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Table of Contents

CHAPTER 9 GEOMETRY: TRANSFORMATIONS, CONGRUENCE, AND SIMILARITY	2
9.0 ANCHOR PROBLEM: CONGRUENCE AND SIMILARITY	8
9.1 RIGID MOTION AND CONGRUENCE	9
9.1a Class Activity: Properties of Translations.....	10
9.1a Homework: Properties of Translations.....	14
9.1b Class Activity: Properties of Reflections.....	16
9.1b Homework: Properties of Reflections	23
9.1c Class Activity: Properties of Rotations.....	26
9.1c Homework: Properties of Rotations.....	30
9.1d Class Activity: Properties of Rotations cont.	32
9.1d Homework: Properties of Rotations cont.....	35
9.1e Class Activity: Congruence	37
9.1e Homework: Congruence	41
9.1f Class Activity: Congruence cont.....	45
9.1f Homework: Congruence cont.....	50
9.1g Self-Assessment: Section 9.1.....	53
SECTION 9.2 DILATIONS AND SIMILARITY	60
9.2a Class Activity: Video Game Animation.....	61
9.2b Class Activity: Properties of Dilations.....	63
9.2b Homework: Properties of Dilations	68
9.2c Class Activity: Dilations cont.	70
9.2c Homework: Dilations cont.....	72
9.2d Class Activity: Problem Solving with Dilations.....	74
9.2d Homework: Review of Dilations.....	77
9.2e Class Activity: Similarity.....	79
9.2e Homework: Similarity	82
9.2f Class Activity: Similarity cont.	84
9.2f Homework: Similarity cont.....	88
9.2g Self-Assessment: Section 9.2.....	93

Chapter 9 Geometry: Transformations, Congruence, and Similarity

Utah Core Standard(s):

- Verify experimentally the properties of rotations, reflections, and translations: (8.G.1)
 - a) Lines are taken to lines, and line segments to line segments of the same length.
 - b) Angles are taken to angles of the same measure.
 - c) Parallel lines are taken to parallel lines.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (8.G.2)
- Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (8.G.3)
- Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (8.G.4)

Academic Vocabulary: transformation, translation, reflection, rotation, rigid motion, image, pre-image, corresponding vertices, corresponding segments, corresponding angles, corresponding parts, coordinate rule, perpendicular bisector, line of reflection, slope, horizontal line, vertical line, clockwise, counterclockwise, center of rotation, angle of rotation, origin, concentric circles, congruent, dilation, center of dilation, scale factor, similar

Chapter Overview:

In this chapter, students explore and verify the properties of translations, reflections, rotations, and dilations. Students learn about the different types of rigid motion (translations, reflections, and rotations), execute them, and write coordinate rules to describe them. They describe the effects of these rigid motions on two-dimensional figures. Students then use this knowledge to determine whether one figure is congruent to another, understanding that two figures are congruent if there is a sequence of rigid motions that takes one figure onto the other. Then, students study dilations, again exploring and verifying the properties of dilations experimentally. They describe and execute dilations. They use this knowledge to determine whether one figure is similar to another, understanding that a two-dimensional figure is similar to another if there is a sequence of rigid motions and dilations that takes one figure onto the other.

Connections to Content:

Prior Knowledge: Up to this point, students have worked with two-dimensional geometric figures, solving real-world and mathematical problems involving perimeter and area. They have classified two-dimensional figures based on their properties. In 7th grade, students scaled figures. Students will rely on work with function and slope from previous chapters in this text in order to investigate the properties of the different transformations and to write coordinate rules to describe transformations. Students also use the skill of writing the equation of a line in order to write the equation for a line of reflection. Students have also been exposed to dilations in Chapter 2 using the properties of dilations to prove that the slope of a line is the same between any two distinct points on a non-vertical line and to derive the equation of a line.

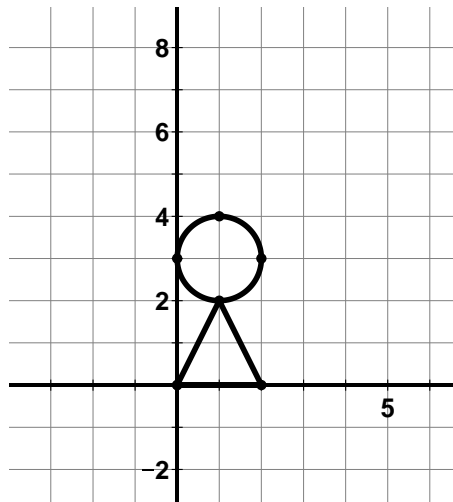
Future Knowledge: In subsequent courses, students will expand on this knowledge, explaining how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Similarly, they will use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

MATHEMATICAL PRACTICE STANDARDS:

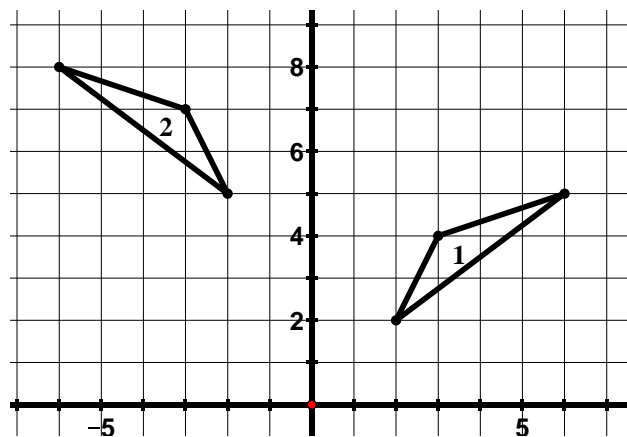


Make sense of problems and persevere in solving them.

Where would you put your center of dilation to enlarge the picture below by a factor of 2 and put the image entirely in the second quadrant? Mark your center of dilation with an O and then do the dilation.



Describe a sequence of rigid motions that would carry triangle 1 onto triangle 2.

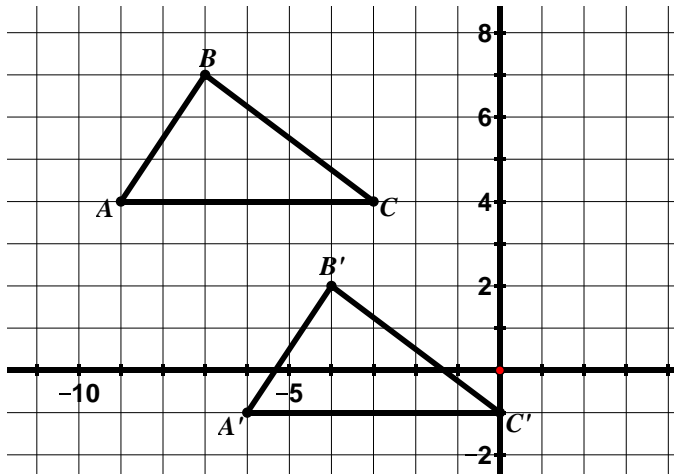


Throughout this chapter, students will see problems with multiple correct answers. For example, in the first problem above, there are many different places to put the center of dilation in order to meet the constraints specified in the problem. Students must use their knowledge of the properties of dilations as an entry point to solving the problem. In the second problem, there are many different sequences of rigid motion that will carry triangle 1 onto triangle 2. Students will have the opportunity to consider the different approaches taken by others, compare the approaches, and identify correspondences between the different approaches.



Reason abstractly and quantitatively.

Write a coordinate rule to describe the translation below.



Throughout the chapter, students write coordinate rules to describe transformations. They also perform transformations described by a coordinate rule.

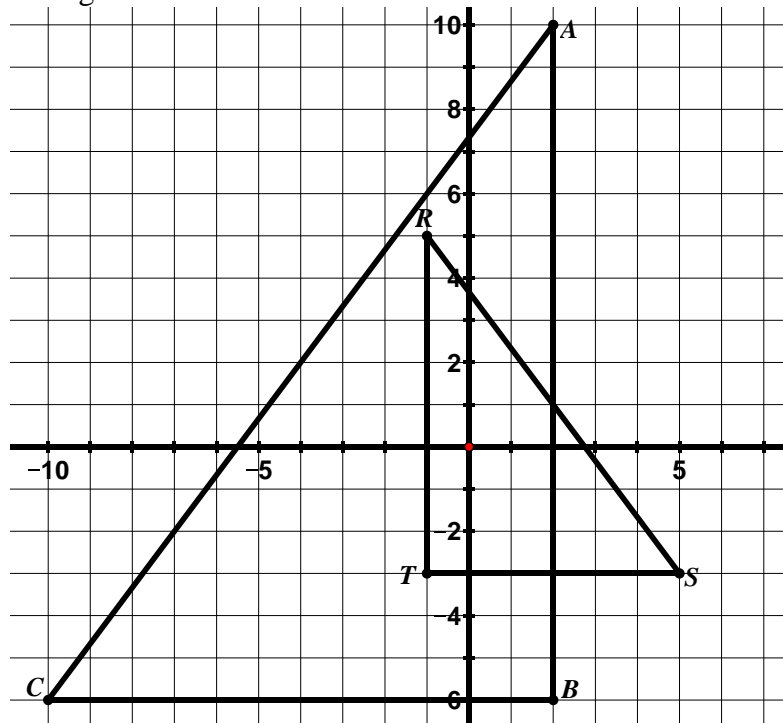
Determine the coordinate rule for a 90° rotation clockwise about the origin. Connect this rule to the slopes of perpendicular lines.

Students are also asked to explain why and how slopes of corresponding segments change under a given transformation, connecting the coordinate rule to the slope change. In this example, students are able to see that a rotation of a figure 90° clockwise can be described by the following coordinate rule $(x, y) \rightarrow (y, -x)$ helping them to understand why the slopes of perpendicular lines are opposite reciprocals of each other.



Construct viable arguments and critique the reasoning of others.

The triangles below are similar.



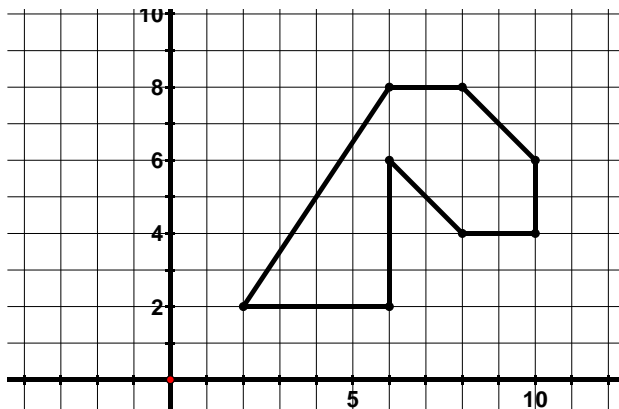
List the sequence of transformations that verifies the similarity of the two figures.

Write a similarity statement for the triangles.

Students will construct an argument that verifies the similarity of these two figures based on their understanding of the definition of similarity in terms of transformational geometry, that is, they must identify a sequence of rigid motions and dilations that takes one figure onto the other. There are different sequences that will accomplish this. Students will justify the sequence they have arrived at and communicate this to classmates. They will also have the opportunity to consider alternative sequences of others and decide whether these sequences do in fact verify the similarity of the two triangles. They will also have the opportunity to identify correspondences between the different sequences.

Animators need your help coming up with the coordinate rule that would reduce the size of the dragon's head to half its size when it enters a cave.

1. Write your proposed coordinate rule in the table below.
2. Write the new coordinates for your rule.
3. Graph the new coordinates.



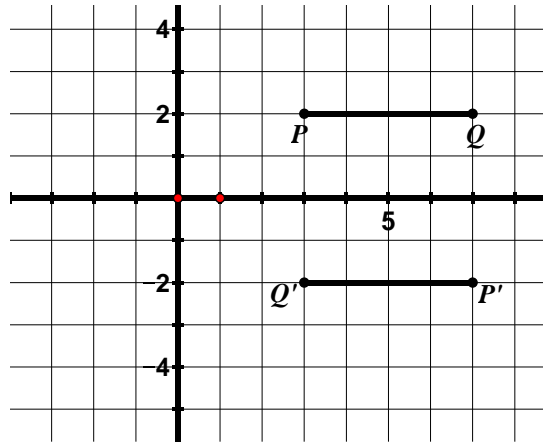
Model with mathematics.

In this problem, students determine how to scale the figure shown above (i.e. make it a different size while maintaining its shape). Scaling is something we see and use constantly in the world around us. Many professionals such as architects and computer animators rely on scaling techniques in order to create scaled down versions of real-life objects.



Use appropriate tools strategically.

Find the angle of rotation (including direction of rotation) and center of rotation for the rotation shown below.

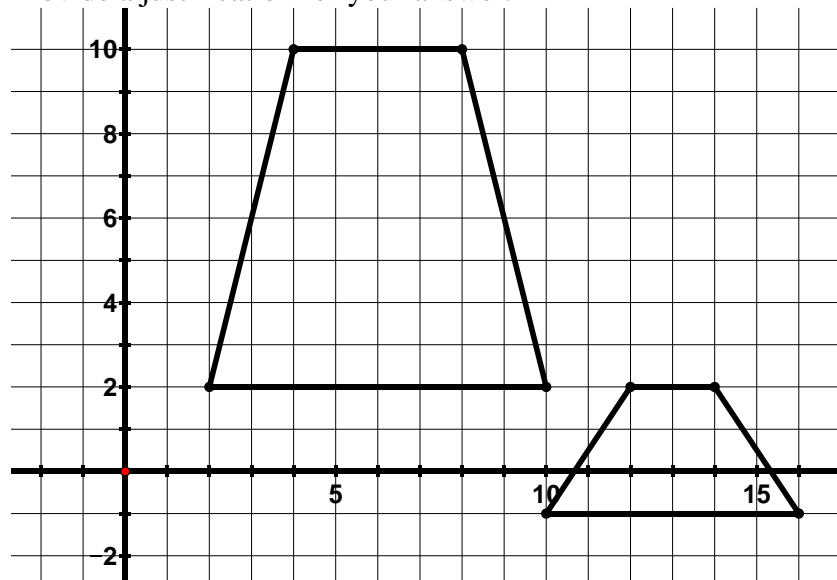


Students use a variety of tools in this chapter. These tools include straight edge, patty (or tracing) paper, compass, protractor, additional graph paper, colored pencils, and dynamic geometry software. In the problem above, one way to find the center of rotation is to trace the figures on patty paper and fold their paper so that P lines up with P' and fold a second time so that Q lines up with Q' . The intersection of these folds is the center of dilation. They can also use the patty paper to find the angle of rotation.



Attend to precision.

Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.

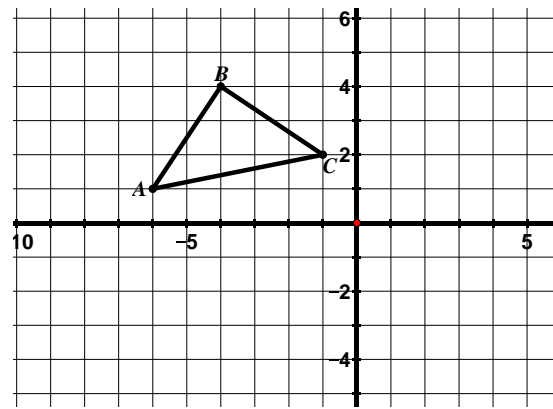


In order to answer this question, students must be clear about their understanding of what establishes congruence and similarity between two figures and they must be able to clearly communicate this to others.



Look for and make use of structure.

Reflect $\triangle ABC$ across the x -axis and label the image.



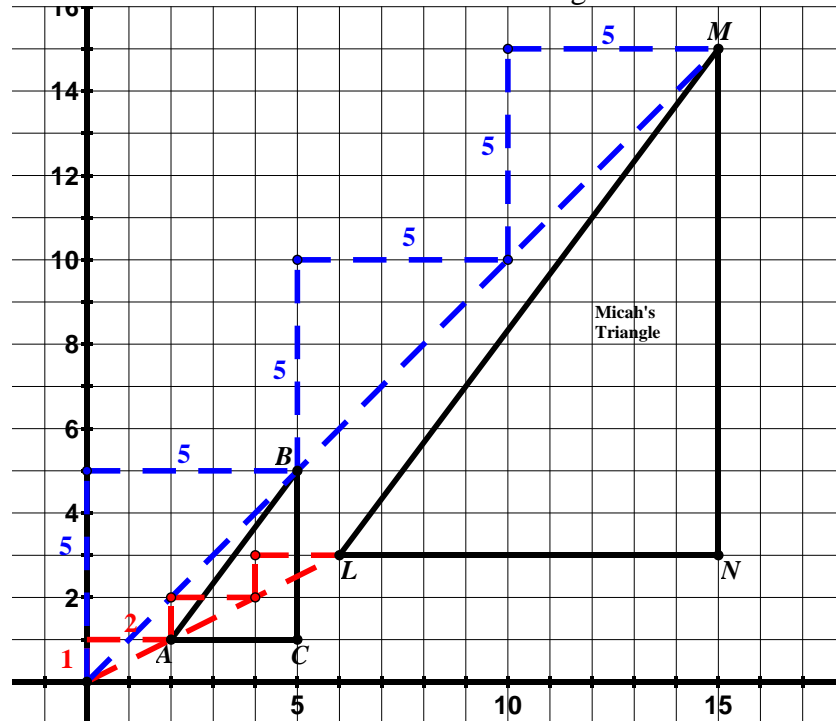
Write a coordinate rule to describe this reflection. Compare the slopes of the corresponding segments of the image and pre-image. What do you notice about the slopes? How does this connect to the coordinate rule $(x, y) \rightarrow (x, -y)$?

In this problem, students determine a coordinate rule to describe a reflection across the x -axis. In doing so, students examine the structure of the ordered pairs, realizing that under a reflection across the x -axis, the x -coordinates remain unchanged while the y -coordinates change sign. Following this, students use the coordinate rule to explain the effect this transformation has on the slope of a segment.



Look for and express regularity in repeated reasoning.

The figure below shows a triangle that has been dilated with a scale factor of 3 and a center of dilation at the origin.

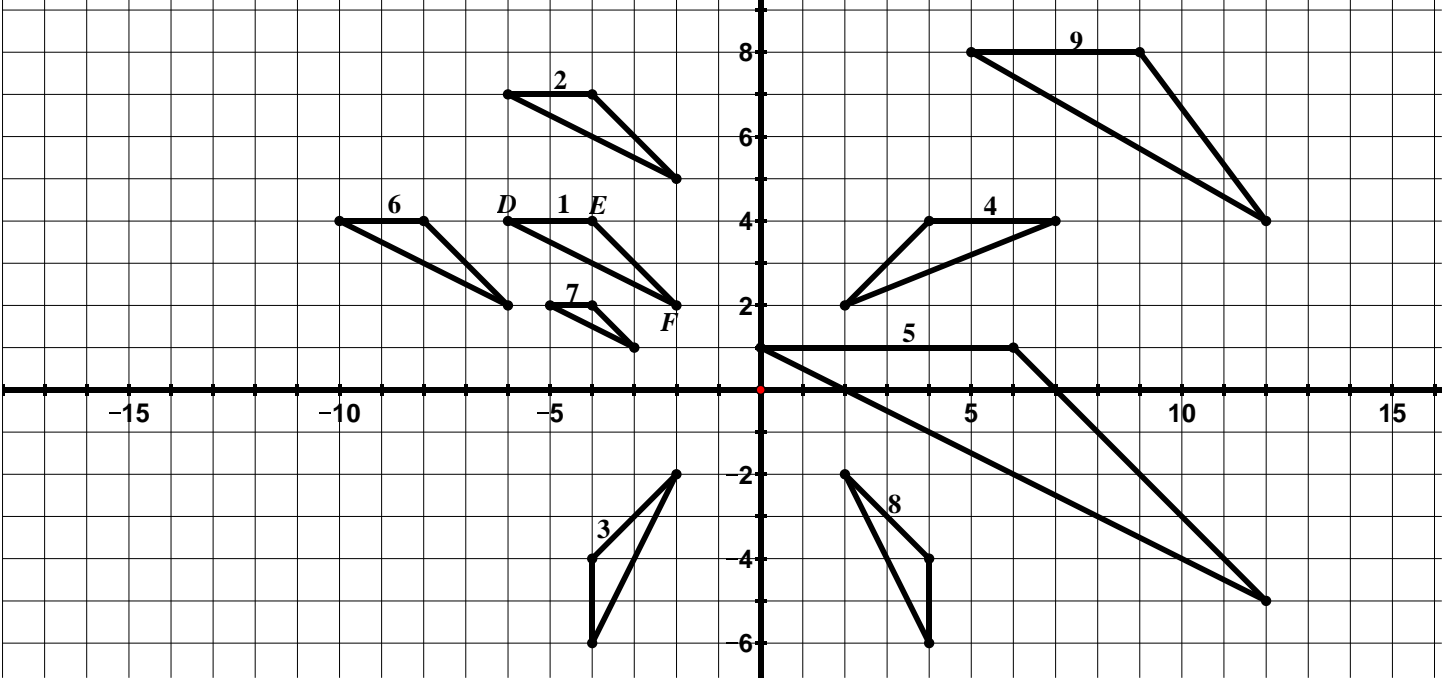


As students solve problems throughout the chapter, they use ideas about slope over and over again to discover the properties of rigid motions and dilations. They also use ideas about slope to translate, reflect, rotate, and dilate figures. In the picture above, Micah is using slope triangles to dilate $\triangle ABC$ with a scale factor of 3 and center of dilation at the origin.

9.0 Anchor Problem: Congruence and Similarity



Directions: Determine whether the triangles pictured below are **congruent** to $\triangle DEF$, **similar** to $\triangle DEF$, or neither congruent nor similar to $\triangle DEF$. Describe a sequence of transformations that support your claims.



9.1 Rigid Motion and Congruence

Section Overview:

In this section, students study the different types of rigid motion: translations, reflections, and rotations. Students begin their study of rigid motion with translations. Students describe translations that have taken place, both in words and with a coordinate rule. Students execute various translations, given a coordinate rule. Then, students summarize the properties of a translation based on the work they have done. These first few lessons also introduce students to some of the vocabulary used in transformational geometry. Next, students turn to reflections, again discovering the properties of reflections (including those over horizontal/vertical lines and the lines $y = x$ and $y = -x$). They write coordinate rules for reflections, connecting these rules to the slopes of the corresponding segments in the image and pre-image. Lastly, students draw lines of reflection and write the equations for these lines. Students then study rotations, with the emphasis on rotations of 90° increments. Students describe the properties of rotations and use these properties to solve problems. They start with rotations where the center of rotation is at the origin, again describing and executing rotations. Then, students study rotations where the center of rotation is not at the origin. Throughout the study of translations, reflections, and rotations, students articulate which properties hold for all of the rigid motions and which are specific to a given rigid motion. Students also perform a sequence of rigid motions and identify sequences of rigid motions that carry one figure to another. Students will then apply this knowledge to determine if two figures are congruent, understanding that two figures are congruent if there is a sequence of rigid motions that takes one figure onto the other.

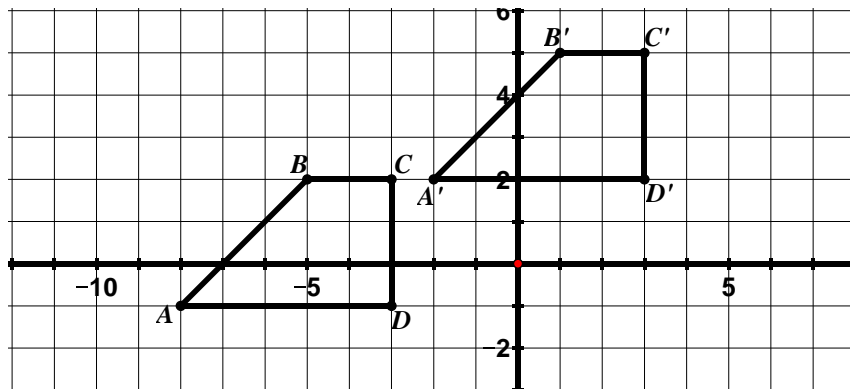
Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Given a pre-image and its image under a translation, describe the translation in words and using a coordinate rule.
2. Perform a translation of a figure given a coordinate rule.
3. Describe the properties of a translation and the effects a translation has on a figure and use this knowledge to solve problems.
4. Given a pre-image and its image under a reflection, describe the reflection in words and using a coordinate rule.
5. Perform a reflection of a figure given a line of reflection.
6. Describe the properties of a reflection and the effects a reflection has on a figure and use this knowledge to solve problems.
7. Find a reflection line for a given reflection and write the equation of the reflection line.
8. Given a pre-image and its image under a rotation, describe the rotation in words and using a coordinate rule (coordinate rule for rotations centered at the origin only).
9. Perform a rotation of a figure given a center of rotation, an angle of rotation, and a direction.
10. Describe the properties of a rotation and the effects a rotation has on a figure and use this knowledge to solve problems.
11. Connect ideas about slopes of perpendicular lines and rotations.
12. Understand what it means for two figures to be congruent.
13. Determine if two figures are congruent based on the definition of congruence.
14. Given two figures that are congruent, describe the sequence of transformations that exhibits the congruence between them.

9.1a Class Activity: Properties of Translations

1. In the grid below, $ABCD$ has been transformed to obtain $A'B'C'D'$.



$ABCD$ is called the **pre-image** and $A'B'C'D'$ is called the **image**. The **pre-image** is the figure prior to the transformation and the **image** is the figure after the transformation. A and A' , B and B' , C and C' , and D and D' are **corresponding vertices**.

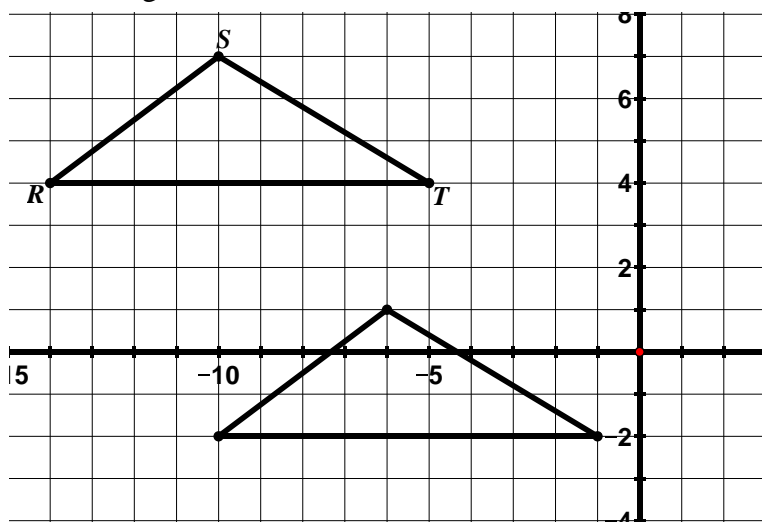
- c. In the table below, write the coordinates for the vertices of the pre-image and image.

Pre-Image	Image
A :	A' :
B :	B' :
C :	C' :
D :	D' :

- a. This type of transformation is called a **translation**. Describe in your own words the movement of a figure that has been translated.
- b. Show on the picture how you would move on the coordinate plane to get from A to A' , B to B' , C to C' , and D to D' .

- d. The coordinate rule for this translation is $(x, y) \rightarrow (x + 6, y + 3)$. Connect this notation to your answer for part b. and to the coordinates of corresponding vertices in the table.

2. In the grid below, $\triangle RST$ has been translated to obtain $\triangle R'S'T'$.



- Label the corresponding vertices of the image on the grid.
- Describe or show on the picture how you would move on the coordinate plane to get from the vertices in the pre-image to the corresponding vertices in the image.

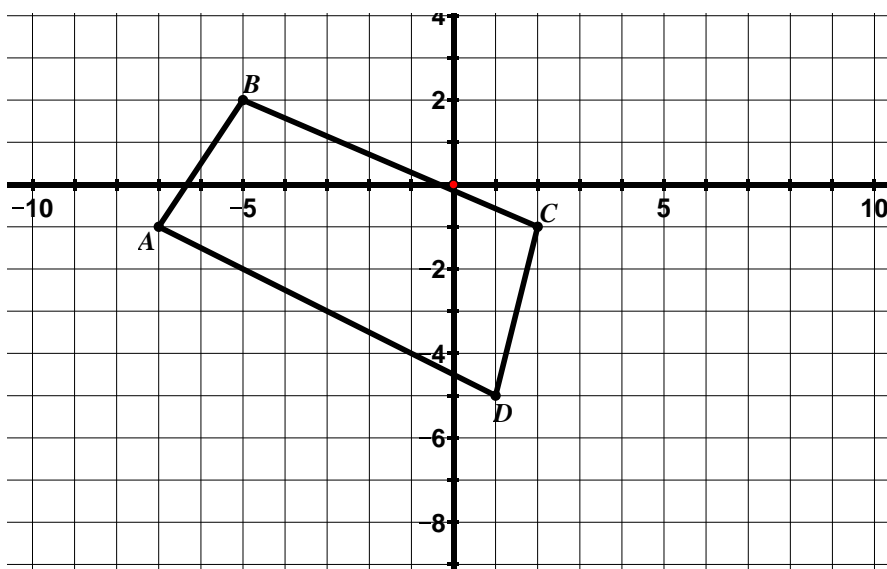
- In the table below, write the coordinates for the vertices of the pre-image and image.

Pre-Image	Image
R:	R':
S:	S':
T:	T':

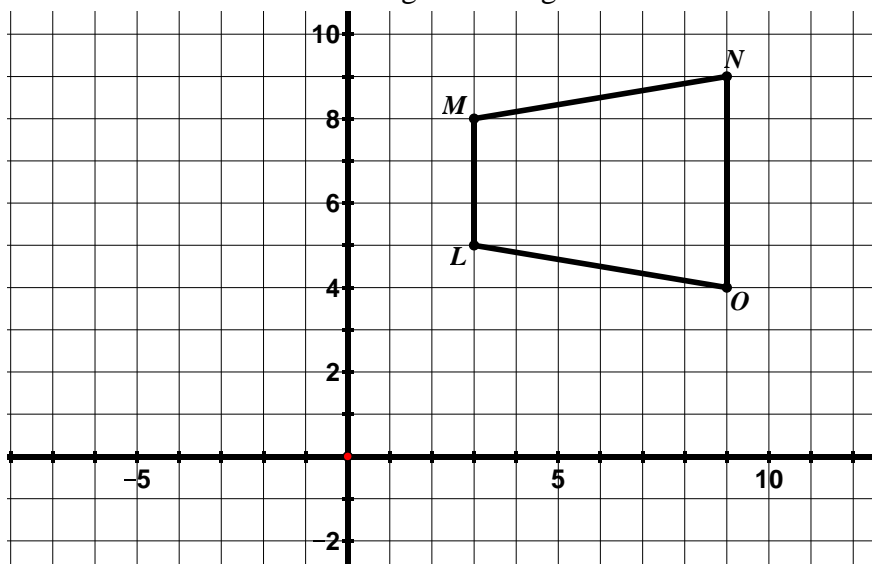
- Write the coordinate rule that describes this translation.

n#

3. Draw and label the image of the figure below for the translation $(x, y) \rightarrow (x + 5, y - 3)$



4. Draw and label the image of the figure below for the translation $(x, y) \rightarrow (x - 7, y)$



Determine the slopes for:

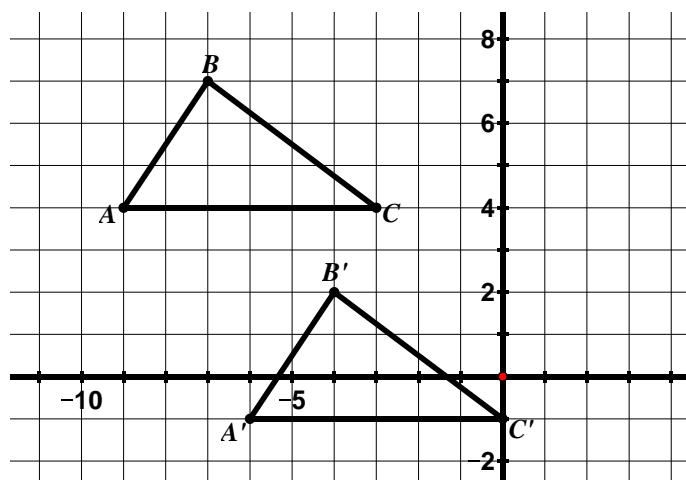
$$\overline{MN}: \quad \overline{M'N'}:$$

$$\overline{NO}: \quad \overline{N'O'}:$$

$$\overline{LO}: \quad \overline{L'O'}:$$

$$\overline{ML}: \quad \overline{M'L'}:$$

5. Write a coordinate rule to describe the translation below. _____



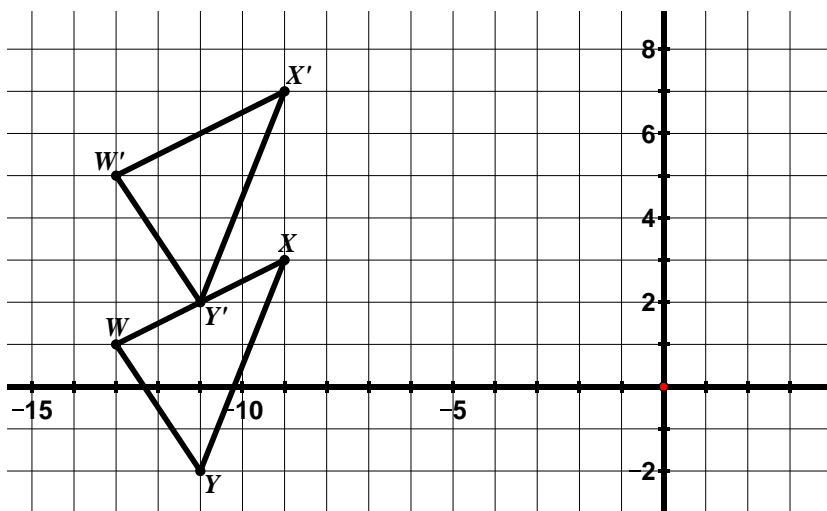
Determine the slopes for:

$$\overline{AB}: \quad \overline{A'B'}:$$

$$\overline{BC}: \quad \overline{B'C'}:$$

$$\overline{CA}: \quad \overline{C'A'}:$$

6. Write a coordinate rule to describe the translation below _____



Determine the slopes for:

$$\overline{WX}: \quad \overline{W'X'}:$$

$$\overline{XY}: \quad \overline{X'Y'}:$$

$$\overline{YW}: \quad \overline{Y'W'}:$$

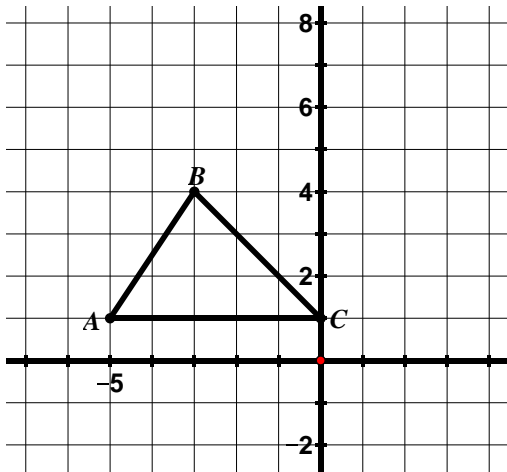
7. Use questions #1 – 6 to explore some **properties of translations** and write your observations below.



9.1a Homework: Properties of Translations

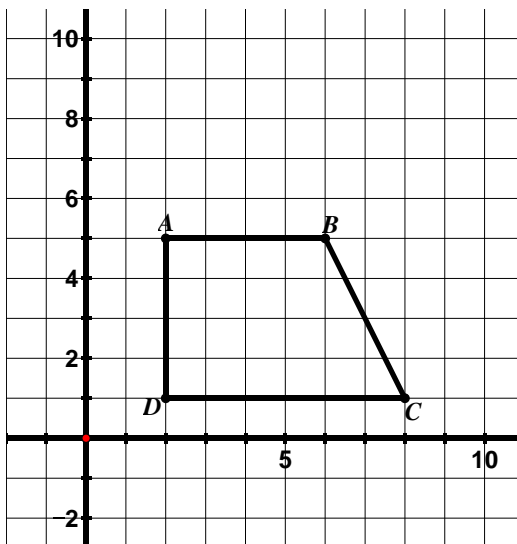
Directions: For #1 – 3, draw and label the image for the coordinate rule given. Then answer the questions.

1. Translate the figure below according to the rule $(x, y) \rightarrow (x + 3, y + 2)$ and label the image.

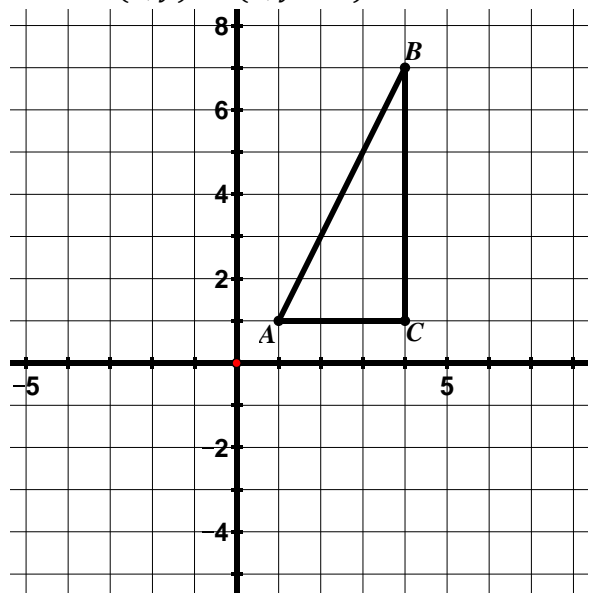


- If the slope of \overline{BC} is -1 , determine the slope of $\overline{B'C'}$ without doing any calculations.
 - If the length of \overline{BC} is $3\sqrt{2}$, determine the length of $\overline{B'C'}$ without doing any calculations.
 - Determine the slopes of \overline{AB} and of $\overline{A'B'}$. What do you notice about the slopes of **corresponding segments of a translated figure**?
- d. Using a ruler, draw a line connecting corresponding vertices in the image and pre-image (A to A' , B to B' , and C to C'). Find the slopes of $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. What do you notice about the slopes of the **segments connecting corresponding vertices** of the image and pre-image of a translated figure?

2. Translate the figure below according to the rule $(x, y) \rightarrow (x - 1, y + 5)$ and label the image.

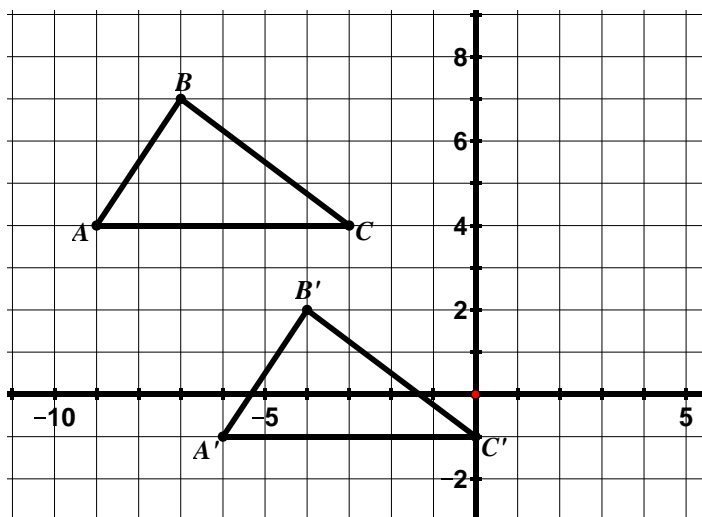


3. Translate the figure below according to the rule $(x, y) \rightarrow (x, y - 4)$ and label the image.

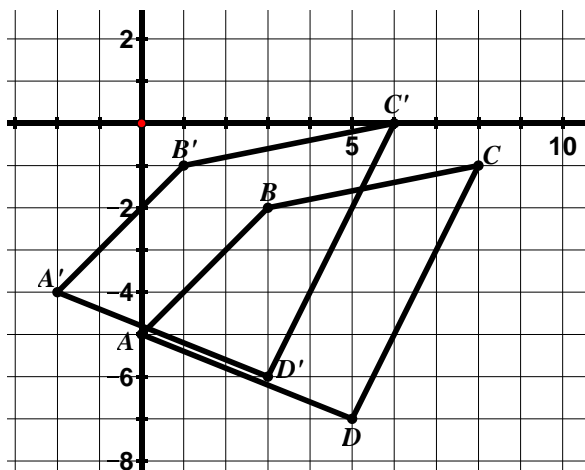


Directions: For #4 – 7, write a coordinate rule to describe the translation. Then answer the questions.

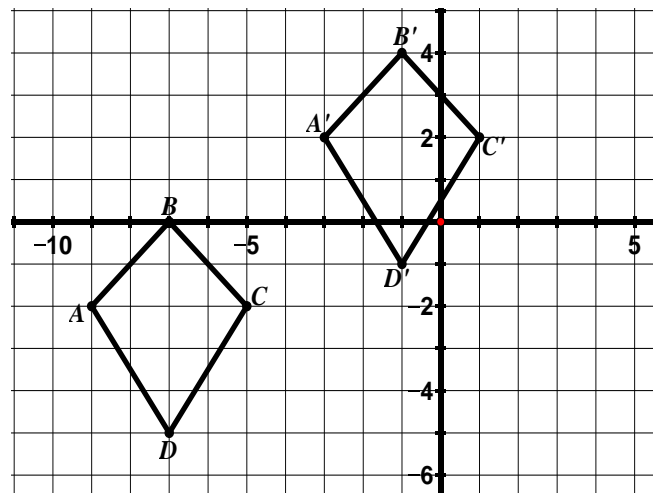
4. Coordinate Rule:



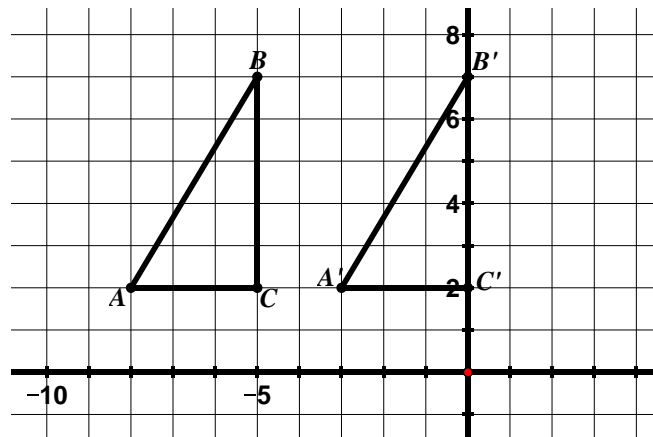
5. Coordinate Rule:



6. Coordinate Rule:



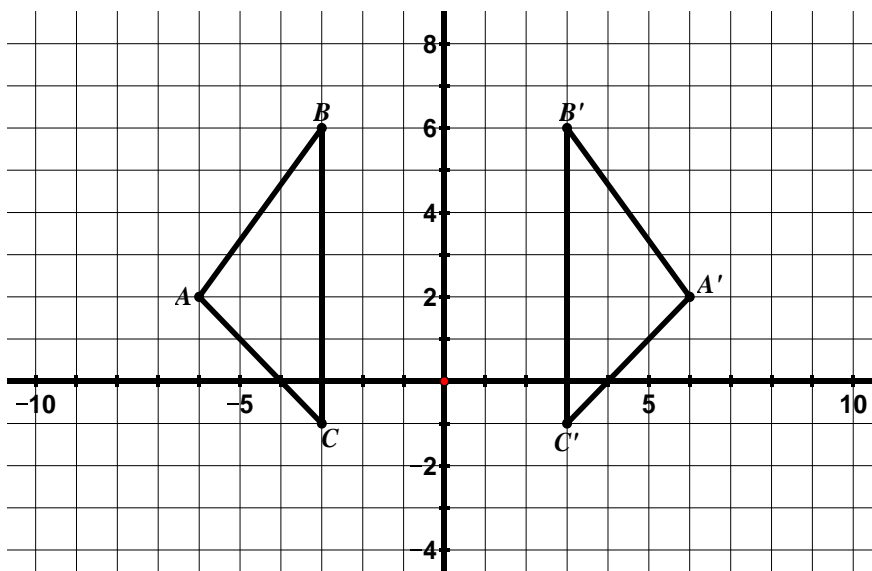
7. Coordinate Rule:



- The slope of $\overline{BB'}$ is $-\frac{5}{3}$. Name two other segments that also have a slope of $-\frac{5}{3}$.
- If the length of $\overline{BB'}$ is $\sqrt{34}$, determine the length of $\overline{CC'}$ without doing any calculations.
- Determine the length of \overline{AC} and of $\overline{A'C'}$.
- Determine the slope of \overline{AC} and of $\overline{A'C'}$.

9.1b Class Activity: Properties of Reflections

1. In the grid below, $\triangle ABC$ has been **reflected** over the y -axis to obtain $\triangle A'B'C'$.



- a. Describe the movement of a figure that has been **reflected**.

- b. In the table below, write the coordinates for the vertices of the pre-image and image.

Pre-Image	Image
A:	A':
B:	B':
C:	C':

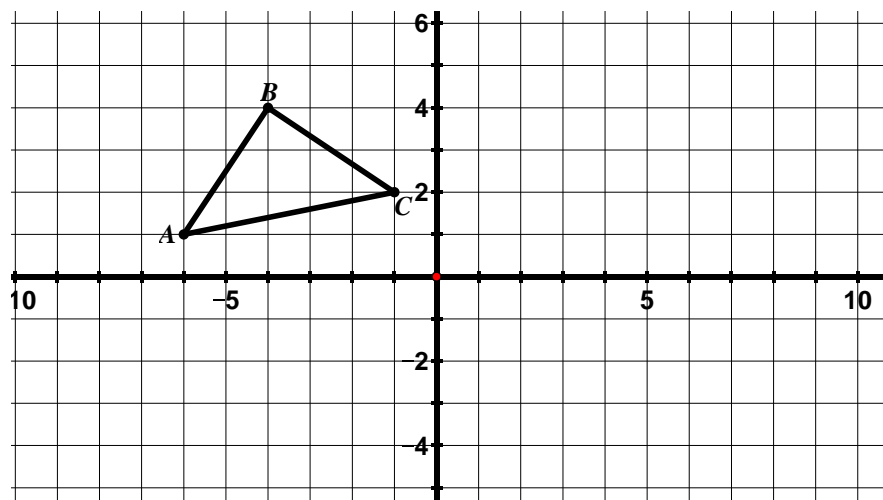
- c. Write a coordinate rule to describe this reflection.

- d. Will this coordinate rule hold true for any figure reflected over the y -axis? Why or why not?



Directions: Draw and label the image of each figure for the reflection given. Then, answer the questions.

2. Reflect $\triangle ABC$ across the x -axis and label the image.




- a. In the table below, write the coordinates for the vertices of the pre-image and image.

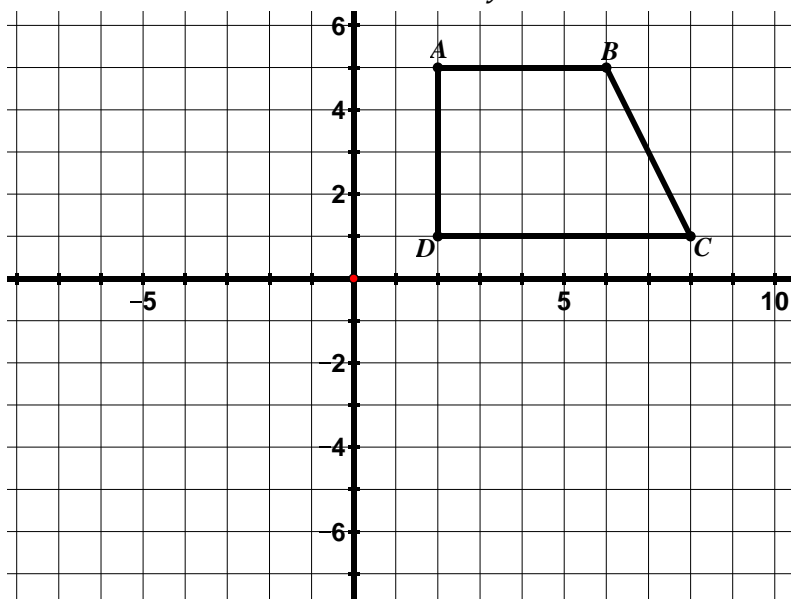
Pre-Image	Image
A:	A':
B:	B':
C:	C':

- b. Write a coordinate rule to describe this reflection.

- c. Will this coordinate rule hold true for any figure reflected over the x -axis? Why or why not?

3. Use questions #1 – 2 to explore some **properties of reflections**.
- Go back to problem #1. Draw a segment connecting B and B', A and A', and C and C'. Make at least two conjectures about the relationship between **the line of reflection and the segments connecting corresponding vertices** in the image and pre-image of a reflection.
 - Do your conjectures hold true in problem #2?
 - Go back to problem #1. For a *translation* we learned that corresponding segments are parallel (have the same slope). Is this property also true for reflections?
 - Now, go to problem #2. Find the slopes of the following segments:
 $\overline{AB} =$ $\overline{AC} =$ $\overline{BC} =$
 $\overline{A'B'} =$ $\overline{A'C'} =$ $\overline{B'C'} =$
 - Compare the slopes of the corresponding segments of the image and pre-image. What do you notice about the slopes? How does this connect to the coordinate rule $(x, y) \rightarrow (x, -y)$? 
 - Examine problems #1 and #2. What do you notice about the **lengths of corresponding segments** in the image and pre-image?

4. Reflect $ABCD$ across the line $y = -1$ and label the image.

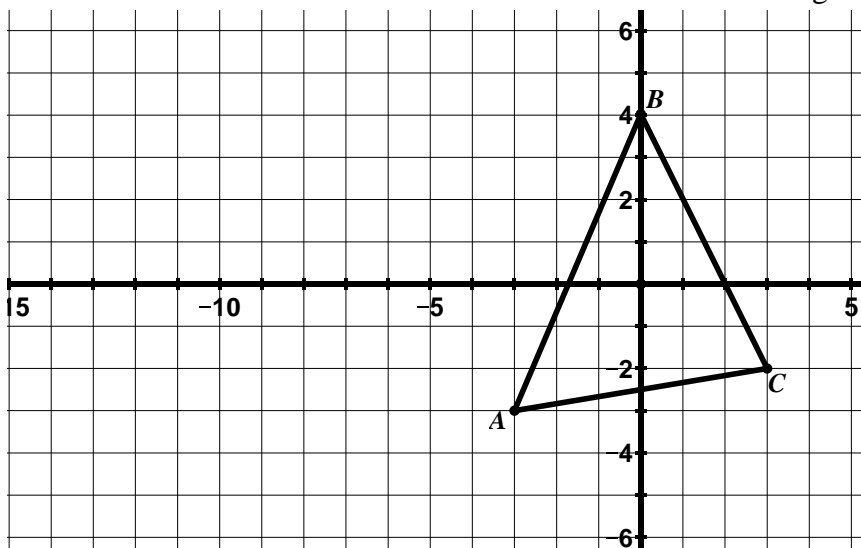


- a. In the table below, write the coordinates for the vertices of the pre-image and image.

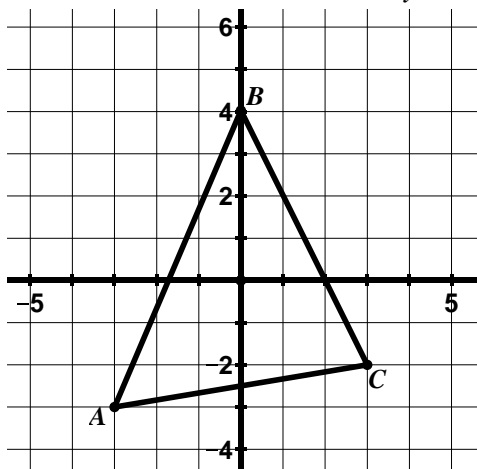
Pre-Image	Image
A:	A':
B:	B':
C:	C':
D:	D':

- b. Write a coordinate rule to describe this reflection.

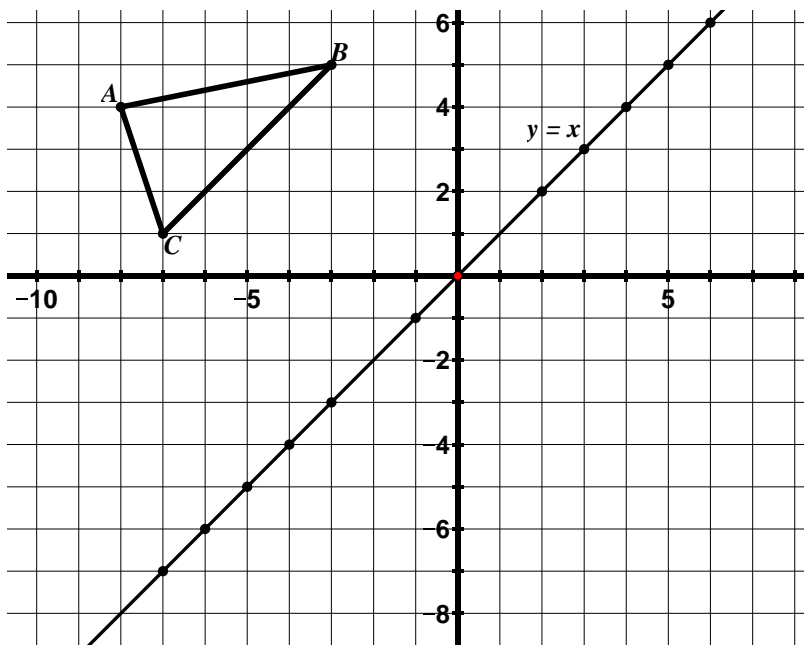
5. Reflect $\triangle ABC$ across the line $x = -5$ and label the image.



6. Reflect $\triangle ABC$ over the y-axis and label the image.



7. Reflect $\triangle ABC$ across the line $y = x$ and label the image.



- a. Describe the method you used to solve this problem.



- b. In the table below, write the coordinates for the vertices of the pre-image and image.

Pre-Image	Image
A:	A':
B:	B':
C:	C':

- c. Write a coordinate rule to describe this reflection.

- d. Will this coordinate rule hold true for any figure reflected over the line $y = x$? Why or why not?


- e. Find the slopes of the following segments:

$$\overline{AB} = \quad \overline{AC} = \quad \overline{BC} =$$

$$\overline{A'B'} = \quad \overline{A'C'} = \quad \overline{B'C'} =$$

- f. Compare the slopes of the corresponding segments of the image and pre-image. What do you notice? How does this connect to the coordinate rule?

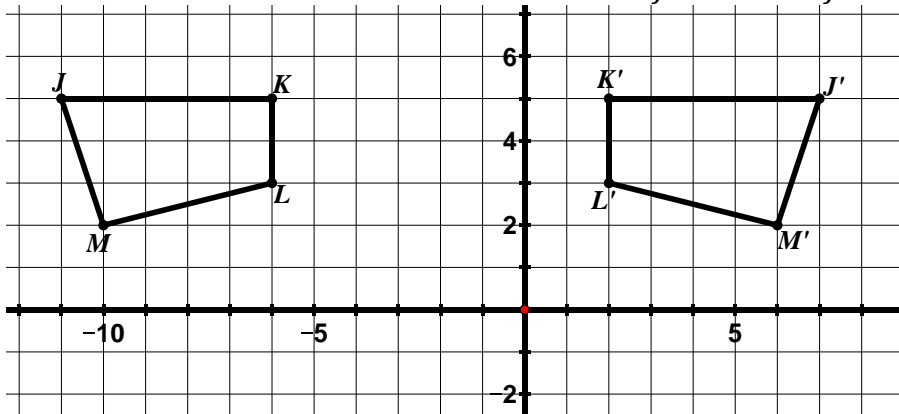
- g. **Bonus:** What is the coordinate rule for a figure reflected across the line $y = -x$?

8. The following table lists the properties of translations discovered in the previous lesson. Put a checkmark in the box if the property is also true for reflections. Add additional statements to the table that are only true for reflections. 

Properties of Translations	Also True for Reflections?
Segments connecting the corresponding vertices of the image and pre-image are the same length.	
Segments connecting the corresponding vertices of the image and pre-image are parallel to each other.	
*Corresponding segments in the image and pre-image are the same length.	
*Corresponding angles in the image and pre-image have the same measure.	
*Parallel lines in the pre-image remain parallel lines in the image.	
Corresponding segments in the image and pre-image have the same slope.	

Directions: For #9 – 11, draw the line of reflection that would reflect one figure onto the other. Then, write the equation for the line of reflection and the coordinate rule that describes the reflection.

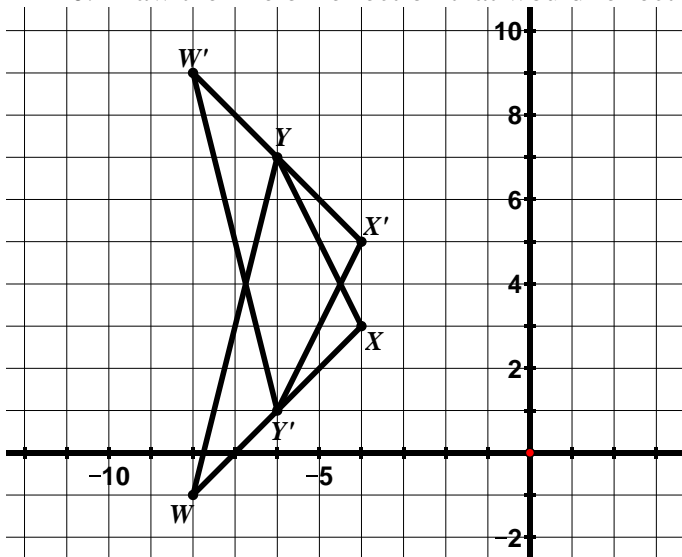
9. Draw the line of reflection that would reflect $\triangle JKL$ onto $\triangle J'K'L'M'$.



a. Write the equation for the line of reflection.

b. Write a coordinate rule for the reflection.

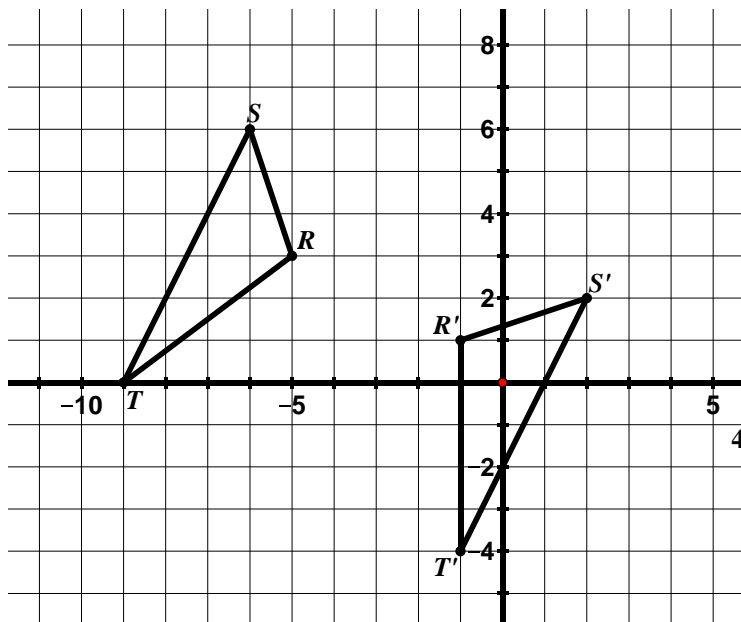
10. Draw the line of reflection that would reflect $\triangle WXY$ onto $\triangle W'X'Y'$.



a. Write the equation for the line of reflection.

b. Write a coordinate rule for the reflection.

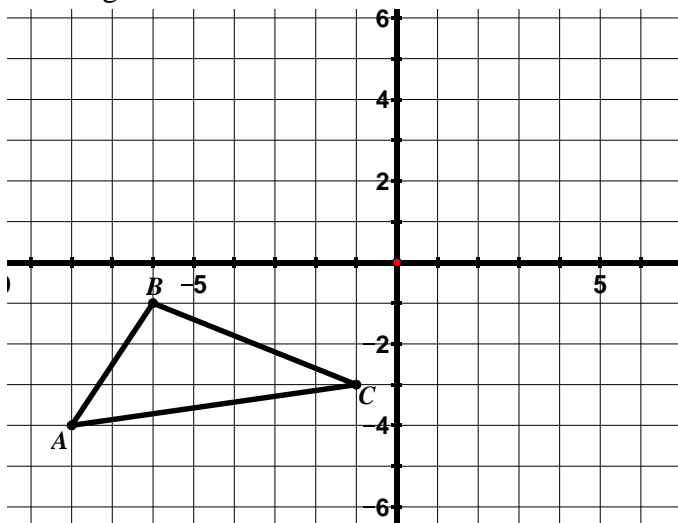
11. Draw the line of reflection that would reflect $\triangle RST$ onto $\triangle R'S'T'$.



- a. Write the equation for the line of reflection.

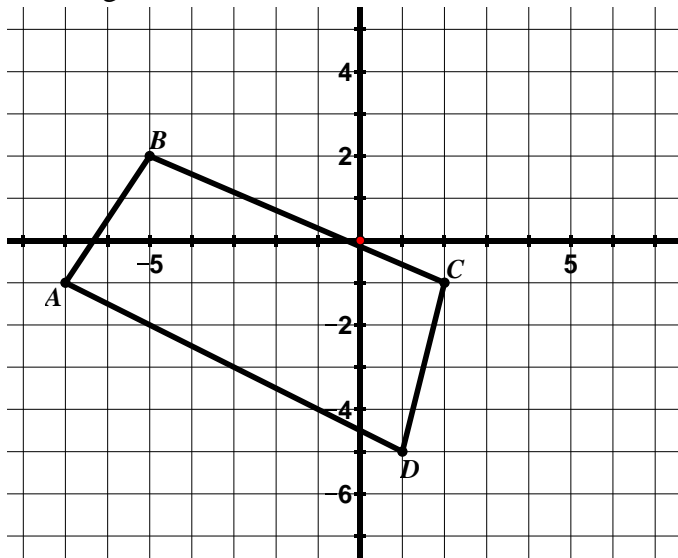
9.1b Homework: Properties of Reflections

1. Reflect $\triangle ABC$ across the x -axis and label the image.



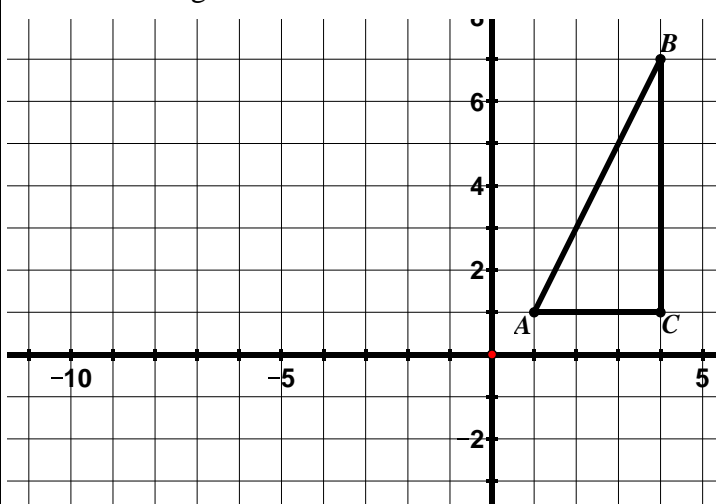
- a. Write a coordinate rule to represent this transformation.

2. Reflect $ABCD$ across the y -axis and label the image.



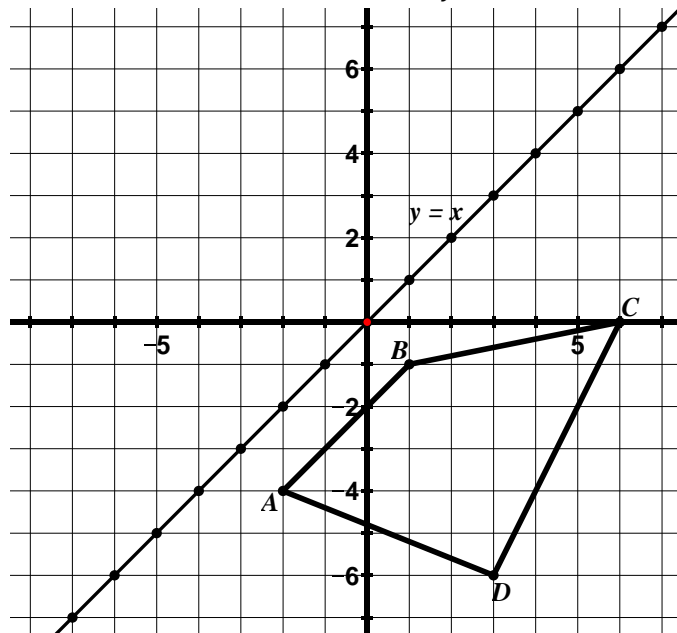
- a. Write a coordinate rule to represent this transformation.

3. Reflect $\triangle ABC$ across the line $x = -3$ and label the image.



- a. Write a coordinate rule to represent this transformation.

4. Reflect $ABCD$ across the line $y = x$.



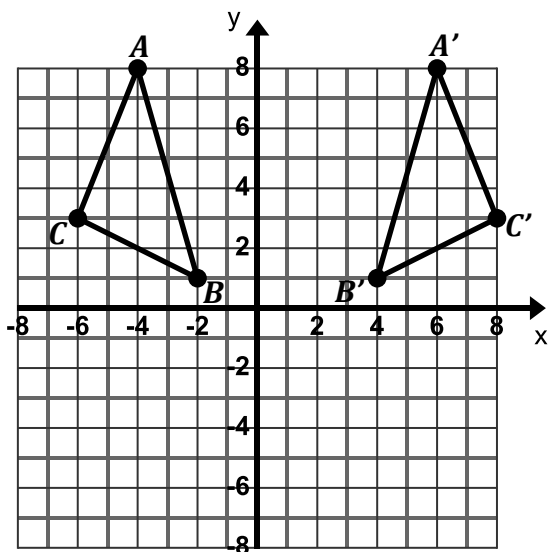
- a. Write a coordinate rule to represent this transformation.

5. For each of the following:

- Draw the line of reflection that would reflect the pre-image onto the image.
- Find the equation for the line of reflection.
- Write a coordinate rule to describe the reflection.

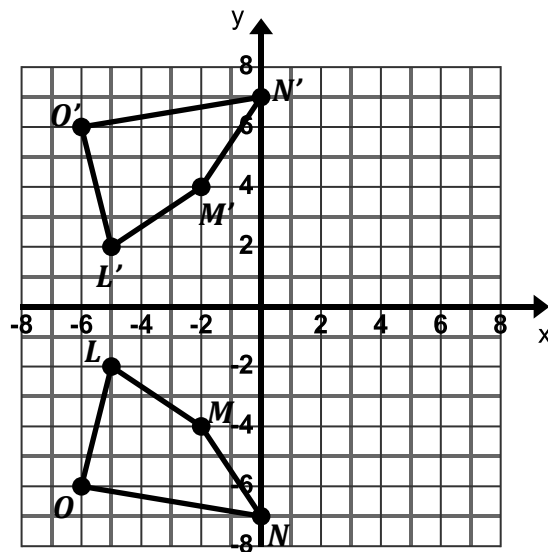
a. Equation of line of reflection: _____

Coordinate Rule: _____



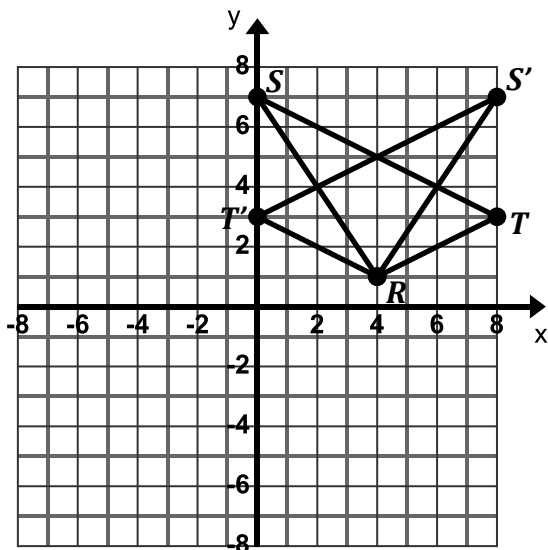
b. Equation of line of reflection: _____

Coordinate Rule: _____



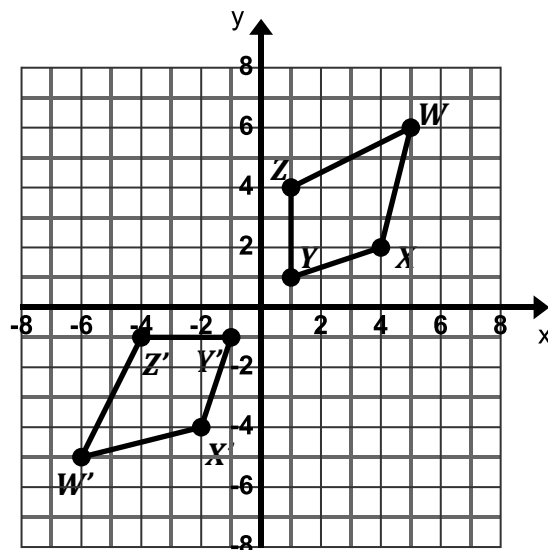
c. Equation of line of reflection: _____

Coordinate Rule: _____

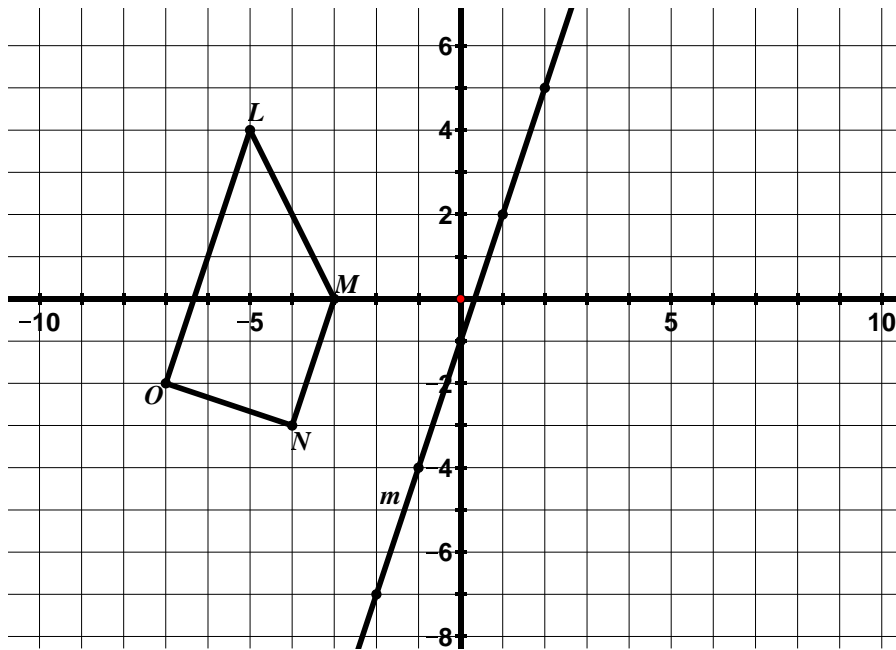


d. Equation of line of reflection: _____

Coordinate Rule: _____



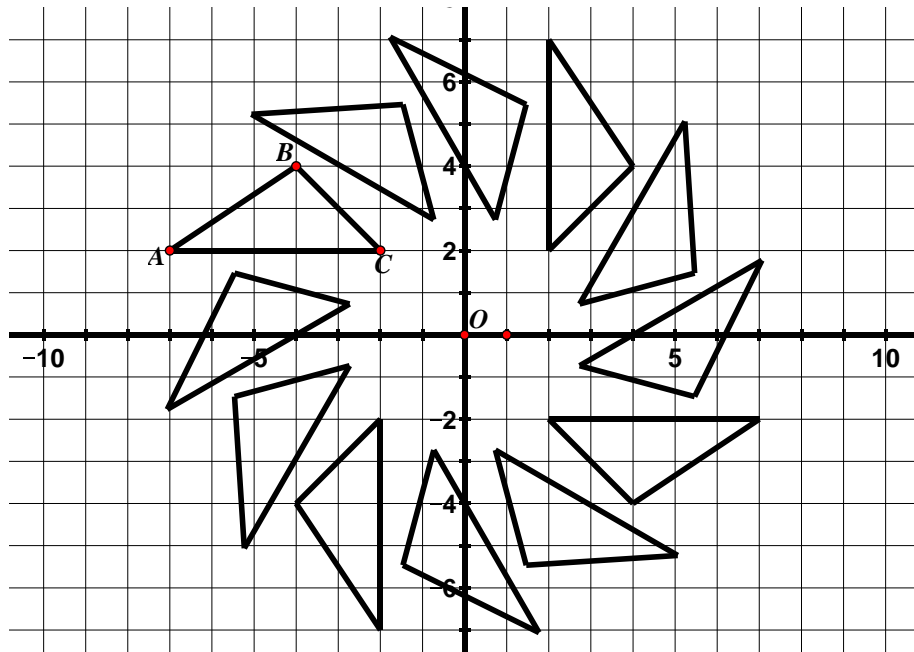
6. Given $LMNO$.




- What is the equation of the line of reflection?
- Based on the slope of the line of reflection, determine what the slope of the segments connecting corresponding points of the image and pre-image should be.
- Reflect $LMNO$ over the line m and label the image.
- What is the relationship between \overline{LO} and \overline{MN} before the transformation? What is the relationship between these two segments after the transformation? Use numerical evidence to support your answer.

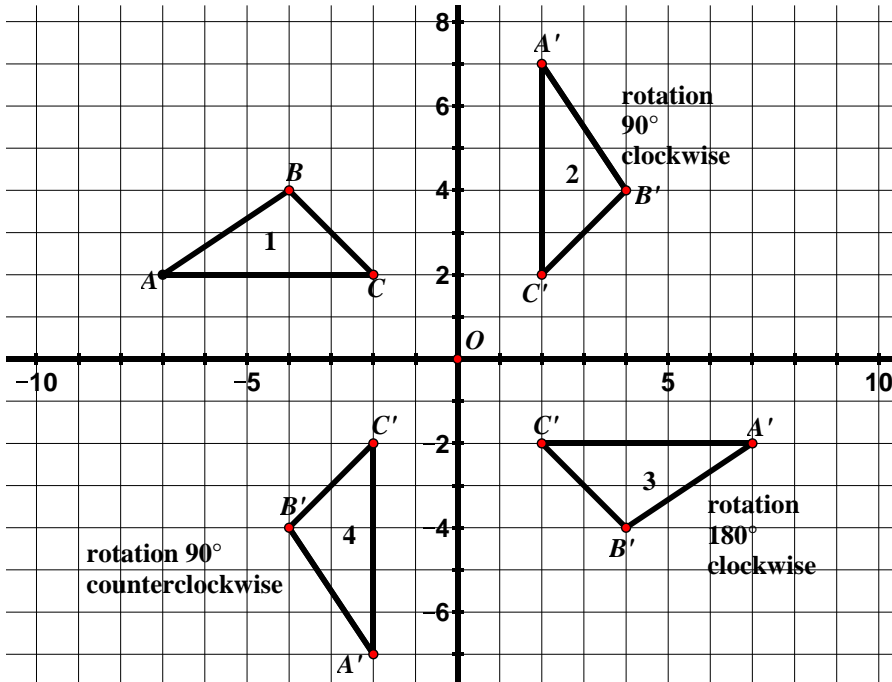
9.1c Class Activity: Properties of Rotations

1. In the grid below, $\triangle ABC$ has been **rotated** counterclockwise with the **center of rotation** at the origin O . This process was repeated several times to create the images shown.



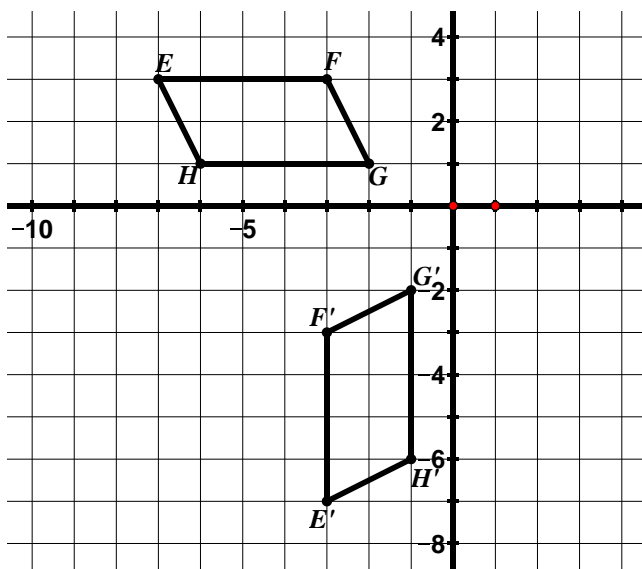
- a. Using tracing paper, trace $\triangle ABC$ and the x -axis. Holding your pencil as an anchor on the origin, rotate the triangle counterclockwise to see how the images were created.
- b. Label the corresponding vertices of the images of $\triangle ABC$.
- c.  Describe the relationship between C and its images to the **center of rotation** O . Do the same for A and its images. Does this relationship to the center of rotation hold true for B and its images?
- d. If there are 360° in one full rotation, determine the angle of rotation from one image to the next in the picture above.

2. The picture from the previous page was modified so that only the images that are increments of 90° rotations of the pre-image $\triangle ABC$ are shown. The center of rotation is the origin O .



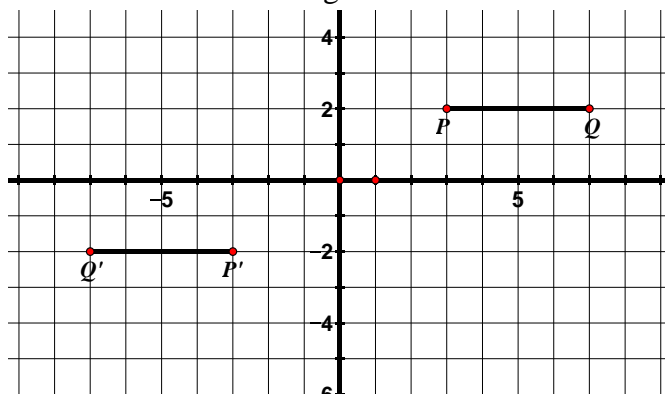
- a. Verify using tracing paper that the descriptions of the rotations are accurate.
- b. The rotation from figure 1 to figure 4 has been described as a rotation 90° counterclockwise. How would you describe this rotation in the clockwise direction?
- c. Consider the rotation from Figure 1 to Figure 2, a rotation 90° clockwise. Find the slopes of the following segments:
- $\overline{AB} =$ $\overline{BC} =$ $\overline{AC} =$
- $\overline{A'B'} =$ $\overline{B'C'} =$ $\overline{A'C'} =$
- d. Use the slopes from the previous question to determine the relationship between corresponding segments in a 90° rotation.
- e. Which segments would you expect to be perpendicular in the rotation from Figure 1 to Figure 4, the rotation 90° counterclockwise? Use slope to support your answer.
- f. Determine the coordinate rule for a 90° rotation clockwise about the origin. Connect this rule to the slopes of perpendicular lines. **n#**
- g. Determine the coordinate rule for a 90° rotation counterclockwise about the origin. Connect this rule to the slopes of perpendicular lines.
- h. Describe what happens in a 180° rotation of a figure. What is the relationship of the corresponding segments?
- i. Determine the coordinate rule for a rotation of 180° . Connect this rule to your answer for part h.

3. For the following rotation, the center of rotation is the origin.



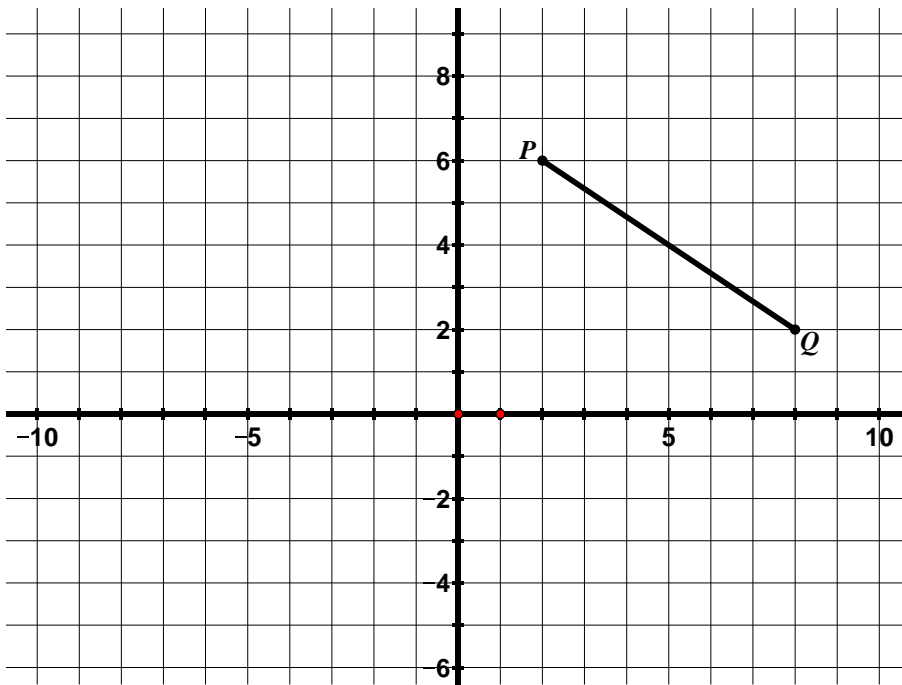
- a. Determine the angle of rotation. Be sure to also indicate a direction of rotation.
- b. If the slope of \overline{EH} is -2 , determine the slope of $\overline{E'H'}$ without doing any calculations.

4. For the following rotation, the center of rotation is the origin.



- a. Determine the angle of rotation. Be sure to also indicate a direction of rotation.


5. Rotate \overline{PQ} 90° counterclockwise with the center of rotation at the origin and label the image.



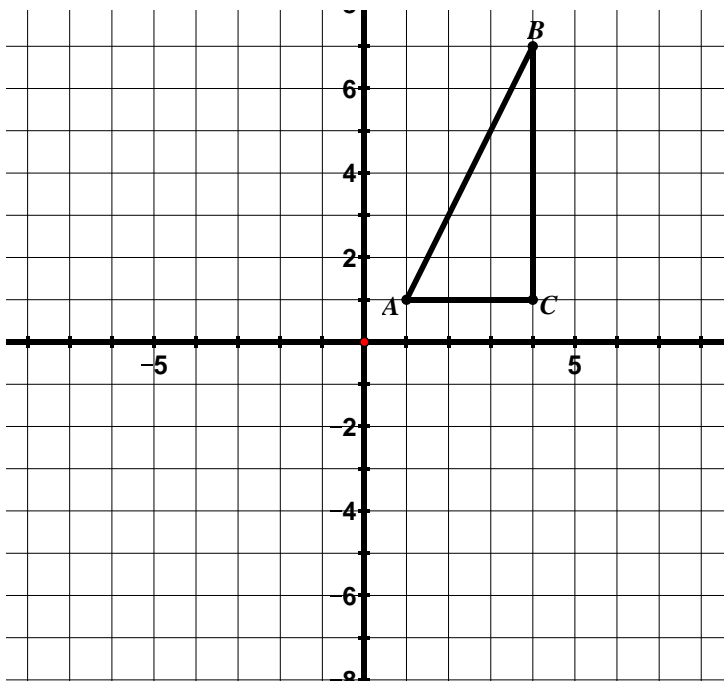
a. How can you verify using slope that your image is in fact a 90° rotation?

b. How can you verify using distance that the center of rotation is the origin?

c. Use a compass to draw the concentric circles of this rotation.

d.  What do the concentric circles prove?

6. Rotate $\triangle ABC$ 180° counterclockwise with the center of rotation at the origin and label the image.

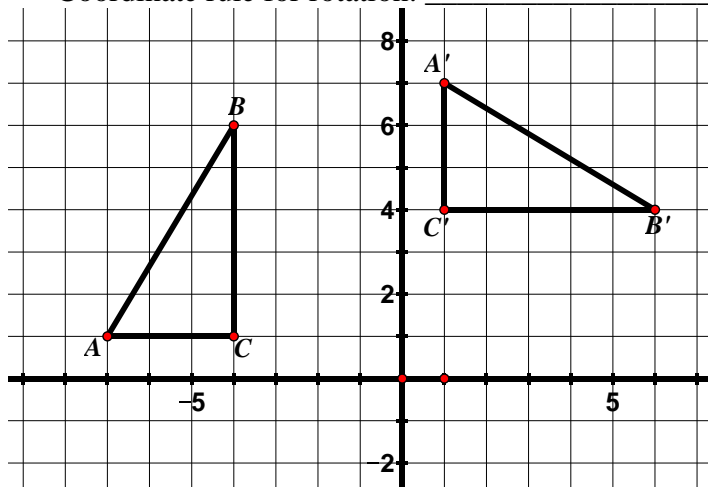


9.1c Homework: Properties of Rotations

Directions: For each of the following rotations, the center of rotation is the origin. Determine the angle of rotation (be sure to also indicate a direction of rotation). Write a coordinate rule for the transformation.

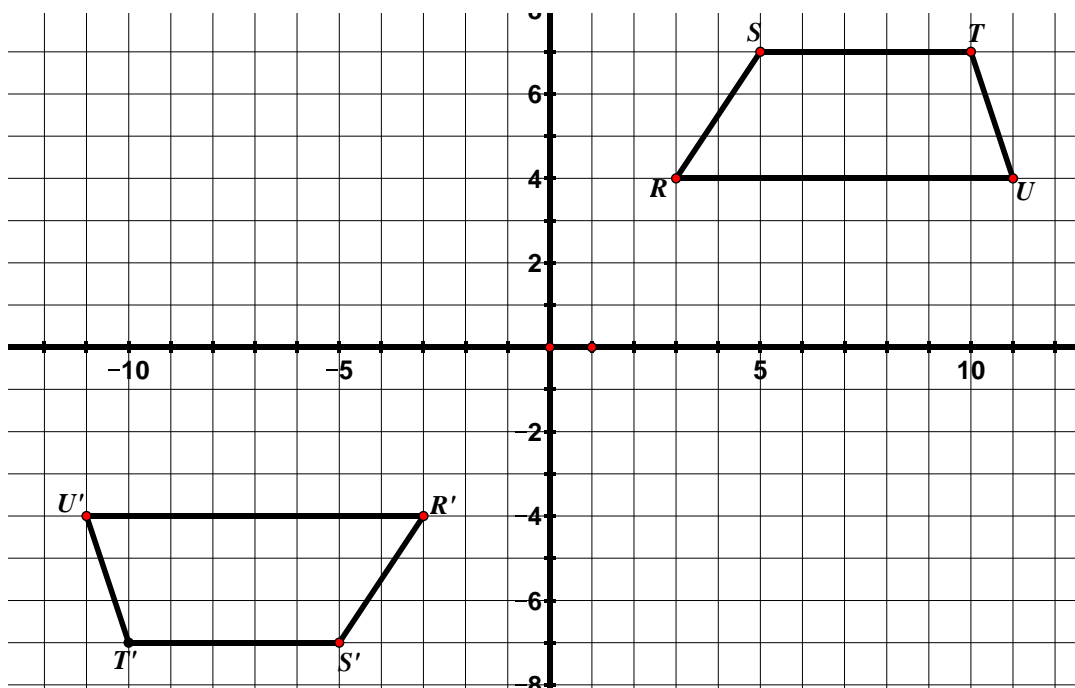
1. Angle of Rotation (including direction of rotation): _____

Coordinate rule for rotation: _____

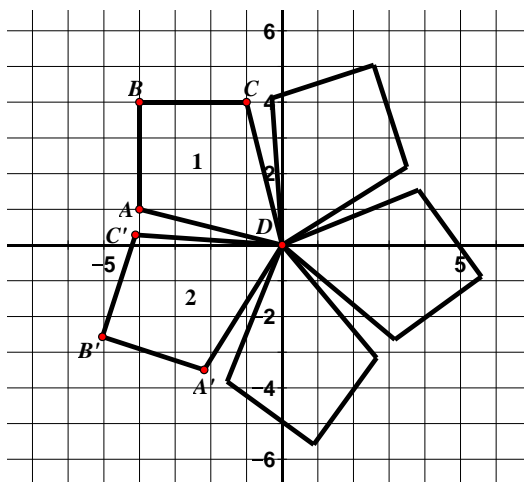


2. Angle of Rotation (including direction of rotation): _____

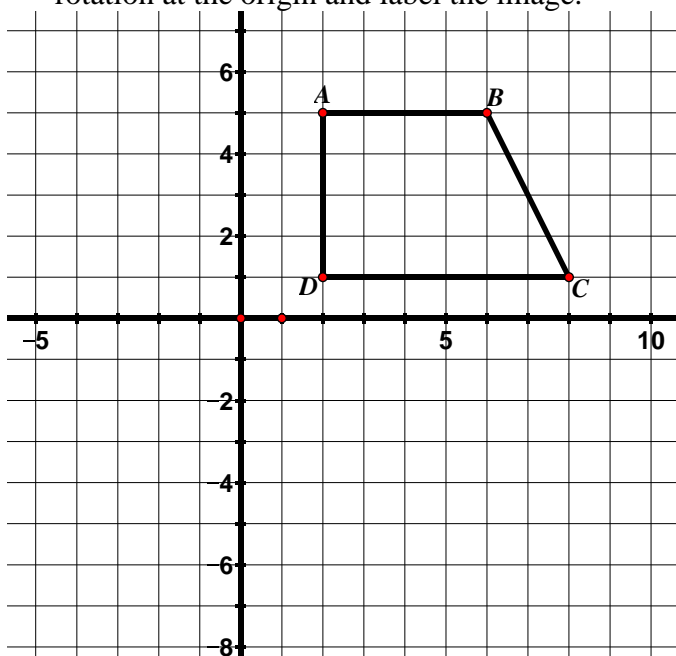
Coordinate rule for rotation: _____



3. Find the angle of rotation from Figure 1 to Figure 2. Be sure to include a direction.

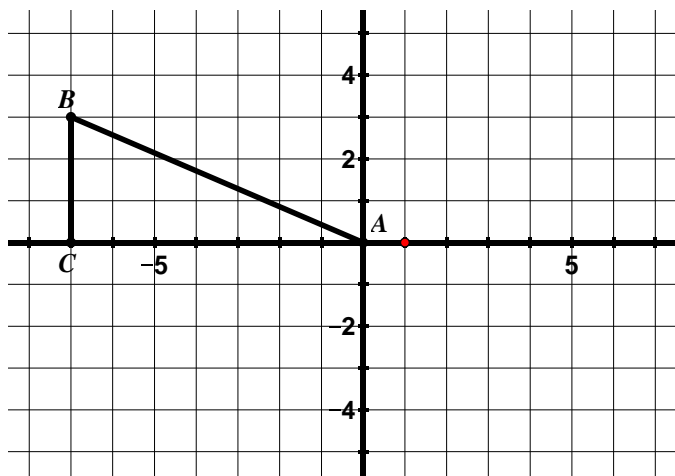


4. Rotate $ABCD$ 90° clockwise with the center of rotation at the origin and label the image.

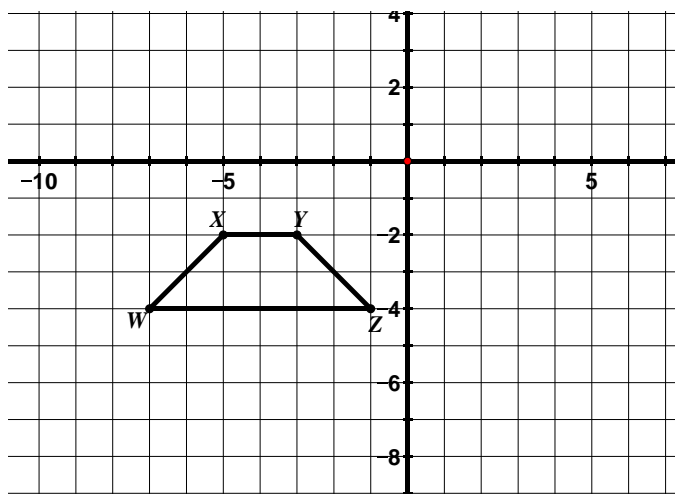


- How can you verify using slope that your image is in fact a 90° rotation?
- How can you verify using distance that the center of rotation is the origin?
- Write a coordinate rule for the rotation.

5. Rotate $\triangle ABC$ 180° clockwise with the center of rotation at the origin.



- Write a coordinate rule for the rotation.
 - Compare the slopes of the segments of the pre-image to the image.
6. Rotate $WXYZ$ 90° counterclockwise with the center of rotation at the origin.



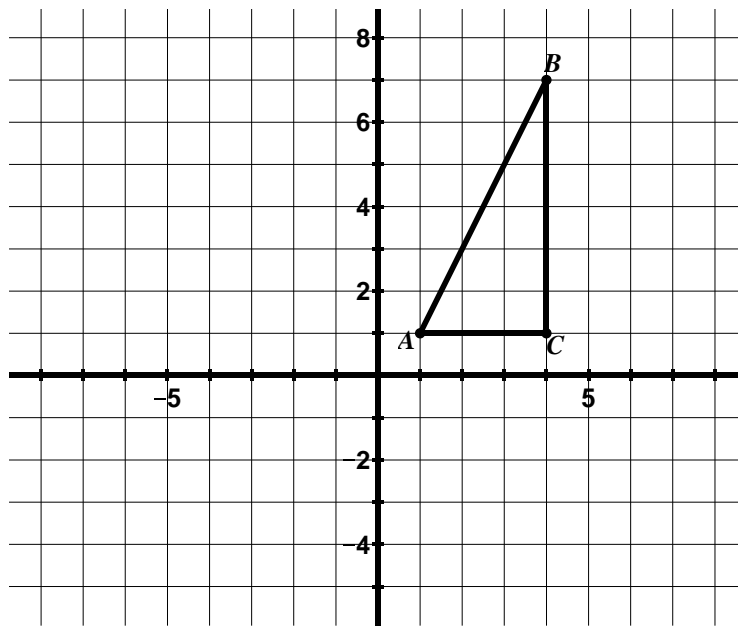
- Compare the slopes of the segments of the pre-image to the image.


9.1d Class Activity: Properties of Rotations cont.

In our prior work with rotations, the center of rotation was always at the origin. Today, we will look at rotations where the center may not be at the origin.

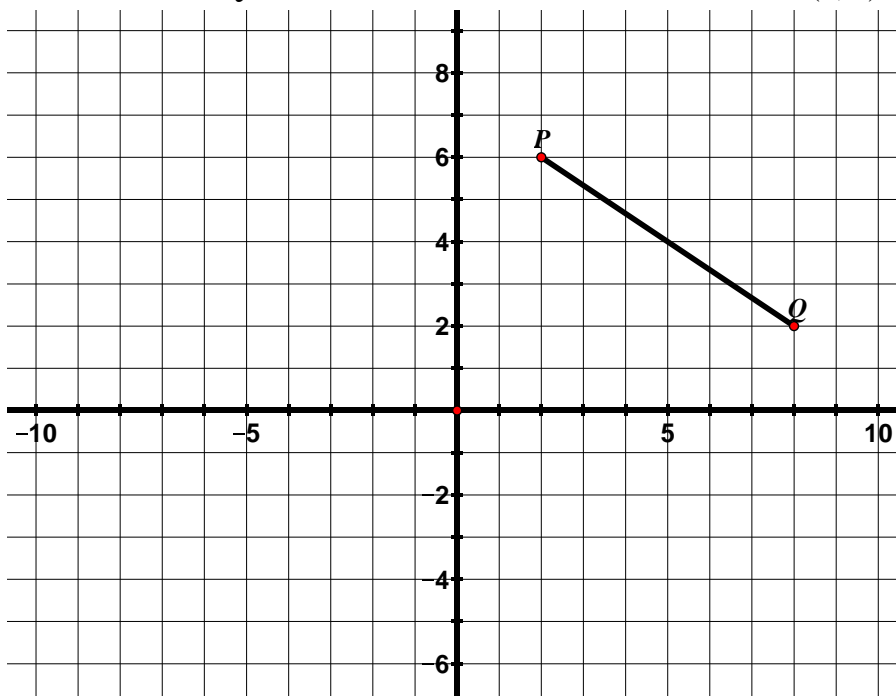
1. **Quiet Write:** In the space below, write everything you have learned about rotations so far.

2. Rotate $\triangle ABC$ 180° counterclockwise with the center of rotation at $(1, 1)$ and label the image.





a. How can you verify that your center of rotation is at $(1, 1)$? 

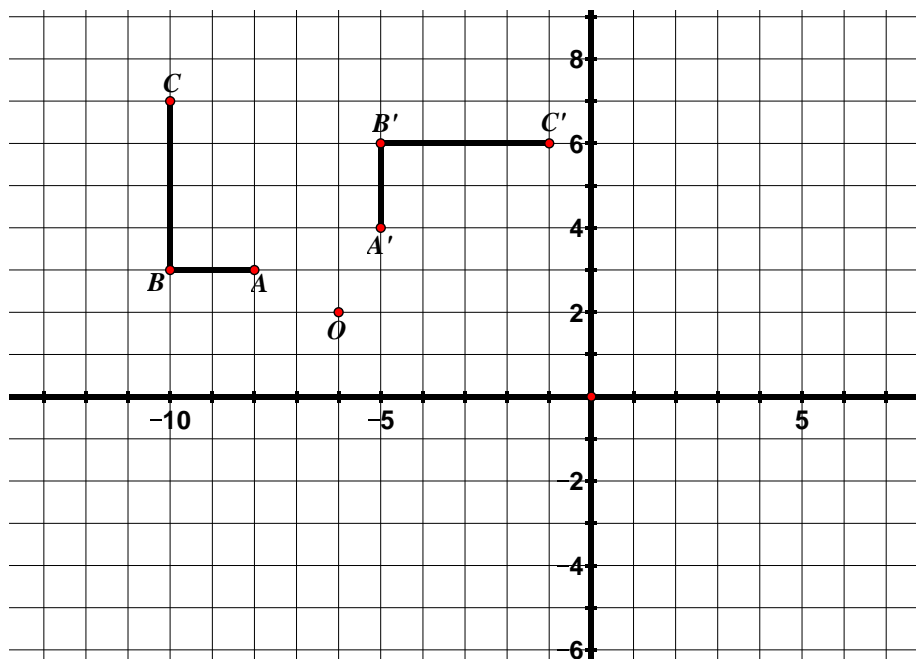
3. Rotate \overline{PQ} 90° clockwise with the center of rotation at $(0, 4)$.



- How can you verify using slope that your image is in fact a 90° rotation?
- How can you verify using distance that the center of rotation is at $(0, 4)$?

4. A teacher asked her students to determine the center of rotation and angle of rotation for the rotation

shown below.  

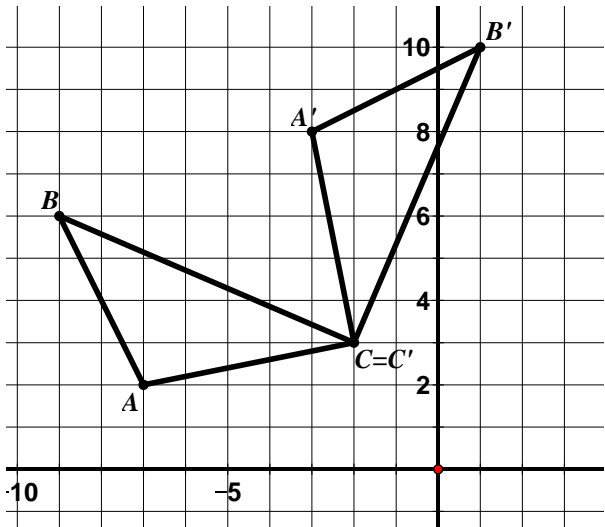


- Aisha described the rotation as a rotation 90° clockwise with the center at $O(-6, 2)$. Do you agree with Aisha? Use the properties of rotations and numerical evidence to support your answer.

Directions: For #5 – 7, find the angle of rotation (including the direction) and the center of rotation.

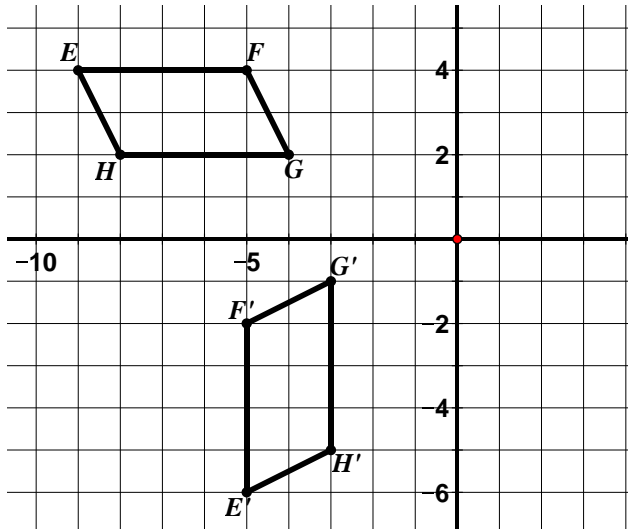
5. Angle of Rotation (including direction of rotation): _____

Center of Rotation: _____



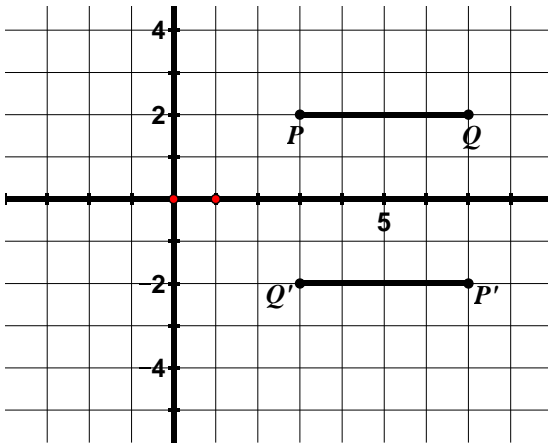
6. Angle of Rotation (including direction of rotation): _____

Center of Rotation: _____



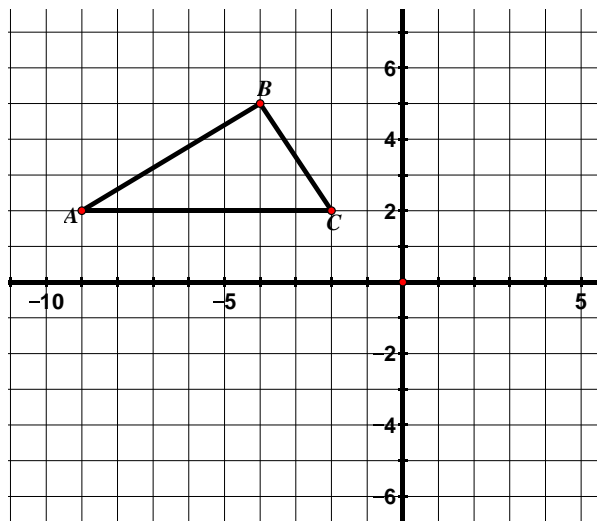
7. Angle of Rotation (including direction of rotation): _____

Center of Rotation: _____

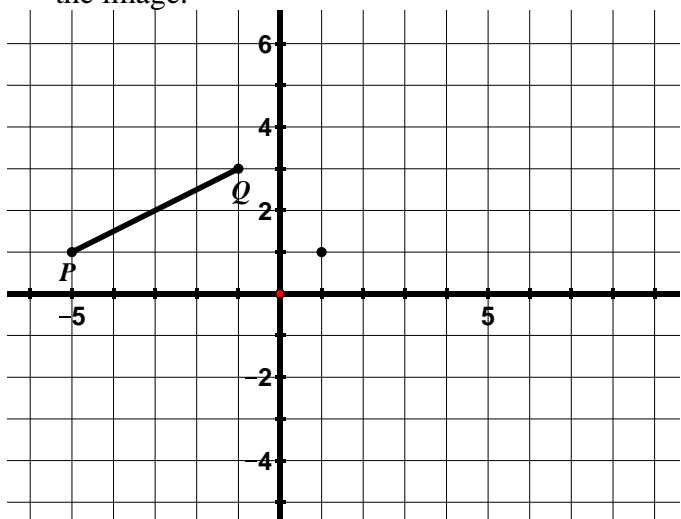


9.1d Homework: Properties of Rotations cont.

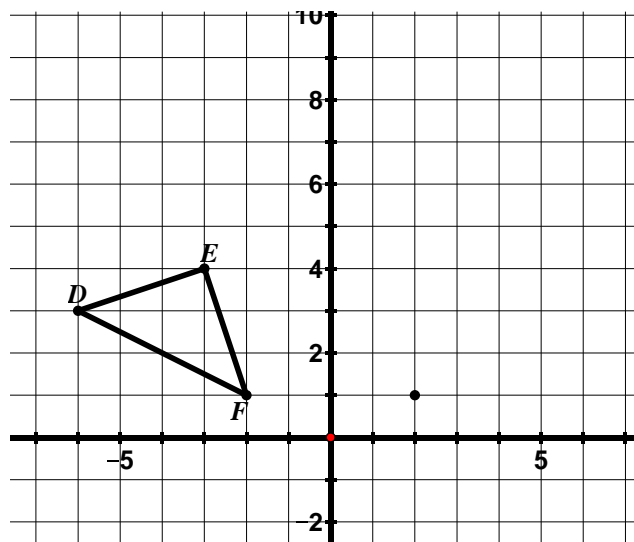
1. Rotate $\triangle ABC$ 90° counterclockwise about C and label the image.



2. Rotate PQ 180° clockwise about $(1, 1)$ and label the image.



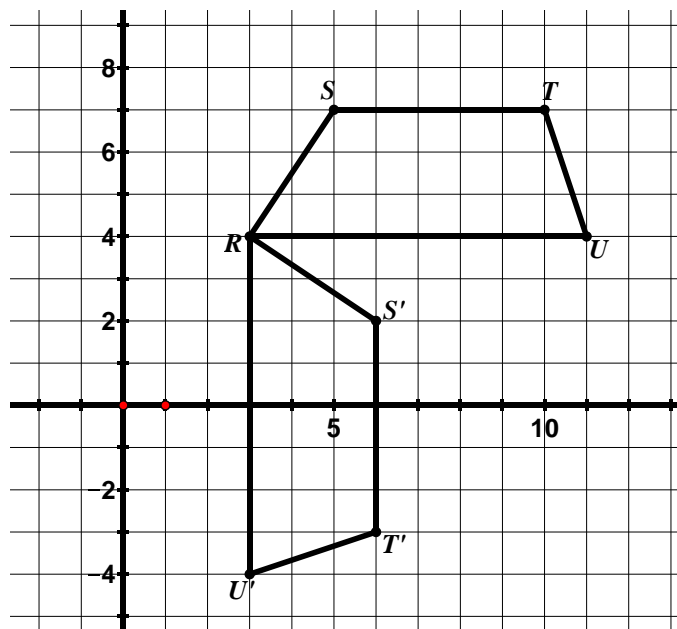
3. Rotate $\triangle DEF$ 90° clockwise about $(2, 1)$ and label the image.



Directions: For #4 – 6, find the angle of rotation (including the direction) and the center of rotation.

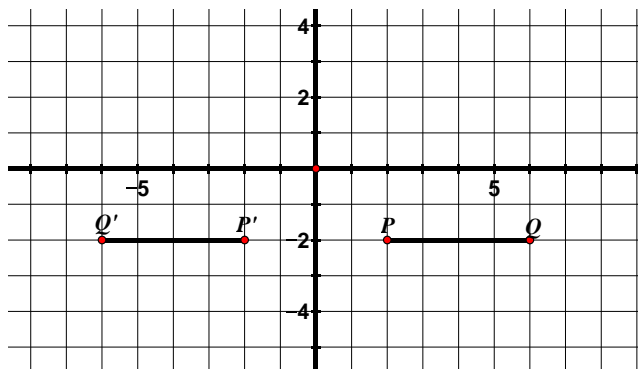
4. Angle of Rotation (including direction of rotation): _____

Center of Rotation: _____



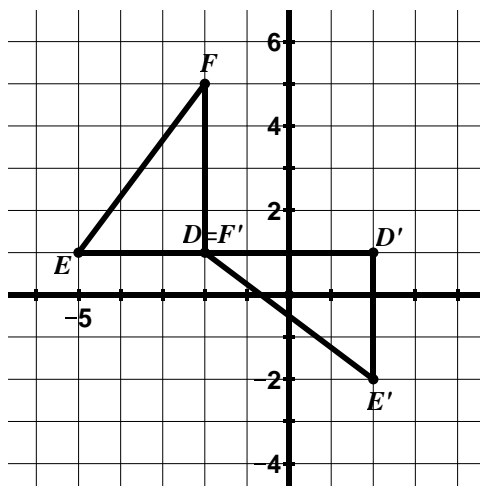
5. Angle of Rotation (including direction of rotation): _____

Center of Rotation: _____



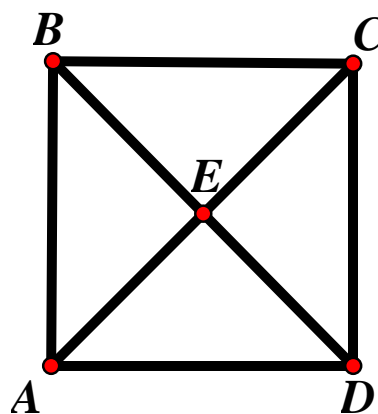
6. Angle of Rotation (including direction of rotation): _____

Center of Rotation: _____



7. $ABCD$ is a square.

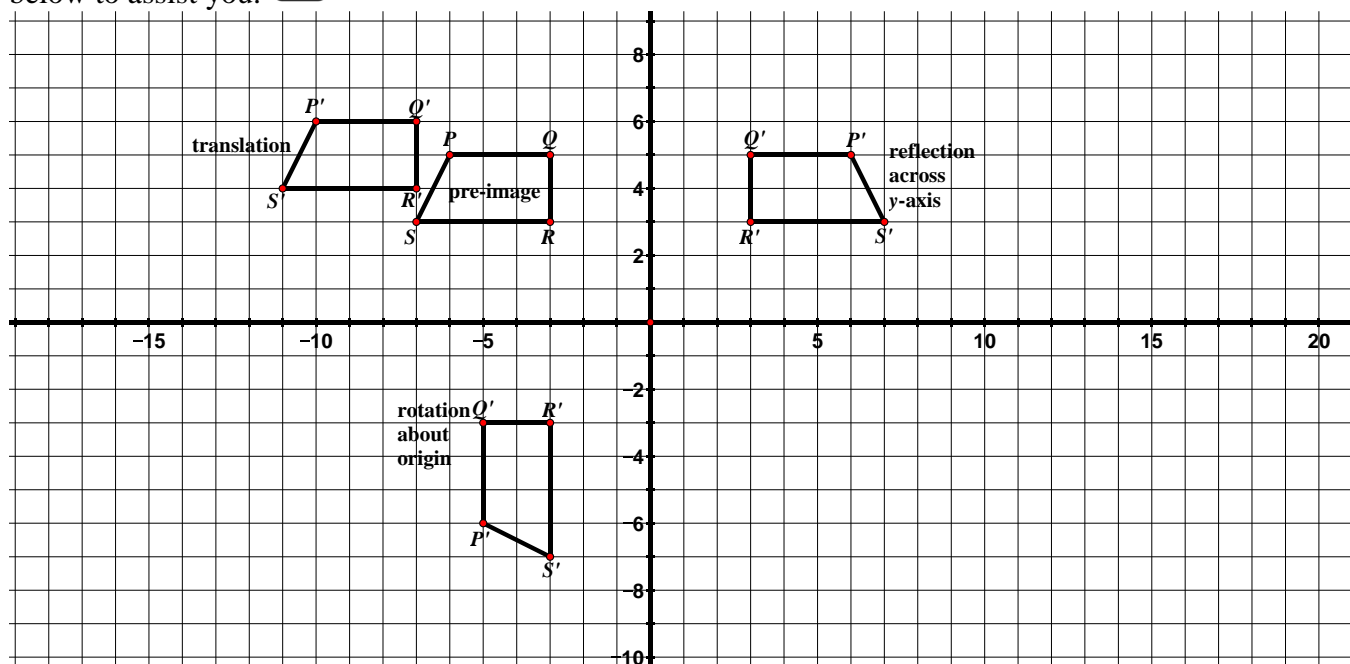
- What is the image of B under a 90° rotation counterclockwise about C ?
- What is the image of B under a 180° rotation about E ?
- Name three different rotations for which the image of A is C .



9.1e Class Activity: Congruence

The following phrases and words are properties or descriptions of one or more of the transformations we have studied so far: translation, reflection, and rotation. Determine which type of transformation(s) the statements describe and write your answer(s) on the line. An example of each type of transformation has been provided

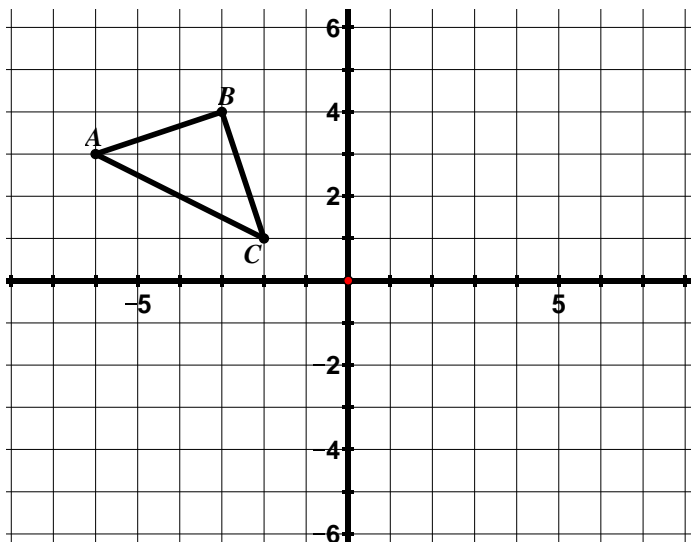
below to assist you.



Property/Description	Type of Transformation(s)
Flip	
Slide	
Turn	
Image has the same orientation as pre-image.	
Specified by a figure, a center of rotation, and an angle of rotation.	
Specified by a figure and a line of reflection.	
Specified by a figure, a distance, and a direction.	
Segments connecting corresponding vertices of image and pre-image are the same length.	
Corresponding image and pre-image vertices lie on the same circle.	
Segments connecting corresponding vertices of image and pre-image are parallel to each other.	
Line of reflection is the perpendicular bisector of all segments connecting corresponding vertices of the image and pre-image.	
Concentric circles	
Orientation of the figure does not change.	
The slopes of corresponding segments may change.	
Corresponding segments in the image and pre-image are the same length.	
Corresponding angles in the image and pre-image have the same measure.	
Parallel lines in the pre-image remain parallel lines in the image.	

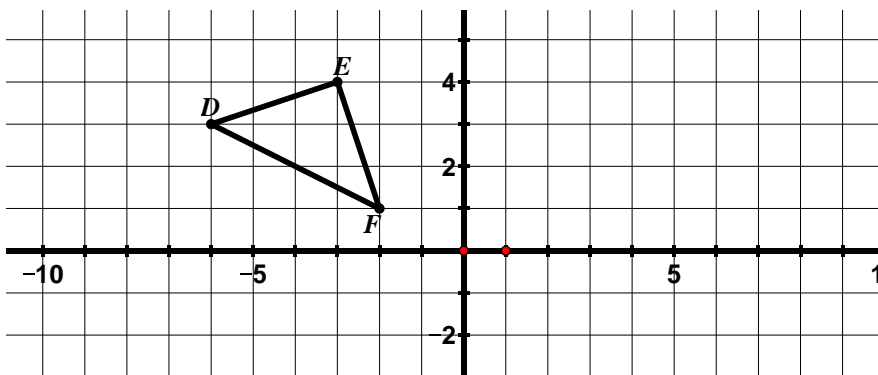
In the examples we have studied so far, we have only performed one transformation on a figure. We can also perform more than one transformation on a figure. In the following problems, you will perform a **sequence of transformations** on a figure.

1. $\triangle ABC$ has been plotted below.



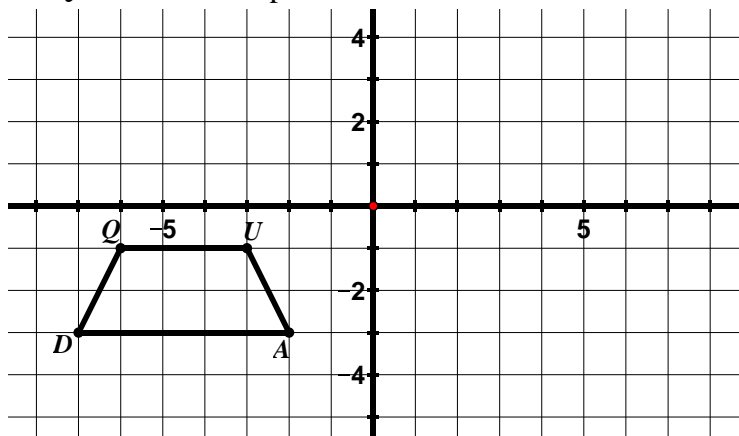
- Reflect $\triangle ABC$ over the y -axis and label the image $\triangle A'B'C'$.
- Reflect $\triangle A'B'C'$ over the x -axis and label the image $\triangle A''B''C''$.
- What **one** transformation is the same as this double reflection?

2. $\triangle DEF$ has been plotted below.



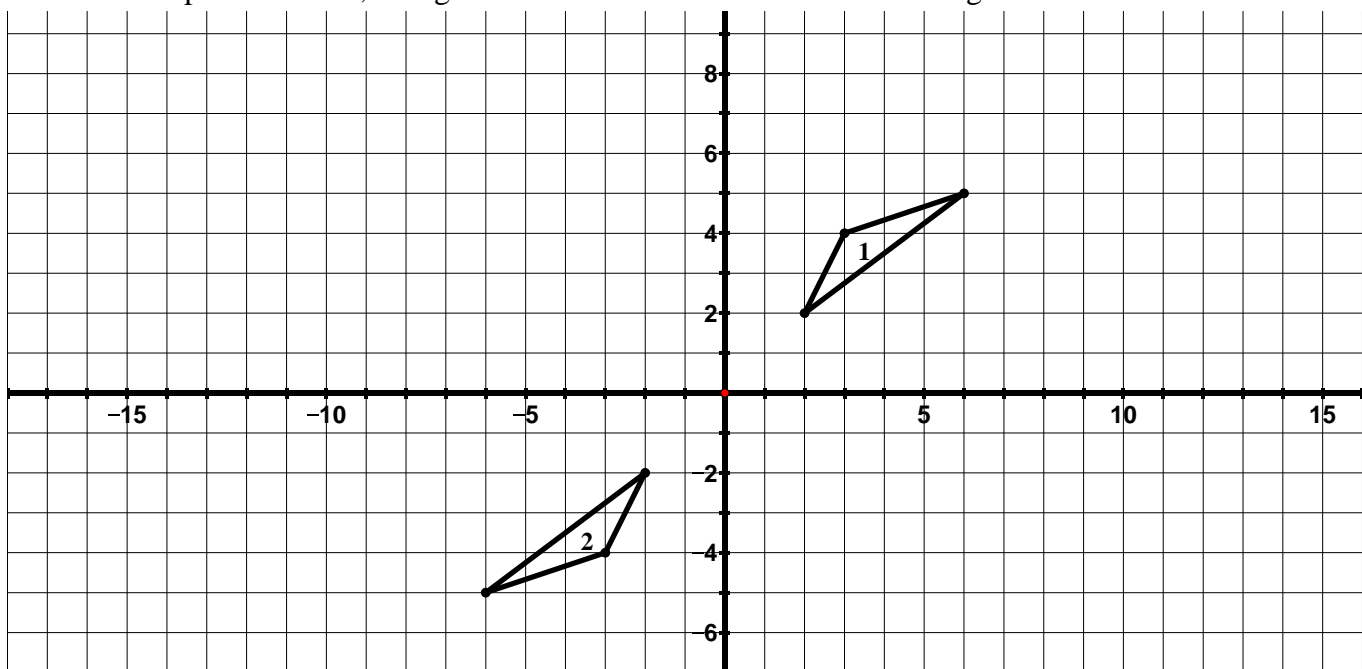
- Reflect $\triangle DEF$ over the line $x = 1$ and label the image $\triangle D'E'F'$.
- Reflect $\triangle D'E'F'$ over the y -axis and label the image $\triangle D''E''F''$.
- What **one** transformation is the same as this double reflection?
- Write a coordinate rule for the transformation of $\triangle DEF$ to $\triangle D''E''F''$.

3. $QUAD$ has been plotted below.



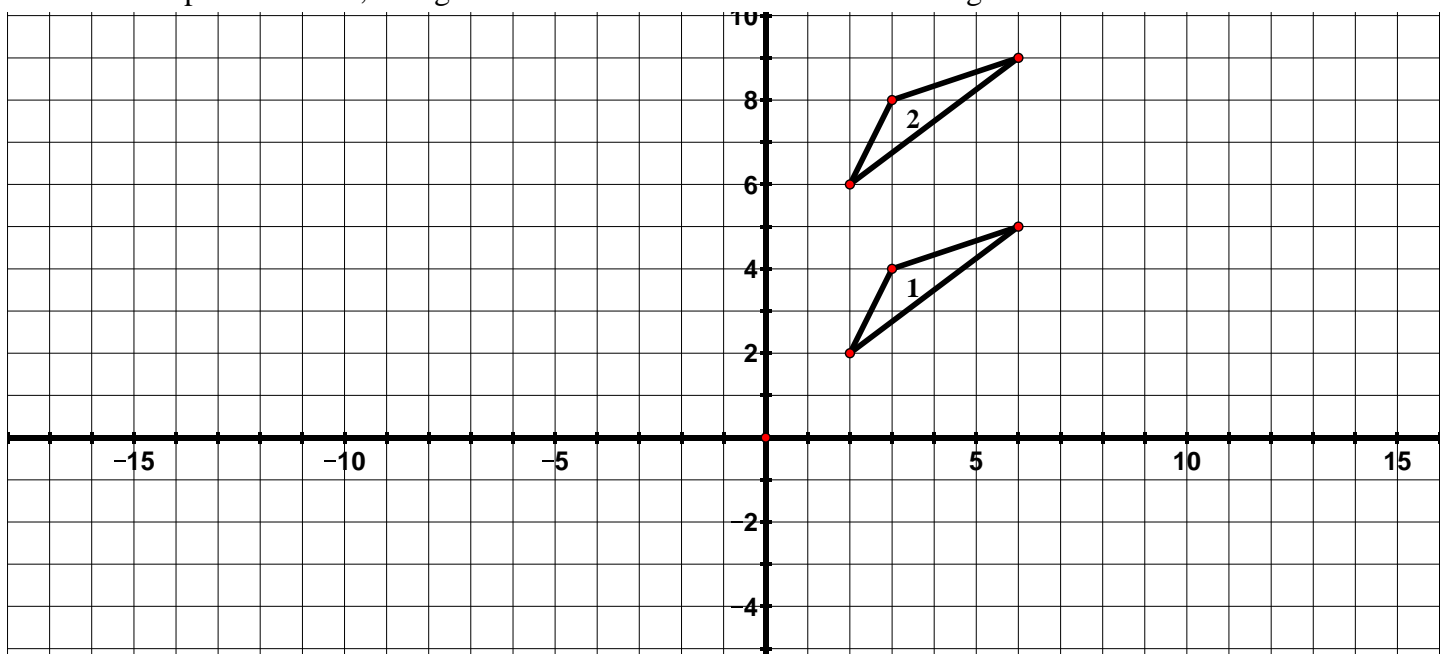
- Reflect $QUAD$ over the x -axis and label the image $Q'U'A'D'$.
- Translate $Q'U'A'D'$ according to the rule $(x, y) \rightarrow (x + 9, y)$ label the image $Q''U''A''D''$.

4. In the picture below, triangle 1 has been transformed to obtain triangle 2.



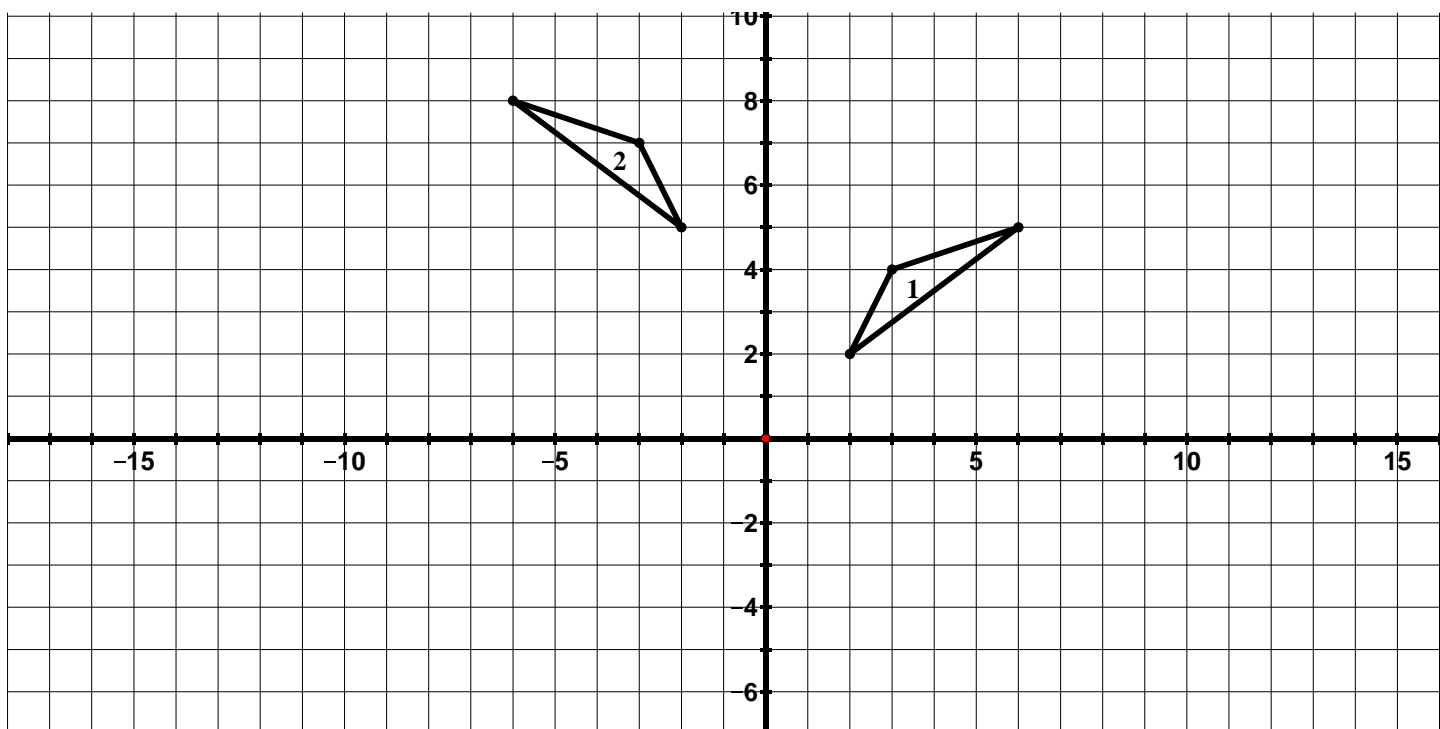
- a. Which **two** transformations in succession would carry triangle 1 onto triangle 2?
- b. Which **one** transformation would carry triangle 1 onto triangle 2?

5. In the picture below, triangle 1 has been transformed to obtain triangle 2.

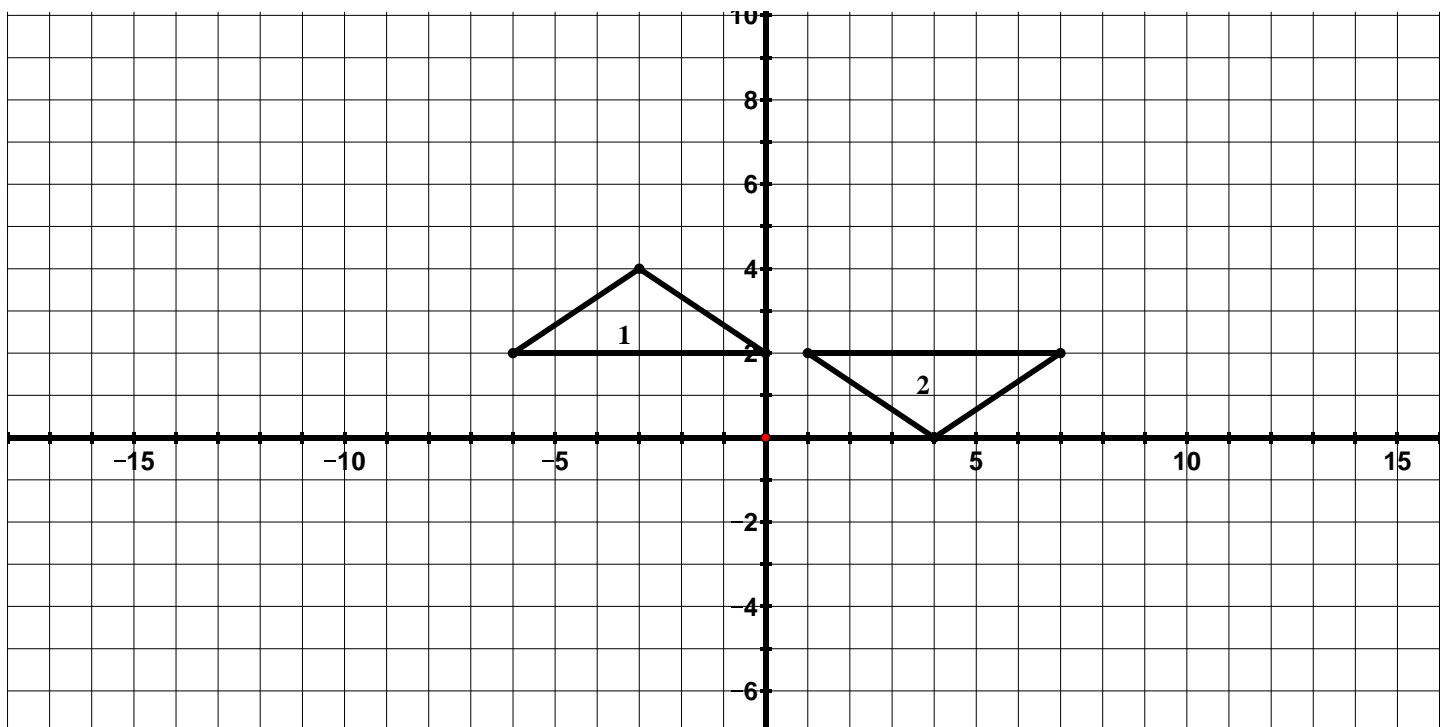


- a. Which **two** transformations in succession would carry triangle 1 onto triangle 2?
- b. Which **one** transformation would carry triangle 1 onto triangle 2?

6. Describe a transformation or sequence of transformations that would carry triangle 1 onto triangle 2.

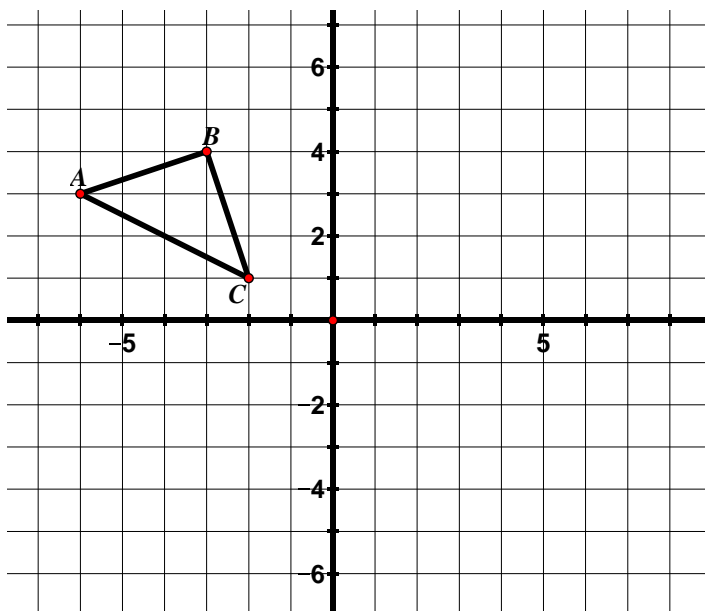



7. Describe a transformation or sequence of transformations that would carry triangle 1 onto triangle 2.



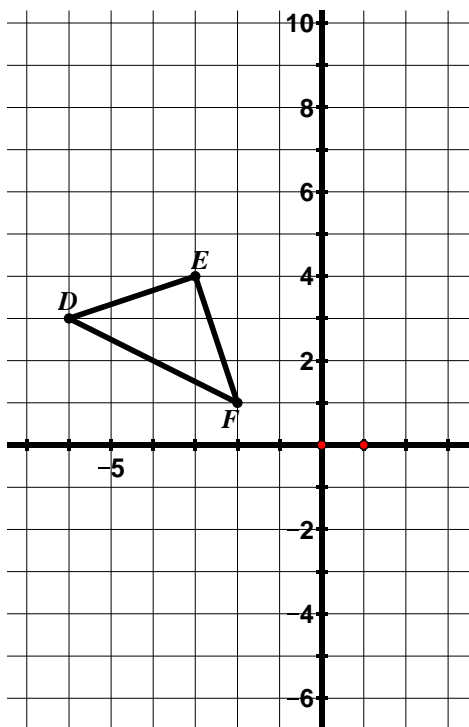
9.1e Homework: Congruence


1. $\triangle ABC$ has been plotted below.



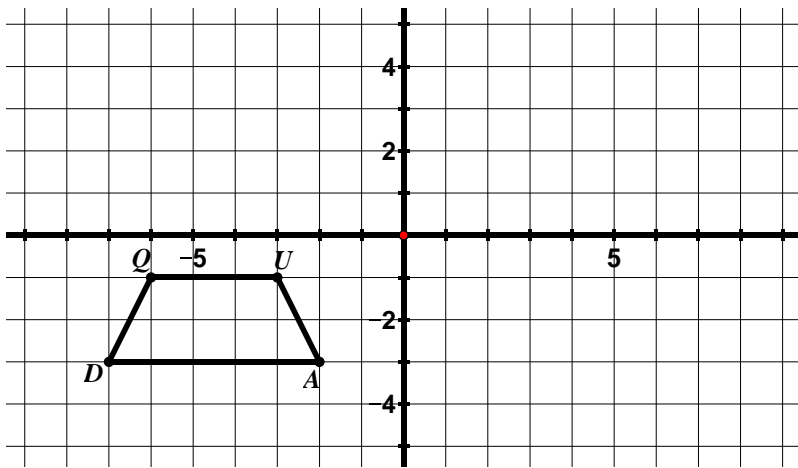
- Reflect $\triangle ABC$ over the x -axis and label the image $A'B'C'$.
- Reflect $\triangle A'B'C'$ over the y -axis and label the image $A''B''C''$.
- What **one** transformation is the same as this double reflection?
- In #1 of your class work, we performed a similar series of transformation; however we reflected over the y -axis first and then the x -axis. Compare these transformations. 


2. $\triangle DEF$ has been plotted below.



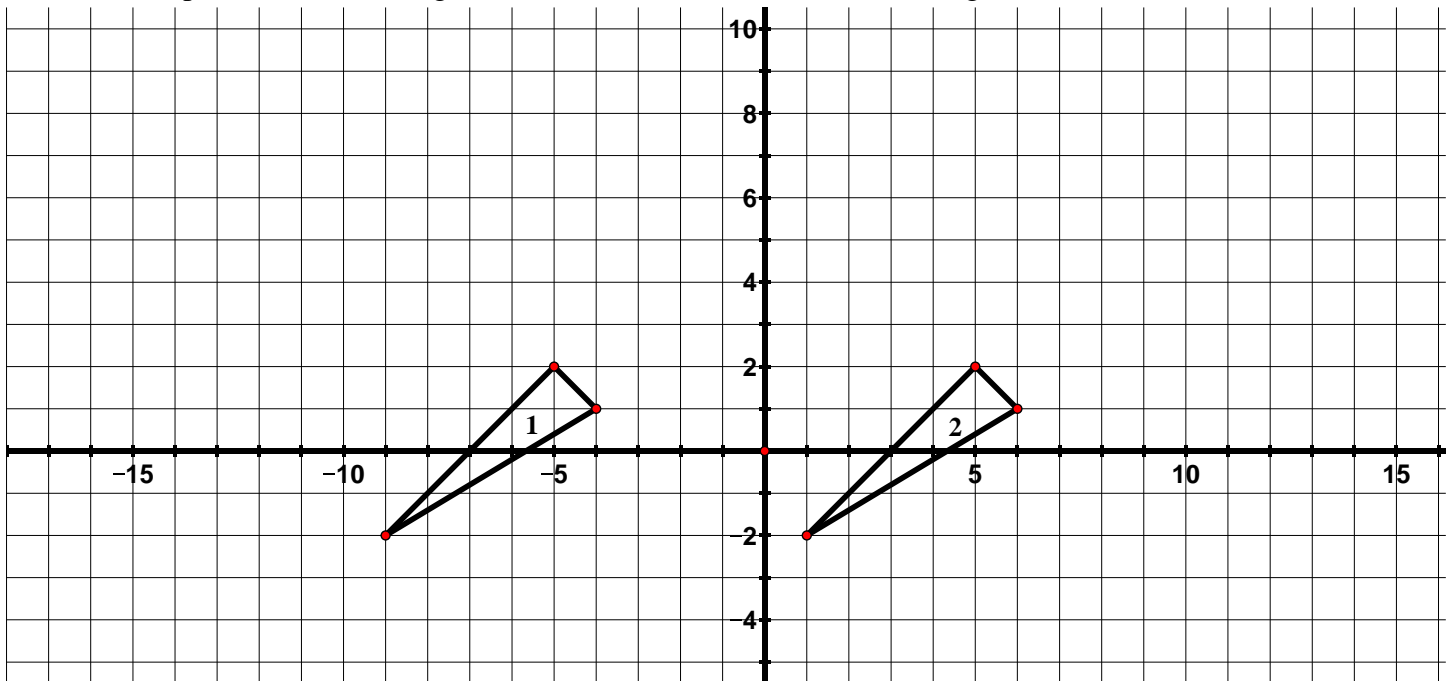
- Reflect $\triangle DEF$ over the line $y = -1$ and label the image $\triangle D'E'F'$.
- Reflect $\triangle D'E'F'$ over the line $y = 2$ and label the image $\triangle D''E''F''$.
- What **one** transformation is the same as this double reflection?
- Write a coordinate rule for the transformation of $\triangle DEF$ to $\triangle D''E''F''$.
- In #2 of your class work, we performed a similar series of transformation; however we reflected over vertical lines. Compare these transformations and write your observations below. 

3. *QUAD* has been plotted below.



- Translate *QUAD* according to the rule $(x, y) \rightarrow (x + 9, y)$.
- Reflect $Q'U'A'D'$ over the x -axis and label the image $Q''U''A''D''$.
- In #3 of your class work, we performed a similar series of transformation; however we did the reflection first and then the translation. Compare these transformations and write your observations below. 

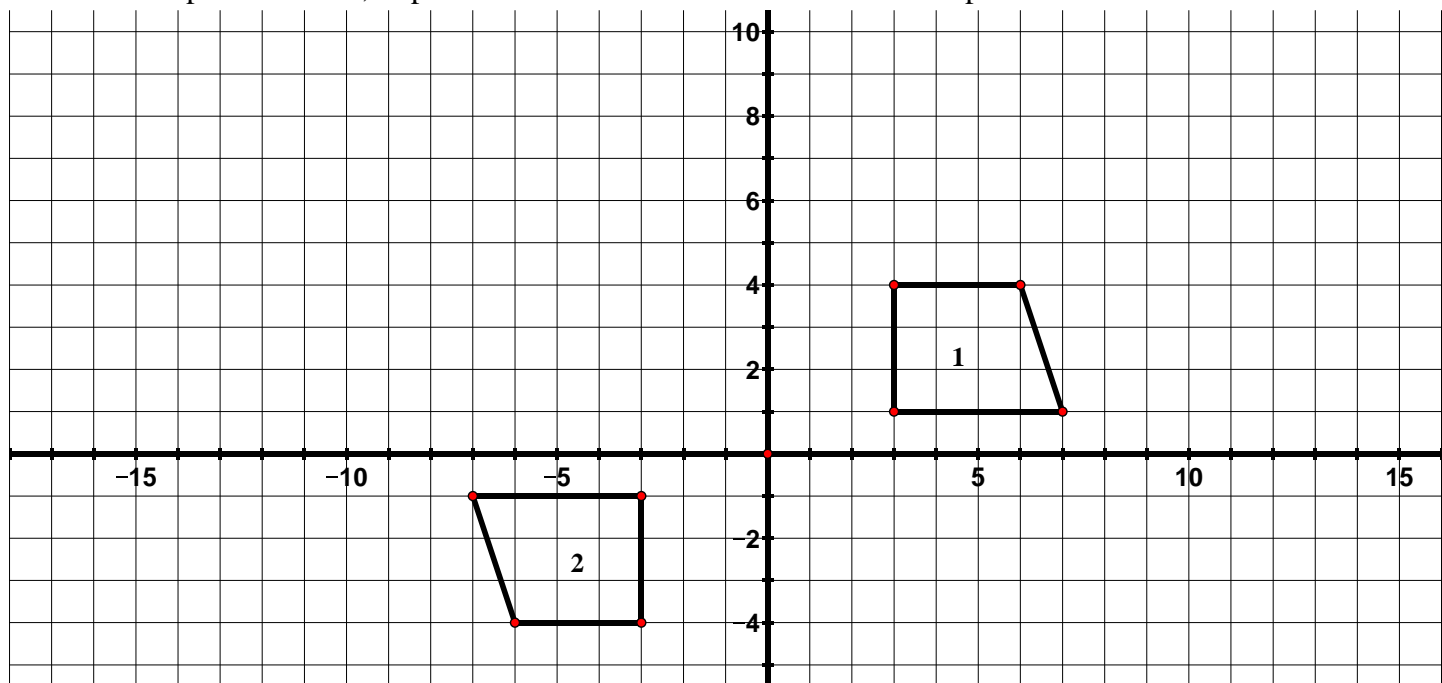
4. In the picture below, triangle 1 has been transformed to obtain triangle 2.



a. Which **two** transformations in succession would carry triangle 1 onto triangle 2?

b. Which **one** transformation would carry triangle 1 onto triangle 2?

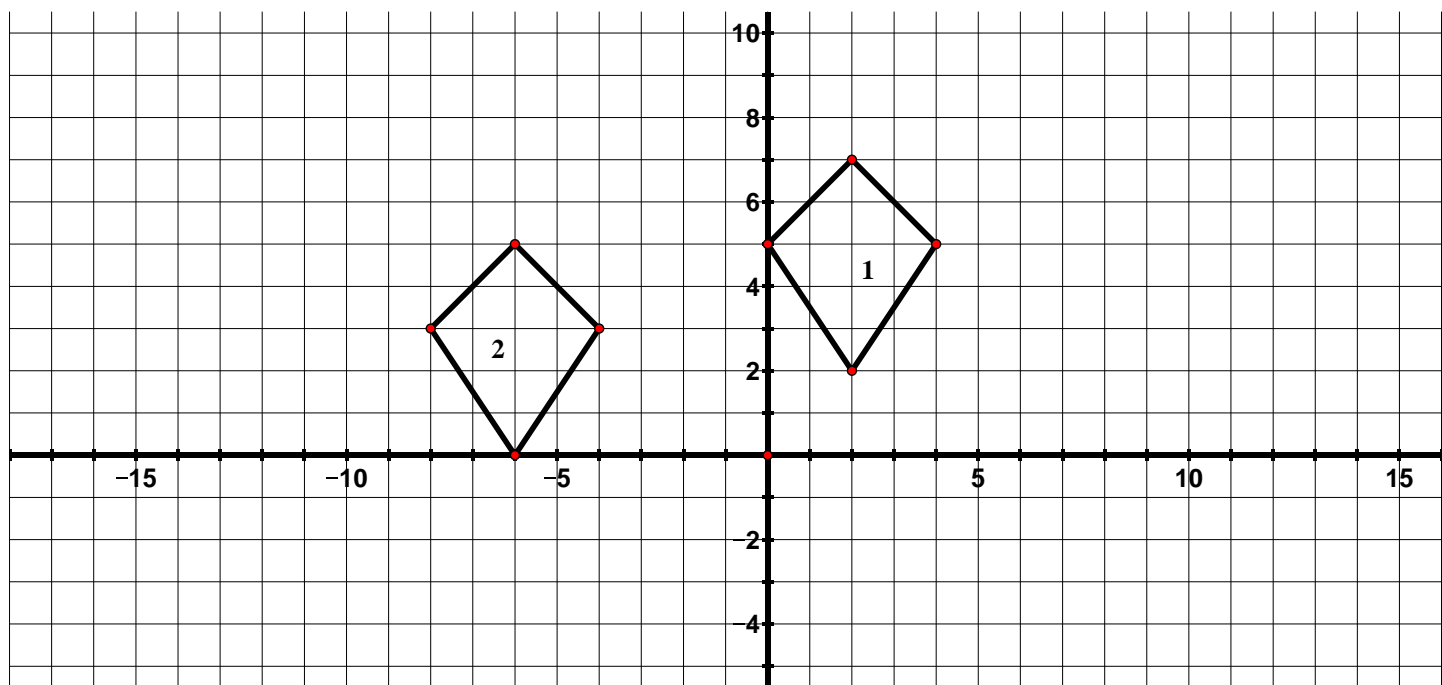
5. In the picture below, trapezoid 1 has been transformed to obtain trapezoid 2.



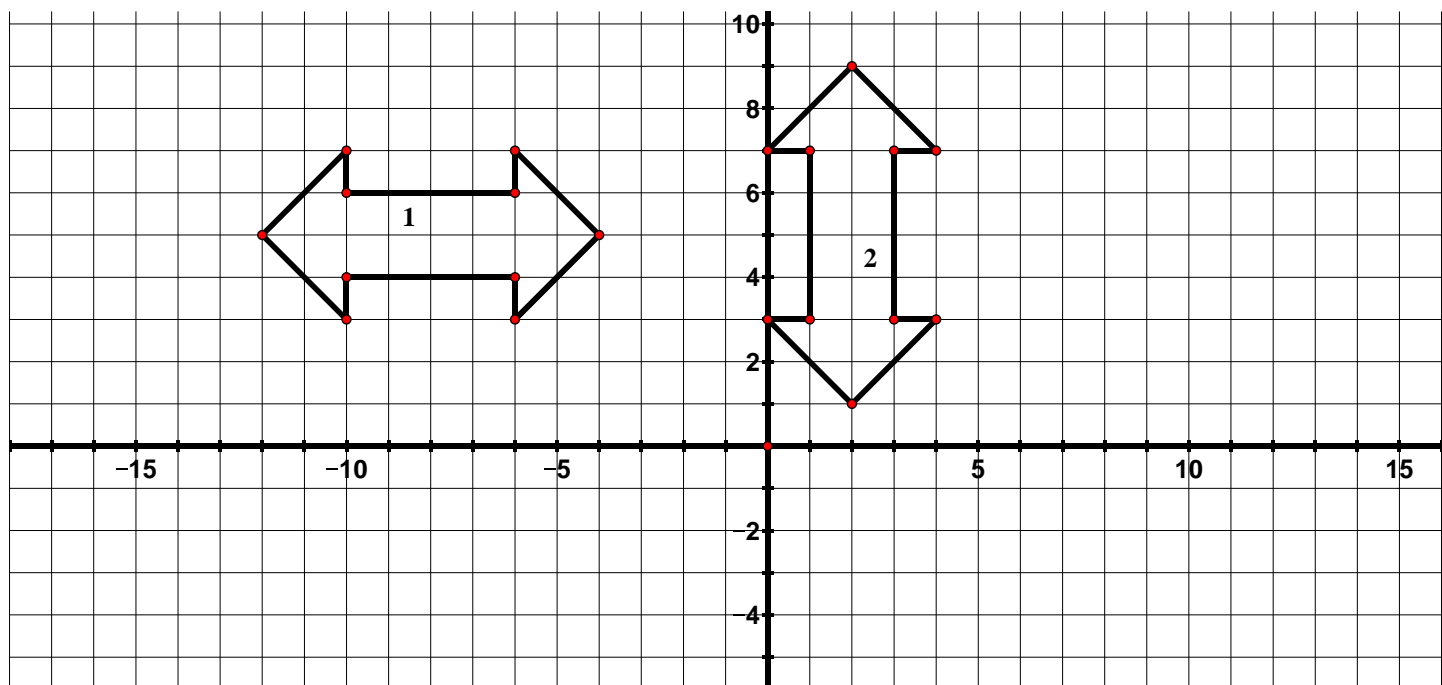
a. Which **two** transformations in succession would carry trapezoid 1 onto trapezoid 2?

b. Which **one** transformation would carry trapezoid 1 onto trapezoid 2?

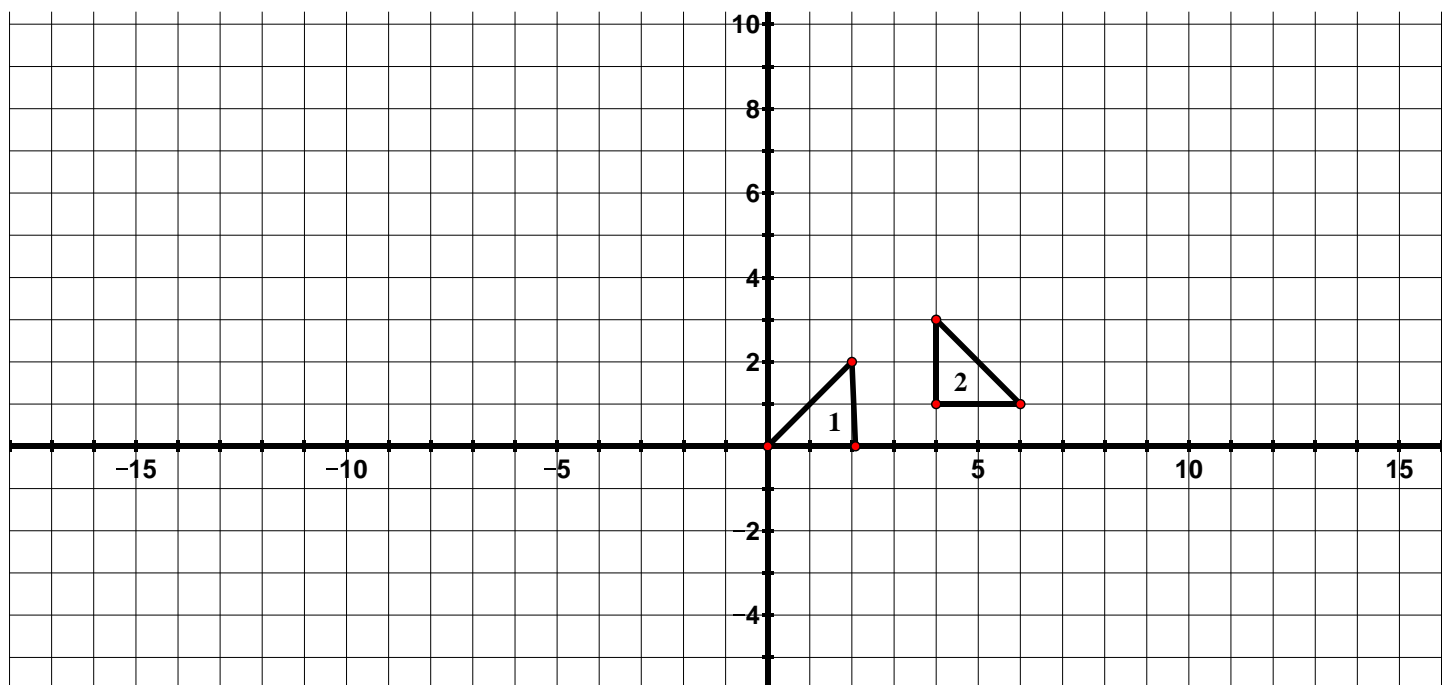
6. Describe a transformation or sequence of transformations that would carry figure 1 onto figure 2.



7. Describe a transformation or sequence of transformations that would carry figure 1 onto figure 2.

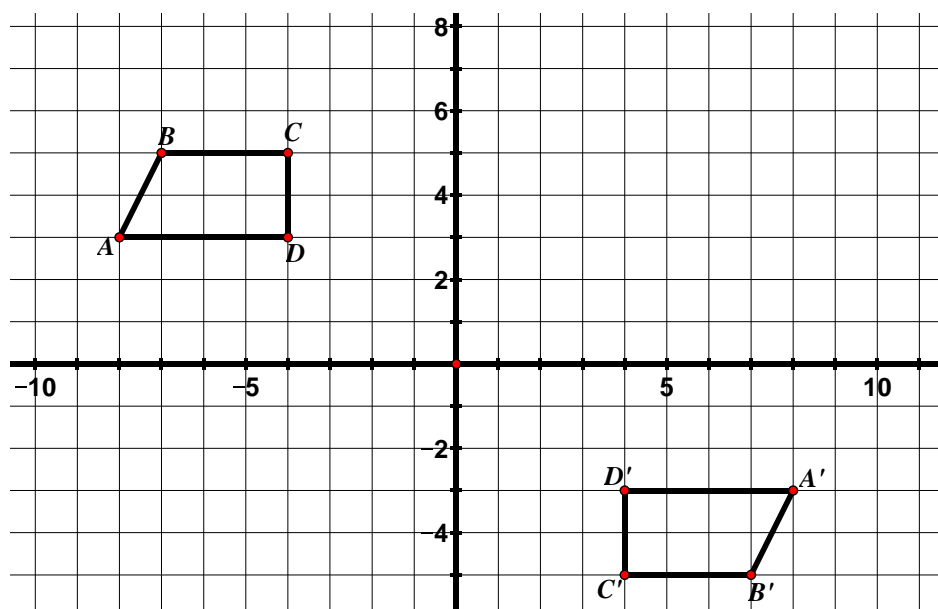


8. Describe a transformation or sequence of transformations that would carry triangle 1 onto triangle 2.



9.1f Class Activity: Congruence cont.

1. Observe the two figures below.

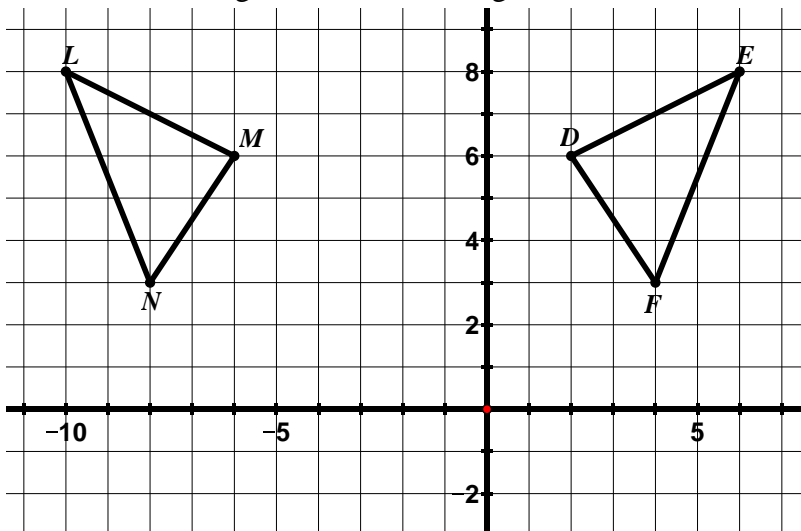


- a. Describe the ways in which the figures are the same and the ways in which they are different.

The two figures above are said to be **congruent**. In 7th grade, you learned that two figures are congruent if they have the **same shape** and are the **same size**. In 8th grade, we define congruence in terms of transformations. A two-dimensional figure is congruent to another if the second can be obtained from the first by a rigid motion (rotation, reflection, or translation) or a sequence of rigid motions. If you can move one of the figures using one of these transformations or a series of these transformations so that it fits exactly on the other one, the two figures are congruent.

- b. In this case, there are several different transformations that will carry one figure onto the other. Describe one transformation (or sequence of transformations) that will carry $ABCD$ onto $A'B'C'D'$.
- c. Can you think of a different transformation (or sequence of transformations) that will carry $ABCD$ onto $A'B'C'D'$?
- d. A translation, reflection, and rotation are described as rigid motions. Describe in your own words what this means.

2. The two figures below are congruent.



a. Describe the transformation or sequence of transformations that will carry $\triangle LMN$ onto $\triangle EDF$.

b. Congruent figures have corresponding parts – their matching sides and angles. For example, in the figure above, \overline{LM} corresponds to \overline{ED} and $\angle D$ corresponds to $\angle M$. List the other corresponding parts below.

\overline{LN} corresponds to _____

$\angle E$ corresponds to _____

\overline{MN} corresponds to _____

$\angle F$ corresponds to _____

We can write a congruence statement for the two triangles. You can denote that two figures are congruent by using the symbol \cong and listing their vertices in corresponding order.

In the example above, we would write this symbolically as $\triangle LMN \cong \triangle EDF$. The order the vertices is written tells us which segments and angles are corresponding in the figures.

Corresponding parts of congruent figures are congruent (corresponding segments have the same length and corresponding angles have the same measure). We can show this symbolically in the following way:

$$\overline{LM} \cong \overline{ED}$$

$$\angle D \cong \angle M$$

$$\overline{LN} \cong \overline{EF}$$

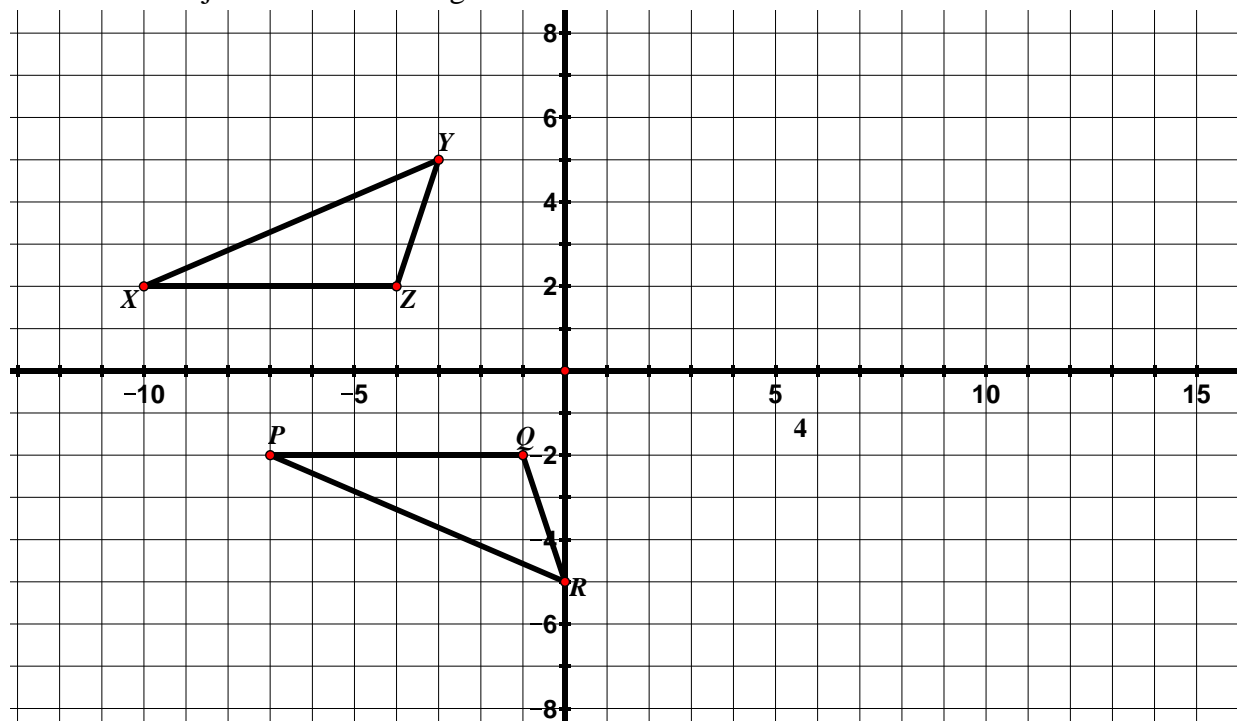
$$\angle E \cong \angle L$$

$$\overline{MN} \cong \overline{DF}$$


$$\angle F \cong \angle N$$

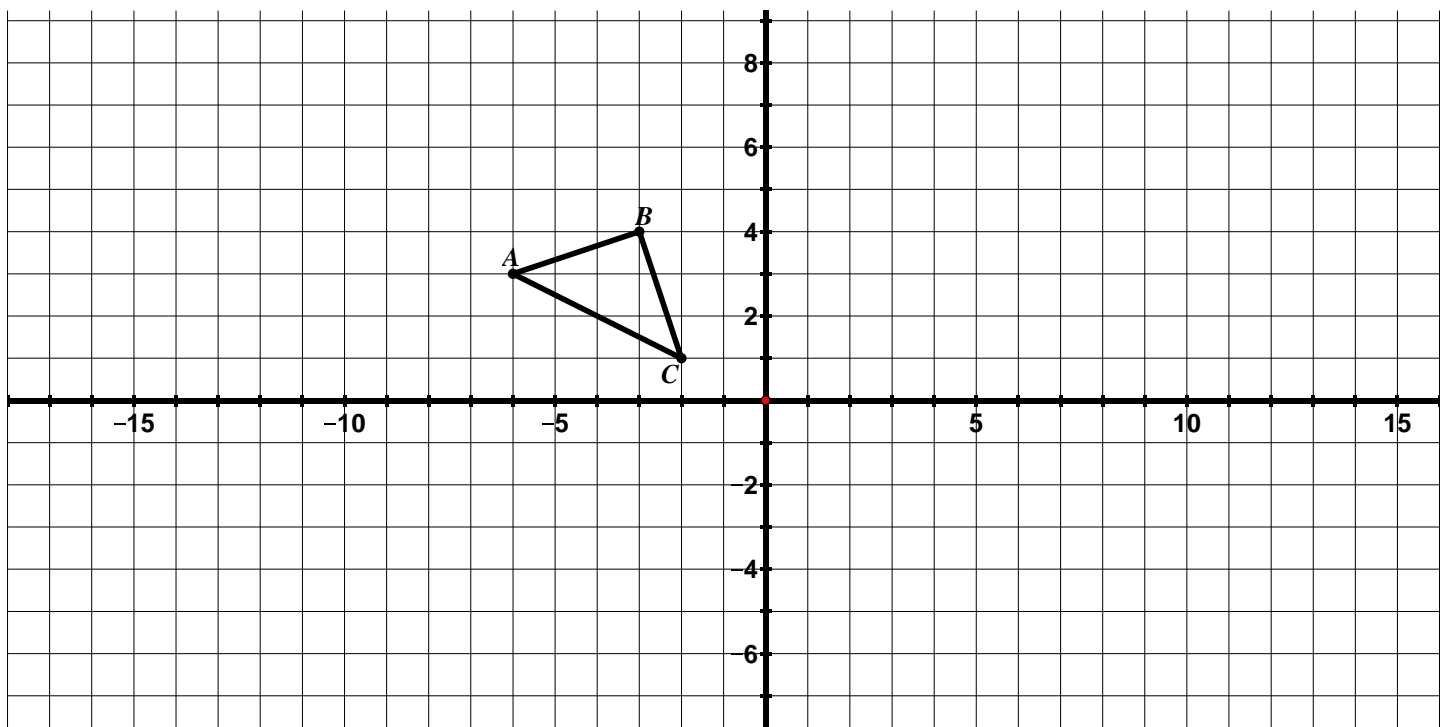
We can also annotate the diagram to show which parts are congruent. Do this on the diagram above.

3. The two objects below are congruent.



- Describe the transformation or sequence of transformations that will carry ΔXYZ onto ΔPRQ .
- List the congruent corresponding parts.
- Write a congruence statement for the triangles.
- Annotate the diagram to show which parts are congruent.

4. Using $\triangle ABC$ in the diagram below as the pre-image, apply the following rules to $\triangle ABC$ and determine whether the resulting image is congruent to $\triangle ABC$. Always start with $\triangle ABC$ as your pre-image. 



- $(x, y) \rightarrow (x, y + 7)$
Is the resulting image congruent to $\triangle ABC$? Why or why not?
- $(x, y) \rightarrow (-x, y)$
Is the resulting image congruent to $\triangle ABC$? Why or why not?
- $(x, y) \rightarrow (x, 2y)$
Is the resulting image congruent to $\triangle ABC$? Why or why not?
- $(x, y) \rightarrow (2x, 2y)$
Is the resulting image congruent to $\triangle ABC$? Why or why not?
- Write your own coordinate rule that is different than the ones above that would result in an image that **is congruent** to $\triangle ABC$. How do you know that the resulting image is congruent to $\triangle ABC$?
- Write your own coordinate rule that is different than the ones above that would result in an image that **is not congruent** to $\triangle ABC$. How do you know that the resulting image is not congruent to $\triangle ABC$?

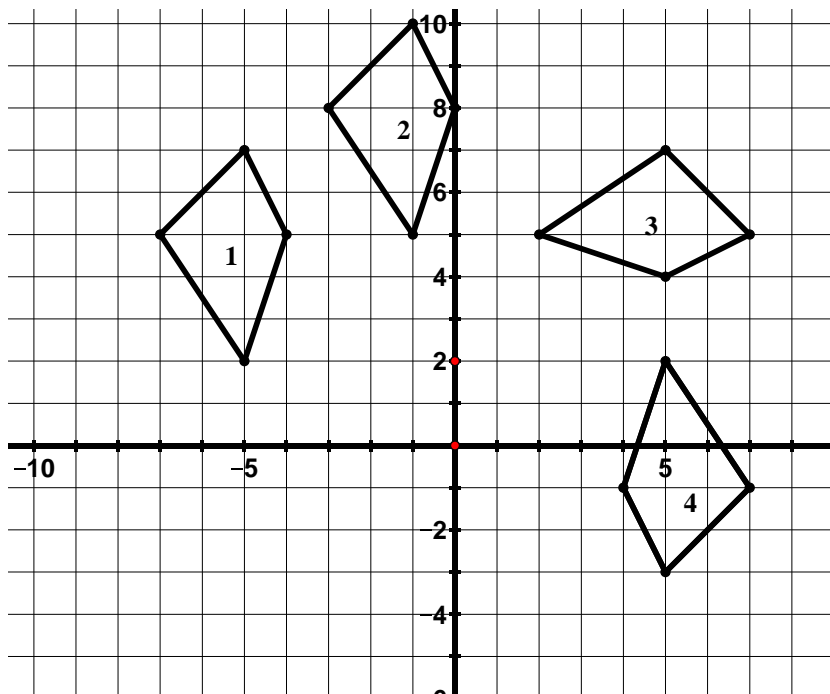



5. Which of the following properties of a figure can change during a rigid motion? Explain.

- a. Interior angles
- b. Slope of a side
- c. Parallel lines in the pre-image
- d. Orientation
- e. Side lengths
- f. Location in the plane
- g. Perimeter
- h. Area

9.1f Homework: Congruence cont.

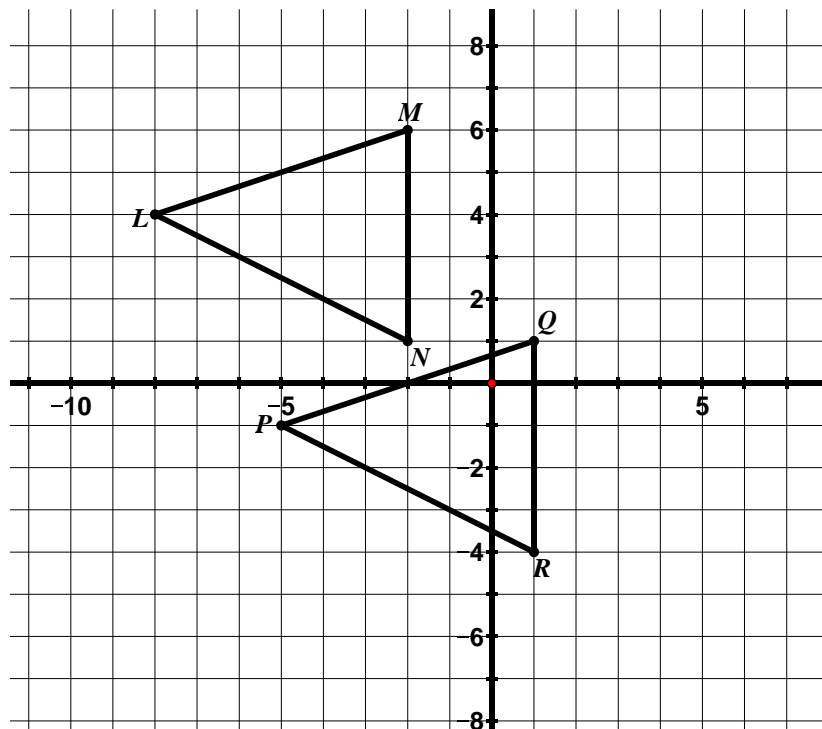
1. Jeff's teacher asked him to create 3 figures that were congruent to figure 1 in the picture below. Jeff created figures 2, 3, and 4.



- a. Use the definition of congruence to determine if Jeff's figures are congruent to figure 1. Explain your answers. 

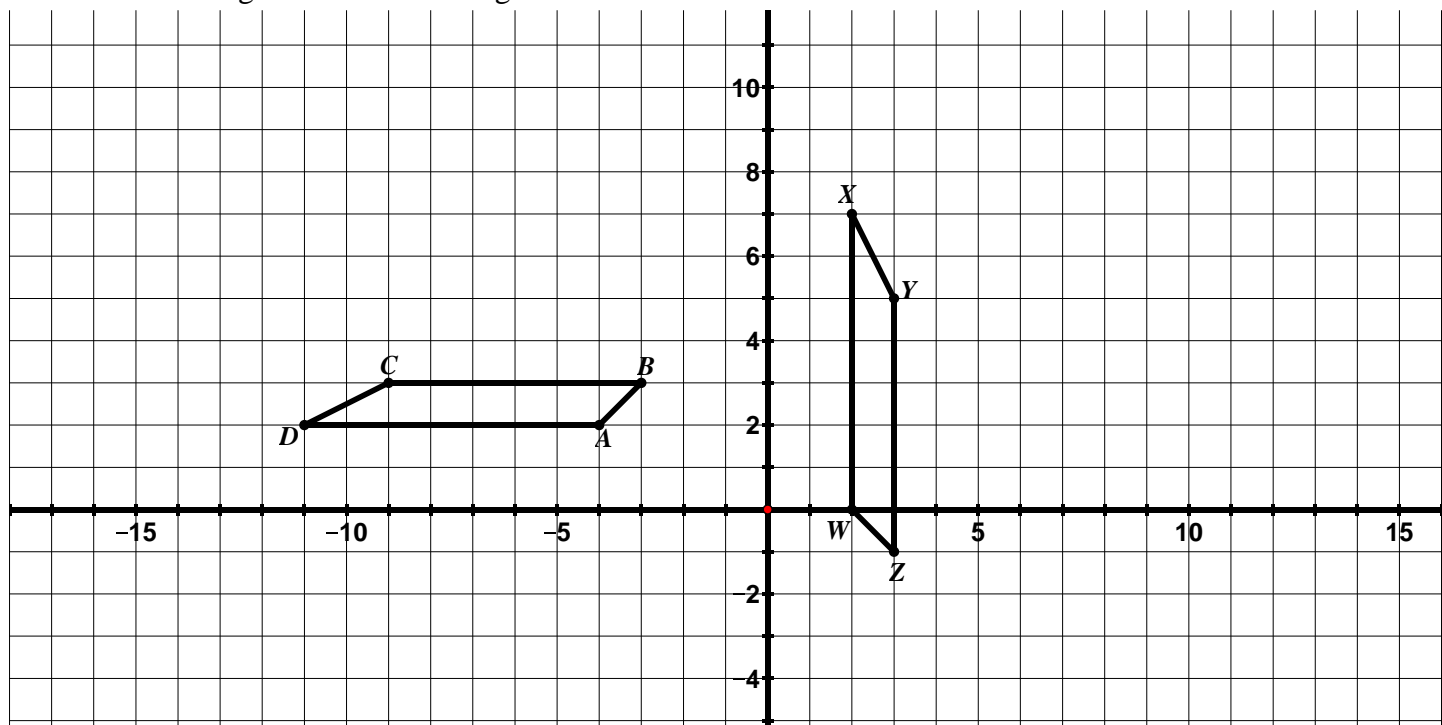
- b. Draw an additional figure that is congruent to figure 1. How do you know your figure is congruent to figure 1?

2. The two figures below are congruent.



- a. Describe the transformation or sequence of transformations that will carry $\triangle LMN$ onto $\triangle PQR$.
- b. List the congruent corresponding parts.
- c. Write a congruence statement for the triangles.

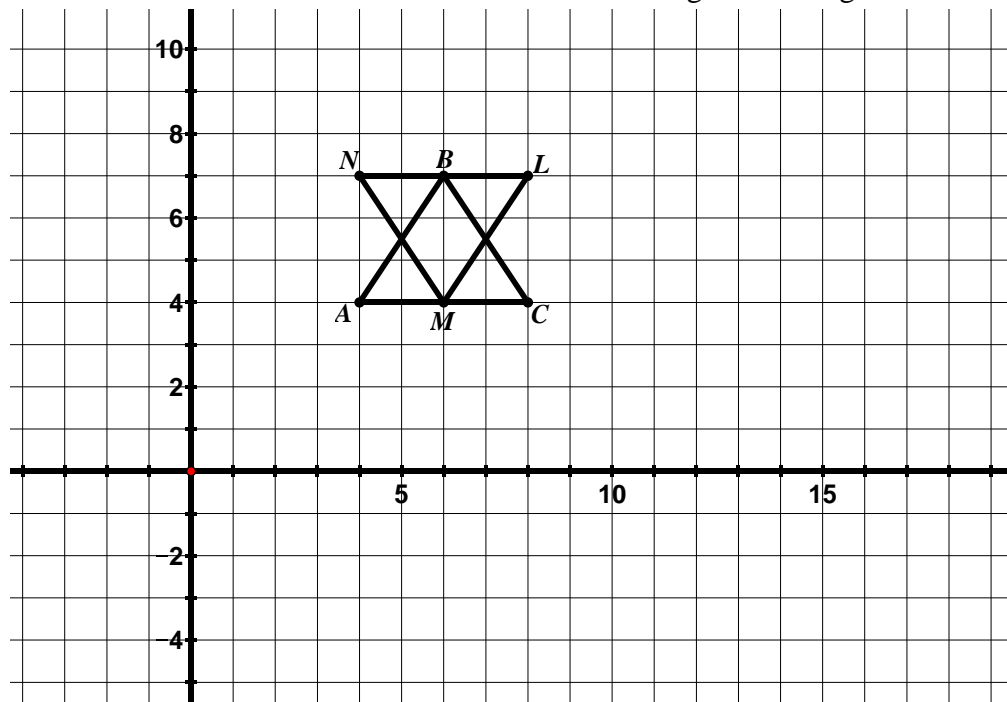
3. The two figures below are congruent.



a. Describe the transformation or sequence of transformations that will carry $ABCD$ onto $WXYZ$.

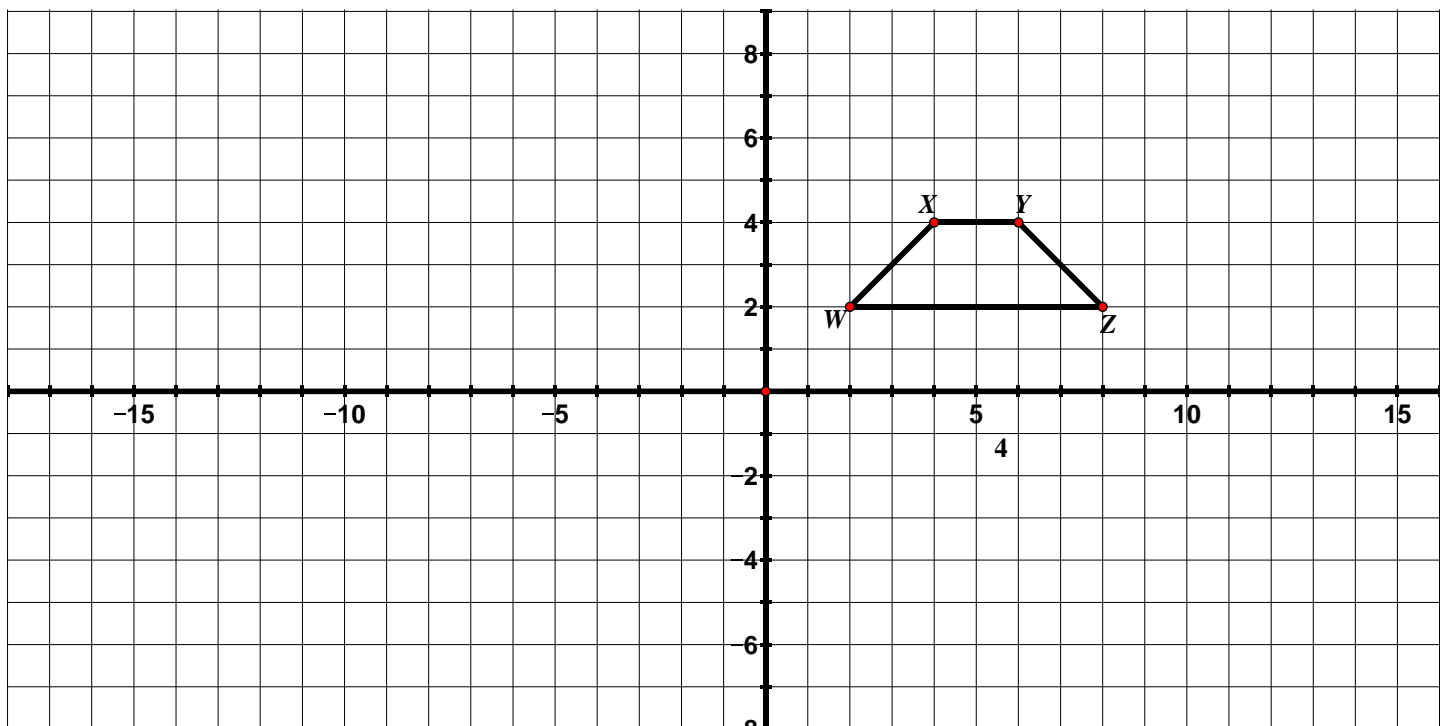
b. Write a congruence statement for the parallelograms.

4. Consider $\triangle ABC$ and $\triangle LMN$ below. The two triangles are congruent.



a. Prove that $\triangle ABC \cong \triangle LMN$.

5. Using $WXYZ$ in the diagram below as the pre-image, apply the following rules to $WXYZ$ and determine whether the resulting image is congruent to $WXYZ$. Always start with $WXYZ$ as your pre-image.



- $(x, y) \rightarrow (x - 2, y + 1)$
Is the resulting image congruent to $WXYZ$? Why or why not?
- $(x, y) \rightarrow (y, x)$
Is the resulting image congruent to $WXYZ$? Why or why not?
- $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$
Is the resulting image congruent to $WXYZ$? Why or why not?
- Write your own coordinate rule that is different than the ones above that would result in an image that **is congruent** to $WXYZ$. How do you know that the resulting image is congruent to $WXYZ$?
- Write your own coordinate rule that is different than the ones above that would result in an image that **is not congruent** to $WXYZ$. How do you know that the resulting image is not congruent to $WXYZ$?

9.1g Self-Assessment: Section 9.1

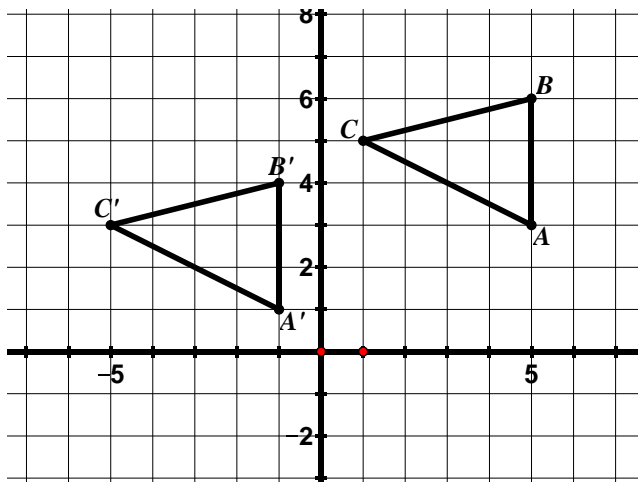
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Given a pre-image and its image under a translation, describe the translation in words and using a coordinate rule.				
2. Perform a translation of a figure given a coordinate rule.				
3. Describe the properties of a translation and the effects a translation has on a figure and use this knowledge to solve problems.				
4. Given a pre-image and its image under a reflection, describe the reflection in words and using a coordinate rule.				
5. Perform a reflection of a figure given a line of reflection.				
6. Describe the properties of a reflection and the effects a reflection has on a figure and use this knowledge to solve problems.				
7. Find a reflection line for a given reflection and write the equation of the reflection line.				
8. Given a pre-image and its image under a rotation, describe the rotation in words and using a				

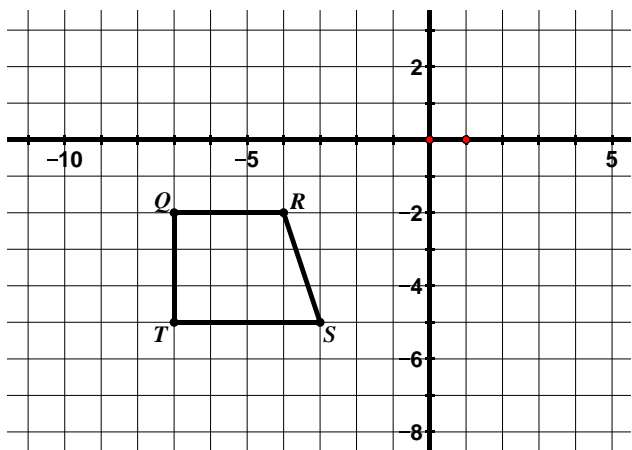
coordinate rule (coordinate rule for rotations centered at the origin only).				
9. Perform a rotation of a figure given a center of rotation, an angle of rotation, and a direction.				
10. Describe the properties of a rotation and the effects a rotation has on a figure and use this knowledge to solve problems.				
11. Connect ideas about slopes of perpendicular lines and rotations.				
12. Understand what it means for two figures to be congruent.				
13. Determine if two figures are congruent based on the definition of congruence.				
14. Given two figures that are congruent, describe the sequence of transformations that exhibits the congruence between them.				

Section 9.1 Sample Problems (For use with self-assessment)

- Describe the following transformation in words and using a coordinate rule.

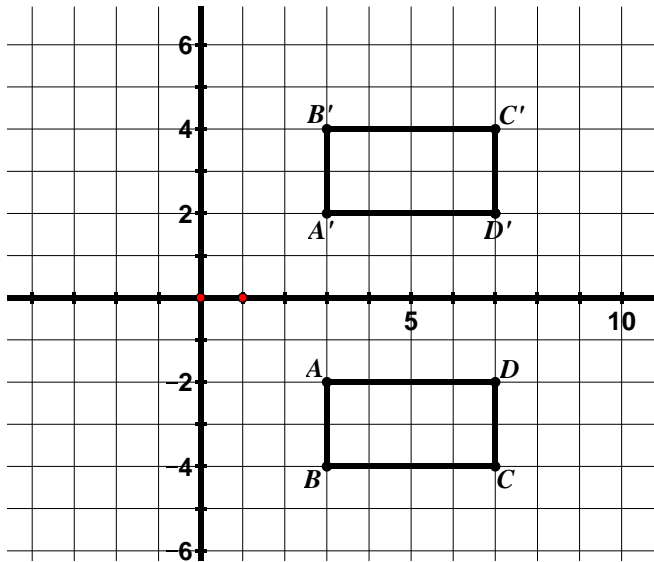


- Translate $QRST$ according to the rule $(x, y) \rightarrow (x + 3, y - 1)$.

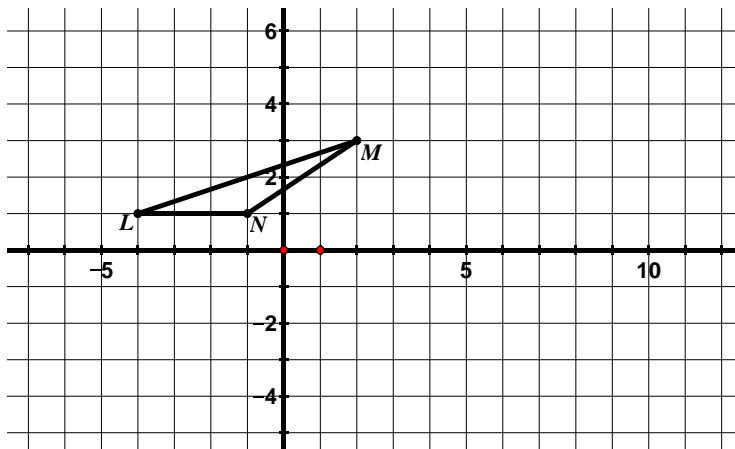


- Which of the following properties are true about a figure that has been translated. Check all that apply.
 - ☐ The slopes of corresponding segments are opposite reciprocals.
 - ☐ Segments connecting corresponding vertices of the image and pre-image are the same length.
 - ☐ The perimeter of the pre-image is smaller than the perimeter of the image.
 - ☐ Segments connecting corresponding vertices of the image and pre-image are parallel.
 - ☐ Corresponding vertices of the image and pre-image lie on the same circle.

4. Describe the following transformation in words and using a coordinate rule.

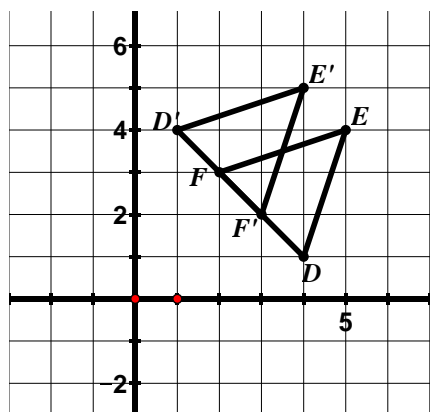


5. Reflect $\triangle LMN$ over the line $x = 2$.

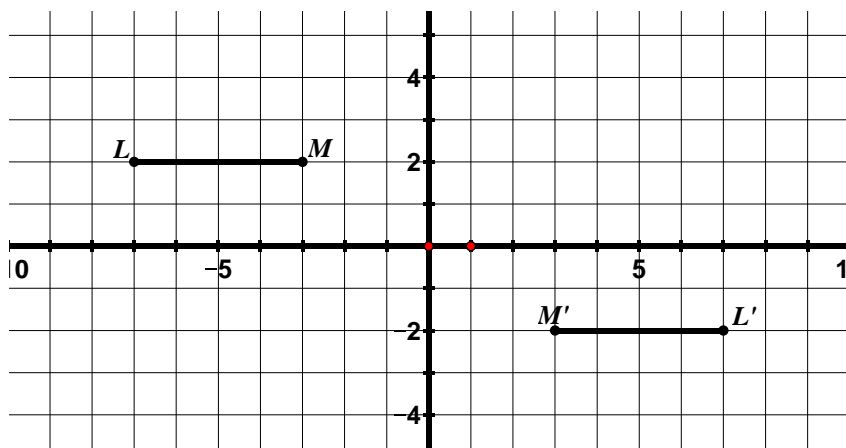


6. Which of the following properties are true about a figure that has been reflected. Check all that apply.
- ☐ The slopes of corresponding segments are the same.
 - ☐ Segments connecting corresponding vertices of the image and pre-image are the same length.
 - ☐ Segments connecting corresponding vertices of the image and pre-image are parallel.
 - ☐ The area of the pre-image is the same as the area of the image.
 - ☐ Corresponding vertices are equidistant from the line of reflection.
 - ☐ The measure of the interior angles of an object may change under a reflection.

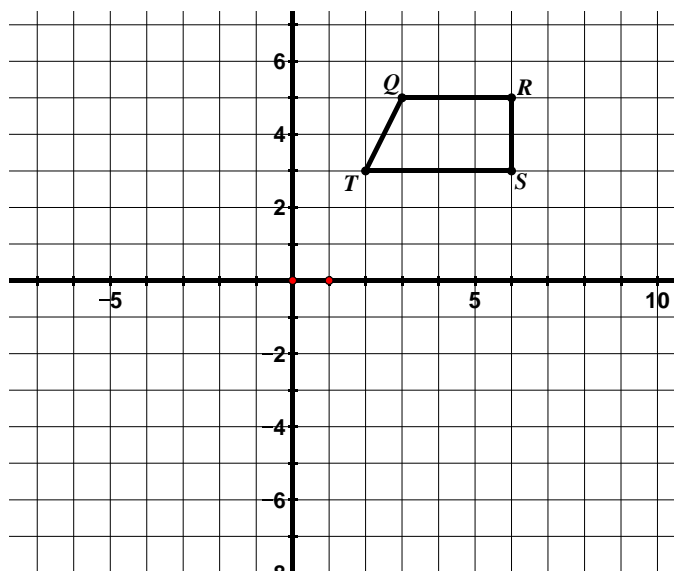
7. $\triangle EFD$ has been reflected to obtain $\triangle E'F'D'$. Write the equation for the line of reflection.



8. Describe the following rotation in words and using a coordinate rule.



9. Rotate $QRST$ 90° clockwise about the origin.



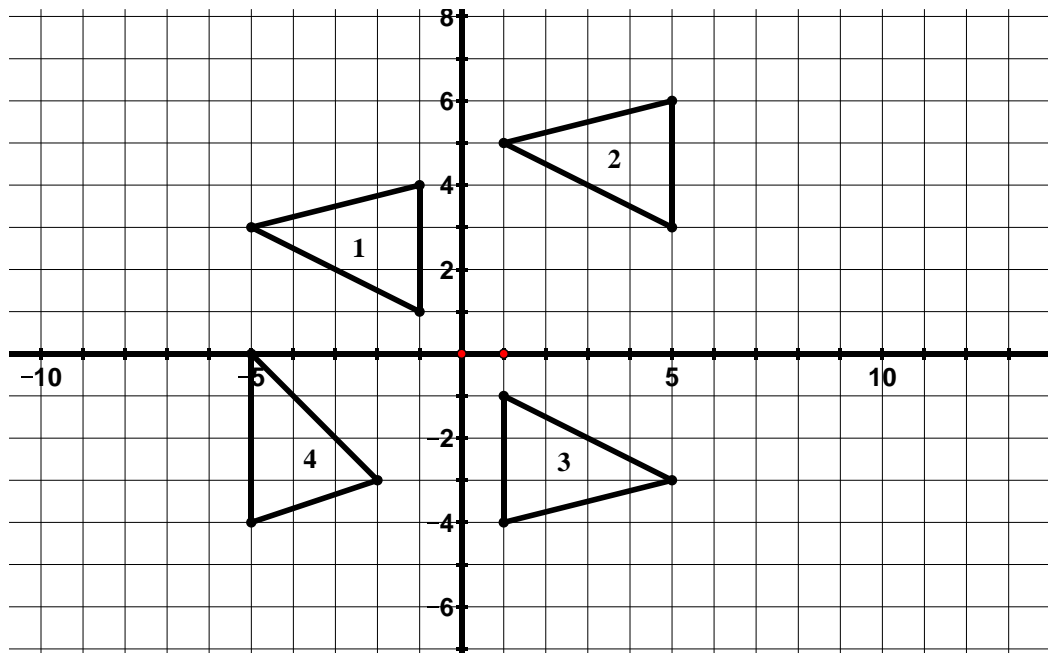
10. Which of the following properties are true about a figure that has been rotated 90° . Check all that apply.

- ☐ The slopes of corresponding segments are opposite reciprocals.
- ☐ Segments connecting corresponding vertices of the image and pre-image are the same length.
- ☐ Segments connecting corresponding vertices of the image and pre-image are parallel.
- ☐ Lines that are parallel in the pre-image are not necessarily parallel in the image.
- ☐ Corresponding segments are perpendicular.
- ☐ Corresponding vertices lie on the same circle.

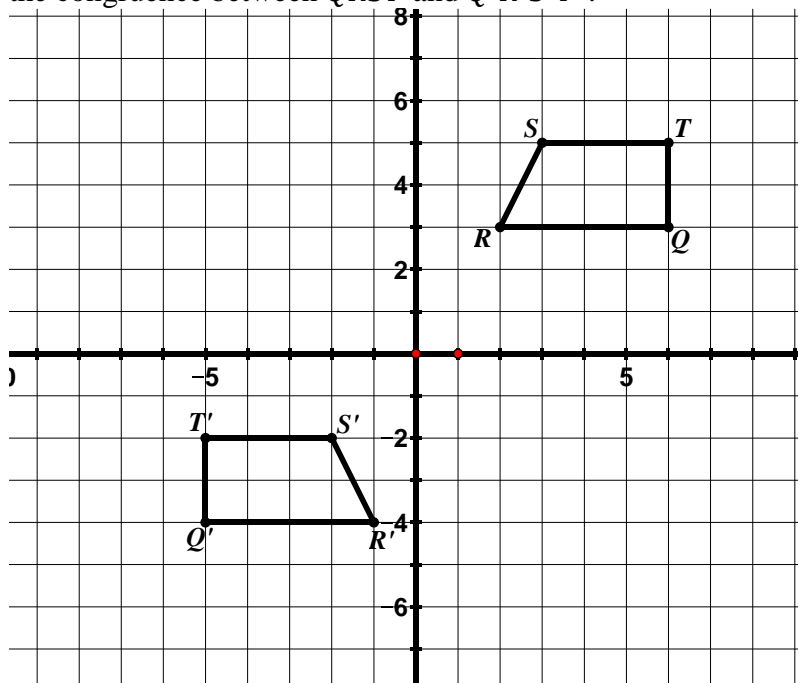
11. In a 90° rotation, what happens to the slopes of the corresponding segments of the image and pre-image? How does the coordinate rule show this? In a 180° rotation, what happens to the slopes of the corresponding segments of the image and pre-image? How does the coordinate rule show this?

12. In your own words, describe what it means for two figures to be congruent.

13. Which of the following triangles are congruent? Justify your answers.



14. $QRST$ is congruent to $Q'R'S'T'$. Describe a transformation or sequence of transformations that exhibits the congruence between $QRST$ and $Q'R'S'T'$.



Section 9.2 Dilations and Similarity

Section Overview:

Students start this section by applying different transformations (given by coordinate rules) to a figure to determine what these rules do to the shape and size of the figure. In this activity, they start to surface ideas about similarity. Students then begin to study dilations in detail. They perform dilations using the slope triangle method and scaling method when given a scale factor and center of dilation. Students determine the scale factor and center of dilation of two figures that have been dilated and write a coordinate rule to describe the dilation. Students continue to refer back to their work in order to describe the properties of dilations. Students will then apply this knowledge to determine if two figures are similar, understanding that a two-dimensional figure is similar to another if there is a sequence of rigid motions and dilations that takes one figure onto the other. In addition, students will be given two figures that are similar and asked to describe a sequence of transformations that exhibits the similarity between them.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Describe the properties of a figure that has been dilated.
2. Perform a dilation given a scale factor and center of dilation.
3. Describe a dilation in words and using a coordinate rule.
4. Determine the center of dilation using the properties of dilations.
5. Understand what it means for two figures to be similar.
6. Determine if two figures are similar.
7. Given two figures that are similar, describe the sequence of transformations that exhibits the similarity between them.

9.2a Class Activity: Video Game Animation

Computer animators are working on designing the head of a dragon for a new video game. The picture below shows the original shape and size of the dragon’s head. However, when the dragon eats a plant, the lengths of the sides of the dragon head double in size. If the dragon eats a cricket, the lengths of the sides of the dragon head triple in size. When the dragon enters a cave, the lengths of the sides shrink to half their original size.



Four different animators submitted the following proposals for how to double the lengths of the sides of the dragon’s head when it eats a plant:

- Animator 1 said to apply the following rule $(x, y) \rightarrow (2x, y)$
- Animator 2 said to apply the following rule $(x, y) \rightarrow (x, 2y)$
- Animator 3 said to apply the following rule $(x, y) \rightarrow (x + 2, y + 2)$
- Animator 4 said to apply the following rule $(x, y) \rightarrow (2x, 2y)$

The chart below shows the coordinates of the dragon’s head when it is its original size. Write the new coordinates for the dragon’s head for the coordinate rules proposed by each of the animators. Then graph each of the animator’s new dragon heads.

Original Size	$(2x, y)$	$(x, 2y)$	$(x + 2, y + 2)$	$(2x, 2y)$
(2, 2)				
(6, 8)				
(8, 8)				
(10, 6)				
(10, 4)				
(8, 4)				
(6, 6)				
(6, 2)				

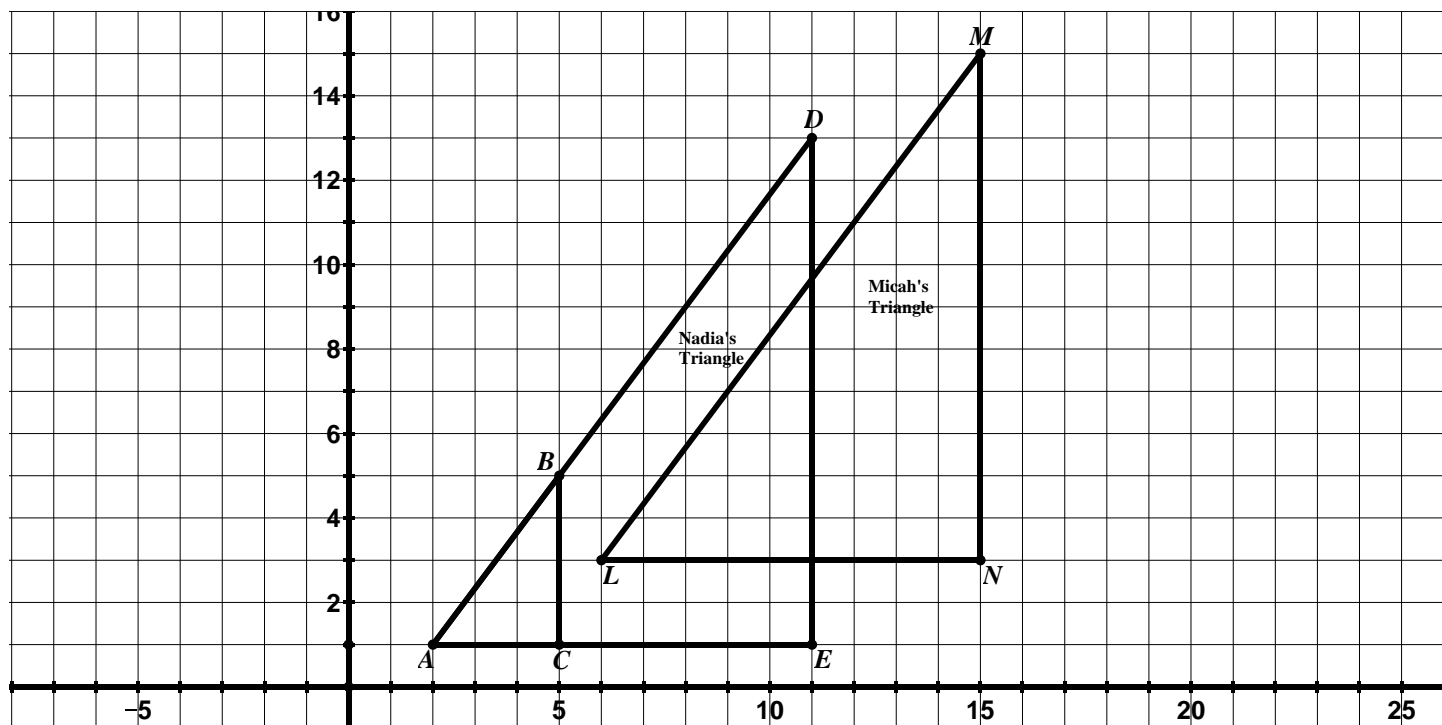
4. Which of the animator's rules results in a dragon that is the same shape as the original but whose side lengths are twice the size? What is the same about this dragon compared to the original dragon? What is different?
5. Describe what the coordinate rules of the other three animators do to the dragon's head.
6. The animators need your help coming up with the coordinate rule that would reduce the size of the dragon's head when it enters a cave (the lengths of the sides should be half their original size).
- Write your proposed coordinate rule in the table below.
 - Write the new coordinates for your rule.
 - Graph the new coordinates.

Original Size	Coordinate Rule:
(2, 2)	
(6, 8)	
(8, 8)	
(10, 6)	
(10, 4)	
(8, 4)	
(6, 6)	
(6, 2)	

- What is the same about this dragon compared to the original dragon? What is different?
- Write the coordinate rule that would triple the size of the dragon's head when it eats a cricket.

9.2b Class Activity: Properties of Dilations

1. Ms. Williams gave her students the grid shown below with $\triangle ABC$ graphed on it. She then asked her students to create a triangle that was the same shape as the original triangle but has sides lengths that are three times larger. Micah created $\triangle LMN$ shown below and Nadia created $\triangle ADE$ shown below.



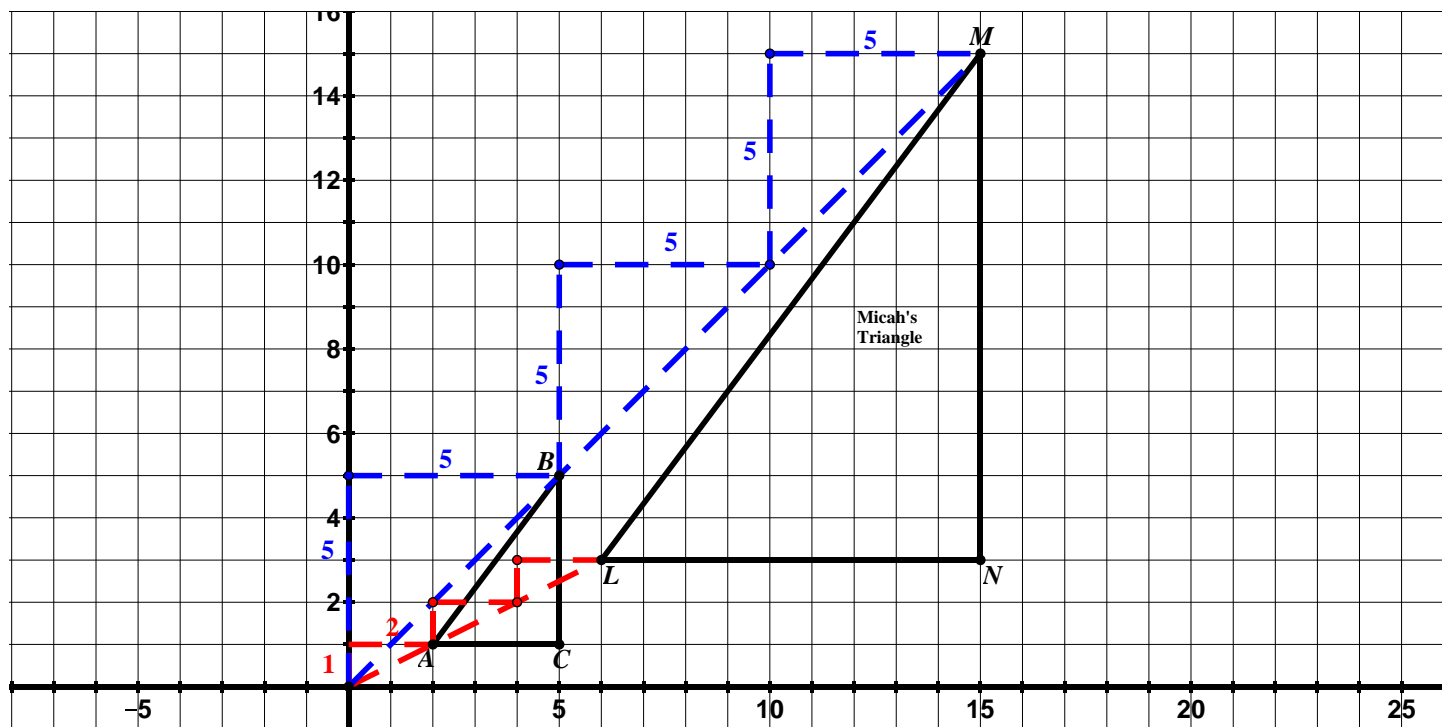
- a. The teacher asked the class who had done the assignment correctly, Nadia or Micah? Iya said they were both correct. Hendrix disagreed and said they could not both be correct because the triangles were not in the same location in the coordinate plane. Who do you agree with and why?



- b. The teacher asked Micah and Nadia to explain the methods they used to create the triangles.

Nadia's Method: Using a ruler, I slid C along the line containing the points A and C until my new segment was three times larger than \overline{AC} and labeled the new point E . I then used my ruler to slide B along the line containing the points A and B as shown below and labeled the new point D . Lastly, I checked to make sure \overline{DE} was 3 times larger than \overline{BC} and it was!

Micah's Method: I noticed that the slope of the line passing through the origin and A had a rise of 1 and a run of 2. Since I wanted the image to be three times larger than $\triangle ABC$, I placed the point that corresponds to A three slope triangles (with a rise of 1 and a run of 2) from the origin. I used the same method to plot the point that corresponds to B . The slope of the line passing through the origin and B has a rise of 5 and a run of 5. Again, I moved three slope triangles with a rise of 5 and a run of 5 from the origin and plotted M . The slope of the line passing through C and the origin has a rise of 1 and a run of 5. I moved three slope triangles with a rise of 1 and a run of 5 from the origin and plotted N .



- c. Compare the two methods used. What is the same about the resulting triangles? What is different? What accounts for the differences in the triangles?



In the previous example, Micah and Nadia both **dilated** $\triangle ABC$. A **dilation** is a transformation that produces an image that is the same shape as the original figure but the image is a different size.

Every dilation has a center of dilation and a scale factor. The **center of dilation** is a fixed point in the plane from which all points are expanded or contracted. The **scale factor** describes the size change from the original figure to the image. We use the letter r to represent scale factor. The dilation is an enlargement if the scale factor is greater than 1 and a reduction if the scale factor is between 0 and 1.

In the example on the previous page, the scale factor for both Nadia and Micah was 3; however Nadia's center of dilation was A: (2, 1) while Micah's was the origin (0, 0).

2. We will use the example on the previous page to examine some of the **properties of dilations**.

- a. Find the following ratios for Nadia's triangle:

$$\frac{AD}{AB} =$$

$$\frac{DE}{BC} =$$

$$\frac{AE}{AC} =$$

- b. Find the following ratios for Micah's triangle:

$$\frac{LM}{AB} =$$

$$\frac{MN}{BC} =$$

$$\frac{LN}{AC} =$$

- c. Complete the following sentence. Under a dilation, **the ratios of the image segments to the corresponding pre-image segments** are...
- d. Complete the following sentence. Under a dilation, **corresponding angles** are...
- e. Complete the following sentence. Under a dilation, **corresponding segments** are...
- f. Complete the following sentence. Under a dilation, **corresponding vertices**...
- g. Complete the following sentence. Under a dilation, **segments connecting corresponding vertices**...

- h. In the table below, list the coordinates of the corresponding vertices in Micah's dilation:

A:	L:
B:	M:
C:	N:

- i. Write a coordinate rule for Micah's dilation using the information in the table above.



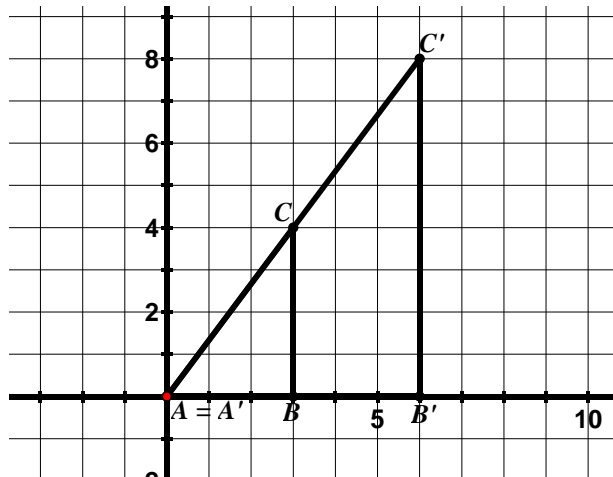
- j. In the table below, list the coordinates of the corresponding vertices in Nadia's dilation:

A:	A:
B:	D:
C:	E:

- k. Write a coordinate rule for Nadia's dilation using the information in the table above. Remember that Nadia's center of dilation is not at the origin. Think about how this shift off the origin will affect the coordinate rule.

Directions: In the following problems, one figure has been dilated to obtain a new figure. Determine the scale factor and center of dilation. Determine the coordinate rule for the dilation when asked.

3. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.

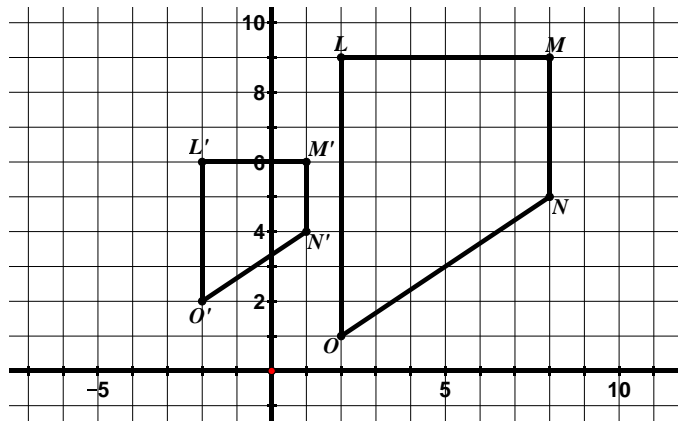


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

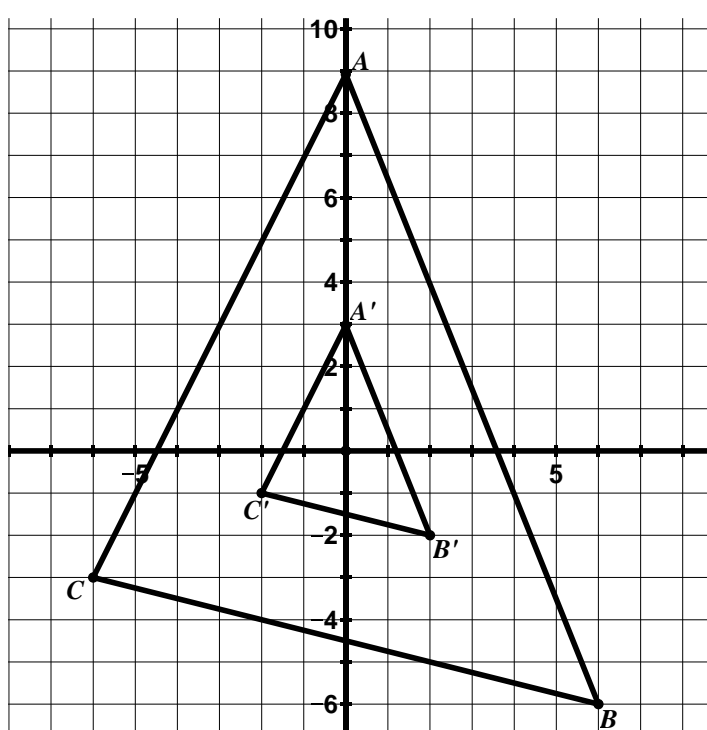
4. $LMNO$ has been dilated to obtain $L'M'N'O'$.



Scale Factor: _____

Center of Dilation: _____

5. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.

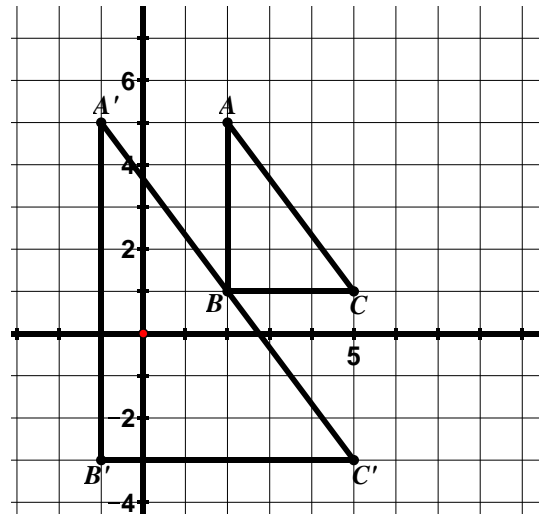


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

6. $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.



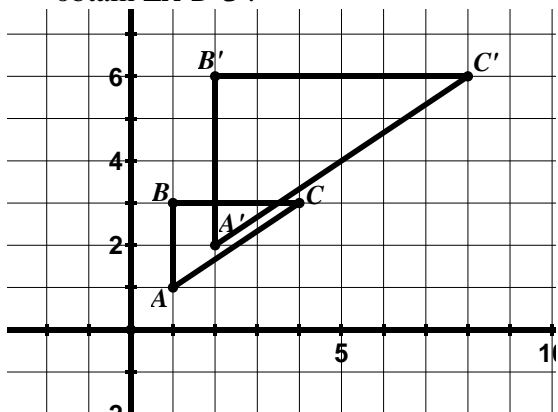
Scale Factor: _____

Center of Dilation: _____

9.2b Homework: Properties of Dilations

Directions: In the following problems, one figure has been dilated to obtain a new figure. Determine the scale factor and center of dilation. Determine the coordinate rule for the dilation when asked.

1. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.

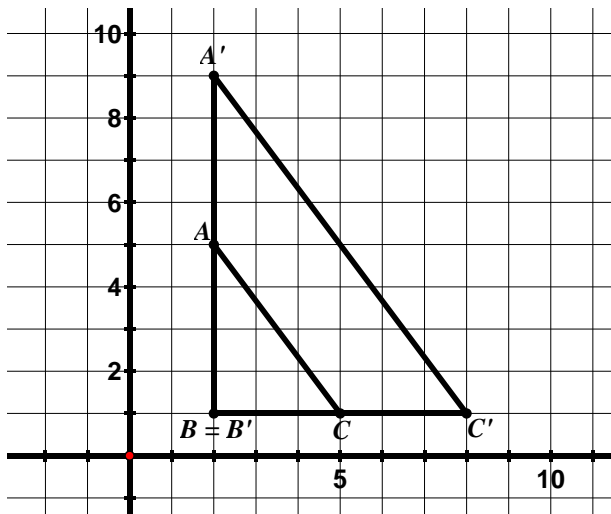


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

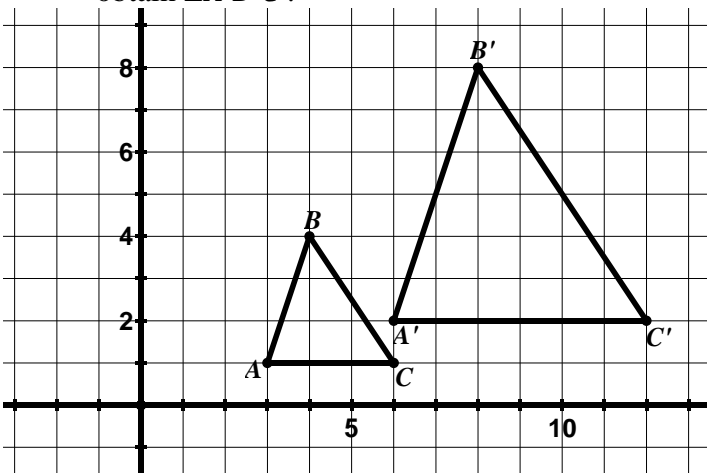
2. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.



Scale Factor: _____

Center of Dilation: _____

3. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.

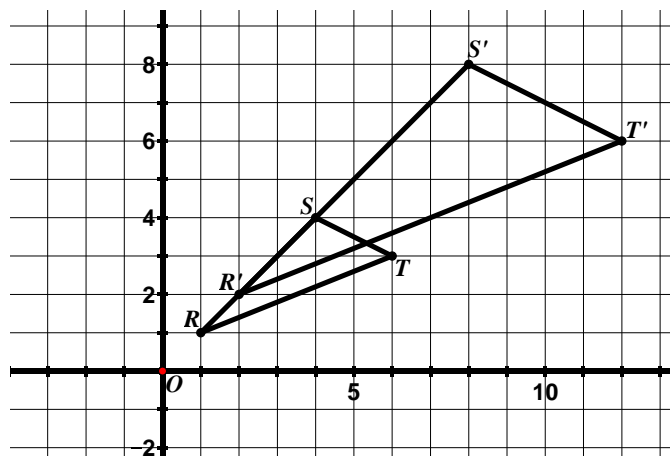


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

4. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.

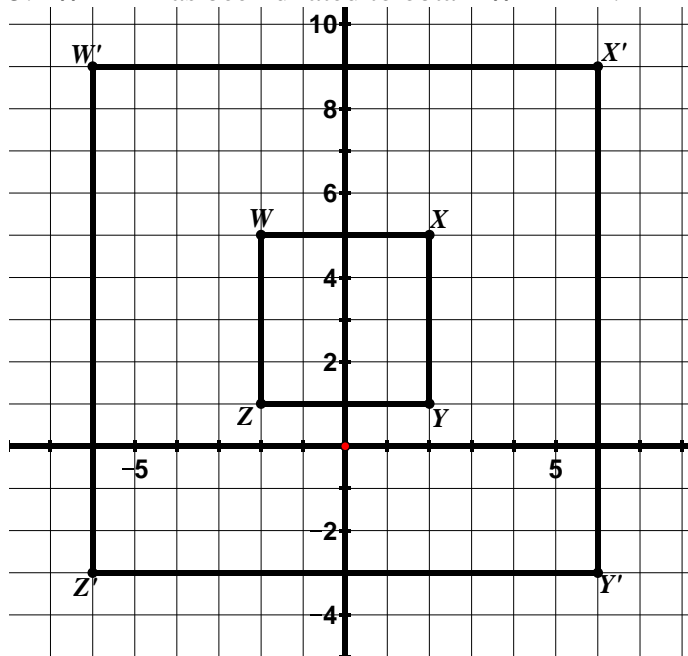


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

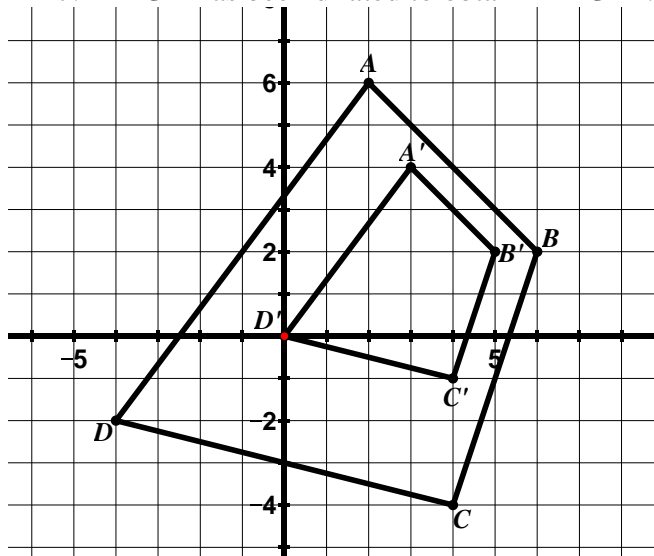
5. $WXYZ$ has been dilated to obtain $W'X'Y'Z'$.



Scale Factor: _____

Center of Dilation: _____

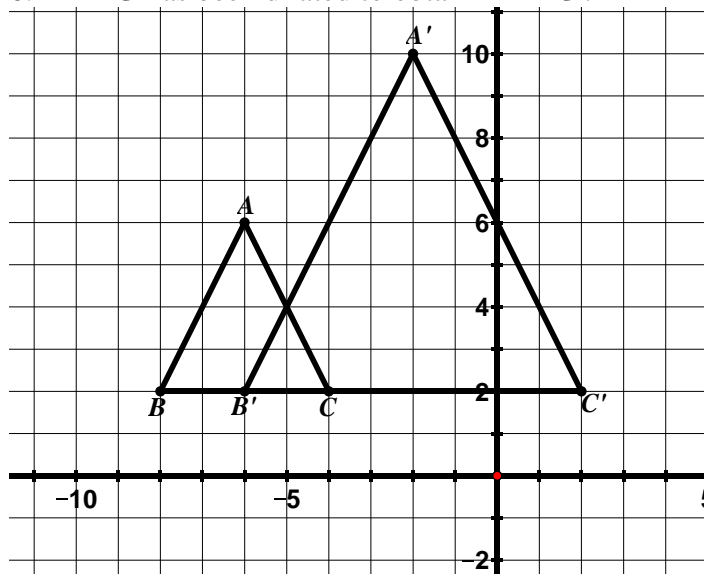
7. $ABCD$ has been dilated to obtain $A'B'C'D'$.



Scale Factor: _____

Center of Dilation: _____

6. $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$.

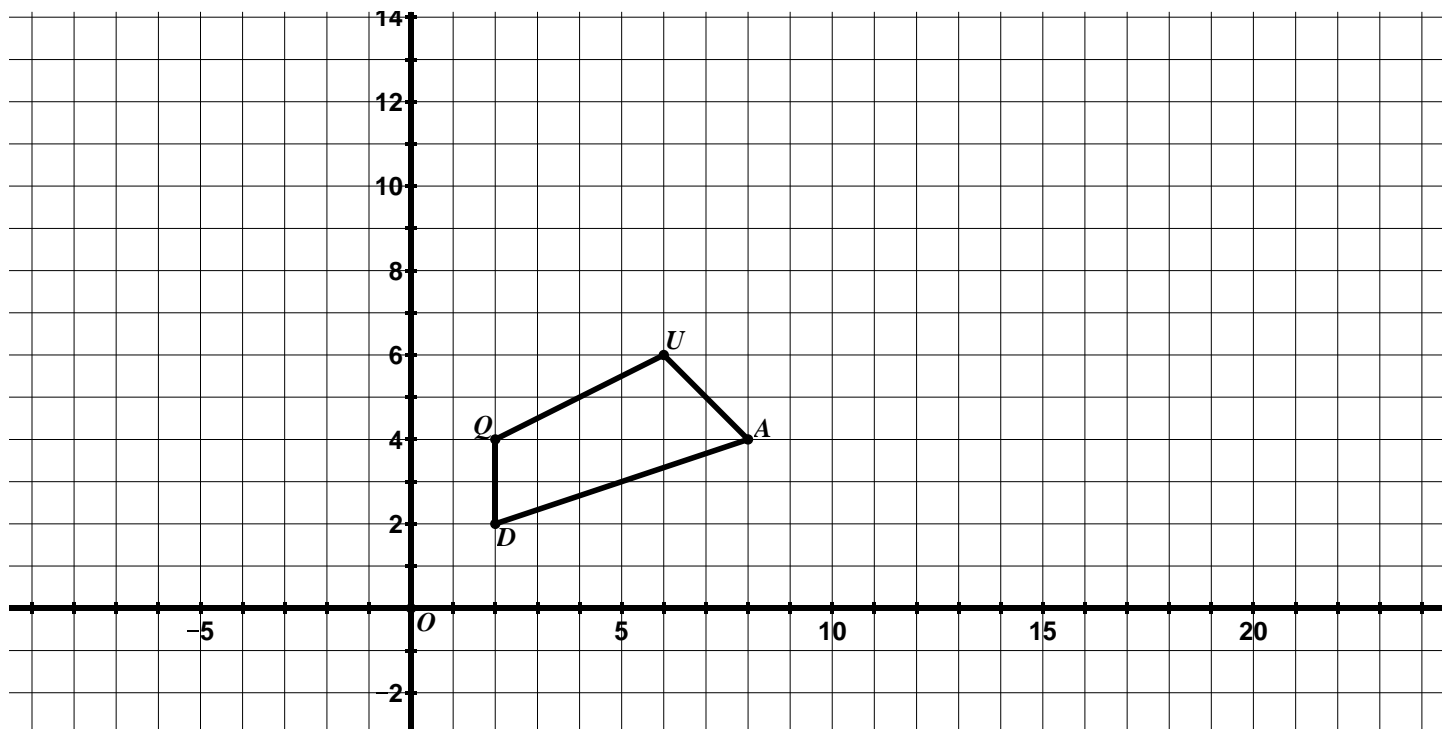


Scale Factor: _____

Center of Dilation: _____

9.2c Class Activity: Dilations cont.

1. $QUAD$ is graphed below.



- a. Create a new quadrilateral whose side lengths are two times larger than the side lengths of $QUAD$ with the center of dilation at the origin and label the image $Q'U'A'D'$. In the space below, describe the method you used to create your new quadrilateral.
- b. Based on what we have learned so far about dilations, what are some different ways you can verify that the side lengths of your new quadrilateral are in fact two times larger than the side lengths of the original?

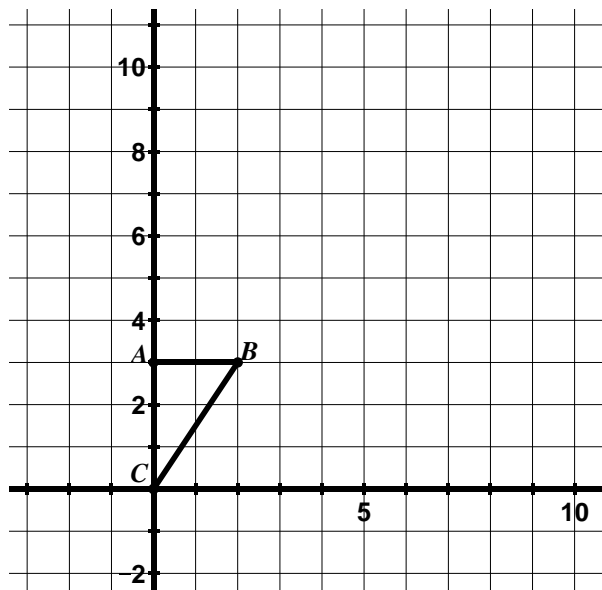


- c. This time, create a quadrilateral whose sides lengths are $\frac{1}{2}$ the size of the side lengths of $QUAD$ with the center of dilation at the origin and label the image $Q''U''A''D''$.

Directions: Find the image of each figure for a dilation with the given center and scale factor. Draw and label the image.

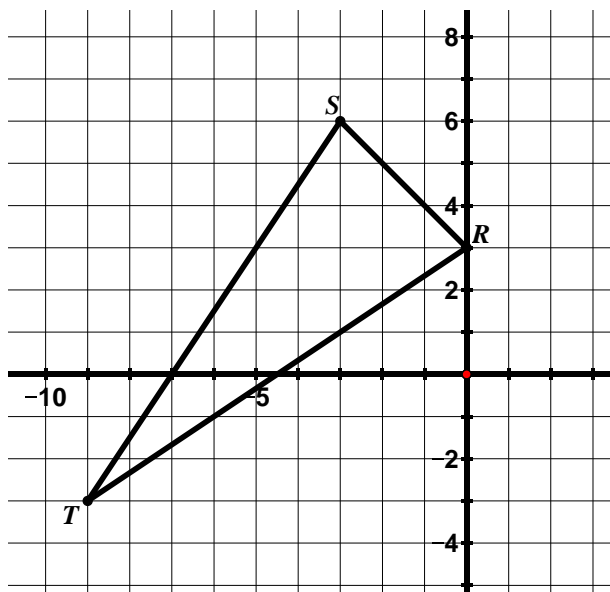
2. $r = 3$

Center of Dilation: C



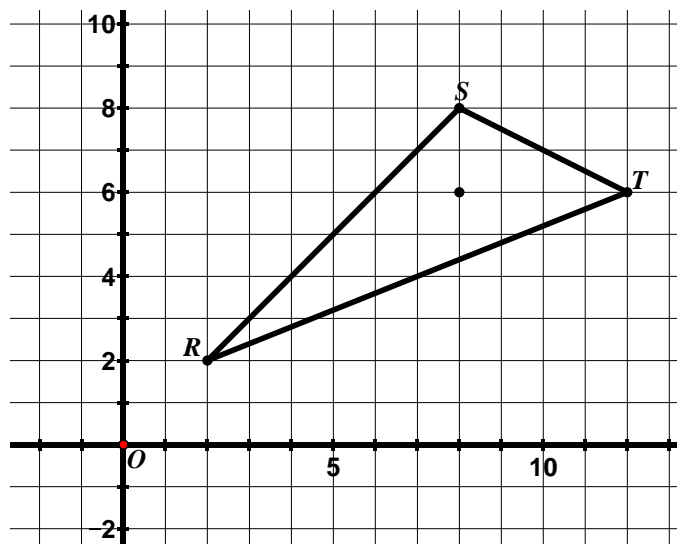
3. $r = \frac{1}{3}$

Center of Dilation: origin



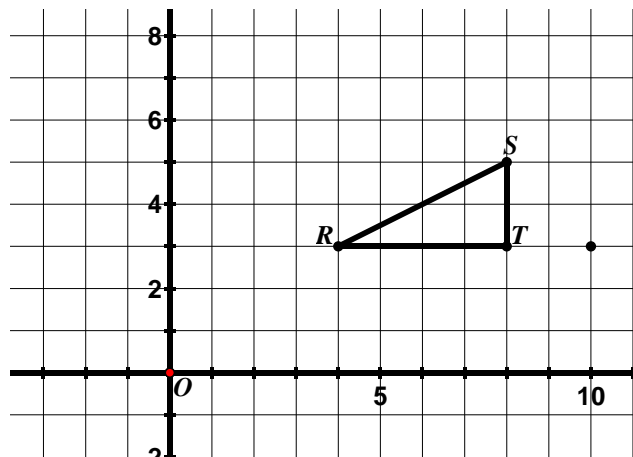
4. $r = \frac{1}{2}$

Center of Dilation: $(8, 6)$.



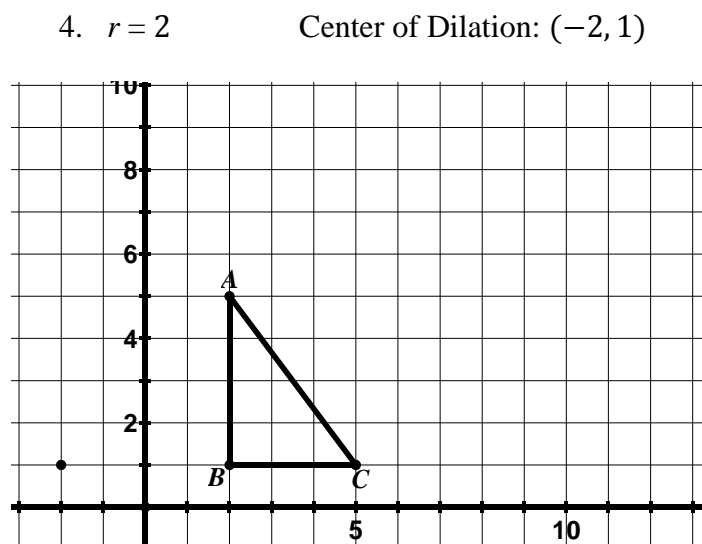
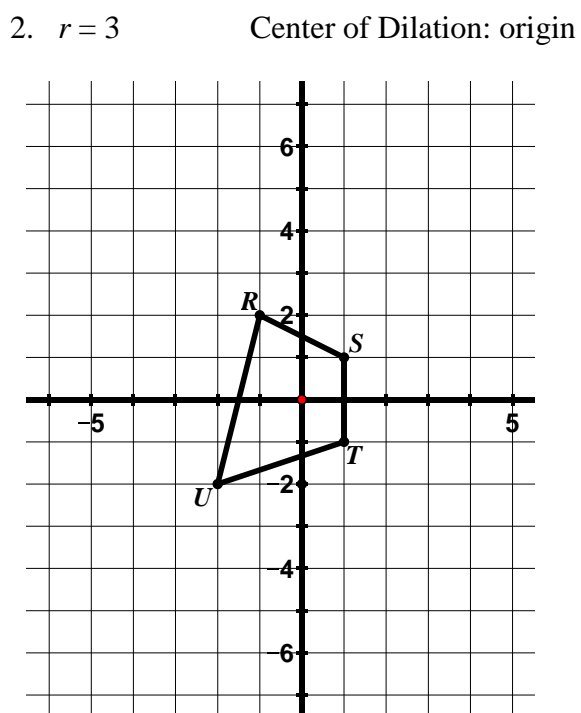
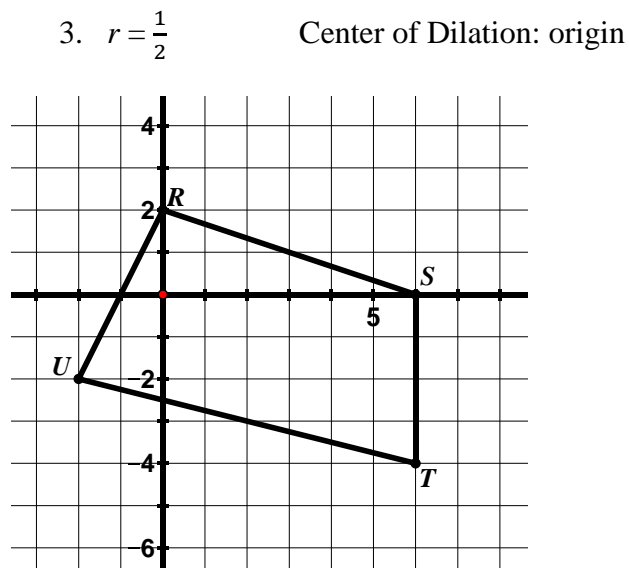
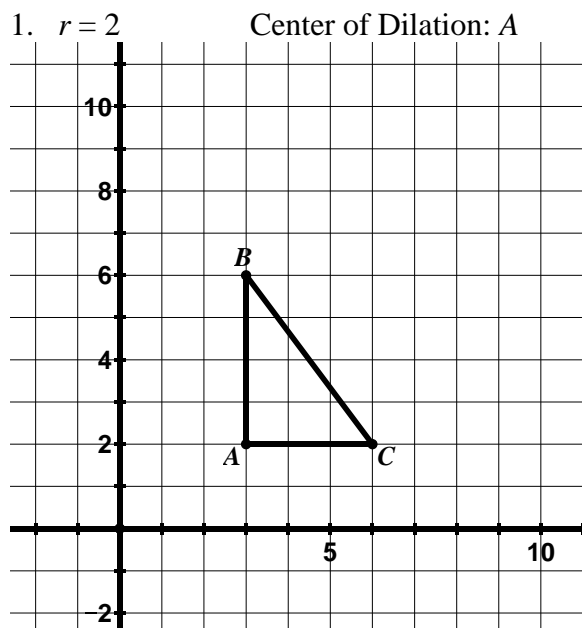
5. $r = 2$

Center of Dilation: $(10, 3)$



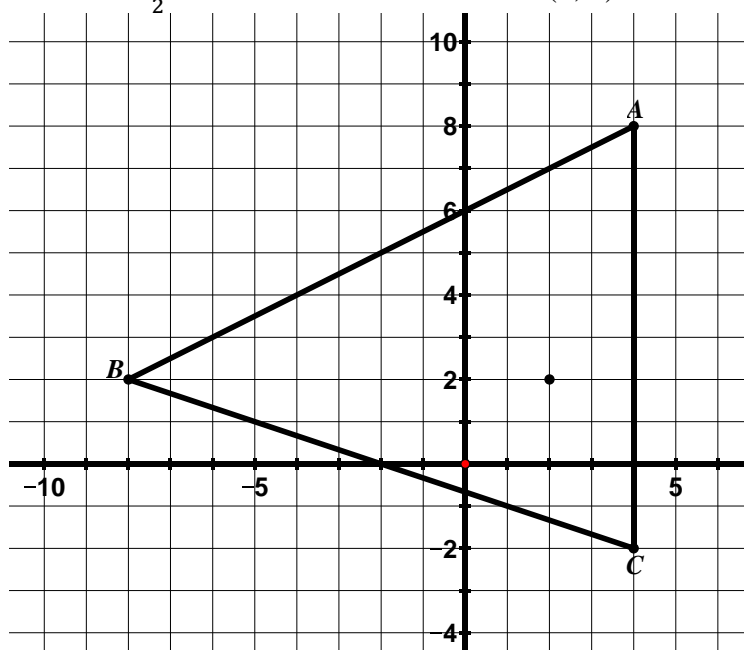
9.2c Homework: Dilations cont.

Directions: Find the image of each figure for a dilation with the given center and scale factor. Draw and label the image.



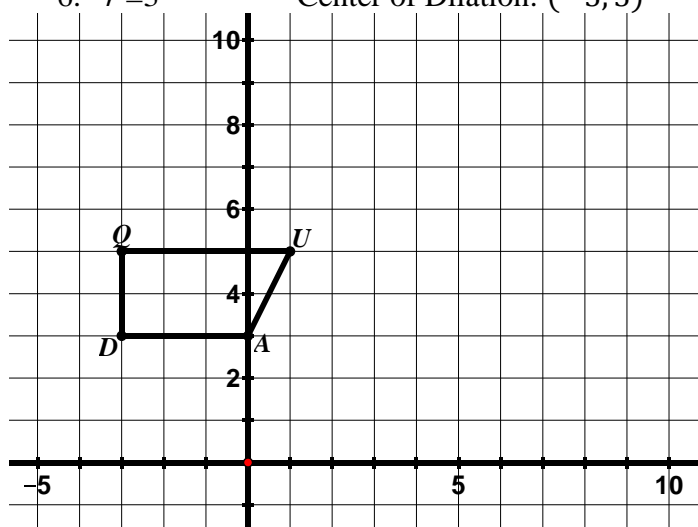
5. $r = \frac{1}{2}$

Center of Dilation: (2, 2)



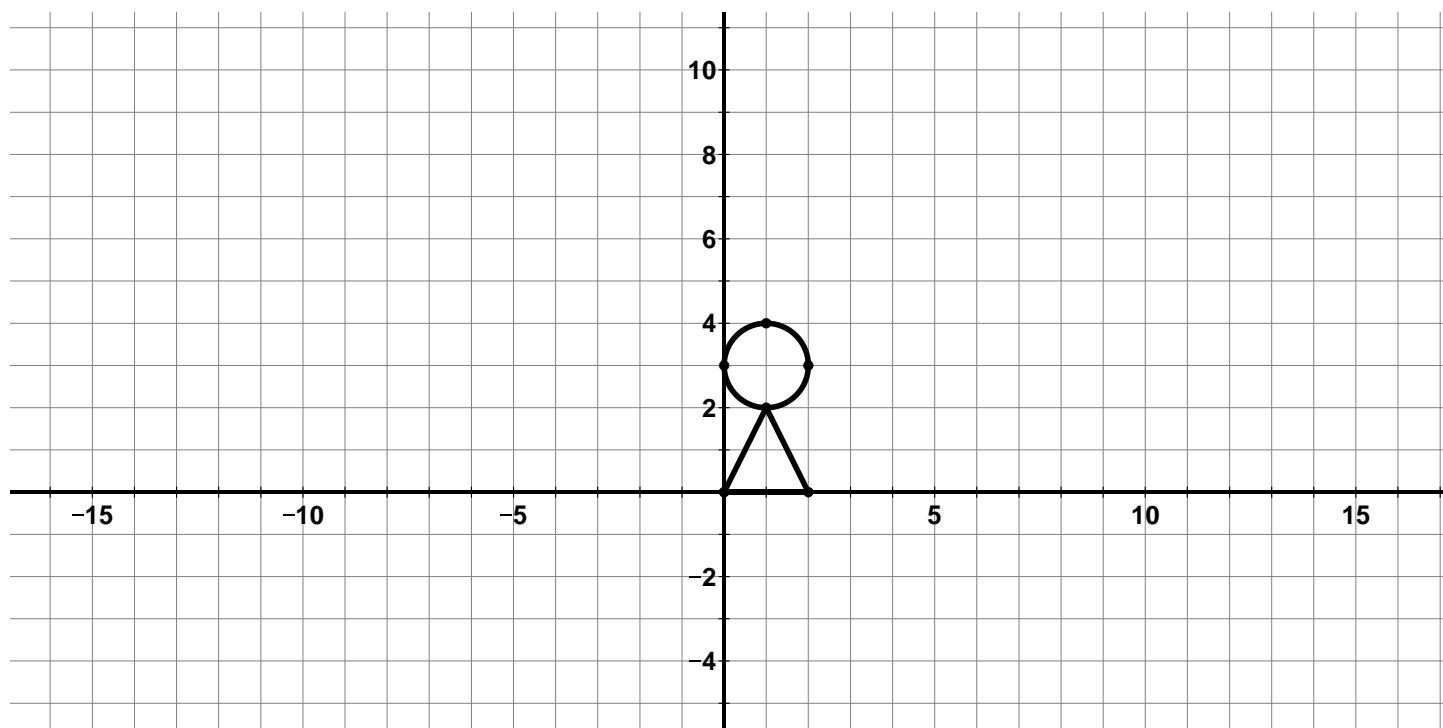
6. $r = 3$

Center of Dilation: (-3, 3)



9.2d Class Activity: Problem Solving with Dilations

1. Where would you put your center of dilation to enlarge the picture below by a factor of 2 and put the image entirely in the second quadrant? Mark your center of dilation with an O and then do the dilation.

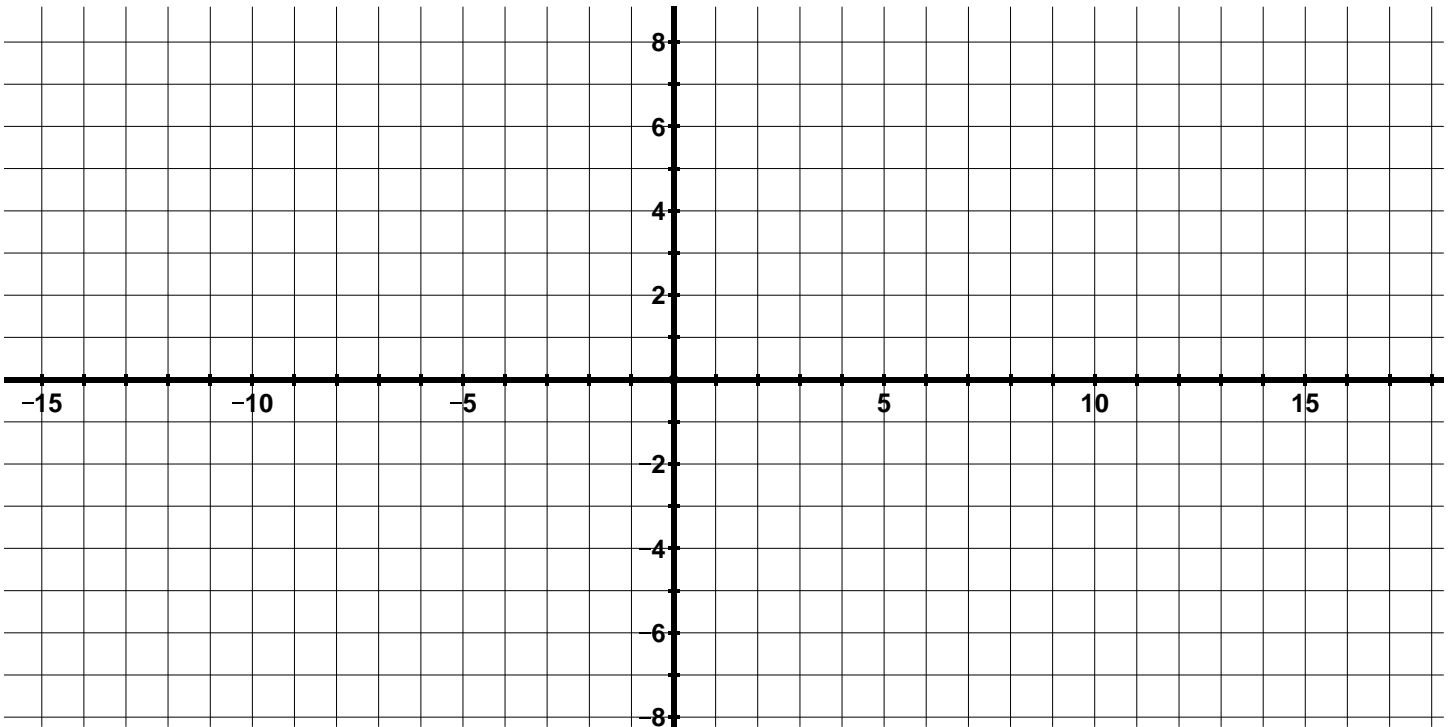


2. A dilation with the center of dilation at the origin maps $\triangle ABC$ to $\triangle A'B'C'$.

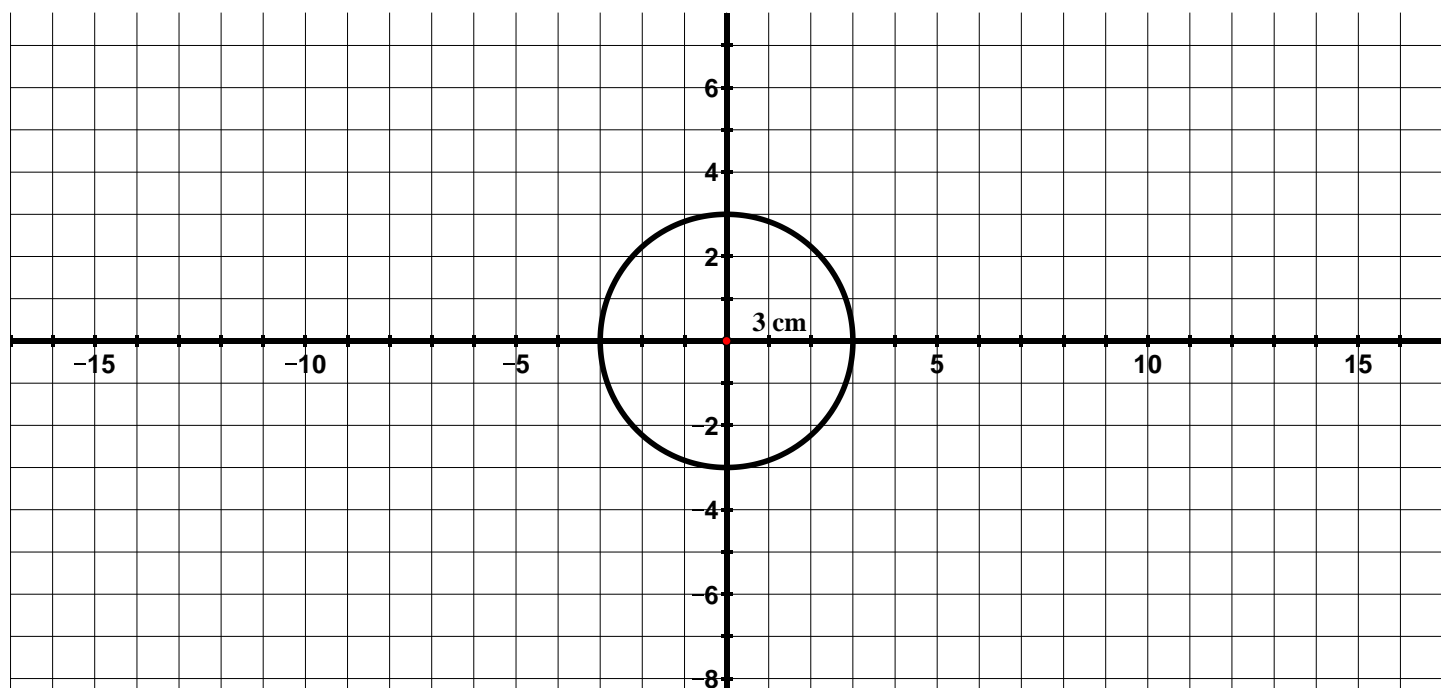
- If $AB = 3$ and $A'B' = 6$, what is the scale factor of the dilation?
- If $B'C' = 8$, what is the length of BC ?
- If $AC = 5$, what is the length of $A'C'$?
- If the slope of \overline{AB} is 0, what is the slope of $\overline{A'B'}$?
- If the slope of $\overline{A'C'}$ is $\frac{4}{3}$, what is the slope of \overline{AC} ?
- Create a picture of this dilation on the grid below using the information from parts $a - e$ and the additional pieces of information below. *Remember that the center of dilation is the origin.*



- The slope of CB is undefined
- A is at the origin



3. A circle with a radius of 3 cm is shown below.

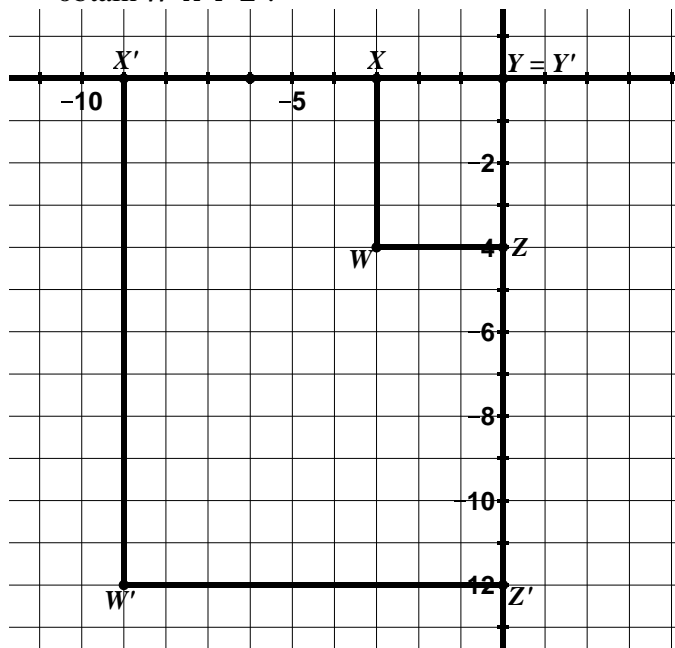


- Determine the length of the radius of a circle whose circumference would be twice as large as the circle pictured above.
- Determine the length of the radius of a circle whose area would be twice as large as the circle pictured above.

9.2d Homework: Review of Dilations

Directions: In the following problems, one figure has been dilated to obtain a new figure. Determine the scale factor and center of dilation. Determine the coordinate rule for the dilation when asked.

1. In the picture below, $WXYZ$ has been dilated to obtain $W'X'Y'Z'$.

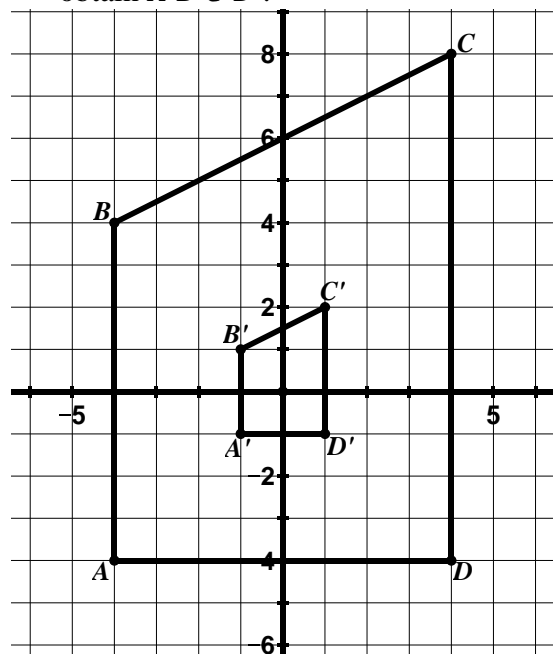


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

2. In the picture below, $ABCD$ has been dilated to obtain $A'B'C'D'$.

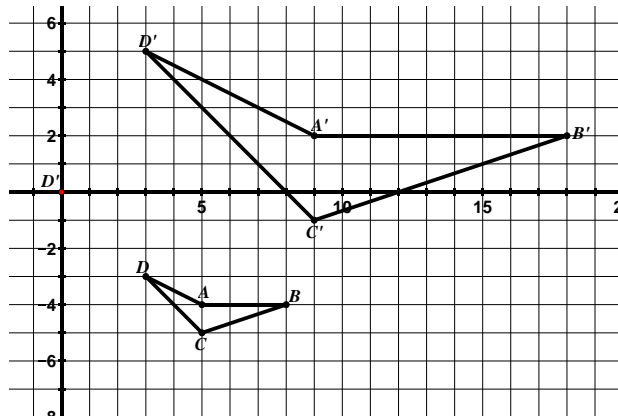


Scale Factor: _____

Center of Dilation: _____

Coordinate Rule: _____

3. $ABCD$ has been dilated to obtain $A'B'C'D'$.



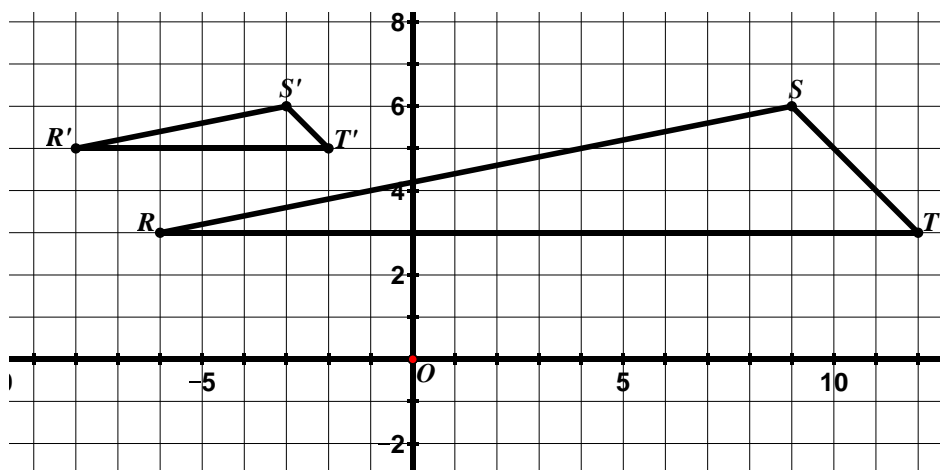
Scale Factor: _____

Center of Dilation: _____

4. $\triangle RST$ has been dilated to obtain $\triangle R'S'T'$.

Scale Factor: _____

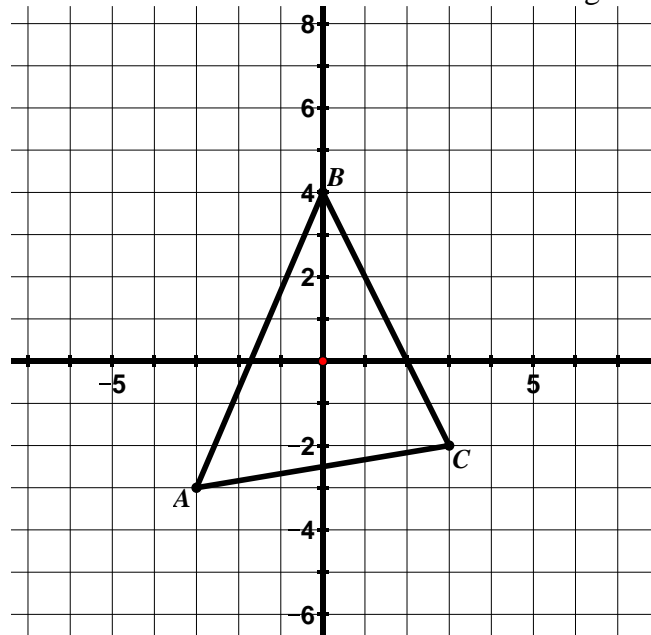
Center of Dilation: _____



Directions: For #5 – 7, find the image of each figure for a dilation with the given center and scale factor. Draw and label the image.

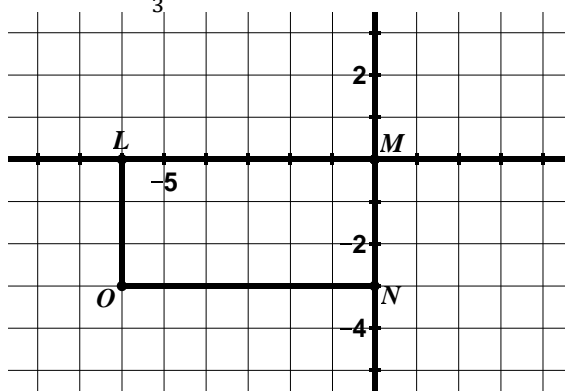
5. $r = 2$

Center of Dilation: origin



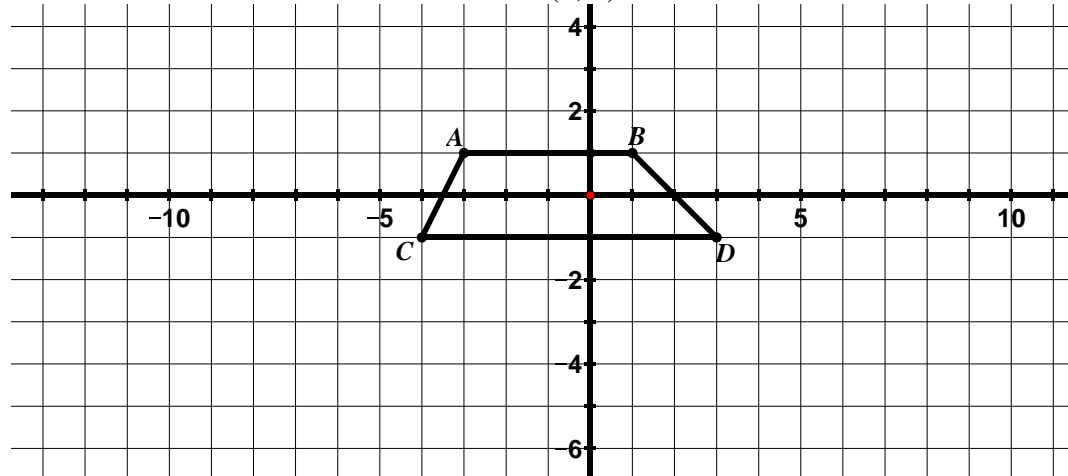
6. $r = \frac{1}{3}$

Center of Dilation: origin



7. $r = 3$

Center of Dilation: (0, 1)



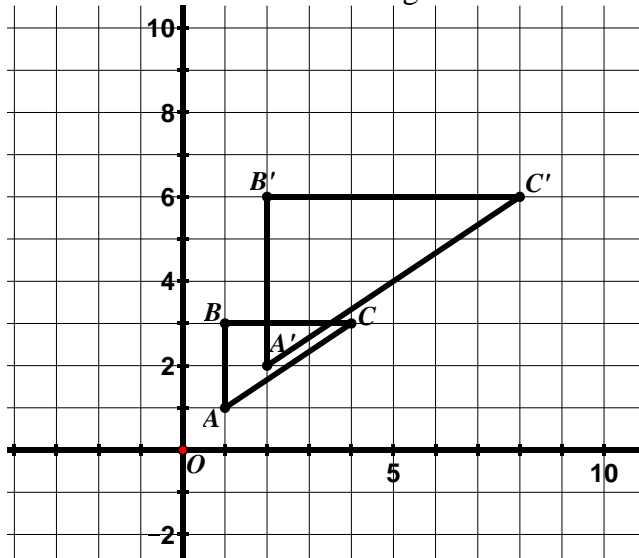
9.2e Class Activity: Similarity

In the first part of the chapter, we discussed congruence. Two figures are **congruent** if one can be obtained from the other by a rigid motion (rotation, reflection, or translation) or a sequence of rigid motions. If you can move one of the figures using one of these transformations or a series of these transformations so that it fits exactly on the other one, the two figures are congruent.

In this section we have seen problems where two figures are similar. In 7th grade, you learned that two figures are similar if they have the **same shape** – they may or may not be the same size. In 8th grade, we define similarity in terms of transformations. Two figures are said to be **similar** if there is a sequence of rigid motions and dilations that take one figure onto the other.

While studying dilations, we have learned that (1) a dilation creates a figure that is the same shape as the original figure but a different size, (2) the measure of corresponding angles is the same and (3) the ratios of corresponding sides are all the same. Since similar figures are produced by a dilation, these properties, as well as some others we observed, also hold true for similar figures.

Let's revisit a problem we have seen before. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$. The center of dilation is the origin and the scale factor is 2.



Because $\triangle A'B'C'$ was produced by a dilation of $\triangle ABC$, the two triangles are similar. We can write a similarity statement for the two triangles. You can denote that two figures are similar by using the symbol \sim and listing their vertices in corresponding order.

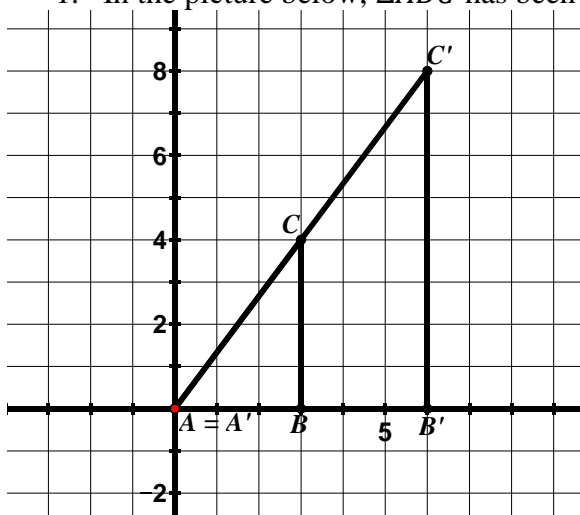
Write a similarity statement for the two triangles.

The order the vertices is written tells us which segments and angles are corresponding in the figures.

When two figures are similar, corresponding angles are congruent and corresponding sides are proportional. Write the congruent statements to represent this.

The ratio of the lengths of the corresponding sides is a similarity ratio. Write the similarity ratio for these two triangles.

1. In the picture below, $\triangle ABC$ has been dilated to obtain $\triangle A'B'C'$. The center of dilation is the origin.



- a. Write a similarity statement for the triangles.

- b. Complete each statement:

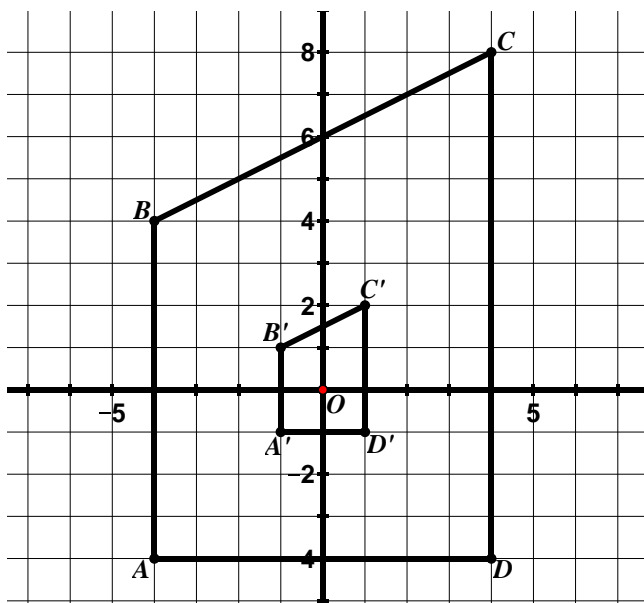
$$m\angle C \cong$$

$$\text{If } m\angle B = 90^\circ, \text{ then } m\angle B' =$$

$$\frac{A'C'}{AC} =$$

$$\frac{A'B'}{AB} = 2$$

2. In the picture below, $ABCD$ has been dilated to obtain $A'B'C'D'$. The center of dilation is the origin.



- a. Write a similarity statement for the trapezoids.

- b. Complete each statement.

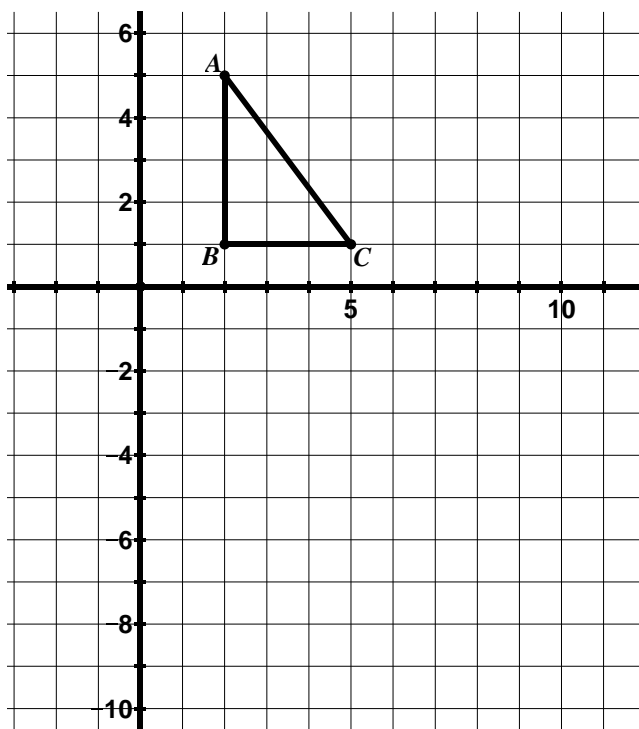
$$m\angle C \cong$$

$$\text{If } m\angle A = 90^\circ, \text{ then } m\angle A' =$$

$$\frac{A'D'}{AD} =$$

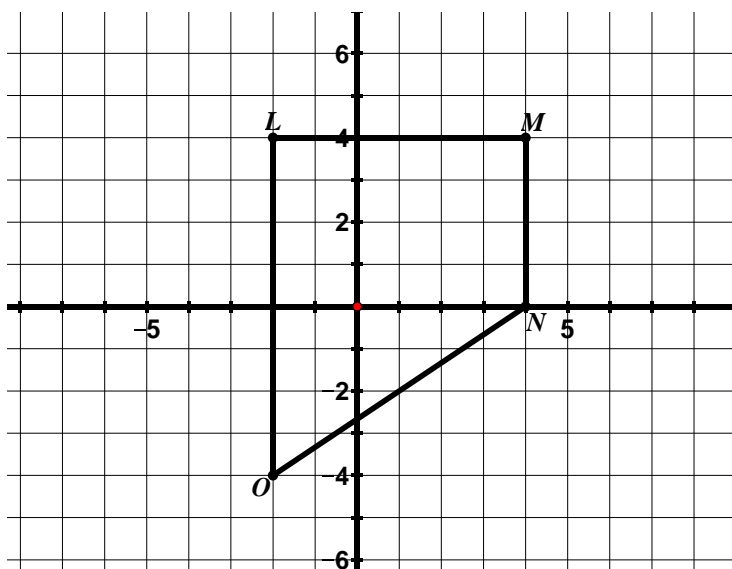
$$\frac{A'B'}{AB} = \frac{1}{4}$$

3. $\triangle ABC$ is graphed on the grid below.



- Reflect $\triangle ABC$ over the x -axis. Label the new triangle $\triangle A'B'C'$.
- Dilate $\triangle A'B'C'$ by a scale factor of 2 with the center of dilation at the origin. Label the new triangle $\triangle A''B''C''$.
- Write a statement that shows the relationship between $\triangle ABC$ and $\triangle A'B'C'$.
- Write a statement that shows the relationship between $\triangle A'B'C'$ and $\triangle A''B''C''$.
- List five other relationships between corresponding angles, sides, etc. that you know to be true based on the relationships in parts c and d.

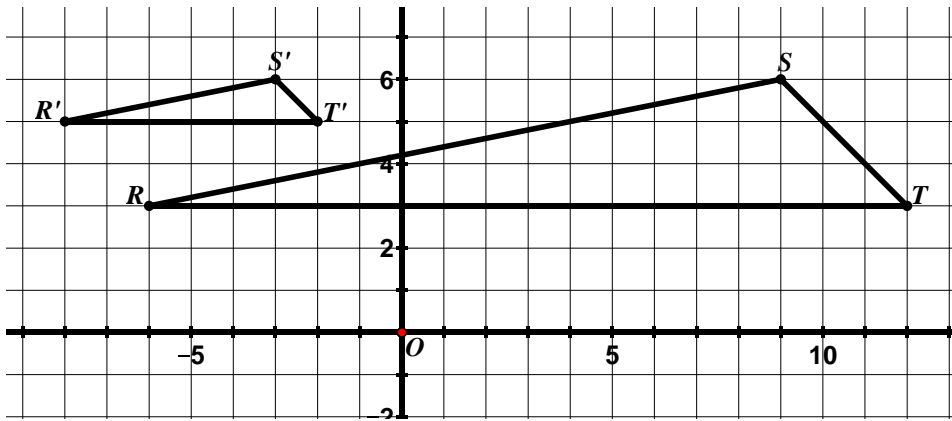
4. $LMNO$ is graphed on the grid below.



- Dilate $LMNO$ by a scale factor of $\frac{1}{2}$ with the center of dilation at the origin. Label the new quadrilateral $L'M'N'O'$.
- Translate $L'M'N'O'$ according to the rule $(x, y) \rightarrow (x - 6, y + 2)$. Label the new quadrilateral $L''M''N''O''$.
- Write a statement that shows the relationship between $LMNO$ and $L'M'N'O'$.
- Write a statement that shows the relationship between $L'M'N'O'$ and $L''M''N''O''$.
- List five other relationships between corresponding angles, sides, etc. that you know to be true based on the relationships in parts c and d.

9.2e Homework: Similarity

1. In the picture below $\triangle RST$ has been dilated to obtain $\triangle R'S'T'$.



a. Write a similarity statement for the triangles.

b. Complete each statement:

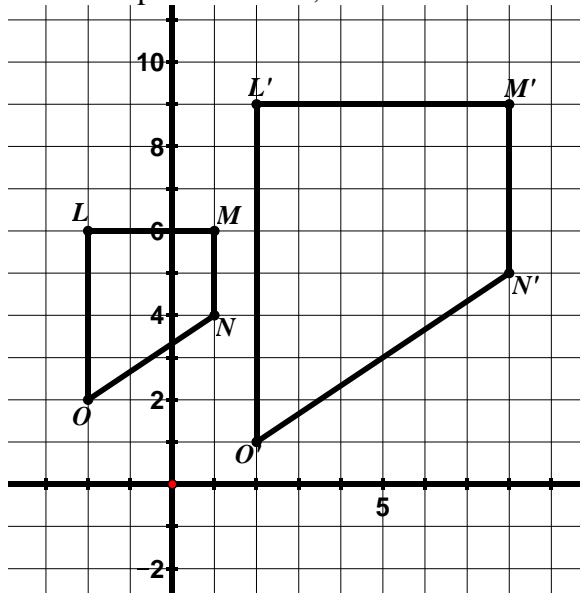
$$m\angle S \cong$$

If $m\angle T = 45^\circ$, then $m\angle T' =$

$$\frac{R'T'}{RT} =$$

$$\frac{R'S'}{RS} = \frac{1}{3}$$

2. In the picture below, $LMNO$ has been dilated to obtain $L'M'N'O'$. The center of dilation is the origin.



a. Write a similarity statement for the triangles.

b. Complete each statement.

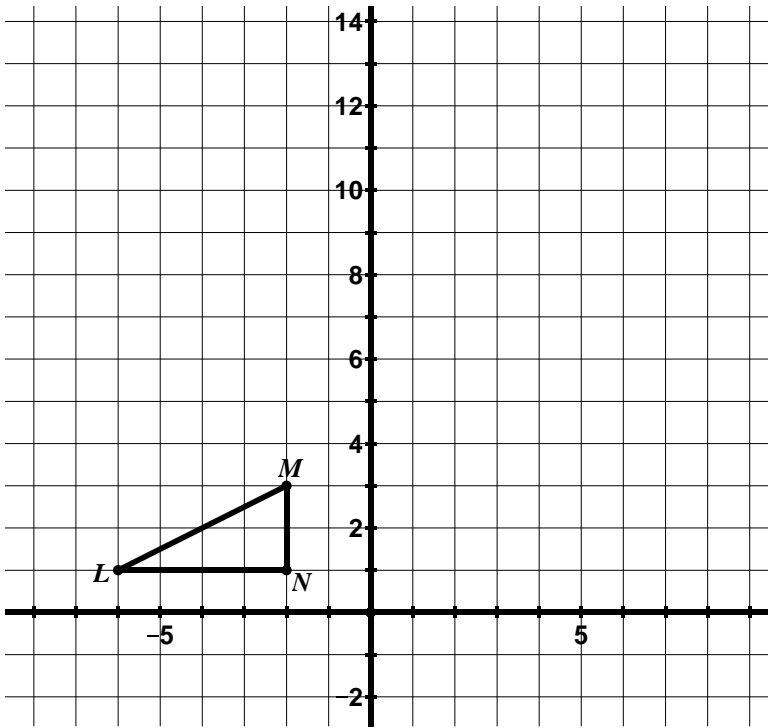
$$m\angle O \cong$$

If $m\angle L = 90^\circ$, then $m\angle L' =$

$$\frac{O'N'}{ON} =$$

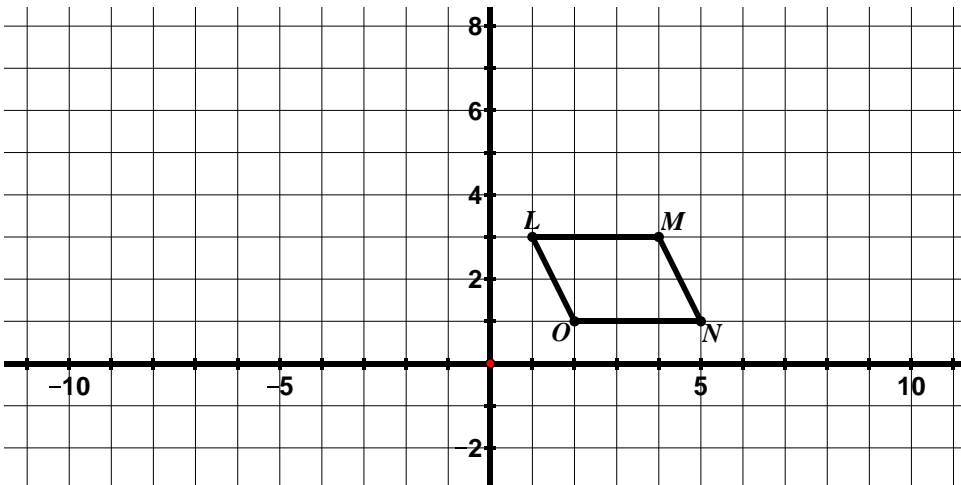
$$\frac{L'O'}{LO} = 2$$

3. $\triangle LMN$ is graphed on the grid below.



- Rotate $\triangle LMN$ 90° clockwise about the origin. Label the new triangle $\triangle L'M'N'$.
- Dilate $\triangle L'M'N'$ by a scale factor of 3 with the center of dilation at $(1, 2)$. Label the new triangle $\triangle L''M''N''$.
- Write a statement that shows the relationship between $\triangle LMN$ and $\triangle L'M'N'$.
- Write a statement that shows the relationship between $\triangle L'M'N'$ and $\triangle L''M''N''$.
- List five other relationships between corresponding angles, sides, etc. that you know to be true based on the relationships in parts c and d.

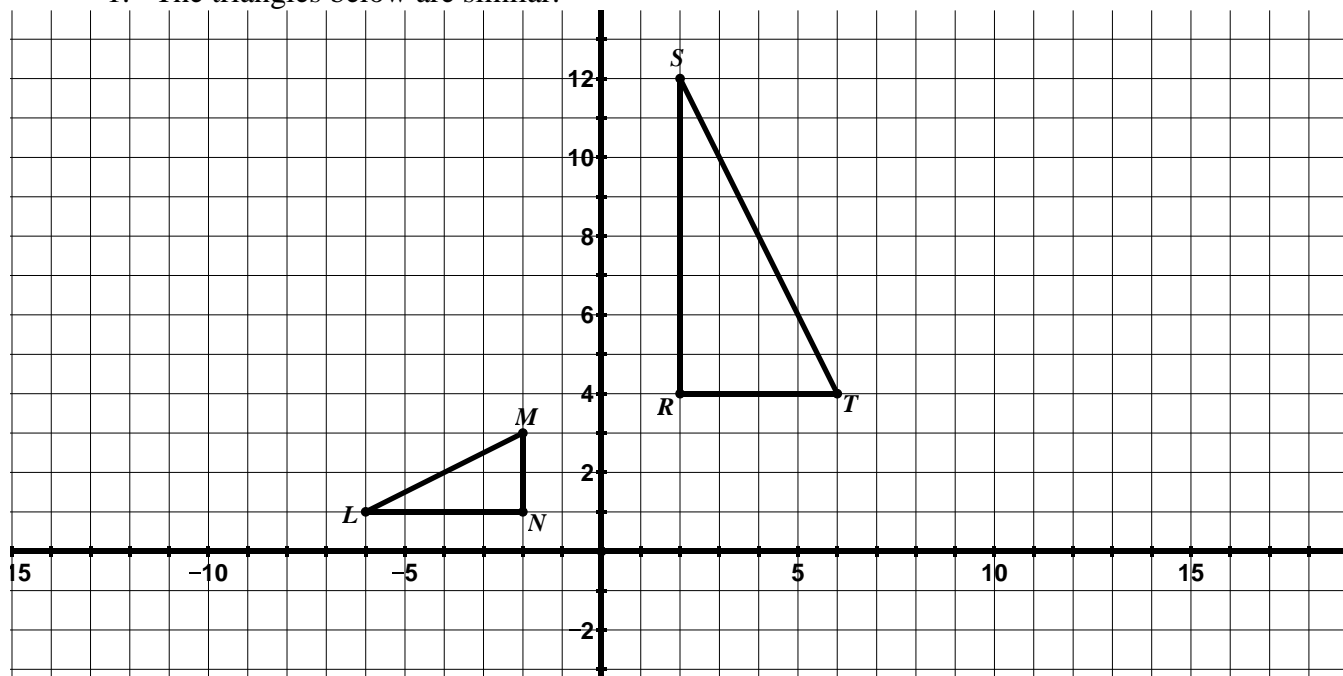
4. $LMNO$ is graphed on the grid below.



- Dilate $LMNO$ by a scale factor of 2 with the center of dilation at the origin. Label the new quadrilateral $L'M'N'O'$.
- Reflect $L'M'N'O'$ across the y-axis. Label the new quadrilateral $L''M''N''O''$.
- Write a statement that shows the relationship between $LMNO$ and $L'M'N'O'$.
- Write a statement that shows the relationship between $L'M'N'O'$ and $L''M''N''O''$.

9.2f Class Activity: Similarity cont.

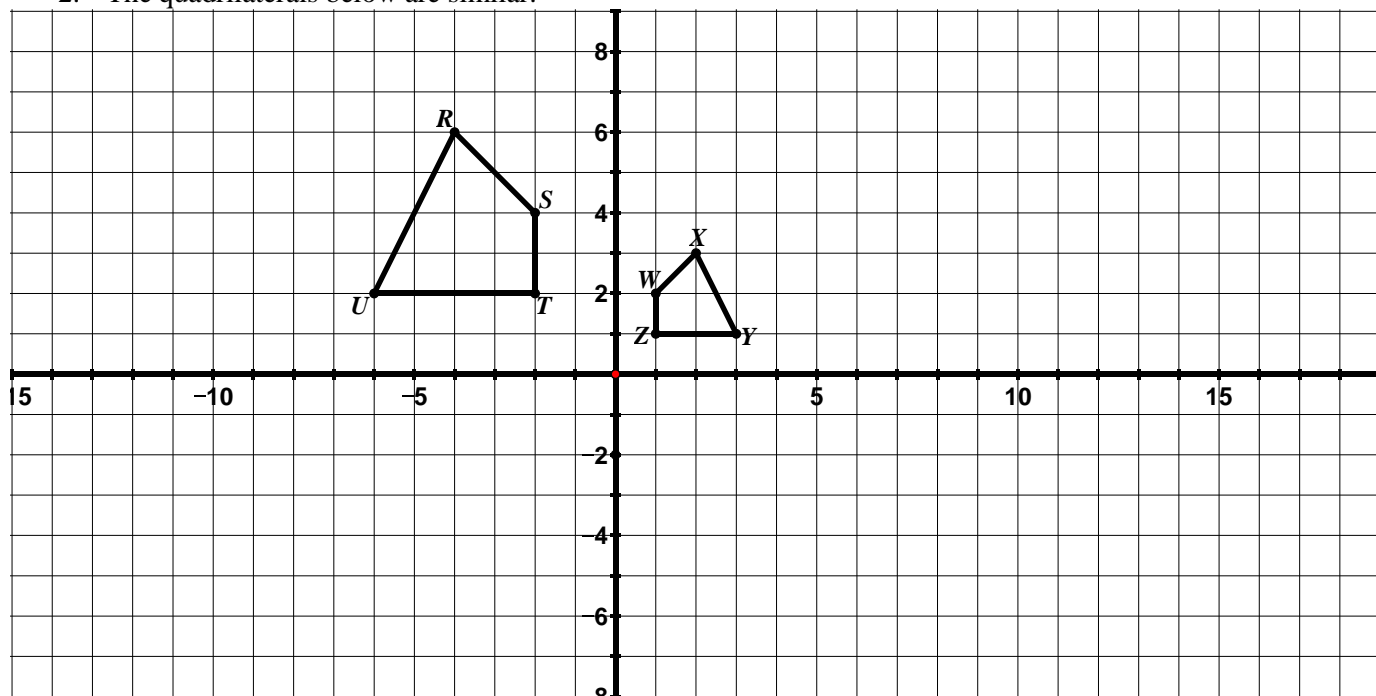
1. The triangles below are similar.



a. List the sequence of transformations that verifies the similarity of the two figures.

b. Write a similarity statement for the triangles.

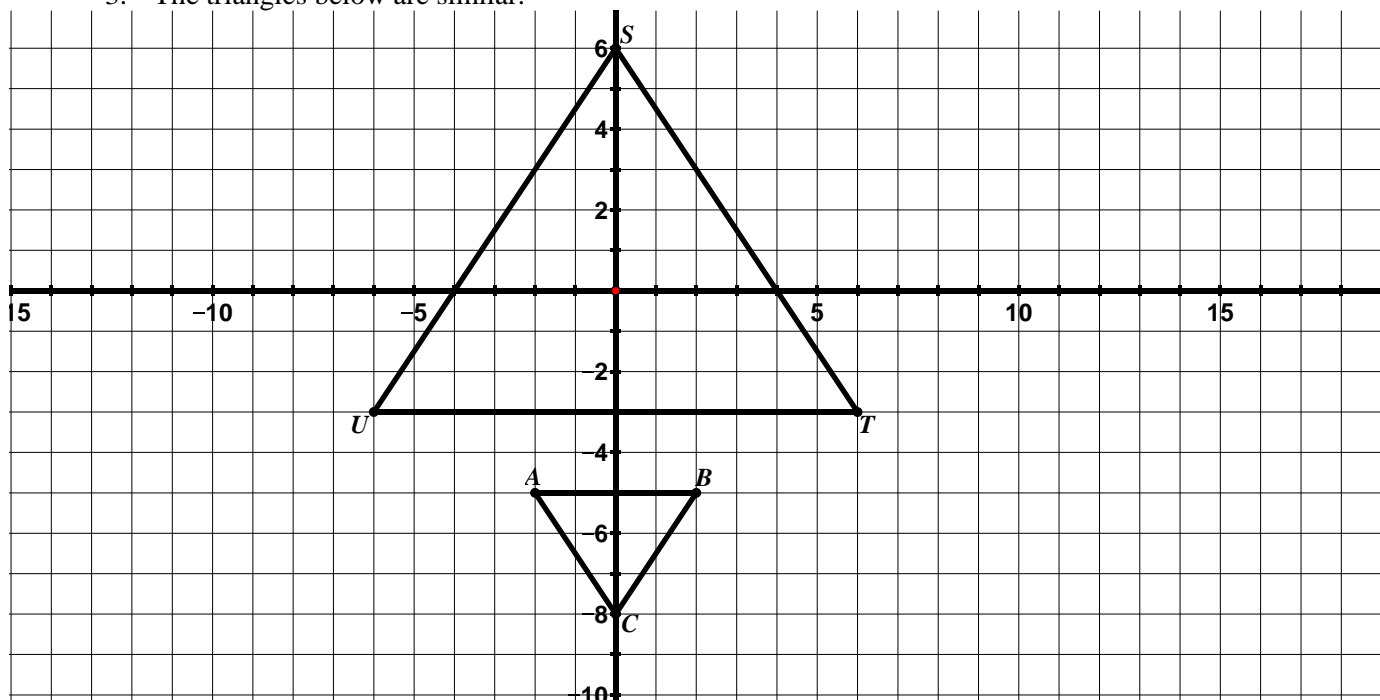
2. The quadrilaterals below are similar.



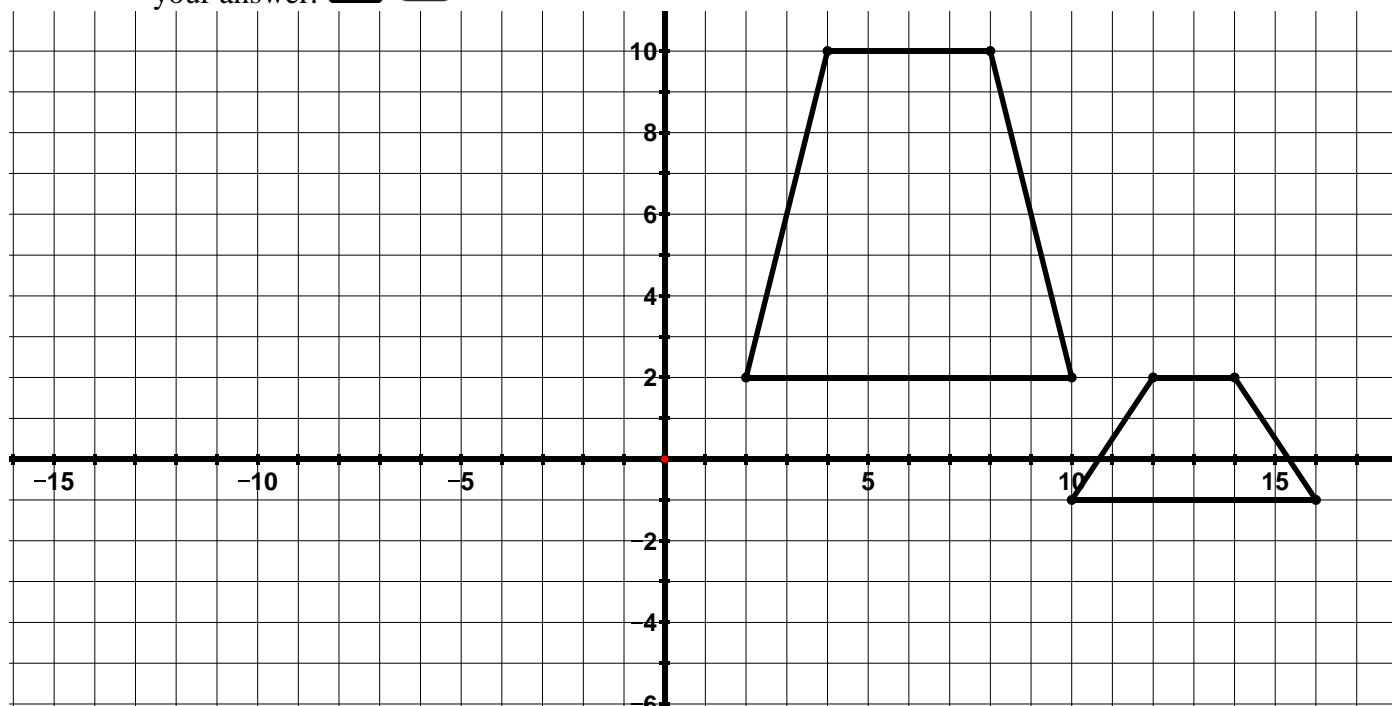
a. List the sequence of transformations that verifies the similarity of the two figures.

b. Write a similarity statement for the quadrilaterals.

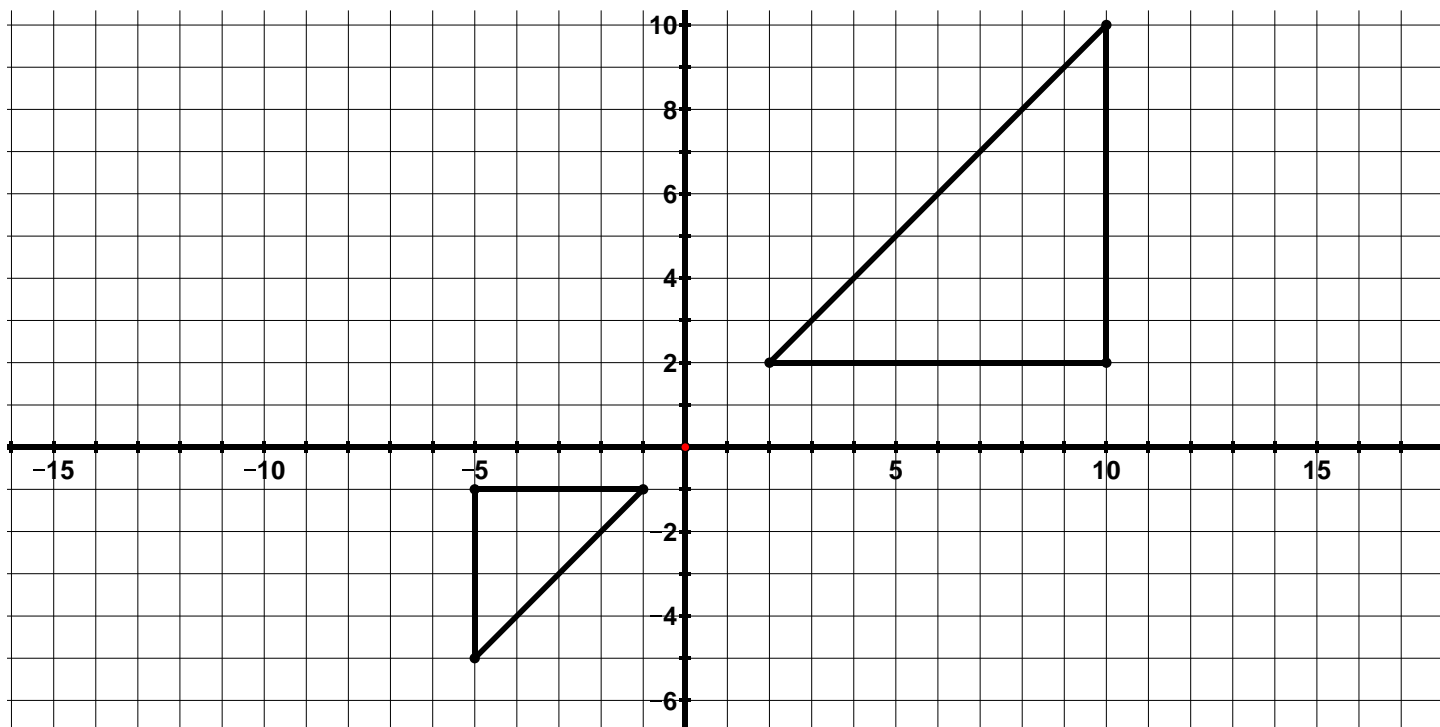
3. The triangles below are similar.



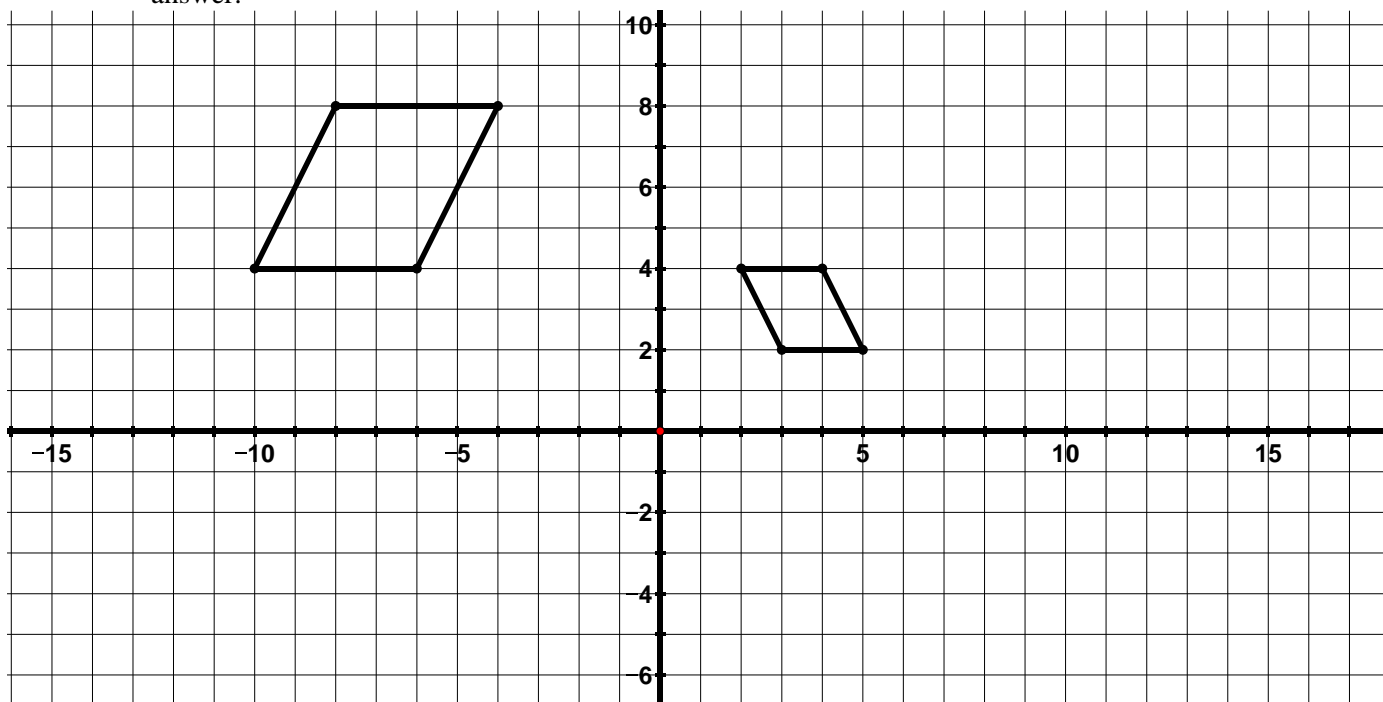
- List the sequence of transformations that verifies the similarity of the two figures.
 - Write a similarity statement for the triangles.
4. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



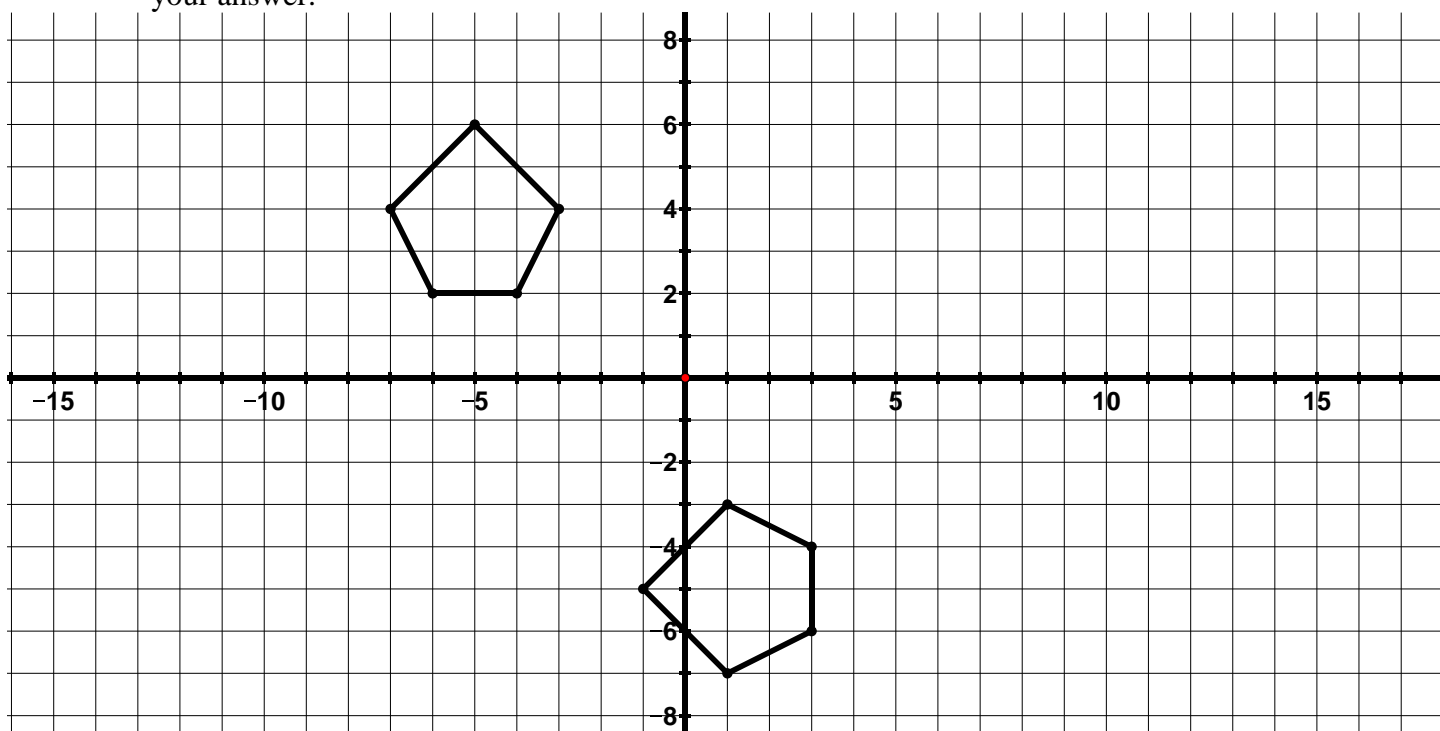
5. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



6. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



7. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



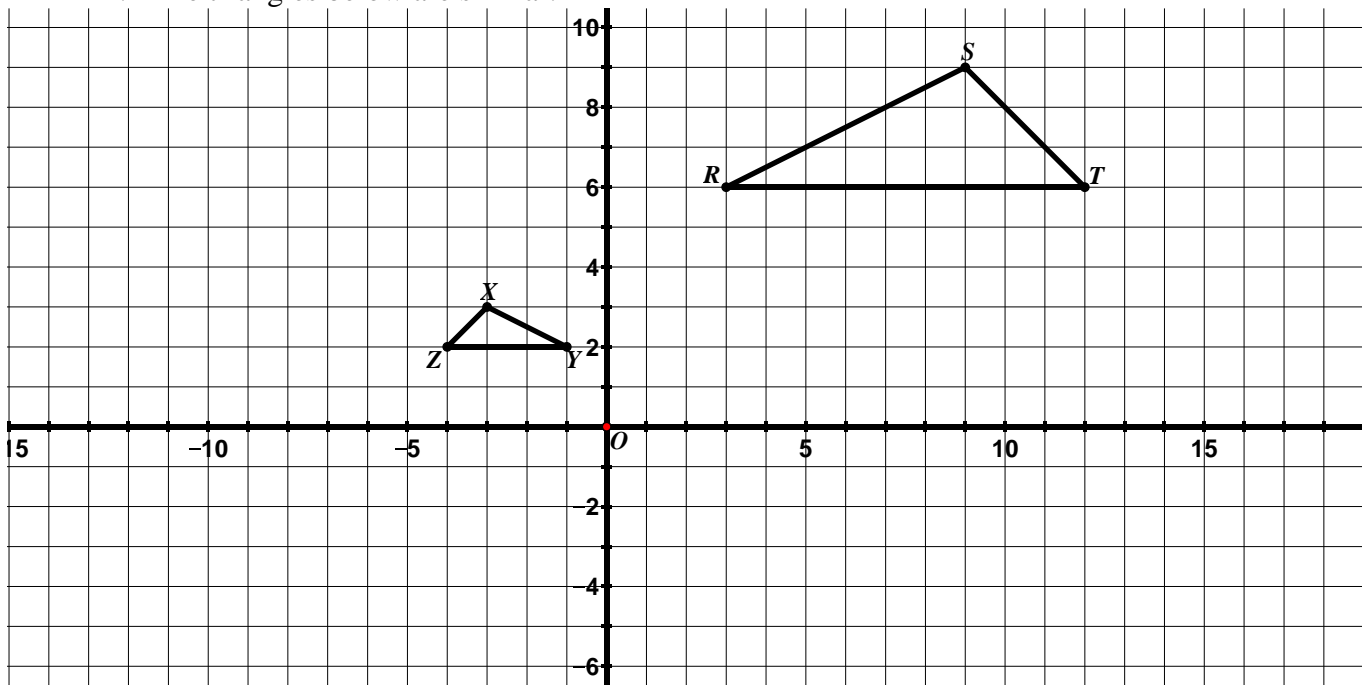
8. Determine whether the following transformations or sequence of transformations will result in similar figures, congruent figures, or neither. Provide a justification for your answer.



- $(x, y) \rightarrow (x - 6, y + 2)$
- $(x, y) \rightarrow (-x, y)$ followed by $(x, y) \rightarrow (2x, 2y)$
- $(x, y) \rightarrow (2x, 3y)$ followed by a reflection across the x -axis
- $(x, y) \rightarrow (x + 5, y + 5)$ followed by a 90° rotation counterclockwise about the origin
- $(x, y) \rightarrow \left(\frac{1}{4}x, \frac{1}{4}y\right)$ followed by $(x, y) \rightarrow (y, x)$
- $(x, y) \rightarrow (x, y + 4)$ followed by a 180° rotation clockwise about the origin

9.2f Homework: Similarity cont.

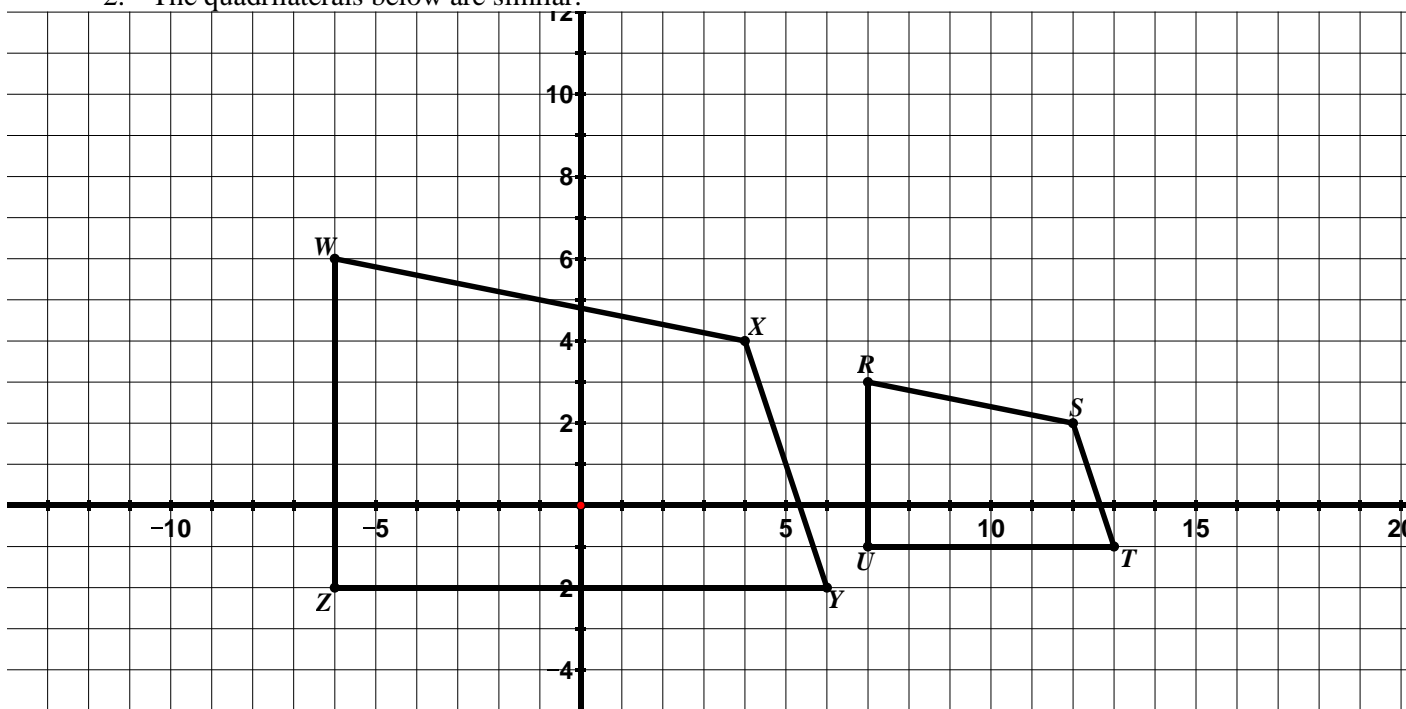
1. The triangles below are similar.



a. List the sequence of transformations that verifies the similarity of the two figures.

b. Write a similarity statement for the triangles.

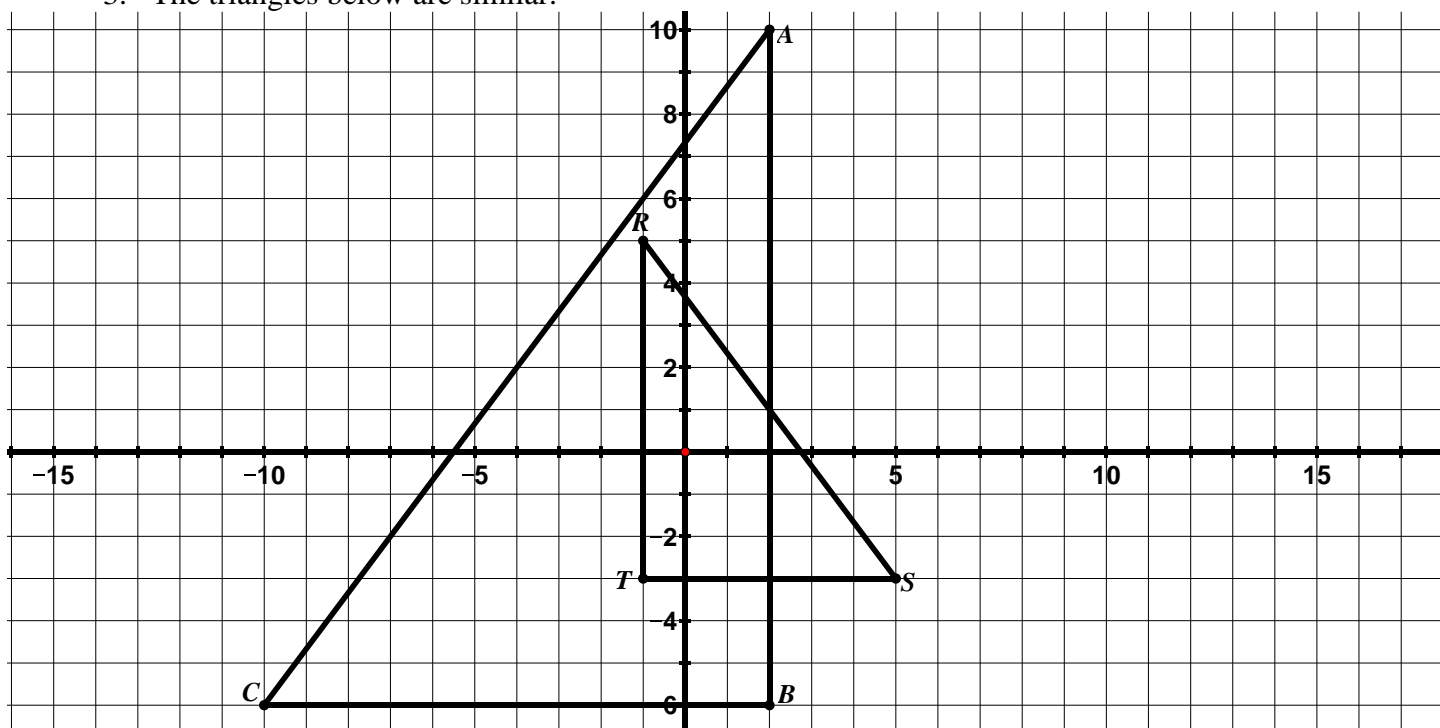
2. The quadrilaterals below are similar.



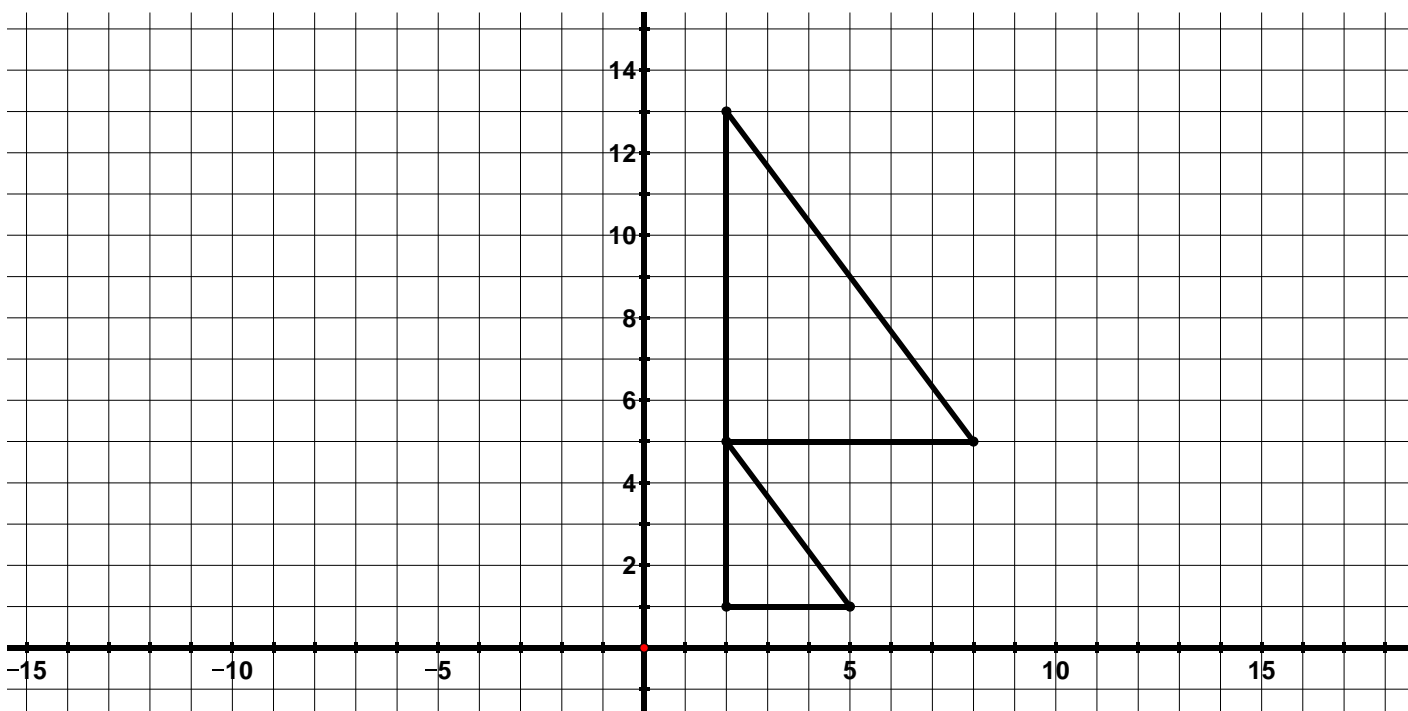
a. List the sequence of transformations that verifies the similarity of the two figures.

b. Write a similarity statement for the quadrilaterals.

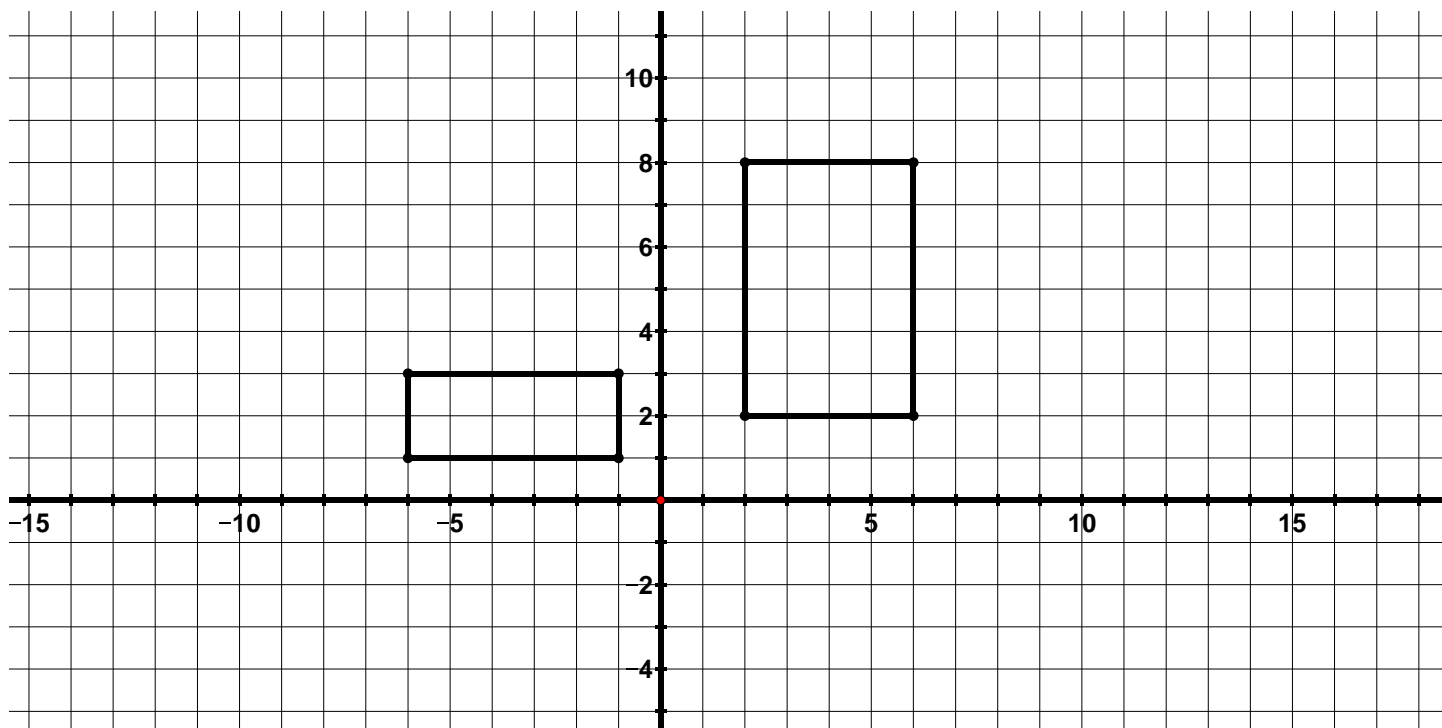
3. The triangles below are similar.



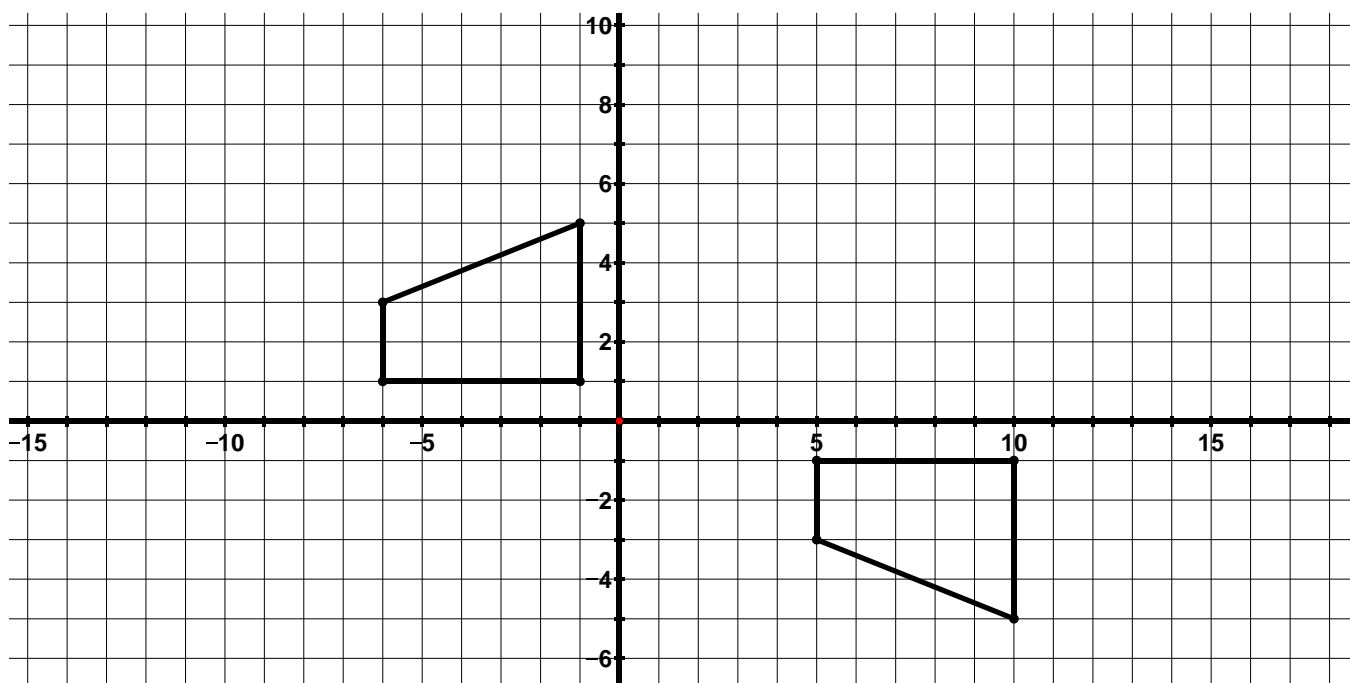
- List the sequence of transformations that verifies the similarity of the two figures.
 - Write a similarity statement for the triangles.
4. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



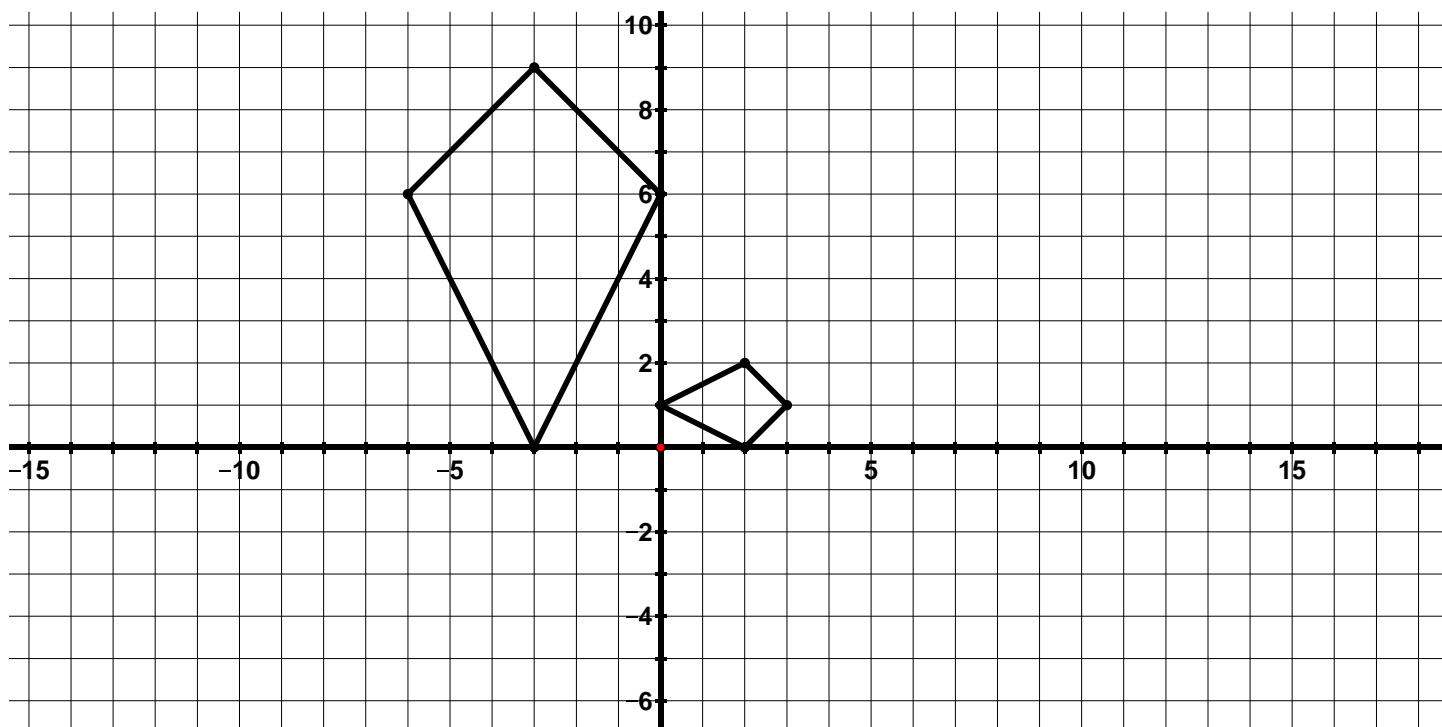
5. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



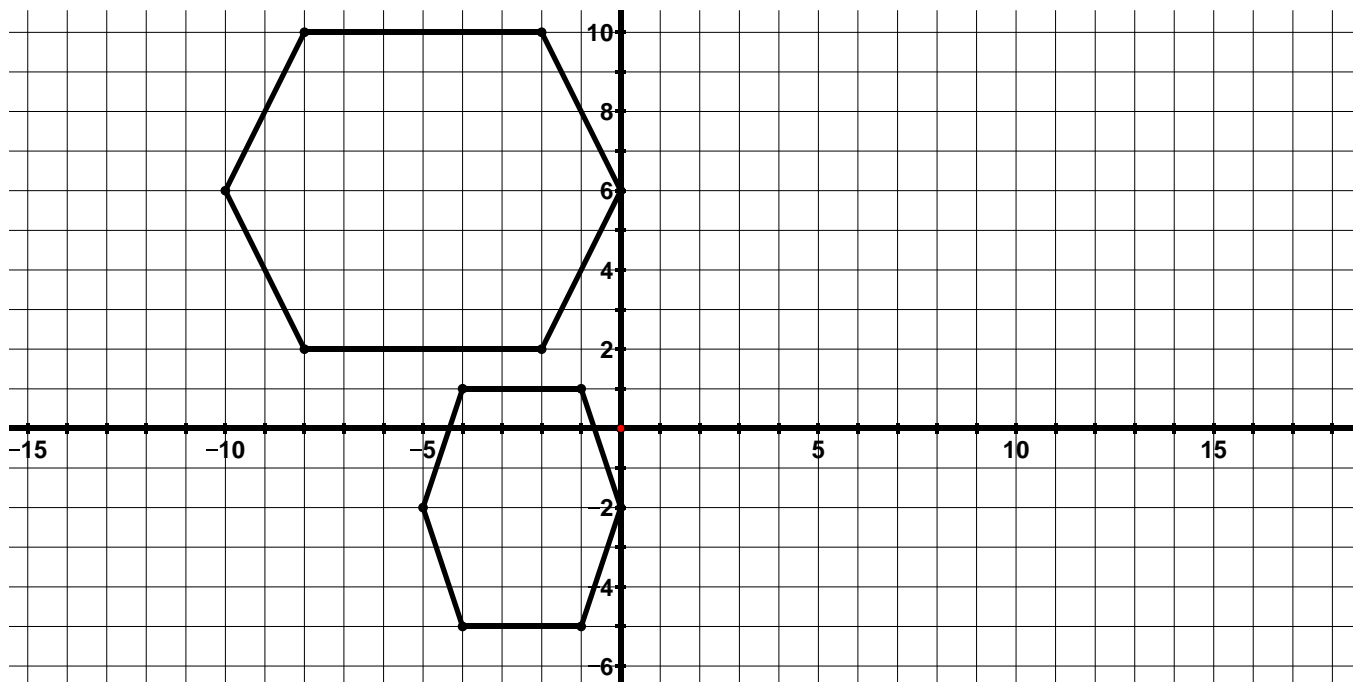
6. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



7. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



8. Determine whether the figures below are congruent, similar, or neither. Provide a justification for your answer.



9. Determine whether the following transformations or sequence of transformations will result in similar figures, congruent figures, or neither.
- a. $(x, y) \rightarrow (x + 6, 6y)$
 - b. $(x, y) \rightarrow (x, -y)$ followed by a 270° rotation clockwise about the origin
 - c. $(x, y) \rightarrow (4x, 4y)$ followed by $(x, y - 2)$
 - d. A 180° rotation counterclockwise about the origin followed by $(x + 2, y + 2)$
 - e. $(x, y) \rightarrow (3x, x + y)$
 - f. $(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$ followed by a reflection across the x -axis
 - g. Write your own transformation or sequence of transformations that will result in two figures that are congruent.
 - h. Write your own transformation or sequence of transformations that will result in two figures that are similar.
 - i. Write your own transformation or sequence of transformations that will result in two figures that are neither congruent nor similar.

9.2g Self-Assessment: Section 9.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems for each standard can be found on the following page(s).

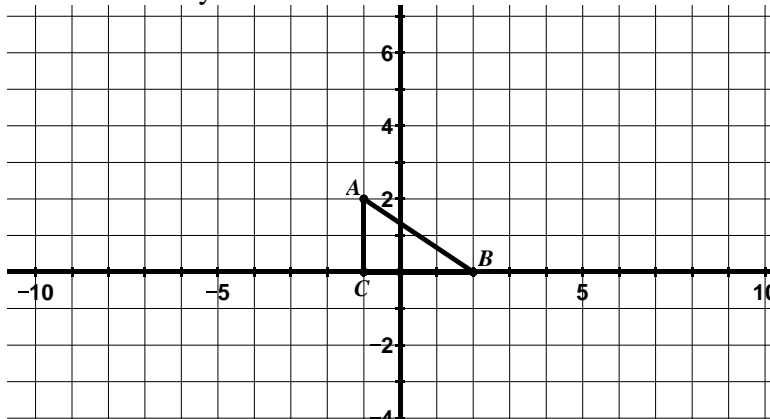
Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Describe the properties of a figure that has been dilated.				
2. Perform a dilation given a scale factor and center of dilation.				
3. Describe a dilation in words and using a coordinate rule.				
4. Determine the center of dilation using the properties of dilations.				
5. Understand what it means for two figures to be similar.				
6. Determine if two figures are similar.				
7. Given two figures that are similar, describe the sequence of transformations that exhibits the similarity between them.				

Section 9.2 Sample Problems (For use with self-assessment)

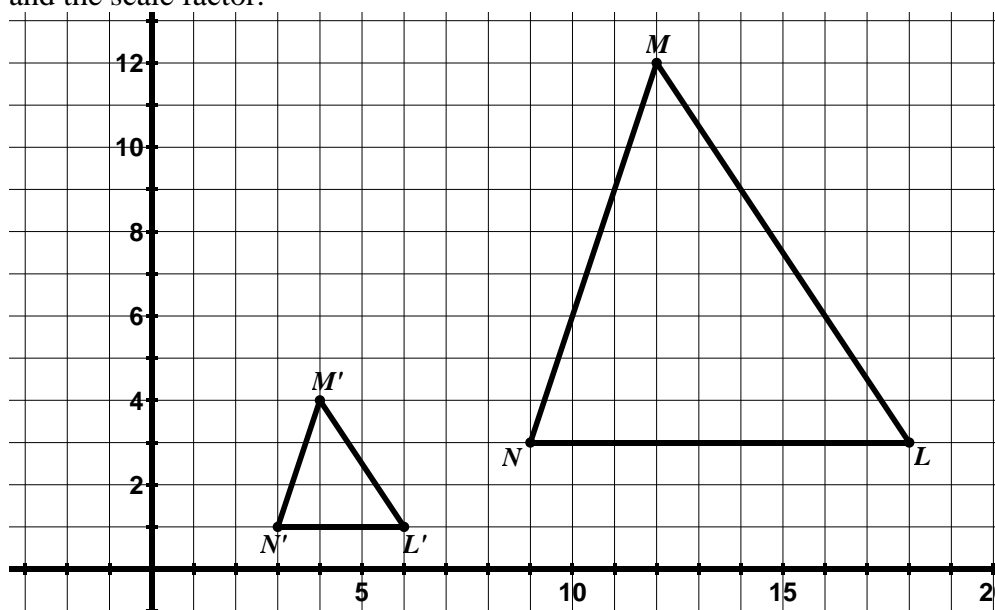
1. Which of the following properties are true about a figure that has only been dilated. Check all that apply.

- ☐ The ratios of the image segments to their corresponding pre-image segments are equal to the scale factor.
- ☐ Corresponding vertices lie on the same circle as the center of dilation.
- ☐ Corresponding vertices lie on the same line as the center of dilation.
- ☐ Corresponding segments are parallel.
- ☐ Corresponding segments are perpendicular.
- ☐ The area of the image is always the same as the area of the pre-image.

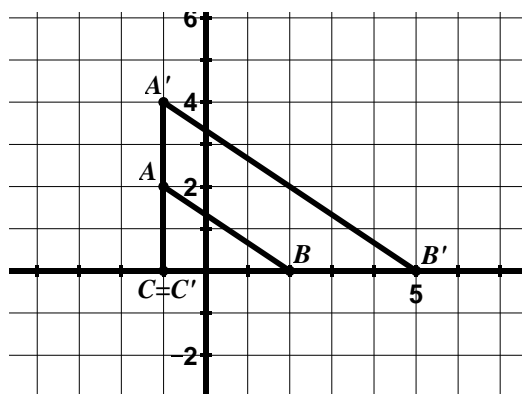
2. Dilate $\triangle ABC$ by a scale factor of 2 with the center of dilation at the origin.



3. Describe the dilation below in words and with a coordinate rule. Be sure to specify the center of dilation and the scale factor.

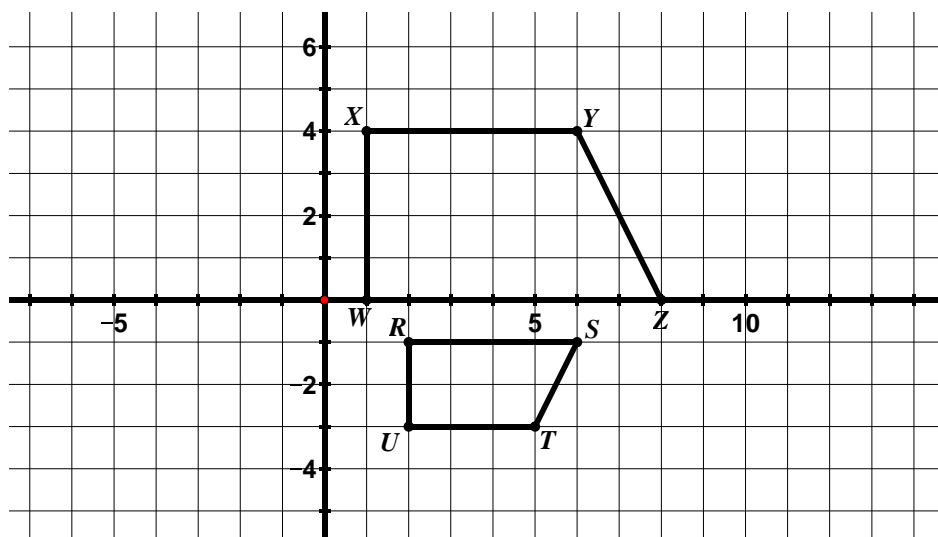


4. $\triangle ABC$ was dilated to produce $\triangle A'B'C'$. Determine the scale factor and center of dilation.

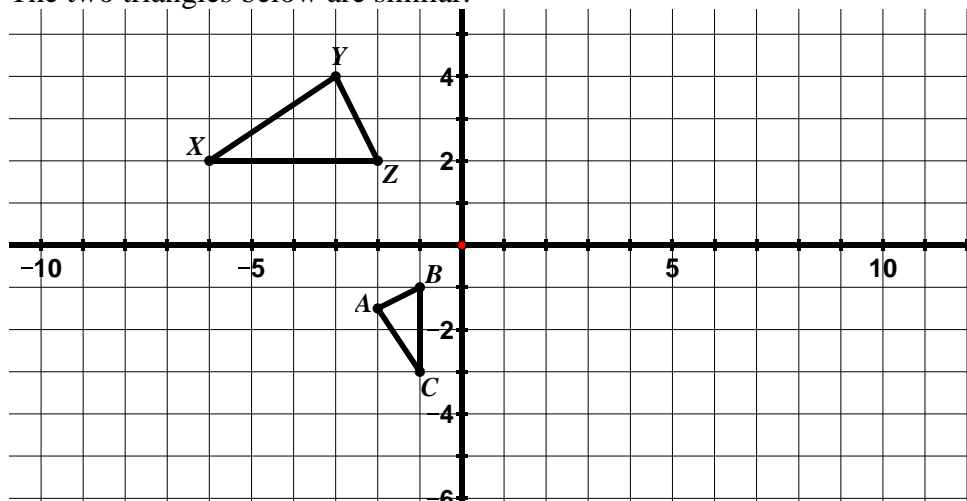


5. Describe in your own words what it means for two figures to be similar.

6. Are the parallelograms shown below similar? Provide a justification for your response.



7. The two triangles below are similar.



- Describe a sequence of transformations that verifies the similarity of the triangles.
- Write a similarity statement for the triangles.
- Determine which angles are congruent.
- Complete the following statements:

$$\frac{BC}{ZX} =$$

$$\frac{ZY}{BA} =$$