares, what are the stars equal to? (11) What if I put in a 5 for the stars, what are the squares equal to? (8) Have students try different values to see that there are an infinite number of ordered pairs that make both equations true.

Directions: Draw a picture of each equation with shapes and then find the value of each shape.

10. $3x + 2y = 41$  +  = 41

$2y = 8$  = 8

Draw a picture that represents each equation. For example, you may use circles to represent x and squares to represent y as shown above. Once you have the pictures, solve using the same process as on the previous pages. In this example, we can use the second equation to see that each square has a value of 4. We can replace the 2 squares in the first equation with 8. Then we are left with $3c + 8 = 41$. Solving this equation, we know that $c = 11$. Putting it back in terms of x and y , $x = 11$ and $y = 4$. Our solution is the ordered pair $(11, 4)$.

11. $2x + y = 9$

$x + y = 5$

12. $x + 3y = 41$

$x + 2y = 32$

$x = 14, y = 9$

13. $2x + 2y = 18$

$2x = y$

Challenge Questions: Find the value of each variable using shapes.

14. $x + 2y = 46$

$$y + 3z = 41$$

$$3z = 27$$

15. $2x + z = 46$

$$3z = 18$$

$$2y + z = 40$$

16. $2x + 2y = 50$

$$2x + y = 42$$

$$y + 2z = 18$$

4.2a Homework: Introduction to Substitution

Directions: Find the value of each shape. Explain how you determined each. Verify your answers.

1.

$$\begin{array}{l} \bigcirc + \bigcirc + \bigcirc + \bigcirc + \square + \square = 34 \\ \bigcirc + \bigcirc + \square + \square = 26 \end{array} \quad \begin{array}{l} \bigcirc = \underline{\textcolor{red}{4}} \\ \square = \underline{\textcolor{red}{9}} \end{array}$$

Students should see that we can replace 2 circles and 2 squares in the first equation with the number 26 leaving us with circle + circle + 26 = 34 or $2c + 26 = 34$. Solving this equation for c , we are left with $c = 4$. Now, we can replace each circle in the second equation with 4 leaving us with $4 + 4 + \text{square} + \text{square} = 26$ or $8 + 2s = 26$. Solving this equation for s , we are left with $s = 9$.

2.

$$\begin{array}{l} \triangle + \triangle + \triangle + \hexagon = 27 \\ \triangle + \triangle + \hexagon = 20 \end{array} \quad \begin{array}{l} \triangle = \underline{\hspace{1cm}} \\ \hexagon = \underline{\hspace{1cm}} \end{array}$$

3.



$$\begin{array}{l} \odot + \odot + \odot + \odot + \smile + \smile = 42 \\ \smile + \smile = \odot + \odot + \odot \end{array} \quad \begin{array}{l} \odot = \underline{\textcolor{red}{6}} \\ \smile = \underline{\textcolor{red}{9}} \end{array}$$

4.

$$\begin{array}{l} \text{drop} + \text{drop} + \text{drop} + \text{drop} + \text{drop} + \text{drop} = \text{cloud} \\ \text{drop} + \text{drop} + \text{drop} + \text{drop} + 8 = \text{cloud} \end{array} \quad \begin{array}{l} \text{drop} = \underline{\hspace{1cm}} \\ \text{cloud} = \underline{\hspace{1cm}} \end{array}$$

Directions: Draw a picture of each equation with shapes and then find the value of each shape.

5. $x + y = 15$

 +  = 15

In this picture, circle = x and square = y .

$y = x + 10$

 =  + 10

One way to solve is to replace the square in the first equation with a circle + 10. This would leave, circle + circle + 10 = 15 or $2x + 10 = 15$. Solving for x , we see that $x = 2.5$. We can then substitute this value into either equation to solve for y .

$x = 2.5, y = 12.5$

6. $y + x = 5$

$x = y - 3$

7. $y = 4x$

$x + y = 5$

8. $2x + y = 7$

$x + y = 1$

9. $3x + 4y = 19$

$$3x + 6y = 33$$

$$x = -3, y = 7$$

10. $5x + 6y = 100$

$$4x + 6y = 92$$

4.2b Class Activity: Substitution Method for Solving Systems of Equations

Directions: Write a system of equations from the shapes. Find the value of each shape.

1.

$$\begin{array}{l} \bigcirc + \square + \square = 30 \\ \square = \bigcirc + \bigcirc \end{array}$$

System of Equations:

$$\begin{array}{l} \bigcirc = \underline{\textcolor{red}{6}} \\ \square = \underline{\textcolor{red}{12}} \end{array}$$

How did you determine the value of each shape?

$$\textcolor{red}{x + 2y = 30}$$

$$\textcolor{red}{y = 2x}$$

2.

$$\begin{array}{l} \triangle + \triangle + \triangle + \bigcirc = 12 \\ \triangle + \triangle + \triangle + \triangle + \bigcirc + \bigcirc + \bigcirc = 26 \end{array}$$

$$\begin{array}{l} \triangle = \underline{\hspace{2cm}} \\ \bigcirc = \underline{\hspace{2cm}} \end{array}$$

System of Equations:

$$\textcolor{red}{3x + y = 12}$$

$$\textcolor{red}{4x + 3y = 26}$$

$$\textcolor{red}{x \text{ (triangle)} = 2; y \text{ (circle)} = 6}$$

This is the first time that students will come across a situation that simply “replacing equal parts” still leaves them with two variables in the remaining equation. From here, we move students toward the step of solving for one of the shapes (variables) and the substituting into the second equation.

To solve any system of linear equations using substitution, do the following:

1. Rewrite one of the equations so that one variable is expressed in terms of the other (solve one of the equations for one of its variables).
2. Substitute the expression from step 1 into the other equation and solve for the remaining variable.
3. Substitute the value from step 2 into the equation from step 1 and solve for the remaining variable.
4. Check the solution in each of the original equations.

Revisit problem #2 from above and use these steps to solve.

At this point, students should begin the transition from the pictorial approach to the algebraic manipulation of the equations.

3.

$$\begin{array}{c} \square + \square + 7 = \bigcirc \\ \square + \square + \square + \bigcirc + \bigcirc = 35 \end{array}$$

- a. Write a system of equations for the picture above.

$$\begin{array}{l} 2x + 7 = y \quad \text{where } x = \text{square and } y = \text{circle} \\ 3x + 2y = 35 \end{array}$$

- b. Solve this system of equations using substitution showing all steps. Check your solution.

(3, 13)

One way to solve this system, is to replace the y in the second equation with the expression $2x + 7$ from the first equation, leaving us with:

$$3x + 2(2x + 7) = 35$$

Now we can solve this equation for x :

$$3x + 4x + 14 = 35$$

$$7x + 14 = 35$$

$$7x = 21$$

$$x = 3$$

Now, we can substitute in 3 for x (into either of the original equations) in order to solve for y . If we use the first equation, we are left with $2(3) + 7 = y$. Solving this equation, $y = 13$. Therefore the solution to our system is the ordered pair (3, 13). In order to check the solution, substitute this ordered pair into both equations and check to see that BOTH equations are true.

4.

$$\begin{array}{c} \star + \star + \star + 1 = \triangle \\ \star + \star + 3 = \triangle \end{array}$$

- a. Write a system of equations for the picture above.

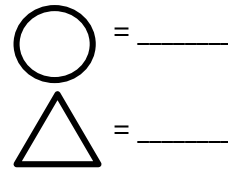
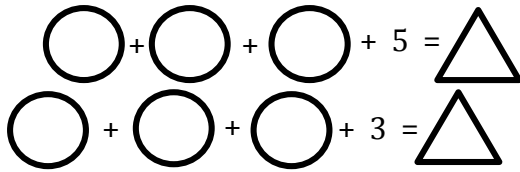
$$3x + 1 = y$$

$$2x + 3 = y$$

- b. Solve this system of equations using substitution showing all steps. Check your solution.

(2, 7)

5.



- a. Write a system of equations for the pictures above.

$$3x + 5 = y$$

$$3x + 3 = y$$

- b. Solve this system of equations using substitution showing all steps. Check your solution.

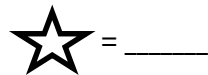
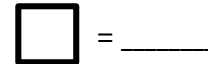
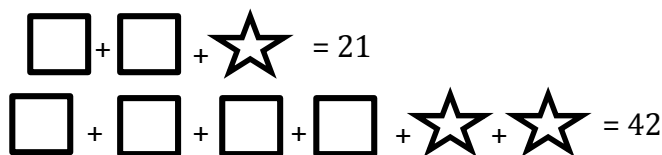
Students will end up with a solving outcome of $a = b$ where a and b are different numbers.

When students solve they will likely get a result of $5 = 3$. Since we know that $5 \neq 3$, there is **no solution** that makes this equation true. Encourage them to examine the structure of the original equations and discuss why it is not possible to take a number, multiply it by 3 and add 5 and then take the same number, multiply by 3 and add 3 and obtain the same result.

- c. Describe what you would see in a graph of this system.

The lines have the same slope but different y-intercepts – they are parallel

6.



- a. Write a system of equations for the pictures above.

$$2x + y = 21$$

$$4x + 2y = 42$$


- b. Solve this system of equations using substitution showing all steps. Check your solution.

Students will end up with a solving outcome of $a = a$ (in this problem they will likely end up with $42 = 42$). Since we know that 42 is always equal to 42, there are **infinitely many solutions**. Again, encourage them to examine the structure of the original equations and discuss why the equations are equivalent.

- c. Describe what you would see in a graph of this system.

The graphs of these equations are the same line.

Directions: Solve each system using the substitution method. When asked, solve the system by graphing in

addition to using the substitution method. 

7. $y = 5x + 4$
 $y = -3x - 12$
 $(-2, -6)$

One way to solve this is to replace the y in the first equation with the expression $-3x - 12$ from the second equation:

$$-3x - 12 = 5x + 4$$

We can now solve this equation for x .

$$-12 = 8x + 4 \quad \text{Add } 3x \text{ to both sides.}$$

$$-16 = 8x \quad \text{Subtract 4 from both sides.}$$

$$-2 = x \quad \text{Divide both sides by 8.}$$

Now that we know that $x = -2$, we can substitute this in for x (into either of the original equations). If we use the second equation, we are left with:

$$y = -3(-2) - 12$$

$$y = 6 - 12$$

$$y = -6$$

8. $y = 6x + 4$
 $y = 6x - 10$
 No solution

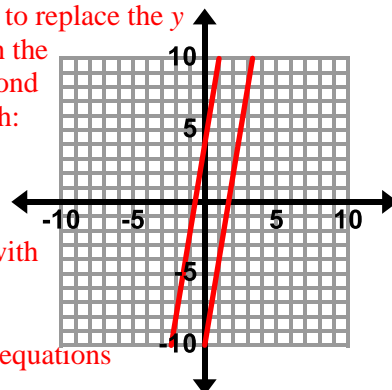
One way to solve this is to replace the y in the first equation with the expression from the second equation, leaving us with:
 $6x - 10 = 6x + 4$.

When we attempt to solve for x , we are left with
 $-10 = 4$

Since we know that $-10 \neq 4$, this system of equations has no solution.

Solve by graphing.

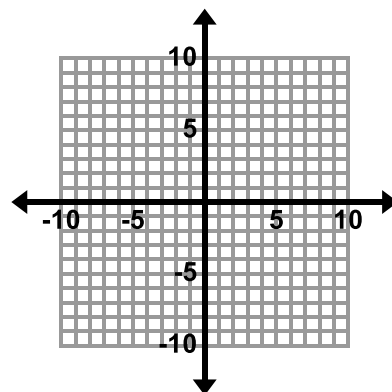
As you can see on the graph above, these lines are parallel, meaning they do not intersect; therefore there is no solution to this system.



9. $y = x + 2$
 $x + 3y = -2$

10. $x = 2y - 4$
 $x + y = 2$

Solve by graphing.



11. $y - x = 5$
 $2x + y = -10$

In order to solve this problem using substitution, you will need to rearrange one of the equations for a single variable. For example, you may solve the first equation for y :

$$y = 5 + x$$

Now you can replace the y in the second equation with $5 + x$ from the first equation:

$$2x + 5 + x = -10$$

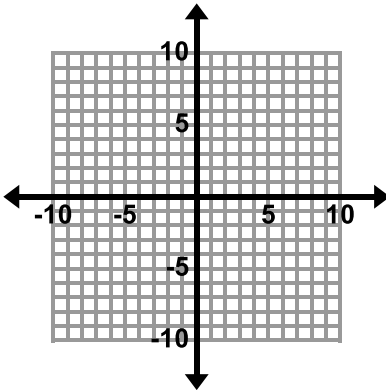
$$3x + 5 = -10$$

$$3x = -15$$

$$x = -5$$

Now substitute in -5 for x and solve for y . The solution to this problem is $(-5, 0)$.

12. $2x + y = 5$
 $y = -5 - 2x$

<p>13. $y + x = 5$ $x = y - 3$</p>	<p>14. $x + y = 4$ $y = -x + 4$ Infinitely many solutions One way to solve this is to replace the y in the first equation with the expression $-x + 4$ from the second equation, leaving us with: $x + (-x) + 4 = 4$ When we solve this equation, we are left with: $4 = 4$ Since 4 always equals 4, there are infinitely many solutions. Another way to solve this problem is to see that the equations are equivalent. If we add x to both sides of the second equation, we are left with $y + x = 4$ which is the exact same equation as the first equation. This means that when we graph these equations, they will be the same line. Any ordered pair that lies on the line will make this system true. Since a line goes on forever, there are infinitely many solutions to this equation. List 2 points that are solutions to this system. Answers will vary but points should be on the line. Possible answers $(5, -1)$, $(2, 2)$, $(0, 4)$</p>
<p>15. $x + 2y = 7$ $2x + 3y = 12$</p>	<p>16. $6x + y = -5$ $-12x - 2y = 10$</p> <p>Solve by graphing.</p> 
<p>17. $x = 2y + 4$ $4y + x = 2$</p>	<p>18. $y = 3x + 2$ $y = -5x$</p>

Directions: The following are examples of real-world problems that can be modeled and solved with systems of linear equations. Answer the questions for each problem.



19. Nettie's Bargain Clothing is having a huge sale. All shirts are \$3 each and all pants are \$5 each. You go to the sale and buy twice as many shirts as pants and spend \$66.

The following system of equations models this situation where s = number of shirts and p = number of pants:

$$s = 2p$$

$$3s + 5p = 66$$

- a. Write in words what each of the equations in the system represents in the context.

$s = 2p$ The number of shirts you bought is twice the number of pants.

$3s + 5p = 66$ Each shirt costs \$3 and each pair of pants is \$5. The total cost is \$66.

- b. Solve this system using substitution to determine how many of each item you bought. Write your answer in a complete sentence.

You bought 6 pairs of pants and 12 shirts. One way to solve this problem is to replace the s in the second equation with the expression $2p$ from the first equation:

$$3(2p) + 5p = 66$$

Solve this equation for p :

$$6p + 5p = 66$$

$$11p = 66$$

$$p = 6$$

Now replace the p in one of the original equations (either one) with 6. If we use the first equation, we are left with:

$$s = 2(6)$$

$$s = 12$$

Encourage students to check that the solution meets the requirements set out in the word problem.

20. Xavier and Carlos have a bet to see who can get more "friends" on a social media site after 1 month. Carlos has 5 more friends than Xavier when they start the competition. After much work, Carlos doubles his amount of friends and Xavier triples his. In the end they have a total of 160 friends together.

The following system of equations models this situation where c = the number of friends Carlos starts with and x = the number of friends Xavier starts with.

$$c = x + 5$$

$$2c + 3x = 160$$

- a. Write in words what each of the equations in the system represents in the context.

$c = x + 5$ _____

$2c + 3x = 160$ _____

- b. Solve this system using substitution to determine how many friends each boy started with. Write your answer in a complete sentence.

4.2b Homework: Substitution Method for Solving Systems of Equations

Directions: Solve each system of linear equations using substitution.

<p>1. $y = 4x$ $x + y = 5$ $(1, 4)$</p> <p>One way to solve this is to replace the y in the second equation with $4x$ from the first equation: $x + 4x = 5$ Solving for x: $5x = 5$ $x = 1$ Now substitute 1 in for x (into either of the original equations). If we use the first equation, we are left with: $y = 4(1)$ $y = 4$</p>	<p>2. $x = -4y$ $3x + 2y = 20$ $(8, -2)$</p> <p>One way to solve this is to replace the x in the second equation with $-4y$ from the first equation: $3(-4y) + 2y = 20$ Solving for y: $-12y + 2y = 20$ $-10y = 20$ $y = -2$ Now substitute -2 in for y (into either of the original equations). If we use the first equation, we are left with: $x = -4(-2)$ $x = 8$</p>	<p>3. $y = x - 1$ $x + y = 3$</p>
<p>4. $3x - y = 4$ $2x - 3y = -9$</p>	<p>5. $x - y = 6$ $2x = 12 + 2y$</p>	<p>6. $2x + 2y = 6$ $x + y = 0$</p>
<p>7. $2x + y = -15$ $y - 5x = 6$</p> <p>To solve using substitution, you will need to rearrange one of the equations to solve for a single variable. For example, you may solve the second equation for y: $y = 6 + 5x$ Now you can replace the y in the first equation with $6 + 5x$ from the second equation and continue to solve for x and y. See #11 from the class work for additional help.</p>	<p>8. $y = 3x + 4$ $y = x - 7$</p>	<p>9. $y = -3x + 6$ $9x + 3y = 18$</p> <p>Infinitely many solutions</p> <p>If we replace the y in the second equation with the expression $-3x + 6$ from the first equation, we have $9x + 3(-3x + 6) = 18$. Solving this equation: $9x + 3(-3x + 6) = 18$ $9x - 9x + 18 = 18$ $18 = 18$ This solving outcome means there are infinitely many solutions. Alternatively, we can observe that the two equations we start with are equivalent. If we solve the second equation for y, we are left with $y = -3x + 6$ (the same as the first equation). See classwork #14 for further explanation.</p>

10. Niona needs \$50 to go on a school trip. She sells necklaces for \$15 each and bracelets for \$5 each. If she raises the money by selling half as many necklaces as bracelets, how many necklaces and bracelets does she sell? Write a system of linear equations that represent this problem.

Define your Unknowns:



System of Equations:

Next to each equation, write in words what the equation represents in the context.

Solve. Write your answer in a complete sentence.

11. A restaurant needs to order stools and chairs. Each stool has 3 legs and each chair has 4 legs. The manager wants to be able to seat 36 people. The restaurant has hard wood floors and the manager doesn't want to scratch them. Therefore, they have ordered 129 plastic feet covers for the bottom of the legs to ensure the stools and chairs don't scratch the floor. How many chairs and how many stools did the restaurant order?
- a. Write a system of equations that matches the verbal descriptions given below if s = number of stools and c = number of chairs.

System of Equations:

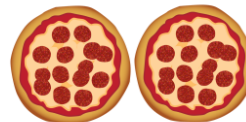
Equation 1: $3s + 4c = 129$ Each stool needs 3 plastic feet covers. Each chair needs 4 plastic feet covers. One hundred twenty-nine plastic feet covers are needed.

Equation 2: There are a total of 36 chairs and stools needed.

- b. Solve the system. Write your answer in a complete sentence.

4.2c Class Activity: Elimination Method for Solving Systems of Linear Equations

1. Ariana and Emily are both standing in line at Papa Joe's Pizza. Ariana orders 4 large cheese pizzas and 1 order of breadsticks. Her total before tax is \$34.46. Emily orders 2 large cheese pizzas and 1 order of breadsticks. Her total before tax is \$18.48. Determine the cost of 1 large cheese pizza and 1 order of breadsticks. **Explain** the method you used for solving this problem.



There are a variety of ways to solve this problem. One possible strategy is that students will observe that the difference in what the women ordered (2 pizzas) accounts for the difference in the amount they spent (\$15.98). That means, each pizza is \$7.99 before tax. Students can then determine that an order of breadsticks costs \$2.50. When students solve the problem in this way, they are informally solving using elimination.

$$4p + b = 34.46$$

$$2p + b = 18.48$$

Subtracting the second equation from the first gives us

$$2p = 15.98$$

$$p = 7.99$$

Substitute back in to determine the value of b .

2. Carter and Sani each have the same number of marbles. Sani's little sister comes in and takes some of Carter's marbles and gives them to Sani. After she has done this, Sani has 18 marbles and Carter has 10 marbles. How many marbles did each of the boys start with? How many marbles did Sani's sister take from Carter and give to Sani?

Again, there are a variety of strategies students may use to solve this problem. One way is to recognize that the total number of marbles is 28 so if the boys had the same amount to start with, they each had 14. Again, if students use a method like this, they are informally solving using elimination.

Sani: $m + x = 18$ where m represents the amount of marbles each boy started with and x represents the number of marbles that Sani's sister took from Carter and gave to Sani.

$$\text{Carter: } m - x = 10$$

Adding the two equations together gives us

$$2m = 28$$

$$m = 14$$

Substitute back in and solve for x : $x = 4$. Each boy started with 14 marbles and Sani's sister took 4 marbles from Carter to give to Sani.

3.

$$\bigcirc + \bigcirc + \bigcirc + \bigcirc + \square = 18$$

$$\bigcirc = \underline{\hspace{2cm}}$$

$$\bigcirc + \bigcirc + \square = 10$$

$$\square = \underline{\hspace{2cm}}$$

- a. Find the value of each shape.

At this point, students may use substitution to determine that the value of the circle is 4 and the value of the square is 2.

- b. Which of the problems from the previous page is this similar to? Compare the strategies you used to solve these problems.

This problem is similar to the pizza problem (one person buys 4 pizzas and 1 order of breadsticks and spends \$18 and another person orders 2 pizzas and 1 order of breadsticks and spends \$10). The difference between the two equations is two circles which account for the difference in the value of the equations (8). That means each circle has a value of 4.

You may choose to write equations for the picture above and show the steps of elimination by subtraction.

4.

$$\bigcirc + \square = 15$$

$$\bigcirc = \underline{\hspace{2cm}}$$

$$\bigcirc - \square = 7$$

$$\square = \underline{\hspace{2cm}}$$

- a. Find the value of each shape.

The value of the circle is 11 and the value of the square is 4.

- b. Which of the problems from the previous page is this similar to? Compare the strategies you used to solve these problems.

This problem is similar to the marble problem.

Write the equations for the picture above and show the steps of elimination by addition.

Directions: Write a system of equations from the shapes. Find the value of each shape. Show the solving actions algebraically. The goal when solving by elimination is to “eliminate” one of the variables by combining the equations. In example 5, if we add the two equations together, the s will be eliminated.

To solve, add these equations together.

5. $\bigcirc + \bigcirc + \square = 14$

$\bigcirc + \bigcirc - \square = 10$

$\bigcirc = \underline{\underline{6}}$

$\square = \underline{\underline{2}}$

Equations:

$$2c + s = 14$$

$$+ 2c - s = 10$$

$$4c = 24$$

$$c = 6$$

c (circle) = 6

Substitute 6 for c in either equation above and solve for s

s (square) = 2

6. $\square + \bigcirc + \bigcirc = 19$

$\bigcirc - \square = 11$

$\bigcirc = \underline{\hspace{2cm}}$

$\square = \underline{\hspace{2cm}}$

Equations:

$$s + 2c = 19$$

$$+ c - s = 11$$

$$3c = 30$$

$$c = 10$$

Substitute 10 for c in either equation above and solve for s

$s = -1$

7. $\square + \square + \bigcirc = 27$

$\bigcirc - \square - \square = 15$

$\bigcirc = \underline{\hspace{2cm}}$

$\square = \underline{\hspace{2cm}}$

Equations:

8. The name of the method you are using to solve the systems of linear equations above is **elimination**. Why do you think this method is called **elimination**?

Directions: Write a system of equations from the shapes. Find the value of each shape. Show the solving actions algebraically.

9.

$$\star + \star - \bigcirc = 8$$

$$\star = \underline{-6}$$

$$\star + \star - \bigcirc - \bigcirc = 4$$

$$\bigcirc = \underline{-4}$$

$$\begin{array}{r} 2s - c = 8 \\ - \quad 2s - 2c = 4 \\ \hline c = 4 \end{array}$$

In this example, adding the equations together will not eliminate either of the variables. The sum of these equations is $4s - 3c = 12$. However if we subtract the second equation from the first, the s will be eliminated, allowing us to solve for c . Once we know the value of c , we can substitute it in to one of the original equations and solve for s .

Examine the c terms in this subtraction problem. Note that when you subtract the second equation from the first, the problem you are solving is $-c - (-2c)$ which is the same as $-c + 2c$ which is equal to c .

10.

$$\triangle + \triangle + \triangle + \triangle + \star + \star = 8$$


$$\triangle = \underline{\hspace{2cm}}$$

$$\triangle + \triangle + \star + \star = -6$$

$$\star = \underline{\hspace{2cm}}$$

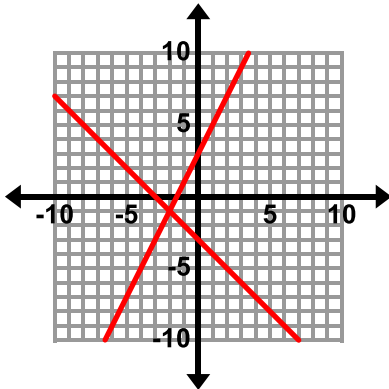
11. How are problems 9 and 10 different from #5 – 7. Describe in your own words how you solved the problems in this lesson.

In problems 5 – 7, you are adding the equations and in #9 and 10 you are subtracting the second equation from the first (or multiplying one equation by -1 and adding the equations together).

Directions: Solve each system of linear equations using **elimination**. Make sure the equations are in the same form first. Graph the first three problems as well as using elimination. 

12. $x + y = -3$
 $2x - y = -3$
 $(-2, -1)$

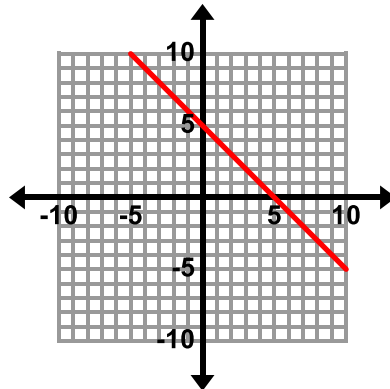
Solve by graphing.



13. $x + y = 5$
 $-x - y = -5$
 Infinitely many solutions

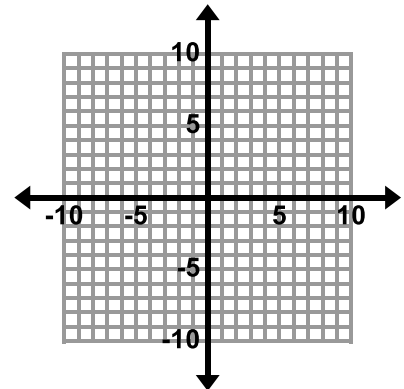
When we add these equations together, the result is $0 = 0$. A solving outcome of $a = a$ means that the original equations are equivalent; therefore there are infinitely many solutions.

Solve by graphing.



14. $x + y = 3$
 $2x - y = -3$

Solve by graphing.



15. $2x - y = -3$
 $3x - y = 1$

16. $2x - y = 9$
 $x + y = 3$

17. $7x - 4y = -30$
 $3x + 4y = 10$

18. $2x + y = 6$ $2x + y = -7$ No solution If we subtract the second equation from the first, the result will be $0 = 13$. A solving outcome of $a = b$ means there are no solutions that make this system true. Examine the structure of the equations to determine why this is true. If we were to graph these lines, they would be parallel.	19. $3x - y = 1$ $x = -y + 3$	20. $x = y + 3$ $x - 2y = 3$
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21. Complete the story for the system of equations shown below if s is number of shirts and p is number of pants. Solve the system and write your solution in a complete sentence.

$$s + p = 18$$

$$5s + 12p = 160$$

Solve:

Story

Jennifer is buying shirts and pants at a sale.

She buys 18... *items total*

Shirts cost \$5 each and pants cost... *\$12*

each.

Jennifer spends... *\$160.*

How many shirts and how many pants did

Jennifer purchase?

Solution (in a complete sentence):

Jennifer buys 8 shirts and 10 pairs of pants.

4.2c Homework: Elimination Method of Solving Linear Systems

Directions: Solve each system of linear equations using **elimination**. Make sure the equations are in the same form first. Choose three problems to solve by graphing as well as using elimination to solve the system. The graphs are located after problem #9. **The goal of elimination is to find a way to “eliminate” one of the variables by either adding the equations together or subtracting one from the other. In example #1 below, if we add the equations together, the y will be eliminated, allowing us to solve for x. Once we know the value of x, we can substitute that value into one of the original equations to solve for y.**

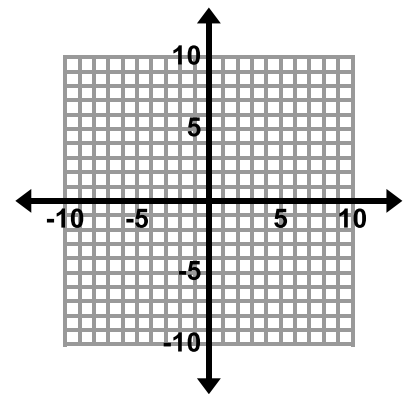
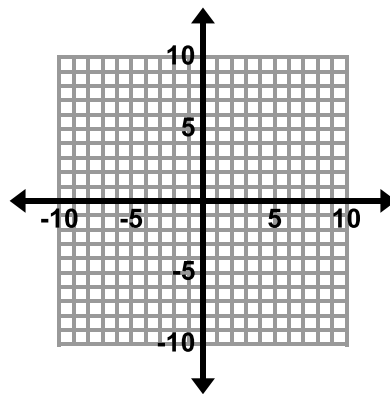
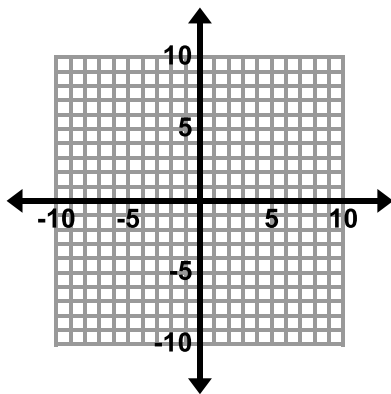
<p>1. $6x - y = 5$ $+ 3x + y = 4$</p> <hr/> <p>$9x = 9$ $x = 1$</p> <p>Using either of the original equations, substitute in 1 for x and solve for y. $3x + y = 4$ $3(1) + y = 4$ $3 + y = 4$ $y = 1$</p> <p>$(1, 1)$</p>	<p>2. $x + 4y = 9$ $-x - 2y = 3$</p>	<p>3. $x + 5y = -8$ $-x - 2y = -13$</p>
<p>4. $2x + y = 7$ $x + y = 1$ $(6, -5)$</p> <p>Adding these equations together will not result in one of the variables being eliminated; however if we subtract the first from the second, the y will be eliminated. The result will be $x = 6$. Now, use either of the original equations, replacing the x with 6, and solve for y.</p>	<p>5. $4x + 3y = 18$ $4x = 8 + 2y$</p>	<p>6. $-5x + 2y = 22$ $3x + 2y = -10$</p>

$$\begin{aligned} 7. \quad &6x - 3y = 36 \\ &5x = 3y + 30 \end{aligned}$$

$$\begin{aligned} 8. \quad &-4x + y = -12 \\ &-y + 6x = 8 \end{aligned}$$

$$\begin{aligned} 9. \quad &x + y = 7 \\ &-x - y = 12 \end{aligned}$$

Refer to class examples for examples of the different solving outcomes and what they mean.



10. An exam worth 145 points contains 50 questions. Some of the questions are worth two points and some are worth five points. How many two-point questions are on the test? How many five-point questions are on the test?

To solve this, first set up a system of equations to represent the problem:

One of the equations represents the total number of questions on the test: $x + y = 50$ where x represents the number of two-point questions and y represents the number of five-point questions on the test.

The other represents the total number of points on the test. Write the second equation for this system and then solve the system.

4.2d Class Activity: Elimination Method Multiply First

1. Solve the following system of linear equations using **elimination**:

$$4x + y = 7$$

$$-2x - 3y = -1$$

If we try to add these equations or subtract the second from the first as we did in the previous section, we notice that none of the variables cancel out. However, if we first multiply the second equation by 2, we can get the x 's to cancel out when we add the equations together.

$$4x + y = 7$$

$$-2x - 3y = -1$$

$\xrightarrow{x2}$

$$\begin{array}{r} 4x + y = 7 \\ + -4x - 6y = -2 \\ \hline -5y = 5 \\ y = -1 \end{array}$$

Now that we know the value of y , plug it back in to one of the original equations to solve for x :

$$4x + (-1) = 7 \rightarrow 4x = 8 \rightarrow x = 2$$

Solution: $(2, -1)$

****Note:** It is important to understand that multiplying (or dividing) all the terms in an equation by the same number results in an equivalent equation. Try graphing the original equation $-2x - 3y = -1$ and the new equation that was produced when we multiplied this equation by 2: $-4x - 6y = -2$. The resulting graphs will be the same line.

To solve any system of linear equations using elimination, do the following:

1. Write both equations in the same form.
2. Multiply the equations by nonzero numbers so that one of the variables will be eliminated if you take the sum or difference of the equations.
3. Take the sum or difference of the equations to obtain a new equation in just one unknown.
4. Solve for the remaining variable.
5. Substitute the value from step 4 back into one of the original equations to solve for the other unknown.
6. Check the solution in each of the original equations.

Directions: Solve each system of linear equations using **elimination**.

$$\begin{array}{r} 2. \quad x + 2y = 15 \xrightarrow{x-5} -5x - 10y = -75 \\ \quad 5x + y = 21 \quad + 5x + y = 21 \\ \hline \end{array}$$

$$-9y = -54 \rightarrow y = 6$$

Substitute 6 in for y into one of the original equations and solve for x :

$$x + 2(6) = 15$$

$$x + 12 = 15$$

$$x = 3$$

There are many different ways to solve this problem using elimination. One way is to multiply the first equation by -5 and add the equations together as shown above. Alternatively, you can multiply the second equation by 2 and subtract it from the first. Try it a few different ways and verify that the result is the same. The solution is $(3, 6)$.

$$\begin{array}{r} 3. \quad -3x + 2y = -8 \\ \quad 6x - 4y = -20 \end{array}$$

$$\begin{array}{r} 4. \quad 2x - 3y = 5 \\ \quad -3x + 4y = -8 \end{array}$$

$$\begin{array}{r} 5. \quad 3x - 2y = 2 \\ \quad 5x - 5y = 10 \end{array}$$




$$\begin{array}{r} 6. \quad 9x + 13y = 10 \\ \quad -9x - 13y = 8 \end{array}$$

$$\begin{array}{r} 7. \quad -16x + 2y = -2 \\ \quad y = 8x - 1 \end{array}$$

If using elimination, make sure these equations are in the same form first. For example, you may put the second equation in standard form by subtracting $8x$ from both sides leaving $-8x + y = -1$.

$$8. \begin{aligned} \frac{1}{2}x + y &= 0 \\ x - y &= -15 \end{aligned}$$

$$9. \begin{aligned} \frac{2}{3}x + \frac{1}{3}y &= 13 \\ -2x + y &= -21 \end{aligned}$$

Directions: Write a system of equations for each word problem below and then solve the system of equations using **elimination**. Write your answer in a complete sentence.   

10. The student officers are buying packs of streamers and balloons to decorate for a school dance. Packs of balloons cost \$3.50 and packs of streamers cost \$2. If the student officers bought a total of 12 packs of decorations and spent \$31.50, how many packs of balloons did they buy? How many packs of streamers did they buy? Write the solution in a complete sentence.

Define your Unknowns:

b = packs of balloons

s = packs of streamers

Equation for Number of Packs of Decorations:

$$b + s = 12$$

Equation for Cost of Decorations:

$$3.5b + 2s = 31.50$$



Solve:

Solution (in a complete sentence):

They bought 5 packs of balloons and 7 packs of streamers.

11. Jayda has a coin collection consisting of nickels and dimes. Write a story that matches the system of equations shown below that describes the coins in Jayda's collection where n is the number of nickels Jayda has and d is the number of dimes Jayda has.

$$n + d = 28$$

$$0.05n + .1d = 2.25$$

Story

Jayda has nickels and dimes in her collection. She has 28 coins worth a total of \$2.25. How many of each type of coin does she have?

Solve:

Solution (in a complete sentence):

Jayda has 17 dimes and 11 nickels.

4.2d Homework: Elimination Method Multiply First

Directions: Solve each system using **elimination**.



Refer to class activity for worked out problems and explanations. Remember, in these problems, you may have one solution, no solution, or infinitely many solutions. When the solving outcome is $a = a$ (i.e. $5 = 5$), there are infinitely many solutions. When the solving outcome is $a = b$ (i.e. $5 = 7$), there are no solutions. Examine the structure of the original equations to determine whether the equation has one, no, or infinitely many solutions before solving.

<p>1. $x + y = 4.5$ $\xrightarrow{\times 2}$ $2x + 2y = 9$ $-2x + 4y = 6$ $\quad +$ $-2x + 4y = 6$</p> <hr/> <p style="text-align: right;">$6y = 15$ $y = 2.5$</p> <p>Substitute 2.5 in for y in either of the original equations and solve for x. The solution is (2, 2.5). There are many different ways to solve this problem using elimination. One way is to multiply the first equation by 2 and then add the equations together as shown above. The goal is to multiply one equation by a factor that will eliminate one of the variables when the equations are added or subtracted.</p>	<p>2. $4x + y = -8$ $3x + 3y = 3$</p>
<p>3. $2x + y = 7$ $4x + 2y = 14$ Infinitely many solutions</p>	<p>4. $2x + 3y = -10$ $-4x + 5y = -2$</p>
<p>5. $x - 2y = \frac{2}{3}$ $-3x + 5y = -2$</p>	<p>6. $-3x - y = -15$ $8x + 4y = 48$ (3, 6)</p>
<p>7. $3x - y = 10$ $2x + 5y = 35$ (5, 5)</p>	<p>8. $x + y = 15$ $-2x - 2y = 30$</p>

Directions: Write a system of equations for each word problem below and then solve the system of equations using **elimination**. Write your answer in a complete sentence.

9. Tickets for a matinee are \$5 for children and \$8 for adults. The theater sold a total of 142 tickets for one matinee. Ticket sales were \$890. How many of each type of ticket did the theater sell? Write the solution in a complete sentence.

See class activity for a similar problem.

Define your Unknowns:

Equation for Number of Tickets Sold:

Equation for Ticket Sales:

Solve:

Solution (in a complete sentence):

10. Jasper has a coin collection consisting of quarters and dimes. He has 50 coins worth \$8.60. How many of each coin does he have? Write the solution in a complete sentence.

See class activity for a similar problem.

Define your Unknowns:

Equation for Number of Coins:

Equation for Value of Coins:

Solve:

Solution (in a complete sentence):

4.2e Class Activity: Revisiting Chickens and Pigs

1. A farmer saw some chickens and pigs in a field. He counted 60 heads and 176 legs. Determine exactly how many chickens and pigs he saw.
 - a. Solve the problem using the methods/strategies studied in this chapter. Solve in as many different ways as you can (graph, substitution, and elimination) and make connections between the strategies.

At this point, students should have the skills to write a system of equations to model this situation and solve this problem graphically and algebraically. Due to the size of the numbers, it is recommended that you show students how to find the point of intersection on a graph using a graphing calculator (or some other form of technology). An important skill for students to learn is how to set the window in the graphing calculator for a given problem.

- b. Which method do you prefer using to solve this problem? Use your preferred method to determine the number of chickens and pigs in a field with 45 heads and 146 legs.
Methods will vary

4.2e Homework: Revisiting Chickens and Pigs

Directions: Solve each of the following problems by writing and solving a system of equations. Use any method you wish to solve. Write your answer in a complete sentence.



1. In 1982, the US Mint changed the composition of pennies from all copper to zinc with copper coating. Pennies made prior to 1982 weigh 3.1 grams. Pennies made since 1982 weigh 2.5 grams. If you have a bag of 1,254 pennies, and the bag weighs 3,508.8 grams, how many pennies from each time period are there in the bag?

$x + y = 1254$ where x is the number of coins made prior to 1982 and y is the number of coins made since 1982

$$3.1x + 2.5y = 3508.8$$

There are 623 pennies made prior to 1982 and 631 pennies made since 1982

2. Blake has some quarters and dimes. He has 20 coins worth a total of \$2.90. How many of each type of coin does he have?
3. Ruby and Will are running a team relay race. Will runs twice as far as Ruby. Together they run 18 miles. How far did each person run?

4. Sarah has \$400 in her savings account and she has to pay \$15 each month to her parents for her cell phone. Darius has \$50 and he saves \$20 each month from his job walking dogs for his neighbor. At this rate, when will Sarah and Darius have the same amount of money? How much money will they each have?

5. The admission fee at a local zoo is \$1.50 for children and \$4.00 for adults. On a certain day, 2,200 people enter the zoo and \$5,050 is collected. How many children and how many adults attended?

6. Dane goes to a fast food restaurant and orders some tacos t and burritos b . Write a story that matches the system of equations shown below that describes the number of items Dane ordered and how many calories he consumed. Solve the system to determine how many tacos and how many burritos Dane ordered and ate.

$$t + b = 5$$

$$170t + 370b = 1250$$



Story

Solve:

Solution (in a complete sentence):

4.2f Class Activity: Solving Systems of Equations Mixed Strategies

Directions: Choose the method you feel is easiest for a given problem (graphing, substitution, or elimination). Place a letter in the box (g, s, or e) for each problem to identify the method you will be using before you solve the system. Solve each system of linear equations. There are blank graphs for you to use after #12.

<p>1. $2x - 3y = 12$ $x = 4y + 1$ $(9, 2)$</p> <p>You can use any method you wish to solve these problems. In this example, it might make sense to use substitution since one of the variables is already solved for. You can replace the x in the first equation with the expression $4y + 1$. Then, solve for y. Once you find the value of y, substitute it into either of the original equations to find the value of x. If you choose to solve using elimination, make sure both equations are in the same form first. See 4.2b for additional help on the substitution method for solving systems.</p>	<p>2. $x + y = 3$ $3x - 4y = -19$ $(-1, 4)$</p> <p>Since both equations are in standard form, it makes sense to solve this system using elimination. One way to do this is to multiply the first equation by 4 and then add the two equations together in an effort to eliminate the y. Once you solve for x, substitute this value into one of the original equations and solve for y. See 4.2c and 4.2d for additional help on the elimination method for solving systems.</p>
<p>3. $y = x - 6$ $y = x + 2$</p>	<p>4. $y - 2x = 1$ $2x + y = 5$</p>
<p>5. $y = 4x - 3$ $y = x + 6$</p>	<p>6. $x - y = 0$ $2x + 4y = 18$</p>

$$7. \begin{aligned} 3y - 9x &= 1 \\ y &= 3x + \frac{1}{3} \end{aligned}$$

☐

$$8. \begin{aligned} x + 2y &= 6 \\ -7x + 3y &= -8 \end{aligned}$$

☐

$$9. \begin{aligned} y &= -x + 5 \\ x - 4y &= 10 \end{aligned}$$

☐

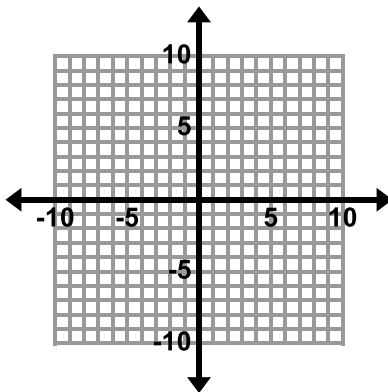
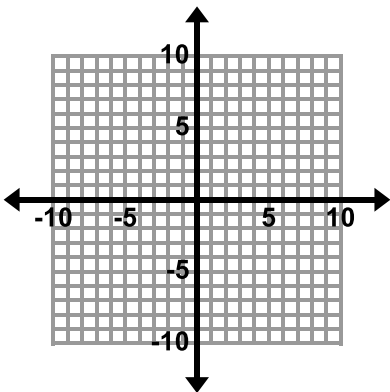
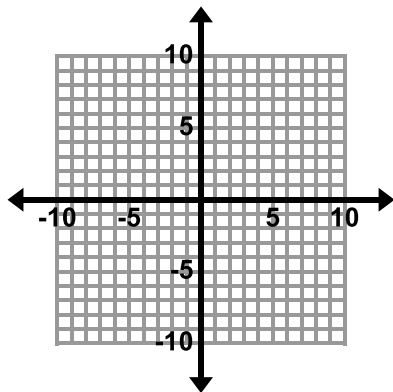
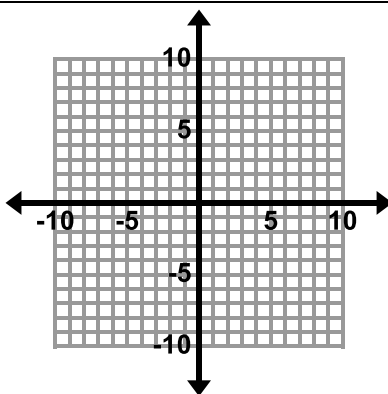
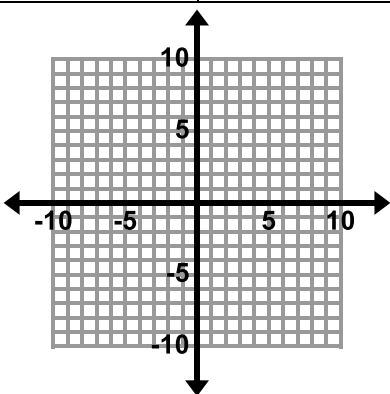
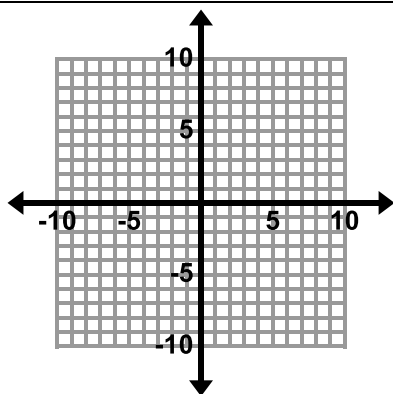
$$10. \begin{aligned} y &= x + 5 \\ y &= 2x - 10 \end{aligned}$$

☐

$$11. \begin{aligned} 3x + 2y &= -5 \\ x - y &= 10 \end{aligned}$$

☐

$$12. \begin{aligned} 2x - 5y &= 6 \\ 2x + 3y &= -2 \end{aligned}$$

☐


4.2f Homework: Solving Systems of Equations Mixed Strategies

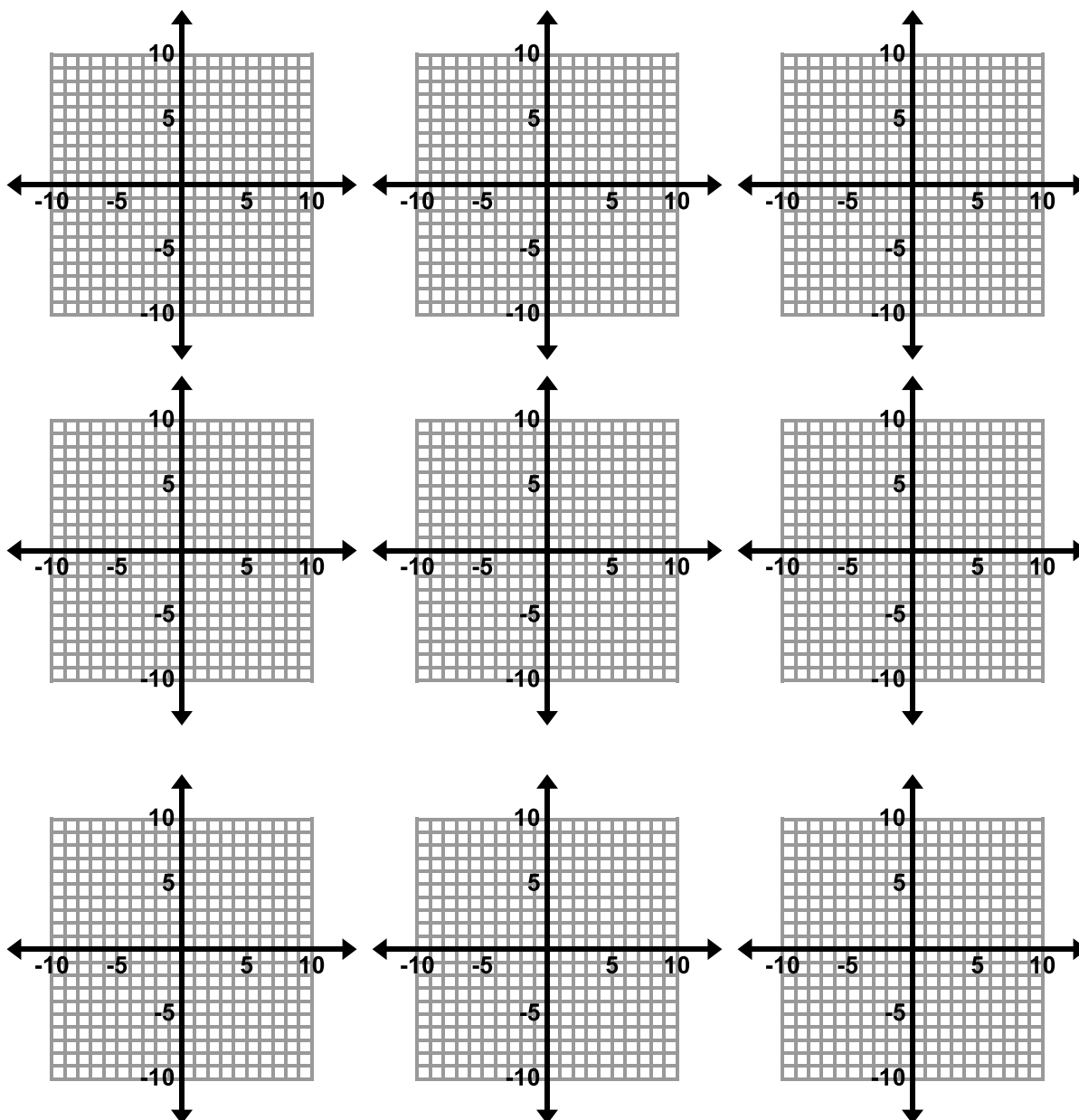
Directions: Choose the method you feel is easiest for a given problem (graphing, substitution, or elimination). Place a letter in the box (g, s, or e) for each problem to identify the method you will be using before you solve the system. Solve each system of linear equations. There are blank graphs for you to use after #8.

<p>1. $y = 4x$ $x + y = 5$ (1, 4)</p> <p>Students can use any method they wish to solve this problem. There are graphs at the end of the assignment if students wish to solve by graphing. Students may also choose to solve this problem by substitution. The y in the second equation can be replaced by the expression $4x$ from the first equation. Then, the second equation can be solved for x. Once the value of x is known, substitute it into either of the original equations to determine the value of y. See 4.2b for additional help on the substitution method for solving systems.</p>	<p>2. $x = -4y$ $3x + 2y = 20$</p>
<p>3. $y = x - 1$ $y = -x + 3$</p>	<p>4. $3x - y = 4$ $2x - 3y = -9$ (3, 5)</p> <p>Since both equations are in standard form, it would make sense to solve this system using elimination. There are many ways to do this. One way is to multiply the first equation by -3 and then add the resulting equation to the second equation. The y will be eliminated, allowing you to solve for x. Once you determine the value of x, substitute it back into one of the original equations to determine the value of y. See 4.2c and 4.2d for additional help on the elimination method for solving systems.</p>
<p>5. $x + 5y = 4$ $3x + y = -2$</p>	<p>6. $y = -x + 10$ $y = 10 - x$</p>

7. $y = 2x$
 $x + y = 12$



8. $y = 2x - 5$
 $4x - y = 7$



4.2g Self-Assessment: Section 4.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
<p>1. Determine which method of solving a system of linear equations may be easier depending on the problem.</p> <p>Sample Problems #1, #2.</p>	I am struggling to determine the best method to use to solve a system of linear equations.	I can determine the best method to use to solve all of the equations in Set A on the following page.	I can determine the best method to use to solve all of the equations in Set A and most of the equations in Set B.	I can determine which method of solving a system of linear equations will be easier depending on the problem.
<p>2. Solve simultaneous linear equations algebraically.</p> <p>Sample Problems #1, #2</p>	I can solve the equations in Set A on the following page.	I can solve all of the equations in Set A and most of the equations in Set B.	I can solve simultaneous linear equations algebraically by using both substitution and elimination methods.	I can solve simultaneous linear equations algebraically by using both substitution and elimination methods. I can also explain why I chose a particular solution method.
<p>3. Create a system of linear equations to model a real world problem, solve the system, and interpret the solution in the context.</p> <p>Sample Problem #3</p>	When faced with word problems similar to those in this chapter, I can match the different pieces of the equations that have been given to me to the story and solve the system of equations.	When faced with word problems similar to those in this chapter, I can identify the important quantities in a practical situation, complete partial expressions and equations <i>that have been given to me</i> , solve the system, and interpret the solution in the context.	When faced with word problems similar to those in this chapter, I can identify the important quantities in a practical situation, <i>write</i> expressions and equations showing the relationship between the quantities, solve the equations, and interpret the solution in the context.	When faced with word problems similar to those in this chapter, I can identify the important quantities in a practical situation, <i>write</i> expressions and equations showing the relationship between the quantities, solve the equations, and interpret the solution in the context. I can explain the reasoning behind each step in the process of arriving at my answer.

Section 4.2 Sample Problems (For use with self-assessment)

1. Determine which method will be easier to use for each of the following problems by placing S (substitution), G (graphing) or E (elimination) in the small box in the corner of each problem.
2. Solve the following systems of equations algebraically.

Set A

1. $y = 2$ $y = 3x + 2$ <input data-bbox="505 394 581 472" type="text"/>	2. $x = y + 2$ $4x - 3y = 11$ <input data-bbox="971 394 1047 472" type="text"/>	3. $x + 2y = 13$ $-x + 4y = 11$ <input data-bbox="1433 394 1510 472" type="text"/>
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Set B

1. $y = -4x + 8$ $5x + 2y = 13$ <input data-bbox="509 833 586 911" type="text"/>	2. $2y = x - 5$ $2y = x + 5$ <input data-bbox="971 833 1047 911" type="text"/>	3. $3x = y - 20$ $-7x + y = 40$ <input data-bbox="1433 833 1510 911" type="text"/>
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3. Tickets to the local basketball arena cost \$54 for lower bowl seats and \$20 for upper bowl seats. A large group purchased 123 tickets at a cost of \$4,262. How many of each type of ticket did they purchase?