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Chapter 7: Rational and Irrational Numbers (3 weeks)

Utah Core Standard(s):

- Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. (8.EE.2)
- Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers, show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. (8.NS.1)
- Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$, is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. (8.NS.2)

Academic Vocabulary: square, perfect square, square root, $\sqrt{}$, cube, perfect cube, cube root, $\sqrt[3]{}$, quadratic equation, cubic equation, inverse operation, decimal expansion, repeating decimal, terminating decimal, rational number, irrational number, truncate, decimal approximation, real number, real number line

Chapter Overview:

In 8th grade, students begin to think more carefully about the real line by asking the question, "Is there a number associated with every point on the line?" Up to this point, students have worked only with rational numbers, numbers they generated from iterations of a whole unit or portions of whole units. Part of their work included identifying a point on the real line associated with each rational number. Students explore the question posed above through an activity that has them constructing the lengths of non-perfect squares, thus introducing students to irrational numbers.

The chapter starts by having students examine the relationship between the area of a square and its side length. This activity introduces students to the idea of what it means to take the square root of a number. Additionally, students begin to surface ideas about the limitations of rational numbers. In the activity, students construct physical lengths of irrational numbers and begin to realize that we cannot find an exact numerical value for these numbers. Students then use the knowledge gained from this activity to simplify square roots and solve simple quadratic and cubic equations (i.e. $x^2 = 40$ and $x^3 = 64$).

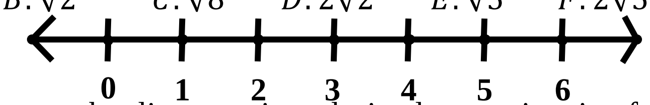
In the last section, students deepen their understanding of what an irrational number is and in the process solidify their understanding of rational numbers. They realize that even though they cannot give an exact numerical value for the lengths of non-perfect squares, they can transfer these lengths to a number line to show the exact location of these numbers. Once the numbers are placed on the real line, students can approximate the value of these irrational numbers and compare their value to rational numbers. At the end of the chapter, students learn additional methods for approximating the value of irrational numbers to desired degrees of accuracy, estimate the value of expressions containing irrational numbers, and compare and order rational and irrational numbers.

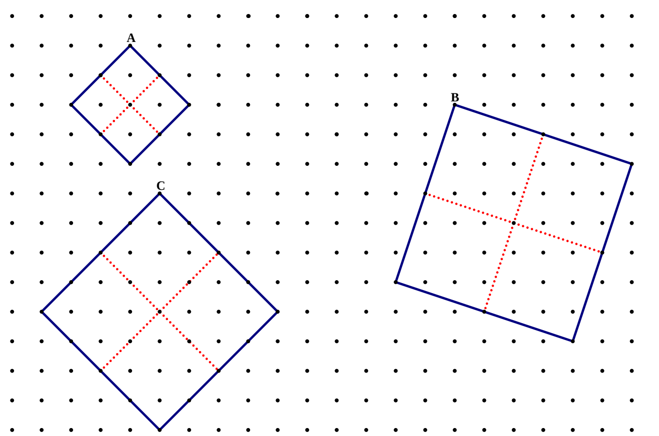
Connections to Content:
Prior Knowledge: Students have worked a great deal with rational numbers up to this point. They have defined and worked with the subsets of rational numbers. They have represented rational numbers on a number line, expressed rational numbers in different but equivalent forms, and operated with rational numbers. Students have also worked a great deal with slope, have an understanding of area, and know how to find the area of polygons and irregular shapes which will help them to access the tilted square material.

Future Knowledge: Later in this book, students will study exponent rules and deepen their understanding of the connection between taking the square root of a number and squaring a number. In subsequent courses, students will continue to extend their knowledge of the number system even further. For example, students will learn about complex numbers as a way to solve quadratic equations that have a negative discriminant. They will also continue to work with irrational numbers, learning how to operate on irrational numbers.

MATHEMATICAL PRACTICE STANDARDS:

	Make sense of problems and persevere in solving them.	A hospital has asked a medical supply company to manufacture intravenous tubing (IV tubing) that has a minimum opening of 7 square millimeters and a maximum opening of 7.1 square millimeters for the rapid infusion of fluids. The medical design team concludes that the radius of the tube opening should be 1.5 mm. Two supervisors review the design team's plans, each using a different estimation for π. **Supervisor 1:** Uses 3 as an estimation for π **Supervisor 2:** Uses 3.1 as an estimation for π The supervisors tell the design team that their designs will not work. The design team stands by their plans and tells the supervisors they are wrong. Who is correct and why? Recall that the formula for the area of a circle is $A = \pi r^2$. *In this problem, students realize the effects of approximating the value of irrational numbers. They must decide which estimation of π is appropriate for the given situation, appreciating that the precision of the estimation may have profound impact on decisions people make in the real world.*
	Reason abstractly and quantitatively.	The decimal $0.\overline{3}$ is a repeating decimal that can be thought of as 0.33333… where the "…" indicates that the 3s repeat forever. If they repeat forever, how can we write this number as a fraction? Here's a trick that will eliminate our repeating 3s. *To solve this problem, students create and solve a system of linear equations. The skills and knowledge they learned about systems of equations become an abstract tool that allows students to write repeating decimals as fractions, proving that they do in fact fit the definition of a rational number.*
	Construct viable arguments and critique the reasoning of others.	**Directions:** The table below contains statements about rational and irrational numbers. If the statement is true, put a check in the box. If the statement is not true, write a correct statement. • You can always use a calculator to determine whether a number is rational or irrational by looking at its decimal expansion. • The number 0.256425642564 ... is rational. • You can build a perfect cube with 36 unit cubes. • If you divide an irrational number by 2, you will still have an irrational number. *Students must have a clear understanding of rational and irrational numbers to assess whether the statements are true or false. If the statement is flawed, students must identify the flaw, and construct a statement that is true. Due to the fact that there are several possible ways to change the statements to make them true, students must communicate their statements to classmates, justify the statements, and question and respond to the statements made by others.*

	Model with mathematics.	People often wonder how far they can see when they're at the top of the tallest buildings such as the Empire State Building, The Sears Tower in Chicago, etc. The farthest distance you can see across flat land is a function of your height above the ground. If *h* is the height in meters of your viewing place, then *d*, the distance in kilometers you can see, can be given by this formula: $d = 3.532\sqrt{h}$ The CN Tower in Toronto, Canada is 555 meters tall. It is near the shore of Lake Ontario, about 50 kilometers across the lake from Niagara Falls. Your friend states that on a clear day, one can see as far as the falls from the top of the Tower. Are they correct? Explain your answer. *The formula shown above is a model for the relationship between the height of a building and the distance one can see. Students use this model along with their knowledge of square roots to solve problems arising in everyday life.*
	Use appropriate tools strategically.	**Directions:** Show the length of the following numbers on the number line below. Use the grid on the following page to construct lengths where needed and transfer those lengths onto the number line. Then answer the questions that follow. **Note:** On the grid, a horizontal or vertical segment joining two dots has a length of 1. On the number line, the unit length is the same as the unit length on the dot grid. $A: \sqrt{25}$ $B: \sqrt{2}$ $C: \sqrt{8}$ $D: 2\sqrt{2}$ $E: \sqrt{5}$ $F: 2\sqrt{5}$  1. Use the number line to write a decimal approximation for $\sqrt{2}$. 2. Would 1.41 be located to the right or to the left of $\sqrt{2}$ on the number line? 3. Describe and show how you can put $-\sqrt{2}$ on the number line. Estimate the value of this expression. 4. Describe and show how you can put $(2+\sqrt{2})$ on the number line. Estimate the value of this expression. 5. Describe and show how you can put $(2-\sqrt{2})$ on the number line. Estimate the value of this expression. 6. Describe and show how you can put $2\sqrt{2}$ on the number line. Estimate the value of this expression. *To solve this problem, students use dot paper to construct physical lengths of irrational numbers. They can then transfer these segments to the number line using patty (or tracing) paper. Once on the number line, students can use these tools (number line, dot paper, patty paper, constructed segments) to approximate the value of given expressions (i.e.* $(2+\sqrt{2})$*).*

	Attend to precision.	Use the following approximations and calculations to answer the questions below. Do not use a calculator. Approximation: π is between 3.14 and 3.15 Calculations: $3.1^2 = 9.61$ $3.2^2 = 10.24$ $3.16^2 = 9.9856$ $3.17^2 = 10.0489$ Put the following numbers in order from **least to greatest**. $\sqrt{10}, 3\frac{1}{10}, 3.\overline{1}, \pi,$ side length of a square with an area of 9 Find a number between $3\frac{1}{10}$ and $3.\overline{1}$. Find a number between 3.1 and $\sqrt{10}$. *This task demands mastery of the topics learned in the chapter. Students must have a very clear understanding of square roots, repeating decimals, and irrational numbers. They must closely analyze the decimal expansions (approximations) of the numbers as well as the calculations given to be able to compare and order the numbers.*
	Look for and make use of structure.	Square A shown below has an area of 8 square units. Determine the following measures: a. The area of one of the smaller squares that makes up Square A b. The side length of one of the smaller squares that makes up Square A c. The side length of the large square A (written 2 different ways)  *This problem allows students to use structure to understand why $\sqrt{8}$ is the same as $2\sqrt{2}$. They can see the equivalence in the concrete model. A square with an area of 8 (see Square A) has a side length of $\sqrt{8}$ units. This side length is comprised of 2 smaller, congruent segments that*

		each measure $\sqrt{2}$ units as they are each the side length of a square with an area of 2. This concrete representation builds a conceptual understanding for students as we then move to the algorithm for simplifying square roots.
	Look for and express regularity in repeated reasoning.	Change the following rational numbers into decimals **without** the use of a calculator. $\frac{1}{7}$ *This problem allows students to understand why the decimal expansion of a rational number either always terminates or repeats a pattern. Working through this problem, and others, students begin to understand that eventually the pattern must repeat because there are only so many ways that the algorithm can go. Once a remainder repeats itself in the division process, the decimal expansion will start to take on a repeating pattern. Students should see this when they begin repeating the same calculations over and over again and conclude they have a repeating decimal.*

7.0 Anchor Problem: Zooming in on the Number Line

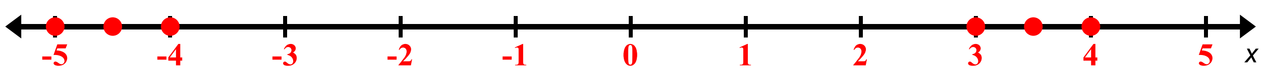
The problems below are a review of skills learned in 6th and 7th grade. In 6th and 7th grade, students used the number line as a model for thinking about numbers. Students learned how to partition the number line into desired lengths in order to associate all rational numbers to a point on the number line. In the last two problems below, students review that we can associate every decimal (a number that can be represented by a fraction whose denominator is a power of 10) to a point on the number line by zooming in on the number line and chopping the intervals into repeated subdivisions of tenths.

In 8th grade, the question becomes, "Do all points on the real number line correspond to a rational number?" The answer of course is no – there are lengths that cannot be represented by fractions. These numbers are called irrational. In 8th grade, students are exposed to a subset of the irrational numbers, the square roots of non-perfect squares. In the lessons that follow, students will construct lengths of non-perfect squares and understand that even though we cannot give an exact decimal value for these numbers, we can show the location of these numbers on the real number line by copying our constructed lengths to the real number line. Students also approximate the value of irrational numbers to increasing levels of accuracy and show the location of their approximations on the number line, zooming in on pieces of the number line until the desired level of accuracy is reached.

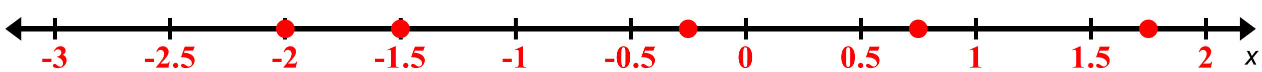
Sample answers are provided.
Another optional activity is to have students explore an interactive number line. A zoomable number line with exploration activities is available at http://www.mathsisfun.com/numbers/number-line-zoom.html.
Additionally, there are other number line tools available for computers and tablets.

Directions: Place the following sets of numbers on the number lines provided and label each point. You will need to decide where to place 0 and the measure of the intervals for each problem.

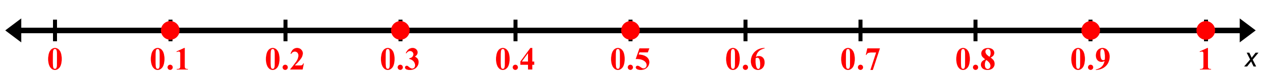
$A: 3$ $B: 4$ $C: 3.5$ $D: -4$ $E: -5$ $F: -4.5$



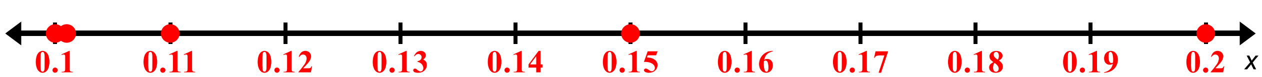
$L: -\frac{1}{4}$ $M: \frac{3}{4}$ $N: -1\frac{1}{2}$ $O: 1.75$ $P: -2$



$V: \frac{1}{10}$ $W: \frac{3}{10}$ $X: \frac{1}{2}$ $Y: \frac{9}{10}$ $Z: \frac{10}{10}$



$H: 0.1$ $I: 0.2$ $J: 0.15$ $K: 0.11$ $L: 0.101$



$A: 3.1$ $B: 3.2$ $C: 3.15$ $D: 3.11$ $E: 3.105$ $F: 3.111$

Directions: Refer to the number line above to answer the questions that follow.
Students do not need to write out all of the answers to the following problems. Many are there for discussion purposes.

1. Are there other numbers you can place between 3.1 and 3.11? If yes, find a number.
   Students really narrow in on the piece of the number line from 3.1 to 3.11. Besides 3.105 that we have already graphed, we can consider 3.101, 3.102, 3.103, etc. We can also divide the interval from 3.1 to 3.11 into thirds, quarters, etc. and name these points. There are many possible answers.
2. Are there other numbers you can place between 3.11 and 3.111? If yes, find a number.
   Many answers, 3.1101, 3.1102, etc. Students can think about taking this tiny segment, zooming in on it, and dividing it into intervals.
3. How are you coming up with the numbers? Are there others? How do you know?
   Listen to student answers as to how they are coming up with numbers. The idea here is that we can partition intervals on the number line in any way we choose to show the location of all rational numbers. We can continue this process over and over showing that the number line is a continuum of numbers and that we can repeat this process of partitioning the number line an infinite number of times.
4. Where would you put $3.\overline{1}$ on the number line and why?
   At this point, students will most likely just be approximating the location of $3.\overline{1}$, knowing that it lies to the right of 3.11. You can talk about zooming in on the piece of the number line from 3.11 to 3.12 and encourage students to be even more specific and see that say $3.\overline{1}$ will lie to the right of 3.111 but to the left of 3.112. This is a good time to review the meaning of the bar to show repeating decimals. Later in the chapter, students will learn how to change repeating decimals into fractions which will allow them to be even more precise in their placement of repeating decimals on the number line.
5. What can you conclude about the real number line based on this activity?
   Listen to what students conclude from this activity. Some possible responses: The number line is a continuum of numbers. We can partition the number line in any way we wish to show the location of all rational numbers on the number line. We can partition segments on the number line an infinite number of times.

Section 7.1: Represent Numbers Geometrically

Section Overview:
In this section, students are exposed to a new set of numbers, irrational numbers. This chapter starts with a review of background knowledge – finding the area of polygons and irregular shapes, using ideas of slope to create segments of equal length, and reviewing the definition of a square. Then students build squares with different areas and express the measure of the side length of these squares, gaining an understanding of what it means to take the square root of a number. Additionally, students start to surface ideas about irrational numbers. Students create squares that are not perfect and realize they cannot find an exact numerical value for the side length of these squares (e.g. a number that when squared results in the area of the square created). Students also simplify square roots, connecting the simplified answer to a physical model. At the end of the section, students use cubes and volume to gain an understanding of what is meant by the cube root of a number.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Understand the relationship between the side length of a square and its area.
2. Understand the relationship between the side length of a cube and its volume.
3. Evaluate the square roots of small perfect squares and the cube roots of small perfect cubes.
4. Simplify square and cube roots.

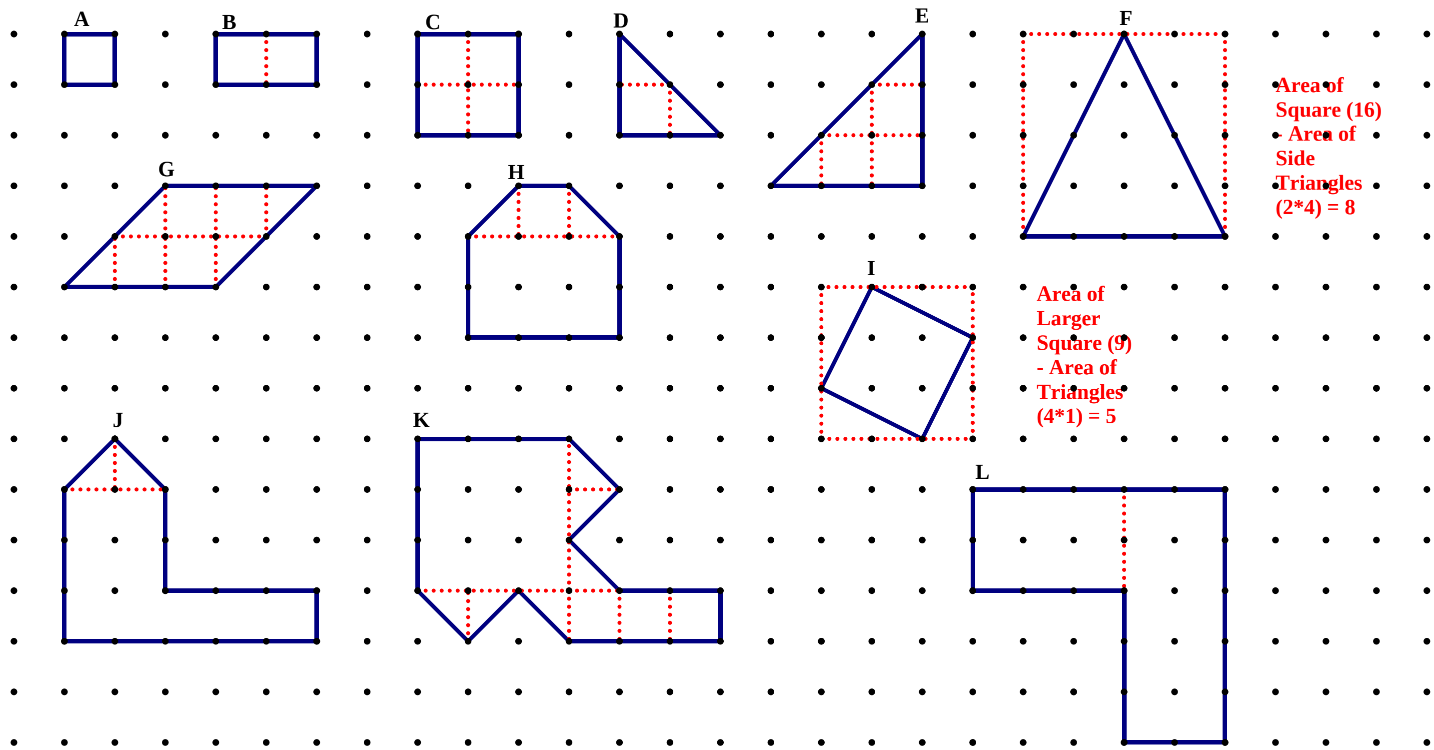
In section 1, **the focus is on understanding the relationship between the side length of a square and its area.** In the process, students construct lengths of irrational numbers and transfer these lengths to the number line (by putting the line segment on the number line with one point at 0). They also begin to see that we cannot find an exact numerical value for these numbers. **It is in section 3** that we define what an irrational number is, a number that cannot be expressed as the ratio of an integer to a natural number.

7.1a Class Activity: Background Knowledge

The purpose of this exercise is to prepare students for 7.1b, where they are finding the area of squares, including tilted squares found in Figure I below. Methods may include: combining and counting squares, decomposing into smaller common shapes, subtraction method, etc.

Activity 1: Finding Area of Irregular Shapes

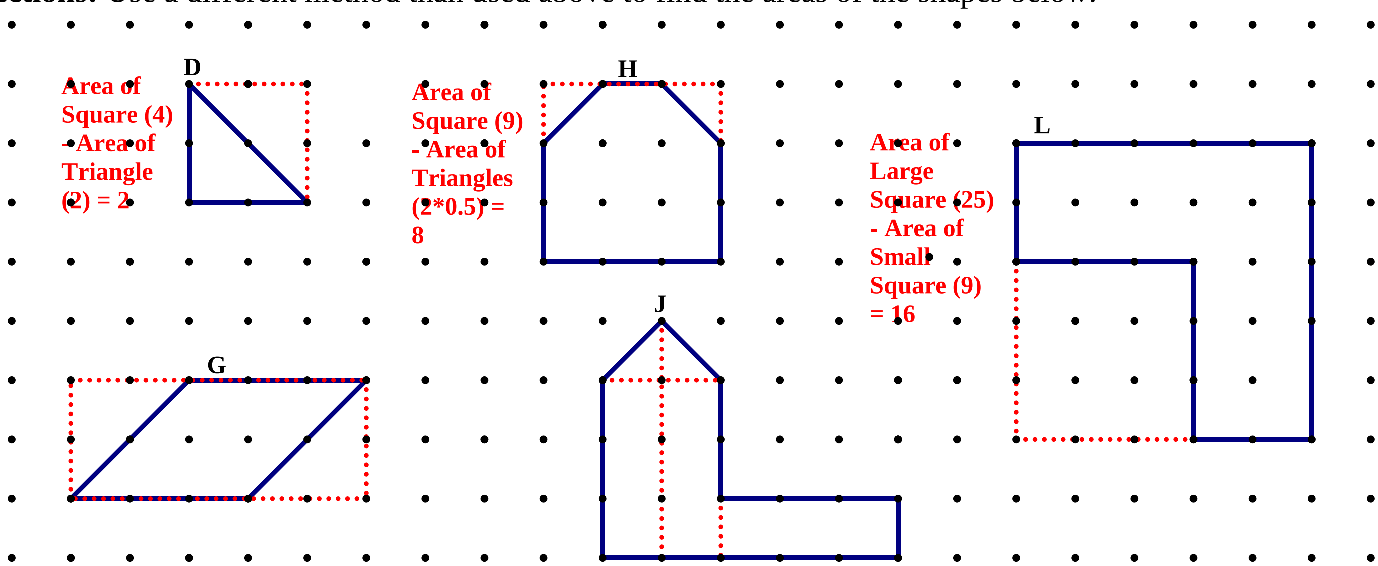
Directions: Find the area of the following shapes. On the grid, a horizontal or vertical segment joining two dots has a length of 1. Put your answers on the lines provided below the grid.



A: ____1____ B: ___2_____ C: ___4_____ D: ___2_____ E: ___4.5_____ F: ___8______

G: ____6____ H: ___8_____ I: ____5_____ J: ____10____K: ____15_____ L: ___16______

Directions: Use a different method than used above to find the areas of the shapes below.

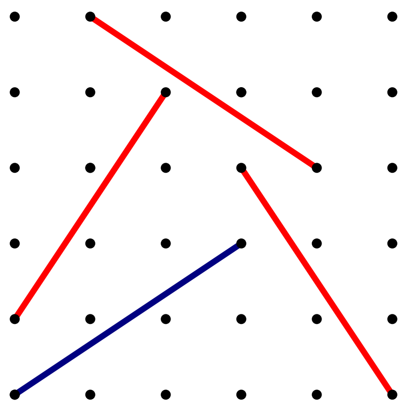


D: ____2____ G: ____6____ H: ____8____ J: ___10_____L: ____16_____

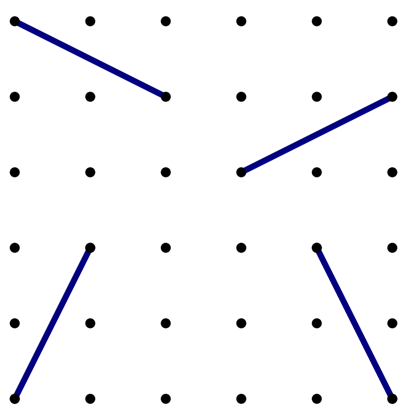
Activity 2: Slopes and Lengths of Segments

1. Using tracing paper, construct 3 additional segments that are the same length as the segment shown below. Your segments cannot be parallel to the segment given and must start and end on a dot on the grid.

The purpose of this activity is that students use ideas about slope in order to construct segments that are equal in length (pairs of segments also happen to be perpendicular). This will help students when constructing their tilted squares. This is also a preview of what is to come in chapter 9 with reflections and rotations. Note that, with the given conditions, every possible answer has to be parallel to one of the 3 segments in red.

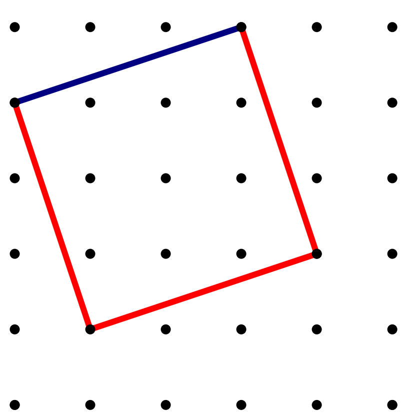


2. Using the ideas from the previous problem and the one below, write down observations you have about the line segments shown on the grid.



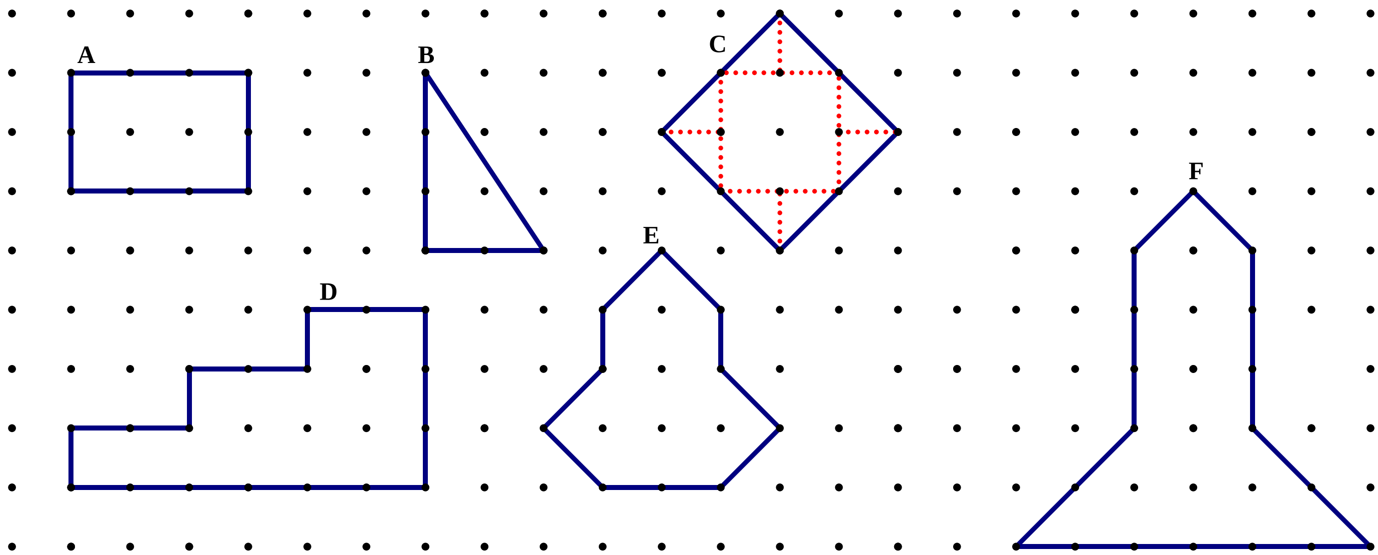
Possible observations: They are the same length. Make the connection to slope: The slope triangles that make up the segments use the same numbers (1 and 2) but in some cases the 1 is the rise and the 2 is the run and in others the 2 is the rise and the 1 is the run. If we imagine this on the coordinate plane, some have positive slopes, some are negative. The segments are perpendicular in pairs.

3. Create a square on the grid below, using the given segment as one of the sides of the square.



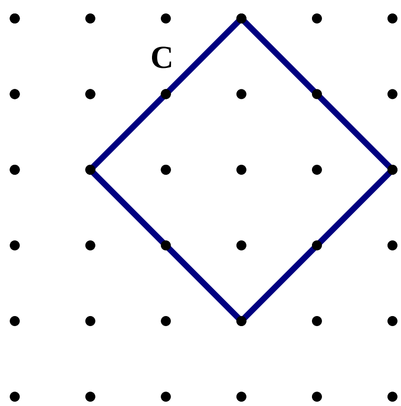
7.1a Homework: Background Knowledge

1. Find the areas of the following shapes. On the grid, a horizontal or vertical segment joining two dots has a length of 1. Put your answers on the lines provided below the grid.

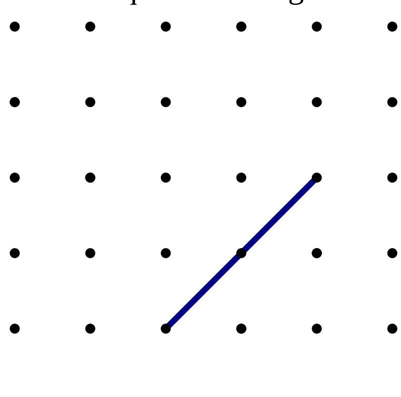


A: ________ B: ____3____ C: ___8_____ D: ________ E: ________ F: _________

2. Show a second method for finding the area of shape C.



3. Create a square on the grid below, using the given segment as one of the sides of the square.



7.1b Class Activity: Squares, Squares, and More Squares

On the following pages of dot paper:

1) Create as many different squares with areas from 1 - 100 as possible. On the grid, a horizontal or vertical segment joining two dots has a length of 1. **Each of the vertices of the square must be on a dot.**

2) Find the area of each square you made and label each square with its area.

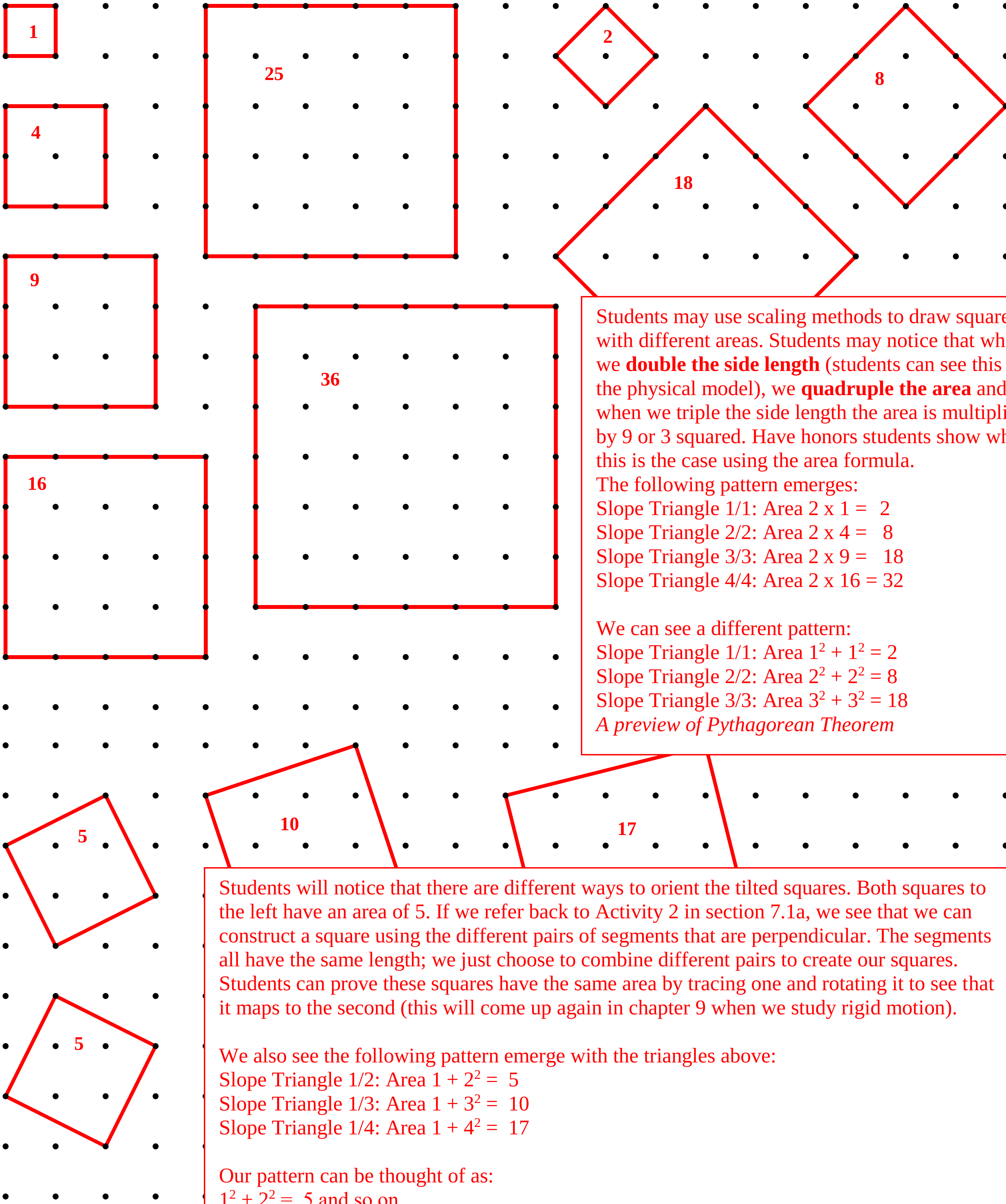
3) Complete the table below using the squares you created. The table has been filled in with the area and side lengths of some of the squares on the following pages.

Area	**Side Length**
1	1
4	2
9	3
25	5
36	6
2	$\sqrt{2}$
8	$\sqrt{8}$ or $2\sqrt{2}$
5	$\sqrt{5}$
17	$\sqrt{17}$

The activities in 7.1a were intended to provide students with the background knowledge and skills necessary to access this lesson.

The following bullets represent the primary mathematical knowledge students should know and understand from doing this lesson.

- If we are given the side length of a square, s, then its area is s^2.
- If we are given the area of a square, A, then its side length is $\sqrt{A}$.
- To find the square root of a number, find a number that when multiplied by itself equals the given number.
- The side length of a perfect square is a whole number.
- We cannot find an exact value for the side length of a non-perfect square; therefore we represent the side length as $\sqrt{A}$.



Students may use scaling methods to draw squares with different areas. Students may notice that when we **double the side length** (students can see this on the physical model), we **quadruple the area** and when we triple the side length the area is multiplied by 9 or 3 squared. Have honors students show why this is the case using the area formula.

The following pattern emerges:

Slope Triangle 1/1: Area 2 x 1 = 2
Slope Triangle 2/2: Area 2 x 4 = 8
Slope Triangle 3/3: Area 2 x 9 = 18
Slope Triangle 4/4: Area 2 x 16 = 32

We can see a different pattern:

Slope Triangle 1/1: Area $1^2 + 1^2 = 2$
Slope Triangle 2/2: Area $2^2 + 2^2 = 8$
Slope Triangle 3/3: Area $3^2 + 3^2 = 18$

A preview of Pythagorean Theorem

Students will notice that there are different ways to orient the tilted squares. Both squares to the left have an area of 5. If we refer back to Activity 2 in section 7.1a, we see that we can construct a square using the different pairs of segments that are perpendicular. The segments all have the same length; we just choose to combine different pairs to create our squares. Students can prove these squares have the same area by tracing one and rotating it to see that it maps to the second (this will come up again in chapter 9 when we study rigid motion).

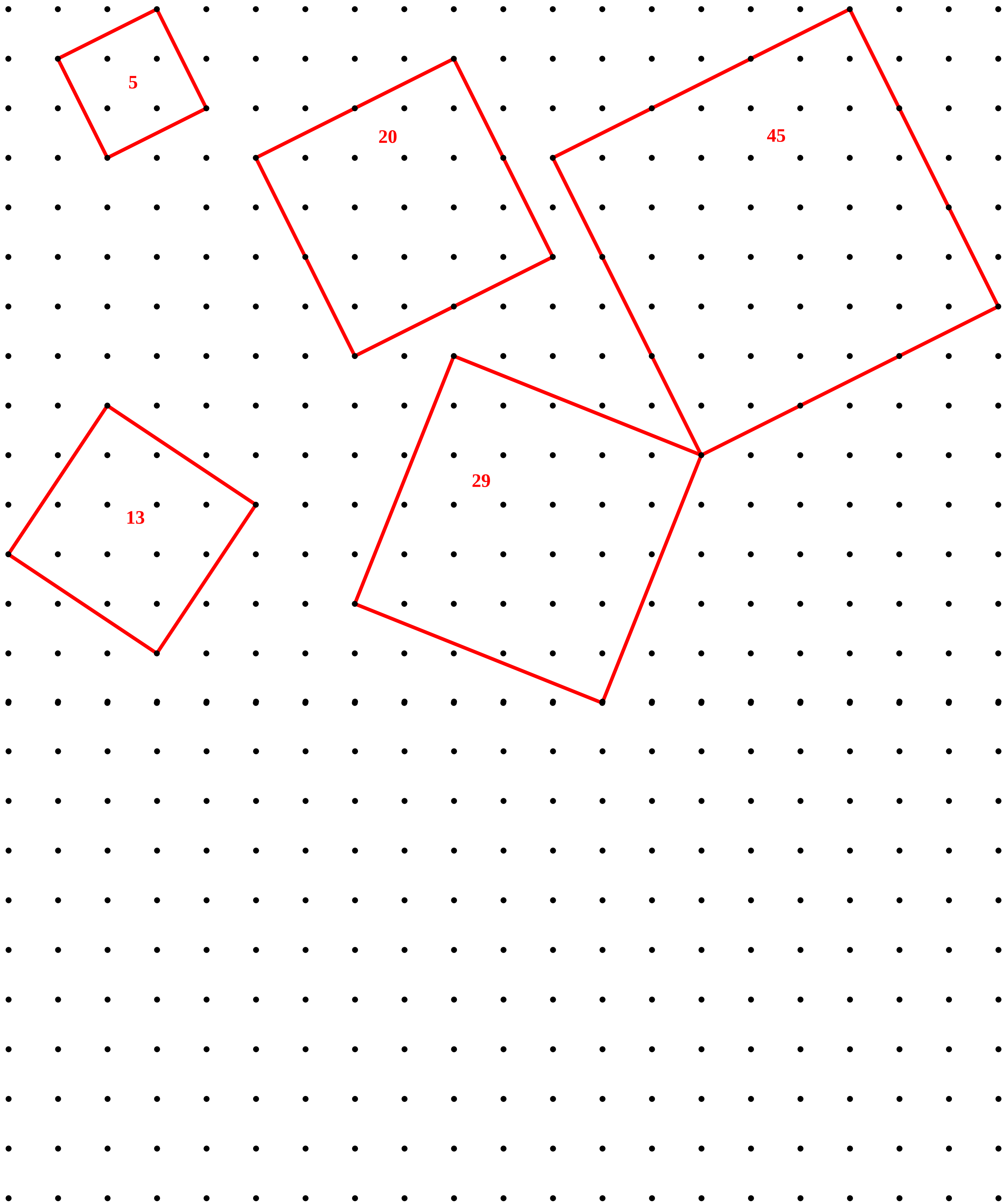
We also see the following pattern emerge with the triangles above:

Slope Triangle 1/2: Area $1 + 2^2 = 5$
Slope Triangle 1/3: Area $1 + 3^2 = 10$
Slope Triangle 1/4: Area $1 + 4^2 = 17$

Our pattern can be thought of as:

$1^2 + 2^2 = 5$ and so on…

A preview of Pythagorean Theorem which will be explored in detail in Chapter 10


5
20
45
13
29

1. Complete the following table…

Area (square units)	Length of Side (units)
1	1
9	3
	5
2	$\sqrt{2}$
5	$\sqrt{5}$
13	$\sqrt{13}$
	$\sqrt{5}$
100	
	20

2. Find the missing measure.

a. A = 121 cm^2 s = 11cm

b. A = 16 m^2 s = 4 m

Directions: Complete the following sentences. **Provide examples to support your statements.**

3. A perfect square is created when…
   A whole number is raised to the second power. Ex. $4^2 = 16$

4. To find the area of a square given the side length of the square…
   Square the side length (examples will vary)
   $A = s^2$

5. To find the side length of a square given the area of the square…
   Take the square root of the area (examples will vary)
   $s = \sqrt{A}$

6. Simplify the following.

a. $\sqrt{36}$
6

b. $\sqrt{121}$
11

c. $\sqrt{16}$

d. $\sqrt{1}$

e. $\sqrt{100}$

f. $\sqrt{49}$

g. $\sqrt{625}$
25

h. $\sqrt{2500}$
50

i. $\sqrt{225}$

Again, we only focus on the positive roots in this chapter. In section 7.2, we will see that there are two roots when solving simple quadratics.

7.1b Homework: Squares, Squares, and More Squares

1. List the first 12 perfect square numbers. The first 3 are 1, 4, 9,

2. What is the side length of a square with an area of 9 units^2? 3

3. What is the area of a square with a side length of 2 units? 4

4. Complete the following table.

Area (square units)	Length of Side (units)
4	2
49	
144	
100	10
2	$\sqrt{2}$
8	
15	$\sqrt{15}$
	$\sqrt{41}$
	1
	9
	s
A	$\sqrt{A}$

5. Find the missing measures of the squares:

a. $A = 47\text{ ft}^2$ $s =$

b. $A =$ $s = 5\text{ in}$

6. Simplify the following:

a. $\sqrt{9}$
3

b. $\sqrt{100}$

c. $\sqrt{64}$
8

d. $\sqrt{4}$

e. $\sqrt{144}$

f. $\sqrt{81}$

g. $\sqrt{400}$

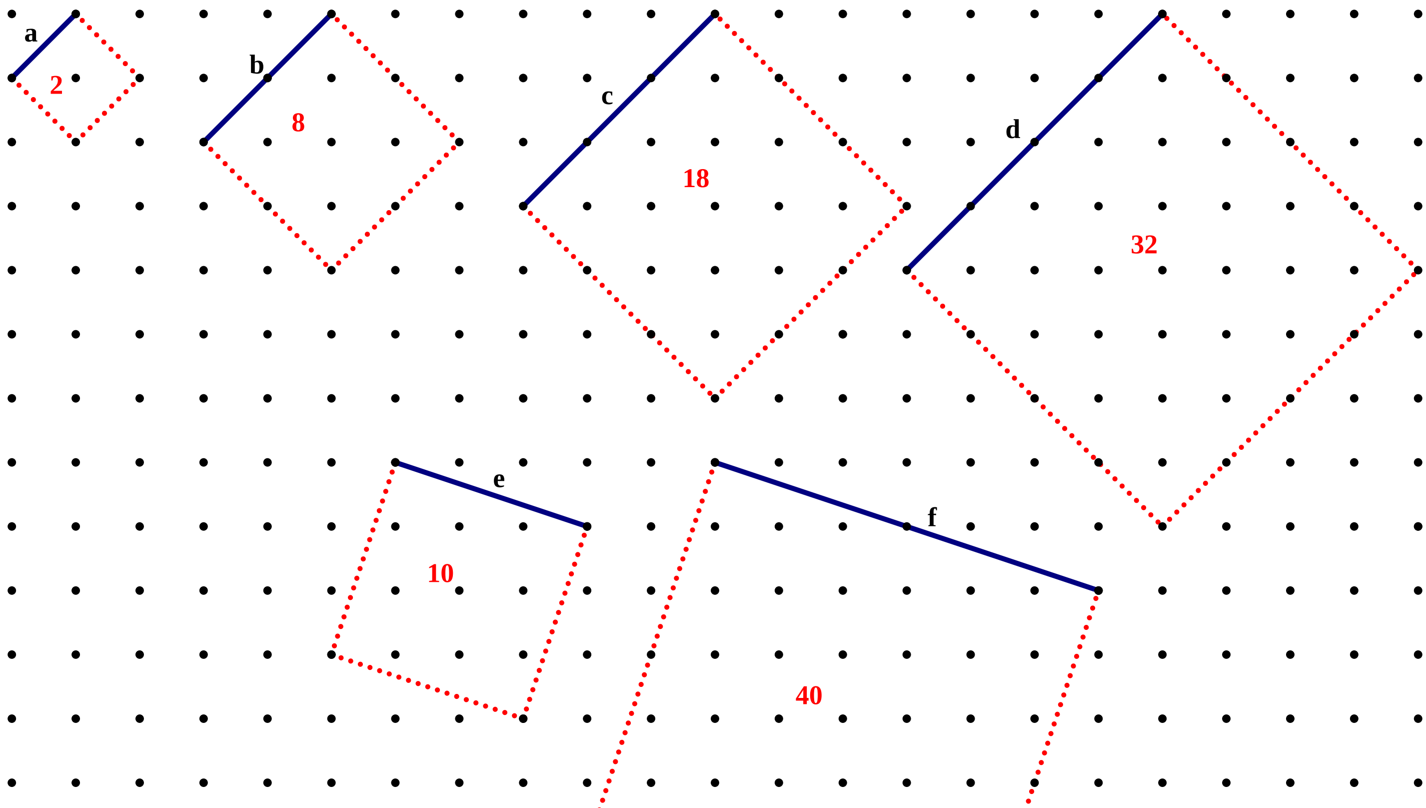
h. $\sqrt{1600}$
40

i. $\sqrt{2500}$

7.1c Class Activity: Squares, Squares, and More Squares Cont.

In the previous sections, we have learned how to simplify square roots of perfect squares. For example, we know that $\sqrt{36} = 6$. What about the square roots of non-perfect squares? How do we know that they are in simplest form? For example, is $\sqrt{5}$ in simplest form? How about $\sqrt{8}$? $\sqrt{147}$? Let's take a look.

1. Determine the lengths of line segments a through f without the use of a ruler. Write your answers in the space provided below each grid.



Students may use a variety of strategies to solve these problems. They may construct the square with the given side length, determine the area of the square, and then determine the side length (as shown above). Once they determine that a) has a side length of $\sqrt{2}$, they may conclude that b) has a side length of $2\sqrt{2}$. The length of the segment in b) is two times longer than a). Put another way, b) is two copies of a). 2 copies of $\sqrt{2}$ can be expressed as $2\sqrt{2}$. c) is three times longer than a) or $3\sqrt{2}$. Students may also call the side lengths $\sqrt{8}$ and $\sqrt{18}$ respectively. We will explore the equivalence of these numbers later in the lesson.

a. ______$\sqrt{2}$__________

b. ______$\sqrt{8}$ or $2\sqrt{2}$___

c. ___$\sqrt{18}$ or $3\sqrt{2}$______

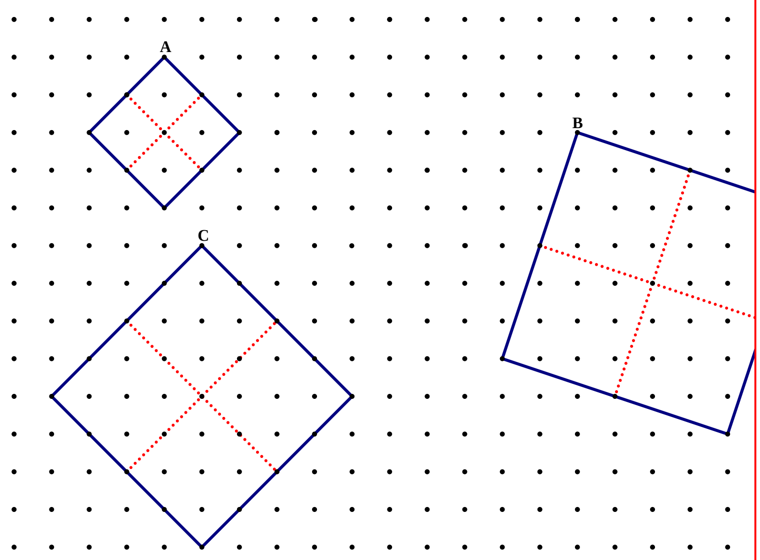
d. ____$\sqrt{32}$ or $4\sqrt{2}$_____________

e. ____$\sqrt{10}$ _____________

f. ____$\sqrt{40}$ or $2\sqrt{10}$ _____________

Directions: Use the squares on the grid below to answer the questions that follow. Each of the large squares A, B, and C has been cut into four smaller squares of equal size.

In this part of the lesson, students see a physical model that shows them that $\sqrt{8}$ and $2\sqrt{2}$ are equivalent but that $2\sqrt{2}$ is the simplified form of $\sqrt{8}$. They also see that $\sqrt{32}$ and $2\sqrt{8}$ and $4\sqrt{2}$ are all equivalent with $4\sqrt{2}$ being the simplified form. This leads into the next section in which students understand what it means to simplify square roots and learn an algorithm for simplifying square roots.



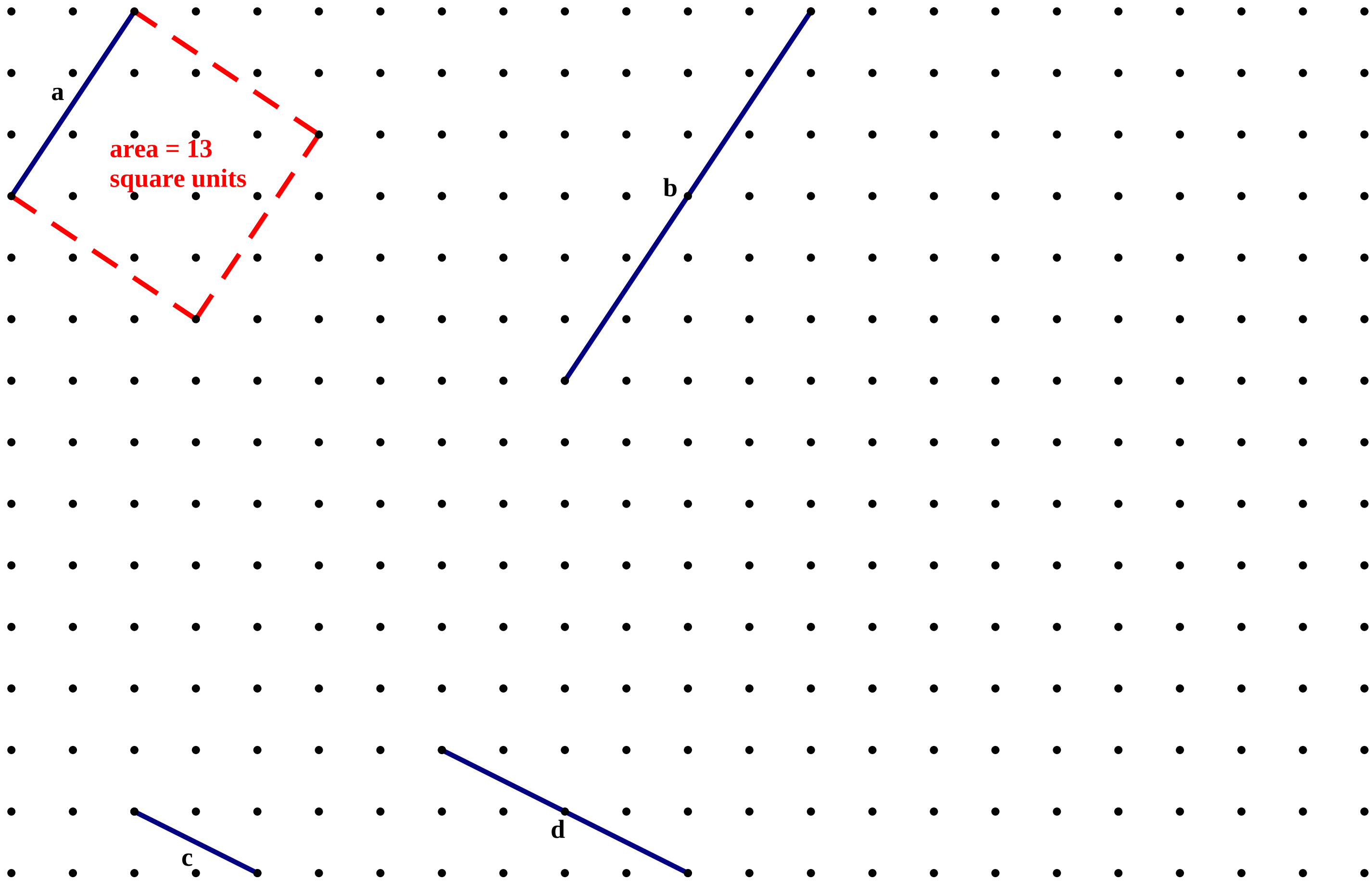
2. Square A has an area of 8 square units. Answer the following questions.
   a. What is the area of one of the smaller squares that makes up Square A? __2________
   b. What is the side length of one of the smaller squares that makes up Square A? __$\sqrt{2}$________
   c. What is the side length of the large square A (written 2 different ways)? __$\sqrt{8}$ or $2\sqrt{2}$________

3. Square B has an area of 40 square units. Answer the following questions.
   a. What is the area of one of the smaller squares that makes up Square B? __________
   b. What is the side length of one of the smaller squares that makes up Square B? __________
   c. What is the side length of the large square B (written two different ways)? _____________

4. Square C has an area of 32 square units. Answer the following questions.
   a. What is the area of one of the smaller squares that makes up Square C? ___8_______
   b. What is the side length of one of the smaller squares that makes up Square C?_$\sqrt{8}$ or $2\sqrt{2}$_____
   c. What is the side length of the large square (written *three* different ways)? __$\sqrt{32}$ or $2\sqrt{8}$ or $4\sqrt{2}$

7.1c Homework: Squares, Squares, and More Squares Cont.

1. Determine the lengths of line segments a through d without the use of a ruler. Write your answers in the space provided below each grid. See class activity #1 for help with this problem.



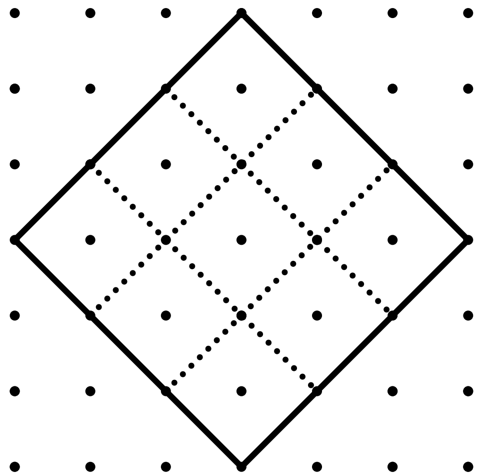
a. ______$\sqrt{13}$____________ c. __________________

b. ________________________ d. __________________

One way to find the lengths of the segments in #1 above is to construct a square off the given side length as shown for a) above. Once you construct the square, you can find the area of the square using the methods outlined in 7.1a and 7.2b. Once we know the area of the square, we can determine the side length of the square (the side length of the square will be the square root of the area of the square).

2. On the grid above, construct a segment that has a length of $\sqrt{45} = 3\sqrt{5}$.

3. Use the square on the grid below to answer the questions that follow.



See class activity #2 – 4 for help with this problem.

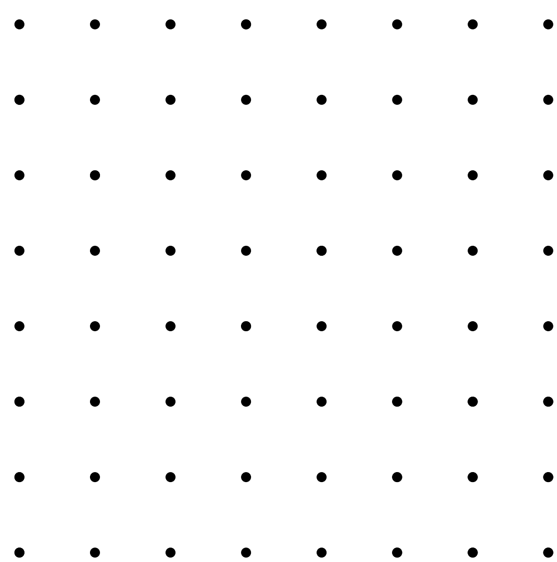
a. What is the area of the larger square? ______________________

b. What is the area of one of the smaller squares? __________2 units^2____________

c. What is the side length of one of the smaller squares? ____________________________

d. What is the side length of the larger square (written in two different ways)?
____

4. On the grid below, construct a segment with a length of $\sqrt{13}$ units. Explain how you know your segment measures $\sqrt{13}$ units.



7.1d Class Activity: Simplifying Square Roots

In this section we will learn two strategies for simplifying square roots of numbers that are not perfect squares. Both strategies are really doing the same thing, but the methods for each are a little different.

<u>Simplifying Square Roots</u>

Think back to the previous lesson. What does it mean to simplify a square root of a non-perfect square? What was the difference between the simplified version of these square roots as opposed to how they looked before they were simplified?

Let's look at some example from the previous lesson:

$$\sqrt{8} = 2\sqrt{2}$$
$$\sqrt{18} = 3\sqrt{2}$$
$$\sqrt{32} = 4\sqrt{2}$$
$$\sqrt{40} = 2\sqrt{10}$$

What observations can you make about the simplified versions of these square roots non-perfect squares? List them here:

Discuss the following with students:

When we simplify a square root of a non-perfect square we factor out as many perfect squares as we can, whatever is left has to stay inside the square root symbol and is called a "surd" which can't be simplified further. We can factor out perfect squares because when we take the square root of a perfect square we get a whole number. When we try to take the square root of non-perfect square we get a decimal that goes on and on without any pattern, so instead we leave it inside to keep it simpler. If we were to just enter the square root into our calculator we would eventually have to round the answer we would get, which is not as accurate. These are the reasons we simplify: accuracy and simplicity.

Two Strategies for Simplifying Square Roots

Strategy 1:

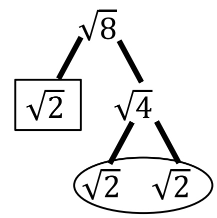
1. Find the greatest perfect square that is a factor of the number inside the square root symbol.
2. Rewrite the number inside the square root symbol as the product of the greatest perfect square and the other factor.
3. Take the square root of the perfect square. Remember: When you take the square root of the perfect square, it is no longer inside the square root symbol.
4. Continue this process until you can no longer find a perfect square other than 1 that is a factor of the number inside the square root symbol.

Strategy 2:

1. Using the factor tree method, factor the number inside the square root symbol.
2. Look for and circle any pairs of numbers among the factors.
3. Put a square around any numbers that are not part of a pair. Re-write the numbers as factors to see that the pairs can be removed, while anything left over must stay under the square root symbol.
4. Remove the pairs and leave any leftover numbers inside the square root symbol. Remember that because we are factoring, all of these numbers are being multiplied, so if you end up with multiple numbers outside or inside the square root symbol, multiply them together.

Examples:

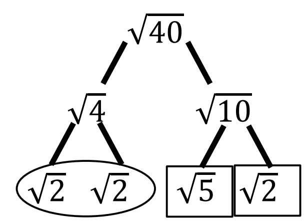
$\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$



$\sqrt{2 \cdot 2 \cdot 2}$ or $\sqrt{2^2} \cdot \sqrt{2}$

$= 2\sqrt{2}$

$\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$



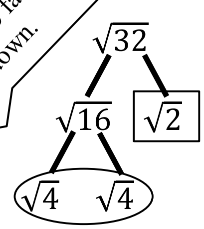
$\sqrt{2 \cdot 2 \cdot 5 \cdot 2}$ or $\sqrt{2^2} \cdot \sqrt{10}$

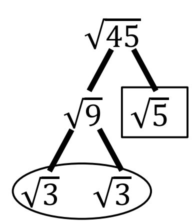
$= 2\sqrt{10}$

$\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

$\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

Hint: You can stop when you find a pair rather than continue to factor it down.





$= 4\sqrt{2}$ $= 3\sqrt{5}$

Now you try…

$\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$

$\sqrt{200} = \sqrt{100 \cdot 2} = \sqrt{10} \cdot \sqrt{2} = 10\sqrt{2}$

$\sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$

$\sqrt{147}$ $7\sqrt{3}$

$\sqrt{128}$ $8\sqrt{2}$

$\sqrt{\frac{1}{4}}$ $\frac{1}{2}$

$\sqrt{\frac{4}{25}}$ $\frac{2}{5}$

$\sqrt{\frac{49}{36}}$ $\frac{7}{6}$

$-\sqrt{36}$ -6

For this problem, find the square root of 36 and then put the negative sign in front. One common error is to think that this problem is the same as $\sqrt{-36}$. $\sqrt{-36}$ does not have a solution. There is no number that when you multiply it by itself is equal to -36. Another way to think of the problem above is that you are taking the opposite of the square root of 36.

$-\sqrt{8}$ $-2\sqrt{2}$

$10\sqrt{96} = 10 \cdot \sqrt{16 \cdot 6} = 10 \cdot \sqrt{16} \cdot \sqrt{6} = 10 \cdot 4\sqrt{6} = 40\sqrt{6}$

$-5\sqrt{45}$ $-15\sqrt{5}$

What happens when we apply this same method with a perfect square?
$\sqrt{100} = \sqrt{25 \cdot 4} = \sqrt{25} \cdot \sqrt{4} = 5 \cdot 2 = 10$

7.1d Homework: Simplifying Square Roots

Directions: Simplify the following square roots.

1. $\sqrt{4}$ = __2___

10. $3\sqrt{12}$ = _________

2. $\sqrt{36}$ = _____

11. $\sqrt{\frac{1}{64}}$ = __$\frac{1}{8}$___

3. $\sqrt{125}$ = _$5\sqrt{5}$____
   $= \sqrt{25 \cdot 5} = \sqrt{25} \cdot \sqrt{5} = 5\sqrt{5}$

12. $\sqrt{\frac{25}{49}}$ = ________

4. $\sqrt{216}$ = ________

13. $-\sqrt{72}$ = _________

5. $\sqrt{80}$ = _______

14. $-\sqrt{100}$ = __-10___

6. $\sqrt{256}$ = _______

15. $-\sqrt{\frac{121}{144}}$ = __________

7. $\sqrt{28}$ = ________

8. $\sqrt{99}$ = __$3\sqrt{11}$___

16. $\sqrt{0.16}$ = ____0.4____

9. $2\sqrt{24}$ = __$4\sqrt{6}$___

$= 2 \cdot \sqrt{4 \cdot 6} = 2 \cdot \sqrt{4} \cdot \sqrt{6} = 2 \cdot 2\sqrt{6} = 4\sqrt{6}$

17. $\sqrt{0.0025}$ = ________

7.1e Class Activity: Creating Cubes

In the previous lessons, we learned how to find the area of a square given the side length and how to find the side length of a square given the area. In this section, we will study how to find the volume of a cube given its side length and how to find the side length of a cube given its volume.

1. Find the volume of the cube to the left. Describe the method(s) you are using.

2. The cube above is called a perfect cube. A cube is considered a perfect cube if you can arrange smaller unit cubes to build a larger cube. In the example above 27 unit cubes were arranged to build the larger cube shown. Can you build additional perfect cubes to fill in the table below? The first one has been done for you for the cube shown above.

Dimensions	**Volume of Cube Exponential Notation (units^3)**	**Volume of Cube (units^3)**	**Side Length (units)**
$3 \times 3 \times 3$	3^3	27 units^3	3 units
$2 \times 2 \times 2$	2^3	8 units^3	2 units

In the previous sections, we learned the following:

- If we are given the side length of a square, s, then its area is s^2.
- If we are given the area of a square, A, then its side length is $\sqrt{A}$.

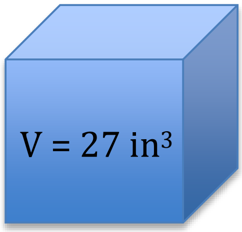
$$A = s^2$$
$$s = \sqrt{A}$$

In this section, we see that:

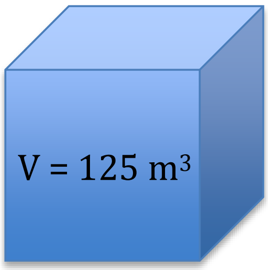
- If we are given the side length of a cube, s, then its volume is s^3.
- If we are given the volume of a cube, V, then its side length is $\sqrt[3]{V}$.
- Explain in your own words what $\sqrt[3]{V}$ means:

$$V = s^3$$
$$s = \sqrt[3]{V}$$

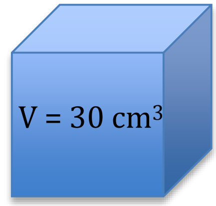
3. Find the side length of the cube: ___3 in___



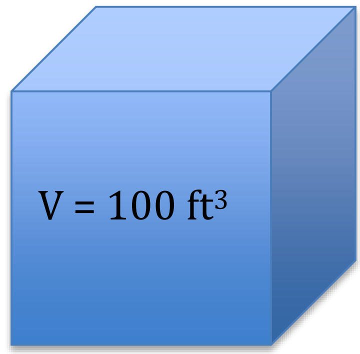
4. Find the side length of the cube: ____5 m__



5. Find the side length of the cube: __$\sqrt[3]{30}$ cm



6. Find the side length of the cube: _$\sqrt[3]{100}$ ft_



Directions: Fill in the following blanks.

7. $\sqrt[3]{27}$ = __3___ because (__3__)^3 = 27

8. $\sqrt[3]{64}$ = _____ because (____)^3 = 64

9. $\sqrt[3]{1}$ = __1___ because (__1__)^3 = 1

10. $\sqrt[3]{125}$ = __5___

11. $\sqrt[3]{343}$ = _____

12. $\sqrt[3]{\frac{1}{216}}$ = __$\frac{1}{6}$___

13. $\sqrt[3]{\frac{1}{1000}}$ = _______

14. $\sqrt[3]{\frac{8}{125}}$ = __$\frac{2}{5}$____

15. $\sqrt[3]{0.001}$ = ___0.1___

16. $\sqrt[3]{0.027}$ = ________

17. $\sqrt[3]{32}$ = ____$2\sqrt[3]{4}$___

18. $\sqrt[3]{135}$ = ________

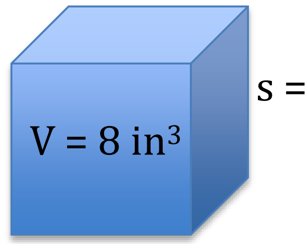
For #17 and 18, students can use the strategies used in the previous section to simplify the cube roots.

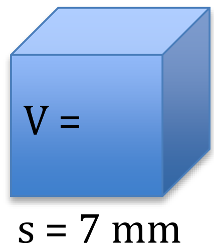
7.1e Homework: Creating Cubes

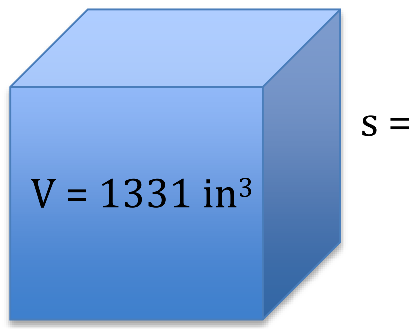
1. Fill in the blanks in the table:

Side Length	Volume
1	1
3	27
4	
	125
6	
$\sqrt[3]{40} = 2\sqrt[3]{5}$	40
	96
$\sqrt[3]{18}$	18
$\sqrt[3]{243}$	
	1000
0.2	
	$\frac{1}{64}$
$\frac{1}{5}$	$\frac{1}{125}$
s	s^3
	V

2. Find the missing measurements:







3. Simplify.

$\sqrt[3]{512}$ 8

$\sqrt[3]{27}$

$\sqrt[3]{729}$

$\sqrt[3]{\frac{27}{64}}$

$\sqrt[3]{24}$ $2\sqrt[3]{3}$

One way to simplify $\sqrt[3]{24}$ is to write the prime factorization of 24: $\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3}$. Since 2 appears 3 times, we can pull this out and leave the 3 under the radical.

$\sqrt[3]{250}$

$\sqrt[3]{128}$

$\sqrt[3]{40}$

$\sqrt[3]{192}$

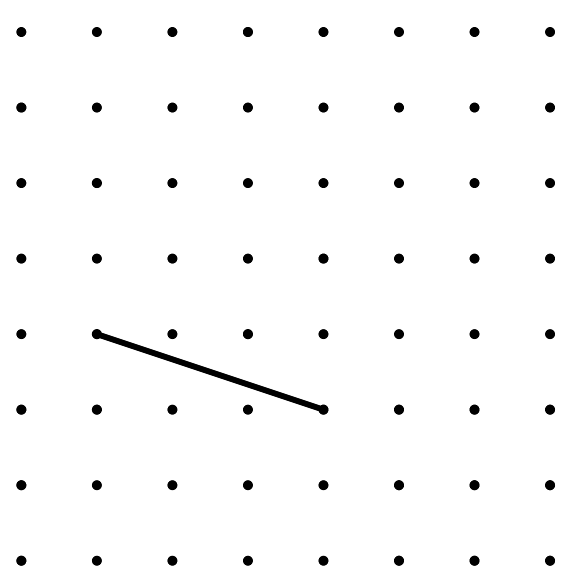
7.1g Self-Assessment: Section 7.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Mastery 3**	**Substantial Mastery 4**
1. Understand the relationship between the side length of a square and its area.				
2. Understand the relationship between the side length of a cube and its volume.				
3. Evaluate the square roots of small perfect squares and the cube roots of small perfect cubes.				
4. Simplify square and cube roots.				

1. Find the following:
    a. The side length of a square with an area of 36 square units.
    b. The side length of a square with an area of 8 square units.
    c. The area of a square with a side length of 5 units.
    d. The area of a square with a side length of $\sqrt{13}$ units.

    e. Find the length of the segment shown below.



2. Find the following:
   a. The side length of a cube with a volume of 125 units^3.

   b. The volume of a cube with a side length of 4 units.

3. Evaluate:
   a. $\sqrt{4}$
   b. $\sqrt{16}$
   c. $\sqrt{81}$
   d. $\sqrt{121}$
   e. $\sqrt[3]{64}$
   f. $\sqrt[3]{125}$
   g. $\sqrt[3]{1000}$

4. Simplify:
   a. $\sqrt{60}$
   b. $-\sqrt{90}$
   c. $2\sqrt{12}$
   d. $\sqrt[3]{81}$
   e. $\sqrt[3]{48}$
   f. $-\sqrt[3]{108}$

Section 7.2: Solutions to Equations Using Square and Cube Roots

Section Overview:
In this section, students will apply their knowledge from the previous section in order to solve simple square and cubic equations. Building on student understanding of how to solve simple linear equations using inverse operations, students will understand that taking the square root of a number is the inverse of squaring a number and taking the cube root is the inverse of cubing a number. Students will express their answers in simplest radical form.

Concepts and Skills to Master:
By the end of this section students should be able to:
1. Solve simple quadratic and cubic equations.

Section 7.2a Teacher Note:
Lead the following discussion with your class before beginning the class activity

Write the equation given below on the board. Then ask the questions that follow.

$$x + 4 = 10$$

1. What does it mean to solve an equation? (Do not describe how to solve the equation but rather what it means to solve an equation.)
Answer: Find a value for *x* so that the equation is true.

2. What is this equation asking us to find? Is there only one solution to this equation?
Answer: A number that when added to 4 results in 10, or "What number added to 4 equals 10?"

3. What is the equation $3x = 12$ asking us to find? Is there only one solution to this equation?
Answer: What number multiplied by 3 equals 12? Yes, there is only one solution.

4. What is the equation $x^2 = 36$ asking us to find? Is there only one solution to this equation?
Answer: What number multiplied by itself equals 36? No there are two solutions, 6 and -6.

5. How is the equation $xy = 36$ different from the one above? What are the possible solutions to this equation?
Answer: There are many solutions to this equation $(4, 9)$, $(9, 4), (36, 1)$, $(12, 3), (6, 6)$. It is different because now you have two variables that can be different numbers.

6. What is the equation $x^2 = 81$ asking us to find? Is there only one solution to this equation?
Answers: What number multiplied by itself equals 81? No, there are two solutions, 9 and -9

7.2a Class Activity: Solve Equations using Square and Cube Roots

In the problems below, we review how to solve some basic equations.

1. Write the inverse operation used to solve each of following equations, then show the steps used to solve the equation.

   a. $x + 3 = 7$
   Subtraction

   b. $-3x = 18$
   Division

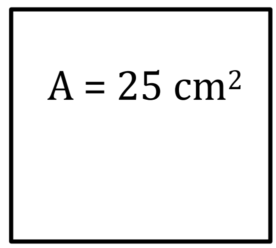
   c. $x - 6 = -14$
   Addition

   d. $\frac{x}{7} = 3$
   Multiplication

Solidify the statements: If $x^2 = a, then\ \sqrt{x^2} = \sqrt{a}\ and\ x = \pm\sqrt{a}$
If $x^3 = a, then\ \sqrt[3]{x^3} = \sqrt[3]{a}\ and\ x = \sqrt[3]{a}$

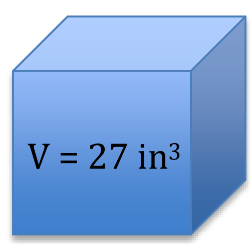
2. What does an inverse operation do? It will "undo" an operation that is performed on a variable; it must be done to both sides.

3. Write and solve an equation to find the side length of a square with an area of 25 cm^2.
   $x^2 = 25$
   $x = 5$



4. Now consider the equation $x^2 = 25$ out of context. Is 5 the only solution? In other words, is 5 the only number that makes this equation true when substituted in for *x*?
   No, -5 is also a solution.

5. Write and solve an equation to find the side length of a cube with a volume of 27 in^3.
   $x^3 = 27$
   $x = 3$



6. Now consider the equation $x^3 = 27$ out of context. Is 3 the only solution? In other words, is 3 the only number that makes this equation true when substituted in for *x*?
   Yes, a common mistake is that students will think that -3 is also a solution. Show them why -3 is not a solution: $(-3)(-3)(-3) = -27$ not positive 27

7. State the inverse operation you would use to solve these equations. Solve each equation.

a. $x^2 = 100$ Square root
$x = \pm\sqrt{10}$

b. $x^2 = 36$ Square root
$x = \pm\sqrt{6}$

c. $x^3 = 27$ Cube root
$x = 3$

8. Solve the equations below. Express your answer in simplest radical form.

a. $x^2 = 64$
$x \pm 8$
To solve this equation, take the square root of both sides:
$$x^2 = 64$$
$$\sqrt{x^2} = \pm\sqrt{64}$$
$$x = \pm 8$$
This notation means the answers are positive 8 and negative 8 – there are two solutions to this equation. We can check by substituting the answers back into the original equation:
For $x = 8$: $x^2 \rightarrow 8^2 \rightarrow (8)(8) = 64$
For $x = -8$: $x^2 \rightarrow (-8)^2 \rightarrow (-8)(-8) = 64$

b. $x^2 = -64$
No solution.
There is not a real number that when multiplied by itself equals -64. A common mistake is for students to say the answer is -8 but remember $(-8)(-8) =$ positive 64, not negative 64.

c. $x^3 = 8$
To solve this equation, take the cube root of both sides:
$x^3 = 8$
$\sqrt[3]{x^3} = 8$
$x = 2$
Again, you can check your answer by substituting it back into the original equation. Why doesn't this have two answers like part a? A common mistake is to think that -2 is also an answer; however $(-2)(-2)(-2) =$ negative 8, not positive 8.

d. $x^3 = -8$
$x = -2$

e. $x^3 = 1$
$x = 1$

f. $x^2 = 9$
$x = \pm 3$

g. $x^2 = 5$
$x = \pm\sqrt{5}$

h. $x^2 = 10$

i. $x^3 = 15$

j. $x^2 = -100$

k. $x^3 = -512$
$x = -8$
Why does this have a solution when part b) does not? Check the answer: $(-8)(-8)(-8) = -512$

l. $x^2 = 8$
$x = \pm 2\sqrt{2}$

m. $x^2 = 45$
$x = \pm 3\sqrt{5}$
Solve this by taking the square root of both sides and then be sure to simplify the radical.

n. $x^3 = 250$
$x = 5\sqrt[3]{2}$

o. $x^3 = 128$

p. $a^2 = \frac{1}{36}$
$a = \pm\frac{1}{6}$

q. $z^3 = \frac{1}{27}$
$z = \frac{1}{3}$

r. $y^2 = 0.16$
$y = 0.4$

s. $x^2 + 16 = 25$
$x = \pm 3$
To solve this, first subtract 16 from both sides of the equation and then take the square root of both sides of the equation.

t. $x^2 - 5 = 59$

u. $10x^2 = 1440$
$x = \pm 12$
To solve this, first divide both sides of the equation by 10 and then take the square root of both sides.

v. $2x^2 = 16$

w. $\frac{y^3}{2} = 32$
$y = 4$

x. $x^2 = p$ where p is a positive rational number
$x = \pm\sqrt{p}$

9. Estimate the solution. Use a calculator to check your estimate.

a. $x^2 = 53$
Students should reason that this is between ± 7 and ± 8 but closer to ± 7, calculator estimate $x \approx \pm 7.28$

b. $a^2 = 15$
Students should reason that this is between ± 3 and ± 4 but closer to ± 4, calculator estimate $x \approx \pm 3.873$

c. $z^3 = 29$

7.2a Homework: Solve Equations using Square and Cube Roots

1. Solve the equations below. Express your answer in simplest radical form. See class activity #8 for help.

a. $x^2 = 121$
$x \pm 11$

b. $x^2 = 81$

c. $y^3 = 125$
$y = 5$

d. $x^3 = 216$

e. $x^3 = -1$

f. $x^2 = 18$
$x = \pm 3\sqrt{2}$

g. $x^2 = -36$

h. $x^2 = 2$

i. $y^3 = 81$
$x = 3\sqrt[3]{3}$
Don't forget to simplify your answer:
$\sqrt[3]{81} = \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3} = 3\sqrt[3]{3}$

j. $x^2 + 12 = 48$

k. $25 + x^2 = 169$

l. $\frac{y^3}{5} = 25$

m. $a^2 = \frac{1}{144}$

n. $z^3 = \frac{1}{8}$
$z = \frac{1}{2}$

o. $y^2 = 0.25$

p. $a^2 = -\frac{1}{36}$
no solution

q. $z^3 = -0.027$
$z = -0.3$

r. $y^3 = \frac{1}{125}$

s. $a^3 = 100$
$a = \sqrt[3]{100}$

t. $a^2 + 576 = 625$

u. $64 + b^2 = 289$

v. $x^3 = p$ where p is a positive rational number

w. Solve for r where A is the area of a circle and r is the radius: $A = \pi r^2$

x. Solve for r where V is the volume of a cylinder, r is the radius, and h is the height: $A = \pi r^2 h$

$$r = \sqrt{\frac{A}{\pi h}}$$

To solve this, first divide both sides by h and π, then take the square root of both sides.

2. Estimate the solution. Use a calculator to check your estimate.

a. $x^2 = 17$
Students should reason that this is between ± 4 and ± 5 but closer to ± 4, calculator estimate $x \approx \pm 4.12$

b. $a^2 = 67$

c. $z^3 = 10$

You are designing a bathroom with the following items in it. Your very odd client has asked that each of these items be a perfect square or cube. Use your knowledge of squares and cubes to write an equation that models the area **or** volume of each item. Then solve the equation to find the side length of each item. The first one has been done for you.

3. Rug 1764 in^2

   Let s equal the length of one side of the rug.
   $s^2 = 1764$
   $\sqrt{s^2} = \sqrt{1764}$
   $s = 42$

   The side length of the rug is 42 inches.

4. Ottoman 3,375 in^3
   Let s equal the length of one side of the ottoman.
   $s^3 = 3,375$
   $\sqrt[3]{s^3} = \sqrt[3]{3375}$
   $s = 15$
   The side length of the ottoman is 15 inches

5. Mirror 1024 cm^2

6. Bar of Soap 27 cm^3

7. Is it probable to have a negative answer for the objects above? Why or why not?

8. Your client tells you that they would like to double the dimensions of the rug. What will happen to the area of the rug if you double the dimensions? Find this new area. What will happen to the area of rug if you triple the dimensions?

9. Your client also tells you that they would like to double the dimensions of the bar of soap. What will happen to the volume of the soap if you double its dimensions? Find this new volume. What will to the volume of the bar of some if you triple the dimensions?
   If you double the dimensions of the bar of soap the volume will be multiplied by a factor or 2^3 or 8. This would make the volume of the new bar of soap 216 cm^3. If you triple the dimensions the volume would be multiplied by a scale factor of 3^3 or 27, making the new volume 729 cm^3.

10. Write and solve an equation of your own that has a power of 2 in it.
    Answers will vary.

11. Write and solve an equation of your own that has a power of 3 in it. Answers will vary.

7.2b Class Activity: Tower Views

1. Use inverse operations to solve the following problems.

a. $\sqrt{x} = 4$
$x = 16$

b. $\sqrt{a} = 9$
$a = 81$

c. $2\sqrt{y} = 4$
$y = 4$

d. $\sqrt[3]{z} + 5 = 13$
$z = 512$

To solve these problems, you must get rid of the square root symbol. This can be achieved by squaring both sides of the equation. Taking the square root of a number and squaring it are inverse operations so they "undo" each other. For example, to solve part a:

$\sqrt{x} = 4$

$\left(\sqrt{x}\right)^2 = 4^2$

$x = 16$

You can check your answer by substituting your answer back into the original equation. For part c. divide both sides by 2 and then square both sides.

People often wonder how far they can see when they're at the top of really tall buildings such as the *Empire State Building, The Sears Tower in Chicago, etc.*

The furthest distance you can see across flat land is a function of your height above the ground.

If h is the height in meters of your viewing place, then d, the distance in kilometers you can see, can be given by this formula: $d = 3.532\sqrt{h}$

2. The equation above can be used to find the distance when you know the height. Rewrite the equation to find height when you know the distance.

$$h = \frac{d^2}{12.475} \text{ or } h = \left(\frac{d}{3.532}\right)^2$$

3. If you were lying down on top of a building that is 100 meters tall, how far could you see? Write an equation to solve this problem. Solve the problem, showing all steps.

Start with your equation – it makes sense to use the original equation (before you rearranged it in #2) because you are trying to find the distance *d* you can see. Substitute in what you know and solve for the unknown:

$d = 3.532\sqrt{h}$

$d = 3.532\sqrt{100}$

$d = 3.532(10)$

$d = 35.32$

You could see approximately 35.32 kilometers from a height of 100 meters.

4. The CN Tower in Toronto, Canada is 555 meters tall. It is near the shore of Lake Ontario, about 50 kilometers across the lake from Niagara Falls. Your friend states that on a clear day, one can see as far as the falls from the top of the Tower. Are they correct? Explain your answer.

Your friend is correct. On a clear day you can see as far as 83 kilometers.

5. The Washington Monument in Washington D.C. is 170 meters tall. How far can one see from its top? Write the equation you need. Show all steps.

6. How high must a tower be in order to see at least 60 kilometers? Write the equation you need. Show all steps.

For this problem, it makes sense to use the rearranged equation from #2 because you are trying to find the height *h*. Again, substitute in what you know and solve for the unknown:

$h = \frac{d^2}{12.475}$

$h = \frac{60^2}{12.475}$

$h = \frac{3600}{12.475}$

$h = 288.577$

You must be on a tower that is at least 288.577 meters high to see 60 kilometers.

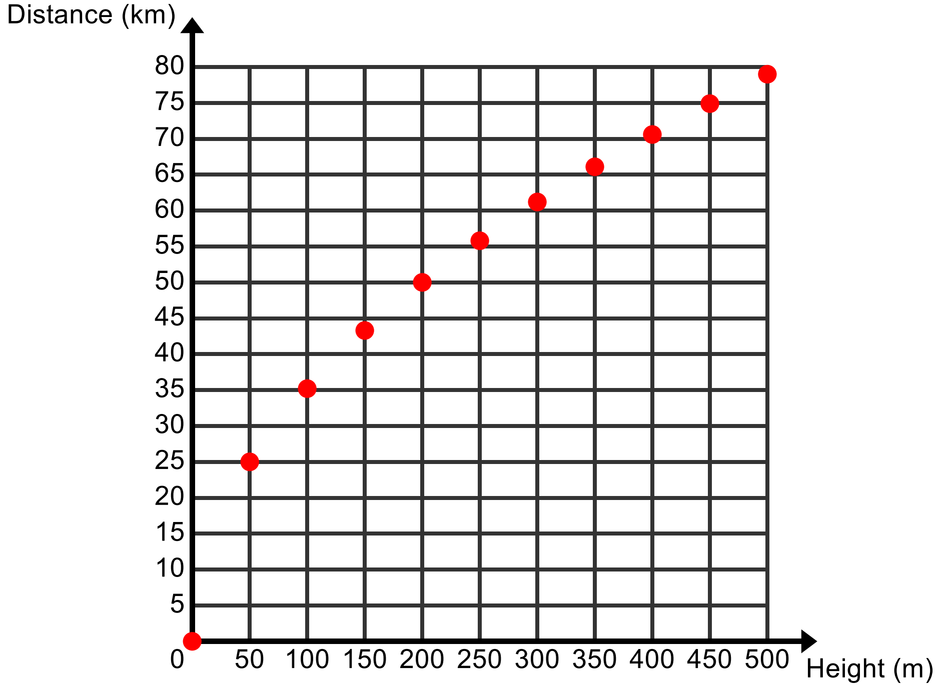
7. Advertising for Queen's Dominion Amusement Park claims you can see 40 kilometers from the top of its observation tower. How high is the tower? Write the equation you need. Show all steps.

8. To enhance understanding of the relation between height and viewing distance, first complete the table below. Express each output value to the nearest whole number; then plot the data points on an appropriately labeled graph. Do not connect the points.

a.

Height (m)	0	50	100	150	200	250	300	350	400	450	500
Distance (km)	0	25	35.2	43.3	50	55.8	61.2	66.1	70.6	74.9	79

b.



c. What kind of association is shown between height and viewing distance?

There is a nonlinear positive association. As the height increases the viewing distance also increases. The relationship between the change in height and change in viewing distance is not constant. Rather, as the height increases, the change in the viewing distance becomes less and less.

Students might want to argue that this is a linear association. Remind them to look at the slope of the line. Ask them if they see a constant rate of change.

7.2b Homework: Driving, Running, and Basketballs

1. Use inverse operations to solve the following problems.

   a. $\sqrt{x} = 5$
      $x = 25$

   b. $3 = \sqrt{a}$

   c. $3\sqrt{y} = 18$

   d. $\sqrt{z} - 3 = 78$

For help with these problems, see class activity #1.

Deven is a civil engineer. He needs to make sure that the design of a curved road ensures the safety of a car driving at the speed limit. The equation $V = \sqrt{2.5r}$ represents the maximum velocity that a car can travel safely on an unbanked curve. V represents the maximum velocity in miles per hour and r represents the radius of the turn in feet.

2. If a curve in the road has a radius of 1690 ft. what is the maximum velocity that a car can safely travel on the curve? To solve this problem, write down the equation from above, substitute in what you know, and solve for the unknown:
   $V = \sqrt{2.5r}$
   $V = \sqrt{2.5 \cdot 1690}$
   $V = \sqrt{4225}$
   $V = 65$
   A car can safely travel 65 mph on the curve.

3. The equation above can be used to find the velocity when you know the radius. Rewrite the equation to find radius if you know the velocity.
   Start with the original equation:
   $V = \sqrt{2.5r}$
   $V^2 = 2.5r$ Square both sides of the equation.
   $\frac{V^2}{2.5} = r$ Divide both sides of the equation by 2.5.

4. If a road is designed for a speed limit of 55 miles per hour, what is the radius of the curve?

5. If a road is designed for a speed limit of 35 miles per hour, what is the radius of the curve?

6. What type of association exists between the radius of the curve and the maximum velocity that a car can travel safely?

Annie is on the track team her coach tells her that the function $S = \pi\sqrt{\frac{9.8l}{7}}$ can be used to approximate the maximum speed that a person can run based off of the length of their leg. S represents the runner's speed in meters per second and l represents the length of the runner's leg in meters.

7. What is the maximum speed that Annie can run if her leg length is 1.12 meters?

   $S = \pi\sqrt{\frac{9.8l}{7}}$

   $S = \pi\sqrt{\frac{9.8(1.12)}{7}}$

   $S = \pi\sqrt{\frac{10.976}{7}}$

   $S = \pi\sqrt{1.568}$

   $S = \pi \cdot 1.252$

   $S = 3.931$

   Annie can run a maximum speed of 3.93 meters per seconds with a leg length of 1.12 meters.

8. The equation given above can be used to find the speed of the runner given their leg length. Rewrite the equation to find the leg length of the runner given their speed.

   $l = .714\left(\frac{s}{\pi}\right)^2$

9. What is the leg length of a runner if their maximum running speed is 2.6 meters per second? Round your answer to the nearest hundredth.

10. What kind of association exists between the length of a person's leg and their maximum running speed? Positive, as the leg length increases the maximum running speed also increases.

11. Is leg length the only thing that affects a runner's maximum speed? Explain your answer.

The surface area of a sphere is found by the equation $A = 4\pi r^2$ where A represents the surface area of the sphere and r represents the radius.

12. A basketball has a radius of 4.7 in, what is its surface area?

13. The equation given above can be used to find the surface area given the radius. Rewrite the equation so that you can find the radius if you are given the surface area.

14. The surface area of a dodge ball is 153.9. in^2. What is the radius of the dodge ball?

7.2c Self-Assessment: Section 7.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Solve simple quadratic and cubic equations.				

1. Solve.
   a. $x^2 = 100$

   b. $x^3 = 64$

   c. $x^2 + 30 = 91$

   d. $x^3 - 9 = 134$

   e. Solve for r. $A = \frac{1}{2}r^2y$

Section 7.3: Rational and Irrational Numbers

Section Overview:

This section begins with a review of the different sets of rational numbers and why a need arose to distinguish them. Students then explore different ways of representing rational numbers, starting with a review of how to change fractions into decimals. During this process, students are reminded that the decimal expansion of all rational numbers either terminates or repeats eventually. From here, students review how to express terminating decimals as fractions and learn how to express repeating decimals as fractions by setting up and solving a system of equations. This skill allows them to show that all decimals that either terminate or repeat can be written as a fraction and therefore fit the definition of a rational number. After this work with rational numbers, students investigate numbers whose decimal expansion does not terminate or repeat: irrational numbers. With this knowledge, students classify numbers as rational and irrational. Students learn different methods for approximating the value of irrational numbers, zooming in to get better and better approximations of the number. They then use these approximations to estimate the value of expressions containing irrational numbers. Lastly, students compare and order rational and irrational numbers.

Concepts and Skills to Master:

By the end of this section, students should be able to:

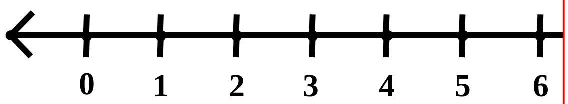
1. Know that real numbers that are not rational are irrational.
2. Show that rational numbers have decimal expansions that either terminate or repeat eventually.
3. Convert a repeating decimal into a fraction.
4. Know that the square root of a non-perfect square is an irrational number.
5. Understand that the decimal expansions of irrational numbers are approximations.
6. Show the location (or approximate location) of real numbers on the real number line.
7. Approximate the value of irrational numbers, zooming in to get better and better approximations.
8. Estimate the value of expressions containing irrational numbers.
9. Compare and order rational and irrational numbers.

An interesting fact: In case you were wondering where the term rational comes from, it comes from the word “ratio”, because rational numbers are those that can be expressed as the ratio of two integers. Irrational, then, means all numbers that are not rational (and therefore cannot be expressed as the ratio of two integers).

7.3 Anchor: Revisiting the Number Line

Directions: Show the length of the following numbers on the number line below. Use the grid on the following page to construct lengths, using tick marks on an index card or tracing paper, and transfer those lengths onto the number line. Then answer the questions that follow. **Note:** On the grid, a horizontal or vertical segment joining two dots has a length of 1. On the number line, the unit length is the same as the unit length on the dot grid.

$A: \sqrt{25}$ $B: \sqrt{2}$ $C: \sqrt{8}$ $D: 2\sqrt{2}$ $E: \sqrt{5}$ $F: 2\sqrt{5}$



Students use the dot paper to construct lengths of irrational numbers as needed and transfer these lengths to the number line. They may realize that in order to construct $2\sqrt{2}$ all they need to do is double the length of $\sqrt{2}$. They may also realize that $\sqrt{8}$ and $2\sqrt{2}$ occupy the same location on the number line.

1. Use the number line to write a decimal approximation for $\sqrt{2}$. Verify .
   Answers will vary – possible answer 1.4

2. Would 1.41 be located to the right or to the left of $\sqrt{2}$ on the number
   left

3. Describe and show how you can put $-\sqrt{2}$ on the number line. Write the decimal approximation for $-\sqrt{2}$.
   Trace the length $\sqrt{2}$, place the right endpoint of the segment on 0, and trace the segment; -1.4

4. Describe and show how you can put $(2 + \sqrt{2})$ on the number line. Estimate the value of this expression.
   Trace the length $\sqrt{2}$, place the left endpoint of the segment on 2, and trace the segment; 3.4

5. Describe and show how you can put $(2 - \sqrt{2})$ on the number line. Estimate the value of this expression.
   Trace the length $\sqrt{2}$, place the right endpoint of the segment on 2, and trace the segment; 0.6

6. Describe and show how you can put $2\sqrt{2}$ on the number line. Estimate the value of this expression.
   Double the length of $\sqrt{2}$; 2.8

7. Use the number line to write a decimal approximation for $\sqrt{5}$.
   Answers will vary – possible answer 2.2

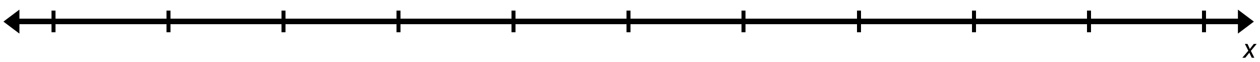
8. Would 2.24 be located to the right or to the left of $\sqrt{5}$ on the number line?
   right

9. Describe and show how you can put $1 + \sqrt{5}$ on the number line. Estimate the value of this expression.
   Trace the length $\sqrt{5}$, place the left endpoint of the segment on 1, and trace the segment; 3.2

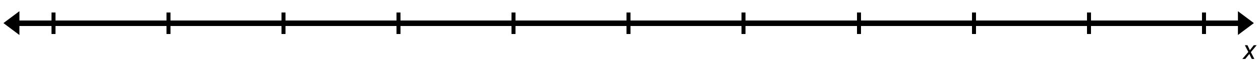
7.3a Class Activity: The Rational Number System

Our number system has evolved over time. On the following pages, you will review the subsets of numbers that are included in the set of **rational numbers**. The following prompts will lead you through a discussion of a student's current understanding which is that of the rational number system. Number lines are provided for you to show how each of these numbers can be associated to a point on the number line.

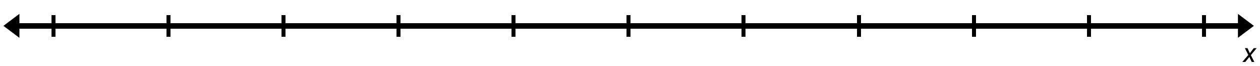
Whole Numbers: Early on, people needed a way to count objects. We call this set of numbers the whole numbers. Talk about how you can construct a number line by marking a point for 0 and then marking a second point to the right of 0 that represents one unit. We can then continue this process and create segments of equal length to associate every whole number to a point on the line.

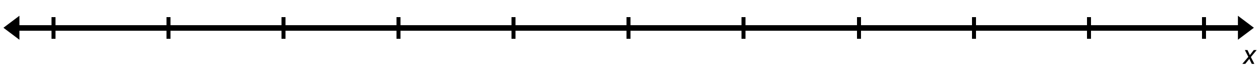


Integers: We need a way to talk about units that are to the left of 0. For example, how can we use a number to represent a temperature of 5 degrees below 0 or that someone is in debt \$25? To the set of whole numbers we add the set of their opposites and call all these numbers the integers. Whole numbers and their opposites are an equal distance from 0. In order to show integers on the number line, we can use our unit length to mark off a succession of equally spaced points on the line that lie to the left of 0. We can now associate every integer to a point on the line.



Rational Numbers: What if I have \$3.25? How would I describe the portion of a pie left if it originally had 8 pieces and 4 of them had been eaten? What if I need a little more than 3 yards of fabric to make a pillow? The need to describe part of a whole gave rise to the set of numbers called **rational numbers**. To associate each **rational number** to a point on the line, divide the unit interval into q parts. If we append p of these together, we get to the point represented by $\frac{p}{q}$. ***There are two number lines here so you can show a few different ways of partitioning the unit interval (i.e. halves, thirds, fifths, tenths). Show positive and negative rational numbers.***





Over the years, you have expanded your knowledge of the number system, gradually incorporating the sets of numbers mentioned above. These sets of numbers are all part of the **rational number system**.

A **rational number** is any number that can be expressed as a quotient $\frac{p}{q}$ of two integers where q does not equal 0.

1. Begin to fill out the table below with different subsets, including equivalent forms, of **rational numbers** you know about so far and give a few examples of each. You will continue to add to this list throughout this section.
As you work through this section, help students build the list below. Some of the numbers listed will be different representations of the same value (i.e. $\frac{1}{2}$ and 0.5).

Subsets of the Rational Numbers	Examples
Natural	
Whole	
Integers	
Fractions	
Terminating Decimals	
Repeating Decimals	
Percent	
Mixed numbers	

2. Change the following rational numbers into decimals **without** the use of a calculator.
Students changed fractions into decimals in 7th grade (see 7th grade, Chapter 1). The point of these problems is not to see how well students know how to divide. The goal is for them to see that the decimal expansion of a fraction will eventually either terminate or repeat. It is recommended that you do these problems as a class and **without** the use of a calculator. This way students can see that there are only so many remainders you can have for a given problem and once a remainder repeats itself, the quotient will start to take on a repeating pattern. They will also see that any fraction with a denominator whose prime factors are only 2 and 5 will always terminate. This is due to the fact that in our decimal system, a decimal has a denominator that (although not explicitly given) is understood to be a power of 10.

a. $\frac{1}{2}$ 0.5	b. $\frac{9}{5}$ 1.8

c. $\frac{3}{8}$ 0.375	d. $\frac{1}{3}$ $0.\overline{3}$
e. $\frac{4}{15}$ $0.2\overline{6}$	f. $\frac{1}{7}$ $0.\overline{142857}$ This is a great problem for discussing why there must always be a repeating pattern because there are only so many ways the division algorithm can go. Before starting the problem, ask students what the possible remainders are when dividing by 7. (0 to 6). For this problem, they may already know that 0 is not a possibility based on the discussion above. Therefore, we can have remainders of 1, 2, 3, 4, 5, and 6. Once one of these reappears, our quotient will start repeating a pattern. Work through the problem with students so that they can see this.

3. What do you notice about the decimal expansion of any rational number? Why is this true?
   See discussion above.

Revisit question #1 on the previous page. Have students add additional representations of rational numbers (i.e. terminating and repeating decimals) to their list.

7.3a Homework: The Rational Number System

1. Change the following rational numbers into decimals **without** the use of a calculator.

a. $\frac{1}{5}$ 0.2	b. $\frac{7}{4}$ 1.75
c. $\frac{5}{8}$	d. $\frac{2}{3}$ $0.\overline{6}$
e. $\frac{2}{9}$	f. $\frac{3}{11}$ $0.\overline{27}$

7.3b Class Activity: Expressing Decimals as Fractions

As we discovered in the previous section, when we converted fractions into decimals, the result was either a **terminating or repeating decimal**.

If we are given a terminating or repeating decimal, we need a method for changing them into a fraction in order to prove that they fit the definition of a rational number.

In 7th grade, you learned how to convert terminating decimals into fractions. Here are a few examples:

$$0.3 = \frac{3}{10}$$

$$0.25 = \frac{25}{100} = \frac{1}{4}$$

$$0.375 = \frac{375}{1000} = \frac{3}{8}$$

$$-2.06 = -2\frac{6}{100} = -2\frac{3}{50} = -\frac{103}{50}$$

Now you try a few…

$$0.4 = \frac{4}{10} \xrightarrow{\div 2} \frac{2}{5}$$

To simplify these fractions, find the greatest common factor and divide both the numerator and denominator by the greatest common factor.

$$0.05 = \frac{5}{100} \xrightarrow{\div 5} \frac{1}{20}$$

$$0.275 = \frac{11}{40}$$

$$1.003 = 1\frac{3}{1000}$$

So, how do we express a repeating decimal as a fraction? For example, how would you convert the repeating decimal $0.\overline{45}$ into a fraction? Try in the space below.

We can use a **system of two linear equations** to convert a repeating decimal into a fraction. Let's look at an example:

Example 1:
The decimal $0.\overline{3}$ is a repeating decimal that can be thought of as 0.33333… where the "…" indicates that the 3s repeat forever. If they repeat forever, how can we write this number as a fraction? Here's a trick that will eliminate our repeating 3s.

Let a represent our number $a = 0.\overline{3}$.
Multiply both sides of the equation by 10 which would give us a second equation $10a = 3.\overline{3}$.

Now we have the following two equations:
$10a = 3.\overline{3}$
$a = 0.\overline{3}$

Let's expand these out:

$10a = 3.333333333333\ldots\ldots$
$a = 0.333333333333\ldots\ldots$

What will happen if we subtract the second equation from the first? Let's try it (remembering to line up the decimals):

$$\begin{array}{rl} & 10a = 3.333333333333\ldots\ldots \\ - & \quad a = 0.333333333333\ldots\ldots \\ \hline & \quad 9a = 3 \end{array}$$

$a = \frac{3}{9}$ (Divide both sides by 9)

$a = \frac{1}{3}$ (Simplify the fraction)

The mathematical foundation walks through how to use substitution to solve these types of problems if you want to show this method to students as well.

Example 2:
The decimal $0.\overline{54}$ is a repeating decimal that can be thought of as 0.54545454… where the "…" indicates that the 54 repeats forever. Let's see how to express this as a fraction.

Let a represent our number $a = 0.\overline{54}$.
Multiply both sides of the equation by 100 this time which would give us a second equation $100a = 54.\overline{54}$.

Now we have the following two equations:
$100a = 54.\overline{54}$
$a = 0.\overline{54}$
Again, let's expand these out:

$100a = 54.5454545454 \ldots \ldots$
$a = \quad 0.5454545454 \ldots \ldots$

Next, subtract the second equation from the first (again, remembering to line up the decimals):

$$
\begin{array}{rl}
 & 100a = 54.5454545454 \ldots \ldots \\
- & \quad a = \quad 0.5454545454 \ldots \ldots \\
\hline
 & 99a = 54
\end{array}
$$

$a = \frac{54}{99}$ (Divide both sides by 99)
$a = \frac{6}{11}$ (Simplify the fraction)

Why do you think we multiplied the second example by 100 instead of 10 as we did in the first example? What would have happened if we had multiplied by 10 in example 2? Try it below and see.
You are not creating a system that will cause the repeating part of the decimal to cancel out – see below.
$10a = 5.4545454 \ldots \ldots$
$a = 0.5454545454 \ldots \ldots$

Discuss how you multiply by 10^a where a is the number of digits that are part of the repeating pattern.

Example 3: Change the decimal $2.\overline{4}$ into a fraction

The decimal $2.\overline{4}$ is a repeating decimal that can be thought of as 2.4444444… where the "…" indicates that the 4s repeat forever.
Let a represent our number $a = 2.\overline{4}$.

$a = \frac{22}{9}$ or $2\frac{4}{9}$

Talk with students about how they only have to deal with changing the decimal piece to a fraction and then they can tack the whole number on – it makes the math a little easier.

Example 4: Change the decimal $3.1\overline{2}$ into a fraction.

$3\frac{11}{90}$

Example 5: Change the decimal $0.\overline{123}$ into a fraction.

In this problem, we will create our second equation by multiplying both sides of the original equation by 1,000 because there are 3 digits that repeat.

$\frac{41}{333}$

Example 6: Change the decimal $4.\overline{1}$ into a fraction.

$4\frac{1}{9}$

Example 7: Change the decimal $2.0\overline{15}$ into a fraction.

In this problem, we will create our second equation by multiplying both sides of the original equation by 100 because there are 2 digits that repeat.

$2\frac{1}{66}$

7.3b Homework: Expressing Decimals as Fractions

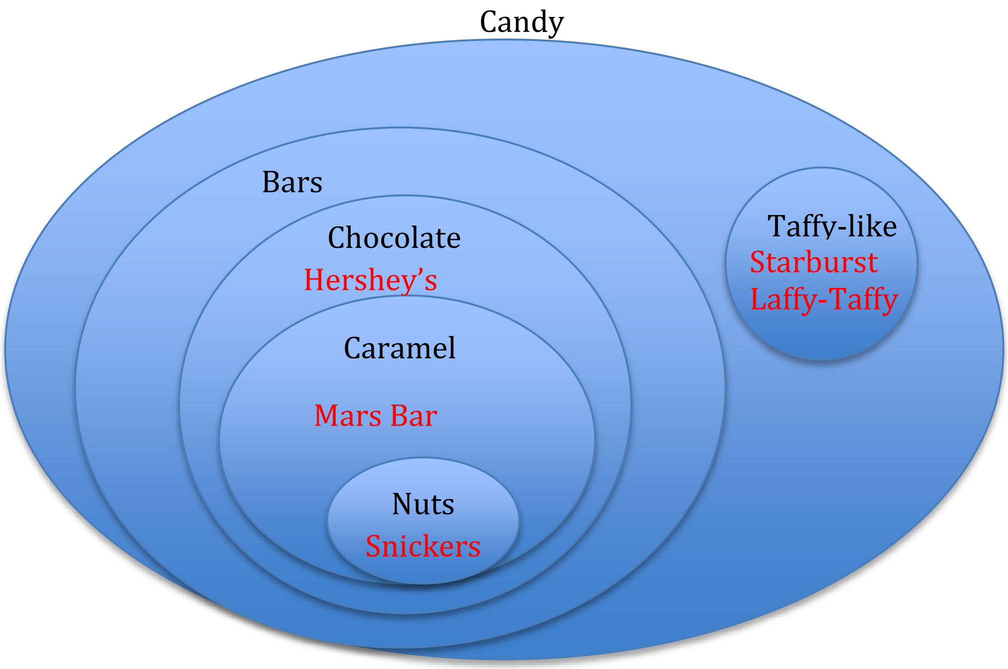
Directions: Circle whether the decimal is terminating or repeating then change the decimals into fractions.

1. 0.6 $\frac{3}{5}$ Terminating Repeating	2. 0.02 Terminating Repeating
3. 1.25 Terminating Repeating	4. -0.064 $-\frac{8}{125}$ Terminating Repeating
5. $0.\overline{5}$ $\frac{5}{9}$ Terminating Repeating	6. $0.\overline{45}$ Terminating Repeating
7. $0.08\overline{3}$ Terminating Repeating	8. $5.2\overline{6}$ $5\frac{4}{15}$ Terminating Repeating
9. -0.24 Terminating Repeating	10. 3.65 Terminating Repeating

11. -0.58 Terminating Repeating $-\frac{29}{50}$	12. $1.1\overline{6}$ Terminating Repeating $\frac{7}{6}$
13. $2.\overline{6}$ Terminating Repeating	14. 0.2 Terminating Repeating
15. $0.\overline{27}$ Terminating Repeating	16. 0.45 Terminating Repeating $\frac{9}{20}$

7.3c Class Activity: Expanding Our Number System

Organize the following candy into the Venn diagram.
Snickers, Hershey's Chocolate Bar, Mars Bar, Laffy-Taffy, Starburst



List the sets of numbers we have learned about so far, including equivalent forms.
Whole numbers, Integers, Fractions, Decimals (repeating and terminating), Natural…….

So are all numbers rational numbers? Are there numbers that cannot be written as a quotient of two integers?

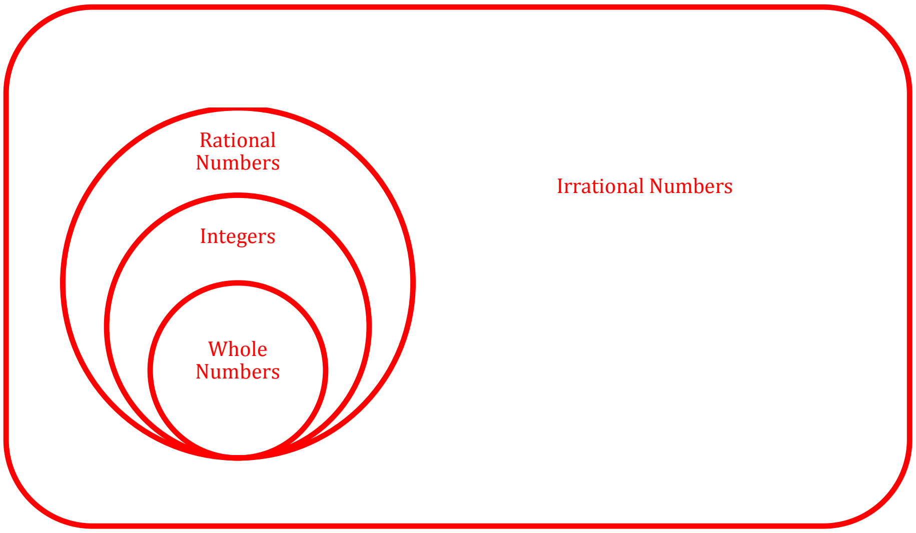
What about $\sqrt{2}$? Can you write $\sqrt{2}$ as a fraction? Why or why not?
The evidence is that the decimal expansion is infinitely long and there is no pattern (as far as we know). Refer to the mathematical foundation for an informal proof.

Numbers like $\sqrt{2}$, which do not have a terminating or repeating decimal expansion are **irrational numbers**. Irrational numbers **cannot** be expressed as a quotient. Discuss with students how the square roots of all non-perfect squares (and the cube roots of all non-perfect cubes) are irrational.

Rational and Irrational numbers together form the set of **real numbers**. Real numbers can be thought of as points on an infinitely long line called the number line. Just like we organized the candy bars in the Venn diagram above we can organize the real number system.

Have students come up with a diagram that represents the real number system and have them compare and contrast diagrams with a neighbor. A sample is shown below. They can also include examples of each set of numbers in their diagrams. Again, re-emphasize that we can show the location of all numbers in the real number system on the real number line.

Real Number System



Directions: Classify the following numbers and provide a justification.

Number	Whole number	Integer	Rational number	Irrational number	Real	Justification **Answers will vary, sample justifications given below**
1. $\frac{2}{3}$			x		x	Number is a fraction
2. 0.25						
3. -2		x	x		x	Number is negative, can be a fraction
4. $\sqrt{5}$				x	x	5 is not a perfect square
5. 10						
6. 0	x	x	x		x	
7. $\sqrt{10}$						
8. $\sqrt{36}$	x	x	x		x	36 is a perfect square so its side length is a whole number
9. $-\sqrt{121}$		x	x		x	Number is a perfect square and then the opposite
10. $2\frac{1}{2}$						
11. $0.08\overline{3}$			x		x	Number is a repeating decimal, number can be written as a fraction

Number	Whole number	Integer	Rational number	Irrational number	Real	Justification **Answers will vary, sample justifications given below**
12. $\frac{10}{13}$						
13. π				x	x	π is a decimal that never repeats or terminates(as far as we know)
14. -3π				x	x	A multiple of an irrational number is irrational
15. $0.26\overline{54}$						
16. $\sqrt[3]{27}$	x	x	x		x	27 is a perfect cube
17. 1.2122122212222…				x	x	This is a predictable pattern but not a repeating pattern.
18. $\sqrt[3]{30}$				x	x	30 is not a perfect cube
19. $\frac{\sqrt{2}}{2}$				x	x	$\sqrt{2}$ is not a perfect square and an irrational number divided by a number is still irrational
20. The side length of a square with an area of 2				x	x	That would make the side length of the square $\sqrt{2}$
21. The side length of a square with an area of 9	x	x	x		x	$\sqrt{9} = 3$
22. The number half-way between 3 and 4			x		x	Number can be written as a fraction
23. The number that represents a loss of 5 yards		x	x		x	Number = -5

7.3c Homework: Expanding Our Number System

See class activity for help and sample problems.

Directions: Classify the following numbers and provide a justification.

Number	Whole number	Integer	Rational number	Irrational number	Real	Justification
1. $\sqrt{2}$				x	x	Number is not a perfect square
2. $\sqrt{1}$	x	x	x		x	Number is a perfect square. $\sqrt{1} = 1$
3. $\frac{1}{3}$						
4. -157		x	x		x	Number is an integer; therefore it can be written as a fraction.
5. $4\frac{1}{9}$						
6. -0.375						
7. $-\sqrt{5}$						
8. $0.\overline{2}$			x		x	Number can be written as a fraction
9. $\sqrt[3]{125}$						
10. $-\sqrt{81}$						
11. $-2.2\overline{4}$						
12. 2π				x	x	π multiplied by a whole number is still irrational
13. The side length of a square with an area of 49						
14. The side length of a square with an area of 1						
15. The side length of the side of a square with an area of 5				x	x	Side length = $\sqrt{5}$ which is not a perfect square
16. The side length of a square with an area of 24						
17. The number half-way between 0 and -1						
18. The number that represents 7 degrees below 0.						

19. Give your own example of a rational number.

Answers will vary, this might be a nice opportunity to discuss with students that 0 is a rational number

20. Give your own example of an irrational number.

Answers will vary

Directions: The table below contains statements about rational and irrational numbers. If the statement is true, put a check in the box. If the statement is not true, write a correct statement.

The corrected statements may vary – have students share out and decide whether they agree with the statements written by their peers. Great way to hit practice standard: Construct viable arguments and critique the reasoning of others.

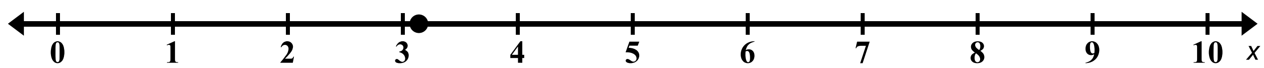
Statement	Check if True or Correct Statement
21. You can show the exact decimal expansion of the side length of a square with an area of 5 square units.	False, corrected statements may vary but possible justifications: The side length of a square with an area of 5 square units is $\sqrt{5}$. $\sqrt{5}$ is an irrational number so you cannot show its exact decimal expansion. A possible corrected statement is: You **cannot** show the exact decimal expansion… Another possible answer is to write in a different area: You can show the exact decimal expansion of the side length of a square with an area of 25 square units (the side length of this square is 5 units) or $\frac{1}{4}$ square units (the side length of this square is $\frac{1}{2}$ units).
22. You can construct and show the length $\sqrt{5}$ on a number line.	✓
23. Square roots of numbers that are perfect squares are rational.	
24. The number 0.256425642564 ... is rational.	
25. You can always use a calculator to determine whether a number is rational or irrational by looking at its decimal expansion.	
26. The number $0.\overline{6}$ is irrational because its decimal expansion goes on forever.	False, The number $0.\overline{6}$ is rational because all repeating decimals can be written as a fraction.
27. The number half-way between 3 and 4 is rational.	
28. You can build a perfect cube with 36 unit cubes.	
29. If you divide an irrational number by 2, you will still have an irrational number.	✓
30. The side length of a cube made of 64 unit blocks is irrational.	

Make up **two** of your own statements that are **true** about rational or irrational numbers.

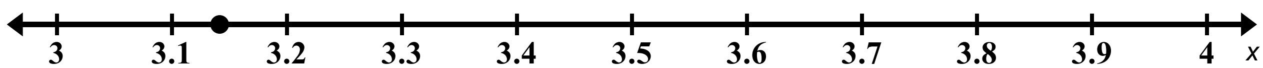
7.3d Class Activity: Approximating the Value of Irrational Numbers

So far, we have seen that we can show the location of an irrational number on the number line. We also know that we cannot show the entire decimal expansion of an irrational number because it is infinitely long and there is no pattern (as far as we know). However, we can come up with good approximations for the numerical value of an irrational number. **It is suggested that the students don't have a calculator but the teacher does any calculations for the class.** Also, make sure that students understand what is happening from one number line to the next. We are zooming in on a piece of the number line in order to make better approximations. In the process, the scale changes.

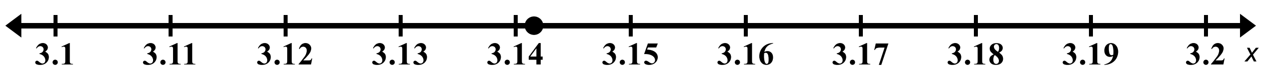
The decimal expansion for π to eight decimal places is 3.14159265… On the number line, we know that π lies somewhere between 3 and 4:



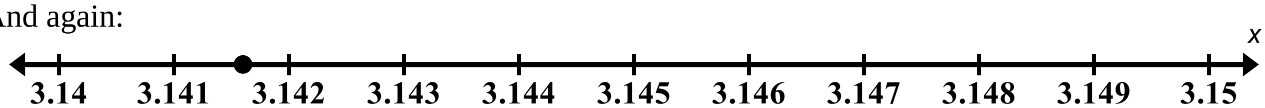
We can zoom in on the interval between 3 and 4 and narrow in on where π lies:



And if we zoom in again on the interval from 3.1 to 3.2:



And again:



We can imagine continuing this process of zooming in on the location of π on the number line, each time narrowing its possible location by a factor of 10.

Once we have an approximation for an irrational number, we can approximate the value of expressions that contain that number.

For example, suppose we were interested in the approximate value of 2π? We can use our approximations of π from above to approximate the value of 2π to different degrees of accuracy:

Because π is between 3 and 4, 2π is between __6___ and __8__.

Because π is between 3.1 and 3.2, 2π is between __6.2___ and _6.4____.

Because π is between 3.14 and 3.15, 2π is between __6.28___ and __6.30___.

Because π is between 3.141 and 3.142, 2π is between __6.282___ and _6.284____.

Check the value of 2π on your calculator. How are we doing with our approximations of 2π?

We can use a method of **guess and check** to give us an estimate of the numerical value of an irrational number that is correct up to as many decimal points as we need.

Directions: Approximate the value of the following irrational numbers to the indicated degrees of accuracy. You can use your calculator for the following questions but do not use the square root key.

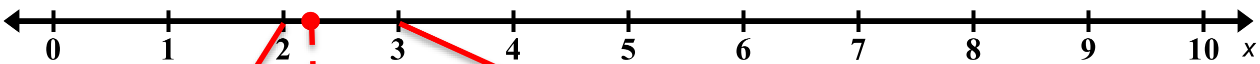
1. Between which two integers does $\sqrt{5}$ lie?

   We know that $\sqrt{5}$ lies between the perfect squares $\sqrt{4}$ and $\sqrt{9}$ so the decimal expansion of $\sqrt{5}$ lies between 2 and 3: $\sqrt{4}$ $\sqrt{5}$ $\sqrt{9}$

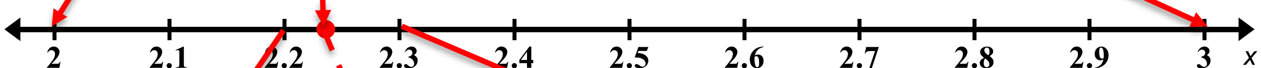
   2 3

   a. Which integer is it closest to? 2

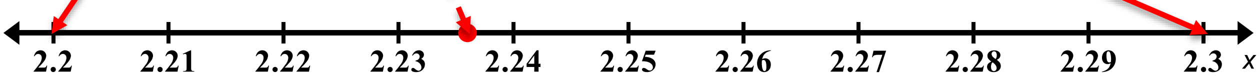
   b. Show its approximate location on the number line below. When students put these numbers on the number line, make sure that they understand that they are just approximating its location but in this case, we know that our point should be closer to 2. On the next number line, we are going to divide the segment from 2 to 3 into 10 equal parts so that we can zoom in further to where our point lives on the interval from 2 to 3. **As you do these problems, make sure students see that we are zooming in on the interval from 2 to 3 and that the length of our intervals change. The arrows shown below may help to illustrate this.**



   c. Now find $\sqrt{5}$ accurate to one decimal place. Show its approximate location on the number line below. Let's narrow in on our approximation. Since we know that our number is closer to 2, it makes sense for our initial guess to be a number less than 2.5. We may even decide to start with 2.1 as a guess. Students should use a calculator to check 2.1^2, 2.2^2, 2.3^2, etc. Once they find the two numbers that $\sqrt{5}$ lies between (2.2 and 2.3), have them approximate its location on the number line below.



   d. Now find $\sqrt{5}$ accurate to two decimal places. Show its approximate location on the number line below. Repeat the process to zoom in even further. We are dividing the interval from 2.2 to 2.3 into 10 equal parts, again narrowing in on the possible location of our point by a factor of 10. $\sqrt{5}$ lies between 2.23 and 2.24. We can continue this process until we get as close as we want in our approximation.



   e. Use your work from above to approximate the value of the expression $2 + \sqrt{5}$ to the nearest whole number. The nearest tenth. The nearest hundredth.

   $2 + \sqrt{5}$ is between 4 and 6 (Students can think of this visually by thinking of taking the segment on the first number line, picking it up, and shifting it so that its left endpoint is at 2 instead of 0.)

   $2 + \sqrt{5}$ is between 4.2 and 4.3

   $2 + \sqrt{5}$ is between 4.23 and 4.24

2. Between which two integers does $\sqrt{15}$ lie?

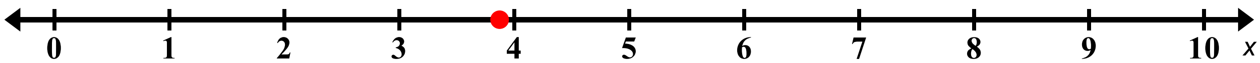
We know that $\sqrt{15}$ lies between the perfect squares $\sqrt{9}$ and $\sqrt{16}$ so the decimal expansion of $\sqrt{15}$ lies between 3 and 4.

$\sqrt{9}$ $\sqrt{15}$ $\sqrt{16}$

3 4

a. Which integer is it closest to? 4

b. Show its approximate location on the number line below.



c. Now find $\sqrt{15}$ accurate to one decimal place. Show its approximate location on the number line below.

Again, narrow in by a factor of 10. Now it makes sense for our initial guess to be greater than 3.5 or we may choose to start with 3.9 as a guess. Students will find that $\sqrt{15}$ lies between 3.8 and 3.9.

d. Now find $\sqrt{15}$ accurate to two decimal places. Show its approximate location on the number line below.

e. Use your work from above to approximate the value of the expression $4\sqrt{15}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

$4\sqrt{15}$ is between 12 and 16

$4\sqrt{15}$ is between 15.2 and 15.6

$4\sqrt{15}$ is between 15.48 and 15.52

3. Repeat the process above to find $\sqrt{52}$ accurate to two decimal places. Place your numbers on the number lines provided each time you increase the degree of accuracy of your estimate.

a. To the nearest **whole number**:

We know that $\sqrt{52}$ lies between the perfect squares $\sqrt{49}$ and $\sqrt{64}$ so the decimal expansion of $\sqrt{52}$ lies between 7 and 8, and it is closer to 7.

$\sqrt{49}$ $\sqrt{52}$ $\sqrt{64}$
7 8

b. To the nearest **tenth**: Narrowing in by a factor of 10, we find that $\sqrt{52}$ lies between 7.2 and 7.3

c. To the nearest **hundredth**: Narrowing in by a factor of 10 again, we find that $\sqrt{52}$ lies between 7.21 and 7.22

d. Use your work from above to approximate the value of $3 + \sqrt{52}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

$3 + \sqrt{52}$ is between 10 and 11
$3 + \sqrt{52}$ is between 10.2 and 10.3
$3 + \sqrt{52}$ is between 10.21 and 10.22

e. Use your work from above to approximate the value of $2\sqrt{52}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

$2\sqrt{52}$ is between 14 and 16
$2\sqrt{52}$ is between 14.2 and 14.6
$2\sqrt{52}$ is between 14.42 and 14.44

#4 – 8 below utilize a quick, computational method for approximating square roots that was developed by Newton and is the method used in calculators to find roots of numbers of all orders. Its general theory forms the basis for most computer algorithms to find solutions of complicated equations. See the mathematical foundation for further discussion and an abstraction of Newton's Method.

4. Pick a positive integer between 1 and 100, call it A_0. Find the average of your number (A_0) and $\frac{100}{\text{your number }(A_0)}$ and call this number A_1. Take the average of A_1 and $\frac{100}{A_1}$ and call this number A_2. Take the average of A_2 and $\frac{100}{A_2}$ and call this number A_3. Repeat this process two more times.
   Students should see that the average stabilizes around 10 (the square root of 100).

5. Pick a different positive integer between 1 and 100 and repeat the process above. What do you notice?

   Regardless of the number chosen, the average stabilizes around 10.

6. Pick a positive integer between 1 and 100, call it A_0. Find the average of your number (A_0) and $\frac{25}{\text{your number }(A_0)}$ and call this number A_1. Take the average of A_1 and $\frac{25}{A_1}$ and call this number A_2. Take the average of A_2 and $\frac{25}{A_2}$ and call this number A_3. Repeat this process two more times.
   Students should see that the average stabilizes around 5 (the square root of 25).

7. Compare the number you picked for #6 with that of a neighbor. Compare your end results. What do you notice?
   Again, regardless of the number chosen, the average stabilizes around 5.

8. Pick a positive integer between 1 and 100, call it A_0. Find the average of your number (A_0) and $\frac{5}{\text{your number }(A_0)}$ and call this number A_1. Take the average of A_1 and $\frac{5}{A_1}$ and call this number A_2. Take the average of A_2 and $\frac{5}{A_2}$ and call this number A_3. Repeat this process two more times. What do you notice?
   Students should see that the average stabilizes around the square root of 5. Students will notice that for each A used, A and 5/A are on opposite sides of sqrt5. Thus it makes sense that the average of A and 5/A is a better approximation to sqrt5 since it lies between A and 5/A. See the mathematical foundation for a more detailed explanation of why this method works.

Directions: Solve the following problems. Again, do not use the square root key on your calculator.

9. A hospital has asked a medical supply company to manufacture intravenous tubing (IV tubing) that has a minimum opening of 7 square millimeters and a maximum opening of 7.1 square millimeters for the rapid infusion of fluids. The medical design team concludes that the radius of the tube opening should be 1.5 mm. Two supervisors review the design team's plans, each using a different estimation for π.

   Supervisor 1: Uses 3 as an estimation for π

   Supervisor 2: Uses 3.1 as an estimation for π

   The supervisors tell the design team that their designs will not work. The design team stands by their plans and tells the supervisors they are wrong. Who is correct and why? Recall that the formula for the area of a circle is $A = \pi r^2$.
   The point of this problem and the one that follows is to help students to see that there are times when we do not need to be very precise in our approximations of irrational numbers and there are others (like the example above) that we need to be very precise in our approximations of irrational numbers.

   Work with students to determine the radius that would have been calculated by each supervisor with the approximation they used for π. What do they think the design team used for their approximation?

10. A square field with an area of 2,000 square ft. is to be enclosed by a fence. Three contractors are working on the project and have decided to purchase slabs of pre-built fencing. The slabs come in pieces that are 5-ft. long.

    - Keith knows that $\sqrt{2000}$ is between 40 and 50. Trying to save as much money as possible, he estimates on the low side and concludes that they will need 160 feet of fencing. Therefore, he concludes they should purchase 32 slabs of the material.

    - Jose also knows that $\sqrt{2000}$ is between 40 and 50 but he is afraid that using Keith's calculations, they will not have enough fencing. He suggests that they should estimate on the high side and buy 200 feet of fencing to be safe. Therefore, he concludes they should purchase 40 slabs of material.

    Keith and Jose begin to argue. Sam jumps in and says, "I have a way to make you both happy – we will purchase enough material to enclose the entire field **and** we will minimize the amount of waste." What do you think Sam's suggestion is and how many slabs will be purchased using Sam's rationale?

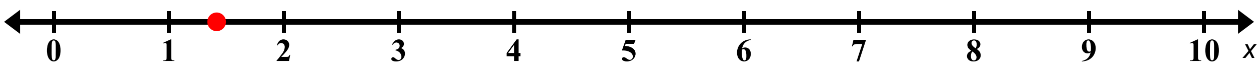
The idea here is – can we do better than the approximations made by Keith and Jose? Encourage students to try to get a better approximation for $\sqrt{2000}$. One possible answer is 45 – this will ensure there is enough material. That would mean that they would need 180 feet of fencing and therefore 36 slabs of material. Discuss why it does not really make sense to try to zoom in on the approximation to the nearest tenth that will provide enough material – 44.8. The slabs are sold in 5-ft increments so it would not make sense to approximate further than the nearest whole number in this case.
Discuss other circumstances that require calculations with irrational numbers and what level of accuracy is desired in different circumstances.

7.3d Homework: Approximating the Value of Irrational Numbers

1. Between which two integers does $\sqrt{2}$ lie?
   1 and 2

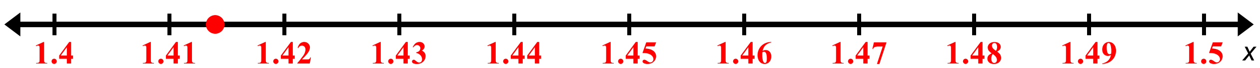
   a. Which integer is it closest to? 1

   b. Show its approximate location on the number line below.



   c. Now find $\sqrt{2}$ accurate to one decimal place. Show its approximate location on the number line below. Between 1.4 and 1.5

   d. Now find $\sqrt{2}$ accurate to two decimal places. Show its approximate location on the number line below. Between 1.41 and 1.42

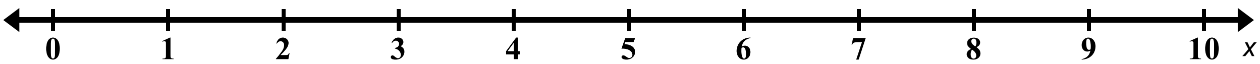


   e. Estimate the value of the expression $2 + \sqrt{2}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.
   $2 + \sqrt{2}$ is between 3 and 4 (When going over homework, show students that you can trace the segment $\sqrt{2}$, pick the segment up, and shift its left endpoint to 2 to estimate the value of this expression.)
   $2 + \sqrt{2}$ is between 3.4 and 3.5
   $2 + \sqrt{2}$ is between 3.41 and 3.42

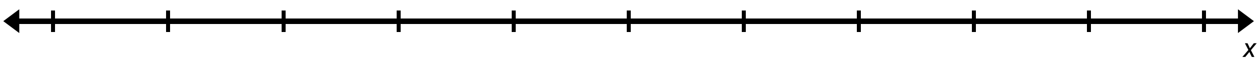
   f. Estimate the value of the expression $2\sqrt{2}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.
   $2\sqrt{2}$ is between 2 and 4 (Show students that you can trace the segment and double its length to estimate the value of this expression.)
   $2\sqrt{2}$ is between 2.8 and 3
   $2\sqrt{2}$ is between 2.82 and 2.84

2. Between which two integers does $\sqrt{40}$ lie?

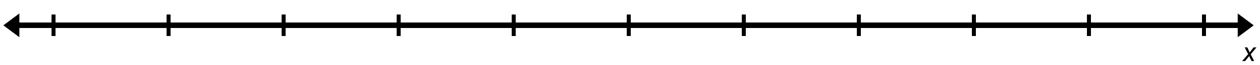
a. Which integer is it closest to?
b. Show its approximate location on the number line below.



c. Now find $\sqrt{40}$ accurate to one decimal place. Show its approximate location on the number line below.



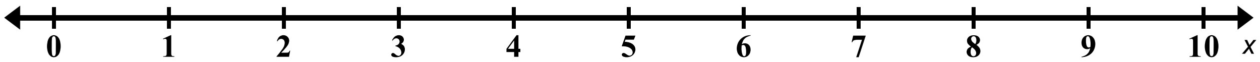
d. Now find $\sqrt{40}$ accurate to two decimal places. Show its approximate location on the number line below.



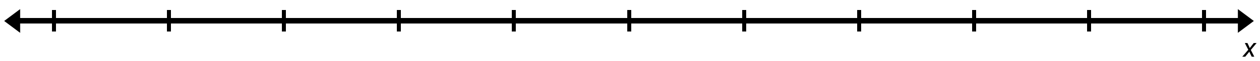
e. Estimate the value of the expression $2\sqrt{40}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

3. Repeat the process above to find $\sqrt{60}$ accurate to two decimal places. Place your numbers on the number lines provided each time you increase the degree of accuracy of your estimate.

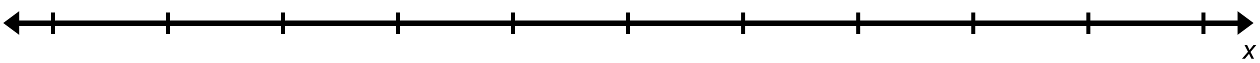
   a. To the nearest **whole number**:



   b. To the nearest **tenth**:



   c. To the nearest **hundredth**:



   d. Use your work from above to approximate the value of $\sqrt{60} - 5$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

   e. Use your work from above to approximate the value of $1 + \sqrt{60}$ to the nearest whole number. To the nearest tenth. To the nearest hundredth.

4. Use the approximations of π on page 60 to estimate the value of the following expressions to increasing levels of accuracy. You can use your calculator but don't use the square key or the π key.

   a. π^2
      Between 9 and 16
      Between 9.61 and 10.24
      Between 9.8596 and 9.9225

   b. 10π

   c. $3 + \pi$

7.3e Class Activity: Comparing and Ordering Real Numbers

These problems may be difficult for students. It is recommended that you do a few as a class to give students strategies for ordering the numbers (i.e. starting by ordering the ones that are easy first and then fitting in the more difficult numbers). Help them to understand how the calculations and estimations can help them. Help them to see how we can view the whole numbers as perfect squares (i.e. in problem 1, we know that 8 and $\sqrt{64}$ are equivalent so we know that $\sqrt{62}$ is smaller than 8 because $\sqrt{62} < \sqrt{64}$.

Directions: Do **not** use a calculator for the following problems. Any calculations you may need are given in the problem.

1. Order the following numbers from **least to greatest**. Note that $8.5^2 = 72.25$.
   $\sqrt{80}, 8, 9, 8.5, \sqrt{62}$
   $\sqrt{62}, 8, 8.5, \sqrt{80}, 9$
2. Order the following numbers from **least to greatest**. Note that $3.5^2 = 12.25$.
   $-\sqrt{13}, -3, -4, -3.5$

   $-4, -\sqrt{13}, -3.5, -3$

   It may be helpful in this problem to think of -3.5 as $-\sqrt{12.25}$.

3. Use the following calculations to answer the questions below.

   $2.2^2 = 4.84$
   $2.3^2 = 5.29$
   $2.23^2 = 4.9729$
   $2.24^2 = 5.0176$

   a. Put the following numbers in order from **least to greatest**.
      $\sqrt{5}, \frac{5}{2}, 2.2,$ the side length of a square with an area of 4
      $2, 2.2, \sqrt{5}, \frac{5}{2}$
   b. Find a number between 2.2 and $\sqrt{5}$. Many possibilities, 2.21, 2.22

   c. Find an irrational number that is smaller than all of the numbers in part a. Many possibilities, $\sqrt{2}$, $\sqrt{3}, -\sqrt{5}$
4. Use the following calculations to answer the questions below.

   $6.48^2 = 41.9904$
   $6.5^2 = 42.25$

   a. Order the following numbers from **least to greatest**.
      $\sqrt{50}, 6, 7, 6.5, \sqrt{42}$
      $6, \sqrt{42}, 6.5, 7, \sqrt{50}$
   b. Find a rational number that is smaller than all of the numbers in part a.

   c. Find an irrational number that is smaller than all of the numbers in part a.

   d. Find a number between $\sqrt{42}$ and 6.5.

5. Use the following calculations to answer the questions below.

$2.44^2 = 5.9536$
$2.45^2 = 6.0025$
$2.449^2 = 5.997601$

a. Order the following numbers from **least to greatest**.
$\sqrt{6}, 2.44, 2.\overline{4}, 2.5,$ the side length of a square with an area of 9

b. Find an irrational number that is between 0 and the smallest number from part a.

c. Find a number that is between 2.44 and $\sqrt{6}$.

6. Use the approximations of π on page 60 and the calculations given below to answer the questions below.
π is between 3 and 4
π is between 3.1 and 3.2
π is between 3.14 and 3.15
π is between 3.141 and 3.142

$3.15^2 = 9.9225$

a. Find a number that is between 3 and π. Many possibilities, 3.12, 3.01

b. Find a number that is between 3.14 and π. Many possibilities, 3.1401

c. Which is larger and why? $(\pi + 5)$ or 8 Since we know that π is a little more than 3, $(\pi + 5)$ is greater than 8

d. Which is larger and why? $(10 - \pi)$ or 7 Again, since we know that π is a little more than 3, $10 - \pi$ is going to be smaller than 7

e. Which is larger and why? 2π or 6.2 2π

f. Which is larger and why? π^2 or 10 10, we can see from the calculations that 3.15^2 is 9.9225. Since π is < 3.15, π^2 is < 10.

7.3e Homework: Comparing and Ordering Real Numbers

Directions: Do **not** use a calculator for the following problems. Any calculations you may need are given in the problem.

1. Give an example of a rational number between $\sqrt{9}$ and $\sqrt{16}$.

2. Give an example of an irrational number between 8 and 9. Many answers. It may help to think of 8 as $\sqrt{64}$ and to think of 9 as $\sqrt{81}$. Possible answers are $\sqrt{65}$ or $\sqrt{70}$. Using a calculator, find the decimal approximation of your number and verify that it is between 8 and 9.

3. Use the following calculations to answer the questions below.

   $1.41^2 = 1.9881$
   $1.42^2 = 2.0164$

   Think about this, if 1.41^2 is smaller than 2 then 1.41 is less than $\sqrt{2}$. If 1.42^2 is larger than 2 then 1.42 is greater than $\sqrt{2}$. What does this tell you about $1.4\overline{2}$?

   a. Order the following numbers from **least to greatest**.
   $\sqrt{2}, 1.41, 1.4, 1\frac{1}{2}, 1.4\overline{2}$

   b. Find a number between 1.4 and $1\frac{1}{2}$.

4. Use the following approximations and calculations to answer the questions below.

   π is between 3.14 and 3.15

   $3.1^2 = 9.61$
   $3.2^2 = 10.24$
   $3.16^2 = 9.9856$
   $3.17^2 = 10.0489$

   a. Order the following numbers from **least to greatest**.
   $\sqrt{10}, 3\frac{1}{10}, 3.\overline{1}, \pi$, side length of a square with an area of 9

   b. Find a number between $3\frac{1}{10}$ and $3.\overline{1}$.

   c. Find a number between 3.1 and $\sqrt{10}$.

5. The number *e* is an important irrational number. In future math classes as well as science and social science, you will see and use this number quite a bit. Use the approximations of *e* and the calculations given below to answer the questions that follow.
e is between 2 and 3
e is between 2.7 and 2.8
e is between 2.71 and 2.72
e is between 2.718 and 2.719

   a. Find a number that is between 2 and *e*. From the data given above, you know that *e* is greater than 2.7; therefore any number between 2 and 2.7 will work. One example is 2.1. There are infinite answers that will work.

   b. Find a number that is between *e* and 2.8.

   c. Which is larger and why? $(e + 10)$ or 13 13 We know that *e* is less than 2.8 so we know that adding 10 to *e* will be less than 13

   d. Which is larger and why? $(6 - e)$ or 4

   e. Which is larger and why? $2e$ or 5.4

   f. Which is larger and why? e^2 or 9

6. Order the following numbers from **least to greatest**. Note that $6.2^2 = 38.44$ and $6.4^2 = 40.96$
$-\sqrt{40}, -7, -6, -6.2, -6.4, -6\frac{1}{2}$

   $-7, -6\frac{1}{2}, -6.4, -\sqrt{40}, -6.2, -6$

7.3f Self-Assessment: Section 7.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Know that real numbers that are not rational are irrational.				
2. Show that rational numbers have decimal expansions that either terminate or repeat.				
3. Convert a repeating decimal into a fraction.				
4. Know that the square root of a non-perfect square is an irrational number.				
5. Understand that the decimal expansions of irrational numbers are approximations.				
6. Show the location (or approximate location) of real numbers on the real number line.				
7. Approximate the value of irrational numbers, zooming in to get better and better approximations.				
8. Estimate the value of expressions containing irrational numbers.				
9. Compare and order rational and irrational numbers.				

1. Circle the numbers that are rational.
   a. -4
   b. -0.34
   c. $\sqrt{7}$
   d. 0
   e. $\frac{1}{2}$
   f. $-\sqrt{11}$
   g. $\sqrt{81}$
   h. $-\sqrt[3]{27}$

2. Change each fraction to a decimal.
   a. $\frac{3}{4}$
   b. $\frac{5}{6}$
   c. $\frac{8}{3}$

3. Change each decimal to a fraction.
   a. $0.\overline{2}$
   b. $1.\overline{34}$
   c. $2.0\overline{1}$

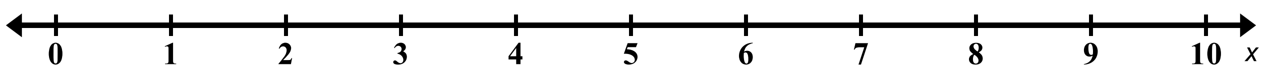
4. Classify the following numbers as rational or irrational and provide a justification.
   a. $\sqrt{10}$
   b. $\sqrt[3]{30}$
   c. $\sqrt{144}$

5. Find the decimal approximation of the following numbers to two decimal places without using the square root key on your calculator.
   a. $\sqrt{22}$

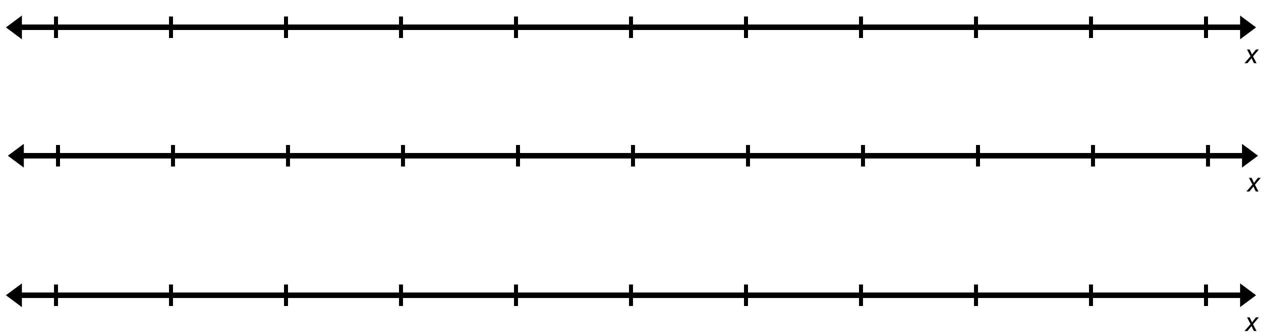
   b. $\sqrt{45}$

   c. $\sqrt{60}$

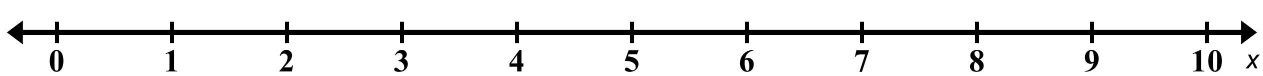
6. Describe how you would plot the following points on the number line shown below.
   2.0, 2.2, 2.24.



Plot the numbers from above on the three number lines shown below, changing the scale of each number line in order to show the location of the points more precisely.



7. Show the approximate location of the following numbers on the number line below.
   $A: \sqrt{3}, B: \sqrt{10}, C: 2\sqrt{5}, D: 3\frac{1}{10}, E: 1.5$



8. Approximate $\sqrt{31}$ to the…
   a. Nearest whole number

   b. Nearest tenth

   c. Nearest hundredth

9. Approximate the value of the following expressions.
   a. $2\sqrt{2}$ if $\sqrt{2} \approx 1.41$

   b. 3π if $\pi \approx 3.14$

   c. $4 + \sqrt{2}$ if $\sqrt{2} \approx 1.41$

10. Order the following numbers from least to greatest.
    $1.2, -2\pi, -3\frac{1}{2}, \sqrt{6}, \frac{4}{3}, -6.28, -\sqrt{2}$