mathematics vision project

Transforming Mathematics Education

SECONDARY MATH ONE

An Integrated Approach

Standard Teacher Notes

MODULE 1

Sequences

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The Mathematics Vision Project

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1.1 Checkerboard Borders

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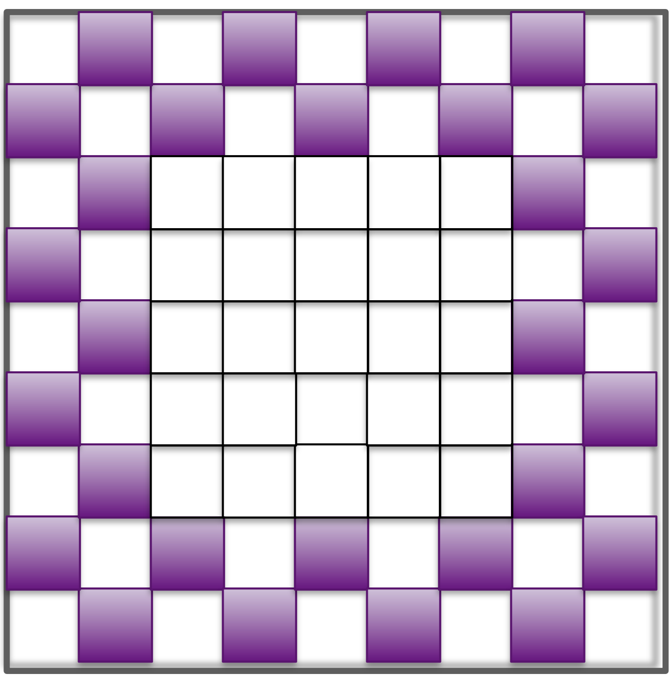
A Develop Understanding Task

In preparation for back to school, the school administration plans to replace the tile in the cafeteria. They would like to have a checkerboard pattern of tiles two rows wide as a surround for the tables and serving carts.

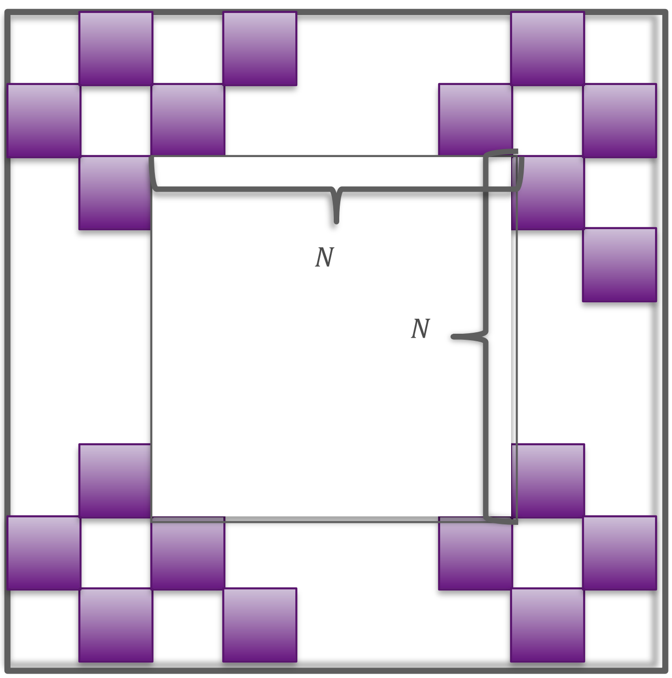
Below is an example of the boarder that the administration is thinking of using to surround a square 5 x 5 set of tiles.

A. Find the number of colored tiles in the checkerboard border. Track your thinking and find a way of calculating the number of colored tiles in the border that is quick and efficient. Be prepared to share your strategy and justify your work.



B. The contractor that was hired to lay the tile in the cafeteria is trying to generalize a way to calculate the number of colored tiles needed for a checkerboard border surrounding a square of tiles with any dimensions. To represent this general situation, the contractor started sketching the square below.

Find an expression for the number of colored border tiles needed for any N x N square center.



1 . 1 Checkerboard Borders – Teacher Notes

A Develop Understanding Task

Purpose:

The focus of this task is on the generation of multiple expressions that connect with the visuals provided for the checkerboard borders. These expressions will also provide opportunity to discuss equivalent expressions and review the skills students have previously learned about simplifying expressions and using variables.

Core Standards Focus:

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Related Standards: A.CED.2, A.REI.1

Standards for Mathematical Practice of Focus in the Task:

SMP 1 – Make sense of problems and persevere in solving them.

SMP 7 – Look for and make use of structure

The Teaching Cycle:

Launch (Whole Class):

After reading and discussing the "Checkerboard Borders" scenario, challenge students to come up with a way to quickly count the number of colored tiles in the border. Have them create numeric expressions that exemplify their process and require students to connect their thinking to the visual representation of the tiles.

The first phase of work should be done individually, allowing students to "see" the problem and patterns in the tiles in their own way. This will provide for more representations to be considered later. After students work individually for a few minutes on part A, have them share with a partner and begin to develop additional ideas as a pair or assist each other in generalizing their strategy for part B.

Explore (Small Group):

For students who don't know where to begin, it may be useful to ask some starter questions like: "How many tiles are there along one side?", "How can you count the tiles in groups rather than one-by-one?"

Press on students to connect their numeric representations to the visual representation. You might ask, "How does that four in your number sentence connect to the visual representation?" Encourage students to mark on the visual or to redraw it so they can demonstrate how they were thinking about the diagram numerically.

Watch for students who calculate the number of border tiles in different ways. Make note of their numeric strategies and the different generalized expressions that are created. The differing strategies and algebraic expressions will be the focus of the discussion at the end, allowing for students to connect back to prior work from previous mathematical experiences and better understand equivalence between expressions and how to properly simplify an algebraic expression. Prompt students to calculate the number of tiles for a given side length using their expression and then to draw the visual model and check for accuracy. Require students to justify why their expression will work for any side length $\boldsymbol{N}$ of the inner square region. Press them to generalize their justifications rather than just repeat the process they have been using. You might ask, "How do you know your expression will work for any side length?", or "What is it about the nature of the pattern that suggests this will always work?", or "What will happen if we look at a side length of six? ten? fifty-three?" Consider these ideas both visually and in terms of the general expression.

Note: Based on the student work and the difficulties they may or may not encounter, a determination will need to be made as to whether a discussion of part A of the task should be held prior to students working on part B. Working with a specific case may facilitate access to the general case for more students. However, if students are ready for whole class discussion of their general representations, then starting there will allow for more time to be spent on making connections between the different expressions, and extending the task to more general representations.

As available, select students to present who found different ways of generalizing. Some possible ways students might "see" the colored tiles grouped are provided after the challenge activity. It would be useful to have at least three different views to discuss and possibly more.

Discuss (Whole Class):

Based on the student work available, you will need to determine the order of the strategies to be presented. A likely progression would start with a strategy that does not provide the most simplified form of the expression. This will promote questioning and understanding from students that may have done it differently and allow for discussion about what each piece of the expression represents. After a couple of different strategies have been shared it might be useful to get the most simplified form of the expression out on the table and then look for an explanation as to how all of the expressions can be equivalent and represent the same thing in so many different ways.

Aligned Ready, Set, Go: Getting Ready 1.1

READY, SET, GO! Name Period Date

READY

Topic: Recognizing Solutions to Equations

The solution to an equation is **the value of the variable** that makes the equation **true**. In the equation $9a + 17 = -21$, "a" is the variable. When $a = 2$, $9a + 17 \neq -19$, because $9(2) + 17 = 35$. Thus $a = 2$ is NOT a solution. However, when $a = -4$, the equation is true $9(-4) + 17 = -19$. Therefore, $a = -4$ must be the solution.

Identify which of the 3 possible numbers is the solution to the equation.

1. $3x + 7 = 13$ $(x = -2;\ x = 2;\ x = 5)$

2. $8 - 2b = -2$ $(b = -3; b = 0; b = 5)$

3. $5 + 4g + 8 = 1$ $(g = -3; g = -1; g = 2)$

4. $6t - 5 + 5t = 105$ $(t = 4; t = 7; t = 10)$

Some equations have two variables. You may recall seeing an equation written like the following: $y = 5x + 2$. We can let x equal a number and then work the problem with this x- value to determine the associated y- value. A solution to the equation must include both the x- value and the y- value. Often the answer is written as an **ordered pair**. The ***x*- value is always first**. Example: (x, y). The order matters!

Determine the y-value of each ordered pair based on the given *x*- value.

5. $y = 6x - 15$; $(8, \quad), (-1, \quad), (5, \quad)$

6. $y = -4x + 9$; $(-5, \quad), (2, \quad), (4, \quad)$

7. $y = 2x - 1$; $(-4, \quad), (0, \quad), (7, \quad)$

8. $y = -x + 9$; $(-9, \quad), (1, \quad), (5, \quad)$

SET

Topic: Using a constant rate of change to complete a table of values

Fill in the table. Then write a sentence explaining how you figured out the values to put in each cell.

9. You run a business making birdhouses. You spend $600 to start your business, and it costs you $5.00 to make each birdhouse.

# of birdhouses	1	2	3	4	5	6	7
Total cost to build							

Explanation:

10. You make a $15 payment on your loan of $500 at the end of each month.

# of months	1	2	3	4	5	6	7
Amount of money owed							

Explanation:

11. You deposit $10 in a savings account at the end of each week.

# of weeks	1	2	3	4	5	6	7
Amount of money saved							

Explanation:

12. You are saving for a bike and can save $10 per week. You have $25 when you begin saving.

# of weeks	1	2	3	4	5	6	7
Amount of money saved							

Explanation:

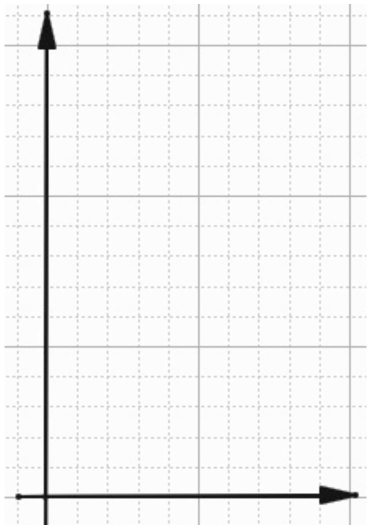
GO

Topic: Graph Linear Equations Given a Table of Values.

Graph the ordered pairs from the tables on the given graphs.

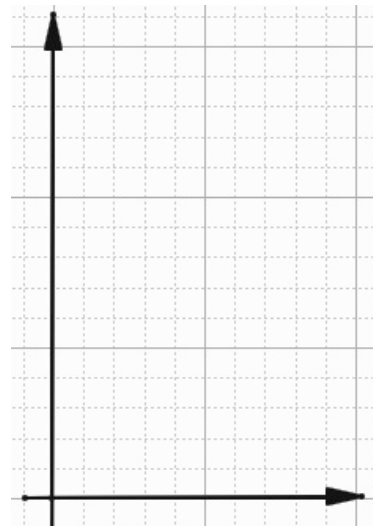
13.

x	y
0	3
2	7
3	9
5	13



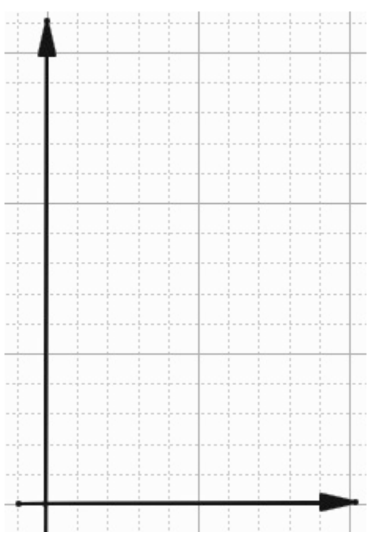
14.

x	y
0	14
4	10
7	7
9	5



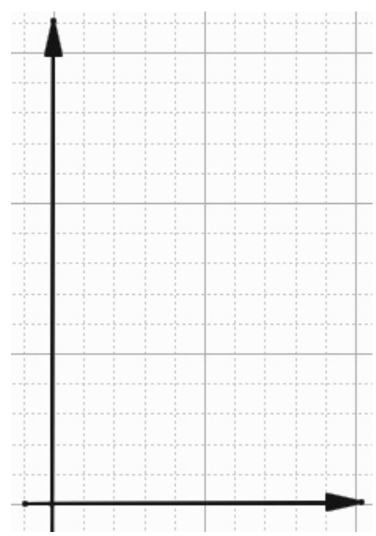
15.

x	y
2	11
4	10
6	9
8	8



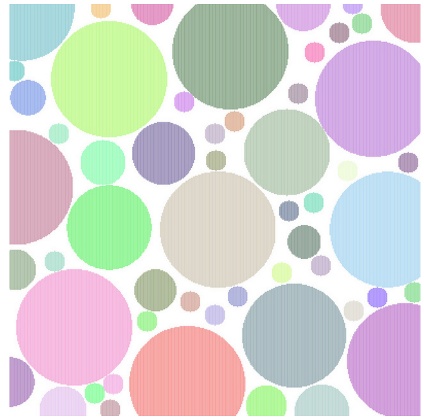
16.

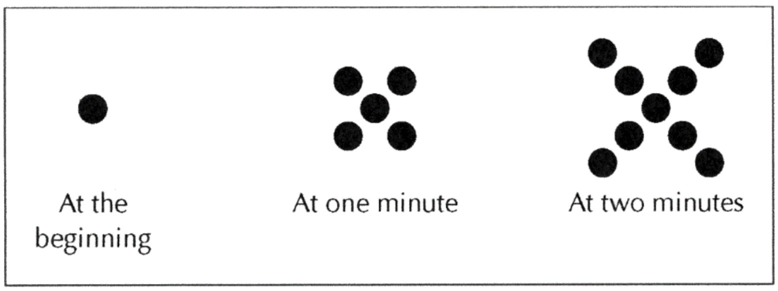
x	y
1	4
2	7
3	10
4	13



1.2 Growing Dots

A Develop Understanding Task





1. Describe the pattern that you see in the sequence of figures above.

2. Assuming the pattern continues in the same way, how many dots are there at 3 minutes?

3. How many dots are there at 100 minutes?

4. How many dots are there at t minutes? Solve the problems by your preferred method. Your solution should indicate how many dots will be in the pattern at 3 minutes, 100 minutes, and t minutes. Be sure to show how your solution relates to the picture and how you arrived at your solution.

1.2 Growing Dots– Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is to develop representations for arithmetic sequences that students can draw upon throughout the module. The visual representation in the task should evoke lists of numbers, tables, graphs, and equations. Various student methods for counting and considering the growth of the dots will be represented by equivalent expressions that can be directly connected to the visual representation.

Core Standards:

F-BF: Build a function that models a relationship between two quantities.

1: Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus on linear and exponential only)

Construct and compare linear, quadratic and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

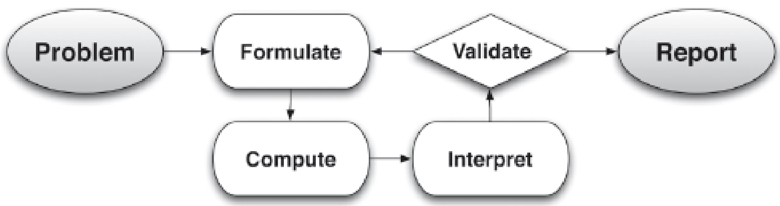
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:



Standards for Mathematical Practice of Focus in the Task:

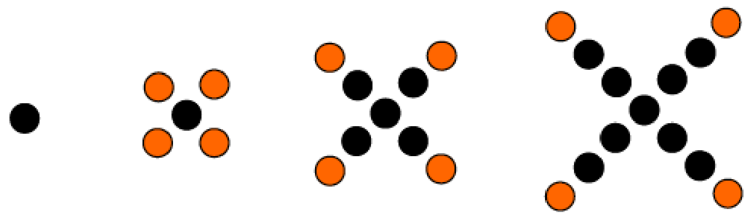
SMP1: Make sense of problems and persevere in solving them.

SMP7: Look for and make use of structure.

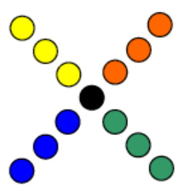
The Teaching Cycle:

Launch (Whole Class): Start the discussion with the pattern on growing dots drawn on the board or projected for the entire class. Ask students to describe the pattern that they see in the dots (Question #1). Students may describe four dots being added each time in various ways, depending on how they see the growth occurring. This will be explored later in the discussion as students write equations, so there should not be any emphasis placed upon a particular way of seeing the growth. Ask students individually to consider and draw the figure that they would see at 3 minutes (Question #2). Then, ask one student to draw it on the board to give other students a chance to check that they are seeing the pattern.

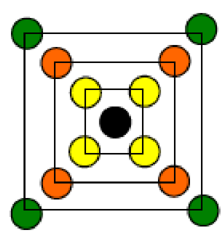
Explore (Small Group or Pairs): Ask students to complete the task. Monitor students as they work, observing their strategies for counting the dots and thinking about the growth of the figures. Some students may think about the figures recursively, describing the growth by saying that the next figure is obtained by placing four dots onto the previous figure as shown:



Some may think of the figure as four arms of length *t.* with a dot in the middle.



Others may use a "squares" strategy, noticing that a new square is added each minute, as shown:



As students work to find the number of dots at 100 minutes, they may look for patterns in the numbers, writing simply 1, 5, 9, . . . If students are unable to see a pattern, you may encourage them to make a table or graph to connect the number of dots with the time:

Time (Minutes)	Number of Dots
0	1
1	5
2	9
3	13
t	

Watch for students that have used a graph to show the number of dots at a given time and to help write an equation. Encourage students to connect their counting strategy to the equation that they write.

For the discussion, select a student for each of the three counting strategies shown, a table, a graph, a recursive equation, and at least one form of an explicit equation.

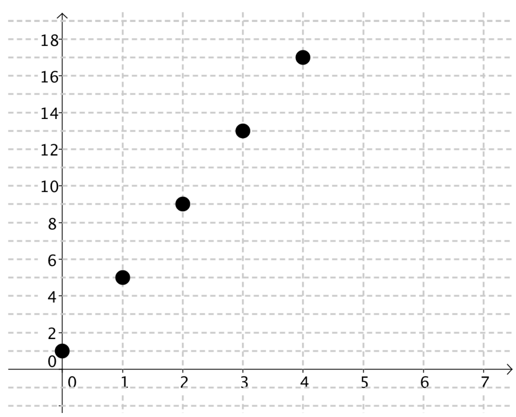
Discuss (Whole Group): Begin the discussion by asking students how many dots that there will be at 100 minutes. There may be some disagreement, typically between 100 and 101. Ask a student that said 101 to explain how they got their answer. If there is general agreement, move on to the discussion of the number of dots at time *t*.

Start by asking a group to chart and explain their table. Ask students what patterns they see in the table. When they describe that the number of dots is growing by 4 each time, add a difference column to the table, as shown.

Time (Minutes)	Number of Dots	Difference
0	1	> 4
1	5	> 4
2	9	> 4
3	13	
…	…	
t		

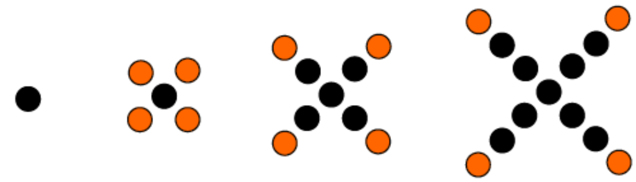
Ask students where they see the difference of 4 occurring in the figures. Note that the difference between terms is constant each time.

Continue the discussion by asking a group to show their graph. Be sure that it is properly labeled, as shown.



Ask students how they see the constant difference of 4 on the graph. They should recognize that the y-value increases by 4 each time, making a line with a slope of 4.

Now, move the discussion to consider the number of dots at time *t,* as represented by an equation. Start with a group that considered the growth as a recursive pattern, recognizing that the next term is 4 plus the previous term. They may represent the idea as: $X + 4$, with X representing the previous term. This may cause some controversy with students that wrote a different formula. Ask the group to explain their work using the figures. It may be useful to rewrite their formula with words, like:



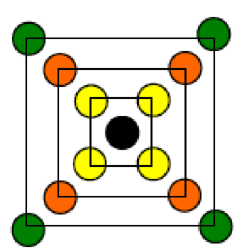
The number of dots in the current figure = the number of dots in the previous figure + 4

Or simply, *Current = previous + 4*

This may be written in function notation as: $f(t) = f(t-1) + 4$. (Although students have some exposure to function notation in grade 8, they have not seen it used to write recursive formulas. You may choose to introduce this notation in later lessons, simply focusing on writing the recursive idea in words as shown above.)

Next ask a group that has used the "four arms strategy" to write and explain their equation. Their equation should be: $f(t) = 4t + 1$. Ask students to connect their equation to the figure. They should articulate that there is 1 dot in the middle and 4 arms, each with t dots. The 4 in the equation shows 4 groups of size t.

Next, ask a group that used the "squares" strategy to describe their equation. They may have written the same equation as the "four arms" group, but ask them to relate each of the numbers in the equation to the figures anyway. In this way of thinking about the figures, there are t groups of 4 dots, plus 1 dot in the middle. Although it is not typically written this way, this counting method would generate the equation $f(t) = t \cdot 4 + 1$.



Now ask students to connect the equations with the table and graphs. Ask them to show what the 4 and the 1 represent in the graph. Ask how they see $4t$ +1 in the table. It may be useful to show this pattern to help see the pattern between the time and the number of dots:

Time (Minutes)	Number of Dots		Difference
0	1	1	
			> 4
1	5	1+4	
			> 4
2	9	1+4+4	
			> 4
3	13	1+4+4+4	
…	…		
t		1+4t	

You may also point out that when the table is used to write a recursive equation like $f(t) = f(t-1) + 4$, you may simply look down the table from one output to the next. When writing an explicit formula like $f(t) = 4t + 1$, it is necessary to look across the rows of the table to connect the input with the output.

Finalize the discussion by explaining that this set of figures, equations, table, and graph represent an arithmetic sequence. An arithmetic sequence can be identified by the constant difference between consecutive terms. Tell students that they will be working with other sequences of numbers that may not fit this pattern, but tables, graphs and equations will be useful tools to represent and discuss the sequences.

Aligned Ready, Set, Go Homework: Sequences 1.2

READY, SET, GO! Name Period Date

READY

Topic: Using function notation

To **evaluate** an equation such as $y = 5x + 1$ when given a specific value for x, replace the variable x with the given value and work the problem to find the value of y.
<u>Example:</u> Find y when x = 2. Replace x with 2. $y = 5(2) + 1 = 10 + 1 = 11$.
Therefore, y = 11 when x = 2. The point (2, 11) is one solution to the equation $y = 5x + 1$. Instead of using x *and* y in an equation, mathematicians often write $f(n) = 5n + 1$ because it can give more information. With this notation, the direction to find $f(2)$, means to replace the value of n with 2 and work the problem to find $f(n)$. The point $(n, f(n))$ is in the same location on the graph as (x, y), where n describes the location along the x–axis, and $f(n)$ is the height of the graph.

Given that $f(n) = 8n - 3$ and $g(n) = 3n - 10$, evaluate the following functions with the indicated values.

1. $f(5) =$
2. $g(5) =$
3. $f(-4) =$
4. $g(-4) =$
5. $f(0) =$
6. $g(0) =$
7. $f(1) =$
8. $g(1) =$

Topic: Looking for patterns of change
Complete each table by looking for the pattern.

9.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	2	4	8	16	32			

10.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	66	50	34	18				

11.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	160	80	40	20				

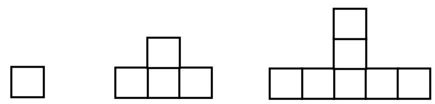
12.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-9	-2	5	12				

SET

Topic: Use variables to create equations that connect with visual patterns.

In the pictures below, each square represents one tile.



Step 1 Step 2 Step 3 Step 4 Step 5

13. Draw Step 4 and Step 5.

The students in a class were asked to find the number of tiles in a figure by describing how they saw the pattern of tiles changing at each step. Match each student's way of describing the pattern with the appropriate equation below. Note that *"s"* represents the step number and "*n*" represents the number of tiles.

(a) $\boldsymbol{n = (2s - 1) + (s - 1)}$ **(b)** $\boldsymbol{n = 3s - 2}$ **(c)** $\boldsymbol{n = s + 2(s - 1)}$

14. ____Dan explained that the middle "tower" is always the same as the step number. He also pointed out that the 2 arms on each side of the "tower" contain one less block than the step number.

15. ____ Sally counted the number of tiles at each step and made a table. She explained that the number of tiles in each figure was always 3 times the step number minus 2.

step number	1	2	3	4	5	6
number of tiles	1	4	7	10	13	16

16. ____ Nancy focused on the number of blocks in the base compared to the number of blocks above the base. She said the number of base blocks were the odd numbers starting at 1. And the number of tiles above the base followed the pattern 0, 1, 2, 3, 4. She organized her work in the table at the right.

Step number	# in base + #on top
1	1 + 0
2	3 + 1
3	5 + 2
4	7 + 3
5	9 + 4

GO

Topic: The meaning of an exponent

Write each expression using an exponent.

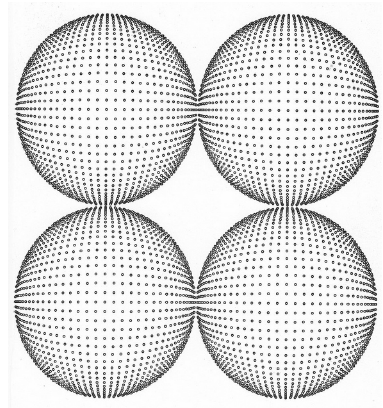
17. $6 \times 6 \times 6 \times 6 \times 6$
18. $4 \times 4 \times 4$
19. $15 \times 15 \times 15 \times 15$
20. $\frac{1}{3} \times \frac{1}{3}$

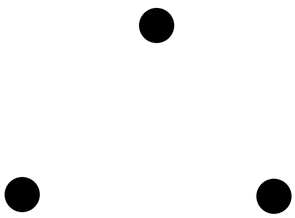
A) Write each expression in expanded form. B) Then calculate the value of the expression.

21. 7^1
22. 3^2
23. 5^3
24. 10^4
25. $7(2)^3$
26. $10(8^2)$
27. $3(5)^4$
28. $16\left(\frac{1}{2}\right)^3$

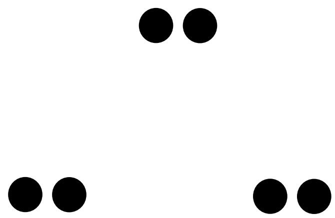
1.3 Growing, Growing Dots

A Develop Understanding Task

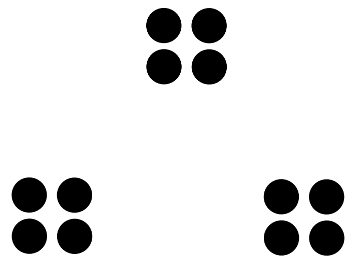




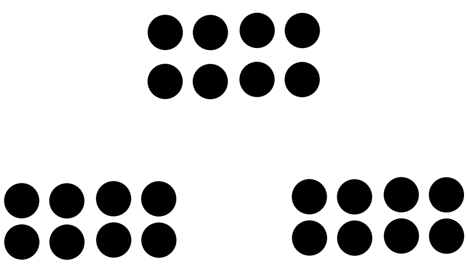
At the beginning



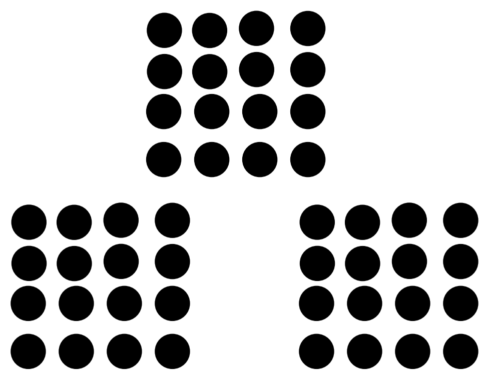
At one minute



At two minutes



At three minutes



At four minutes

1. Describe and label the pattern of change you see in the above sequence of figures.

2. Assuming the sequence continues in the same way, how many dots are there at 5 minutes?

3. Write a recursive formula to describe how many dots there will be after t minutes.

4. Write an explicit formula to describe how many dots there will be after t minutes.

1.3 Growing, Growing Dots – Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is to develop representations for geometric sequences that students can draw upon throughout the module. The visual representation in the task should evoke lists of numbers, tables, graphs, and equations. Various student methods for counting and considering the growth of the dots will be represented by equivalent expressions that can be directly connected to the visual representation.

Core Standards:

F-BF: Build a function that models a relationship between two quantities.

1: Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Linear, Quadratic, and Exponential Models* (Secondary Mathematics I focus on linear and exponential only)

Construct and compare linear, quadratic and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

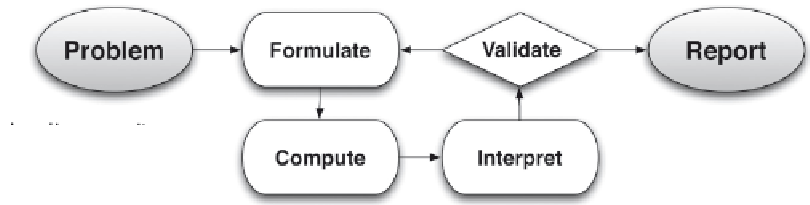
c. Recognize situations in which one quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:



Standards for Mathematical Practice of Focus in the Task:

SMP1 – Make sense of problems and persevere in solving them.

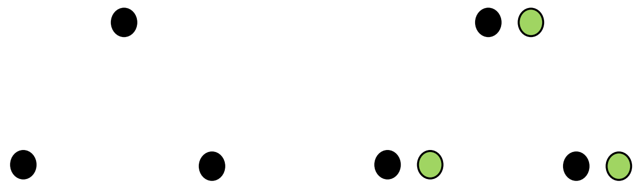
SMP6 – Attend to precision.

The Teaching Cycle:

Launch (Whole Class): Start the discussion with the pattern of growing dots drawn on the board or projected for the entire class. Ask students to describe the pattern that they see in the dots (Question #1). Students may describe an increasing number of triangles being added each time or seeing three groups that each have an increasing number of dots each time, depending on how they see the growth occurring. This will be explored later in the discussion as students write equations, so there should not be any emphasis placed upon a particular way of seeing the growth. Ask students individually to consider and draw the figure that they would see at 5 minutes (Question #2). Then, ask one student to draw it on the board to give other students a chance to check that they are seeing the pattern correctly. Remind students of the work they did yesterday to write explicit and recursive formulas. These are new terms that should be reinforced at the beginning to clarify the instructions for questions 3 and 4.

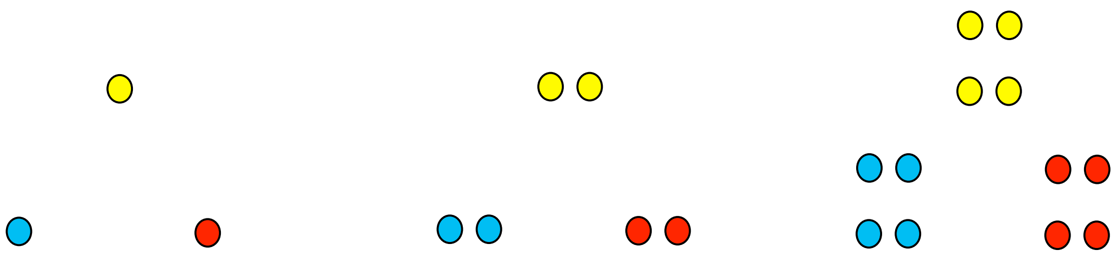
Explore (Small Group or Pairs): Ask students to complete the task. Monitor students as they work, observing their strategies for counting the dots and thinking about the growth of the figures.

Some students may think about the figures recursively, describing the growth by saying that the next figure is obtained doubling the previous figure as shown:



$t = 0$ $t = 1$

Some may think of the figure as three groups that are each doubling, as shown below.



$t = 0$ $t = 1$ $t = 2$

As students work to find the formulas, they may look for patterns in the numbers, writing simply 3, 6, 12, 24, 48. If students are unable to see a pattern, you may encourage them to make a table or graph to connect the number of dots with the time:

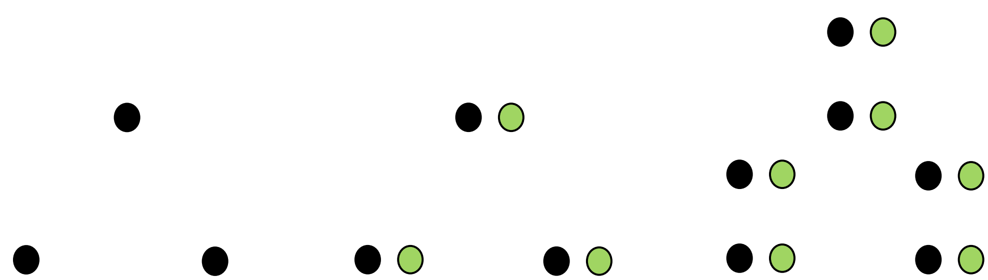
Time (Minutes)	Number of Dots
0	3
1	6
2	12
3	24
4	48

Watch for students that have used a graph to show the number of dots at a given time and to help write an equation. Encourage students to connect their counting strategy to the equation that they write.

For the discussion, select a student for each of the counting strategies shown, a table, a graph, a recursive equation, and at least one form of an explicit equation. Have two large charts showing the dot figures prepared in advance for students to use in explaining their counting strategies.

Discuss (Whole Group):

Begin the discussion with the group that saw the pattern as doubling the previous figure each time. Ask them to explain how they thought about the pattern and how they annotated the figures.



3 6 12

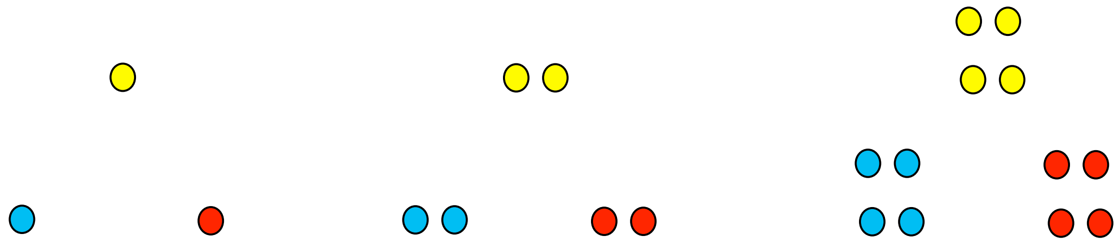
Often, students who are using this strategy will think of the number of dots, without thinking of the relationship between the number of dots and the time. If they don't mention the time at this point, be careful to point out the relationship with time when the next group presents a strategy that connects the time and the number of dots.

Ask students to describe the pattern they see and record their words:

Next figure = 2 × Previous figure

Support students in representing this idea algebraically as: $f(0) = 3, f(t) = 2f(t-1)$ and help them to understand that this formula expresses the idea that a way to find a term at time t is to double the previous term, starting with 3 at time 0.

Next, ask the group that saw this pattern of growth to explain the way they saw the pattern of growth.



$t = 0$ $t = 1$ $t = 2$

Ask for a table that shows the relationship between time and the number of dots. Ask students what patterns they see in the table. Ask students to add a difference column to the table, like they did in Growing Dots. Students may be surprised to see the difference between terms repeating the pattern in the number of dots. Ask students if they see a common difference between terms. Explain that since there is no common difference, it is not an arithmetic sequence.

Time (Minutes)	Number of Dots
0	3
1	6
2	12
3	24
4	48

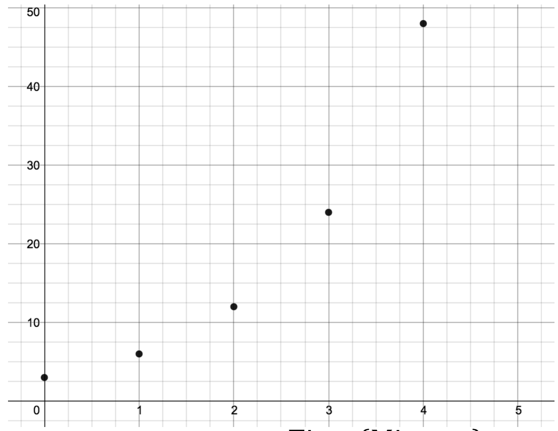
Difference
> 3
> 6
> 12
> 24

At this point, it can be pointed out that since you get the next term by doubling the previous term, there is a common ratio between terms. Demonstrate that:

$$\frac{6}{3} = \frac{12}{6} = \frac{24}{12} = 2$$

The common ratio between terms is the identifying feature of a geometric sequence, another special type of number sequence. Continue the discussion by asking a group to show their graph. Ask the class what they predict the graph to look like. Why would we not expect the graph to be a line? Be sure the graph is properly labeled, as shown.

Number of dots



Time (Minutes)

Now, move the discussion to consider the number of dots at time *t,* as represented by an explicit equation. Ask a group to show their explicit formula for the number of dots at time *t,* which is: $f(t) = 3 \cdot 2^t$.

Now ask students to connect the equations with the table and graphs. Ask them to show what the 2 and the 3 represent in the graph. Ask how they see $3 \cdot 2^t$ in the table. It may be useful to show the connection to the table to help demonstrate the pattern between the time and the number of dots:

Time (Minutes)	Number of Dots	
0	3	3
1	6	3·2
2	12	3·2·2
3	24	3·2·2·2
4	48	3·2·2·2·2
…	…	
t	$3 \cdot 2^t$	

Difference

> 3
> 6
> 12
> 24

You may also remind students that when the table is used to write a recursive equation such as:

$f(0) = 3, f(t) = 2f(t-1)$, one may simply look down the table from one output to the next. When writing an explicit formula such as $f(t) = 3 \cdot 2^t$, it is necessary to look across the rows of the table to connect the input with the output.

Finalize the discussion by explaining that this set of figures, equations, table, and graph represent a geometric sequence. A geometric sequence can be identified by the constant ratio between consecutive terms. Tell students that they will continue to work with sequences of numbers using tables, graphs and equations to identify and represent geometric and arithmetic sequences.

Aligned Ready, Set, Go Homework: Sequences 1.3

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Interpreting function notation

A) Use the given table to identify the indicated value for *n*. B) Then using the value for *n* that you determined in A, use the table to find the indicated value for B.

n	1	2	3	4	5	6	7	8	9	10
$f(n)$	-8	-3	2	7	12	17	22	27	32	37

1. $A)$ When $f(n) = 12$, what is the value of n?

   $B)$ What is the value of $f(n-1)$?

2. $A)$ When $f(n) = 17$, what is the value of n?

   $B)$ What is the value of $f(n-1)$?

3. $A)$ When $f(n) = 32$, what is the value of n?

   $B)$ What is the value of $f(n+1)$?

4. $A)$ When $f(n) = 2$, what is the value of n?

   $B)$ What is the value of $f(n+3)$?

5. $A)$ When $f(n) = 27$, what is the value of n?

   $B)$ What is the value of $f(n-6)$?

6. $A)$ When $f(n) = -8$, what is the value of n?

   $B)$ What is the value of $f(n+9)$?

SET

Topic: Comparing explicit and recursive equations

Use the given information to decide which equation will be the easiest to use to find the indicated value. Find the value and explain your choice.

7. Explicit equation: y = 3x + 7

   Recursive: $now = previous\ term + 3$

term #	1	2	3	4
value	10	13	16	

Find the value of the 4th term. _______________

Explanation:

8. Explicit equation: y = 3x + 7

   Recursive: $now = previous\ term + 3$

term #	1	2	…	50
value	10	13	…	

Find the value of the 50th term. _______________

Explanation:

<table>
<tr>
<td>9. The value of the 8th term is 78.
The sequence is increasing by 10 at each step.

Explicit equation: y = 10x – 2
Recursive: $now = previous\ term + 10$

Find the 20th term. ____________________
Explanation:</td>
<td>10. The value of the 8th term is 78.
The sequence is increasing by 10 at each step.

Explicit equation: y = 10x – 2
Recursive: $now = previous\ term + 10$

Find the 9th term. ____________________
Explanation:</td>
</tr>
<tr>
<td>11. The value of the 4th term is 80.
The sequence is being doubled at each step.

Explicit equation: $y = 5(2^x)$
Recursive: $now = previous\ term\ * 2$

Find the value of the 5th term. ____________
Explanation:</td>
<td>12. The value of the 4th term is 80.
The sequence is being doubled at each step.

Explicit equation: $y = 5(2^x)$
Recursive: : $now = previous\ term\ * 2$

Find the value of the 7th term. ____________
Explanation:</td>
</tr>
</table>

GO

Topic: Evaluating Exponential Equations

Evaluate the following equations when x = { 1, 2, 3, 4, 5 }. Organize your inputs and outputs into a table of values for each equation. Let x be the input and y be the output.

13. $y = 4^x$

x input	*y* output
1	
2	
3	
4	
5	

14 $y = (-3)^x$

x input	*y* output
1	
2	
3	
4	
5	

15. $y = -3^x$

x input	*y* output
1	
2	
3	
4	
5	

16. $y = 10^x$

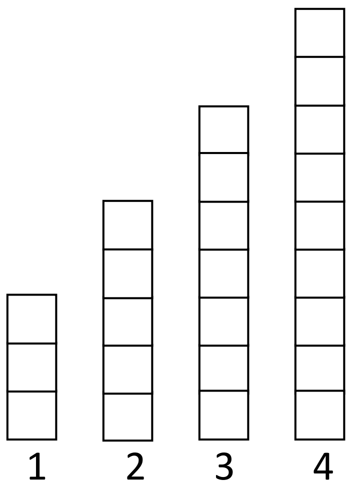
x input	*y* output
1	
2	
3	
4	
5	

17. If $f(n) = 5^n$, *what is the value of* $f(4)$?

1.4 Scott's Workout

A Solidify Understanding Task

Scott has decided to add push-ups to his daily exercise routine. He is keeping track of the number of push-ups he completes each day in the bar graph below, with day one showing he completed three push-ups. After four days, Scott is certain he can continue this pattern of increasing the number of push-ups he completes each day.



1. How many push-ups will Scott do on day 10?

2. How many push-ups will Scott do on day *n*?

3. Model the number of push-ups Scott will complete on any given day. Include both explicit and recursive equations.

4. Aly is also including push-ups in her workout and says she does more push-ups than Scott because she does fifteen push-ups every day. Is she correct? Explain.

1.4 Scott's Push-Ups – Teacher Notes

A Solidify Understanding Task

Purpose:

This task is to solidify understanding that arithmetic sequences have a constant difference between consecutive terms. The task is designed to generate tables, graphs, and both recursive and explicit formulas. The focus of the task should be to identify how the constant difference shows up in each of the representations and defines the functions as an arithmetic sequence.

Standards Focus:

F-BF: Build a function that models a relationship between two quantities.

1: Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus on linear and exponential only)

Construct and compare linear, quadratic and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

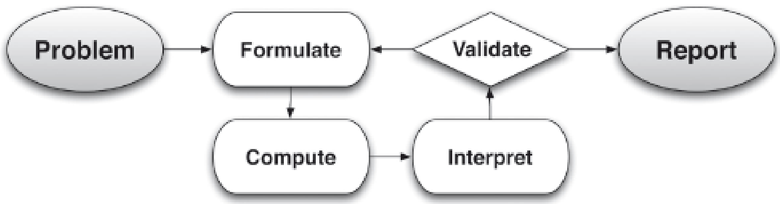
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:



Standards for Mathematical Practice of Focus in the Task:

SMP2 - Reason abstractly and quantitatively.

SMP7 – Look for and make use of structure.

The Teaching Cycle:

Launch (Whole Class): Remind students of the work they have done previously with Growing Dots and Growing, Growing Dots. Read Scott's Workout with the students and ask a student to add the number of push-ups that Scott will do on the fifth day to the diagram for the class. Ask students what they are observing about the pattern. Allow just a few responses so that you know that students understand the task, but avoid giving away the work of the task.

Explore (Small Group): Monitor student thinking as they work by moving from one group to another. Encourage students to use tables, graphs, and recursive and explicit equations as they work on the task. Listen to students and identify different groups to present and explain their work on one representation each. If students are having difficulty writing the equation, ask them to be sure that they have the other representations first.

Discuss (Whole Class): When the various groups are prepared to present, start the discussion with a table. Be sure that the columns of the table are labeled. After students have presented their table, ask students to identify the difference between consecutive terms and mark the table so that it looks like this:

n Days	$f(n)$ Push-ups
1	3
2	5
3	7
4	9
5	11
…	…
n	$3+2(n-1)$

Difference between terms

> 2

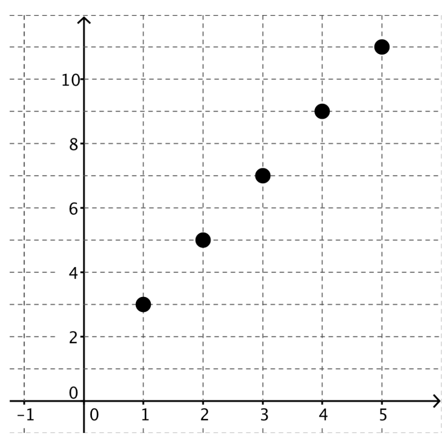
> 2

> 2

Ask if the sequence is arithmetic or geometric based upon the table. Students should be able to identify that it is arithmetic because there is a constant difference between consecutive terms.

Next, ask the students to present the graph. The graph should be labeled and look like this:

Number of Push-ups



Number of Days

Ask students where they see the difference between terms from the table on the graph. Identify that for each day, the number of push-ups increases by 2, so for each increase of 1 in the x value, the y value increases by 2. The students should recognize this as a slope of 2. Ask why the points in the graph are not connected. Students should be able to answer that the push-ups are assumed to be done all at once in the day. A continuous graph would suggest that the push-ups were happening for the entire time shown on the graph.

Next, ask students for their recursive equations. Students may have written any of these equations:

Number of push-ups today = Number of push-ups yesterday + 2

Or:

Next term = Previous term + 2

Ask how they see this equation in their table and their graph. On the table, they should point out that as you move from one row to the next, you add 2 to the previous term. They should be able to demonstrate a similar idea on the graph as you move from one y-value to the next.

Ask if anyone has written a recursive equation in function form. If no one has written the equation in function form, explain that the more formal method of writing the equation is: $f(1) = 3, f(n) = f(n-1) + 2$. This form still denotes the idea that the current term is 2 more than the previous term. Ask students how they can identify that this is an arithmetic sequence using the recursive equation. The answer should be that the constant difference of 2 between terms shows up in the equation as adding 2 to get the next term. Also note that to use a recursive formula you have to know the previous term. That means that when you make a recursive formula for an arithmetic sequence, you need to provide the first term as part of the formula.

Conclude the discussion with the explicit equation, $f(n) = 3 + 2(n-1)$. Although this equation could be simplified, it is useful to consider it in this form. Ask students how they used the table to write this equation. How does this formula show the constant difference between terms? Also ask, "If you are looking for the 10th term, what number will you multiply by 2?" Help students to connect that the $(n-1)$ in the formula tells them that the number they multiply by 2 is one less than the term they are looking for. Can they explain that using the table or graph?

Conclude the lesson by asking students to compare recursive and explicit formulas. What information do you need to use either type of formula? What are the advantages of each? What ideas about arithmetic sequences are highlighted in each?

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Use function notation to evaluate equations.

Evaluate the given equation for the indicated function values.

1. $f(n) = 5n + 8$
   $f(4) =$
   $f(-2) =$

2. $f(n) = -2n + 1$
   $f(10) =$
   $f(-1) =$

3. $f(n) = 6n - 3$
   $f(-5) =$
   $f(0) =$

4. $f(n) = -n$
   $f(9) =$
   $f(-11) =$

5. $f(n) = 5^n$
   $f(2) =$
   $f(3) =$

6. $f(n) = 3^n$
   $f(4) =$
   $f(1) =$

7. $f(n) = 10^n$
   $f(6) =$
   $f(0) =$

8. $f(n) = 2^n$
   $f(0) =$
   $f(5) =$

SET

Topic: Finding terms for a given sequence

Find the next 3 terms in each sequence. Identify the constant difference. Write a recursive function and an explicit function for each sequence. Circle where you see the common difference in both functions. (The first number is the 1st term, not the 0th term).

9. A) 3 , 8 , 13 , 18 , 23 , _____ , _____ , _____ , … B) Common Difference: __________
   C) Recursive Function: ______________________ D) Explicit Function: ________________________

10. A) 11 , 9 , 7 , 5 , 3 , _____ , _____ , _____ , … B) Common Difference: __________
    C) Recursive Function: ______________________ D) Explicit Function: ________________________

11. A) 3 , 1.5 , 0 , -1.5 , -3 , _____ , _____ , _____ , … B) Common Difference: __________
    C) Recursive Function: ______________________ D) Explicit Function: ________________________

GO

Topic: Reading a graph

Olaf is a mountain climber. The graph shows Olaf's location on the mountain beginning at noon. Use the information in the graph to answer the following questions.

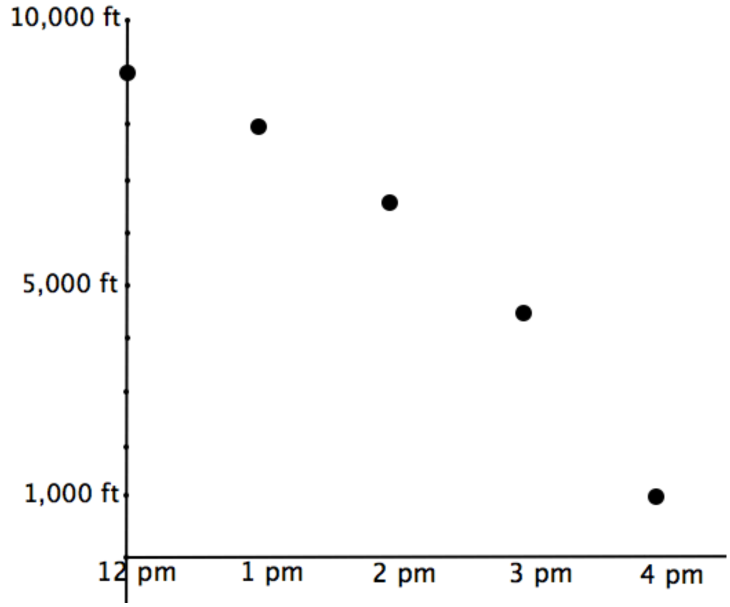
12. What was Olaf's elevation at noon?

13. What was his elevation at 2 pm?

14. How many feet had Olaf descended from noon until 2 pm?

15. Olaf reached the base camp at 4 pm. What is the elevation of the base camp?

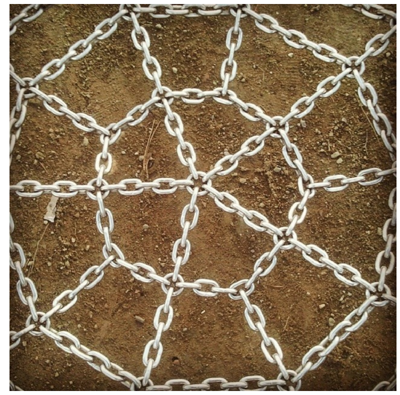
16. During which hour was Olaf descending the mountain the fastest? Explain how you know.



17. Is the value of $f(n)$ the time or the elevation?

1.5 Don't Break the Chain

A Solidify Understanding Task



CC BY Thomas Kohler
https://flic.kr/p/iV6hUJ

Maybe you've received an email like this before:

> Hi! My name is Bill Weights, founder of Super Scooper Ice Cream. I am offering you a gift certificate for our signature "Super Bowl" (a $4.95 value) if you forward this letter to 10 people.
>
> When you have finished sending this letter to 10 people, a screen will come up. It will be your Super Bowl gift certificate. Print that screen out and bring it to your local Super Scooper Ice Cream store. The server will bring you the most wonderful ice cream creation in the world—a Super Bowl with three yummy ice cream flavors and three toppings!
>
> This is a sales promotion to get our name out to young people around the country. We believe this project can be a success, but only with your help. Thank you for your support.
>
> Sincerely,
>
> Bill Weights
> Founder of Super Scooper Ice Cream

These chain emails rely on each person that receives the email to forward it on. Have you ever wondered how many people might receive the email if the chain remains unbroken? To figure this out, assume that it takes a day for the email to be opened, forwarded, and then received by the next person. On day 1, Bill Weights starts by sending the email out to his 8 closest friends. They each forward it to 10 people so that on day 2 it is received by 80 people. The chain continues unbroken.

1. How many people will receive the email on day 7?

2. How many people with receive the email on day *n*? Explain your answer with as many representations as possible.

3. If Bill gives away a Super Bowl that costs $4.95 to every person that receives the email during the first week, how much will he have spent?

1.5 Don't Break the Chain– Teacher Notes

A Solidify Understanding Task

Purpose:

This task is to solidify understanding that geometric sequences have a constant ratio between consecutive terms. The task is designed to generate tables, graphs, and both recursive and explicit formulas. The focus of the task should be to identify how the constant ratio shows up in each of the representations.

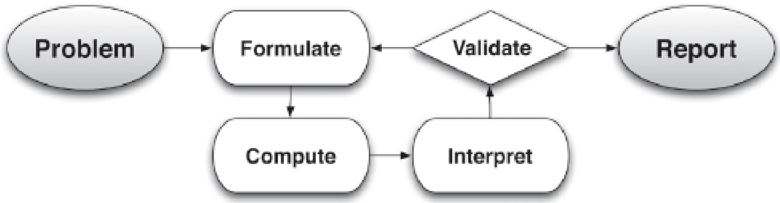
Core Standards:
F-BF: Build a function that models a relationship between two quantities.
1: Write a function that describes a relationship between two quantities.*
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus on linear and exponential only)
Construct and compare linear, quadratic and exponential models and solve problems.
1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
c. Recognize situations in which one quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:



Standards for Mathematical Practice of Focus in the Task:

SMP 8 – Look for and express regularity in repeated reasoning.

SMP 5 – Use appropriate tools strategically.

The Teaching Cycle:

Launch (Whole Class): Remind students of the work they have done previously with Growing Dots and Growing, Growing Dots. Without handing the task to students, read "Don't Break the Chain" past the end of the letter to the point where it says that Bill starts by sending the letter to 8 people. Ask students how many people will receive the email the next day if the chain is unbroken. Ask students what they are observing about the pattern. You may wish to draw a diagram that shows the first 8 and then each of them sending the letter to 10 more so that students see that this is not a situation where they are simply adding 10 each time. Be sure to label the diagram to show that there were 8 people on day 1 and 80 on day 2 to avoid confusion about the time. Have students work on the task in small groups (2-4 students per group). Access to graphing calculators will facilitate graphing the function.

Explore (Small Group): Monitor student thinking as they work by moving from one group to another. Encourage students to use tables, graphs, and recursive and explicit equations as they work on the task. Listen to students and identify different groups to present and explain their work on one representation each. If students are having difficulty writing the equations, ask them to be sure that they have the other representations first. The focus of the task is questions 1 and 2. The third question requires students to sum the first 7 terms of the sequence. While it gives an interesting result, it is an extension for those who finish quickly. When the class is finished with the first two questions, call the class back for a whole group discussion.

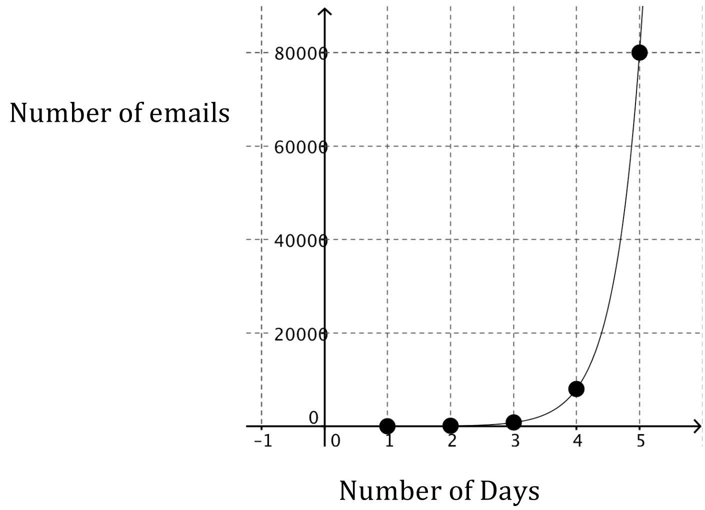
Discuss (Whole Class): Start the discussion with a table. Be sure that the columns of the table are labeled. After students have presented their table, ask students to identify the difference between consecutive terms and mark the table so that it looks like this:

n Days	$f(n)$ # of emails
1	8
2	80
3	800
4	8000
5	80,000
…	…

Difference
> 72
> 720
> 7,200

Ask if the sequence is arithmetic, based upon the table. Students should be able to identify that it is not arithmetic because there is no constant difference between consecutive terms. Ask, "What other patterns do you see in the table?" Students should notice that the next term is obtained by multiplying by 10. Ask students to test if there is a constant ratio between consecutive terms. They should recognize the ratio of 10. Confirm that the constant ratio between consecutive terms means that this is a geometric sequence.

Next, ask the students to present the graph. The graph should be labeled and look like the graph below, but with only the points in the table. (The curve has been shown here as continuous only to see an accurate placement of the points.):



Ask students what was difficult about creating the graph. They will probably point out that the y-values were increasing so quickly that it was difficult to scale the graph. Ask if the points on the graph make a line. Help them to see that the points make a curve; the graph is linear only when there is a constant difference between terms. Identify that for each day, the number of emails multiplies by 10, so for each increase of 1 in the x value, the y value is 10 times bigger. Ask students which graph this most resembles: Growing Dots or Growing, Growing Dots. Help them to identify the characteristic shape of an increasing exponential function.

Next, ask students for their recursive equations. Students may have written any of these equations:

Number of emails today = 10 * Number of emails yesterday

Or: Next term = 10*Previous term

Ask how they see this equation in their table and their graph. On the table, they should point out that as you move from one row to the next, you multiply the previous term by 10. They should be able to demonstrate a similar idea on the graph as you move from one y-value to the next.

Ask if anyone has written their recursive equation in function form. If no one has written the equation in function form, explain that the more formal method of writing the equation is: $f(1) = 8, f(n) = 10f(n-1)$. Ask students how this formula is different than the recursive formula for Scott's workout, an arithmetic sequence. Ask students how they can identify that this is a geometric sequence using the recursive equation. The answer should be that the constant ratio of 10 between terms shows up in the equation as multiplying by 10 to get the next term. Also note that to use a recursive formula you have to know the previous term. That means that when you have a recursive formula for an arithmetic sequence, you need to provide the first term as part of the formula.

Conclude the discussion with the explicit formula, $f(n) = 8(10^{n-1})$. Many students may find it difficult to write the pattern that they see in this form. First, it may be hard to see that they need to use an exponent, and then it may be hard to see how the exponent relates to the term number. It may be helpful to refer back to the table, adding the third column shown here.

n Days	$f(n)$ Number of emails	$f(n)$ Number of emails	$f(n)$ Number of emails
1	8	8	$8(10^0)$
2	80	$8 \cdot 10$	$8(10^1)$
3	800	$8 \cdot 10 \cdot 10$	$8(10^2)$
4	8,000	$8 \cdot 10 \cdot 10 \cdot 10$	$8(10^3)$
5	80,000	$8 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	$8(10^4)$
…	…	…	
n			$8(10^{n-1})$

Ask students to notice how many times 10 is used as a power and compare it to the number of days. Remind them that to write an explicit formula they need to read across the table and relate the *n* value to *f(n).* You may wish to ask students to help you complete a fourth column that shows the powers of 10. Help students to connect that the $(n - 1)$ in the formula tells them the number of times they multiply by 10 is one less than the term they are looking for. Ask students to relate the formula to graph.

Conclude the lesson by asking students to compare the recursive and explicit formulas for arithmetic and geometric sequences. How are the recursive formulas for geometric and arithmetic sequences alike? How are they different? How are the explicit formulas for geometric and arithmetic sequence alike? How are they different?

Aligned Ready, Set, Go Homework: Sequences 1.5

READY, SET, GO! **Name** **Period** **Date**

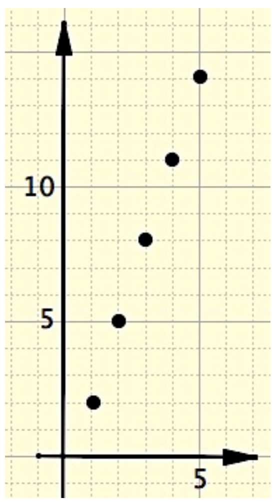
READY

Topic: Rates of change in a table and a graph

The same sequence is shown in both a table and a graph. Indicate on the table where you see the rate of change of the sequence. Then draw on the graph where you see the rate of change.

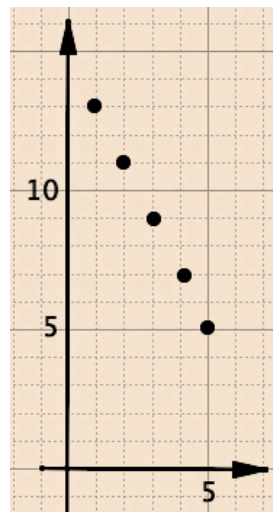
1.

n	$f(n)$
1	2
2	5
3	8
4	11
5	14



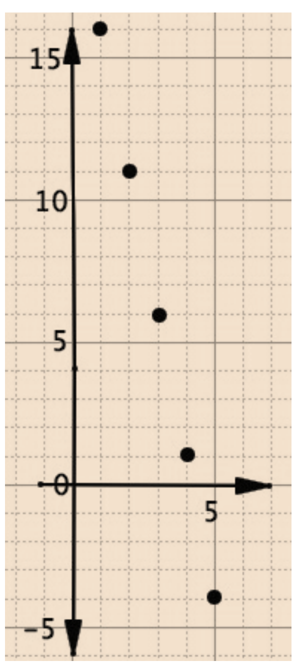
2.

n	$f(n)$
1	13
2	11
3	9
4	7
5	5



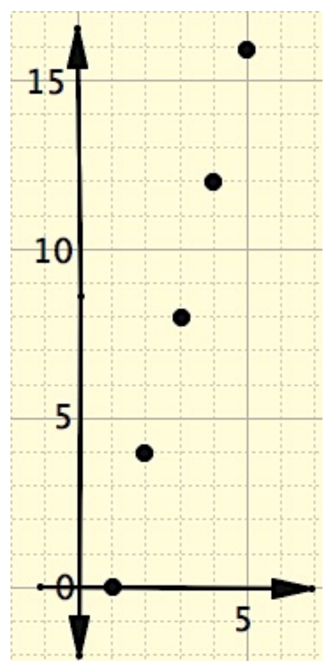
3.

n	$f(n)$
1	16
2	11
3	6
4	1
5	-4



4.

n	$f(n)$
1	0
2	4
3	8
4	12
5	16

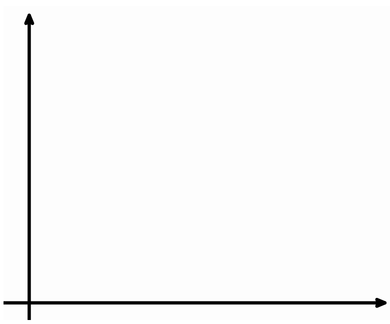


SET

Topic: Recursive and explicit functions of **geometric** sequences

Below you are given various types of information. Write the recursive and explicit functions for each geometric sequence. Finally, graph each sequence, making sure you clearly label your axes.

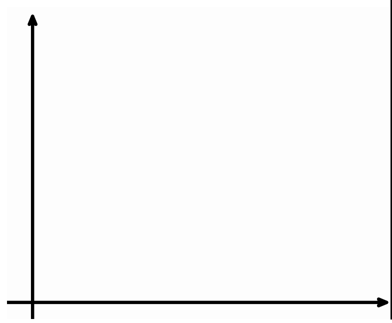
5. 2 , 4 , 8 , 16 , …



Recursive:________________________________

Explicit:__________________________________

6.

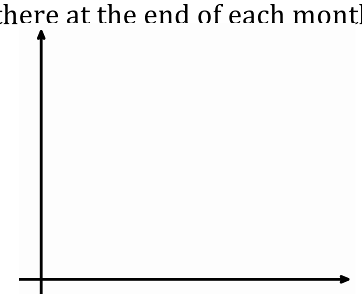


Time (days)	Number of cells
1	3
2	6
3	12
4	24

Recursive:________________________________

Explicit:__________________________________

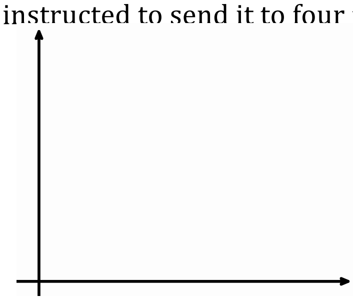
7. Claire has $300 in an account. She decides she is going to take out half of what's left in there at the end of each month.



Recursive:________________________________

Explicit:__________________________________

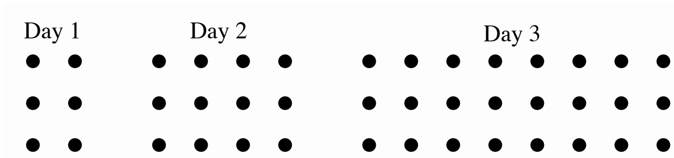
8. Tania creates a chain letter and sends it to four friends. Each day each friend is then instructed to send it to four friends and so forth.



Recursive:________________________________

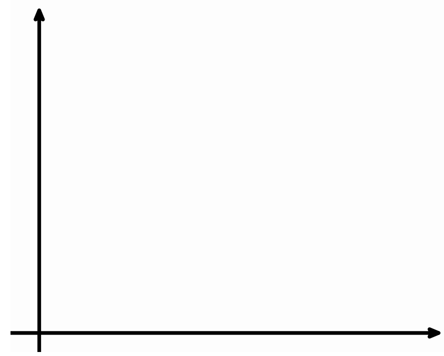
Explicit:__________________________________

9.



Recursive:______________________________

Explicit:_________________________________

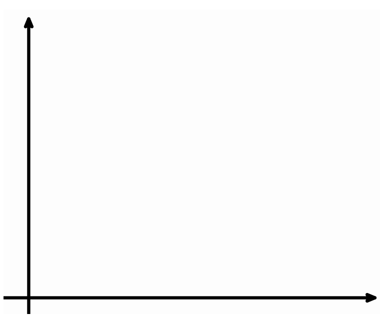


GO

Topic: Recursive and explicit functions of **arithmetic** sequences

Below you are given various types of information. Write the recursive and explicit functions for each arithmetic sequence. Finally, graph each sequence, making sure you clearly label your axes.

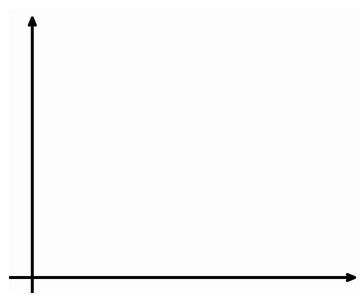
10. 2 , 4 , 6 , 8 , …



Recursive:______________________________

Explicit:_________________________________

11.



Time (days)	Number of cells
1	3
2	6
3	9
4	12

Recursive:______________________________

Explicit:_________________________________

12. Claire has $300 in an account. She decides she is going to take out $25 each month.

Recursive:______________________________

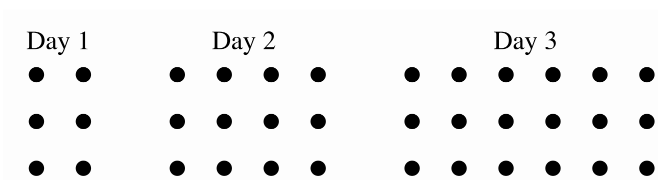
Explicit:________________________________

13. Each day Tania decides to do something nice for 2 strangers. What is the relationship between the number people helped and days?

Recursive:______________________________

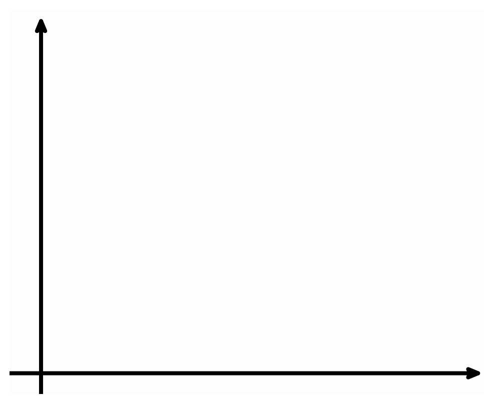
Explicit:________________________________

14.



Recursive:______________________________

Explicit:________________________________



1.6 Something to Chew On

A Solidify Understanding Task

The Food-Mart grocery store has a candy machine like the one pictured here. Each time a child inserts a quarter, 7 candies come out of the machine. The machine holds 15 pounds of candy. Each pound of candy contains about 180 individual candies.

1. Represent the number of candies in the machine for any given number of customers. About how many customers will there be before the machine is empty?

2. Represent the amount of money in the machine for any given number of customers.

3. To avoid theft, the store owners don't want to let too much money collect in the machine, so they take all the money out when they think the machine has about $25 in it. The tricky part is that the store owners can't tell how much money is actually in the machine without opening it up, so they choose when to remove the money by judging how many candies are left in the machine. About how full should the machine look when they take the money out? How do you know?

1.6 Something to Chew On – Teacher Notes

A Solidify Understanding Task

Purpose:

This task introduces a decreasing arithmetic sequence to further solidify the idea that arithmetic sequences have a constant difference between consecutive terms. Again, connections should be made among all representations: table, graph, recursive and explicit formulas. The emphasis should be on comparing increasing and decreasing arithmetic sequences through the various representations.

Core Standards:

F-BF: Build a function that models a relationship between two quantities.

1: Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus in linear and exponential only)
Construct and compare linear, quadratic and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

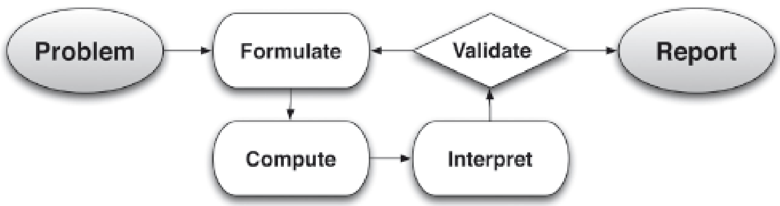
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:



Standards for Mathematical Practice of Focus in the Task:

SMP4 - Model with mathematics.

SMP2 – Reason abstractly and quantitatively.

The Teaching Cycle:

Launch (Whole class): Before handing out the task, ask students to define an arithmetic sequence. Later we will say that it is a linear function with the domain of positive integers. Right now, expect students to identify the constant rate of change or constant difference between consecutive terms. Ask students to give a few examples of arithmetic sequences. Since the only sequences they have seen up to this point have been increasing, expect them to add a number to get to the next term. Then, wonder out loud whether or not it would be an arithmetic sequence if a number is subtracted to get the next term. Don't answer the question or solicit responses.

Read the opening paragraph of the task and be sure that all students understand how the candy machines work; when a quarter is inserted, 7 candies come out. Read the first prompt in the task and discuss what it means to "Represent the number of candies in the machine for a given number of customers." Explain that their representations should include tables, graphs, and equations. As soon as students understand the task, set them to work.

Explore (Small groups): To begin the task, students will need to decide how to set up their tables or graphs. Tables should be set with customers as the independent variable and the number of candies as the dependent variable. One way to represent candies versus customers is to calculate how many candies in the machine when it is full and then build tables and graphs by subtracting 7 for each customer. It would be appropriate to have graphing calculators available for this task.

The second prompt (#2) is similar to other increasing geometric sequences that students have previously modeled in Scott's Workout and Growing Dots. Again, encourage as many representations as possible. When you find that most students are finished with number 1 and 2 it probably a good time to start the discussion. Number 3 is an extension provided for differentiation, but not the focus of the task for most students.

Monitor the group work with particular focus on the work in #1. Have one group prepared to present the table and one group present the graph. Another group can present both forms of the equations from #1. You may choose to have the presenters begin to draw their tables and graphs while the other groups finish their work. Also select just one group to do all of problem #2 on the board for comparison.

Discuss (Whole Group): Start the discussion by repeating the question that was stated in the launch: Can you form an arithmetic sequence by subtracting a number from each term to get the next term? Ask the group to present the table that they made for #1, which should look something like this, although students may not have included the first difference at the side. If not, add it in as part of the discussion.

# of Customers	# of Candies
0	2700
1	2693
2	2686
3	2679
4	2672
5	2665
6	2658
7	2651
8	2644
9	2637
10	2632
….	….
n	$2700 - 7n$

Difference

> -7

> -7

> -7

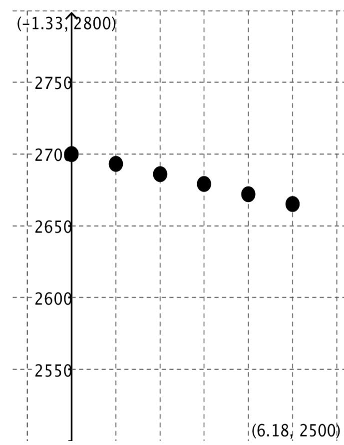
> -7

> -7

Ask students what they notice about the table. Does the table show a constant difference between terms? What is the constant difference? Does this table represent an arithmetic sequence?

Next, discuss the graph, which should be properly labeled and look something like this:

Number of candies



Number of customers

The constant difference between terms has been demonstrated in the table. Now ask students how this graph of an arithmetic sequence is like the other arithmetic sequences that we have studied in Scott's Workout and Growing Dots. They should identify that the points form a line and the graph is not continuous. (There is no need to emphasize the discrete nature of the sequence since it is a focus in the next module.) How is this graph different? It is decreasing at a constant rate, rather than increasing at a constant rate.

Ask students to compare the recursive formulas for both #1 and #2. Encourage them to use function notation only, so that their formulas look like:

1. $f(0) = 2700, f(n) = f(n-1) - 7$ 2. $f(1) = .25, f(n) = f(n-1) + .25$

Ask students, "Based on the recursive formula, is #2 an arithmetic sequence? Why or why not?" Expect students to answer that +.25 shows that each term is increasing by a constant amount.

Now ask students to compare the explicit formulas for both #1 and #2. In function notation, they should be like:

1. $f(n) = 2700 - 7n$ 2. $f(n) = .25n$

Ask students how they can identify an arithmetic sequence from an explicit equation. Re-emphasize the definition of an arithmetic sequence as a sequence that has a common difference between consecutive terms. The common difference can be either positive or negative. You may wish to end the discussion by working with students to complete the chart given in the Intervention Activity.

Aligned Ready, Set, Go Homework: Sequences 1.6

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Finding the common difference

Find the missing terms for each arithmetic sequence and state the common difference.

1. 5, 11, ______ , 23, 29, ______...

Common Difference = ________

2. 7, 3, -1, ______ , ______ , -13...

Common Difference = ________

3. 8, ______ , ______ , 47, 60...

Common Difference = ________

4. 0, ______ , ______ , 2, $\frac{8}{3}$ ...

Common Difference = ________

5. 5, ______ , ______ , ______ , 25...

Common Difference = ________

6. 3, ______ , ______ , ______ , -13 ...

Common Difference = ________

SET

Topic: Writing the recursive function

Two consecutive terms in an arithmetic sequence are given. Find the recursive function.

7. If $f(3) = 5 \text{ and } f(4) = 8$ ...

$f(5)$ = ______. $f(6)$ = ______. Recursive Function: ________________________________

8. If $f(2) = 20 \text{ and } f(3) = 12$ ...

$f(4)$ = ______. $f(5)$ = ______. Recursive Function: ________________________________

9. If $f(5) = 3.7 \text{ and } f(6) = 8.7$ ...

$f(7)$ = ______. $f(8)$ = ______. Recursive Function: ________________________________

Two consecutive terms in a <u>geometric</u> sequence are given. Find the recursive function.

10. If $f(3) = 5\ and\ f(4) = 10$ ...

$f(5)$ = ______. $f(6)$ = ______. Recursive Function: ____

11. If $f(2) = 20\ and\ f(3) = 10$ ...

$f(4)$ = ______. $f(5)$ = ______. Recursive Function: ____

12. If $f(5) = 20.58\ and\ f(6) = 2.94$ ...

$f(7)$ = ______. $f(8)$ = ______. Recursive Function: ____

GO

Topic: Evaluating using function notation

Find the indicated values of $f(n)$.

13. $f(n) = 2^n$ Find $f(5)\ and\ f(0)$.

14. $f(n) = 5^n$ Find $f(4)\ and\ f(1)$.

15. $f(n) = (-2)^n$ Find $f(3)\ and\ f(0)$.

16. $f(n) = -2^n$ Find $f(3)\ and\ f(0)$.

17. In what way are the problems in #15 and #16 different?

18. $f(n) = 3 + 4(n - 1)$ Find $f(5)$ and $f(0)$.

19. $f(n) = 2(n - 1) + 6$ Find $f(1)$ and $f(6)$.

1.7 Chew On This

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A Solidify Understanding Task

Mr. and Mrs. Gloop want their son, Augustus, to do his homework every day. Augustus loves to eat candy, so his parents have decided to motivate him to do his homework by giving him candies for each day that the homework is complete. Mr. Gloop says that on the first day that Augustus turns in his homework, he will give him 10 candies. On the second day he promises to give 20 candies, on the third day he will give 30 candies, and so on.

1. Write both a recursive and an explicit formula that shows the number of candies that Augustus earns on any given day with his father's plan.

2. Use a formula to find how many candies Augustus will get on day 30 in this plan.

Augustus looks in the mirror and decides that he is gaining weight. He is afraid that all that candy will just make it worse, so he tells his parents that it would be ok if they just give him 1 candy on the first day, 2 on the second day, continuing to double the amount each day as he completes his homework. Mr. and Mrs. Gloop like Augustus' plan and agree to it.

3. Model the amount of candy that Augustus would get each day he reaches his goals with the new plan.

4. Use your model to predict the number of candies that Augustus would earn on the 30^{th} day with this plan.

5. Write both a recursive and an explicit formula that shows the number of candies that Augustus earns on any given day with this plan.

Augustus is generally selfish and somewhat unpopular at school. He decides that he could improve his image by sharing his candy with everyone at school. When he has a pile of 100,000 candies, he generously plans to give away 60% of the candies that are in the pile each day. Although Augustus may be earning more candies for doing his homework, he is only giving away candies from the pile that started with 100,000. (He's not that generous.)

6. How many pieces of candy will be left on day 4? On day 8?

7. Model the amount of candy that would be left in the pile each day.

8. How many days will it take for the candy to be gone?

1.7 Chew On This – Teacher Notes

A Solidify Understanding Task

Purpose:

The purpose of this task is to solidify and extend the idea that geometric sequences have a constant ratio between consecutive terms to include sequences that are decreasing ($0 < r < 1$). The common ratio in one geometric sequence is a whole number and in the other sequence it is a percent. This task contains an opportunity to compare the growth of arithmetic and geometric sequences. This task also provides practice in writing and using formulas for arithmetic sequences.

Core Standards:

F.BF.1 Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

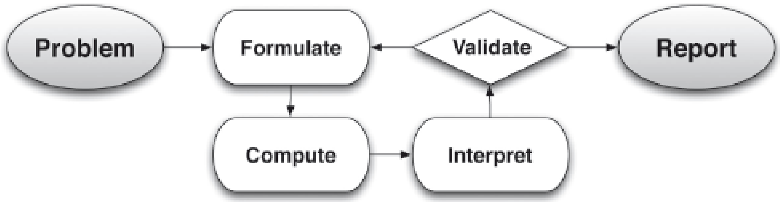
a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

c. Recognize situations in which one quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.IF.5 Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:



Standards for Mathematical Practice of Focus in the Task:

SMP7 – Look for and make use of structure.

SMP6 – Attend to precision.

The Teaching Cycle:

Launch (Whole Group):

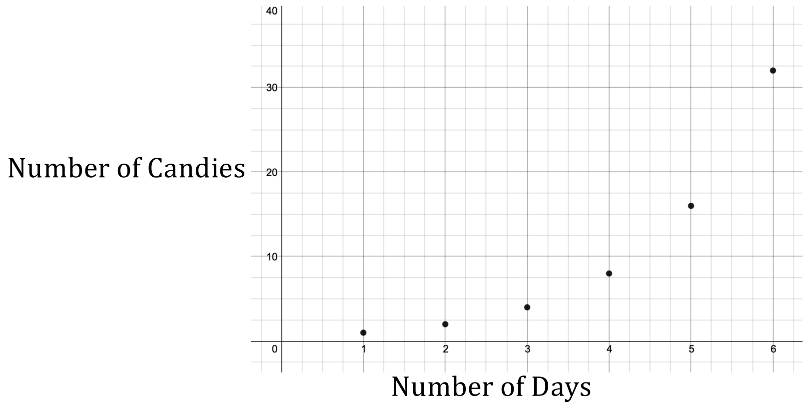
Begin by reading the first part of the task, leading to questions 1 and 2. Be sure that students understand the problem situation and then ask what type of sequence would model this situation and why? Students should be able to identify that this is an arithmetic sequence because the number of candies increases by 10 each day that Augustus meets his goals. Ask students to work on #1 individually and then call on students to share their responses. The formulas should be:

$f(n) = 10n$ and $f(1) = 10, f(n) = f(n-1) + 10$

Read #2 together and ask students which formula will be easier to use to figure out how many candies there will be on day 30. Confirm that the explicit formula will be better for this purpose since we don't know the 29th term so that it can be used in the recursive formula.

Read the information provided to answer questions 3, 4, and 5. After checking to see that students understand the problem situation, ask them to answer #3 and #4 individually. This will be a good assessment opportunity to see what students understand about their previous work, since this is very similar to the geometric sequences that they have already modeled. After they have had some time, they should be able to produce properly labeled tables, graphs and equations like:

# of Days	# of Candies
1	1
2	2
3	4
4	8
5	16
…	…
n	2^n



Common ratio, $r = 2$

Explicit formula: $f(n) = 2^{n-1}$

Recursive formula: $f(1) = 1,\ f(n) = 2f(n-1)$

When students have shared their work, ask them to predict which plan will give Augustus more candies on day 30 and why? Then have student use a formula to find the number of candies on day 30 and compare it to what they found in #2. They will probably be surprised to find that in the second plan, Augustus will have 536,870,912. Ask students, "Why the huge difference in results? What is it about a geometric sequence like this that makes it grow so quickly?"

Explore (Small Group):

Before allowing students to work in their groups on the last part of the problem, check to be sure they understand the problem situation. The pile of candy starts with 100,000. When 60% of the candy is removed on day 1 there are 40,000 candies. Students need to be clear that they are keeping track of the number of candies that remain in the pile, not the number of candies that are removed. There are several different strategies for figuring this out, but that should be part of the conversation in the small groups. As you monitor student work at the beginning of the task, make sure that they are finding some way to calculate the number of candies left in the pile. Let students work in their groups to decide how to organize and analyze this information. Students have experience in creating tables and graphs in these types of situations, although you should check to be sure that they have scaled their graphs appropriately. Watch for groups that recognize are able to write formulas and recognize that this is a geometric sequence.

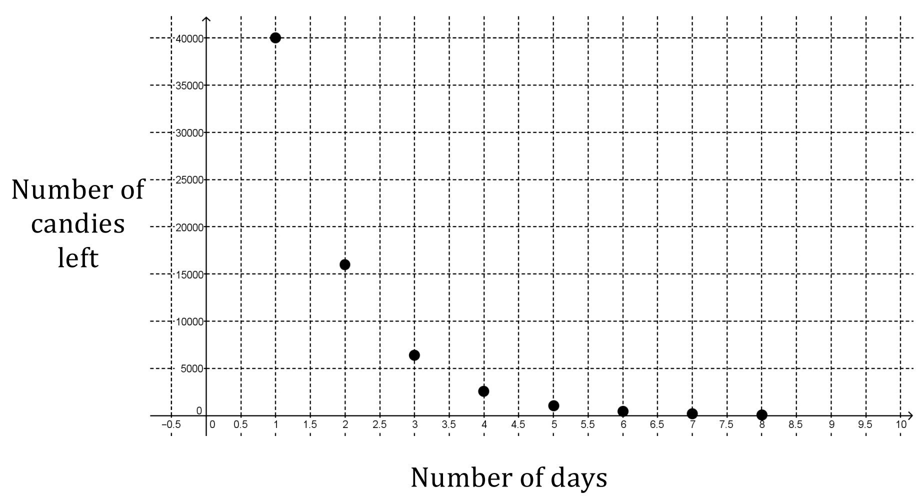
Discuss (Whole Class):

Start the discussion with students presenting a table. Ask students if they check for a common difference or common ratio between terms. Note that the common ratio to the side of the table. Ask students what kind of sequence this must be, given a common ratio between terms. Ask, "How is this geometric sequence different than others we have seen?"

The following table and graph show some of the models that students should produce.

Number of Days	Number of Candies Left
1	40000
2	16000
3	6400
4	2560
5	1024
6	410
7	164
8	66

Common ratio, r =0.4



Continue the discussion with the graph of the sequence.

How is the graph different than other geometric sequences we have seen? What is it about the sequence that causes the behavior of the graph?

Next, ask for the recursive formula. If students have identified the common ratio, they should be able to write: $f(1) = 40{,}000$, $f(n) = .4f(n-1)$. Students that calculated each term of the sequence by subtracting 60% of the previous term may have struggled with this formula. Emphasize the usefulness of the table in finding the common ratio for writing formulas.

The explicit formula, $f(n) = 100{,}000(0.4^n)$ will probably be difficult for students to produce. Help them to recall that in past geometric sequences that the number in the formula that is raised to a power is the common ratio. Then they need to think about the exponent—is it n or (n-1)? Discuss ways that they can test the exponent. Finally, they need to consider the multiplier, in this case 100,000. Where does it come from in the problem, both in this case and in previous geometric sequence problems like Growing, Growing Dots? Help students to generalize to make their work in writing explicit formulas easier.

You may choose to wrap up the discussion by completing the intervention activity with the class.

Aligned Ready, Set, Go Homework: Sequences 1.7

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Distinguishing between arithmetic and geometric sequences

Find the missing values for each arithmetic or geometric sequence. Underline whether it has a constant difference or a constant ratio. State the value of the constant difference or ratio. Indicate if the sequence is arithmetic or geometric by circling the correct answer.

1. 5, 10, 15, ____ , 25, 30, ____...

Common difference or ratio?

Common Difference/ratio = ________

Arithmetic or geometric?

2. 20, 10, ____ , 2.5, ____,...

Common difference or ratio?

Common Difference/ratio = ________

Arithmetic or geometric?

3. 2, 5, 8, ____ , 14, ____, ...

Common difference or ratio?

Common Difference/ratio = ________

Arithmetic or geometric?

4. 30, 24 , ____ , 12, 6, ...

Common difference or ratio?

Common Difference/ratio = ________

Arithmetic or geometric?

SET

Topic: Recursive and explicit equations

Determine whether the given information represents an arithmetic or geometric sequence. Then write the recursive and the explicit equation for each.

5. 2, 4, 6, 8, ...

Arithmetic or geometric?

Recursive:
____

Explicit:
____

6. 2, 4, 8, 16, ...

Arithmetic or geometric?

Recursive:
____

Explicit:
____

7.

Time (in days)	Number of dots
1	3
2	7
3	11
4	15

Arithmetic or geometric?

Recursive:

Explicit:

8.

Time (in days)	Number of cells
1	5
2	8
3	12.8
4	20.48

Arithmetic or geometric?

Recursive:

Explicit:

9. Michelle likes chocolate but it causes acne. She chooses to limit herself to three chocolate bars every 5 days. (So, she eats part of a bar each day.)

Arithmetic or geometric?

Recursive:

Explicit:

10. Scott decides to add running to his exercise routine and runs a total of one mile his first week. He plans to double the number of miles he runs each week.

Arithmetic or geometric?

Recursive:

Explicit:

11. Vanessa has $60 to spend on rides at the state fair. Each ride costs $4.

Arithmetic or geometric?

Recursive:

Explicit:

12. Cami invested $6,000 into an account that earns 10% interest each year. (Hint: Make a table of values to help yourself.)
Arithmetic or geometric?

Recursive:

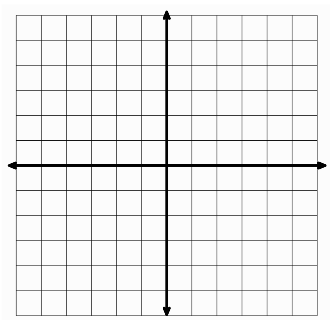
Explicit:

GO

Topic: Graphing and counting slope between two points.

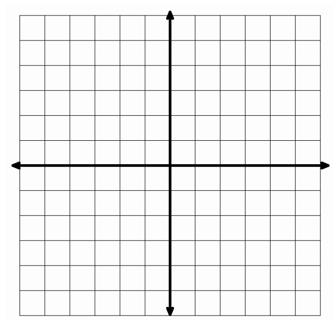
For the following problems two points and a slope are given. Plot and label the 2 points on the graph. Draw the line segment between them. Then sketch on the graph how you count the slope of the line by moving up or down and then sideways from one point to the other.

13. A(2 , -1) and B(4 , 2)



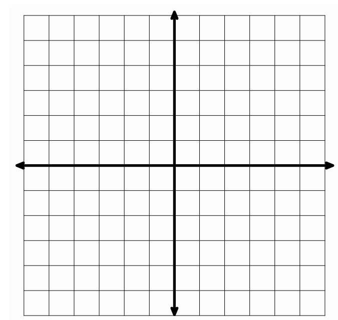
Slope: $m = \frac{3}{2}$

14. H(-2 , 1) and K(2 , 5)



Slope: $m = 1\ or\ \frac{1}{1}$

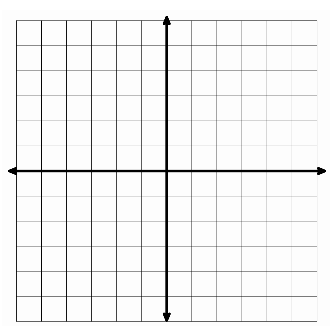
15. P(0 , 0) and Q(3 , 6)



Slope: $m = 2\ or\ \frac{2}{1}$

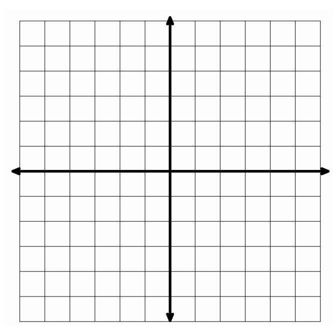
For the following problems, two points are given. Plot and label these points on the graph. Then count the slope.

16. C(-3 , 0) and D(0 , 5)



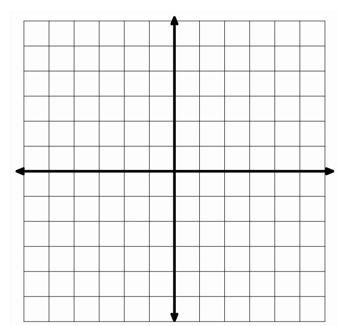
Slope: $m =$

17. E(-2 , -1) and N(-4 , 4)



Slope: $m =$

18. S(0 , 3) and W(1 , 6)



Slope: $m =$

1.8 What Comes Next? What Comes Later?

A Practice Understanding Task

For each of the following tables,

- describe how to find the next term in the sequence,
- write a recursive rule for the function,
- describe how the features identified in the recursive rule can be used to write an explicit rule for the function, and
- write an explicit rule for the function.
- identify if the function is arithmetic, geometric or neither

Example:

x	y
0	5
1	8
2	11
3	14
4	?
…	…
n	?

- To find the next term: add 3 to the previous term
- Recursive rule: $f(0) = 5, f(n) = f(n-1) + 3$
- To find the n^{th} term: start with 5 and add 3 n times
- Explicit rule: $f(n) = 5 + 3n$
- Arithmetic , geometric, or neither? Arithmetic

Function A

1. How to find the next term: ________________________________
2. Recursive rule: ______________________________________
3. To find the n^{th} term: ____________________________________
4. Explicit rule: ____
5. Arithmetic, geometric, or neither? _________________________

x	y
1	5
2	10
3	20
4	40
5	?
…	…
n	?

Function B

6. How to find the next term: ______________________________

7. Recursive rule: ______________________________

8. To find the n^{th} term: ______________________________

9. Explicit rule: ______________________________

10. Arithmetic, geometric, or neither? ______________________________

x	*y*
1	-8
2	-17
3	-26
4	-35
5	-44
6	-53
…	…
n	

Function C

11. To find the next term: ______________________________

12. Recursive rule: ______________________________

13. To find the n^{th} term: ______________________________

14. Explicit rule: ______________________________

15. Arithmetic, geometric, or neither? ______________________________

x	*y*
1	2
2	6
3	18
4	54
5	162
6	486
…	…
n	

Function D

16. To find the next term: ______________________________

17. Recursive rule: ______________________________

18. To find the n^{th} term: ______________________________

19. Explicit rule: ______________________________

20. Arithmetic, geometric, or neither? ______________________________

x	***y***
1	3
2	15
3	27
4	39
5	51
6	?
…	…
n	?

Function E

21. To find the next term: ______________________________

22. Recursive rule: __________________________________

23. To find the n^{th} term: ____________________________

24. Explicit rule: ___________________________________

25. Arithmetic, geometric, or neither? ___________________

x	y
0	1
1	$1\frac{3}{5}$
2	$2\frac{1}{5}$
3	$2\frac{4}{5}$
4	$3\frac{2}{5}$
5	4
…	…
n	

Function F

26. To find the next term: ______________________________

27. Recursive rule: __________________________________

28. To find the n^{th} term: ____________________________

29. Explicit rule: ___________________________________

30. Arithmetic, geometric, or neither? ___________________

x	y
0	3
1	4
2	7
3	12
4	19
5	?
…	…
n	?

Function G

31. To find the next term: _____________________________

32. Recursive rule: _________________________________

33. To find the n^{th} term: ___________________________

34. Explicit rule: __________________________________

35. Arithmetic, geometric, or neither? ___________________

x	y
1	10
2	2
3	$\frac{2}{5}$
4	$\frac{2}{25}$
5	$\frac{2}{125}$
6	$\frac{2}{625}$
…	…
n	

Function H

36. To find the next term: ______________________________

37. Recursive rule: ______________________________

38. To find the n^{th} term: ______________________________

39. Explicit rule: ______________________________

40. Arithmetic, geometric, or neither? ______________________

x	y
1	-1
2	0.2
3	-0.04
4	0.008
5	-0.0016
6	0.00032
…	…
n	

1.8 What Comes Next? What Comes Later? – Teacher Notes

A Practice Understanding Task

Purpose:

The purpose of this task is to practice writing recursive and explicit formulas for arithmetic and geometric sequences from a table. This task also provides practice in using tables to identify when a sequence is arithmetic, geometric, or neither. The task extends students' experiences with sequences to include geometric sequences with alternating signs, and more work with fractions and decimal numbers in the sequences.

Core Standards:

F.BF.1: Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F.LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

F.LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Standards for Mathematical Practice of Focus in the Task:

SMP7 – Look for and make use of structure.

SMP8 – Look for and express regularity in repeated reasoning.

The Teaching Cycle:

Launch (Whole Class): Remind students of the work done in previous tasks in this module. Ask, "What strategies are you working on for writing formulas for arithmetic and geometric sequences?"

Students may offer some ideas that will be helpful in this task. Hand out the task and work through the example with the class. It will be important to clarify the two prompts that ask students to “describe” what they are seeing in the table. Writing a verbal description before writing the formula can promote deeper understanding.

Explore (Small Group): Monitor student work on the task. Functions A and B should be familiar and accessible for most students. Function F is more challenging; although students may recognize a pattern, it is neither arithmetic nor geometric. It is not necessary for students to write the formulas, since this is an unfamiliar function type. If you find that a group is entirely stuck on the problem, you might ask them to move on to some of the others and go back to finish Function F. Watch their work on Function H to be sure that they consider the alternating sign in their formula. If they haven’t included it in the formula, ask them how their formulas will produce the negative sign on some of the terms. Allow them to work to find a way to include the negative sign in their formula. Watch for misconceptions that might make for a productive discussion and listen for students generalizing strategies for writing equations. Identify groups to present their work for Functions B and E that have shown the first difference on their tables and can articulate how they have used the common difference or common ratio to write the equations.

Discuss (Whole Class): Start the discussion with the student presentation of their work on Function D. Their work will probably look something like:

x	y	Difference
1	-8	
2	-17	> -9
3	-26	> -9
4	-35	> -9
5	-44	
6	-53	
…	…	
n		

Function B

To find the next term: Add -9 to the previous term

Recursive rule: $f(1) = -8, f(n) = f(n-1) - 9$

To find the n^{th} term: Start with -8 and add -9 *n-1* times

Explicit rule: $f(n) = -8 - 9(n-1)$

Arithmetic, geometric, or neither? Arithmetic

Ask students how they were able to use the common difference of -9 in the formulas. Ask how the first term, -8, shows up in each formula.

Next, ask a group to show their work with Function C. It should look something like this:

x	y
1	2
2	6
3	18
4	54
5	162
6	486

Difference

> 4

> 12

> 36

Common ratio 3

Function C

To find the next term: Multiply the previous term by 3

Recursive rule: $f(1) = 2,\ f(n) = 3f(n-1)$

To find the n^{th} term: start with 2 and multiply by 3 $(n\text{-}1)$ times

Explicit rule: $f(n) = 2(3)^{n-1}$

Arithmetic, geometric, or neither? Geometric

Ask students how they were able to use the common ratio of 3 in the formulas. Then, ask how the first term, 2, appears in the formulas.

If time allows, you may wish to have some of the others presented. If not, move to asking students to generalize their thinking about writing formulas for arithmetic and geometric functions with the following chart:

	How do you use:	
	Common ratio or difference?	First term?
Arithmetic Sequence – Recursive Formula		
Arithmetic Sequence – Explicit Formula		
Geometric Sequence – Recursive Formula		
Geometric Sequence – Explicit Formula		

Ask if there are any other strategies that they have found for writing formulas and record them beneath the chart. Leave the chart in view for students to use in upcoming tasks.

Aligned Ready, Set, Go Homework: Sequences 1.8

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Common Ratios

Find the common ratio for each <u>geometric</u> sequence.

1. 2, 4, 8, 16…

2. $\frac{1}{2}$, 1, 2, 4, 8…

3. -5, 10, -20, 40…

4. 10, 5, 2.5, 1.25…

SET

Topic: Recursive and explicit equations

Fill in the blanks for each table; then write the recursive and explicit equation for each sequence.

5. Table 1

x	1	2	3	4	5
y	5	7	9		

Recursive: ____________________ Explicit: ____________________

6. Table 2

x	*y*
1	-2
2	-4
3	-6
4	
5	

Recursive:

Explicit:

7. Table 3

x	*y*
1	3
2	9
3	27
4	
5	

Recursive:

Explicit

8. Table 4

x	*y*
1	27
2	9
3	3
4	
5	

Recursive:

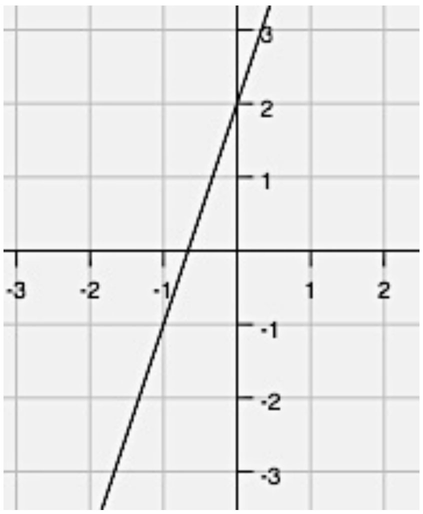
Explicit:

GO

Topic: Writing equations of lines given a graph.

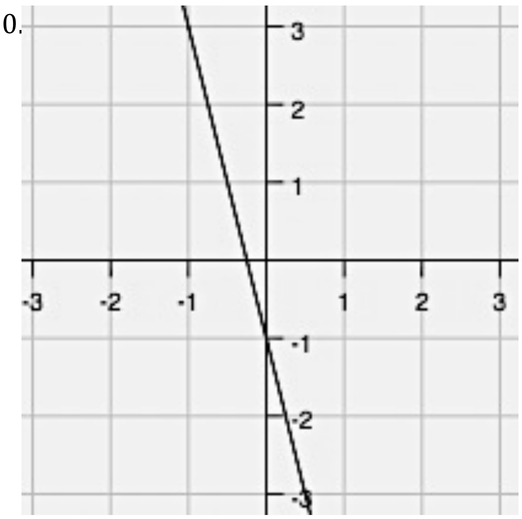
Write each equation of the line in $y = mx + b$ form. Name the value of m and b.
Recall that m is the slope or rate of change and b is the y-intercept.

9.



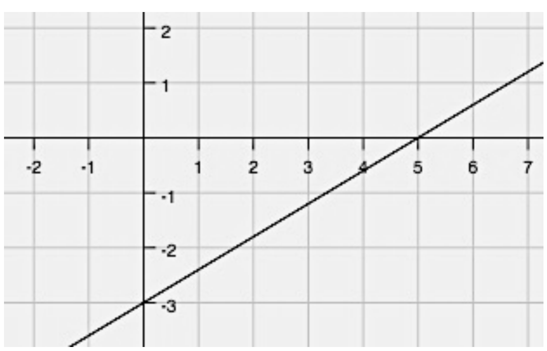
$m =$ $b =$ Equation:

10.



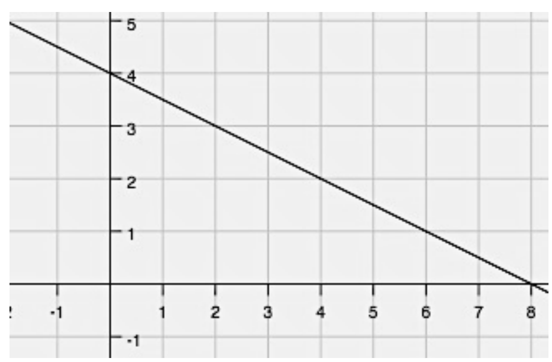
$m =$ $b =$ Equation:

11.



$m =$ $b =$ Equation:

12.



$m =$ $b =$ Equation:

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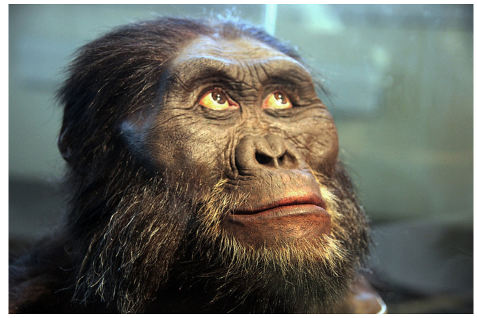
1.9 What Does It *Mean*?

A Solidify Understanding Task



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Each of the tables below represents an arithmetic sequence. Find the missing terms in the sequence, showing your method.

1.

x	1	2	3
y	5		11

2.

x	1	2	3	4	5
y	18				-10

3.

x	1	2	3	4	5	6	7
y	12						-6

4. Describe your method for finding the missing terms. Will the method always work? How do you know?

Here are a few more arithmetic sequences with missing terms. Complete each table, either using the method you developed previously or by finding a new method.

5.

x	1	2	3	4
y	50			86

6.

x	1	2	3	4	5	6
y	40					10

7.

x	1	2	3	4	5	6	7	8
y	-23							5

8. The missing terms in an arithmetic sequence are called "arithmetic means". For example, in the problem above, you might say, "Find the 6 arithmetic means between -23 and 5". Describe a method that will work to find arithmetic means and explain why this method works.

1.9 What Does It *Mean*? – Teacher Notes

A Solidify Understanding Task

Purpose:

The purpose of this task is to solidify student understanding of arithmetic sequences by finding missing terms in the sequence. Students will draw upon their previous work in using tables and writing explicit formulas for arithmetic sequences. The task will also reinforce their fluency in solving equations in one variable.

Core Standards:

A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters.

Clusters with Instructional Notes: Solve equations and inequalities in one variable.
Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.

Standards for Mathematical Practice of Focus in the Task:

SMP7 – Look for and make use of structure.

The Teaching Cycle:

Launch (Whole Class): Refer students to the previous work with arithmetic sequences. In the past, we have been given terms and asked to write formulas. In this task, we will be using what we know about sequences to find missing terms. A brief discussion of the chart that was created as part of the "What Come Next, What Comes Later" task may be useful in drawing students' attention to the role of the first term and the common difference in writing explicit formulas.

Explore Part 1 (Small Group): Give students the task and be sure that they understand the instructions. Tell them that they are looking for a strategy for solving these problems that works all the time, so they should be paying attention to their methods. The task builds in complexity from the problems on first page to the problems on second page. Because the problems on the first side are all asking students to find an odd number of terms, most students will probably average the first and last terms to find the middle term, and then average the next two, and so on until the table is completed. Some students may choose to write an equation and use it to find the missing terms. Others may use guess and check to find the common difference to get to each term. Watch for the various strategies and note the students that you will select for the discussion of the first part. Let students work on the task until they complete the first page, and then call them back for a short discussion.

Discuss (Whole Class): Begin the discussion by asking a student that used guess and check to explain his/her strategy. Ask the class what understanding of arithmetic sequences is necessary to use this strategy? They should answer that it relies on the common difference and that it can be added to a term to get the next term. This is using the reasoning that we use to write recursive formulas. Next, ask the student that used an averaging method to explain his/her method. Ask the class why this method works. Now ask the class what the advantages are to each of these methods. Will they both work every time? If students haven't noticed that all of these examples are working with an odd number of missing terms, ask them what would happen if you needed to use one of these strategies to find an even number of missing terms. Don't discuss the answer as a class, just use it as a teaser for the next section.

Explore (Small Group): Have students work on side 2 of the task, with the same instructions about keeping track of their strategies. Monitor students as they work noting the strategies that they are trying. Some will continue to try guess and check, although it becomes more difficult. You might encourage them to look for a more reliable method, just in case the numbers are too messy to guess at. The method of taking a simple average will not work well for these problems either, so students may become stuck. You might ask them what they would need to know and how they would use it to fill in the missing terms. When they know that they need the difference between terms, then ask if they can think of a way to find it without guessing. This may help them to think of writing an equation to find the difference. Listen to students as they explain their thinking about the formula and be prepared to ask students to present their various strategies.

Discuss (Whole Group): Since most students will use the same method for all three problems, start the discussion by talking about the second problem. A couple of useful ideas that could emerge from the discussion follow in the section below:

x	1	2	3	4	5	6
y	40					10

One way to think about setting up an equation to find the difference is to say: I know that I need to start at 40 and add the difference *(d)*, 5 times to get to 10. So, I wrote the equation:

$$40 + 5d = 10$$

A second common approach is to subtract the first term from the last term and divide by the number of "jumps" to get the common difference. This is really just finding the slope by finding the change in y divided by the change in x, although students probably won't recognize it as such. This can be written:

$$d = \frac{f(n) - f(1)}{n - 1}$$

This is worth pointing out if it arises, since students often find an analogous pattern in geometric sequences in the next task.

A third strategy is to think more about the explicit formulas that we have written in the past as a general formula: $f(n) = f(1) + d(n - 1)$. Students probably won't use this notation, but might say something like, "We learned in the last task that when you write an explicit formula to get any term, you multiply the common difference by 1 less than the term you're looking for and add the first term." If they apply the idea like a general formula they would write:

$$10 = 40 + d(6 - 1)$$

Point out that this strategy is closely related to the previous strategy. Notice how the two formulas are really just the same formula rearranged?

Formula from second strategy: $d = \frac{f(n)-f(1)}{n-1}$

Formula from third strategy: $f(n) = f(1) + d(n-1)$.

Either way they think about it, students can get the common difference and add it to the term they know to get the next term. Ask students if the strategy of writing an equation works with the three problems on side 1 and give them a chance to convince themselves that it will.

Finally, ask students to amend, edit, or add to their strategy and explanation on the last question. Conclude the discussion by having a few students share what they have written with the class.

Aligned Ready, Set, Go Homework: Sequences 1.9

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Comparing arithmetic and geometric sequences

1. How are arithmetic and geometric sequences similar?

2. How are they different?

SET

Topic: Finding missing terms in an Arithmetic sequence

Each of the tables below represents an arithmetic sequence. Find the missing terms in the sequence, showing your method.

3. Table 1

x	1	2	3
y	3		12

4. Table 2

x	y
1	2
2	
3	
4	26

5. Table 3

x	y
1	24
2	
3	6
4	

6. Table 4

x	y
1	16
2	
3	
4	4
5	

GO

Topic: Sequences

Determine the recursive and explicit equations for each. (if the sequence is not arithmetic or geometric, identify it as neither and don't write the equations).

7. 5, 9, 13, 17,… This sequence is: Arithmetic , Geometric , Neither

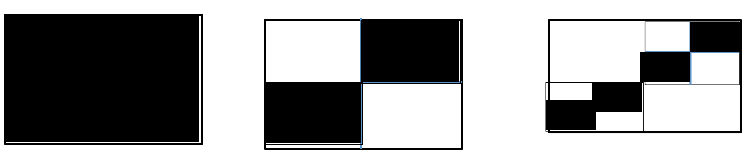
Recursive Equation: ______________________ Explicit Equation: ________________________

8. 60, 30, 0, -30 ,… This sequence is: Arithmetic , Geometric , Neither

Recursive Equation: ______________________ Explicit Equation: ________________________

9. 60, 30, 15, $\frac{15}{2}$, ,… This sequence is: Arithmetic , Geometric , Neither

Recursive Equation: ______________________ Explicit Equation: ________________________

10. 

(The number of black tiles above) This sequence is: Arithmetic , Geometric , Neither

Recursive Equation: ______________________ Explicit Equation: ________________________

11.. 4, 7, 12, 19, ,… This sequence is: Arithmetic , Geometric , Neither

Recursive Equation: ______________________ Explicit Equation: ________________________

1.10 Geometric Meanies

A Practice Understanding Task

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Each of the tables below represents a geometric sequence. Find the missing terms in the sequence, showing your method.

Table 1

x	1	2	3
y	3		12

Is the missing term that you identified the only answer? Why or why not?

Table 2

x	1	2	3	4
y	7			875

Are the missing terms that you identified the only answers? Why or why not?

Table 3

x	1	2	3	4	5
y	6				96

Are the missing terms that you identified the only answers? Why or why not?

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Table 4

x	1	2	3	4	5	6
y	4					972

Are the missing terms that you identified the only answers? Why or why not?

A. Describe your method for finding the geometric means.

B. How can you tell if there will be more than one solution for the geometric means?

1.10 Geometric Meanies – Teacher Notes

A Practice Understanding Task

Purpose:

The purpose of this task is to solidify student understanding of geometric sequences to find missing terms in the sequence. Students will draw upon their previous work in using tables and writing explicit formulas for geometric sequences.

Core Standards:
A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters.

Clusters with Instructional Notes: Solve equations and inequalities in one variable.
Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents,
such as $5^x = 125$ or $2^x = 1/16$.

Standards for Mathematical Practice of Focus in the Task:

SMP 7 – Look for and make use of structure.

The Teaching Cycle:
Launch (Whole Class): Explain to students that today's puzzles involve finding missing terms between two numbers in a geometric sequence. These numbers are called "geometric means". Ask them to recall the work that they have done previously with arithmetic means. Ask, "What information do you think will be useful for finding geometric means? Some may remember that the first term and the common difference were important for arithmetic means; similarly the first term and the common ratio may be important for geometric means.

Ask, "How do you predict the methods for finding arithmetic and geometric means to be similar?" Ideally, students would think to use the common ratio to multiply to get the next term in the same way that they added the common difference to get the next term. They may also think about writing the equation using the number of "jumps" that it takes to get from the first term to the next term that they know.

Ask, "How do you think the methods will be different?" Ideally, students will recognize that they have to multiply rather than simply add. They may also think that the equations have exponents in them because the formulas that they have written for geometric sequences have exponents in them.

Explore (Small Group): The problems in this task get larger and require students to solve equations of increasingly higher order. Some students will start with a guess and check strategy, which may work for many of these. Even if it is working, you may choose to ask them to work on a strategy that is more consistent and will work no matter what the numbers are. Students are likely to try the same strategies as they used for arithmetic sequences. These strategies will be successful if they think to multiply by the common ratio, rather than add the common difference. If they are writing equations, they may have similar thinking to this:

Table 2

x	1	2	3	4
y	7			875

"I need to start at 7 and multiply by a number, r, to get to the second term, then multiply by r again to get to the third term, and by r again to end up at 875. So, I will write the equation:

$$7 \cdot r \cdot r \cdot r = 875 \text{ or } 7r^3 = 875$$

Some students may have difficulty solving the equation that they write. It may be helpful to get them to divide by 7 and then think about the number that can be raised to the third power to get 125. Calculators will also be helpful if students understand how to take roots of numbers. All of the numbers used in this task are straightforward, making them accessible for most students to think about without a calculator. Watch for students that can explain the strategy shown above, along with other productive strategies so that you can highlight their work in the discussion.

Along with completing the table, students are asked if the solution they found is the only solution. This is a reminder for them to think of both the positive and negative solution if the exponent is even. As you are monitoring student work, watch to see if they are answering this prompt appropriately. If not, you may want to ask a question like, "I see that you have $r^2 = 4$, $r = 2$. Is there another number that you can multiply by itself to get 4?"

Discuss (Whole Class): Start the discussion with Table 1. Select a student that guessed at the common ratio and ask how he/she figured it out and then how the common ratio was used to find the missing term. Since most guessers only get the positive ratio (2) , you will probably have students that want to add the negative solution (-2) also. Great! Ask how they figured the solution out and to verify for the class that a common ratio of -2 also produces a term that works in the sequence. Some students may have simply reasoned that -2 would work as a common ratio. Be sure to select a student that has written an equation like: $3r^2 = 12$. Use this as an opportunity for students to see that the algebraic solution to this equation is $r = \pm 2$.

Next, ask a student to write and explain an equation for Table 2, as shown above. As they solve the equation, $7r^3 = 875$, $r^3 = 125$, ask students if there is more than one number that can be cubed to obtain 125? They should decide that 5 is a solution, but -5 is not. Have a student demonstrate using the common ratio that was found to complete the table.

Now that students have worked a couple of problems as a class, they have had a chance to check their work, ask them to reconsider the generalizations at the end of the task. Give students a few minutes to modify their responses to questions A and B. Then, ask a few students to share their procedure for finding geometric means and knowing the number of solutions with the entire class.

Students may notice that one of the strategies they are using is analogous to the process they were using to find the common difference in an arithmetic sequence. With the arithmetic sequence, the process led to the slope formula. With a geometric sequence, the process becomes:

- Divide the last term by the first term
- Take the n-1 root of the result to get the common ratio (Students will probably not use this terminology)

If this occurs in class, it may be productive to compare how to "undo" an arithmetic sequence and a geometric sequence. The differences result from the different nature of the two sequence types; arithmetic sequences are additive and geometric sequences are multiplicative.

Aligned Ready, Set, Go Homework: Sequences 1.10

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Arithmetic and geometric sequences

For each set of sequences, find the first five terms. Then compare the growth of the arithmetic sequence and the geometric sequence. Which grows faster? When?

1. Arithmetic sequence: $f(1) = 2$, common difference, $d = 3$
   Geometric sequence: $g(1) = 2$, common ratio, $r = 3$

Arithmetic	Geometric
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

a) Which value do you think will be more, $f(100)$ or $g(100)$? b) Why?

2. Arithmetic sequence: $f(1) = 2$, common difference, $d = 10$
   Geometric sequence: $g(1) = 128$, common ratio, $r = \frac{1}{2}$

Arithmetic	Geometric
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

a) Which value do you think will be more, $f(100)$ or $g(100)$? b) Why?

3. Arithmetic sequence: $f(1) = 20, d = 10$
   Geometric sequence: $g(1) = 2, r = 2$

Arithmetic	Geometric
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

a) Which value do you think will be more, $f(100)$ or $g(100)$? b) Why?

4. Arithmetic sequence: $f(1) = 50$, common difference, $d = -10$
Geometric sequence: $g(1) = 1$, common ratio, $r = 2$

Arithmetic	Geometric
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

a) Which value do you think will be more, $f(100)$ or $g(100)$? b) Why?

5. Arithmetic sequence: $f(1) = 64$, common difference, $d = -2$
Geometric sequence: $g(1) = 64$, common ratio, $r = \frac{1}{2}$

Arithmetic	Geometric
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

a) Which value do you think will be more, $f(100)$ or $g(100)$? b) Why?

6. Considering arithmetic and geometric sequences, would there ever be a time that a geometric sequence does not out grow an arithmetic sequence in the long run as the number of terms of the sequences becomes really large? Explain.

SET

Topic: Finding missing terms in a geometric sequence

Each of the tables below represents a *geometric* sequence. Find the missing terms in the sequence. Show your method.

7. Table 1

x	1	2	3
y	3		12

8. Table 2

x	y
1	2
2	
3	
4	54

9. Table 3

x	y
1	5
2	
3	20
4	

10. Table 4

x	y
1	4
2	
3	
4	
5	324

GO

Topic: Writing the explicit equations of a geometric sequence

Given the following information, determine the explicit equation for each geometric sequence.

11. $f(1) = 8, common\ ratio\ r = 2$

12. $f(1) = 4, f(n) = 3f(n-1)$

13. $f(n) = 4f(n-1);\ f(1) = \frac{5}{3}$

14. Which geometric sequence above has the greatest value at $f(100)$?

1.11 I Know ... What Do You Know?

A Practice Understanding Task

In each of the problems below I share some of the information that I know about a sequence. Your job is to add all the things that you know about the sequence from the information that I have given. Depending on the sequence, some of the things you may be able to figure out for the sequence are:

- a table;
- a graph;
- an explicit equation;
- a recursive formula;
- the constant ratio or constant difference between consecutive terms;
- any terms that are missing;
- the type of sequence;
- a story context.

Try to find as many as you can for each sequence, but you must have at least 4 things for each.

1. I know that: the recursive formula for the sequence is $f(1) = -12,\ f(n) = f(n-1) + 4$

   What do you know?

2. I know that: the first 5 terms of the sequence are 0, -6, -12, -18, -24 . . .

   What do you know?

3. I know that: the explicit formula for the sequence is $f(n) = -10(3)^n$

   What do you know?

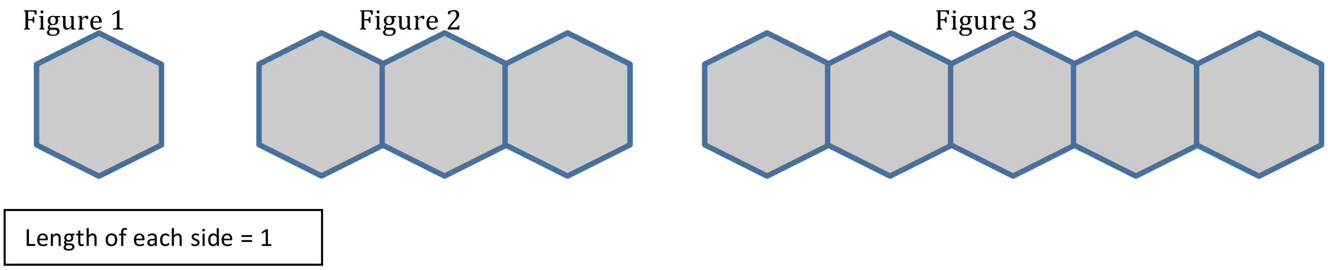
4. I know that: The first 4 terms of the sequence are 2, 3, 4.5, 6.75 . . .

   What do you know?

5. I know that: the sequence is arithmetic and $f(3) = 10$ and $f(7) = 26$

   What do you know?

6. I know that: the sequence is a model for the perimeter of the following figures:



What do you know?

7. I know that: it is a sequence where $f(1) = 5$ and the constant ratio between terms is -2.

What do you know?

8. I know that: the sequence models the value of a car that originally cost \$26,500, but loses 10% of its value each year.

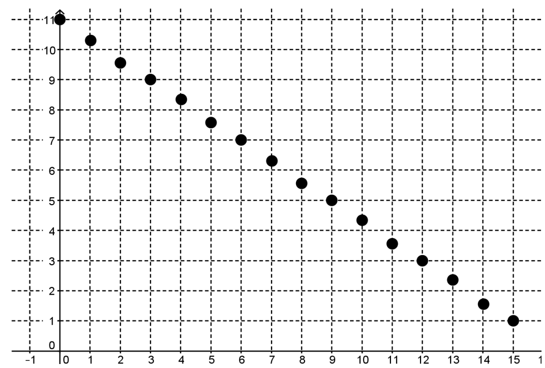
What do you know?

9. I know that: the first term of the sequence is -2, and the fifth term is $-\frac{1}{8}$.

   What do you know?

10. I know that: a graph of the sequence is:

   What do you know?



1.11 I Know… What Do You Know? – Teacher Notes

A Practice Understanding Task

Purpose:

The purpose of this task is to practice working with geometric and arithmetic sequences. This is the final task in the module and is intended to help students develop fluency in using various representations for sequences. This task could be used as a performance assessment.

Core Standards:

F-LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Standards for Mathematical Practice of Focus in the Task:

SMP 6 – Attend to precision.

SMP7 – Look for and make use of structure.

The Teaching Cycle:

Launch (Whole Class): The task gives students some information about a sequence and asks them to complete the remaining information. Ask students what are some things that we can know about a sequence. This question should generate a list like:

- A table with 4 terms of the sequence
- A graph of the sequence
- An explicit formula for the sequence
- A recursive formula for the sequence
- The common ratio or common difference between terms
- Whether the sequence is geometric, arithmetic or neither

If the class doesn't identify all of these representations, add the missing ones to the list. Hand out the task and tell students that they will be trying to figure out all the things that can be known from the information given.

Explore (Small Group): Monitor students as they work. After most students have completed a substantial portion of the task, begin assigning one group per problem to make a large chart and present it to the class. Each chart should have all the information listed above about the sequence that they have been assigned. (In each case, some of the information was given.) Ask students to be prepared to discuss how they found each piece of information.

Discuss (Whole Group): Ask each group to present their work with the sequence one at a time. Encourage the groups that are not presenting to be checking their work and asking questions to the presenters. In the interest of time, you may choose just a few groups to present, possibly #6 and #8, and then have the other groups post their work. You could then organize a "gallery stroll" in which students went from one chart to the next, comparing their work to the chart and formulating questions for the groups. When they have had a chance to go to each chart, facilitate a discussion where students can ask their questions to the group that developed the chart.

Aligned Ready, Set, Go Homework: Sequences 1.11

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Comparing linear equations and arithmetic sequences

1. Describe the similarities and differences between linear equations and arithmetic sequences.

Similarities	Differences

SET

Topic: Representations of arithmetic sequences

Use the given information to complete the other representations for each arithmetic sequence.

2. **Recursive Equation:**

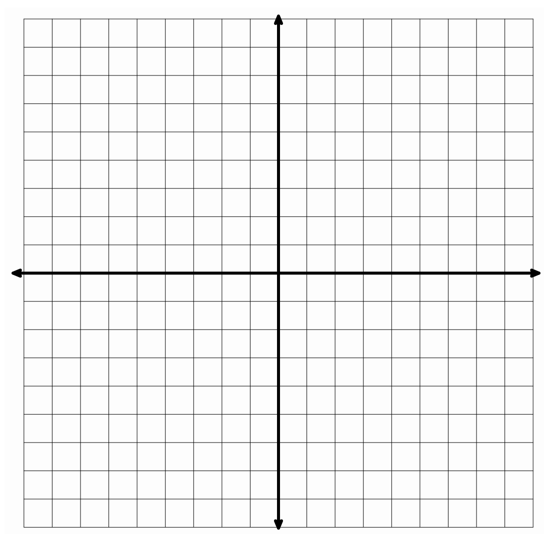
Explicit Equation:

Table

Days	Cost
1	8
2	16
3	24
4	32

Create a context

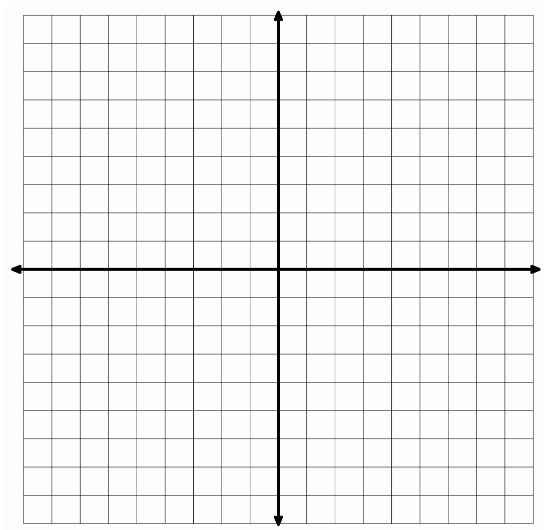
Graph



3. **Recursive Equation:** $f(1) = 4, \quad f(n) = f(n-1) + 3$

Explicit Equation:

Graph



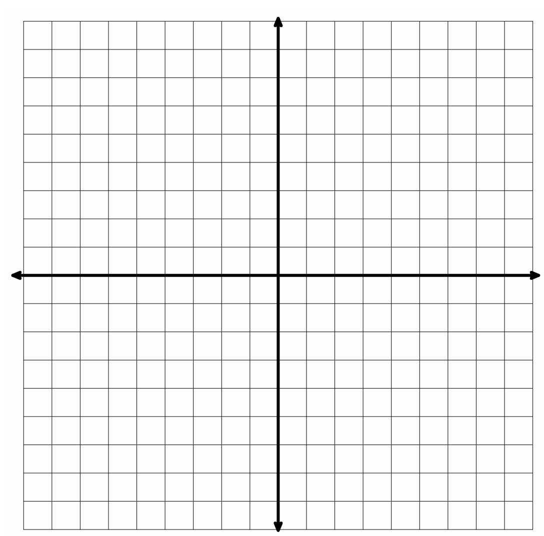
Table

Create a context

4. **Recursive Equation:**

Graph

Explicit Equation: $f(n) = 4 + 5(n-1)$



Table

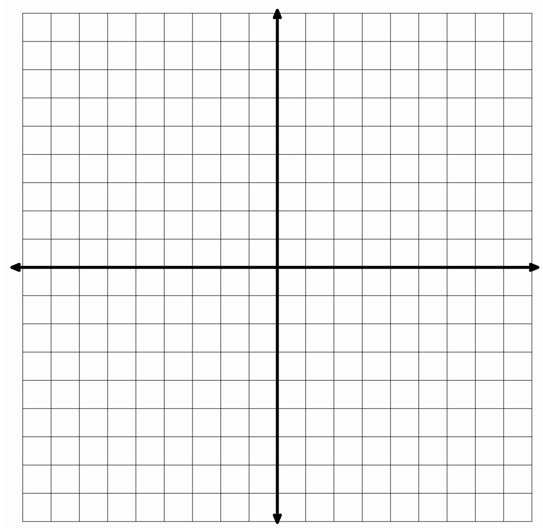
Create a context

5. **Recursive Equation:**

Graph

Explicit Equation:

Table



Create a context

Janet wants to know how many seats are in each row of the theater. Jamal lets her know that each row has 2 seats more than the row in front of it. The first row has 14 seats.

GO

Topic: Writing explicit equations

Given the recursive equation for each arithmetic sequence, write the explicit equation.

6. $f(n) = f(n-1) - 2; f(1) = 8$

7. $f(n) = 5 + f(n-1); f(1) = 0$

8. $f(n) = f(n-1) + 1;\ f(1) = \frac{5}{3}$