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# Chapter 7: Rational and Irrational Numbers (3 weeks)

## Utah Core Standard(s):

- Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where  $p$  is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that  $\sqrt{2}$  is irrational. (8.EE.2)
- Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers, show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. (8.NS.1)
- Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,  $\pi^2$ ). For example, by truncating the decimal expansion of  $\sqrt{2}$ , show that  $\sqrt{2}$  is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. (8.NS.2)

**Academic Vocabulary:** square, perfect square, square root,  $\sqrt{\quad}$ , cube, perfect cube, cube root,  $\sqrt[3]{\quad}$ , quadratic equation, cubic equation, inverse operation, decimal expansion, repeating decimal, terminating decimal, rational number, irrational number, truncate, decimal approximation, real number, real number line

## Chapter Overview:

In 8<sup>th</sup> grade, students begin to think more carefully about the real line by asking the question, “Is there a number associated with every point on the line?” Up to this point, students have worked only with rational numbers, numbers they generated from iterations of a whole unit or portions of whole units. Part of their work included identifying a point on the real line associated with each rational number. Students explore the question posed above through an activity that has them constructing the lengths of non-perfect squares, thus introducing students to irrational numbers.

The chapter starts by having students examine the relationship between the area of a square and its side length. This activity introduces students to the idea of what it means to take the square root of a number. Additionally, students begin to surface ideas about the limitations of rational numbers. In the activity, students construct physical lengths of irrational numbers and begin to realize that we cannot find an exact numerical value for these numbers. Students then use the knowledge gained from this activity to simplify square roots and solve simple quadratic and cubic equations (i.e.  $x^2 = 40$  and  $x^3 = 64$ ).




In the last section, students deepen their understanding of what an irrational number is and in the process solidify their understanding of rational numbers. They realize that even though they cannot give an exact numerical value for the lengths of non-perfect squares, they can transfer these lengths to a number line to show the exact location of these numbers. Once the numbers are placed on the real line, students can approximate the value of these irrational numbers and compare their value to rational numbers. At the end of the chapter, students learn additional methods for approximating the value of irrational numbers to desired degrees of accuracy, estimate the value of expressions containing irrational numbers, and compare and order rational and irrational numbers.



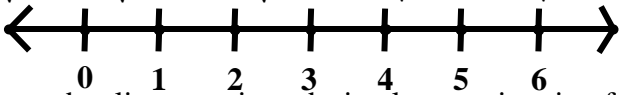
**Connections to Content:**

Prior Knowledge: Students have worked a great deal with rational numbers up to this point. They have defined and worked with the subsets of rational numbers. They have represented rational numbers on a number line, expressed rational numbers in different but equivalent forms, and operated with rational numbers. Students have also worked a great deal with slope, have an understanding of area, and know how to find the area of polygons and irregular shapes which will help them to access the tilted square material.

Future Knowledge: Later in this book, students will study exponent rules and deepen their understanding of the connection between taking the square root of a number and squaring a number. In subsequent courses, students will continue to extend their knowledge of the number system even further. For example, students will learn about complex numbers as a way to solve quadratic equations that have a negative discriminant. They will also continue to work with irrational numbers, learning how to operate on irrational numbers.

## MATHEMATICAL PRACTICE STANDARDS:

	<p><b>Make sense of problems and persevere in solving them.</b></p>	<p>A hospital has asked a medical supply company to manufacture intravenous tubing (IV tubing) that has a minimum opening of 7 square millimeters and a maximum opening of 7.1 square millimeters for the rapid infusion of fluids. The medical design team concludes that the radius of the tube opening should be 1.5 mm. Two supervisors review the design team's plans, each using a different estimation for <math>\pi</math>.</p> <p><b>Supervisor 1:</b> Uses 3 as an estimation for <math>\pi</math></p> <p><b>Supervisor 2:</b> Uses 3.1 as an estimation for <math>\pi</math></p> <p>The supervisors tell the design team that their designs will not work. The design team stands by their plans and tells the supervisors they are wrong. Who is correct and why? Recall that the formula for the area of a circle is <math>A = \pi r^2</math>.</p> <p><i>In this problem, students realize the effects of approximating the value of irrational numbers. They must decide which estimation of <math>\pi</math> is appropriate for the given situation, appreciating that the precision of the estimation may have profound impact on decisions people make in the real world.</i></p>
	<p><b>Reason abstractly and quantitatively.</b></p>	<p>The decimal <math>0.\overline{3}</math> is a repeating decimal that can be thought of as 0.33333... where the "..." indicates that the 3s repeat forever. If they repeat forever, how can we write this number as a fraction? Here's a trick that will eliminate our repeating 3s.</p> <p><i>To solve this problem, students create and solve a system of linear equations. The skills and knowledge they learned about systems of equations become an abstract tool that allows students to write repeating decimals as fractions, proving that they do in fact fit the definition of a rational number.</i></p>
	<p><b>Construct viable arguments and critique the reasoning of others.</b></p>	<p><b>Directions:</b> The table below contains statements about rational and irrational numbers. If the statement is true, put a check in the box. If the statement is not true, write a correct statement.</p> <ul style="list-style-type: none"> <li>You can always use a calculator to determine whether a number is rational or irrational by looking at its decimal expansion.</li> <li>The number 0.256425642564 ... is rational.</li> <li>You can build a perfect cube with 36 unit cubes.</li> <li>If you divide an irrational number by 2, you will still have an irrational number.</li> </ul> <p><i>Students must have a clear understanding of rational and irrational numbers to assess whether the statements are true or false. If the statement is flawed, students must identify the flaw, and construct a statement that is true. Due to the fact that there are several possible ways to change the statements to make them true, students must communicate their statements to classmates, justify the statements, and question and respond to the statements made by others.</i></p>

	<p><b>Model with mathematics.</b></p>	<p>People often wonder how far they can see when they're at the top of the tallest buildings such as the Empire State Building, The Sears Tower in Chicago, etc. The farthest distance you can see across flat land is a function of your height above the ground. If <math>h</math> is the height in meters of your viewing place, then <math>d</math>, the distance in kilometers you can see, can be given by this formula: <math>d = 3.532\sqrt{h}</math></p> <p>The CN Tower in Toronto, Canada is 555 meters tall. It is near the shore of Lake Ontario, about 50 kilometers across the lake from Niagara Falls. Your friend states that on a clear day, one can see as far as the falls from the top of the Tower. Are they correct? Explain your answer.</p> <p><i>The formula shown above is a model for the relationship between the height of a building and the distance one can see. Students use this model along with their knowledge of square roots to solve problems arising in everyday life.</i></p>
	<p><b>Use appropriate tools strategically.</b></p>	<p><b>Directions:</b> Show the length of the following numbers on the number line below. Use the grid on the following page to construct lengths where needed and transfer those lengths onto the number line. Then answer the questions that follow. <b>Note:</b> On the grid, a horizontal or vertical segment joining two dots has a length of 1. On the number line, the unit length is the same as the unit length on the dot grid.</p> <p>A: <math>\sqrt{25}</math>   B: <math>\sqrt{2}</math>   C: <math>\sqrt{8}</math>   D: <math>2\sqrt{2}</math>   E: <math>\sqrt{5}</math>   F: <math>2\sqrt{5}</math></p>  <ol style="list-style-type: none"> <li>1. Use the number line to write a decimal approximation for <math>\sqrt{2}</math>.</li> <li>2. Would 1.41 be located to the right or to the left of <math>\sqrt{2}</math> on the number line?</li> <li>3. Describe and show how you can put <math>-\sqrt{2}</math> on the number line. Estimate the value of this expression.</li> <li>4. Describe and show how you can put <math>(2 + \sqrt{2})</math> on the number line. Estimate the value of this expression.</li> <li>5. Describe and show how you can put <math>(2 - \sqrt{2})</math> on the number line. Estimate the value of this expression.</li> <li>6. Describe and show how you can put <math>2\sqrt{2}</math> on the number line. Estimate the value of this expression.</li> </ol> <p><i>To solve this problem, students use dot paper to construct physical lengths of irrational numbers. They can then transfer these segments to the number line using patty (or tracing) paper. Once on the number line, students can use these tools (number line, dot paper, patty paper, constructed segments) to approximate the value of given expressions (i.e. <math>(2 + \sqrt{2})</math>).</i></p>



**Attend to precision.**

Use the following approximations and calculations to answer the questions below. Do not use a calculator.

Approximation:  $\pi$  is between 3.14 and 3.15

Calculations:

$$3.1^2 = 9.61$$

$$3.2^2 = 10.24$$

$$3.16^2 = 9.9856$$

$$3.17^2 = 10.0489$$

Put the following numbers in order from **least to greatest**.

$\sqrt{10}$ ,  $3\frac{1}{10}$ ,  $3.\bar{1}$ ,  $\pi$ , side length of a square with an area of 9

Find a number between  $3\frac{1}{10}$  and  $3.\bar{1}$ .

Find a number between 3.1 and  $\sqrt{10}$ .

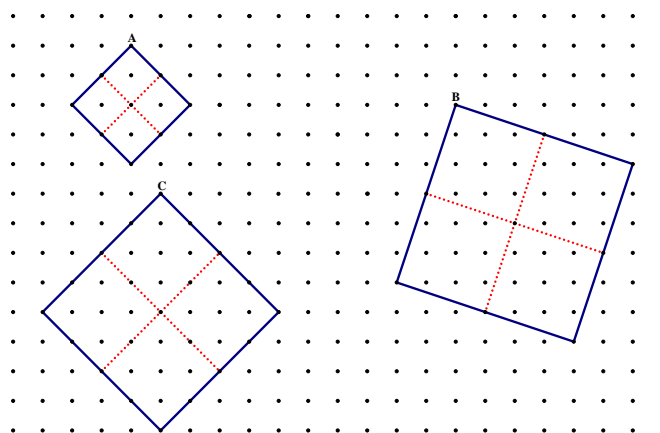
*This task demands mastery of the topics learned in the chapter. Students must have a very clear understanding of square roots, repeating decimals, and irrational numbers. They must closely analyze the decimal expansions (approximations) of the numbers as well as the calculations given to be able to compare and order the numbers.*



**Look for and make use of structure.**


Square A shown below has an area of 8 square units. Determine the following measures:

- The area of one of the smaller squares that makes up Square A
- The side length of one of the smaller squares that makes up Square A
- The side length of the large square A (written 2 different ways)



*This problem allows students to use structure to understand why  $\sqrt{8}$  is the same as  $2\sqrt{2}$ . They can see the equivalence in the concrete model. A square with an area of 8 (see Square A) has a side length of  $\sqrt{8}$  units. This side length is comprised of 2 smaller, congruent segments that*



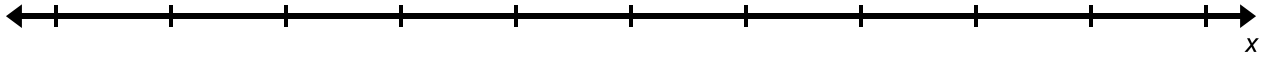
		<p><i>each measure <math>\sqrt{2}</math> units as they are each the side length of a square with an area of 2. This concrete representation builds a conceptual understanding for students as we then move to the algorithm for simplifying square roots.</i></p>
	<p><b>Look for and express regularity in repeated reasoning.</b></p>	<p>Change the following rational numbers into decimals <b>without</b> the use of a calculator.</p> $\frac{1}{7}$ <p><i>This problem allows students to understand why the decimal expansion of a rational number either always terminates or repeats a pattern. Working through this problem, and others, students begin to understand that eventually the pattern must repeat because there are only so many ways that the algorithm can go. Once a remainder repeats itself in the division process, the decimal expansion will start to take on a repeating pattern. Students should see this when they begin repeating the same calculations over and over again and conclude they have a repeating decimal.</i></p>

## 7.0 Anchor Problem: Zooming in on the Number Line

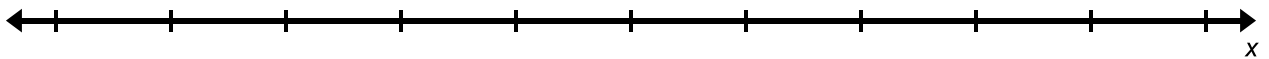


**Directions:** Place the following sets of numbers on the number lines provided and label each point. You will need to decide where to place 0 and the measure of the intervals for each problem.

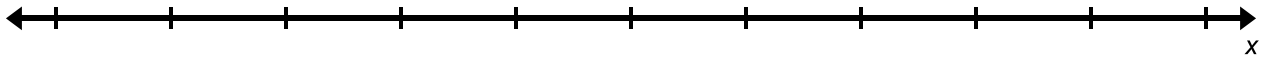
$A: 3$        $B: 4$        $C: 3.5$        $D: -4$        $E: -5$        $F: -4.5$



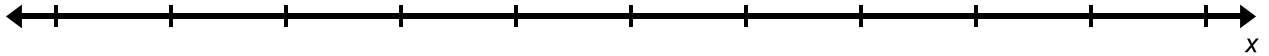
$L: -\frac{1}{4}$        $M: \frac{3}{4}$        $N: -1\frac{1}{2}$        $O: 1.75$        $P: -2$



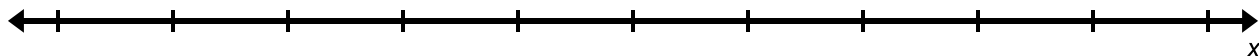
$V: \frac{1}{10}$        $W: \frac{3}{10}$        $X: \frac{1}{2}$        $Y: \frac{9}{10}$        $Z: \frac{10}{10}$



$H: 0.1$        $I: 0.2$        $J: 0.15$        $K: 0.11$        $L: 0.101$



A: 3.1      B: 3.2      C: 3.15      D: 3.11      E: 3.105      F: 3.111



**Directions:** Refer to the number line above to answer the questions that follow.

1. Are there other numbers you can place between 3.1 and 3.11? If yes, find a number.
2. Are there other numbers you can place between 3.11 and 3.111? If yes, find a number.
3. How are you coming up with the numbers? Are there others? How do you know?
4. Where would you put  $3.\overline{1}$  on the number line and why?
5. What can you conclude about the real number line based on this activity?

## Section 7.1: Represent Numbers Geometrically

### Section Overview:

In this section, students are exposed to a new set of numbers, irrational numbers. This chapter starts with a review of background knowledge – finding the area of polygons and irregular shapes, using ideas of slope to create segments of equal length, and reviewing the definition of a square. Then students build squares with different areas and express the measure of the side length of these squares, gaining an understanding of what it means to take the square root of a number. Additionally, students start to surface ideas about irrational numbers. Students create squares that are not perfect and realize they cannot find an exact numerical value for the side length of these squares (e.g. a number that when squared results in the area of the square created). Students also simplify square roots, connecting the simplified answer to a physical model. At the end of the section, students use cubes and volume to gain an understanding of what is meant by the cube root of a number.

### Concepts and Skills to Master:

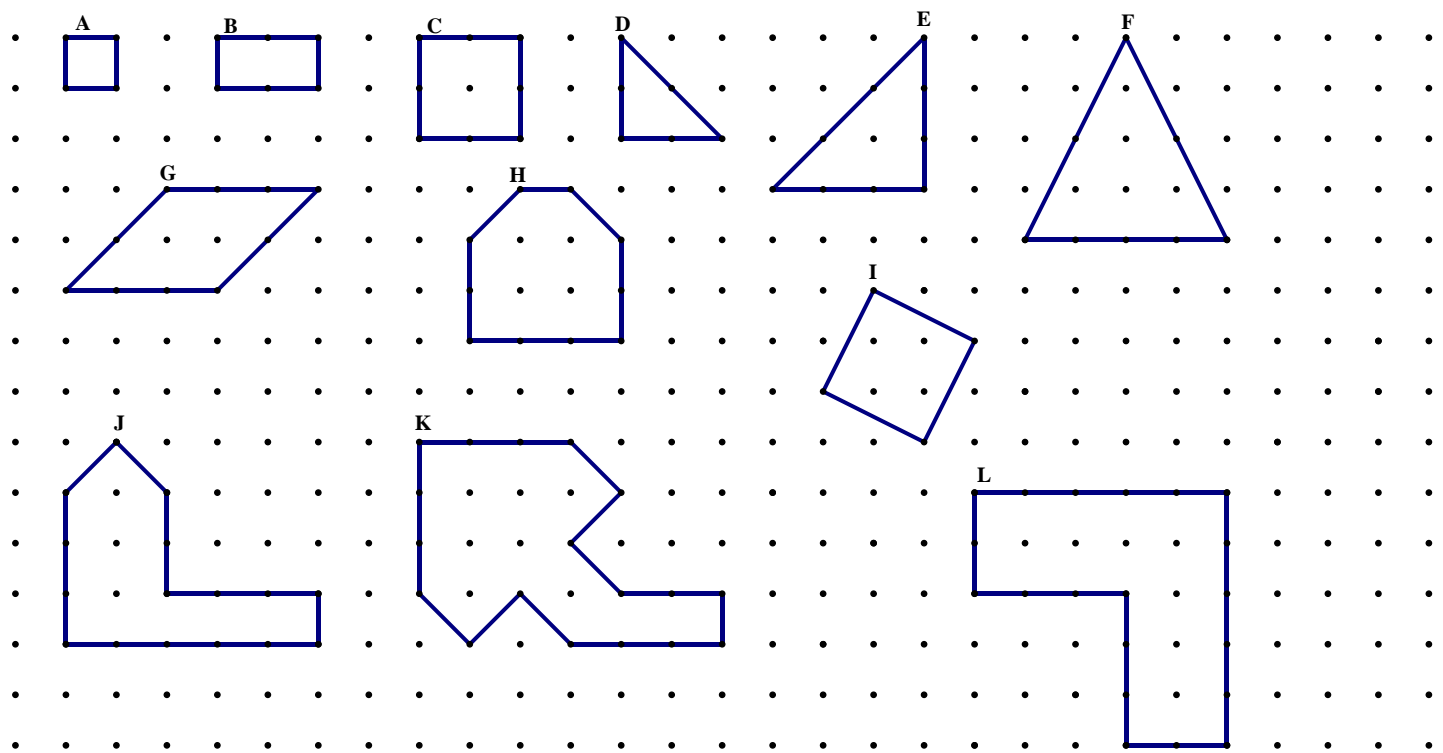
*By the end of this section, students should be able to:*

1. Understand the relationship between the side length of a square and its area.
2. Understand the relationship between the side length of a cube and its volume.
3. Evaluate the square roots of small perfect squares and the cube roots of small perfect cubes.
4. Simplify square and cube roots.

7.1a Class Activity: Background Knowledge

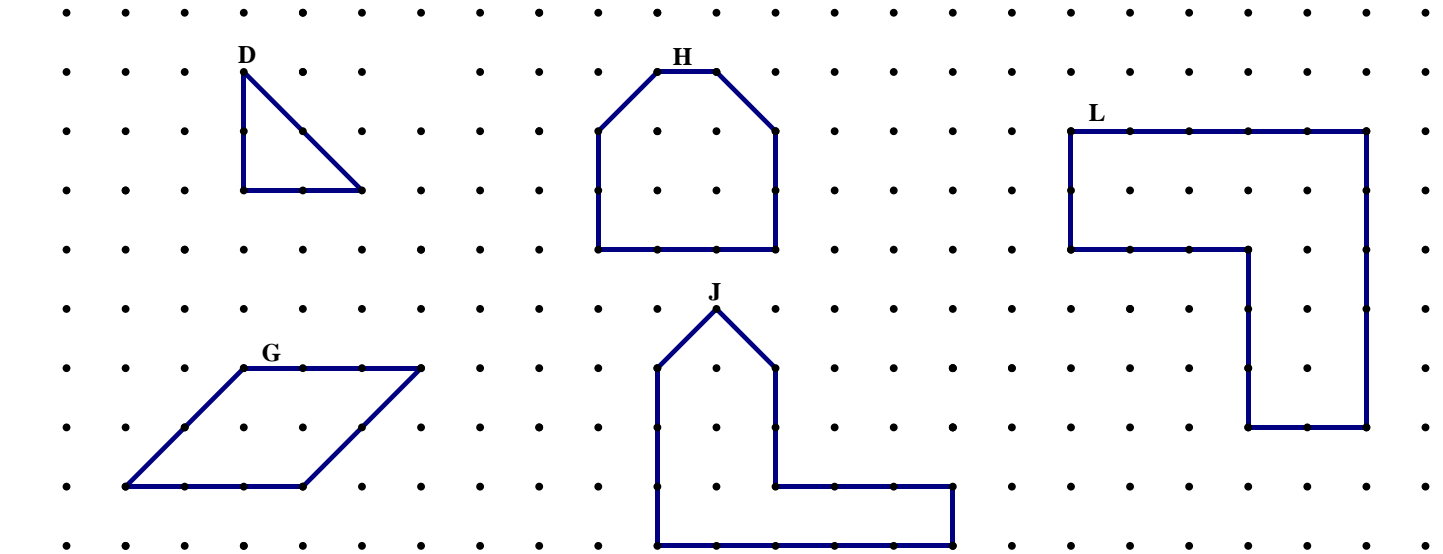
Activity 1: Finding Area of Irregular Shapes

**Directions:** Find the area of the following shapes. On the grid, a horizontal or vertical segment joining two dots has a length of 1. Put your answers on the lines provided below the grid.



A: \_\_\_\_\_ B: \_\_\_\_\_ C: \_\_\_\_\_ D: \_\_\_\_\_ E: \_\_\_\_\_ F: \_\_\_\_\_  
G: \_\_\_\_\_ H: \_\_\_\_\_ I: \_\_\_\_\_ J: \_\_\_\_\_ K: \_\_\_\_\_ L: \_\_\_\_\_

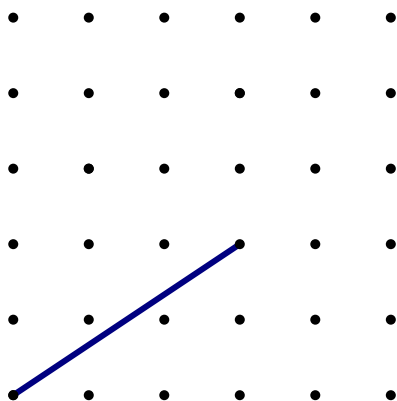
**Directions:** Use a different method than used above to find the areas of the shapes below.



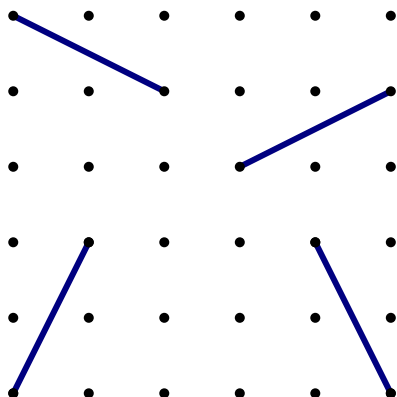
D: \_\_\_\_\_ G: \_\_\_\_\_ H: \_\_\_\_\_ J: \_\_\_\_\_ L: \_\_\_\_\_

## Activity 2: Slopes and Lengths of Segments

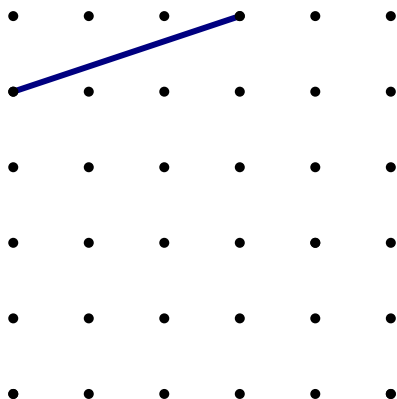
1. Using tracing paper, construct 3 additional segments that are the same length as the segment shown below. Your segments cannot be parallel to the segment given and must start and end on a dot on the grid.



2. Using the ideas from the previous problem and the one below, write down observations you have about the line segments shown on the grid.

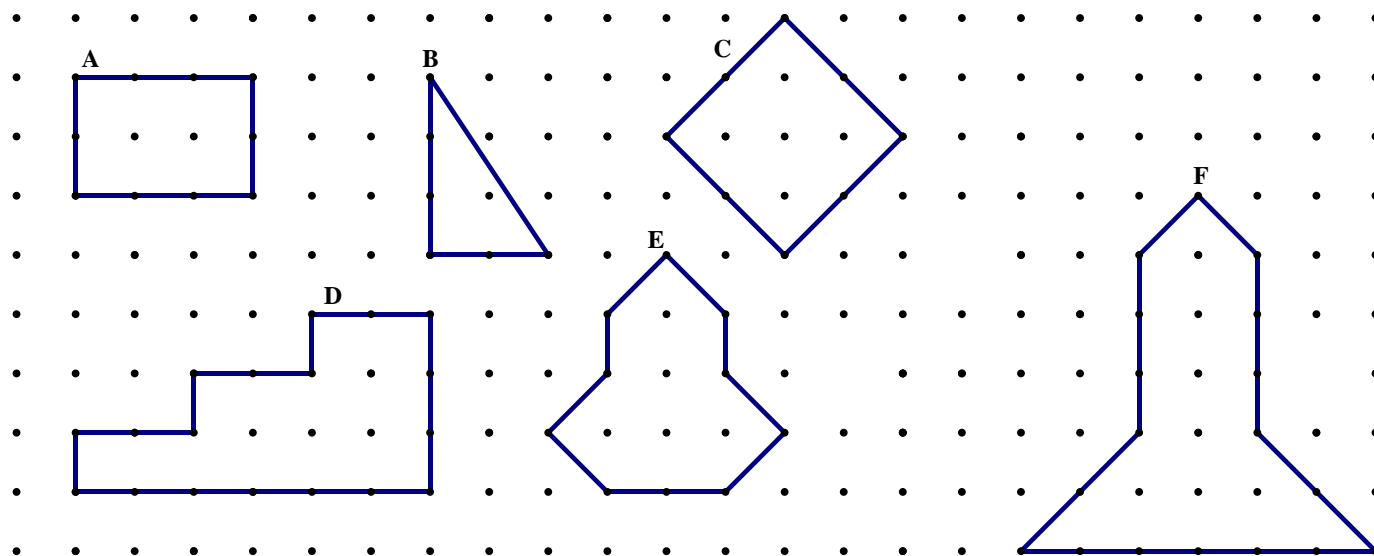


3. Create a square on the grid below, using the given segment as one of the sides of the square.



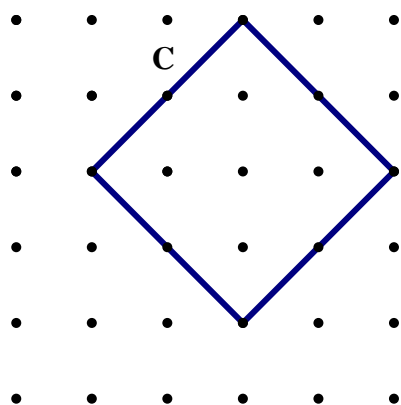
## 7.1a Homework: Background Knowledge

- Find the areas of the following shapes. On the grid, a horizontal or vertical segment joining two dots has a length of 1. Put your answers on the lines provided below the grid.

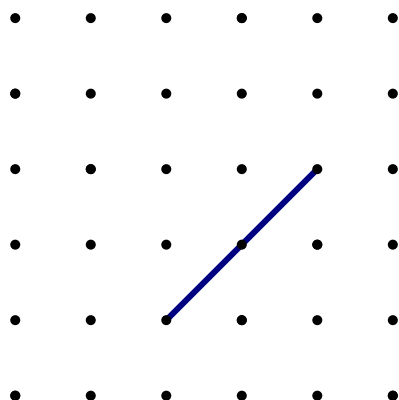


A: \_\_\_\_\_ B: \_\_\_\_\_ C: \_\_\_\_\_ D: \_\_\_\_\_ E: \_\_\_\_\_ F: \_\_\_\_\_

- Show a second method for finding the area of shape C.



- Create a square on the grid below, using the given segment as one of the sides of the square.



## 7.1b Class Activity: Squares, Squares, and More Squares

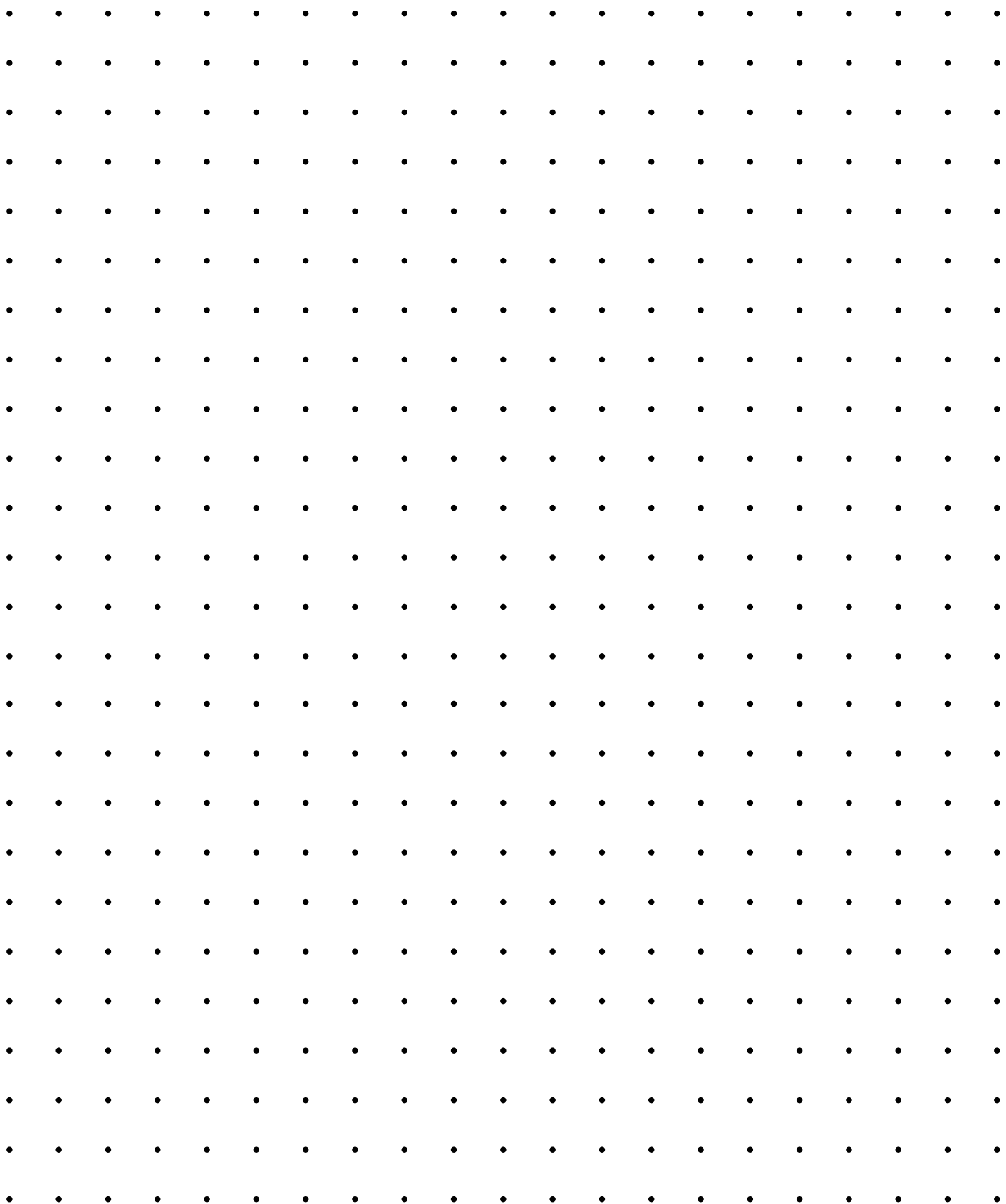


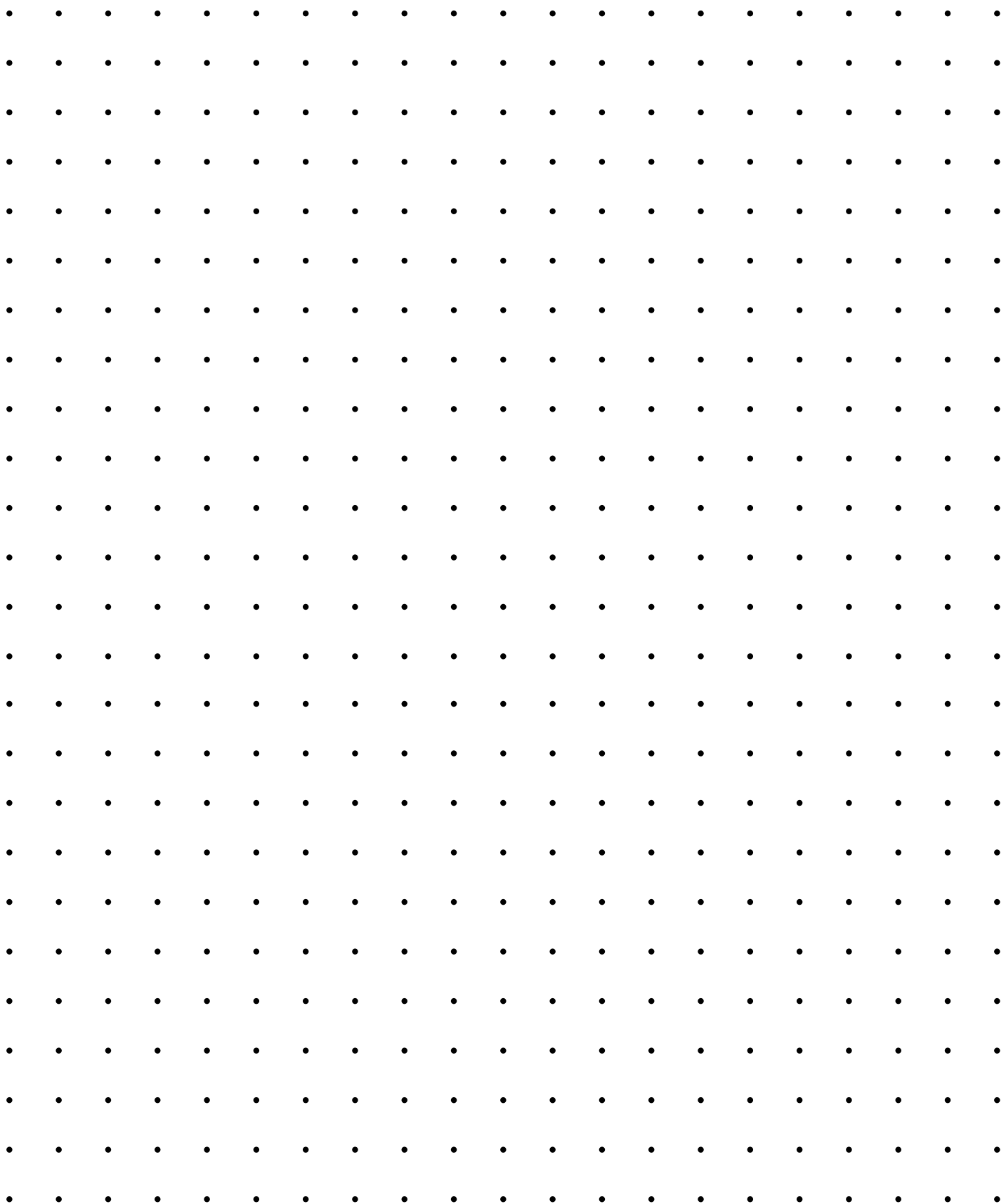


On the following pages of dot paper:

- 1) Create as many different squares with areas from 1 - 100 as possible. On the grid, a horizontal or vertical segment joining two dots has a length of 1. **Each of the vertices of the square must be on a dot.**
- 2) Find the area of each square you made and label each square with its area.
- 3) Complete the table below using the squares you created.

Area	Side Length

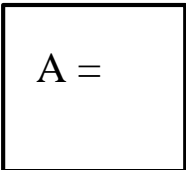


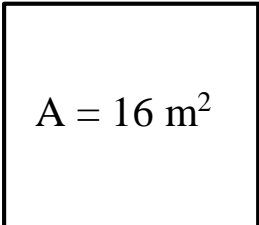


1. Complete the following table...

Area (square units)	Length of Side (units)
1	
9	
	5
2	
5	
	$\sqrt{13}$
	$\sqrt{5}$
100	
	20

2. Find the missing measure.

a.   $A =$   $s = 11\text{cm}$

b.   $A = 16\text{ m}^2$   $s =$

**Directions:** Complete the following sentences. **Provide examples to support your statements.**

3. A perfect square is created when...

4. To find the area of a square given the side length of the square...

5. To find the side length of a square given the area of the square...

6. Simplify the following.

a.  $\sqrt{36}$

d.  $\sqrt{1}$

g.  $\sqrt{625}$

b.  $\sqrt{121}$

e.  $\sqrt{100}$

h.  $\sqrt{2500}$

c.  $\sqrt{16}$

f.  $\sqrt{49}$

i.  $\sqrt{225}$

## 7.1b Homework: Squares, Squares, and More Squares

1. List the first 12 perfect square numbers.
2. What is the side length of a square with an area of 9 units<sup>2</sup>?
3. What is the area of a square with a side length of 2 units?

4. Complete the following table.

Area (square units)	Length of Side (units)
4	
49	
144	
	10
2	
8	
	$\sqrt{15}$
	$\sqrt{41}$
	1
	9
	$s$
$A$	

6. Simplify the following:

a.  $\sqrt{9}$

b.  $\sqrt{100}$

c.  $\sqrt{64}$

d.  $\sqrt{4}$

e.  $\sqrt{144}$

f.  $\sqrt{81}$

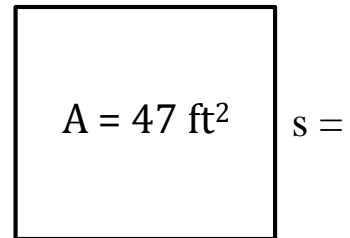
g.  $\sqrt{400}$

h.  $\sqrt{1600}$

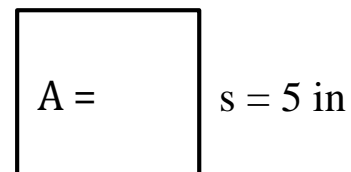
i.  $\sqrt{2500}$

5. Find the missing measures of the squares:

a.



b.

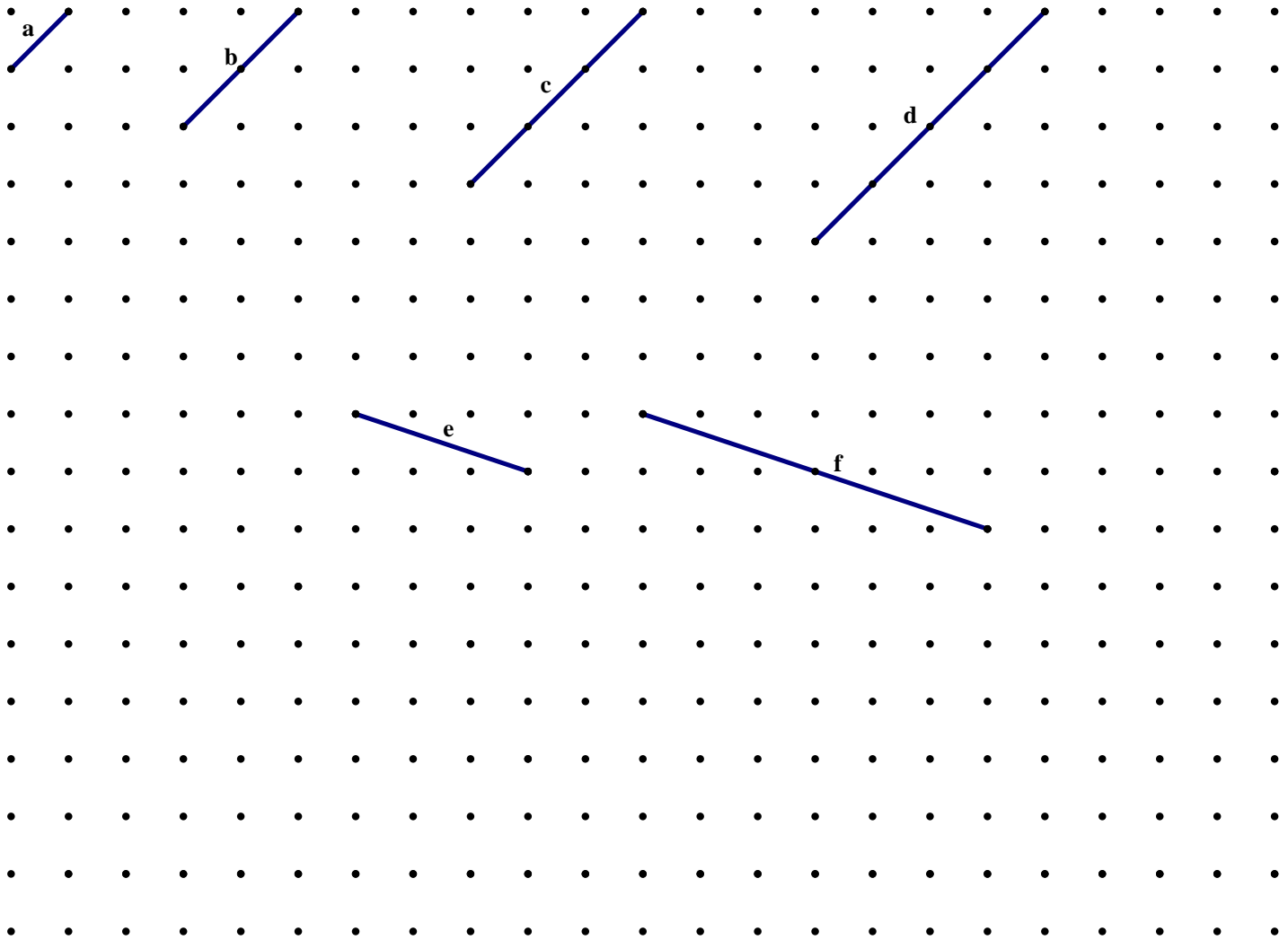


### 7.1c Class Activity: Squares, Squares, and More Squares Cont.



In the previous sections, we have learned how to simplify square roots of perfect squares. For example, we know that  $\sqrt{36} = 6$ . What about the square roots of non-perfect squares? How do we know that they are in simplest form? For example, is  $\sqrt{5}$  in simplest form? How about  $\sqrt{8}$ ?  $\sqrt{147}$ ? Let's take a look.

- Determine the lengths of line segments a through f without the use of a ruler. Write your answers in the space provided below each grid.



a. \_\_\_\_\_

b. \_\_\_\_\_

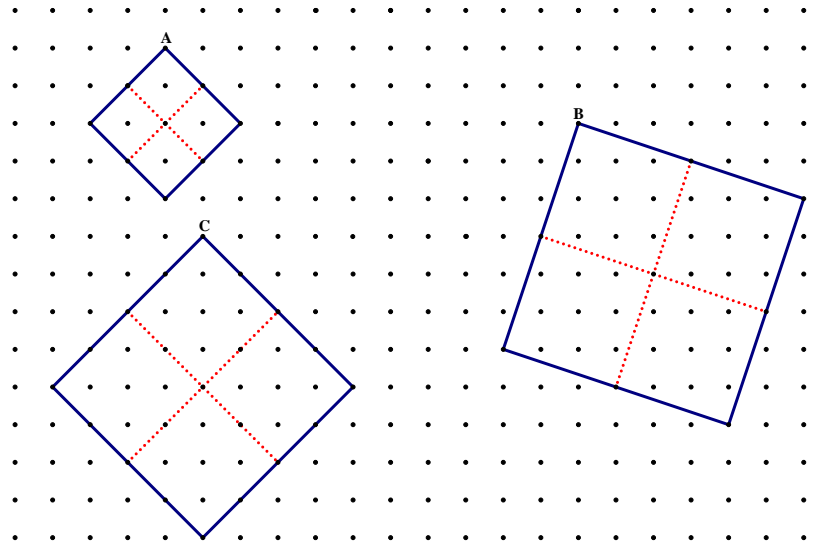
c. \_\_\_\_\_

d. \_\_\_\_\_

e. \_\_\_\_\_

f. \_\_\_\_\_

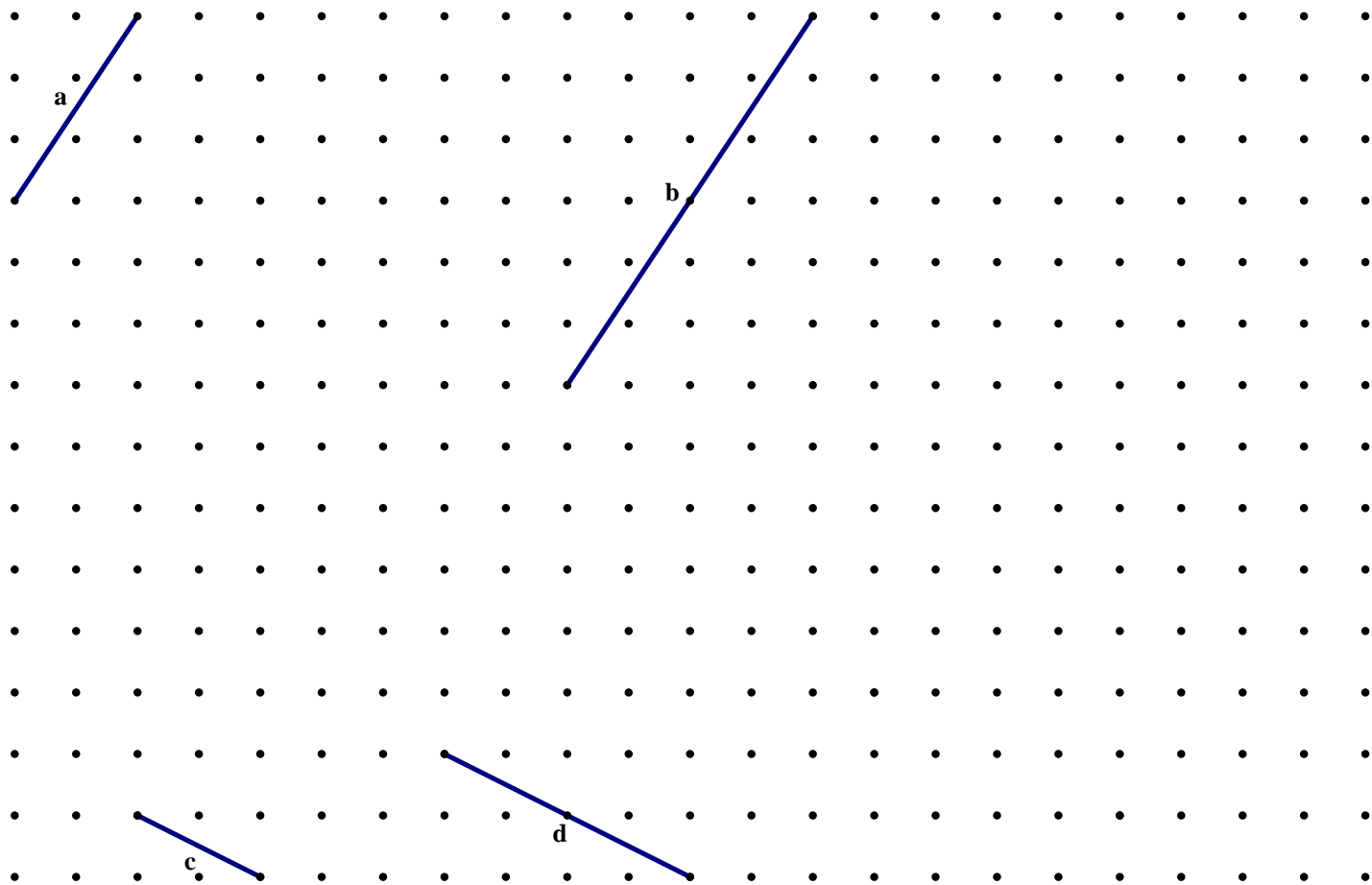
**Directions:** Use the squares on the grid below to answer the questions that follow. Each of the large squares A, B, and C has been cut into four smaller squares of equal size.



2. Square A has an area of 8 square units. Answer the following questions.
  - a. What is the area of one of the smaller squares that makes up Square A? \_\_\_\_\_
  - b. What is the side length of one of the smaller squares that makes up Square A? \_\_\_\_\_
  - c. What is the side length of the large square A (written 2 different ways)? \_\_\_\_\_
  
3. Square B has an area of 40 square units. Answer the following questions.
  - a. What is the area of one of the smaller squares that makes up Square B? \_\_\_\_\_
  - b. What is the side length of one of the smaller squares that makes up Square B? \_\_\_\_\_
  - c. What is the side length of the large square B (written two different ways)? \_\_\_\_\_
  
4. Square C has an area of 32 square units. Answer the following questions.
  - a. What is the area of one of the smaller squares that makes up Square C? \_\_\_\_\_
  - b. What is the side length of one of the smaller squares that makes up Square C? \_\_\_\_\_
  - c. What is the side length of the large square (written *three* different ways)? \_\_\_\_\_

7.1c Homework: Squares, Squares, and More Squares Cont.

1. Determine the lengths of line segments a through f without the use of a ruler. Write your answers in the space provided below each grid.



a. \_\_\_\_\_

c. \_\_\_\_\_

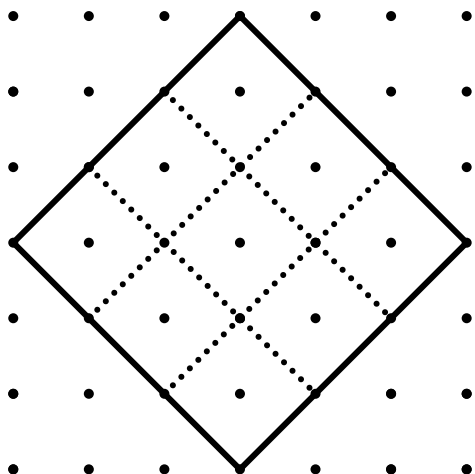
b. \_\_\_\_\_

d. \_\_\_\_\_

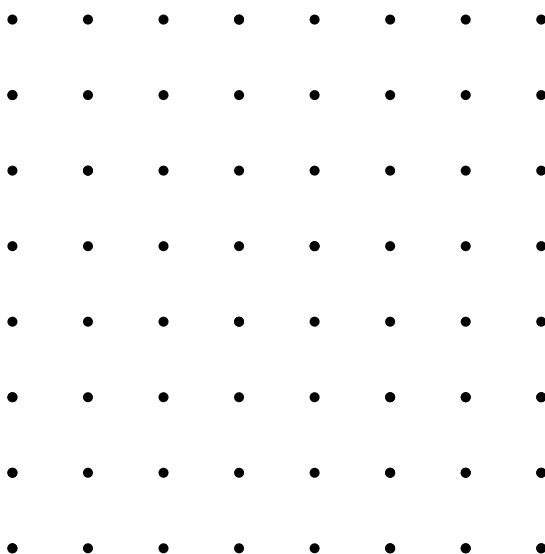
2. On the grid above, construct a segment that has a length of  $\sqrt{45} = 3\sqrt{5}$ .



3. Use the square on the grid below to answer the questions that follow.



- What is the area of the larger square? \_\_\_\_\_
  - What is the area of one of the smaller squares? \_\_\_\_\_
  - What is the side length of one of the smaller squares? \_\_\_\_\_
  - What is the side length of the larger square (written in two different ways)? \_\_\_\_\_
4. On the grid below, construct a segment with a length of  $\sqrt{13}$  units. Explain how you know your segment measures  $\sqrt{13}$  units.



### 7.1d Class Activity: Simplifying Square Roots



In this section we will learn two strategies for simplifying square roots of numbers that are not perfect squares. Both strategies are really doing the same thing, but the methods for each are a little different.

#### Simplifying Square Roots

Think back to the previous lesson. What does it mean to simplify a square root of a non-perfect square? What was the difference between the simplified version of these square roots as opposed to how they looked before they were simplified?

Let's look at some example from the previous lesson:

$$\begin{aligned}\sqrt{8} &= 2\sqrt{2} \\ \sqrt{18} &= 3\sqrt{2} \\ \sqrt{32} &= 4\sqrt{2} \\ \sqrt{40} &= 2\sqrt{10}\end{aligned}$$

What observations can you make about the simplified versions of these square roots non-perfect squares? List them here:

## Two Strategies for Simplifying Square Roots

### Strategy 1:

1. Find the greatest perfect square that is a factor of the number inside the square root symbol.
2. Rewrite the number inside the square root symbol as the product of the greatest perfect square and the other factor.
3. Take the square root of the perfect square.  
Remember: When you take the square root of the perfect square, it is no longer inside the square root symbol.
4. Continue this process until you can no longer find a perfect square other than 1 that is a factor of the number inside the square root symbol.

### Examples:

$$\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$$

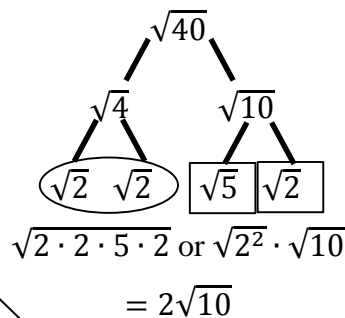
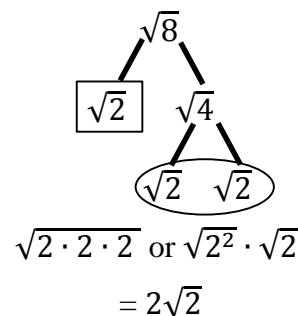
$$\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$$

$$\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$$

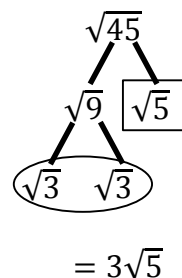
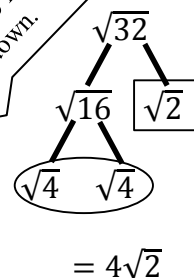
$$\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$$

### Strategy 2:

1. Using the factor tree method, factor the number inside the square root symbol.
2. Look for and circle any pairs of numbers among the factors.
3. Put a square around any numbers that are not part of a pair. Re-write the numbers as factors to see that the pairs can be removed, while anything left over must stay under the square root symbol.
4. Remove the pairs and leave any leftover numbers inside the square root symbol.  
Remember that because we are factoring, all of these numbers are being multiplied, so if you end up with multiple numbers outside or inside the square root symbol, multiply them together.



Hint: You can stop when you find a pair rather than continue to factor it down.



**Now you try...**

$$\sqrt{50}$$

$$\sqrt{\frac{1}{4}}$$

$$\sqrt{200}$$

$$\sqrt{\frac{4}{25}}$$

$$\sqrt{72}$$

$$\sqrt{\frac{49}{36}}$$

$$\sqrt{147}$$

$$-\sqrt{36}$$

$$\sqrt{128}$$

$$-\sqrt{8}$$

$$10\sqrt{96}$$

$$-5\sqrt{45}$$

What happens when we apply this same method with a perfect square?

$$\sqrt{100} = \sqrt{25 \cdot 4} = \sqrt{25} \cdot \sqrt{4} = 5 \cdot 2 = 10$$

### 7.1d Homework: Simplifying Square Roots

**Directions:** Simplify the following square roots.

1.  $\sqrt{4} = \underline{\hspace{2cm}}$

10.  $3\sqrt{12} = \underline{\hspace{2cm}}$

2.  $\sqrt{36} = \underline{\hspace{2cm}}$

11.  $\sqrt{\frac{1}{64}} = \underline{\hspace{2cm}}$

3.  $\sqrt{125} = \underline{\hspace{2cm}}$

12.  $\sqrt{\frac{25}{49}} = \underline{\hspace{2cm}}$

4.  $\sqrt{216} = \underline{\hspace{2cm}}$

13.  $-\sqrt{72} = \underline{\hspace{2cm}}$

5.  $\sqrt{80} = \underline{\hspace{2cm}}$

14.  $-\sqrt{100} = \underline{\hspace{2cm}}$

6.  $\sqrt{256} = \underline{\hspace{2cm}}$

15.  $-\sqrt{\frac{121}{144}} = \underline{\hspace{2cm}}$

7.  $\sqrt{28} = \underline{\hspace{2cm}}$

16.  $\sqrt{0.16} = \underline{\hspace{2cm}}$

8.  $\sqrt{99} = \underline{\hspace{2cm}}$

9.  $2\sqrt{24} = \underline{\hspace{2cm}}$

17.  $\sqrt{0.0025} = \underline{\hspace{2cm}}$

7.1e Class Activity: Creating Cubes



In the previous lessons, we learned how to find the area of a square given the side length and how to find the side length of a square given the area. In this section, we will study how to find the volume of a cube given its side length and how to find the side length of a cube given its volume.



1. Find the volume of the cube to the left. Describe the method(s) you are using.
2. The cube above is called a perfect cube. A cube is considered a perfect cube if you can arrange smaller unit cubes to build a larger cube. In the example above 27 unit cubes were arranged to build the larger cube shown. Can you build additional perfect cubes to fill in the table below? The first one has been done for you for the cube shown above.

Dimensions	Volume of Cube Exponential Notation (units <sup>3</sup> )	Volume of Cube (units <sup>3</sup> )	Side Length (units)
3 × 3 × 3	3 <sup>3</sup>	27 units <sup>3</sup>	3 units

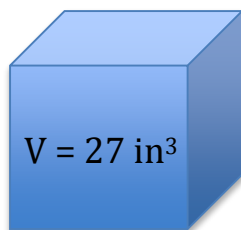
- In the previous sections, we learned the following:
- If we are given the side length of a square,  $s$ , then its area is  $s^2$ .
  - If we are given the area of a square,  $A$ , then its side length is  $\sqrt{A}$ .

$$A = s^2$$
$$s = \sqrt{A}$$

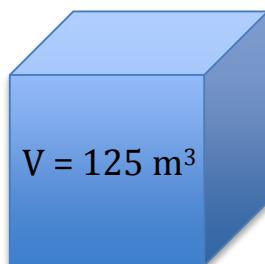
- In this section, we see that:
- If we are given the side length of a cube,  $s$ , then its volume is  $s^3$ .
  - If we are given the volume of a cube,  $V$ , then its side length is  $\sqrt[3]{V}$ .
  - Explain in your own words what  $\sqrt[3]{V}$  means:

$$V = s^3$$
$$s = \sqrt[3]{V}$$

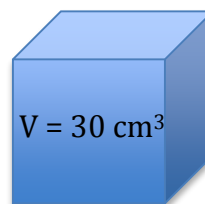
3. Find the side length of the cube: \_\_\_\_\_



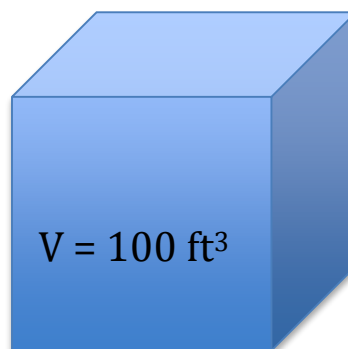
4. Find the side length of the cube: \_\_\_\_\_



5. Find the side length of the cube: \_\_\_\_\_



6. Find the side length of the cube: \_\_\_\_\_



**Directions:** Fill in the following blanks.

7.  $\sqrt[3]{27} = \underline{\hspace{1cm}}$  because  $(\underline{\hspace{1cm}})^3 = 27$

8.  $\sqrt[3]{64} = \underline{\hspace{1cm}}$  because  $(\underline{\hspace{1cm}})^3 = 64$

9.  $\sqrt[3]{1} = \underline{\hspace{1cm}}$  because  $(\underline{\hspace{1cm}})^3 = 1$

10.  $\sqrt[3]{125} = \underline{\hspace{1cm}}$

11.  $\sqrt[3]{343} = \underline{\hspace{1cm}}$

12.  $\sqrt[3]{\frac{1}{216}} = \underline{\hspace{1cm}}$

13.  $\sqrt[3]{\frac{1}{1000}} = \underline{\hspace{1cm}}$

14.  $\sqrt[3]{\frac{8}{125}} = \underline{\hspace{1cm}}$

15.  $\sqrt[3]{0.001} = \underline{\hspace{1cm}}$

16.  $\sqrt[3]{0.027} = \underline{\hspace{1cm}}$

17.  $\sqrt[3]{32} = \underline{\hspace{1cm}}$

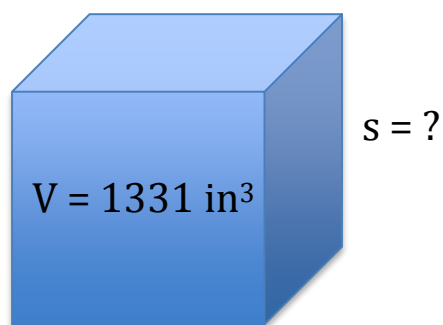
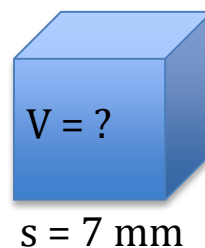
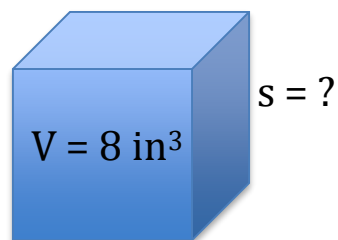
18.  $\sqrt[3]{135} = \underline{\hspace{1cm}}$

## 7.1e Homework: Creating Cubes

1. Fill in the blanks in the table:

Side Length	Volume
1	
	27
4	
	125
6	
	40
	96
$\sqrt[3]{18}$	
$\sqrt[3]{243}$	
	1000
0.2	
	$\frac{1}{64}$
	$\frac{1}{125}$
$s$	
	$V$

2. Find the missing measurements:



3. Simplify.

$$\sqrt[3]{512}$$

$$\sqrt[3]{27}$$

$$\sqrt[3]{729}$$

$$\sqrt[3]{\frac{27}{64}}$$

$$\sqrt[3]{24}$$

$$\sqrt[3]{250}$$

$$\sqrt[3]{128}$$

$$\sqrt[3]{40}$$

$$\sqrt[3]{192}$$

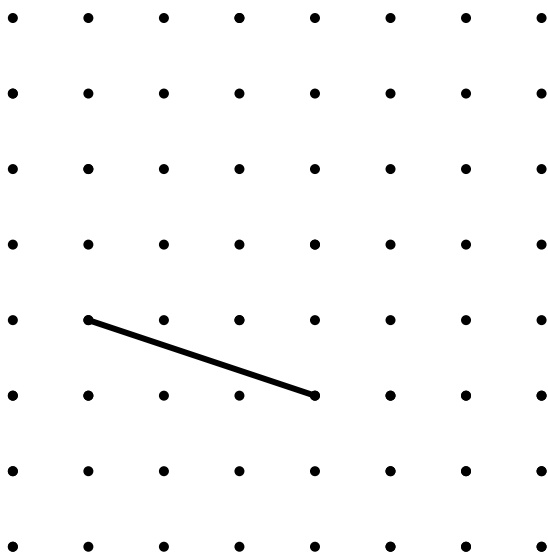


## 7.1g Self-Assessment: Section 7.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Understand the relationship between the side length of a square and its area.				
2. Understand the relationship between the side length of a cube and its volume.				
3. Evaluate the square roots of small perfect squares and the cube roots of small perfect cubes.				
4. Simplify square and cube roots.				

1. Find the following:
  - a. The side length of a square with an area of 36 square units. \_\_\_\_\_
  - b. The side length of a square with an area of 8 square units. \_\_\_\_\_
  - c. The area of a square with a side length of 5 units. \_\_\_\_\_
  - d. The area of a square with a side length of  $\sqrt{13}$  units. \_\_\_\_\_
  - e. Find the length of the segment shown below.



2. Find the following:

a. The side length of a cube with a volume of 125 units<sup>3</sup>. \_\_\_\_\_

b. The volume of a cube with a side length of 4 units. \_\_\_\_\_

3. Evaluate:

a.  $\sqrt{4}$

b.  $\sqrt{16}$

c.  $\sqrt{81}$

d.  $\sqrt{121}$

e.  $\sqrt[3]{64}$

f.  $\sqrt[3]{125}$

g.  $\sqrt[3]{1000}$

4. Simplify:

a.  $\sqrt{60}$

b.  $-\sqrt{90}$

c.  $2\sqrt{12}$

d.  $\sqrt[3]{81}$

e.  $\sqrt[3]{48}$

f.  $-\sqrt[3]{108}$

## Section 7.2: Solutions to Equations Using Square and Cube Roots

### Section Overview:

In this section, students will apply their knowledge from the previous section in order to solve simple square and cubic equations. Building on student understanding of how to solve simple linear equations using inverse operations, students will understand that taking the square root of a number is the inverse of squaring a number and taking the cube root is the inverse of cubing a number. Students will express their answers in simplest radical form.

### Concepts and Skills to Master:

*By the end of this section students should be able to:*

1. Solve simple quadratic and cubic equations.

## 7.2a Class Activity: Solve Equations using Square and Cube Roots

In the problems below, we review how to solve some basic equations.



1. Write the inverse operation used to solve each of following equations, then show the steps used to solve the equation.

a.  $x + 3 = 7$

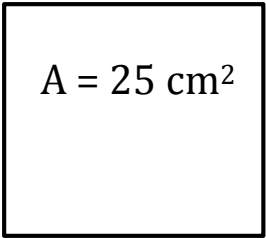
b.  $-3x = 18$

c.  $x - 6 = -14$

d.  $\frac{x}{7} = 3$

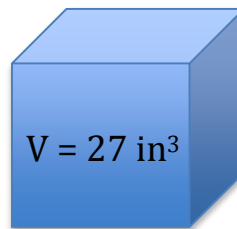
2. What does an inverse operation do?

3. Write and solve an equation to find the side length of a square with an area of  $25 \text{ cm}^2$ .


$$A = 25 \text{ cm}^2$$

4. Now consider the equation  $x^2 = 25$  out of context. Is 5 the only solution? In other words, is 5 the only number that makes this equation true when substituted in for  $x$ ?

5. Write and solve an equation to find the side length of a cube with a volume of  $27 \text{ in}^3$ .



6. Now consider the equation  $x^3 = 27$  out of context. Is 3 the only solution? In other words, is 3 the only number that makes this equation true when substituted in for  $x$ ?

7. State the inverse operation you would use to solve these equations. Solve each equation.

a.  $x^2 = 100$

b.  $x^2 = 36$

c.  $x^3 = 27$

8. Solve the equations below. Express your answer in simplest radical form.

a.  $x^2 = 64$

b.  $x^2 = -64$

c.  $x^3 = 8$

d.  $x^3 = -8$

e.  $x^3 = 1$

f.  $x^2 = 9$

g.  $x^2 = 5$

h.  $x^2 = 10$

i.  $x^3 = 15$

j.  $x^2 = -100$

k.  $x^3 = -512$

l.  $x^2 = 8$

m.  $x^2 = 45$

n.  $x^3 = 250$

o.  $x^3 = 128$

p.  $a^2 = \frac{1}{36}$

q.  $z^3 = \frac{1}{27}$

r.  $y^2 = 0.16$

s.  $x^2 + 16 = 25$

t.  $x^2 - 5 = 59$

u.  $10x^2 = 1440$

v.  $2x^2 = 16$

w.  $\frac{y^3}{2} = 32$

x.  $x^2 = p$  where  $p$  is a positive rational number

9. Estimate the solution. Use a calculator to check your estimate.

a.  $x^2 = 53$

b.  $a^2 = 15$

c.  $z^3 = 29$

## 7.2a Homework: Solve Equations using Square and Cube Roots

1. Solve the equations below. Express your answer in simplest radical form.

a.  $x^2 = 121$

b.  $x^2 = 81$

c.  $y^3 = 125$

d.  $x^3 = 216$

e.  $x^3 = -1$

f.  $x^2 = 18$

g.  $x^2 = -36$

h.  $x^2 = 2$

i.  $y^3 = 81$

j.  $x^2 + 12 = 48$

k.  $25 + x^2 = 169$

l.  $\frac{y^3}{5} = 25$

m.  $a^2 = \frac{1}{144}$

n.  $z^3 = \frac{1}{8}$

o.  $y^2 = 0.25$

p.  $a^2 = -\frac{1}{36}$

q.  $z^3 = -0.027$

r.  $y^3 = \frac{1}{125}$

s.  $a^3 = 100$

t.  $a^2 + 576 = 625$

u.  $64 + b^2 = 289$

v.  $x^3 = p$  where  $p$  is a positive rational number

w. Solve for  $r$  where  $A$  is the area of a circle and  $r$  is the radius:  $A = \pi r^2$

x. Solve for  $r$  where  $V$  is the volume of a cylinder,  $r$  is the radius, and  $h$  is the height:  $A = \pi r^2 h$

2. Estimate the solution. Use a calculator to check your estimate.

a.  $x^2 = 17$

b.  $a^2 = 67$

c.  $z^3 = 10$

You are designing a bathroom with the following items in it. Your very odd client has asked that each of these items be a perfect square or cube. Use your knowledge of squares and cubes to write an equation that models the area **or** volume of each item. Then solve the equation to find the side length of each item. The first one has been done for you.

3. Rug  $1764 \text{ in}^2$

4. Ottoman  $3,375 \text{ in}^3$

Let  $s$  equal the length of one side of the rug.

$$s^2 = 1764$$

$$\sqrt{s^2} = \sqrt{1764}$$

$$s = 42$$

The side length of the rug is 42 inches.

5. Mirror  $1024 \text{ cm}^2$

6. Bar of Soap  $27 \text{ cm}^3$

7. Is it probable to have a negative answer for the objects above? Why or why not?
8. Your client tells you that they would like to double the dimensions of the rug. What will happen to the area of the rug if you double the dimensions? Find this new area. What will happen to the area of rug if you triple the dimensions?
9. Your client also tells you that they would like to double the dimensions of the bar of soap. What will happen to the volume of the soap if you double its dimensions? Find this new volume. What will to the volume of the bar of some if you triple the dimensions?
10. Write and solve an equation of your own that has a power of 2 in it.
11. Write and solve an equation of your own that has a power of 3 in it.

## 7.2b Class Activity: Tower Views

1. Use inverse operations to solve the following problems.

a.  $\sqrt{x} = 4$

b.  $\sqrt{a} = 9$

c.  $2\sqrt{y} = 4$

d.  $\sqrt[3]{z} + 5 = 13$



People often wonder how far they can see when they're at the top of really tall buildings such as the Empire State Building, The Sears Tower in Chicago, etc.

The furthest distance you can see across flat land is a function of your height above the ground.

If  $h$  is the height in meters of your viewing place, then  $d$ , the distance in kilometers you can see, can be given by this formula:  $d = 3.532\sqrt{h}$

2. The equation above can be used to find the distance when you know the height. Rewrite the equation to find height when you know the distance.
3. If you were lying down on top of a building that is 100 meters tall, how far could you see? Write an equation to solve this problem. Solve the problem, showing all steps.
4. The CN Tower in Toronto, Canada is 555 meters tall. It is near the shore of Lake Ontario, about 50 kilometers across the lake from Niagara Falls. Your friend states that on a clear day, one can see as far as the falls from the top of the Tower. Are they correct? Explain your answer.
5. The Washington Monument in Washington D.C. is 170 meters tall. How far can one see from its top? Write the equation you need. Show all steps
6. How high must a tower be in order to see at least 60 kilometers? Write the equation you need. Show all steps.
7. Advertising for Queen's Dominion Amusement Park claims you can see 40 kilometers from the top of its observation tower. How high is the tower? Write the equation you need. Show all steps.

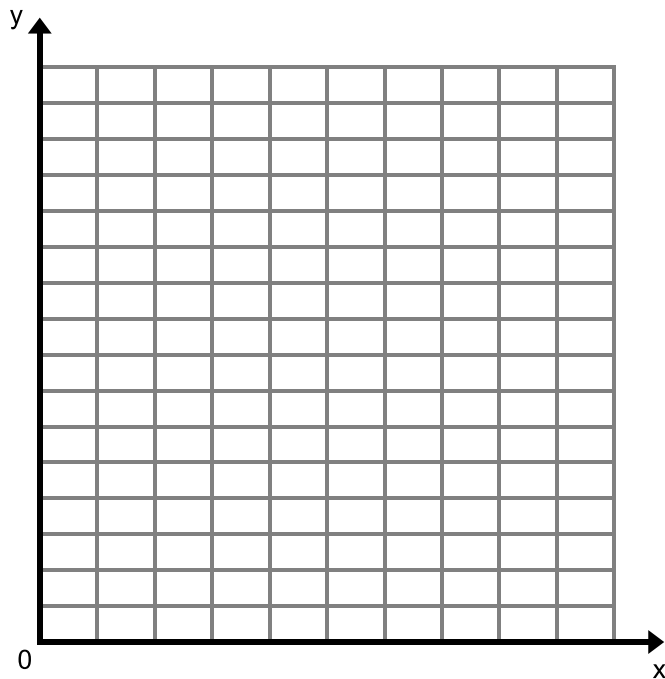


8. To enhance understanding of the relation between height and viewing distance, first complete the table below. Express each output value to the nearest whole number; then plot the data points on an appropriately labeled graph. Do not connect the points.

a.

Height (m)	0	50	100	150	200	250	300	350	400	450	500
Distance (km)											

b.



- c. What kind of association is shown between height and viewing distance?

## 7.2b Homework: Driving, Running, and Basketballs

1. Use inverse operations to solve the following problems.

a.  $\sqrt{x} = 5$

b.  $3 = \sqrt{a}$

c.  $3\sqrt{y} = 18$

d.  $\sqrt{z} - 3 = 78$

Deven is a civil engineer. He needs to make sure that the design of a curved road ensures the safety of a car driving at the speed limit. The equation  $V = \sqrt{2.5r}$  represents the maximum velocity that a car can travel safely on an unbanked curve.  $V$  represents the maximum velocity in miles per hour and  $r$  represents the radius of the turn in feet.

2. If a curve in the road has a radius of 1690 ft. what is the maximum velocity that a car can safely travel on the curve?
3. The equation above can be used to find the velocity when you know the radius. Rewrite the equation to find radius if you know the velocity.
4. If a road is designed for a speed limit of 55 miles per hour, what is the radius of the curve?
5. If a road is designed for a speed limit of 35 miles per hour, what is the radius of the curve?
6. What type of association exists between the radius of the curve and the maximum velocity that a car can travel safely?

Annie is on the track team her coach tells her that the function  $S = \pi \sqrt{\frac{9.8l}{7}}$  can be used to approximate the maximum speed that a person can run based off of the length of their leg.  $S$  represents the runner's speed in meters per second and  $l$  represents the length of the runner's leg in meters.

7. What is the maximum speed that Annie can run if her leg length is 1.12 meters?
8. The equation given above can be used to find the speed of the runner given their leg length. Rewrite the equation to find the leg length of the runner given their speed.
9. What is the leg length of a runner if their maximum running speed is 2.6 meters per second? Round your answer to the nearest hundredth.
10. What kind of association exists between the length of a person's leg and their maximum running speed?
11. Is leg length the only thing that affects a runner's maximum speed? Explain your answer.

The surface area of a sphere is found by the equation  $A = 4\pi r^2$  where  $A$  represents the surface area of the sphere and  $r$  represents the radius.

12. A basketball has a radius of 4.7 in, what is its surface area?
13. The equation given above can be used to find the surface area given the radius. Rewrite the equation so that you can find the radius if you are given the surface area.
14. The surface area of a dodge ball is 153.9. in<sup>2</sup>. What is the radius of the dodge ball?

## 7.2c Self-Assessment: Section 7.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Solve simple quadratic and cubic equations.				

1. Solve.

a.  $x^2 = 100$

b.  $x^3 = 64$

c.  $x^2 + 30 = 91$

d.  $x^3 - 9 = 134$

e. Solve for  $r$ .  $A = \frac{1}{2}r^2y$

## Section 7.3: Rational and Irrational Numbers

### Section Overview:

This section begins with a review of the different sets of rational numbers and why a need arose to distinguish them. Students then explore different ways of representing rational numbers, starting with a review of how to change fractions into decimals. During this process, students are reminded that the decimal expansion of all rational numbers either terminates or repeats eventually. From here, students review how to express terminating decimals as fractions and learn how to express repeating decimals as fractions by setting up and solving a system of equations. This skill allows them to show that all decimals that either terminate or repeat can be written as a fraction and therefore fit the definition of a rational number. After this work with rational numbers, students investigate numbers whose decimal expansion does not terminate or repeat: irrational numbers. With this knowledge, students classify numbers as rational and irrational. Students learn different methods for approximating the value of irrational numbers, zooming in to get better and better approximations of the number. They then use these approximations to estimate the value of expressions containing irrational numbers. Lastly, students compare and order rational and irrational numbers.

### Concepts and Skills to Master:

*By the end of this section, students should be able to:*

1. Know that real numbers that are not rational are irrational.
2. Show that rational numbers have decimal expansions that either terminate or repeat eventually.
3. Convert a repeating decimal into a fraction.
4. Know that the square root of a non-perfect square is an irrational number.
5. Understand that the decimal expansions of irrational numbers are approximations.
6. Show the location (or approximate location) of real numbers on the real number line.
7. Approximate the value of irrational numbers, zooming in to get better and better approximations.
8. Estimate the value of expressions containing irrational numbers.
9. Compare and order rational and irrational numbers.

### 7.3 Anchor: Revisiting the Number Line



**Directions:** Show the length of the following numbers on the number line below. Use the grid on the following page to construct lengths, using tick marks on an index card or tracing paper, and transfer those lengths onto the number line. Then answer the questions that follow. **Note:** On the grid, a horizontal or vertical segment joining two dots has a length of 1. On the number line, the unit length is the same as the unit length on the dot grid.

$A: \sqrt{25}$

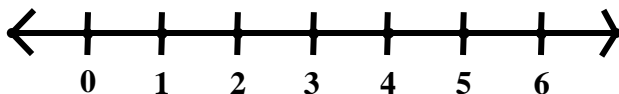
$B: \sqrt{2}$

$C: \sqrt{8}$

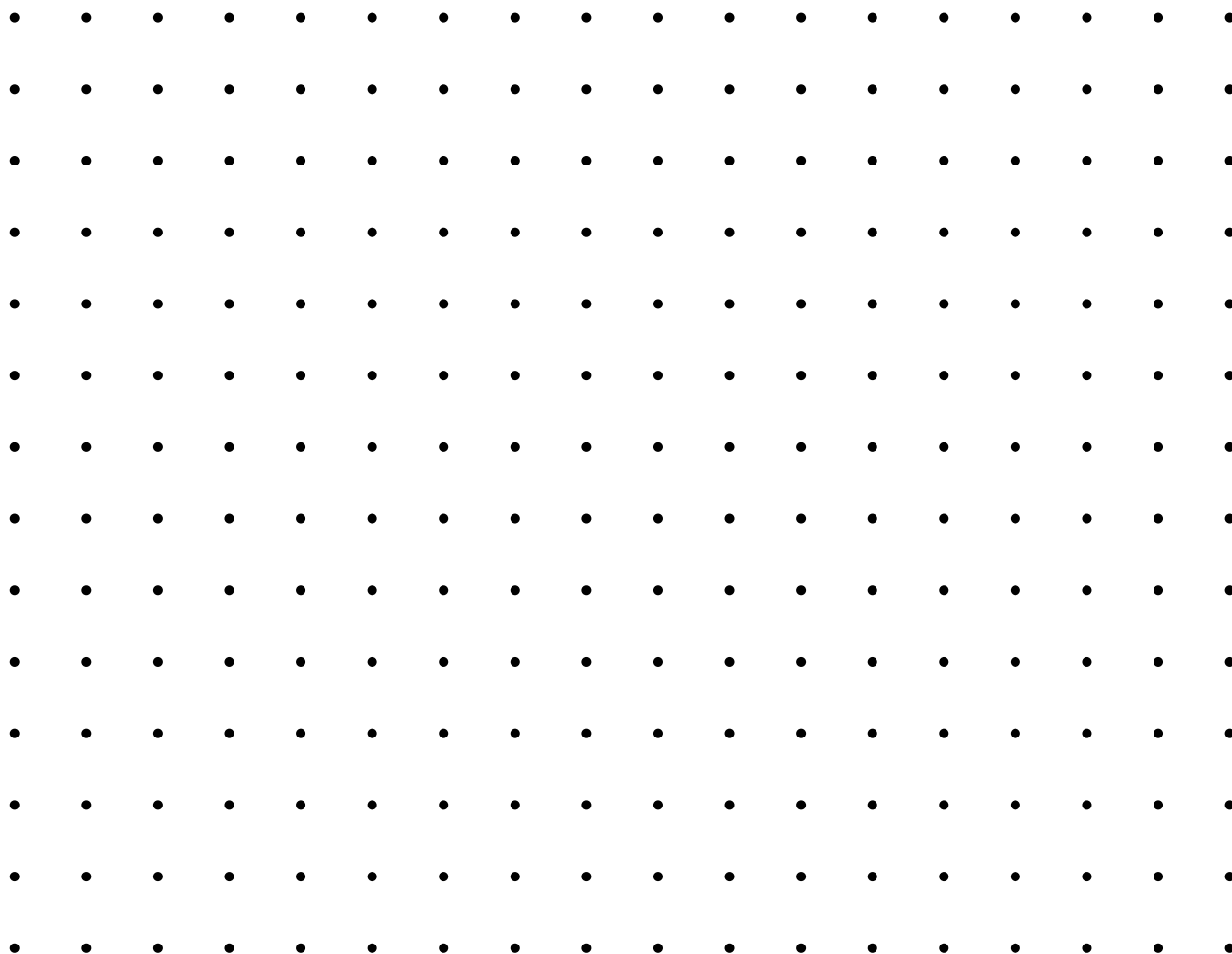
$D: 2\sqrt{2}$

$E: \sqrt{5}$

$F: 2\sqrt{5}$



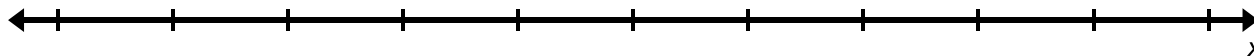
1. Use the number line to write a decimal approximation for  $\sqrt{2}$ . Verify your estimate with a calculator.
2. Would 1.41 be located to the right or to the left of  $\sqrt{2}$  on the number line?
3. Describe and show how you can put  $-\sqrt{2}$  on the number line. Write the decimal approximation for  $-\sqrt{2}$ .
4. Describe and show how you can put  $(2 + \sqrt{2})$  on the number line. Estimate the value of this expression.
5. Describe and show how you can put  $(2 - \sqrt{2})$  on the number line. Estimate the value of this expression.
6. Describe and show how you can put  $2\sqrt{2}$  on the number line. Estimate the value of this expression.
7. Use the number line to write a decimal approximation for  $\sqrt{5}$ .
8. Would 2.24 be located to the right or to the left of  $\sqrt{5}$  on the number line?
9. Describe and show how you can put  $1 + \sqrt{5}$  on the number line. Estimate the value of this expression.



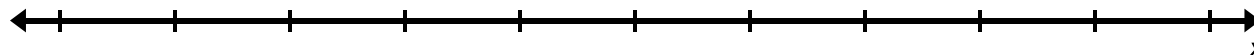
### 7.3a Class Activity: The Rational Number System

Our number system has evolved over time. On the following pages, you will review the subsets of numbers that are included in the set of **rational numbers**.

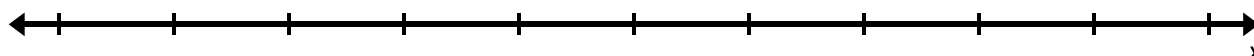
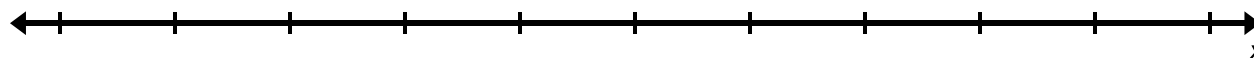
**Whole Numbers:**



**Integers:**



**Rational Numbers:**



Over the years, you have expanded your knowledge of the number system, gradually incorporating the sets of numbers mentioned above. These sets of numbers are all part of the **rational number system**.

A **rational number** is any number that can be expressed as a quotient  $\frac{p}{q}$  of two integers where  $q$  does not equal 0.



1. Begin to fill out the table below with different subsets, including equivalent forms, of **rational numbers** you know about so far and give a few examples of each. You will continue to add to this list throughout this section.

Subsets of the Rational Numbers	Examples

2. Change the following rational numbers into decimals **without** the use of a calculator.

a. $\frac{1}{2}$	b. $\frac{9}{5}$
------------------	------------------

c. $\frac{3}{8}$	d. $\frac{1}{3}$
e. $\frac{4}{15}$	f. $\frac{1}{7}$

3. What do you notice about the decimal expansion of any rational number? Why is this true?



### 7.3a Homework: The Rational Number System

1. Change the following rational numbers into decimals **without** the use of a calculator.

a. $\frac{1}{5}$	b. $\frac{7}{4}$
c. $\frac{5}{8}$	d. $\frac{2}{3}$
e. $\frac{2}{9}$	f. $\frac{3}{11}$

### 7.3b Class Activity: Expressing Decimals as Fractions

As we discovered in the previous section, when we converted fractions into decimals, the result was either a **terminating or repeating decimal**.

If we are given a terminating or repeating decimal, we need a method for changing them into a fraction in order to prove that they fit the definition of a rational number.

In 7<sup>th</sup> grade, you learned how to convert terminating decimals into fractions. Here are a few examples:

$$0.3 = \frac{3}{10}$$

$$0.25 = \frac{25}{100} = \frac{1}{4}$$

$$0.375 = \frac{375}{1000} = \frac{3}{8}$$

$$-2.06 = -2\frac{6}{100} = -2\frac{3}{50} = -\frac{103}{50}$$

*Now you try a few...*

$$0.4 =$$

$$0.05 =$$

$$0.275 =$$

$$1.003 =$$

**So, how do we express a repeating decimal as a fraction?** For example, how would you convert the repeating decimal  $0.\overline{45}$  into a fraction? Try in the space below.

We can use a **system of two linear equations** to convert a repeating decimal into a fraction. Let's look at an example:

**Example 1:**

The decimal  $0.\overline{3}$  is a repeating decimal that can be thought of as  $0.33333\dots$  where the “...” indicates that the 3s repeat forever. If they repeat forever, how can we write this number as a fraction? Here's a trick that will eliminate our repeating 3s.



Let  $a$  represent our number  $a = 0.\overline{3}$ .

Multiply both sides of the equation by 10 which would give us a second equation  $10a = 3.\overline{3}$ .

Now we have the following two equations:

$$10a = 3.\overline{3}$$

$$a = 0.\overline{3}$$

Let's expand these out:

$$10a = 3.3333333333 \dots$$

$$a = 0.3333333333 \dots$$

What will happen if we subtract the second equation from the first? Let's try it (remembering to line up the decimals):

$$\begin{array}{r} 10a = 3.3333333333 \dots \\ - \quad a = 0.3333333333 \dots \\ \hline \end{array}$$

$$9a = 3$$

$$a = \frac{3}{9} \quad (\text{Divide both sides by 9})$$

$$a = \frac{1}{3} \quad (\text{Simplify the fraction})$$

**Example 2:**

The decimal  $0.\overline{54}$  is a repeating decimal that can be thought of as  $0.54545454\dots$  where the “...” indicates that the 54 repeats forever. Let’s see how to express this as a fraction.

Let  $a$  represent our number  $a = 0.\overline{54}$ .

Why do you think we multiplied the second example by 100 instead of 10 as we did in the first example? What would have happened if we had multiplied by 10 in example 2? Try it below and see.

**Example 3:** Change the decimal  $2.\overline{4}$  into a fraction

The decimal  $2.\overline{4}$  is a repeating decimal that can be thought of as  $2.4444444\dots$  where the “...” indicates that the 4s repeat forever.

Let  $a$  represent our number  $a = 2.\overline{4}$ .

**Example 4:** Change the decimal  $3.\overline{12}$  into a fraction.

**Example 5:** Change the decimal  $0.\overline{123}$  into a fraction.

**Example 6:** Change the decimal  $4.\overline{1}$  into a fraction.

**Example 7:** Change the decimal  $2.\overline{015}$  into a fraction.

### 7.3b Homework: Expressing Decimals as Fractions

**Directions:** Circle whether the decimal is terminating or repeating then change the decimals into fractions.

1. 0.6      Terminating      Repeating	2. 0.02      Terminating      Repeating
3. 1.25      Terminating      Repeating	4. $-0.0\overline{64}$ Terminating      Repeating
5. $0.\overline{5}$ Terminating      Repeating	6. $0.\overline{45}$ Terminating      Repeating
7. $0.08\overline{3}$ Terminating      Repeating	8. $5.2\overline{6}$ Terminating      Repeating
9. $-0.24$ Terminating      Repeating	10. $3.\overline{65}$ Terminating      Repeating

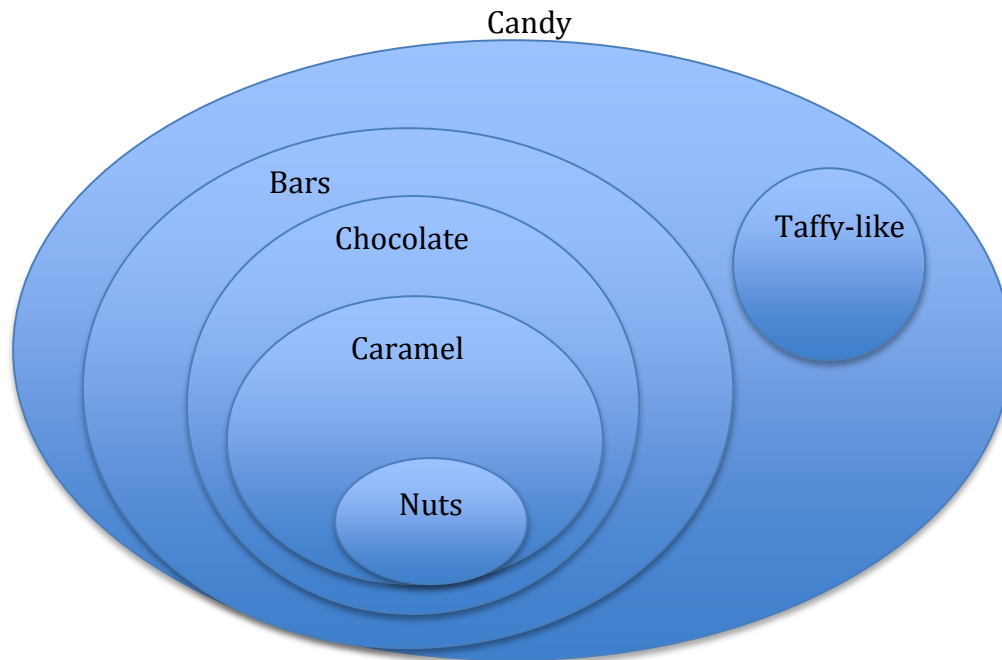


11. $-0.58$ Terminating Repeating	12. $1.1\overline{6}$ Terminating Repeating
13. $2.\overline{6}$ Terminating Repeating	14. $0.2$ Terminating Repeating
15. $0.\overline{27}$ Terminating Repeating	16. $0.45$ Terminating Repeating

### 7.3c Class Activity: Expanding Our Number System

Organize the following candy into the Venn diagram.

Snickers, Hershey's Chocolate Bar, Mars Bar, Laffy-Taffy, Starburst



List the sets of numbers we have learned about so far, including equivalent forms.

So are all numbers rational numbers? Are there numbers that cannot be written as a quotient of two integers?

What about  $\sqrt{2}$ ? Can you write  $\sqrt{2}$  as a fraction? Why or why not?

Numbers like  $\sqrt{2}$ , which do not have a terminating or repeating decimal expansion are **irrational numbers**. Irrational numbers **cannot** be expressed as a quotient.

Rational and Irrational numbers together form the set of **real numbers**. Real numbers can be thought of as points on an infinitely long line called the number line. Just like we organized the candy bars in the Venn diagram above we can organize the real number system.

## Real Number System



**Directions:** Classify the following numbers and provide a justification.

Number	Whole number	Integer	Rational number	Irrational number	Real	Justification
1. $\frac{2}{3}$						
2. 0.25						
3. -2						
4. $\sqrt{5}$						
5. 10						
6. 0						
7. $\sqrt{10}$						
8. $\sqrt{36}$						
9. $-\sqrt{121}$						
10. $2\frac{1}{2}$						
11. $0.08\bar{3}$						

Number	Whole number	Integer	Rational number	Irrational number	Real	Justification
12. $\frac{10}{13}$						
13. $\pi$						
14. $-3\pi$						
15. $0.26\overline{54}$						
16. $\sqrt[3]{27}$						
17. $1.2122122212222\dots$						
18. $\sqrt[3]{30}$						
19. $\frac{\sqrt{2}}{2}$						
20. The side length of a square with an area of 2						
21. The side length of a square with an area of 9						
22. The number half-way between 3 and 4						
23. The number that represents a loss of 5 yards						



### 7.3c Homework: Expanding Our Number System

**Directions:** Classify the following numbers as rational or irrational and provide a justification.

Number	Whole number	Integer	Rational number	Irrational number	Real	Justification
1. $\sqrt{2}$						
2. $\sqrt{1}$						
3. $\frac{1}{3}$						
4. $-157$						
5. $4\frac{1}{9}$						
6. $-0.375$						
7. $-\sqrt{5}$						
8. $0.\bar{2}$						
9. $\sqrt[3]{125}$						
10. $-\sqrt{81}$						
11. $-2.2\bar{4}$						
12. $2\pi$						
13. The side length of a square with an area of 49						
14. The side length of a square with an area of 1						
15. The side length of the side of a square with an area of 5						
16. The side length of a square with an area of 24						
17. The number half-way between 0 and -1						
18. The number that represents 7 degrees below 0.						

19. Give your own example of a rational number.

20. Give your own example of an irrational number.

**Directions:** The table below contains statements about rational and irrational numbers. If the statement is true, put a check in the box. If the statement is not true, write a correct statement.  

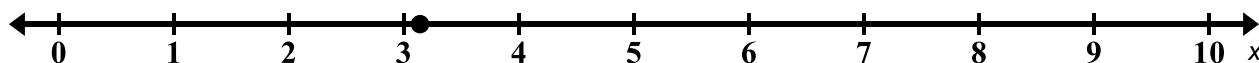
Statement	Check if True or Correct Statement
21. You can show the exact decimal expansion of the side length of a square with an area of 5 square units.	
22. You can construct and show the length $\sqrt{5}$ on a number line.	
23. Square roots of numbers that are perfect squares are rational.	
24. The number 0.256425642564 ... is rational.	
25. You can always use a calculator to determine whether a number is rational or irrational by looking at its decimal expansion.	
26. The number $0.\overline{6}$ is irrational because its decimal expansion goes on forever.	
27. The number half-way between 3 and 4 is rational.	
28. You can build a perfect cube with 36 unit cubes.	
29. If you divide an irrational number by 2, you will still have an irrational number.	
30. The side length of a cube made of 64 unit blocks is irrational.	

Make up **two** of your own statements that are **true** about rational or irrational numbers.

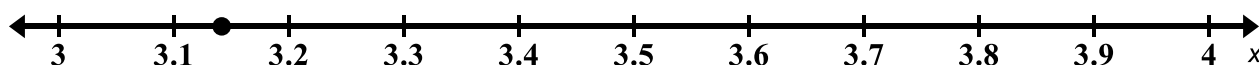
### 7.3d Class Activity: Approximating the Value of Irrational Numbers

So far, we have seen that we can show the location of an irrational number on the number line. We also know that we cannot show the entire decimal expansion of an irrational number because it is infinitely long and there is no pattern (as far as we know). However, we can come up with good approximations for the numerical value of an irrational number.

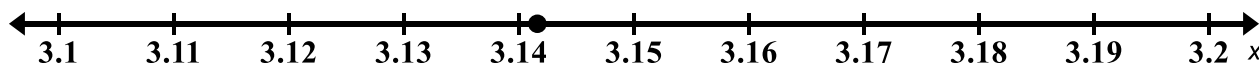
The decimal expansion for  $\pi$  to eight decimal places is 3.14159265... On the number line, we know that  $\pi$  lies somewhere between 3 and 4:



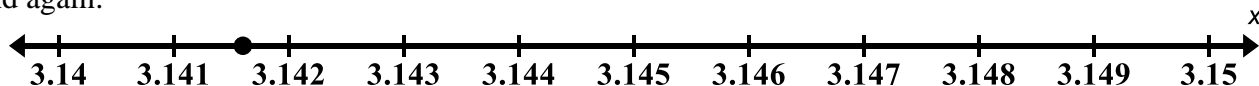
We can zoom in on the interval between 3 and 4 and narrow in on where  $\pi$  lies:



And if we zoom in again on the interval from 3.1 to 3.2:



And again:



We can imagine continuing this process of zooming in on the location of  $\pi$  on the number line, each time narrowing its possible location by a factor of 10.

Once we have an approximation for an irrational number, we can approximate the value of expressions that contain that number.

For example, suppose we were interested in the approximate value of  $2\pi$ ? We can use our approximations of  $\pi$  from above to approximate the value of  $2\pi$  to different degrees of accuracy:

Because  $\pi$  is between 3 and 4,  $2\pi$  is between \_\_\_\_\_ and \_\_\_\_\_.

Because  $\pi$  is between 3.1 and 3.2,  $2\pi$  is between \_\_\_\_\_ and \_\_\_\_\_.

Because  $\pi$  is between 3.14 and 3.15,  $2\pi$  is between \_\_\_\_\_ and \_\_\_\_\_.

Because  $\pi$  is between 3.141 and 3.142,  $2\pi$  is between \_\_\_\_\_ and \_\_\_\_\_.

Check the value of  $2\pi$  on your calculator. How are we doing with our approximations of  $2\pi$ ?

We can use a method of **guess and check** to give us an estimate of the numerical value of an irrational number that is correct up to as many decimal points as we need.

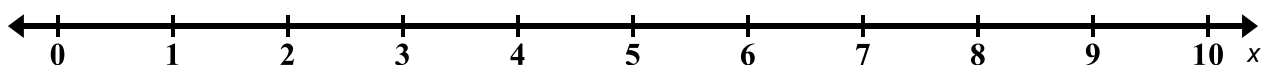


**Directions:** Approximate the value of the following irrational numbers to the indicated degrees of accuracy. You can use your calculator for the following questions but do not use the square root key.

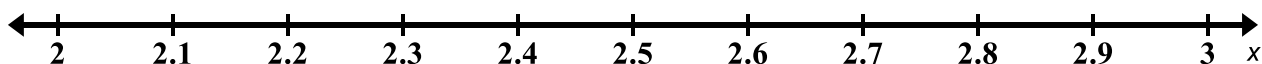
1. Between which two integers does  $\sqrt{5}$  lie?

- a. Which integer is it closest to?

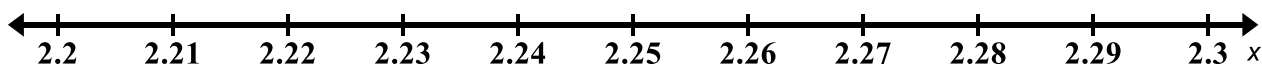
- b. Show its approximate location on the number line below.



- c. Now find  $\sqrt{5}$  accurate to one decimal place. Show its approximate location on the number line below.



- d. Now find  $\sqrt{5}$  accurate to two decimal places. Show its approximate location on the number line below.



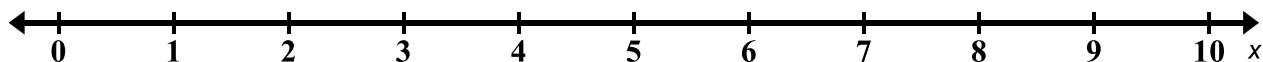
- e. Use your work from above to approximate the value of the expression  $2 + \sqrt{5}$  to the nearest whole number. The nearest tenth. The nearest hundredth.



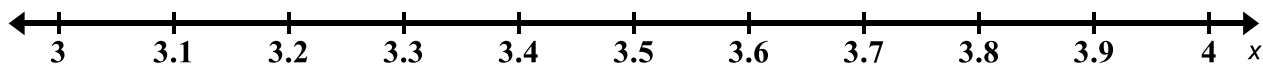
2. Between which two integers does  $\sqrt{15}$  lie?

a. Which integer is it closest to?

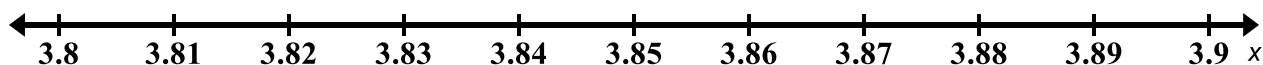
b. Show its approximate location on the number line below.



c. Now find  $\sqrt{15}$  accurate to one decimal place. Show its approximate location on the number line below.



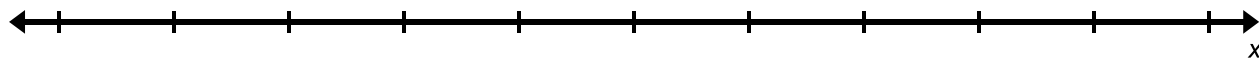
d. Now find  $\sqrt{15}$  accurate to two decimal places. Show its approximate location on the number line below.



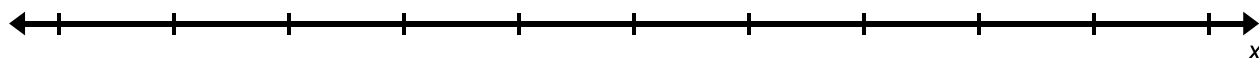
e. Use your work from above to approximate the value of the expression  $4\sqrt{15}$  to the nearest whole number. To the nearest tenth. To the nearest hundredth.

3. Repeat the process above to find  $\sqrt{52}$  accurate to two decimal places. Place your numbers on the number lines provided each time you increase the degree of accuracy of your estimate.

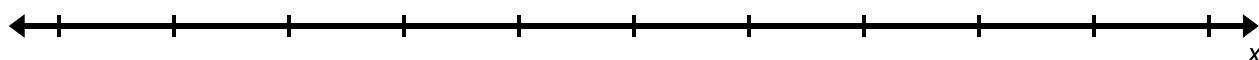
a. To the nearest **whole number**:



b. To the nearest **tenth**:



c. To the nearest **hundredth**:



- d. Use your work from above to approximate the value of  $3 + \sqrt{52}$  to the nearest whole number. To the nearest tenth. To the nearest hundredth.
- e. Use your work from above to approximate the value of  $2\sqrt{52}$  to the nearest whole number. To the nearest tenth. To the nearest hundredth.

4. Pick a positive integer between 1 and 100, call it  $A_0$ . Find the average of your number ( $A_0$ ) and  $\frac{100}{\text{your number } (A_0)}$  and call this number  $A_1$ . Take the average of  $A_1$  and  $\frac{100}{A_1}$  and call this number  $A_2$ . Take the average of  $A_2$  and  $\frac{100}{A_2}$  and call this number  $A_3$ . Repeat this process two more times.
  
5. Pick a different positive integer between 1 and 100 and repeat the process above. What do you notice?
  
6. Pick a positive integer between 1 and 100, call it  $A_0$ . Find the average of your number ( $A_0$ ) and  $\frac{25}{\text{your number } (A_0)}$  and call this number  $A_1$ . Take the average of  $A_1$  and  $\frac{25}{A_1}$  and call this number  $A_2$ . Take the average of  $A_2$  and  $\frac{25}{A_2}$  and call this number  $A_3$ . Repeat this process two more times.
  
7. Compare the number you picked for #6 with that of a neighbor. Compare your end results. What do you notice?
  
8. Pick a positive integer between 1 and 100, call it  $A_0$ . Find the average of your number ( $A_0$ ) and  $\frac{5}{\text{your number } (A_0)}$  and call this number  $A_1$ . Take the average of  $A_1$  and  $\frac{5}{A_1}$  and call this number  $A_2$ . Take the average of  $A_2$  and  $\frac{5}{A_2}$  and call this number  $A_3$ . Repeat this process two more times. What do you notice?

**Directions:** Solve the following problems. Again, do not use the square root key on your calculator.



9. A hospital has asked a medical supply company to manufacture intravenous tubing (IV tubing) that has a minimum opening of 7 square millimeters and a maximum opening of 7.1 square millimeters for the rapid infusion of fluids. The medical design team concludes that the radius of the tube opening should be 1.5 mm. Two supervisors review the design team's plans, each using a different estimation for  $\pi$ .

**Supervisor 1:** Uses 3 as an estimation for  $\pi$

**Supervisor 2:** Uses 3.1 as an estimation for  $\pi$

The supervisors tell the design team that their designs will not work. The design team stands by their plans and tells the supervisors they are wrong. Who is correct and why? Recall that the formula for the area of a circle is  $A = \pi r^2$ .

10. A square field with an area of 2,000 square ft. is to be enclosed by a fence. Three contractors are working on the project and have decided to purchase slabs of pre-built fencing. The slabs come in pieces that are 5-ft. long.
- Keith knows that  $\sqrt{2000}$  is between 40 and 50. Trying to save as much money as possible, he estimates on the low side and concludes that they will need 160 feet of fencing. Therefore, he concludes they should purchase 32 slabs of the material.
  - Jose also knows that  $\sqrt{2000}$  is between 40 and 50 but he is afraid that using Keith's calculations, they will not have enough fencing. He suggests that they should estimate on the high side and buy 200 feet of fencing to be safe. Therefore, he concludes they should purchase 40 slabs of material.

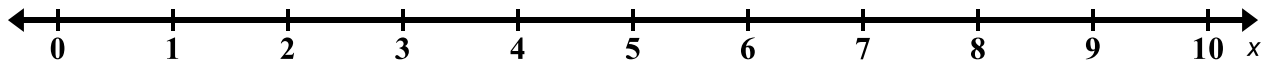
Keith and Jose begin to argue. Sam jumps in and says, "I have a way to make you both happy – we will purchase enough material to enclose the entire field **and** we will minimize the amount of waste." What do you think Sam's suggestion is and how many slabs will be purchased using Sam's rationale?

### 7.3d Homework: Approximating the Value of Irrational Numbers

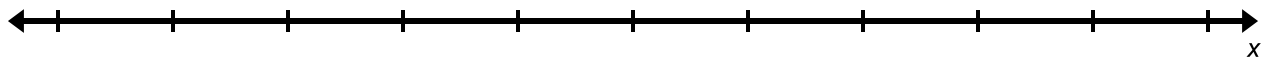
1. Between which two integers does  $\sqrt{2}$  lie?

a. Which integer is it closest to?

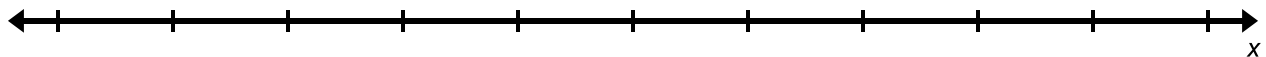
b. Show its approximate location on the number line below.



c. Now find  $\sqrt{2}$  accurate to one decimal place. Show its approximate location on the number line below.



d. Now find  $\sqrt{2}$  accurate to two decimal places. Show its approximate location on the number line below.

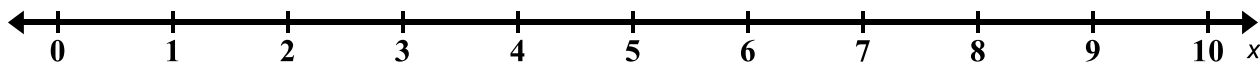


e. Estimate the value of the expression  $2 + \sqrt{2}$  to the nearest whole number. To the nearest tenth. To the nearest hundredth.

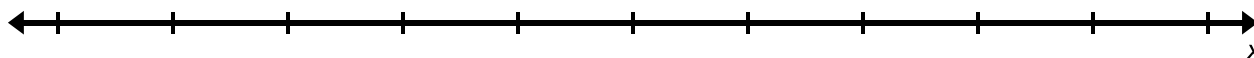
f. Estimate the value of the expression  $2\sqrt{2}$  to the nearest whole number. To the nearest tenth. To the nearest hundredth.

2. Between which two integers does  $\sqrt{40}$  lie?

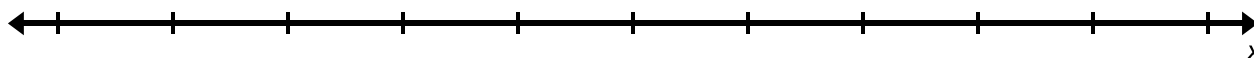
- Which integer is it closest to?
- Show its approximate location on the number line below.



- Now find  $\sqrt{40}$  accurate to one decimal place. Show its approximate location on the number line below.



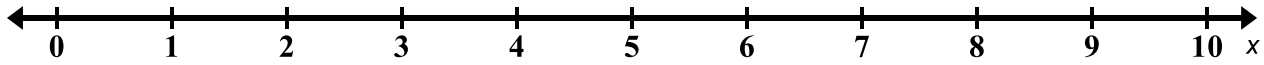
- Now find  $\sqrt{40}$  accurate to two decimal places. Show its approximate location on the number line below.



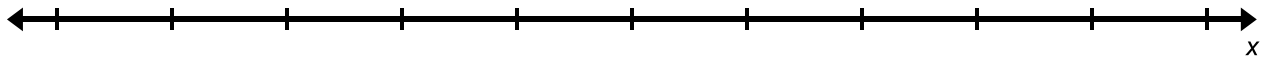
- Estimate the value of the expression  $2\sqrt{40}$  to the nearest whole number. To the nearest tenth. To the nearest hundredth.

3. Repeat the process above to find  $\sqrt{60}$  accurate to two decimal places. Place your numbers on the number lines provided each time you increase the degree of accuracy of your estimate.

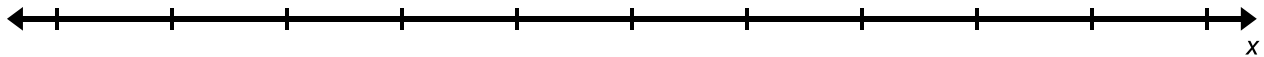
a. To the nearest **whole number**:



b. To the nearest **tenth**:



c. To the nearest **hundredth**:



- d. Use your work from above to approximate the value of  $\sqrt{60} - 5$  to the nearest whole number. To the nearest tenth. To the nearest hundredth.

- e. Use your work from above to approximate the value of  $1 + \sqrt{60}$  to the nearest whole number. To the nearest tenth. To the nearest hundredth.

4. Use the approximations of  $\pi$  on page 60 to estimate the value of the following expressions to increasing levels of accuracy. You can use your calculator but don't use the square key or the  $\pi$  key.

a.  $\pi^2$

b.  $10\pi$

c.  $3 + \pi$



### 7.3e Class Activity: Comparing and Ordering Real Numbers

**Directions:** Do **not** use a calculator for the following problems. Any calculations you may need are given in the problem.



1. Order the following numbers from **least to greatest**. Note that  $8.5^2 = 72.25$ .  
 $\sqrt{80}$ , 8, 9, 8.5,  $\sqrt{62}$

2. Order the following numbers from **least to greatest**. Note that  $3.5^2 = 12.25$ .  
 $-\sqrt{13}$ , -3, -4, -3.5

3. Use the following calculations to answer the questions below.

$$2.2^2 = 4.84$$

$$2.3^2 = 5.29$$

$$2.23^2 = 4.9729$$

$$2.24^2 = 5.0176$$

- a. Put the following numbers in order from **least to greatest**.

$\sqrt{5}$ ,  $\frac{5}{2}$ , 2.2, the side length of a square with an area of 4

- b. Find a number between 2.2 and  $\sqrt{5}$ .

- c. Find an irrational number that is smaller than all of the numbers in part a.

4. Use the following calculations to answer the questions below.

$$6.48^2 = 41.9904$$

$$6.5^2 = 42.25$$

- a. Order the following numbers from **least to greatest**.

$\sqrt{50}$ , 6, 7, 6.5,  $\sqrt{42}$

- b. Find a rational number that is smaller than all of the numbers in part a.

- c. Find an irrational number that is smaller than all of the numbers in part a.

- d. Find a number between  $\sqrt{42}$  and 6.5.

5. Use the following calculations to answer the questions below.

$$2.44^2 = 5.9536$$

$$2.45^2 = 6.0025$$

$$2.449^2 = 5.997601$$

a. Order the following numbers from **least to greatest**.

$\sqrt{6}$ , 2.44,  $2.\bar{4}$ , 2.5, the side length of a square with an area of 9

b. Find an irrational number that is between 0 and the smallest number from part a.

c. Find a number that is between 2.44 and  $\sqrt{6}$ .

6. Use the approximations of  $\pi$  on page 60 and the calculations given below to answer the questions below.

$\pi$  is between 3 and 4

$\pi$  is between 3.1 and 3.2

$\pi$  is between 3.14 and 3.15

$\pi$  is between 3.141 and 3.142

$$3.15^2 = 9.9225$$

a. Find a number that is between 3 and  $\pi$ .

b. Find a number that is between 3.14 and  $\pi$ .

c. Which is larger and why?  $(\pi + 5)$  or 8

d. Which is larger and why?  $(10 - \pi)$  or 7

e. Which is larger and why?  $2\pi$  or 6.2

f. Which is larger and why?  $\pi^2$  or 10

### 7.3e Homework: Comparing and Ordering Real Numbers

**Directions:** Do **not** use a calculator for the following problems. Any calculations you may need are given in the problem.

1. Give an example of a rational number between  $\sqrt{9}$  and  $\sqrt{16}$ .
2. Give an example of an irrational number between 8 and 9.
3. Use the following calculations to answer the questions below.

$$1.41^2 = 1.9881$$

$$1.42^2 = 2.0164$$

- a. Order the following numbers from **least to greatest**.

$$\sqrt{2}, 1.41, 1.4, 1\frac{1}{2}, 1.4\bar{2}$$

- b. Find a number between 1.4 and  $1\frac{1}{2}$ .

4. Use the following approximations and calculations to answer the questions below.

$\pi$  is between 3.14 and 3.15

$$3.1^2 = 9.61$$

$$3.2^2 = 10.24$$

$$3.16^2 = 9.9856$$

$$3.17^2 = 10.0489$$

- a. Order the following numbers from **least to greatest**.

$$\sqrt{10}, 3\frac{1}{10}, 3.\bar{1}, \pi, \text{side length of a square with an area of 9}$$

- b. Find a number between  $3\frac{1}{10}$  and  $3.\bar{1}$ .

- c. Find a number between 3.1 and  $\sqrt{10}$ .

5. The number  $e$  is an important irrational number. In future math classes as well as science and social science, you will see and use this number quite a bit. Use the approximations of  $e$  and the calculations given below to answer the questions that follow.

$e$  is between 2 and 3

$e$  is between 2.7 and 2.8

$e$  is between 2.71 and 2.72

$e$  is between 2.718 and 2.719

- Find a number that is between 2 and  $e$ .
  - Find a number that is between  $e$  and 2.8.
  - Which is larger and why?  $(e + 10)$  or 13
  - Which is larger and why?  $(6 - e)$  or 4
  - Which is larger and why?  $2e$  or 5.4
  - Which is larger and why?  $e^2$  or 9
6. Order the following numbers from **least to greatest**. Note that  $6.2^2 = 38.44$  and  $6.4^2 = 40.96$
- $-\sqrt{40}, -7, -6, -6.2, -6.4, -6\frac{1}{2}$

### 7.3f Self-Assessment: Section 7.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

<b>Skill/Concept</b>	<b>Minimal Understanding 1</b>	<b>Partial Understanding 2</b>	<b>Sufficient Mastery 3</b>	<b>Substantial Mastery 4</b>
1. Know that real numbers that are not rational are irrational.				
2. Show that rational numbers have decimal expansions that either terminate or repeat.				
3. Convert a repeating decimal into a fraction.				
4. Know that the square root of a non-perfect square is an irrational number.				
5. Understand that the decimal expansions of irrational numbers are approximations.				
6. Show the location (or approximate location) of real numbers on the real number line.				
7. Approximate the value of irrational numbers, zooming in to get better and better approximations.				
8. Estimate the value of expressions containing irrational numbers.				
9. Compare and order rational and irrational numbers.				

1. Circle the numbers that are rational.

a.  $-4$

b.  $-0.34$

c.  $\sqrt{7}$

d.  $0$

e.  $\frac{1}{2}$

f.  $-\sqrt{11}$

g.  $\sqrt{81}$

h.  $-\sqrt[3]{27}$

2. Change each fraction to a decimal.

a.  $\frac{3}{4}$

b.  $\frac{5}{6}$

c.  $\frac{8}{3}$

3. Change each decimal to a fraction.

a.  $0.\bar{2}$

b.  $1.\overline{34}$

c.  $2.0\bar{1}$

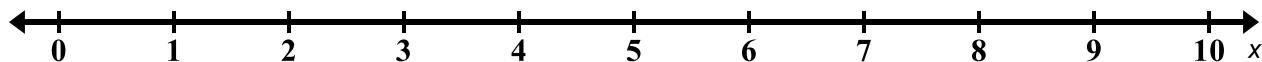
4. Classify the following numbers as rational or irrational and provide a justification.

a.  $\sqrt{10}$

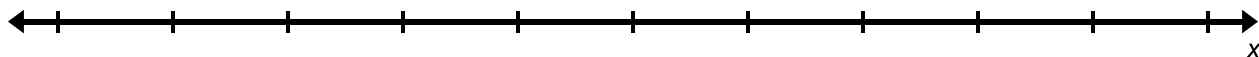
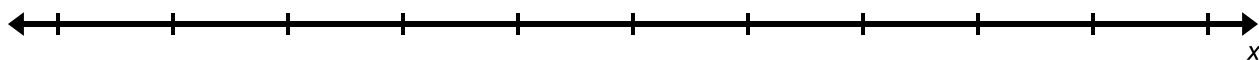
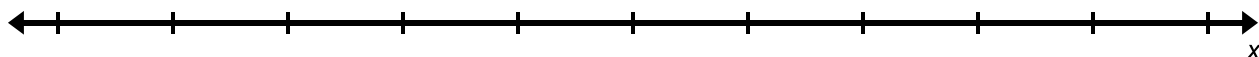
b.  $\sqrt[3]{30}$

c.  $\sqrt{144}$

5. Find the decimal approximation of the following numbers to two decimal places without using the square root key on your calculator.
- $\sqrt{22}$
  - $\sqrt{45}$
  - $\sqrt{60}$
6. Describe how you would plot the following points on the number line shown below.  
2.0, 2.2, 2.24.

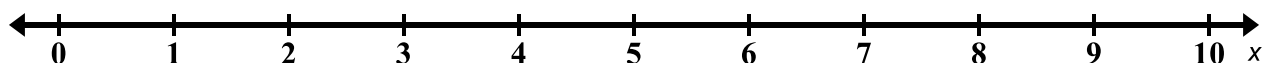


Plot the numbers from above on the three number lines shown below, changing the scale of each number line in order to show the location of the points more precisely.



7. Show the approximate location of the following numbers on the number line below.

$A: \sqrt{3}, B: \sqrt{10}, C: 2\sqrt{5}, D: 3\frac{1}{10}, E: 1.5$



8. Approximate  $\sqrt{31}$  to the...
- a. Nearest whole number
  - b. Nearest tenth
  - c. Nearest hundredth
9. Approximate the value of the following expressions.
- a.  $2\sqrt{2}$  if  $\sqrt{2} \approx 1.41$
  - b.  $3\pi$  if  $\pi \approx 3.14$
  - c.  $4 + \sqrt{2}$  if  $\sqrt{2} \approx 1.41$
10. Order the following numbers from least to greatest.
- $1.2, -2\pi, -3\frac{1}{2}, \sqrt{6}, \frac{4}{3}, -6.28, -\sqrt{2}$