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Chapter 5: Functions (3 weeks)

Utah Core Standard(s):

- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (8.F.1)
- Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line. (8.F.3)
- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (8.F.4)
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. (8.F.2)
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (8.F.5)

Academic Vocabulary: function, input, output, relation, mapping, independent variable, dependent variable, linear, nonlinear, increasing, decreasing, constant, discrete, continuous, intercepts

Chapter Overview: In this chapter, the theme changes from that of solving an equation for an unknown number, to that of "function" that describes a relationship between two variables. Students have been working with many functional relationships in previous chapters; in this chapter we take the opportunity to formally define function. In a function, the emphasis is on the relationship between two varying quantities where one value (the output) depends on another value (the input). We start the chapter with an introduction to the concept of function and provide students with the opportunity to explore functional relationships algebraically, graphically, numerically in tables, and through verbal descriptions. We then make the distinction between linear and nonlinear functions. Students analyze the characteristics of the graphs, tables, equations, and contexts of linear and nonlinear functions, solidifying the understanding that linear functions grow by equal differences over equal intervals. Finally, students use functions to model relationships between quantities that are linearly related. Students will also describe attributes of a function by analyzing a graph and create a graphical representation given the description of the relationship between two quantities.

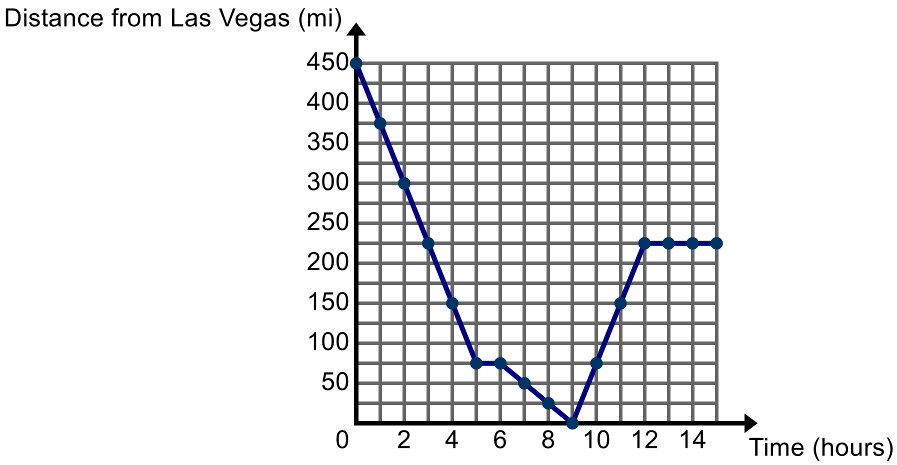
Connections to Content:

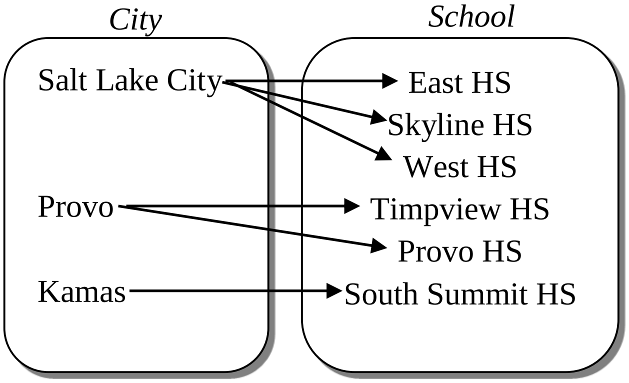
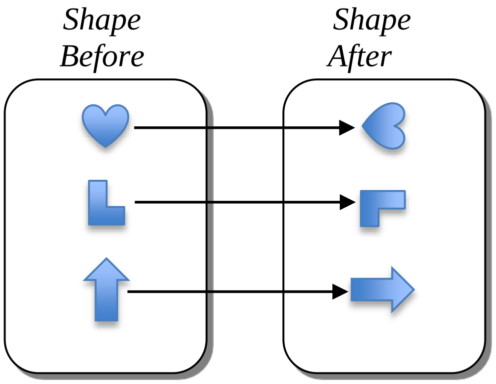
Prior Knowledge: Up to this point, students have been working with linear equations. They know how to solve, write, and graph equations. In this chapter, students make the transition to function. In the realm of functions, we begin to interpret symbols as variables that range over a whole set of numbers. Functions describe situations where one quantity determines another. In this chapter, we seek to understand the relationship between the two quantities and to construct a function to model the relationship between two quantities that are linearly related.

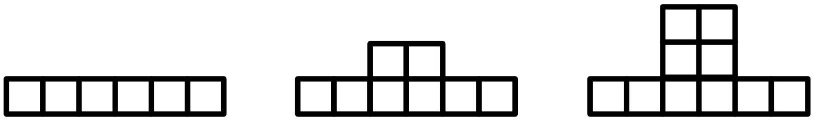
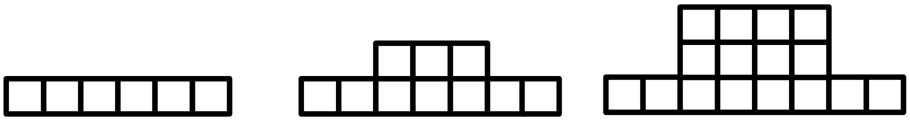
Future Knowledge: This chapter builds an understanding of what a function is and gives students the opportunity to interpret functions represented in different ways, identify the key features of functions, and construct functions for quantities that are linearly related. This work is fundamental to future coursework where students will apply these concepts, skills, and understandings to additional families of functions.

MATHEMATICAL PRACTICE STANDARDS

	Make sense of problems and persevere in solving them.	On Tamara's first day of math class, her teacher asked the students to shake hands with everyone in the room to introduce themselves. There are 26 students total in the math class. Can you determine the number of handshakes that took place in Tamara's math class on the first day of class? Can the relationship between number of students and the number of handshakes exchanged be modeled by a linear function? Justify your answer. *As students grapple with this problem, they will start to look for entry points to its solution. They may consider a similar situation with fewer students. They may construct a picture, table, graph, or equation. They may even act it out, investigating the solution with a concrete model. Once they have gained entry into the problem, students may look for patterns and shortcuts that will help them to arrive at a solution either numerically or algebraically.*
n#	**Reason abstractly and quantitatively.**	Nazhoni has completed her Driver's Education Training and is at the DMV (Division of Motor Vehicles) waiting in line to get her license to drive. She entered the DMV at 12:50 and pulled a number 17 to reserve a spot in line. Nazhoni notices that all of the employees at the DMV are still at lunch when she arrives. Once the employees return they start with number 1. There is digital sign showing the number for the person who is at the counter being helped. Nazhoni jots down some information on a piece of scratch paper as she is waiting in line. #5 was called to the counter at 1:25 pm #10 was called to the counter at 2:00 pm I have to leave by 2:45 pm in order to pick up my sister from school on time. Will Nazhoni make it to the front of the line in time to pick up her sister from school? *In order to solve this problem, students must make sense of the quantities involved in this situation and the relationship between the quantities. Students may first investigate this problem numerically, determining the average wait time between each person called to the counter. Students may also abstract this situation and construct a function to model the amount of time Nazhoni will have to wait based on the number she draws.*

💬	**Construct viable arguments and critique the reasoning of others.**	Compare and contrast the relationship of the gumball machines at Vincent Drug and Marley's Drug Store. If needed revise your conjecture about what kind of relationship makes a function and what disqualifies a relationship from being a function. *As students create, modify, and formulate their definition of a function they are constructing a viable argument that describes their thoughts on what a function is and what it is not. They make conjectures and build a logical progression of statements to explore the truth about their conjectures. They can share their definitions with others and decide whether they make sense and compare others' thoughts and ideas to their own.*
📈	**Model with mathematics.**	*Throughout this chapter, students will apply the mathematics they have learned to solve problems arising in everyday life, society, and workplace. The following problems give students the opportunity to use functions to model relationships between two quantities.* Steve is a lifeguard at a local community pool. Each day at noon, he records the temperature and the number of people in the pool. Do you think the relationship between temperature and number of people in the pool is linear? Why or why not? Two thousand, five hundred students attend a local high school. School starts at 8 am and ends at 2:30 pm. Many students stay after school for clubs, sports, etc. The school has a one-hour lunch at noon and seniors are allowed to leave campus for lunch. Sketch a graph of the number of cars in the student parking lot from 6 am to 4 pm. Ben and his family took a road trip to visit their cousins. The graph below shows their journey. Label the key features of the graph. Write a story about the graph. 

	Use appropriate tools strategically.	**Directions:** Determine whether the situations you choose can be modeled by a linear function or not. Provide evidence to support your claim. Show your work in the space below. Mr. Cortez drove at a constant rate for 5 hours. At the end of 2 hours he had driven 90 miles. After 5 hours, he had driven 225 miles. Can the relationship between time and distance driven be modeled by a linear function? Provide evidence to support your claim. Round 1 of a tennis tournament starts with 64 players. After each round, half the players have lost and are eliminated from the tournament. Therefore, in round 2 there are 32 players, in round 3 there are 16 players and so on. Can the relationship between round number and number of players remaining be modeled by a linear function? Provide evidence to support your claim. *The first step in constructing a function to model the relationship between two quantities is to determine what type of model is a potential fit for the data. At this point, student knowledge of the rate of change of a linear function is a tool the students rely on to determine whether the relationship between two quantities can be modeled by a linear function.*
	Attend to precision.	Determine whether each representation describes a function.   Is letter grade a function of percentage scored on a test? *In order to determine whether or not a given representation describes a function, students must be precise in their understanding of what a function is.*

<table>
<tr>
<td></td>
<td>Look for and make use of structure.</td>
<td>
<p>Examine the patterns below. Can the relationship between stage number and number of blocks in a stage be modeled by a linear function? Provide at least 2 pieces of evidence to support your answer.</p>

<p>Current Display Stage 2 Stage 3</p>

<p>Current Display Stage 2 Stage 3</p>
<p><i>While examining the patterns above, students may see that a linear pattern exhibits growth in one direction while the second pattern shown exhibits growth in two directions. These geometric representations give insight into the structure of a linear equation (and a quadratic equation which will be studied in subsequent courses).</i></p>
<p>Circle the letter next to each equation if it represents a linear function.
$2x + 4y = 16$ $y = x^2 + 5$
$y = x(x + 2)$ $xy = 24$</p>
<p><i>The equations above are a sampling of the types of functions students will encounter in this chapter. By the end of the chapter, students will solidify their understanding of the structure of a linear function and will surface ideas about the structure of additional types of functions that will be studied in subsequent courses.</i></p>
</td>
</tr>
<tr>
<td></td>
<td>Look for and express regularity in repeated reasoning.</td>
<td>
<p>Emily’s little brother painted on her math homework. She knows the data in each of the tables below represents a linear function. Help Emily determine what number is hidden behind the blob of paint.</p>
<table>
<tr><td><i>x</i></td><td>10</td><td>20</td><td>30</td><td>40</td></tr>
<tr><td><i>y</i></td><td>8</td><td>13</td><td></td><td>23</td></tr>
</table>
<p><i>Slope is a calculation that is repeated in a linear relationship. In order to solve this problem and similar problems, students must understand that linear functions grow by equal differences over equal intervals and apply this knowledge in order to complete the table.</i></p>
</td>
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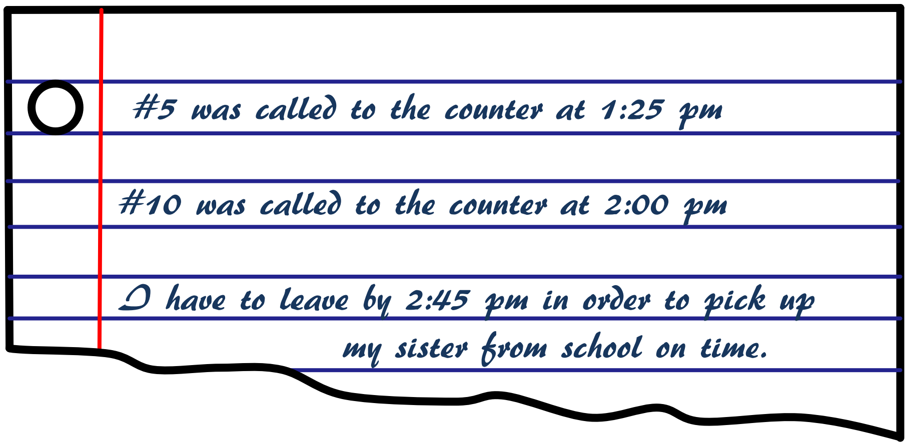
5.0: Anchor Problem: Waiting at the DMV

1. Nazhoni has completed her Driver's Education Training and is at the DMV (Division of Motor Vehicles) waiting in line to get her license to drive. She entered the DMV at 12:50 and pulled a number 17 to reserve a spot in line. Nazhoni notices that all of the employees at the DMV are still at lunch when she arrives. Once the employees return they start with number 1. There is digital sign showing the number for the person who is at the counter being helped. Nazhoni jots down some information on a piece of scratch paper as she is waiting in line.



#5 was called to the counter at 1:25 pm

#10 was called to the counter at 2:00 pm

I have to leave by 2:45 pm in order to pick up my sister from school on time.

   a. Use the picture of the scratch paper above to estimate what time it will be when Nazhomi will make it to the front of the line. **(Note: Assume that each person takes the same amount of time while being helped at the counter)**

Between numbers 5 and 10 thirty-five minutes have elapsed. This means it took 35 minutes for 5 people to be helped. If every person takes the same amount of time at the counter that means that it takes 7 minutes/person at the counter. If Nazhoni is number 17 and they were on number 10 at 2:00 that means they still need to serve 7 more people before they get to Nazhoni. At 7 minutes per person will it will take 49 minutes past 2:00 until they call her name.

$$7\ people \cdot \frac{7\ minutes}{person} = 49\ minutes$$

49 minutes past 2:00 pm is 2:49 pm. That means that Nazhoni will have her number called at 2:49.

   b. Will Nazhoni make it to the front of the line in time to pick up her sister from school?

No, she will make it to the front of the line at 2:49 and she must account for the 7 minutes at the counter. She will not be able to leave the DMV until 2:56.

   c. What time did the employees return from lunch and begin working.

Number 1 was the first number they called when they got back. We also know that at 1:25 pm they called number 5. Between the time they started working and 1:25 pm they helped 4 people. It takes 7 minutes for them to help someone at the counter which means that 28 minutes had elapsed between the time the employees returned from lunch and 1:25 pm. This means that they employees returned from their lunch break at 12:57 pm.

   d. Write an equation that represents the amount of time Nazhoni would have to wait dependent on the number she draws when she enters the DMV at 12:50.

Let n equal the number that a Nazhoni draws upon entering the DMV at 12:50 pm and t equal the number of minutes Nazhoni will have to wait until she can leave the DMV. Since the employees did not start working until 12:57 that is 7 minutes after 12:50. If each person takes 7 minutes at the counter the equation would be;

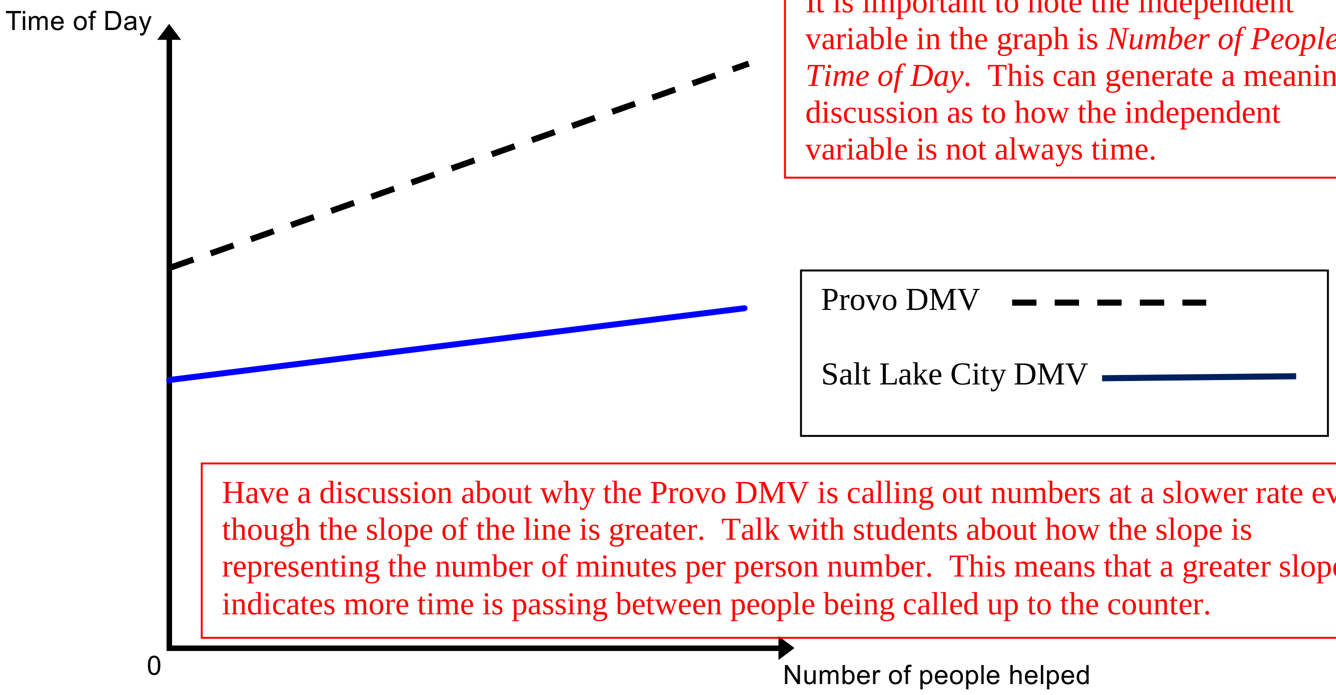
$t = 7n + 7$

In this equation wait time is a function of the number that Nazhoni draws at 12:50.

This problem was adapted from a task on Illustrative Mathematics.

2. The DMV in Provo and Salt Lake opened their doors for the day at the same time. The graphs below show the time of day as a function of the number of people called to the counter. Write down as many differences between the two DMVs as you can based upon the graphs.

It is important to note the independent variable in the graph is *Number of People* not *Time of Day*. This can generate a meaningful discussion as to how the independent variable is not always time.



Have a discussion about why the Provo DMV is calling out numbers at a slower rate even though the slope of the line is greater. Talk with students about how the slope is representing the number of minutes per person number. This means that a greater slope indicates more time is passing between people being called up to the counter.

Based upon the graph it can be inferred that the Provo DMV did not start calling out numbers immediately after they opened. The Provo DMV also calls out numbers at a slower rate than the Salt Lake City DMV because the slope, which corresponds to "the number of minutes per person number" is greater. This means that it takes a greater amount of minutes per person number at the Provo DMV line (dashed line) which makes them slower.

3. Do you think it is realistic that it takes the exact same amount of time for each person at the DMV? Explain.

No, it is not likely that each person would take the same amount of time because there are several different reasons why people go to the DMV and some business will take longer than others.

4. The following table shows more realistic data for the waiting time at the DMV: Is there a constant rate of change for this data? If not, is the data still useful? What can be inferred about the information given from the table?

Time	# Being Helped
12:58	30
1:25	33
2:00	37
2:08	38
2:50	44
3:30	49

There is not a constant rate of change for this data; however the data is still useful. Upon analyzing the wait time over the different intervals you can get a general idea of the "average" wait time at the DMV. Given this data, 8 to 8.5 minutes may be a good estimate of the average wait time.

Section 5.1: Define Functions

Section Overview:
This section begins by using a context to introduce a relation that represents a function and one that is not a function. By analyzing several situations students derive their own definition of a function. They also create their own representations of relations that are functions and those that are not functions. In the next lesson a candy machine analogy is used to help students further their understanding of a function as a rule that assigns to each input exactly one output. Students then play the function machine game and discover the rule that generates the output for a given input. As the section progresses, students are given different representations of relationships (i.e. table, graph, mapping, story, patterns, equations, and ordered pairs) and must determine if the representation describes a function. In the last lesson, students determine the dependent and independent variables in a functional relationship, understanding that the roles of the variables are often interchangeable depending on what one is interested in finding.

Concepts and Skills to be Mastered:
By the end of this section students should be able to:

1. Understand that a function is a rule that assigns to each input exactly one output.
2. Determine whether a given relation defines a function given different representations (i.e., table, graph, mapping, story, patterns, equations, and ordered pairs).
3. Determine the independent and dependent variable in a functional relationship.

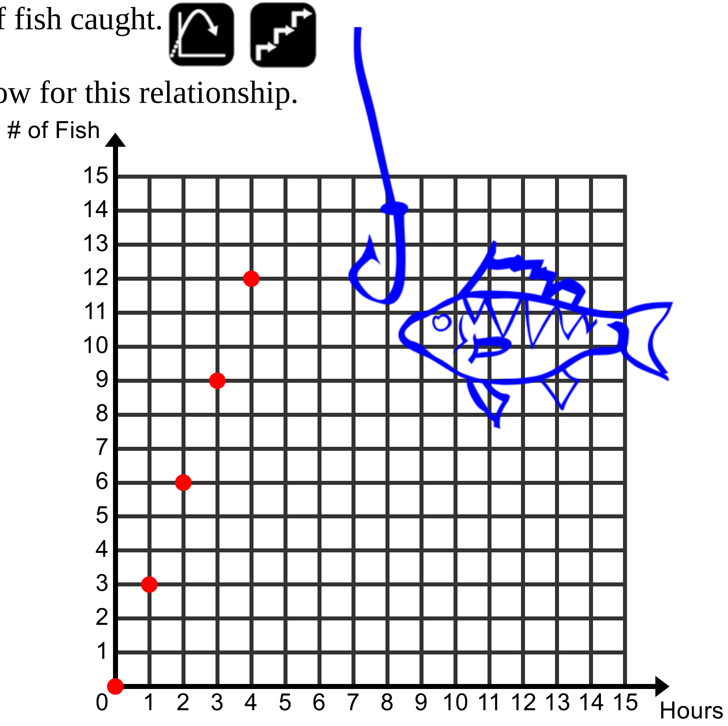
It is important to mention that students have studied many relationships that are functions already. In this chapter we take the opportunity to formally define function and discuss features of functions. As you work through the chapter, refer back to examples of functional relationships from chapters 2, 3, and 4 as appropriate.

5.1a Class Activity: Introduction to Functions

1. Jason is spending the week fishing at the Springville Fish Hatchery. Each day he catches 3 fish for each hour he spends fishing. This relationship can be modeled by the equation $y = 3x$, where x = number of hours spent fishing and y = the number of fish caught.

   a. Complete the graph and table below for this relationship.

Number of hours spent fishing x	Number of fish caught y
3	9
2	6
1	3
2	6
0	0
4	12

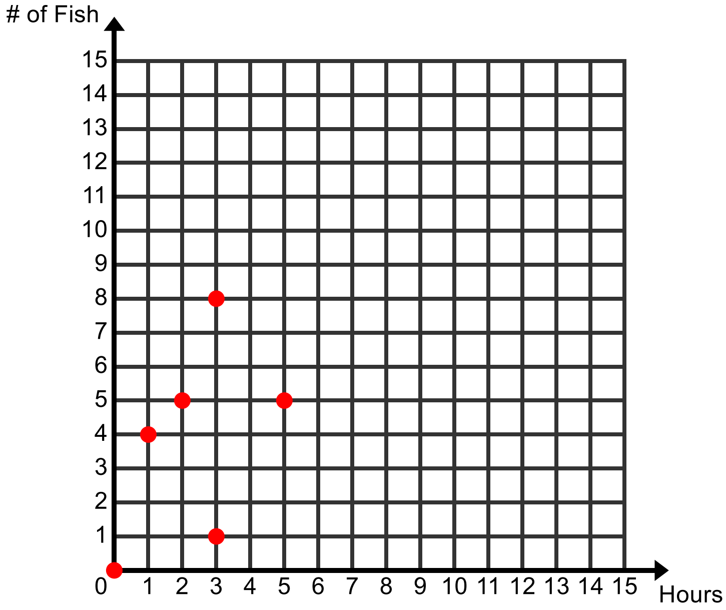


The situation above is an example of a **function**. We would say that *the number of fish caught is a **function** of the number of hours Jason spends fishing.*

2. Sean is also spending the week fishing; however he is fishing in the Bear River. Each day he records how many hours he spends fishing and how many fish that he caught. The table of values below shows this relationship.

   a. Complete the graph for this relationship.

Number of hours spent fishing x	Number of fish caught y
1	4
0	0
2	5
3	1
3	8
5	5

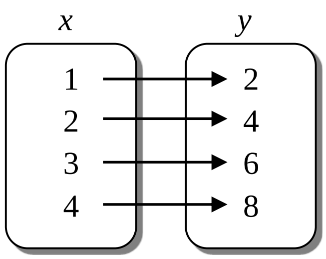


This situation is an example of a relation that is **not a function**. The number of fish that Sean catches is **not** a function of the number of hours he spends fishing.

3. Compare and contrast the relationship for Jason’s week spent fishing and Sean’s week spent fishing. Make a <u>conjecture</u> (an educated guess) about what kind of relationship makes a function and what disqualifies a relation from being a function.

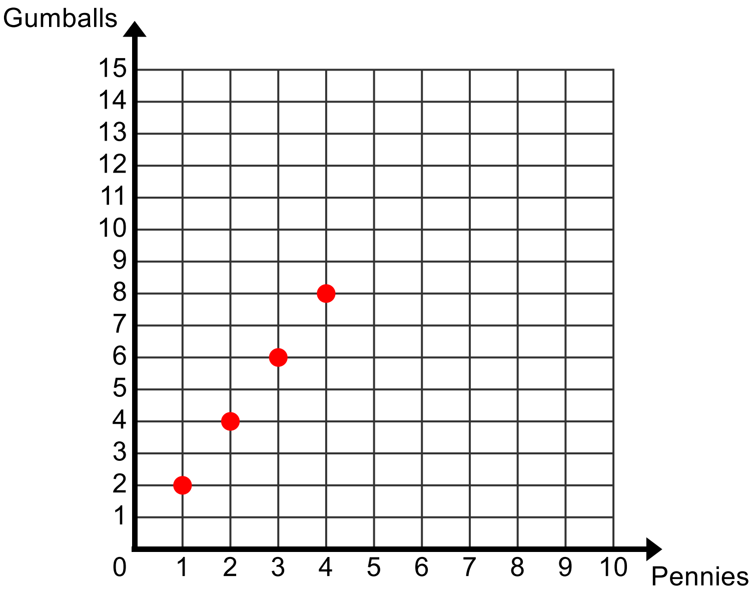
This is where you want your students to start to distinguish that there is not a functional relationship described by Sean’s week. Talk about how the y value (or the number of fish caught) cannot be determined for a given x value (number of hours spent fishing) for Sean’s week. However, if we consider Jason’s situation, the number of fish he catches is a function of the amount of time he spends fishing. You may ask, if I tell you how long Jason is fishing, can you tell me how many fish he will catch? The answer is yes. The number of fish Jason catches is a function of the amount of time he spends fishing. What if I tell you how long Sean spent fishing – can you tell me how many fish he will catch? The answer is no so the number of fish Sean catches is not a function of the amount of time he spends fishing.

4. Vanessa is buying gumballs at Vincent’s Drug Store. The mapping below shows the relationship between number of pennies, or x, she puts into the machine and the number of gumballs she gets out, or y.



   a. Complete the graph and table below for this relationship.

Number of pennies x	Number of gumballs y
1	2
2	4
3	6
4	8

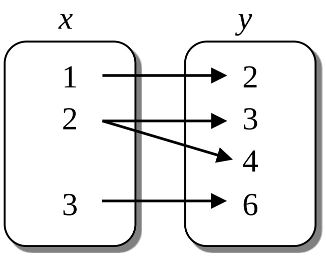


   b. Write an equation that models this relationship.

$$y = 2x$$

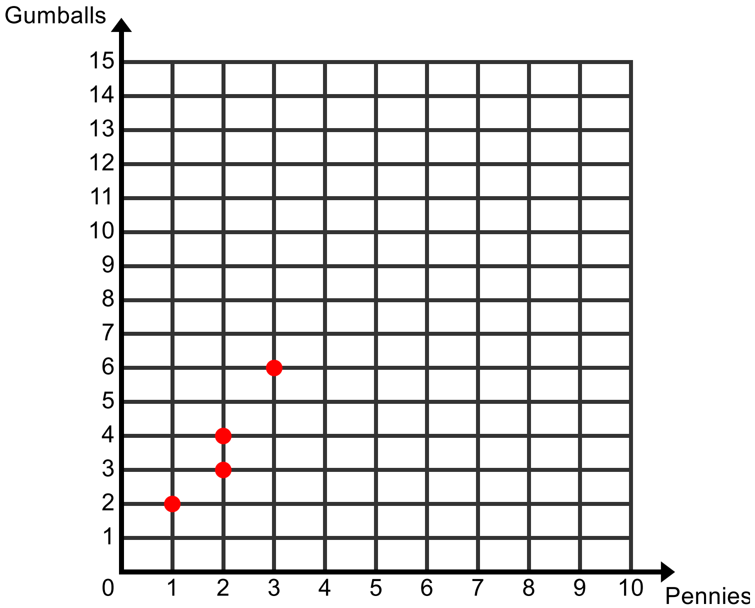
This is also an example of a **function.** We would say that *the number of gumballs received is a* ***function*** *of the number of pennies put in the machine.*

5. Kevin is across town at Marley's Drug Store. The mapping below relates the number of pennies he puts into the machine and how many gumballs he get outs.



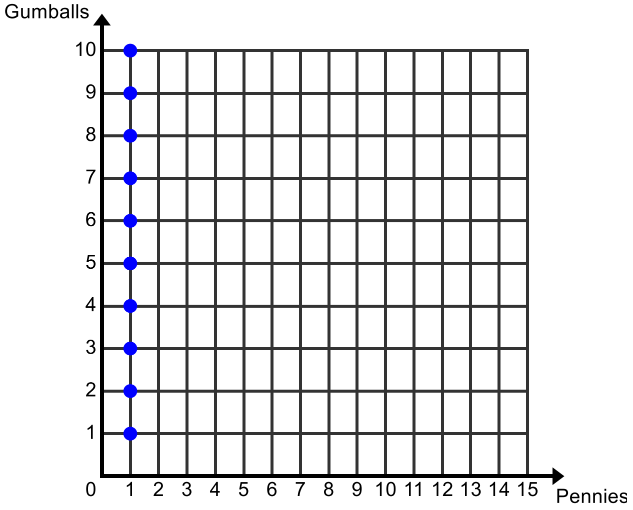
a. Complete the graph and table below for this relationship.

Number of pennies *x*	Number of gumballs *y*
1	2
2	3
2	4
3	6



This situation is an example of a relation that is **not a function**.

6. Cody is at Ted's Drug Store. The graph below relates the number of pennies he puts into the machine on different occasions and how many gumballs he gets out.



a. Explain how this gumball machine works. Each time Cody puts one penny in the machine he gets a different number of gumballs.

b. In this example, is the number of gumballs received a function of the amount of money put in? Explain your answer. This is not a function because Cody gets a different output each time he puts a penny in the machine. The machine is not "functioning". If I told you how many pennies I put in, would you be able to tell me how many gumballs would come out?

7. Compare and contrast the relationship of the gumball machines at the different drugstores. If needed revise your conjecture about what kind of relationship makes a function and what disqualifies a relationship from being a function.

See student answer

Students begin to use *repeated reasoning* at this point to formulate their definition of a function. They see that in relations that are not functions there is more than one *y*-value for every *x*-value. Likewise in the relations that are functions there is only one unique *y*-value for every *x*-value.

Below is a formal definition of a function. As you read it compare it to the conjecture you made about what makes a relation a function.

Given two variables, *x* and *y*, *y* is a function of *x* if there is a rule that determines one unique *y* value for a given *x* value.

Refer back to the first two examples. When Jason went fishing, he caught a unique number of fish based on the number of hours he spent fishing. If you know the number of hours Jason fishes for, you can determine the number of fish he will catch; therefore the number of fish he catches is a function of the number of hours he spends fishing. On the other hand, when Sean is fishing, it is not possible to determine the number of fish he catches based on the number of hours he fishes. On one day, he fished for three hours and caught one fish and on another day he fished for three hours and caught eight fish. There are two different *y* values assigned to the *x* value of 3 hours. In Sean's situation, the number of fish he catches is **not** a function of the number of hours he spends fishing.

Likewise, the gumball machine at Vincent's Drug Store represents a function because each penny inserted into the gumball machine generates a unique amount of gumballs. If you know how many pennies are inserted into the gumball machine at Vincent's, you can determine how many gumballs will come out. However, the gumball machine at Marley's Drug Store is **not** a function because there is not a unique number of gumballs generated based on the number of pennies you put in. One time 2 pennies were inserted and 4 gumballs came out and at another time 2 pennies were inserted and 3 gumballs came out. You are unable to determine the number of gumballs that will come out based on how many pennies are put into the machine.

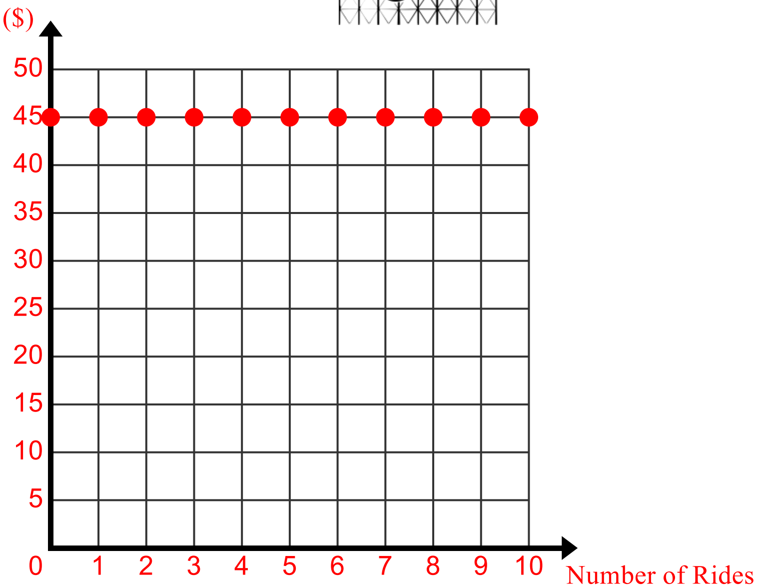
8. Explain in your own words why the number of gumballs received at Ted's Drug store is not a function of the amount of money put in. Be specific and give examples to support your reasoning.

The gumball machine at Ted's Drug store is not a function because for the given *x* value of 1 penny there is not a unique y value or number of gumballs. For example, when Cody inserts 1 penny on one occasion he gets one gumball but when he inserts one gumball on a different occasion he gets 9 gumballs.

9. The cost for entry into a local amusement park is $45. Once inside, you can ride an unlimited number of rides.

   a. Complete the graph and table below for this relationship.

Number of rides x	Amount spent (dollars) y
0	45
1	45
2	45
3	45
4	45



   b. Is the amount one spends a function of the number of rides he/she goes on? Why or why not? Yes, each input generates a unique output. It is a common error to say that this is not a function because we see the same output given different inputs. Ask students, “If I tell you how many rides I went on, can you tell me how much I spent?” The answer is yes so it is a function.

10. The table below show the number of hours Owen plays his favorite video game and the number of points he scores.

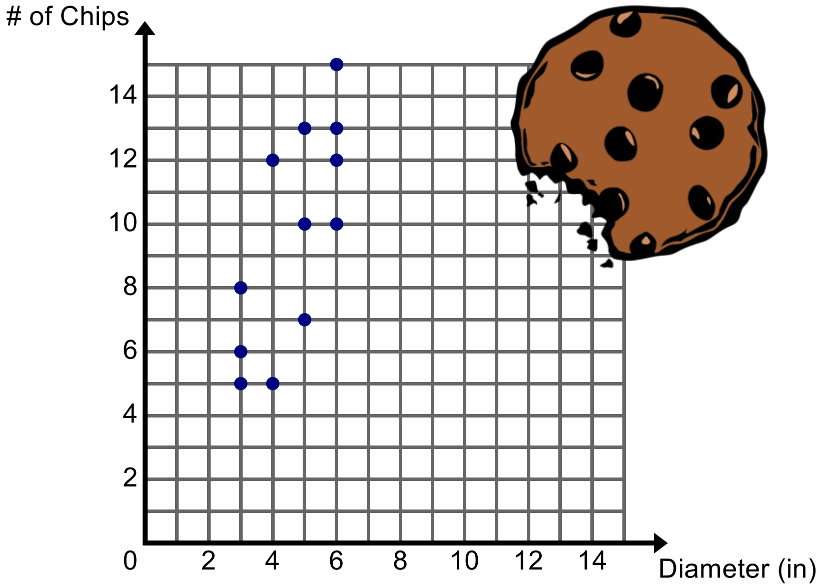
Time Spent Playing (hours)	Number of Points Scored
1	5,000
1	5,550
1	6,500
2	11,300
2	12,400
3	15,000

   a. Is the number of points Owen scores a function of the amount of time he spends playing? Why or why not? No, this is not a function. There are different output values (number of points scored) for the same input value (time spent playing).

5.1a Homework: Introduction to Functions

See class activity for several examples similar to the problems in this homework.

1. Betty’s Bakery makes cookies in different sizes measured by the diameter of the cookie in inches. Curious about the quality of their cookies, Betty and her assistant randomly chose cookies of different sizes and counted the number of chocolate chips in each cookie. The graph below shows the size of each cookie and the number of chocolate chips it contains.



Diameter of Cookie (in) x	# of Chocolate Chips y
3	5
3	8
4	12

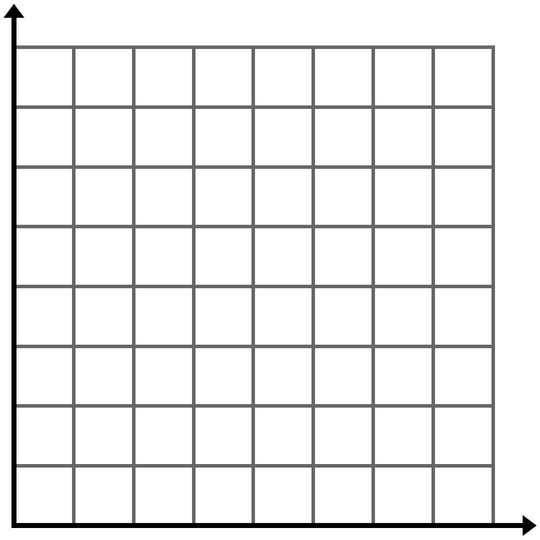
   a. Complete the table to the right of the graph. A few answers have been given in the table above.

   b. Is the number of chocolate chips in a cookie a function of the diameter of the cookie? Why or why not? No, there are different outputs for the same input. For example, a cookie with a diameter of 3 inches (the input) yields three different outputs: 5 chocolate chips, 6 chocolate chips, and 8 chocolate chips. If one is told the diameter of a cookie, it is not possible to determine the number of chocolate chips the cookie has; therefore the number of chocolate chips in a cookie is not a function of the diameter of the cookie.

2. The number of tires y in the parking lot at Hank’s Honda Dealership can be modeled by the equation $y = 4x$ where x represents the number of cars in the parking lot.

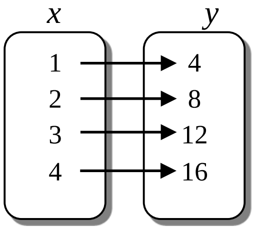
   a. Complete the table and graph below for this relationship. A few ordered pairs have been given in the table. Complete the table and corresponding graph. Be sure to label the axes of the graph.

Number of cars x	Number of tires y
0	0
1	4



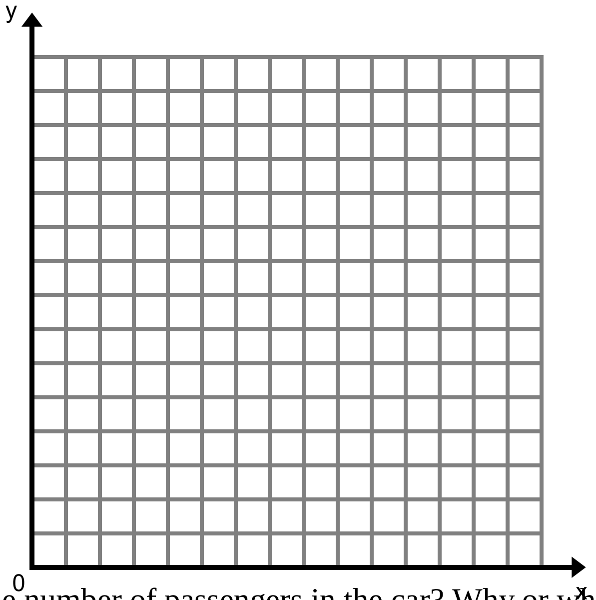
   b. Is the number of tires a function of the number of cars? Why or why not?

3. The cost for cars entering a scenic by-way toll road in Wyoming is given by the mapping below. In this relation y is the dollar amount to enter the by-way and x is the number of passengers in the car.



a. Complete the graph and table below for this relationship.

Number of passengers x	Amount per car (dollars) y



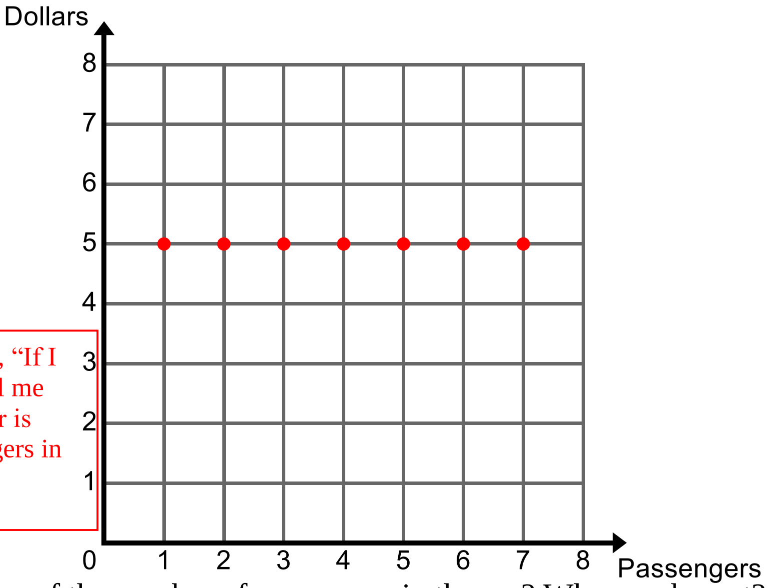
b. Is the amount spent per car a function of the number of passengers in the car? Why or why not?

4. The cost for cars entering a scenic by-way toll road in Utah is $5 regardless of the number of passengers in the car.

a. Complete the graph and table below for this relationship.

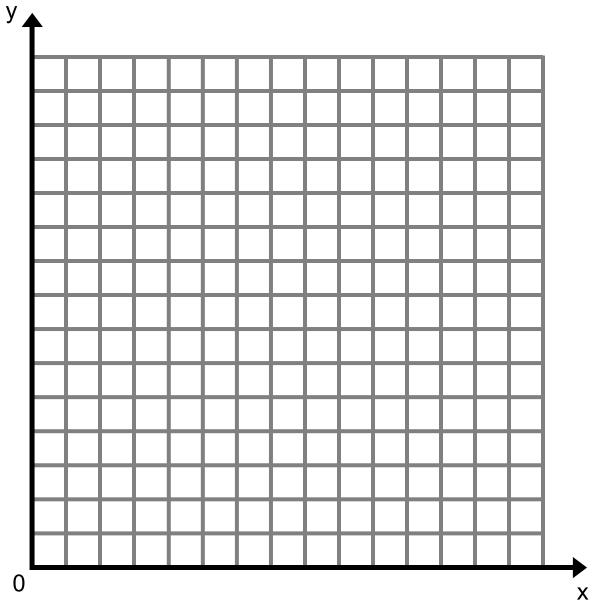
Number of passengers x	Amount per car y
1	5
2	5
3	5
4	5
5	5

Yes, for each input there is a unique output. Ask students, “If I tell you about many passengers are in the car, can you tell me how much they will have to pay at the toll?” If the answer is yes, then there is a rule that relates the number of passengers in the car to the amount you have to pay.



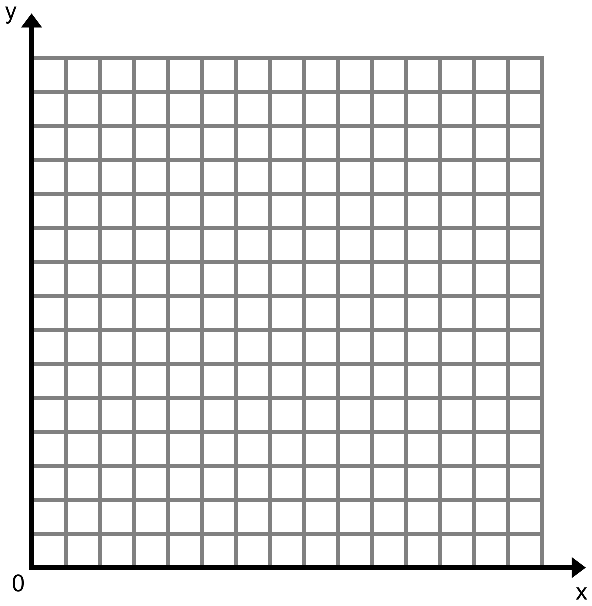
b. Is the amount spent per car a function of the number of passengers in the car? Why or why not?
See box above for answer and explanation

5. Create your own context or story that represents a relation that is a function.
   a. Story: There are many different situations that represent relations that are functions. For example, consider a can of tennis balls with three balls in it. The total number of tennis balls one has is a function of the number of cans of tennis balls that person has. Put another way, if I told you the number of cans of tennis balls I have, would you be able to tell me the number of tennis balls I have? The answer is yes so the number of tennis balls one has is a function of the number of cans of tennis balls one has.
   b. Complete the graph and table below for this relationship.



   c. Explain why this relation is a function.

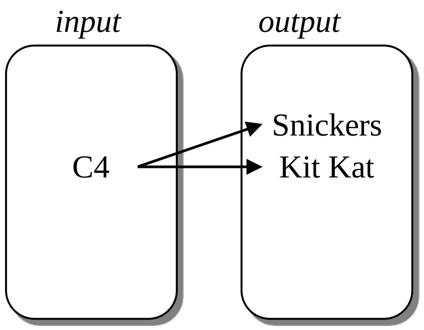
6. Create your own context or story that represents a relation that is **not** a function.
   a. Story:
   There are many different situations that represent relations that are not functions. For example, the distance a person can run is likely not a function of the amount of time the person runs for. Consider three different people running for 10 minutes. One person may run 1.8 miles, another may run 1 mile, and a third may run 0.8 miles. There are several different outputs (distance run) for a given input (time spent running).
   b. Complete the graph and table below for this relationship.



   c. Explain why this relation is not a function.

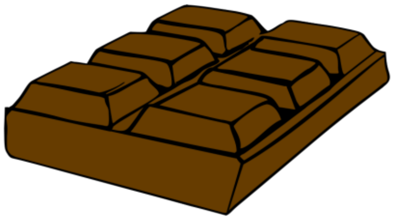
5.1b Class Activity: Function Machine

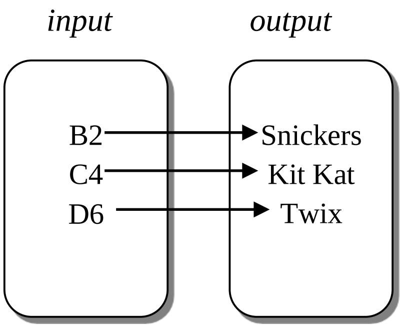
One way to think about the *x* and *y* variables in a functional relationship are as input (*x*) and output (*y*) values. To better understand how input and output values are related in a function consider the following analogy.
The Candy Machine Analogy: When you buy candy from a vending machine, you push a button (your input) and out comes your candy (your output). Let's pretend that C4 corresponds to a Snickers bar. If you input C4, you would expect to get a Snickers bar as your output. If you entered C4 and sometimes the machine spits out a Snickers and other times it spits out a Kit Kat bar, you would say the machine is "not functioning" – one input (C4) corresponds to two different outputs (Snickers and Kit Kat).



Not a function!

Let's look at what a diagram might look like for a machine that is "functioning" properly:



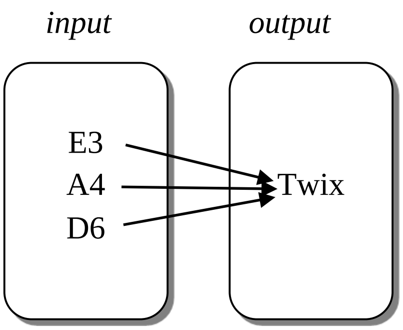


A Function!

In this situation, each input corresponds to exactly one output. The candy bar that comes out of the machine is dependent on the button you push. We call this variable the **dependent variable**. The button you push is the **independent variable**.

Let's look at one more scenario with the candy machine. There are times that different inputs will lead to the same output. In the case of the candy machines, companies often stock popular items in multiple locations in the machine. This can be represented by the following diagram:

It is important to point out the difference between this example and the first example. This is a function because for each input there is only one output. However in the previous example, there are different outputs for the same input. This requires students to attend to precision in the definition of a function.

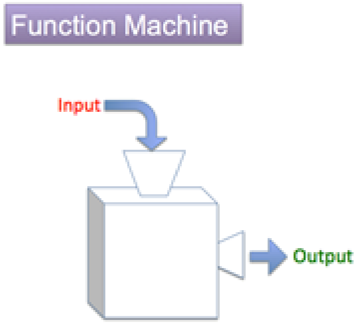


A Function!

Even though the different inputs correspond to the same output, our machine is still "functioning" properly. This still fits the special requirement of a function – each input corresponds to exactly one output.

THE FUNCTION MACHINE:

In this activity, you will give your teacher a number. He/she will perform some operations on the number, changing it to a new number. Your goal is to figure out what rule/function is being applied to the number. Use the tables below to keep track of the numbers you give your teacher (inputs) and the numbers your teacher gives you back (outputs). Once you figure out the function, write it in the space below the table.



In this activity, students give the teacher a number (input), the teacher applies a function/rule, and gives the students a new number (output). Their job is to figure out the rule/function being applied to the input. It may help students to first say the rule verbally and then work toward the symbolic representation/equation. For example, in table 1, students would say, “the output is two less than the input” or “the output is equal to the input minus 2”.

The examples given below are samples of what teachers may choose to do in class. The actual examples teachers do in class may differ from those shown below.

INPUT #	OUTPUT #
0	–2
2	0
5	3
10	8

Function: $y = x - 2$

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #
15	5
3	1
1	$\frac{1}{3}$
–21	–7

Function: $y = \frac{x}{3}$

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #
0	10
–1	10
5	10
7	10
½	10

Function: $y = 10$

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #
0	–4
1	–1
2	2
3	5
4	8
5	11

Function: $y = 3x - 4$

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

Now try it on your own or with a partner. Write the function for each of the following relations. Use the words input and output in your written function equation. Then write the function as an equation using *x* and *y*. The first one has been done for you. There is space for you to verify your function is correct.

1.	**Double the input increased by one will get the output.**	
Input x	Function: $y = 2x + 1$	Output y
8	$2(8) + 1 = 17$	17
0	$2(0) + 1 = 1$	1
–3	$2(-3) + 1 = 1$	–5
2		5
1		3
–1		–1
7		15

2.	The output is equal to 5 less than the input	
Input x	Function: $y = x - 5$	Output y
3		–2
0		–5
10		5
2		–7
5		0
1		–4
–4		–9

3.	The output is 5 times the input	
Input x	Function: $y = 5x$	Output y
5		25
–2		–10
0		0
1		5
–4		–20
–9		–45
3		15

4.	The output is 2 more than twice the input	
Input x	Function: $y = 2x + 2$	Output y
–3		–4
–2		–2
–1		0
0		2
1		4
2		6
3		8

5.	The output is 10 less than twice the input	
Input x	Function: $y = 2x - 10$	Output y
4		–2
–3		–16
0		–10
1		–8
7		4
9		8
5		0

6.		
Input x	Function: cannot be determined, this is not a function	Output y
2		3
5		–9
2		7
5		10
4		9
2		0
–1		–3

5.1b Homework: Function Machine

Directions: Write the function for each of the following relations. See class activity, pg. 21, for several examples with answers.

1.		
Input *x*	Function: $y = x - 2.5$	Output *y*
5		2.5
–2		–4.5
0		–2.5
1		–1.5
–4		–6.5
–9		–11.5
3		0.5

2.		
Input *x*	Function:	Output *y*
2		½
–4		–1
0		0
1		¼
–9		–9/4
24		6
8		2

3.		
Input *x*	Function: $y = 3x + 3$	Output *y*
–4		–9
–3		–6
–2		–3
0		3
1		6
2		9
3		12

4.		
Input *x*	Function:	Output *y*
2		3
–4		–3
0		1
1		2
–9		–8
–17		–16
10		11

5.		
Input *x*	Function:	Output *y*
–4		16
–3		9
–2		4
–1		1
0		0
1		1
2		4

6.		
Input *x*	Function:	Output *y*
1		3
6		–8
1		7
5		10
6		8
2		0
–1		–3

7. Were you able to find a function for number 6? If so, write it down. If not, explain why.

Directions: Create your own function machines, fill in the values for each input and its corresponding output. #8 is a sample answer. The rule being applied is $4x - 1$. Take any input, apply the rule, and write the corresponding output.

8.		
Input	Function:__$4x - 1$________	Output
–2	$4(-2) - 1$	–9
–1	$4(-1) - 1$	–5
0	$4(0) - 1$	–1
1	$4(1) - 1$	3
2	$4(2) - 1$	7
3	$4(3) - 1$	11
4	$4(4) - 1$	15

9.		
Input	Function:________________	Output

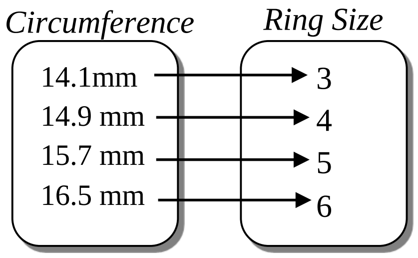
10. Create a machine that is **not** a function. Explain why your machine is "dysfunctional".

Input	Function:________________ =	Output

5.1c Class Activity: Representations of a Function

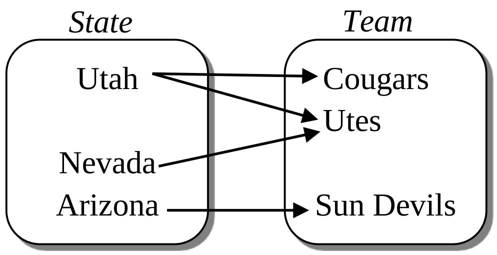
Functions can also be described by non-numeric relations. A **mapping** is a representation of a function that helps to better understand non-numeric relations. Study each relation and its mapping below. Then decide if the relation represents a function. Explain your answer.

1. **Input:** circumference of finger
Output: ring size



Function? Explain. Yes, for every input there is a unique output.

2. **Input:** state a person lives in
Output: the team they root for in college football



Function? Explain. No, for the input of Utah there is more than one output. Someone who lives in Utah could root for the Utes and the Cougars.

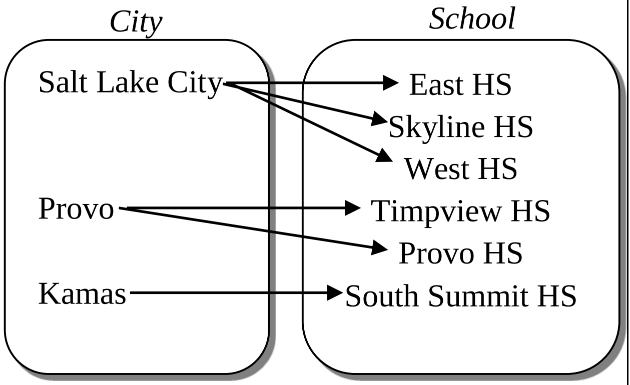
3. Write the ordered pairs (circumference, ring size) that correspond to problem #1.

$\{(14.1, 3), (14.9, 4), (15.7, 5), (16.5,6)\}$

4. Write the ordered pairs (state a person lives in, team they root for) that correspond to problem #2.

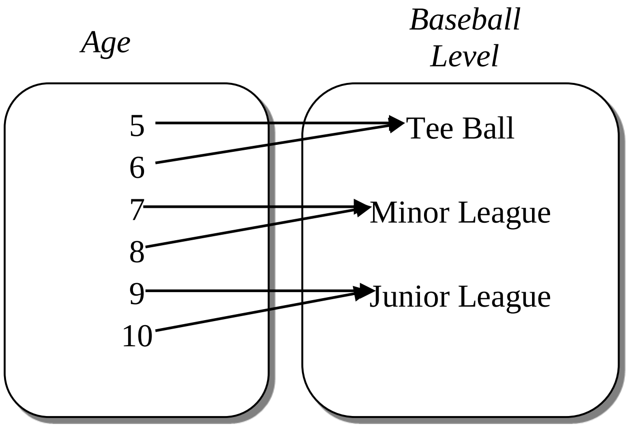
{(Utah, Cougars), (Utah, Utes), (Nevada, Utes),
(Arizona, Sun Devils)}

5. **Input:** city student lives in
Output: high school they go to



Function? Explain. No, if you were told the city someone lived in, it would not be possible to determine which high school they attend

6. **Input:** Age
Output: Level of Baseball Team



Function? Explain. Yes, for each input there is a unique output

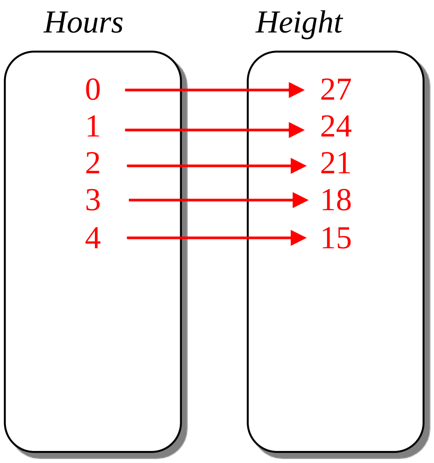
As we have seen, there are many ways to represent a relation or function. In the following problems, you will be given one representation of a relation and asked to create additional representations. Then, you will be asked to determine whether the relation represents a function or not.

7. **Story:** A candle is 27 centimeters high and burns 3 centimeters per hour. An equation that models this relation is $c = 27 - 3h$ where c is the height of the candle in centimeters and h is the number of hours the candle has been burning.
   a. Express this relation as a table, mapping, graph, and set of ordered pairs.

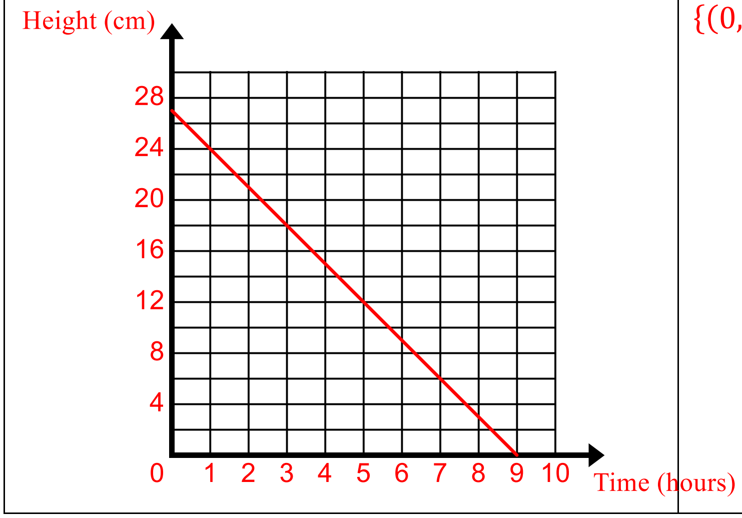
Table

Time (hours) h	Height (cm) c
0	27
1	24
2	21
3	18
4	15

Mapping



Graph



Set of Ordered Pairs

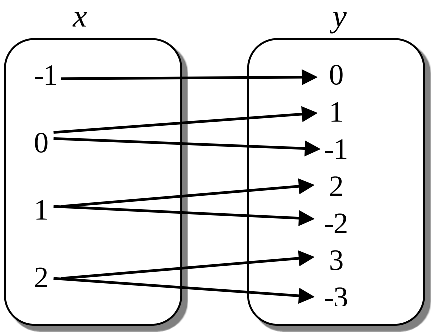
$\{(0, 27), (1, 24), (2, 21), (3, 18), (4, 15)\}$

   b. Is the height of the candle a function of the amount of time it has been burning? Explain.

This is a function, for every input there is only one output. The height of the candle depends on the amount of time the candle has been burning.

As students work through this lesson encourage them to explain how one can determine if a relation is a function or not based on the different representations of a function. You may also ask them if there is a representation that makes it easier for them to determine whether two quantities are/are not in a functional relationship.

8. **Mapping:**

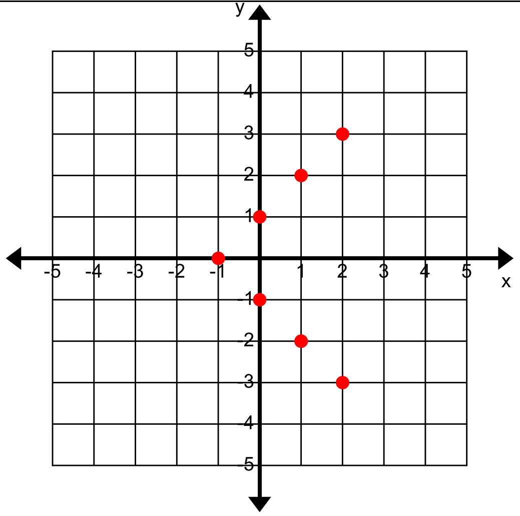


a. Express this relation as a table, graph, and set of ordered pairs.

Table

x	y
-1	0
0	1
0	-1
1	2
1	-2
2	3
2	-3

Graph



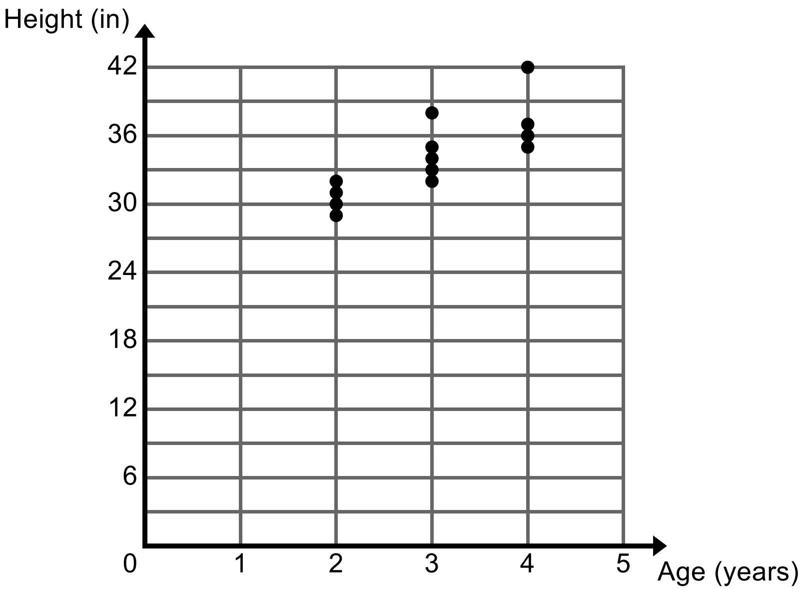
Set of Ordered Pairs

$\{(-1,0),(0,1),(0,-1),(1,2),(1,-2),(2,3),(2,-3)\}$

b. Is this relation a function? Explain.

No this is not a function, for several of the *x* values or inputs, there are 2 different *y* values or outputs. For example, the input of 0 corresponds to the outputs 1 and –1.

9. **Graph:** Discovery Place Preschool gathered data on the age of each student (in years) and the child's height (in inches). The graph below displays the data they gathered.

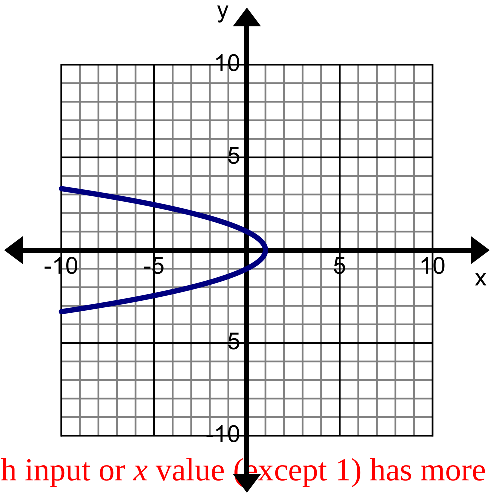


a. Is a child's height a function of the child's age? Explain. No, there are several different outputs (heights) for the same input (age)

Directions: Determine if each relation or situation defines a function. Justify your answer. It may help to make an additional representation of the relation.

10. {(30, 2), (45, 3), (32, 1.5), (30, 4), (41, 3.4)}
No, for the input of 30 there are two outputs, 2 and 4.

11.

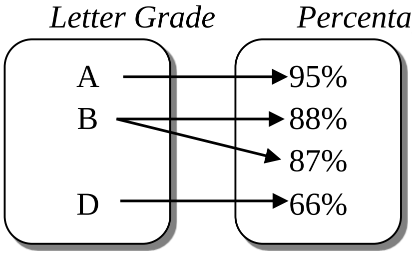


No, each input or x value (except 1) has more than one output or y-value. Have students list the ordered pairs for $x = -3$

12. $x = 2$
No, there are an infinite number of y-values for $x =$ 2; consider creating a mapping of this situation and comparing it to a mapping of $y = 2$.

13. $3x + 6y = 18$
Yes, this graph is a line

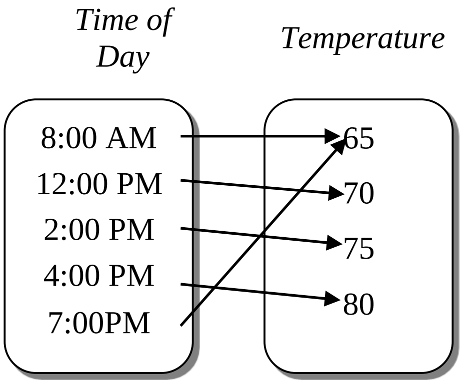
14.



No for the input of B there is more than one output, 88% and 87%. Ask students, if I told you the letter grade I earned on a test, would you be able to tell me the percentage I earned? Also, use the language, “percentage is not a function of letter grade.”

15. Is letter grade a function of percentage scored on a test? It may help students to make a mapping of this relationship. Yes, for each input there is a unique output. For example, if I tell you that I scored an 88% on the test, can you tell me the letter grade? Would there be two different letter grades associated with 88%?

16.



Yes, for each input there is only one output. It cannot be both 65˚ at 8 am and 70˚. Temperature is a function of time of day.

17. Is time of day a function of the temperature? Again, make a mapping to help – you may decide to use the mapping in #16, switching the input and output. No, time of day is not a function of temperature. There are multiple times of day that could have the same temperature.

18.

Length of Radius (cm)	Length of Diameter (cm)
0.5	1
1	2
1.5	3
2	4

Yes, ask students for the rule in this situation, the diameter of a circle is always twice the radius

19. **Input:** name of city in the U.S.
Output: state city is in
Hint: There are 16 states in the United States that have a city called Independence.

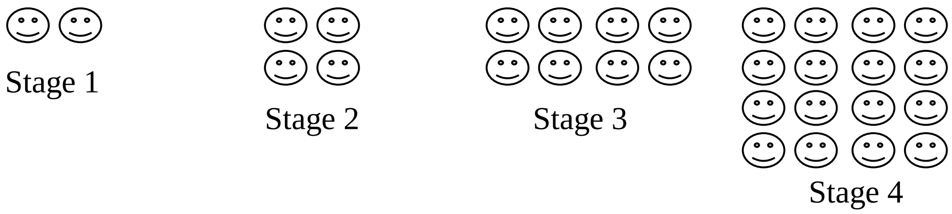
Not a function, create a mapping to help, have students research other city names that appear in multiple states

5.1c Homework: Representations of a Function

See class activity for *several* examples with answers and explanations.

1. Use the pattern below to answer the questions that follow.

Pattern:

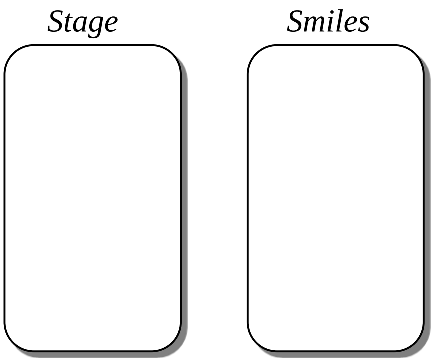


a. Express this relation as a table, mapping, and graph.

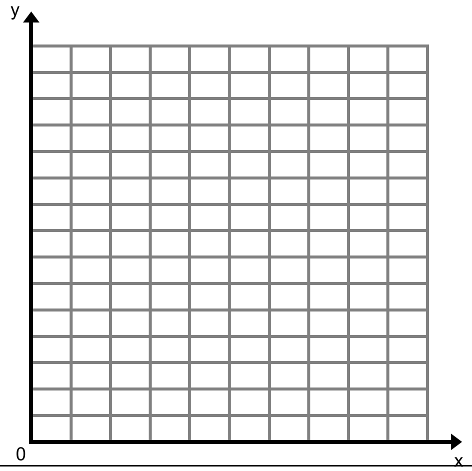
Table

Stage number	Number of Smiles

Mapping



Graph

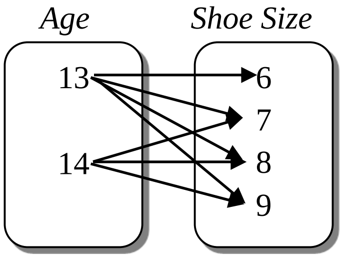


Set of Ordered Pairs

b. Is this relation a function? Explain how you know.

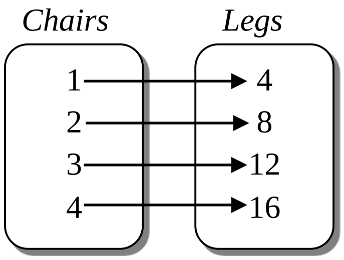
Directions: Determine if each relation or situation defines a function. Justify your answer.

2. **Input:** age
   Output: shoe size



Function? Explain.

3. **Input:** number of chairs
   Output: number of legs



Function? Explain.

4. List the ordered pairs that correspond to #2: (age, shoe size).

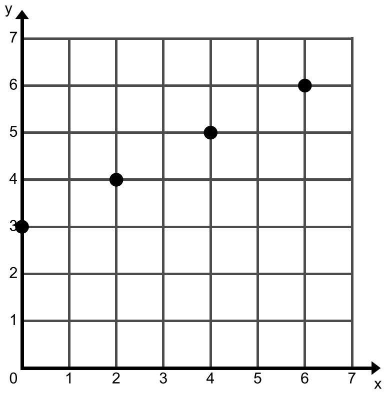
5. List the ordered pairs that correspond to #3: (number of chairs, number of legs)

6.

x	*y*
0.2	1.5
0.4	1.25
0.6	1.5
0.8	1.25

Function? Explain. Yes, this data describes a function

7.

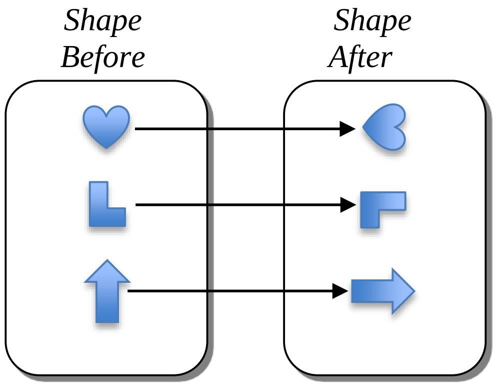


Function? Explain.

8. A car is traveling at a constant rate of 60 mph. Is the car's distance traveled a function of the number of hours the car has been driving?

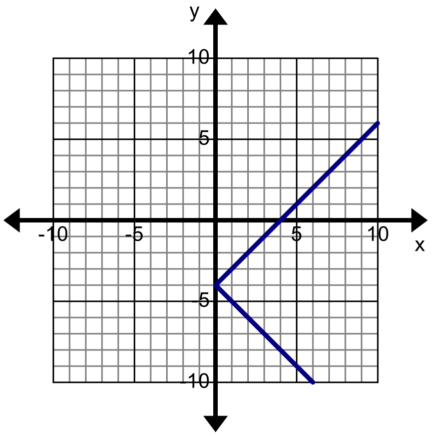
Function? Explain. Encourage students to make a representation of this problem in the space below (graph, equation, table, mapping, etc.)

9.



Function? Explain. Yes, put in a shape, the rule is rotate it 90˚ clockwise

10.



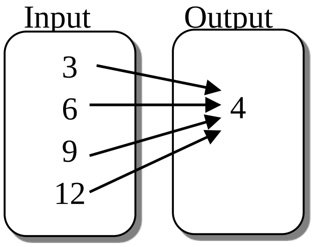
Function? Explain. No, have students write some ordered pairs to confirm this.

11.

Number of People	1	3	4	7
Cost	4	8	10	16

Function? Explain.

12.



Function? Explain.

13. $y = \frac{1}{3}x + 4$

Function? Explain. Yes, again students can sketch a graph or make a table of values in order to consider a different representation of this relation

14. You know your cousin lives at the zip code 12345 so you type it in Google to find your cousin's full address. Is address a function of zip code?

Function? Explain. No, you will find more than one address that relates to the zip code. The zip code is the input and there is more than one address or output for each zip code.

15. You know your cousin's cellular phone number is (123) 456-7890 so you dial that number to call him. Is person being called a function of phone number dialed?

Function? Explain. Yes, the only person that relates to the phone number (123) 456-7890 is your cousin. The phone number is the input and the only output for that phone number is your cousin.

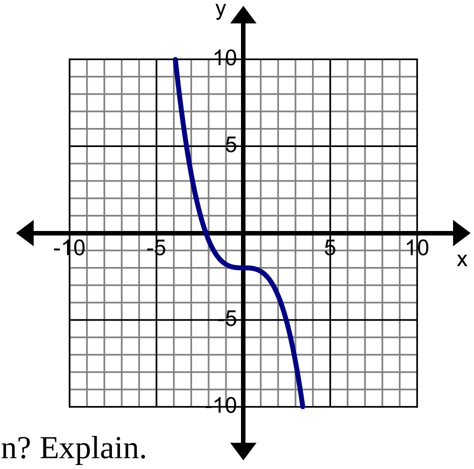
16. You call the post office and ask for the zip code for the city of Salt Lake City, UT. Is zip code a function of the name of the city?

Function? Explain.

17. $y = -5$

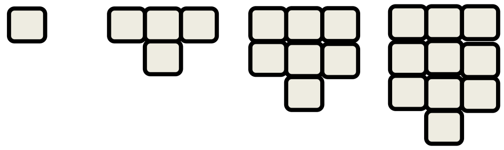
Function? Explain.

18.



Function? Explain.

19.



Stage 1 Stage 2 Stage 3 Stage 4

Function? Explain.

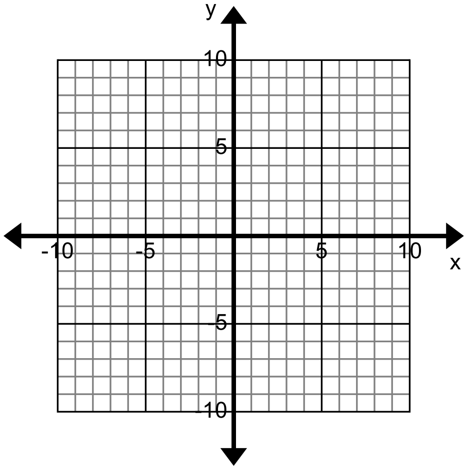
20. $\{(2, -10), (5, -25), (8, -40), (-5, 25)\}$

Function? Explain.

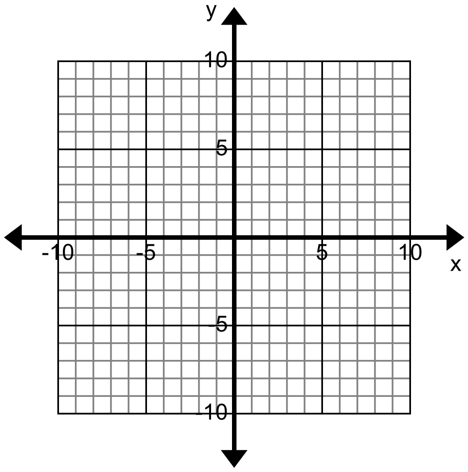
21. $\{(-1, 0), (1, 2), (1, 4), (5, 2)\}$

Function? Explain. No, for the *x* value of 1 there are two *y* values, 2 and 4.

22. Draw a graph of a relation that is a function. Explain how you know.



23. Draw a graph of a relation that is **not** a function. Explain how you know.

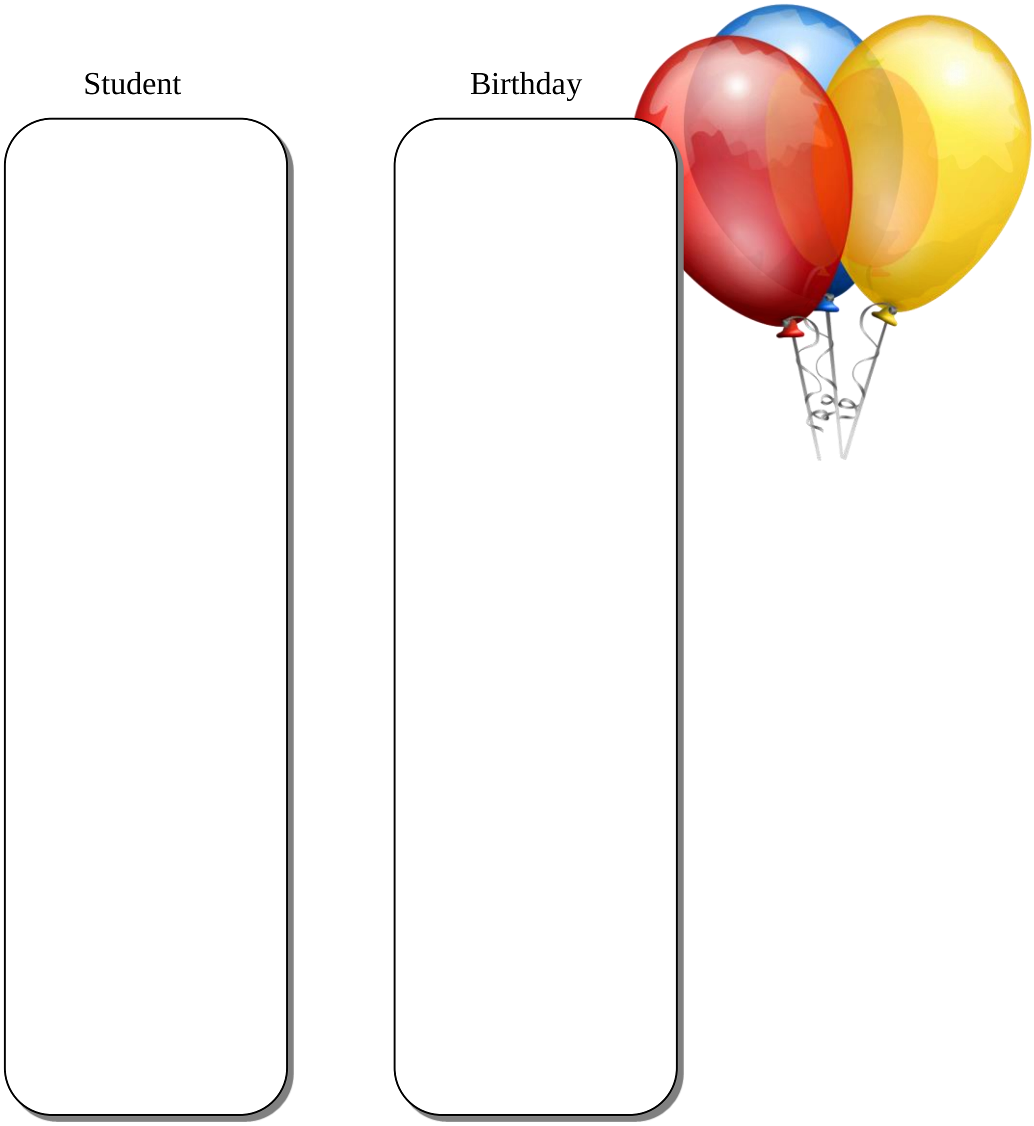


24. Make a mapping of a relation that is a function. Explain how you know.

25. Creat a set of ordered pairs that do **not** represent a function. Explain how you know.

5.1d Class Activity: Birthdays

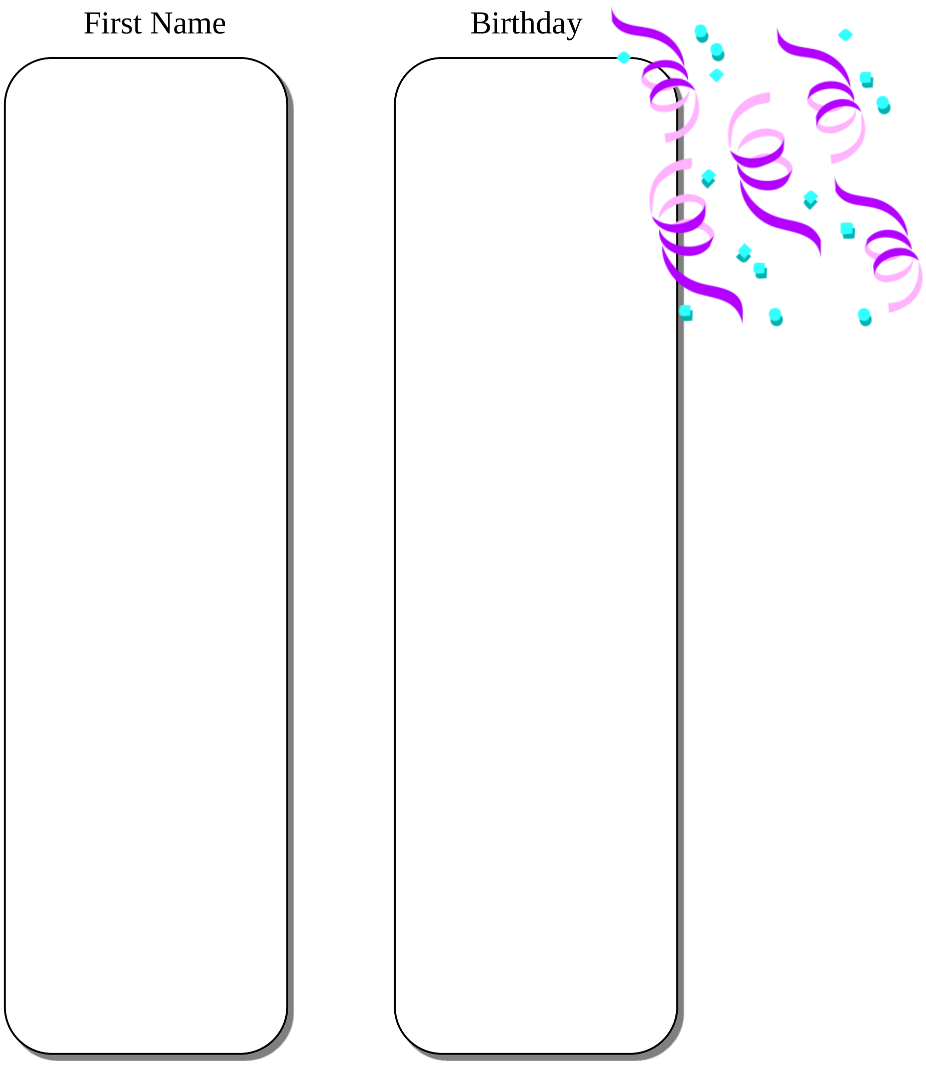
1. Make a mapping that shows the students in your class and their birthdays.



a. Is the birth date of a student a function of the individual student? Justify your answer.

Yes, this relation is a function. You will likely find that two students share the same birthday. It may also occur that you have two people with the same first and last name in your class. If this is the case, students should find a way to provide each student with a unique identifier. For example, if you have two Sams, you might start by putting the first initial of their last name: Sam M. and Sam J. If both Sams have last names that start with an M, you might write out the entire last name: Sam Martin and Sam Meade. What if you have two students named Sam Martin? Sam Martin 1 and Sam Martin 2. The point is that each person is a unique individual so we need to find a way to distinguish each person from every other person. Birthday will always be a function of individual because each person is unique.

2. Make a mapping that shows the first name of the students in your class and their birthdays.



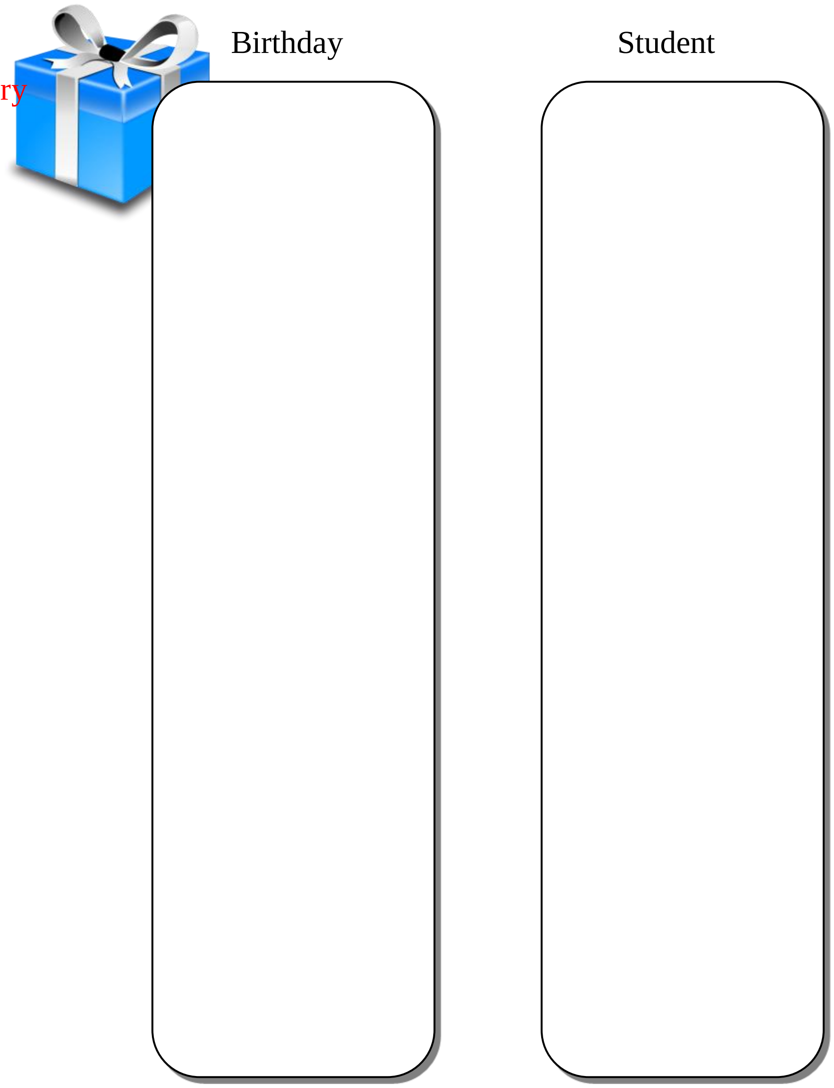
b. Is the birth date of a student a function of the student's first name? Justify your answer.

This relation may or may not be a function depending on the first names of the students in your class. If two or more students share the same first name and have different birthdays, the relation will not be a function. If you do not have two people with the same name, you may consider expanding your domain to include all students in the school, state, etc.

How is this different from the previous problem? Names are not unique; however people are.

3. Make a mapping, switching the input to be birthday and the output to be student.

Answers will vary



a. Is student a function of birth date in your class? Justify your answer.
This relation may or may not generate a function. If more than one student shares a birthday with someone else in the class it will not be a function. Otherwise it will be a function. Again, you may consider asking students, what if we were to consider all of the students in the school, state, etc.?

5.1d Homework: Birthdays

Directions: Determine if each relation or situation defines a function. Make an additional representation of the relation to help you. Justify your answer. Refer to 5.1d class activity and 5.1c class activity for examples, answers, and explanations.

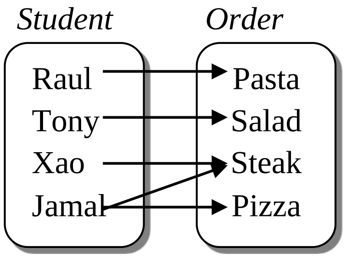
a. Is a student’s ID number a function of his/her first name? Consider all students in your school.

Function? Explain. If two students share the same first name this relation will not be a function

b. Is a student’s first name a function of his/her student ID number? Consider all students in your school.

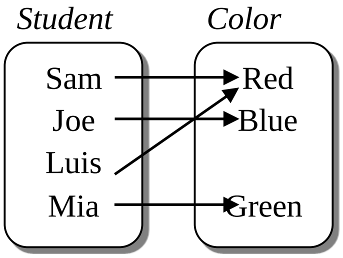
Function? Explain. Yes, each student has a unique ID

c.



Function? Explain.

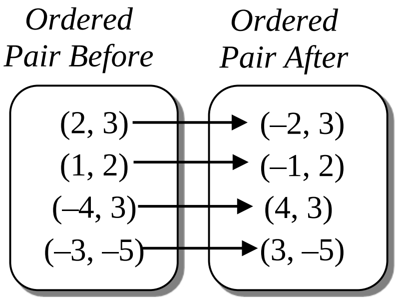
d.



Function? Explain.

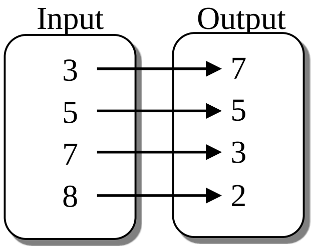
e. Les surveys the students in his class to determine if shoe size is a function of last name of the student? What would have to be true about the names of the students in the class if Les found that shoe size is **not** a function of the last name of the student?

f.



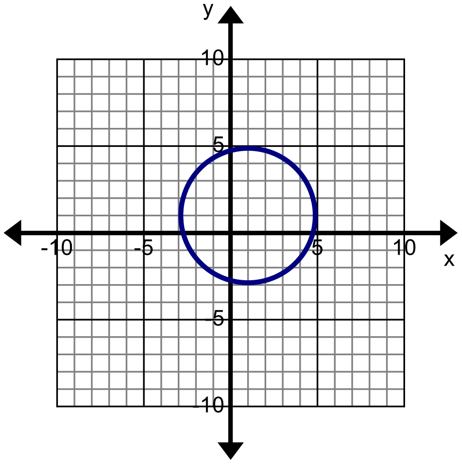
Function? Explain. Yes, this is a function. The input is an ordered pair, the rule is reflect across *y*-axis, the output is a different ordered pair. Students may not use this language – they may say that the rule is to take the opposite of the *x*-value in the ordered pair.

g.



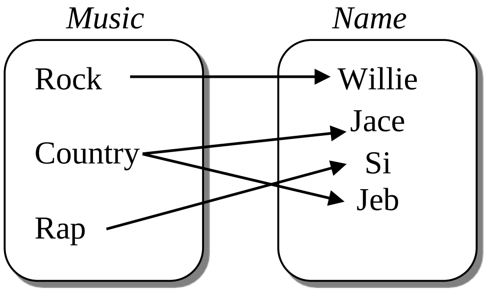
Function? Explain.

h.



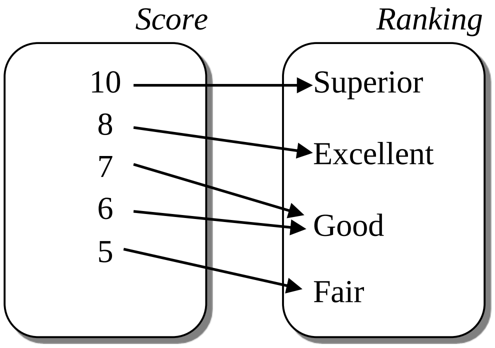
Function? Explain.

i. **Input:** favorite type of music
Output: name



Function? Explain. No, for the input of Country Music there is more than one output, Jace and Jeb.

j. **Input:** a pianist's overall score in a music competition
Output: ranking



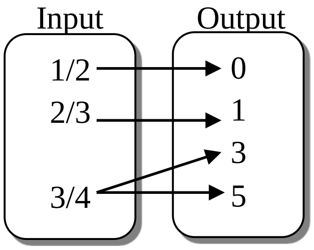
Function? Explain. Yes, for every input there is only one output.

k.

Input	**Output**
25	14
30	13
30	12
35	11

Function? Explain.

l.



Function? Explain.

5.1e Class Activity: More About Functions

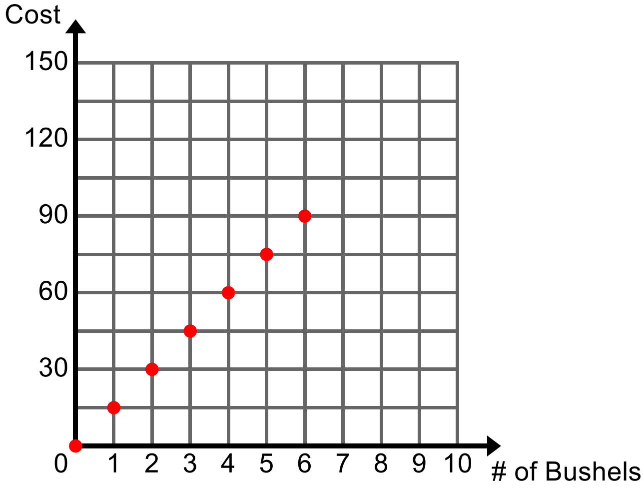
1. Paradise Valley Orchards has the banner shown hanging from their store window. Sally is trying to determine how much she will spend depending on how many bushels of apples she purchases.

   a. Write an equation that gives the amount Sally will spend y depending on how many bushels of apples x she purchases. $y = 15x$

   b. Complete the graph and table below for this relationship.

Number of Bushels x	Amount Spent (dollars) y
0	0
1	15
2	30
3	45
4	60

c.



We know from the previous lessons, that the relationship between number of bushels purchased and amount spent is an example of a function. The equation above gives us a rule for how to determine the amount of money spent based on the number of bushels purchased.

In a functional relationship represented with an equation, the **independent variable** represents the input or x-value of the function and the **dependent variable** represents the output or y-value of the function. In a function, the **dependent variable** is determined by or depends on the **independent variable**. In our example above the **independent variable** is the number of bushels purchased and the **dependent variable** is the amount of money spent. The amount of money one spends **depends** on the number of bushels one purchases. Another way to say this is that the amount of money spent is a function of the number of bushels purchased.

If we think of our input machine, we are inputing the number of bushels purchased and the machine takes that number and multiplies it by 15 to give us our output which is the amount of money we will spend.

This is a good time to point out to students that the independent variable is graphed on the x-axis and the dependent variable is graphed on the y-axis.

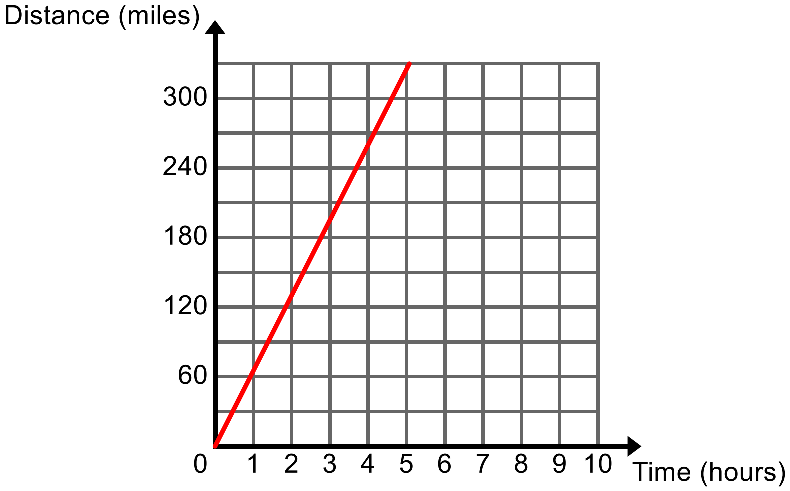
2. Miguel is taking a road trip and is driving at a constant speed of 65 mph. He is trying to determine how many miles he can drive based on how many hours he drives.

   a. Identify the **independent variable** in this situation: ______time driving__________________

   b. Identify the **dependent variable** in this situation: ______distance traveled__________________

   c. Complete the graph and table below for this relationship. Make sure you label the columns and axes in your table and graph.

time driving x	distance traveled y
0	0
1	65
2	130
3	195
4	260



   d. Write an equation that represents this situation: ____$y = 65x$__________________________

   e. In this situation ___distance traveled______ is a function of __time driving________________.

An important note about independent and dependent variables is that often times the dependent and independent variables are interchangeable in a situation and are determined by what we are trying to find. For most of these problems, the wording was carefully scripted in an effort to make it clear which variable is the dependent variable and which is the independent variable. Still, it may be difficult for students to decipher. Additionally, the variable may be "hidden". For example, when considering the length of a workout, the variable is time. When considering how far someone can run, the variable is distance. Students must make sense of the problems and attend to the quantities involved.

3. The drama club is selling tickets to the Fall Ball. They use $2 from each ticket sale for food and decorations.

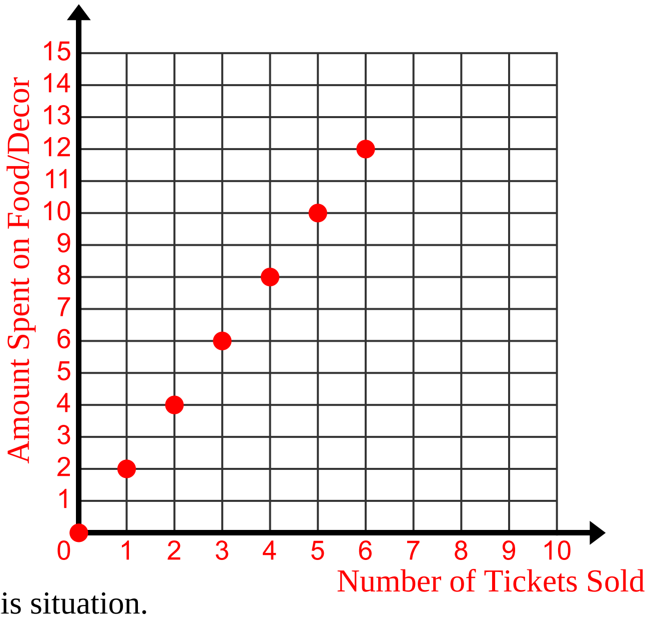
   a. Identify the **independent variable** in this situation: ______number of tickets sold__________

   b. Identify the **dependent variable** in this situation: ______amount spent on decor___________

   c. Create a table, graph, and equation for this function.

# of tickets sold x	Amount spent on decor y
0	0
1	2
2	4
3	6

Equation: __$y = 2x$_____________________



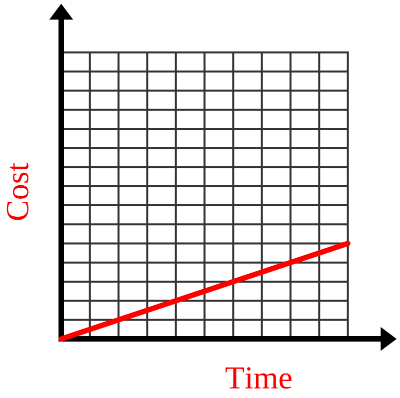
   d. Complete the following sentence for this situation.

   ___Amount spent on food and decor_____ is a function of _number of tickets sold__________.

4. The average cost of a movie ticket has steadily increased over time.

   a. Identify the dependent and independent variables in this functional relationship.
   I = time; D = cost of movie ticket

   b. Sketch a possible graph of this situation.



5. Susan is reading her history text book for an upcoming test. She can read 5 pages in 10 minutes. Susan is interested in determining how many pages she can read based on how long she reads for.

   a. Identify the **independent variable** in this situation: ______time__________

   b. Identify the **dependent variable** in this situation: ______number of pages read___________

   c. Create a representation (table, graph, equation) of this function in the space below.
   Representations will vary. Possible representation: $y = \frac{1}{2}x$

6. Chris is also reading his history text book for an upcoming test and can also read 5 pages in 10 minutes. However, Chris is interested in determining how long it will take him to read based on how many pages he has to read.

   a. Identify the **independent variable** in this situation: ______number of pages to read__________

   b. Identify the **dependent variable** in this situation: ______time___________

   c. Create a representation (table, graph, equation) of this function in the space below.
   Representations will vary. Possible representation: $y = 2x$

Directions: Each of the following situations represents a functional relationship between two quantities. Determine the dependent variable and the independent variable. The first one has been done for you.

It is often easier to find the dependent variable first. Ask students, "What quantity depends on the other?" In some situations, the variable is "hidden." For example, in #8, the variable is time in minutes.

D **I**
7. In warm climates, the average amount of electricity used rises as the daily average temperature increases and falls as the daily average temperature decreases.

D **I**
8. The number of calories you burn increases as the number of minutes that you walk increases.

D **I**
9. The air pressure inside a tire increases with the temperature.

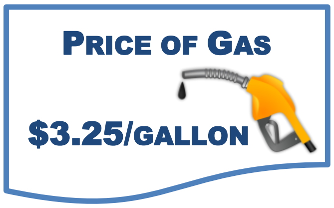
I **D**
10. As the amount of rain decreases, so does the water level of the river.

D **I**
11. The total number of jars of pickles that a factory can produce depends on the number of pickles they receive.

D **I**
12. The weight of the box increases as the number of books placed inside the box increases.

5.1e Homework: More About Functions

1. Shari is filling up her gas tank. She wants to know how much it will cost to put gas in her car. The sign below shows the cost for gas at Grizzly's Gas-n-Go.



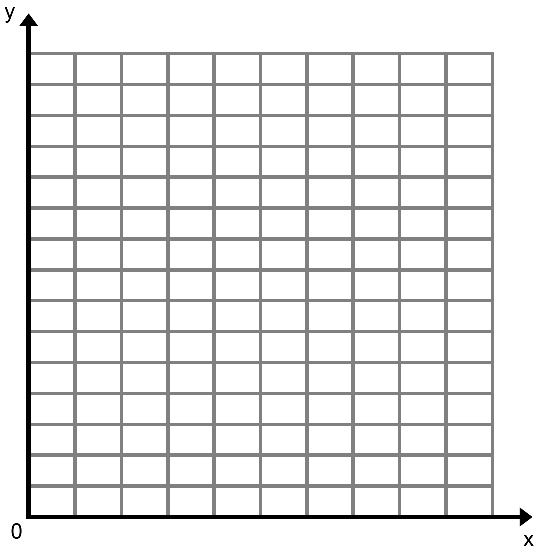
   a. Identify the **independent variable** in this situation: ________________________

   b. Identify the **dependent variable** in this situation: ________________________

   c. Complete the graph and table below for this relationship. Make sure you label the columns and axes in your table and graph.

gallons of gas x	cost y
0	0
1	3.25

A few ordered pairs have been given in the table.



   d. Write an equation that represents this situation: ______________________________

   e. In this situation ____cost__________________ is a function of _____gallons of gas____.

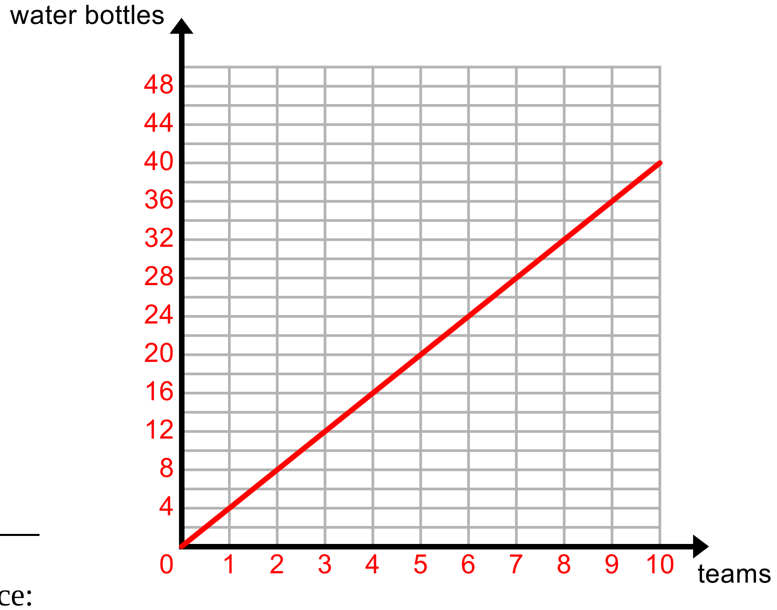
2. Peter is the event planner for a team race taking place in Park City, UT. He needs to determine how many bottles of water to have ready at the finish line of the race so that each participant in the race receives a bottle of water. There are 4 people on a team.

   a. Identify the **independent variable** in this situation: ________________________

   b. Identify the **dependent variable** in this situation: ____________________________________

   c. Create a table, graph, and equation of this situation.

x	*y*



Equation: ____________________

   d. Complete the following sentence:

_______________________ is a function of ____________________________________.

Directions: Each of the following situations represents a functional relationship between two quantities. Underline the two quantities. Put an I above the independent variable and a D above the dependent variable.

I D
3. As the size of your family increases so does the cost of groceries.

4. The value of your car decreases with age.

I D
5. The greater the distance a sprinter has to run the more time it takes to finish the race.

6. A car has more gas in its tank can drive a farther distance.

7. A child's wading pool is being inflated. The pool's size increases at a rate of 2 cubic feet per minute.

8. The total number of laps run depends on the length of each workout.

9. A tree grows 15 **D** feet in 10 **I** years.

10. There are 5 inches of **D** water in a bucket after a 2 ½ **I** hour rain storm.

11. Jenny has 30 coins she has collected over 6 years.

12. Sally's track coach wants to know **D** how far she can run based on the amount of **I** time she runs for.

13. Whitney is training for a half marathon. She wants to know how long it will take her to run based on how far she has to run for.

14. Write your own relationship that contains an independent and dependent variable.

5.1f Self-Assessment: Section 5.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Mastery 3**	**Substantial Mastery 4**
1. Understand that a function is a rule that assigns to each input exactly one output.				
2. Determine whether a given relation defines a function given different representations (i.e., table, graph, mapping, story, patterns, equations, and ordered pairs).				
3. Determine the independent and dependent variables in a functional relationship.				

1. Define function in your own words. Provide examples to support your definition.

2. Do the representations below define a function? Why or why not?

a.

Stage 1 Stage 2 Stage 3 Stage 4 Stage 5

Is the number of hearts in a stage a function of the stage number?

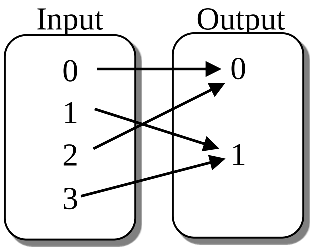
Is the stage number a function of the number of hearts in a stage?

b. Maria is draining her hot tub at a rate of 5.5 gallons per minute. Is the amount of water left in the pool a function of the amount of time she has been draining it?

c.

x	***y***
10	1
10	2
10	3
10	4

d.



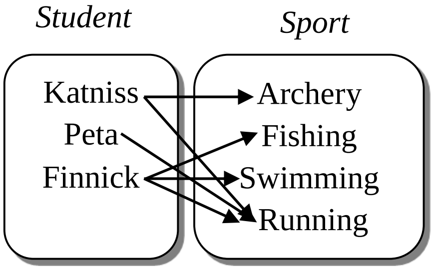
e. Is state capitol a function of state name? Consider states in the United States.

f. Is a person’s weight a function of the person’s age?

g. Is the amount of time it takes a person to run a marathon a function of the person's age?

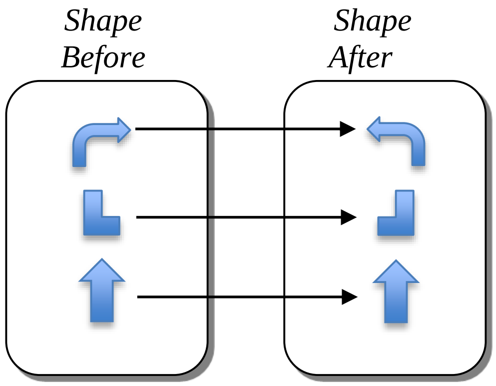
h. $\{(1,1),(2,1),(3,1),(4,1),(5,1)\}$

i.

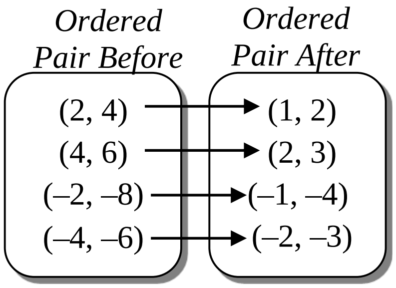


j. $y = \frac{1}{4}x - 2$

k.



l.



3. In each of the following situations an independent variable is given. Determine a possible dependent variable that would create a functional relationship.
   a. The amount of gas remaining in a tank

   b. Time

   c. Number of people

   d. Number of t-shirts

   e. Circumference of head

Section 5.2: Explore Linear and Nonlinear Functions

Section Overview:
This section focuses on the characteristics that separate linear from nonlinear functions. Students will analyze the different representations of a function (graph, table, equation, and context) to determine whether or not the representations suggest a linear relationship between the two variables. In the process of studying non-linear functions, students will solidify their understanding of how a linear function grows (changes).

Concepts and Skills to Master:
By the end of this section, students should be able to:

- Distinguish between linear and nonlinear functions given a context, table, graph, or equation.

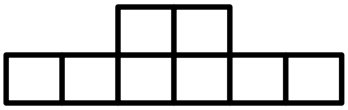
5.2a Class Activity: Display Designs

Complete Foods, a local grocery store, has hired three different companies to come up with a display for food items that are on sale each week. They currently have a display that is 6 boxes wide as shown below.

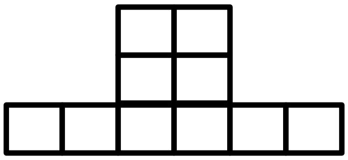
They would like the center part of the display to be taller than the outside pieces of the display to showcase their “mega deal of the week”. The following are the designs that two different companies submitted to Complete Foods, using the current display as their starting point.

Design Team 1:

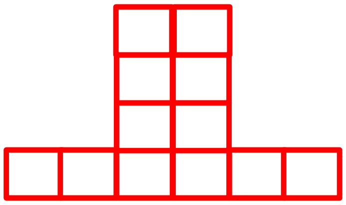
Current Display (Stage 1)



Stage 2



Stage 3



Stage 4

1. Draw Stage 4 of this design. Describe how you went about drawing stage 4.

2 additional boxes are added to the center column each time

2. Can the relationship between stage number and number of blocks in a stage in this pattern be modeled by a linear function? Provide at least **2** pieces of evidence to support your answer.

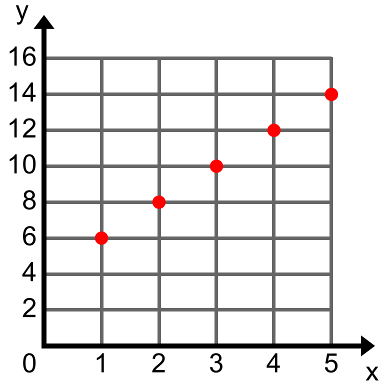
Yes, pieces of evidence include:

In the model, the same number of blocks is being added each time

Stage	**Number of Squares**
1	6
2	8
3	10
4	12
5	14

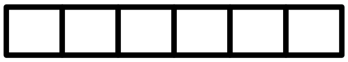
In the table, first different is constant

The graph is a line

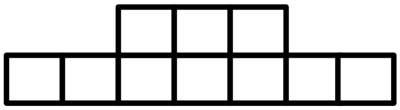


The equation that models this relationship is $y = 4 + 2x$

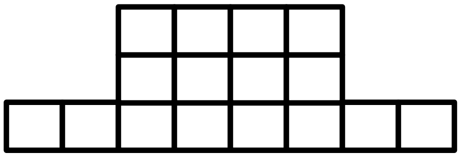
Design Team 2:



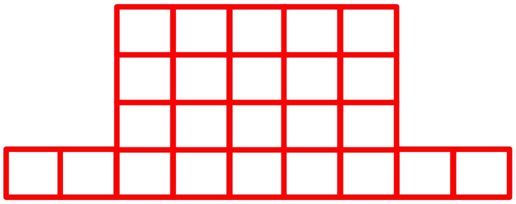
Current Display (Stage 1)



Stage 2



Stage 3



Stage 4

An interesting thing to note about the growth of each of these patterns is that the linear pattern grows in one direction – in the previous example the center column is growing taller. However for this pattern, which is quadratic, the pattern grows in two directions simultaneously – the center rectangle is getting wider and taller at the same time.

3. Draw Stage 4 of this design. Describe how you went about drawing stage 4.

Answers will vary. One way of seeing this is that each time you add an additional block to the base, then the width of the center rectangle column increases by 1 block and the height of the center rectangle increases by 1 block

4. Can the relationship between stage number and number of blocks in a stage in this pattern be modeled by a linear function? Provide at least **2** pieces of evidence to support your answer.

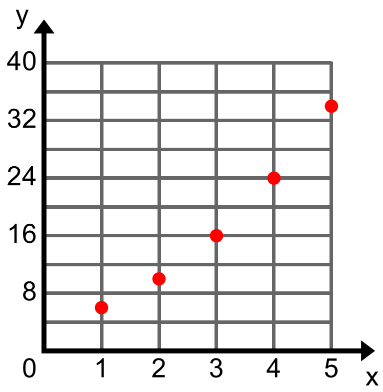
No, pieces of evidence include:

In the model, the number of blocks being added each time is not constant

In the table, the first difference is not constant.

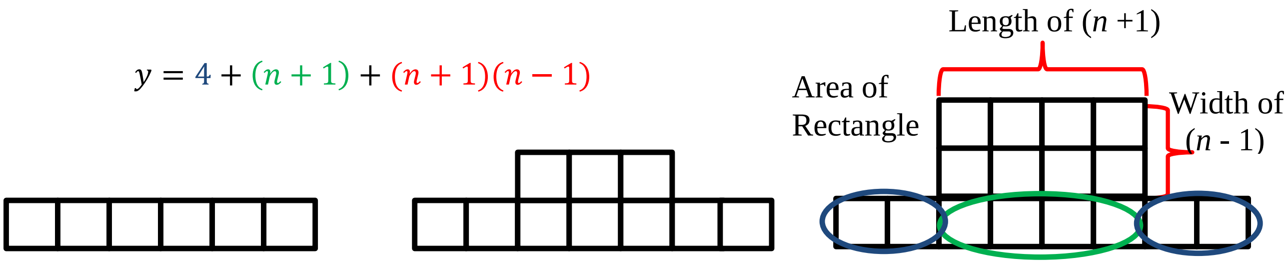
Stage	**Number of Squares**
1	6
2	10
3	16
4	24
5	34

The graph is not a line



The equation that models this relationship is $y = x^2 + x + 4$. It is not expected that students will be able to come up with this equation in Grade 8; however you may challenge your honors' students to try to do this. On equation that models this situation is shown below. It has been color-coded to show how the equation connects to the geometric model.

$$y = 4 + (n + 1) + (n + 1)(n - 1)$$

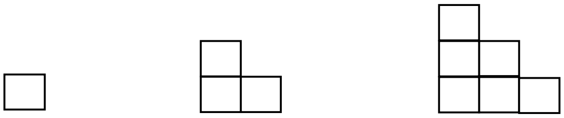


Current Display (Stage 1) **Stage 2** **Stage 3**

5.2a Homework: More Patterns – Are They Linear?

Directions: For each of the following patterns, draw the next stage and determine whether relationship between the stage number and the number of blocks in a stage can be represented by a linear function. Justify your answer.

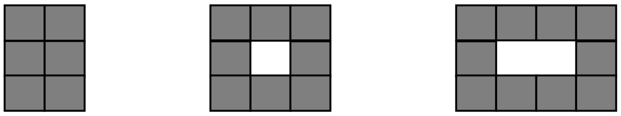
1.



Stage 1 Stage 2 Stage 3 Stage 4

a. Is this pattern linear? ________ no _________
   Justification: Answers will vary (see class work for possible justifications)

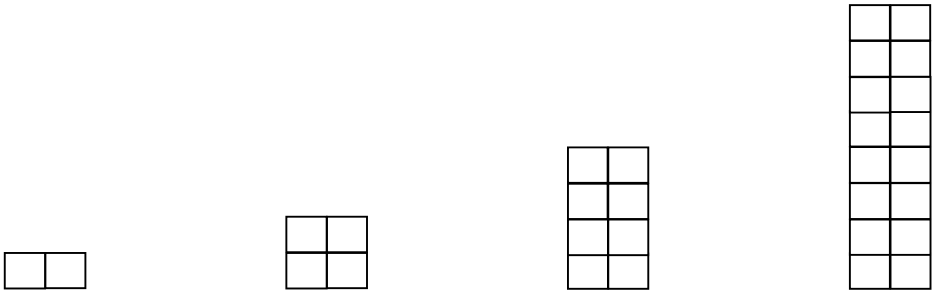
2. Consider the gray tiles only



Stage 1 Stage 2 Stage 3 Stage 4

a. Is this pattern linear? _________________________
b. Justification:

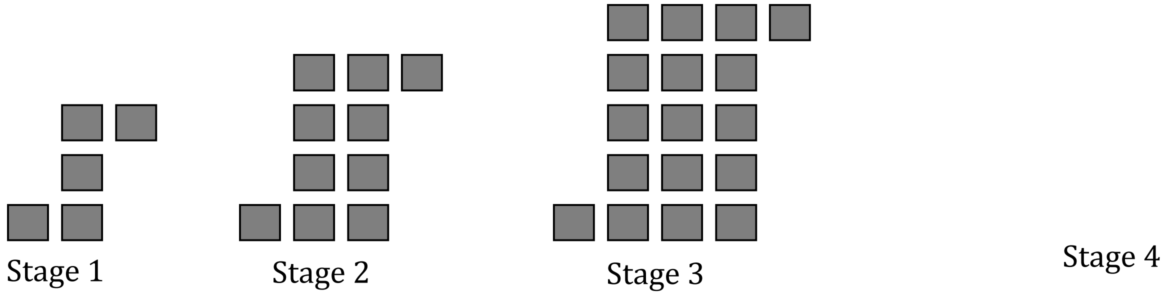
3.



Stage 1 Stage 2 Stage 3 Stage 4 Stage 5

a. Is this pattern linear? _________________________
b. Justification:

4.

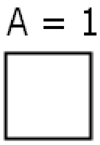


a. Is this pattern linear? ______________________
b. Justification:

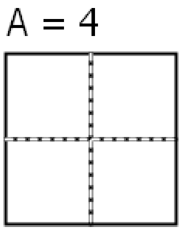
5. Make up your own pattern that is **not** linear. Prove that your pattern is not linear with at least **2** pieces of evidence.

5.2b Class Activity: Linear and NonLinear Functions in Context

1. Consider the **area** of a square as a function of the side length of the square.
   a. Draw pictures to represent these squares. The first two have been drawn for you.



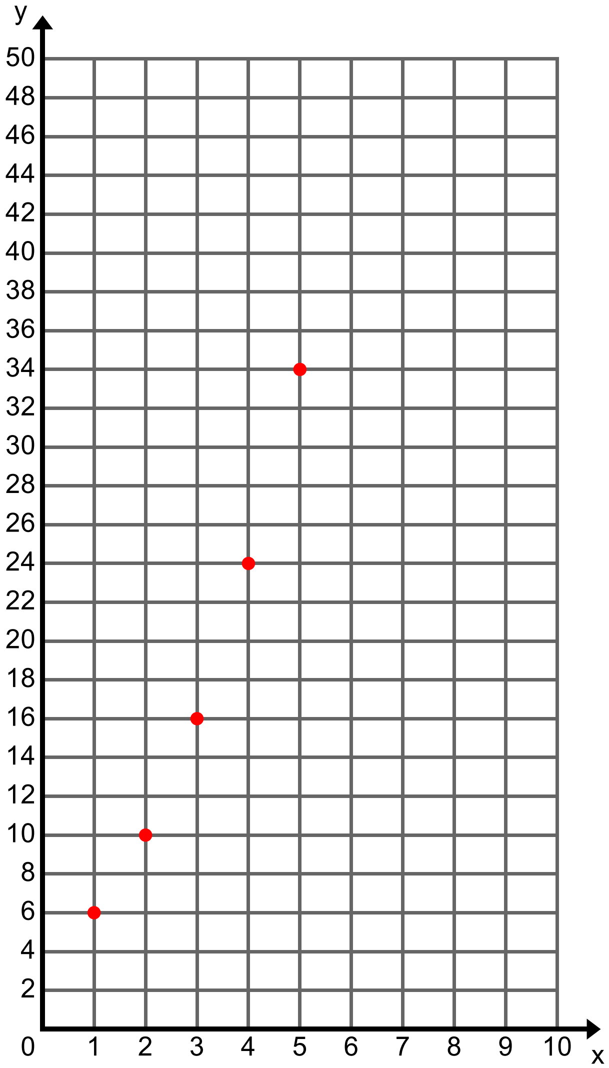
Side length = 1



Side length = 2

   b. Complete the graph and table for this function.

side length	area
1	1
2	4
3	9
4	16
5	25

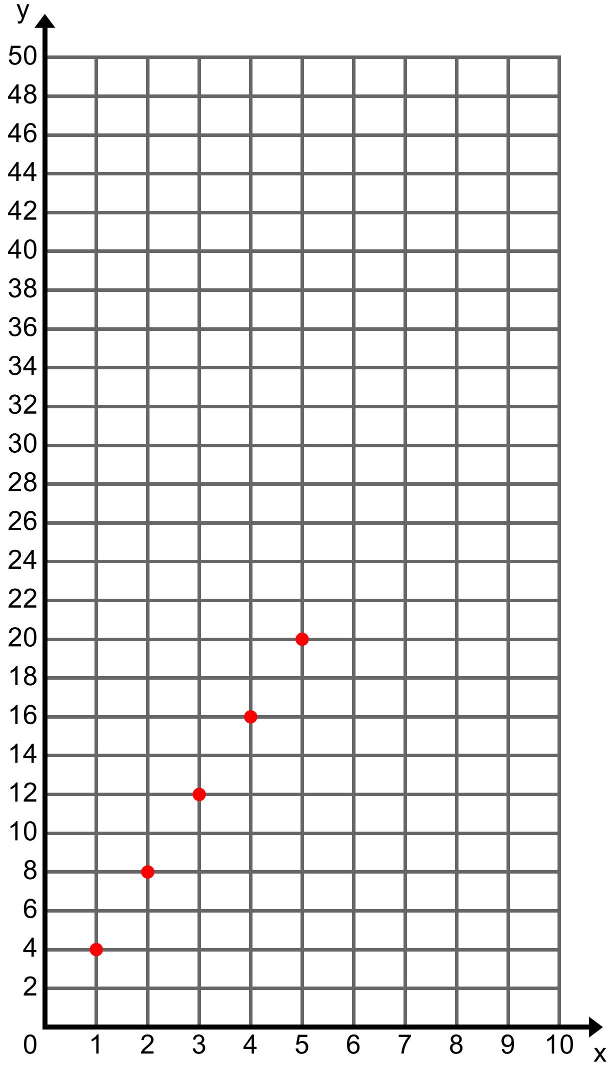


   c. What is the dependent variable? The independent variable?
   Area is the dependent variable;
   side length is the independent variable
   d. Write an equation to model A as a function of s.
   $A = s^2$
   e. Does this graph pass through the point (8, 64)? Explain how you know.
   Yes, this ordered pair satisfies the equation.
   f. What does the point (8, 64) represent in this context?
   A square with a side length of 8 has an area of 64.
   g. List three more ordered pairs that this graph passes through.
   Possible answers: (6, 36); (7, 49); (10, 100)
   h. Is this function linear? Explain or show on the graph, table, and equation why or why not?
   No; graph is not a line, in the table the first difference is not constant, and the equation is a second degree equation

2. Consider the **perimeter** of a square as a function of the side length of the square.

a. Complete the graph and table for this function.

side length	perimeter
1	4
2	8
3	12
4	16
5	20



b. What is the dependent variable? The independent variable?
Perimeter is the dependent variable;
side length is the independent variable

c. Write an equation to model P as a function of s.
$P = 4s$

d. Find another ordered pair that the graph passes through.
Possible answers: (6, 24); (10, 40); (100, 400)

e. What does the point (10, 40) represent in this context?
(10, 40) – a square with a side length of 10 has a perimeter of 40.

f. Is this function linear? Explain or show on the graph, table, and equation why or why not?
Yes, first difference constant, graph is a line, equation in the form $y = mx + b$

5.2b Homework: Linear and NonLinear Functions in Context

1. The following tables show the distance traveled by three different cars over five seconds.

Car 1	
Time (s)	Distance (ft.)
1	4
2	7
3	10
4	13
5	16

Car 2	
Time (s)	Distance (ft.)
1	2
2	5
3	10
4	17
5	26

Car 3	
Time (s)	Distance (ft.)
1	3
2	5
3	9
4	17
5	33

a. Consider the relationship between time and distance traveled for each car. Which of the tables of data can be modeled by a linear function? Which ones cannot be modeled by a linear function? Justify your answer.

b. For any of the data sets that can be modeled by a linear function, write a function that models the distance traveled *D* as a function of time *t*.

c. What is the dependent variable in this situation? The independent variable?

d. Which car is traveling fastest? Justify your answer.

2. Hermione argues that the table below represents a linear function. Is she correct? How do you know?

x	2	4	8	16
y	1	3	5	7

Hermione is not correct. The *y*-values are not growing by equal differences **over equal intervals**.

3. Emily's little brother painted on her math homework. She knows the data in each of the tables below represents a linear function. Help Emily determine what number is hidden behind the blob of paint.

a.

x	10	20	30	40
y	8	13	●	23

18

b.

x	–2	0	2	3
y	–5	●	7	10

c.

x	0	1	●	6
y	0	3	9	18

Directions: Choose **3** of the following situations. Determine whether the situations you choose can be modeled by a linear function or not. Provide evidence to support your claim. Show your work in the space below.

*For the answers below, students can provide various pieces of evidence (constant/changing rate of change; first difference is the table is constant/not constant; graph is/is not a line; form of the equation).

4. Mr. Cortez drove at a constant rate for 5 hours. At the end of 2 hours he had driven 90 miles. After 5 hours, he had driven 225 miles.
   a. What is the dependent variable in this relationship? ___distance________________
   b. What is the independent variable in this relationship? __time___________________
   c. Can this relationship be modeled by a linear function? Provide evidence to support your claim. Yes, this relationship can be modeled by a linear equation. The key phrase in this example is "constant rate". This means that the distance being covered each hour is the same. Mr. Cortez is driving at a rate of 45 mph. In other words, each hour he travels 45 miles. Create a table, graph, or equation of this situation all of which will confirm that this is a linear relationship.

5. Round 1 of a tennis tournament starts with 64 players. After each round, half the players have lost and are eliminated from the tournament. Therefore, in round 2 there are 32 players, in round 3 there are 16 players and so on.

   a. What is the dependent variable in this relationship? ________________________
   b. What is the independent variable in this relationship? ________________________
   c. Can this relationship be modeled by a linear function? Provide evidence to support your claim.

6. A rock is dropped from a cliff that is 200 feet above the ground. The table below represents the height of the rock (in feet) with respect to time (in seconds).

Time (s)	**Height (ft.)**
0	200
1	184
2	136
3	56

   a. What is the dependent variable in this relationship? ___height________________
   b. What is the independent variable in this relationship? __time__________________
   c. Can this relationship be modeled by a linear function? Provide evidence to support your claim. No, the distance the rock falls each second is not constant. For example, in the time interval from 0 – 1 seconds, the rock falls a distance of 16 feet; in the time interval from 1 – 2 seconds, it falls 48 feet; and in the time interval from 2 – 3 seconds, it falls 80 feet. The distance the rock falls is increasing at an increasing rate. This is caused by acceleration due to gravity. Create a graph of this situation or show on the table that the first difference is not constant.

7. A student comes to school with the flu and infects three other students within an hour before going home. Each newly infected student passes the virus to three new students in the next hour. This pattern continues until all students in the school are infected with the virus.
   a. What is the dependent variable in this relationship? ______________________________
   b. What is the independent variable in this relationship? _______________________
   c. Can this relationship be modeled by a linear function? Provide evidence to support your claim.

8. A piece of paper is cut into two equal sections. Each new piece is cut into two additional pieces of equal size. This pattern continues until it is no longer possible to cut the paper any more.
   a. What is the dependent variable in this relationship? ___# of pieces of paper____________
   b. What is the independent variable in this relationship? ___# of cuts__________________
   c. Can this relationship be modeled by a linear function? Provide evidence to support your claim.

   Perform this experiment and create a table of your results. Use the table to determine if the relationship is linear or not.

5.2d Class Activity: Different Types of Functions

The goal of this activity is that students explore the tables, graphs, and equations of non-linear functions and make connections between the representations. It is not important that they know the different functions shown below only that they recognize when a graph, table, or equation defines a non-linear function.

1. Sketch the general appearance of the graph of the equation $y = mx + b$.
   a. What do m and b represent?
   b. What makes the graph linear?

   Linear functions grow by equal differences over equal intervals;
   The rate of change is constant
2. Complete the table of values for the functions shown in the table below. Using the table of values,

   predict what the graphs of the equations will look like. Compare the tables to the table for $y = x$.

$y = x$	
x	y
-2	-2
-1	-1
0	0
1	1
2	2

$y = \lvert x \rvert$	
x	y
-2	2
-1	1
0	0
1	1
2	2

$y = x^2$	
x	y
-2	4
-1	1
0	0
1	1
2	4

$y = \frac{1}{x}$	
x	y
-2	$-1/2$
-1	-1
0	und.
1	1
2	$1/2$

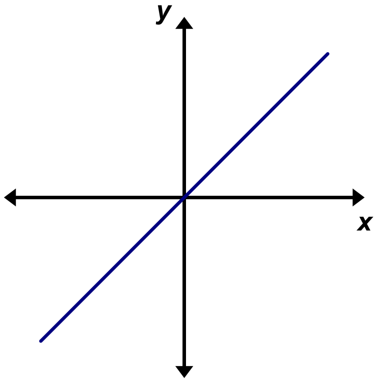
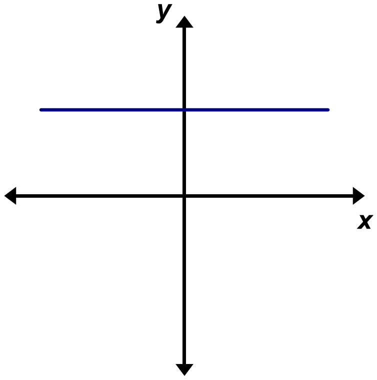
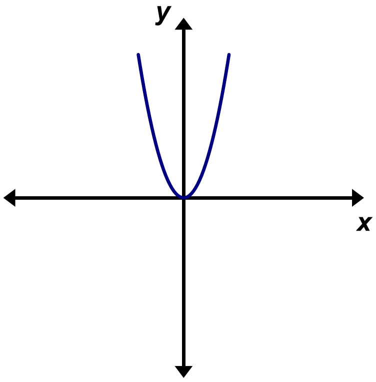
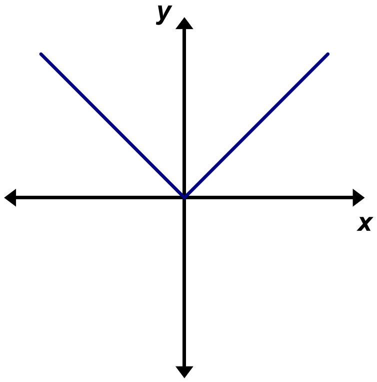
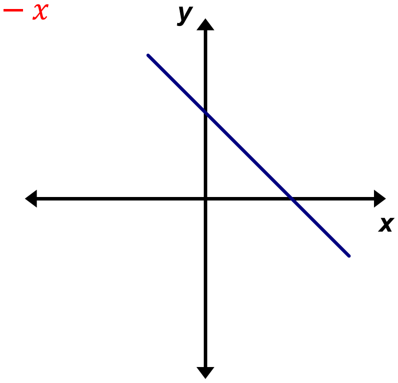
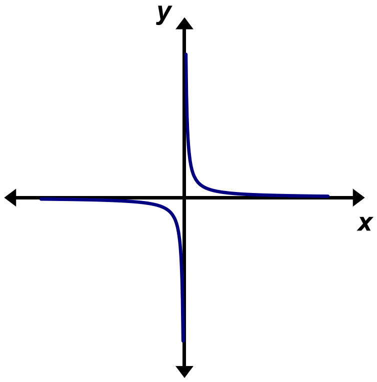
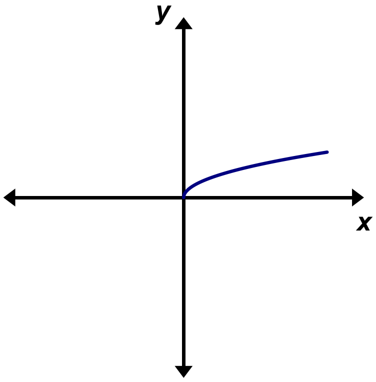
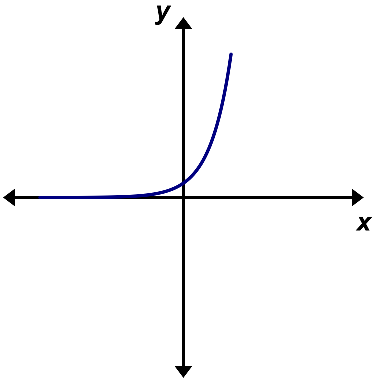
$y = \sqrt{x}$	
x	y
0	0
1	1
2	1.4
3	1.7
4	2

$y = 2^x$	
x	y
0	1
1	2
2	4
3	8
4	16

$x + y = 6$	
x	y
-2	8
-1	7
0	6
1	5
2	4

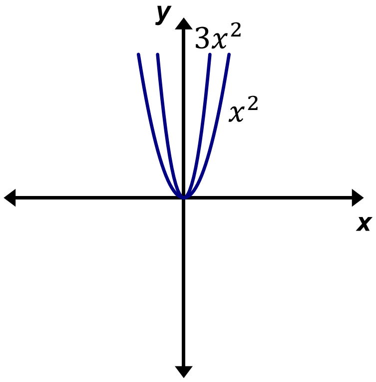
$y = 6$	
x	y
-2	6
-1	6
0	6
1	6
2	6

3. Use the tables of values from the previous page to match each equation to its graph below. Write the equation to the left of the graph. The first one has been done for you.

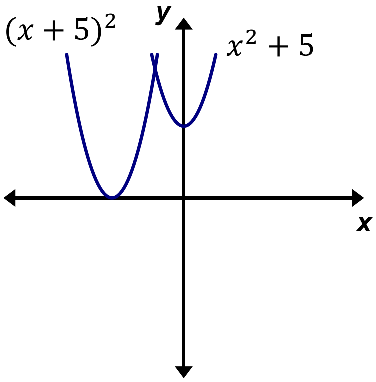
$y = x$  	$y = 6$ 
$y = x^2$  	$y = \|x\|$ 
$y = 6 - x$  	$y = \frac{1}{x}$ 
$y = \sqrt{x}$  	$y = 2^x$ 

4. Compare each of the graphs on the previous page to the graph of $y = x$. What is the same? What is different? Discuss with a class mate. Answers will vary but encourage students to talk about where the graphs intersect the axes (for example both $y = x$ and $y = x^2$ cross at the origin) both go through the point (1, 1); however $y = x$ goes through $(-1, -1)$ while $y = x^2$ goes through $(-1, 1)$.

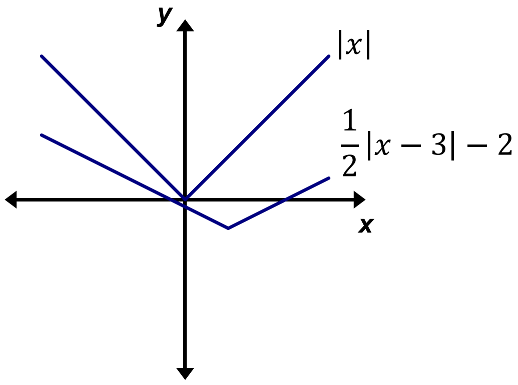
5. The graphs of x^2 and $3x^2$ are shown below. Compare these graphs. What is the same? What is different?
The goal of these questions is not to explore transformations in detail. The goal is that students see that the graph has the same basic shape and that it is non-linear. That way, any time students see an equation with an x^2 term in it, they will know it is non-linear. In this example, the transformation has stretched the graph.



6. The graphs of $y = x^2 + 5$ and $y = (x + 5)^2$ are shown below. Compare these graphs to the graph of $y = x^2$. What is the same? What is different?
See note in #5, basic shape, each graph is a translation of $y = x^2$



7. The graphs of $y = |x|$ and $y = \frac{1}{2}|x - 3| - 2$ are shown below. Compare these graphs. What is the same? What is different?
See note in #5, same basic shape



8. Describe the basic structure of an equation that defines a **linear function**. Think about the different forms a linear equation might take. Provide examples of the different forms.
Have students think about the linear equations they have studied so far. Have them make conjectures about the structure of a linear equation ($y = mx + b$). Have them start to think about the different forms of a linear equation. For example, is $3x + y = -12$ a linear equation? Yes. Is $y - 2 = \frac{1}{2}(x - 4)$ a linear equation? Yes. What qualifies these as linear equations?

9. Describe attributes you see in the equations that define **nonlinear functions**. Provide examples.
Have students look at the non-linear equations from above. What makes them non-linear? If they say something like squaring *x*, ask, "What if it is *x* raised to the power of 3 (or 4, or -1, or ½)?" Use the graphing calculator to check conjectures they have. Discuss equations that may cause confusion:

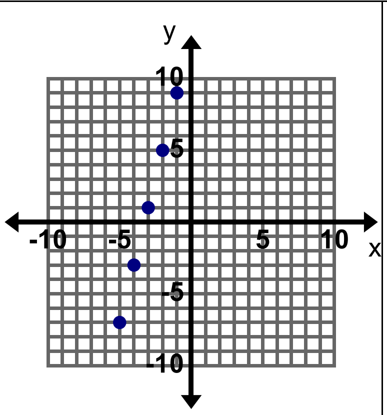
$y = \frac{2}{x}$ vs. $y = \frac{x}{2}$ $\quad$ $x + y = 10$ vs. $xy = 10$

$y = \frac{2}{x}$ is not linear; however $y = \frac{x}{2}$ is linear with a *y*-intercept of $(0,0)$ and a slope of $\frac{1}{2}$.

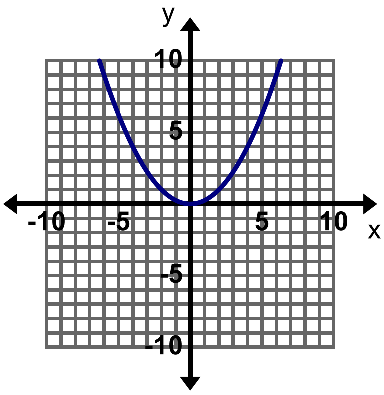
5.2d Homework: Different Types of Functions

1. Circle the letter next to the graph if it represents a linear function. A linear function grows by equal differences over equal intervals. The slope along a line is constant. In the examples below, if the graph is a line, then it is linear. The points do not need to be connected. For example, A below is a linear function. The slope of the line passing through the points given is 4.

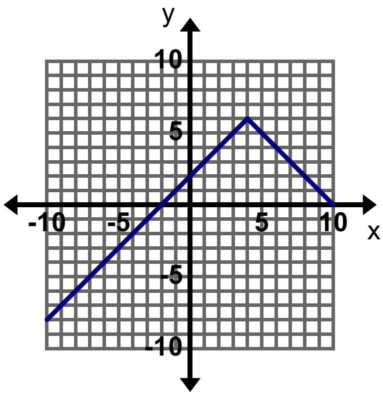
A



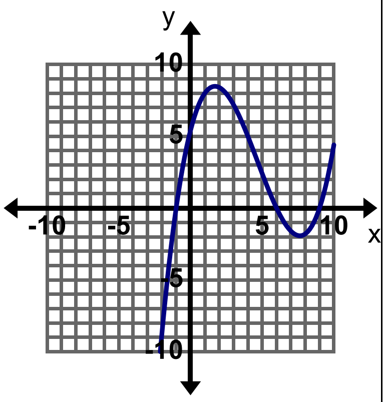
B



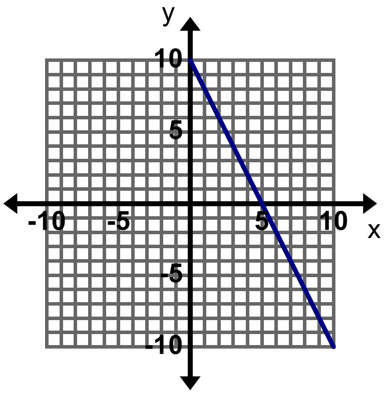
C



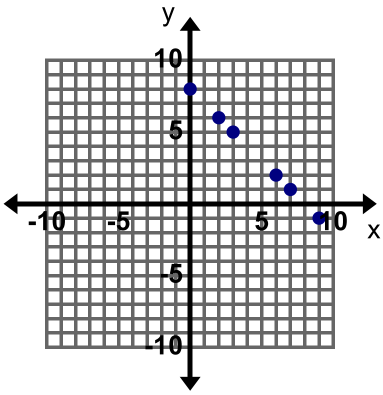
D



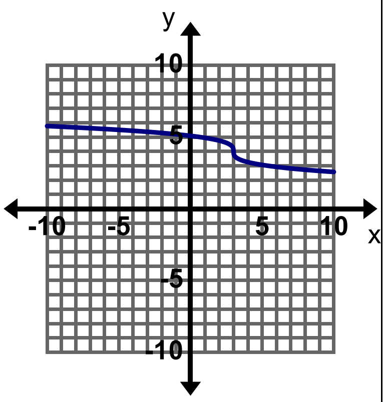
E



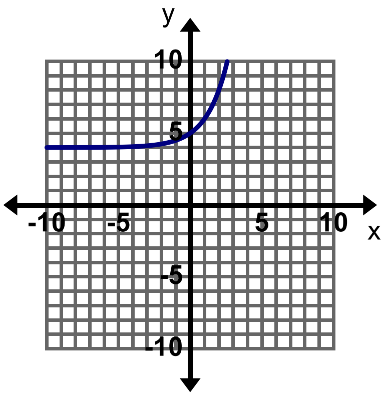
F



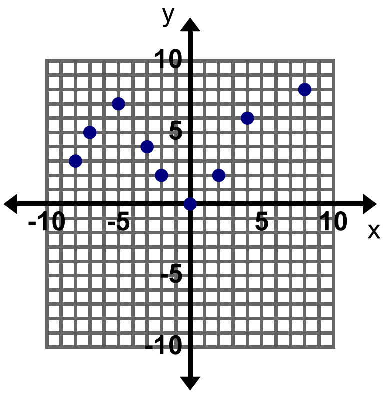
G



H



I



Bonus: Do any of the graphs of nonlinear functions have a shape similar to the ones studied in class? Make a prediction about the basic structure of the equations of these functions.

2. Circle the letter next to the table if the data represents a linear function. Again, linear functions grow by equal differences over equal intervals. Examining table A below, the difference between successive terms is +5. However C is not linear. The difference between the first terms is +4, the difference between the next terms is +8, and the difference between the next terms is +16. Since the difference is not constant, the data is not linear. Be careful with I. At first glance it appears as though the differences between the y values are not constant (+4, +8, +16); however take note of the fact that the differences between the x-terms are not the same in this problem. Determine the slope using multiple sets of points in this problem. If the slope is constant, then the data represents a linear relationship.

A	***x***	***y***	**B**	***x***	***y***	**C**	***x***	***y***	**D**	***x***	***y***
	0	–5		0	15		0	4		0	98
	1	0		1	12.5		1	8		1	98
	2	5		2	10		2	16		2	98
	3	10		3	7.5		3	32		3	98

E	***x***	***y***	**F**	***x***	***y***	**G**	***x***	***y***	**H**	***x***	***y***
	–3	3		1	2		2.1	4		0	2
	–2	2		3	4		2.2	5		4	6
	2	2		6	6		2.3	6		8	18
	3	3		10	8		2.4	7		12	54

I	***x***	***y***	**J**	***x***	***y***	**K**	***x***	***y***	**L**	***x***	***y***
	3	20		10	–20		5	0		15.1	4.2
	6	24		30	–40		10	–1		16.7	12.2
	12	32		50	–60		15	–2		18.3	20.2
	24	48		70	80		20	–3		19.9	28.2

3. Circle the letter next to each equation if it represents a linear function. See class activity. Be aware that many linear equations take the form $y = mx + b$; however at times we see linear equations in a different form (i.e. Standard Form $Ax + By = C$)). For example, problem A below is a linear equation written in standard form. If students are unsure, they can create graphs or tables of the equations below to determine whether or not the equations represent a linear relationship.

A	$2x + 4y = 16$	**B**	$y = \|2x\| + 5$	**C**	$y = x^2 + 5$	**D**	$y = 5 \cdot 3^x$
E	$y = \frac{4}{x} + 3$	**F**	$y = \frac{x}{4} + 3$	**G**	$y = \sqrt{4x}$	**H**	$x^2 + y^2 = 25$
I	$xy = 24$	**J**	$2x + y = 6$	**K**	$y = -\frac{2}{3}x$	**L**	$y = 8$
M	$y = \frac{2}{3}x$	**N**	$y = x^3$	**O**	$3x - y = 2$	**P**	$y = x(x + 2)$

Bonus: Can you predict the basic shape of any of the graphs of the nonlinear equations in #3?

5.2e Class Activity: Matching Representations of Functions

Matching Activity: Match the following representations together. Each representation will have a

1) a story,
2) an equation,
3) a table of values, and
4) a graph.

After you have matched the representations, **label** the axes of the graphs on the graph cards, **answer** the questions asked in the word problems on the story cards, and **identify** the dependent and independent variable in each story.

Story	Equation	Table	Graph
DD	V	C	O
Z	T	G	P
AA			
FF			
EE			
Y			
BB			
CC			

A

x	0	3	6	9	12	15
y	6	7	8	9	10	11

B

x	0	1	2	3	4	5
y	6	100	162	192	190	156

C

x	0	4	8	12	16	20
y	0	200	200	200	224	248

D

x	0	1	2	3	4	5
y	6	12	24	48	96	192

E

x	0	5	10	15	20	25
y	6	7	8	9	10	11

F

x	0	1	2	3	4	5
y	200	196	192	188	184	180

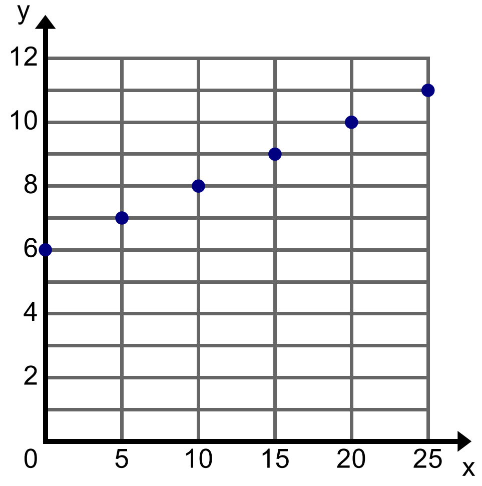
G

x	0	1	2	3	4	5
y	6	206	406	606	806	1006

H

x	0	1	2	3	4	5
y	6	9	12	15	18	21

I


y
12
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4
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0
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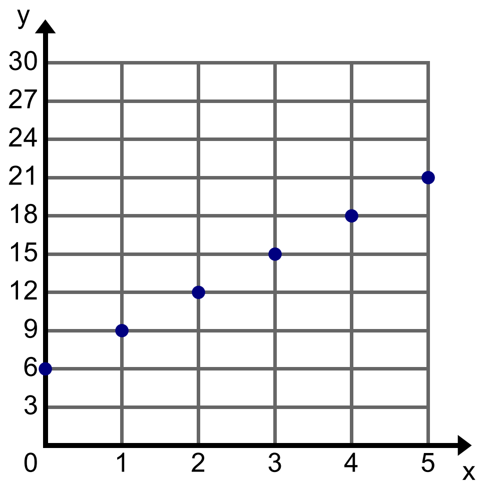
15
20
25
x

J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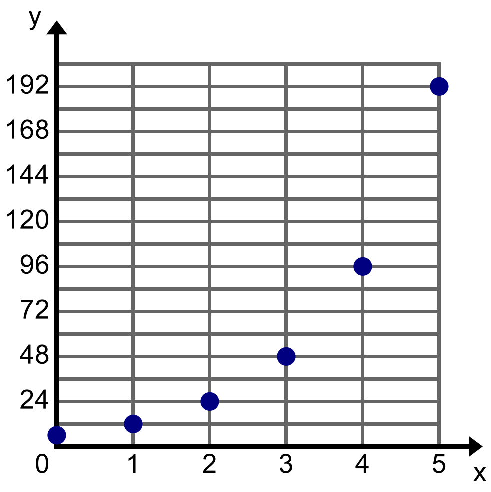
K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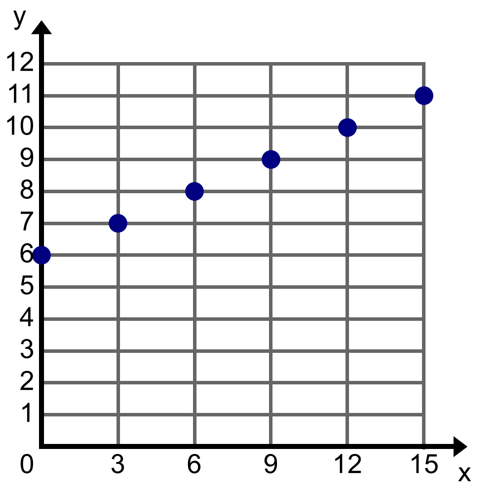
168
144
120
96
72
48
24
0
1
2
3
4
5
x

L


y
12
11
10
9
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7
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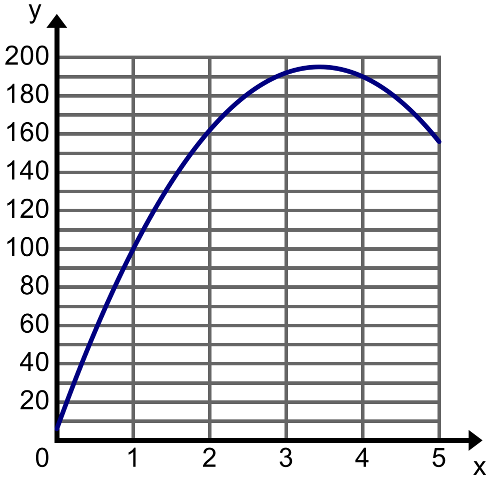
4
3
2
1
0
3
6
9
12
15
x

M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140
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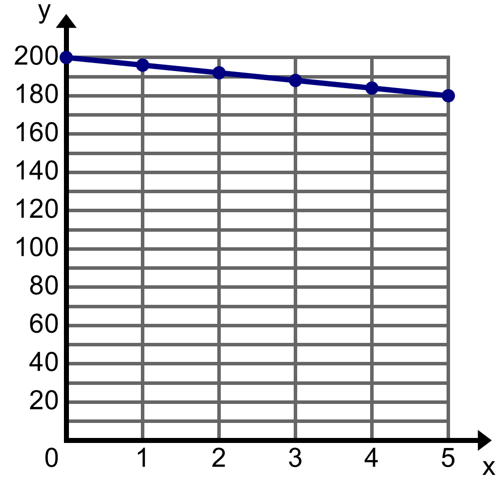
N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80
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0
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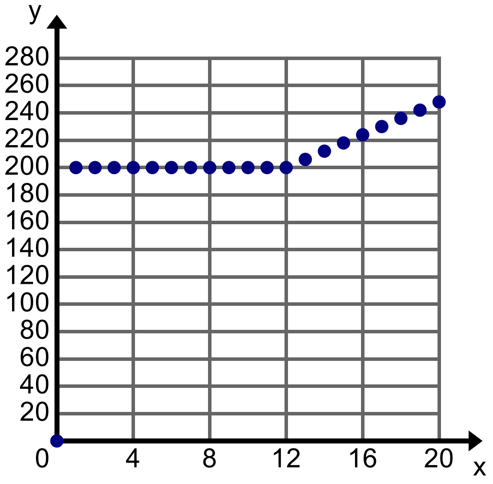
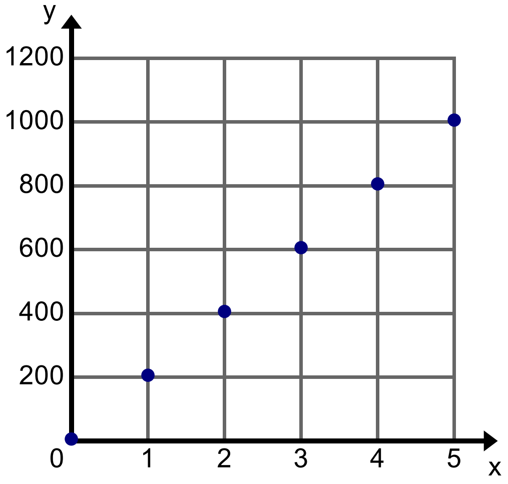
5
x

O


y
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
4
8
12
16
20
x

P


y
1200
1000
800
600
400
200
0
1
2
3
4
5
x

Q

$$y = -4x + 200$$

R

$$y = -16x^2 + 110x + 6$$

S

$$y = 3x + 6$$

T

$$y = 200x + 6$$

U

$$y = 6 \cdot 2^x$$

V

$$y = \begin{cases} 200, & \text{if } 0 < x \leq 12 \\ 200 + 6x, & \text{if } x > 12 \end{cases}$$

W

$$y = \frac{1}{5}x + 6$$

X

$$y = \frac{1}{3}x + 6$$

Y

A certain bacteria reproduces by binary fission every hour. This means that one bacterium grows to twice its size, replicates its DNA, and splits in 2. If 6 of these bacterium are placed in a petri dish, how many will there by after 5 hours?

Dependent Variable:
Independent Variable:

Z

The state is building a road 4.5 km long from point A to point B. Six meters of the road have already been completed when a new crew starts the job. It takes the crew 3 weeks to complete 600 meters of the road. How much of the road will be completed after 5 weeks if the crew works at a constant rate? 1,006 km

Dependent Variable: amount of road completed
Independent Variable: time

AA

Talen loves to help his mom clean to earn money for his cash box. He currently has $6 in his cash box. He earns $1 for every 3 jobs he does. How much money will Talen have if he does 15 jobs?

Dependent Variable:
Independent Variable:

BB

Josh is draining a swimming pool at a constant rate of 4 gallons per minute. If the swimming pool starts will 200 gallons of water, how many gallons will remain after 5 minutes?

Dependent Variable:
Independent Variable:

CC

Kendall's mom and dad have agreed to sponsor her in a school walk-a-thon to raise money for soccer uniforms. Her mom is donating $6 to her. Her dad is donating $3 for each mile she walks. How much money will she collect if she walks 5 miles?

Dependent Variable:
Independent Variable:

DD

The Planetarium charges $200 for a birthday party for up to 12 guests. Each additional guest is $6. How much will it cost for a birthday party with 20 guests?
$248

Dependent Variable: cost
Independent Variable: # of guests

EE

Suppose a rocket is fired from a platform 6 ft. off the ground into the air vertically with an initial speed of 110 feet/second. Where will the rocket be after 5 seconds? *Note:* The gravitational force of the earth on the rocket is -16 ft./sec^2.

Dependent Variable:
Independent Variable:

FF

Suzy is helping her mom fill Easter eggs with jelly beans for a community egg hunt. Before they get started, Suzy eats 6 jelly beans. Her mom tells her that after that she can eat 1 jelly bean for every 5 eggs she fills. How many jelly beans total did Suzy eat if she filled 25 eggs?

Dependent Variable:
Independent Variable:

5.2e Homework: Matching Representations of Functions

Directions: Create each of the following representations. Answers will vary. Refer to 5.2d and 5.2e class activities and 5.2d homework for help.

1. a table of data that represents a linear function	2. a table of data that represents a nonlinear function
3. a graph that represents a linear function	4. a graph that represents a nonlinear function
5. an equation that defines a linear function	6. an equation that defines a nonlinear function
7. a context (story) that can be modeled by a linear function	8. a context (story) that can be modeled by a nonlinear function

5.2f Self-Assessment: Section 5.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Mastery 3**	**Substantial Mastery 4**
1. Distinguish between linear and nonlinear functions given a context, table, graph, or equation.				

1. On Tamara’s first day of math class, her teacher asked the students to shake hands with everyone in the room to introduce themselves. There are 26 students total in the math class. Can the relationship between number of people and number of handshakes exchanged be modeled by a linear function? Why or why not? Can you determine the number of handshakes that took place in Tamara’s math class on the first day of class? Justify your answer.

Section 5.3: Model and Analyze a Functional Relationship

Section Overview:
In this section, students will analyze functional relationships between two quantities given different representations. For relationships that are linear, students will construct a function to model the relationship between the quantities. Students will also compare properties of linear functions represented in different ways, examining rates of change and intercepts, and using this information to solve problems. Next, students will learn about some of the key features of graphs of functions and apply this knowledge in order to describe qualitatively the functional relationship between two quantities. Lastly, students will sketch graphs that display key features of a function given a verbal description of the relationship between two quantities.

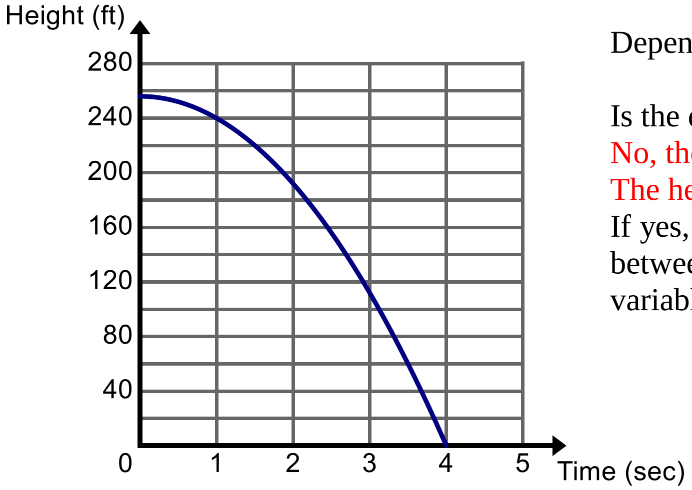
Concepts and Skills to Master:
By the end of this section, students should be able to:

- Determine whether the relationship between two quantities can be modeled by a linear function and construct a function to model a linear relationship between two quantities.
- Compare properties of linear functions (rates of change and intercepts) and use this information to solve problems.
- Identify and interpret key features of a graph that models a relationship between two quantities.
- Sketch a graph that displays key features of a function that has been described verbally.

5.3a Class Activity: Constructing Linear Functions

Directions: Identify the dependent and independent variable in the following situations. Determine whether the situations are linear or nonlinear. **For the situations that are linear,** construct a function that models the relationship between the two quantities. Be sure to define your variables.

1. An object is dropped from a bridge into the water below. The graph below shows the height of the object (in feet) with respect to time (in seconds). Consider the relationship between the height of the object and time.

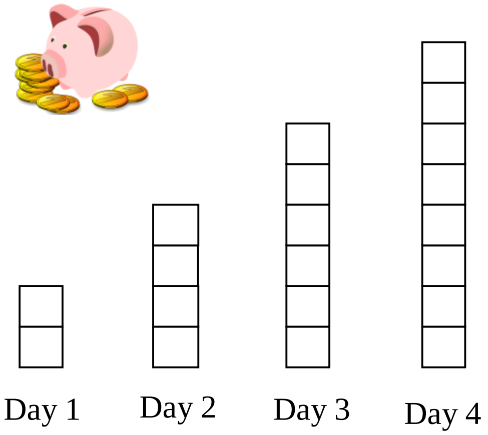


Independent Variable: time

Dependent Variable: height

Is the data linear? Why or why not?
No, the graph is not a line
The height is decreasing at an increasing rate.
If yes, construct a function to model the relationship between the two quantities. Be sure to define your variables.

2. Owen is earning pennies each day that he makes his bed in the morning. On the first day, Owen's mom gives him 2 pennies. On the second day, Owen's mom gives him 4 pennies, on the third day 6 pennies, on the fourth day 8 pennies, and so on. Owen makes his bed every day and this pattern continues. The model below shows how many pennies Owen earns each day (each box represents 1 penny). Consider the relationship between **the number of pennies received on a given day** and the **day number**.



Independent Variable: day

Dependent Variable: # of blocks on a given day

Is the data linear? Why or why not?
Yes, the number of blocks (or pennies) being added each day is 2.

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.
d = day number
p = # of pennies earned on that day

$p = 2d$

3. Refer back to #2 and Owen earning pennies. Consider the relationship between the **total number of pennies** Owen has earned and the **day number**.

Day	# of Pennies Added That Day	Sum of Pennies
1	2	2
2	4	6
3	6	12
4	8	20
5	10	30

Independent Variable: day

Dependent Variable: total number of pennies

Is the data linear? Why or why not?
No, the number of pennies being added to the total each day is changing. Students may also show that the first difference of the sum of the pennies is not constant as shown in the table to the left.

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

4. Carbon-14 has a half-life of 5,730 years. The table below shows the amount of carbon-14 that will remain after a given number of years. Consider the relationship between number of years and amount of carbon-14 remaining.

# of Years	**Milligrams of Carbon-14**
0	8
5,730	4
11,460	2
17,190	1
22,920	$\frac{1}{2}$

Independent Variable: # of Years

Dependent Variable: Carbon-14 Remaining

Is the data linear? Why or why not?
No, the first difference is not constant. Encourage students to think about the relationship between the outputs in this case (the ratio of terms is constant).

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

5. The table below shows the amount of time a recipe recommends you should roast a turkey at 325° F dependent on the weight of the turkey in pounds. Consider the relationship between cooking time and weight of the turkey.

Weight of Turkey (lbs.)	12	13	14	15
Cooking Time (hours)	4	$4\frac{1}{3}$	$4\frac{2}{3}$	5

Independent Variable: weight

Dependent Variable: cooking time

Is the data linear? Why or why not?
Yes, the data changes by equal differences over equal intervals; constant rate of change of $\frac{1}{3}$

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.
w = weight of turkey in pounds
t = cooking time in hours

$$t = \frac{1}{3}w$$

6. Steve is a lifeguard at a local community pool. Each day at noon, he records the temperature and the number of people in the pool. Do you think the relationship between temperature and number of people in the pool is linear? Why or why not?

Accept all answers that students can justify. Logically, it would make sense that as the temperature increases, the number of people in the pool increases. You may consider having students come up with some plausible data and plotting the points with students.

If the data resembles a line but is not a perfect line, you can start to have conversations about real world data and the fact that real world data does not usually lie on a straight line but we can still use linear models as approximations. This is frontloading students for chapter 6 where they will be writing best fit linear functions.

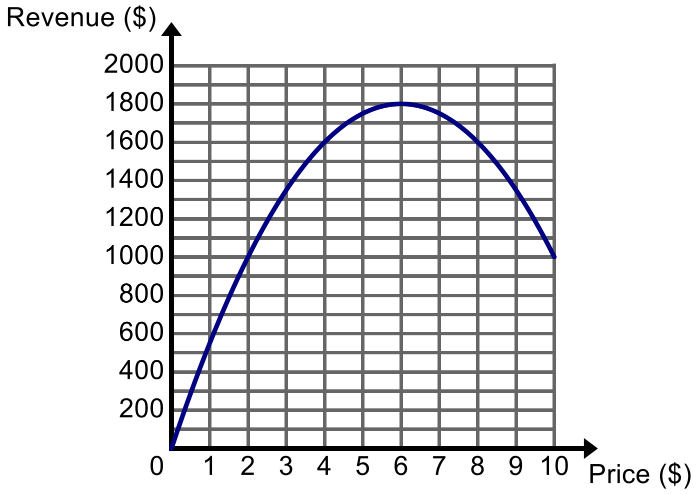
Some students may argue that the relationship is not linear because as the temperature increases, the number of people getting into the pool increases at an increasing rate. For example, you may expect to see a much bigger jump in the number of people getting into the pool when the temperature changes from 95 to 100 than you would a change from 70 to 75 degrees.

5.3a Homework: Constructing Linear Functions

Directions: Determine whether the situations represented below are linear or nonlinear. **For the situations that are linear,** construct a function that models the relationship between the two quantities. Be sure to define your variables.

See class activity for examples with answers and explanations.

1. The graph below shows the amount of revenue a company will make selling t-shirts dependent on the price of each t-shirt. Consider the relationship between price of each shirt and revenue made.



Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

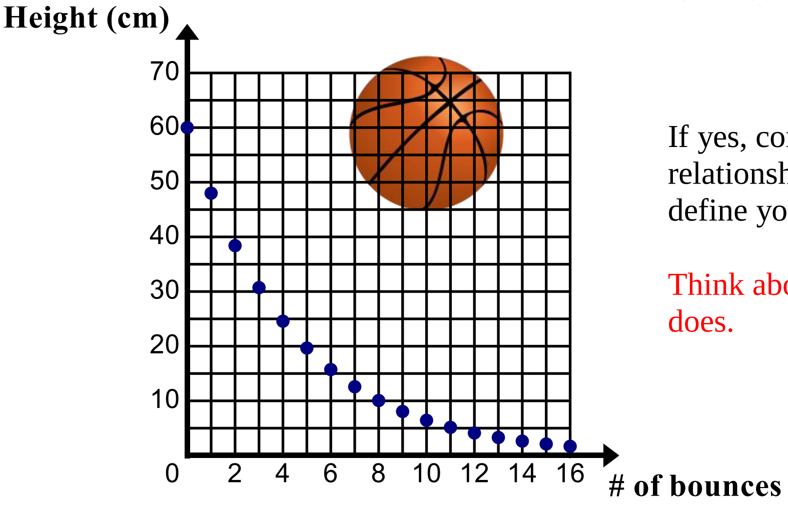
2. When Camilo opened his email this morning he had 140 unread emails. The table below shows the number of remaining unread emails Camilo has in his inbox. Assume that Camilo does not receive any new emails while he is reading his email. Consider the relationship between time and the number of unread emails.

Time (hours)	# of Unread Emails
0	180
0.5	160
1	140
2	100
2.5	80
4	20
4.5	0

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two quantities. Be sure to define your variables.

3. Suppose you drop a basketball from a height of 60 inches. The graph below shows the height of the object after *b* bounces.



Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

Think about why this graph looks the way that it does.

4. Justine and her family are floating down a river. After 1 hour, they have floated 1.25 miles, after 4 hours they have floated 5 miles, and after 6 hours they have floated 7.5 miles. Is the relationship between time (in hours) and distance (in miles) linear? Why or why not? If it is linear, write a function that models the relationship between the two quantities.

Yes, they float at a constant rate of 1.25 miles/hr.
t = time in hours
d = distance in miles
$d = 1.25t$

Extension Questions: What physical features of the river might explain the linearity? What features of a river would cause nonlinearity?

5. You and your friends go to a BMX dirt-biking race. For one of the events, the competitors are going off a jump. The winner of the event is the competitor that gets the most air (or jumps the highest). Do you think the relationship between the weight of the bike and the height of the jump can be modeled by a linear relationship? Why or why not?

6. Homes in a certain neighborhood sell for $117 per square foot. Can the relationship between the number of square feet in the home and the sale price of the home be modeled by a linear function? Why or why not? If it can be modeled by a linear function, write a function that models the relationship between the two quantities.

7. Suppose a certain bank pays 4% interest at the end of each year on the money in an account. When Devon was born, his parents put $100 in the account and will leave it there until he goes to college. Is the relationship between time (in years) and the amount of money in the account (in dollars) linear or not? Why or why not? If it is linear, write a function that models the relationship between the two quantities.

No, students can show that the rate of change in a table is not constant

5.3b Class Activity: Comparing Linear Functions

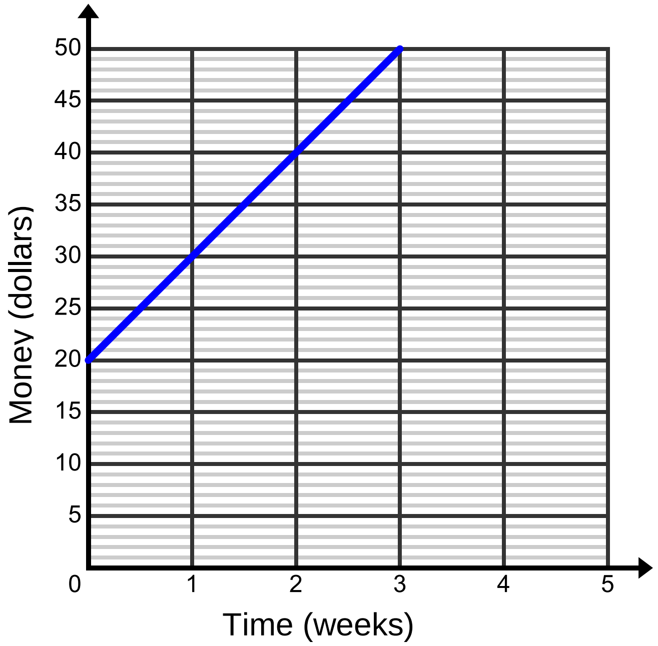
1. Who will have $100 first, George or Mark?

George has $20 and is saving $15 every week.

George (saving $15 per week vs. Mark who is only saving $10 per week)

As students work through this lesson, you may ask consider having them construct functions for some of the representations where the equation is not given if you feel they need more practice doing this.

Mark starts with $20. His savings are shown on the graph below.



2. Put the cyclists in order from slowest to fastest. (Note variables: x = time in seconds, y = meters traveled)

Cyclist A:

Time (x)	Distance (y)
2	1
4	2
6	3

Cyclist B:

Bob has cycled 12 meters in the past 6 seconds.

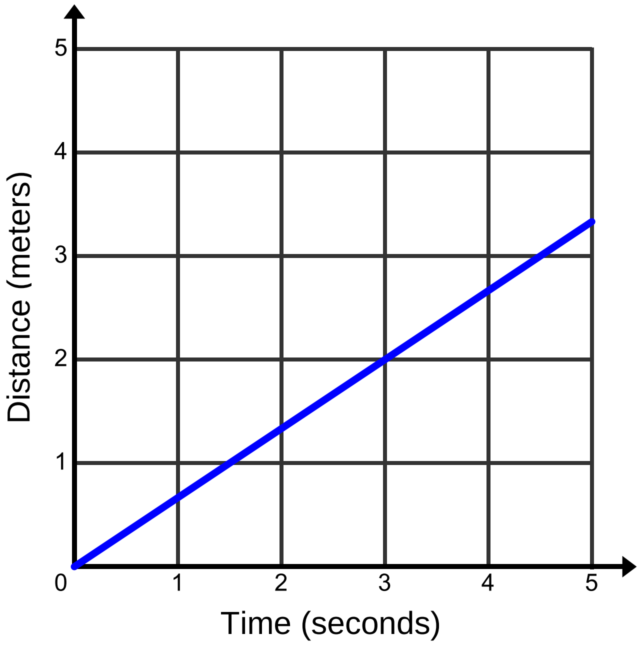
Cyclist C:

$$y = \frac{1}{3}x$$

C (1/3 m/s), A (0.5 m/s), D (1.5 m/s), B (2 m/s)

Note: The formula for calculating rate is $\frac{\text{distance}}{\text{time}}$.

Cyclist D:

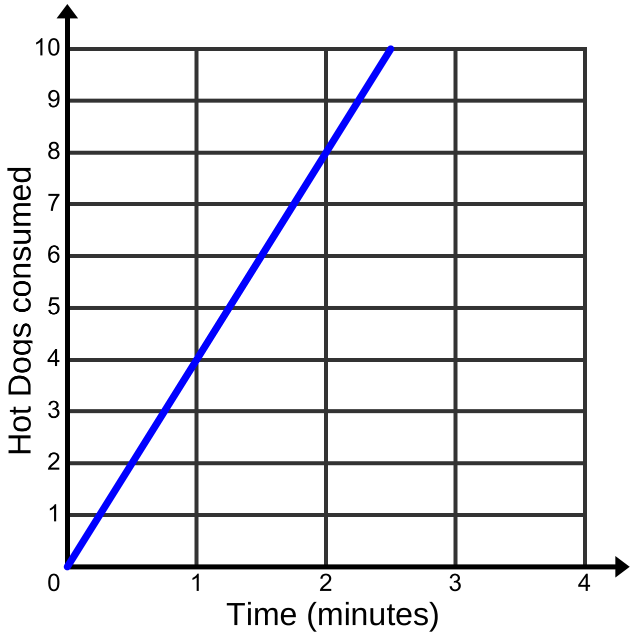


3. Assume the rates below will remain constant. Who will win the hot dog eating contest? Why?

Helga, who has eaten 18 hot dogs in 5 minutes.

Pablo will win – he eats 4 hotdogs/minute while Helga eats 3.6 hotdogs/minute.

Pablo whose eating record is shown below.



4. Based on the information below, which bathtub will be empty first? Why?

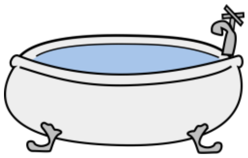
Bathtub A:

Starts with 25 gallons and is draining 1.5 gallons a minute.

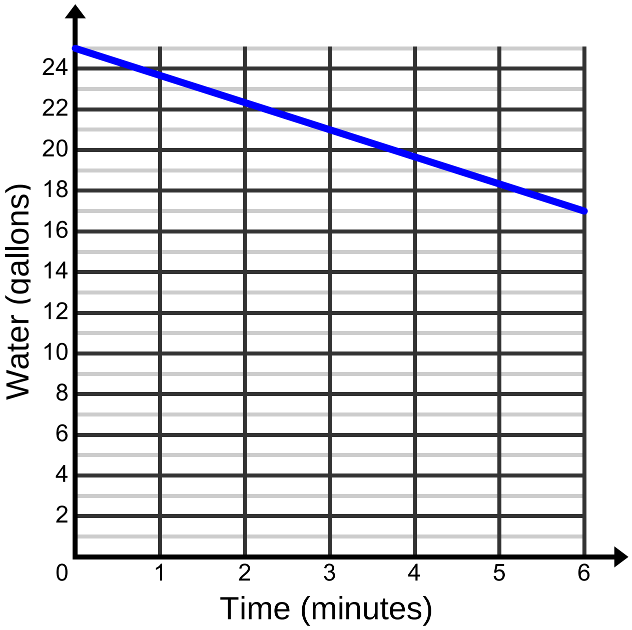
Bathtub B:

Minutes	Gallons
0	25
3	20
6	15

Bathtub B will empty first. All bathtubs start with 25 gallons of water and bathtub B drains at the fastest rate (B: $1.\overline{6}$ gal/min; A: 1.5 gal/min; C: $1.\overline{3}$ gal/min).



Bathtub C:



5. Put the cars in order from fastest to slowest. (Note variables: x = time in hours, y = miles traveled). Assume all cars travel at a constant rate.

Car A:

$y = 65x$

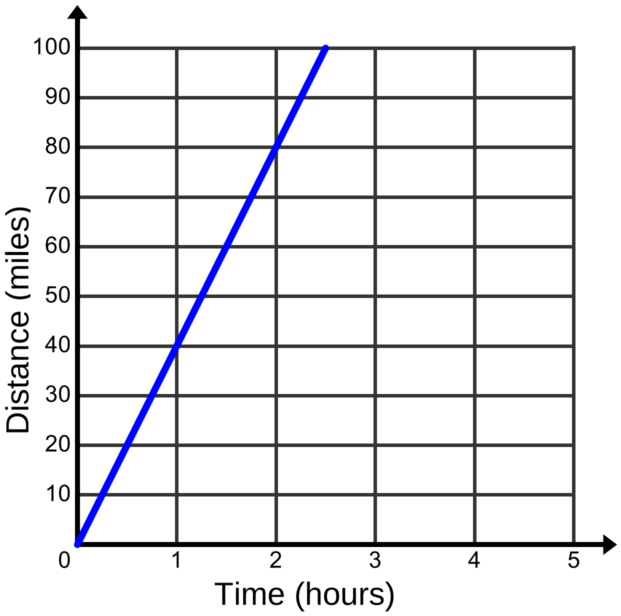
Car B:

The Bakers completed the 64-mile trip to Salt Lake City in 50 minutes.

Car C:

Hours	Miles
2	150
4	300

Car D:



Car B (76.8 mph); Car C (75 mph); Car A: (65 mph); Car D (40 mph)

6. Put the exercises below in order from burns the most calories to burns the least calories. (Note variables: x = time in minutes, y = calories burned). Assume the rate at which you burn calories in each of the exercises is constant.

Cycling:

$y = 10.8x$

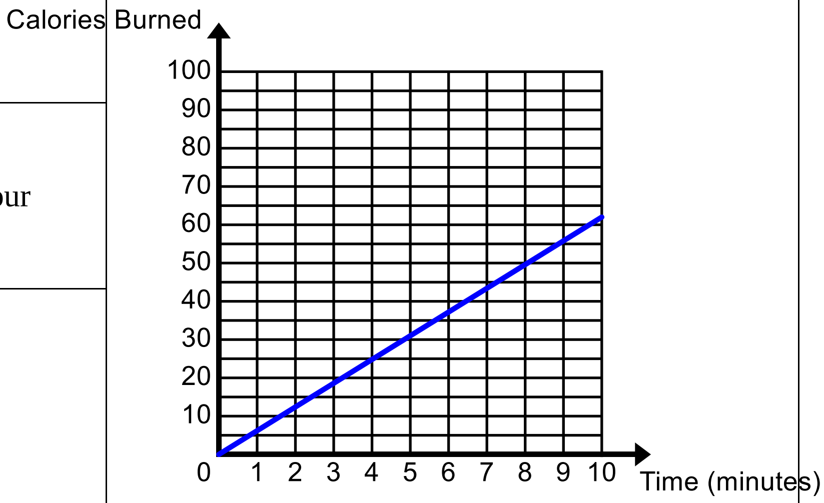
Yoga:

Meredith burned 429 calories in a 1-hour long yoga class.

Running:

Minutes	Calories Burned
10	137
15	205.5
25	342.5

Swimming:



Running (13.7 cal/min), cycling (10.8 cal/min), yoga (7.15 cal/min), swimming (6.25 cal/min)

7. Four families are meeting up in Disneyland. Each family starts driving from home. The representations below show the distance each family is from Disneyland over time. (Note variables: x = time in hours, y = distance from Disneyland.) Assume the families drive to Disneyland at a constant rate.

Family A:

$y = 95 - 55x$

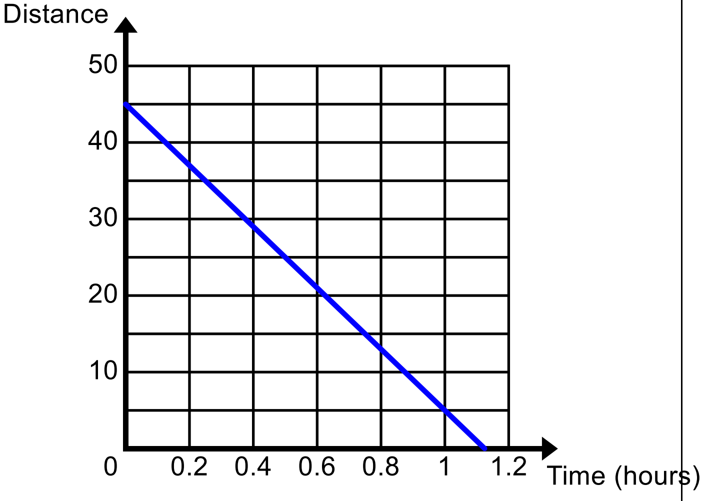
Family B:

Family B lives 120 miles from Disneyland and drives 60 mph.

Family C:

Hours	Distance from Disneyland
0	80
1.5	5

Family D:



a. Which family lives the closest to Disneyland? Family D, from the graph, we see that Family D lives 45 miles from Disneyland (Family A lives 95 miles from Disneyland, Family B lives 120 miles from Disneyland, and Family C lives 80 miles from Disneyland)

b. Which family lives the farthest from Disneyland? Family B

c. Which family is traveling at the fastest speed? Family B (Family B travels 60 mph, Family A travels 55 mph, Family C travels $\frac{75}{1.5}$ or 50 mph). We don't know the exact speed at which Family D travels but we know that their speed is less than 45 mph – they live 45 miles away and it takes them over an hour to get there.

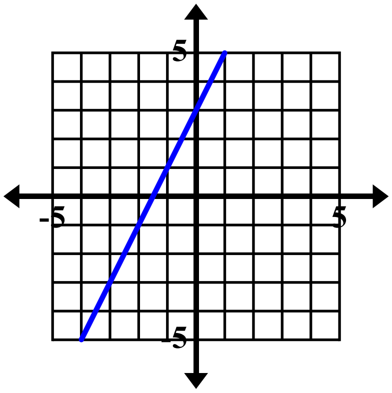
d. Which family is traveling at the slowest speed? Family D

e. Who will get to Disneyland first? Family D. To determine the time it takes each family to get to Disneyland, divide the total distance traveled by the speed. It takes Family A approximately 1.7 hours ($\frac{95}{55}$), Family B 2 hours ($\frac{120}{60}$), Family C 1.6 hours ($\frac{80}{50}$), and we can see on the graph that it takes Family D under 1.2 hours.

f. Who will get to Disneyland last? Family B

Directions: For each problem, circle the representation with the greatest rate of change. Put a star by the representation with the greatest *y*-intercept. Assume all representations have a constant rate of change.

<table>
<tr>
<td>8. [✓ $y = 2 + 3.5x$]

In this problem, the equation has a slope of 3.5 and a *y*-intercept of $(0, 2)$. The table has a slope of 3 and a *y*-intercept of $(0, 5)$. The graph has a slope of 2 and a *y*-intercept of $(0, 3)$. Remember, when an equation is in slope-intercept form, the slope is the number in front of *x*. To find slope from a table, use the formula $\frac{y_2 - y_1}{x_2 - x_1}$ and to find the slope on a graph determine the $\frac{\text{rise}}{\text{run}}$. Remember that the *y*-intercept is the point where the line crosses the *y*-axis (at this point $x = 0$).</td>
<td>☆<table><tr><th>x</th><th>y</th></tr><tr><td>1</td><td>8</td></tr><tr><td>5</td><td>20</td></tr><tr><td>7</td><td>26</td></tr></table></td>
<td> </td>
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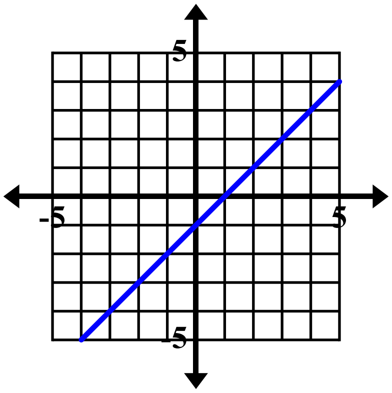
<tr>
<td>9. [✓ $y = \frac{3}{2}x$]</td>
<td> </td>
<td>☆ $(0,1)(1, 2.2)$</td>
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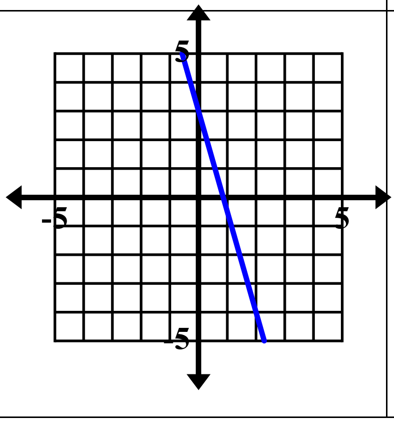
<td>10. </td>
<td>☆<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>4</td></tr><tr><td>2</td><td>-2</td></tr><tr><td>5</td><td>-11</td></tr></table></td>
<td>[✓ $(0,3)(2, -5)$]</td>
</tr>
</table>

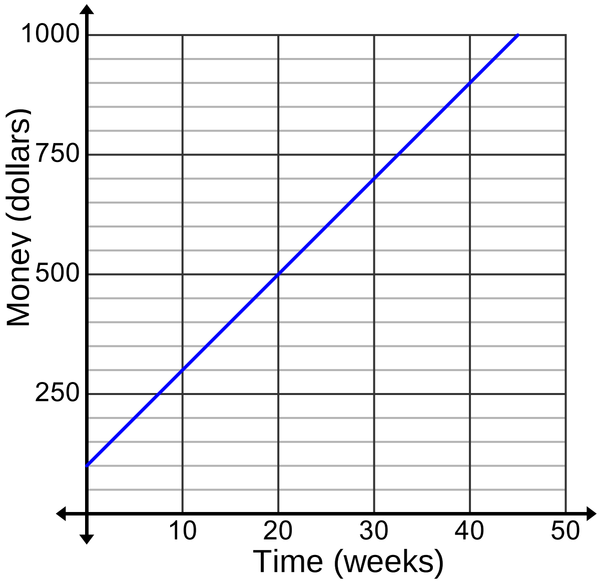
5.3b Homework: Comparing Linear Functions

1. Who will have $1,000 first, Becky or Olga?

Becky has $100 and is saving $10 every week.

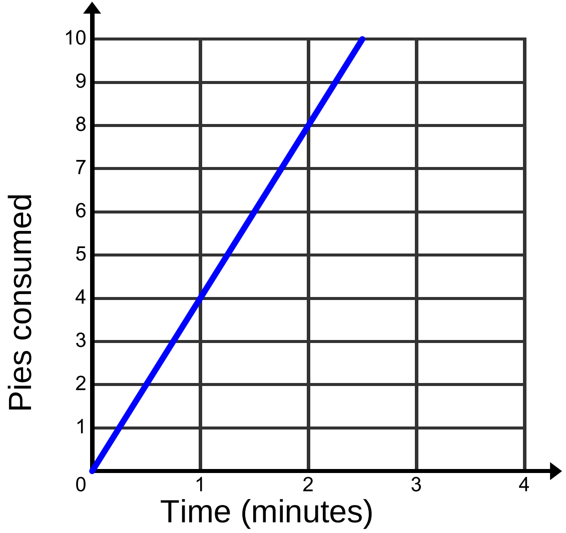
Olga (she saves $20 per week while Becky only saves $10). They both start with $100

Olga's information is shown on the graph below.



2. Assume the rates below will remain constant. Who will win the pie eating contest? Why?

Joe, whose information is shown below.



Donna, who has eaten 11 pies in 2.5 minutes.

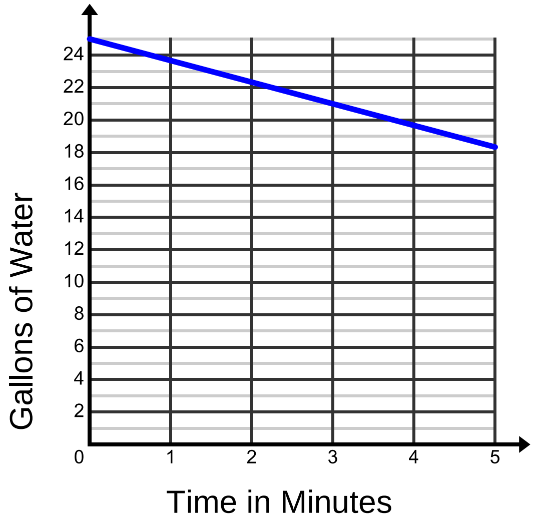
3. Based on the information below, which hot water heater will use up the available hot water first?

Water Heater A:
Starts with 50 gallons and drains 1.5 gallons a minute.

Water Heater B:

Time in Minutes	Gallons
0	30
2	27.5
4	25

Water Heater C:



Heater A (1.5 gal/min) Heater C is 4/3 gal/min and Heater B is 1.25 gal/min; Heater C will use up the available hot water first – its water will be gone in 18.75 minutes, B in 24 minutes, and A in 33 minutes

4. Put the cars in order from fastest to slowest. (Note variables: x = time in hours, y = miles traveled.)

Car A:
$y = 35x$

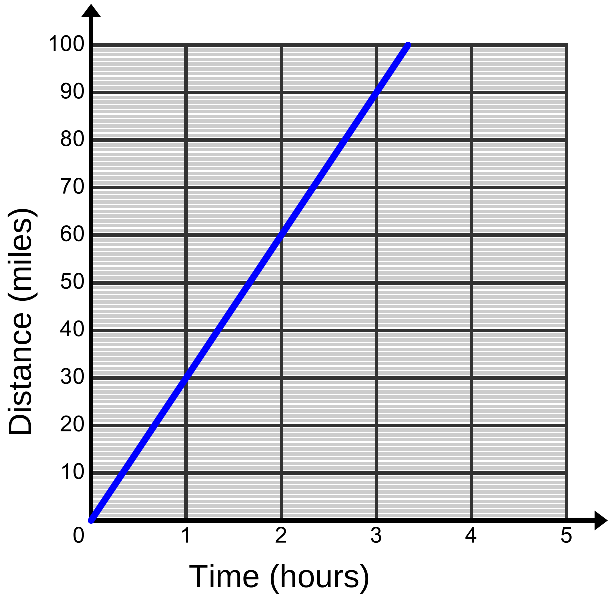
Car B:

The Andersons completed the 24 mile trip to Salt Lake City in 30 minutes.

Car C:

Time (hours)	Distance (miles)
0.5	18
2.5	90
4	144

Car D:



5. Put the runners in order from slowest to fastest. (Note variables: x = time in minutes, y = miles traveled.)

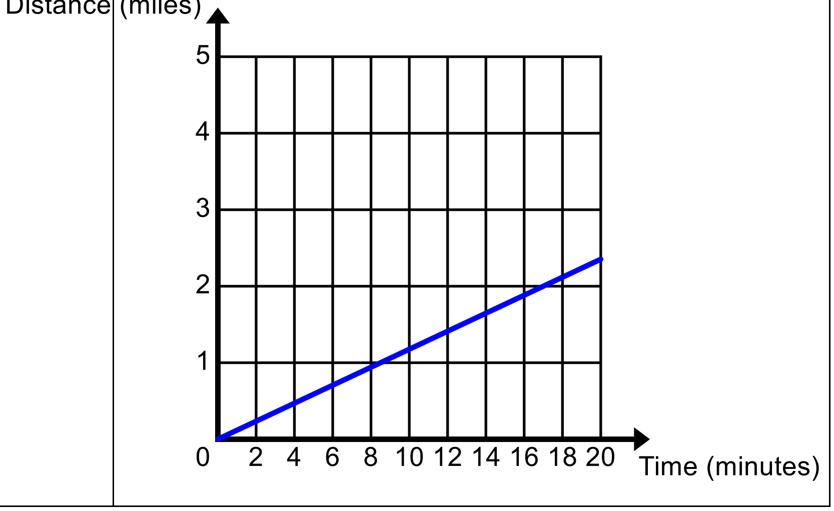
Ellen ran 1 mile in the last 10 minutes

Samantha: $y = \frac{2}{13}x$

Jason:

Time (minutes)	Distance (miles)
3	0.5
6	1
9	1.5

Dale:



One way to approach this problem is to determine how long it takes each person to run one mile. The runners in order from slowest to fastest: Ellen (10 minutes/mile); Dale (8.5 minutes/mile); Samantha (6.5 minutes/mile); Jason (6 minutes/mile)

6. Use the representations below to answer the questions that follow. (Note variables: x = time in weeks, y = amount of money remaining.)

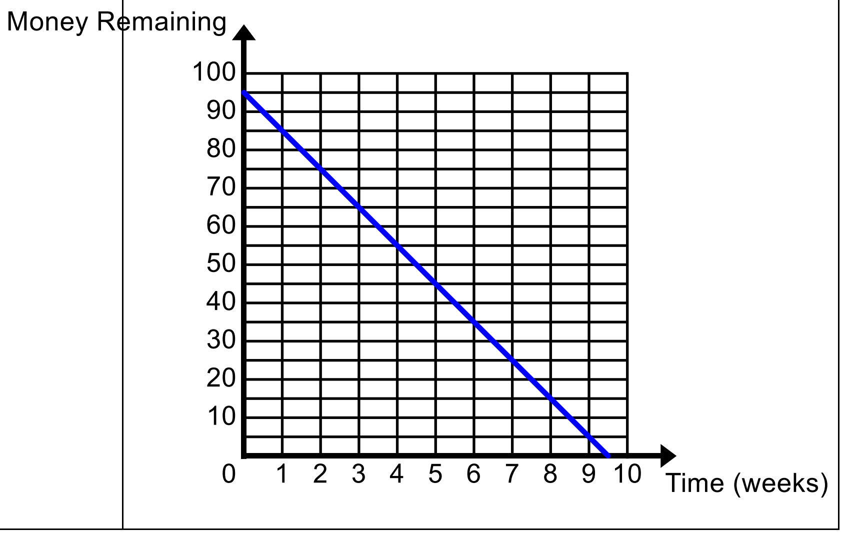
Imiko starts with \$60 and spends \$2 per week

Henry: $y = 80 - 5x$

Garek:

Time (weeks)	Amount of Money Remaining
2	59
5	35
7	19

Roslyn:



a. Who starts with the most money?

b. Who is spending his/her money at the fastest rate?

c. Who will run out of money first at the current rate of spending?

7. Jack, George, Lucy, and Anna are playing games on their iPads. The representations below show the battery life remaining on each child's iPad over time. (Note variables: x = time in hours, y = battery life remaining as a percent.) Use these representations to answer the questions that follow.

Jack:

$y = 78 - 10x$

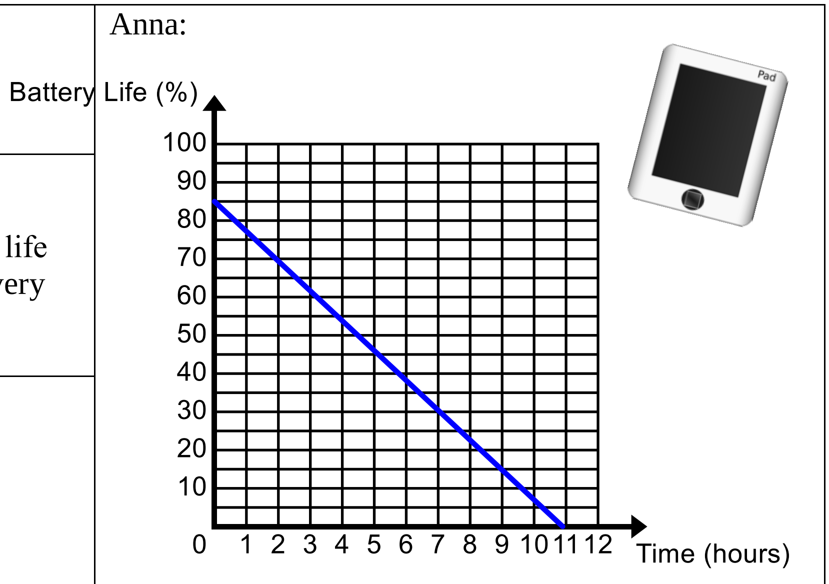
George:

George's iPad started with 92% battery life and is using 12.5% of the battery life every hour.

Lucy:

Hours	Battery Life Remaining (%)
0	76
2	60
3	52

Anna:



a. Whose iPad had the most battery life when the kids started playing?

b. Whose iPad is using the battery at the fastest rate? At the slowest rate?

c. Who will run out of battery life first?

d. Whose will be able to play their iPad for the longest amount of time?

Directions: For each problem, circle the representation with the greatest rate of change. Put a star by the representation with the greatest *y*-intercept. Assume all representations have a constant rate of change.

<table>
<tr>
<td>8. [✓ $y = x$]</td>
<td>☆
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>1</td><td>1.5</td></tr>
<tr><td>2</td><td>2</td></tr>
<tr><td>3</td><td>2.5</td></tr>
</table>
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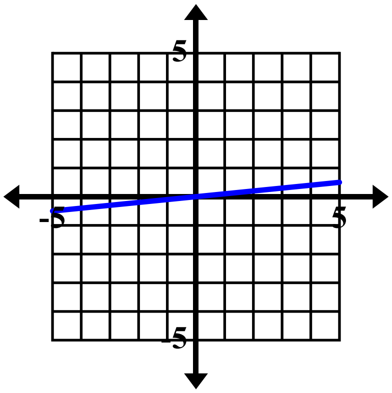
<td>
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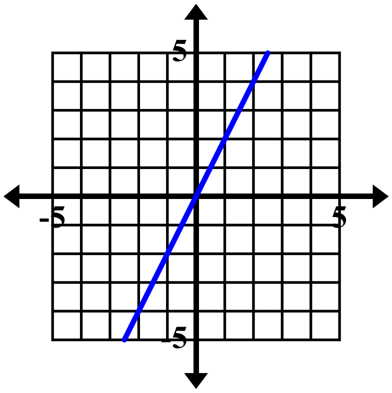
<tr>
<td>9. $y = \frac{7}{4}x + 2$</td>
<td>$(1,1.5)(2,3)$</td>
<td>
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<tr>
<td>10. $y = -0.5x$</td>
<td>
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>2</td><td>-2</td></tr>
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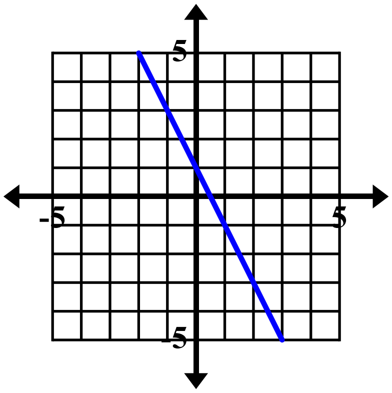
<td>
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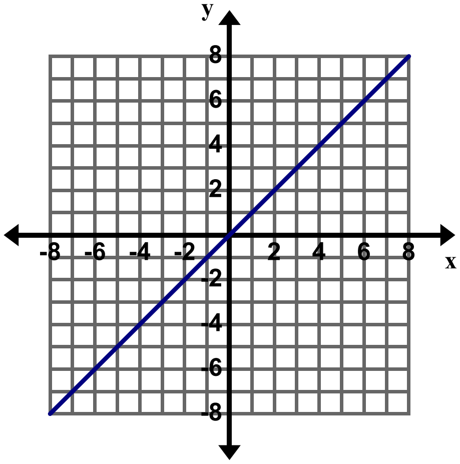
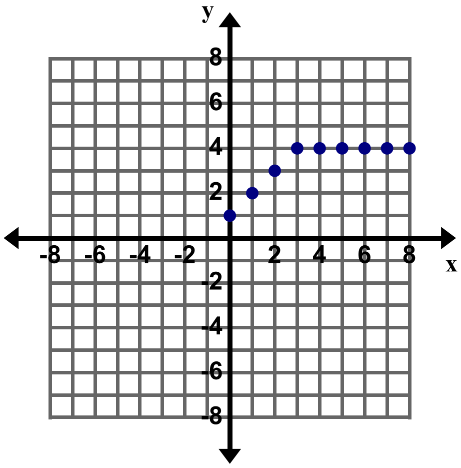
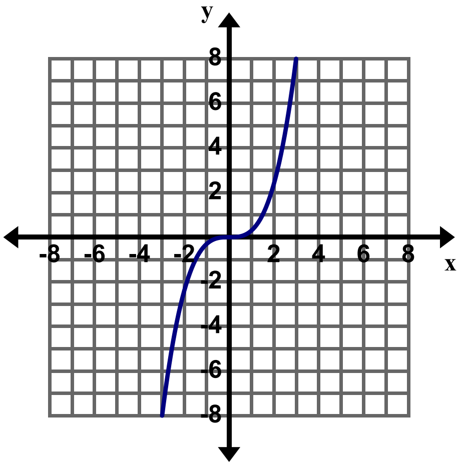
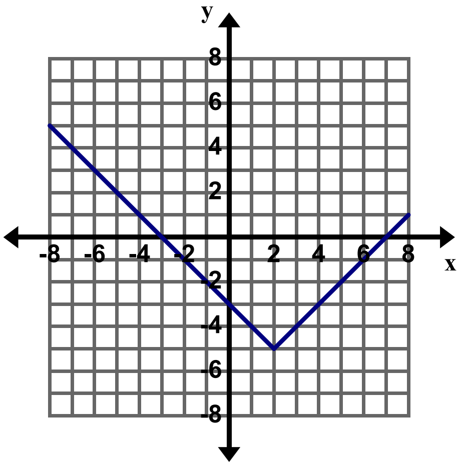
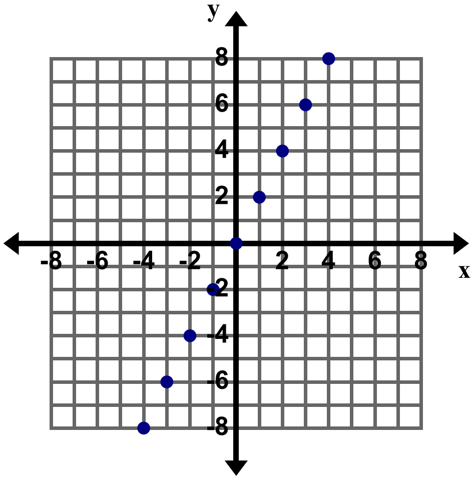
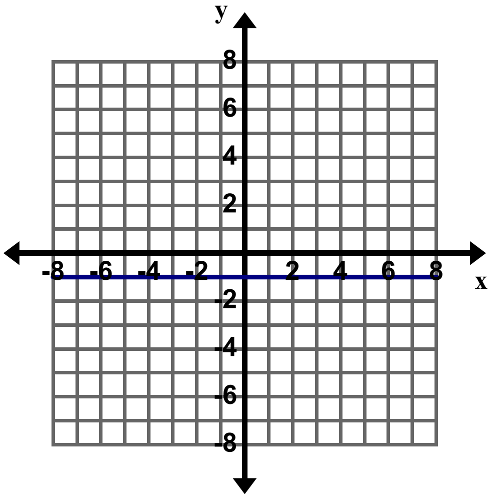
</table>

5.3c Class Activity: Features of Graphs

1. Cut out each graph. Sort the graphs into groups and be able to explain why you grouped the graphs the way you did. In the table that follows, name your groups, describe your groups, and list the graphs that are in your group.

A  	**B** 
C  	**D** 
E  	**F** 

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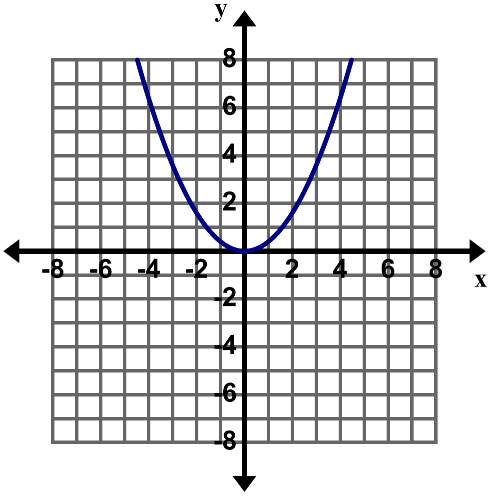
y
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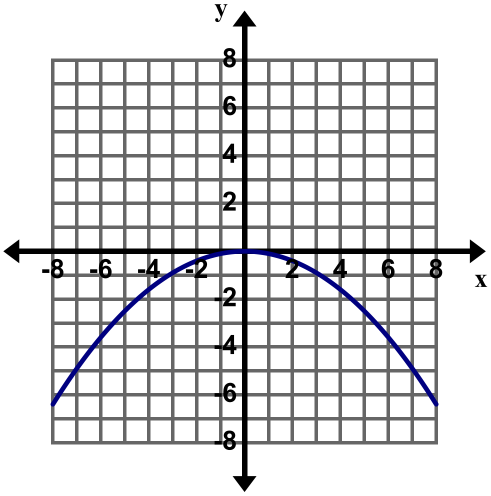
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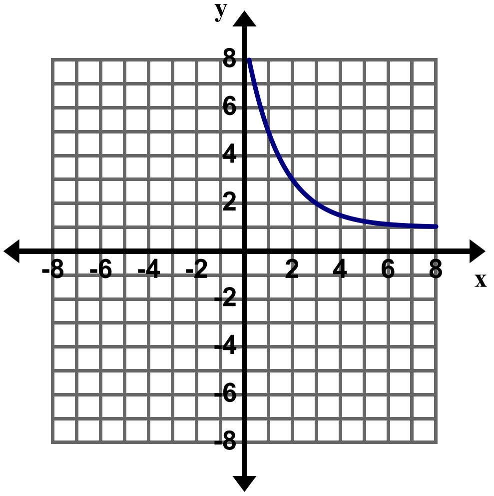
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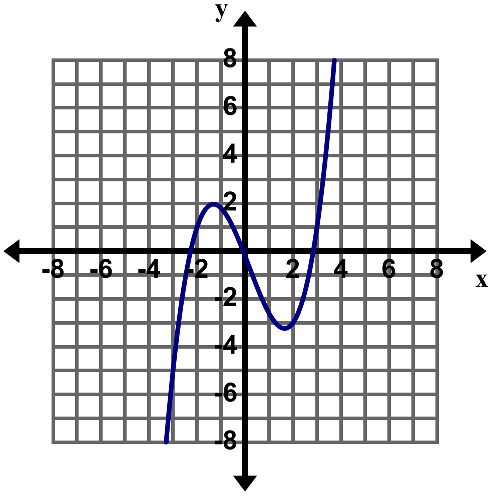
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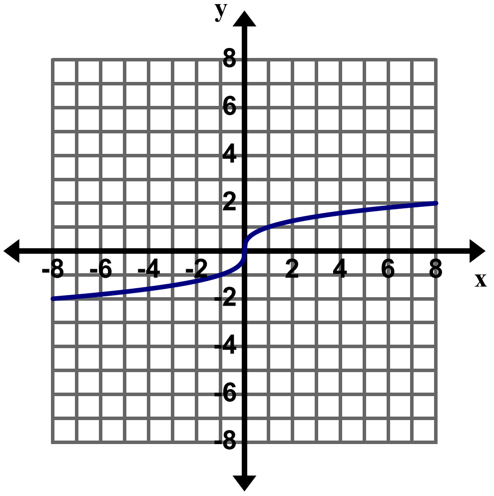
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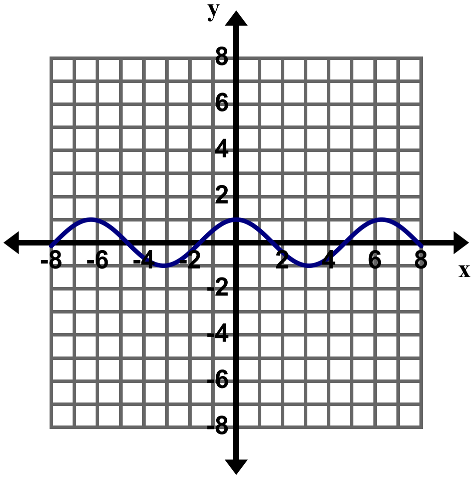
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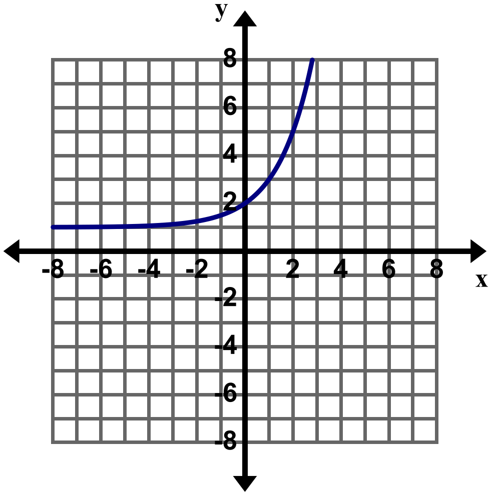
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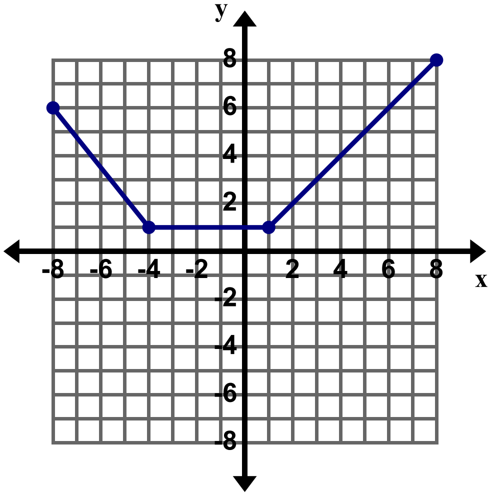
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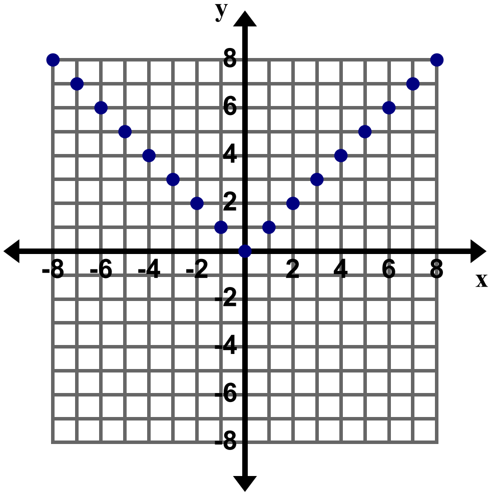
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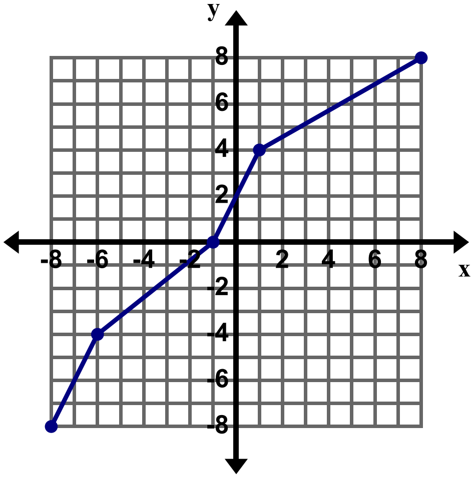
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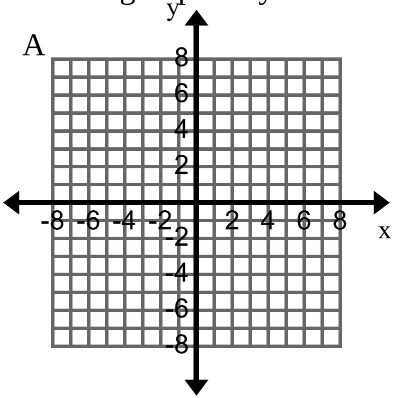
Your Groups There are no right or wrong answers in this activity; however students must be able to justify their groupings.

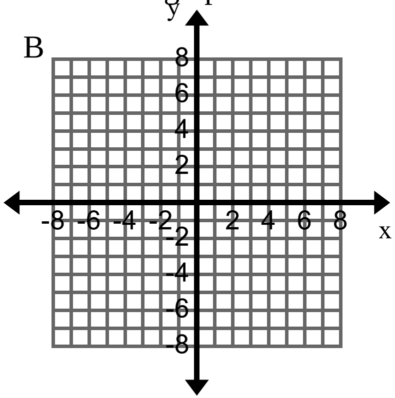
Name of the group	Description of the group	Graphs in the group
A.		
B.		
C.		
D.		
E.		
F.		

2. For each group that you created, draw another graph that would fit in that group.

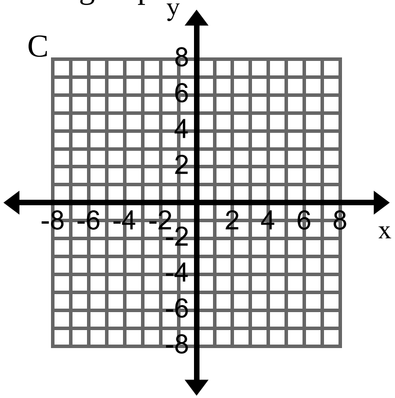
A



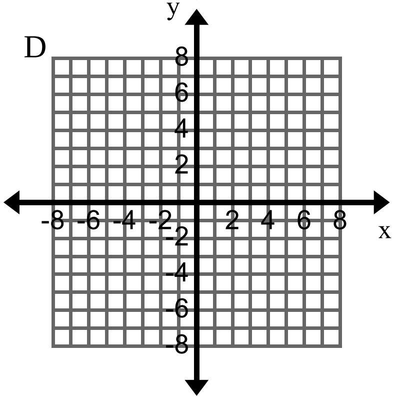
B



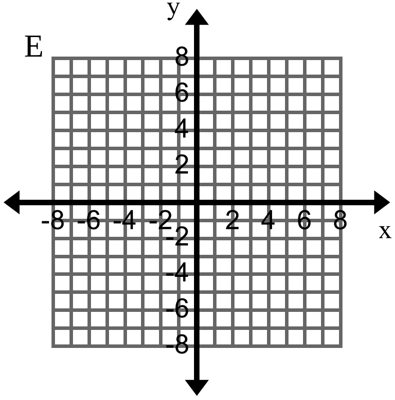
C



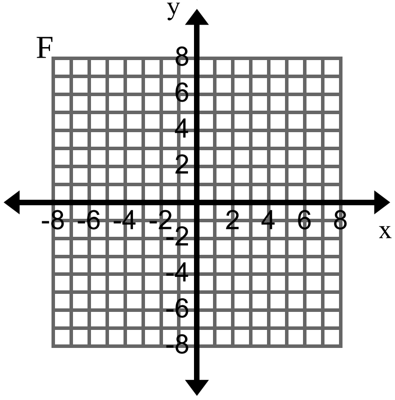
D



E



F



3. Lucy grouped hers as follows:

Increasing on the entire graph: **A, C, E, K, M, P**

Decreasing on the entire graph: **I**

Constant on the entire graph: **F**

Increasing on some parts of the graph, decreasing on some parts of the graph: **D, G, H, J, L, O**

Increasing on some parts of the graph, decreasing on some parts of the graph, constant on some parts of the graph: **N**

Increasing on some parts of the graph, constant on some parts of the graph: **B**

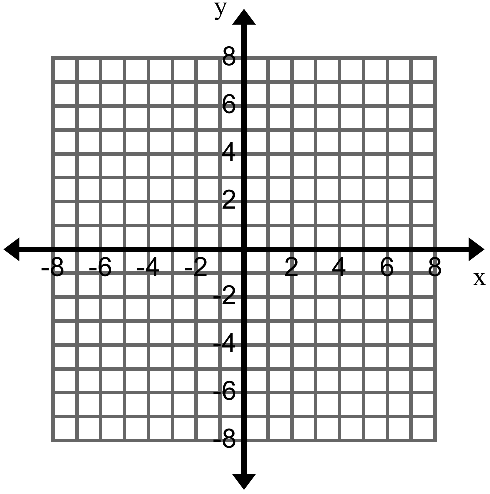
4. Define **increasing**, **decreasing**, and **constant** in your own words.

Increasing: The values of y increase as x increases (values of y get larger as you move from left to right)

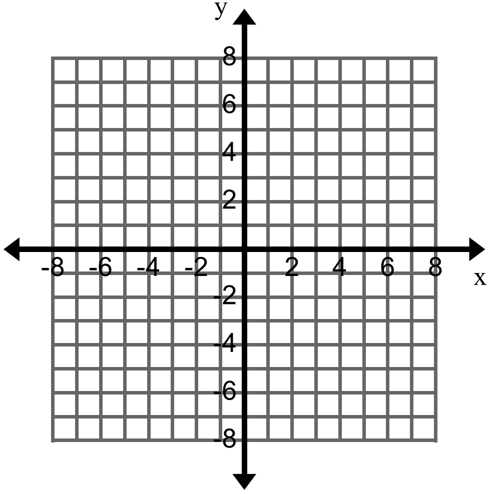
Decreasing: The values of y decrease as x increases (values of y get smaller as you move from left to right)

Constant: The values of y are neither increasing nor decreasing as x increases

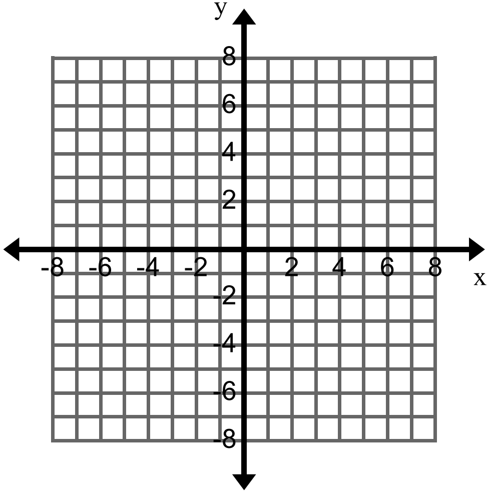
5. Draw an example of a graph that is increasing.



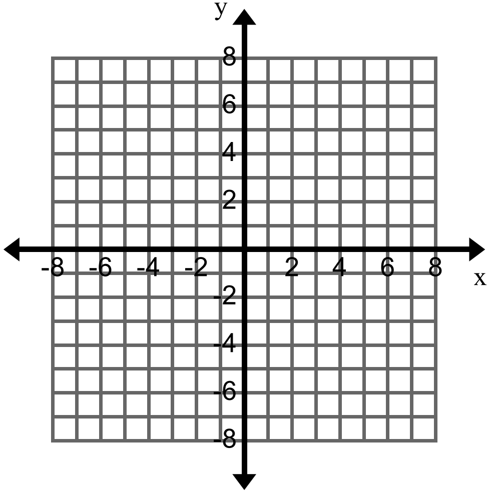
6. Draw an example of a graph that is decreasing.



7. Draw an example of a graph that is constant.



8. Draw an example of a graph that is increasing then decreasing.

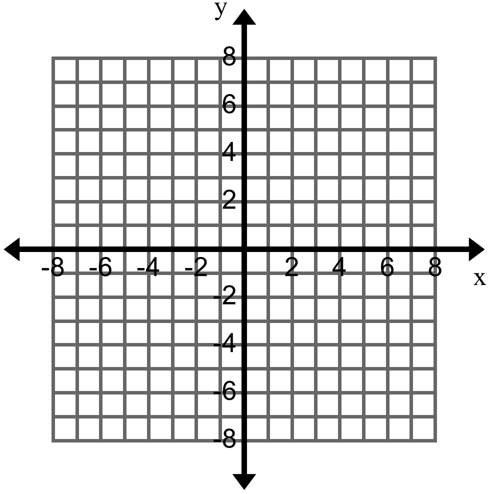


9. Ellis grouped hers as follows:

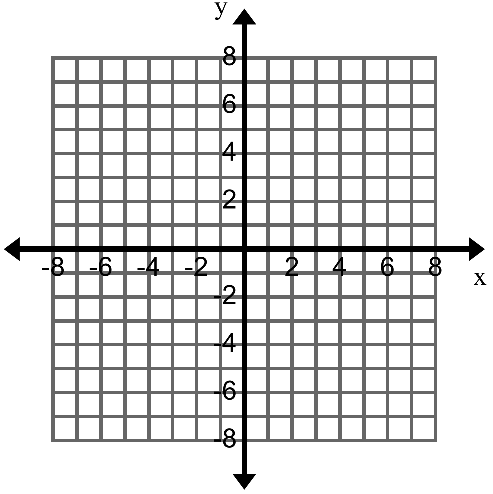
Discrete: **B, E, O**
Continuous: **A, C, D, F, G, H, I, J, K, L, M, N, P**

10. Define **discrete** and **continuous** in your own words. Can you think of a real world situation that has a discrete graph? Why doesn’t it make sense to connect the points in this situation?
In a discrete situation, the data points are not connected. In a continuous situation, the data points are connected. An example of a discrete situation would be number of adult movie tickets purchased and cost (you cannot purchase (1/2) of a movie ticket).

11. Draw an example of a graph that is discrete.



12. Draw an example of a graph that is continuous.



13. Grace grouped hers as follows:

Linear: **A, E, F**

Nonlinear: **C, G, H, I, J, K, L, M**

Made up of pieces of different linear functions: **B, D, N, O, P**

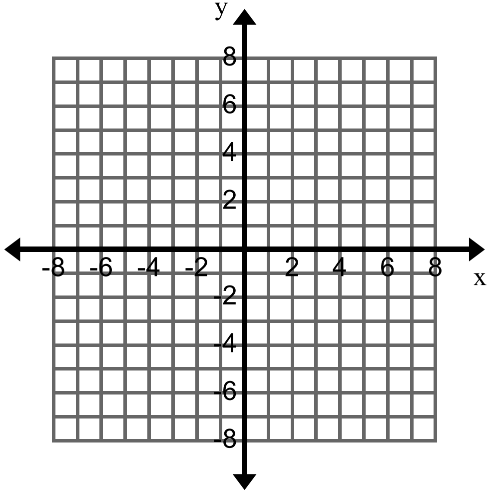
14. Define **linear** in your own words.
Graph is a line (constant slope)

15. Define **nonlinear** in your own words.
The graph is curved (the rate of change is not constant)

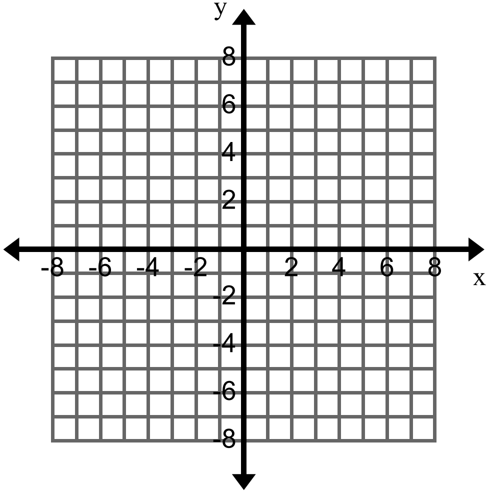
5.3c Homework: Features of Graphs

Directions: Draw a graph with the following features. Answers will vary

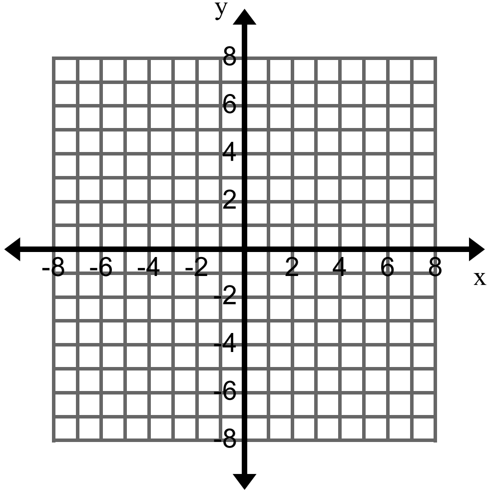
1. increasing, linear, continuous



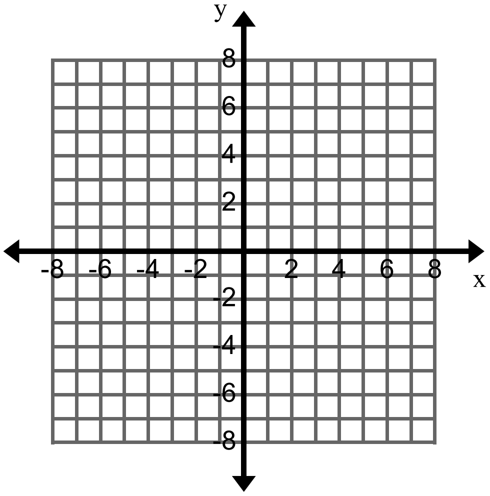
2. Decreasing, linear, and discrete



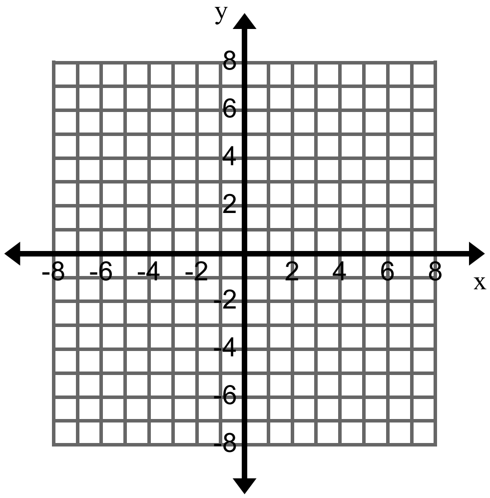
3. Increasing then decreasing, continuous



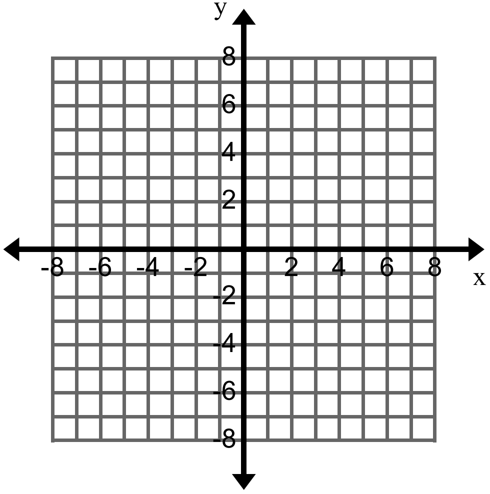
4. Constant, discrete



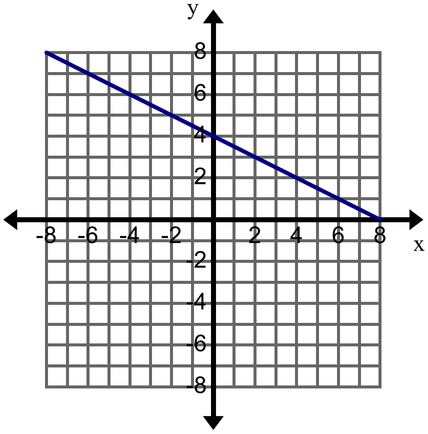
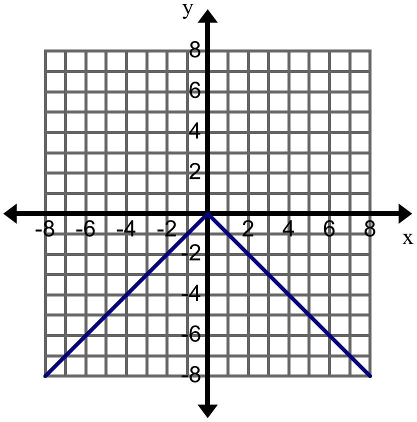
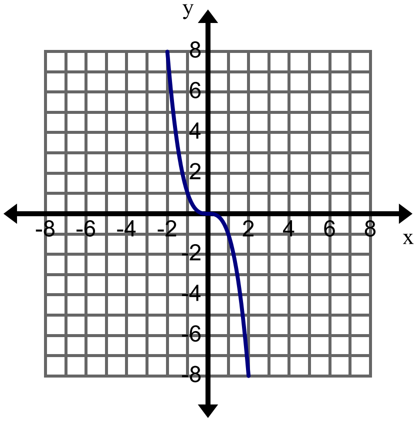
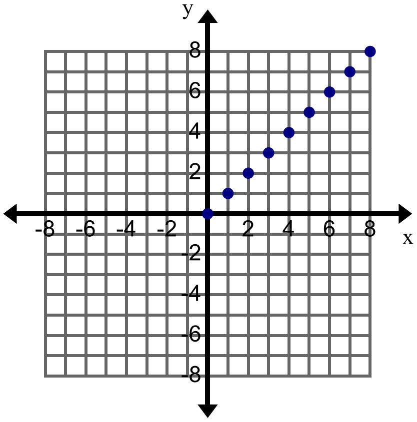
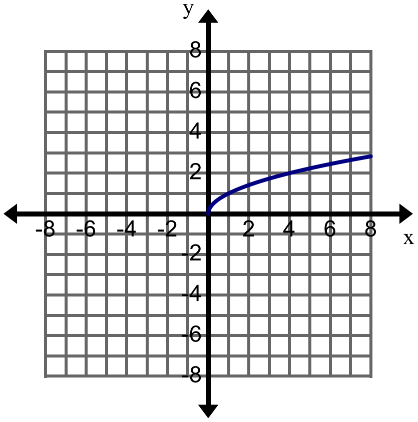
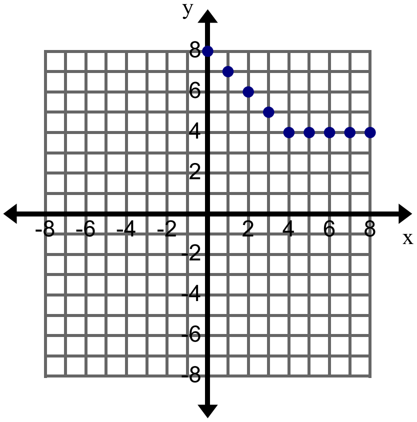
5. Decreasing, then constant, then increasing, continuous



6. Increasing and nonlinear

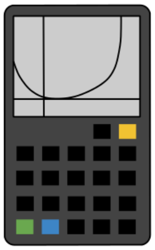


Directions: Describe the features of each of the following graphs (increasing/decreasing/constant; discrete/continuous; linear/nonlinear). Label on the graph where it is increasing, decreasing, or constant. Identify the intercepts of the graph.

7. 	8. 
Features: decreasing, linear, continuous	Features: increasing then decreasing, made up of 2 different linear functions, continuous
9. 	10. 
Features:	Features:
11. 	12. 
Features:	Features: decreasing then constant, discrete, made up of 2 different linear functions

5.3d Class Activity: CBR Activity

You will be using the *DIST MATCH application in the CBR^{TM} Ranger* program on the TI 73 (or other) graphing calculators. Instructions for CBR/calculator use:

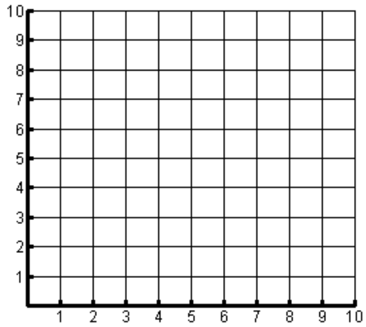
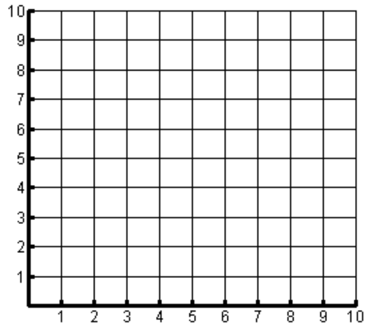
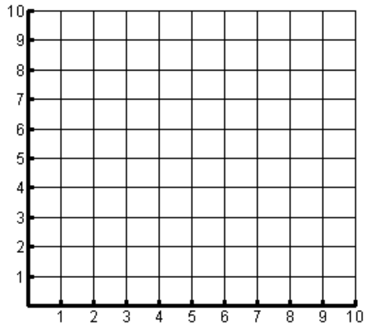


- Firmly attach the TI 73 to the CBR Ranger.
- Choose the APPS button on the TI 73.
- Choose 2: CBL/CBR.
- Choose 3: RANGER.
- Choose 3: APPLICATIONS.
- Choose 2: FEET.
- Choose 1: DIST MATCH. Get your first graph onto the calculator screen.

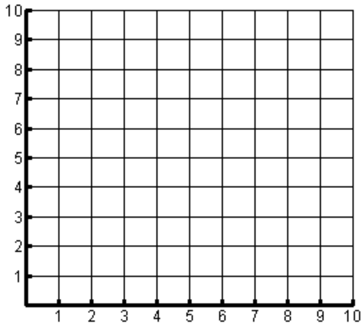
1. Try to match the graph given to you in the program. You will reproduce the graph by walking. Then trace the graph onto the grids below.

Be sure to model a few examples with your class before you begin in teams!

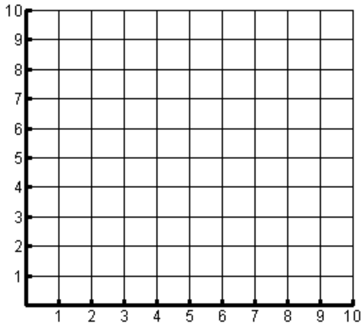
a. Get a graph to match ready in the calculator.
b. Decide how far away from the sensor you should stand to begin.
c. Talk through the walk that will make a graph match. (how far away to begin, walk forward or backward, how fast to move forward or backward, how long to walk forward or backward, when to change directions or speed, etc.)
d. You may wish to write the story of the graph first (before you walk it)—see below.
e. Have a member of your group hold the CBR so that the CBR sensor is up and directed toward the person that is walking
f. Have a group member press start on the calculator. Then walk toward or away from the sensor trying to make your walk match the graph on the calculator screen.
g. Each member of your group should walk to match at least one graph on the calculator.
h. Sketch each graph below. Write the story for the graph.

Graph 1: 	Graph 2: 	Graph 3: 
Story:	Story:	Story:

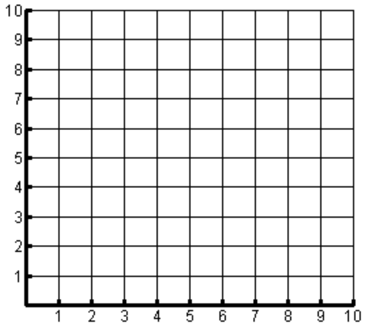
Graph 4:



Graph 5:



Graph 6:



Story:

Story:

Story:

Extra for Experts

If you finish early try to create the following graphs, write a description/story that matches the graph.

1. A line that rises at a steady rate.
   Story:

2. A line that falls at a steady rate.
   Story:

3. A horizontal line
   Story:

4. A “V”
   Story:

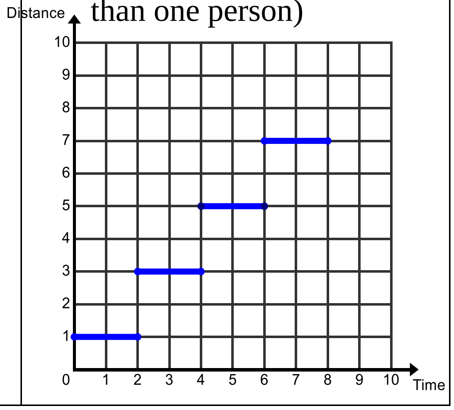
5. A “U
   Story:

6. An “M”
   Story:

7. Try creating an O. Are you successful? Why or why not?

8. Name a letter you could graph using the CBR. Name a letter you cannot graph using the CBR. Explain your choices.

9. Try creating this graph. (Hint: It will take more than one person)

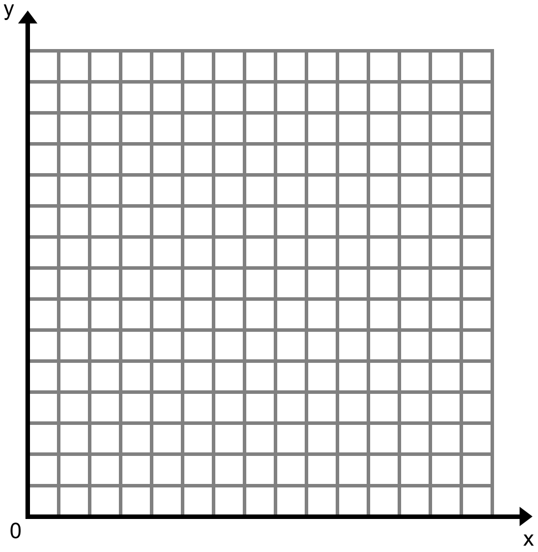


5.3d Homework: Stories and Graphs

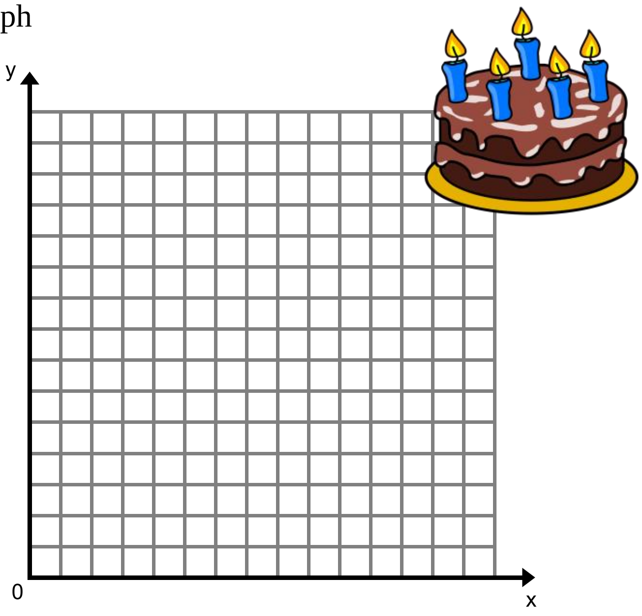
Directions: Sketch graphs to match the stories.

1. Before School

Create a graph to match the story below (distance in feet, time in minutes). (Note: This graph will show distance traveled related to time passing—consider the student to be continually moving forward.)

Story:	Graph
A student walks through the halls before school. He/she begins at the front door, stops to talk to at least three different friends, stops at his/her locker, stops in the office. Be sure to label your graph with the different pieces of information from the story.	

2. Birthday Cake
   a. Write a story about your family eating a birthday cake. You want to talk about amount of cake eaten related to passing time.
   b. Create the graph to tell the same story. You decide on the labels.

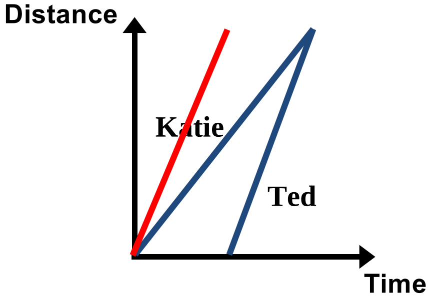
Story:	Graph
	

3. Make up stories to go with the following graphs. In this problem, distance represents the **distance from school**. Include in the stories specific details about starting points and slopes. Answer the additional questions. There are many different acceptable stories and answers.

a. Tell the story of this graph.

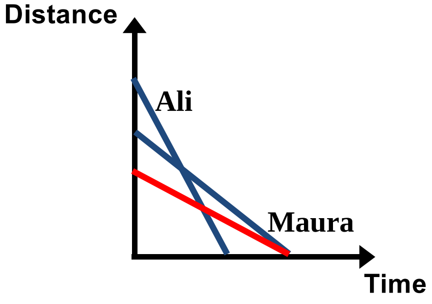
Ted leaves school after Katie. He walks at a faster rate and they meet up.

Draw and label a line that represents Izzy who started at the same time as Katie but walked away from school at a faster rate than Katie.



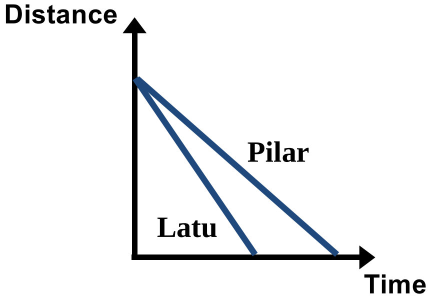
b. Tell the story of this graph.

Draw and label a line that represents Gabi who walks slower than both Ali and Maura. Lines will vary – one possible line is that she lives closer and gets there at the same time as Maura, this means she is traveling at a slower rate.



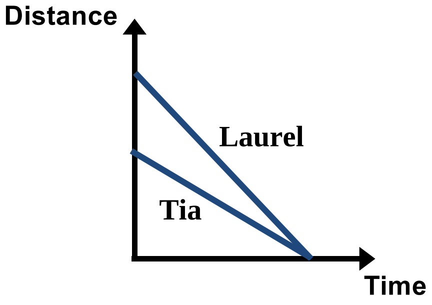
c. Tell the story of this graph.

Draw and label a line that represents Carmen who starts the same distance from school as Pilar and Latu but gets to school faster than both of them.



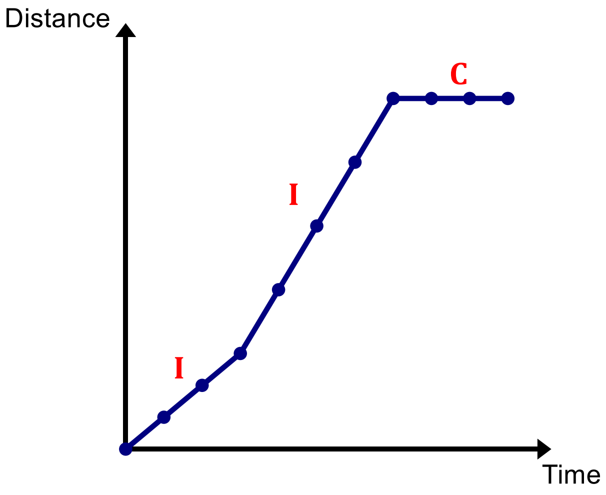
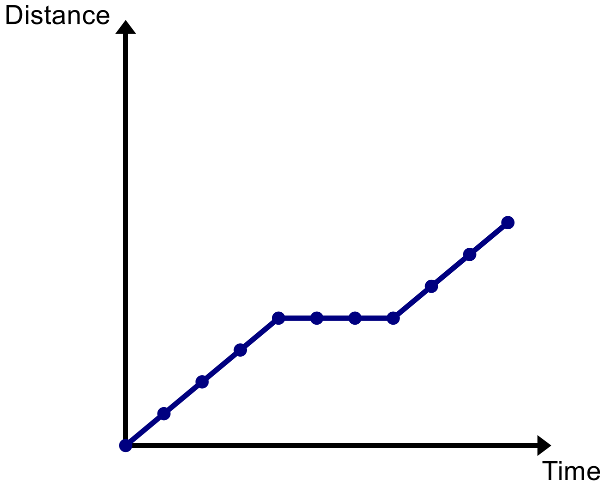
d. Tell the story of this graph.

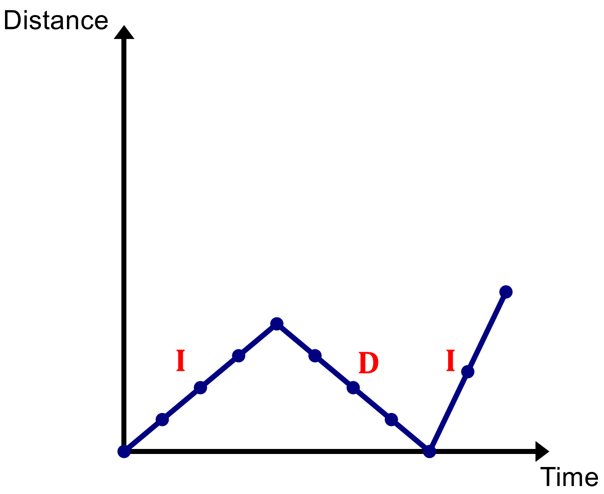
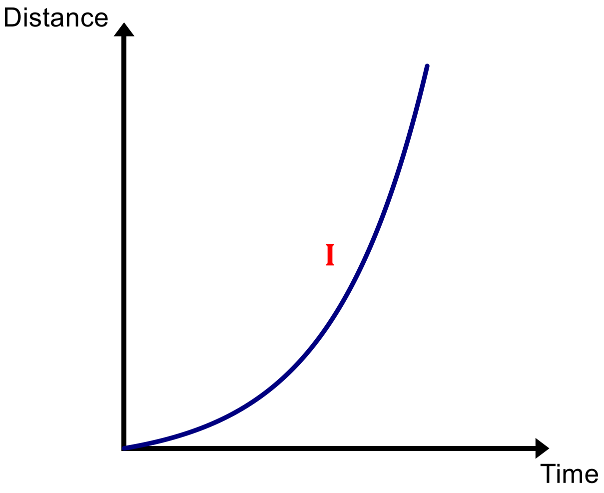
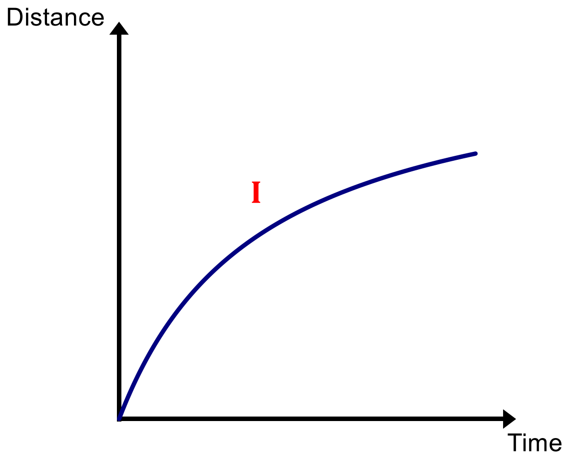
Draw and label a line that represents Colin who lives closer to school, leaves at the same time as Laurel and Tia to walk to school, and arrives at the same time as Laurel and Tia.



5.3e Class Activity: School's Out

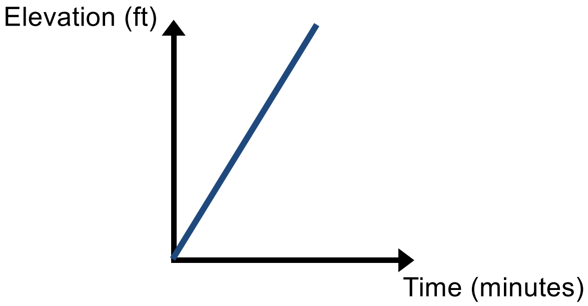
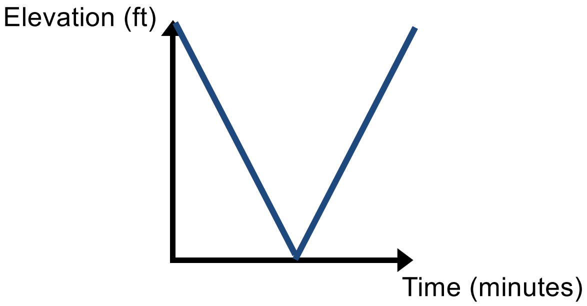
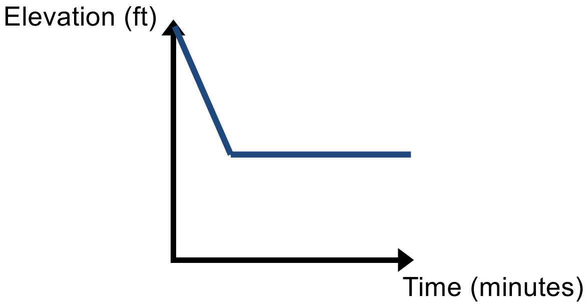
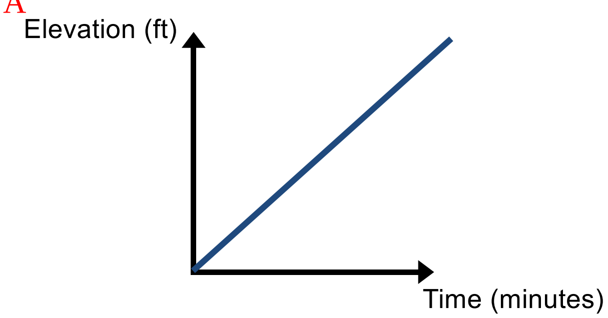
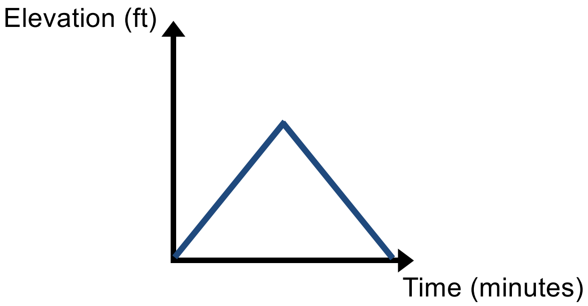
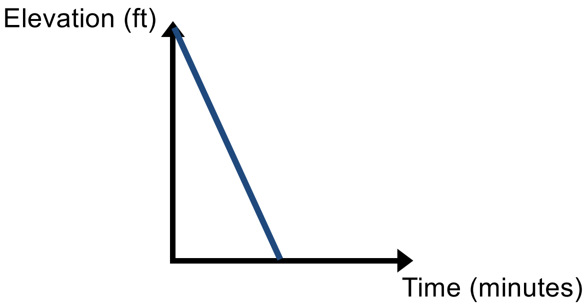
Directions: The following graphs tell the story of five different students leaving school and walking home. Label the key features of the graph. Write a story for each graph describing the movement of each of the students. Have students label the key features of the graphs (i.e. increasing, decreasing, constant, etc.)

Abby's Journey Home:  	Stories will vary but there are key features that students should include in their stories. Possible story: Abby leaves school and starts walking home at a constant rate. Then, she realizes she is late, increases her rate, and starts running home. Then, she is home (or stops somewhere). You can tell she is stopped because her distance does not change as time passes.
Beth's Journey Home:  	

Chad's Journey Home:  	Chad starts walking home. He realizes he forgot his math book and turns around and walks back to school. Realizing he is going to be late, he runs home.
Drew's Journey Home:  	Drew starts home slowly and then increases his speed on his journey home. Notice that this graph is not linear, meaning that Drew is not moving at a constant rate. Extension Question: Can Drew sustain the pace suggested by this graph?
Eduardo's Journey Home:  	Eduardo starts home quickly and then decreases his speed on his journey home. Notice that this graph is not linear, meaning that Eduardo is not moving at a constant rate.

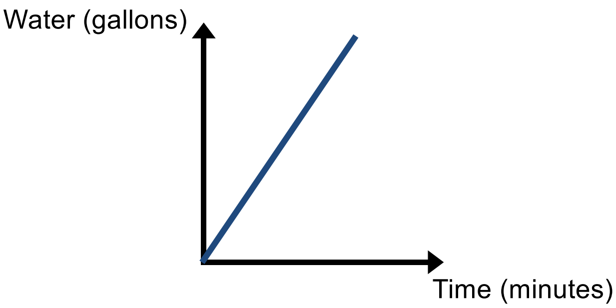
5.3e Homework: School's Out

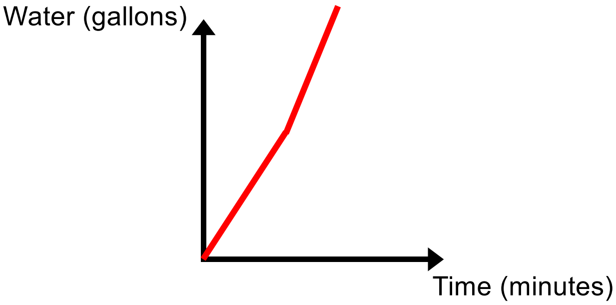
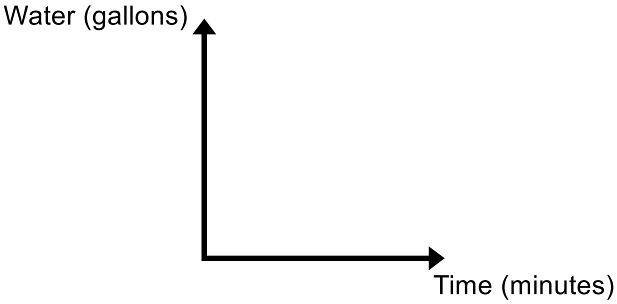
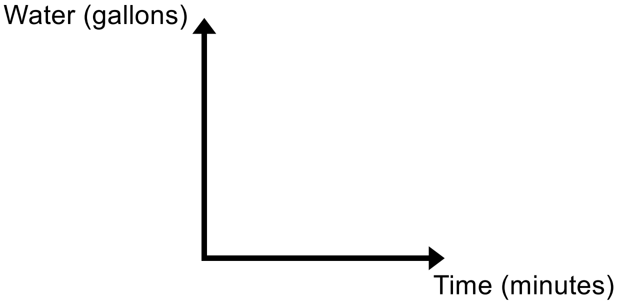
1. The graphs below show Estefan's elevation (height above the ground) over time as he is playing around on a flight of stairs. Assume the bottom of the stairs has an elevation of 0 feet. Match each story (shown below the graphs) to a graph by writing the letter of the story under each graph.

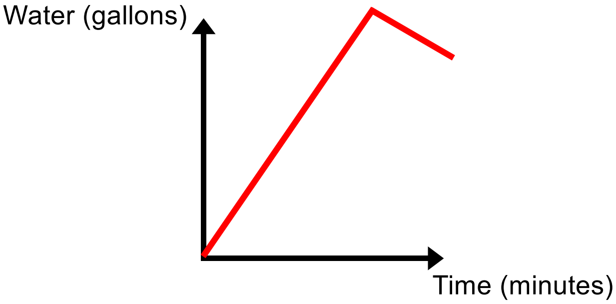
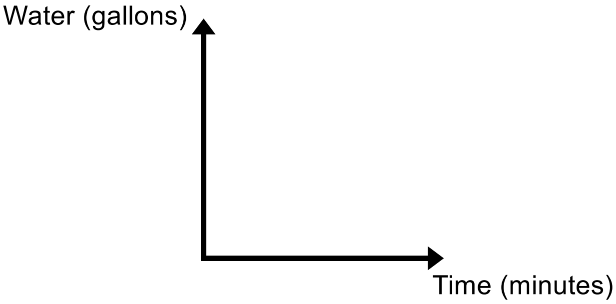
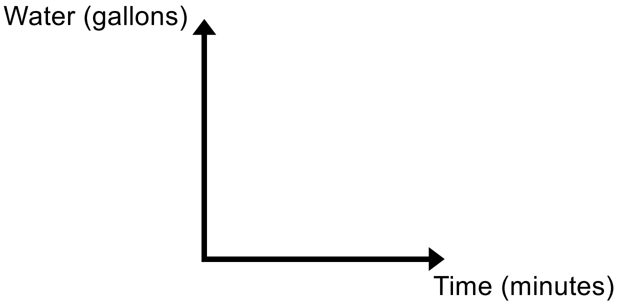
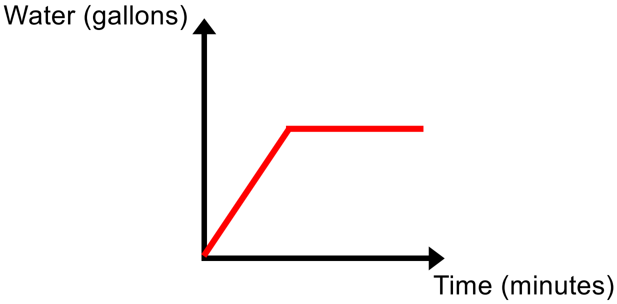
	F 
	A 
	

Story A: Estefan starts at the bottom of the stairs and walks up the stairs at a constant rate.
Story B: Estefan starts at the bottom of the stairs and sprints up the stairs at a constant rate.
Story C: Estefan starts at the bottom of the stairs, runs half-way up the stairs, turns around and runs back down the stairs.
Story D: Estefan starts at the top of the stairs and sprints down the stairs until he reaches the bottom.
Story E: Estefan starts at the top of the stairs, sprints down the stairs, and stops when he is half-way down the stairs.
Story F: Estefan starts at the top of the stairs, runs down to the bottom, turns around and runs back up to the top of the stairs.

2. The graph below tells the story of Kelii filling up her empty swimming pool with a hose at a constant rate. Create new graphs based on the changes described below.



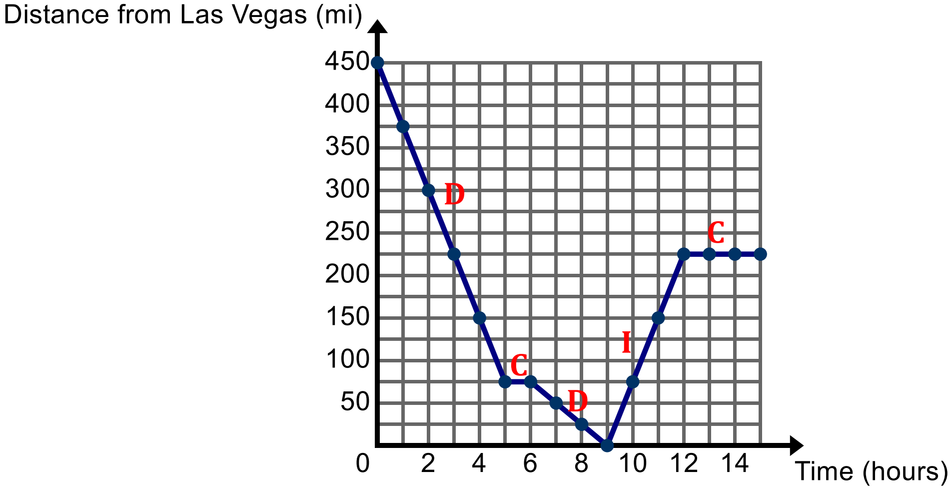
a. After Kelii has been filling the pool for a few minutes, her friend decides to help her and puts a second hose in the pool.	
b. After Kelii has been filling the pool for a few minutes, the hose gets a hole in it so the water is coming out at a slower rate.	
c. The pool started with some water in it.	

d. Kelii fills the pool up and realizes that she filled it up too much so she drains some of the water out at a constant rate.	
e. Kelii fills up the pool, her brother drops his ice cream into the pool, so she drains the water back out at a constant rate until the pool is empty.	
f. Kelii starts filling the pool, stops to go eat lunch, and then comes back out and starts filling up the pool again.	
g. Kelii fills the pool half-way and decides that is enough.	

5.3f Class Activity: From Graphs to Stories

1. Ben and his family took a road trip to visit their cousins. The graph below shows their journey. Label the key features of the graph.

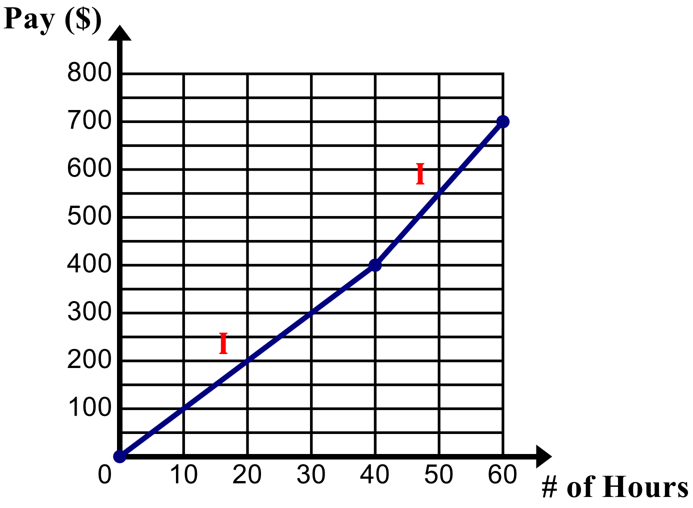
Have students label the key features of the graphs (i.e. increasing, decreasing, constant, etc.)



   a. Tell the story of the graph.

Ben and his family live 450 miles from Vegas. They are traveling toward Vegas at a constant rate of 75 mph for the first 5 hours. Then, they stop for lunch for an hour when they are 75 miles outside of Las Vegas. After lunch, they travel at a constant rate of 25 mph for 3 hours and then they are in Vegas. They pass through Vegas, driving away from Las Vegas at a constant rate of 75 mph for 3 hours. After 12 hours, they are at their cousins' house which is 225 miles from Vegas (or they have stopped somewhere to rest for the night).

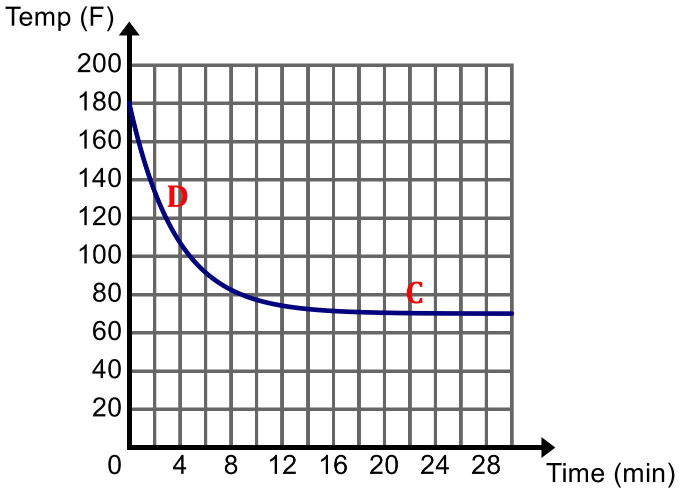
2. The graph below shows the amount Sally makes based on how many hours she works in one week. Label the key features of the graph.



   a. Tell the story of the graph.

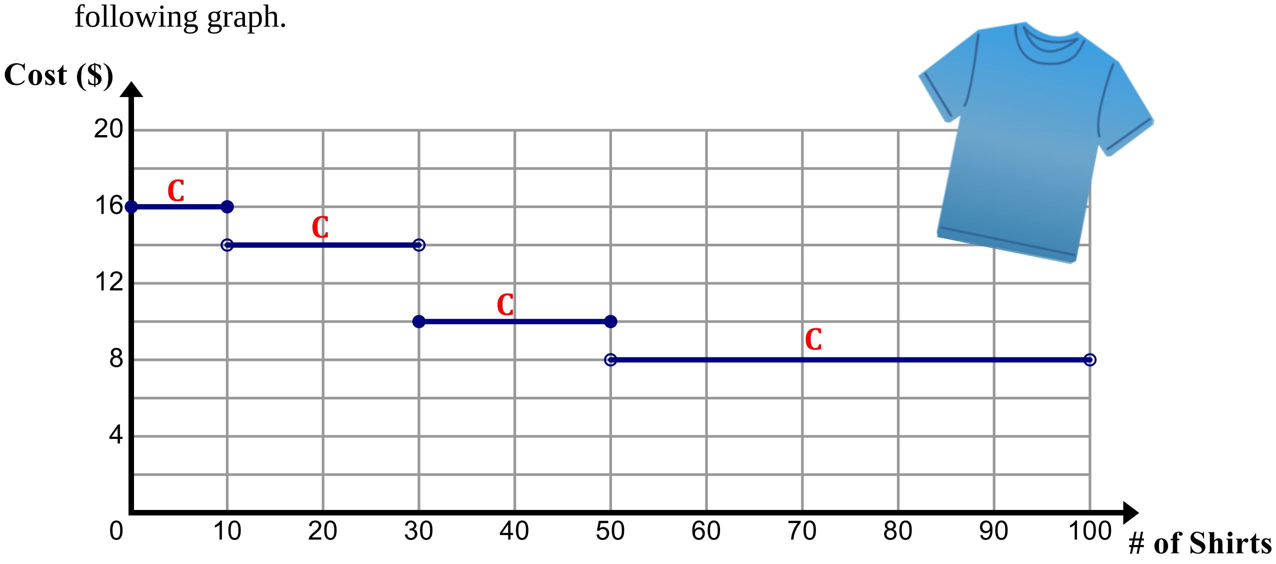
For the first 40 hours that Sally works, she gets paid $10/hr. After 40 hours, Sally gets paid $15/hr. On this particular week, Sally worked 60 hours and made $700

3. Cynthia is doing research on how hot coffee is when it is served. The graph below shows the temperature of a coffee (in ˚F) as a function of time (in minutes) since it was served. Label the key features of the graph.



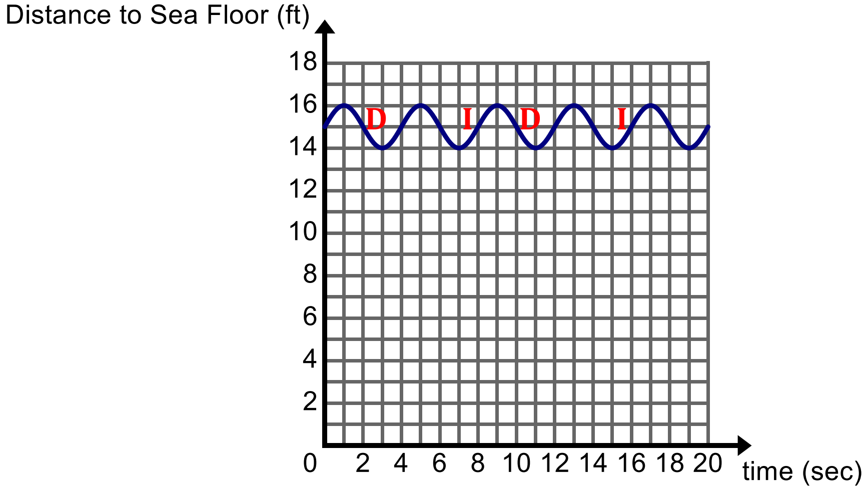
a. Tell the story of the graph.
When the coffee is served, it is 180˚ F. The temperate of the coffee drops quickly at first and then the rate at which the temperature is dropping starts to decrease. At approximately, 18 minutes, the temperature of the coffee has reached room temperature (70˚) where it stays.

4. Jorge is the team captain of his soccer team. He would like to order shirts for the team and is looking into how much it will cost. He called Custom T's to ask about pricing and the manager sent him the following graph.



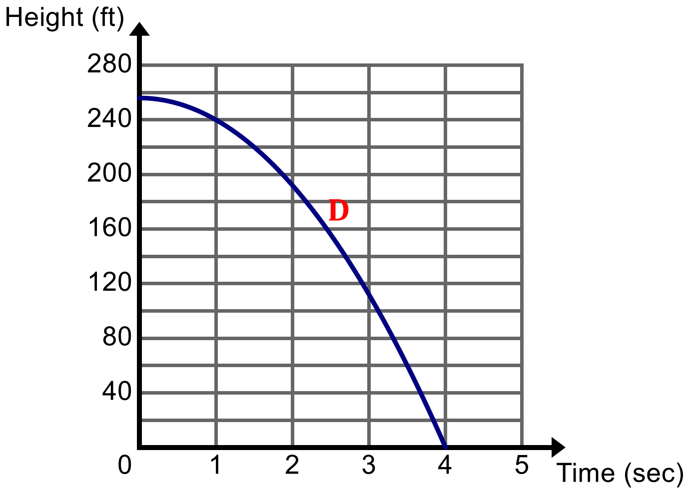
a. Tell the story of the graph.
If Jorge orders 0 – 10 shirts, he will pay $16 per shirt; 11 – 29 shirts, he will pay $14 per shirt; 30 – 50 shirts, he will pay $10/shirt; and 51 or more shirts, he will pay $8.

5. A boat is anchored near a dock. The graph below shows the distance from the bottom of the boat to the sea floor over a period of time.



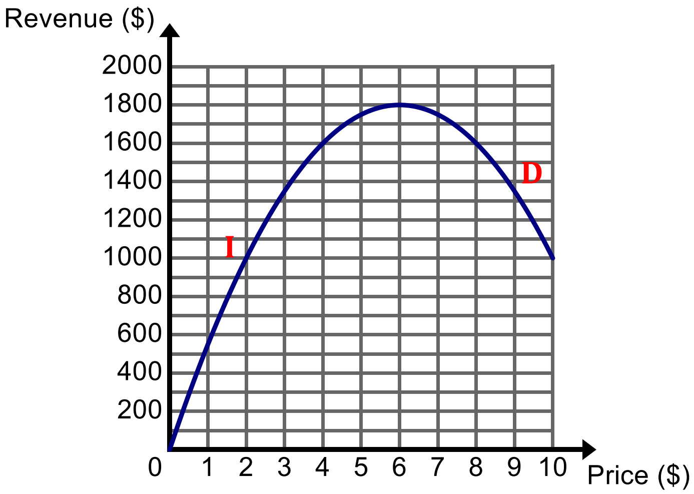
a. Tell the story of the graph.
At time 0, the bottom of the boat is 15 feet above the sea floor. After one second, the boat is 16 feet above the sea floor, then it drops down to 14 feet above the sea floor. The boat could be bobbing up and down in the water due to waves coming in.

6. An object is dropped from a bridge into the water below. The graph below shows the height of the object (in feet) with respect to time (in seconds). Consider the relationship between the height of the object and time.



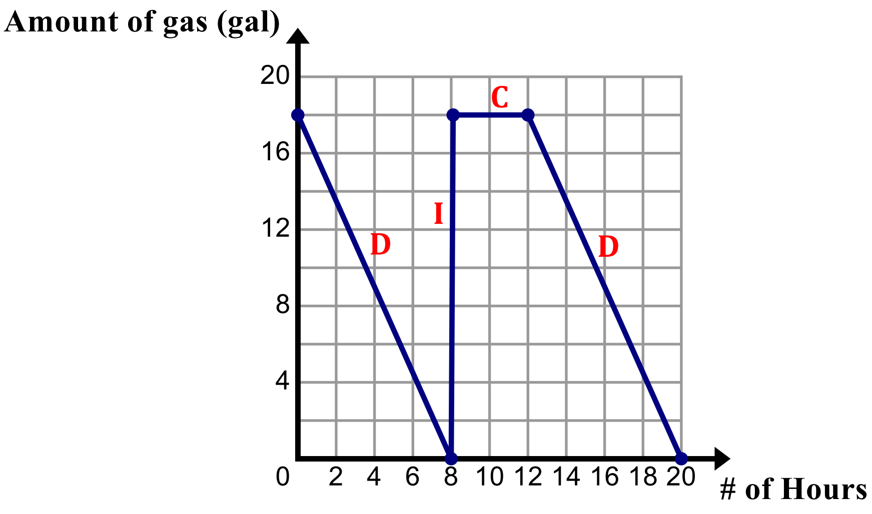
a. Tell the story of the graph.
The object was dropped from a height of approximately 258 feet. The object reaches the ground in 4 seconds. As the object falls, its speed increases (acceleration due to gravity).

7. The graph below shows the amount of revenue a company will make selling t-shirts dependent on the price of each t-shirt.



   a. Tell the story of the graph.
   The company will maximize revenue when they sell the shirts for $6. At $6, the company will make a revenue of $1800. For a price less than $6, they may sell more shirts; however because the price is lower, their revenue will also be lower. For a price greater than $6, they will likely sell fewer shirts, also making their revenue lower.

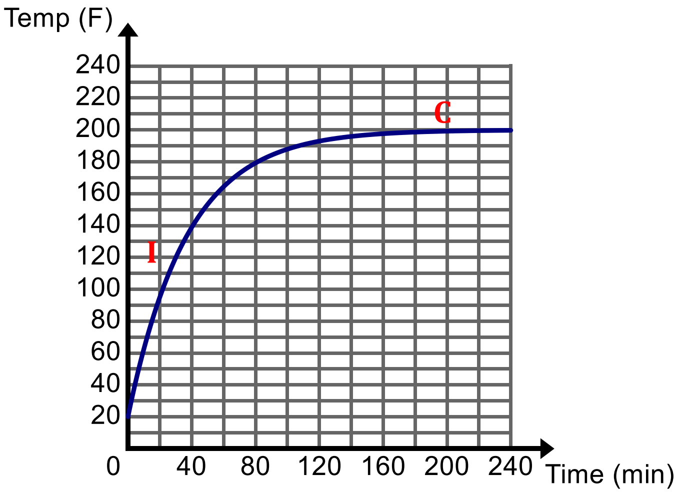
8. The graph below shows the amount of gas remaining in a vehicle over time.



   a. Tell the story of the graph.
   A car starts with 18 gallons of gas in its tank. The owner is driving the car for 8 hours and gas is being used at a rate of 2.25 gallons/hour. At 8 hours, the tank is empty. The owner fills the car with gas and lets it sit for 2 hours. Then, the owner starts driving the car again, using gas at a rate of 2.25 gallons/hour. After 8 hours of driving, it is empty again.

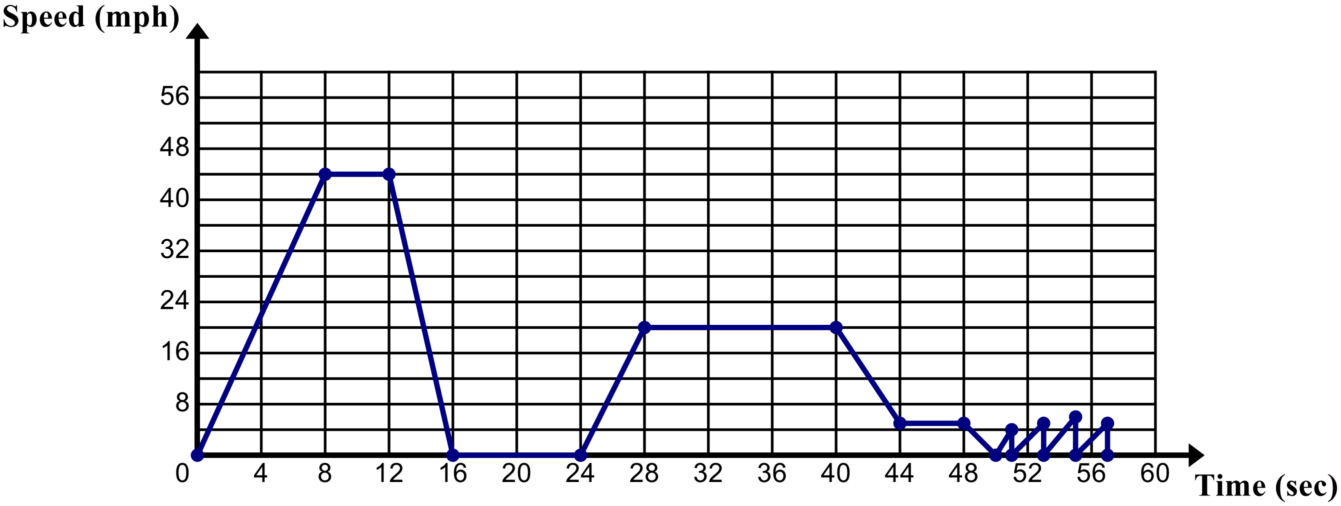
5.3f Homework: From Graphs to Stories

1. Tessa is cooking potatoes for dinner. She puts some potatoes in an oven pre-heated to 200˚ F. The graph below shows the temperature of the potatoes over time. Label the key features of the graph. The *y*-intercept of the graph is (0, 20).

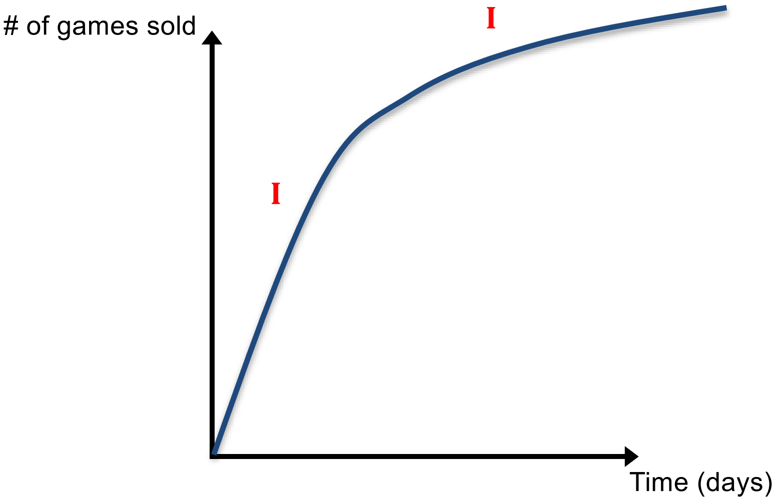


   a. Tell the story of the graph.

2. Steve is driving to work. The graph below shows Steve's speed over time. Label the key features of the graph to tell the story of the speed of Steve's car over time. Use words like accelerating, decelerating, driving at a constant speed, stopped. You can abbreviate these words using the first letter of each word (i.e. A for accelerating, D for decelerating, C for driving at a constant speed, S for stopped). Explain what might be happening at the end of the graph. Be sure to discuss how this graph is different from a time/distance graph. Many students will think that Steve is stopped anywhere they see a horizontal line. Ask, "What does it mean when your speed is not changing vs. when your distance is not changing? What does it mean to increase your speed vs. increase your distance? Decrease your speed? What does it mean when your speed is 0?" At the end of the graph, Steve is likely in stop-and-go traffic.

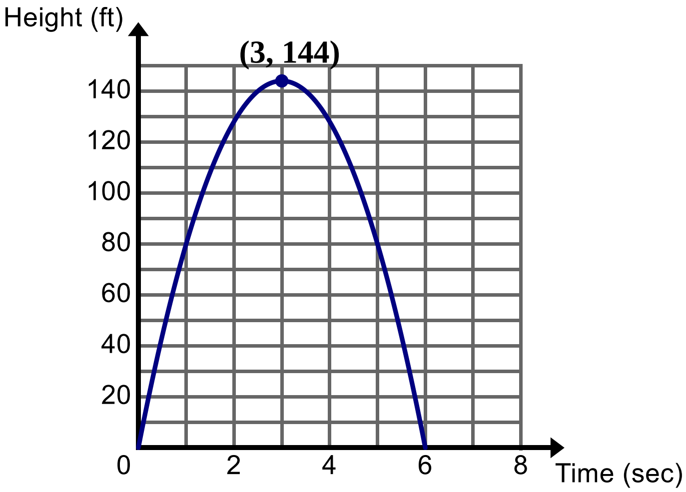


3. Microsoft is releasing the most anticipated new Xbox game of the summer. The graph below shows the total number of games sold as a function of the number of days since the game was released.



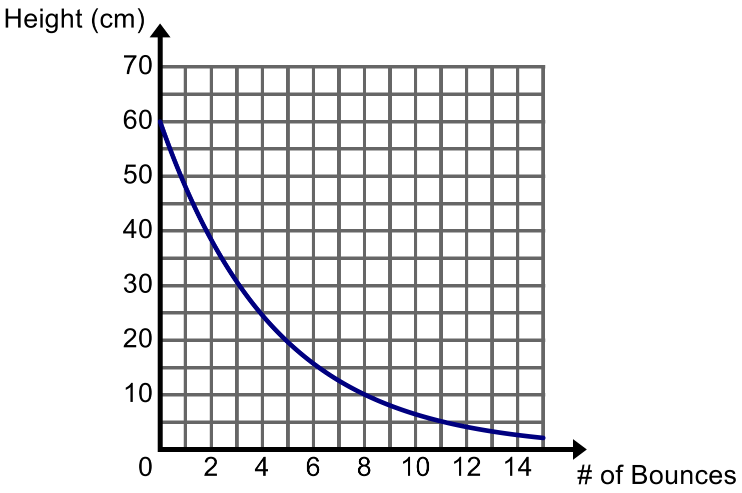
a. Tell the story of the graph.
When the game is first released, games are being sold at a high rate. As more time goes by, the rate at which the games are being sold begins to decrease. Discuss with students that this graph is always increasing, the number of games sold is increasing over time even if it is doing so at a decreasing rate.

4. A toy rocket is launched straight up in the air from the ground. It leaves the launcher with an initial velocity of 96 ft. /sec. The graph below shows the height of the rocket in feet with respect to time in seconds. Label the key features of the graph.



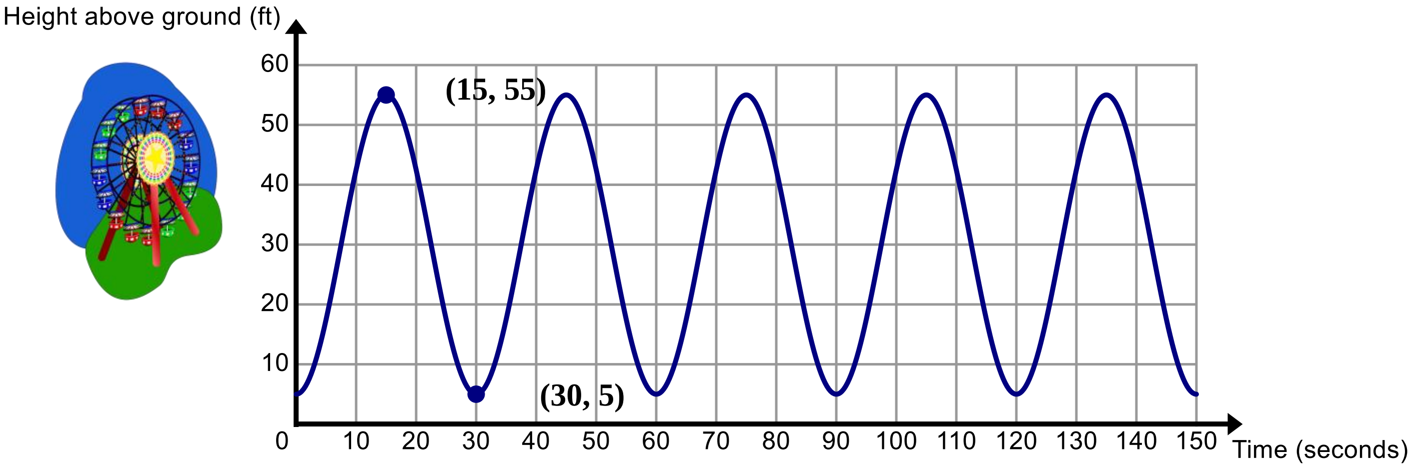
a. Tell the story of the graph.

5. Suppose you drop a basketball from a height of 60 inches. The graph below shows the height of the object after *b* bounces.



a. Tell the story of the graph.

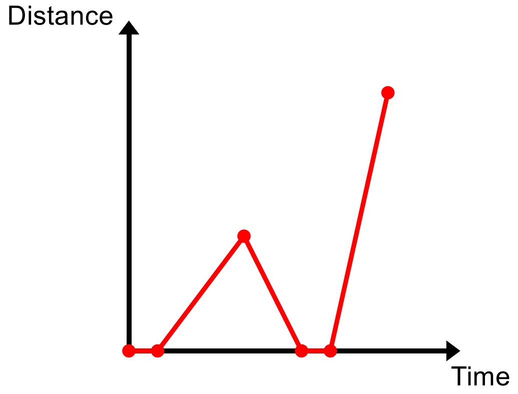
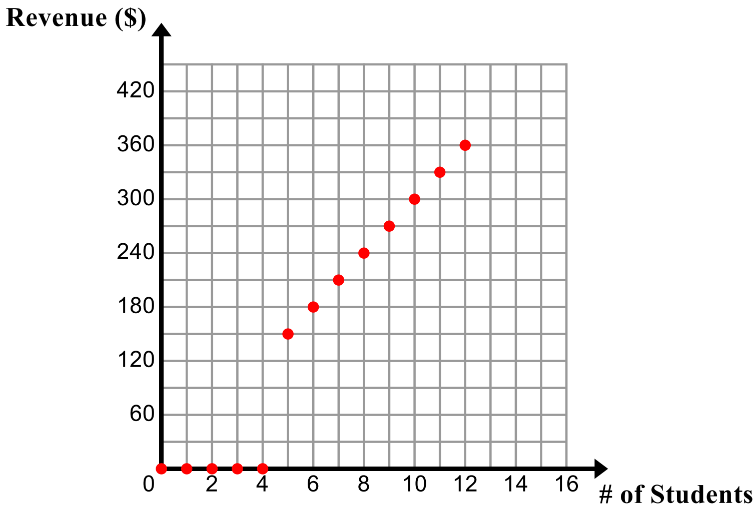
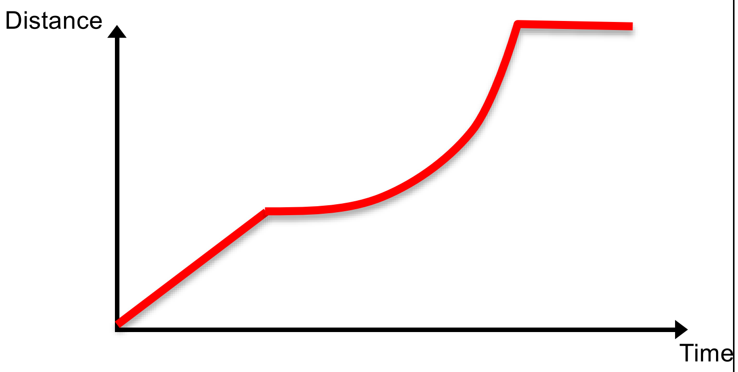
6. You are riding a Ferris wheel. The graph below shows your height (in feet) above the ground as you ride the Ferris wheel.



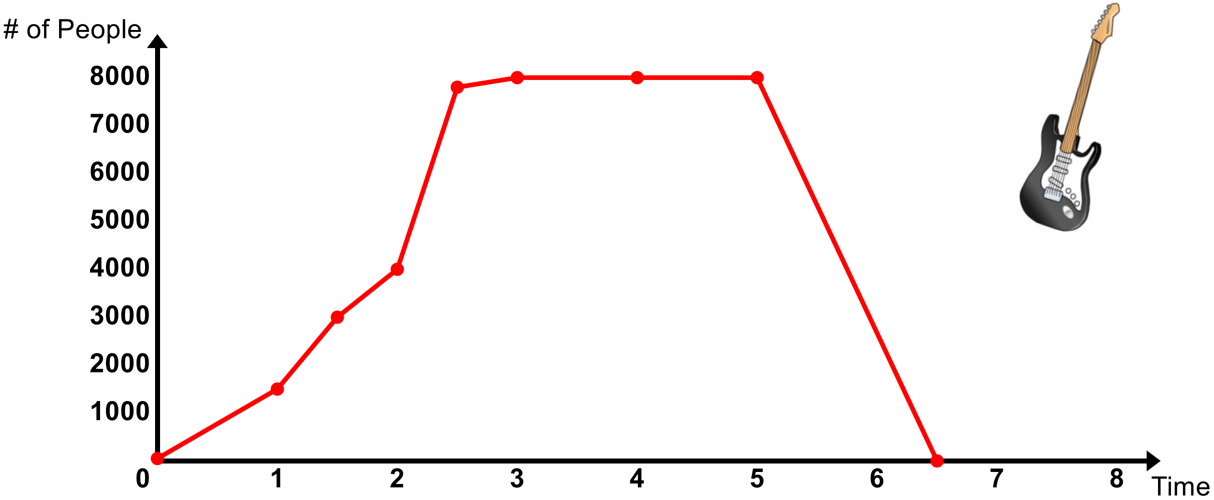
a. Tell the story of the graph.
The gondola is 5 feet off the ground. When a rider is at the top of the Ferris wheel, the rider is 55 feet off the ground. It takes 30 seconds to make one full revolution on the Ferris wheel.

5.3g Class Activity: From Stories to Graphs

Directions: Sketch a graph to match each of the following stories. Label key features of your graph.
Graphs may vary. Possible graphs are given.

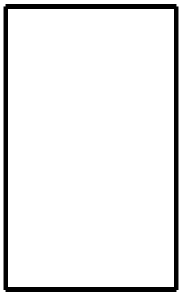
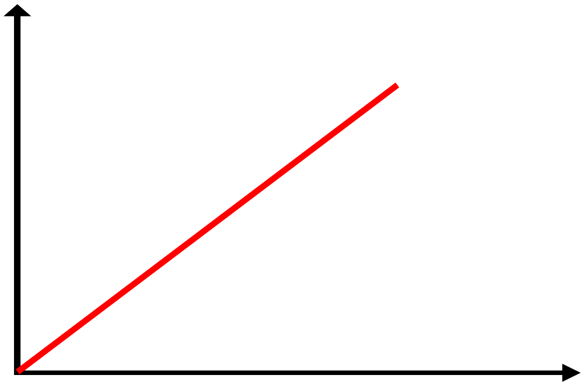
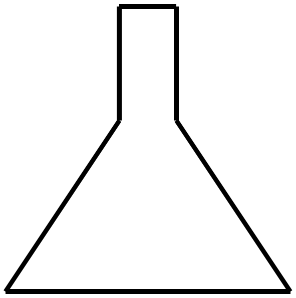
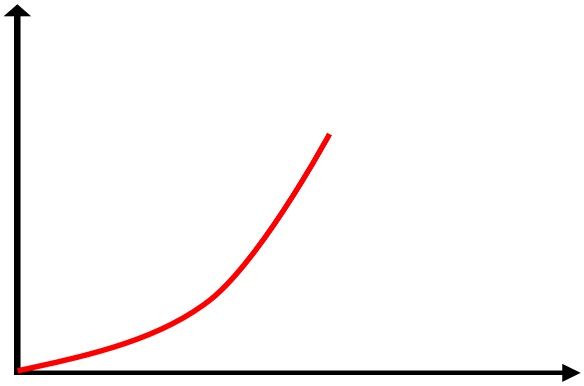
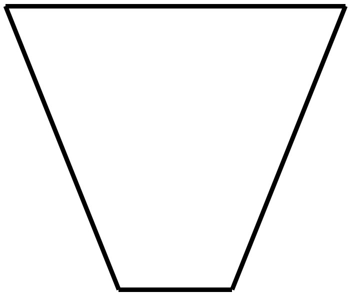
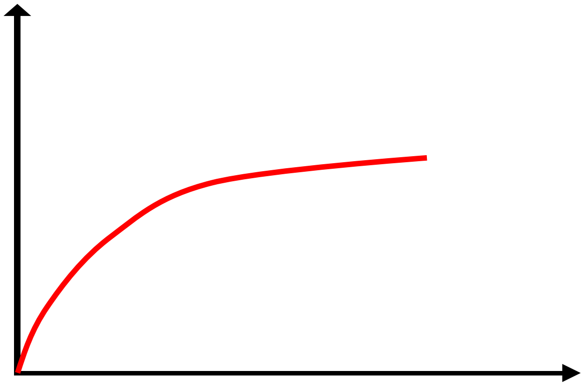
Story	Graph
1. Zach walks home from school each day. Sketch a graph of Zach's distance from school as a function of time since the bell rang if the following happens: When the bell rings, Zach runs to his locker to grab his books and starts walking home. When he is about halfway home, he realizes that he forgot his math book so he turns around and runs back to school. After retrieving his math book, he realizes that he is going to be late so he sprints home.	
2. Solitude is offering a ski clinic for teens. The cost of the class is $30 per student. A minimum of 5 students must sign up in order for Solitude to hold the class. The maximum number of students that can participate in the class is 12. Sketch a graph that shows the revenue Solitude will bring in dependent on the number of students that take the class.	
3. A biker is riding up a hill at a constant speed. Then he hits a downhill and coasts down the hill, picking up speed as he descends. At the bottom of the hill, he gets a flat tire. Sketch a graph that shows the distance traveled by the biker as a function of time.	

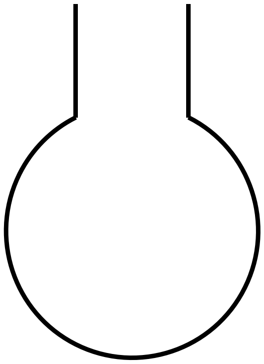
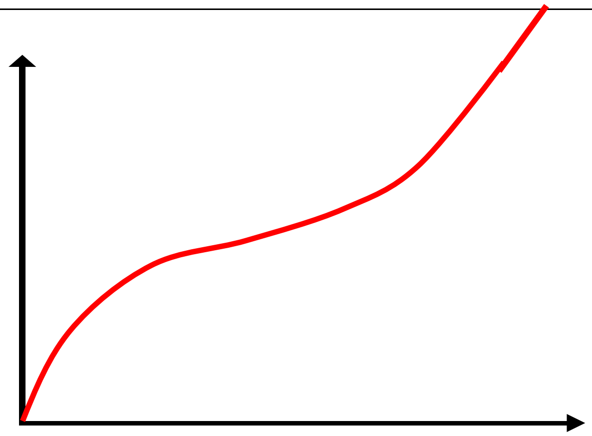
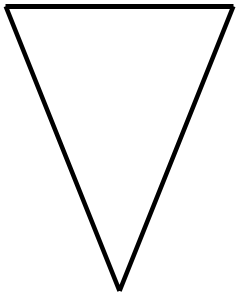
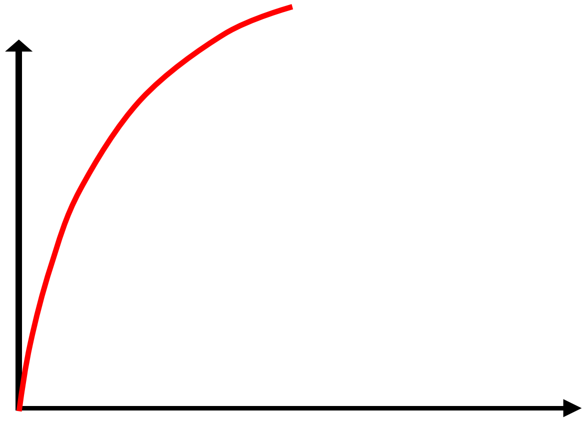
4. A concert for a popular rock group is sold out. The arena holds 8,000 people. The rock group is scheduled to take the stage at 8 pm. A band that is not very well known is opening for the rock band at 6:30 pm. The rock band is scheduled to play for 2 hours and the staff working the concert have been told that the arena must be cleared of people by 11:30 pm. Sketch a graph of the number of people in the arena from 5 pm to midnight. Time 0 on the grid below is 5 pm.



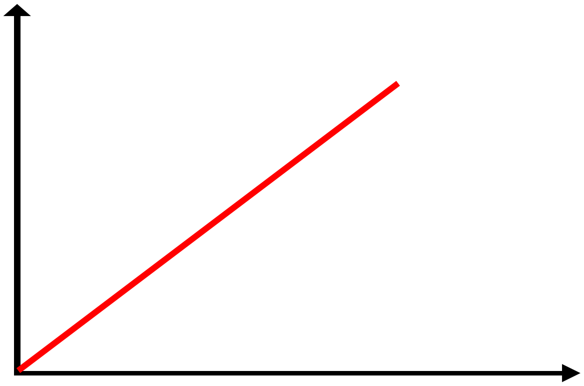
5. A parking garage charges $5 per hour and has a maximum cost of $40 for 12 hours. Sketch a graph of the total cost depending on how many hours a car is in the garage.

6. Your science teacher has the beakers shown below. He is going to fill them with water from a faucet that runs at a constant rate. Your job is to sketch a graph of the height of the water in each of the beakers over time. If students struggle, make this a matching activity by drawing the graphs on the board.

Beaker	**Graph of the height of the water over time**
 **Beaker 1**	
 **Beaker 2**	
 **Beaker 3**	

 **Beaker 4**	
 **Beaker 5**	

7. Now consider the volume of the water in each of the beakers over time. Sketch a graph of the volume of the water in each of the beakers over time.

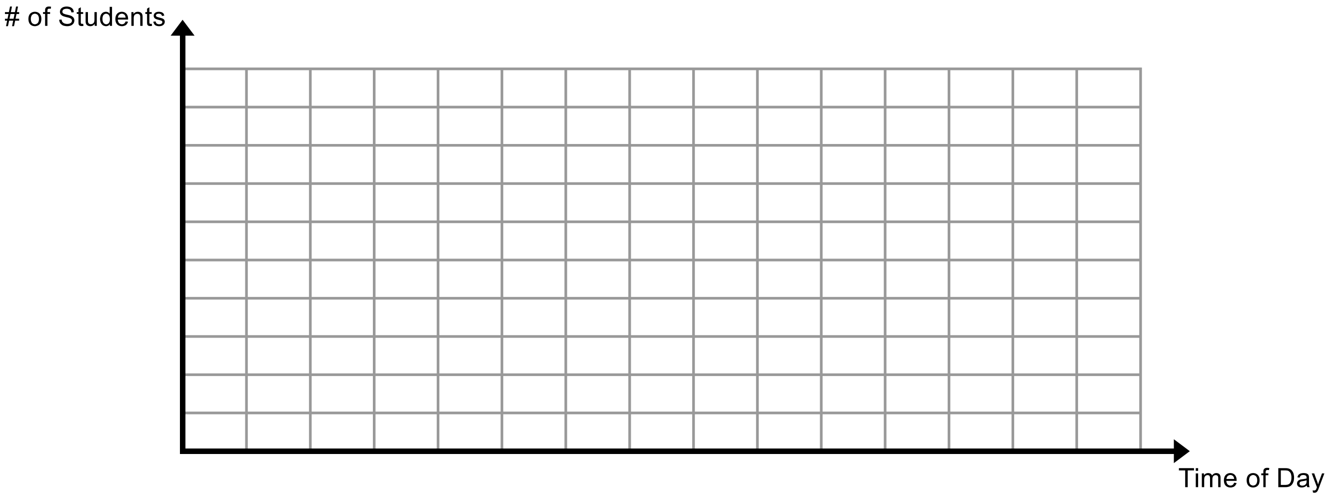


Since the faucet is flowing at a concept rate, the volume of water in all of the flasks is increasing at a constant rate over time.

5.3g Homework: From Stories to Graphs

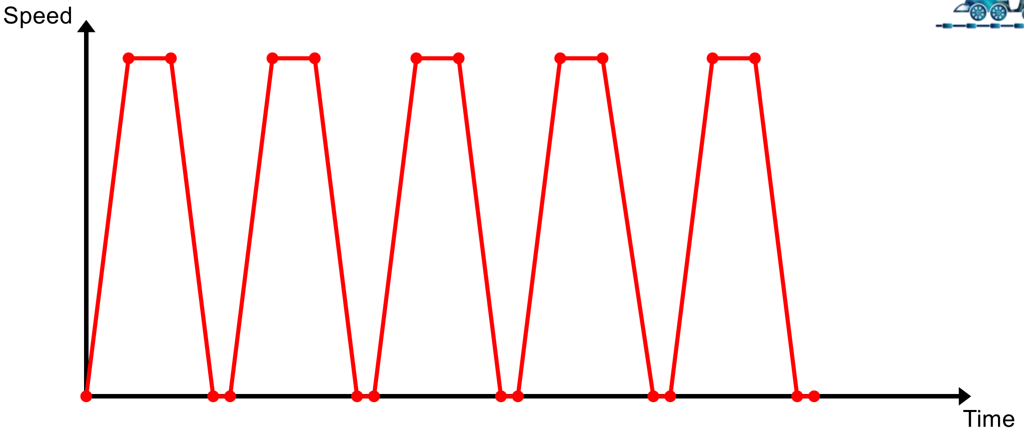
Directions: Sketch a graph for each of the stories below.

1. Sketch a graph of the number of students in the cafeteria as a function of time throughout the school day at your school. Tell the story of your graph.
   Graphs will vary depending on your school's start time, lunch times, end time, etc.

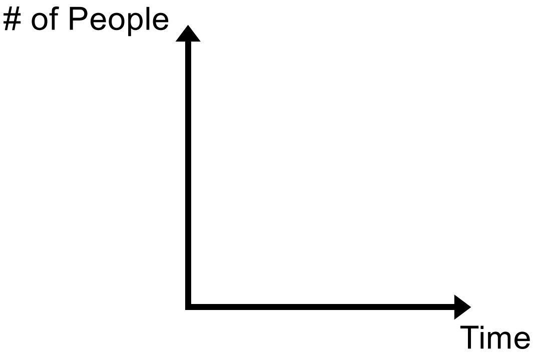


2. Two thousand, five hundred students attend a local high school. School starts at 8 am and ends at 2:30 pm. Many students stay after school for clubs, sports, etc. The school has a one-hour lunch at noon and seniors are allowed to leave campus for lunch. Sketch a graph of the number of cars in the student parking lot from 6 am to 4 pm. Time 0 on the grid below is 6 am.

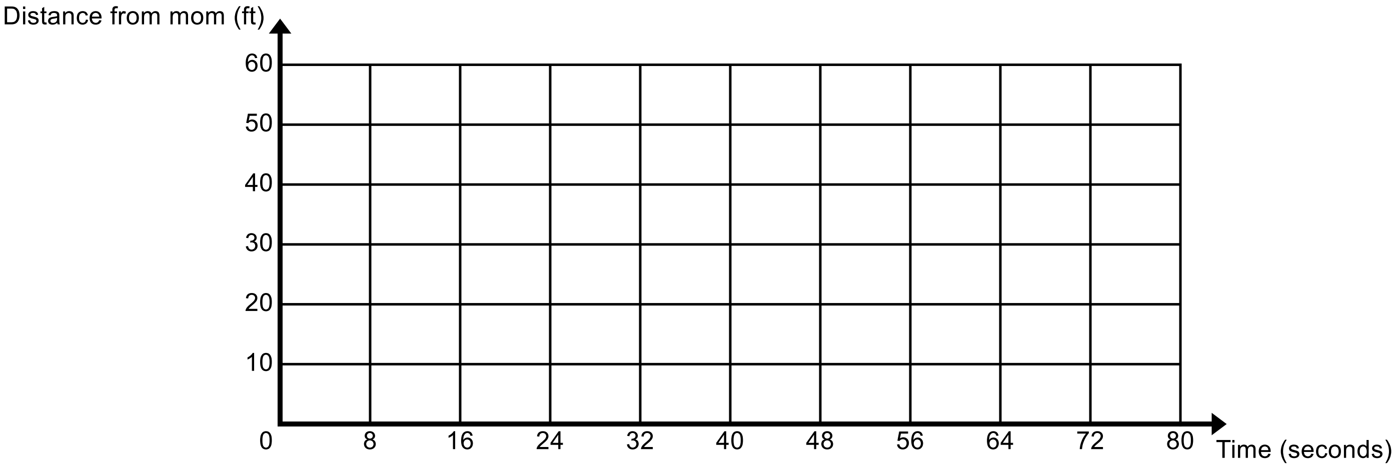
3. A train that takes passengers from downtown back home to the suburbs makes 5 stops. The maximum speed at which the train can travel is 40 mph. Sketch a graph of the speed of the train a function of time since leaving the downtown train station.



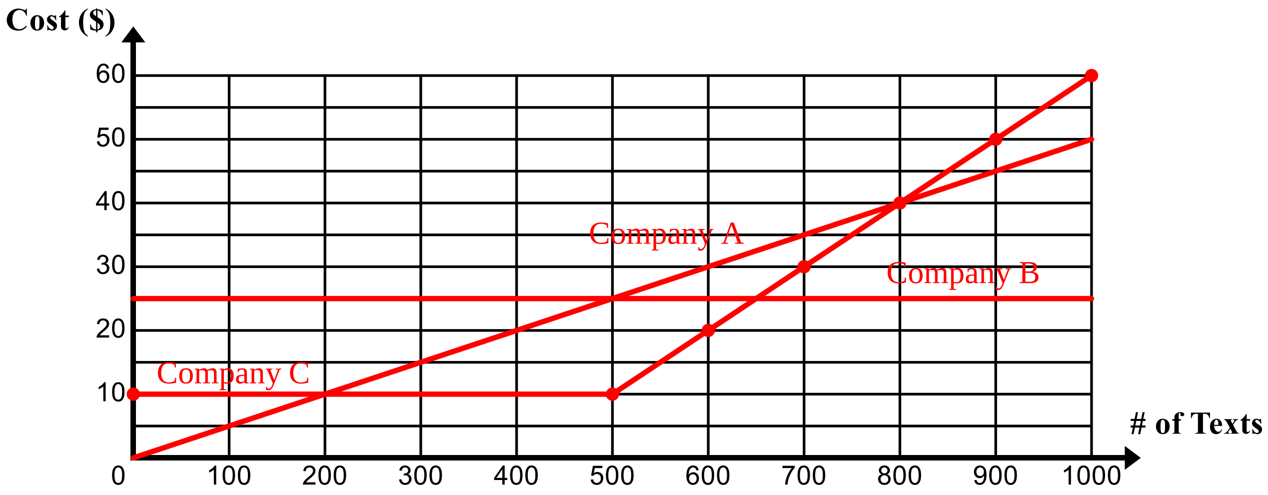
4. Sketch the graph of the total number of people that have seen the hit movie of the summer as a function of the time since opening day of the movie.



5. A little girl is going around on a merry-go-round. Her mom is standing at the entrance to the ride. Sketch a graph of the distance the little girl is from her mom as she goes around if the minimum distance she is from her mom during the ride is 5 feet and the maximum distance she is from her mom is 45 feet. Assume it takes 16 seconds to make one full revolution on the merry-go-round.



6. Yvonne is researching cell phone plans. Company A offers charges $0.05 for each text message sent. Company B offers unlimited texting for $25 per month. Company C charges $10 per month for up to 500 text messages and an additional $0.10 for each text message over 500. Sketch and label a graph that shows the relationship between number of texts sent and total monthly cost for each of the plans.



5.3h Self-Assessment: Section 5.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Mastery 3**	**Substantial Mastery 4**
1. Determine whether the relationship between two quantities can be modeled by a linear function. Construct a function to model a linear relationship between two quantities.				
2. Compare properties of linear functions (rates of change and intercepts) and use this information to solve problems.				
3. Identify and interpret key features of a graph that models a relationship between two quantities.				
4. Sketch a graph that displays key features of a function that has been described verbally.				

For use with skill/concept #1

1. Which of the following representations/situations can be modeled by the function $y = 2x + 10$? Circle all that apply.
    a. A pool has 10 gallons of water in it and water is being added to the pool at a rate of 2 gallons per minute.
    b. A pool has 2 gallons of water in it and water is being added to the pool at a rate of 10 gallons per minute.
    c. There are 10 bacterium in a petri dish. Each hour, the number of bacteria in the dish doubles.
    d. There are currently 10 shoes on the shelf in a store. The owner is adding boxes with pairs of shoes inside to the shelf.
    e. Penny has 10 pennies in a jar. Each day, she adds 2 pennies to the jar.

2. Create 3 different representations (a table, graph, and context) that can be modeled by the function $y = 4x$.

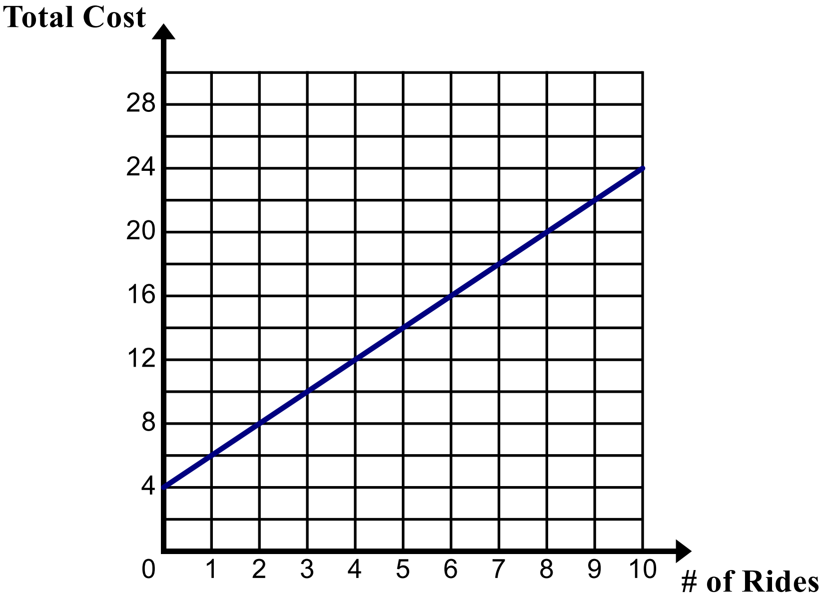
3. Circle the letter of the representations that can be modeled by a linear function. Construct a linear function for those that are linear.

   a.

Radius *(in)*	Area *(in²)*
1	3.14
2	12.56
3	28.26
4	50.24
5	78.50

   b. The cost of a frozen yogurt at Callie's Custard Shop is \$4.50. Each additional topping is \$0.25.

   c. The graph below shows the total cost dependent on the number of rides taken.



   d. Nick receives a 3% raise every year.

   e. A plane starts is descent from an elevation of 35,000 feet. The table below shows the elevation of the plane as it is descending.

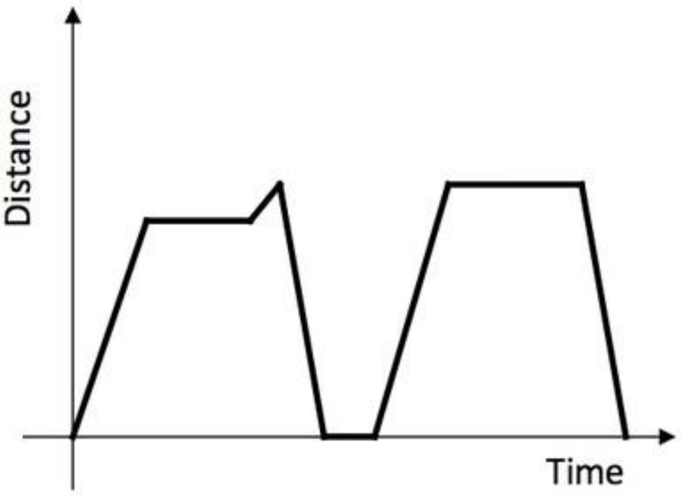
Time (min.)	Elevation (ft.)
0	35,000
3	27,500
5	22,500
6	20,000

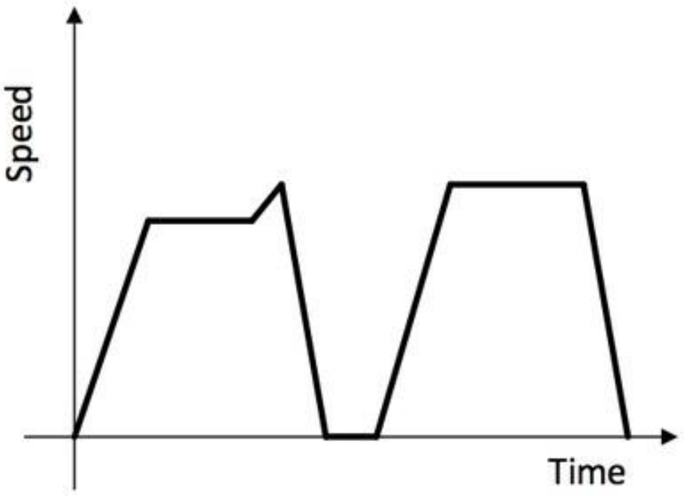
For use with skill/concept #2

1. Maya and her brother each brought a seedling plant home from the store. The plants are both growing at a constant rate. Maya's plant was 8 cm. tall 2 weeks after she brought it home and 20 cm. tall 8 weeks after she brought it home. The height h of her brother's plant in centimeters t weeks after he brought it home can be modeled by the equation $h = \frac{3}{2}t + 6$. Which plant is growing at a faster rate? Which plant was taller when they brought the plants home?

For use with skill/concept #3

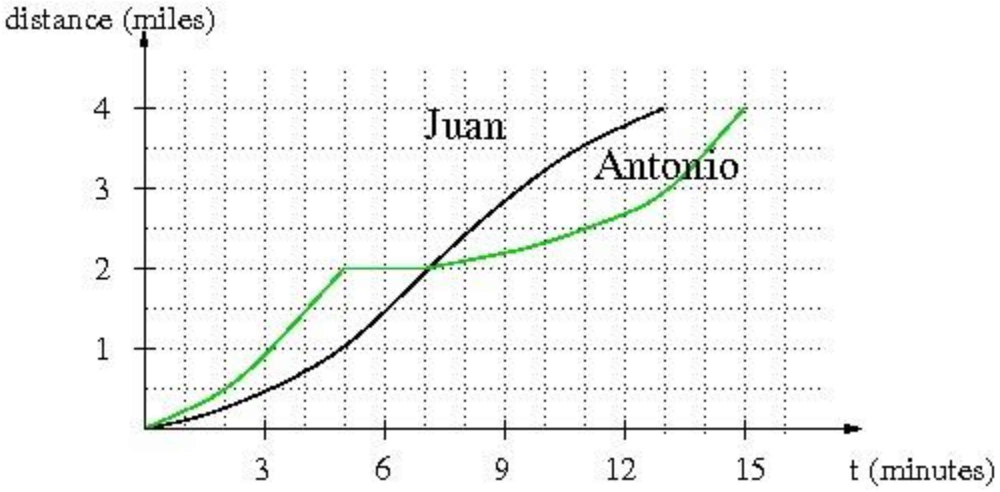
1. Below are two graphs that look the same. Note that the first graph shows the distance of a car from home as a function of time and the second graph shows the speed of a different car as a function of time. Describe what someone who observes the car's movement would see in each case.





This is an Illustrative Mathematics Task: https://www.illustrativemathematics.org/illustrations/632

2. Antonio and Juan are in a 4-mile bike race. The graph below shows the distance of each racer (in miles) as a function of time (in minutes).

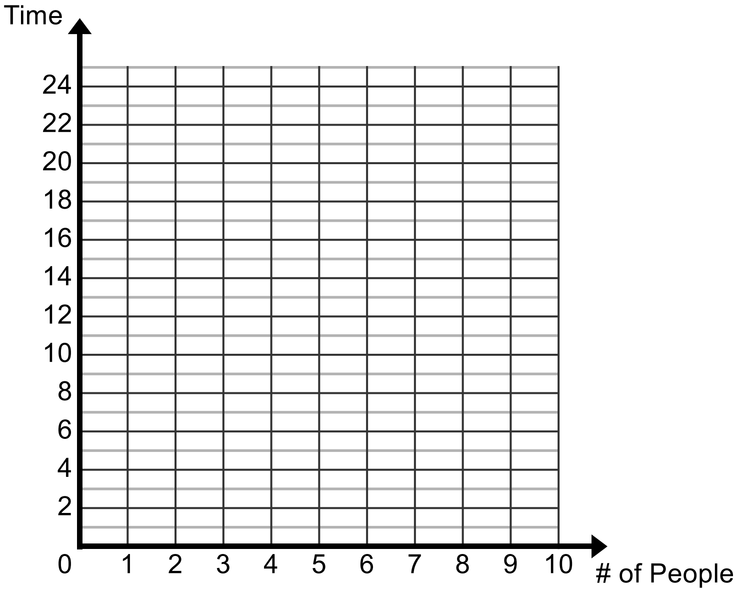


   a. Who wins the race? How do you know?
   b. Imagine you were watching the race and had to announce it over the radio. Write a little story describing the race.

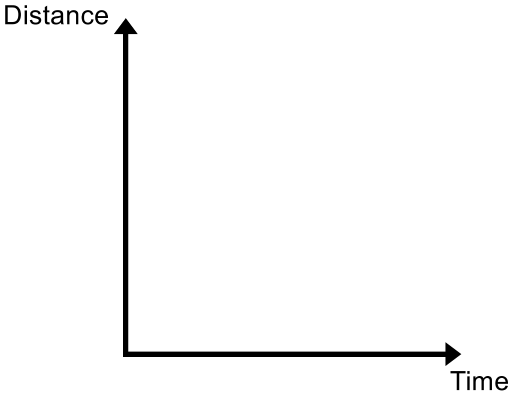
This is an Illustrative Mathematics Task: https://www.illustrativemathematics.org/illustrations/633

For use with skill/concept #4

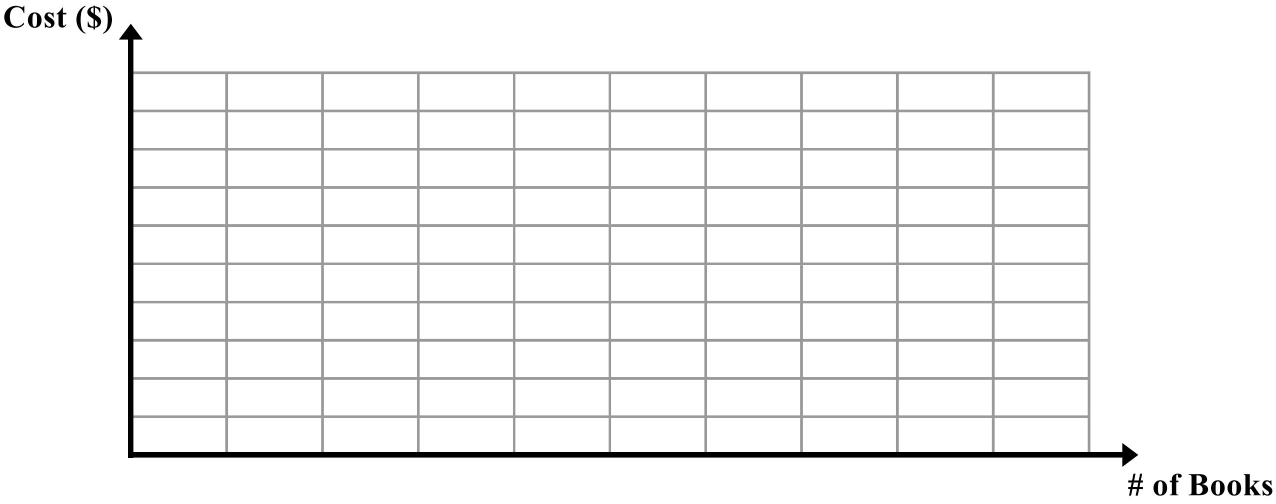
1. It will take Rick 24 hours to paint a fence in his backyard. Rick is trying to get some friends to help him paint the fence. Sketch a graph of the amount of time it will take to paint the fence dependent on how many friends Rick gets to help.



2. Sketch a graph of Carrie's distance from home. Carrie starts at home, walks to the neighbors to play, stays at the neighbors to play, then runs home.



3. Sketch the graph of the total cost of ordering books dependent on the number ordered given the following criteria: it costs \$110 per book if you order 0 – 50 books, \$90 if you order 51 – 100 books, and \$75 if you order more than 100 books.



4. Sketch a graph of your energy level during the day from the time you wake up until the time you go to sleep at night. Label key features and events of the day.
5. Sketch a graph of the distance the second hand of a clock is from the number 6 as it moves around the clock.