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Chapter 8: Integer Exponents, Scientific Notation and Volume (4 weeks)

Utah Core Standard(s):

- Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.* (8.EE.1)
- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.* (8.EE.3)
- Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurement of very large and very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. (8.EE.4)
- Know the formulas for volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. (8.G.9)

Vocabulary: base, cone, cylinder, diameter, estimate, exponent, exponential form, hemisphere, scientific notation, pi, power, powers of ten, radius, scientific notation, slant height, sphere, standard form, volume

Chapter Overview:

Students begin this chapter with the study of integer exponents. They represent repeated multiplication in exponential form and begin to explore the properties of integer exponents as another method for transforming expressions. Students explore problems and patterns that lead them to properties related to negative exponents and an exponent of 0. Through the investigation of these properties they learn to generate equivalent expressions in a quick and efficient way. Their study is then turned to using their knowledge of exponents, place value, and powers of ten to express a number in scientific notation. This notation is used to denote very small and very large numbers. Students learn to change numbers from standard form to scientific notation and vice versa. They also learn to perform operations with numbers in scientific notation. This enables them to work with and analyze real world situations where large and small quantities exist. Finally, students study volume and how exponents play a role in the formulas for the volume of a cylinder, cone, and sphere. They use these formulas to solve a variety of problems related to the volume of these three-dimensional objects.

Connections to Content:

Prior Knowledge:

Prior to 8th grade students have explained patterns in the number of zeros of the product when multiplying a number by a power of ten. They have also analyzed the patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. In addition they have used whole number exponents to denote powers of 10. They extend this knowledge to base numbers other than 10 in 8th grade as they generate the exponent rules. In addition to rewriting numbers as powers of ten to better understand multiplication and place value students have previously rewritten numbers in a variety of ways to perform an indicated skill, i.e., 24 as $2 \times 2 \times 2 \times 3$ and 18 as $2 \times 3 \times 3$, to reduce $\frac{18}{24}$ to $\frac{3}{4}$, or to find the LCM of 72 or GCF of 6. Similarly, in 8th grade, students express numbers as decimals or in scientific notation in order to compare and estimate very large and small quantities. Finally, students must gather their knowledge of area and volume from 6th and 7th grade as they begin their work with volumes of cylinders, cones, and spheres.

Future Knowledge:

A solid foundation with exponent laws and rules will help students significantly as they begin to transform more complicated expressions in high school mathematics courses. For example, in Secondary II they will extend the laws of exponents to rational exponents. In high school students will explore exponential functions and the work they do in 8th grade begins to familiarize them with how exponents work algebraically and how exponential behavior is exhibited. Scientific notation will be used in a variety of contexts in high school mathematics and science courses. By studying the volume formulas for cylinders, cones, and spheres in 8th grade, students will be prepared to investigate informal arguments and proofs, specifically Cavalieri's Principle for the derivation of these formulas in Secondary II.

Many of the section lessons in this chapter may take more than one day to complete depending on the length of your class period. Think about the needs of your students and pace the lessons and homework assignments according to their needs.

MATHEMATICAL PRACTICE STANDARDS:

	Reason abstractly and quantitatively.	1. If you fill the hemisphere with water or other filling material, predict what fraction of the cylinder is filled by the volume of one hemisphere. 2. Now try it, what fraction of the cylinder is filled by the volume of one hemisphere? 3. Write down the equation for the volume of the cylinder below the cylinder, be sure to write your height in terms of the radius or r. 4. Manipulate the equation for the volume of the cylinder to show the volume of the hemisphere. 5. Now double your formula to find the formula for the volume of a sphere. <i>Students derive the formula for the volume of a sphere by physically comparing the volume of a sphere to that of a cylinder. Based on the physical differences discovered, students manipulate the formula for the volume of a cylinder in order to derive the formula for the volume of a sphere. They use a similar process to derive the formula for the volume of a cone. They must algebraically interpret these changes as they reason abstractly about the dimensions dealt with. They can then make the appropriate changes as they manipulate the formulas.</i>
	Look for and make use of structure.	Discuss the multiplication problem $(5 \times 3)(2 \times 8)$ with your class. Write your thoughts below. Of course it most natural to just multiply 15 times 16. But could you rewrite the problem as $(5 \times 2)(3 \times 8)$ or $(5 \times 8)(2 \times 3)$? Is the answer the same? Why can you do this? Rewrite this problem $(5.1 \times 10^5)(6.8 \times 10^3)$ like the problem above (group the powers of 10 together). Then solve the problem (use exponent properties) and write the solution. <i>Looking for structure is a big part of this chapter. The example above is showing how the structure of a number written in scientific notation can aid in completing basic operations of very large and small numbers in a fast and efficient way. The students will also make use of structure when they look at how exponents are used to represent repeated multiplication. This in turn points toward the discovery and understanding of the exponent properties and rules.</i>
	Attend to precision.	Analyze the pairs of expressions given below; discuss the similarities and differences between them. $(-4)^2$ and -4^2 $(-4)^3$ and -4^3 <i>Students might bring up the fact that the only thing that is different about the first set of expressions are the parentheses. The parentheses are important because they indicate that there are two copies of negative 4. If you expand this expression you get $(-4)(-4) = 16$. The expression -4^2 indicates that there is a coefficient of negative 1. Upon expansion, you get $-4^2 = (-1)(4)(4) = (-1)(16) = -16$. Students will get different answers even though the expressions are similar. They must attend to precision. In the second set of expressions they both equal -64. This is because of the odd exponent. Throughout the chapter students must attend to precision constantly as they grapple with the notation used with exponents and as they decipher what these special notations are communicating to them.</i>

	Look for and express regularity in repeated reasoning.	Complete columns 2 and 3 in the table below by writing each expression in expanded form and in exponential form. As you are simplifying try to discover a shortcut or rule that you could use to simplify more quickly. Test your shortcut in the last column and compare your answers. Product of Powers <table><tr><th>Column 1</th><th>Column 1 rewritten in expanded form or as repeated multiplication.</th><th>Column 2 rewritten in exponential form.</th><th>Test your shortcut and compare.</th></tr><tr><td>$5^3 \cdot 5^2$</td><td></td><td></td><td></td></tr><tr><td>$b^6 \cdot b^2$</td><td></td><td></td><td></td></tr><tr><td>$y^2 y^{10} y^4$</td><td></td><td></td><td></td></tr></table> Explain how to multiply exponents with the same base without using expanded form. Algebraic Rule for the Product of Powers(explain your rule using symbols): <i>Throughout section 8.1 students will expand expressions repeatedly in order to discover the rules and properties of integer exponents.</i>	Column 1	Column 1 rewritten in expanded form or as repeated multiplication.	Column 2 rewritten in exponential form.	Test your shortcut and compare.	$5^3 \cdot 5^2$				$b^6 \cdot b^2$				$y^2 y^{10} y^4$			
Column 1	Column 1 rewritten in expanded form or as repeated multiplication.	Column 2 rewritten in exponential form.	Test your shortcut and compare.															
$5^3 \cdot 5^2$																		
$b^6 \cdot b^2$																		
$y^2 y^{10} y^4$																		
	Use appropriate tools strategically.	You are asked to enter the following expression into your scientific calculator. $\left(\frac{2}{3}\right)^4$ Which of the following is not a correct way to enter the expression into a scientific calculator? <table><tr><td>a. 2 ÷ 3 y^x 4</td><td>b. (2 ÷ 3) y^x 4</td><td>c. 2 y^x 4 ÷ 3 y^x 4</td></tr></table> <i>Using a calculator appropriately is an important component of this chapter. Students often mistake how to enter exponents into the calculator correctly. They must also learn how to enter and interpret a number expressed in scientific notation on a calculator.</i>	a. 2 ÷ 3 y^x 4	b. (2 ÷ 3) y^x 4	c. 2 y^x 4 ÷ 3 y^x 4													
a. 2 ÷ 3 y^x 4	b. (2 ÷ 3) y^x 4	c. 2 y^x 4 ÷ 3 y^x 4																
	Make sense of problems and persevere in solving them.	<i>Gas 'N' Go Convenience Stores</i> claim that 10% of Utahans fuel up at their stores each week. Decide whether their claim is true using the following information. - There are about 2.85×10^6 people in Utah. - There are 2.18×10^2 <i>Gas 'N' Go</i> stores in Utah. - Each stations serves gasoline to about 1.2×10^3 people each week. <i>There are several questions in this chapter where students are given the opportunity to problem solve. In the example above students must find an entry point into the problem by determining that they need to figure out what 10% of the Utah population is. They must analyze what is given to them and how they can use it to reach their intended goal. In addition to reaching an outcome they can analyze their solution to see if it makes sense given the context.</i>																



Construct viable arguments and critique the reasoning of others.

Edward, Pattie, and Mitch were each simplifying the same exponential expression. Their work is shown below. Determine who has simplified the expression correctly. If they did not simplify the expression correctly, identify the mistake, explain it, and fix it.

Edward

$$\left(\frac{3x^4}{6x} \right)^2 = \left(\frac{2x^4}{x} \right)^2 = (2x^3)^2 = 2x^6$$

Pattie

$$\left(\frac{3x^4}{6x} \right)^2 = \frac{3^2 \cdot x^8}{6^2 \cdot x^2} = \frac{9x^8}{36x^2} = \frac{x^6}{4}$$

Mitch

$$\left(\frac{3x^4}{6x} \right)^2 = \left(\frac{x^4}{2x} \right)^2 = \left(\frac{x^3}{2} \right)^2 = \frac{x^6}{4}$$

In the problem above students must critique the work of others. As they do so they solidify their understanding of different exponent properties. They also analyze common misconceptions and mistakes that are made when simplifying expressions with exponents. As they examine two problems that are correct but simplified differently they see that it is possible to arrive at the same answer in a variety of ways. This enhances their understanding of the structure of these expressions and how they are composed.



Model with mathematics.

A silo is a storage bin that is a cylinder with a hemisphere on top. A farmer has a silo with a base radius of 30 feet and a storage height of 100 feet. The “storage height” is the part which can be filled with grain - it is just the cylinder. A cubic foot of grain weighs 62 lbs.

- Draw and label a picture of the silo.
- How many pounds of grain can the farmer store in the silo?
- How high (including the hemispherical top) is the silo?
- 1000 square feet of wheat produces 250 pounds of grain. The farmer’s wheat field is 3,500 ft by 20,000 ft. Is the silo large enough to hold the grain? By how much? Explain your answer.
- If the farmer decides to fill the silo all the way to the top of the hemisphere how many cubic feet of grain can he store?

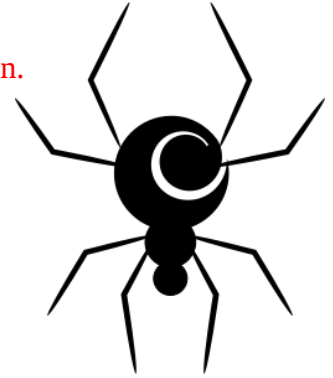
While working with volume students use geometry to model a variety of situations. In the problem above, a silo is modeled with a hemisphere and cylinder. Student’s use this geometric model to answer questions about the silo. They use a mathematical formula to model the volume of the silo to determine how much grain it will hold and the height of the silo.

8.0 Anchor Problem: Spiders



1. The genetically altered spider that turned Peter Parker into Spider Man with a single bite was about 0.035 ounces. If Spider Man weighs roughly 185 pounds how many spiders does it take to have the same mass as Spider Man?

It would take roughly 84,600 spiders in order to equal the mass of Spider Man.



2. Spider Man fights evil villains in New York City. The size of New York City is roughly 1,214,450,000 square meters. It is estimated that on average there are approximately 1,308 spiders per square meter of land. Use this information to determine how many spiders are in New York City?

There are approximately 1,589,000,000,000 spiders in New York City.

Bonus: Determine how many spiders there are in your city or even a bedroom in your basement.

This task is intended to introduce students to very large and very small numbers. In this chapter students will learn how to use exponents to express numbers in scientific notation. At this point let students try to answer the questions using any method they can. After doing section 8.2, where students will learn how to express a number in scientific notation, return to this task and have them answer the questions using scientific notation. They will see how using scientific notation as a tool when appropriate can help them to better make sense of and calculate quantities with ease and efficiency.

Section 8.1: Integer Exponents

Section Overview:

This section begins with an overview of the structure of exponents and how an exponential expression represents repeated multiplication as opposed to expressions that represent repeated addition. Using the structure of an exponential expression special properties or rules are discovered in this section. These exponent properties and rules aid in simplifying exponential expressions. Students will informally prove why an exponent of zero equals one and also look at the definition of negative exponents, that is, $x^{-1} = \frac{1}{x}$. Once students become familiar with these properties and rules they use them to simplify more complex exponential expressions.

Concepts and Skills to Master:

By the end of this section students should be able to:

1. Apply the properties of integer exponents to simplify algebraic and numerical expressions.

Be sure to reference the teacher notes for section 8.1. They will give you important guidelines as to how group and class discussions will aid in the discovery of exponent properties. The “Find, Fix, and Justify” activities are meant to help students solidify the properties of exponents by reasoning about how algebraic expressions are expanded and can be simplified through basic algebraic properties. As they look for mistakes in algebraic reasoning and argue or justify their thinking a deeper understanding of how the expressions are constructed emerges and solidifies how expanding an expression can lead to a shortcut or rule.

8.1a Class Activity: Get Rich Quick

Mario and Tony both want you to come and drive Go-Karts for their team. They will pay you in gold coins. Each one makes an offer:

Mario: I will give you 3 gold coins on the first day. Then, every day after that, I will pay you 3 times as much as I paid you the day before.

Tony: I will give you 3 gold coins on the first day. Then, every day after that, I will pay you 3 more coins than I paid you the day before.

1. Who would you rather work for? Use the table below to help you decide.

Mario's Deal	Daily Wage	Tony's Deal	Daily Wage
Monday	$3 = 3$	Monday	$3 = 1(3)$
Tuesday	$9 = 3 \cdot 3 = 3^2$	Tuesday	$6 = 3 + 3 = 2(3)$
Wednesday	$27 = 3 \cdot 3 \cdot 3 = 3^3$	Wednesday	$9 = 3 + 3 + 3 = 3(3)$
Thursday	$81 = 3 \cdot 3 \cdot 3 \cdot 3 = 3^4$	Thursday	$12 = 3 + 3 + 3 + 3 = 4(3)$
Friday	$243 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$	Friday	$15 = 3 + 3 + 3 + 3 + 3 = 5(3)$
Total Earnings	$3 + 9 + 27 + 81 + 243 = 363$	Total Earnings	$3 + 6 + 9 + 12 + 15 = 45$

2. For whom would you rather work and why?

Mario is the better deal, you will make more money.

3. Rewrite your earnings for each day as repeated multiplication or repeated addition.

See above.



4. You have used exponents previously to represent whole numbers in expanded form as powers of ten. Complete the following table to remind yourself how exponents are used. The first couple of rows have been done for you.

10^1	means	10	which is equal to	10
10^2	means	$10 \cdot 10$	which is equal to	100
10^3	means	$10 \cdot 10 \cdot 10$	which is equal to	1,000
10^5	means	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	which is equal to	10,000
10^8	means	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	which is equal to	100,000,000
10^{10}	means	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	which is equal to	10,000,000,000

5. Now rewrite your earnings for each day using exponents or multiplication. Be ready to discuss the effect that an exponent has on a number; think about the difference between repeated multiplication and repeated addition. **See above.**

Lead the following discussion with your class.

You know from previous grades that when you **add** the number 2 to itself 5 times, you can use multiplication to write this in an abbreviated form. $2 + 2 + 2 + 2 + 2 = 5 \times 2$

You can use exponents to help you to know how many times you must **multiply** the base number by itself. 2^5 means to multiply the number 2 by itself 5 times. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

It is important for students to understand the difference between repeated multiplication and repeated addition. Show an example such as, $x^5 = x \cdot x \cdot x \cdot x \cdot x$ and $5x = x + x + x + x + x$. Point out how x^5 shows “multiply 5 x’s” and how $5x$ shows “add five x’s.”

Also review the parts of an exponent. The number that is a power is called the *exponent*. It indicates how many times the base number is being multiplied. The *base number* is the number that is being multiplied. In the example given below 2 is the *base number* and 5 is the *exponent*.

$$\begin{array}{ccc} & \nearrow 2^5 \nwarrow & \\ & \text{Exponent} & \\ \text{Base number} & & \end{array}$$

In general, for any number x , and any whole number n ,

$$x^n = \underbrace{(x \cdot x \cdot x \cdots x)}_{n \text{ times}}$$

The expression x^n is read x raised to the n th power. In this expression n is the exponent and x is the base number.

6. Write each expression given below in exponential form.

a. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 6^5$	b. $\frac{9}{7} \cdot \frac{9}{7} \cdot \frac{9}{7} = \left(\frac{9}{7}\right)^3$	c. $\underbrace{(4 \cdot 4 \cdot 4 \cdots 4)}_{15 \text{ times}} = 4^{15}$
d. $(-2)(-2)(-2)(-2) = (-2)^4$	e. $\left(\frac{-2}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{-2}{3}\right) = \left(\frac{-2}{3}\right)^3$	f. $x \cdot x \cdot x \cdot x \cdot x \cdot x = x^6$
g. $\frac{1}{x} \cdot \frac{1}{x} \cdot \frac{1}{x} \cdot \frac{1}{x} \cdot \frac{1}{x} \cdot \frac{1}{x} \cdot \frac{1}{x} = \left(\frac{1}{x}\right)^7$	h. $2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 2^2 \cdot 3^3$	i. $11 \cdot 11 \cdot 11 \cdot x \cdot x = 11^3 \cdot x^2 = 1,331x^2$
j. $x \cdot x \cdot x \cdot y = x^3y$	k. $a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot c \cdot c = a^3b^3c^2$	l. $\underbrace{(r \cdot r \cdot r \cdots r)}_{20 \text{ times}} \underbrace{(q \cdot q \cdot q \cdots q)}_{9 \text{ times}} = r^{20}q^9$

7. Examine problem *j.* above. What exponent does the variable y have? Do you have to write the exponent?

The exponent for the variable y is 1, you do not have to write the exponent of 1.

Explain to students that we do not need to write the "1" for an exponent of x just like we do not need to write that x has a coefficient of 1. $x = 1x^1$

8. Notice the use of parentheses in problems *d.* and *e.* above. Why do you think they are they used?
The parentheses are used to differentiate between a negative sign and a subtraction sign.

9. Evaluate each exponential expression by first re-writing it using multiplication.

$(-3)^2$	means	$(-3)(-3)$	which is equal to	9
$(-3)^3$	means	$(-3)(-3)(-3)$	which is equal to	-27
$(-3)^4$	means	$(-3)(-3)(-3)(-3)$	which is equal to	81
$(-3)^5$	means	$(-3)(-3)(-3)(-3)(-3)$	which is equal to	-243
$(-3)^6$	means	$(-3)(-3)(-3)(-3)(-3)(-3)$	which is equal to	363
$(-3)^7$	means	$(-3)(-3)(-3)(-3)(-3)(-3)(-3)$	which is equal to	-2,187

10. Describe one pattern that you notice in the table above.

If the exponent is even the answer is positive. If the exponent is odd then the answer is negative.

11. Examine the pairs of expressions given below; discuss the similarities and differences between them.

$$(-4)^2 \text{ and } -4^2$$

Students might bring up some of the following ideas. The only thing that is different about these expressions are the parentheses. The parentheses are important because they indicate that there are two copies of negative 4. If you expand this expression you get $(-4)(-4) = 16$. The expression -4^2 indicates that there is a coefficient of negative 1. Upon expansion you get $-4^2 = (-1)(4)(4) = (-1)(16) = -16$. You will get different answers even though the expressions are similar.

$$(-4)^3 \text{ and } -4^3$$

In this case both expressions equal -64 . This is because of the odd exponent.

Emphasize how students must attend to precision when dealing with negative numbers and exponents.

March Madness, the NCAA basketball tournament, has the form of a single-elimination tournament. In such a tournament, we start with a certain number of teams, and we pair them off into games; each team plays a game. This is called the first round. All the losers in the first round are eliminated; in the second round all the winning teams are paired off into games, and all the second round losers are eliminated. This process continues until only two teams remain; this is the final round and the winner is the champion of the tournament.

For the March Madness problem consider showing students a bracket for the current year's tournament and explain to them the following.

Since there are two teams in the final round, there had to be four teams in the semifinal round, and thus eight teams in the preceding round and so forth. So, for a single elimination tournament to work, with no teams ever idle, that we start with a number of teams that is a power of two, and that exponent is the number of rounds. So, for example, if we start with 16 teams, since $16 = 2^4$, there are 4 rounds and $8 + 4 + 2 + 1 = 15$ games.

12. March Madness starts with 64 teams. How many rounds are there?

Since $64 = 2^6$, there are six rounds.



13. How many teams are in the second round? In any round?

Each round eliminates half the remaining teams, so there are 32 teams in the second round, 16 in the third, and so forth.

14. How many games total are played? There are $32 + 16 + 8 + 4 + 2 + 1 = 63$ games. Another way of counting is that there are 63 teams that are NOT champions, and each game produces one non-champion.

8.1a Homework: Get Rich Quick

1. Write each expression in exponential form

a. $\underbrace{(5 \cdot 5 \cdot 5 \cdots 5)}_{17 \text{ times}} = 5^{17}$

b. $(-4)(-4)(-4)(-4) = (-4)^4$

c. $(3.7)(3.7)(3.7) = (3.7)^3$

d. $\frac{5}{8} \cdot \frac{5}{8} \cdot \frac{5}{8} \cdot \frac{5}{8} \cdot \frac{5}{8} \cdot \frac{5}{8} = \left(\frac{5}{8}\right)^6$

e. $x \cdot x \cdot x \cdot x \cdot y \cdot y = x^4 y^2$

f. $3 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot b = 3a^5 b$

2. Upon taking a VERY good job, Manuel is given one of the following two options for his retirement plan.

Option A: \$10 the first year, then every year after that you will get 10 times as much as the year before.

Option B: \$100,000 the first year and then every year after that you will get \$100,000 more than the year before.

- a. What option should he choose? Justify your answer? Use the table to decide.

It depends on how long Manuel plans on working for the company. If he plans on working there for 5 years or less it would be better to go with Option B. But if he plans on working there for more than 5 years it would be smarter for Manuel to choose Option A. This plan is showing exponential growth and repeated multiplication. Even though he initially does not get very much with Option A the money grows very quickly.

Option A	Yearly Retirement	Option B	Yearly Retirement
Year 1	$10 = 10^1$	Year 1	100,000
Year 2	$100 = 10 \cdot 10 = 10^2$	Year 2	$200,000 = 100,000 + 100,000$
Year 3	$1,000 = 10 \cdot 10 \cdot 10 = 10^3$	Year 3	$300,000 = 100,000 + 100,000 + 100,000$
Year 4	$10,000 = 10 \cdot 10 \cdot 10 \cdot 10 = 10^4$	Year 4	$400,000 = \underbrace{100,000 + \dots + 100,000}_{4 \text{ times}}$
Year 5	$100,000 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5$	Year 5	$500,000 = \underbrace{100,000 + \dots + 100,000}_{5 \text{ times}}$
Year 6	$1,000,000 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^6$	Year 6	$600,000 = \underbrace{100,000 + \dots + 100,000}_{6 \text{ times}}$

- b. Write each yearly retirement amount as repeated multiplication or repeated addition.

See table above.

- c. Write each yearly retirement amount in exponential form for Option A.

See table above.

- d. Extension: Write an equation or function that could be used to find the amount of retirement Manuel would receive for any given year for Option A or Option B

Option A: $y = 10^x$, where x is the numbers of years and y is the amount of money given in retirement that year.

Option B: $y = 100,000x$, where x is the numbers of years and y is the amount of money given in retirement that year.

3. Write an exponential expression with (-1) as its base that will result in a positive product.

Answers will vary; the expression must have an even exponent.

4. Write an exponential expression with (-1) as its base that will result in a negative product.

Answers will vary; the expression must have an odd exponent.

5. Rewrite each number in exponential form, with the number two as its base.

a. $8 = 2^3$

b. $32 = 2^5$

c. $128 = 2^7$

6. Pablo wrote $(-2)^5 = -32$. Is he correct; why or why not?

Pablo is correct because, $(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$.

7. Chantal wrote $-6^2 = 36$. Is she correct; why or why not?

Chantal is not correct because, $-6^2 = (-1) \cdot 6^2 = (-1) \cdot 36 = -36$.

A candy maker is making taffy. He starts with one long piece of taffy and cuts it into 3 pieces. He then takes each resulting piece and cuts it into three pieces. He then takes each of these resulting pieces and cuts it into three pieces. He continues this process.

8. Use exponents to represent the number of pieces of taffy the candy maker has after the first 4 rounds of cuts.

$3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81$

It is interesting to point out that as the powers of 3 are generated a pattern emerges among the last digit produced. The numbers 3,9,7,1 repeat themselves. This is similar to the repetitions that take place when identifying terminating and repeating decimals.

$3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243, 3^6 = 729$

9. How many pieces of taffy will the candy maker have after 8 rounds of cuts?

$3^8 = 6,561$. The candy maker will have 6,561 pieces of taffy after 8 rounds of cuts.

10. The candy maker gets a special order for 243 pieces of peppermint flavored taffy. How many rounds of cuts will he have to make to get this many pieces?

The candy maker will have to make 5 rounds of cuts to get 243 pieces of taffy.

8.1b Class Activity: Find, Fix, and Justify and Exponent Properties

Part 1: Find, Fix, and Justify



The following statements are **incorrect**. For each of the statements do the following:

- **Find** the mistake(s) in each statement.
- **Fix** the mistake.
- **Justify** your reasoning. You may use pictures if needed.

1. In the expression below; 4 is called the base number and 5 is called the exponent.
- $$5^4$$

This statement is incorrect because 5 is the base number and 4 is the exponent or power.

Base number $\swarrow$ x^a $\nwarrow$ Exponent

Review with students the parts of an exponential expression. The exponent indicates the number of times that the base is multiplied by itself.

2. $2^6 = 12$

$$2^6 \neq 12$$

$$2^6 = 64$$

This is true because,

$$2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64.$$

Clear up the misconception shown in the problem by reviewing with students that $2 \times 6 = 2 + 2 + 2 + 2 + 2 + 2 = 12$. Point out the differences between the two expressions. Also, point out similarities 2×6 means add six twos and 2^6 means multiply six twos.

3. $2^5 = 5^2$

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32 \text{ and}$$

$$5^2 = 5 \cdot 5 = 25. \text{ Since } 32 \neq 25 \text{ then } 2^5 \neq 5^2.$$

This is another example where you can discuss the difference between the exponent and base number.

4. $3x = x^3$

$$3x = x + x + x \text{ and } x^3 = x \cdot x \cdot x, \text{ therefore } 3x \neq x^3.$$

Reemphasize the difference between repeated multiplication and repeated addition. Confusion may arise here if you look at this statement as an equation to solve rather than an identity. The equation is true if $x = \sqrt{3}$, $-\sqrt{3}$, or 0. If you think of this geometrically, the first expression represents a rectangle with a length of 3 and a width of x ; whereas the second expression represents the volume of a cube with side length x .

5. $(-2)^3 = 8$

$$(-2)^3 = (-2)(-2)(-2)$$

$$= 4(-2)$$

$$= -8$$

$$\text{Thus } (-2)^3 = -8.$$

Be sure to discuss how an exponent affects a negative number. Recall how they can determine the sign of the final outcome by noting whether the exponent is even or odd.

6. $-7^2 = 49$

$$-7^2 = (-1)(7)(7) = -49 \text{ or}$$

$$-7^2 = (-7)(7) = -49$$

Discuss with students how this example differs from number 5. The notation is important here and they must *attend to precision*.

As each problem is discussed with the class, make changes or add notes to your work above if needed. Use the space below to write down important notes about exponents.

One way of implementing this lesson in your class is to give students about 5 minutes to work on these six problems. When they are finished have them discuss their answers with a neighbor before moving on to a group discussion. Ask for people to come to the board to show and justify how they fixed the mistake in each statement. Lead a class discussion for each statement, discuss the multiple representations for these statements such as repeated multiplication and clear up misconceptions.

You can add in the optional statement; $x = x^0$. This can generate a lot of discussion that you may not want to get into at this point, however it would be good to discuss that this statement is incorrect because a variable with no power listed can be assumed to have a power of 1. It is a common mistake for students to assume that it has a power of 0. A discussion around an exponent of 0 will come later on in Section 8.1e.

Part 2: Exponent Properties

You are going to further investigate expressions with exponents by combining them through multiplication and division. There are special properties that help to transform exponential expressions with a shortcut; they are called Exponent Properties or Rules. The problems given below show the special properties that hold true for all exponential expressions.



Complete columns 2 and 3 in the table below by writing each expression in expanded form and in exponential form. The first one has been done for you. As you are simplifying try to discover a shortcut or rule that you could use to simplify more quickly. Test your shortcut in the last column and compare your answers.

Product of Powers

Column 1	Column 1 rewritten in expanded form or as repeated multiplication.	Column 2 rewritten in exponential form.	Test your shortcut and compare.
$5^3 \cdot 5^2$	$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$	$5^5 = 3125$	$5^3 \cdot 5^2 = 5^{3+2} = 5^5 = 3125$
$b^6 \cdot b^2$	$b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b$	b^8	$b^6 \cdot b^2 = b^{6+2} = b^8$
$y^2 y^{10} y^4$	$(y \cdot y)(y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y)(y \cdot y \cdot y \cdot y)$	y^{16}	$y^2 y^{10} y^4 = y^{2+10+4} = y^{16}$
$3x^2 y \cdot 2x^2 y^2$	$3(x \cdot x)(y) \cdot 2(x \cdot x)(y \cdot y)$	$6x^4 y^3$	$3x^2 y \cdot 2x^2 y^2 = 3 \cdot 2x^{2+2} y^{1+2} = 6x^4 y^3$

Explain how to multiply exponents with the same base without using expanding.

To multiply exponents with the same base, add the exponents, keep the base number the same.

Algebraic Rule for the Product of Powers (explain your rule using symbols):

$$x^a \cdot x^b = x^{a+b}$$

You can have students work on the tables independently, with a partner, or in a group. Be sure to formalize the rules together as a class.

Simplify each expression:

a. $x^5 x^3 = x^8$

b. $x^{40} x^3 = x^{43}$

c. $a^3 b^2 \cdot a^6 b^3 = a^9 b^5$

d. Make up your own problem that requires the Product of Powers to simplify.

See student answer.

Discuss problem e. below. Some students might find it helpful to get into the habit of adding an exponent of 1 to all numbers/variables that do not show an exponent before beginning to simplify the expression. Also talk about the role of the coefficients and how the commutative property allows you to simply multiply them together.

Remind students how the parentheses () and the multiplication dot (·) indicate the same operation.

You might want to ask “Why do you think this property is called the Product of Powers?”

e. How would you simplify $a \cdot a^3$. A number that does not show any exponent has an exponent of 1; thus $a \cdot a^3 = a^1 \cdot a^3 = a^{1+3} = a^4$.

Try these problems:

f. $ab^5 \cdot 8a^2 b = 8a^3 b^6$

g. $(2xy)(4x^2 y^3 z) = 8x^3 y^4 z$

The specific names of the rules do not need to be emphasized. They are given to aid in communicating about the rules. More important than the naming of the properties is that students are able to examine and interpret the structure of these expressions in order to write an equivalent expression.

Quotient of Powers

	Column 1 rewritten in expanded form or as repeated multiplication.	Column 2 rewritten in exponential form.	Test your shortcut and compare.
$\frac{3^5}{3^2}$	$\frac{\cancel{3} \cdot \cancel{3} \cdot 3 \cdot 3 \cdot 3}{\cancel{3} \cdot \cancel{3}} = \frac{3 \cdot 3 \cdot 3}{1}$	3^3	$\frac{3^5}{3^2} = 3^{5-2} = 3^3 = 27$
$\frac{a^4}{a^3}$	$\frac{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot a}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a}} = \frac{a}{1}$	a	$\frac{a^4}{a^3} = a^{4-3} = a^1 = a$
$\frac{b^3}{b^5}$	$\frac{\cancel{b} \cdot \cancel{b} \cdot \cancel{b}}{\cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot b \cdot b} = \frac{1}{b \cdot b}$	$\frac{1}{b^2}$	$\frac{b^3}{b^5} = b^{3-5} = b^{-2}$
$\frac{4^5 \cdot x^7}{4^2 \cdot x^4}$	$\frac{(\cancel{4} \cdot \cancel{4} \cdot 4 \cdot 4 \cdot 4)(\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x \cdot x \cdot x)}{(\cancel{4} \cdot \cancel{4})(\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x)} = \frac{4 \cdot 4 \cdot 4 \cdot x \cdot x \cdot x}{1}$	$4^3 x^3 = 64x^3$	$\frac{4^5 \cdot x^7}{4^2 \cdot x^4} = 4^{5-2} \cdot x^{7-4} = 4^3 \cdot x^3 = 64x^3$

Explain how to divide exponents with the same base without expanding.

To divide exponents with the same base subtract the exponents and keep the base number the same.

Algebraic Rule for the Quotient of Powers:

$$\frac{a^m}{a^p} = a^{m-p}$$

Students may want to talk about the negative exponent. In example *d*, you can certainly start having conversations about negative exponents and talk about how the expression can also

Simplify each expression:

a. $\frac{6c^5}{c^2}$ $6c^3$

b. $\frac{a^7 b^{10}}{ab^3 c^2}$ $\frac{a^6 b^7}{c^2}$

c. $\frac{x^{20} y^{15}}{x^{15} y^{14}}$ $x^5 y$

d. $\frac{6x^2}{2x^4}$ $3x^{-2}$

e. $\frac{8s^{12} t^4 u^3}{2s^{10} t^4 u}$ $4s^2 u^2$

f. $\frac{2m^2 n^5}{8m^5 n^2}$ $\frac{m^{-3} n^3}{4}$

be simplified to $\frac{3}{x^2}$. It is true that in order to be truly simplified the expression should not contain a negative exponent but until negative exponents are addressed in section 8.1e they can leave the expression with a negative exponent.

As students write out the expanded form of the expression remind them that when you have the same value on the top and bottom of the fraction they reduce to one.

You may want to ask, “Why do you think that this rule is called the Quotient of Powers Property?”

Emphasize with students that on problem *d*, a common mistake is to try and subtract the 6 and 2. Remind students that these coefficients are not exponents and should be simplified through division and not subtraction.

8.1b Homework: Product of Powers and Quotient of Powers Properties

1. For parts a. through c. write each expression in expanded form. Then write the simplified expression in exponential form.

Product of Powers

#	Expression	Expanded Form	Exponential Form
a.	x^4x^6	$(x \cdot x \cdot x \cdot x)(x \cdot x \cdot x \cdot x \cdot x \cdot x)$	x^{10}
b.	$a^3b^2 \cdot a^3b^5$	$(a \cdot a \cdot a \cdot b \cdot b)(a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b)$	a^6b^7
c.	$(3abc)(2a^2b)$	$(3 \cdot a \cdot b \cdot c)(2 \cdot a \cdot a \cdot b)$	$6a^3b^2c$

Simplify each expression.

d. x^6x^{12} x^{18}	e. $y^{50}y^{200}$ y^{250}	f. $a^{13}b^5 \cdot a^3b^{20}$ $a^{16}b^{25}$	g. $5a^6 \cdot -4ab^7$ $-20a^7b^7$	h. $3y^2 \cdot x^4$ $3x^4y^2$
If you do not get through the Quotient of Powers rule in class be selective in the problems that you assign for homework. You may have to save some of the problems for the following day.				

2. For parts a. through c. write each expression in expanded form. Then write the simplified expression in exponential form.

Quotient of Powers

#	Expression	Expanded Form	Exponential Form
a.	$\frac{4^3}{4^2}$	$\frac{\cancel{4} \cdot \cancel{4} \cdot 4}{\cancel{4} \cdot \cancel{4}} = \frac{4}{1}$	$4^1 = 4$
b.	$\frac{x^2}{x^6}$	$\frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x^4}$	$\frac{1}{x^4} = x^{-4}$
c.	$\frac{a^4b^7}{a^3b^6}$	$\frac{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot a \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot b}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b}} = \frac{ab}{1}$	ab

Simplify each expression.

d. $\frac{b^{20}}{b^5}$ b^{15}	e. $\frac{c^{200}}{c^{146}}$ c^{54}	f. $\frac{p^{10}r^{20}}{p^3r^{10}}$ p^7r^{10}	g. $\frac{s^2t^3}{t^7}$ s^2t^{-4}	h. $\frac{5^4a^4b^2}{5^3ab^2}$ 5^1a^3
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3. Simplify each expression.

Mixed Practice

a. $(2xy^2)(4x^2y)$ $8x^3y^3$	b. $(-4x^2t^3)(-6r^5x^2t)$ $24r^5x^4t^4$	c. $\frac{yz^2 \cdot y^3z}{x^2yz}$ $\frac{y^3z^2}{x^2}$	d. $3x^2\left(\frac{1}{2}y^3\right)$ $\frac{3}{2}x^2y^3$
e. $\frac{15x^3}{5x^3}$ 3	f. $\frac{x^4 \cdot x^3}{x^9}$ x^{-2}	g. $\frac{-ab^5a^4}{a^3bc^5}$ $\frac{-a^2b^4}{c^5}$	h. $\frac{4^2x^2y^4x}{x^2} \cdot \frac{1}{2y}$ $8y^3x$

8.1c Class Activity: Power of a Power, Power of a Product, and Power of a Quotient



Complete columns 2 and 3 in the table below by writing each expression in expanded form and in exponential form. As you are simplifying try to discover a shortcut or rule that you could use to simplify more quickly. Test your shortcut in the last column and compare your answers.

Power of a Power

	Column 1 rewritten in expanded form or as repeated multiplication.	Column 2 rewritten in exponential form.	Test your shortcut and compare.
$(3^3)^2$	$(3^3)(3^3) = (3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3)$	3^6	$3^{3 \cdot 2} = 3^6$
$(a^2)^4$	$(a^2)(a^2)(a^2)(a^2) = (a \cdot a)(a \cdot a)(a \cdot a)(a \cdot a)$	a^8	$a^{2 \cdot 4} = a^8$
$[(b^3)^2]^2$	$(b^3)^2 \cdot (b^3)^2 = (b^3)(b^3)(b^3)(b^3) = (b \cdot b \cdot b)(b \cdot b \cdot b)(b \cdot b \cdot b)(b \cdot b \cdot b)$	b^{12}	$b^{3 \cdot 2 \cdot 2} = b^{12}$
$[(x^4)^{10}]^5$	This is too long to expand, find a short cut.		$x^{4 \cdot 10 \cdot 5} = x^{200}$

Explain how to find the power of a power without expanding.

To find the power of a power multiply the exponents and keep the base number the same.

Algebraic Rule for the Power of a Power:

$$(a^m)^p = a^{m \cdot p}$$

Simplify each expression:

a. $(a^3)^5 = a^{15}$

b. $(b^7)^{11} = b^{77}$

c. $[(c^5)^6]^7 = c^{210}$

d. $[(-d^2)^{10}]^3 = (-d)^{60}$

e. Explain the difference between $(x^2)^3$ and $(x^2)(x^3)$.

The expression $(x^2)^3$ represents x^2 multiplied by itself 3 times. It simplifies to x^6 . The expression $(x^2)(x^3)$ represents x^2 times x^3 . It simplifies to x^5 .

Explain to your students that there are several different expansions for the expressions above. It would be helpful to show multiple expansions to see that you get the same simplified expression with all methods.

Think of a way to remember when to add the exponents and when to multiply them.

Power of a Product

	Column 1 rewritten in expanded form or as repeated multiplication.	Column 2 rewritten in exponential form.	Test your shortcut and compare.
$(ab)^4$	$(ab)(ab)(ab)(ab) = a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b$	a^4b^4	$(ab)^4 = a^{1 \cdot 4}b^{1 \cdot 4}$
$(abc)^3$	$(abc)(abc)(abc) = a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot c \cdot c \cdot c$	$a^3b^3c^3$	$(abc)^3 = a^{1 \cdot 3}b^{1 \cdot 3}c^{1 \cdot 3}$
$(a^2b)^3$	$(a^2b)(a^2b)(a^2b) = a^2 \cdot a^2 \cdot a^2 \cdot b \cdot b \cdot b$	a^6b^3	$(a^2b)^3 = a^{2 \cdot 3}b^{1 \cdot 3} = a^6b^3$
$(4xy^2)^4$	$(4xy^2)(4xy^2)(4xy^2)(4xy^2)$	$4^4x^4y^8$	$4^{1 \cdot 4}x^{1 \cdot 4}y^{2 \cdot 4} = 4^4x^4y^8$ $= 256x^4y^8$
$(3^2a^{10}b^{11})^4$	This is too long to expand, find a short cut.		$3^{2 \cdot 4}a^{10 \cdot 4}b^{11 \cdot 4} = 3^8a^{40}b^{44}$ $= 6,561a^{40}b^{44}$

Explain how to find the power of a product without expanding.

To find the power of a product find the power of each factor and multiply.

Algebraic Rule for the Power of a Product:

$$(ab)^m = a^mb^m$$

Simplify each expression:

a. $(ab)^5 = a^5b^5$

b. $(ab^3)^6 = a^6b^{18}$

c. $(2y^2z)^3 = 2^3y^6z^3 = 8y^6z^3$

d. $(3^3x^2yz^2)^4 = 3^{12}x^8y^4z^8 = 531,441x^8y^4z^8$

e. Create two different expressions that simplify to 4^6 .

Answers may include $(4^2)^3 = 4^6$.
 $4^3 \cdot 4^3 = 4^6$

f. Explain the difference between finding $(xy)^2$ and $(x + y)^2$. Use the values $x = 2$ and $y = 3$ in your explanation. Upon expanding each expression you get $(xy)(xy)$ and $(x + y)(x + y)$. If you substitute $x = 2$ and $y = 3$ into each expression you get $(2 \cdot 3)(2 \cdot 3) = 6 \cdot 6 = 36$ and $(2 + 3)(2 + 3) = 5 \cdot 5 = 25$

g. Which value is greater, $(x^1)(x^4)$ or x^5 ? Explain. They are equal because $(x^1)(x^4) = x^{1+4} = x^5$.

Power of a Quotient

	Column 1 rewritten in expanded form or as repeated multiplication.	Column 2 rewritten in exponential form.	Test your shortcut and compare.
$\left(\frac{a}{b}\right)^3$	$\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right) = \frac{a \cdot a \cdot a}{b \cdot b \cdot b}$	$\frac{a^3}{b^3}$	$\left(\frac{a}{b}\right)^3 = \frac{a^{1 \cdot 3}}{b^{1 \cdot 3}} = \frac{a^3}{b^3}$
$\left(\frac{x}{5}\right)^2$	$\left(\frac{x}{5}\right)\left(\frac{x}{5}\right) = \frac{x \cdot x}{5 \cdot 5}$	$\frac{x^2}{5^2}$	$\left(\frac{x}{5}\right)^2 = \frac{x^{1 \cdot 2}}{5^{1 \cdot 2}} = \frac{a^2}{5^2} = \frac{a^2}{25}$
$\left(\frac{xy}{z}\right)^4$	$\left(\frac{xy}{z}\right)\left(\frac{xy}{z}\right)\left(\frac{xy}{z}\right)\left(\frac{xy}{z}\right) = \frac{x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y}{z \cdot z \cdot z \cdot z}$	$\frac{x^4 y^4}{z^4}$	$\left(\frac{xy}{z}\right)^4 = \frac{x^{1 \cdot 4} y^{1 \cdot 4}}{z^{1 \cdot 4}} = \frac{x^4 y^4}{z^4}$
$\left(\frac{a^3}{b^5}\right)^{10}$	<i>This is too long to expand, find a short cut.</i>		$\left(\frac{a^3}{b^5}\right)^{10} = \frac{a^{3 \cdot 10}}{b^{5 \cdot 10}} = \frac{a^{30}}{b^{50}}$

Explain how to find the power of a quotient without expanding.

To find the power of a quotient, find the power of the numerator and the power of the denominator.

Algebraic Rule for the Power of a Quotient:

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Simplify each expression:

a. $\left(\frac{x}{y}\right)^6 = \frac{x^6}{y^6}$

b. $\left(\frac{5}{x^4}\right)^2 = \frac{5^2}{x^{4 \cdot 2}} = \frac{25}{x^8}$

c. $\left(\frac{4x^2}{y}\right)^6 = \frac{4^6 x^{2 \cdot 6}}{y^6} = \frac{4096x^{12}}{y^6}$

d. $\left(\frac{2^2 x^3 y^2}{x}\right)^2 = \frac{2^{2 \cdot 2} x^{3 \cdot 2} y^{2 \cdot 2}}{x^2} = \frac{16x^6 y^4}{x^2} = 16x^5 y^4$

e. $\left(\frac{3a^5 \cdot 2a^4}{4a^3}\right)^5 = \frac{3^5 a^{5 \cdot 5} \cdot 2^5 a^{4 \cdot 5}}{4^5 a^{3 \cdot 5}} = \frac{243a^{30}}{32}$

On practice problem d. let students show that there are multiple ways to simplify the expression. Tell students that it might be helpful to write in an exponent of 1 for variables that do not have exponents. This will remind them to “distribute” the exponent outside of the parentheses to every number inside the parentheses. It is also helpful for some students to always work from the inside of the parentheses to the outside. This means to simplify everything they can inside the parentheses and then “distribute” the exponent that is outside the parentheses into the simplified expression.

You can talk about how the Power of Power property and the Power of a Quotient property really are the same thing.

You may not get through this activity in one class period. Assign the problems from the aligned homework assignment based on how many properties you are able to cover in class. Then assign the remaining problems the following day. Also be sure to summarize the rules together as a class before starting the homework assignment.

8.1c Homework: Power of a Power, Power of a Product, and Power of a Quotient

- For parts a. through c. write each expression in expanded form. Then write the simplified expression in exponential form.

Power of a Power

#	Expression	Expanded Form	Exponential Form
a.	$(x^3)^4$	$(x^3)(x^3)(x^3)(x^3)$	x^{12}
b.	$(2^3)^5$	$(2^3)(2^3)(2^3)(2^3)(2^3)$	$2^{15} = 32,768$
c.	$[(a^2)^3]^2$	$[(a^2)(a^2)(a^2)][(a^2)(a^2)(a^2)]$	a^{12}

Simplify each expression.

d. $(r^5)^8$ r^{40}	e. $(y^{50})^4$ y^{200}	f. $(-4^5)^3$ $-1,073,741,824$	g. $[(a^3)^{10}]^5$ a^{150}	h. $(k^9)^5(k^3)^2$ k^{51}
Take some time to talk about how simplify expressions like the one in part f with the calculator.				

- For parts a. through c. write each expression in expanded form. Then write the simplified expression in exponential form.

Power of a Product

#	Expression	Expanded Form	Exponential Form
a.	$(xy^3)^4$	$(xy^3)(xy^3)(xy^3)(xy^3)$	x^4y^{12}
b.	$(x^3y^4)^2$	$(x^3y^4)(x^3y^4)$	x^6y^8
c.	$(2a^3c^2)^3$	$(2a^3c^2)(2a^3c^2)(2a^3c^2)$	$8a^9c^6$

Simplify each expression.

d. $(a^6b)^7$ $a^{42}b^7$	e. $(a^2b^5)^5$ $a^{10}b^{25}$	f. $(2ab^4)^6$ $64a^6b^{24}$	g. $(6^2ab^3a^3)^4$ $1,679,616a^{16}b^{12}$	h. $(-3r^4)^4 \cdot (r^5)^2$ $81r^{26}$
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It might be helpful for students to write the following at the top of their homework assignment. "When in doubt, write it out!" This means that if they do not know what property to use they can always expand the expression and simplify.

3. For parts a. through d. write each expression in expanded form. Then write the simplified expression in exponential form.

Power of a Quotient

#	Expression	Expanded Form	Exponential Form
a.	$\left(\frac{3}{5}\right)^2$	$\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)$	$\left(\frac{9}{25}\right)$
b.	$\left(\frac{c}{d}\right)^3$	$\left(\frac{c}{d}\right)\left(\frac{c}{d}\right)\left(\frac{c}{d}\right)$	$\frac{c^3}{d^3}$
c.	$\left(\frac{x^2}{y}\right)^3$	$\left(\frac{x^2}{y}\right)\left(\frac{x^2}{y}\right)\left(\frac{x^2}{y}\right)$	$\frac{x^6}{y^3}$
d.	$\left(\frac{2x}{3y}\right)^4$	$\left(\frac{2x}{3y}\right)\left(\frac{2x}{3y}\right)\left(\frac{2x}{3y}\right)\left(\frac{2x}{3y}\right)$	$\frac{16x^4}{81y^4}$

Simplify each expression.

e.	$\left(\frac{2}{a}\right)^{12}$ $\frac{4096}{a^{12}}$	f.	$\left(\frac{2ab}{a}\right)^4$ $16b^4$	g.	$\left(\frac{s^2}{t^3}\right)^5$ $\frac{s^{10}}{t^{15}}$	h.	$\left(\frac{4xy^2}{2y}\right)^3$ $8x^3y^3$
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4. Simplify each expression.

Mixed practice

a.	$4z^5 \cdot 2y^7 \cdot 3z^5y$ $24z^{10}y^8$	b.	$(3ab)^4$ $81a^4b^4$	c.	$\frac{48z^2 \cdot y^3x^4r^5}{12z^2yxr^3}$ $4y^2x^3r^2$
d.	$(4a^3 \cdot 3b^7)^2$ $144a^6b^{14}$	i.	$\left(\frac{4x^6}{2b^3}\right)^3$ $\frac{8x^{18}}{b^9}$	e.	$\left(\frac{2a^4 \cdot 3a}{3a^2}\right)^2$ $4a^6$
f.	$(-3xyz)^6$ $729x^6y^6z^6$	j.	$\frac{27a^8b^4a^6}{18a^{10}b}$ $\frac{3a^4b^3}{2}$	g.	$\left(\frac{3a^4}{b}\right)\left(\frac{6ab^3}{a^2}\right)^2$ $108a^2b^5$

8.1d Class Activity: Find, Fix, and Justify

The following statements are **incorrect**. For each of the statements do the following:

- **Find** the mistake(s) in each statement.
- **Fix** the mistake.
- **Justify** your reasoning. You may use pictures if needed.

1. $x^2 \cdot x^4 = x^8$ $x^2 \cdot x^4$ $= (x \cdot x) \cdot (x \cdot x \cdot x \cdot x)$ $= x^6$	2. $a^3b^2 \cdot a^4b^5 = a^5b^9$ $a^3b^2 \cdot a^4b^5$ $= (a \cdot a \cdot a)(b \cdot b) \cdot (a \cdot a \cdot a \cdot a)(b \cdot b \cdot b \cdot b \cdot b)$ $= a^7b^7$	3. $a^2b^5 \cdot ab^3 = a^2b^8$ $a^2b^5 \cdot ab^3$ $= (a \cdot a)(b \cdot b \cdot b \cdot b \cdot b) \cdot (a)(b \cdot b \cdot b)$ $= a^3b^8$
4. $\frac{x^7}{x^4} = x^{11}$ $\frac{x^7}{x^4} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = \frac{x^3}{1} = x^3$	5. $\frac{a^6}{a^2} = a^3$ $\frac{a^6}{a^2} = \frac{\cancel{a} \cdot \cancel{a} \cdot a \cdot a \cdot a \cdot a}{\cancel{a} \cdot \cancel{a}} = \frac{a^4}{1} = a^4$	6. $\frac{b^3}{b^9} = b^6$ $\frac{b^3}{b^9} = \frac{\cancel{b} \cdot \cancel{b} \cdot \cancel{b}}{\cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b} = \frac{1}{b^6}$
7. $\frac{12x^6y^3}{2x^2y^2} = 10x^4y$ $\frac{12x^6y^3}{2x^2y^2} \neq 10x^4y$ $= \frac{\cancel{2} \cdot 6 \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x \cdot x \cdot \cancel{y} \cdot \cancel{y} \cdot y}{\cancel{2} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{y}}$ $= \frac{6x^4}{1} = 6x^4y$	8. $(y^3)^4 = y^7$ $(y^3)^4 \neq y^7$ $= y^3 \cdot y^3 \cdot y^3 \cdot y^3$ $= y^{3+3+3+3}$ $= y^{12}$	9. $(tw)^3 = tw^3$ $(tw)^3 \neq tw^3$ $= tw \cdot tw \cdot tw$ $= t^3w^3$

One way that you can implement this lesson is to give students about 10 minutes to work on these 9 problems. Students must use their knowledge about exponential expressions and exponent properties to find mistakes. This activity is meant to solidify their understanding of how exponential expressions are constructed. As students analyze the structure of the expression their understanding of the exponent rule becomes more concrete. When they are finished have them discuss their answers with a neighbor before moving onto a group discussion. Ask for people to come to the board to show and justify how they fixed the mistake in each statement.

Some of the following statements are **correct** and some are **incorrect**. If the statement is correct justify why it is correct by expanding the expression. If the statement is incorrect:

- Find the mistake(s) in each statement.
- Fix the mistake.
- Justify your reasoning.

10. $[(y^2)^3]^2 = y^{12}$ Correct $\left[(y^2)^3 \right]^2 = y^{12}$ $= [y^2 \cdot y^2 \cdot y^2]^2$ $= [y^2 \cdot y^2 \cdot y^2] \cdot [y^2 \cdot y^2 \cdot y^2]$ $= y^{12}$	11. $(a^2b^3)^2 = a^4b^6$ Correct $(a^2b^3)^2 = a^4b^6$	12. $(5y^3)^3 = 5y^9$ Not Correct $(5y^3)^3 \neq 5y^9$ $= 5^{1 \cdot 3} y^{3 \cdot 3}$ $= 5^3 y^9$ $= 125y^9$
13. $\left(\frac{3}{5}\right)^2 = \frac{9}{5}$ Not Correct $\left(\frac{3}{5}\right)^2 \neq \frac{9}{5}$ $= \frac{3^2}{5^2} = \frac{9}{25}$	14. $\left(\frac{c}{d}\right)^3 = \frac{c^3}{d^3}$ Correct $\left(\frac{c}{d}\right)^3 = \frac{c^3}{d^3}$	15. $\left(\frac{2x}{yz}\right)^3 = \frac{8x^3}{y^3z^3}$ Correct $\left(\frac{2x}{yz}\right)^3 = \frac{8x^3}{y^3z^3}$
16. $\left(\frac{a^2}{b^3}\right)^4 = \frac{a^8}{b^3}$ Not Correct $\left(\frac{a^2}{b^3}\right)^4 \neq \frac{a^8}{b^3} = \frac{a^8}{b^{12}}$	17. $\left(\frac{2x^2}{x^2y^3}\right)^2 = \frac{4}{y^6}$ Correct $\left(\frac{2x^2}{x^2y^3}\right)^2 = \frac{4}{y^6}$	18. $3^2 + 3^4 = 3^6$ Not Correct $3^2 + 3^4 \neq 3^6$ $= 3 \cdot 3 + 3 \cdot 3 \cdot 3 \cdot 3$ $= 9 + 81 = 90$

19. Write three expressions equivalent to $3^2 \cdot 9^2$

Answers may include

$$3 \cdot 3 \cdot 9 \cdot 9, 3^2 \cdot 3 \cdot 3 \cdot 9, 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

$$3^2 [(3^2)^2], 3^2 \cdot 3^4$$

20. Write three exponential expressions equivalent to 400. Answers may include

$$2^4 \cdot 5^2, 2^2 \cdot 10^2, 4 \cdot 100$$

21. $\frac{3^a}{3^b} = 3^2$. Find numbers a and b that satisfy the equation. Can you find different numbers for a and b ?

Answers will vary. Note that a or b can be positive or negative or even rational numbers.

22. Consider the equation, $x^y = y^x$, where x and y are two different whole numbers. Find the value for x and y .

$$x = 2 \text{ and } y = 4 \text{ or } x = 4 \text{ and } y = 2$$

23. You are asked to enter the following expression into your scientific calculator.

$$(4^2)^3$$

Which of the following is a correct way to enter the expression into a scientific calculator?

a. 4 y^x 5	b. 4 y^x 6	c. 4 y^x 2 x 3	d. 4 x 2 x 3
---	---	--	--

8.1d Homework: Find, Fix , and Justify.

1. Edward, Pattie, and Mitch were each simplifying the same exponential expression. Their work is shown below. Determine who has simplified the expression correctly. If they did not simplify the expression correctly, identify the mistake, explain it, and fix it.

Edward

$$\left(\frac{3x^4}{6x} \right)^2 = \left(\frac{2x^4}{x} \right)^2 = (2x^3)^2 = 2x^6$$

Edward is not correct when he simplified the constants he wrote the 2 in the numerator and not the denominator. He also did not square the 2 in the last step.

Patti

$$\left(\frac{3x^4}{6x} \right)^2 = \frac{3^2 \cdot x^8}{6^2 \cdot x^2} = \frac{9x^8}{36x^2} = \frac{x^6}{4}$$

Patti and Mitch are both correct. They simplified the expression differently but both arrived at the right answer.

Mitch

$$\left(\frac{3x^4}{6x} \right)^2 = \left(\frac{x^4}{2x} \right)^2 = \left(\frac{x^3}{2} \right)^2 = \frac{x^6}{4}$$

2. Find the value of 1^8 , 1^9 , 1^{10} , and 1^0 . What can you say about the value of any power of 1?

They all equal 1. One to any power is 1.

3. What is the area of a square with a side length of $3a^5$?

$9a^{10}$

4. What is the area of a rectangle with a length of $12x^3$ units and a width of $6x^2$ units?

$72x^5$ units

5. You are asked to enter the following expression into your scientific calculator.

$$(5^2)^5$$

Which of the following is **not a correct** way to enter the expression into a scientific calculator?

a. 5 y^x 2 y^x 5	b. 5 y^x 10	c. 25 y^x 5	d. 5 y^x 7
--	--	--	---

6. You are asked to enter the following expression into your scientific calculator.

$$\left(\frac{2}{3}\right)^4$$

Which of the following **not a correct** way to enter the expression into a scientific calculator?

a. 2 ÷ 3 y^x 4	b. (2 ÷ 3) y^x 4	c. 2 y^x 4 ÷ 3 y^x 4
--	--	---

7. Given the statement, $3^a \cdot 3^b = 3^{10}$. Find two numbers for a and b that satisfy the equation. Can you find different numbers for a and b ?

Answers can include any values where $a + b = 10$.

8. Given the statement, $(3^a)^b = 3^{30}$. Find two numbers for a and b that satisfy the equation. Can you find different numbers for a and b ?

Answers can include any values where $a \cdot b = 30$.

9. Make up your own problem that requires two Properties of Exponents to simplify. Be sure to show your answer.

Answers will vary.

10. Make up your own problem that requires two different properties than the ones you used in number 9. Be sure to show your answer.

Answers will vary.

11. On Tuesday, you invited 2 friends to your party. On Wednesday, each of these friends invited 2 other friends. This pattern continued Thursday and Friday. How many people were invited on Friday? Write the answer as a power. How many people were invited in all? Explain your reasoning.

On Friday 16 or 2^4 people were invited to the party. Altogether 30 people were invited.

8.1e Class Activity: Zero and Negative Exponents

The table below lends itself to a lot of discovery and discussion. It will be used to help explain the meaning of negative exponent and numbers with a zero exponent. This class activity will most likely take more than one day to complete.

- Complete the table below. (Hint: Use the patterns in the Powers of 10 section to help with the Powers of 2 section.)



POWERS of 10				POWERS of 2	
Words	Number (as decimals)	Number (as Fractions)	Number (with Powers)	Number	Number (with Powers)
a. One Million	1,000,000	$\frac{1,000,000}{1}$	10^6	64	2^6
b. One Hundred Thousand	100,000	$\frac{100,000}{1}$	10^5	32	2^5
c. Ten Thousand	10,000	$\frac{10,000}{1}$	10^4	16	2^4
d. One Thousand	1,000	$\frac{1,000}{1}$	10^3	8	2^3
e. One hundred	100	$\frac{100}{1}$	10^2	4	2^2
f. Ten	10	$\frac{10}{1}$	10^1	2	2^1
g. One	1	$\frac{1}{1}$	10^0	1	2^0
h. One Tenth	.1	$\frac{1}{10}$	$\frac{1}{10^1}$ or 10^{-1}	$\frac{1}{2}$	2^{-1}
i. One Hundredth	.01	$\frac{1}{100}$	$\frac{1}{10^2} = 10^{-2}$	$\frac{1}{4}$	2^{-2}
j. One Thousandth	.001	$\frac{1}{1,000}$	$\frac{1}{10^3} = 10^{-3}$	$\frac{1}{8}$	2^{-3}
k. One ten thousandth	.0001	$\frac{1}{10,000}$	$\frac{1}{10^4} = 10^{-4}$	$\frac{1}{16}$	2^{-4}
l. One One Hundredth Thousandth	.00001	$\frac{1}{100,000}$	$\frac{1}{10^5} = 10^{-5}$	$\frac{1}{32}$	2^{-5}
m. One Millionth	.000001	$\frac{1}{1,000,000}$	$\frac{1}{10^6} = 10^{-6}$	$\frac{1}{64}$	2^{-6}

- Complete this sentence: Any number with a zero exponent is...1

See teacher note at the bottom of page 28 for discussion surrounding 0^0 .
- Explain what happens to the size of the numbers as you move up the column from 10^1 .
They get bigger or you multiply by a factor of 10.
- Explain what happens to the size of the numbers as you move down the column from 10^1 .
They get smaller or you divide by a factor of 10.
- Write 5^{-2} as a fraction. $\frac{1}{5^2}$ Write x^{-6} as a fraction. $\frac{1}{x^6}$

Zero Exponent Property

Quotient	Expanded Form	Simplified	Thus...	Any fraction that has the same numerator and denominator equals 1.
$\frac{x^5}{x^5}$	$\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x}$	$\frac{1}{1} = 1$	$\frac{x^5}{x^5} = 1$	
$\frac{4^3}{4^3}$	$\frac{4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4}$	$\frac{1}{1} = 1$	$\frac{4^3}{4^3} = 1$	
$\frac{(ab)^2}{(ab)^2}$	$\frac{a \cdot b \cdot a \cdot b}{a \cdot b \cdot a \cdot b}$	$\frac{1}{1} = 1$	$\frac{(ab)^2}{(ab)^2} = 1$	

Quotient	Simplify using the Quotient Rule	Thus...	Zero Exponent Property	Any number to the zero power equals 1. This cell will need to be summarized together as a class.
$\frac{x^5}{x^5}$	$x^{5-5} = x^0$	$\frac{x^5}{x^5} = x^0$	Since $\frac{x^5}{x^5} = x^0$ and $\frac{x^5}{x^5} = 1$, then $x^0 = 1$	
$\frac{4^3}{4^3}$	$4^{3-3} = 4^0$	$\frac{4^3}{4^3} = 4^0$	Since $\frac{4^3}{4^3} = 4^0$ and $\frac{4^3}{4^3} = 1$, then $4^0 = 1$	
$\frac{(ab)^2}{(ab)^2}$	$(ab)^{2-2} = (ab)^0$	$\frac{(ab)^2}{(ab)^2} = (ab)^0$	Since $\frac{(ab)^2}{(ab)^2} = (ab)^0$ and $\frac{(ab)^2}{(ab)^2} = 1$, then $(ab)^0 = 1$	

Algebraic Rule for a Zero Exponent:

$$x^0 = 1$$

Simplify each expression:

a. $a^0 = 1$

b. $(240)^0 = 1$

c. $\left(\frac{ab}{c}\right)^0 = 1$

d. $\left(\frac{5^2 x^3 y^2}{z}\right)^0 = 1$

e. $\left(\frac{a^0 b^4}{c}\right)^2 = \frac{b^8}{c^2}$

f. $(3x^3 y^4 x^0 y)^3 = 27x^9 y^{15}$

The tables given above may be challenging for your students to complete on their own. Give Honor's students the opportunity to grapple with it and see what they come up with. You will want to summarize the learning that takes place here. That is, through the Quotient Rule and the fact that a fraction with the same numerator and denominator simplifies to 1; we can show that $x^0 = 1$.

The statement given in number 2 on page 29 presents a problem if we are dealing with 0^0 . Check out the links below to help you give clarity as you discuss this with your students.

<http://www.askamathematician.com/2010/12/q-what-does-00-zero-raised-to-the-zeroth-power-equal-why-do-mathematicians-and-high-school-teachers-disagree/>

<https://www.math.hmc.edu/funfacts/ffiles/10005.3-5.shtml>

Negative Exponent Rule

Quotient	Column 1 rewritten in expanded form or as repeated multiplication.	Column 1 simplified using the Quotient Rule.	Compare your answers; one written as a fraction and the other in exponent form.
$\frac{3^3}{3^5}$	$\frac{3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \frac{1}{3^2} = \frac{1}{9}$	$3^{3-5} = 3^{-2}$	$\frac{1}{3^2} = 3^{-2}$
$\frac{a^4}{a^7}$	$\frac{a \cdot a \cdot a \cdot a}{a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a} = \frac{1}{a^3}$	$a^{4-7} = a^{-3}$	$\frac{1}{a^3} = a^{-3}$
$\frac{a^3b^6}{a^6b^{10}}$	$\frac{a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b}{a \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b} = \frac{1}{a^3b^4}$	$a^{3-6}b^{6-10} = a^{-3}b^{-4}$	$\frac{1}{a^3b^4} = a^{-3}b^{-4}$
$\frac{4^{15}a^{30}}{4^{20}a^{50}}$	<i>This is too long to expand, find a short cut.</i>	$4^{15-20}a^{30-50} = 4^{-5}a^{-20}$	$\frac{1}{4^5a^{20}} = 4^{-5}a^{-20}$

Explain what a negative exponent means:

a^{-n} is the reciprocal of a^n . Likewise a^n is the reciprocal of a^{-n} .

Algebraic Rule for Negative Exponents:

$$a^{-n} = \frac{1}{a^n}$$

Discuss with students that at this point in order for the expression to be completely simplified it must not contain a negative exponent.

Simplify each expression:

a. $4^{-1} \frac{1}{4^1} = \frac{1}{4}$

b. $x^{-2} \frac{1}{x^2}$

c. $x^{-3}y^{-4} \frac{1}{x^3y^4}$

d. $a^{-3}b^4 \frac{b^4}{a^3}$

e. $\frac{1}{a^{-5}} a^5$

f. $\frac{a^2bc^{-3}}{a^3b^{-4}} \frac{b^5}{ac^3}$

g. $4n^{-2} \cdot 3n^3$ **$12n$**

h. $\frac{m^5n}{m^2n^3} \frac{m^3}{n^2}$

i. $(m^{-3})^2 \frac{1}{m^6}$

1. Simplify each of the following and order them from least to greatest.

$(-6)^4$ $(-6)^0$ -6^4 $(-6)^{-4}$ **-1296 , $\frac{1}{1296}$, 1 , 1296**

2. What is the difference between $-r^3$, r^{-3} and $(-r)^3$?

$-r^3 = -(r)(r)(r)$, $r^{-3} = \frac{1}{r^3}$, and $(-r)^3 = (-r)(-r)(-r)$.

3. What is the difference between $6t^{-2}$ and $(6t)^{-2}$?

$6t^{-2} = 6 \frac{1}{t^2} = \frac{6}{t^2}$ and $(6t)^{-2} = \frac{1}{(6t)^2} = \frac{1}{36t^2}$.

4. What is the difference between $\left(\frac{5}{3}\right)^{-3}$ and $\frac{5^{-3}}{3}$?

$\left(\frac{5}{3}\right)^{-3} = \left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3} = \frac{27}{125}$ and $\frac{5^{-3}}{3} = \frac{1}{3 \cdot 5^3} = \frac{1}{3 \cdot 125} = \frac{1}{375}$.

For expressions f. through i. above point out the several different ways to simplify each expression; for example, in part g. they can apply the product rule first and add the exponents or they could use the negative exponent rule and move n^{-2} to the denominator first and then use the Quotient Rule to finish simplifying. It might be helpful to tell students that the sign of the exponent changes as you move the number between the numerator and denominator of the fraction. For number 2 ask students to evaluate each expression for $r = 2$. Ask; what value of f would yield the same result when evaluated?

8.1e Homework: Zero Exponent and Negative Exponents

Directions: Simplify each expression. The simplified expression should not include any negative exponents.

#	Expression	Expanded Form	Simplified Solution
1.	$m^4 \cdot 2m^{-3}$	$m \cdot m \cdot m \cdot m$ $\cdot 2 \cdot \cancel{m^{-1}} \cdot \cancel{m^{-1}} \cdot \cancel{m^{-1}}$	$2m$
2.	$4r^{-3}r^2$	$4 \cdot \cancel{r^{-1}} \cdot \cancel{r^{-1}} \cdot \cancel{r^{-1}} \cdot r \cdot r = 4r^{-1}$	$\frac{4}{r}$
3.	$\frac{c^4d}{cd^4}$	$\frac{\cancel{c} \cdot c \cdot c \cdot c \cdot \cancel{d}}{\cancel{c} \cdot d \cdot d \cdot d \cdot \cancel{d}}$	$\frac{c^3}{d^3}$
4.	$\frac{3w^3}{21w^5}$	$\frac{\cancel{3} \cdot \cancel{w} \cdot \cancel{w} \cdot \cancel{w}}{\cancel{3} \cdot 7 \cdot w \cdot w \cdot \cancel{w} \cdot \cancel{w} \cdot \cancel{w}}$	$\frac{1}{7w^2}$
5.	$\frac{xyz^2}{x^2yz}$	$\frac{\cancel{x} \cdot \cancel{y} \cdot \cancel{z} \cdot z}{x \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{z}}$	$\frac{z}{x}$
6.	$(4x^0)^4$	$(4 \cdot x^0)(4 \cdot x^0)(4 \cdot x^0)(4 \cdot x^0) = (4 \cdot 1)(4 \cdot 1)(4 \cdot 1)(4 \cdot 1) = 4 \cdot 4 \cdot 4 \cdot 4$	256
7.	$\frac{4x^0y^{-2}z^{-3}}{4xy^{-1}}$	$\frac{\cancel{4} \cdot x^0 \cdot \cancel{y^{-1}} \cdot y^{-1} \cdot z^{-1} \cdot z^{-1} \cdot z^{-1}}{\cancel{4} \cdot x \cdot \cancel{y^{-1}}^{-1}} = \frac{1}{x \cdot y \cdot z \cdot z \cdot z}$	$\frac{1}{xyz^3}$

Directions: Use the properties of exponents to simplify each expression.

8. $\frac{x^{-3}}{x} \cdot \frac{1}{x^4}$

9. $(x^0)^2 \cdot 1$

10. $3^{-4} \cdot \frac{1}{81}$

11. $(x^2)^0 \cdot 1$

12. $2x^{-3}y^{-3} \cdot 2x^{-1}y^3 \cdot \frac{4}{x^4}$

13. $(2x^2)^{-4} \cdot \frac{1}{16x^8}$

14. $(4r^0)^4 \cdot 256$

15. $(4xy)^{-1} \cdot \frac{1}{4xy}$

16. $\frac{r^3}{2r^3} \cdot \frac{1}{2}$

17. $\frac{3m^{-4}}{m^3} \cdot \frac{3}{m^7}$

18. $\frac{m^4}{2m^4} \cdot \frac{1}{2}$

19. $\frac{x^{-1}}{4x^4} \cdot \frac{1}{4x^5}$

8.1f Class Activity: Properties of Exponents Game and Mixed Practice

Directions: Complete each exponent property by filling in the space.

$$1. a^b \cdot a^c = a^{b+c}$$

$$2. \frac{a^b}{a^c} = a^{b-c}$$

$$3. (a^b)^c = a^{b \cdot c}$$

$$4. (ab)^c = a^c b^c$$

$$5. \left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

$$6. a^0 = 1$$

$$7. a^{-1} = \frac{1}{a}$$

Make a class set of the following puzzle and cut it out; direct students to assemble the puzzle so that each expression touches or lines up with its simplified form. Have fast finishers simplify the expressions on the border of the puzzle that do not match up with the simplified expressions.

Once you have received the exponent puzzle from your teacher assemble the puzzle so that each expression touches or lines up with its simplified form.

$(a^{-7})^1$ $x^6 x^7$ $(a^{-3} b^{-3})^0$	1 $\frac{x^5 y^2}{y^9 x^3}$ $\frac{2k^3 k^2}{k^{-3}}$	$2k^8$ $\frac{4x^0 y^{-2} z^3}{4x}$ $\frac{3x^5 y^{-1} z}{x^{-4} y^0 z}$	$(a^{-7})^1$ $\frac{y^9}{y^4}$ $\frac{3x^4 y^2}{y^3 z^2}$
$x^2 y^0 (3x^{-2})^1$ $(3y)^2$ $x^4 y^3 (2y^2)^0$	$x^4 y^3$ $(2x^4 y^{-3})^{-1}$ $\frac{x^2}{y^7}$	1 $\frac{2x^3}{(x^{-1})^3}$ $\frac{z^3}{x^2 y^2}$	y^5 $2x^6$ $(x^4 y^2)^6$
$(5x^3)^3 (-xy)^2$ $3y^2$ $(5x^2 y^2)(-xy)^3$	$2^{-1} x^{-4} y^3$ $-5x^5 y^5$ $(2^{-3} x^4 y^{-3})^2$	25 $\frac{x^4}{10^2}$ $\frac{2x^2}{(2x^2)^{-1}}$	$(-7x^4 y^3)^{-1}$ $\frac{1}{2x^4}$ $\frac{y^2}{(x^4)^{-2}}$

8.1f Homework: Properties of Exponents Mixed Practice

Directions: Simplify each expression. Assume that no denominator is equal to zero.

1. $x^3 \cdot x^5$ x^8

2. $a^{15} \cdot a^{11}$ a^{26}

3. $3y^8 \cdot 2y^2$ $6y^{10}$

4. $5j^4(-9j^5)$ $-45j^9$

5. $(x^5)^2$ x^{10}

6. $(a^3)^6$ a^{18}

7. $(h^4)^3$ h^{12}

8. $[(k^5)^2]^3$ k^{30}

9. $(xy)^7$ x^7y^7

10. $(4gz)^2$ $16g^2z^2$

11. $(-2a^4wy^4)^3$ $-8a^{12}w^3y^{12}$

12. $-3(km^4)^4$ $-3k^4m^{16}$

13. $\frac{d^8}{d^4}$ d^4

14. $\frac{t^9}{t^3}$ t^6

15. $\frac{a^5b^3}{a^2d} \cdot \frac{a^3b^3}{d}$

16. $\frac{x^3y^2z}{x^2y^2}$ xz

$$17. \left(\frac{4}{5}\right)^2 \frac{16}{25}$$

$$18. \left(\frac{2}{3}\right)^4 \frac{16}{81}$$

$$19. \left(\frac{x}{3}\right)^3 \frac{x^3}{27}$$

$$20. \left(\frac{c}{b}\right)^{15} \frac{c^{15}}{b^{15}}$$

$$21. x^{-4} \frac{1}{x^4}$$

$$22. \frac{3}{c^{-2}} 3c^2$$

$$23. \frac{s^3}{s^{-4}} s^7$$

$$24. \frac{6p^{-2}}{p^2} \frac{6}{p^4}$$

$$25. 5^0 \quad 1$$

$$26. \left[\left(\frac{33y^{17}z}{12a^{115}b} \right)^5 \right]^0 \quad 1$$

$$27. [(-2^3)^3]^2 \quad 262,144$$

$$28. (bc^3)(b^4c^3) \quad b^5c^6$$

$$29. \frac{(u^{-3}v^3)^2}{(u^3v)^{-3}} u^3v^9$$

$$30. \frac{9a^2b^7c^3}{2a^5b^4c^5} \frac{9b^3}{2a^3c^2}$$

$$31. \left(\frac{1}{2}w^3\right)^2 (w^4)^2 \frac{w^{14}}{4}$$

$$32. \left(\frac{-18x^0a^3}{6(x^{-2})(x^3a^2)} \right)^2 \frac{9a^2}{x^2}$$

8.1g Self-Assessment: Section 8.1

The indicators given in this rubric are sample indicators.

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Apply the properties of integer exponents to simplify algebraic and numerical expressions. <i>See sample problem #1</i>	I can apply the properties of integer exponents to simplify 1 of the expressions given.	I can apply the properties of integer exponents to simplify 2 of the expressions given.	I can apply the properties of integer exponents to simplify 3 of the expressions given.	I can apply the properties of integer exponents to simplify all 4 of the expressions given.

Sample problem #1

Use the properties of exponents to simplify each expression.

a. $\frac{3}{r^{-4}}$

$3r^4$

b. $(xy^3)^2(x^2y)^4$

$x^{10}y^{10}$

c. $\frac{9a^2b^4c^2}{3a^5bc^4}$

$\frac{3b^3}{a^3c^2}$

d. $\left(\frac{1}{2}r^2\right)^2 \cdot (s^0r)^3$

$\frac{r^7}{4}$

Section 8.2: Scientific Notation

Section Overview:

The first lesson in this section is a review of place value and powers of ten. In order to properly access and understand how scientific notation works students must have a solid foundation of place value and how powers of ten show the relationship between digits that are next to each other in a multi-digit number. They will use this foundation to compare numbers written as a single digit times an integer power of ten by a scale factor. After reviewing place value students learn to change numbers between standard form and scientific notation in order to estimate very large and very small quantities. Students also learn how to operate with numbers in scientific notation so they can compare and express how many times as much one number is to another. As they are problem solving with scientific notation they must choose units of appropriate size for measurements of very large or very small quantities. Finally, students will use scientific notation to solve in a variety of contexts that require the use of very large and very small numbers.

Concepts and Skills to master:

By the end of this section students should be able to:

1. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
2. Convert a number between scientific notation and standard form.
3. Perform operations with numbers expressed in scientific notation.
4. Choose units of appropriate size for measurements of very large or very small quantities.
5. Interpret scientific notation that has been generated by technology.
6. Use scientific notation to problem solve with really small and really large numbers.

8.2a Class Activity: Place Value and Powers of Ten

Do not let students use a calculator with this lesson.

- Fill in the top row of the place value chart below with the name of the place value that each number digit given refers to. See page 27 if you need a reminder.

Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundredth Thousandths
1	5	8	8	4	6	3	1	0	8	.	7	3	4	1	1
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}

The problems below are not intended to teach scientific notation. They are meant to access a student's background knowledge of how to write and estimate numbers using powers of ten. See standards 5.NBT 1 and 2 from 5th grade math and standard 8.EE.3.

- Fill in the power of ten in the last row in the table above that corresponds with each place value. Some of them have been filled in for you.

See table above.

- What is the value of the 6 in the chart above?

60,000

- What is the value of 7 in the chart above?

0.7 or seven tenths.

- Write each number given below as a single digit times an integer power of 10. The first one has been done for you.

a. $30,000 = 3 \times 10^4$	b. $700 = 7 \times 10^2$	c. $0.0005 = 5 \times 10^{-4}$
d. $0.0000003 = 3 \times 10^{-7}$	e. $8,000,000,000,000 = 8 \times 10^{12}$	f. $2 = 2 \times 10^0$

6. Estimate each number by writing it as a single digit times an integer power of 10.

a. The land area of Canada is 499,000,000 square kilometers. 5×10^8	b. An adult blue whale can eat 41,000,000 krill in one day. 4×10^7	c. The distance between Pluto and Earth is 4,670,000,000 miles. 5×10^9
d. The diameter of the average human body cell is 0.00012 meters. 1×10^{-4}	e. A sample of sand from a beach has 21,450,000 grains of sand. 2×10^7	f. The width of the diameter of a piece of fishing line is 0.000017 meters. 2×10^{-5}

7. Rearrange **all** the digits and decimal below to build a number with the given conditions. If needed write these numbers on a separate piece of paper and cut them out to rearrange them.

3	0	7	8	.
---	---	---	---	---

- Build the largest number
 8730
- Build the smallest number
 $.0378$
- Build a number less than 8
Sample answer; 7.308
- Build a different number less than 7
Sample answer; 3.078
- Build a number between 70 and 80
Sample answer; 70.83
- Build a number than rounds to 70
Sample answer; 70.38
- Build a number that is between 7000 and 8000
Sample answer; 7830
- Build a number that is closest to 3
 3.078
- Build a number that is between 0.7 and 0.8
Sample answer; 0.7830

The purpose of this lesson is to refresh your students' knowledge of place value and powers of 10. This knowledge is a key component to truly understanding the structure of scientific notation and how it is comprised of powers of 10. A good foundation in place value will help with performing operations in scientific notation.

You could definitely bring in some base ten blocks to investigate many of the questions in this section.

8. Fill in the missing entries in the tables below. You may have to find and follow a pattern.

0.004	×	10	=	0.04
0.04	×	10	=	0.4
0.4	×	10	=	4
4	×	10	=	40
40	×	10	=	400
400	×	10	=	4,000

1280	×	$\frac{1}{10}$	=	128
128	×	$\frac{1}{10}$	=	12.8
12.8	×	$\frac{1}{10}$	=	1.28
1.28	×	$\frac{1}{10}$	=	0.128
0.128	×	$\frac{1}{10}$	=	0.0128
0.0128	×	$\frac{1}{10}$	=	0.00128

9. Use the tables above to fill in the missing words in the statement below.

In a multi-digit number, a digit in one place represents ten times as much as it represents in the place to its right and one-tenth of what it represents in the place to its left.

10. Explain the relationship between the two 3's in the number 533,271.

The 3 in the ten thousands place is ten times as much as the 3 in the thousands place. The 3 in the thousands place is one-tenth of the 3 in the ten thousands place.

The statement in number 9 is Standard 5.NBT.A.1 from 5th grade math. It is a prerequisite standard for students to fully understand scientific notation which is why we are reviewing it here.

11. Daxton's Candy (*adapted from an Illustrative Mathematics Task*)

- Daxton has a digital scale. He puts a Marshmallow Peep on the scale and it reads 6.2 grams. How much would you expect 10 Marshmallow Peeps to weigh? Why?
Ten Marshmallow Peeps will weigh 62 grams. You can move the decimal one space to the right to multiply by ten.
- Daxton takes the marshmallows off the scale. He then puts on 10 jellybeans and the scale reads 12.0 grams. How much would you expect 1 jellybean to weigh? Why?
One jellybean will weigh 1.2 grams. You can move the decimal one place to the left to divide by ten.
- Daxton then takes off the jellybeans and puts on 10 cinnamon bunnies. The scale reads 88.2 grams. How much would you expect 1,000 cinnamon bunnies to weigh? Why? One thousand cinnamon bunnies will weigh 8820 grams. Since 1,000 is 100 times as much as 10 you want to multiply by 100. You do this by moving the decimal two places to the right to multiply by 100.
- Estimate how many jellybeans equal one cinnamon bunny.
One cinnamon bunny equals approximately 7.3 jelly beans

12. If one package of cereal costs \$2.46, then,
- 10 will cost \$24.60
 - 100 will cost \$246
 - 1,000 will cost \$2,460
 - 1/10 of the package will cost \$0.25
 - 1/100 of the package will cost \$0.03

13. Ten thousand is how many times bigger than 100? (Hint: Remember the statement you completed on the previous page.)

10,000 is 100 times bigger than 100.

Spend some time investigating this problem. Use the statement on the previous page to help students recognize that every additional digit or place value changes the number by a scale factor of 10. Since there are two additional digits of 0 in 10,000 compared to 100, then the number changes by a factor of $10 \cdot 10 = 10^2 = 100$. Thus 10,000 is 100 times bigger than 100.

14. Now write 100 and 10,000 as a single digit times an integer power of ten.

1×10^2 and 1×10^4

Use the powers of ten to determine how many times bigger 10,000 is than 100.

The difference between the powers of ten is $10^2 = 100$, so 10,000 is 100 times bigger than 100.

Students should answer the remaining problems by using what they know about powers of ten. They are technically also using scientific notation. However, the focus should be on how additional powers of ten increase the value of a number by a scale factor of 10. This is addressing standard 8.EE.3 which emphasizes writing numbers as a **single digit** times an integer power of ten to estimate and make comparisons.

15. Write each number as a single digit times an integer power of ten to determine how many times bigger one is compared to the other.

a. 1,000,000 and 100 1×10^6 and 1×10^2 1,000,000 is $10^4 = 10,000$ times bigger than 100.	b. 10,000 and 10 1×10^4 and 1×10^1 10,000 is $10^3 = 1,000$ times bigger than 10.	c. 0.0001 and 0.000001 1×10^{-4} and 1×10^{-6} 0.0001 is $10^2 = 100$ times bigger than 0.000001.
d. 0.001 and 0.00000001 1×10^{-3} and 1×10^{-8} 0.001 is $10^5 = 100,000$ times bigger than 0.00000001.	e. 10 and 0.01 1×10^1 and 1×10^{-2} 10 is $10^3 = 1,000$ times bigger than 0.01.	f. 200,000 and 2,000 2×10^5 and 2×10^3 200,000 is $10^2 = 100$ times bigger than 2,000.

16. Write each number as a single digit times an integer power of ten to determine how many times bigger one is compared to the other.

a. 60,000,000 and 30,000 6×10^7 and 3×10^4 60,000,000 is $2 \times 10^3 = 2 \times 1,000 = 2,000$ times bigger than 30,000.	b. 70,000 and 200 7×10^4 and 2×10^2 700,000 is $= 3.5 \times 10^2 = 350$ times bigger than 200.	c. 0.0004 and 0.0000003 4×10^{-4} and 3×10^{-7} 0.0004 is $1.3 \times 10^3 = 1.3 \times 1,000 = 1,300$ times bigger than 0.0000003.
d. How many hundreds are in a thousand? There are 10 hundreds in a thousand.	e. How many thousands are in a million? A thousand thousands are in a million.	f. How many pennies are in \$100? There are 10,000 pennies in \$100.

Again, the focus here is not to do operations in scientific notation but rather for students to use place value and powers of ten to make comparisons. For example, on part g. in the table above a student might reason that the difference between a place value of 10^7 and 10^4 is 10^3 . Also 3 goes into 6 two times. Thus a scale factor of 2×10^3 defines how many times bigger sixty million is than 30,000.

If needed estimate each number given in the problems below by writing it in the form of a single digit times an integer power of 10 and use your estimates to approximate each answer.

17. Company A's net profit for the year was \$323,000. Company B's net profit for the year was \$49,500,000. Approximately how many times larger is Company B's profit than Company A's profit?
 5×10^7 and 3×10^5 ; Company B made more than 200 times as much money as Company A.

18. A species of bacteria is 0.00013 decimeters long. A virus is 0.000000012 decimeters long. Approximately how many times longer is the bacteria than the virus?
 1×10^{-4} and 1×10^{-8} ; The bacteria is 10,000 times larger than the virus.

19. The E. coli bacterium is about 0.0000005 meters wide. A hair is about 0.000017 meters wide. Approximately how many times longer the hair is than the bacteria?
 5×10^{-7} and 2×10^{-5} ; The hair is more than 40 times wider than the bacteria.

20. The mass of the earth is about 6×10^{24} kilograms. The mass of Mercury is about 3×10^{23} kilograms. Approximately how many times larger is the mass of Earth than the mass of Mercury?
 Earth is about 20 times larger than Mercury.

Problems similar to the ones above will be visited later on after students have worked with scientific notation in a variety of ways. They will learn that by changing these numbers into scientific notation and dividing they can easily and accurately answer these questions.

8.2a Homework: Place Value and Powers of Ten

1. Write each number given below as a single digit times an integer power of 10.

a. $400,000 = 4 \times 10^5$	b. $70 = 7 \times 10^1$	c. $0.0008 = 8 \times 10^{-4}$
d. $0.000002 = 2 \times 10^{-6}$	e. $90,000,000,000,000 = 9 \times 10^{13}$	f. $0.4 = 4 \times 10^{-1}$

2. Estimate each number by writing it as a single digit times an integer power of 10.

a. The land area of Australia is 2,967,892 square kilometers. 3×10^6	b. The cornea of an eye is 0.00058 meters thick. 6×10^{-4}	c. The distance between Jupiter and Earth is 365,000,000 miles. 4×10^8
d. A large earthquake slowed the rotation of the Earth, making the length of a day 0.00000268 seconds shorter. 3×10^{-6}	e. An average cell phone has 918,440 germs on it. 9×10^5	f. The thickness of a piece of paper is 0.001076 meters 1×10^{-3}

3. Rearrange **all** the digits and decimal below to build a number with the given conditions. If needed write these numbers on a separate piece of paper and cut them out to rearrange them.

7	4	0	2	.
---	---	---	---	---

- Build the largest number
 7420
- Build the smallest number
 $.0247$
- Build a number less than 7
Sample answer; 4.027
- Build a different number less than 6
Sample answer; 2.704
- Build a number between 70 and 80
Sample answer; 74.02
- Build a number than rounds to 70
Sample answer; 70.42
- Build a number that is between 7000 and 8000
Sample answer; 7402
- Build a number that is closest to 4
 4.027
- Build a number that is between 0.7 and 0.8
Sample answer; 0.724

4. At the store 4 pounds of salmon cost \$48, how much would 40 pounds of the same salmon cost?
 40 pounds of the same salmon will cost ten times as much as 4 pounds of salmon. 48 times 10 is \$480.
5. Fill in the blanks.
- $\$12.50 = \underline{\hspace{1cm}}10\hspace{1cm} \times \1.25
 - $\$125 = \underline{\hspace{1cm}}100\hspace{1cm} \times \1.25
 - $\$1,250 = \underline{\hspace{1cm}}1,000\hspace{1cm} \times \1.25

6. The following problem is given to Miranda:

$$23.10 \times 100$$

She states that since she is multiplying by 100 she simply must add two zeros to 23.10. Explain the error in Miranda's thinking and explain how to find the correct answer without using a calculator or long multiplication. **Multiplying by 100 indicates moving the decimal two places to the right; this will give you 2,310.**

7. Write each number as a single digit times an integer power of ten to determine how many times bigger one is compared to the other.

a. 8,000,000 and 300 8×10^6 and 3×10^2 8,000,000 is $2.5 \times 10^4 = 25,000$ times bigger than 300.	b. How many ten dollar bills are in \$100,000? 1×10^5 and 1×10^1 There are 1,000 ten dollar bills in \$100,000	c. 0.007 and 0.000005 7×10^{-3} and 5×10^{-6} 0.007 is $1.4 \times 10^3 = 1,400$ times bigger than 0.000005.
d. 9,000,000 and 300 9×10^6 and 3×10^2 9,000,000 is $3 \times 10^4 = 3 \times 10,000 = 30,000$ times bigger than 300.	e. 70,000 and 200 7×10^4 and 2×10^2 70,000 is $3.5 \times 10^2 = 350$ times bigger than 200.	f. How many millions are in a trillion? 1×10^{12} and 1×10^6 A million millions are in a trillion.

If needed estimate each number given in the problems below by writing it in the form of a single digit times an integer power of 10 and use your estimates to approximate each answer.

8. Website A gets 5,000,000 hits in one day. Website B gets 4,000 hits in one day. Approximately how many times more hits does Website A get than Website B?
 5×10^6 and 4×10^3 ; Website A gets more than 1,000 times more hits than Website B.
9. Picoplankton can be as small as 0.00002 centimeters. Microplankton can be as small as 0.002 centimeters. Approximately how many times larger are Microplankton than Picoplankton?
 2×10^{-5} and 2×10^{-3} ; The Microplankton is 100 times bigger than the Picoplankton.
10. The population of the United States is estimated to be 3×10^8 while the population of the Earth is estimated to be 7×10^9 . Approximately how many times larger is the population of the Earth compared to the population of the United States.
 7×10^9 and 3×10^8 ; The population of the earth is more than 29 times larger than the population of earth

8.2b Class Activity: Scientific Notation Part 1

Think about the following question: Why do you text?



Most of us would agree that texting is a fast and efficient way of communicating. In fact, texting allows us to abbreviate many common phrases. Mathematicians and scientists have a way of expressing really large and really small numbers in a fast and efficient way; it is called **Scientific Notation**. Just like texting allows you to communicate quickly, scientific notation is a special way of writing a number that would otherwise be tedious to write if it were left in standard form.

Scientific notation is very similar to writing a number as the product of a single digit and an integer power of ten, a skill they learned in the previous section.

The four expressions written below represent the same number. Write the number in **Standard Form** on a sheet of paper or mini-white board.

$$\begin{aligned} 500,000 \times 10^{-1} \\ 0.5 \times 10^5 \\ 5 \times 10^4 \\ 5,000 \times 10 \end{aligned}$$

You may need to remind them that a number written in standard form is writing a number using the digits 0-9; this is the form in which they are used to writing numbers.

Discussion: What expression did you use to write the number in standard form? Why?

Check the other expressions to see if they give you the same number.

Are all of these expressions written in scientific notation? (It is important to note that all of these expressions are written in Scientific Notation. However, the expression, 5×10^4 , is the only expression written in what is called “*Normalized Scientific Notation*”. This particular way of writing an expression is often the preferred and only accepted form of scientific notation.)

At this point it is okay that students do not formally know how to change a number from standard form into scientific notation. This discussion is meant to help students begin to surface their ideas and background knowledge about scientific notation. Encourage students to share what they do know about converting between the two forms.

Which of the expressions given below is greater?



$$\begin{aligned} 6 \times 10^{-5} \\ 3 \times 10^{-2} \end{aligned}$$

Once students have decided which expression they think is bigger ask them to discuss their answer with a neighbor. Then move to a group discussion, a common mistake is to ignore the exponents and assume that since 6 is twice as big as 3 the first expression is bigger. Or that since the first expression has an exponent of -5 then it is bigger (even though -5 is smaller than -2). Begin to identify patterns. Ask them what they know about negative exponents. Tell students to write down key ideas in the blank space on this page.

Matching Activity

(Adapted from a MARS task found at: <http://map.mathshell.org/materials/download.php?fileid=1221>)

Cut out the cards and arrows given below. Work with a partner to match the number written in standard form with the number given in scientific notation. Do not worry about the Object and Arrow Cards right now. If there appears to be no match then write the corresponding number on the blank card to make a match. Lay your cards next to each other on your desk and be ready to defend and discuss your answers.

Standard Form	Scientific Notation	Object
0.0001 m	1×10^{-4}	Thickness of a sheet of paper
0.006 m	6×10^{-3} Blank Card	Length of an ant
0.15 m	1.5×10^{-1} m	Length of a pencil
2 m Blank Card	2×10^0 m	Height of the average NBA basketball player
20 m	2.0×10^1 m	Height of a Red Maple tree
60 m	6.0×10^1 m	Wingspan of a Boeing 777 aircraft
300 m	3.00×10^2 m	Length of a cruise ship
8,000 m	8×10^3 m	Height of a mountain
400,000,000 m	4×10^8 m	Distance to the moon from Earth.
$0.006 \times 25 = 0.15$		$20 \times (2 \times 10^7) = 400,000,000$
$60 \times 5 = 300$		$0.15 \times (2 \times 10^3) = 300$

As students are matching the cards they should begin to formalize some general procedures for converting between standard form and scientific notation. After the whole class has finished matching the cards ask someone to explain their procedures, ask them about the cards with a negative exponent. It might be useful to draw the positive number line, mark the point 1, and identify points to the left of 1 as having negative exponents, and points to the right as having positive. Do they prefer to go from standard form to scientific notation or vice versa? Does the exponent always indicate the number of zeros in standard form?

Write the definition for Scientific Notation together as a class.

A number that is in Scientific Notation takes on the form $a \times 10^n$ where a is called the *significant figure* and $1 \leq a < 10$ and n is an integer. The number after the $\times$, or 10^n , is called the *order of magnitude*.

Refer to the Chapter 8 Mathematical Foundation and look at the discussion on significant figures and order of magnitude so you can talk about a context where these terms are applied and important.

The cards on the previous page are mixed up in the student version of the workbook so that they will not know the answers. Provide students with an envelope or paper clip to store their cards for use in a later section.

8.2b Homework: Scientific Notation Part 1

1. The table below includes numbers written in standard form or scientific notation. Change the numbers written in scientific notation into standard form and vice versa. Use a calculator if needed.

Scientific Notation			Standard Form	Scientific Notation			Standard Form
Calculator Notation	Exponent Notation		Standard Number	Calculator Notation	Exponent Notation		Standard Number
Follow the Pattern							
a.	10^0	10^0	1	2×10^0	2×10^0		2
b.	10^1	10^1	10	2×10^1	2×10^1		20
c.	10^2	10^2	100	2×10^2	2×10^2		200
d.	10^3	10^3	1,000	2×10^3	2×10^3		2,000
e.	10^4	10^4	10,000	2×10^4	2×10^4		20,000
Watch for Patterns							
f.	$4 \cdot 10^3$	4×10^3	4,000	$4.2 \cdot 10^3$	4.2×10^3		4,200
g.	$6 \cdot 10^5$	6×10^5	600,000	$6.9 \cdot 10^5$	6.9×10^5		690,000
h.	$7 \cdot 10^8$	7×10^8	700,000,000	$7.12 \cdot 10^8$	7.12×10^8		712,000,000
i.	$8.1 \cdot 10^3$	8.1×10^3	8,100	$8.1 \cdot 10^4$	8.1×10^4		81,000
j.	$4 \cdot 10^9$	4×10^9	4,000,000,000				

2. From the table above, write two things you learned about scientific notation.
See student answer.
3. Complete the following statements:
- In scientific notation, as the exponent power goes up by 1, the standard number's decimal is...
moved one place to the right.
 - In scientific notation, as the exponent power goes down by 1, the standard number's decimal is...
moved one place to the left.

8.2c Class Activity: Scientific Notation Part 2

Recall the definition for scientific notation

A number that is in Scientific Notation takes on the form $a \times 10^n$ where a is called the *significant figure* and $1 \leq a < 10$ and n is an integer. The number after the $\times$, or 10^n , is called the *order of magnitude*.

1. Change these LARGE scientific notation numbers to standard notation and vice versa. Make up a number for the blank cells.

Scientific Notation	Standard Notation	Scientific Notation	Standard Notation
a. 6.345×10^8	634500000	e. 5.32×10^3	5,320
b. 8.04×10^4	80400	f. 4.2×10^5	420,000
c. 4.26×10^5	426000	g. 9.04×10^9	9,040,000,000
d. See student answer	See student answer	h. See student answer	See student answer

2. Now try these SMALL numbers. See if you can figure out the method (one example is given). Make up a number for the blank cells.

Scientific Notation	Standard Notation	Scientific Notation	Standard Notation
Example: 3.2×10^{-3}	0.0032	Example: 5.4×10^{-6}	0.0000054
a. 4.2×10^{-8}	0.000000042	e. 7.5×10^{-4}	0.00075
b. 8.12×10^{-7}	0.000000812	f. 4.005×10^{-3}	0.004005
c. 7.625×10^{-3}	0.007625	g. 9.2×10^{-10}	0.00000000092
d. See student answer	See student answer	h. See student answer	See student answer

3. Express 4,532,344 in scientific notation with 3 significant figures.

4.53×10^6

4. Express 0.00045323 in scientific notation with 2 significant figures.

4.5×10^{-4}

5. Type the following into a calculator: 5,555,555,555 multiplied by 5,555,555,555. What does the answer say?

3.086E19

Be mindful that different calculators display a number in scientific notation in different ways. Be sure to address the differences that may arise.

Some calculators can give you answers in scientific notation. Other calculators have different ways of displaying scientific notation. One way they can display scientific notation is $3.08\text{E}19$. This means 3.08×10^{19} .

6. Write this number in standard form.

$30,800,000,000,000,000$

7. A calculator gives you an answer of $5.025\text{E}-3$, write this number in scientific notation and standard form.

5.02×10^{-3}

.00502

8. A calculator gives you an answer of $9.22\text{E}8$. Write this number in scientific notation and standard form.

9.22×10^8

922,000,000

9. Enter the following problems into your calculator, write the answer in scientific notation and standard form. Express your answer with three significant figures.

a. $(3 \times 10^5) + (5.45 \times 10^6)$

5.75×10^6

5,750,000

b. $(3.2 \times 10^{-2}) - (5.4 \times 10^2)$

-5.39×10^2

-539.968

c. $(2 \times 10^8)(1.4 \times 10^{-3})$

2.80×10^5

280,000

10. Explain why the numbers 402.2×10^{21} and 0.217×10^4 are not written in scientific notation.

In order for a number to be written in scientific notation the value of the significant figure must be greater than or equal to one and less than 10. In the examples given above the significant figure does not fit this criteria.

11. Observe the numbers given below, if the number is written in scientific notation circle it. If it is not written in scientific notation change it to scientific notation. You will need to think about how many spaces you will have to move the decimal and how that will affect the exponent. 

a. 348×10^8 3.48×10^{10}	b. 0.004026×10^9 4.026×10^6	c. 0.00742×10^{-3} 7.42×10^{-6}
d. 45.5×10^{-6} 4.55×10^{-5}	e. 6.05×10^4	f. 3.03554×10^{-7}

Students often struggle with writing numbers in scientific notation if they are not originally in standard form. Talk to them about what direction they are moving the decimal point and how it will affect the order of magnitude. Students might opt to just change the number to standard form and then into scientific notation.

12. As of September 2014 Facebook was worth \$2,000,000,000. Write this number in scientific notation.

2×10^9

13. The diameter of a human hair is 0.000099 meters long. Write this number in scientific notation.

9.9×10^{-5}

You might want to review the metric system before completing the problems below. The metric system is similar to scientific notation in that it uses powers of ten to express the same number in many different ways.

14. A computer at a radio station stores all of the station's music digitally. The computer can display the amount of time it will take to play through its entire library of music. The DJ can choose if she wants to display this total amount of playing time in seconds, minutes, hours, and years. The radio station has about 7,000 songs on the computer that have an average playing time of 3 minutes for each song.

- a. Calculate the total amount of music in minutes that is on the radio station's computer. Write this number in scientific notation.

$7,000 \times 3 = 21,000$ minutes

2.1×10^4 minutes

- b. If the D.J. is planning a playlist for the entire week, should she display the total amount of time in seconds, minutes, hours, days, or years? Convert the playing time into your desired unit of time.

Days would probably be the best unit of time to choose. The total playing time is 14.58 days. The D.J. needs to know if she has enough music to cover the seven days in a week.

15. The mass of a snowflake is approximately 0.000003 kilograms.

- a. Write this number in scientific notation.

3.0×10^{-6} kilograms

- b. If you are only concerned about the mass of one snowflake circle the unit below that would best represent this quantity. Convert the mass of the snowflake to your chosen unit of measurement.

Milligrams

Grams

Kilograms

The mass of the snowflake is 3 milligrams.

- c. Suppose there are approximately 1,000,000 snowflakes in one giant snowball. What unit should you choose to represent the weight of the snowball? Find the mass of the snowball with your chosen unit.

The snowballs weight 3000 grams.

- d. A snowplow is removing snow from a parking lot and dumping it into a dump truck. What unit of measurement would be most appropriate to represent the weight of the snow in the truck?

Kilograms would be the best unit of measurement for the weight of the snow in the truck.

16. A seafloor spreads at a rate of 10 centimeters per year. If you collect data on the spread of the sea floor each week what unit of measurement would be most appropriate to use? Convert the rate at which the seafloor spreads to your chosen unit of measurement.

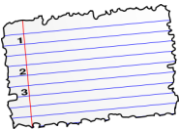
Millimeters

Centimeters

Meters

Millimeters per week would be most appropriate because 10 centimeters per year yields .83 centimeters per month and about .2 centimeters per week. This is approximately 2 millimeters.

17. Change each number below to scientific notation then fill in the blank with the best unit of measure from the column to the right.

a. The E. coli bacteria has a width of 0.0005 <u>millimeters</u>. 5×10^{-4} 	millimeters kilometers meters
b. The acceleration of a bullet is 1,700,000 <u>meters/second²</u>. 1.7×10^6 	meters/second² nanometers/second² miles/second²
c. The thickness of a piece of paper is 0.1 <u>millimeters</u>. 1×10^{-1} 	feet millimeters meters
d. The mass of a dust particle is 0.753 <u>nanograms</u>. 7.53×10^{-1} 	nanograms grams decagrams
e. The consumption of cereal in the United States is 1,350,000,000 <u>kilograms</u>. 1.35×10^9 	nanograms centigrams kilograms
f. The net worth of the richest person in the United States is 46,000,000,000 <u>dollars</u>. 4.6×10^{10} 	pennies dollars nickels
g. The size of a drop of water is .002083 <u>ounces</u>. 2.08×10^{-3} 	pounds ounces tons

Ask them to think about what they are measuring and what kind of unit describes this measurement. A great extension would be to change these numbers to different metric units. For example, in part d ask students how many grams 0.753 nanograms is equivalent to (7.53×10^{-13} g).

8.2c Homework: Scientific Notation

1. Change these LARGE scientific notation numbers to standard notation and vice versa. **Make a number up for the blank cells.**

Scientific Notation	Standard Notation	Scientific Notation	Standard Notation
a. 1×10^{12}	1,000,000,000,000	e. 4.56×10^3	4,560
b. 9.3×10^6	9,300,000	f. 1.22×10^6	1,220,000
c. 7.832×10^{10}	78,320,000,000	g. 1.405×10^9	1,405,000,000
d. See student answer	See student answer	h. See student answer	See student answer

2. Now try these SMALL numbers. **Make a number up for the blank cells.**

Scientific Notation	Standard Notation	Scientific Notation	Standard Notation
a. 5×10^{-4}	0.0005	e. 6.5×10^{-3}	0.0065
b. 6.8×10^{-7}	0.00000068	f. 5.005×10^{-3}	0.005005
c. 3.065×10^{-8}	0.00000003065	g. 7.09×10^{-9}	0.00000000709
d. See student answer	See student answer	h. See student answer	See student answer

3. Change the numbers below into scientific notation.

a. -0.00036
 -3.6×10^{-4}

b. 0.00036
 3.6×10^{-4}

c. $36,000$
 3.6×10^4

d. $-36,000$
 -3.6×10^4

4. Express the numbers below in scientific notation with 3 significant figures.

a. $4,651,284$
 4.65×10^6

b. 0.0005643411
 5.64×10^{-4}

5. A calculator gives you an answer of $4.02\text{E}-6$, write this number in scientific notation and standard form.

4.02×10^{-6}
.00000402

6. A calculator gives you an answer of $2.21\text{E}7$, write this number in scientific notation and standard form.

2.21×10^7
22,100,000

7. Enter the following problems into your calculator, write the answer in scientific notation and standard form. Express your answer with three significant figures.

a. $(2 \times 10^4) + (1.35 \times 10^7)$ 1.35×10^7 13,520,000	b. $(3.2 \times 10^{-8}) - (5.4 \times 10^{-9})$ 2.66×10^{-8} 0.0000000266	c. $(2 \times 10^{11})(1.4 \times 10^{-3})$ 2.8×10^8 280000000
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8. The nucleus of a cell has a diameter of 1 micrometer that is equivalent to 0.000001 meters. Change this number to scientific notation.

1×10^{-6}

9. The length of a DNA nucleotide building block is about 1 nanometer that is 0.000000001 meters. Change this number to scientific notation.

1×10^{-9}

10. Teenagers spend \$13 billion on clothing each year. Change this number to scientific notation. (Go back and look at your place value chart if you don't know how many zeros a billion has.)

1.3×10^{10}

11. A bakery is making cakes for a huge weeklong city celebration. The recipe for each cake calls for 96 grams of sugar. Each cake serves 12 people and the city plans on serving 1500 slices of cake per day for 7 days.

- a. How many total cakes does the bakery need to make?

The bakery needs to make 875 cakes for the celebration.

- b. If the bakery wants to know how much sugar to purchase for the entire event choose the best unit of measurement that would be the most appropriate to use. Find the amount of sugar needed based on the measurement you chose.

Grams

Kilograms

Tons

The bakery needs to purchase 84 kilograms of sugar.

- c. Rosa is very health conscious and wants to know how much sugar is in her piece of cake. Determine the amount of sugar in one piece of cake and label your answer with the appropriate unit of measure.

Each slice of cake has 8 grams of sugar.

Extension: The diameter of an electron is 2.85×10^{-15} kilometers. If you are only concerned about the diameter of one electron circle the unit below that would best represent this quantity. Convert the diameter of the electron to your chosen unit of measurement.

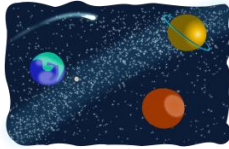
Nanometers

Meters

Kilometers

You should change the unit to 2.85×10^{-4} nanometers.

12. Change each number below to scientific notation then fill in the blank with the best unit of appropriate size from the column to the right.

a. The diameter of the Milky Way is 100,000 _____ light years _____. 1×10^5		feet miles light years
b. The wavelength of the shortest electromagnetic waves is 0.01 _____ millimeters _____. 1×10^{-2}		meters decimeters millimeters
c. The speed of a Banana Slug is 0.00023 _____ meters/second _____. 2.3×10^{-4}		meters/second kilometers/second miles/second
d. The area of the Antarctic Icecaps is 34,000,000 _____ kilometers² _____. 3.4×10^7		millimeters² kilometers² inches²
e. The mass of a train is 72,200,000 _____ kilograms _____. 7.22×10^7		grams centigrams kilograms
f. The world's petroleum production is 3,214,000,000,000 _____ liters _____. 3.214×10^{12}		cups milliliters liters

8.2d Class Activity: Multiplying and Dividing with Scientific Notation

In a previous section you were asked how many millions are in a trillion. Scientific notation can help you answer this question with ease.

1. Begin by writing these two numbers in standard form and then changing them to scientific notation.

	Standard Form	Scientific Notation
One Million	1,000,000	1×10^6
One Trillion	1,000,000,000,000	1×10^{12}

2. What operation should you use if you want to compare these numbers? (Hint: Remember it is asking how many millions are in a trillion.)

In order to determine how many times one number fits into another number you must divide.

3. Write this problem out with the correct operation using scientific notation.

$$1 \times 10^{12} \div 1 \times 10^6 = \frac{1 \times 10^{12}}{1 \times 10^6}$$

When numbers are written in scientific notation the problem above can be solved rather quickly. The problems below will help you practice the skills you will need to do this. You will return to the problem above on the next page.

4. Discuss with a partner what properties of exponents you will use to help simplify the problems below.

Use these properties to simplify each expression. **The Power of a Product and Power of a Quotient**

Properties can be used to simplify the expressions below.

a. $10^4 \times 10^3$ 10^7	b. $10^{-3} \times 10^5$ 10^2	c. $\frac{10^6}{10^3} 10^3$	d. $10^4 \div 10^6 10^{-2}$
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5. Discuss the multiplication problem $(5 \times 3)(2 \times 8)$ with your class. Write your thoughts below.

Of course it most natural to just multiply 15 times 16. But could you rewrite the problem as $(5 \times 2)(3 \times 8)$ or $(5 \times 8)(2 \times 3)$ or even $5 \times 3 \times 2 \times 8 = 5 \times 3 \times 2^4$? Is the answer the same? Why can you do this? You can rewrite this expression because of the commutative property of multiplication.

6. Rewrite this problem $(5.1 \times 10^5)(6.8 \times 10^3)$ like the problem above (group the powers of 10 together). Then solve the problem (use exponent properties) and write the solution.

$$(5.1 \times 10^5)(6.8 \times 10^3) = (5.1 \times 6.8)(10^5 \times 10^3) = 34.688 \times 10^8$$

Give students time to experiment with the problems below. Writing the factors in a different order is not the only method for simplifying these expressions. Let your students change the numbers to standard form and then multiply or divide them. This will let them see how efficient the "re-ordering" method is.

7. Use the same method to evaluate the problems below.

a. $(6.9 \times 10^2)(3.5 \times 10^5)$ Rewrite the problem: $(6.9 \times 3.5)(10^2 \times 10^5)$ Problem solution: $24.15 \times 10^7 = 2.415 \times 10^8$	b. Solve the problem: $(1.9 \times 10^3)(2.4 \times 10^6) =$ 4.56×10^9	c. Solve the problem: $\frac{(7.2 \times 10^5)}{(3.6 \times 10^2)} = 2 \times 10^3$
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8. Find each product or quotient. Write your answer in scientific notation.

a. $\frac{2.3958 \times 10^3}{1.98 \times 10^7}$ 1.21×10^{-4}	b. $(7 \times 10^5)(3.5 \times 10^{-3})$ 2.45×10^3	c. $\frac{3.006 \times 10^8}{7.3 \times 10^3}$ 4.12×10^4
d. What is 3 millionths multiplied by 7 ten-thousandths? 2.1×10^{-9}	e. $(3.1 \times 10^{-3}) \times 562.1$ 1.743×10^0	f. How much is 40% of 140 million? 5.6×10^7
g. $\frac{30}{1.2 \times 10^5}$ 2.5×10^{-4}	h. $(5 \times 10^5)(0.4)$ 2×10^5	i. What percent of (1.3×10^6) is (6.5×10^5) ? 50%
Encourage students to use a calculator as they operate on the decimals. They can then find the power of ten in their head.		

9. Return back to the problem at the beginning of the section. If we want to figure out how many millions are in a trillion what operation will help us achieve this? **Division**

a. Use the method discovered above to perform this operation.

$$\frac{(1 \times 10^{12})}{(1 \times 10^6)} = 1 \times 10^6 = 1,000,000. \text{ There are one million millions in a trillion.}$$

b. Now try it to find out how many thousands are in a trillion.

$$\frac{(1 \times 10^{12})}{(1 \times 10^3)} = 1 \times 10^9 = 1,000,000,000. \text{ There are one billion thousands in a trillion.}$$

Use scientific notation to answer each question

10. In the world, approximately 1,146,000,000 people speak Chinese as their first language, while, 341,000,000 people speak English as their first language. Approximately how many times more people speak Chinese than English as their first language?

$$\frac{1 \times 10^9}{3 \times 10^8} = .333 \times 10^1 \approx 3.3$$

The number of people that speak Chinese as their first language is about 3.3 times as many as the number of people that speak English as their first language.

Students might be interested to note that Hindi (the official language of India and Pakistan) is the second most common language with 3.66×10^8 people speaking it as a first language.

11. The thickness of a dollar bill is .010922 cm. The thickness of a dime is .135 cm. How many times thicker is a dime compared to a dollar bill?

$$\frac{1.35 \times 10^{-1}}{1.0922 \times 10^{-2}} = 1.23 \times 10^1 ; \text{ A dime is about 12 times thicker than a dollar bill.}$$

12. A millipede's leg is 4.23×10^{-3} cm long.

- a. How long is the millipede's leg in standard form?

0.00423 cm.

- b. Despite its name a millipede does not really have 1000 legs. If it did, what would the length be if you could line up all the legs of a 1,000 leg millipede end to end?

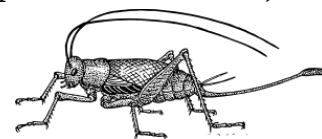
$$(4.23 \times 10^{-3})(1 \times 10^3) = 4.23 \times 10^0 = 4.23$$

The millipede's legs lined up end to end will be 4.23 cm.

13. A cricket weighs 3.88×10^{-2} ounces. How many crickets are in a pound (a pound has 16 ounces)?

$$\frac{1.6 \times 10^1}{3.88 \times 10^{-2}} = 4.12 \times 10^2$$

There are approximately 412 crickets in a pound.



14. Determine the thickness of one sheet of paper in this book. Show your work and solution using scientific notation. Express your answer in units of appropriate size. See student answer.

This would be a great problem to do the last five minutes of class on an exit slip.

8.2d Homework: Multiplying and Dividing with Scientific Notation

1. Write each number in scientific notation.

a. 0.0006033×10

6.033×10^{-3}

b. 0.000142×10^{-4}

1.42×10^{-8}

c. 322×10^5

3.22×10^7

d. 13.5×10^{-7}

1.35×10^{-6}

2. Find the product or quotient for the following. Negative exponents are acceptable.

a. $10^{-4} \times 10^2$

10^{-2}

b. $10^{-5} \times 10^{-2}$

10^{-7}

c. $10^3 \div 10^5$

10^{-2}

d. $10^4 \div 10^{-2}$

10^6

3. Find each product or quotient. Write your answer in scientific notation.

a. $(7.2 \times 10^{-4}) \times (2.8 \times 10^{-3})$ 2.016×10^{-6}	b. $\frac{2.35 \times 10^8}{4.3 \times 10^3}$ 5.47×10^4	c. $(8.4 \times 10^6) \times (1.3 \times 10^6)$ 1.092×10^{13}
d. $\frac{3.1748 \times 10^4}{2.07 \times 10^8}$ 1.534×10^{-4}	e. $(5 \times 10^6)(4.5 \times 10^{-4})$ 2.25×10^3	f. $\frac{1.005 \times 10^7}{6.3 \times 10^2}$ 1.6×10^4
g. What is 4 millionths multiplied by 5 ten-thousandths? 2×10^{-9}	h. $(4.2 \times 10^{-3}) \times 44,462.1$ 1.867×10^2	i. How much is 30% of 170 million? 5.1×10^7

j. $\frac{3.15 \times 10^{-8}}{6.1 \times 10^2}$ 5.16×10^{-11}	k. $\frac{1.3 \times 10^{-4}}{0.3 \times 10^{-1}}$ 4.33×10^{-3}	l. 145,000 is what percent of 4 million? 3.6%
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4. In a class action lawsuit, 4,000 claimants were offered an \$800 million settlement. How much is that per claimant? Change the numbers into scientific notation to calculate.

$$\frac{8 \times 10^8}{4 \times 10^3} = 2 \times 10^5 = 200,000$$

Each claimant will get \$200,000.



5. A cable company earned \$125 million in one year. The next year they earned \$312.5 million dollars. Estimate how many times bigger their profit was the second year compared to the first year.

$$\frac{3.125 \times 10^8}{1.25 \times 10^8} = 2.5 \times 10^0 = 2.5$$

The cable company made about 2.5 times more money the second year compared to the first year.

6. There are about 6.022×10^{23} atoms of hydrogen in a mole of hydrogen. How many hydrogen atoms are in 3.5×10^3 moles of hydrogen?

$$(6.022 \times 10^{23})(3.5 \times 10^3) = 21.077 \times 10^{26} = 2.1077 \times 10^{27}$$

There are 2.1077×10^{27} atoms in 3.5×10^3 moles of hydrogen.

7. During the year 2013 approximately 7.07×10^9 pennies were minted (made by the U.S. Mint). In the year 2000 approximately 1.43×10^{10} were minted. Estimate how many times more pennies were minted in the year 2000 compared to the year 2013. Give a possible explanation for the decline.

$$\frac{1.43 \times 10^{10}}{7.07 \times 10^9} = 0.2 \times 10^1 = 2$$

There were about twice as many pennies minted in the year 2000 than in 2013. This could possibly be due to the fact that more people pay for things electronically (debit or credit card) now.

8.2e Class Activity: More Operations with Scientific Notation

- Will the method for multiplying and dividing numbers in scientific notation work for adding and subtracting numbers in scientific notation? **No, since you are combining the operations of addition or subtraction with multiplication the commutative property does not hold.**



Lead a discussion with students:

Consider the numbers 5,000,000 and 2,000,000. If you were to add these numbers you would most likely add $5 + 2$, which equals 7. Then, since they both have six zeros you would add six zeros to get 7,000,000. The same can be done with numbers in scientific notation.

- Rewrite 5,000,000 and 2,000,000 in scientific notation.

$$5,000,000 = 5 \times 10^6$$

$$2,000,000 = 2 \times 10^6$$

Lead a discussion with students:

Notice that they both have the same exponent. Similarly, if you add these numbers in scientific notation you can simply add $5+2$ to get 7 and keep the 10^6 . You get 7×10^6 .

- Test the method you learned above to see if it works for subtraction. First subtract 2,000,000 from 5,000,000. Then change the numbers to scientific notation and subtract them using the method above to see if you get the same answer.

$$5,000,000 - 2,000,000$$

$$= 3,000,000$$

$$5 \times 10^6 - 2 \times 10^6 = 3 \times 10^6$$

You may want to give your students a few more examples to experiment with for them to see the pattern and formulate a method for adding and subtracting numbers with the same power of ten. You can also think about the example above like adding like terms. You have 5 millions and 2 millions when you add them together to get 7 millions. Just like when you add $5x$ and $2x$ you get $7x$. Or similarly, when you add $\frac{1}{7}$ and $\frac{3}{7}$ you get $\frac{4}{7}$.

- Write in your own words how to add or subtract numbers in scientific notation that have the same exponent or order of magnitude.

See student answer.

After you have given students an opportunity to write their method in their own words; summarize a method together as a class. Make sure that everyone understands that they can perform the desired operation on the significant digits and keep the order of magnitude the same. Of course this will only work when the powers of ten are the same. We will discuss what to do if they are not the same shortly.

- Find each sum or difference. Write your answer in scientific notation.

a. $(3.45 \times 10^3) + (6.11 \times 10^3)$ 9.56×10^3	b. $(8.96 \times 10^7) - (3.41 \times 10^7)$ 5.55×10^7	c. $(6.43 \times 10^0) + (4.39 \times 10^0)$ 1.082×10^1
d. $(1.23 \times 10^{-4}) + (8.04 \times 10^{-4})$ 9.27×10^{-4}	e. $(4.5 \times 10^{11}) - (3.2 \times 10^{11})$ 1.3×10^{11}	f. $(6.1 \times 10^{-8}) - (3.2 \times 10^{-8})$ 2.9×10^{-8}

6. You might be wondering what to do if the numbers do not have the same order of magnitude. Write down your ideas of how you might be able to add or subtract these numbers. Be ready to share your ideas with the class. **See student answer.**

Talk about student ideas from number 6 by using an example and leading a discussion. Encourage students to use the space provided to write the example and their ideas.

- Try adding 4 million and 6 billion. Start by writing these numbers in standard form and adding them with long addition.

$$4,000,000 + 6,000,000,000 = 6,004,000,000$$

- Write these numbers in scientific notation and add them using the method on the previous page to see if it works for addition and subtraction.

$$(4 \times 10^6) + (6 \times 10^9) = 10 \times 10^7$$

- Why does this method not work for the problem above? How could you change your numbers so that this method will work?

The numbers must have the same order of magnitude for the method on the previous page to work. You can change the order of magnitude on a number written in scientific notation by moving the decimal in one or both of the numbers so that your powers of ten are the same.

To add or subtract numbers in scientific notation:

- Make sure they have the same exponent or order of magnitude. If they don't, move the decimal so they do.
- Add or subtract the significant figures and keep the order of magnitude the same.
- Write your final answer in scientific notation.

$$(a \times 10^n) + (b \times 10^n) = (a + b) \times 10^n$$

$$(a \times 10^n) - (b \times 10^n) = (a - b) \times 10^n$$

Try it out with the problems given below.

7. Find each sum or difference. Write your answer in scientific notation.

a. $(4.12 \times 10^6) + (3.94 \times 10^4)$ $(412 \times 10^4) + (3.94 \times 10^4) =$ $415.94 \times 10^4 = 4.1594 \times 10^6$	b. $(4.23 \times 10^3) - (9.56 \times 10^2)$ $(42.3 \times 10^2) - (9.56 \times 10^2) =$ 3.274×10^3	c. $(3.4 \times 10^{-3}) + (4.57 \times 10^{-2})$ $(3.4 \times 10^{-3}) + (4.57 \times 10^{-2}) =$ $(.34 \times 10^{-2}) + (4.57 \times 10^{-2}) =$ 4.91×10^{-2}
d. $(3.67 \times 10^3) - (1.6 \times 10^{-1})$ $(3.67 \times 10^3) - (1.6 \times 10^{-1}) =$ $(36700 \times 10^{-1}) - (1.6 \times 10^{-1}) = 36698.4 \times 10^{-1}$ $= 3.66984 \times 10^3$	e. $(8.41 \times 10^{-5}) - (7.9 \times 10^{-6})$ $(8.41 \times 10^{-5}) - (7.9 \times 10^{-6}) =$ $(84.1 \times 10^{-6}) - (7.9 \times 10^{-6}) = 76.2 \times$ $= 7.62 \times 10^{-5}$	f. $(6.91 \times 10^{-2}) + (2.4 \times 10^2)$ $(6.91 \times 10^{-2}) + (2.4 \times 10^2) =$ $(.000691 \times 10^2) + (2.4 \times 10^2) =$ 2.401×10^2

Problem Solving (use scientific notation where possible)

8. The earth is 9.3×10^7 miles from the sun. Pluto is 3.67×10^9 miles from the sun. How far is it to Pluto from Earth? (Hint: Draw and label a picture.)

$$(3.67 \times 10^9) - (9.3 \times 10^7) =$$

$$(367 \times 10^7) - (9.3 \times 10^7) = 357.7 \times 10^7$$

$$= 3.577 \times 10^9$$

The distance from Earth to Pluto is 3.577×10^9 miles.

9. Pretend a new planet has been found in the far reaches of the universe.
- a. You know the earth is 9.3×10^7 miles from the sun and the planet you are interested in is 7.3×10^{12} miles beyond the sun in the opposite direction of the earth. What is the distance to the planet from Earth? (Hint: Draw and label a picture)
- The distance from earth to the new planet is approximately 7.3×10^{12} miles.
- b. Using the distance you found above and the fact that light travels at 5.88×10^{12} miles in one light year. Determine how many light years it will take for light to travel to this planet from Earth. It will take approximately 1.24 light years for light to travel to this planet.

8.2e Homework: More Operations in Scientific Notation

1. Find each sum or difference. Write your answer in scientific notation.

a. $(2.3 \times 10^3) + (6.2 \times 10^3)$
 8.5×10^3

b. $(9.8 \times 10^2) + (2.72 \times 10^4)$
 2.818×10^4

c. $0.456 + (2.3 \times 10^5)$
 2.3×10^5

d. $(7.23 \times 10^7) - (6.08 \times 10^6)$
 6.622×10^7

e. $(2.3 \times 10^5) - (2.01 \times 10^6)$
 -1.78×10^6

f. $(8.9 \times 10^{-7}) + (9.6 \times 10^{-6})$
 1.049×10^{-5}

g. What is ten thousand plus 125,000?
 1.35×10^5

h. What is the difference between 4 hundredths and 8 ten thousandths?
 3.9×10^{-2}

i. $(1.6 \times 10^{-4}) - (9.6 \times 10^{-3})$
 -9×10^{-3}

2. The areas of 4 major oceans on the Earth are shown in the table below. Estimate how many square miles the oceans cover all together.

All of the oceans cover 1.3014×10^8 square miles.

Ocean	Area (sq miles)
Arctic	5.44×10^6
Atlantic	3.18×10^7
Indian	2.89×10^7
Pacific	6.40×10^7

3. Estimate how many more square miles the Atlantic Ocean covers than the Arctic Ocean. The Atlantic Ocean is approximately 2.636×10^7 square miles bigger than the Arctic Ocean.
4. The surface area of the earth is 19.69×10^7 square miles. Find the percentage of Earth that is covered by the oceans listed above.
Approximately 66% of the Earth is covered by the oceans listed above.
5. The mass of the Sun is about 1.98×10^{30} kg. The mass of the Earth is about 5.97×10^{24} kg. Estimate how many times bigger the mass of the Sun is than the mass of the Earth. The mass of the Sun is approximately 3.32×10^5 times the mass of the Earth.
6. A neutron has a mass of 1.67×10^{-27} kg and an electron has a mass of 9.11×10^{-31} kg. Determine how many times smaller the mass of the electron is than the mass of the neutron. The mass of the electron is approximately 1.83×10^3 times smaller than the neutron.

8.2f Class Activity: Matching, Ordering, and Problem Solving with Scientific Notation.

Return to the cards that you cut out in the matching activity in section 8.2b.

1. Rematch each Standard Form card with its Scientific Notation card. Don't worry about the Object and Arrow Cards right now.
2. Order your matches on your desk from least to greatest.
3. Collect all the Object Cards and match each Object Card with its numerical value. Note that a meter is about the length from the tip of your nose to the tip of your finger if you hold out your arm to the side of your body at a right angle. Check to see if the order that you placed your measurement cards in makes sense with the heights of each object.
4. Collect all the Arrow Cards and place them between a pair of measurement/object cards to estimate how much bigger one object is than the other. Do this for as many pairs as possible.

Extension: Once you have completed the four tasks above mount your cards on a poster board showing all of the corresponding matches with the arrow cards comparing the objects. Draw a picture next to each object and display it in the classroom.

5. In the table below, sort the numbers given in the first column into the correct cells to help you order the numbers.

Numbers	Positive Numbers	Numbers Greater than 1	Greatest
1.68×10^{-1}	3.78×10^6	3.78×10^6	3.78×10^6
-3.403×10^{-2}	3.39×10^{-1}		3.39×10^{-1}
-4.53×10^2	1.68×10^{-1}	Numbers between 0 and 1	1.68×10^{-1}
-7.21×10^2	1.68×10^{-2}	3.39×10^{-1}	1.68×10^{-2}
3.78×10^6		1.68×10^{-1}	-3.403×10^{-2}
3.39×10^{-1}	Negative Numbers	1.68×10^{-2}	-2.11×10^1
1.68×10^{-2}	-3.403×10^{-2}	Numbers between 0 and -1	-4.53×10^2
-2.11×10^1	-2.11×10^1	-3.403×10^{-2}	-7.21×10^2
	-4.53×10^2	Numbers Less than -1	
	-7.21×10^2	-2.11×10^1	
		-4.53×10^2	
		-7.21×10^2	Least

There is an interactive website where you can do the above activity on the computer. It is found at http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.EXPO&lesson=html/video_interactives/exponents/exponentsInteractive.html

For numbers 6 and 7 order the numbers from least to greatest.

6. -2.3×10^4 , 5.6×10^{-1} , -1.6×10^{-4}
 -2.3×10^4 , -1.6×10^{-4} , 5.6×10^{-1}

7. -4.3×10^{-3} , -1.5×10^{-4} , 7.4×10^{-4}
 -4.3×10^{-3} , -1.5×10^{-4} , 7.4×10^{-4}

Consider how to structure your class as they work on #9 below. You could split your class into group and have them work in pairs within their group. Each pair pick two problems from section 1 and the problem in section 2.

8. Write one million in as many ways as you can.

Answers may include, but are not limited to $1,000,000$, 10^6 , 1×10^6 , $\frac{1,000,000}{1}$.

9. To continue working with very large numbers, problem solve to answer the following questions. Be prepared to explain your problem solving process and solution.

Section 1	Section 2
a. How long is a million days in years? A million days is 2.74×10^3 years. b. How long is a million days in hours? A million days in hours is 2.4×10^7 hours. c. How far is a million inches in miles? A million inches 15.8 miles. d. If you laid a million one-dollar bills end to end, how far would they reach? The dollar bills would reach 96.91 miles.	e. At one time, McDonald's had sold more than a billion hamburgers (far more now). If it were possible to eat a hamburger every minute of every day (day and night) without stopping, how long would it take to eat a billion hamburgers? Express your answer in appropriate units of time. It will take 1902.58 years to eat a billion hamburgers. 

8.2f Homework: Matching, Ordering, and Problem Solving with Scientific Notation

1. In the table below, sort the numbers given in the first column into the correct cells to help you order the numbers.

Numbers	Positive Numbers	Numbers Greater than 1	Greatest
-4.57×10^2	4.65×10^7	4.65×10^7	4.65×10^7
7.36×10^2	7.36×10^2	7.36×10^2	7.36×10^2
-1.403×10^{-3}	3.44×10^{-2}	Numbers between 0 and 1	3.44×10^{-2}
4.65×10^7	3.44×10^{-3}	3.44×10^{-2}	3.44×10^{-3}
3.44×10^{-3}		3.44×10^{-3}	-1.403×10^{-3}
-5.21×10^2	Negative Numbers	Numbers between 0 and -1	-1.14×10^1
3.44×10^{-2}	-1.14×10^1	-1.403×10^{-3}	-4.57×10^2
-1.14×10^1	-4.57×10^2		-5.21×10^2
	-5.21×10^2	Numbers Less than -1	
	-1.403×10^{-3}	-1.14×10^1	
		-4.57×10^2	
		-5.21×10^2	Least

For numbers 2 and 3 order the numbers from least to greatest.

2. $-4.3 \times 10^4, 4.2 \times 10^{-1}, 4.6 \times 10^{-4}$
 $-4.3 \times 10^4, 4.6 \times 10^{-4}, 4.2 \times 10^{-1}$
3. $1.4 \times 10^{-4}, -2.3 \times 10^{-2}, -1.5 \times 10^4$
 $-1.5 \times 10^4, -2.3 \times 10^{-2}, 1.4 \times 10^{-4}$

As you work on the problems below, try to think about how you might use scientific notation to help you. Be prepared to explain your methods and solutions.

4. Calculate the following in relationship to your age on your next birthday. Write your answer in scientific notation. **Answers given below are for a 14 year old.**

a. How many days have you been alive? A fourteen year old had been alive for 5.11×10^3 days.	b. How many hours have you been alive? A fourteen year old has been alive for 1.2264×10^5 hours.
c. How many minutes have you been alive? A fourteen year old has been alive for approximately 7.36×10^6 minutes.	d. How many seconds have you been alive? A fourteen year old has been alive for approximately 4.42×10^8 seconds.

Extension:

Counting one number per second how long does it take to count to...

- a. ...a million in minutes?

It takes approximately 1.67×10^4 minutes to count to a million.

- b. ...a million in hours?

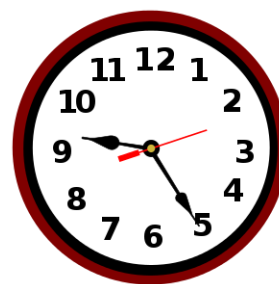
It takes approximately 277 hours to count to a million.

- c. ...a million in days?

It takes approximately 11.5 days to count to a million.

- d. ...a million in weeks?

It takes approximately 1.6 weeks to count to a million.



Talk about the fact that a quantitative fact can be expressed in a number of different ways. We choose how we represent something often to meet a specific purpose. For example it's pretty impressive that it would take 11.5 days to count to a million; it makes it seem so much larger. You could also ask-given this information, how many days would it take to count to a billion or even a trillion (1000, or 1,000,000 times as long as 1,157 or 1,157,000 days).

8.2g Class Activity: Problem Solving with Scientific Notation



Task 1: Taxes and the National Debt

We read in the newspapers that the United States has a 15 trillion dollar debt. Assume that there are 300 million working people in the United States.

- a. Estimate the national debt per person?

The national debt per person is about \$50,000.

Be sure to do these problems before class so that you can have an idea of what questions students might have.

For some students you will need to provide some problem solving strategies for them.

Ask them probing questions such as: What information are you given? What do you need to know? What operation should you use?

Tameka works at a retail store. Assume the following statements apply to her wages.

- Tameka has a job at which she earns \$10 per hour.
- 18% of her pay check goes to federal taxes.
- All of these taxes go towards paying off the national debt.
- Tameka works 2×10^3 hours a year.

- b. Estimate how many hours will she have to work to pay off her share of the national debt.

Tameka will have to work approximately 27,788 hours to pay for her portion of the national debt.

- c. Estimate how many years will it take Tameka to pay off her portion of the national debt.

Tameka will have to work approximately 13.89 or 14 years to pay of her portion of the national debt.

Task 2: Computers

On the computer a *byte* is a unit of information. A typical document contains many tens of thousands of bytes, and so it is useful to use the words below to describe storage capacity for items related to a computer.

1 kilobyte=1000 bytes

1 megabyte=1000 kilobytes

1 gigabyte=1000 megabytes

1 terabyte=1000 gigabytes

Scientific Notation

1 kilobyte= 1×10^3 bytes

1 megabyte= 1×10^3 kilobytes

1 gigabyte= 1×10^3 megabytes

1 terabyte= 1×10^3 gigabytes

- a. Rewrite each of these terms using scientific notation (use the space given above).

- b. Calculate how many bytes are in each of these terms. Write your answer in scientific notation.

1 kilobyte= 1×10^3 bytes

1 gigabyte= $(1 \times 10^3)(1 \times 10^6) = 1 \times 10^9$ bytes

1 megabyte= $(1 \times 10^3)(1 \times 10^3) = 1 \times 10^6$ bytes

1 terabyte= $(1 \times 10^9)(1 \times 10^3) = 1 \times 10^{12}$ bytes

- c. My computer has a memory (storage capacity) of 16 gigabytes, how many bytes of memory is this?

The computer has 1.6×10^{10} or 16,000,000,000 bytes of memory.

- d. How many computers like the one above do you need to have in order to get 1 terabyte of memory?

You will need approximately 63 computers like the one above to get 1 terabyte of memory.

Remind students that they will need to have the same units in order to compare the storage of the computer with the amount of storage that the novel takes up.

- e. An online novel consists of about 250 megabytes. How many novels can I store on my 16 gigabyte computer? You can store approximately 64 novels on the 16 gigabyte computer.

8.2g Homework: Problem Solving with Scientific Notation

Task 1: Gasoline

Gas 'N' Go Convenience Stores claim that 10% of Utahans fuel up at their stores each week. Decide whether their claim is true using the following information. Explain your answer.



- There are about 2.85×10^6 people in Utah.
- There are 2.18×10^2 Gas 'N' Go stores in Utah.
- Each station serves gasoline to about 1.2×10^3 people each week.

Gas 'N' Go's claims are not true, their stores do not serve 10% of the population in Utah. 10% of Utah's population can be found by finding 10% of 2.85×10^6 .

$$(.10 \times 10^0)(2.85 \times 10^6) = .285 \times 10^6 = 285,000$$

Thus 10% of the population is 285,000. Now we need to see how many people Gas 'N' Go serves each week by multiplying the number of stores by the number of people each store serves a week.

$$(2.18 \times 10^2)(1.2 \times 10^3) = 2.616 \times 10^5 = 261,600$$

Since 261,000 is less 285,000 the claims are not true.

Task 2: Time

Many chemical and physical changes happen in extremely small periods of time. For that reason the following vocabulary is used.

Scientific Notation

1 microsecond = 1000 nanoseconds

1 microsecond = 1×10^3 nanoseconds

1 millisecond = 1000 microseconds

1 millisecond = 1×10^3 microseconds

1 second = 1000 milliseconds

1 second = 1×10^3 milliseconds

a. Rewrite each of these terms using scientific notation (use the space given above).

b. How many nanoseconds are in a millisecond?

There are $(1 \times 10^3)(1 \times 10^3) = 1 \times 10^6$ nanoseconds in a millisecond.

c. How many nanoseconds are in a second?

There are $(1 \times 10^6)(1 \times 10^3) = 1 \times 10^9$ nanoseconds in a second.

d. How many nanoseconds are in a hour?

There are $(1 \times 10^9)(\underbrace{3.6 \times 10^3}_{\text{seconds in an hour}}) = 3.6 \times 10^{12}$ nanoseconds in an hour.

Online conversion calculators are very easy to find. They are a great tool for students to check their answers. However, be sure that they show their work so that you know that they did not just use the conversion calculator to answer each question.

Extension:

e. I can download a byte of information in a nanosecond. How long will it take to download a typical book (250 megabytes)? Express your answer in appropriate measures of time.

It will take .25 seconds to download a 250 megabyte book.

f. How long will it take to download the Library of Congress (containing 35 million books)? Express your answer in appropriate measures of time. It will take approximately 101.3 days to download the library of congress.

8.2h Self-Assessment: Section 8.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. <i>See sample problem #1</i>				
2. Convert a number between scientific notation and standard form. <i>See sample problem #2</i>				
3. Perform operations with numbers expressed in scientific notation. <i>See sample problem #3</i>				
4. Choose units of appropriate size for measurements of very large or very small quantities. <i>See sample problem #4</i>				
5. Interpret scientific notation that has been generated by technology. <i>See sample problem #5</i>				
6. Use scientific notation to problem solve with really small and really large numbers. <i>See sample problem #6</i>				

Sample Problem #1

Every day there is an estimated 329,000 smart phones bought in the United States.¹ Every day there is an estimated 12,000 smart phones lost or stolen in the United States.² Approximately how many times more smart phones are bought than are lost or stolen.

3×10^5 and 1×10^4

There are approximately 30 times more smart phones bought each day then are lost or stolen.

Sample Problem #2

Change the numbers below into scientific notation.

a. $3,450,000,000 = 3.45 \times 10^9$

b. $0.00000000455 = 4.55 \times 10^{-9}$

Change the number given below into standard form.

c. $6.03 \times 10^8 = 603,000,000$

d. $1.2 \times 10^{-6} = 0.0000012$

Sample Problem #3

Perform the indicated operation for each problem below.

a. $(3.13 \times 10^8) + (2.9 \times 10^9) = 3.213 \times 10^9$

b. $2.54 \times 10^{-4} - 3.2 \times 10^{-5} = 2.22 \times 10^{-4}$

c. $(3 \times 10^6)(5.6 \times 10^{-8}) = 1.68 \times 10^{-1}$

d. $\frac{1.0004 \times 10^8}{7.2 \times 10^2} = 1.4 \times 10^5$

Sample Problem #4

Fill in the blank with a unit of appropriate size from the column to the right.

a. The mass of trash produced by New York City in one day is 1.2×10^7 _____kilograms_____.	kilograms nanograms grams
b. The period of the sun's orbit around the galaxy is 2.4×10^8 _____years_____.	seconds hours years
c. The area of the Earth's land surface is 1.49×10^8 _____kilometers ² _____.	millimeters ² meters ² kilometers ²

1-<http://appleinsider.com/articles/14/02/20/apples-iphone-led-2013-us-consumer-smartphone-sales-with-45-share---npd>,
<http://www.latimes.com/business/technology/la-fi-tn-45-million-smartphones-lost-stolen-2013-20140417-story.html>,
<http://techcrunch.com/2013/09/19/gartner-102b-app-store-downloads-globally-in-2013-26b-in-sales-17-from-in-app-purchases/>

Sample Problem #5

- a. A calculator gives you an answer of $3.023\text{E}-3$, write this number in scientific notation and standard form.
 3.023×10^{-3}
0.003023
- b. A calculator give an you answer of $9.2 \cdot 10^5$, write this number in scientific notation and standard form.
 9.2×10^5
920,000

Sample Problem #6

In the year 2013 the U.S. mint produced 2.112×10^9 dimes.

- a. Estimate the value of this money?
 $\frac{2.112 \times 10^9}{1 \times 10^1} = 2.112 \times 10^8 = 211,200,000$. This is approximately 211 million dollars worth of dimes.
- b. Every second 175 cups of coffee are bought at America's most popular coffee shop.² The average cup of coffee at this particular shop costs \$1.85. At this rate how long will it take for America to spend the 211 million dollars worth of dimes produced in 2013 on coffee at this shop? Express your answer using appropriate units of time.
It will take America about 7.54 days or little over a week to spend 211 million dollars on coffee at this coffee shop.

²http://www.boston.com/business/articles/2011/09/17/starbucks_looks_for_way_to_encourage_paper_cup_recycling/

Section 8.3: Volume of Cylinders, Cones, and Spheres

Section Overview:

Throughout this section, students are solving real-world and mathematical problems involving volumes of cylinders, cones, and spheres. Students begin by deriving the volume of a cylinder, relying on their knowledge from previous grades that the volume of a right three-dimensional object can be found by taking the area of its base and multiplying it by the height. Students then use the formula for the volume of a cylinder to arrive at the formulas for the volumes of a cone and sphere. Using concrete models of these three-dimensional objects, students physically compare the volume of a cone to the volume of a cylinder. Students then manipulate the formula for the volume of a cylinder to reflect these differences, arriving at the formula for the volume of a cone. They use a similar process to derive the formula for the volume of a sphere. Once students understand where these formulas come from, they apply them to solve real-world problems, knowing when and how to use the formulas.

Concepts and Skills to be mastered:

By the end of this section students should be able to:

1. Find the volume of a cylinder, cone, and sphere given a radius and height.
2. Find a missing measurement (height, radius, or diameter) for a cylinder, cone, or sphere given the volume.
3. Use the formulas for the volumes of cylinders, cones, and spheres to solve a variety of real-world problems.

It is particularly important to read the Mathematical Foundation for this section. It gives a more detailed and historical discussion of how the formulas for the volume of a cylinder, cone, and sphere are derived, specifically, Cavalieri's principle and the packing of three (polyhedral) cones in a cube.

There are some great tasks found at the following links that are related to the content in this section. The Popcorn task would be a good one to do with cylinders and the Meatball task would be good to do with spheres. You can even use them as an assessment.

<http://threeacts.mrmeyer.com/popcornpicker/>
<http://www.101qs.com/2352-meatballs>

8.3a Class Activity: Wet or Dry (This activity is optional)



We have been discussing exponents throughout this chapter. You have learned how to simplify expressions with exponents in them and have looked at how expressing numbers in scientific notation can better help us deal with numbers that are really big and really small. Exponents are also used to find the volume of a three-dimensional object.

1. Describe what volume is. Compare it to finding perimeter or area.

Answer may include that volume describes the amount of space that an object takes up or hold.
Common misconceptions might include that volume describes that area.

To help us better understand how important it is to know how to find the volume of a three-dimensional object do the following activity.

2. Choose two different sizes of cylindrical cans to use for this activity. Measure the diameter and the height of each can **in centimeters**.

Can 1: Diameter _____ Height _____

Can 2: Diameter _____ Height _____

3. As a group determine the volume of each can. Show your work below or explain how you found the volume of your cans. Make sure that your units are correct. Once you have found the volume in cubic centimeters change your answer to millimeters. (Hint: One cubic centimeter is the same as one milliliter.)

Can 1

Can 2

You could provide several different empty soup cans for students to choose from. Be sure to calculate the volume of the can before you do the activity and remove any labels that may state the volume. In order to measure the water you will need a graduated cylinder and a bucket of water, it is best to do this part of the activity outside. Do not let students use the graduated cylinder to calculate the volume. This activity is intended for students to start thinking critically about volume and how it is calculated. Do not tell the students how to find the volume of a cylinder or provide the formula. It definitely brings out the need to attend to precision.

Select one of your cans and bring it up to the teacher with your calculation for the volume of the can. Also, select one member of the team to test your calculations.

This will be done by having the teacher fill the can with water over a team member's head. If they calculate too much the water will overflow onto their head and if their calculations are too low then all the water will get dumped on their head.

4. Which can did your team choose and why did you choose this can?

See student answer.

5. How close were your calculations to the actual volume of the can?

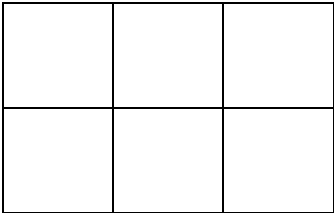
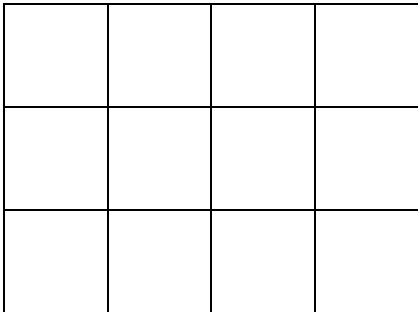
See student answer.

6. What would you do differently if you could recalculate the volume of your can?

See student answer.

8.3b Class Activity: Volume of Cylinders

1. Gunner just started his summer job doing swimming pool maintenance. He has a variety of things to do for each pool. For each item below fill in the missing measurement in the space provided for each pool.
 - a. He needs to build a fence around each of the swimming pools below. If each unit represents one meter determine how much fencing he needs for each pool. Write your answer below each pool in the appropriate spot. **See below.**
 - b. Gunner now has to cover each pool. Determine how much material he will need to cover each pool. Write your answer below each pool in the appropriate spot. **See below.**
 - c. After Gunner has put up a fence and knows how much material he needs to cover the pools he needs to fill the pools back up with water. Determine how much water he would need to fill each pool to a depth of one meter. Write your answer below each pool in the appropriate spot. **See below.**
 - d. Now determine of much water he would need to fill each pool to a depth of 2 meters. Continue filling in the chart to 10 meters deep for each pool. **See below.**

Pool #1	Pool #2
	
Perimeter: 10m Area: 6m^2 1 meter deep volume: $1 \cdot 6 = 6\text{m}^3$ 2 meter deep volume: $2 \cdot 6 = 12\text{m}^3$ 3 meter deep volume: $3 \cdot 6 = 18\text{m}^3$ 4 meter deep volume: $4 \cdot 6 = 24\text{m}^3$ 10 meter deep volume: $10 \cdot 6 = 60\text{m}^3$	Perimeter: 14m Area: 12m^2 1 meter deep volume: $1 \cdot 12 = 12\text{m}^3$ 2 meter deep volume: $2 \cdot 12 = 24\text{m}^3$ 3 meter deep volume: $3 \cdot 12 = 36\text{m}^3$ 4 meter deep volume: $4 \cdot 12 = 48\text{m}^3$ 10 meter deep volume: $10 \cdot 12 = 120\text{m}^3$

2. Describe how to find the volume of the pool for any given depth. **To find the volume of the pool for any given depth, multiply the area of the pool by the depth of the pool.**

Students have dealt with the formulas for the volume of prisms and pyramids in previous grades. If you feel that your students basic knowledge of volume is lacking consider using colored linking cubes in this activity. Have your students build the various volumes for Pools 1 and 2. The cubes will allow them to see how volume is composed of the area of the base times the depth or height of the pool.

Be sure to solidify the following before moving on.

Volume of a *right* geometric solid is often described by the equation $V = Bh$ where V is the volume, B is the area of the base and h is the height. This was solidified in 7th grade, in 6th grade students used $V = lwh$ to describe the volume of prism. Be sure to point out the relationship between the two forms, why the B is capitalized and the h is not, and how the orientation of the figure does not change its base.

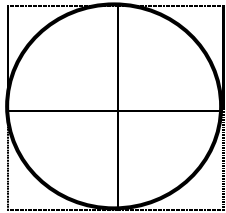
3. Explain how the formula $V = Bh$ helps you find the volume.



The equation $V = Bh$ describes multiplying the area of the base of the solid by the height.

4. Gunner has one more pool to work on. Use what you know about the formula above to fill in the missing information for Pool #3. Recall that each unit represents 1 meter.

Pool #3



Perimeter: $C = 2\pi r = 2\pi \cdot 1 \approx 6.283 \text{ m}$

Area: $A = \pi r^2 = \pi \cdot 1^2 = \pi \text{ m}^2$

1 meter deep volume: $1\pi \approx 3.142 \text{ m}^3$

2 meter deep volume: $2\pi \approx 6.283 \text{ m}^3$

3 meter deep volume: $3\pi \approx 9.425 \text{ m}^3$

4 meter deep volume: $4\pi \approx 12.566 \text{ m}^3$

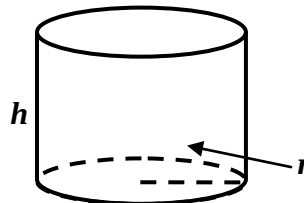
10 meter deep volume: $10\pi \approx 31.416 \text{ m}^3$

5. What type of three-dimensional object is Pool #4?

The pool is a cylinder.

6. Use the picture given below to describe how to find the volume of a Cylinder. Be sure to describe each part of the formula and how it relates to the formula $V = Bh$.

A cylinder is a solid obtained by taking a circle in a plane (called the base) and drawing it out in a direction perpendicular to the base for a distance h (called the height).

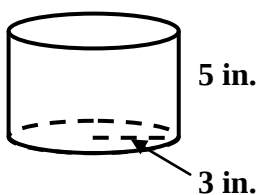


The volume of a cylinder is found by multiplying the area of the base by the height.

$V = \pi r^2 h$
 Area of the base $\rightarrow \pi r^2$ $\leftarrow$ Height of the cylinder h

Directions: Find the volume for each cylinder described below. If needed draw and label a picture.

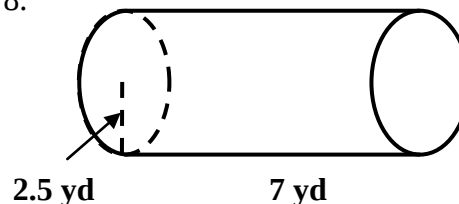
7.



$$V = 45\pi \text{ in}^3$$

$$V \approx 141.37 \text{ in}^3$$

8.



$$V = 43.75\pi \text{ yd}^3$$

$$V \approx 137.45 \text{ yd}^3$$

9. Cylinder with a Radius = 21 mm and Height = 19 mm. $V = 8,379\pi \text{ mm}^3$ $V \approx 26,323.41 \text{ mm}^3$	10. Cylinder with a Diameter = 8.8 cm and Height = 9 cm. $V = 174.24\pi \text{ cm}^3$ $V \approx 547.39 \text{ cm}^3$
Point out that in order to find the exact volume you must write your answer in terms of π. Have a discussion about irrational numbers, talking about how we can see exactly how much is contained, but we cannot write it as a ratio of whole numbers. This is similar to what we did in Chapter 7 when we were asked to describe the length of the side of a square. Ask students to show the approximate volume and the exact volume for some of the problems.	

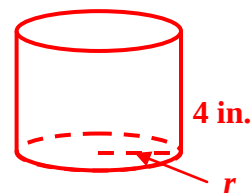
Directions: Find the missing measurement for each cylinder described below.

11. The volume of a cylinder is 117.1 cubic feet, and its height is 15 ft. Find the diameter of the base of the cylinder. $d \approx 3.15 \text{ ft}$	12. The volume of a cylinder is 4,224.8 cubic millimeters, it has a diameter of 16.4 mm, find the height of the cylinder. $h \approx 20 \text{ mm}$ Extension: Find the circumference of the base of the cylinder. $C = 2\pi r = 2\pi \cdot 8.2 \approx 51.522 \text{ mm}$
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Directions: For each problem given below draw and label a picture that describes each cylinder. Then solve the problem.

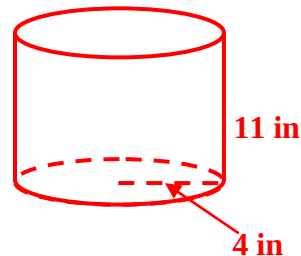
13. An ice cream company wants to package a pint of ice cream in a circular cylinder that is 4 inches high. A pint is 16 fluid ounces and 1 fluid ounce is 1.8 cubic inches. What does the radius of the base circle have to be?

$V = 16(1.8) = 28.8 \text{ in}^3$ thus $28.8 = \pi r^2 \cdot 4$. Upon solving for r you get $r \approx 1.51 \text{ in}$.



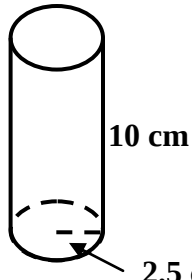
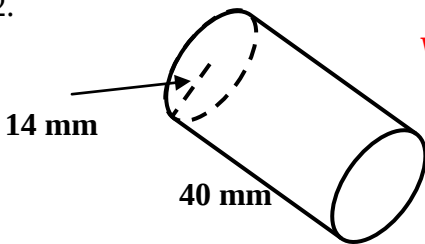
14. For a science project, Hassan put a can out to collect rainwater. The can was 11 inches tall and had a diameter of 8 inches. If it rained exactly 20 cubic inches each day, how many days did it take to fill the can?

The volume of the can is $V = \pi(4)^2 \cdot 11 = 552.92 \text{ in}^3$. Since 552.92 divided by 20 is 27.6, it will take approximately 28 days to fill the can.



8.3b Homework: Volume of Cylinders

Directions: Find the volume for each cylinder described below. If needed draw and label a picture.

1.  10 cm 2.5 cm $V = 196.35\text{cm}^3$	2.  14 mm 40 mm $V = 24630.086\text{mm}^3$
3. Cylinder with a radius of 2 ft and a height of 7 ft. The volume of the cylinder is 88ft^3.	4. Cylinder with a diameter of 2.7 m and a height of 30 m. The volume of the cylinder is 171.8m^3.

Find the missing measurement for each cylinder described below.

5. The volume of a cylinder is 63.6 cubic inches, and its height is 9 inches. Find the diameter of the base of the cylinder. The diameter of the base of the cylinder is 3 inches. Extension: Find the circumference of the base of the cylinder. $C = 2\pi r = 2\pi(0.75) = 4.71\text{ in.}$	6. The volume of a cylinder is 8,685.9 cubic ft, it has a diameter of 19.2 ft, find the height of the cylinder. The height of the cylinder is 30 ft.
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Directions: For each problem given below draw and label a picture that describes each cylinder. Then solve the problem.

7. What is the volume of Keisha's thermos if it has a radius of 2.5 in at the opening and 10 in for a height?
- The volume of Keisha's thermos is 196.35in^3 .

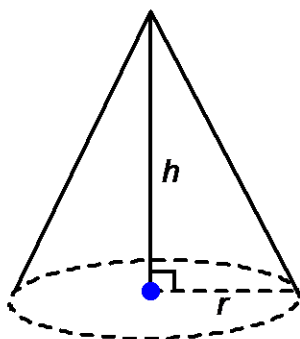
8. Mr. Riley bought 2 cans of paint to paint his garage. Each can had a radius of 5.5 inches and a height of 8 inches. How many cubic inches of paint did he buy in all?

Mr. Riley has 1520.53 cubic inches of paint.

8.3c Class Activity: Volume of Cones

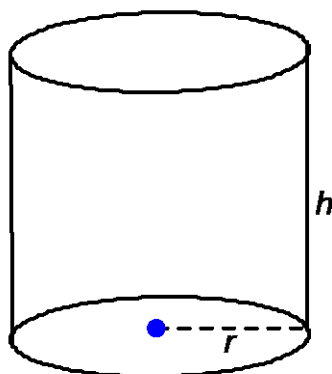


Recall from seventh grade, that a cone is a three-dimensional figure with a circular base. A curved surface connects the base and the vertex.

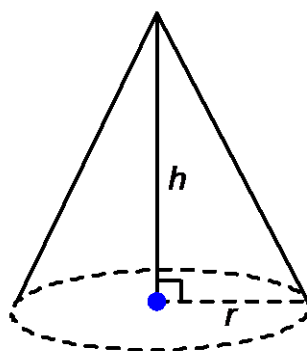


In order to derive the equations for a cone and sphere you will want to use special manipulatives to compare their volumes to a cylinder. You will need a cylinder, cone, and hemisphere that all have the same radius and height. To find the formula for the sphere the height of the cylinder needs to be equal to its radius as well as the radius of the hemisphere. You can reference utahmiddleschoolmath.org to find where to get these resources. Once again, it is highly recommended that you reference the Mathematical Foundation for this section for a more comprehensive explanation of the derivation of these formulas.

The cylinder and cone given below have the same height and their bases are congruent.



$$V = \pi r^2 h$$



$$V = \frac{1}{3} \pi r^2 h$$

- Predict how the volume of the cone compares to the volume of the cylinder.
See student answer.
- If you fill the cone with water or other filling material, predict how many cones of water will fit into the cylinder.
See student answer.

The Cube/Tetrahedron construction depicted in the Mathematical Foundation is a great way to see the relationship between the volume of a cone and cylinder.

- Now try it. How many cones fit into the cylinder?
Approximately 3 cones fit into one cylinder of water.

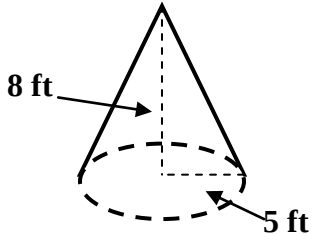
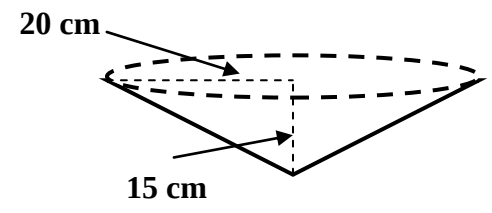
Ask students to write down the equation for the volume of the cylinder below the cylinder.

- About what fraction of the cylinder is filled by the volume of one cone?
One third of the cylinder will be filled by the volume of the cone.
- Manipulate the equation for the volume of the cylinder to show the volume of the cone.

The volume of the cone is equal to one third of the volume of the cylinder. Thus $V = \frac{1}{3} \pi r^2 h$

- Explain in your own words how the volume of a cone compares to the volume of a cylinder. Describe the parts of the formula for the volume of a cone. Write this formula below the cone in the picture above.
See student answer.

Directions: Find the volume for each cone described below. If needed draw and label a picture.

7.  The volume of the cone is $V = \frac{200}{3} \pi \text{ ft}^3$ $V \approx 209.4 \text{ ft}^3$	8.  The volume of the cone is $V = 2000\pi \text{ cm}^3$ $V \approx 6283.19 \text{ cm}^3$
9. A cone with a radius of 8.4 feet and a height of 5.5 feet. The volume of the cone is approximately 406.4 ft^3.	10. A cone with a diameter of 9 meters and a height of 4.2 meters. The volume of the cylinder approximately 89.1 m^3.

Directions: Find the missing measurement for each cylinder described below. Round your answer to the nearest tenth.

11. The volume of a cone is 122.8 cubic inches, and its height is 4.5 inches. Find the diameter of the base of the cone. The diameter of the base of the cone is 10.2 inches.	12. The volume of a cone is 188.5 cubic ft, it has a diameter of 12 ft, find the height of the cylinder. The height of the cone is 5 ft.
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For each problem given below draw and label and picture that describes each cylinder. Then solve the problem.

13. Salt and sand mixtures are often used on icy roads. When the mixture is dumped from a truck into the staging area, it forms a cone-shaped mound with a diameter of 10 feet and a height of 6 feet. What is the volume of the salt-sand mixture?

The volume of the salt-sand mixture is 157.08 cubic feet.

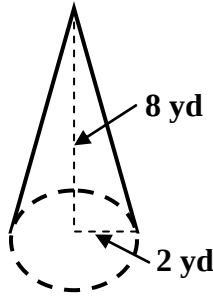
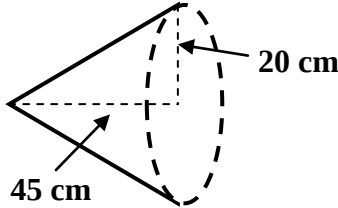
14. A glass in the shape of a cone has a diameter of 8 cm. If the glass has a volume of 200 ml (or 200 cubic centimeters), what is the greatest depth that a liquid can be poured into the glass? Explain.

The greatest depth the liquid could be is 11.94 cm.

There is a great experiment related to the Towers of Hanoi in the Mathematical Foundation for this section. It would be a good activity to do in an Honors Class.

8.3c Homework: Volume of Cones

Directions: Find the volume for each cone described below. If needed draw and label a picture.

1.  $V = \frac{32}{3}\pi \text{ yd}^3$ $V \approx 33.51 \text{ yd}^3$	2.  $V = 6000\pi \text{ cm}^3$ $V \approx 18,849.6 \text{ cm}^3$ The volume of the cone is approximately 18,849.6 cubic centimeters.
3. A cone with a radius of 40 feet and a height of 100 feet. $V = \frac{1,600,000}{3}\pi \text{ ft}^3$ $V \approx 1,675,515 \text{ ft}^3$	4. A cone with a diameter of 4.2 meters and a height of 5 meters. $V = 7.35\pi \text{ m}^3$ $V \approx 23.1 \text{ m}^3$ The volume of the cone is 23.1 m³.

Directions: Find the missing measurement for each cone described below.

5. The volume of a cone is 37.7 cubic inches, and its height is 4 inches. Find the diameter of the base of the cone. The diameter of the base of the cone is approximately 6 inches.	6. The volume of a cone is 628.3 cubic ft, it has a diameter of 20 ft, find the height of the cone. The height of the cone is approximately 6 ft.
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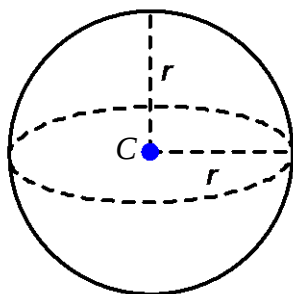
Directions: For each problem given below draw and label a picture that describes each cylinder. Then solve the problem.

7. The American Heritage Center at the University of Wyoming is a conical building. If the height is 77 feet, and the area of the base is about 38,000 square feet, find the volume of air that the heating and cooling systems would have to accommodate.
The volume of the air is about 975,333.3 cubic feet.
8. A stalactite, a geological formation, in the Endless Caverns in Virginia is cone-shaped. It is 4 feet long and has a diameter at its base of 1.5 feet.
 - a. Assuming that the stalactite forms a perfect cone, find the volume of the stalactite.
The volume of the stalactite is 2.4 cubic feet.
 - b. The stalactite is made of calcium carbonate, which weighs 131 pounds per cubic foot. What is the weight of the stalactite? The stalactite weighs 314.4 pounds.

8.3d Class Activity Volume of Spheres

Recall that a sphere is a set of points in space that are a distance of r away from a point C , called the center of the sphere.

n#



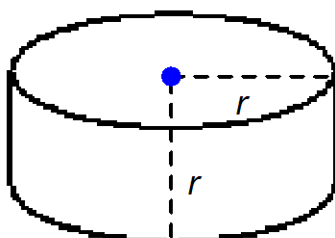
$$V = \frac{4}{3}\pi r^3$$

Just like you compared the volume of a cone to the volume of a cylinder to find the formula for the volume of a cone you are going to compare the volume of a sphere to the volume of a cylinder.

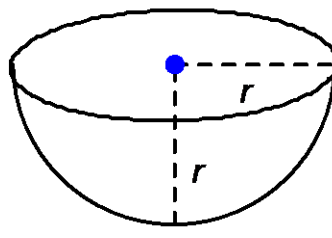
In this case it will be easier to deal with a **hemisphere** and then double the formula. Remember that a hemisphere is half of a sphere. If we can find the formula for the volume of a hemisphere we can simply double all of our numbers to get the volume of a sphere.

Be sure to use the manipulatives mentioned in the previous section as you do this lesson with your students.

The cylinder and hemisphere given below have the same radius and the height of the cylinder is also the same as its radius.



$$V = \pi r^2 \cdot r = \pi r^3$$



$$V = \frac{2}{3}\pi r^3$$

A possible extension could be will the volume of an inscribed hemisphere be more or less than the inscribed cone? Explain your answer.

1. Predict how the volume of the hemisphere compares to the volume of the cylinder. Which one holds more volume?

See student answer. You can discuss how the hemisphere will have less volume than the cylinder because if you were to put the hemisphere inside of the cylinder it would fit perfectly except the empty space at the bottom of the hemisphere.

2. If you fill the hemisphere with water or other filling material, predict what fraction of the cylinder is filled by the volume of one hemisphere.

See student answer.

3. Now try it, what fraction of the cylinder is filled by the volume of one hemisphere?

Upon experimentation, students will find that roughly $\frac{2}{3}$ of the cylinder is filled with the volume of the hemisphere.

It may be helpful to mark the cylinder into thirds with a non-permanent marker so that students can see the water/material level.

4. Write down the formula for the volume of the cylinder below the cylinder, be sure to write your height in terms of the radius or r . See above. Because the height and the radius are the same in the cylinder we use r to represent the height.

5. Manipulate the equation for the volume of the cylinder to show the volume of the hemisphere.

The volume of the hemisphere is $\frac{2}{3}$ of the cylinder. This is shown in the equation, $V = \frac{2}{3} \pi r^3$.

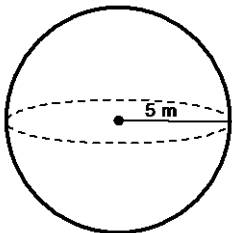
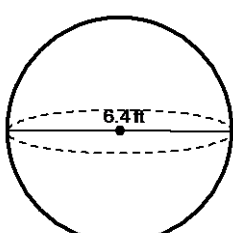
6. In number 10 you found the volume for a hemisphere. Adjust this formula to find the volume of a sphere.

$$V = \left(\frac{2}{3} \pi r^3 \right) \cdot 2 = \frac{4}{3} \pi r^3$$

7. Explain in your own words how the volume of a sphere compares to the volume of a cylinder. Describe the parts of the formula for the volume of a sphere. Write this formula below the sphere in the picture on the previous page. See student answer.

Once again refer to the Mathematical Foundation as it illustrates this and other connections between these solids and their volumes. It explains how the volume of a cylinder is the sum of a cone and hemisphere.

Directions: Find the volume for each sphere described below. If needed draw and label a picture.

8.  The volume of the sphere is 523.6 m^3	9.  The volume of the sphere is 137.3 ft^3
10. A sphere with a radius of 1.3 yds. The volume of the sphere is 9.2 yd^3.	11. A sphere with a diameter of 25 inches The volume of the sphere is $8,181.2 \text{ in}^3$.

Directions: Find the missing measurement for each sphere described below. Round your answer to the nearest tenth.

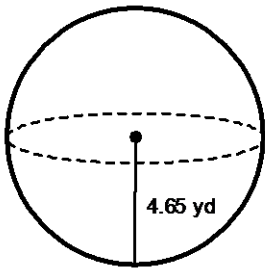
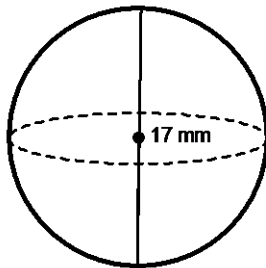
12. The volume of a sphere is 6882.3 in^3; find the diameter of the sphere. The diameter of the sphere is 23.6 inches.	13. The volume of a sphere is 1436.8 ft^3; find the radius of the sphere. The radius of the sphere is 7 ft.
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Directions: For each problem given below draw and label a picture that describes each sphere. Then solve the problem.

14. If a golf ball has a diameter of 4.3 centimeters and a tennis ball has a diameter of 6.9 centimeters, find the difference between the volumes of the two balls.
The volume of a tennis ball is roughly 130.38 cubic centimeters bigger than the volume of the golfball.
15. Kauri pours the water out of a cylindrical flower vase with a height of 5 inches and a radius of 4 inches into a spherical flower vase. The spherical vase has a radius of 4 inches. Will the water overflow? If so, by how much? If not, how much space is left in the spherical vase? The water will not overflow; there will be approximately 16.8 cubic inches of space left in the spherical vase.

8.3d Homework: Volume of Spheres

Directions: Find the volume for each sphere described below. If needed draw and label a picture.

1.  The volume of the sphere is approximately 421.2 yd^3	2.  The volume of the sphere is $2,572.4 \text{ mm}^3$
3. A sphere with a radius of 10 yards. The volume of the sphere is $4,188.8 \text{ yd}^3$.	4. A sphere with a diameter of 60 inches The volume of the sphere is $113,097.3 \text{ in}^3$.

Directions: Find the missing measurement for each sphere described below.

5. The volume of a sphere is 113.1 cm^3; find the diameter of the sphere. The diameter of the sphere is 6 cm.	6. The volume of a sphere is 4,188.8 cubic feet; find the radius of the sphere. The radius of the sphere is 10 ft.
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Directions: For each problem given below draw and label a picture that describes each sphere. Then solve the problem.

7. The diameter of the moon is 3,476 kilometers. Approximate the volume of the moon.
The volume of the moon is approximately $2.2 \times 10^{10} \text{ km}^3$.
8. Find the volume of the empty space in a cylindrical tube of three tennis balls. The diameter of each ball is 2.5 inches. The cylinder is 2.5 inches in diameter and is 7.5 inches tall.
There is approximately 12.27 cubic inches of empty space in the cylinder.

8.3e Class Activity: Volume of Cylinders, Cones, and Spheres



Task 1: Silos

A silo is a storage bin that is a cylinder with a hemisphere on top. A farmer has a silo with a base radius of 30 feet and a storage height of 100 feet. The “storage height” is the part which can be filled with grain - it is just the cylinder. A cubic foot of grain weighs 62 lbs.

- a. Draw and label a picture of the silo

See drawing.

Depending on the needs of your class you may decide to do just some of these problems. You could assign different problems to different groups or assign certain groups to become the “expert” teacher for a certain problem and have them teach the class as a whole. This gives students the opportunity to critique the reasoning of others.

- b. How many pounds of grain can the farmer store in the silo?

The farmer can store approximately 17,530,087 pounds of grain.

- c. How high (including the hemispherical top) is the silo?

The height of the silo is 130 feet.

- d. One thousand square feet of wheat produces 250 pounds of grain. The farmer’s wheat field is 3,500 feet by 20,000 feet. Is the silo large enough to hold the grain? By how much? Explain your answer.

The silo is large enough to hold the wheat by 30,087 lbs.

- e. If the farmer decides to fill the silo all the way to the top of the hemisphere how many cubic feet of grain can he store?

If the farmer fills the silo all the way to the top of the hemisphere he can hold 339,292 cubic feet of grain.

A snow cone consists of a cone filled with flavored shaved ice topped with hemisphere of flavored shaved ice. The cone is 4 inches long and the top has a diameter of 3 inches.

- 8WB8-85

Task 3: Pipes

Which will carry the most water? Explain your answer.

- Two pipes each 100 cm tall. One with a 3 cm radius and the other with a 4 cm radius
- One pipe that is also 100 cm tall with a 5 cm radius.

The two pipes will carry a total of 7854 cubic centimeters of water while the one pipe will also carry 7854 cubic centimeters of water. Thus they both carry the same amount of water.

Task 4: Fruit

A cantaloupe a diameter of 23 centimeters and a Clementine orange has a diameter of 7 centimeters. Predict how many times bigger the cantaloupe is than the orange. Then calculate the volume of each fruit to determine how many times bigger the cantaloupe is than the orange.

The volume of the cantaloupe is approximately $6,370.62 \text{ cm}^3$ and the volume of the orange is approximately 179.59 cm^3 . The cantaloupe is approximately 35.5 times bigger than the orange.

8.3e Homework: Volume of Cylinders, Cones, and Spheres

Task 1: Containers

A cylindrical glass 7 cm in diameter and 10 cm tall is filled with water to a height of 9 cm. If a ball 5 cm in diameter is dropped into the glass and sinks to the bottom, will the water in the glass overflow? If it does overflow, how much water will be lost? Explain and justify your response.

The volume of the water is 346.4 cubic centimeters. The volume of the sphere is 65.45 cubic centimeters. The total volume of the water and the sphere is 411.8 cubic centimeters. The volume of the cylinder is 384.9 cubic centimeters. Since the volume of the water and sphere is greater than the volume of the cylinder that means that the water will overflow by 26.9 cubic centimeters.

Task 2: Ice Cream

Izzi's Ice Cream Shoppe is about to advertise giant spherical scoops of ice cream 8 cm in diameter! Izzi wants to be sure there is enough ice cream and wonders how many scoops can be obtained from each cylindrical container of ice cream. The containers are 20 cm in diameter and 26 cm tall.

- a. Draw and label a picture of the ice cream containers and the scoop of ice cream.
See student answer.
- b. Determine the number of scoops of ice cream one container will give her?
Izzi will be able to get about 30 scoops of ice cream from each container.
- c. Ingrid purchases one of these famous giant scoops of ice cream but does not get to it fast enough and the ice cream melts! The radius of the cone and the ice cream (sphere) is 4 cm and the height of the cone is 10 cm. Will all of the melted ice cream fit inside the cone? The volume of the cone is 167.6 cubic centimeters and the volume of the ice cream sphere is 268 cubic centimeters. Thus the ice cream will not fit into the cone when melted.
- d. If it does fit, how much more ice cream will fit in the cone? If it doesn't fit, how many cubic centimeters of ice cream does she need to eat before it melts in order to make it fit?
Ingrid needs to eat 100.5 cubic centimeters of ice cream before it melts so that the cone does not overflow.

8.3f Class Activity: Banana Splits

Materials: graph paper, string, rulers, pen or pencil, banana, ice cream scoop

Use any of the materials on your table to approximate the volume of your banana and one scoop of ice cream. Be prepared to show and explain all your methods and your results.

1. What is your estimate for the volume of the peeled banana (include units)? _____

Show how you found this volume.

There are many ways in which students might go about finding the volume. The most common will probably be cutting the bananas with string into several cylinders and two cones. They can then find the volume of all of the cylinders and cones and add them together.

Be sure to set a standard unit of measurement for your class; either inches or centimeters. Try to get a bunch of bananas that are roughly the same size. If you would like to find the most accurate measurement for the volume of the banana to compare your students answers to; water displacement will probably work best. Recall that one cubic centimeter equals 1 milliliter. You could also talk about how the more cylinders they cut their banana into the more accurate they will be on calculating the volume.

2. What is your estimate for the volume of one scoop of ice cream (include units)? _____

Show how you found this volume.

To find the volume of the scoop of ice cream use the radius of the ice cream scoop to calculate the volume of the sphere of ice cream.

Make sure that the ice cream scoop that you provide is one that is a perfect hemisphere.

3. Comment on each of the other groups' methods and results. Compare their strategies and their results to yours. **See student answer.**

4. How do you think you could have a more accurate approximation for the volume of the banana?

See student answer.

5. How do you think you could have a more accurate approximation for the volume of the scoop of ice cream?

See student answer.

6. If you make your banana split sundae with one banana, 3 scoops of ice cream, and 2 Tbsp chocolate syrup, what will be the total volume of your sundae?

(Hint: $1 \text{ Tbsp} \approx 14.8 \text{ cm}^3$ and $1 \text{ in}^3 \approx 16.4 \text{ cm}^3$)

Answers will vary based off of the size of bananas that you provide your class.

8.3g Self-Assessment: Section 8.3

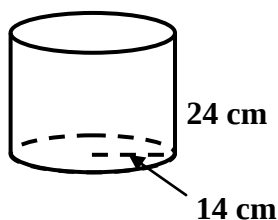
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Find the volume of a cylinder, cone, and sphere given a radius and height. <i>See sample problem #1</i>				
2. Find a missing measurement (height, radius, or diameter) for a cylinder, cone, or sphere given the volume. <i>See sample problem #2</i>				
3. Use the formulas for the volumes of cylinders, cones, and spheres to solve a variety of real-world problems. <i>See sample problem #3</i>				

Sample Problem #1

Find the volume for each object described below. Find the exact volume and the approximate volume rounded to the nearest hundredth.

- a. The cylinder pictured below.



$$V = 4,704\pi \text{ cm}^3$$

$$V \approx 14,778.05 \text{ cm}^3$$

- b. A cone with a radius of 3 ft and a height of 10 ft.

$$V = 30\pi \text{ ft}^3$$

$$V \approx 94.25 \text{ ft}^3$$

- c. A glass tree ornament is a gold sphere. The diameter of the ornament is 4 inches.

$$V = \frac{32}{3}\pi \text{ in}^3$$

$$V \approx 33.51 \text{ in}^3$$

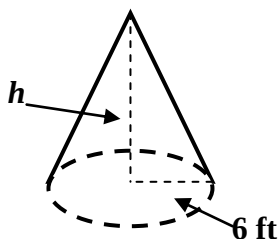
Sample Problem #2

Find the missing measurement for each object described below. Draw and label a picture if needed.

- a. The volume of a regular can of soda pop is approximately 23.7 in^3 . The height of the can is 4.83 inches. Find the diameter of the can.

The diameter of the can is approximately 2.5 inches.

- b. The volume of the cone below is approximately 377 ft^3 . Find the height of the cone.



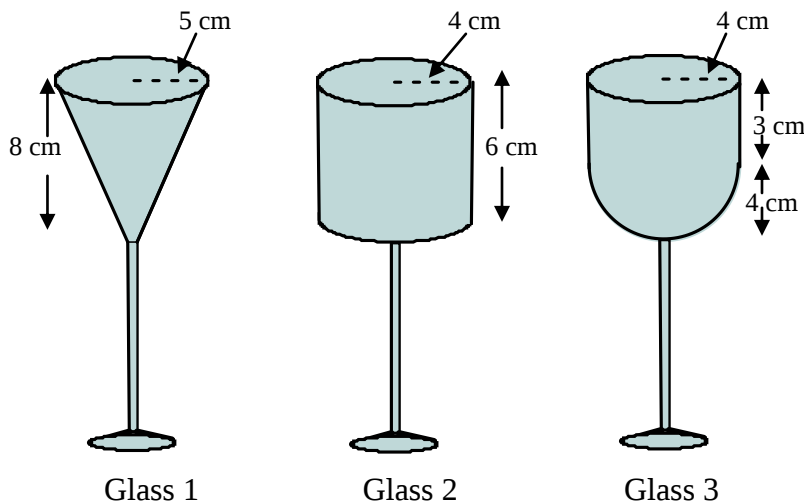
The height of the cone is approximately 10 ft.

- c. A sphere has a volume of 113.1 mm^3 . Find the radius of the sphere.

The radius of the sphere is 3 millimeters.

Sample Problem #3

Suzy is throwing a party and is choosing from the glasses below to serve her punch. Use the information below to answer the questions that follow.



- The shape of Glass 1 is a cone with a radius of 5 cm and a height of 8 cm.
- The shape of Glass 2 is a cylinder with a radius of 4 cm and a height of 6 cm.
- The shape of Glass 3 is a hemisphere with radius of 4 cm with a cylinder on top of it with a radius of 4 cm and a height of 3 cm.

- a. Suzy wants to choose the glass that has the smallest volume so that she doesn't have to use as much punch. Find the volume of each glass to determine which glass she should choose.

The volume of Glass 1 is $V = \frac{200}{3} \pi \text{ cm}^3$.
 $V \approx 209.44 \text{ cm}^3$

The volume of Glass 2 is $V = 96\pi \text{ cm}^3$.
 $V \approx 301.59 \text{ cm}^3$

The volume of Glass 3 is $V = \frac{272}{3} \pi \text{ cm}^3$.
 $V \approx 284.83 \text{ cm}^3$

Suzy should choose Glass 1 because it has the smallest volume and will hold the least amount of punch.

- b. Suzy really wants to use the cylinder shaped glass. What would the approximate height of the cylinder shaped glass need to be to hold the same amount of punch as the cone shaped glass. Would this be practical?

The height of the cylinder would need to be about 41.1 cm. It would not be practical to have a glass that is this tall.