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Transforming Mathematics Education

SECONDARY MATH ONE

An Integrated Approach

Standard Teacher Notes

MODULE 6

Transformations & Symmetry

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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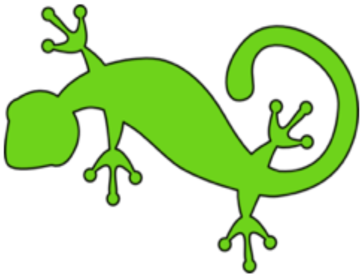
6. 1 Leaping Lizards!



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A Develop Understanding Task

Animated films and cartoons are now usually produced using computer technology, rather than the hand-drawn images of the past. Computer animation requires both artistic talent and mathematical knowledge.

Sometimes animators want to move an image around the computer screen without distorting the size and shape of the image in any way. This is done using geometric transformations such as translations (slides), reflections (flips), and rotations (turns), or perhaps some combination of these. These transformations need to be precisely defined, so there is no doubt about where the final image will end up on the screen.

So where do you think the lizard shown on the grid on the following page will end up using the following transformations? (The original lizard was created by plotting the following anchor points on the coordinate grid, and then letting a computer program draw the lizard. The anchor points are always listed in this order: tip of nose, center of left front foot, belly, center of left rear foot, point of tail, center of rear right foot, back, center of front right foot.)

Original lizard anchor points:

{(12,12), (15,12), (17,12), (19,10), (19,14), (20,13), (17,15), (14,16)}

Each statement below describes a transformation of the original lizard. Do the following for each of the statements:

- plot the anchor points for the lizard in its new location
- connect the **pre-image** and **image** anchor points with line segments, or circular arcs, whichever best illustrates the relationship between them

Lazy Lizard

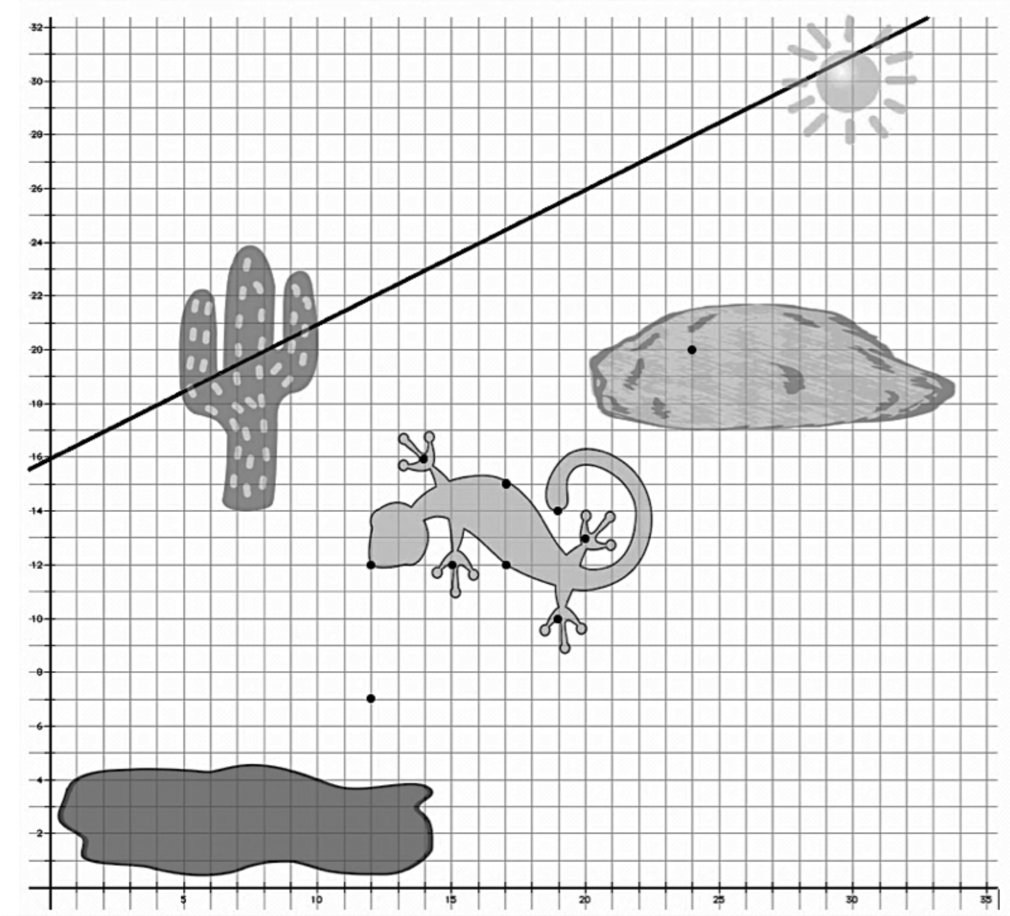
Translate the original lizard so the point at the tip of its nose is located at (24, 20), making the lizard appears to be sunbathing on the rock.

Lunging Lizard

Rotate the lizard 90° about point A (12,7) so it looks like the lizard is diving into the puddle of mud.

Leaping Lizard

Reflect the lizard about given line $y = \frac{1}{2}x + 16$ so it looks like the lizard is doing a back flip over the cactus.



6. 1 Leaping Lizards! – Teacher Notes

A Develop Understanding Task

Purpose: This task provides an opportunity for formative assessment of what students already know about the three rigid-motion transformations: translations, reflections, and rotations. As students engage in the task they should recognize a need for precise definitions of each of these transformations so that the final image under each transformation is a unique figure, rather than an ill-defined sketch. The exploration and subsequent discussion described below should allow students to begin to identify the essential elements in a precise definition of the rigid-motion transformations, *e.g., translations move points a specified distance along parallel lines; rotations move points along a circular arc with a specified center and angle, and reflections move points across a specified line of reflection so that the line of reflection is the perpendicular bisector of each line segment connecting corresponding pre-image and image points.*

In addition to the work with the rigid-motion transformations, this task also surfaces thinking about the slope criteria for determining when lines are parallel or perpendicular. In a translation, the line segments connecting pre-image and image points are parallel, having the same slope. In a 90° rotation, the line segments connecting pre-image and image points are perpendicular, having opposite reciprocal slopes. Likewise, in a reflection, the line segments connecting pre-image and image points are perpendicular to the line of reflection.

Finally, this task reminds students that rigid-motion transformations preserves distance and angle measures within a shape—implying that the figures forming the pre-image and image are congruent. Students will be attending to two different categories of distances—the lengths of line segments that are used in the definitions of the transformations, and the lengths of the congruent line segments that are contained within the pre-image and image figures themselves. Students may determine that these lengths are preserved by counting units of "rise" and "run", or by using the

Pythagorean Theorem. Ultimately, this work will lead to the development of the distance formula in future tasks.

Core Standards Focus:

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Related Standards: G.CO.2, G.CO.6, G.GPE.5

Teacher Note: Students' previous experiences with rigid motions may have surfaced intuitive ways of thinking about these transformations, but such informal definitions will not support students in proving geometric properties based on a transformational approach. Experiences with sliding, flipping and turning rigid objects will have provided experimental evidence that rigid-motion transformations preserve distance and angle within a shape, such that,

- Lines are taken to lines, and line segments to line segments of the same length.
- Angles are taken to angles of the same measure.
- Parallel lines are taken to parallel lines.

Students who have used technology to translate, rotate or reflect objects may not have attended to the essential features that define such transformations. For example, a student can mark a mirror line and click on a button to reflect an object across the mirror line without noting the relationship between the pre-image and image points relative to the line of reflection. Consequently, research shows that students harbor many misconceptions about the placement of an image after a transformation—erroneous assumptions such as:

- one of the sides of a reflected image must coincide with the line of reflection
- the center of a rotation must be located at a point on the pre-image (e.g., a vertex point) or at the origin
- a pre-image point and corresponding image point do not need to be the same distance away from the center of the rotation

Watch for these misconceptions as students engage in this task.

Standards for Mathematical Practice of focus in the task:

SMP 1 – Make sense of problems and persevere in solving them
SMP 5 – Use appropriate tools strategically
SMP 7 – Look for and make use of structure

Additional Resources for Teachers:

An enlarged copy of the image on the second page of the task can be found at the end of this set of teacher notes. This image can be printed for use with students who may be accessing the task on a computer or tablet.

The Teaching Cycle:

Launch (Whole Class):

Set the stage for the work of this learning cycle by discussing the ideas of computer animation as outlined in the first few paragraphs of this task. As part of the launch ask students why they think we need only keep track of a few anchor points, since the image of the lizard consists of infinitely many points, in addition to the eight points that are listed. The issue to be raised here is that rigid-motion transformations preserve distance and angle (properties that have been established in Math 8). Therefore a software animation program could draw features of the lizard, such as the toes on each of the feet, by starting at an anchor point and using predetermined angle and distance measures to locate other points on the toes. Make sure students pay attention to the order in which each of the anchor points should be listed after completing each of the transformations. This will help students pay attention to individual pairs of pre-image and image points.

Provide multiple tools for students to do this work, such as transparencies or tracing paper, protractors, rulers, and compasses. The coordinate grid on which the images are drawn is also a tool for doing this work, but initially students may not recognize the usefulness of the grid as a way of carrying out the transformations, but rather just as a way of designating the location of the points after the transformation is complete. Technology tools may obscure the ideas being surfaced in the task, so it is best to use the tools described, which will allow students to pay attention to the details of their work.

It is intended that students should work on the transformations in the order listed in the task.

Explore (Small Group):
This task provides a great opportunity to pre-assess what students know about each of the rigid-motion transformations, so don't worry if not all students are locating the final images correctly. Pay attention to the misconceptions that may arise (see teacher note).

If students use transparencies (or tracing paper) to copy the original lizard and then locate the image by sliding, turning or flipping the transparency, you will want to make sure they also think about these movements relative to the coordinate grid. Ask, "How could you have used the coordinate grid to locate this same set of points?" Focusing students' attention on the coordinate grid will facilitate connecting the details that need to be articulated in the definitions of the rigid-motion transformations to coordinate geometry ideas, such as using slope to determine if lines are parallel or perpendicular. In this task, these ideas are surfaced and informally explored. In subsequent tasks these ideas are made more explicit and eventually justified.

Students should be fairly successful translating "Lazy Lizard", since the point at the tip of the nose moves up 8 units and right 12 units, every anchor point must move the same. Watch for two different strategies to emerge: some students may move each point up 8, right 12; others may move one point to the correct location, and then duplicate the relative positions of the points in the pre-image to locate points in the image—thereby preserving distance and angle between the points in the pre-image and those same points in the image.

To get started on "Lunging Lizard", you may want to direct students' attention to the point at the tip of the lizard's nose, which lies on a vertical line, 5 units above the center of rotation. Ask students where this point would end up after rotating 90° counterclockwise. Watch for students who are attending to the 90° angle of rotation by drawing line segments from the center of rotation to the image and corresponding pre-image points. Also watch for how students determine that an image point is the same distance away from the center of rotation as its corresponding pre-image point: do they measure with a ruler, do they draw concentric circles centered at (12, 7), do they count the rise and run from (12, 7) to a point on the lizard and then use a related way of counting rise and run to locate the image point—intuitively using the Pythagorean Theorem to keep the same distance, or do they ignore distance altogether?

For "Leaping Lizard", watch for students who may have noticed that an image point and its corresponding pre-image point are equidistant from the line of reflection. Listen for how they justify that these distances are the same: do they measure with a ruler, do they fold the paper along the line of reflection, do they count the rise and run from the pre-image to the line of reflection and then from the line of reflection to the image point—intuitively using the Pythagorean Theorem to keep the same distance. Also watch for students who notice that the line segments connecting the image points to their corresponding pre-image points are all parallel to each other—perhaps even noticing that all of these line segments have a slope of -2.

Discuss (Whole Class):

If students have not all located the same set of points for the images of the transformations, have students discuss whether this is reasonable or not. Inform students, "That transformations are like functions—any set of points that form a pre-image should have a unique set of points that form the image that is the result of the transformation. If we have not obtained unique images, then we have not recognized the precise nature of these transformations. That is the goal of our work today, to notice what is important about each transformation so the images produced by the transformation are precisely defined."

Discuss strategies for locating the images of the anchor points for each transformation. Here is a suggested list of a sequence of ideas to be presented, if available. While we will not be writing precise definitions for the transformations until the task *Leap Year*, it is important that the ideas of distance and direction (e.g., along a parallel line, perpendicular to a line, or along a circle) emerge during this discussion. If not all of the suggested strategies are available in the student work, at least make sure the debrief of each transformation does focus on both distance and direction. If either idea is missing, ask additional questions to prompt for it. For example, "How did you know how far away from the center point (or the reflecting line) this image point should be?" Also, be aware of the tasks that follow in this learning cycle—not everything needs to be neatly wrapped up in this discussion.

Debriefing the translation:

- Have a student present who used a transparency or tracing paper to get a set of image points that the whole class can agree upon.
- Next, have a student present who moved each anchor point up 8, right 12 units.
- Finally, have a student present who moved one anchor point up 8, right 12 units and then used the relative positions of the points in the original figure to locate related points in the image figure. Discuss that this is possible because translations preserve distance, angle and parallelism.

Debriefing the rotation:

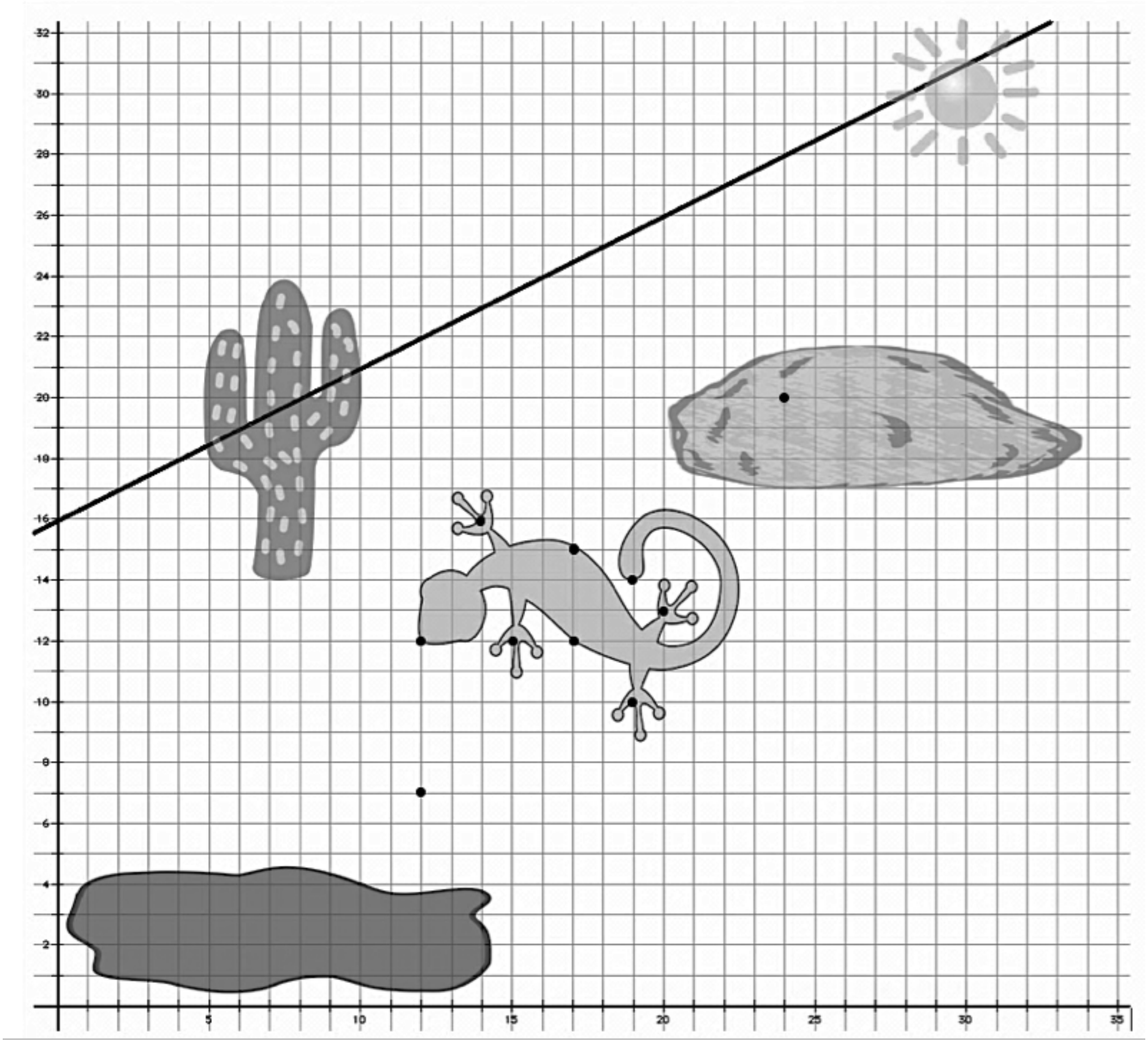
- Have a student present who used a transparency or tracing paper to get a set of image points that the whole class can agree upon.
- Next, have a student present who used a protractor to measure 90° and a ruler to measure distances from the center of rotation. Draw in the line segments between (12, 7) and the corresponding image and pre-image points, using a different color for each image/pre-image pair. This will highlight the 90° angle of rotation, centered at (12, 7).

- Next, have a student present who drew concentric circles (or arcs) to show that pairs of image/pre-image points are the same distance from (12, 7) because they lie on the same circle.
- Finally, have a student present who showed that image/pre-image points are the same distance from (12, 7) by using the Pythagorean Theorem, or some strategy that is intuitively equivalent.

Debriefing the reflection:

- Have a student present who used a transparency or tracing paper to get a set of image points that the whole class can agree upon.
- Next, have a student present who used a ruler to measure distances from the line of reflection.
- If available, have a student describe how they determined these distances from the line of reflection using the Pythagorean Theorem, or some strategy that is intuitively equivalent.
- Next, have a student present who noticed that the segments connecting pairs of image/pre-image points are parallel, perhaps by pointing out that they have the same slope.
- Finally, have a student present who might argue that the segments connecting pairs of image/pre-image points are perpendicular to the line of reflection.

Aligned Ready, Set, Go: Transformation and Symmetry 6.1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READY, SET, GO! Name Period Date

READY

Topic: Pythagorean Theorem

For each of the following right triangles determine the measure of the missing side. Leave the measures in exact form if irrational.

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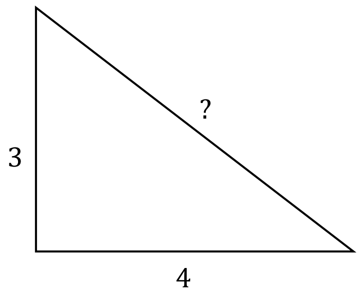

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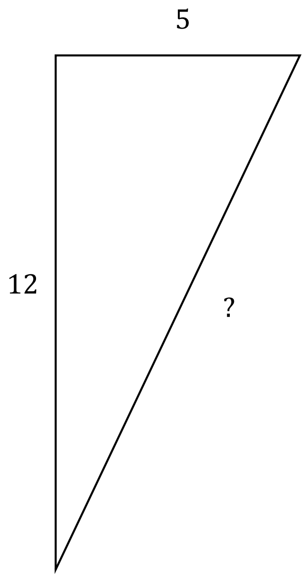

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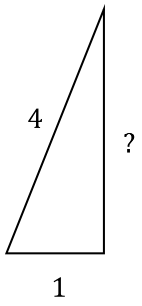

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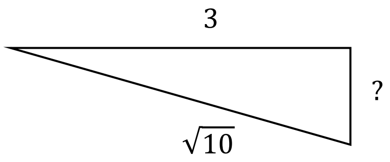

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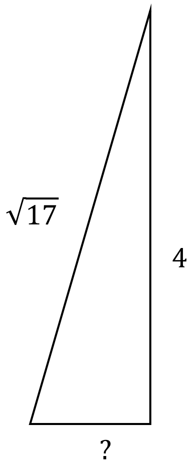

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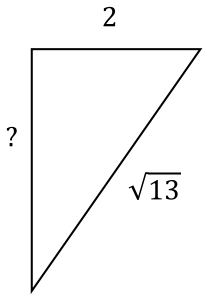


SET

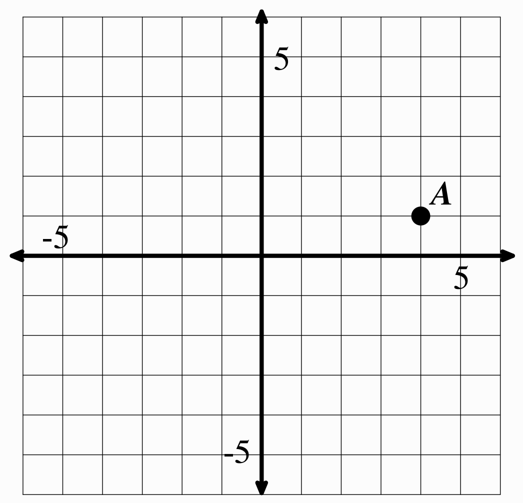
Topic: Transformations.

Transform points as indicated in each exercise below.

7a. Rotate point A around the origin 90° clockwise, label as A’

b. Reflect point A over x-axis, label as A’’

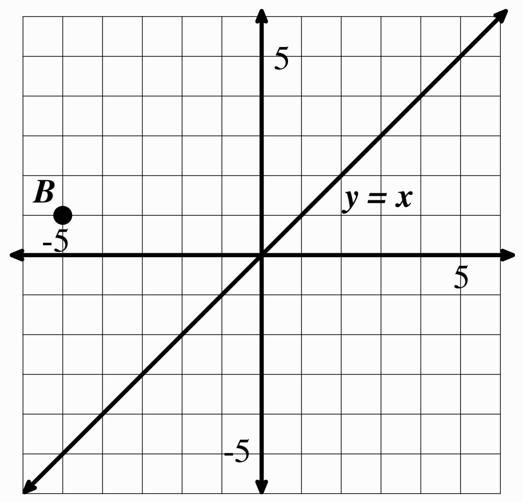
c. Apply the rule $(x-2\,,y-5)$, to point A and label A’’’



8a. Reflect point B over the line $y\ =\ x$, label as B’

b. Rotate point B 180° about the origin, label as B’’

c. Translate point B the point up 3 and right 7 units, label as B’’’

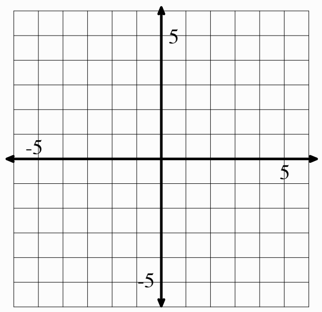


GO

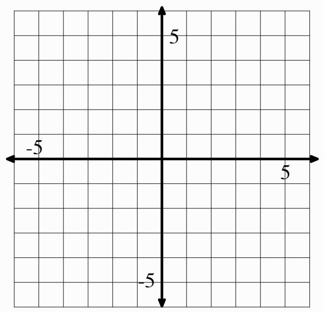
Topic: Graphing linear equations.

Graph each function on the coordinate grid provided. Extend the line as far as the grid will allow.

9. $f(x) = 2x - 3$

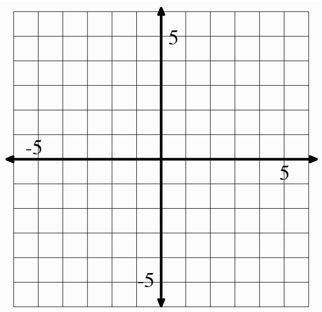


10. $g(x) = -2x - 3$

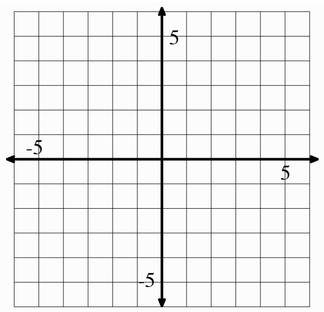


11. What similarities and differences are there between the functions *f(x)* and *g(x)*?

12. $h(x) = \frac{2}{3}x + 1$

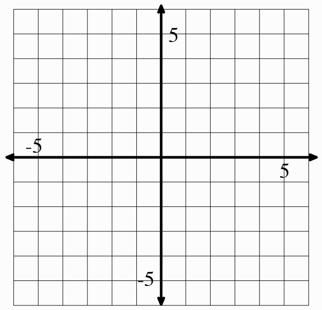


13. $k(x) = -\frac{3}{2}x + 1$

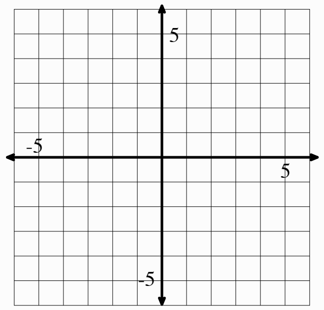


14. What similarities and differences are there between the equations *h(x)* and *k(x)*?

15. $a(x) = x + 1$



16. $b(x) = x - 3$



17. What similarities and differences are there between the equations *a(x)* and *b(x)*?

6. 2 Is It Right?

A Solidify Understanding Task

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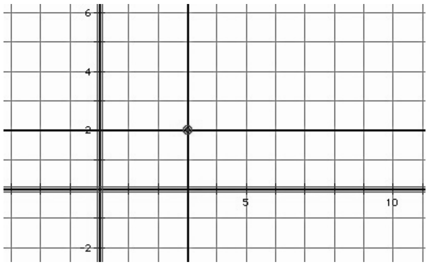
In *Leaping Lizards* you probably thought a lot about perpendicular lines, particularly when rotating the lizard about a given center a 90° angle or reflecting the lizard across a line.

In previous tasks, we have made the observation that *parallel lines have the same slope*. In this task we will make observations about the slopes of perpendicular lines. Perhaps in *Leaping Lizards* you used a protractor or some other tool or strategy to help you make a right angle. In this task we consider how to create a right angle by attending to slopes on the coordinate grid.

We begin by stating a fundamental idea for our work: *Horizontal and vertical lines are perpendicular.* For example, on a coordinate grid, the horizontal line $y = 2$ and the vertical line $x = 3$ intersect to form four right angles.

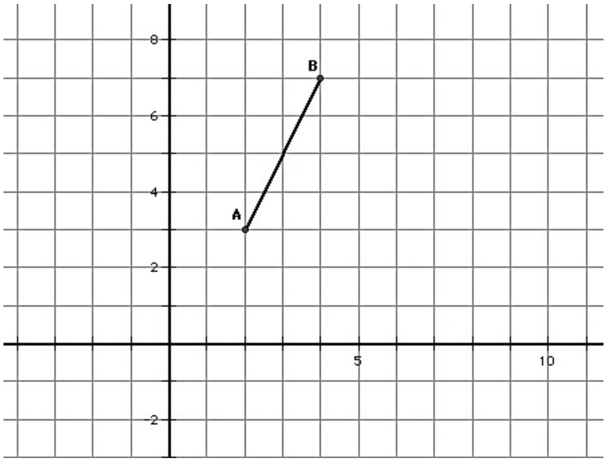


But what if a line or line segment is not horizontal or vertical? How do we determine the slope of a line or line segment that will be perpendicular to it?

<u>Experiment 1</u>

1. Consider the points A (2, 3) and B (4, 7) and the line segment, $\overline{AB}$, between them. What is the slope of this line segment?

2. Locate a third point C (x, y) on the coordinate grid, so the points A (2, 3), B (4, 7) and C (x, y) form the vertices of a right triangle, with $\overline{AB}$ as its hypotenuse.



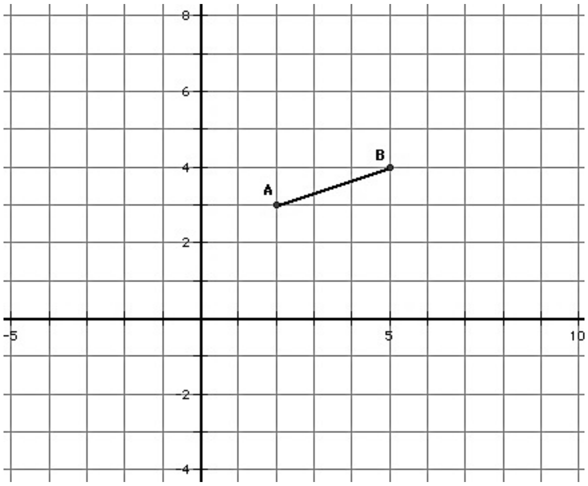
3. Explain how you know that the triangle you formed contains a right angle?

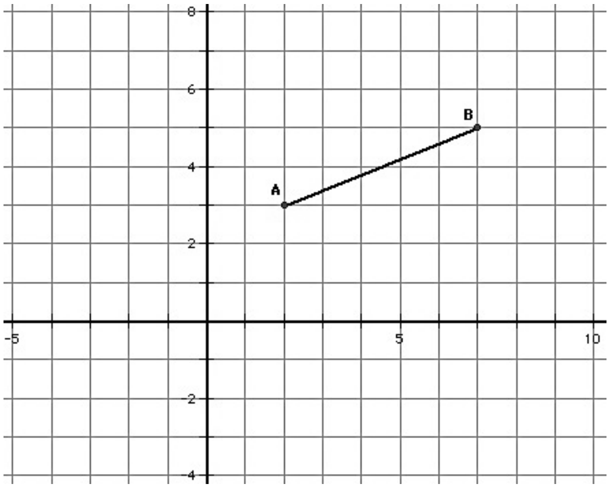
4. Now rotate this right triangle 90° about the vertex point (2, 3). Explain how you know that you have rotated the triangle 90°.

5. Compare the slope of the hypotenuse of this rotated right triangle with the slope of the hypotenuse of the pre-image. What do you notice?

Experiment 2

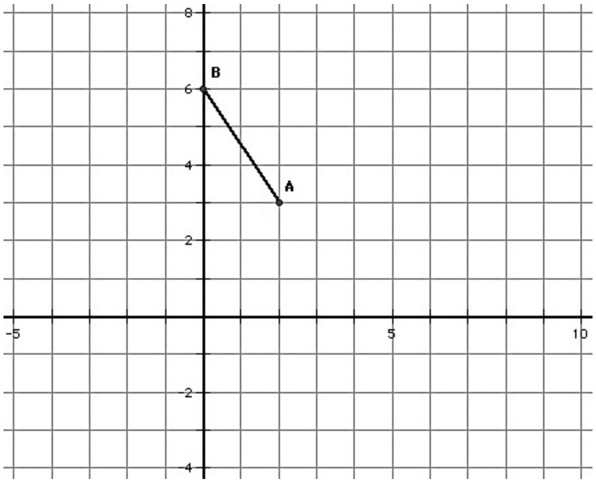
Repeat steps 1-5 above for the points A (2, 3) and B (5, 4).





Experiment 3

Repeat steps 1-5 above for the points A (2, 3) and B (7, 5).



Experiment 4

Repeat steps 1-5 above for the points A (2, 3) and B (0, 6).

Based on experiments 1-4, state an observation about the slopes of perpendicular lines.

While this observation is based on a few specific examples, can you create an argument or justification for why this is always true? (Note: You will examine a formal proof of this observation in a future module.)

6. 2 Is It Right? – Teacher Notes

A Solidify Understanding Task

Purpose: In this task students make a conjecture about the slopes of perpendicular lines. This observation will be formally proved in a later task, but the representation-based argument presented in this task suggests that this is a reasonable generalization across all cases. In the previous task, students encountered the idea of perpendicularity when they were asked to rotate the lizard 90°, and they may also have noticed the idea when they considered the relationship between the line of reflection and the points on the reflected image of the lizard. Students might have used the square corner of a piece of paper or a protractor to measure these right angles. In this task they consider how the coordinate grid can be used to determine if two lines are perpendicular. This is one example of the connections that can be made between coordinate geometry and transformations. It is powerful for students to be able to draw upon two different representational systems to think about the same ideas.

In this task, students also consider a definition of perpendicular that is related to reflections: *Two lines are perpendicular if they meet to form congruent adjacent angles*.

Core Standards Focus:

G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Related Standards: G.GPE.4

Standards for Mathematical Practice of focus in the task:

SMP 7 – Look for and make use of structure

SMP 8 – Look for and express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Class):

Students already have a sense of what it means for two lines to be perpendicular. Have them state their "definitions" and list their ideas on the board (e.g., they meet at right angles, they form 90° angles, if one is horizontal then the other is vertical, etc.) Then propose the following definition of perpendicular lines: *Lines are perpendicular if they meet to form congruent adjacent angles*. Ask students how this definition connects to the ideas they have recorded on the board about perpendicular lines (e.g., since the two adjacent angles form a straight angle measuring 180°, if the adjacent angles are also congruent they must each measure 90°; by definition, a right angle measures 90°). Ask students how they might justify this definition of perpendicular lines using transformations. Students might suggest folding or reflecting half of the line onto the other half, creasing the fold along the perpendicular line or using the perpendicular line as the line of reflection. Since reflections preserve angle measure, the image angle and its adjacent pre-image angle are congruent.

With these ideas about perpendicular lines activated for students, turn their attention to the experiments outlined in the task. Inform students that they are going to make a conjecture about lines that are perpendicular on a coordinate grid. For the sake of these experiments, we will agree that horizontal and vertical lines are perpendicular.

As part of the launch, make sure that students can accurately plot point C to form the third vertex of a right triangle ABC with segment AB as the hypotenuse. There are two positions where C can be located so that the legs of the right triangle lie along horizontal and vertical lines, thus guaranteeing that we have a right angle at C.

Explore (Small Group):

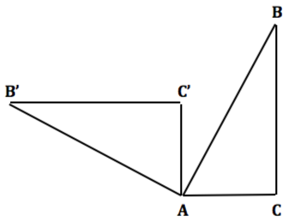
The exploration asks students to rotate a right triangle 90° around a vertex located at one of the acute angles. Since the legs of the right triangle are oriented along horizontal and vertical lines, students will know that they have rotated the triangle 90° when the leg adjacent to point A in the pre-image right triangle is oriented in the other direction—horizontal or vertical—in the resulting image right triangle. (See diagrams below)

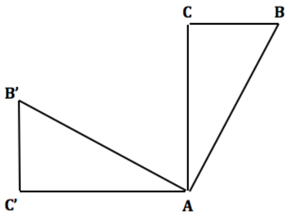
Watch for students to plot point C in a correct location, and listen for their arguments as to how they know they have rotated the right triangle 90°. Ask students how they know that the hypotenuse of the rotated right triangle is perpendicular to the hypotenuse of the original right triangle as a result of this rotation. It may be helpful to have students use a compass to connect points B and C with their image points along circles, centered at A—particularly if this was a strategy used to analyze the rotation in the previous task.

Make sure that students are recording the slopes of the hypotenuse of the pre-image right triangle and its resulting image right triangle correctly.

Discuss (Whole Class):

Post samples of student work that illustrate what happens when C is located below B and to the right of A, as well as what happens when C is located above A and to the left of B, as in the following diagram:





Have students articulate how they know when they have rotated the triangle 90° about point A by having them identify the horizontal/vertical relationship between leg AC and its image. Have them justify why this means that the hypotenuse of the rotated right triangle is also perpendicular to the hypotenuse of the original right triangle. Then have them state a conjecture about the slope of perpendicular lines based on these four examples. Remind students that four examples do not prove a conjecture, and have them suggest how they might generalize this work—that is, does the visual representation of the perpendicular hypotenuses hold, regardless of the size of the right triangle or the measures of the acute angles in the right triangle? This is an informal argument for an idea that will be formally proven in a task in a later module.

Aligned Ready, Set, Go: Transformations and Symmetry 6.2

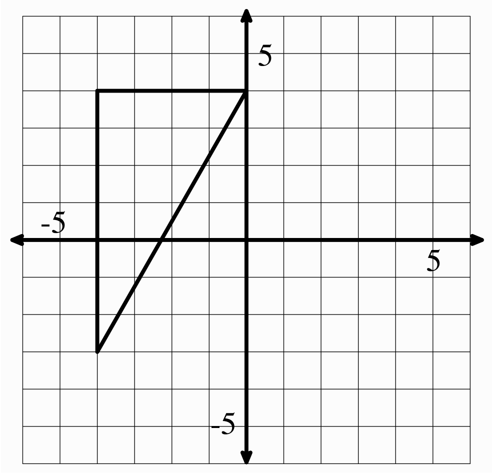
READY, SET, GO! Name Period Date

READY

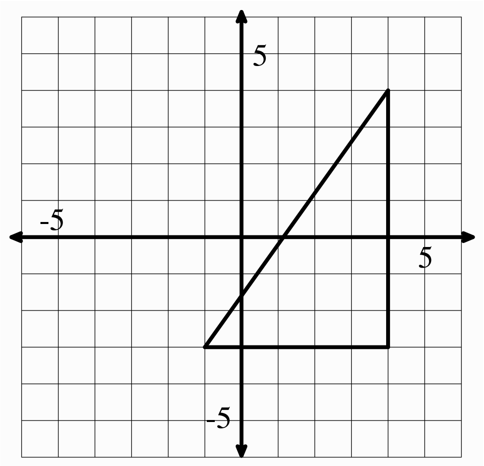
Topic: Finding Distance using Pythagorean Theorem

Use the coordinate grid to find the length of each side of the triangles provided. Give answers in exact form and where necessary rounded to the nearest hundredth.

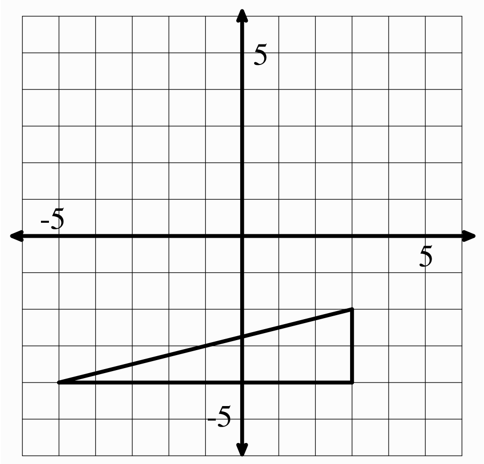
1.



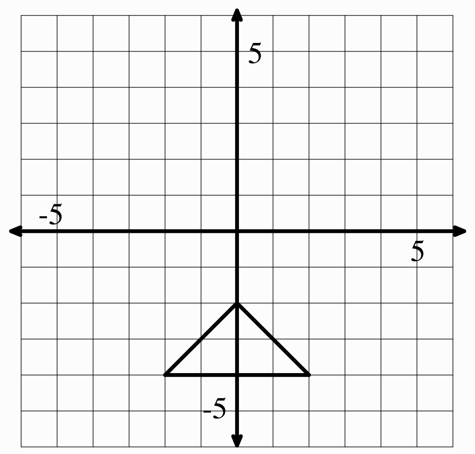
2.



3.



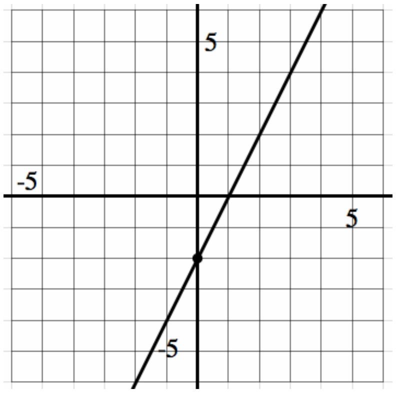
4.



SET

Topic: Slopes of parallel and perpendicular lines.

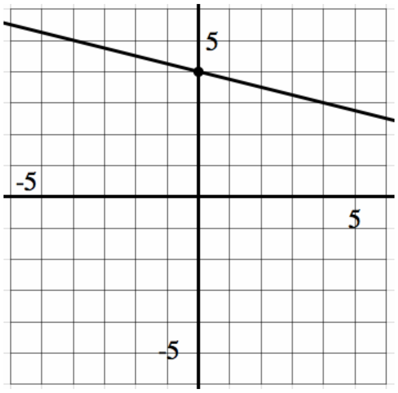
5. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

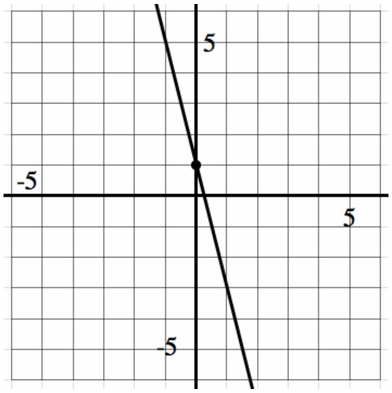
6. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

7. Graph a line *parallel* to the given line.

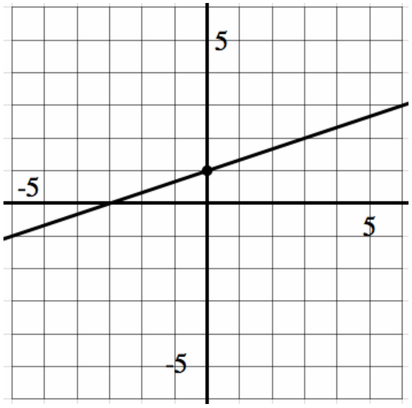


Equation for given line:

Equation for new line:

---

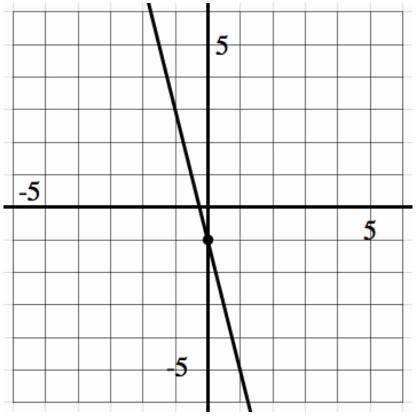
8. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

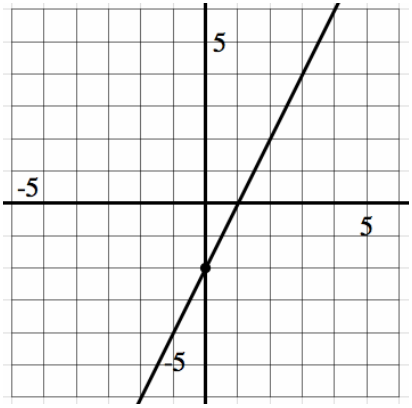
9. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

10. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

GO

Topic: Solve the following equations.

Solve each equation for the indicated variable.

11. $3(x-2)=5x+8$; Solve for x.

12. $-3+n=6n+22$; Solve for n.

13. $y-5=m(x-2)$; Solve for x.

14. $Ax+By=C$; Solve for y.

6. 3 Leap Frog

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http://openclipart.org/detail/33781/architetto

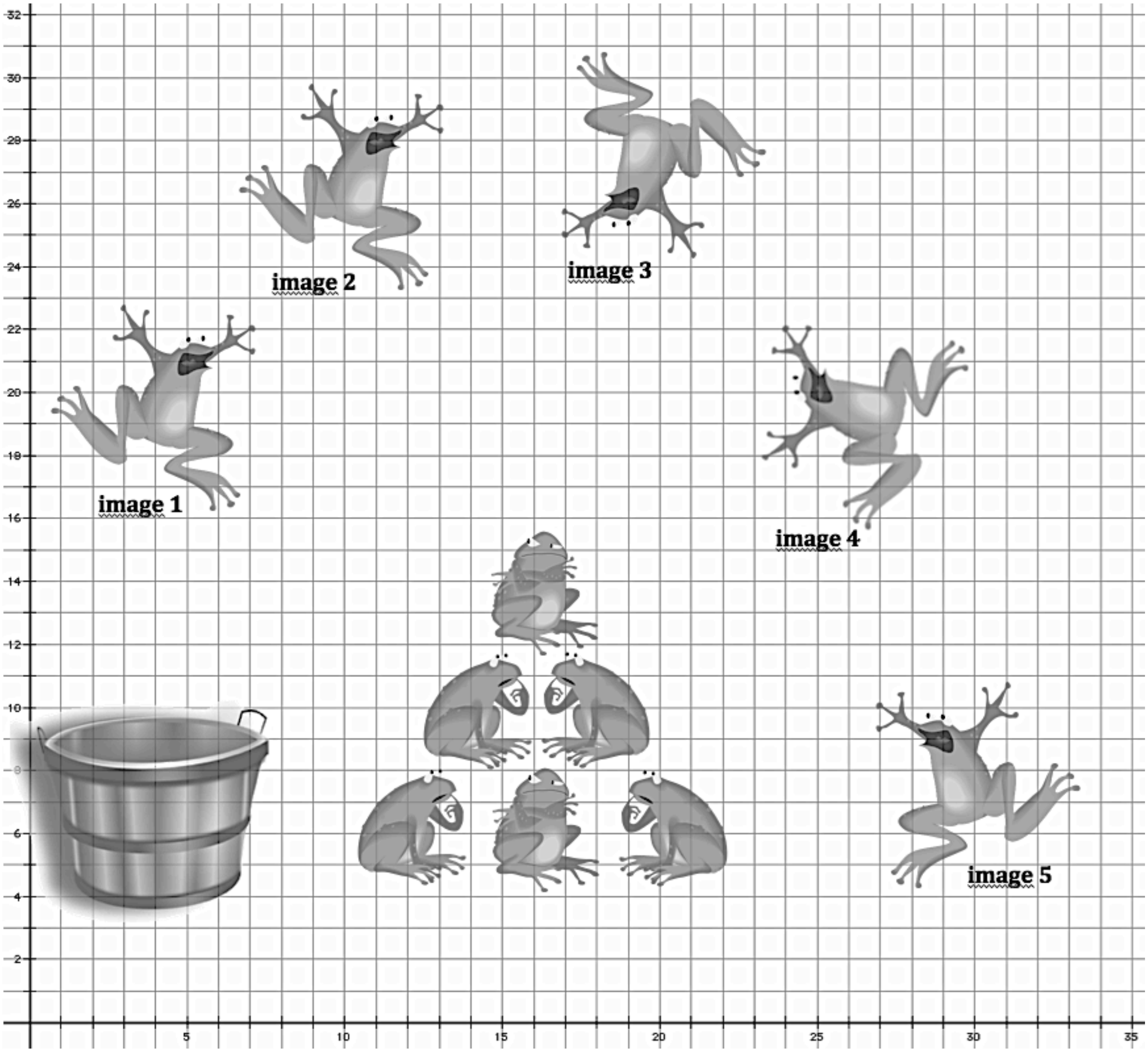
A Solidify Understanding Task

Josh is animating a scene in which a troupe of frogs is auditioning for the Animal Channel reality show, "The Bayou's Got Talent". In this scene the frogs are demonstrating their "leap frog" acrobatics act. Josh has completed a few key images in this segment, and now needs to describe the transformations that connect various images in the scene.

For each pre-image/image combination listed below, describe the transformation that moves the pre-image to the final image.

- If you decide the transformation is a rotation, you will need to give the center of rotation, the direction of the rotation (clockwise or counterclockwise), and the measure of the angle of rotation.
- If you decide the transformation is a reflection, you will need to give the equation of the line of reflection.
- If you decide the transformation is a translation you will need to describe the "rise" and "run" between pre-image points and their corresponding image points.
- If you decide it takes a combination of transformations to get from the pre-image to the final image, describe each transformation in the order they would be completed.

Pre-image	Final Image	Description
image 1	image 2	
image 2	image 3	
image 3	image 4	
image 1	image 5	
image 2	image 4	



images this page:
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CC0 http://openclipart.org/detail/33979/architetto
CC0 http://openclipart.org/detail/33985/architetto
CC0 http://openclipart.org/detail/170806/hatalar205

6. 3 Leap Frog – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to continue solidifying the definitions of the rigid-motion transformations. Building on the work started in *Leaping Lizards*, students describe translations, rotations and reflections in terms of the important features that define these transformations, such as the distance from the center of rotation or from the line of reflection. Students will also surface the idea that it may take a combination of transformations to move from one image to another.

Core Standards Focus:

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.5 Specify a sequence of transformations that will carry a given figure onto another.

Related Standards: G.CO.1, G.CO.2, G.CO.6

Standard for Mathematical Practice of focus in the task:

SMP 7 – Look for and make use of structure

The Teaching Cycle:

Launch (Whole Class):

Remind students that in *Leaping Lizards* they were trying to find where an image would end up after undergoing a specified transformation. In that task the transformation was described and they had to locate the image. In this task the work is reversed: the image and its pre-image are specified, and they have to describe the transformation that carries one onto the other. Review with students the bulleted list of features they should include in a description of each type of transformation. Also point out the possibility that it may take more than one transformation to get from one figure to another. In such a case, they are to describe each transformation in the sequence of transformations they would perform.

Explore (Small Group):

Since no anchor points are specified, students will need to select a few points to pay attention to, however they should intuitively recognize that all of the points on the pre-image get transformed in the same way. (Technically every point in the plane moves to a new location except for the points on a line of reflection or the point at the center of a rotation.) Students might focus on points that lie on lattice points of the grid, such as the center of the frog's tongue or the end of the middle toe on the left foot.

Listen for ways that students are describing the transformations, and that they are noting the important characteristics of each transformation, such as finding both the center and the angle of rotation for figures that have been rotated. The last two pre-image/image pairs will require more than one transformation—a reflection and translation in the 4th pair, and a translation and rotation in the 5th pair—however, there are many possible combinations of transformations that will work. These last two problems are setting up the work of a later task, *Can You Get There From Here*, and it is not necessary at this time that all students successfully finish these last two problems.

Discuss (Whole Class):

Select students to share their descriptions on each of the five pre-image/image pairs. Emphasize the essential features of each transformation in these discussions: every point in a translation moves the same distance along parallel lines; every point in a rotation moves through a specified angle along circular arcs that all have the same center point; and, the line of reflection is the perpendicular bisector of the segments connecting image and pre-image points in a reflection. Students will write formal definitions of the three rigid-motion transformations in the next task, so in this discussion it is sufficient for students to be noticing these details and supporting their claims as they analyze the transformations that carry one figure onto another.

There are multiple sequences of transformations that work for the 4th and 5th pre-image/image pairs. If available, share a couple of descriptions for each. For example, to get from image 1 to image 5 you might reflect image 1 about some vertical line, then translate the reflected image until it coincides with image 5; or, you might translate image 1 until you can identify a mirror line in which to reflect the image to get it to coincide with image 5.

Aligned Ready, Set, Go: Transformations and Symmetry 6.3

6.3

READY, SET, GO! Name Period Date

READY

Topic: Rotations and Reflections of figures.

In each problem there will be a pre-image and several images based on the give pre-image. Determine which of the images are rotations of the given pre-image and which of them are reflections of the pre-image. If an image appears to be created as the result of a rotation and a reflection then state both. (Compare all images to the pre-image.)

1.

Pre-Image

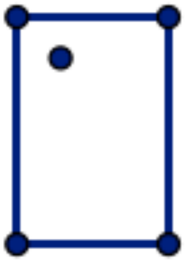


Image A

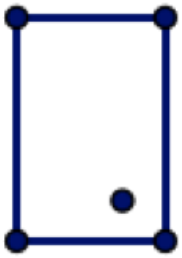


Image B

Image C

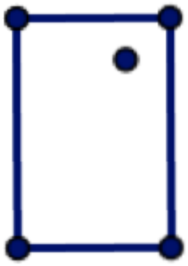


Image D

2.

Pre-Image

Image A

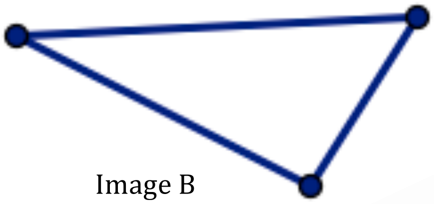


Image B

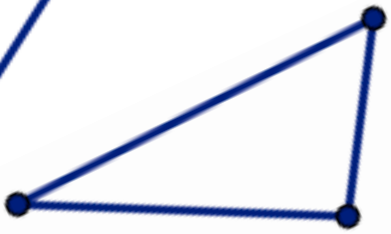


Image D

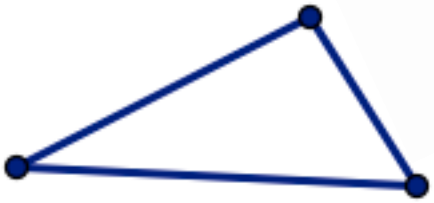


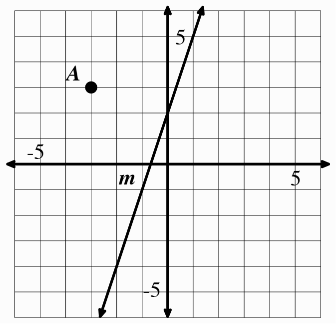
Image C

SET

Topic: Reflecting and rotating points.

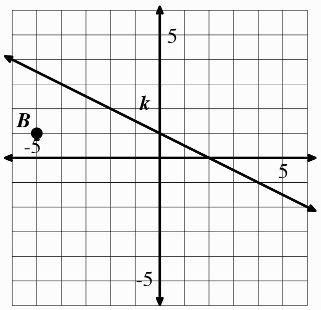
On each of the coordinate grids there is a labeled point and line. Use the line as a line of reflection to reflect the given point and create its reflected image over the line of reflection. (Hint: points reflect along paths perpendicular to the line of reflection. Use perpendicular slope!)

3.



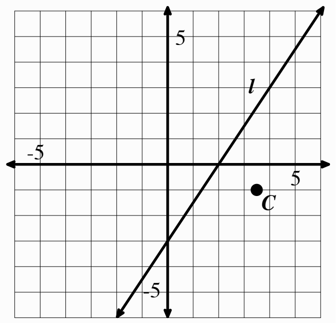
Reflect point ***A*** over line ***m*** and label the image ***A'***

4.



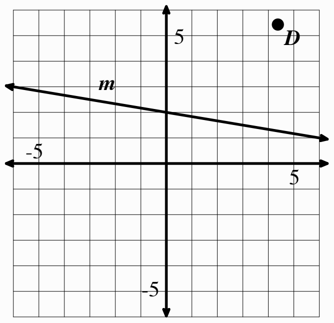
Reflect point ***B*** over line ***k*** and label the image ***B'***

5.



Reflect point ***C*** over line ***l*** and label the image ***C'***

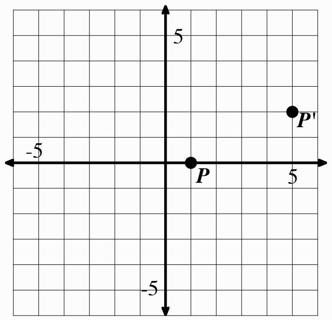
6.



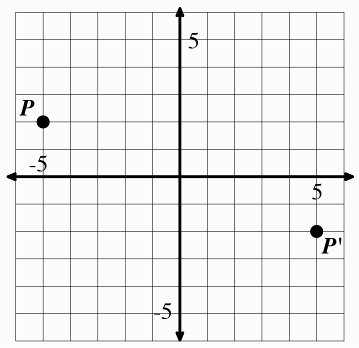
Reflect point ***D*** over line ***m*** and label the image ***D'***

For each pair of point, *P* and *P'* draw in the line of reflection that would need to be used to reflect *P* onto *P'*. Then find the equation of the line of reflection.

7.

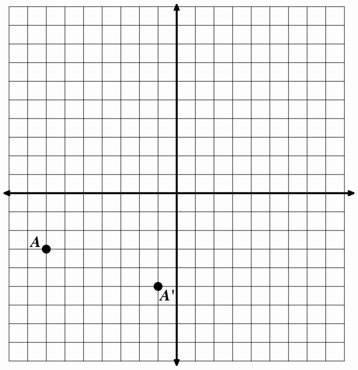


8.

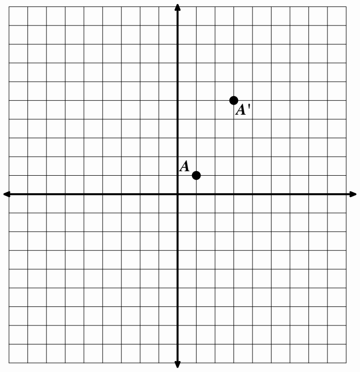


For each pair of point, A and A' draw in the line of reflection that would need to be used to reflect A onto A'. Then find the equation of the line of reflection.

9.



10.



GO

Topic: Slopes of parallel and perpendicular lines and finding slope and distance between two points.

For each linear equation <u>write the slope</u> of a line parallel to the given line.

11. $y = -3x + 5$

12. $y = 7x - 3$

13. $3x - 2y = 8$

For **each linear equation <u>write the slope</u> of a line perpendicular to the given line.**

14. $y = -\frac{2}{7}x + 5$

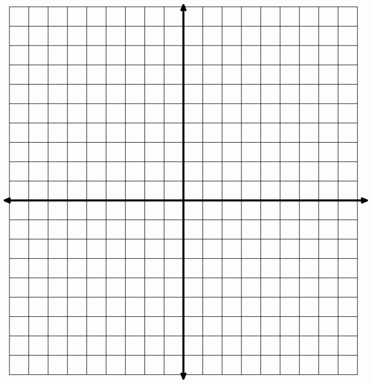
15. $y = \frac{1}{5}x - 4$

16. $3x + 5y = -15$

Find the *slope* between each pair of points. Then, using the Pythagorean Theorem, find the *distance* between each pair of points. You may use the graph to help you as needed.

17. (-2, -3) (1, 1)
a. Slope: b. Distance:

18. (-7, 5) (-2, -7)
a. Slope: b. Distance:



6. 4 Leap Year

A Practice Understanding Task

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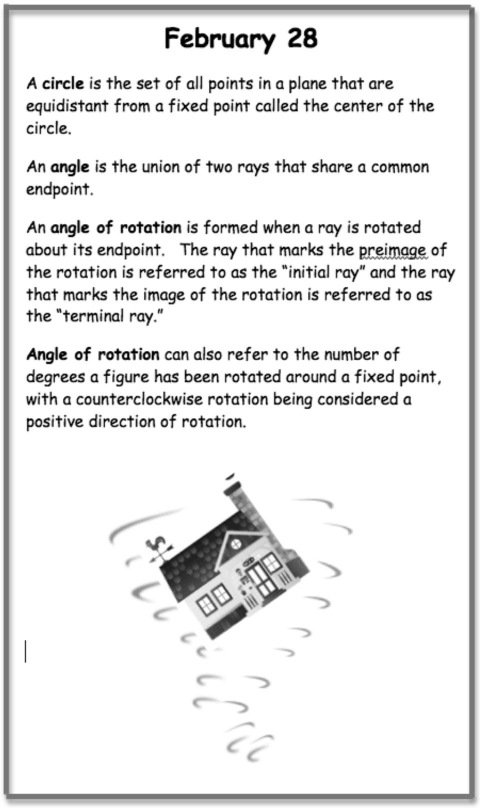
Carlos and Clarita are discussing their latest business venture with their friend Juanita. They have created a daily planner that is both educational and entertaining. The planner consists of a pad of 365 pages bound together, one page for each day of the year. The planner is entertaining since images along the bottom of the pages form a flip-book animation when thumbed through rapidly. The planner is educational since each page contains some interesting facts. Each month has a different theme, and the facts for the month have been written to fit the theme. For example, the theme for January is astronomy, the theme for February is mathematics, and the theme for March is ancient civilizations. Carlos and Clarita have learned a lot from researching the facts they have included, and they have enjoyed creating the flip-book animation.

The twins are excited to share the prototype of their planner with Juanita before sending it to printing. Juanita, however, has a major concern. "Next year is leap year," she explains, "you need 366 pages." So now Carlos and Clarita have the dilemma of needing to create an extra page to insert between February 28 and March 1. Here are the planner pages they have already designed.



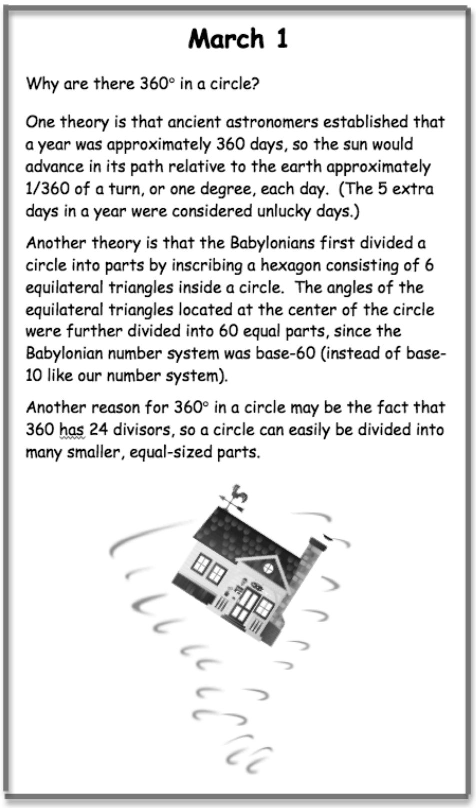
February 28

A **circle** is the set of all points in a plane that are equidistant from a fixed point called the center of the circle.

An **angle** is the union of two rays that share a common endpoint.

An **angle of rotation** is formed when a ray is rotated about its endpoint. The ray that marks the preimage of the rotation is referred to as the "initial ray" and the ray that marks the image of the rotation is referred to as the "terminal ray."

Angle of rotation can also refer to the number of degrees a figure has been rotated around a fixed point, with a counterclockwise rotation being considered a positive direction of rotation.



March 1

Why are there 360° in a circle?

One theory is that ancient astronomers established that a year was approximately 360 days, so the sun would advance in its path relative to the earth approximately 1/360 of a turn, or one degree, each day. (The 5 extra days in a year were considered unlucky days.)

Another theory is that the Babylonians first divided a circle into parts by inscribing a hexagon consisting of 6 equilateral triangles inside a circle. The angles of the equilateral triangles located at the center of the circle were further divided into 60 equal parts, since the Babylonian number system was base-60 (instead of base-10 like our number system).

Another reason for 360° in a circle may be the fact that 360 has 24 divisors, so a circle can easily be divided into many smaller, equal-sized parts.

Part 1

Since the theme for the facts for February is mathematics, Clarita suggests that they write formal definitions of the three rigid-motion transformations they have been using to create the images for the flip-book animation.

How would you complete each of the following definitions?

1. A translation of a set of points in a plane . . .

2. A rotation of a set of points in a plane . . .

3. A reflection of a set of points in a plane . . .

4. Translations, rotations and reflections are rigid motion transformations because . . .

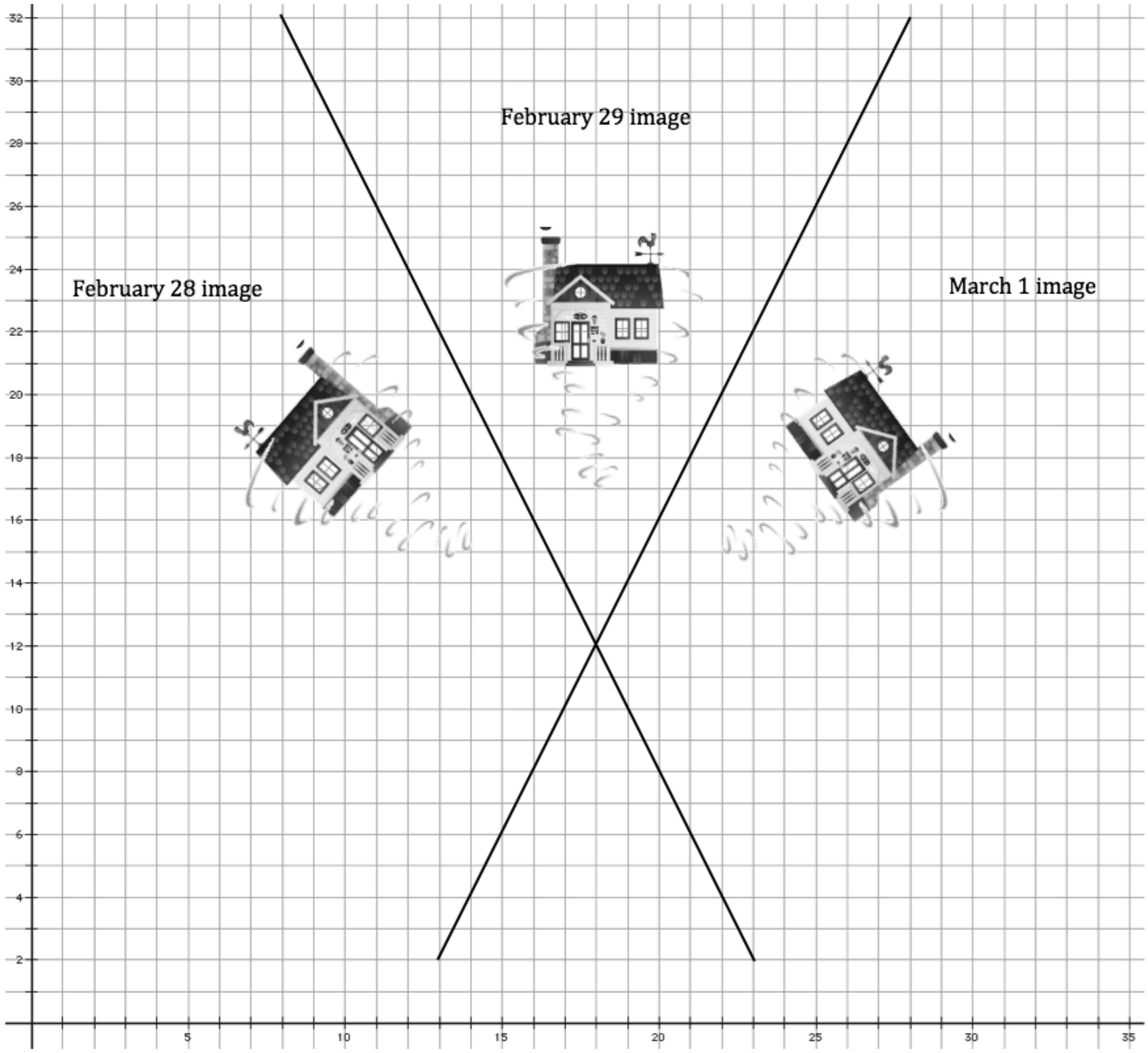
Carlos and Clarita used these words and phrases in their definitions: perpendicular bisector, center of rotation, equidistant, angle of rotation, concentric circles, parallel, image, pre-image, preserves distance and angle measures within the shape. Revise your definitions so that they also use these words or phrases.

Part 2

In addition to writing new facts for February 29, the twins also need to add another image in the middle of their flip-book animation. The animation sequence is of Dorothy's house from the Wizard of Oz as it is being carried over the rainbow by a tornado. The house in the February 28 drawing has been rotated to create the house in the March 1 drawing. Carlos believes that he can get from the February 28 drawing to the March 1 drawing by reflecting the February 28 drawing, and then reflecting it again.

Verify that the image Carlos inserted between the two images that appeared on February 28 and March 1 works as he intended. For example,

- What convinces you that the February 29 image is a reflection of the February 28 image about the given line of reflection?
- What convinces you that the March 1 image is a reflection of the February 29 image about the given line of reflection?
- What convinces you that the two reflections together complete a rotation between the February 28 and March 1 images?



Images this page:

http://openclipart.org/detail/168722/simple-farm-pack-by-viscious-speed

www.clker.com/clipart.tornado-gray

6. 4 Leap Year – Teacher Notes

A Practice Understanding Task

Purpose: In this task students will write precise definitions for the three rigid-motion transformations, based on the observations they have made in the previous tasks in this learning cycle. To prepare students for writing their own definitions, they will study the language used in the definitions given for *circle, angle,* and *angle of rotation.* They will also write a definition for the word *degree* based on the information given. In part 2 of this task students will use their definitions to justify that multiple images have been correctly drawn based on specified transformations. As part of this task students will also explore the idea that two consecutive reflections produce a rotation when the lines of reflection are not parallel.

Core Standards Focus:
G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Related Standards: G.CO.5, G.CO.6

Standards for Mathematical Practice of focus in the task:

SMP 6 – Attend to precision

SMP 7 – Look for and make use of structure

Additional Resources for Teachers:
An answer key for the questions in the task can be found as a separate page at the end of these teacher notes. It is recommended that you work through the task yourself before consulting the answer key to develop a better sense of how your students might engage in the task.

The Teaching Cycle:

Launch (Whole Class):
Read and clarify the definition of *circle* given on the February 28 calendar page. Ask how the words used identify only the points on a circle and omit every other point in the plane from being identified as points belonging to the circle. Ask what would happen if the words "in a plane" were removed from the definition.

Ask students about the images formed in their minds from the two definitions for *angle* and *angle of rotation*. Make sure they can use either of these definitions to describe an angle. Then have students read the historical notes considering why there are 360° in a circle. (What does this really mean—where are each of the 360 degrees located "in the circle"?) Based on this discussion, ask students to write a definition for the word degree. Press for something like, "A degree is the measure of an angle of rotation that is equal to 1/360 of a complete rotation around a fixed point."

After emphasizing the precision of language in a formal definition, have partners write definitions for each of the three rigid-motion transformations. Let students know that you will be formalizing these definitions as a class before working on part 2 of the task.

Explore (Small Group), part 1:

Give students time to consider all three definitions. Listen for language about distance and direction in each definition. Students who finish their three definitions can also work on writing a definition for *rigid-motion transformation,* which should include the idea that rigid-motion transformations preserve distance and angle measurements.

Discuss (Whole Class), part 1:

As a whole class, write definitions that students agree will define each transformation with precision. The essential elements of each definition are as follows:

- Translation: *translations move points the same distance and direction along lines that are parallel to each other*
- Rotation: *rotations move points the same direction along concentric circles and through the same angle of rotation*
- Reflection: *reflections move points across a specified line of reflection so that the line of reflection is the perpendicular bisector of each line segment connecting corresponding pre-image and image points*

Explore (Small Group), part 2:

In part 2, students should justify their answers to the three questions by using the definitions they wrote for the three rigid-motion transformations. Here are some additional prompting questions, if students are not attending to the definitions:

- What evidence can you provide that the first given line is a line of reflection? How can we convince ourselves that the line is the perpendicular bisector of the line segments connecting pre-image and image points of the first two drawings of Dorothy's house? (Hint: You may want to use the Pythagorean theorem and also think about slopes.)
- What evidence can you provide that the second line is the perpendicular bisector of the line segments connecting pre-image and image points of the last two drawings of Dorothy's house?
- Where is the center of this rotation located? What evidence can you provide that pre-image and image points are equidistant from the center of rotation?

Discuss (Whole Class), part 2:

The focus of this discussion is on using the definitions to justify the transformations that Carlos claims to have used. Therefore, students will need to verify that corresponding vertex points on the image and pre-image satisfy the conditions defining the particular transformation. For example, is the line of reflection the perpendicular bisector of each line segment joining a vertex point on the image with its corresponding vertex point on the pre-image? How did students verify this? (Note: the question of determining if a point is a midpoint of a line segment may come up in this discussion. If so, allow students to discuss how they think they might find the coordinates of the midpoint of a segment when they know the coordinates of the endpoints.)

Once students are convinced that these two reflections produced a rotation you might ask them to consider if this would always be the case, and what makes them think this might be so, or under what conditions it might not be so.

Aligned Ready, Set, Go: Transformations and Symmetry 6.4

READY, SET, GO! Name Period Date

READY

Topic: Defining polygons and their attributes

For each of the geometric words below write a definition of the object that addresses the essential elements.

1. Quadrilateral:

2. Parallelogram:

3. Rectangle:

4. Square:

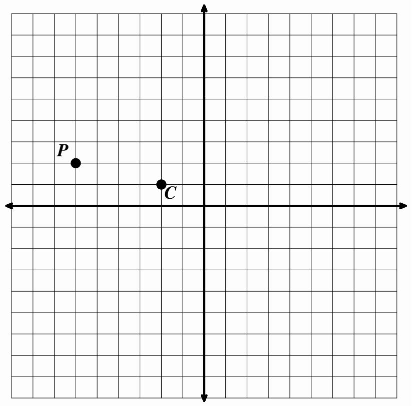
5. Rhombus:

6. Trapezoid:

SET

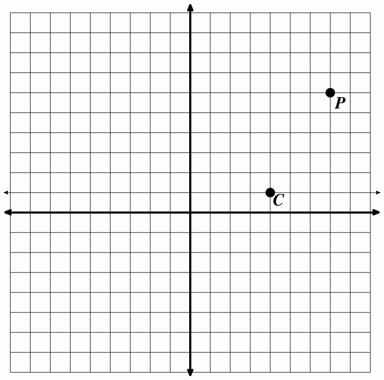
Topic: Reflections and rotations, composing reflections to create a rotation.

7.



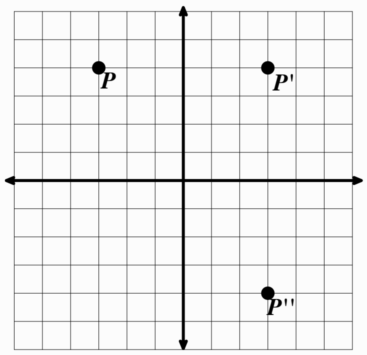
Use the center of rotation point ***C*** and rotate point ***P*** clockwise around it 90^0. Label the image ***P'***. With point ***C*** as a center of rotation also rotate point ***P*** 180^0. Label this image ***P''***.

8.



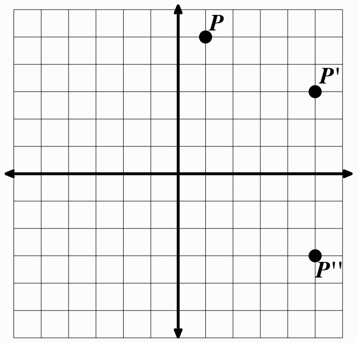
Use the center of rotation point ***C*** and rotate point ***P*** clockwise around it 90⁰. Label the image ***P’***.
With point ***C*** as a center of rotation also rotate point ***P*** 180⁰. Label this image ***P’’***.

9.



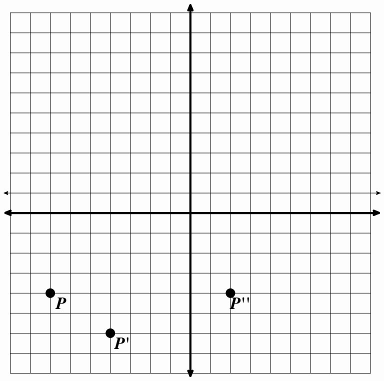
a. What is the equation for the line for reflection that reflects point ***P*** onto ***P’***?
b. What is the equation for the line of reflections that reflects point ***P’*** onto ***P’’*** ?
c. Could ***P’’*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

10.



a. What is the equation for the line for reflection that reflects point ***P*** onto ***P’***?
b. What is the equation for the line of reflections that reflects point ***P’*** onto ***P’’*** ?
c. Could ***P’’*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

11.



a. What is the equation for the line for reflection that reflects point ***P*** onto ***P’***?
b. What is the equation for the line of reflections that reflects point ***P’*** onto ***P’’*** ?
c. Could ***P’’*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

GO

Topic: Rotations about the origin.

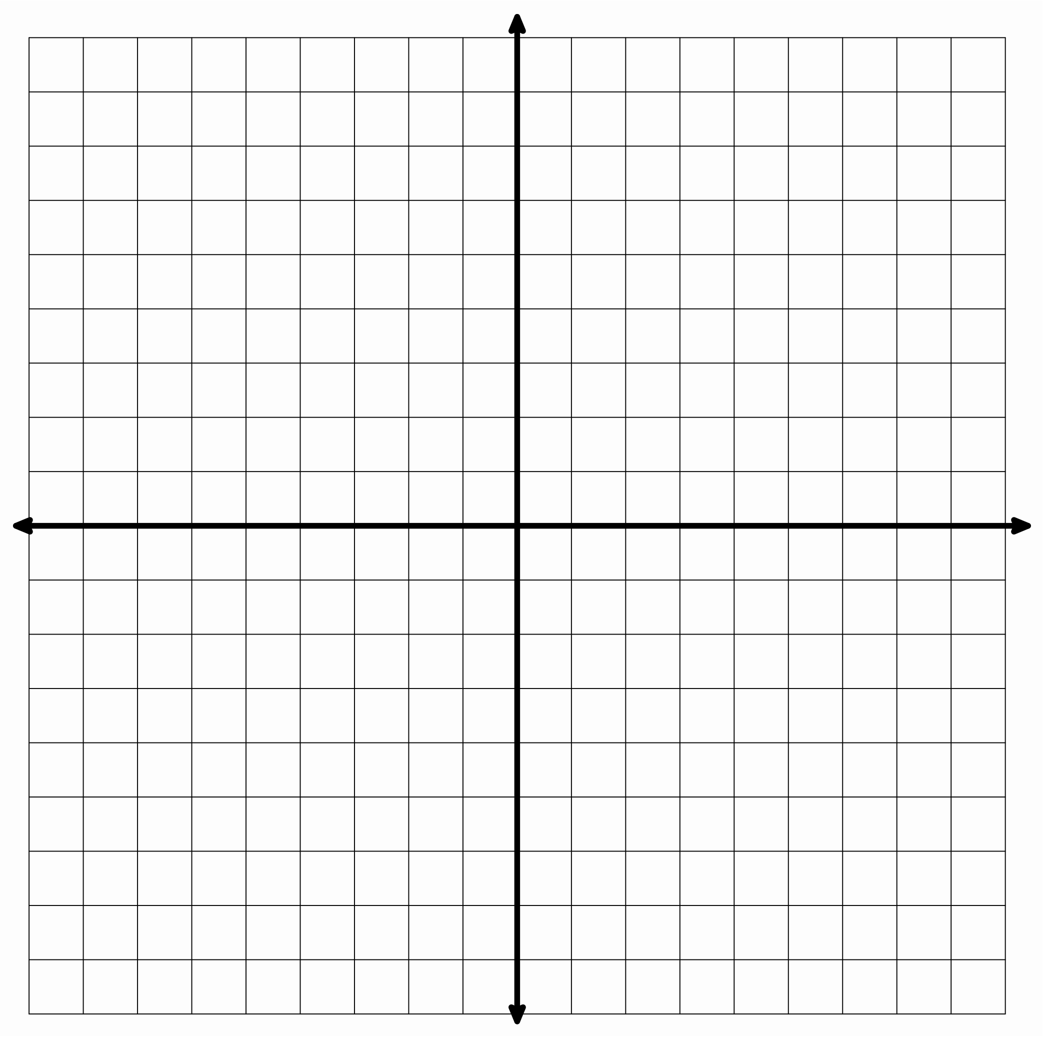
Plot the given coordinate and then perform the indicated rotation in a clockwise direction around the origin, the point (0 , 0), and plot the image created. State the coordinates of the image.

12. Point ***A*** (4, 2) rotate 180^0
Coordinates for Point ***A'*** (___ , ___)

13. Point ***B*** (-5, -3) rotate 90^0 clockwise
Coordinates for Point ***B'*** (___ , ___)

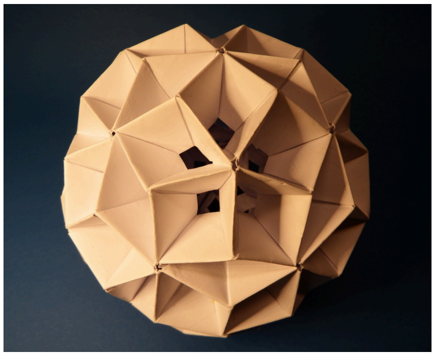
14. Point ***C*** (-7, 3) rotate 180^0
Coordinates for Point ***C'*** (___ , ___)

15. Point ***D*** (1, -6) rotate 90^0 clockwise
Coordinates for Point ***D'*** (___ , ___)



6. 5 Symmetries of Quadrilaterals

A Develop Understanding Task



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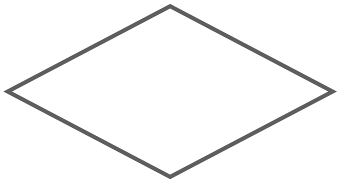
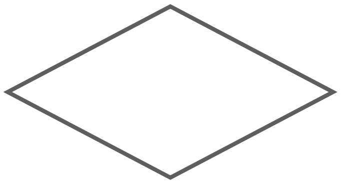
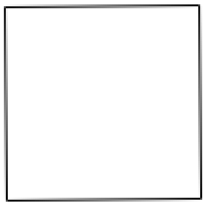
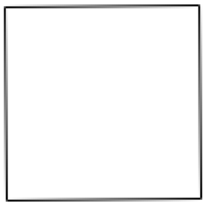
A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**.

Every four-sided polygon is a **quadrilateral**. Some quadrilaterals have additional properties and are given special names like squares, parallelograms and rhombuses. A **diagonal** of a quadrilateral is formed when opposite vertices are connected by a line segment. Some quadrilaterals are symmetric about their diagonals. Some are symmetric about other lines. In this task you will use rigid-motion transformations to explore line symmetry and rotational symmetry in various types of quadrilaterals.

For each of the following quadrilaterals you are going to try to answer the question, "Is it possible to reflect or rotate this quadrilateral onto itself?" As you experiment with each quadrilateral, record your findings in the following chart. Be as specific as possible with your descriptions.

Defining features of the quadrilateral	**Lines of symmetry that reflect the quadrilateral onto itself**	**Center and angles of rotation that carry the quadrilateral onto itself**
A **rectangle** is a quadrilateral that contains four right angles.		
A **parallelogram** is a quadrilateral in which opposite sides are parallel.		

A **rhombus** is a quadrilateral in which all sides are congruent.		
A **square** is both a rectangle and a rhombus.		

A **trapezoid** is a quadrilateral with one pair of opposite sides parallel. Is it possible to reflect or rotate a trapezoid onto itself?

Draw a trapezoid based on this definition. Then see if you can find:

- any lines of symmetry, or
- any centers of rotational symmetry,

that will carry the trapezoid you drew onto itself.

If you were unable to find a line of symmetry or a center of rotational symmetry for your trapezoid, see if you can sketch a different trapezoid that might possess some type of symmetry.

6. 5 Symmetries of Quadrilaterals – Teacher Notes

A Develop Understanding Task

Purpose: In this learning cycle, students focus on classes of geometric figures that can be carried onto themselves by a transformation—figures that possess a line of symmetry or rotational symmetry. In this task the idea of "symmetry" is surfaced relative to finding lines that reflect a figure onto itself, or determining if a figure has rotational symmetry by finding a center of rotation about which a figure can be rotated onto itself. This work is intended to be experimental (e.g., folding paper, using transparencies, using technology, measuring with ruler and protractor, etc.), with the definitions of reflection and rotation being called upon to support students' claims that a figure possesses some type of symmetry. The particular classes of geometric figures considered in this task are various types of quadrilaterals.

Core Standards Focus:

G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure.

Related Standards: G.CO.4, G.CO.5

Standards for Mathematical Practice:

SMP 7 – Look for and make use of structure

Additional Resources for Teachers:

A copy of the chart from the task can be found at the end of this set of teacher notes. This chart can be printed for use with students who may be accessing the task on a computer or tablet. In addition, each of the quadrilaterals has been provided on a master copy that can be reproduced and distributed to students. Students can cut out the individual quadrilaterals in order to manipulate them, such as folding them along a diagonal or rotating them about a point.

The Teaching Cycle:

Launch (Whole Class):

Discuss the concept of symmetry in terms of finding a rigid-motion transformation that carries a geometric figure onto itself. Help students recognize that two such types of symmetries exist: a line of symmetry might exist that reflects a figure onto itself, or a center of rotation might exist about which a figure might be rotated onto itself. Also remind students of the definitions of "quadrilateral" and "diagonal" giving in the introduction of the task.

Students are to experiment with the various types of quadrilaterals listed in the task to determine if they can find any lines of symmetry or centers and angles of rotation that will carry the given quadrilateral onto itself. You will need to decide what tools to make available for this investigation. For example, you could provide cut-outs of each of the figures which would allow students to find lines of symmetry by folding the figures onto themselves (note that a handout of the set of figures is provided at the end of the teacher notes). This would also be a good task to support using dynamic geometry software programs, such as Geometer's Sketchpad or Geogebra. If you use technology, students will need to be provided with a set of well-constructed quadrilaterals, so they can focus on searching for lines of symmetry and centers of rotation, rather than on the construction of the geometric figures themselves.

Explore (Small Group):

Since students are dealing with classes of quadrilaterals, rather than individual quadrilaterals, in addition to finding the line of symmetry or the center and angle of a rotation, they should also provide some type of justification as to how they know that this symmetry exists for all members of the class. The given definitions for each quadrilateral should support making such an argument. For example, if students say that the diagonal of a square is a line of symmetry, they might note that distance and angle are preserved by this reflection since adjacent sides of a square are congruent and opposite angles of a square are both right angles. Try to press students to move away from basing their decisions about lines of symmetry or centers of rotation simply on intuition and "it looks like it works" type of justification, and towards arguments based on the definitions of the rigid-motion transformations and the defining properties of the geometric figures.

Look for students who find all of the lines of symmetry or describe all of the possible rotations that might exist for each type of quadrilateral. For example, a square has two different types of lines of symmetry: the diagonals, and the lines passing through the midpoints of opposite sides. Hence, there are four lines of symmetry in a square. The point where the two diagonals of a square intersect locates the center of rotation for describing the rotational symmetry of a square. A square can be rotated 90°, 180°, 270° or 360° about this center of rotation. Each reflection or rotation carries a segment onto another segment of the same length, or a right angle onto another right angle, due to the defining properties of a square.

Watch for misconceptions that might arise, such as the diagonals of a parallelogram being identified as lines of reflection. Experimentation with technology or paper folding will disprove this conjecture, but it is important to have students describe why they initially thought it was true, and how they might convince themselves that this conjecture isn't true based on the definition of a reflection.

Discuss (Whole Class):

Start the discussion by asking if the diagonals of a parallelogram are lines of symmetry for the parallelogram. Ask students how they know a diagonal is not a line of symmetry. If their only arguments are experimental in nature (e.g., "if you fold it on the diagonal, opposite vertices don't match up" or "when I used the diagonal as a mirror line in GSP it didn't work"), press for an explanation based on the essential ideas of a reflection (e.g., "since adjacent sides of a parallelogram aren't necessarily congruent, we can't find a line of reflection that will reflect a side of a parallelogram onto an adjacent side so that distance is preserved").

Ask students how they determined where the center of rotation is located in various classes of quadrilaterals. This should lead to a discussion about the point of intersection of the diagonals.

Ask students which type of quadrilateral has the most types of symmetry, and why this might be so.

Aligned Ready, Set, Go: Transformations and Symmetry 6.5

6.5

READY, SET, GO! Name Period Date

READY

Topic: Polygons, definition and names

1. What is a polygon? Describe in your own words what a polygon is.

2. Fill in the names of each polygon based on the number of sides the polygon has.

Number of Sides	Name of Polygon
3	
4	
5	
6	
7	
8	
9	
10	

SET

Topic: Kites, Lines of symmetry and diagonals.

3. One quadrilateral with special attributes is a kite. Find the geometric definition of a kite and write it below along with a sketch. (You can do this fairly quickly by doing a search online.)

4. Draw a kite and draw all of the lines of reflective symmetry and all of the diagonals.

Lines of Reflective Symmetry **Diagonals**

5. List all of the rotational symmetry for a kite.

6. Are lines of symmetry also diagonals in a polygon? Explain.

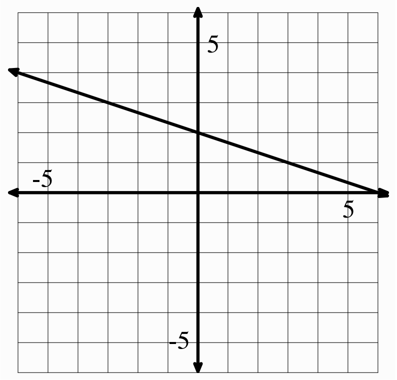
6. Are all diagonals also lines of symmetry in a polygon? Explain.

7. Which quadrilaterals have diagonals that are not lines of symmetry? Name some and draw them.

8. Do parallelograms have diagonals that are lines of symmetry? If so, draw and explain. If not draw and explain.

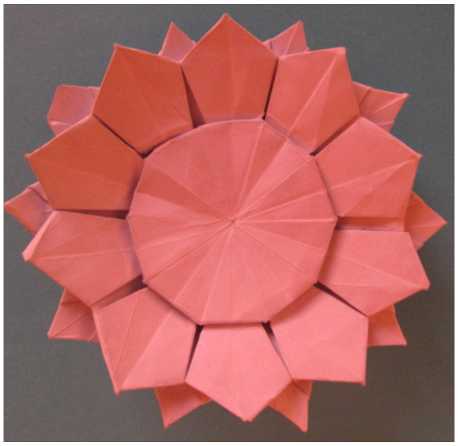
GO

Topic: Equations for parallel and perpendicular lines.

<table>
<tr><th></th><th>Find the equation of a line PARALLEL to the given info and through the indicated y-intercept.</th><th>Find the equation of a line PERPENDICULAR to the given line and through the indicated y-intercept.</th></tr>
<tr><td>9. Equation of a line:
$y = 4x + 1.$</td><td>a. Parallel line through point (0, -7):</td><td>b. Perpendicular to the line line through point (0 , -7):</td></tr>
<tr><td>10. Table of a line:
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>3</td><td>-8</td></tr>
<tr><td>4</td><td>-10</td></tr>
<tr><td>5</td><td>-12</td></tr>
<tr><td>6</td><td>-14</td></tr>
</table></td><td>a. Parallel line through point (0 , 8):</td><td>b. Perpendicular to the line through point (0 , 8):</td></tr>
<tr><td>11. Graph of a line:

</td><td>a. Parallel line through point (0 , -9):</td><td>b. Perpendicular to the line through point (0 , -9):</td></tr>
</table>

6.6 Symmetries of Regular Polygons

A Solidify Understanding Task



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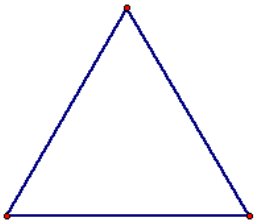
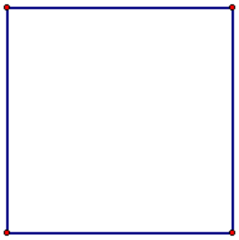
A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**. A **diagonal of a polygon** is any line segment that connects non-consecutive vertices of the polygon.

For each of the following regular polygons, describe the rotations and reflections that carry it onto itself: (be as specific as possible in your descriptions, such as specifying the angle of rotation)

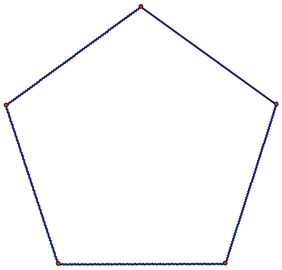
1. An equilateral triangle



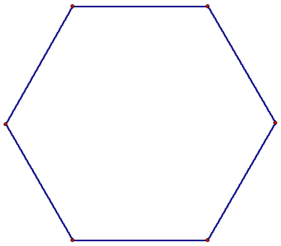
2. A square



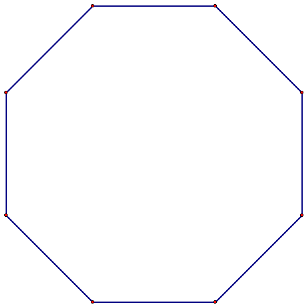
3. A regular pentagon



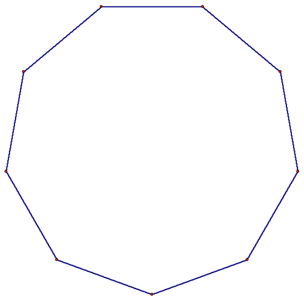
4. A regular hexagon



5. A regular octagon



6. A regular nonagon



What patterns do you notice in terms of the number and characteristics of the lines of symmetry in a regular polygon?

What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon?

6.6 Symmetries of Regular Polygons – Teacher Notes

A Solidify Understanding Task

Purpose: In this task, students continue to focus on classes of geometric figures that can be carried onto themselves by a transformation—figures that possess a line of symmetry or rotational symmetry. Students solidify the idea of "symmetry" relative to finding lines that reflect a figure onto itself, or determining if a figure has rotational symmetry by finding a center of rotation about which a figure can be rotated onto itself. They also look for and describe the structure that determines if a figure possesses some type of symmetry. This work can be experimental (e.g., folding paper, using transparencies, using technology, measuring with ruler and protractor, etc.), or theoretical, with the definitions of reflection and rotation being called upon to support students' claims that a figure possesses some type of symmetry.

The particular classes of geometric figures considered in this task are various types of regular polygons, and students will look for patterns in the types and numbers of lines of symmetry a regular polygon with an odd number of sides possesses, versus those with an even number of sides. They should also note a pattern between the smallest angle of rotation that carries a regular polygon onto itself and the number of sides of the polygon.

Core Standards Focus:
G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure.

Related Standards: G.CO.4, G.CO.5

Standards for Mathematical Practice of focus in the task:

SMP 7 – Look for and make use of structure

SMP 6 – Attend to precision

SMP 8 – Look for and express regularity in repeated reasoning

Additional Resources for Teachers:

Images of each of the regular polygons have been provided on a master copy that can be reproduced and distributed to students. Students can cut out the individual polygons in order to manipulate them, or they can trace them on tracing paper in order to fold them along a diagonal or rotate them about a point.

The Teaching Cycle:

Launch (Whole Class):

Based on the level of student thinking that exists in your class, decide if you want the work of this task to be experimental (e.g., using tools, cut-outs and/or dynamic geometric software), or theoretical, making decisions based on the definitions of reflections and rotations. Introduce the task as being similar in nature to the previous task. If you have decided to provide experimental tools, outline what is available for students (note that a handout of the figures is provided at the end of the teacher notes, if needed.) You might also choose to just hand out the task and let students decide if they want to draw upon reasoning or tools to support their work. Or, you might want to press students to analyze the figures using reasoning based on the definitions of reflection and rotation.

Before having students start on the task, read the last two questions together, and point out that this is the goal of the task: to look for patterns in the number and characteristics of the lines of symmetry in a regular n-gon, and to look for patterns that describe the nature of the rotational symmetry in a regular n-gon. Encourage students to keep these goals in mind as they explore.

Explore (Small Group):

Listen for how students are determining the types of symmetry that exists for each regular polygon, and make sure they are identifying both types of symmetry—lines of symmetry and rotational symmetry. For rotational symmetry, make sure they are identifying all possible angles of rotation. As you observe students work, you may want to suggest additional tools for exploration, or set tools aside if you feel students are capable of noting the symmetries by reasoning with the definitions of rotation and reflection and the properties of regular polygons.

It is important for students to examine the last two questions in small groups, before moving to a whole class discussion. Keep reminding students of these goals, even before they have examined all of the listed regular polygons. That way, students will be able to attend to the presentations during the discussion, even if they haven't examined the complete set of polygons.

Discuss (Whole Class):

The discussion should focus on the last two questions, and students might draw upon specific examples of regular polygons to support their conjectures.

Ask students to state a conjecture as to the number of lines of symmetry in a regular n-gon, and to provide some justification of their conjecture. If students have not noticed any patterns in the number of lines of symmetry, make a table on the board consisting of "number of sides" as the input and "number of lines of symmetry" as the output. Ask what they may have noticed about the types of lines of symmetry in a regular polygon with an even number of sides versus an odd number of sides. What accounts for the differences in the ways they located the lines of symmetry?

Students should notice that in a regular polygon with an odd number of sides they can only draw lines of symmetry (or locate a crease line that folds the polygon onto itself) by using the lines that pass through a vertex point and the midpoint of the opposite side. Since such a line can be drawn through each vertex, a regular n-gon with an odd number of sides will possess n lines of symmetry.

In a regular polygon with an even number of sides you can draw (or fold) a line of reflection through opposite vertices. Since only one line of symmetry exists for each pair of opposite vertices, there are $n/2$ such lines of symmetry. You can also draw lines of symmetry through the midpoints of opposite sides of the polygon—the sides that are parallel to each other. Since only one line of symmetry exists for each pair of opposite sides, there are also $n/2$ such lines of symmetry. Consequently, a regular polygon with an even number of sides also has $n/2 + n/2 = n$ lines of symmetry, but for different reasons than in the case of regular polygons with an odd number of sides. Make sure that the arguments for n lines of symmetry in any regular polygon are based on the structure of the geometric figures themselves, and not just on the pattern observed in the table.

Turn the focus of the discussion to the second question: What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon? Again it may be helpful to create a table with the input representing "number of sides" and output representing "the smallest angle of rotation". Point out that every regular polygon can be rotated onto itself by rotating 360° about the point of intersection of the diagonals of the polygon. How might the smallest angle of rotation be related to this 360° rotation? You might draw the line segments between a pair of consecutive vertices and the center of rotation and ask, "What is the measure of this angle, and how do you know?" Students should notice that the smallest angle of rotation in a regular n-gon is $360°/n$ and they should be able to justify why this is so. They should also note that any whole-number multiple of this smallest angle of rotation is also an angle of rotation for the polygon.

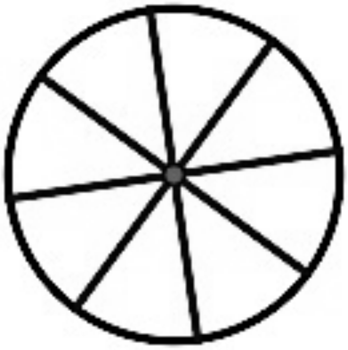
Aligned Ready, Set, Go: Transformations and Symmetry 6.6

READY, SET, GO! Name Period Date

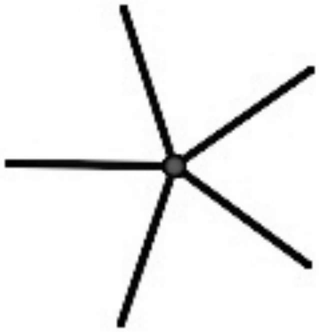
READY

Topic: Rotational symmetry, connected to fractions of a turn and degrees.

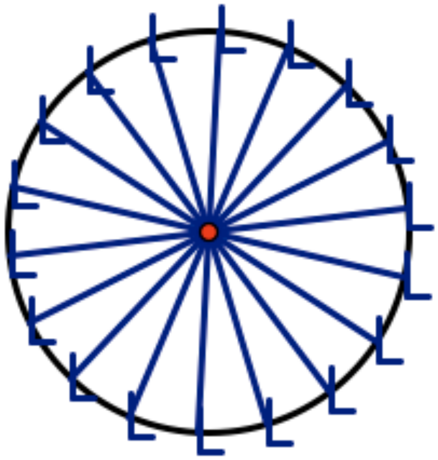
1. What fraction of a turn does the wagon wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



2. What fraction of a turn does the propeller below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



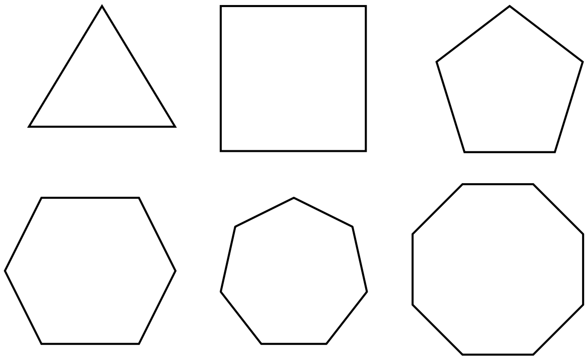
3. What fraction of a turn does the model of a Ferris wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



SET

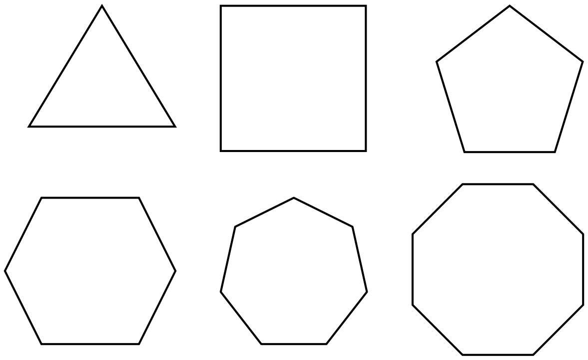
Topic: Finding angles of rotational symmetry for regular polygons, lines of symmetry and diagonals

4. Draw the lines of symmetry for each regular polygon, fill in the table including an expression for the number of lines of symmetry in a ***n***-sided polygon.



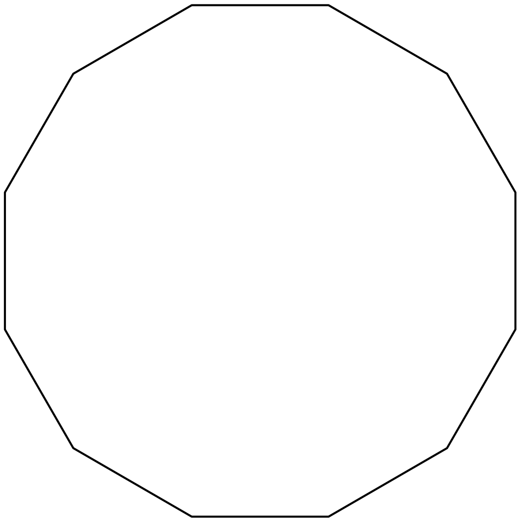
Number of Sides	Number of lines of symmetry
3	
4	
5	
6	
7	
8	
n	

5. Draw all of the diagonals in each regular polygon. Fill in the table and find a pattern, is it linear, exponential or neither? How do you know? Attempt to find an expression for the number of diagonals in a ***n***-sided polygon.



Number of Sides	Number of diagonals
3	
4	
5	
6	
7	
8	
n	

6. Find the angle(s) of rotation that will carry the 12 sided polygon below onto itself.



7. What are the angles of rotation for a 20-gon? How many lines of symmetry (lines of reflection) will it have?

8. What are the angles of rotation for a 15-gon? How many line of symmetry (lines of reflection) will it have?

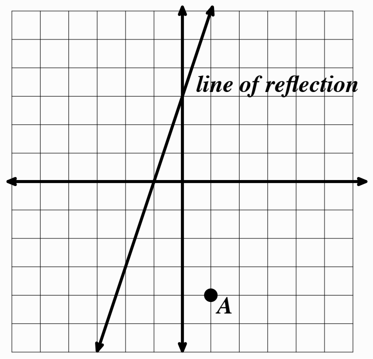
9. How many sides does a regular polygon have that has an angle of rotation equal to 18^0? Explain.

10. How many sides does a regular polygon have that has an angle of rotation equal to 20^0? How many lines of symmetry will it have?

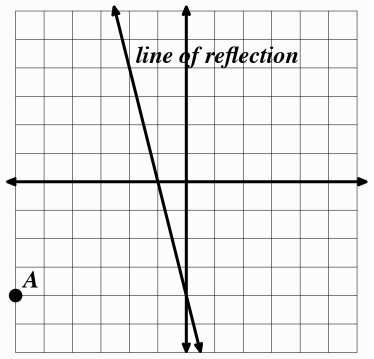
GO

Topic: Reflecting and rotating points on the coordinate plane.
(The coordinate grid, compass, ruler and other tools may be helpful in doing this work.)

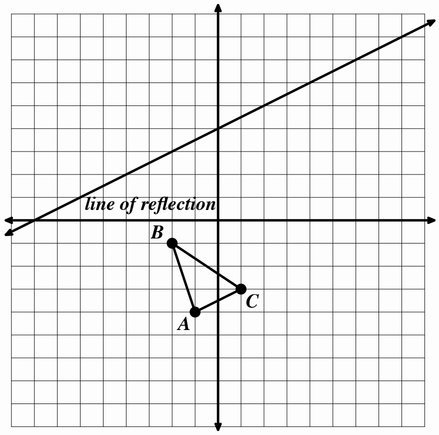
9. Reflect point ***A*** over the line of reflection and label the image ***A'***.



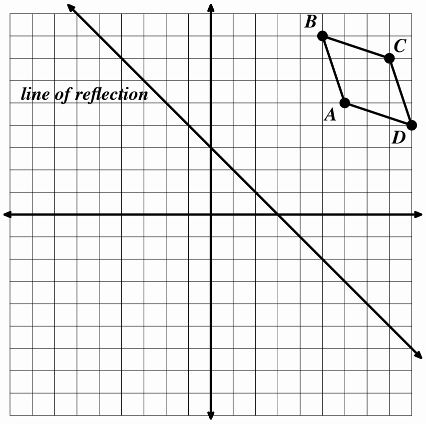
10. Reflect point ***A*** over the line of reflection and label the image ***A'***.



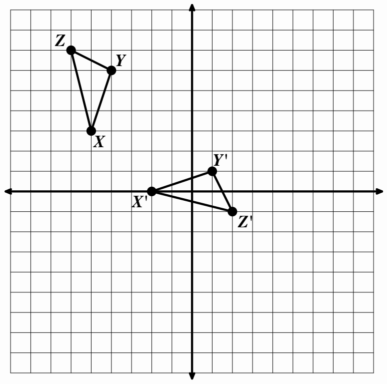
11. Reflect triangle ***ABC*** over the line of reflection and label the image ***A'B'C'***.



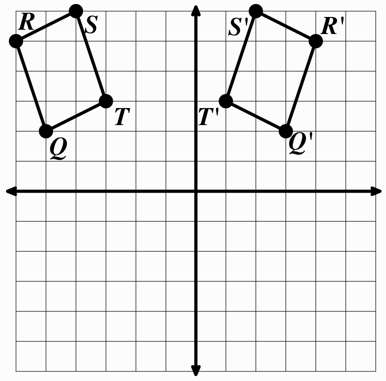
12. Reflect parallelogram ***ABCD*** over the line of reflection and label the image ***A'B'C'D'.***



13. Given triangle ***XYZ*** and its image ***X'Y'Z'*** draw the line of reflection that was used.



14 Given parallelogram ***QRST*** and its image ***Q'R'S'T'*** draw the line of reflection that was used.



6.7 Quadrilaterals—Beyond Definition

A Practice Understanding Task

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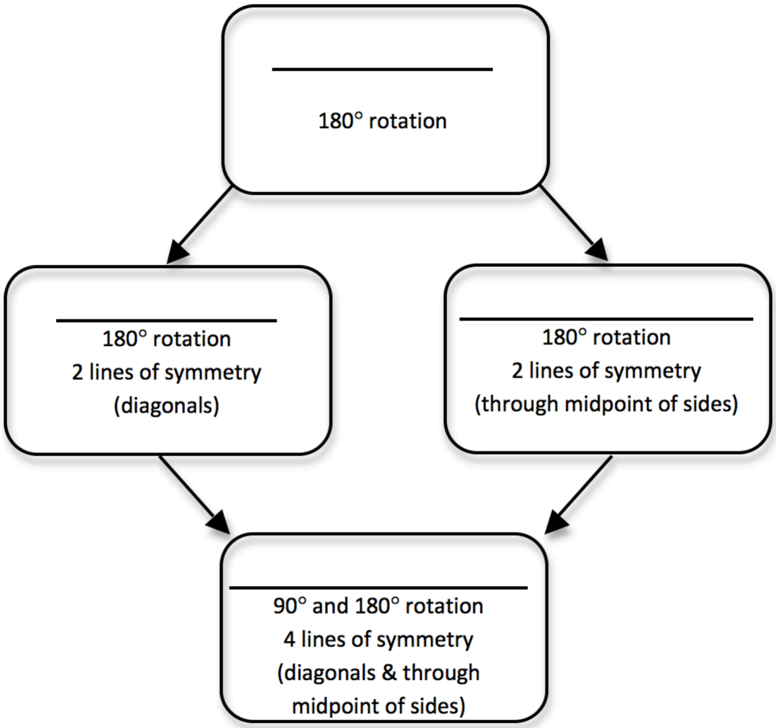
We have found that many different quadrilaterals possess lines of symmetry and/or rotational symmetry. In the following chart, write the names of the quadrilaterals that are being described in terms of their symmetries.



What do you notice about the relationships between quadrilaterals based on their symmetries and highlighted in the structure of the above chart?

Based on the symmetries we have observed in various types of quadrilaterals, we can make claims about other features and properties that the quadrilaterals may possess.

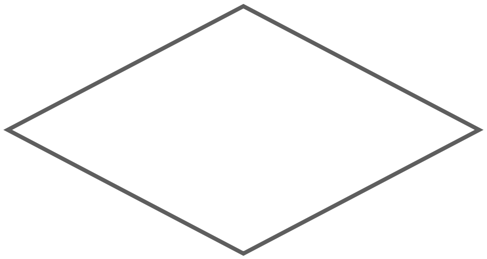
1. A **rectangle** is a quadrilateral that contains four right angles.

Based on what you know about transformations, what else can we say about rectangles besides the defining property that "all four angles are right angles?" Make a list of additional properties of rectangles that seem to be true based on the transformation(s) of the rectangle onto itself. You will want to consider properties of the sides, the angles, and the diagonals. Then justify why the properties would be true using the transformational symmetry.

2. A **parallelogram** is a quadrilateral in which opposite sides are parallel.

Based on what you know about transformations, what else can we say about parallelograms besides the defining property that "opposite sides of a parallelogram are parallel?" Make a list of additional properties of parallelograms that seem to be true based on the transformation(s) of the parallelogram onto itself. You will want to consider properties of the sides, angles and the diagonals. Then justify why the properties would be true using the transformational symmetry.

3. A **rhombus** is a quadrilateral in which all four sides are congruent.

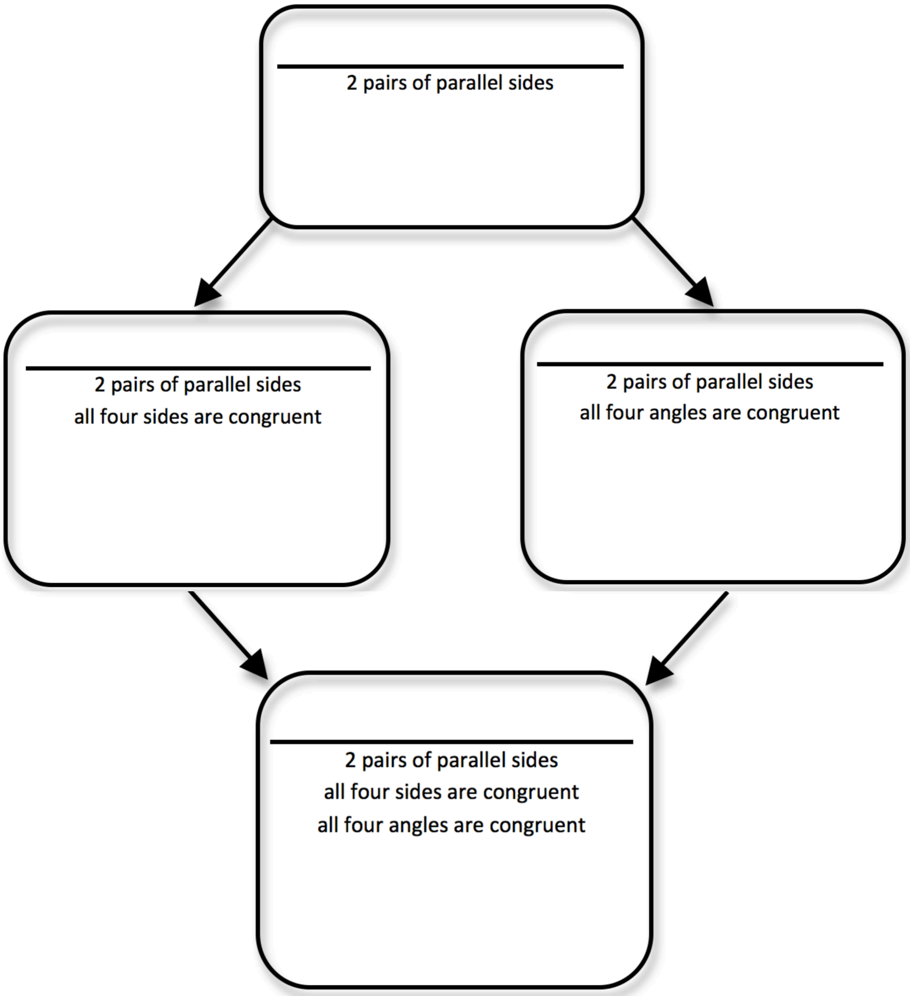


Based on what you know about transformations, what else can we say about a rhombus besides the defining property that "all sides are congruent?" Make a list of additional properties of rhombuses that seem to be true based on the transformation(s) of the rhombus onto itself. You will want to consider properties of the sides, angles and the diagonals. Then justify why the properties would be true using the transformational symmetry.

4. A **square** is both a rectangle and a rhombus.

Based on what you know about transformations, what can we say about a square? Make a list of properties of squares that seem to be true based on the transformation(s) of the squares onto itself. You will want to consider properties of the sides, angles and the diagonals. Then justify why the properties would be true using the transformational symmetry.

In the following chart, write the names of the quadrilaterals that are being described in terms of their features and properties, and then record any additional features or properties of that type of quadrilateral you may have observed. Be prepared to share reasons for your observations.



What do you notice about the relationships between quadrilaterals based on their characteristics and the structure of the above chart?

How are the charts at the beginning and end of this task related? What do they suggest?

6.7 Quadrilaterals—Beyond Definition – Teacher Notes

A Practice Understanding Task

Purpose: This task allows students to extend their work with symmetries of quadrilaterals and practice making conjectures about geometric figures that are based on reasoning with the definitions of reflection and rotation. The work of this task will be revisited in Mathematics II, where students will be asked to create formal proofs for the conjectures they are making in this task about the properties of different types of quadrilaterals. Therefore, while this is classified as a practice understanding task, the mathematics students should be practicing is making and justifying conjectures about geometric figures based on the definitions of rigid-motion transformations, rather than practicing knowledge about the specific properties of different types of quadrilaterals. Whatever properties about sides, angles and diagonals of quadrilaterals students surface is sufficient for this task.

Core Standards Focus:

G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure.

Related Standards: G.CO.11

Standards for Mathematical Practice of focus in the task:

SMP 3 – Construct viable arguments and critique the reasoning of others

SMP 7 – Look and make use of structure

Additional Resources for Teachers:

Cutouts or tracking paper for the quadrilaterals from 6.5 may be used again in this task. An answer key for the questions in the task can also be found as a separate page at the end of these teacher notes. It is recommended that you work through the task yourself before consulting the answer key to develop a better sense of how your students might engage in the task.

The Teaching Cycle:

Launch (Whole Class):

Give students a few minutes to examine the chart on the first page of the task. They should summarize their work with symmetries of quadrilaterals by identifying the types of quadrilaterals that possess the symmetries being described in each portion of the chart. Discuss the question: What do you notice about the relationships between quadrilaterals based on their symmetries and highlighted in the structure of the above chart? Help students notice that special quadrilaterals inherit the symmetries of all categories of quadrilaterals to which they belong.

Remind students that in the previous task they were able to make some conjectures about properties of regular polygons based on features that revealed themselves when they looked at the symmetries of the polygons. In this task they will return to quadrilaterals and see what conjectures they might make about relationships between sides, angles and diagonals of different types of quadrilaterals based on the symmetries of the quadrilateral. Have students practice making conjectures by working on problem 1: what else might be true about a rectangle—in addition to being a quadrilateral with four right angles—and how can they justify these observations based on the definitions of reflection and rotation.

After a few minutes summarize what groups have observed and share the arguments they might make to support their claims. Don't bring up properties that students have not observed on their own, but you might prompt further discussion by asking if students noticed anything about the diagonals of a rectangle that could be justified based on lines of symmetry or rotational symmetry. If they haven't already noticed anything about the diagonals suggest that they could add thinking about the diagonals to their list of things to pay attention to during their exploration.

Now that students have a sense of the work that is expected of them on this task, assign them to work on making conjectures about the remaining quadrilaterals.

Explore (Small Group):

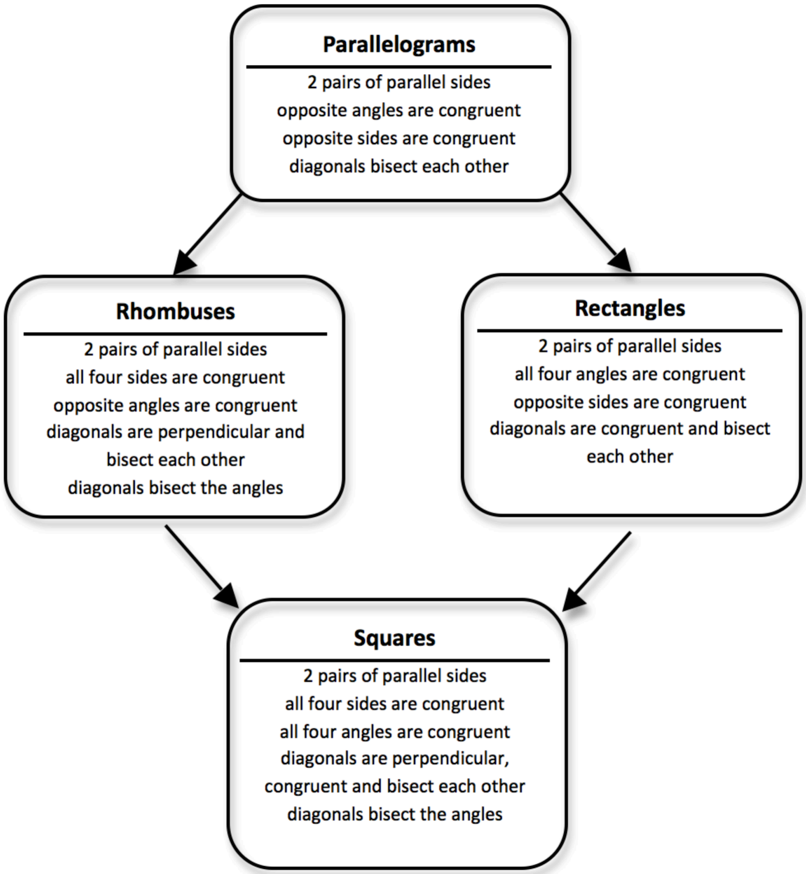
Listen for the types of conjectures students are making about each quadrilateral. Press them to look for more conjectures by asking questions like, "Is there anything you can say about opposite sides? Opposite angles? Adjacent sides? Adjacent angles? The diagonals and the way they interact with the sides and angles and with each other?"

Whenever students state a conjecture ask them why they think that conjecture is true—is it based on intuitive guessing, experimentation with tools, or on reasoning with the definitions of reflection and rotation? Press for justifications that are based on reasoning with transformations.

Discuss (Whole Class):

Since the purpose of this task is to practice making conjectures based on symmetry, there are no specific conjectures that need to be highlighted. Select students to share conjectures for which they have some justification based on transformations. As conjectures are shared and discussed, have students list those conjectures in the appropriate places on the chart at the end of the task. (They will first need to label the portions of the chart based on the defining properties of the different types of quadrilaterals. As the chart evolves, you will want to relate the chart at the beginning of this task to the chart at the end as a way of acknowledging the role that symmetry plays in the inherited properties that quadrilaterals possess based on the different categories of quadrilaterals to which they belong.)

Possible lists of properties of quadrilaterals that may surface are summarized in the following chart. However, not all of these properties need to be discussed.



Parallelograms

2 pairs of parallel sides
opposite angles are congruent
opposite sides are congruent
diagonals bisect each other

Rhombuses

2 pairs of parallel sides
all four sides are congruent
opposite angles are congruent
diagonals are perpendicular and bisect each other
diagonals bisect the angles

Rectangles

2 pairs of parallel sides
all four angles are congruent
opposite sides are congruent
diagonals are congruent and bisect each other

Squares

2 pairs of parallel sides
all four sides are congruent
all four angles are congruent
diagonals are perpendicular, congruent and bisect each other
diagonals bisect the angles

Aligned Ready, Set, Go: Transformations and Symmetry 6.7

6.7

READY, SET, GO! Name Period Date

READY

Topic: Defining congruence and similarity.

1. What do you know about two figures if they are congruent?

2. What do you need to know about two figures to be convinced that the two figures are congruent?

3. What do you know about two figures if they are similar?

4. What do you need to know about two figures to be convinced that the two figures are similar?

SET

Topic: Classifying quadrilaterals based on their properties.

Using the information given determine the most accurate classification of the quadrilateral.

5. Has 180^0 rotational symmetry.

6. Has 90^0 rotational symmetry.

7. Has two lines of symmetry that are diagonals.

8. Has two lines of symmetry that are not diagonals.

9. Has congruent diagonals.

10. Has diagonals that bisect each other.

11. Has diagonals that are perpendicular.

12. Has congruent angles.

GO

Topic: Slope and distance.

Find the *slope* between each pair of points. Then, using the Pythagorean Theorem, find the *distance* between each pair of points. Distances should be provided in the most exact form.

13. (-3 , -2) , (0 , 0)

a. Slope: b. Distance:

14. (7 , -1) , (11 , 7)

a. Slope: b. Distance:

15. (-10 , 13) , (-5 , 1)

a. Slope: b. Distance:

16. (-6 , -3) , (3 , 1)

a. Slope: b. Distance:

17. (5 , 22) , (17 , 28)

a. Slope: b. Distance:

18. (1 , -7) , (6 , 5)

a. Slope: b. Distance:

S