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Chapter 5: Functions (3 weeks)

Utah Core Standard(s):

- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (8.F.1)
- Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line. (8.F.3)
- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (8.F.4)
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. (8.F.2)
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (8.F.5)

Academic Vocabulary: function, input, output, relation, mapping, independent variable, dependent variable, linear, nonlinear, increasing, decreasing, constant, discrete, continuous, intercepts

Chapter Overview: In this chapter, the theme changes from that of solving an equation for an unknown number, to that of "function" that describes a relationship between two variables. Students have been working with many functional relationships in previous chapters; in this chapter we take the opportunity to formally define function. In a function, the emphasis is on the relationship between two varying quantities where one value (the output) depends on another value (the input). We start the chapter with an introduction to the concept of function and provide students with the opportunity to explore functional relationships algebraically, graphically, numerically in tables, and through verbal descriptions. We then make the distinction between linear and nonlinear functions. Students analyze the characteristics of the graphs, tables, equations, and contexts of linear and nonlinear functions, solidifying the understanding that linear functions grow by equal differences over equal intervals. Finally, students use functions to model relationships between quantities that are linearly related. Students will also describe attributes of a function by analyzing a graph and create a graphical representation given the description of the relationship between two quantities.

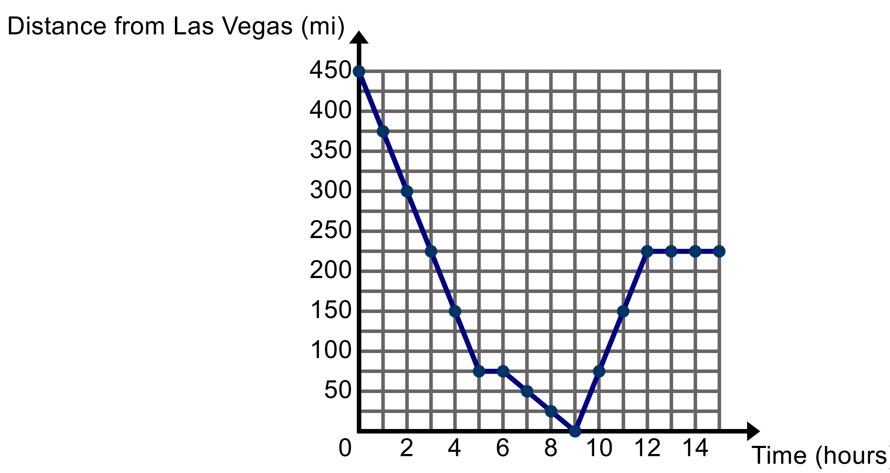
Connections to Content:

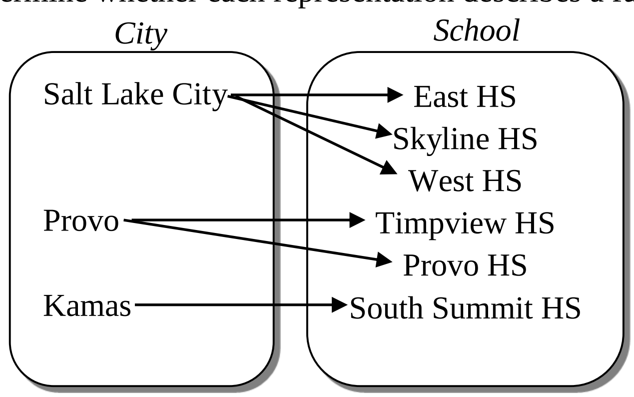
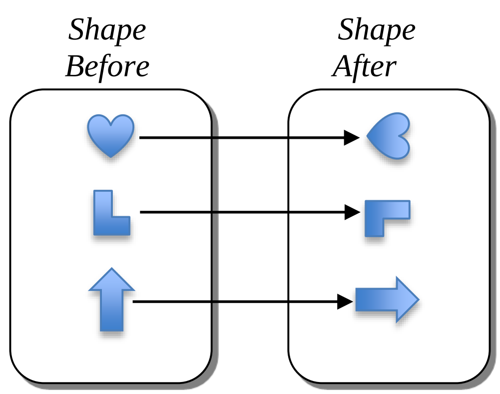
<u>Prior Knowledge</u>: Up to this point, students have been working with linear equations. They know how to solve, write, and graph equations. In this chapter, students make the transition to function. In the realm of functions, we begin to interpret symbols as variables that range over a whole set of numbers. Functions describe situations where one quantity determines another. In this chapter, we seek to understand the relationship between the two quantities and to construct a function to model the relationship between two quantities that are linearly related.

<u>Future Knowledge</u>: This chapter builds an understanding of what a function is and gives students the opportunity to interpret functions represented in different ways, identify the key features of functions, and construct functions for quantities that are linearly related. This work is fundamental to future coursework where students will apply these concepts, skills, and understandings to additional families of functions.

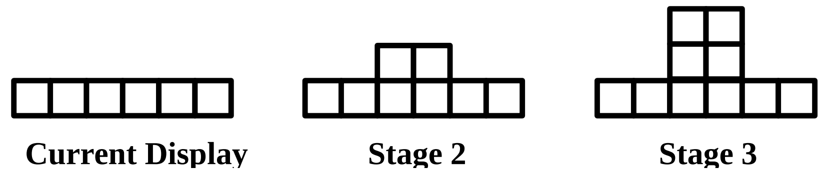
MATHEMATICAL PRACTICE STANDARDS

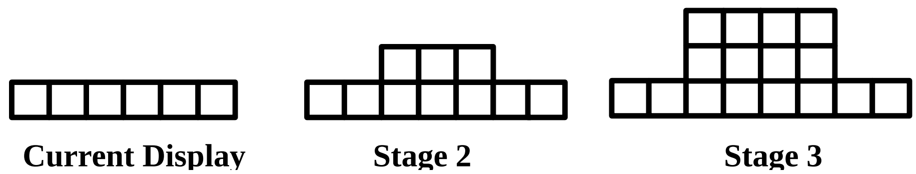
	Make sense of problems and persevere in solving them.	On Tamara's first day of math class, her teacher asked the students to shake hands with everyone in the room to introduce themselves. There are 26 students total in the math class. Can you determine the number of handshakes that took place in Tamara's math class on the first day of class? Can the relationship between number of students and the number of handshakes exchanged be modeled by a linear function? Justify your answer. *As students grapple with this problem, they will start to look for entry points to its solution. They may consider a similar situation with fewer students. They may construct a picture, table, graph, or equation. They may even act it out, investigating the solution with a concrete model. Once they have gained entry into the problem, students may look for patterns and shortcuts that will help them to arrive at a solution either numerically or algebraically.*
	Reason abstractly and quantitatively.	Nazhoni has completed her Driver's Education Training and is at the DMV (Division of Motor Vehicles) waiting in line to get her license to drive. She entered the DMV at 12:50 and pulled a number 17 to reserve a spot in line. Nazhoni notices that all of the employees at the DMV are still at lunch when she arrives. Once the employees return they start with number 1. There is digital sign showing the number for the person who is at the counter being helped. Nazhoni jots down some information on a piece of scratch paper as she is waiting in line. #5 was called to the counter at 1:25 pm #10 was called to the counter at 2:00 pm I have to leave by 2:45 pm in order to pick up my sister from school on time. Will Nazhoni make it to the front of the line in time to pick up her sister from school? *In order to solve this problem, students must make sense of the quantities involved in this situation and the relationship between the quantities. Students may first investigate this problem numerically, determining the average wait time between each person called to the counter. Students may also abstract this situation and construct a function to model the amount of time Nazhoni will have to wait based on the number she draws.*

	Construct viable arguments and critique the reasoning of others.	Compare and contrast the relationship of the gumball machines at Vincent Drug and Marley's Drug Store. If needed revise your conjecture about what kind of relationship makes a function and what disqualifies a relationship from being a function. *As students create, modify, and formulate their definition of a function they are constructing a viable argument that describes their thoughts on what a function is and what it is not. They make conjectures and build a logical progression of statements to explore the truth about their conjectures. They can share their definitions with others and decide whether they make sense and compare others' thoughts and ideas to their own.*
	Model with mathematics.	*Throughout this chapter, students will apply the mathematics they have learned to solve problems arising in everyday life, society, and workplace. The following problems give students the opportunity to use functions to model relationships between two quantities.* Steve is a lifeguard at a local community pool. Each day at noon, he records the temperature and the number of people in the pool. Do you think the relationship between temperature and number of people in the pool is linear? Why or why not? Two thousand, five hundred students attend a local high school. School starts at 8 am and ends at 2:30 pm. Many students stay after school for clubs, sports, etc. The school has a one-hour lunch at noon and seniors are allowed to leave campus for lunch. Sketch a graph of the number of cars in the student parking lot from 6 am to 4 pm. Ben and his family took a road trip to visit their cousins. The graph below shows their journey. Label the key features of the graph. Write a story about the graph. 

	Use appropriate tools strategically.	**Directions:** Determine whether the situations you choose can be modeled by a linear function or not. Provide evidence to support your claim. Show your work in the space below. Mr. Cortez drove at a constant rate for 5 hours. At the end of 2 hours he had driven 90 miles. After 5 hours, he had driven 225 miles. Can the relationship between time and distance driven be modeled by a linear function? Provide evidence to support your claim. Round 1 of a tennis tournament starts with 64 players. After each round, half the players have lost and are eliminated from the tournament. Therefore, in round 2 there are 32 players, in round 3 there are 16 players and so on. Can the relationship between round number and number of players remaining be modeled by a linear function? Provide evidence to support your claim. *The first step in constructing a function to model the relationship between two quantities is to determine what type of model is a potential fit for the data. At this point, student knowledge of the rate of change of a linear function is a tool the students rely on to determine whether the relationship between two quantities can be modeled by a linear function.*
	Attend to precision.	Determine whether each representation describes a function.   Is letter grade a function of percentage scored on a test? *In order to determine whether or not a given representation describes a function, students must be precise in their understanding of what a function is.*

<table>
<tr>
<td></td>
<td>Look for and make use of structure.</td>
<td>Examine the patterns below. Can the relationship between stage number and number of blocks in a stage be modeled by a linear function? Provide at least 2 pieces of evidence to support your answer.





While examining the patterns above, students may see that a linear pattern exhibits growth in one direction while the second pattern shown exhibits growth in two directions. These geometric representations give insight into the structure of a linear equation (and a quadratic equation which will be studied in subsequent courses).

Circle the letter next to each equation if it represents a linear function.
$2x + 4y = 16$ $y = x^2 + 5$
$y = x(x + 2)$ $xy = 24$

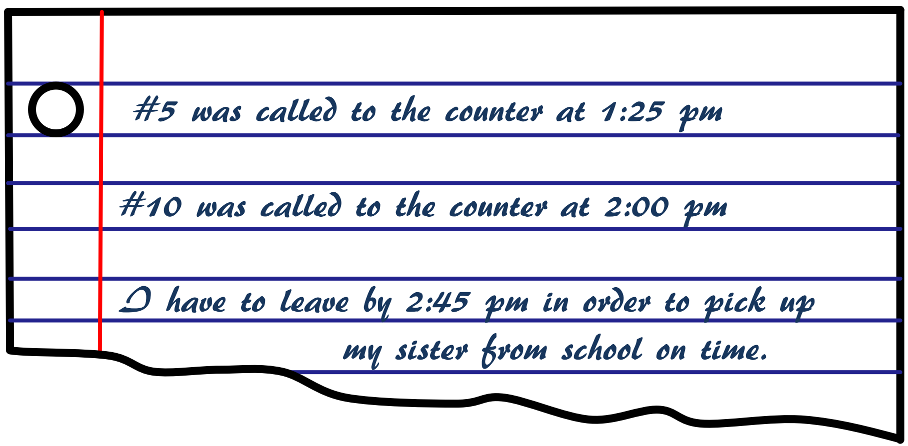
The equations above are a sampling of the types of functions students will encounter in this chapter. By the end of the chapter, students will solidify their understanding of the structure of a linear function and will surface ideas about the structure of additional types of functions that will be studied in subsequent courses.</td>
</tr>
<tr>
<td></td>
<td>Look for and express regularity in repeated reasoning.</td>
<td>Emily's little brother painted on her math homework. She knows the data in each of the tables below represents a linear function. Help Emily determine what number is hidden behind the blob of paint.

<table>
<tr><td>x</td><td>10</td><td>20</td><td>30</td><td>40</td></tr>
<tr><td>y</td><td>8</td><td>13</td><td></td><td>23</td></tr>
</table>

Slope is a calculation that is repeated in a linear relationship. In order to solve this problem and similar problems, students must understand that linear functions grow by equal differences over equal intervals and apply this knowledge in order to complete the table.</td>
</tr>
</table>

5.0: Anchor Problem: Waiting at the DMV

1. Nazhoni has completed her Driver's Education Training and is at the DMV (Division of Motor Vehicles) waiting in line to get her license to drive. She entered the DMV at 12:50 and pulled a number 17 to reserve a spot in line. Nazhoni notices that all of the employees at the DMV are still at lunch when she arrives. Once the employees return they start with number 1. There is digital sign showing the number for the person who is at the counter being helped. Nazhoni jots down some information on a piece of scratch paper as she is waiting in line.



   a. Use the picture of the scratch paper above to estimate what time it will be when Nazhomi will make it to the front of the line. **(Note: Assume that each person takes the same amount of time while being helped at the counter)**

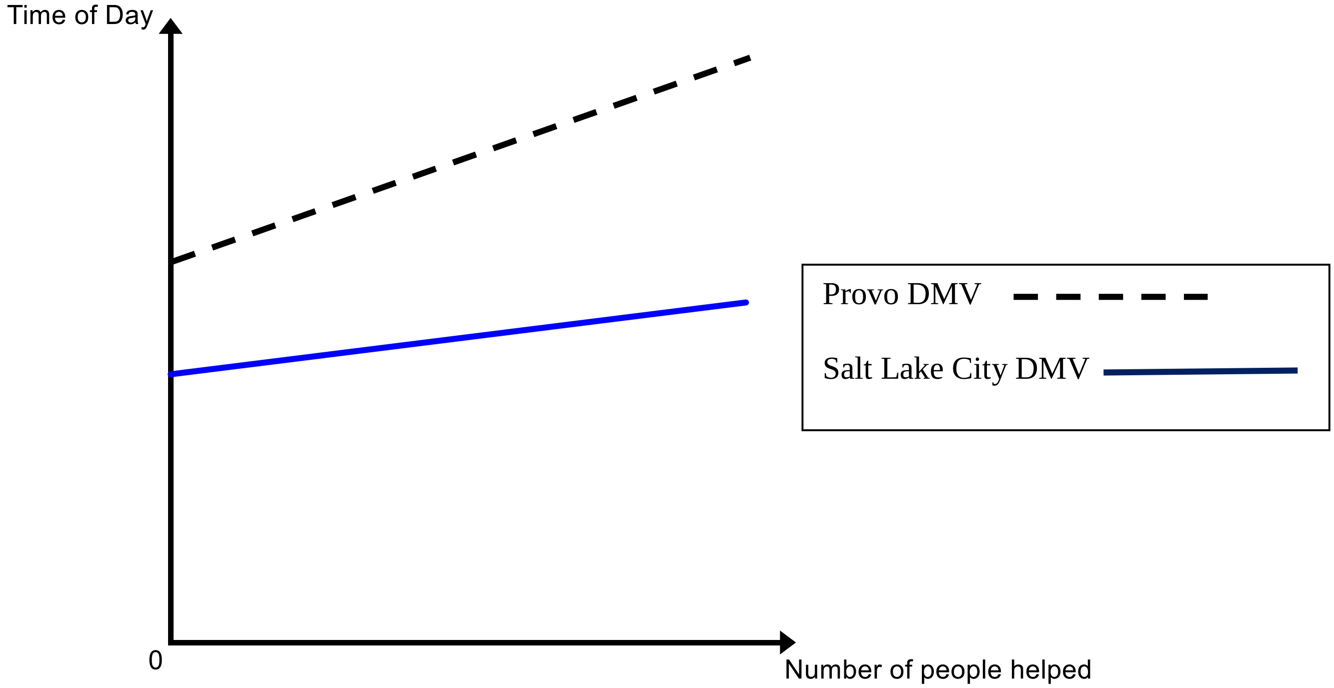
   b. Will Nazhoni make it to the front of the line in time to pick up her sister from school?

   c. What time did the employees return from lunch and begin working.

   d. Write an equation that represents the amount of time Nazhoni would have to wait dependent on the number she draws when she enters the DMV at 12:50.

This problem was adapted from a task on Illustrative Mathematics.

2. The DMV in Provo and Salt Lake opened their doors for the day at the same time. The graphs below show the time of day as a function of the number of people called to the counter. Write down as many differences between the two DMVs as you can based upon the graphs.



3. Do you think it is realistic that it takes the exact same amount of time for each person at the DMV? Explain.

4. The following table shows more realistic data for the waiting time at the DMV: Is there a constant rate of change for this data? If not, is the data still useful? What can be inferred about the information given from the table?

Time	# Being Helped
12:58	30
1:25	33
2:00	37
2:08	38
2:50	44
3:30	49

Section 5.1: Define Functions

Section Overview:
This section begins by using a context to introduce a relation that represents a function and one that is not a function. By analyzing several situations students derive their own definition of a function. They also create their own representations of relations that are functions and those that are not functions. In the next lesson a candy machine analogy is used to help students further their understanding of a function as a rule that assigns to each input exactly one output. Students then play the function machine game and discover the rule that generates the output for a given input. As the section progresses, students are given different representations of relationships (i.e. table, graph, mapping, story, patterns, equations, and ordered pairs) and must determine if the representation describes a function. In the last lesson, students determine the dependent and independent variables in a functional relationship, understanding that the roles of the variables are often interchangeable depending on what one is interested in finding.

Concepts and Skills to be Mastered:
By the end of this section students should be able to:

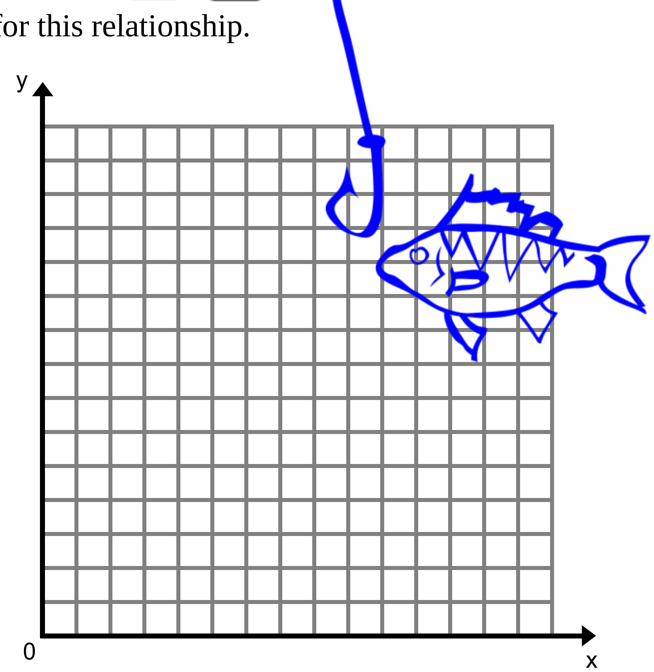
1. Understand that a function is a rule that assigns to each input exactly one output.
2. Determine whether a given relation defines a function given different representations (i.e., table, graph, mapping, story, patterns, equations, and ordered pairs).
3. Determine the independent and dependent variable in a functional relationship.

5.1a Class Activity: Introduction to Functions

1. Jason is spending the week fishing at the Springville Fish Hatchery. Each day he catches 3 fish for each hour he spends fishing. This relationship can be modeled by the equation $y = 3x$, where x = number of hours spent fishing and y = the number of fish caught. 

   a. Complete the graph and table below for this relationship.

Number of hours spent fishing x	Number of fish caught y
3	
2	
1	
2	
0	
4	

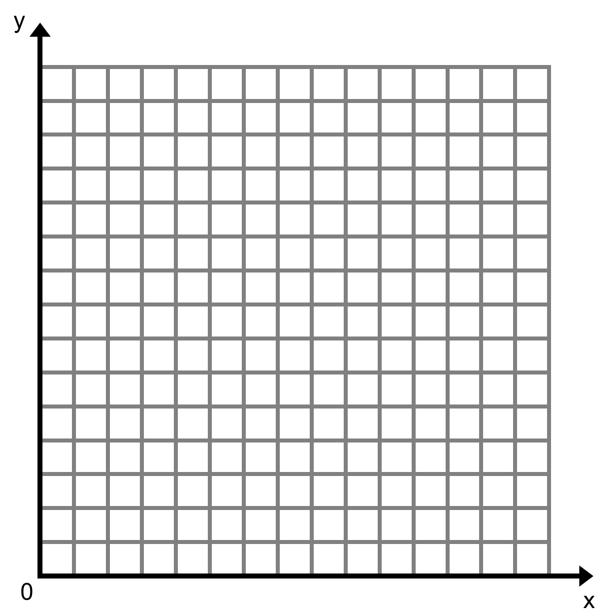


The situation above is an example of a **function**. We would say that *the number of fish caught is a **function** of the number of hours Jason spends fishing.*

2. Sean is also spending the week fishing; however he is fishing in the Bear River. Each day he records how many hours he spends fishing and how many fish that he caught. The table of values below shows this relationship.

   a. Complete the graph for this relationship.

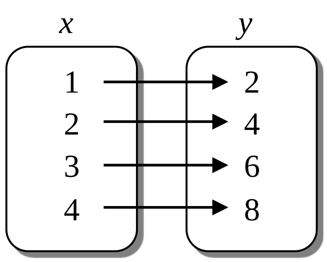
Number of hours spent fishing x	Number of fish caught y
1	4
0	0
2	5
3	1
3	8
5	5



This situation is an example of a relation that is **not a function**. The number of fish that Sean catches is **not** a function of the number of hours he spends fishing.

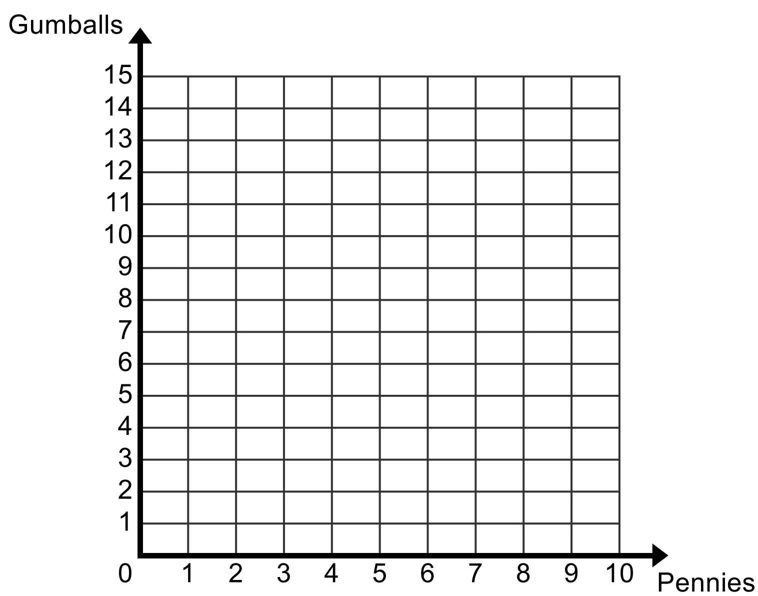
3. Compare and contrast the relationship for Jason's week spent fishing and Sean's week spent fishing. Make a <u>conjecture</u> (an educated guess) about what kind of relationship makes a function and what disqualifies a relation from being a function.

4. Vanessa is buying gumballs at Vincent's Drug Store. The mapping below shows the relationship between number of pennies, or *x*, she puts into the machine and the number of gumballs she gets out, or *y*.



   a. Complete the graph and table below for this relationship.

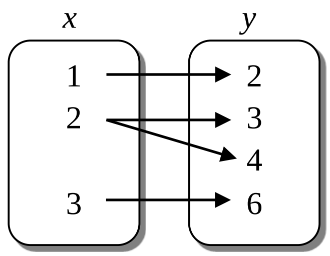
Number of pennies *x*	Number of gumballs *y*



   b. Write an equation that models this relationship.

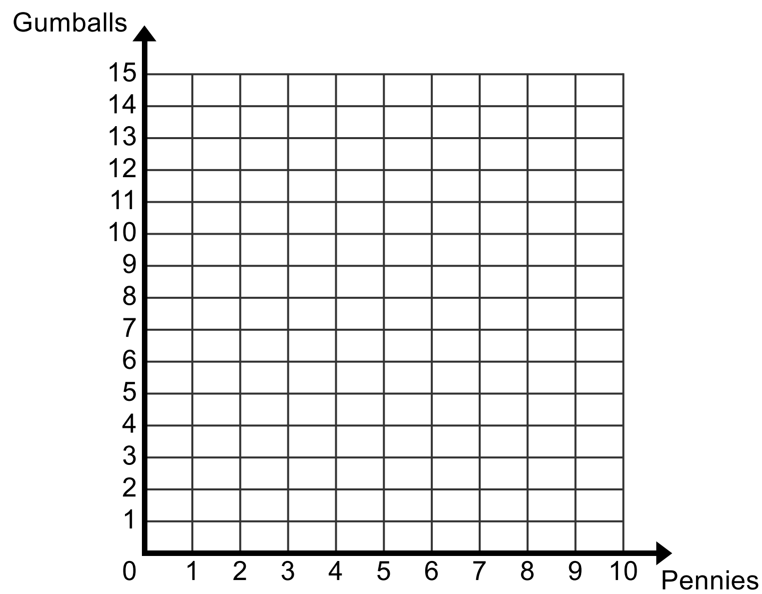
This is also an example of a **function.** We would say that *the number of gumballs received is a* ***function*** *of the number of pennies put in the machine.*

5. Kevin is across town at Marley's Drug Store. The mapping below relates the number of pennies he puts into the machine and how many gumballs he get outs.



a. Complete the graph and table below for this relationship.

Number of pennies *x*	Number of gumballs *y*

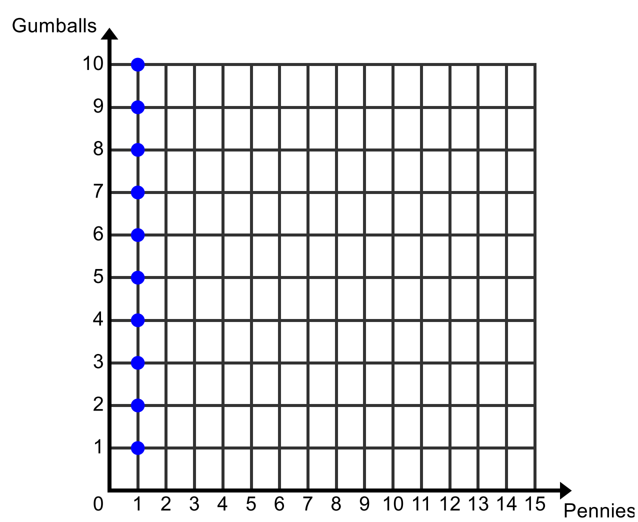


This situation is an example of a relation that is **not a function**.

6. Cody is at Ted's Drug Store. The graph below relates the number of pennies he puts into the machine on different occasions and how many gumballs he gets out.

a. Explain how this gumball machine works.

b. In this example, is the number of gumballs received a function of the amount of money put in? Explain your answer.



7. Compare and contrast the relationship of the gumball machines at the different drugstores. If needed revise your conjecture about what kind of relationship makes a function and what disqualifies a relationship from being a function.

Below is a formal definition of a function. As you read it compare it to the conjecture you made about what makes a relation a function.

Given two variables, *x* and *y*, *y* is a function of *x* if there is a rule that determines one unique *y* value for a given *x* value.

Refer back to the first two examples. When Jason went fishing, he caught a unique number of fish based on the number of hours he spent fishing. If you know the number of hours Jason fishes for, you can determine the number of fish he will catch; therefore the number of fish he catches is a function of the number of hours he spends fishing. On the other hand, when Sean is fishing, it is not possible to determine the number of fish he catches based on the number of hours he fishes. On one day, he fished for three hours and caught one fish and on another day he fished for three hours and caught eight fish. There are two different y values assigned to the x value of 3 hours. In Sean's situation, the number of fish he catches is **not** a function of the number of hours he spends fishing.

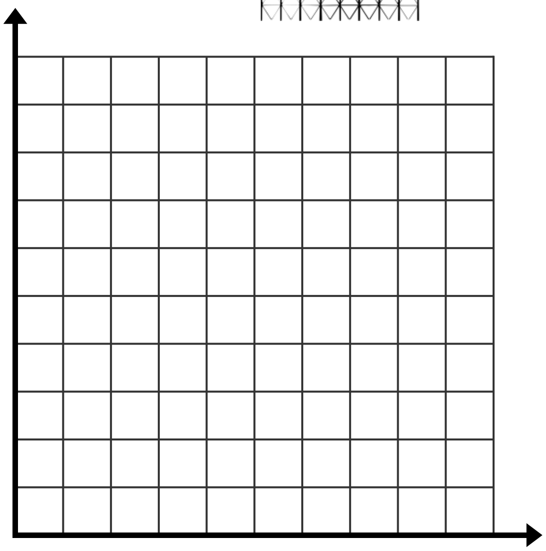
Likewise, the gumball machine at Vincent's Drug Store represents a function because each penny inserted into the gumball machine generates a unique amount of gumballs. If you know how many pennies are inserted into the gumball machine at Vincent's, you can determine how many gumballs will come out. However, the gumball machine at Marley's Drug Store is **not** a function because there is not a unique number of gumballs generated based on the number of pennies you put in. One time 2 pennies were inserted and 4 gumballs came out and at another time 2 pennies were inserted and 3 gumballs came out. You are unable to determine the number of gumballs that will come out based on how many pennies are put into the machine.

8. Explain in your own words why the number of gumballs received at Ted's Drug store is not a function of the amount of money put in. Be specific and give examples to support your reasoning.

9. The cost for entry into a local amusement park is $45. Once inside, you can ride an unlimited number of rides.

a. Complete the graph and table below for this relationship.

Number of rides x	Amount spent (dollars) y



b. Is the amount one spends a function of the number of rides he/she goes on? Why or why not?

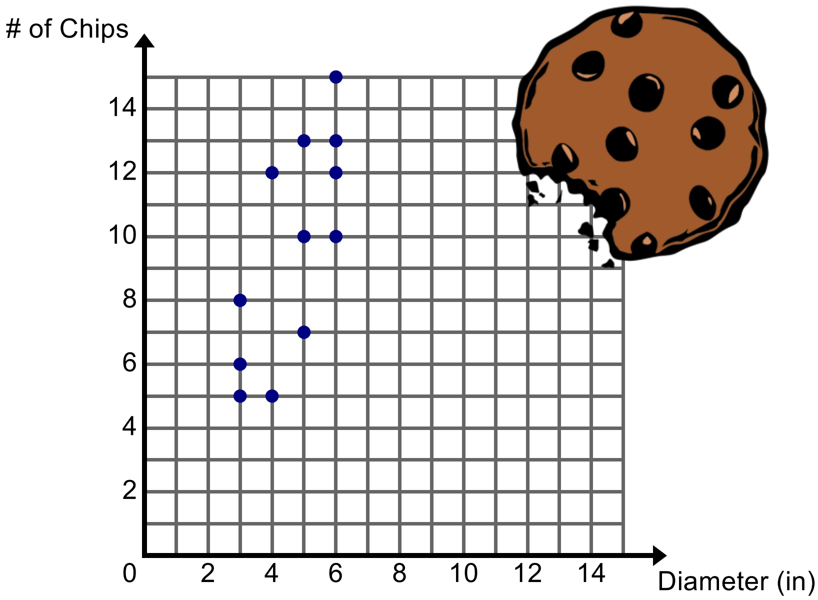
10. The table below show the number of hours Owen plays his favorite video game and the number of points he scores.

Time Spent Playing (hours)	Number of Points Scored
1	5,000
1	5,550
1	6,500
2	11,300
2	12,400
3	15,000

a. Is the number of points Owen scores a function of the amount of time he spends playing? Why or why not?

5.1a Homework: Introduction to Functions

1. Betty's Bakery makes cookies in different sizes measured by the diameter of the cookie in inches. Curious about the quality of their cookies, Betty and her assistant randomly chose cookies of different sizes and counted the number of chocolate chips in each cookie. The graph below shows the size of each cookie and the number of chocolate chips it contains.

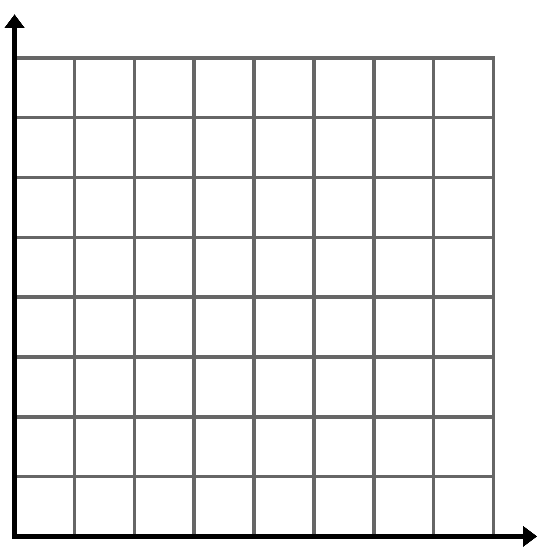


Diameter of Cookie (in) x	# of Chocolate Chips y

   a. Complete the table to the right of the graph.
   b. Is the number of chocolate chips in a cookie a function of the diameter of the cookie? Why or why not?

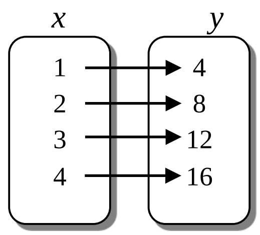
2. The number of tires y in the parking lot at Hank's Honda Dealership can be modeled by the equation $y = 4x$ where x represents the number of cars in the parking lot.
   a. Complete the table and graph below for this relationship.

Number of cars x	Number of tires y



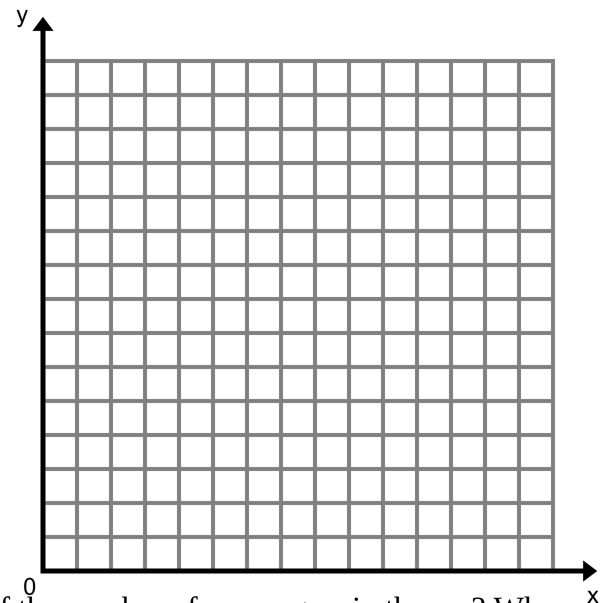
   b. Is the number of tires a function of the number of cars? Why or why not?

3. The cost for cars entering a scenic by-way toll road in Wyoming is given by the mapping below. In this relation *y* is the dollar amount to enter the by-way and *x* is the number of passengers in the car.



a. Complete the graph and table below for this relationship.

Number of passengers *x*	Amount per car (dollars) *y*

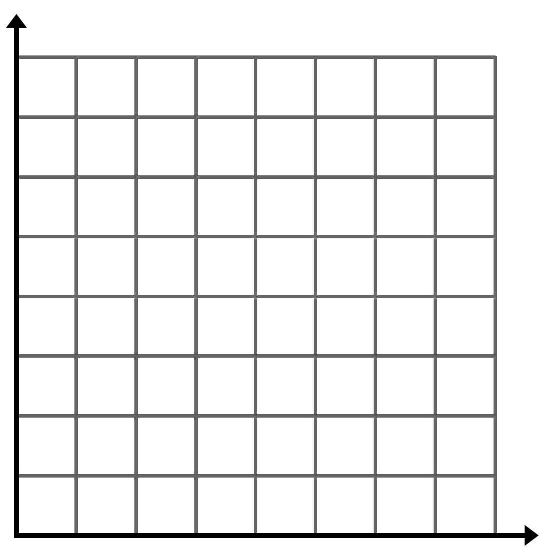


b. Is the amount spent per car a function of the number of passengers in the car? Why or why not?

4. The cost for cars entering a scenic by-way toll road in Utah is $5 regardless of the number of passengers in the car.

a. Complete the graph and table below for this relationship.

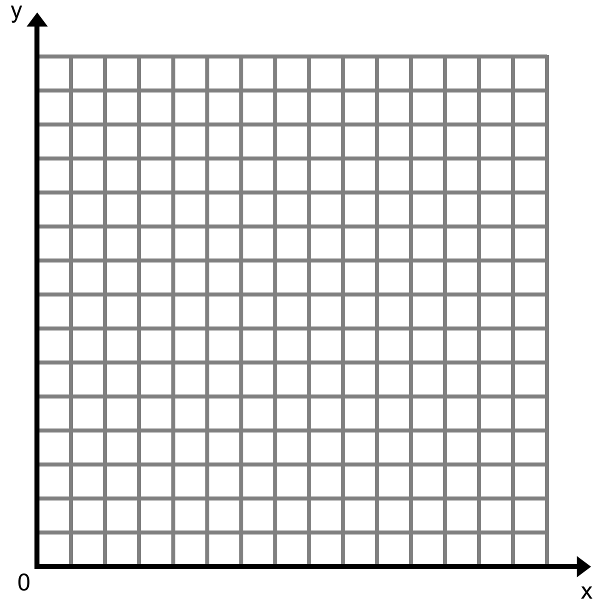
Number of passengers *x*	Amount per car *y*



b. Is the amount spent per car a function of the number of passengers in the car? Why or why not?

5. Create your own context or story that represents a relation that is a function.
   a. Story:

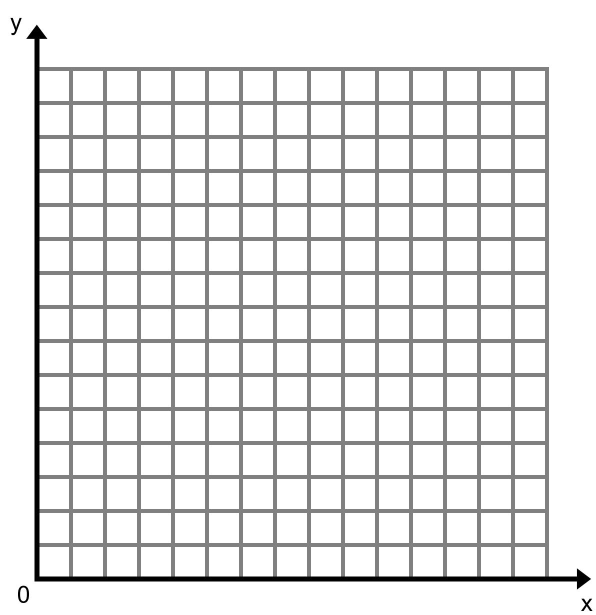
   b. Complete the graph and table below for this relationship.



   c. Explain why this relation is a function.

6. Create your own context or story that represents a relation that is **not** a function.
   a. Story:

   b. Complete the graph and table below for this relationship.

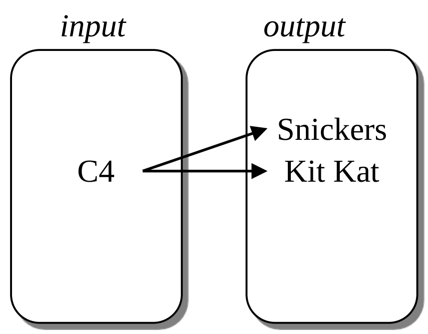


   c. Explain why this relation is not a function.

5.1b Class Activity: Function Machine

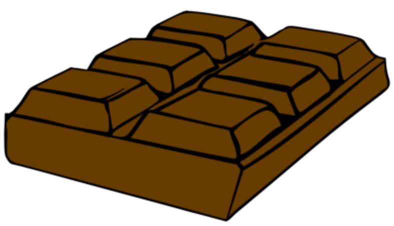
One way to think about the *x* and *y* variables in a functional relationship are as input (*x*) and output (*y*) values. To better understand how input and output values are related in a function consider the following analogy.

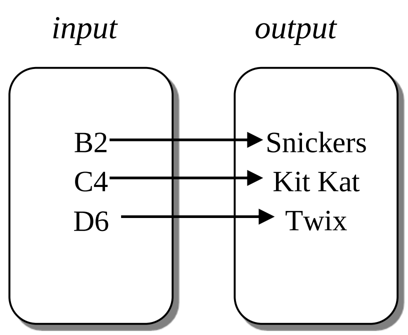
The Candy Machine Analogy: When you buy candy from a vending machine, you push a button (your input) and out comes your candy (your output). Let's pretend that C4 corresponds to a Snickers bar. If you input C4, you would expect to get a Snickers bar as your output. If you entered C4 and sometimes the machine spits out a Snickers and other times it spits out a Kit Kat bar, you would say the machine is "not functioning" – one input (C4) corresponds to two different outputs (Snickers and Kit Kat).



Not a function!

Let's look at what a diagram might look like for a machine that is "functioning" properly:

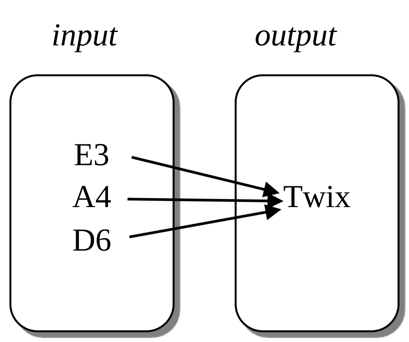




A Function!

In this situation, each input corresponds to exactly one output. The candy bar that comes out of the machine is dependent on the button you push. We call this variable the **dependent variable**. The button you push is the **independent variable**.

Let's look at one more scenario with the candy machine. There are times that different inputs will lead to the same output. In the case of the candy machines, companies often stock popular items in multiple locations in the machine. This can be represented by the following diagram:

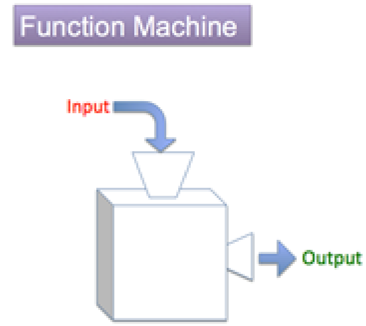


A Function!

Even though the different inputs correspond to the same output, our machine is still "functioning" properly. This still fits the special requirement of a function – each input corresponds to exactly one output.

THE FUNCTION MACHINE:

In this activity, you will give your teacher a number. He/she will perform some operations on the number, changing it to a new number. Your goal is to figure out what rule is being applied to the number. Use the tables below to keep track of the numbers you give your teacher (inputs) and the numbers your teacher gives you back (outputs). Once you figure out the function, write it in the space below the table.



INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

INPUT #	OUTPUT #

Function:

Now try it on your own or with a partner. Write the function for each of the following relations. Use the words input and output in your written function equation. Then write the function as an equation using x and y. The first one has been done for you. There is space for you to verify your function is correct.

1.	**Double the input increased by one will get the output.**	
Input x	Function: $y = 2x + 1$	Output y
8	$2(8) + 1 = 17$	17
0	$2(0) + 1 = 1$	1
–3	$2(-3) + 1 = 1$	–5
2		5
1		3
–1		–1
7		15

2.		
Input x	Function:	Output y
3		–2
0		–5
10		5
2		–7
5		0
1		–4
–4		–9

3.		
Input x	Function:	Output y
5		25
–2		–10
0		0
1		5
–4		–20
–9		–45
3		15

4.		
Input x	Function:	Output y
–3		–4
–2		–2
–1		0
0		2
1		4
2		6
3		8

5.		
Input x	Function:	Output y
4		–2
–3		–16
0		–10
1		–8
7		4
9		8
5		0

6.		
Input x	Function:	Output y
2		3
5		–9
2		7
5		10
4		9
2		0
–1		–3

5.1b Homework: Function Machine

Directions: Write the function for each of the following relations.

1.		
Input *x*	Function:	Output *y*
5		2.5
–2		–4.5
0		–2.5
1		–1.5
–4		–6.5
–9		–11.5
3		0.5

2.		
Input *x*	Function:	Output *y*
2		½
–4		–1
0		0
1		¼
–9		–9/4
24		6
8		2

3.		
Input *x*	Function:	Output *y*
–4		–9
–3		–6
–2		–3
0		3
1		6
2		9
3		12

4.		
Input *x*	Function:	Output *y*
2		3
–4		–3
0		1
1		2
–9		–8
–17		–16
10		11

5.		
Input *x*	Function:	Output *y*
–4		16
–3		9
–2		4
–1		1
0		0
1		1
2		4

6.		
Input *x*	Function:	Output *y*
1		3
6		–8
1		7
5		10
6		8
2		0
–1		–3

7. Were you able to find a function for number 6? If so, write it down. If not, explain why.

Directions: Create your own function machines, fill in the values for each input and its corresponding output.

8.		
Input	Function:________________	Output

9.		
Input	Function:________________	Output

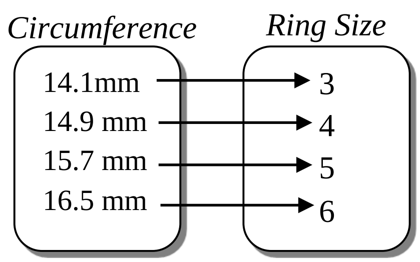
10. Create a machine that is **not** a function. Explain why your machine is "dysfunctional".

Input	Function:________________ =	Output

5.1c Class Activity: Representations of a Function

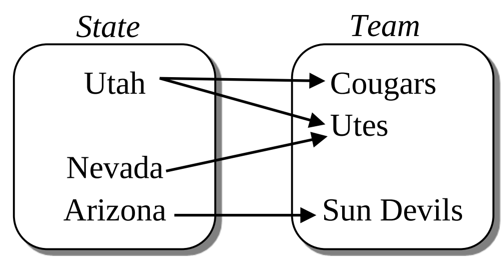
Functions can also be described by non-numeric relations. A **mapping** is a representation of a function that helps to better understand non-numeric relations. Study each relation and its mapping below. Then decide if the relation represents a function. Explain your answer.

1. **Input:** circumference of finger
   Output: ring size



Function? Explain.

2. **Input:** state a person lives in
   Output: the team they root for in college football

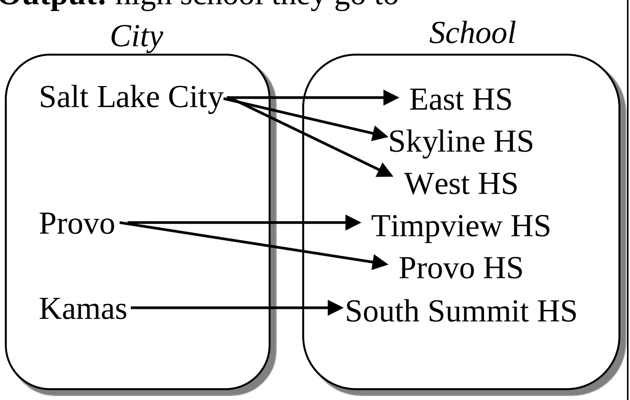


Function? Explain.

3. Write the ordered pairs (circumference, ring size) that correspond to problem #1.

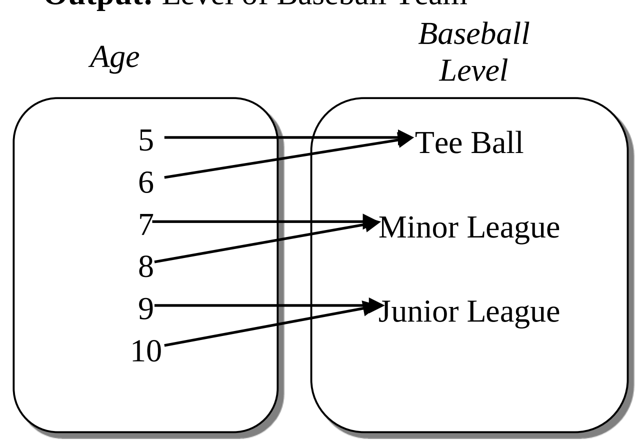
4. Write the ordered pairs (state a person lives in, team they root for) that correspond to problem #2.

5. **Input:** city student lives in
   Output: high school they go to



Function? Explain.

6. **Input:** Age
   Output: Level of Baseball Team



Function? Explain.

As we have seen, there are many ways to represent a relation or function. In the following problems, you will be given one representation of a relation and asked to create additional representations. Then, you will be asked to determine whether the relation represents a function or not.

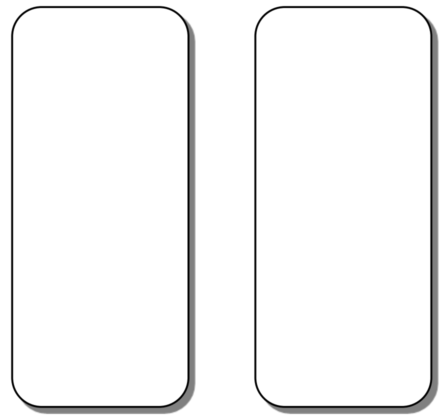
7. **Story:** A candle is 27 centimeters high and burns 3 centimeters per hour. An equation that models this relation is $c = 27 - 3h$ where c is the height of the candle in centimeters and h is the number of hours the candle has been burning.
   a. Express this relation as a table, mapping, graph, and set of ordered pairs.

Table

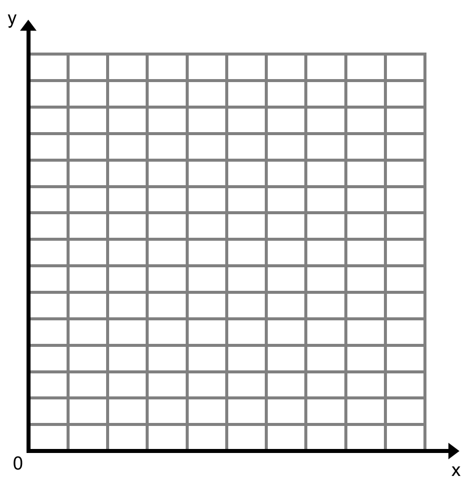
Time (hours) h	Height (cm) c

Mapping

Hours *Height*



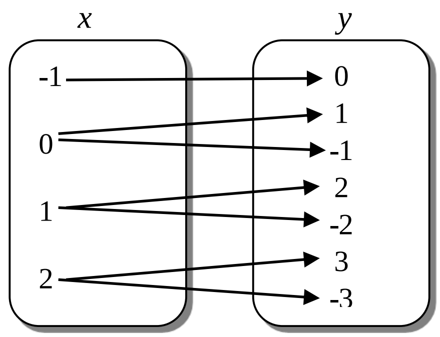
Graph



Set of Ordered Pairs

   b. Is the height of the candle a function of the amount of time it has been burning? Explain.

8. **Mapping:**

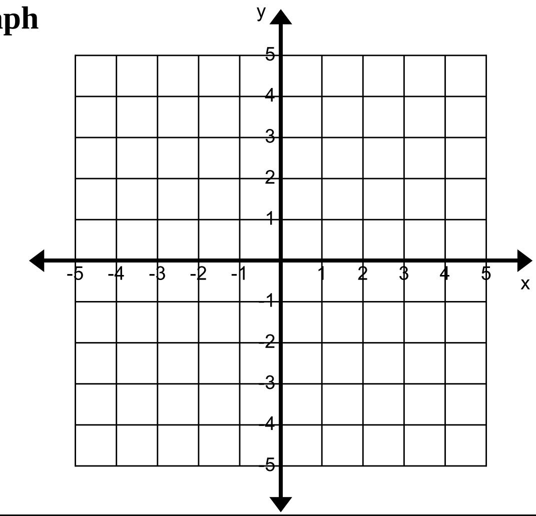


a. Express this relation as a table, graph, and set of ordered pairs.

Table

x	*y*

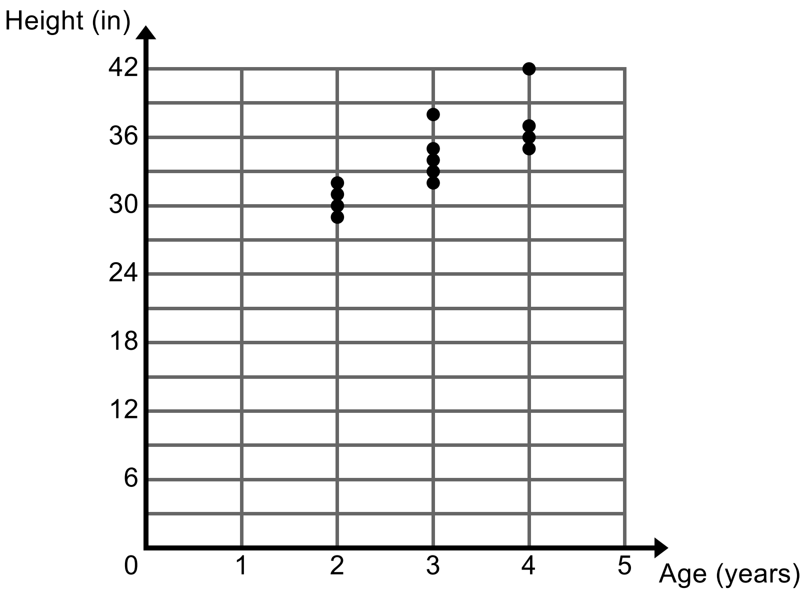
Graph



Set of Ordered Pairs

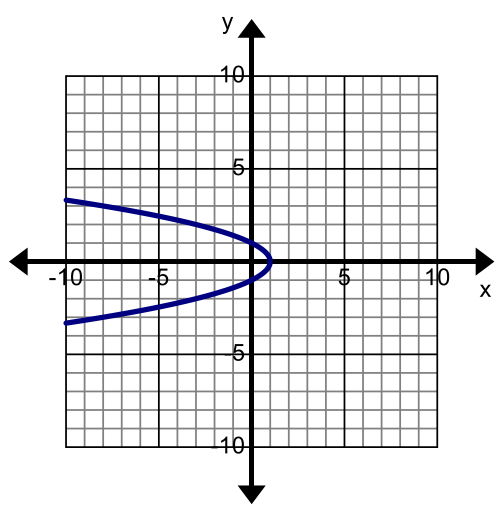
b. Is this relation a function? Explain.

9. **Graph:** Discovery Place Preschool gathered data on the age of each student (in years) and the child's height (in inches). The graph below displays the data they gathered.

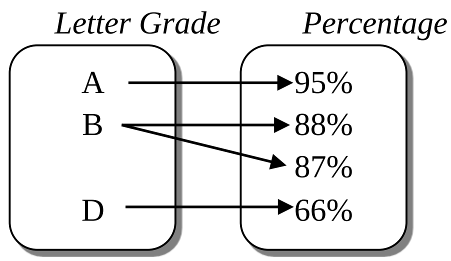


a. Is a child's height a function of the child's age? Explain.

Directions: Determine if each relation or situation defines a function. Justify your answer. It may help to make an additional representation of the relation.

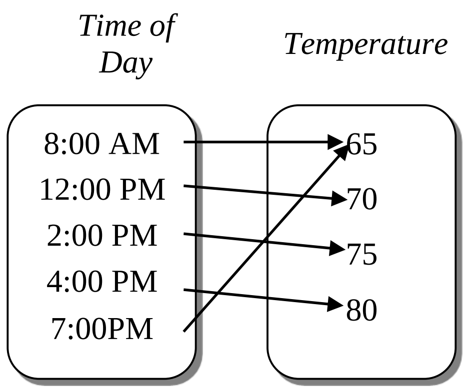
10. {(30, 2), (45, 3), (32, 1.5), (30, 4), (41, 3.4)}	11. 
12. $x = 2$	13. $3x + 6y = 18$

14.



15. Is letter grade a function of percentage scored on a test?

16.



17. Is time of day a function of the temperature?

18.

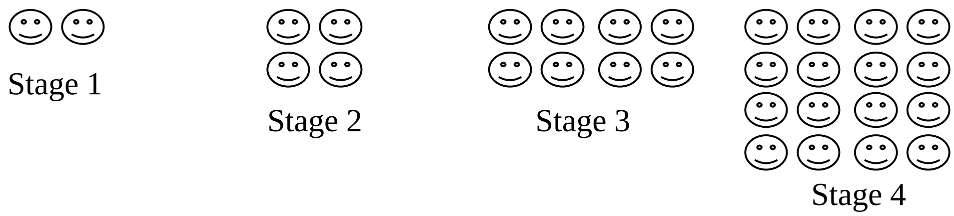
Length of Radius (cm)	Length of Diameter (cm)
0.5	1
1	2
1.5	3
2	4

19. **Input:** name of city in the U.S.
Output: state city is in
Hint: There are 16 states in the United States that have a city called Independence.

5.1c Homework: Representations of a Function

1. Use the pattern below to answer the questions that follow.

Pattern:

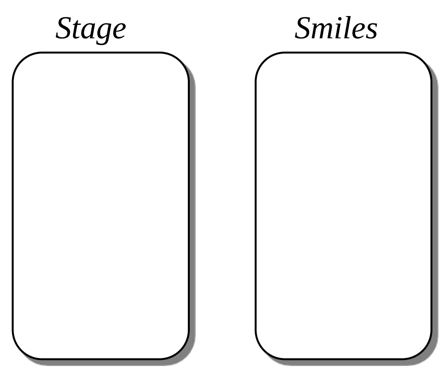


a. Express this relation as a table, mapping, and graph.

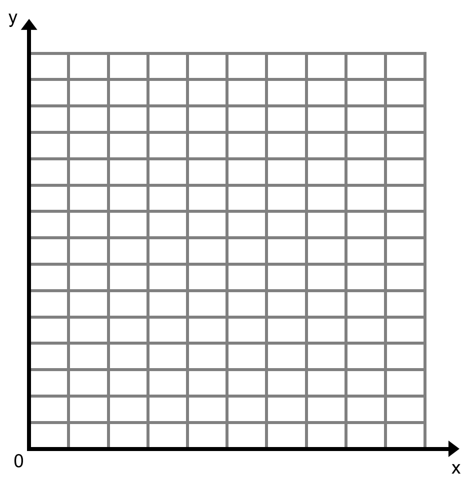
Table

Stage number	Number of Smiles

Mapping



Graph

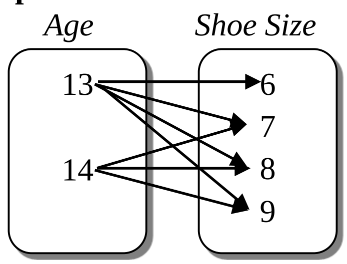


Set of Ordered Pairs

b. Is this relation a function? Explain how you know.

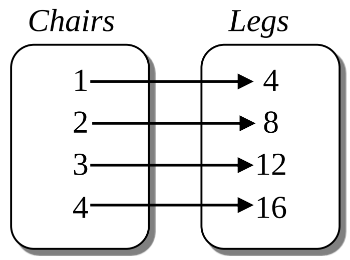
Directions: Determine if each relation or situation defines a function. Justify your answer.

2. **Input:** age
Output: shoe size



Function? Explain.

3. **Input:** number of chairs
Output: number of legs



Function? Explain.

4. List the ordered pairs that correspond to #2: (age, shoe size).

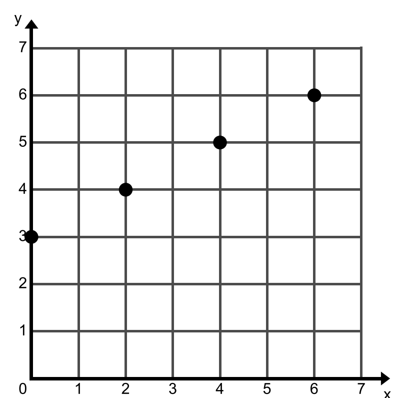
5. List the ordered pairs that correspond to #3: (number of chairs, number of legs)

6.

x	***y***
0.2	1.5
0.4	1.25
0.6	1.5
0.8	1.25

Function? Explain.

7.

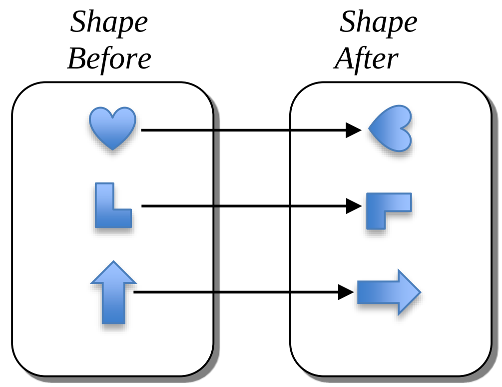


Function? Explain.

8. A car is traveling at a constant rate of 60 mph. Is the car's distance traveled a function of the number of hours the car has been driving?

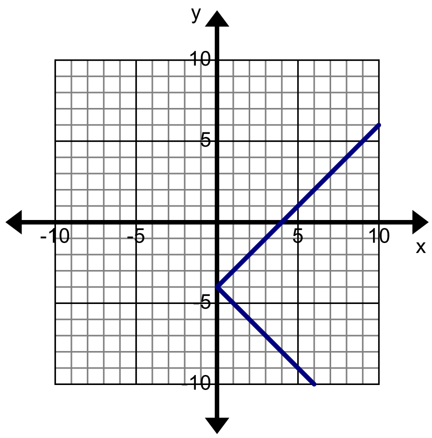
Function? Explain.

9.



Function? Explain.

10.



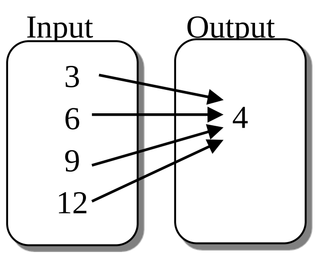
Function? Explain.

11.

Number of People	1	3	4	7
Cost	4	8	10	16

Function? Explain.

12.



Function? Explain.

13. $y = \frac{1}{3}x + 4$

Function? Explain.

14. You know your cousin lives at the zip code 12345 so you type it in Google to find your cousin's full address. Is address a function of zip code?

Function? Explain.

15. You know your cousin's cellular phone number is (123) 456-7890 so you dial that number to call him. Is person being called a function of phone number dialed?

Function? Explain.

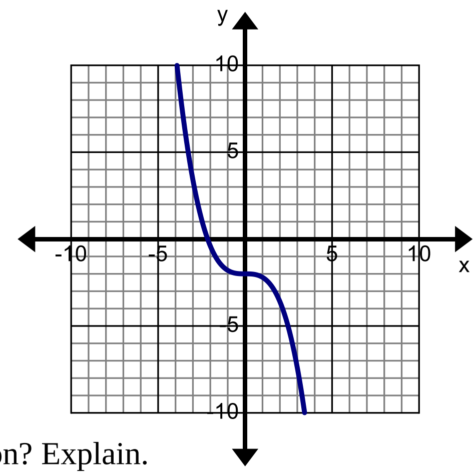
16. You call the post office and ask for the zip code for the city of Salt Lake City, UT. Is zip code a function of the name of the city?

Function? Explain.

17. $y = -5$

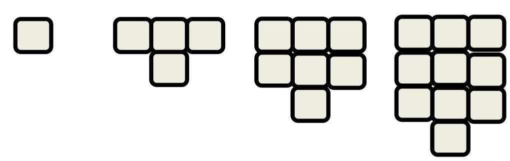
Function? Explain.

18.



Function? Explain.

19.



Stage 1 Stage 2 Stage 3 Stage 4

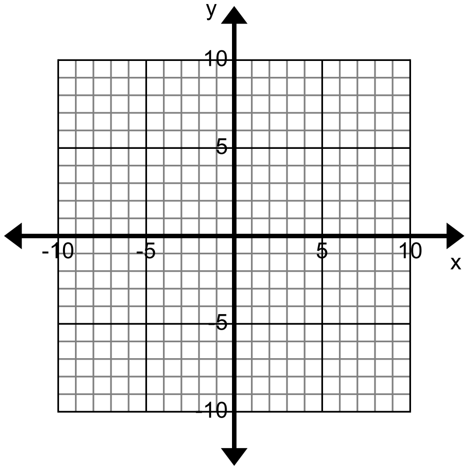
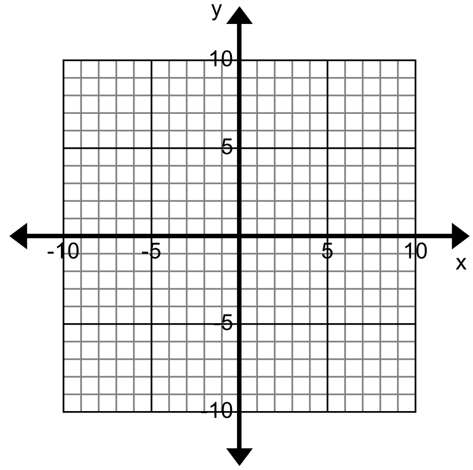
Function? Explain.

20. $\{(2,-10),(5,-25),(8,-40),(-5,25)\}$

Function? Explain.

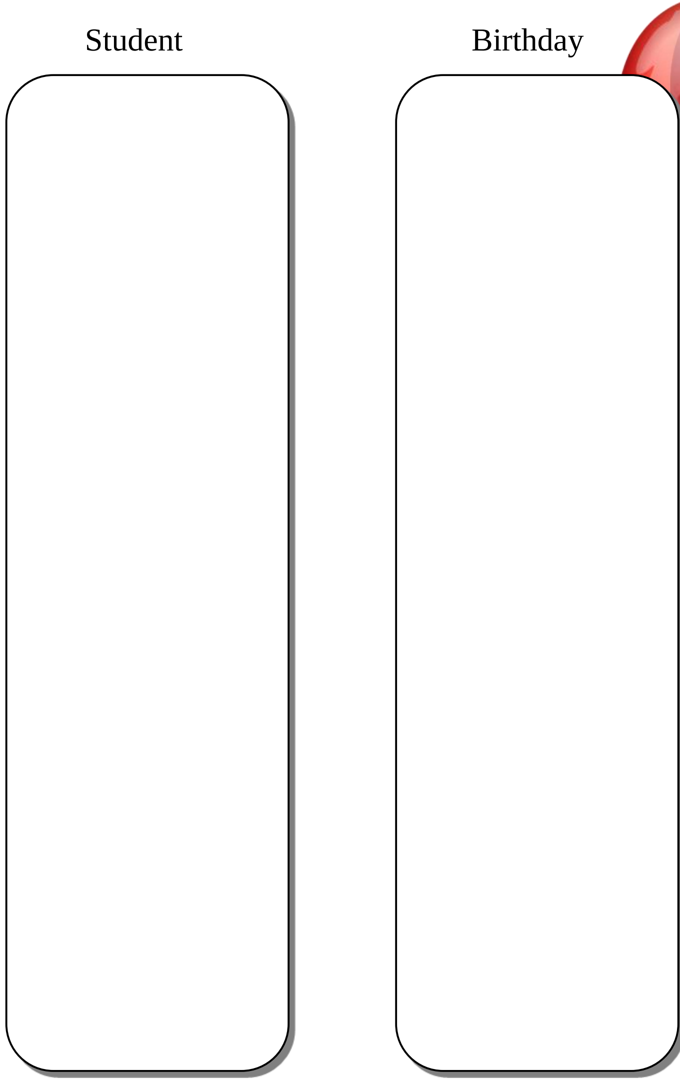
21. $\{(-1,0),(1,2),(1,4),(5,2)\}$

Function? Explain.

22. Draw a graph of a relation that is a function. Explain how you know. 	23. Draw a graph of a relation that is **not** a function. Explain how you know. 
24. Make a mapping of a relation that is a function. Explain how you know.	25. Creat a set of ordered pairs that do **not** represent a function. Explain how you know.

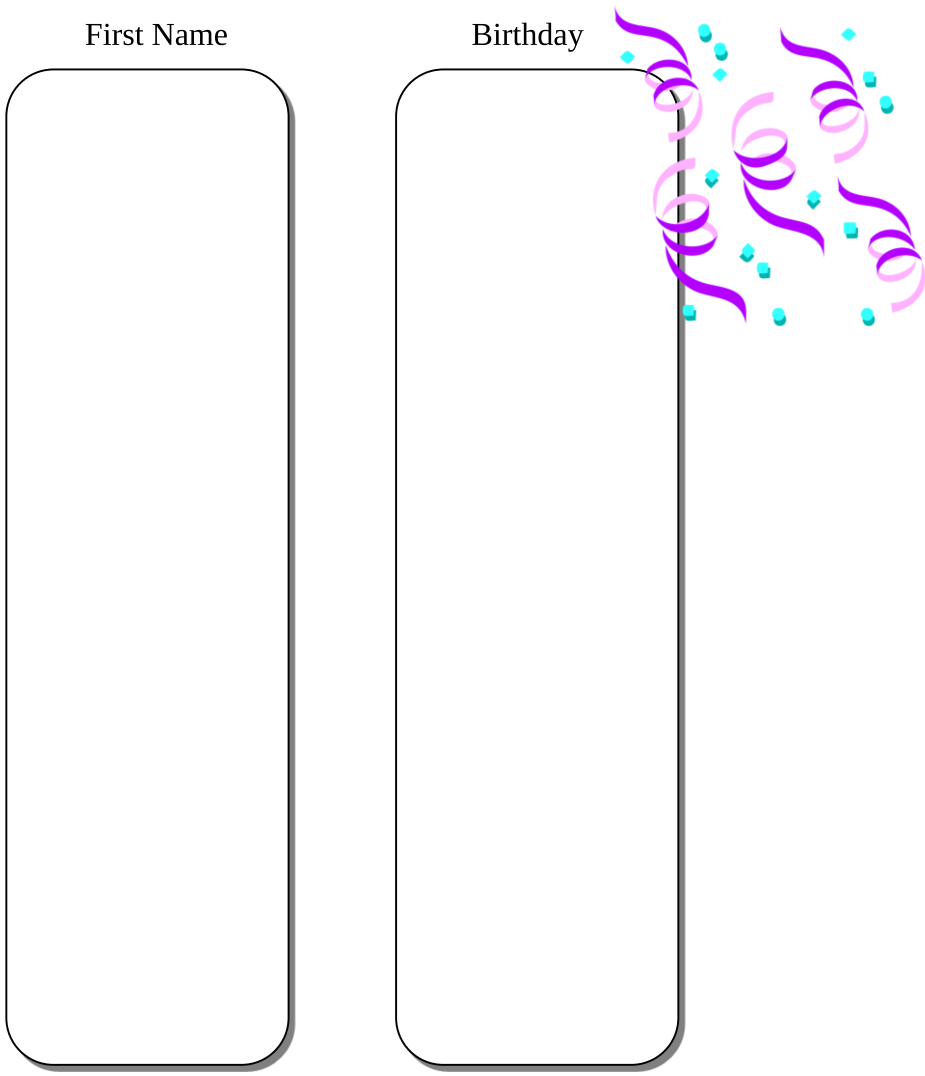
5.1d Class Activity: Birthdays

1. Make a mapping that shows the students in your class and their birthdays.



   a. Is the birth date of a student a function of the individual student? Justify your answer.

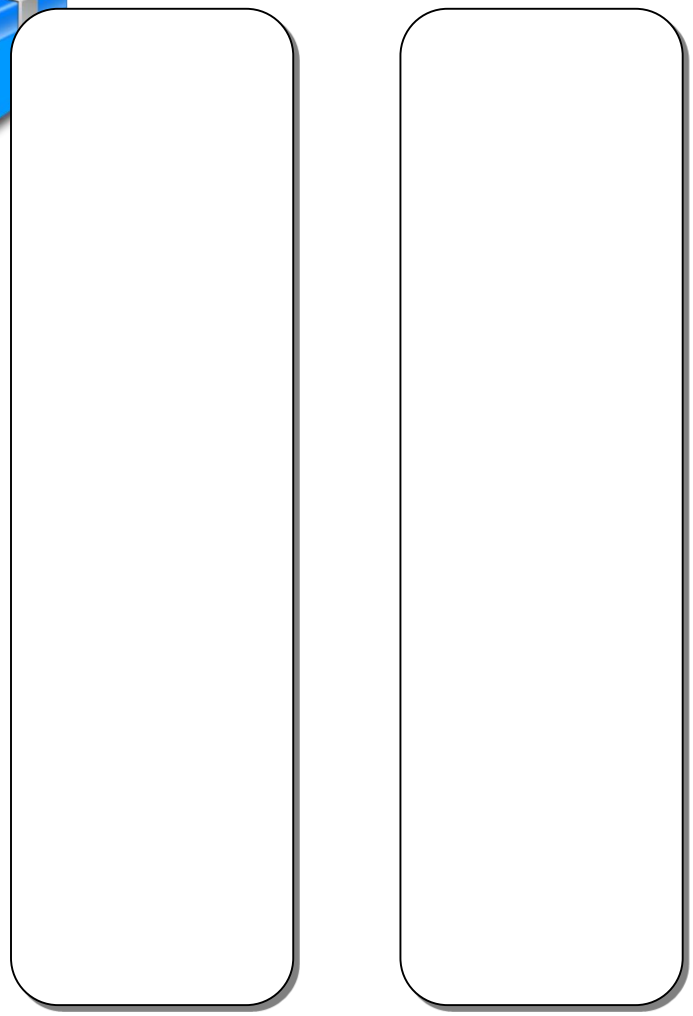
2. Make a mapping that shows the first name of the students in your class and their birthdays.



b. Is the birth date of a student a function of the student's first name? Justify your answer.

3. Make a mapping, switching the input to be birthday and the output to be student.

Birthday Student



a. Is student a function of birth date in your class? Justify your answer.

5.1d Homework: Birthdays

Directions: Determine if each relation or situation defines a function. Make an additional representation of the relation to help you. Justify your answer.

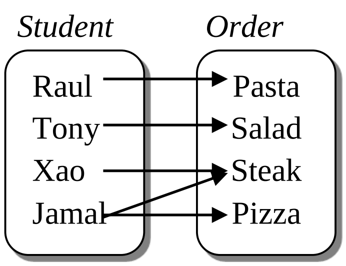
a. Is a student's ID number a function of his/her first name? Consider all students in your school.

Function? Explain.

b. Is a student's first name a function of his/her student ID number? Consider all students in your school.

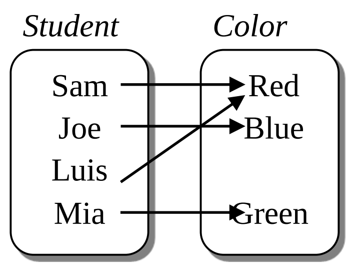
Function? Explain.

c.



Function? Explain.

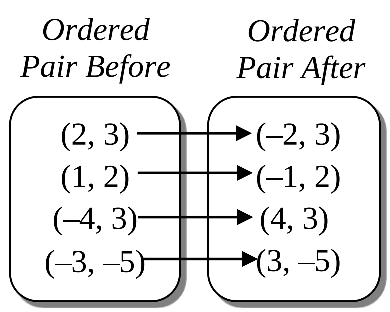
d.



Function? Explain.

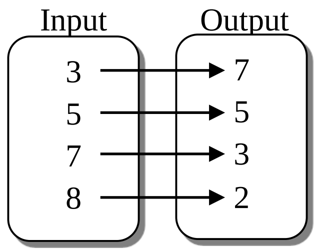
e. Les surveys the students in his class to determine if shoe size is a function of last name of the student? What would have to be true about the names of the students in the class if Les found that shoe size is **not** a function of the last name of the student?

f.



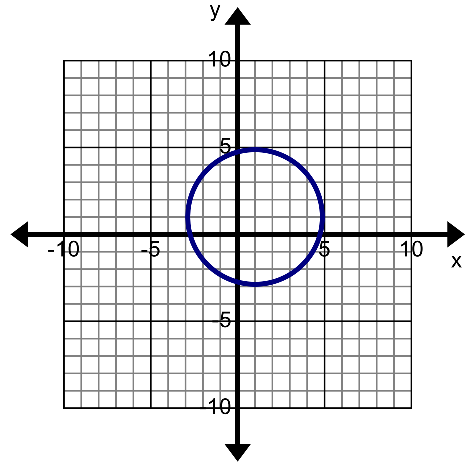
Function? Explain.

g.



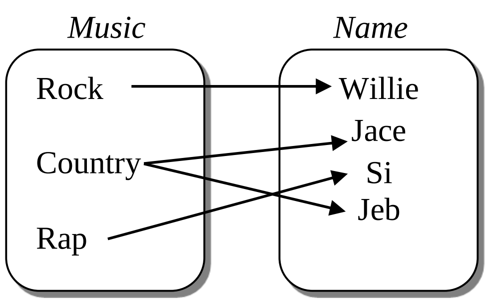
Function? Explain.

h.



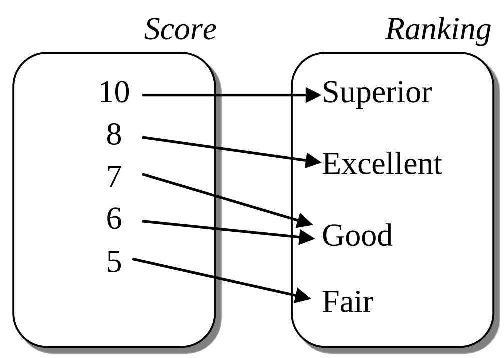
Function? Explain.

i. **Input:** favorite type of music
Output: name



Function? Explain.

j. **Input:** a pianist's overall score in a music competition
Output: ranking



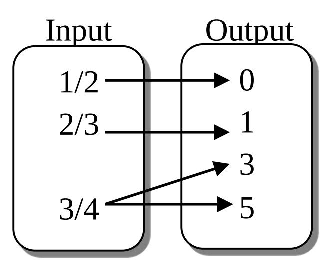
Function? Explain.

k.

Input	**Output**
25	14
30	13
30	12
35	11

Function? Explain.

l.



Function? Explain.

5.1e Class Activity: More About Functions

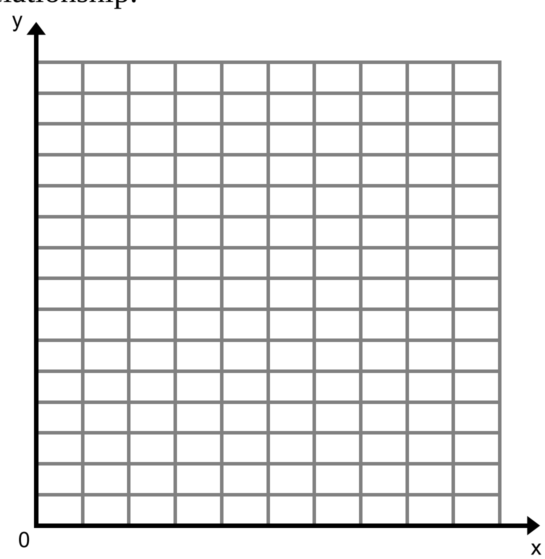
1. Paradise Valley Orchards has the banner shown hanging from their store window. Sally is trying to determine how much she will spend depending on how many bushels of apples she purchases.

   a. Write an equation that gives the amount Sally will spend y depending on how many bushels of apples x she purchases.

   b. Complete the graph and table below for this relationship.

Number of Bushels x	Amount Spent (dollars) y

c.



We know from the previous lessons, that the relationship between number of bushels purchased and amount spent is an example of a function. The equation above gives us a rule for how to determine the amount of money spent based on the number of bushels purchased.

In a functional relationship represented with an equation, the **independent variable** represents the input or x-value of the function and the **dependent variable** represents the output or y-value of the function. In a function, the **dependent variable** is determined by or depends on the **independent variable**. In our example above the **independent variable** is the number of bushels purchased and the **dependent variable** is the amount of money spent. The amount of money one spends **depends** on the number of bushels one purchases. Another way to say this is that the amount of money spent is a function of the number of bushels purchased.

If we think of our input machine, we are inputing the number of bushels purchased and the machine takes that number and multiplies it by 15 to give us our output which is the amount of money we will spend.

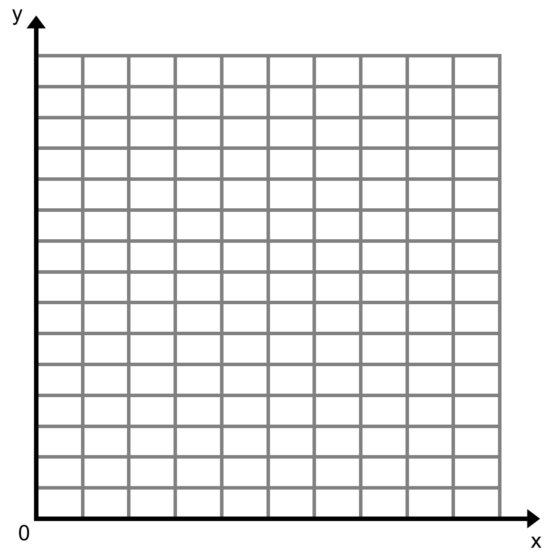
2. Miguel is taking a road trip and is driving at a constant speed of 65 mph. He is trying to determine how many miles he can drive based on how many hours he drives.

   a. Identify the **independent variable** in this situation: ______________________________

   b. Identify the **dependent variable** in this situation: _____________________________

   c. Complete the graph and table below for this relationship. Make sure you label the columns and axes in your table and graph.

x	*y*



   d. Write an equation that represents this situation: ______________________________

   e. In this situation _______________________ is a function of __________________________.

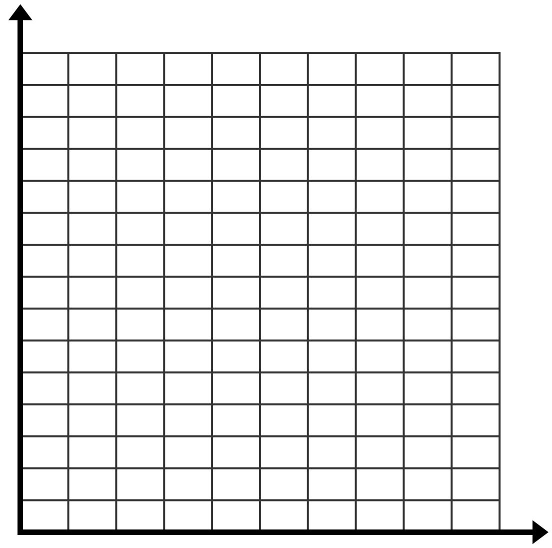
3. The drama club is selling tickets to the Fall Ball. They use $2 from each ticket sale for food and decorations.

   a. Identify the **independent variable** in this situation: ___________________________

   b. Identify the **dependent variable** in this situation: __________________________

   c. Create a table, graph, and equation for this function.

x	y



   Equation: _______________________

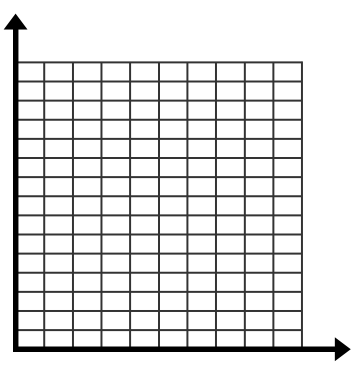
   d. Complete the following sentence for this situation.

   _________________________________ is a function of _________________________________.

4. The average cost of a movie ticket has steadily increased over time.

   a. Identify the dependent and independent variables in this functional relationship.

   b. Sketch a possible graph of this situation.



5. Susan is reading her history text book for an upcoming test. She can read 5 pages in 10 minutes. Susan is interested in determining how many pages she can read based on how long she reads for.

    a. Identify the **independent variable** in this situation: ______________________

    b. Identify the **dependent variable** in this situation: ____________________________

    c. Create a representation (table, graph, equation) of this function in the space below.

6. Chris is also reading his history text book for an upcoming test and can also read 5 pages in 10 minutes. However, Chris is interested in determining how long it will take him to read based on how many pages he has to read.

    a. Identify the **independent variable** in this situation: _____________________________

    b. Identify the **dependent variable** in this situation: ____________________________

    c. Create a representation (table, graph, equation) of this function in the space below.

Directions: Each of the following situations represents a functional relationship between two quantities. Determine the dependent variable and the independent variable. The first one has been done for you.

7. In warm climates, the average amount of electricity used rises as the daily average temperature increases and falls as the daily average temperature decreases.

8. The number of calories you burn increases as the number of minutes that you walk increases.

9. The air pressure inside a tire increases with the temperature.

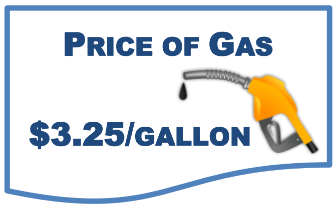
10. As the amount of rain decreases, so does the water level of the river.

11. The total number of jars of pickles that a factory can produce depends on the number of pickles they receive.

12. The weight of the box increases as the number of books placed inside the box increases.

5.1e Homework: More About Functions

1. Shari is filling up her gas tank. She wants to know how much it will cost to put gas in her car. The sign below shows the cost for gas at Grizzly's Gas-n-Go.

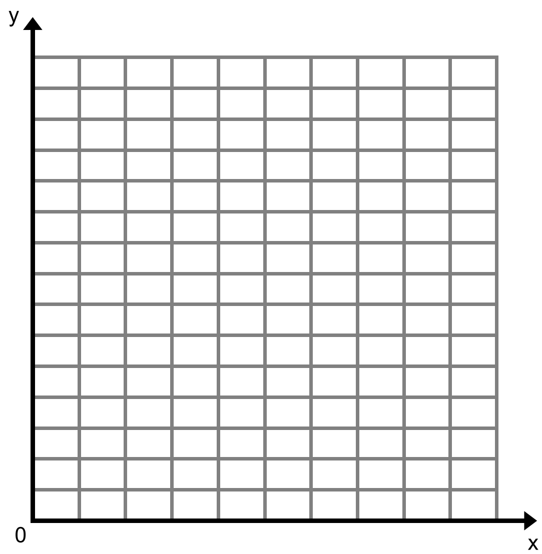


   a. Identify the **independent variable** in this situation: ___________________________________

   b. Identify the **dependent variable** in this situation: ___________________________________

   c. Complete the graph and table below for this relationship. Make sure you label the columns and axes in your table and graph.

x	y



   d. Write an equation that represents this situation: _____________________________________

   e. In this situation ____________________________ is a function of ______________________.

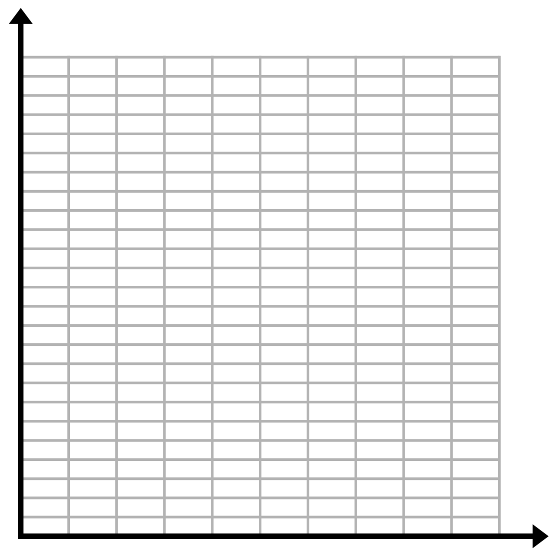
2. Peter is the event planner for a team race taking place in Park City, UT. He needs to determine how many bottles of water to have ready at the finish line of the race so that each participant in the race receives a bottle of water. There are 4 people on a team.

    a. Identify the **independent variable** in this situation: ________________________________

    b. Identify the **dependent variable** in this situation: __________________________________

    c. Create a table, graph, and equation of this situation.

x	*y*



Equation: _______________________

    d. Complete the following sentence:

    __________________________ is a function of ____________________________________.

Directions: Each of the following situations represents a functional relationship between two quantities. Underline the two quantities. Put an I above the independent variable and a D above the dependent variable.

3. As the size of your family increases so does the cost of groceries.

4. The value of your car decreases with age.

5. The greater the distance a sprinter has to run the more time it takes to finish the race.

6. A car has more gas in its tank can drive a farther distance.

7. A child's wading pool is being inflated. The pool's size increases at a rate of 2 cubic feet per minute.

8. The total number of laps run depends on the length of each workout.

9. A tree grows 15 feet in 10 years.

10. There are 5 inches of water in a bucket after a 2 ½ hour rain storm.

11. Jenny has 30 coins she has collected over 6 years.

12. Sally's track coach wants to know how far she can run based on the amount of time she runs for.

13. Whitney is training for a half marathon. She wants to know how long it will take her to run based on how far she has to run for.

14. Write your own relationship that contains an independent and dependent variable.

5.1f Self-Assessment: Section 5.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Mastery 3**	**Substantial Mastery 4**
1. Understand that a function is a rule that assigns to each input exactly one output.				
2. Determine whether a given relation defines a function given different representations (i.e., table, graph, mapping, story, patterns, equations, and ordered pairs).				
3. Determine the independent and dependent variables in a functional relationship.				

1. Define function in your own words. Provide examples to support your definition.

2. Do the representations below define a function? Why or why not?

a.

Stage 1 Stage 2 Stage 3 Stage 4 Stage 5

Is the number of hearts in a stage a function of the stage number?

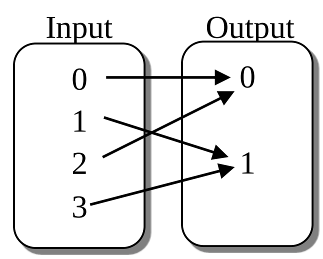
Is the stage number a function of the number of hearts in a stage?

b. Maria is draining her hot tub at a rate of 5.5 gallons per minute. Is the amount of water left in the pool a function of the amount of time she has been draining it?

c.

x	y
10	1
10	2
10	3
10	4

d.



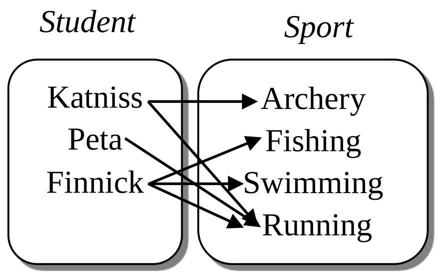
e. Is state capitol a function of state name? Consider states in the United States.

f. Is a person's weight a function of the person's age?

g. Is the amount of time it takes a person to run a marathon a function of the person's age?

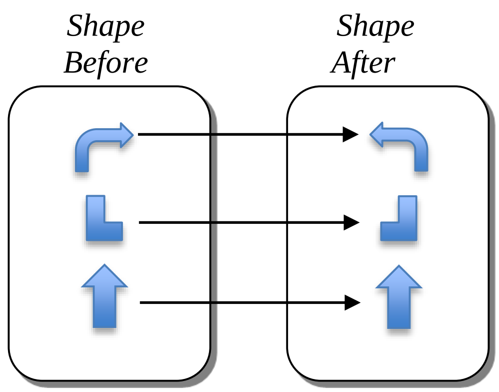
h. $\{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1)\}$

i.

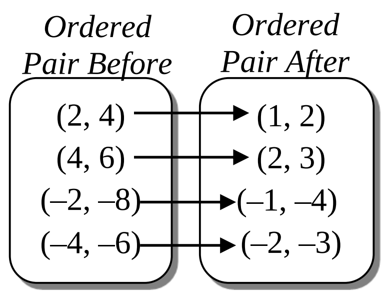


j. $y = \frac{1}{4}x - 2$

k.



l.



3. In each of the following situations an independent variable is given. Determine a possible dependent variable that would create a functional relationship.
   a. The amount of gas remaining in a tank

   b. Time

   c. Number of people

   d. Number of t-shirts

   e. Circumference of head

Section 5.2: Explore Linear and Nonlinear Functions

Section Overview:
This section focuses on the characteristics that separate linear from nonlinear functions. Students will analyze the different representations of a function (graph, table, equation, and context) to determine whether or not the representations suggest a linear relationship between the two variables. In the process of studying non-linear functions, students will solidify their understanding of how a linear function grows (changes).

Concepts and Skills to Master:
By the end of this section, students should be able to:

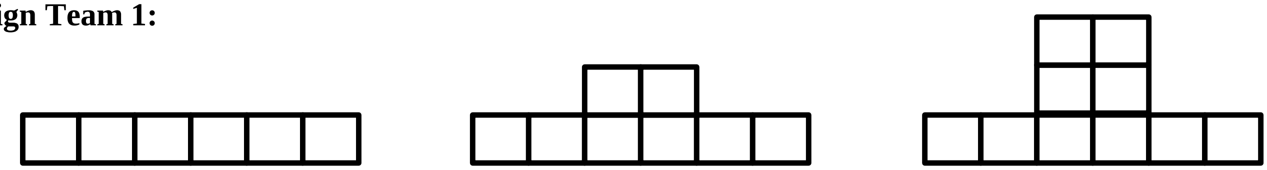
- Distinguish between linear and nonlinear functions given a context, table, graph, or equation.

5.2a Class Activity: Display Designs

Complete Foods, a local grocery store, has hired three different companies to come up with a display for food items that are on sale each week. They currently have a display that is 6 boxes wide as shown below.

They would like the center part of the display to be taller than the outside pieces of the display to showcase their "mega deal of the week". The following are the designs that two different companies submitted to Complete Foods, using the current display as their starting point.

Design Team 1:



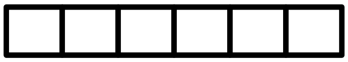
Current Display (Stage 1) **Stage 2** **Stage 3**

Stage 4

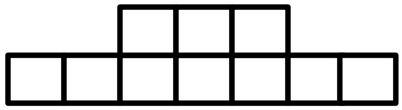
1. Draw Stage 4 of this design. Describe how you went about drawing stage 4.

2. Can the relationship between stage number and number of blocks in a stage in this pattern be modeled by a linear function? Provide at least **2** pieces of evidence to support your answer.

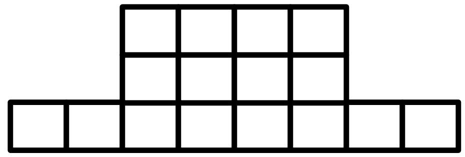
Design Team 2:



Current Display (Stage 1)



Stage 2



Stage 3

Stage 4

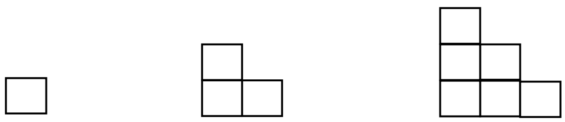
3. Draw Stage 4 of this design. Describe how you went about drawing stage 4.

4. Can the relationship between stage number and number of blocks in a stage in this pattern be modeled by a linear function? Provide at least **2** pieces of evidence to support your answer.

5.2a Homework: More Patterns – Are They Linear?

Directions: For each of the following patterns, draw the next stage and determine whether relationship between the stage number and the number of blocks in a stage can be represented by a linear function. Justify your answer.

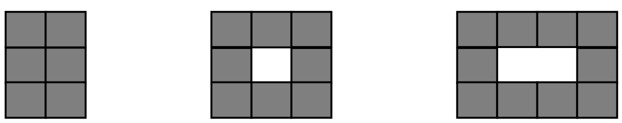
1.



Stage 1 Stage 2 Stage 3 Stage 4

a. Is this pattern linear? __________________
   Justification:

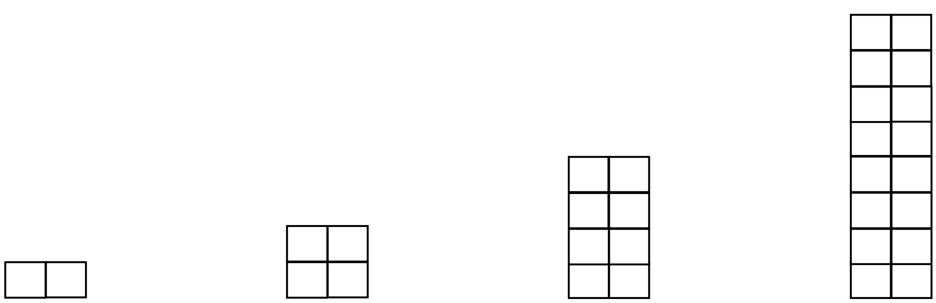
2. Consider the gray tiles only



Stage 1 Stage 2 Stage 3 Stage 4

a. Is this pattern linear? ___________________________
b. Justification:

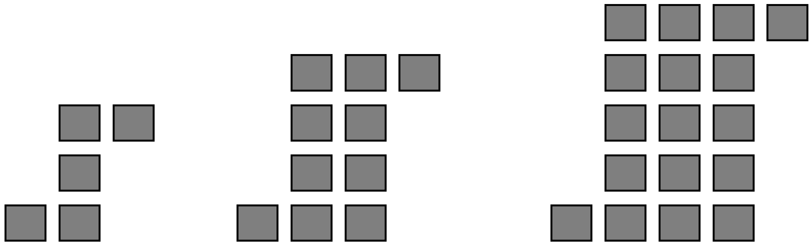
3.



Stage 1 Stage 2 Stage 3 Stage 4 Stage 5

a. Is this pattern linear? ________________________
b. Justification:

4.



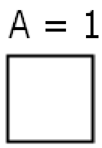
Stage 1 Stage 2 Stage 3 Stage 4

a. Is this pattern linear? ______________________
b. Justification:

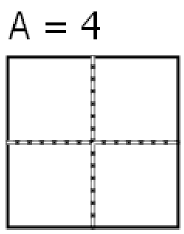
5. Make up your own pattern that is **not** linear. Prove that your pattern is not linear with at least **2** pieces of evidence.

5.2b Class Activity: Linear and NonLinear Functions in Context

1. Consider the **area** of a square as a function of the side length of the square.
   a. Draw pictures to represent these squares. The first two have been drawn for you.



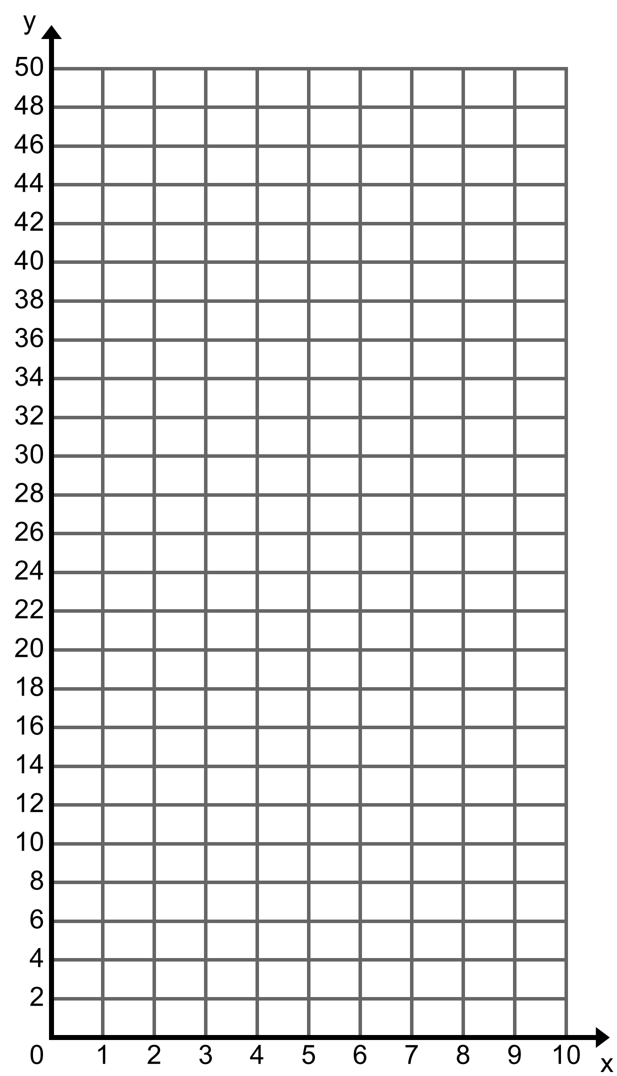
Side length = 1



Side length = 2

   b. Complete the graph and table for this function.

side length	area
1	
2	
3	
4	
5	



   c. What is the dependent variable? The independent variable?

   d. Write an equation to model *A* as a function of *s*.

   e. Does this graph pass through the point (8, 64)? Explain how you know.

   f. What does the point (8, 64) represent in this context?

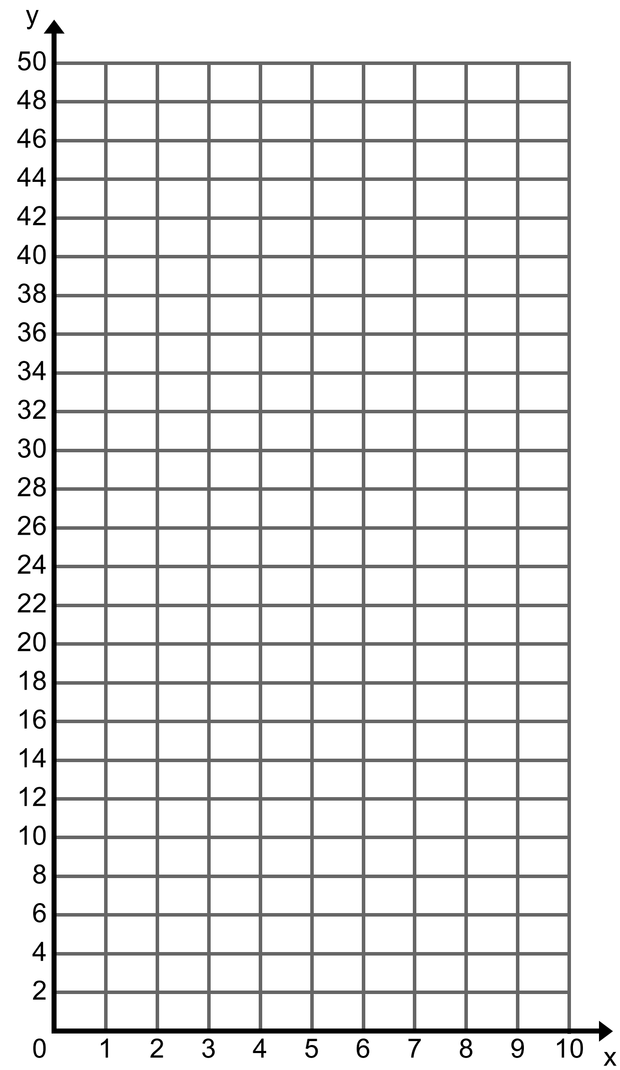
   g. List three more ordered pairs that this graph passes through.

   h. Is this function linear? Explain or show on the graph, table, and equation why or why not?

2. Consider the **perimeter** of a square as a function of the side length of the square.

a. Complete the graph and table for this function.

side length	perimeter
1	
2	
3	
4	
5	



b. What is the dependent variable? The independent variable?

c. Write an equation to model *P* as a function of *s*.

d. Find another ordered pair that the graph passes through.

e. What does the point (10, 40) represent in this context?

f. Is this function linear? Explain or show on the graph, table, and equation why or why not?

5.2b Homework: Linear and NonLinear Functions in Context

1. The following tables show the distance traveled by three different cars over five seconds.

Car 1	
Time (s)	Distance (ft.)
1	4
2	7
3	10
4	13
5	16

Car 2	
Time (s)	Distance (ft.)
1	2
2	5
3	10
4	17
5	26

Car 3	
Time (s)	Distance (ft.)
1	3
2	5
3	9
4	17
5	33

   a. Consider the relationship between time and distance traveled for each car. Which of the tables of data can be modeled by a linear function? Which ones cannot be modeled by a linear function? Justify your answer.

   b. For any of the data sets that can be modeled by a linear function, write a function that models the distance traveled D as a function of time t.

   c. What is the dependent variable in this situation? The independent variable?

   d. Which car is traveling fastest? Justify your answer.

2. Hermione argues that the table below represents a linear function. Is she correct? How do you know?

x	2	4	8	16
y	1	3	5	7

3. Emily's little brother painted on her math homework. She knows the data in each of the tables below represents a linear function. Help Emily determine what number is hidden behind the blob of paint.

a.

x	10	20	30	40
y	8	13		23

b.

x	–2	0	2	3
y	–5		7	10

c.

x	0	1		6
y	0	3	9	18

Directions: Choose **3** of the following situations. Determine whether the situations you choose can be modeled by a linear function or not. Provide evidence to support your claim. Show your work in the space below.

4. Mr. Cortez drove at a constant rate for 5 hours. At the end of 2 hours he had driven 90 miles. After 5 hours, he had driven 225 miles.
   a. What is the dependent variable in this relationship? ___________________________

   b. What is the independent variable in this relationship? ___________________________

   c. Can this relationship be modeled by a linear function? Provide evidence to support your claim.

5. Round 1 of a tennis tournament starts with 64 players. After each round, half the players have lost and are eliminated from the tournament. Therefore, in round 2 there are 32 players, in round 3 there are 16 players and so on.

   a. What is the dependent variable in this relationship? ___________________________

   b. What is the independent variable in this relationship? ___________________________

   c. Can this relationship be modeled by a linear function? Provide evidence to support your claim.

6. A rock is dropped from a cliff that is 200 feet above the ground. The table below represents the height of the rock (in feet) with respect to time (in seconds).

Time (s)	**Height (ft.)**
0	200
1	184
2	136
3	56

   a. What is the dependent variable in this relationship? __________________________

   b. What is the independent variable in this relationship? __________________________

   c. Can this relationship be modeled by a linear function? Provide evidence to support your claim.

7. A student comes to school with the flu and infects three other students within an hour before going home. Each newly infected student passes the virus to three new students in the next hour. This pattern continues until all students in the school are infected with the virus.
    a. What is the dependent variable in this relationship? ___________________________

    b. What is the independent variable in this relationship? __________________________

    c. Can this relationship be modeled by a linear function? Provide evidence to support your claim.

8. A piece of paper is cut into two equal sections. Each new piece is cut into two additional pieces of equal size. This pattern continues until it is no longer possible to cut the paper any more.
    a. What is the dependent variable in this relationship? ______________________________

    b. What is the independent variable in this relationship? _____________________________

    c. Can this relationship be modeled by a linear function? Provide evidence to support your claim.

5.2d Class Activity: Different Types of Functions

1. Sketch the general appearance of the graph of the equation $y = mx + b$.
   a. What do m and b represent?
   b. What makes the graph linear?

2. Complete the table of values for the functions shown in the table below. Using the table of values, predict what the graphs of the equations will look like. Compare the tables to the table for $y = x$.

$y = x$	
x	y
−2	
−1	
0	
1	
2	

$y = \|x\|$	
x	y
−2	
−1	
0	
1	
2	

$y = x^2$	
x	y
−2	
−1	
0	
1	
2	

$y = \frac{1}{x}$	
x	y
−2	
−1	
0	
1	
2	

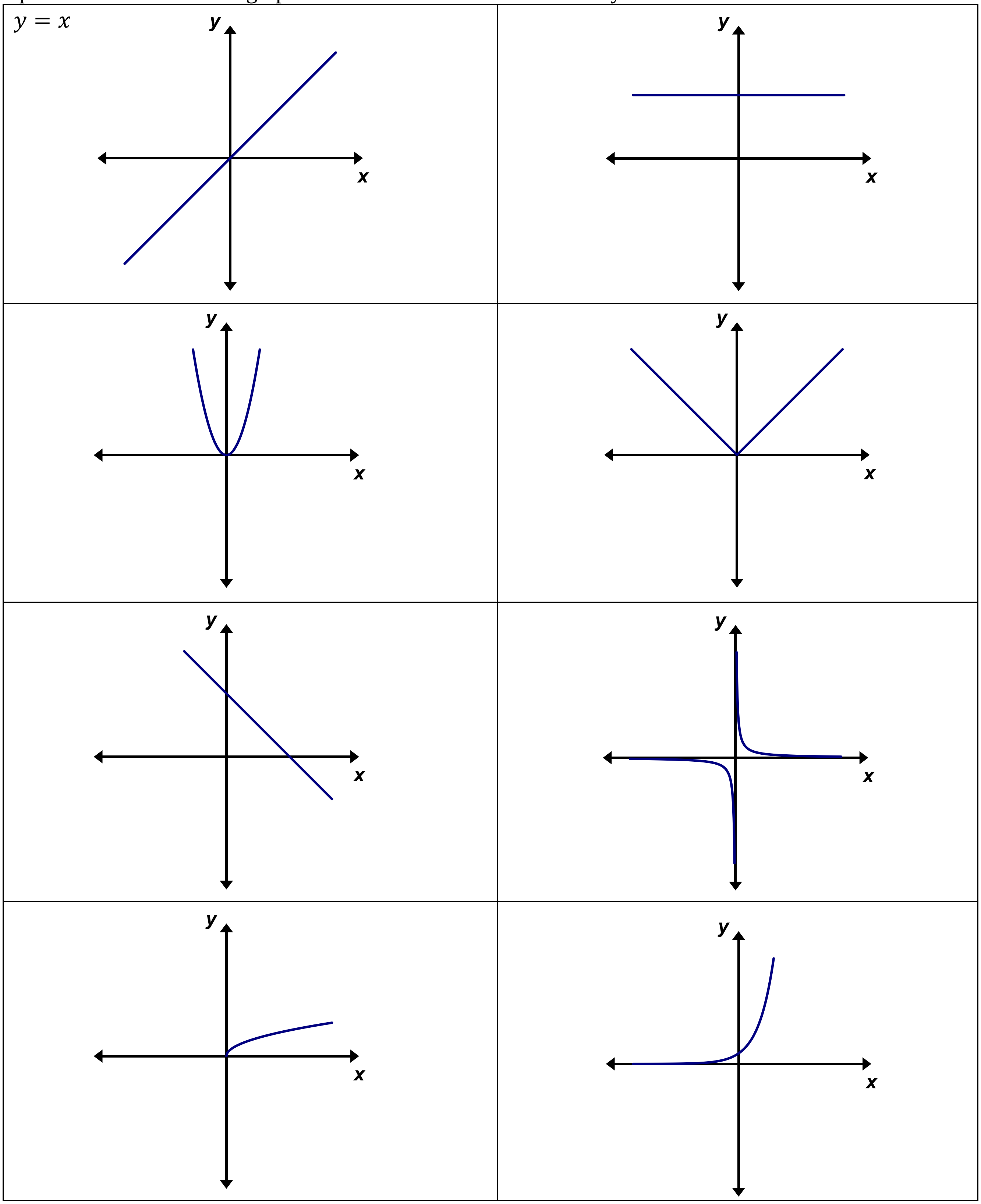
$y = \sqrt{x}$	
x	y
0	
1	
2	
3	
4	

$y = 2^x$	
x	y
0	
1	
2	
3	
4	

$x + y = 6$	
x	y
−2	
−1	
0	
1	
2	

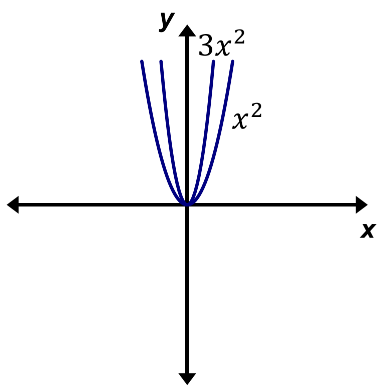
$y = 6$	
x	y
−2	
−1	
0	
1	
2	

3. Use the tables of values from the previous page to match each equation to its graph below. Write the equation to the left of the graph. The first one has been done for you.

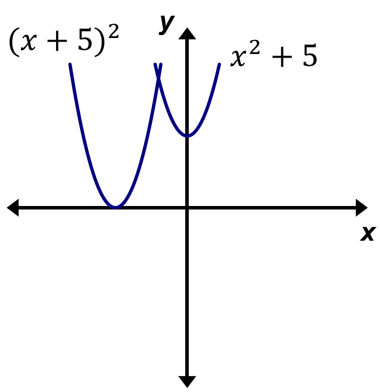


4. Compare each of the graphs on the previous page to the graph of $y = x$. What is the same? What is different? Discuss with a class mate.

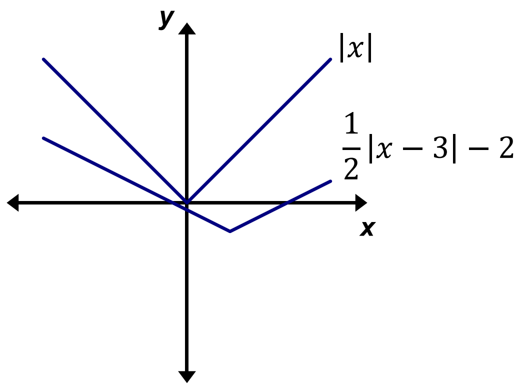
5. The graphs of $x^2$and $3x^2$ are shown below. Compare these graphs. What is the same? What is different?



6. The graphs of $y = x^2 + 5$ and $y = (x + 5)^2$ are shown below. Compare these graphs to the graph of $y = x^2$. What is the same? What is different?



7. The graphs of $y = |x|$ and $y = \frac{1}{2}|x - 3| - 2$ are shown below. Compare these graphs. What is the same? What is different?

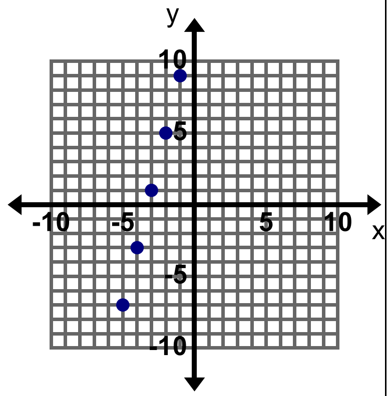
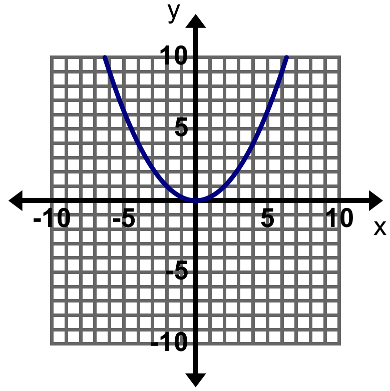
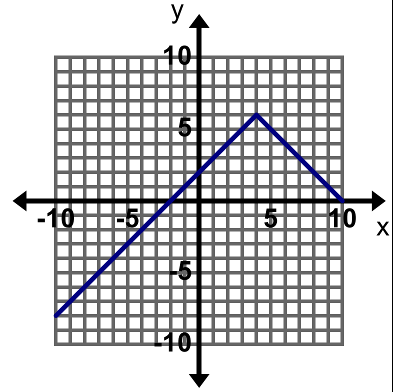
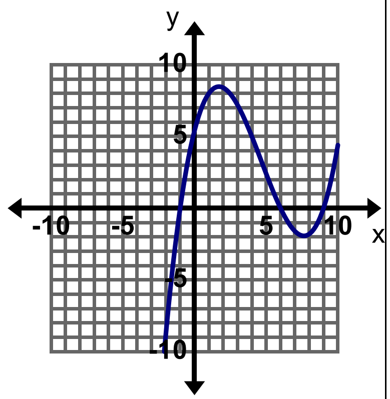
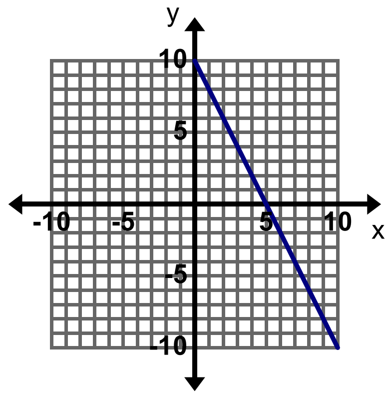
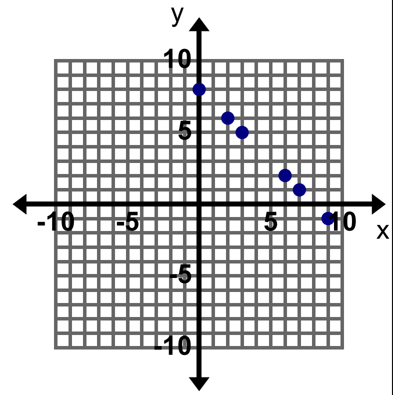
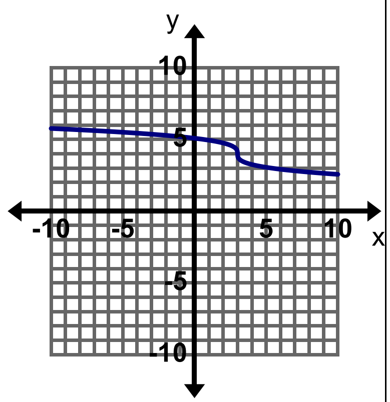
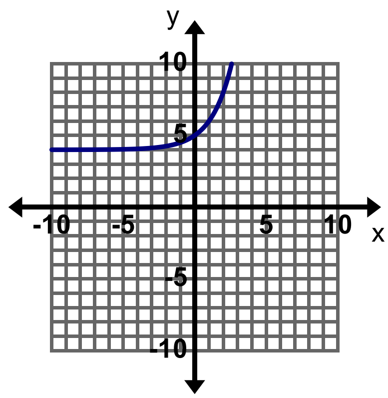
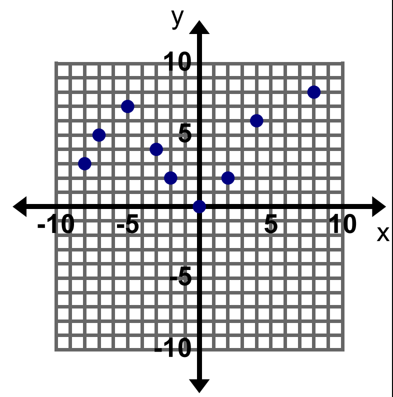


8. Describe the basic structure of an equation that defines a **linear function**. Think about the different forms a linear equation might take. Provide examples of the different forms.

9. Describe attributes you see in the equations that define **nonlinear functions**. Provide examples.

5.2d Homework: Different Types of Functions

1. Circle the letter next to the graph if it represents a linear function.

A		**B**		**C**	
D		**E**		**F**	
G		**H**		**I**	

Bonus: Do any of the graphs of nonlinear functions have a shape similar to the ones studied in class? Make a prediction about the basic structure of the equations of these functions.

2. Circle the letter next to the table if the data represents a linear function.

A	**B**	**C**	**D**
x: 0, 1, 2, 3; *y*: –5, 0, 5, 10	*x*: 0, 1, 2, 3; *y*: 15, 12.5, 10, 7.5	*x*: 0, 1, 2, 3; *y*: 4, 8, 16, 32	*x*: 0, 1, 2, 3; *y*: 98, 98, 98, 98

A

x	*y*
0	–5
1	0
2	5
3	10

B

x	*y*
0	15
1	12.5
2	10
3	7.5

C

x	*y*
0	4
1	8
2	16
3	32

D

x	*y*
0	98
1	98
2	98
3	98

E

x	*y*
–3	3
–2	2
2	2
3	3

F

x	*y*
1	2
3	4
6	6
10	8

G

x	*y*
2.1	4
2.2	5
2.3	6
2.4	7

H

x	*y*
0	2
4	6
8	18
12	54

I

x	*y*
3	20
6	24
12	32
24	48

J

x	*y*
10	–20
30	–40
50	–60
70	80

K

x	*y*
5	0
10	–1
15	–2
20	–3

L

x	*y*
15.1	4.2
16.7	12.2
18.3	20.2
19.9	28.2

3. Circle the letter next to each equation if it represents a linear function.

A	$2x + 4y = 16$	**B**	$y = \|2x\| + 5$	**C**	$y = x^2 + 5$	**D**	$y = 5 \cdot 3^x$
E	$y = \frac{4}{x} + 3$	**F**	$y = \frac{x}{4} + 3$	**G**	$y = \sqrt{4x}$	**H**	$x^2 + y^2 = 25$
I	$xy = 24$	**J**	$2x + y = 6$	**K**	$y = -\frac{2}{3}x$	**L**	$y = 8$
M	$y = \frac{2}{3}x$	**N**	$y = x^3$	**O**	$3x - y = 2$	**P**	$y = x(x + 2)$

Bonus: Can you predict the basic shape of any of the graphs of the nonlinear equations in #3?

5.2e Class Activity: Matching Representations of Functions

Matching Activity: Match the following representations together. Each representation will have a

1) a story,
2) an equation,
3) a table of values, and
4) a graph.

After you have matched the representations, **label** the axes of the graphs on the graph cards, **answer** the questions asked in the word problems on the story cards, and **identify** the dependent and independent variable in each story.

Story	Equation	Table	Graph
DD			
Z			
AA			
FF			
EE			
Y			
BB			
CC			

A

x	0	3	6	9	12	15
y	6	7	8	9	10	11

B

x	0	1	2	3	4	5
y	6	100	162	192	190	156

C

x	0	4	8	12	16	20
y	0	200	200	200	224	248

D

x	0	1	2	3	4	5
y	6	12	24	48	96	192

E

x	0	5	10	15	20	25
y	6	7	8	9	10	11

F

x	0	1	2	3	4	5
y	200	196	192	188	184	180

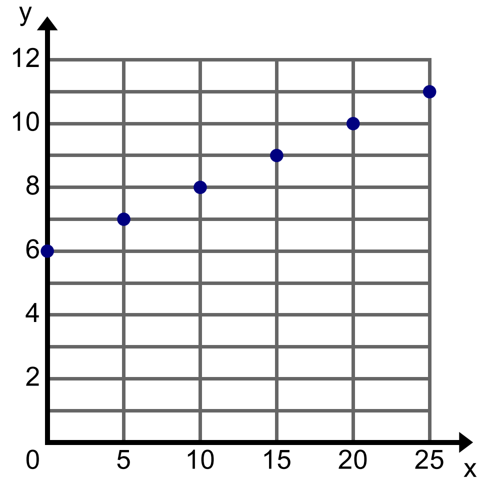
G

x	0	1	2	3	4	5
y	6	206	406	606	806	1006

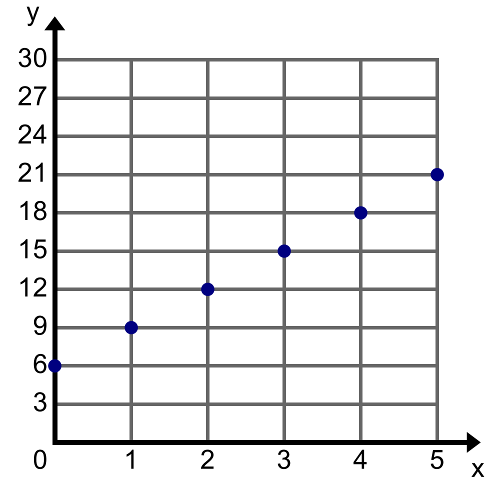
H

x	0	1	2	3	4	5
y	6	9	12	15	18	21

I


y
12
10
8
6
4
2
0
5
10
15
20
25
x

J


y
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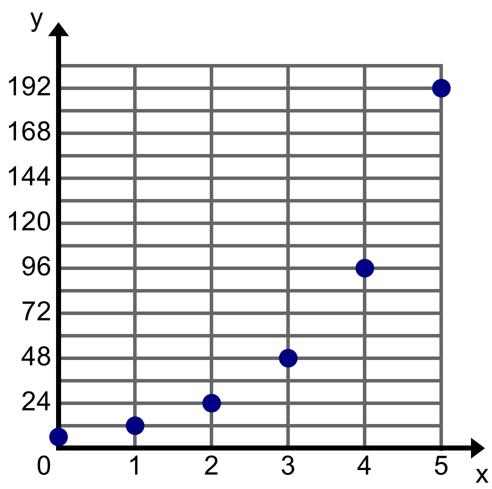
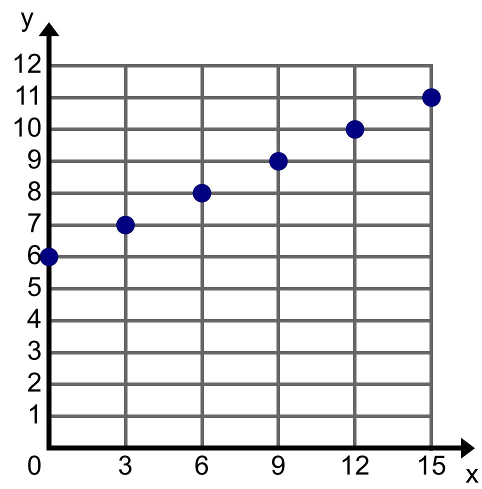
27
24
21
18
15
12
9
6
3
0
1
2
3
4
5
x

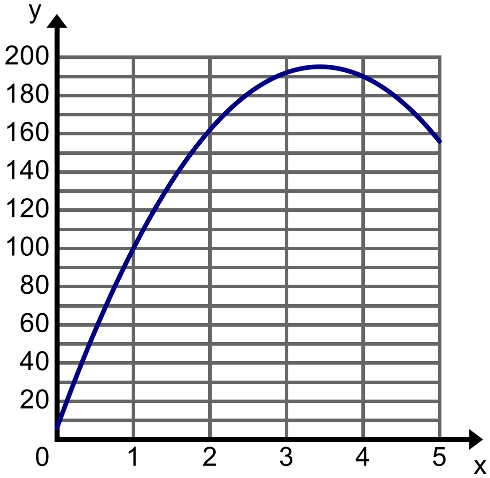
K


y
192
168
144
120
96
72
48
24
0
1
2
3
4
5
x

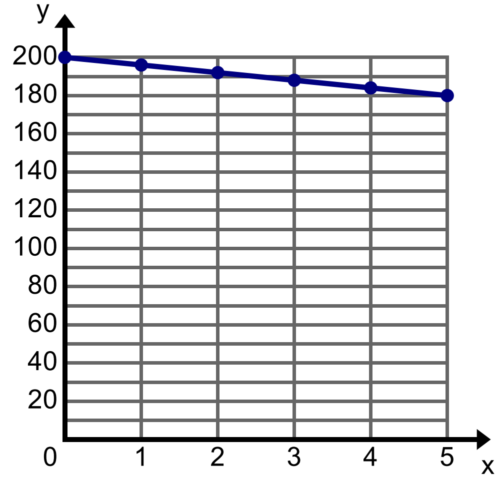
L


y
12
11
10
9
8
7
6
5
4
3
2
1
0
3
6
9
12
15
x

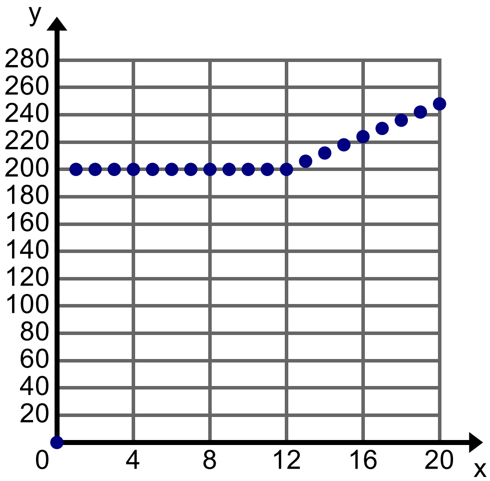
M


y
200
180
160
140
120
100
80
60
40
20
0
1
2
3
4
5
x

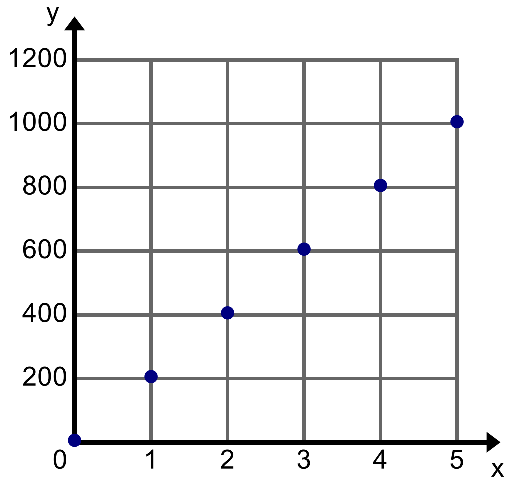
N


y
200
180
160
140
120
100
80
60
40
20
0
1
2
3
4
5
x

O


y
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
4
8
12
16
20
x

P


y
1200
1000
800
600
400
200
0
1
2
3
4
5
x

Q

$y = -4x + 200$

R

$y = -16x^2 + 110x + 6$

S

$y = 3x + 6$

T

$y = 200x + 6$

U

$y = 6 \cdot 2^x$

V

$y = \begin{cases} 200, & \text{if } 0 < x \leq 12 \\ 200 + 6x, & \text{if } x > 12 \end{cases}$

W

$y = \frac{1}{5}x + 6$

X

$y = \frac{1}{3}x + 6$

Y

A certain bacteria reproduces by binary fission every hour. This means that one bacterium grows to twice its size, replicates its DNA, and splits in 2. If 6 of these bacterium are placed in a petri dish, how many will there by after 5 hours?

Dependent Variable:
Independent Variable:

Z

The state is building a road 4.5 km long from point A to point B. Six meters of the road have already been completed when a new crew starts the job. It takes the crew 3 weeks to complete 600 meters of the road. How much of the road will be completed after 5 weeks if the crew works at a constant rate?

Dependent Variable:
Independent Variable:

AA

Talen loves to help his mom clean to earn money for his cash box. He currently has $6 in his cash box. He earns $1 for every 3 jobs he does. How much money will Talen have if he does 15 jobs?

Dependent Variable:
Independent Variable:

BB

Josh is draining a swimming pool at a constant rate of 4 gallons per minute. If the swimming pool starts will 200 gallons of water, how many gallons will remain after 5 minutes?

Dependent Variable:
Independent Variable:

CC

Kendall's mom and dad have agreed to sponsor her in a school walk-a-thon to raise money for soccer uniforms. Her mom is donating $6 to her. Her dad is donating $3 for each mile she walks. How much money will she collect if she walks 5 miles?

Dependent Variable:
Independent Variable:

DD

The Planetarium charges $200 for a birthday party for up to 12 guests. Each additional guest is $6. How much will it cost for a birthday party with 20 guests?

Dependent Variable:
Independent Variable:

EE

Suppose a rocket is fired from a platform 6 ft. off the ground into the air vertically with an initial speed of 110 feet/second. Where will the rocket be after 5 seconds? *Note:* The gravitational force of the earth on the rocket is -16 ft./sec^2.

Dependent Variable:
Independent Variable:

FF

Suzy is helping her mom fill Easter eggs with jelly beans for a community egg hunt. Before they get started, Suzy eats 6 jelly beans. Her mom tells her that after that she can eat 1 jelly bean for every 5 eggs she fills. How many jelly beans total did Suzy eat if she filled 25 eggs?

Dependent Variable:
Independent Variable:

5.2e Homework: Matching Representations of Functions

Directions: Create each of the following representations.

1. a table of data that represents a linear function	2. a table of data that represents a nonlinear function
3. a graph that represents a linear function	4. a graph that represents a nonlinear function
5. an equation that defines a linear function	6. an equation that defines a nonlinear function
7. a context (story) that can be modeled by a linear function	8. a context (story) that can be modeled by a nonlinear function

5.2f Self-Assessment: Section 5.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Mastery 3**	**Substantial Mastery 4**
1. Distinguish between linear and nonlinear functions given a context, table, graph, or equation.				

1. On Tamara’s first day of math class, her teacher asked the students to shake hands with everyone in the room to introduce themselves. There are 26 students total in the math class. Can the relationship between number of people and number of handshakes exchanged be modeled by a linear function? Why or why not? Can you determine the number of handshakes that took place in Tamara’s math class on the first day of class? Justify your answer.

Section 5.3: Model and Analyze a Functional Relationship

Section Overview:
In this section, students will analyze functional relationships between two quantities given different representations. For relationships that are linear, students will construct a function to model the relationship between the quantities. Students will also compare properties of linear functions represented in different ways, examining rates of change and intercepts, and using this information to solve problems. Next, students will learn about some of the key features of graphs of functions and apply this knowledge in order to describe qualitatively the functional relationship between two quantities. Lastly, students will sketch graphs that display key features of a function given a verbal description of the relationship between two quantities.

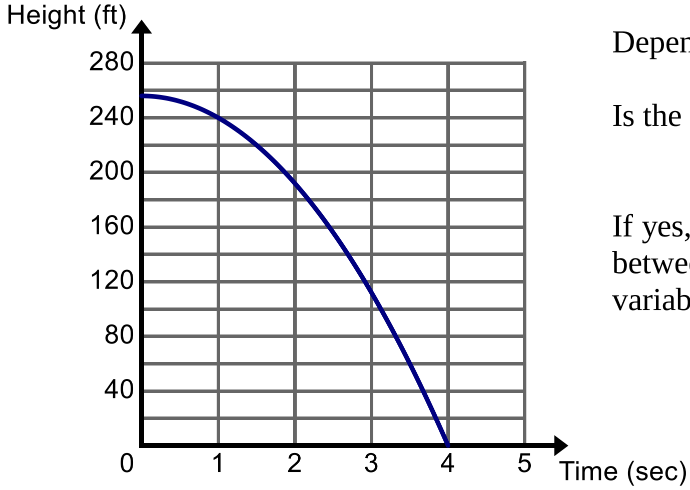
Concepts and Skills to Master:
By the end of this section, students should be able to:

- Determine whether the relationship between two quantities can be modeled by a linear function and construct a function to model a linear relationship between two quantities.
- Compare properties of linear functions (rates of change and intercepts) and use this information to solve problems.
- Identify and interpret key features of a graph that models a relationship between two quantities.
- Sketch a graph that displays key features of a function that has been described verbally.

5.3a Class Activity: Constructing Linear Functions

Directions: Identify the dependent and independent variable in the following situations. Determine whether the situations are linear or nonlinear. **For the situations that are linear,** construct a function that models the relationship between the two quantities. Be sure to define your variables.

1. An object is dropped from a bridge into the water below. The graph below shows the height of the object (in feet) with respect to time (in seconds). Consider the relationship between the height of the object and time.



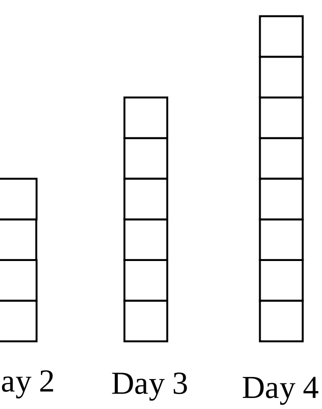
Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two quantities. Be sure to define your variables.

2. Owen is earning pennies each day that he makes his bed in the morning. On the first day, Owen's mom gives him 2 pennies. On the second day, Owen's mom gives him 4 pennies, on the third day 6 pennies, on the fourth day 8 pennies, and so on. Owen makes his bed every day and this pattern continues. The model below shows how many pennies Owen earns each day (each box represents 1 penny). Consider the relationship between **the number of pennies received on a given day** and the **day number**.



Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

3. Refer back to #2 and Owen earning pennies. Consider the relationship between the <u>**total**</u> **number of pennies** Owen has earned and the **day number**.

Day	# of Pennies Added That Day	Sum of Pennies

Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

4. Carbon-14 has a half-life of 5,730 years. The table below shows the amount of carbon-14 that will remain after a given number of years. Consider the relationship between number of years and amount of carbon-14 remaining.

# of Years	**Milligrams of Carbon-14**
0	8
5,730	4
11,460	2
17,190	1
22,920	$\frac{1}{2}$

Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

5. The table below shows the amount of time a recipe recommends you should roast a turkey at 325˚ F dependent on the weight of the turkey in pounds. Consider the relationship between cooking time and weight of the turkey.

Weight of Turkey (lbs.)	12	13	14	15
Cooking Time (hours)	4	$4\frac{1}{3}$	$4\frac{2}{3}$	5

Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

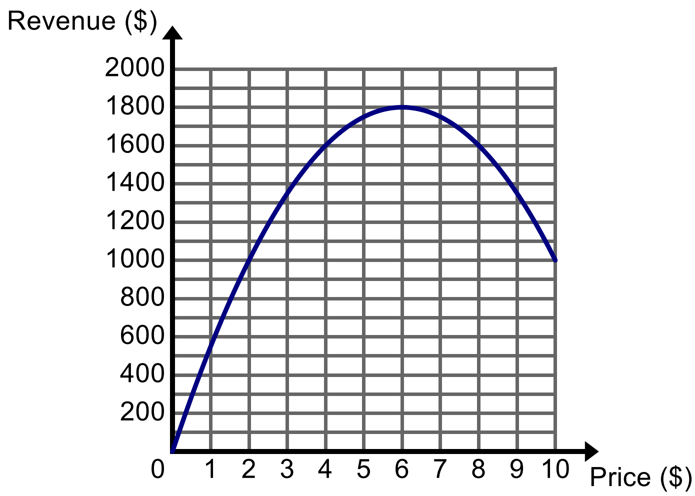
If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

6. Steve is a lifeguard at a local community pool. Each day at noon, he records the temperature and the number of people in the pool. Do you think the relationship between temperature and number of people in the pool is linear? Why or why not?

5.3a Homework: Constructing Linear Functions

Directions: Determine whether the situations represented below are linear or nonlinear. **For the situations that are linear,** construct a function that models the relationship between the two quantities. Be sure to define your variables.

1. The graph below shows the amount of revenue a company will make selling t-shirts dependent on the price of each t-shirt. Consider the relationship between price of each shirt and revenue made.



Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

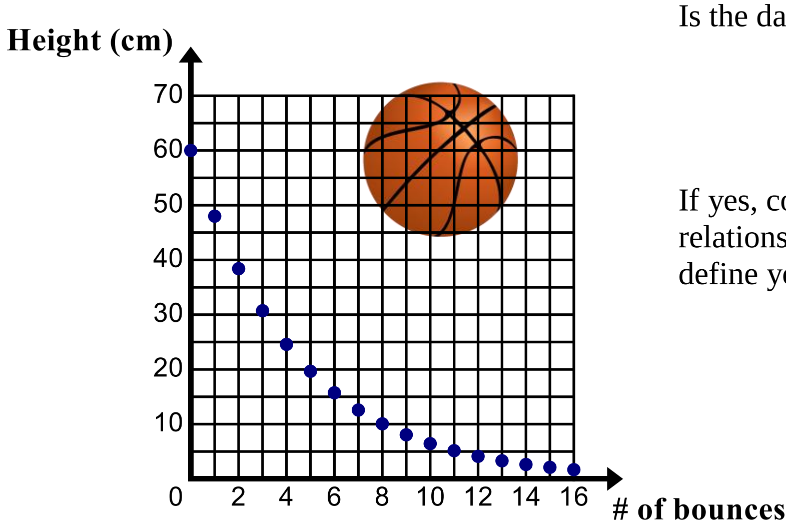
2. When Camilo opened his email this morning he had 140 unread emails. The table below shows the number of remaining unread emails Camilo has in his inbox. Assume that Camilo does not receive any new emails while he is reading his email. Consider the relationship between time and the number of unread emails.

Time (hours)	# of Unread Emails
0	180
0.5	160
1	140
2	100
2.5	80
4	20
4.5	0

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two quantities. Be sure to define your variables.

3. Suppose you drop a basketball from a height of 60 inches. The graph below shows the height of the object after *b* bounces.



Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

4. Justine and her family are floating down a river. After 1 hour, they have floated 1.25 miles, after 4 hours they have floated 5 miles, and after 6 hours they have floated 7.5 miles. Is the relationship between time (in hours) and distance (in miles) linear? Why or why not? If it is linear, write a function that models the relationship between the two quantities.

5. You and your friends go to a BMX dirt-biking race. For one of the events, the competitors are going off a jump. The winner of the event is the competitor that gets the most air (or jumps the highest). Do you think the relationship between the weight of the bike and the height of the jump can be modeled by a linear relationship? Why or why not?

6. Homes in a certain neighborhood sell for $117 per square foot. Can the relationship between the number of square feet in the home and the sale price of the home be modeled by a linear function? Why or why not? If it can be modeled by a linear function, write a function that models the relationship between the two quantities.

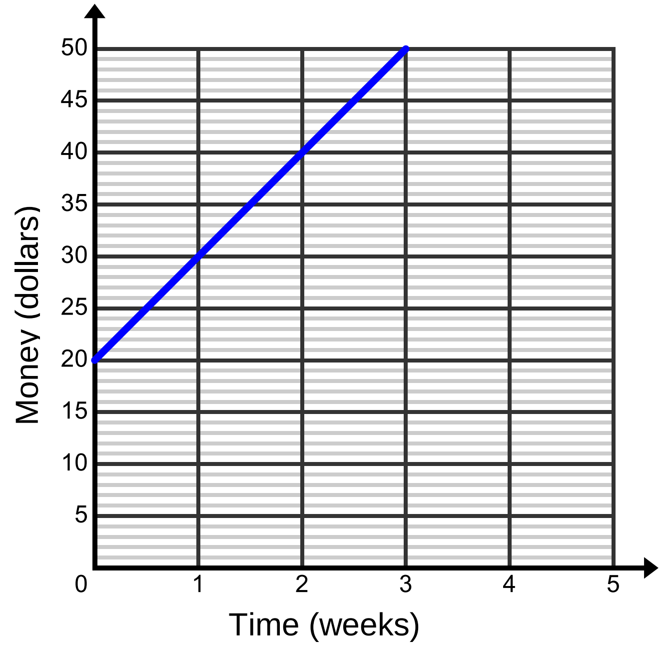
7. Suppose a certain bank pays 4% interest at the end of each year on the money in an account. When Devon was born, his parents put $100 in the account and will leave it there until he goes to college. Is the relationship between time (in years) and the amount of money in the account (in dollars) linear or not? Why or why not? If it is linear, write a function that models the relationship between the two quantities.

5.3b Class Activity: Comparing Linear Functions

1. Who will have \$100 first, George or Mark?

George has \$20 and is saving \$15 every week.

Mark starts with \$20. His savings are shown on the graph below.



2. Put the cyclists in order from slowest to fastest. (Note variables: *x* = time in seconds, *y* = meters traveled)

Cyclist A:

Time (*x*)	Distance (*y*)
2	1
4	2
6	3

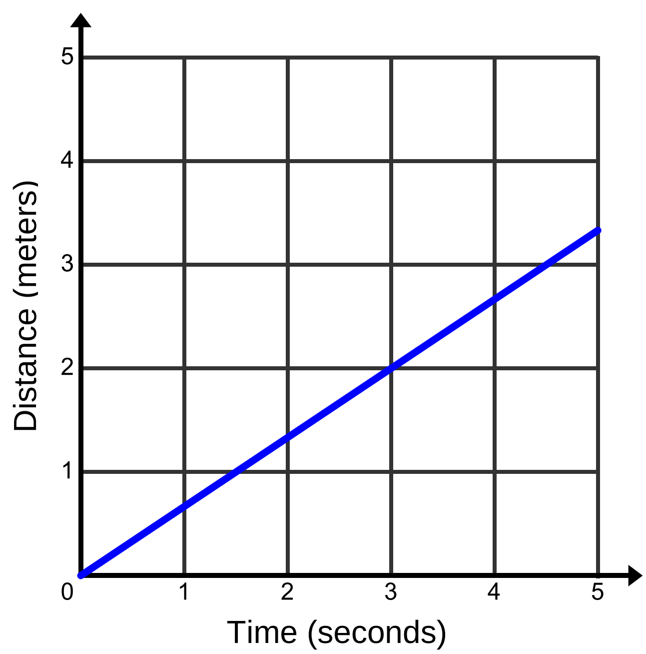
Cyclist B:

Bob has cycled 12 meters in the past 6 seconds.

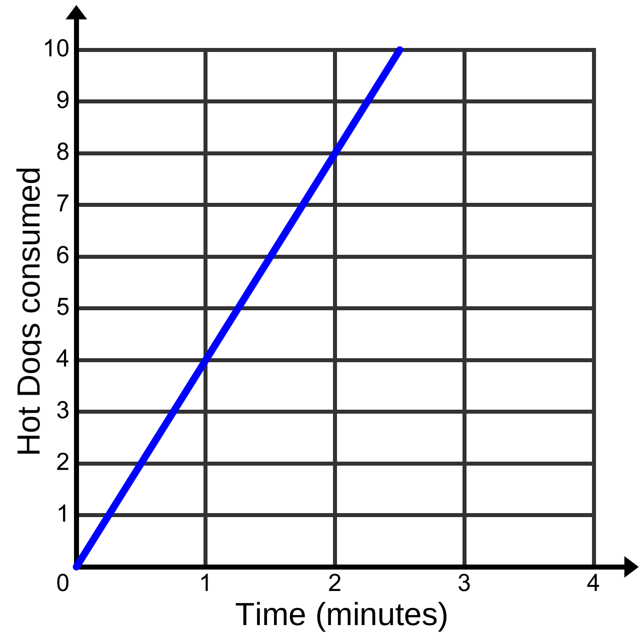
Cyclist C:

$y = \frac{1}{3}x$

Cyclist D:



3. Assume the rates below will remain constant. Who will win the hot dog eating contest? Why?

Helga, who has eaten 18 hot dogs in 5 minutes.	Pablo whose eating record is shown below. 

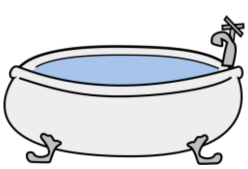
4. Based on the information below, which bathtub will be empty first? Why?

Bathtub A:

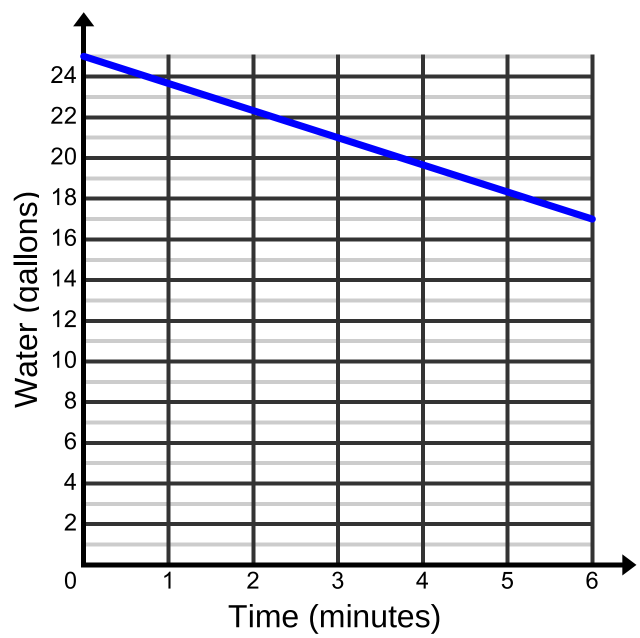
Starts with 25 gallons and is draining 1.5 gallons a minute.

Bathtub B:

Minutes	Gallons
0	25
3	20
6	15



Bathtub C:



5. Put the cars in order from fastest to slowest. (Note variables: x = time in hours, y = miles traveled). Assume all cars travel at a constant rate.

Car A:

$y = 65x$

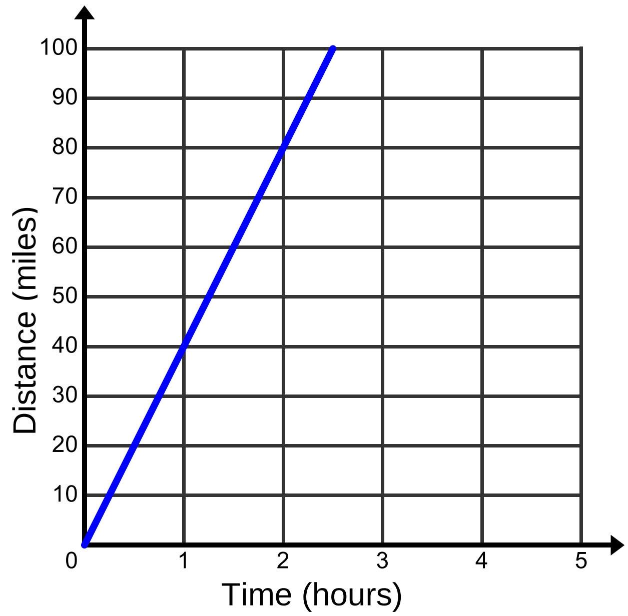
Car B:

The Bakers completed the 64-mile trip to Salt Lake City in 50 minutes.

Car C:

Hours	Miles
2	150
4	300

Car D:



6. Put the exercises below in order from burns the most calories to burns the least calories. (Note variables: x = time in minutes, y = calories burned). Assume the rate at which you burn calories in each of the exercises is constant.

Cycling:

$y = 10.8x$

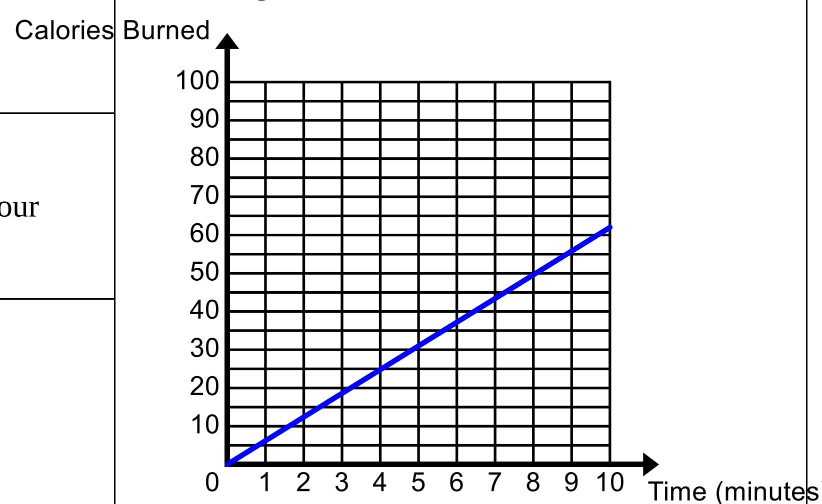
Yoga:

Meredith burned 429 calories in a 1-hour long yoga class.

Running:

Minutes	Calories Burned
10	137
15	205.5
25	342.5

Swimming:



7. Four families are meeting up in Disneyland. Each family starts driving from home. The representations below show the distance each family is from Disneyland over time. (Note variables: x = time in hours, y = distance from Disneyland.) Assume the families drive to Disneyland at a constant rate.

Family A:

$y = 95 - 55x$

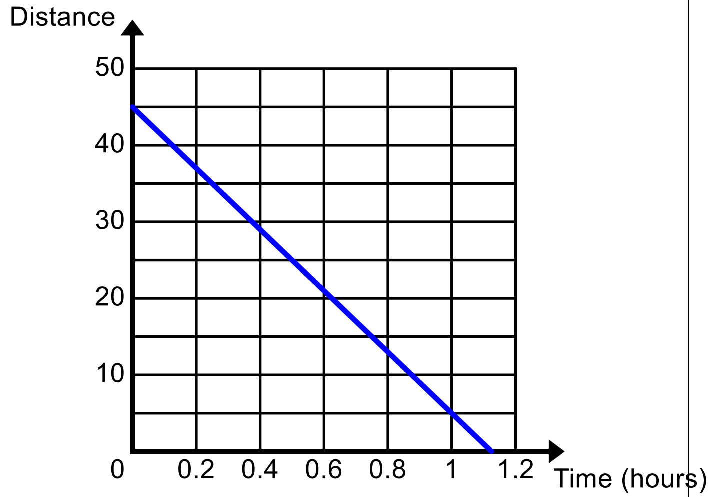
Family B:

Family B lives 120 miles from Disneyland and drives 60 mph.

Family C:

Hours	Distance from Disneyland
0	80
1.5	5

Family D:



a. Which family lives the closest to Disneyland?

b. Which family lives the farthest from Disneyland?

c. Which family is traveling at the fastest speed?

d. Which family is traveling at the slowest speed?

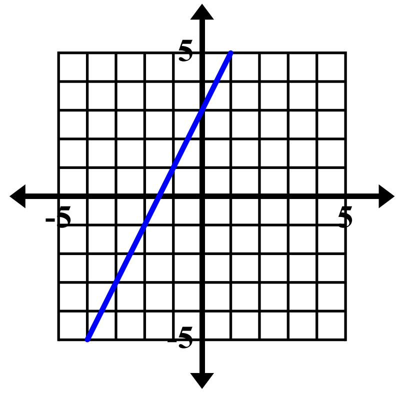
e. Who will get to Disneyland first?

f. Who will get to Disneyland last?

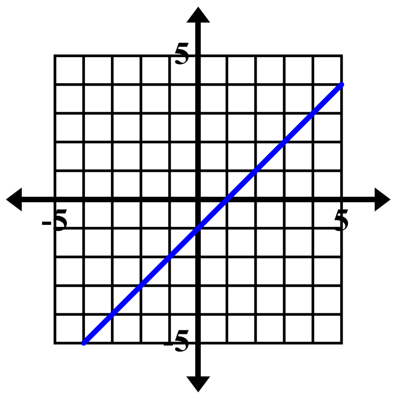
Directions: For each problem, circle the representation with the greatest rate of change. Put a star by the representation with the greatest *y*-intercept. Assume all representations have a constant rate of change.

8. $y = 2 + 3.5x$

x	***y***
1	8
5	20
7	26

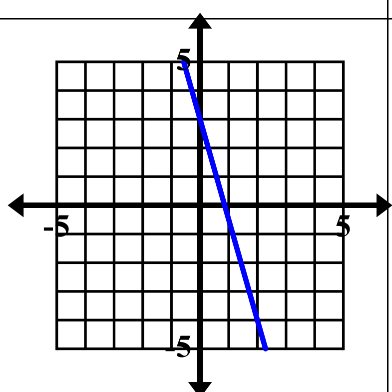


9. $y = \frac{3}{2}x$



$(0,1)(1,2.2)$

10.

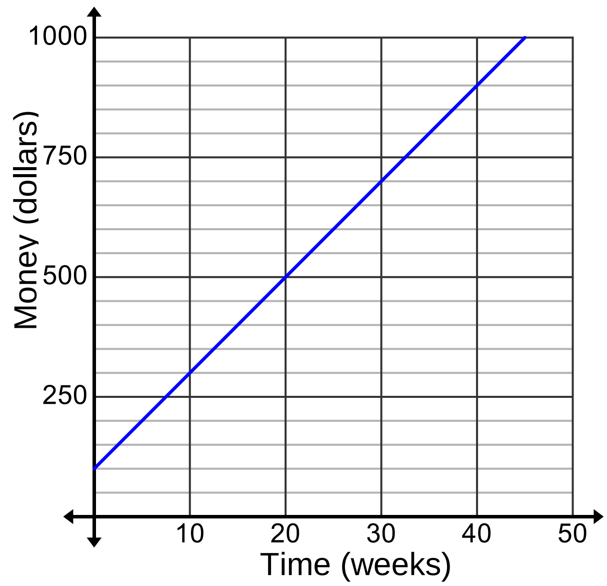


x	***y***
0	4
2	-2
5	-11

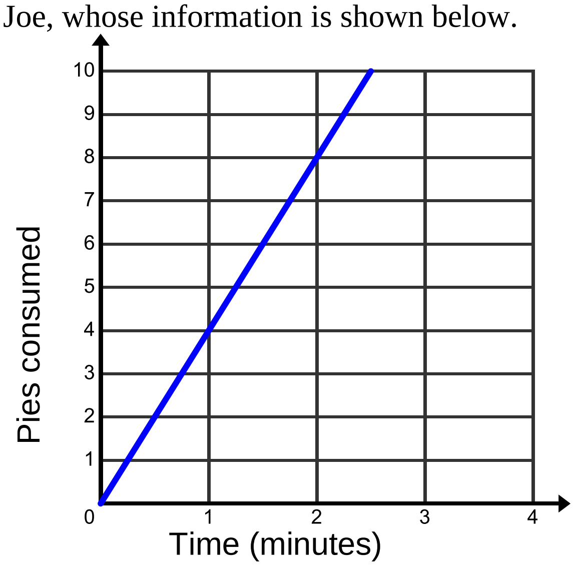
$(0,3)(2,-5)$

5.3b Homework: Comparing Linear Functions

1. Who will have $1,000 first, Becky or Olga?

Becky has $100 and is saving $10 every week.	Olga's information is shown on the graph below. 

2. Assume the rates below will remain constant. Who will win the pie eating contest? Why?

Joe, whose information is shown below. 	Donna, who has eaten 11 pies in 2.5 minutes.

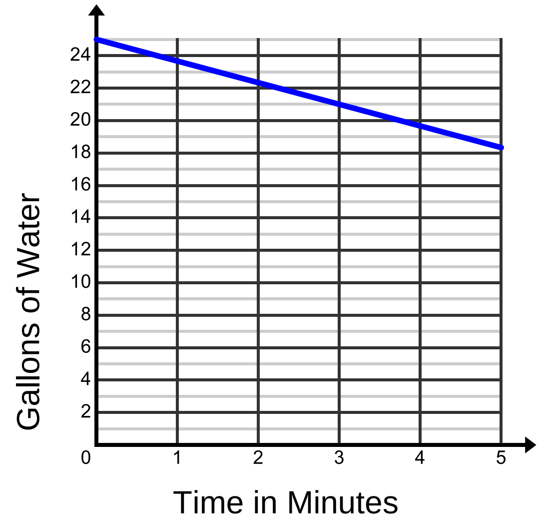
3. Based on the information below, which hot water heater will use up the available hot water first?

Water Heater A:
Starts with 50 gallons and drains 1.5 gallons a minute.

Water Heater B:

Time in Minutes	Gallons
0	30
2	27.5
4	25

Water Heater C:



4. Put the cars in order from fastest to slowest. (Note variables: x = time in hours, y = miles traveled.)

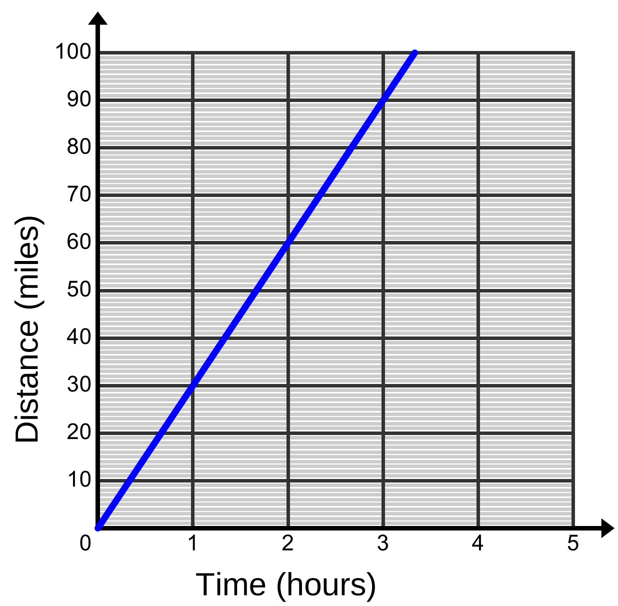
Car A:
$y = 35x$

Car B:
The Andersons completed the 24 mile trip to Salt Lake City in 30 minutes.

Car C:

Time (hours)	Distance (miles)
0.5	18
2.5	90
4	144

Car D:



5. Put the runners in order from slowest to fastest. (Note variables: x = time in minutes, y = miles traveled.)

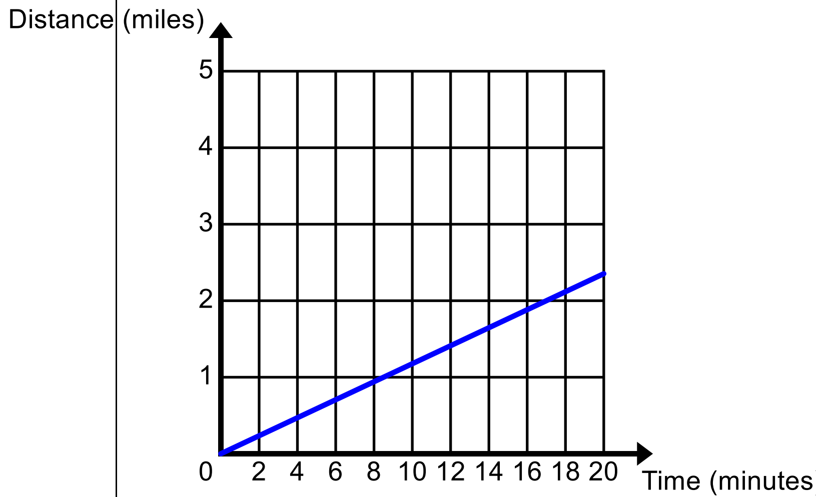
Ellen ran 1 mile in the last 10 minutes

Samantha: $y = \frac{2}{13}x$

Jason:

Time (minutes)	Distance (miles)
3	0.5
6	1
9	1.5

Dale:



6. Use the representations below to answer the questions that follow. (Note variables: x = time in weeks, y = amount of money remaining.)

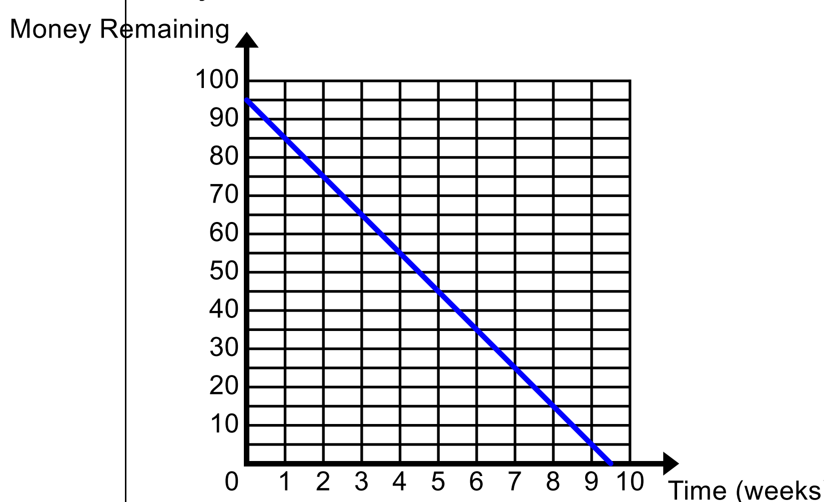
Imiko starts with $60 and spends $2 per week

Henry: $y = 80 - 5x$

Garek:

Time (weeks)	Amount of Money Remaining
2	59
5	35
7	19

Roslyn:



a. Who starts with the most money?

b. Who is spending his/her money at the fastest rate?

c. Who will run out of money first at the current rate of spending?

7. Jack, George, Lucy, and Anna are playing games on their iPads. The representations below show the battery life remaining on each child's iPad over time. (Note variables: x = time in hours, y = battery life remaining as a percent.) Use these representations to answer the questions that follow.

Jack:

$y = 78 - 10x$

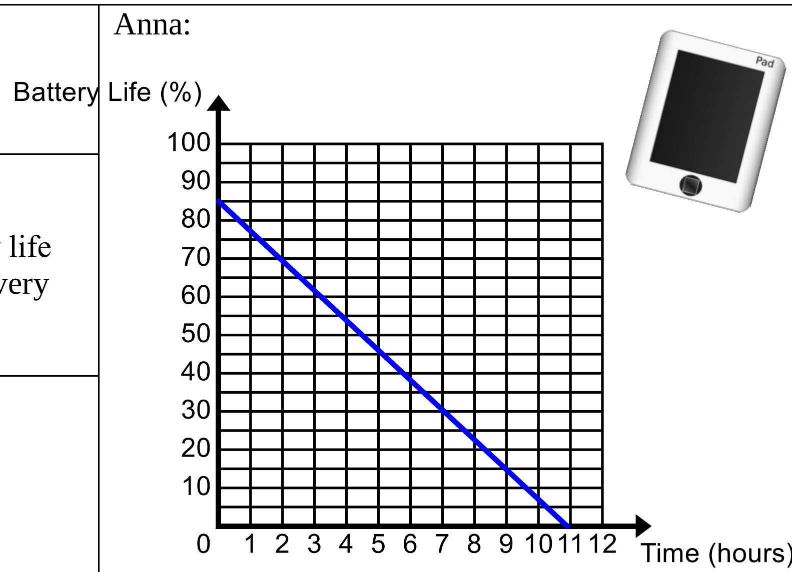
George:

George's iPad started with 92% battery life and is using 12.5% of the battery life every hour.

Lucy:

Hours	Battery Life Remaining (%)
0	76
2	60
3	52

Anna:



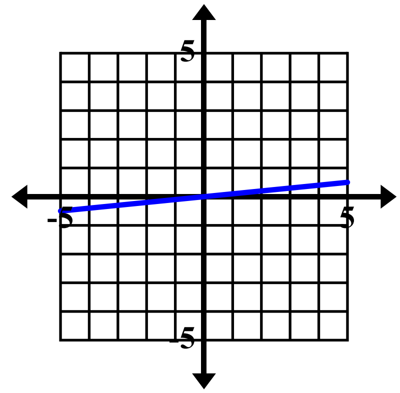
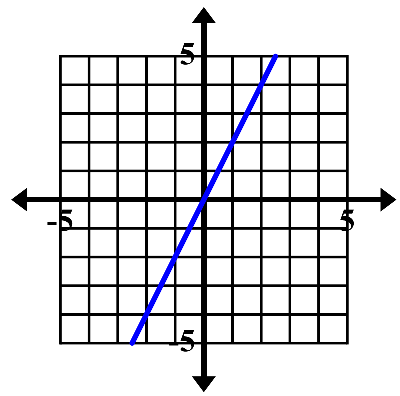
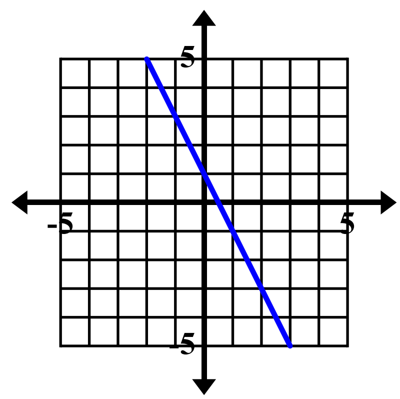
a. Whose iPad had the most battery life when the kids started playing?

b. Whose iPad is using the battery at the fastest rate? At the slowest rate?

c. Who will run out of battery life first?

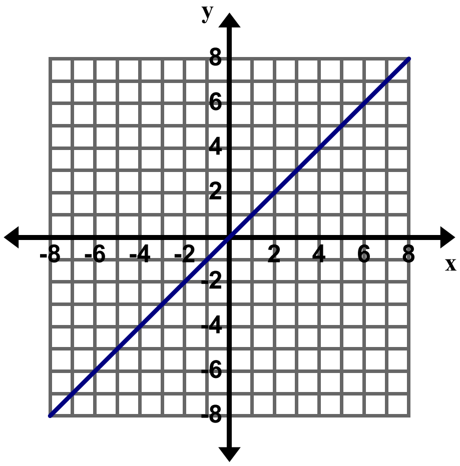
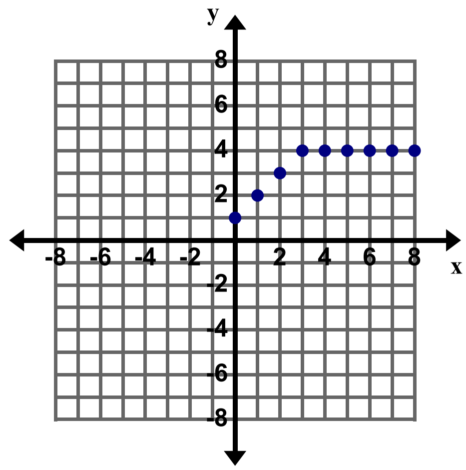
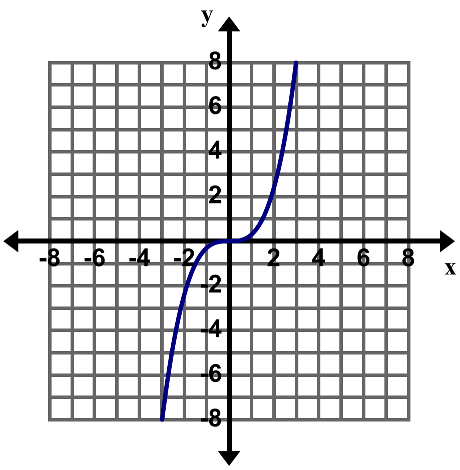
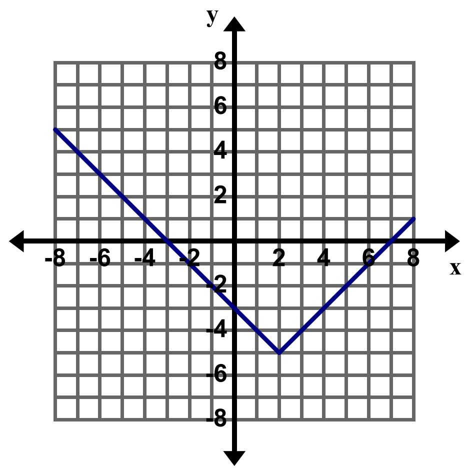
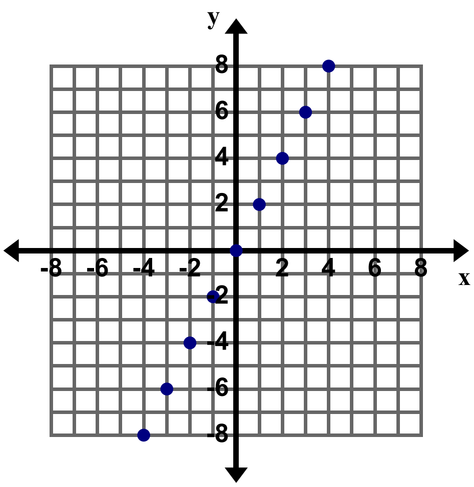
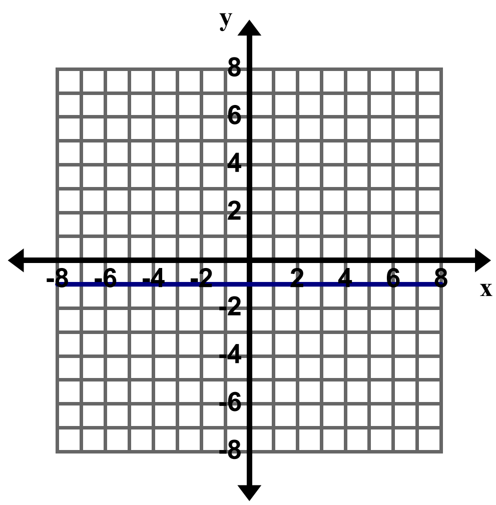
d. Whose will be able to play their iPad for the longest amount of time?

Directions: For each problem, circle the representation with the greatest rate of change. Put a star by the representation with the greatest y-intercept. Assume all representations have a constant rate of change.

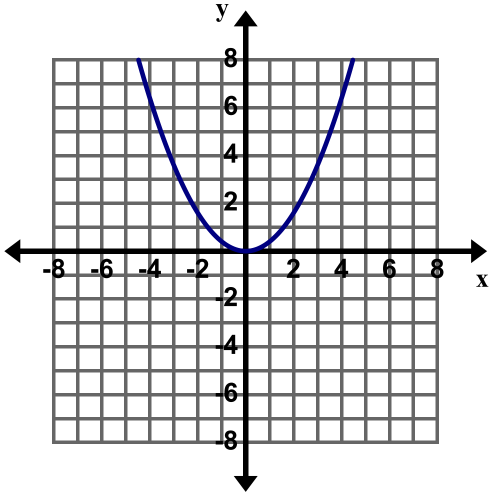
<table>
<tr>
<td>8. $y = x$</td>
<td><table><tr><th>x</th><th>y</th></tr><tr><td>1</td><td>1.5</td></tr><tr><td>2</td><td>2</td></tr><tr><td>3</td><td>2.5</td></tr></table></td>
<td>
</td>
</tr>
<tr>
<td>9. $y = \frac{7}{4}x + 2$</td>
<td>$(1, 1.5)(2, 3)$</td>
<td>
</td>
</tr>
<tr>
<td>10. $y = -0.5x$</td>
<td><table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>0</td></tr><tr><td>2</td><td>-2</td></tr><tr><td>5</td><td>-5</td></tr></table></td>
<td>
</td>
</tr>
</table>

5.3c Class Activity: Features of Graphs

1. Cut out each graph. Sort the graphs into groups and be able to explain why you grouped the graphs the way you did. In the table that follows, name your groups, describe your groups, and list the graphs that are in your group.

A 	**B** 
C 	**D** 
E 	**F** 

G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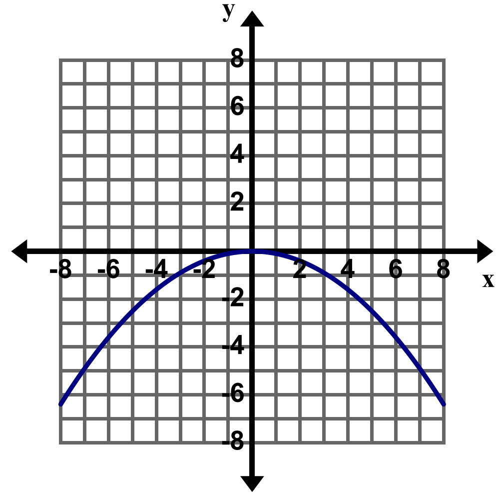
8
6
4
2
-2
-4
-6
-8
-8 -6 -4 -2 2 4 6 8
x

H


y
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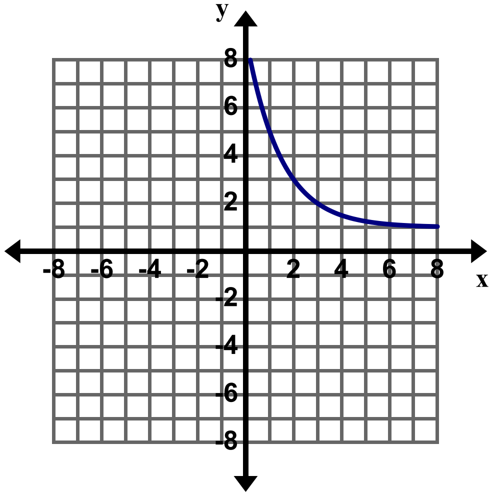
4
2
-2
-4
-6
-8
-8 -6 -4 -2 2 4 6 8
x

I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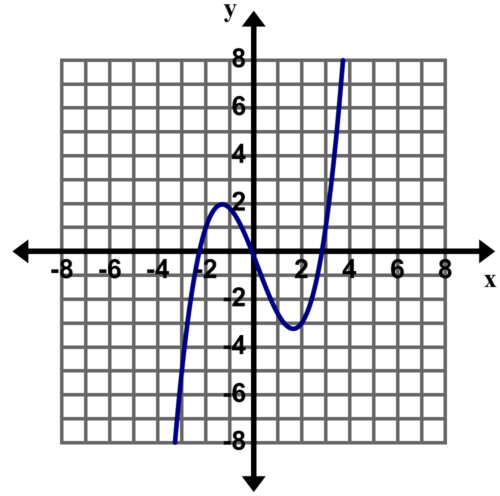
-2
-4
-6
-8
-8 -6 -4 -2 2 4 6 8
x

J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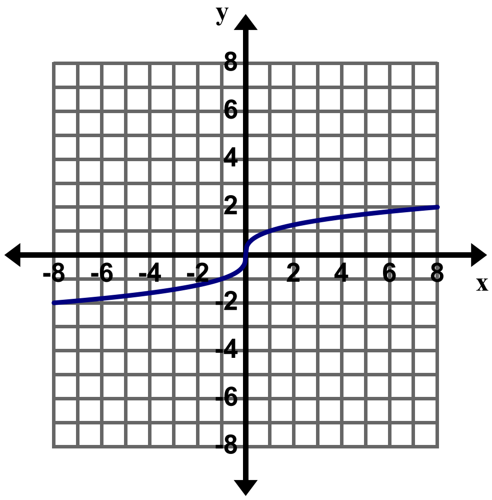
-6
-8
-8 -6 -4 -2 2 4 6 8
x

K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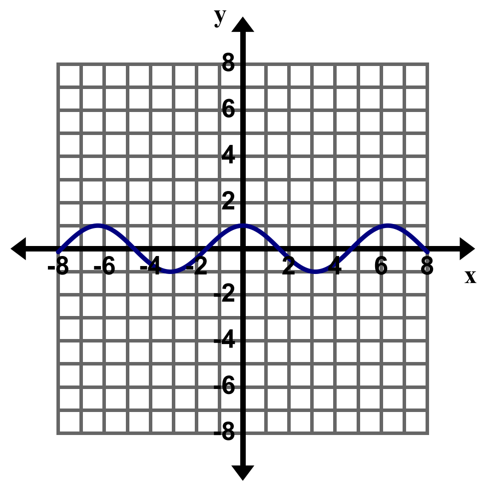
2
-2
-4
-6
-8
-8 -6 -4 -2 2 4 6 8
x

L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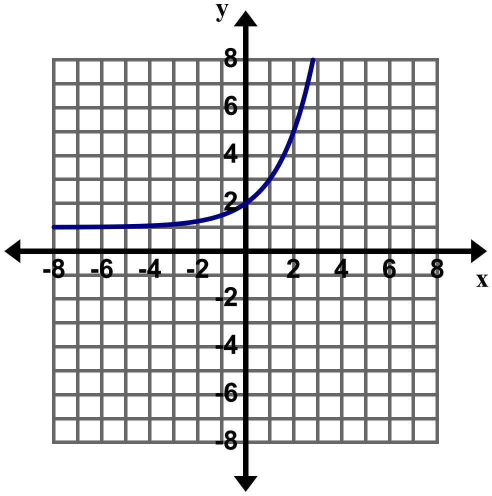
4
2
-2
-4
-6
-8
-8 -6 -4 -2 2 4 6 8
x

M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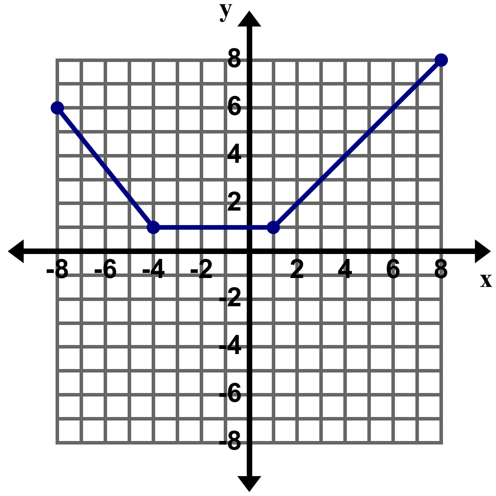
2
-8 -6 -4 -2 2 4 6 8
x
-2
-4
-6
-8

N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2
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x
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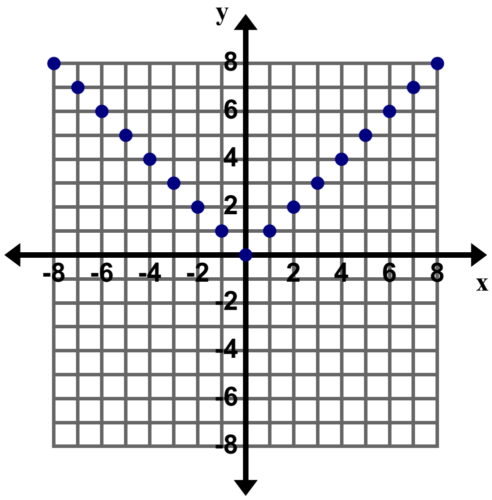
-4
-6
-8

O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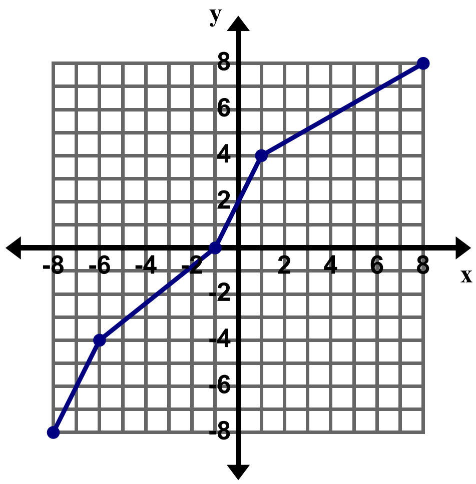
6
4
2
-8 -6 -4 -2 2 4 6 8
x
-2
-4
-6
-8

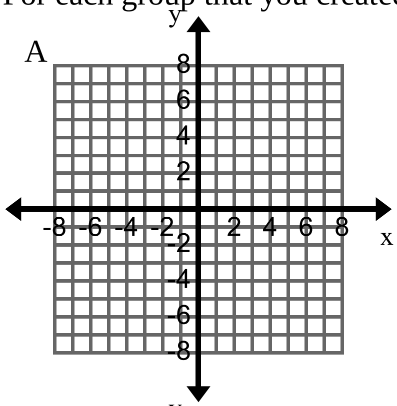
P


y
8
6
4
2
-8 -6 -4 -2 2 4 6 8
x
-2
-4
-6
-8

Your Groups

Name of the group	Description of the group	Graphs in the group
A.		
B.		
C.		
D.		
E.		
F.		

2. For each group that you created, draw another graph that would fit in that group.

A


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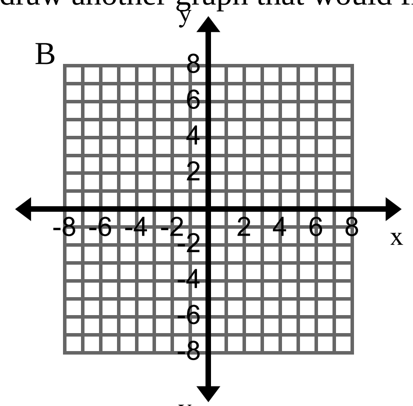
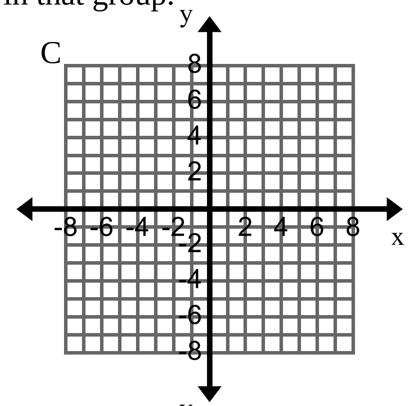
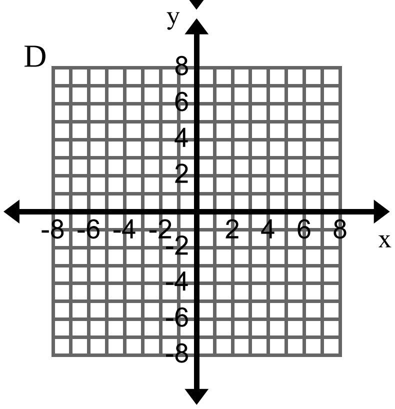
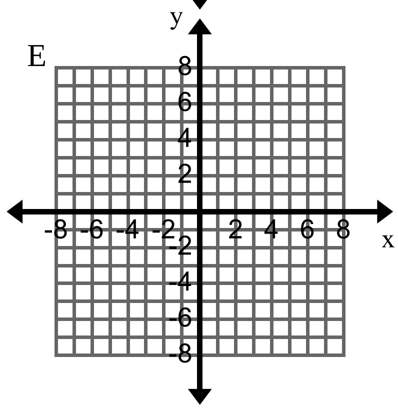
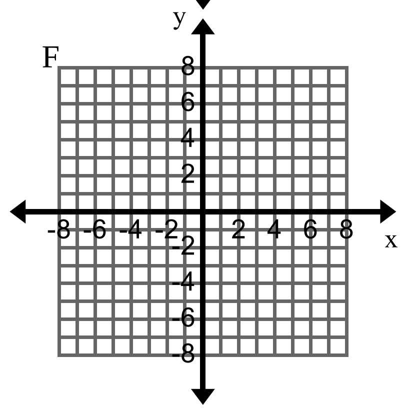


C


D


E


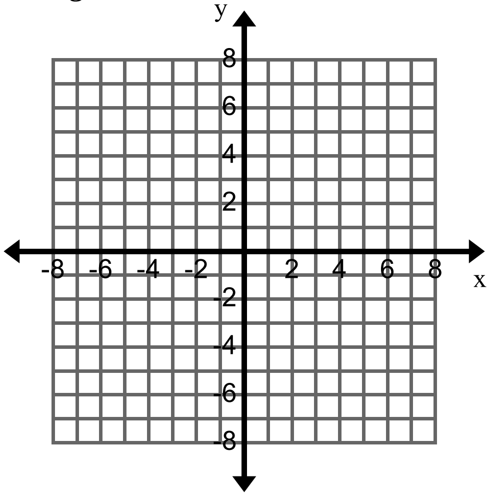
F


3. Lucy grouped hers as follows:

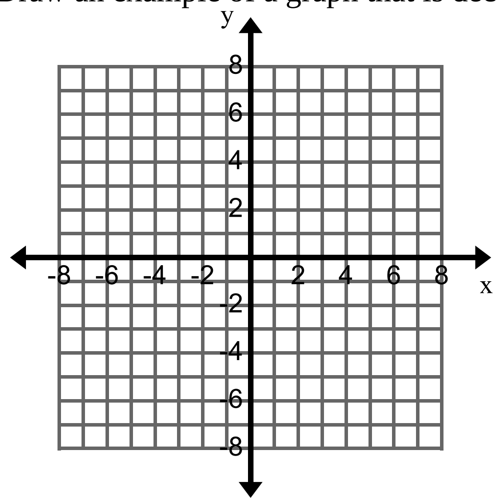
Increasing on the entire graph: **A, C, E, K, M, P**
Decreasing on the entire graph: **I**
Constant on the entire graph: **F**
Increasing on some parts of the graph, decreasing on some parts of the graph: **D, G, H, J, L, O**
Increasing on some parts of the graph, decreasing on some parts of the graph, constant on some parts of the graph: **N**
Increasing on some parts of the graph, constant on some parts of the graph: **B**

4. Define **increasing**, **decreasing**, and **constant** in your own words.

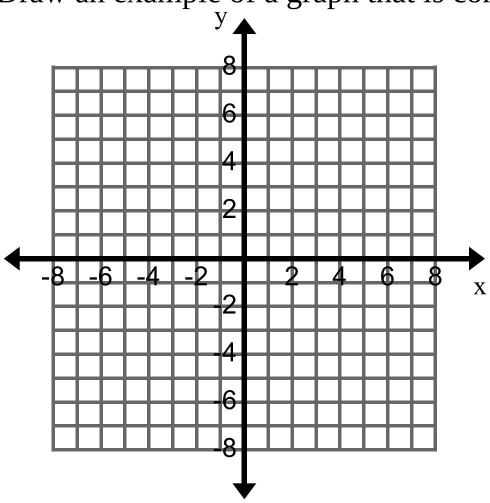
5. Draw an example of a graph that is increasing.



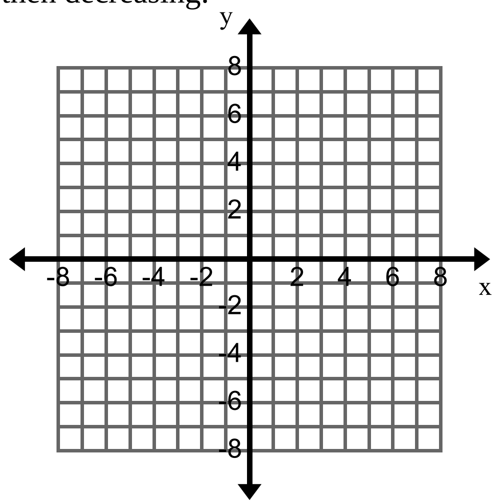
6. Draw an example of a graph that is decreasing.



7. Draw an example of a graph that is constant.



8. Draw an example of a graph that is increasing then decreasing.

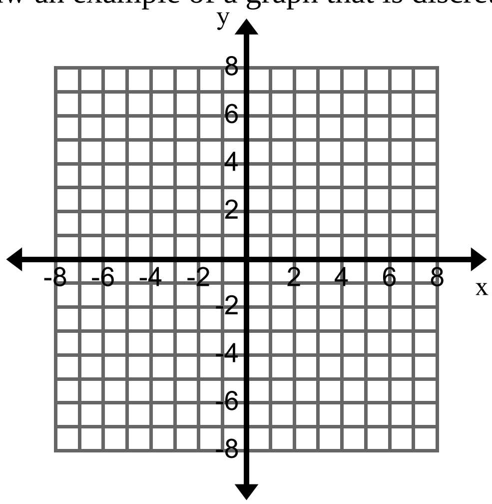


9. Ellis grouped hers as follows:

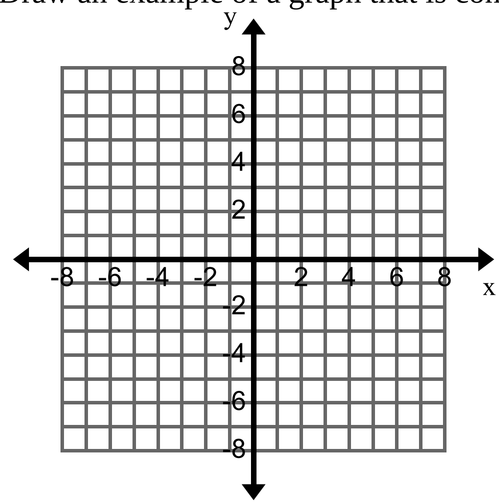
Discrete: **B, E, O**
Continuous: **A, C, D, F, G, H, I, J, K, L, M, N, P**

10. Define **discrete** and **continuous** in your own words. Can you think of a real world situation that has a discrete graph? Why doesn't it make sense to connect the points in this situation?

11. Draw an example of a graph that is discrete.



12. Draw an example of a graph that is continuous.



13. Grace grouped hers as follows:

Linear: **A, E, F**
Nonlinear: **C, G, H, I, J, K, L, M**
Made up of pieces of different linear functions: **B, D, N, O, P**

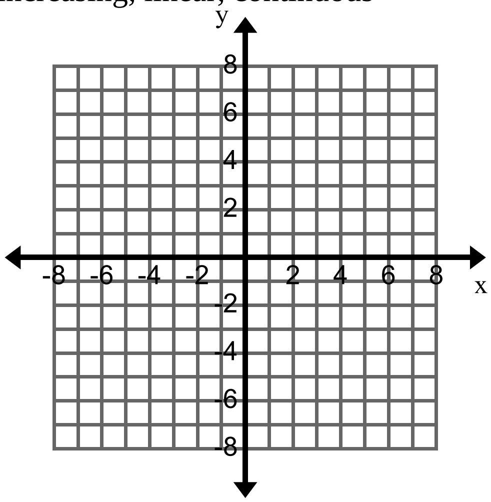
14. Define **linear** in your own words.

15. Define **nonlinear** in your own words.

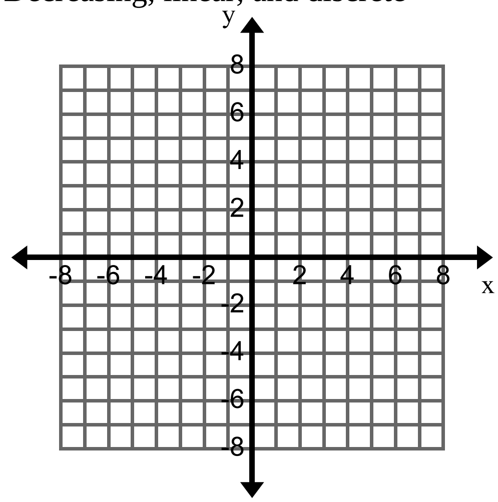
5.3c Homework: Features of Graphs

Directions: Draw a graph with the following features.

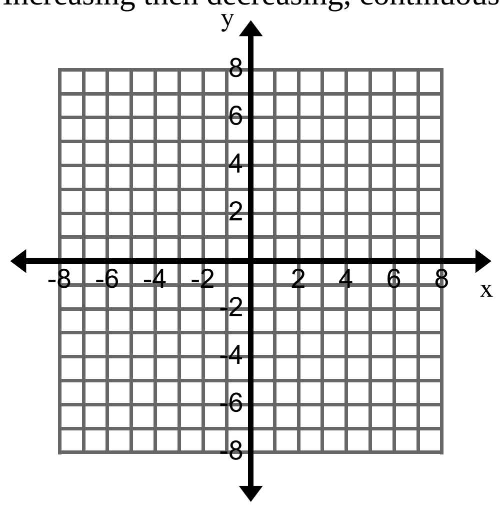
1. increasing, linear, continuous



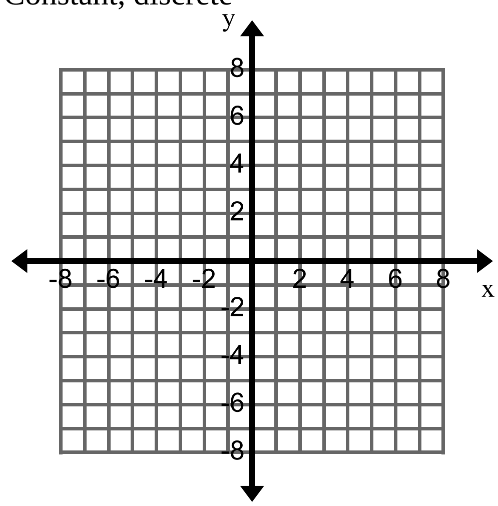
2. Decreasing, linear, and discrete



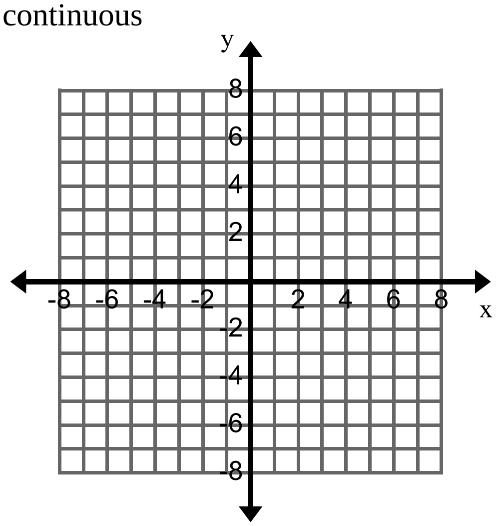
3. Increasing then decreasing, continuous



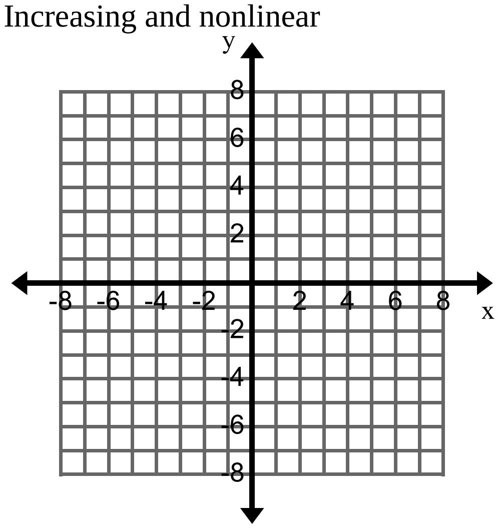
4. Constant, discrete



5. Decreasing, then constant, then increasing, continuous

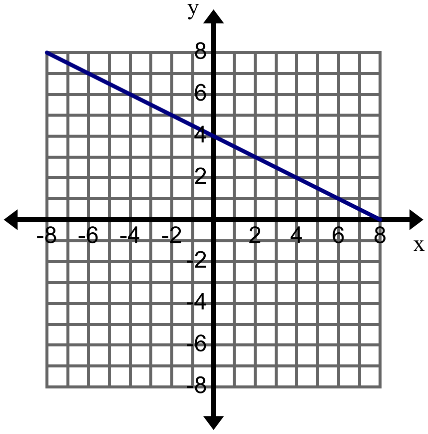


6. Increasing and nonlinear



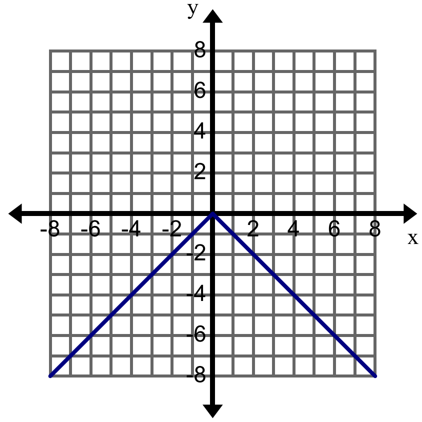
Directions: Describe the features of each of the following graphs (increasing/decreasing/constant; discrete/continuous; linear/nonlinear). Label on the graph where it is increasing, decreasing, or constant. Identify the intercepts of the graph.

7.



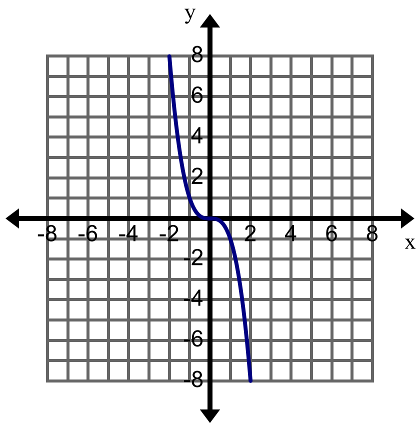
Features:

8.



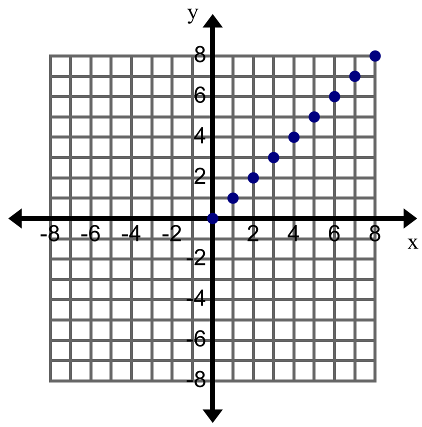
Features:

9.



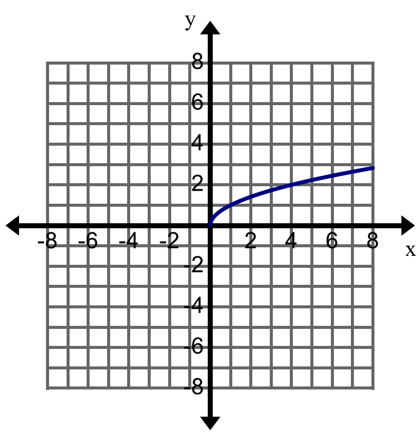
Features:

10.



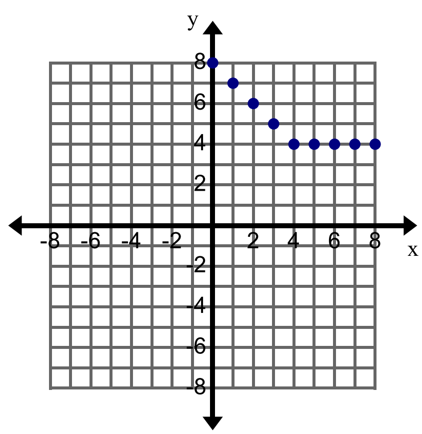
Features:

11.



Features:

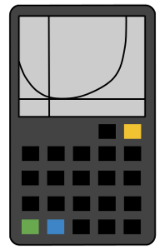
12.



Features:

5.3d Class Activity: CBR Activity

You will be using the *DIST MATCH application in the CBR*[TM] *Ranger* program on the TI 73 (or other) graphing calculators. Instructions for CBR/calculator use:

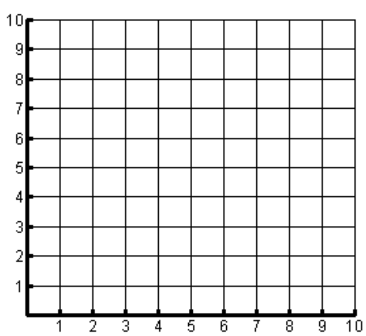
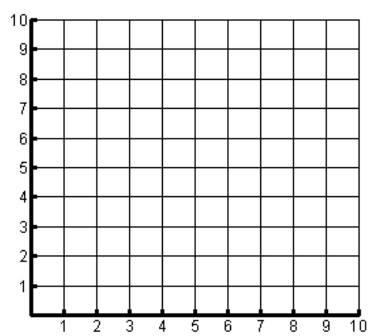
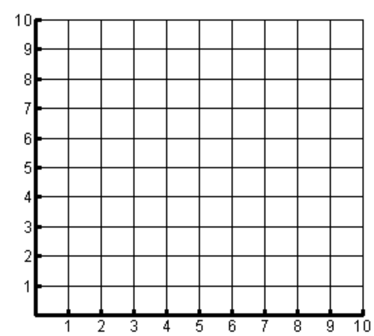


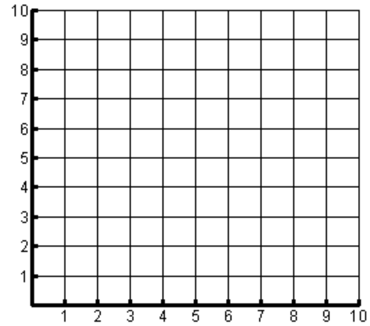
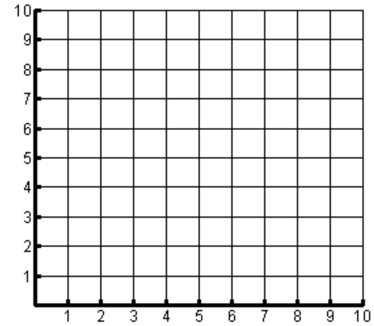
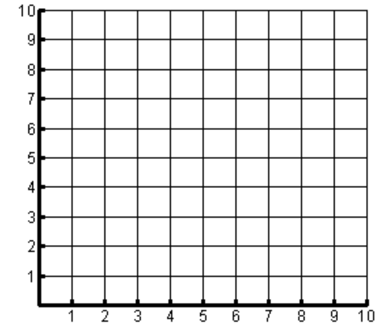
- Firmly attach the TI 73 to the CBR Ranger.
- Choose the APPS button on the TI 73.
- Choose 2: CBL/CBR.
- Choose 3: RANGER.
- Choose 3: APPLICATIONS.
- Choose 2: FEET.
- Choose 1: DIST MATCH. Get your first graph onto the calculator screen.

1. Try to match the graph given to you in the program. You will reproduce the graph by walking. Then trace the graph onto the grids below.

<u>Be sure to model a few examples with your class before you begin in teams!</u>

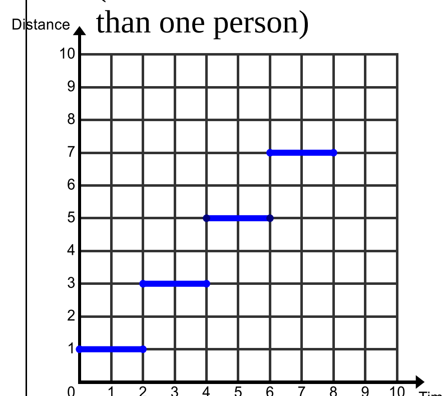
a. Get a graph to match ready in the calculator.
b. Decide how far away from the sensor you should stand to begin.
c. Talk through the walk that will make a graph match. (how far away to begin, walk forward or backward, how fast to move forward or backward, how long to walk forward or backward, when to change directions or speed, etc.)
d. You may wish to write the story of the graph first (before you walk it)—see below.
e. Have a member of your group hold the CBR so that the CBR sensor is up and directed toward the person that is walking
f. Have a group member press start on the calculator. Then walk toward or away from the sensor trying to make your walk match the graph on the calculator screen.
g. Each member of your group should walk to match at least one graph on the calculator.
h. Sketch each graph below. Write the story for the graph.

Graph 1: 	Graph 2: 	Graph 3: 
Story:	Story:	Story:

Graph 4:	Graph 5:	Graph 6:
		
Story:	Story:	Story:

Extra for Experts

If you finish early try to create the following graphs, write a description/story that matches the graph.

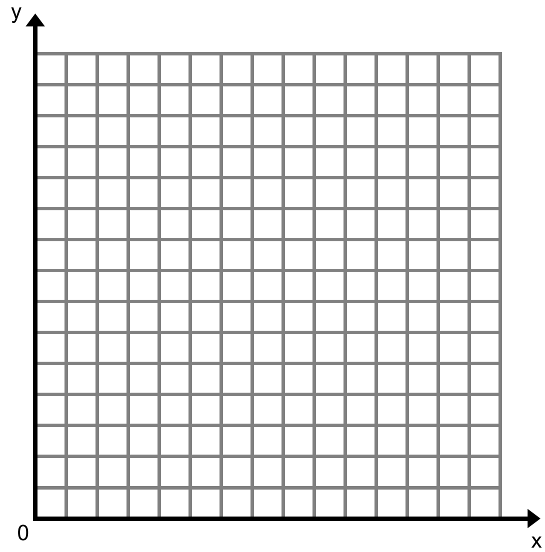
1. A line that rises at a steady rate. Story:	2. A line that falls at a steady rate. Story:	3. A horizontal line Story:
4. A "V" Story:	5. A "U Story:	6. An "M" Story:
7. Try creating an O. Are you successful? Why or why not?	8. Name a letter you could graph using the CBR. Name a letter you cannot graph using the CBR. Explain your choices.	9. Try creating this graph. (Hint: It will take more than one person) 

5.3d Homework: Stories and Graphs

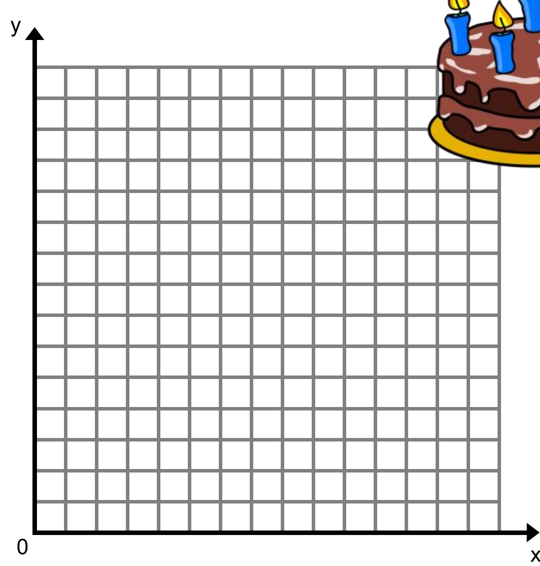
Directions: Sketch graphs to match the stories.

1. Before School

   Create a graph to match the story below (distance in feet, time in minutes). (Note: This graph will show distance traveled related to time passing—consider the student to be continually moving forward.)

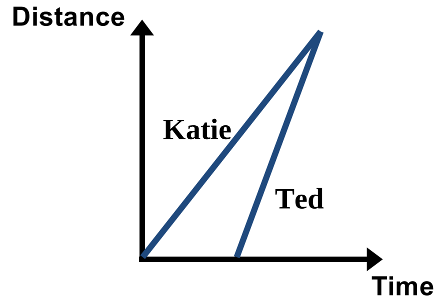
Story:	Graph
A student walks through the halls before school. He/she begins at the front door, stops to talk to at least three different friends, stops at his/her locker, stops in the office.	

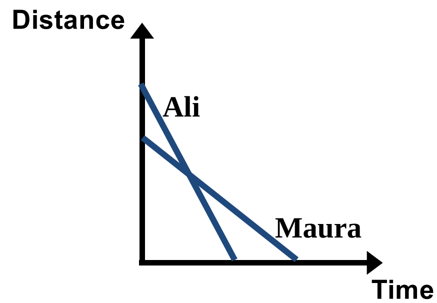
2. Birthday Cake
   a. Write a story about your family eating a birthday cake. You want to talk about amount of cake eaten related to passing time.
   b. Create the graph to tell the same story. You decide on the labels.

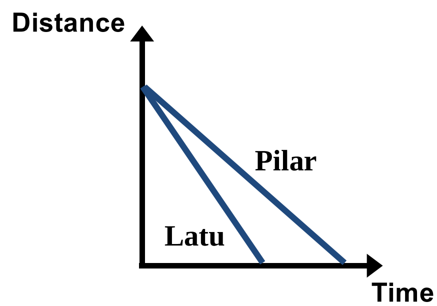
Story:	Graph
	

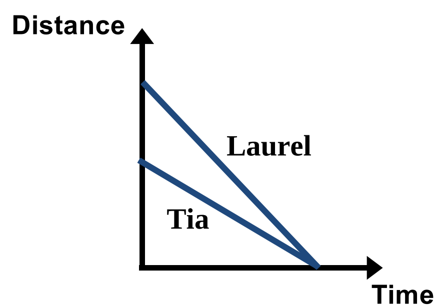
3. Make up stories to go with the following graphs. In this problem, distance represents the **distance from school**. Include in the stories specific details about starting points and slopes. Answer the additional questions.

<table>
<tr>
<td>a. Tell the story of this graph.

Draw and label a line that represents Izzy who started at the same time as Katie but walked away from school at a faster rate than Katie.</td>
<td>
</td>
</tr>
<tr>
<td>b. Tell the story of this graph.

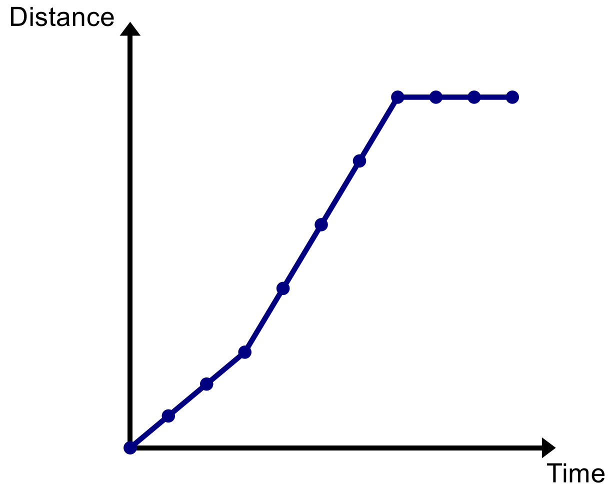
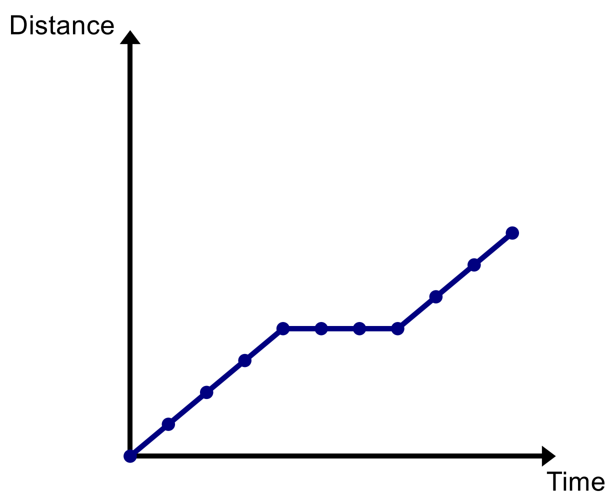
Draw and label a line that represents Gabi who walks slower than both Ali and Maura.</td>
<td>
</td>
</tr>
<tr>
<td>c. Tell the story of this graph.

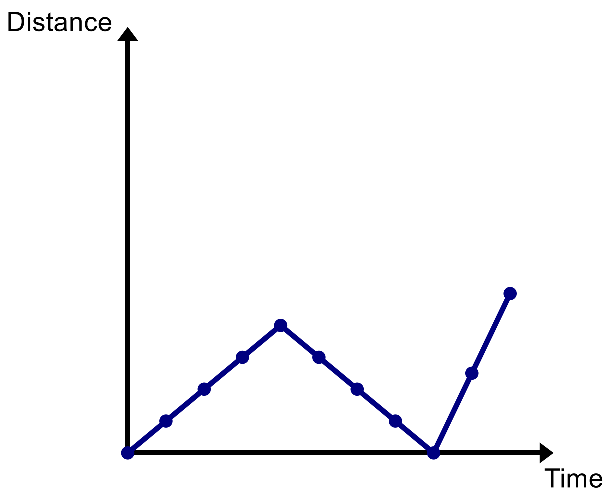
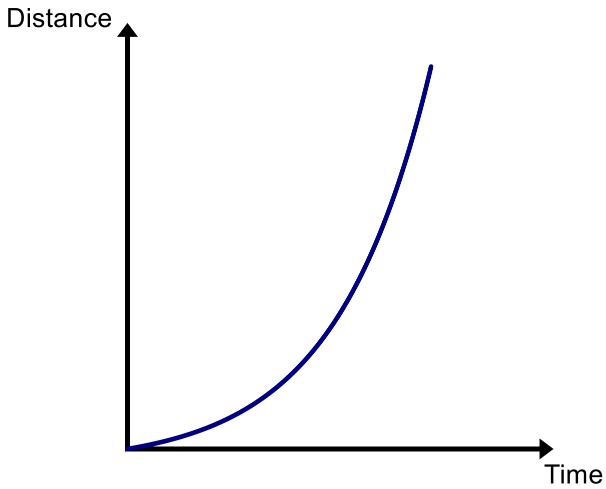
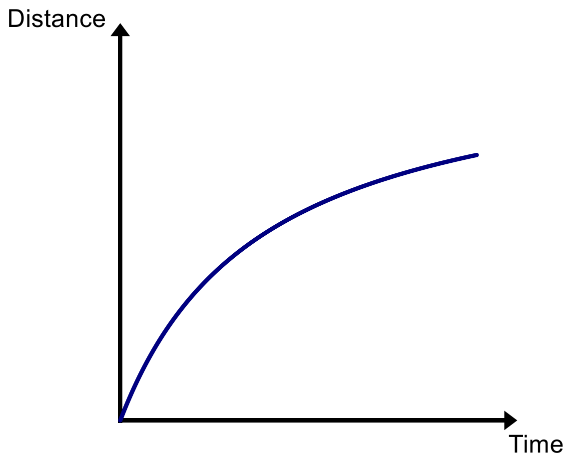
Draw and label a line that represents Carmen who starts the same distance from school as Pilar and Latu but gets to school faster than both of them.</td>
<td>
</td>
</tr>
<tr>
<td>d. Tell the story of this graph.

Draw and label a line that represents Colin who lives closer to school, leaves at the same time as Laurel and Tia to walk to school, and arrives at the same time as Laurel and Tia.</td>
<td>
</td>
</tr>
</table>

5.3e Class Activity: School’s Out

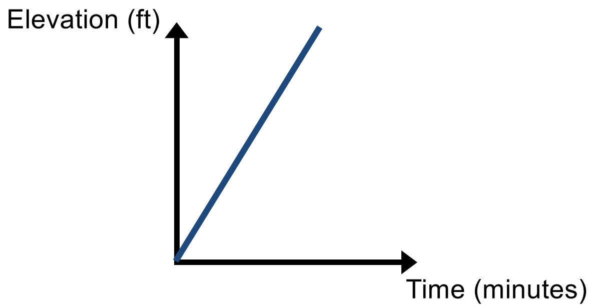
Directions: The following graphs tell the story of five different students leaving school and walking home. Label the key features of the graph. Write a story for each graph describing the movement of each of the students.

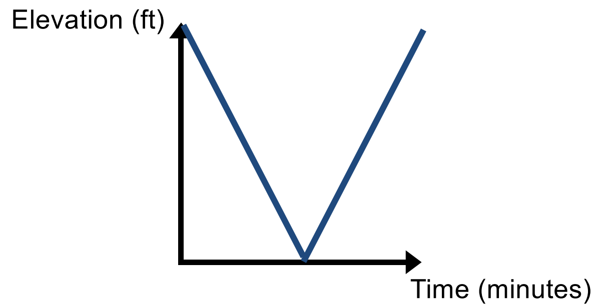
Abby’s Journey Home: 	
Beth’s Journey Home: 	

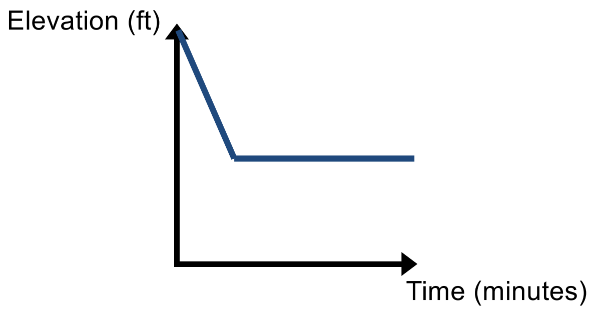
Chad's Journey Home:  	
Drew's Journey Home:  	
Eduardo's Journey Home:  	

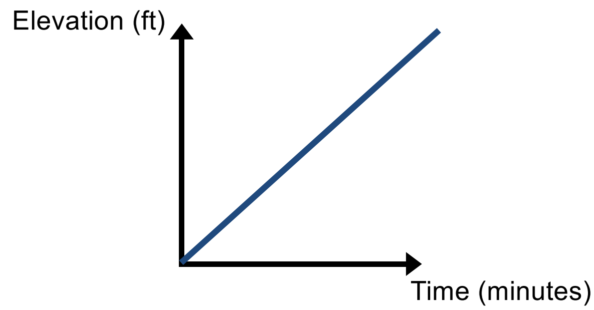
5.3e Homework: School's Out

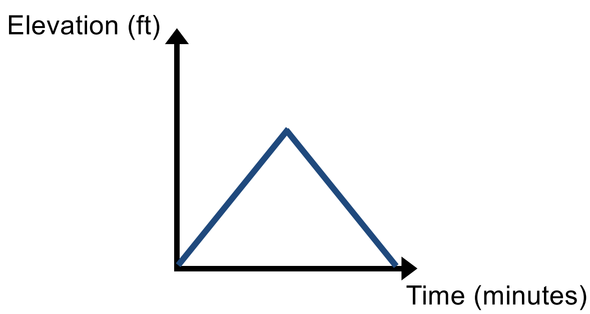
1. The graphs below show Estefan's elevation (height above the ground) over time as he is playing around on a flight of stairs. Assume the bottom of the stairs has an elevation of 0 feet. Match each story (shown below the graphs) to a graph by writing the letter of the story under each graph.

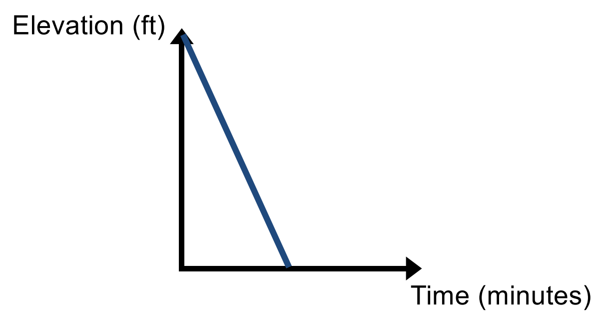






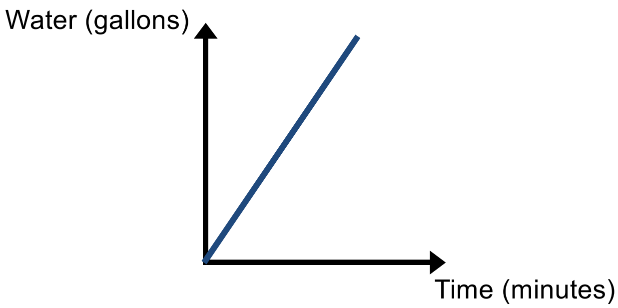


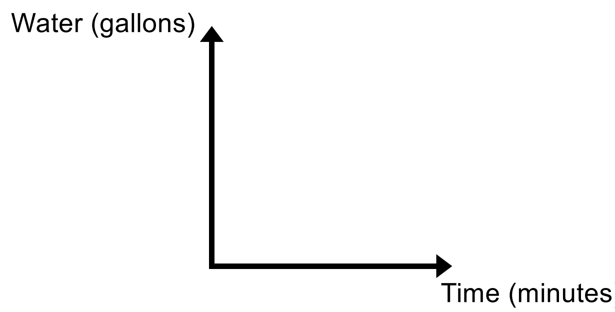
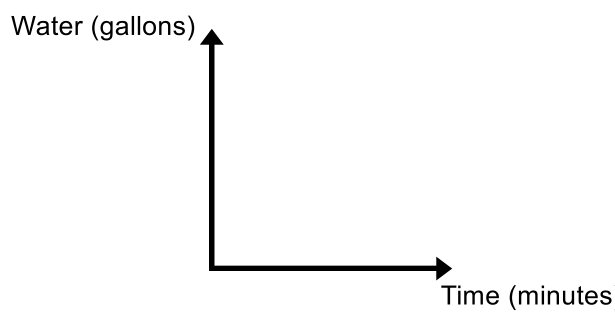
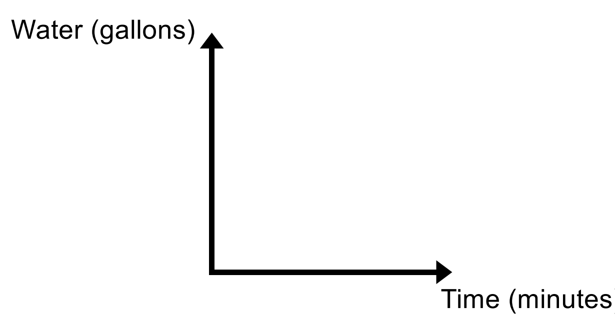




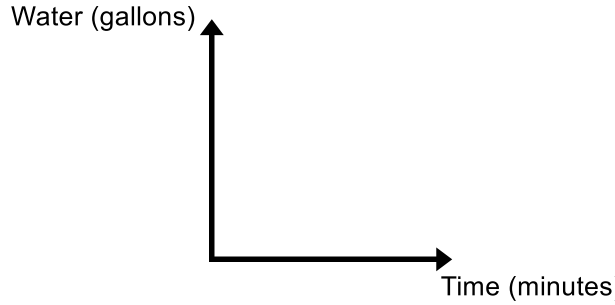
Story A: Estefan starts at the bottom of the stairs and walks up the stairs at a constant rate.
Story B: Estefan starts at the bottom of the stairs and sprints up the stairs at a constant rate.
Story C: Estefan starts at the bottom of the stairs, runs half-way up the stairs, turns around and runs back down the stairs.
Story D: Estefan starts at the top of the stairs and sprints down the stairs until he reaches the bottom.
Story E: Estefan starts at the top of the stairs, sprints down the stairs, and stops when he is half-way down the stairs.
Story F: Estefan starts at the top of the stairs, runs down to the bottom, turns around and runs back up to the top of the stairs.

2. The graph below tells the story of Kelii filling up her empty swimming pool with a hose at a constant rate. Create new graphs based on the changes described below.

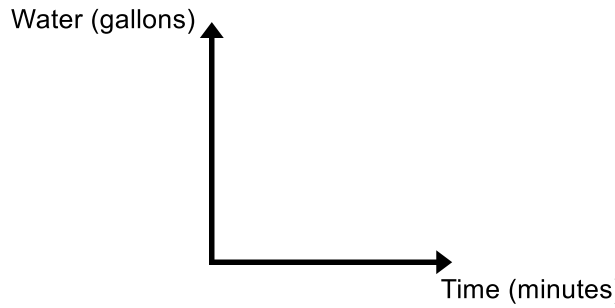


a. After Kelii has been filling the pool for a few minutes, her friend decides to help her and puts a second hose in the pool.	
b. After Kelii has been filling the pool for a few minutes, the hose gets a hole in it so the water is coming out at a slower rate.	
c. The pool started with some water in it.	

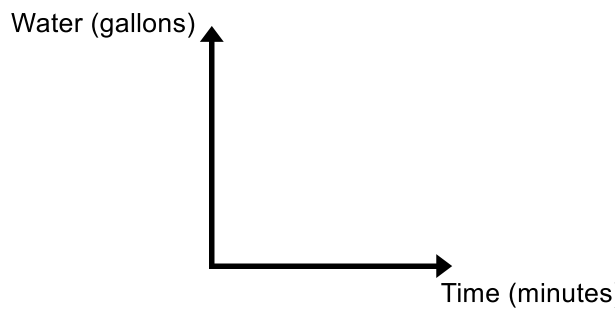
d. Kelii fills the pool up and realizes that she filled it up too much so she drains some of the water out at a constant rate.



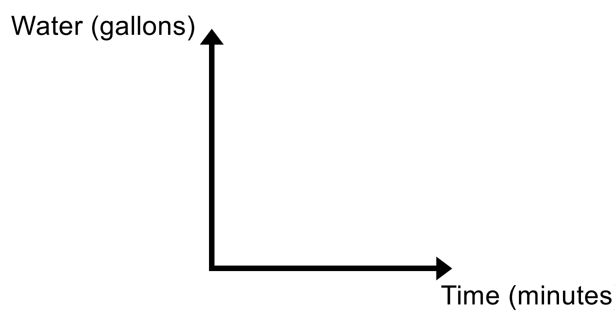
e. Kelii fills up the pool, her brother drops his ice cream into the pool, so she drains the water back out at a constant rate until the pool is empty.



f. Kelii starts filling the pool, stops to go eat lunch, and then comes back out and starts filling up the pool again.

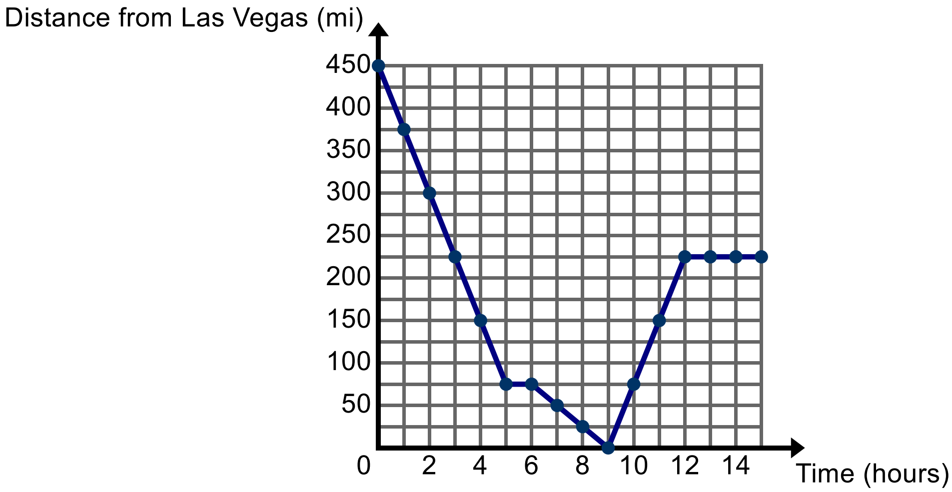


g. Kelii fills the pool half-way and decides that is enough.



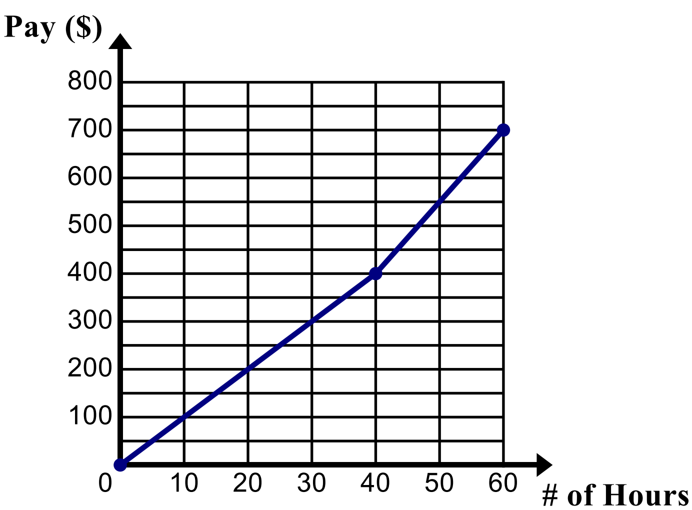
5.3f Class Activity: From Graphs to Stories

1. Ben and his family took a road trip to visit their cousins. The graph below shows their journey. Label the key features of the graph.



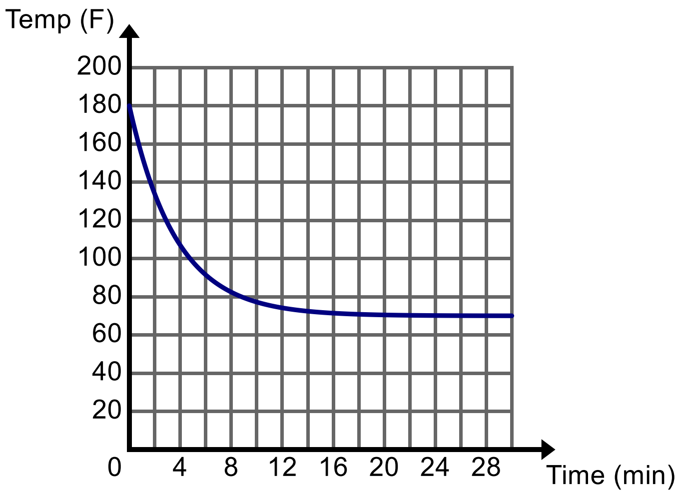
   a. Tell the story of the graph.

2. The graph below shows the amount Sally makes based on how many hours she works in one week. Label the key features of the graph.



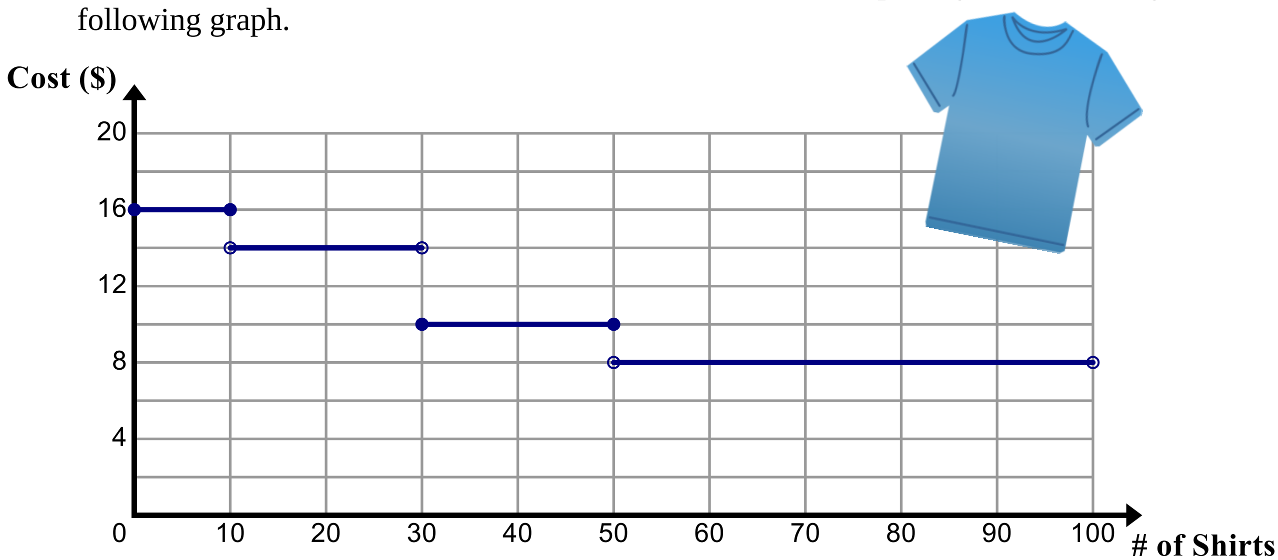
   a. Tell the story of the graph.

3. Cynthia is doing research on how hot coffee is when it is served. The graph below shows the temperature of a coffee (in ˚F) as a function of time (in minutes) since it was served. Label the key features of the graph.



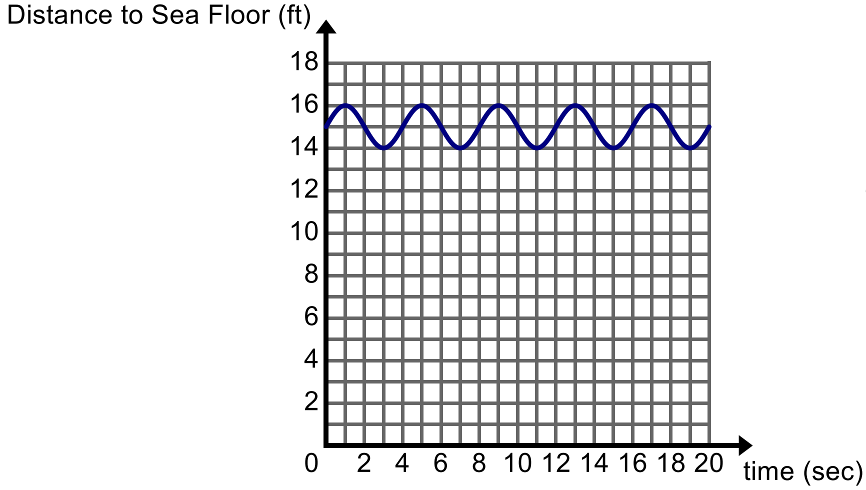
a. Tell the story of the graph.

4. Jorge is the team captain of his soccer team. He would like to order shirts for the team and is looking into how much it will cost. He called Custom T's to ask about pricing and the manager sent him the following graph.



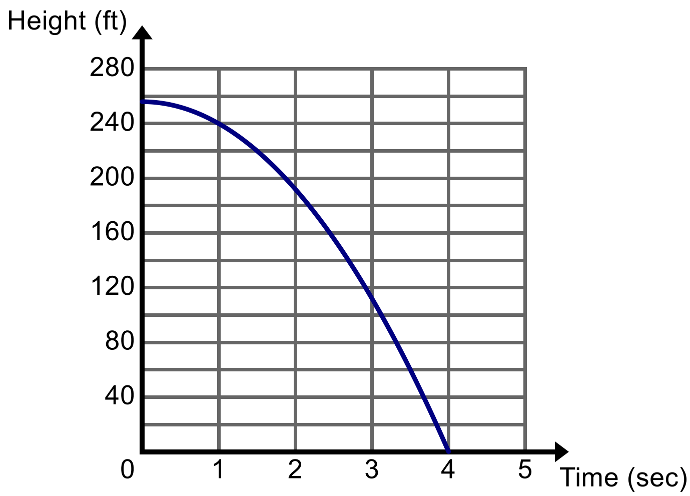
a. Tell the story of the graph.

5. A boat is anchored near a dock. The graph below shows the distance from the bottom of the boat to the sea floor over a period of time.



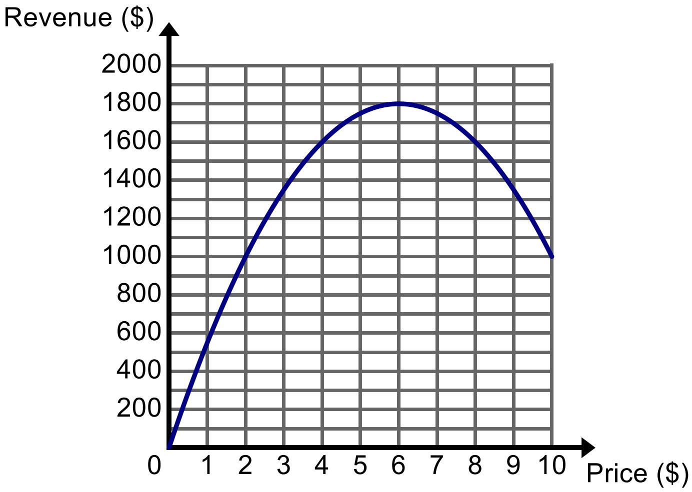
a. Tell the story of the graph.

6. An object is dropped from a bridge into the water below. The graph below shows the height of the object (in feet) with respect to time (in seconds). Consider the relationship between the height of the object and time.



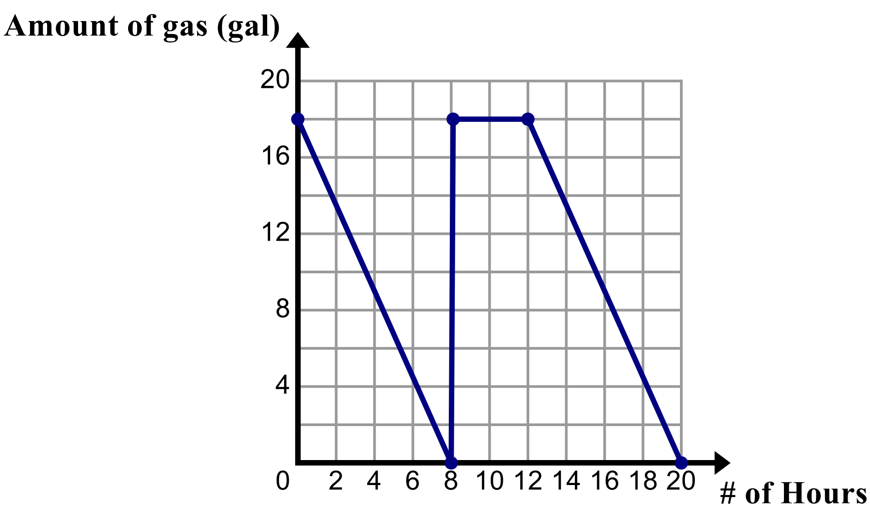
a. Tell the story of the graph.

7. The graph below shows the amount of revenue a company will make selling t-shirts dependent on the price of each t-shirt.



a. Tell the story of the graph.

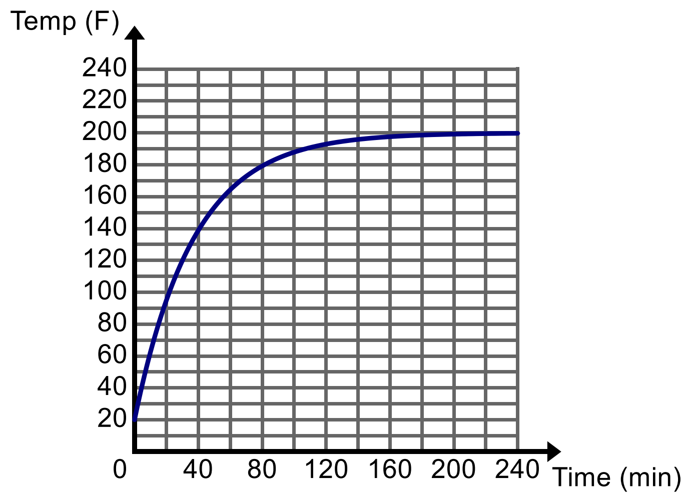
8. The graph below shows the amount of gas remaining in a vehicle over time.



a. Tell the story of the graph.

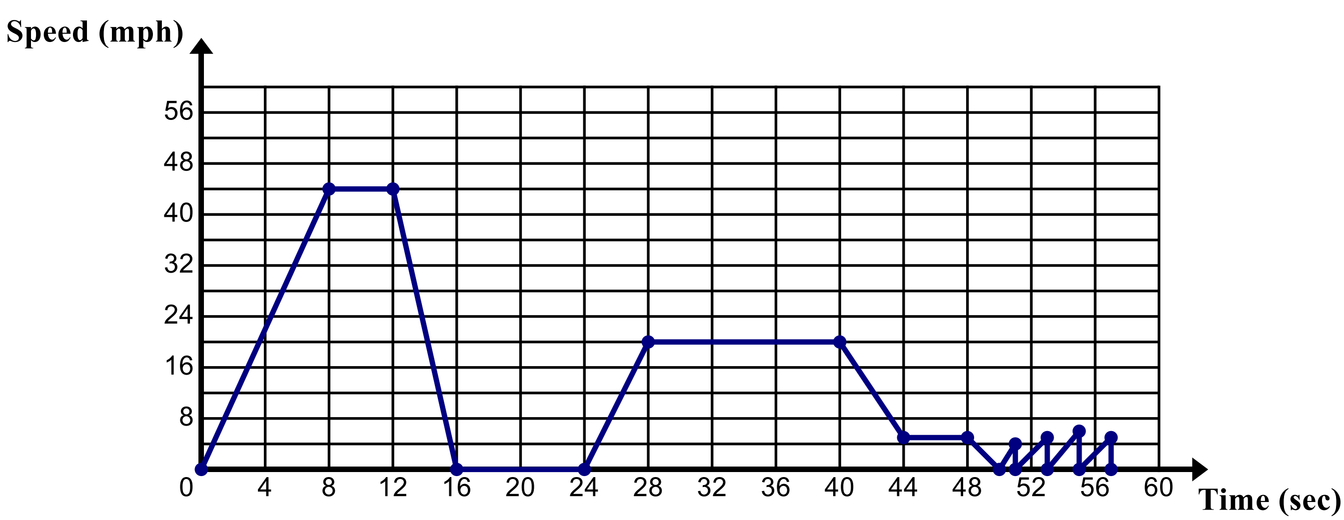
5.3f Homework: From Graphs to Stories

1. Tessa is cooking potatoes for dinner. She puts some potatoes in an oven pre-heated to 200˚ F. The graph below shows the temperature of the potatoes over time. Label the key features of the graph. The *y*-intercept of the graph is (0, 20).

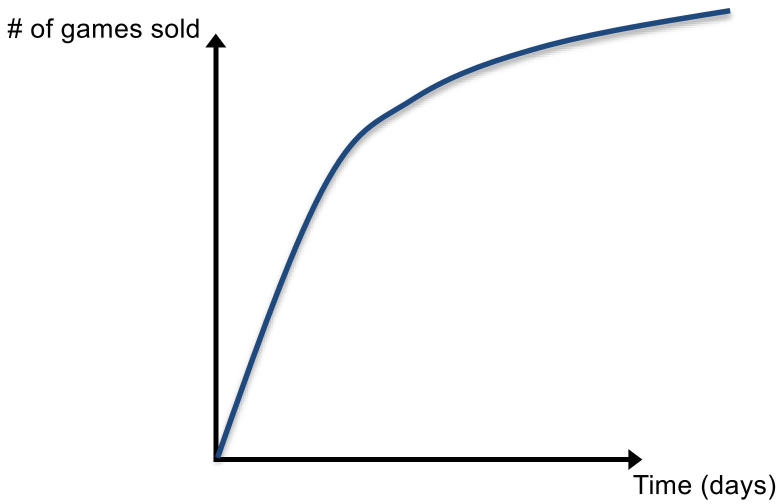


   a. Tell the story of the graph.

2. Steve is driving to work. The graph below shows Steve's speed over time. Label the key features of the graph to tell the story of the speed of Steve's car over time. Use words like accelerating, decelerating, driving at a constant speed, stopped. You can abbreviate these words using the first letter of each word (i.e. A for accelerating, D for decelerating, C for driving at a constant speed, S for stopped). Explain what might be happening at the end of the graph.

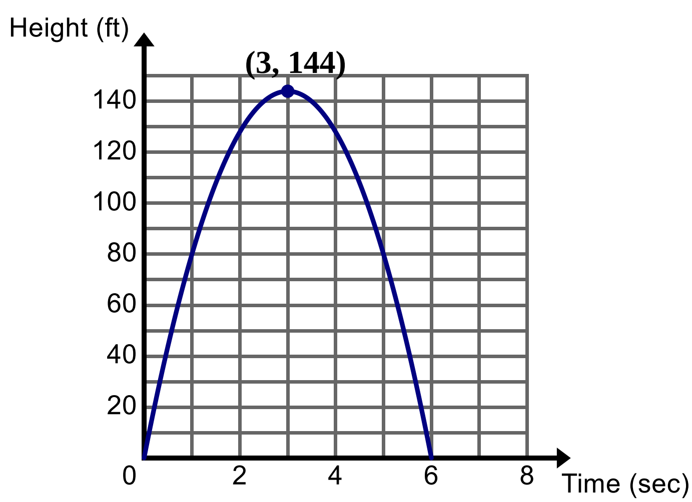


3. Microsoft is releasing the most anticipated new Xbox game of the summer. The graph below shows the total number of games sold as a function of the number of days since the game was released.



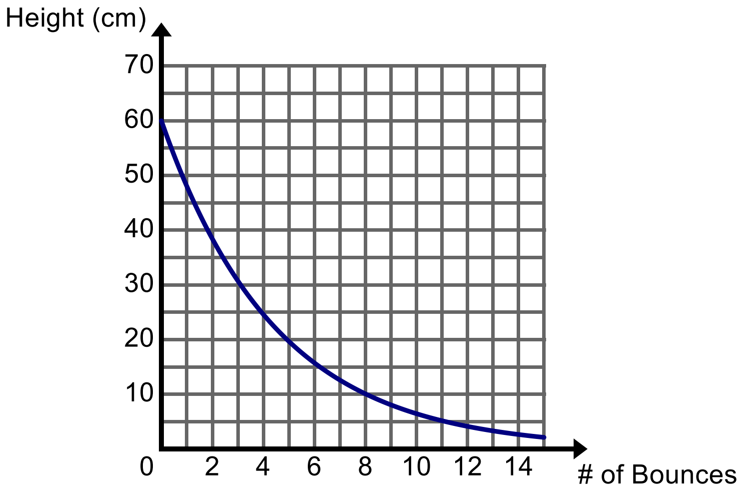
   a. Tell the story of the graph.

4. A toy rocket is launched straight up in the air from the ground. It leaves the launcher with an initial velocity of 96 ft. /sec. The graph below shows the height of the rocket in feet with respect to time in seconds. Label the key features of the graph.



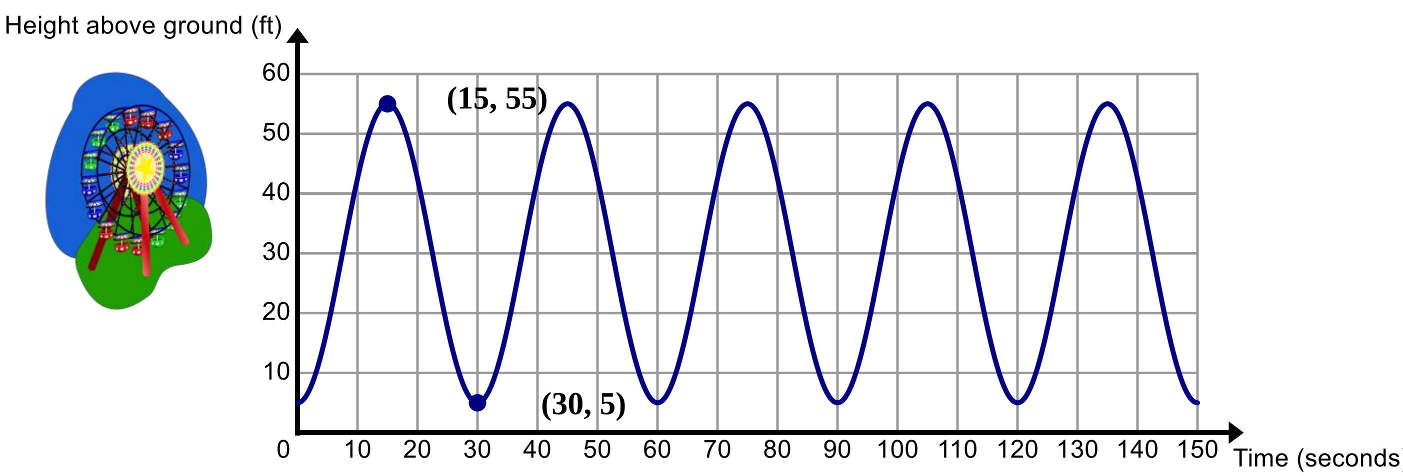
   a. Tell the story of the graph.

5. Suppose you drop a basketball from a height of 60 inches. The graph below shows the height of the object after *b* bounces.



a. Tell the story of the graph.

6. You are riding a Ferris wheel. The graph below shows your height (in feet) above the ground as you ride the Ferris wheel.



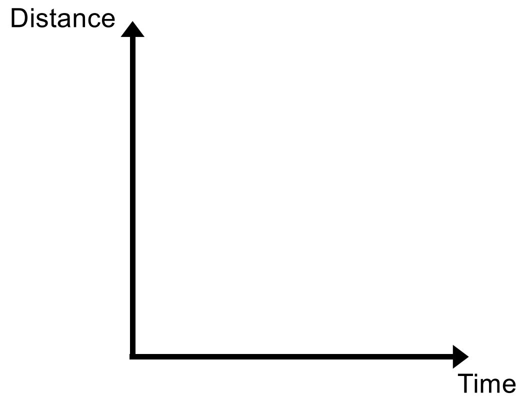
a. Tell the story of the graph.

5.3g Class Activity: From Stories to Graphs

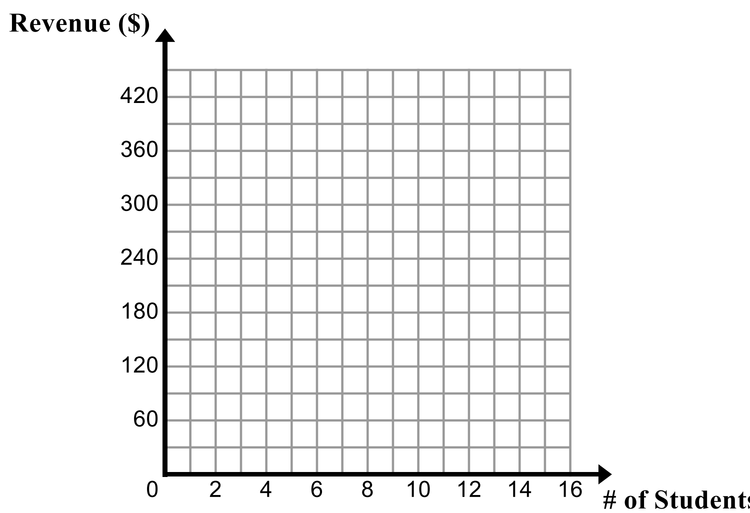
Directions: Sketch a graph to match each of the following stories. Label key features of your graph.

Story | **Graph**

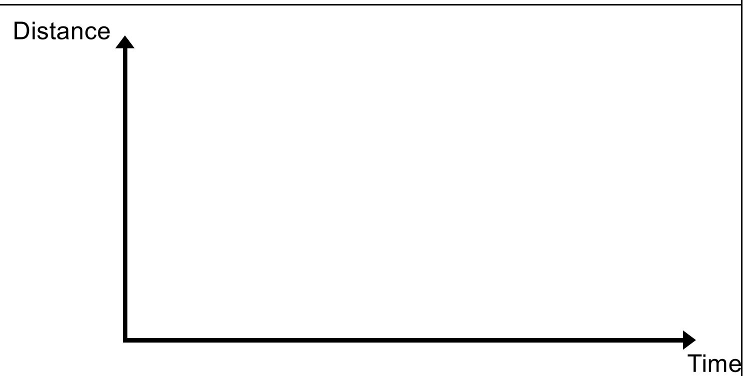
1. Zach walks home from school each day. Sketch a graph of Zach's distance from school as a function of time since the bell rang if the following happens: When the bell rings, Zach runs to his locker to grab his books and starts walking home. When he is about halfway home, he realizes that he forgot his math book so he turns around and runs back to school. After retrieving his math book, he realizes that he is going to be late so he sprints home.



2. Solitude is offering a ski clinic for teens. The cost of the class is $30 per student. A minimum of 5 students must sign up in order for Solitude to hold the class. The maximum number of students that can participate in the class is 12. Sketch a graph that shows the revenue Solitude will bring in dependent on the number of students that take the class.

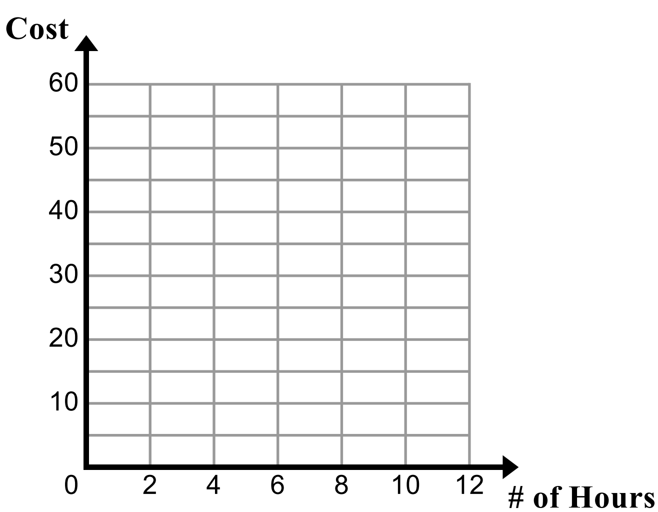


3. A biker is riding up a hill at a constant speed. Then he hits a downhill and coasts down the hill, picking up speed as he descends. At the bottom of the hill, he gets a flat tire. Sketch a graph that shows the distance traveled by the biker as a function of time.

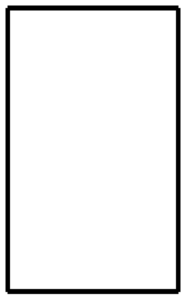
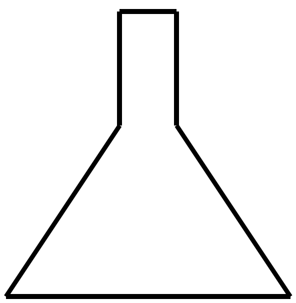
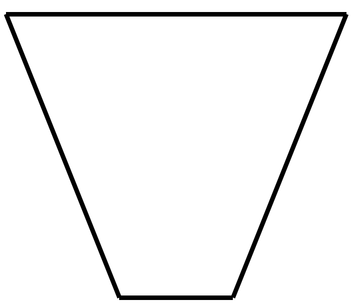


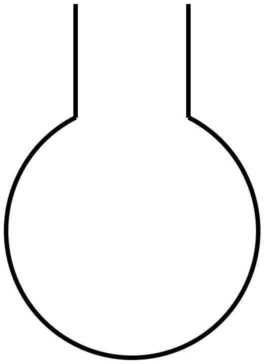
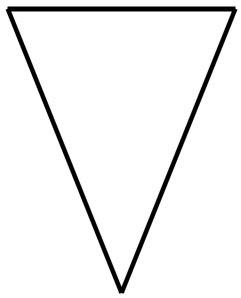
4. A concert for a popular rock group is sold out. The arena holds 8,000 people. The rock group is scheduled to take the stage at 8 pm. A band that is not very well known is opening for the rock band at 6:30 pm. The rock band is scheduled to play for 2 hours and the staff working the concert have been told that the arena must be cleared of people by 11:30 pm. Sketch a graph of the number of people in the arena from 5 pm to midnight. Time 0 on the grid below is 5 pm.

5. A parking garage charges $5 per hour and has a maximum cost of $40 for 12 hours. Sketch a graph of the total cost depending on how many hours a car is in the garage.



6. Your science teacher has the beakers shown below. He is going to fill them with water from a faucet that runs at a constant rate. Your job is to sketch a graph of the height of the water in each of the beakers over time.

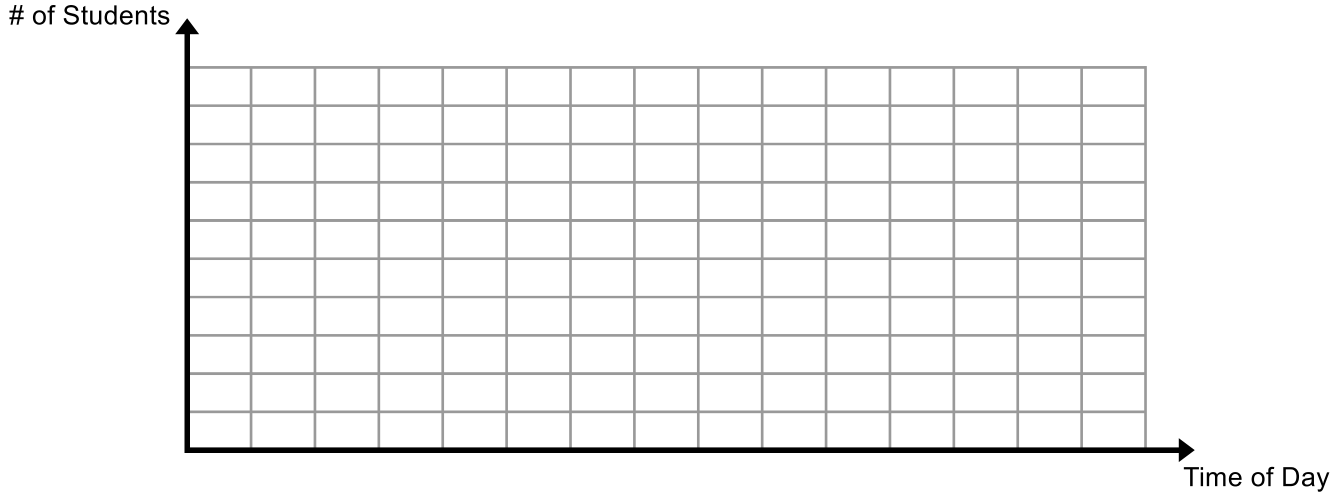
Beaker	Graph of the height of the water over time
 **Beaker 1**	
 **Beaker 2**	
 **Beaker 3**	

 **Beaker 4**	
 **Beaker 5**	

7. Now consider the volume of the water in each of the beakers over time. Sketch a graph of the volume of the water in each of the beakers over time.

5.3g Homework: From Stories to Graphs
Directions: Sketch a graph for each of the stories below.

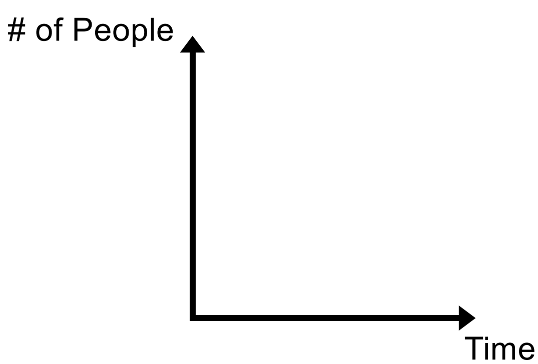
1. Sketch a graph of the number of students in the cafeteria as a function of time throughout the school day at your school. Tell the story of your graph.



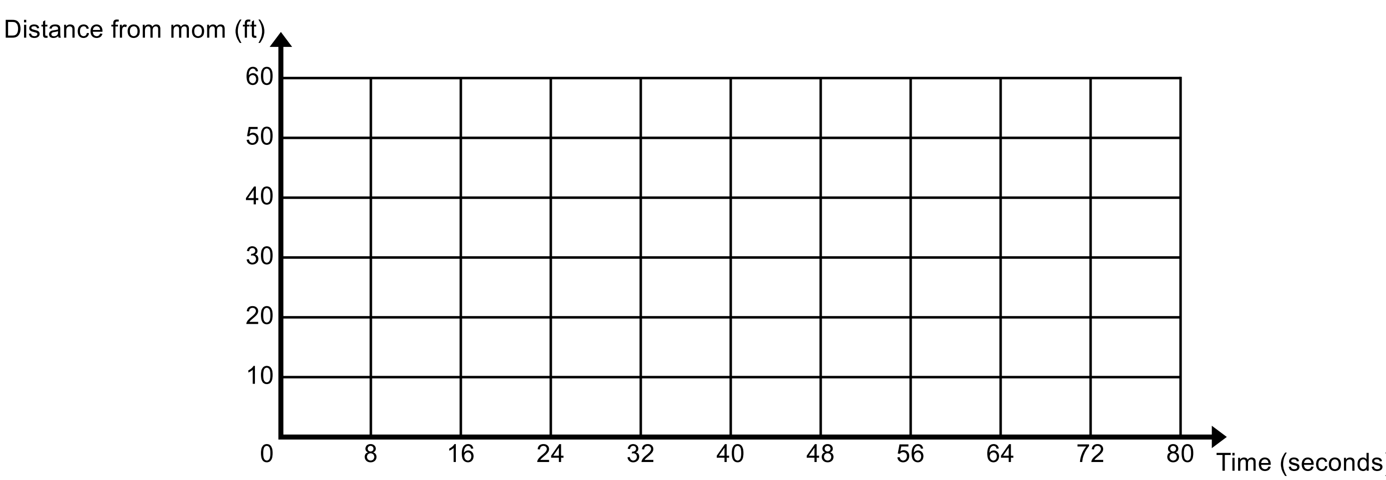
2. Two thousand, five hundred students attend a local high school. School starts at 8 am and ends at 2:30 pm. Many students stay after school for clubs, sports, etc. The school has a one-hour lunch at noon and seniors are allowed to leave campus for lunch. Sketch a graph of the number of cars in the student parking lot from 6 am to 4 pm. Time 0 on the grid below is 6 am.

3. A train that takes passengers from downtown back home to the suburbs makes 5 stops. The maximum speed at which the train can travel is 40 mph. Sketch a graph of the speed of the train a function of time since leaving the downtown train station.

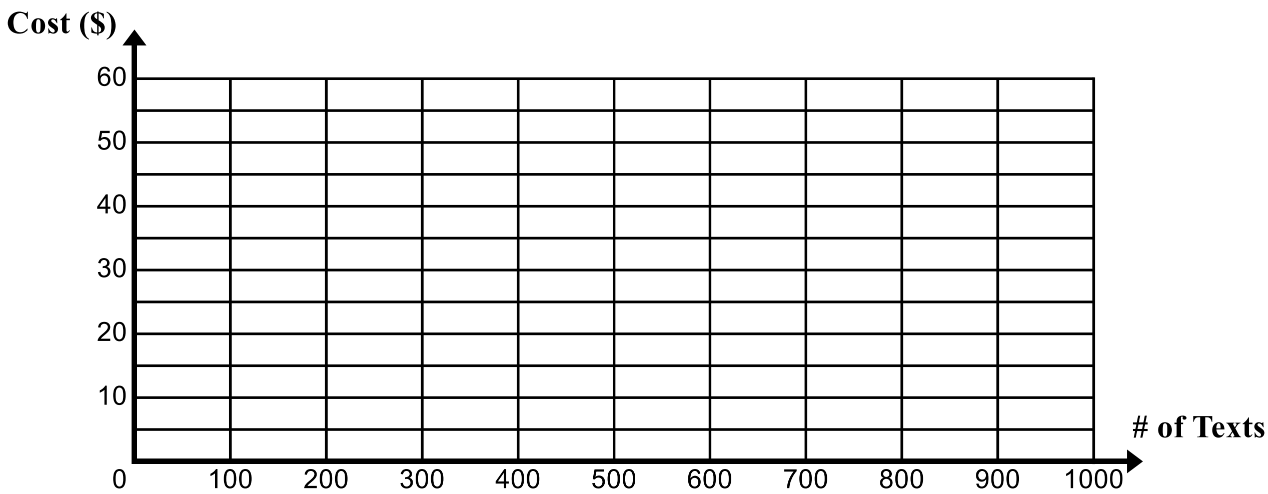
4. Sketch the graph of the total number of people that have seen the hit movie of the summer as a function of the time since opening day of the movie.



5. A little girl is going around on a merry-go-round. Her mom is standing at the entrance to the ride. Sketch a graph of the distance the little girl is from her mom as she goes around if the minimum distance she is from her mom during the ride is 5 feet and the maximum distance she is from her mom is 45 feet. Assume it takes 16 seconds to make one full revolution on the merry-go-round.



6. Yvonne is researching cell phone plans. Company A offers charges $0.05 for each text message sent. Company B offers unlimited texting for $25 per month. Company C charges $10 per month for up to 500 text messages and an additional $0.10 for each text message over 500. Sketch and label a graph that shows the relationship between number of texts sent and total monthly cost for each of the plans.



5.3h Self-Assessment: Section 5.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Determine whether the relationship between two quantities can be modeled by a linear function. Construct a function to model a linear relationship between two quantities.				
2. Compare properties of linear functions (rates of change and intercepts) and use this information to solve problems.				
3. Identify and interpret key features of a graph that models a relationship between two quantities.				
4. Sketch a graph that displays key features of a function that has been described verbally.				

For use with skill/concept #1

1. Which of the following representations/situations can be modeled by the function $y = 2x + 10$? Circle all that apply.
    a. A pool has 10 gallons of water in it and water is being added to the pool at a rate of 2 gallons per minute.
    b. A pool has 2 gallons of water in it and water is being added to the pool at a rate of 10 gallons per minute.
    c. There are 10 bacterium in a petri dish. Each hour, the number of bacteria in the dish doubles.
    d. There are currently 10 shoes on the shelf in a store. The owner is adding boxes with pairs of shoes inside to the shelf.
    e. Penny has 10 pennies in a jar. Each day, she adds 2 pennies to the jar.

2. Create 3 different representations (a table, graph, and context) that can be modeled by the function $y = 4x$.

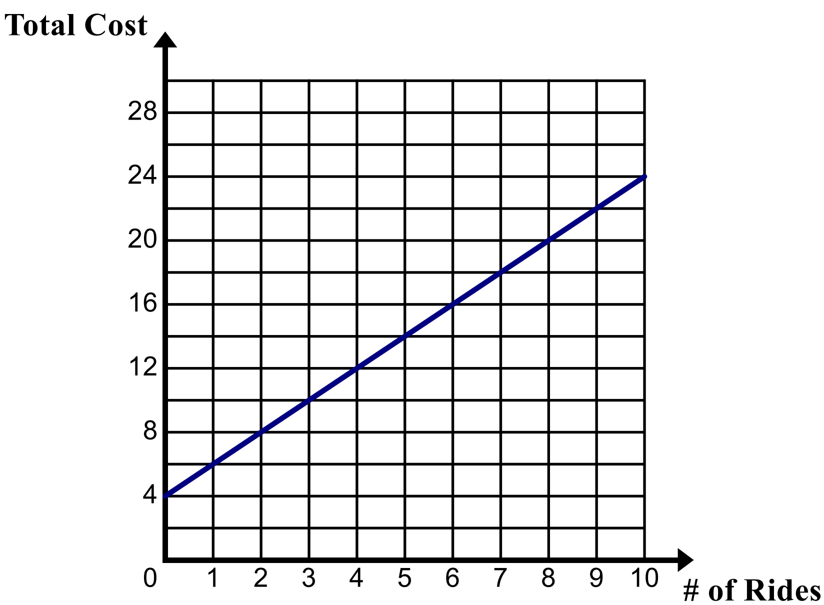
3. Circle the letter of the representations that can be modeled by a linear function. Construct a linear function for those that are linear.

a.

Radius *(in)*	Area *(in^2)*
1	3.14
2	12.56
3	28.26
4	50.24
5	78.50

b. The cost of a frozen yogurt at Callie's Custard Shop is \$4.50. Each additional topping is \$0.25.

c. The graph below shows the total cost dependent on the number of rides taken.



d. Nick receives a 3% raise every year.

e. A plane starts is descent from an elevation of 35,000 feet. The table below shows the elevation of the plane as it is descending.

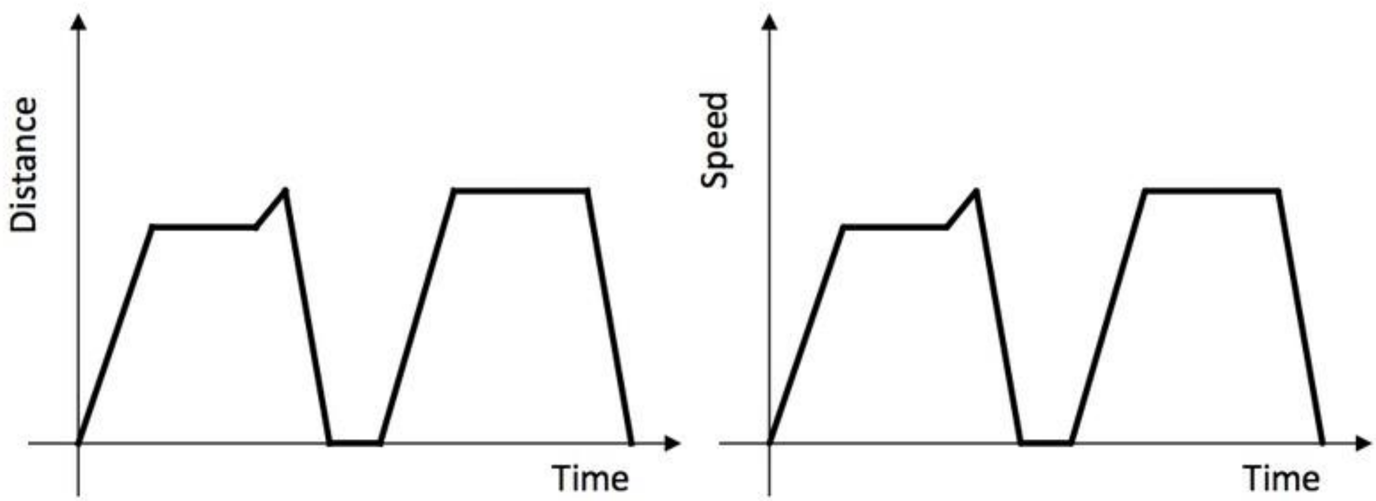
Time (min.)	Elevation (ft.)
0	35,000
3	27,500
5	22,500
6	20,000

For use with skill/concept #2

1. Maya and her brother each brought a seedling plant home from the store. The plants are both growing at a constant rate. Maya's plant was 8 cm. tall 2 weeks after she brought it home and 20 cm. tall 8 weeks after she brought it home. The height h of her brother's plant in centimeters t weeks after he brought it home can be modeled by the equation $h = \frac{3}{2}t + 6$. Which plant is growing at a faster rate? Which plant was taller when they brought the plants home?

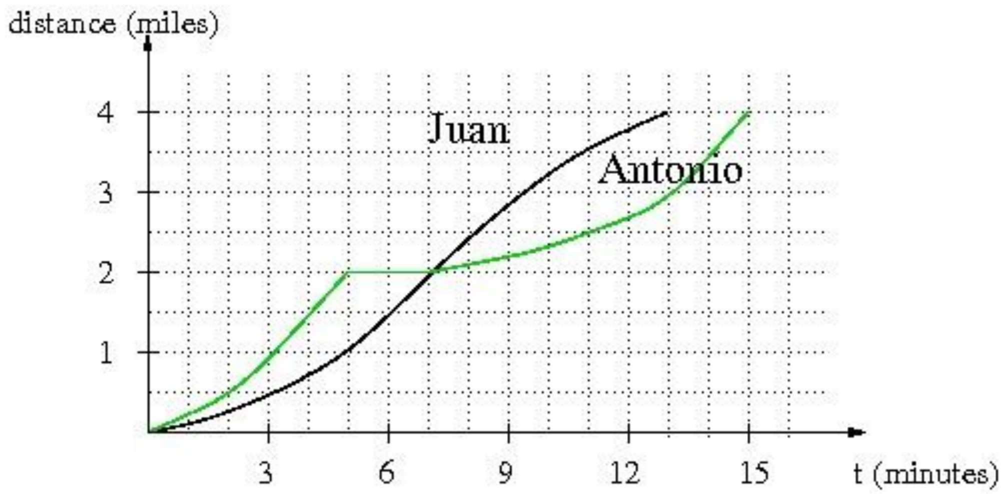
For use with skill/concept #3

1. Below are two graphs that look the same. Note that the first graph shows the distance of a car from home as a function of time and the second graph shows the speed of a different car as a function of time. Describe what someone who observes the car's movement would see in each case.



This is an Illustrative Mathematics Task: https://www.illustrativemathematics.org/illustrations/632

2. Antonio and Juan are in a 4-mile bike race. The graph below shows the distance of each racer (in miles) as a function of time (in minutes).

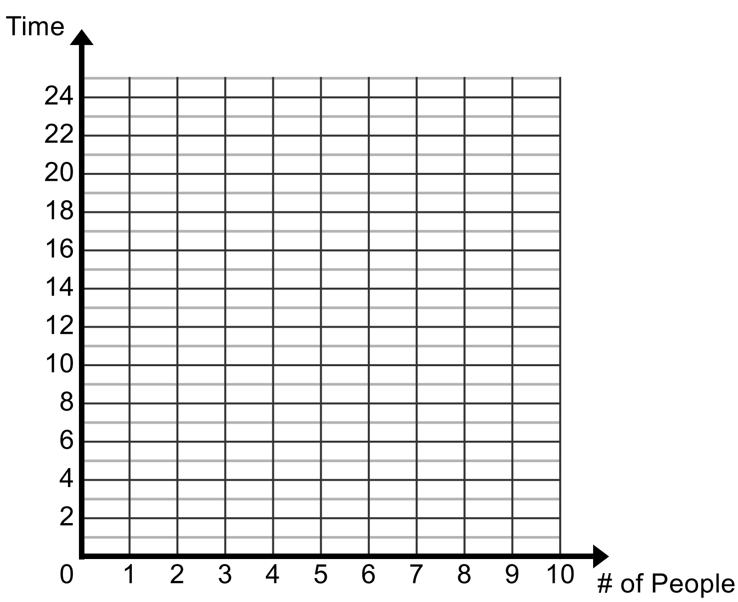


a. Who wins the race? How do you know?
b. Imagine you were watching the race and had to announce it over the radio. Write a little story describing the race.

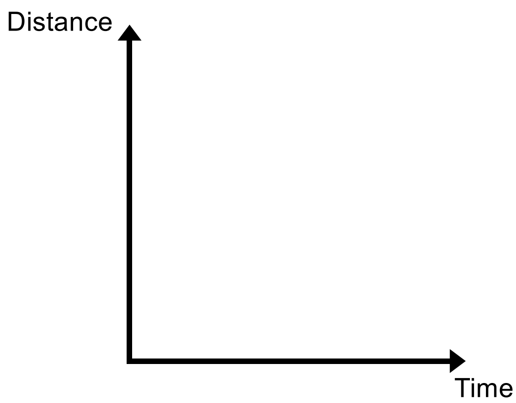
This is an Illustrative Mathematics Task: https://www.illustrativemathematics.org/illustrations/633

For use with skill/concept #4

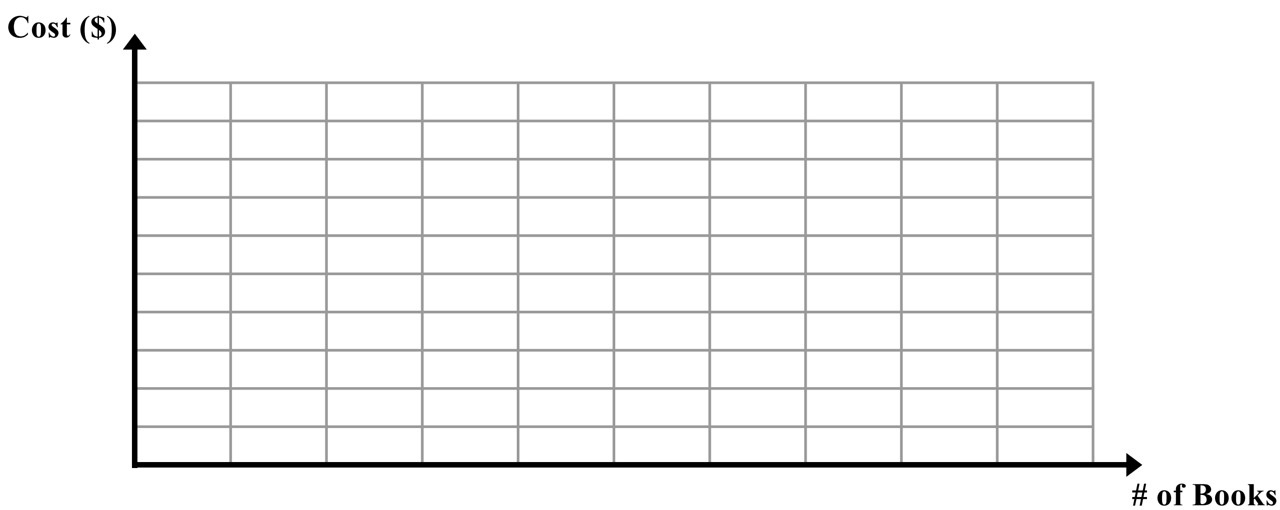
1. It will take Rick 24 hours to paint a fence in his backyard. Rick is trying to get some friends to help him paint the fence. Sketch a graph of the amount of time it will take to paint the fence dependent on how many friends Rick gets to help.



2. Sketch a graph of Carrie's distance from home. Carrie starts at home, walks to the neighbors to play, stays at the neighbors to play, then runs home.



3. Sketch the graph of the total cost of ordering books dependent on the number ordered given the following criteria: it costs $110 per book if you order 0 – 50 books, $90 if you order 51 – 100 books, and $75 if you order more than 100 books.



4. Sketch a graph of your energy level during the day from the time you wake up until the time you go to sleep at night. Label key features and events of the day.
5. Sketch a graph of the distance the second hand of a clock is from the number 6 as it moves around the clock.