mathematics vision project

Transforming Mathematics Education

SECONDARY MATH TWO

An Integrated Approach

Standard Teacher Notes

MODULE 5

Geometric Figures

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The Mathematics Vision Project

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5.1 How Do You Know That?

A Develop Understanding Task

You probably know that the sum of the interior angles of any triangle is 180°. (If you didn't know that, you do now!) But an important question to ask yourself is, "How do you know that?"

We know a lot of things because we *accept it on authority*—we believe what other people tell us; things such as the distance from the earth to the sun is 93,020,000 miles or that the population of the United States is growing about 1% each year. Other things are just defined to be so, such as the fact that there are 5,280 feet in a mile. Some things we accept as true based on experience or repeated experiments, such as the sun always rises in the east, or "I get grounded every time I stay out after midnight." In mathematics we have more formal ways of deciding if something is true.

Experiment #1

1. Cut out several triangles of different sizes and shapes. Tear off the three corners (angles) of the triangle and arrange the vertices so they meet at a single point, with the edges of the angles (rays) touching each other like pieces of a puzzle. What does this experiment reveal about the sum of the interior angles of the triangles you cut out, and how does it do so?

2. Since you and your classmates have performed this experiment with several different triangles, does it guarantee that we will observe this same result for *all* triangles? Why or why not?

Experiment #2

Perhaps a different experiment will be more convincing. Cut out another triangle and trace it onto a piece of paper. It will be helpful to color-code each vertex angle of the original triangle with a different color. As new images of the triangle are produced during this experiment, color-code the corresponding angles with the same colors.

- Locate the midpoints of each side of your cut out triangle by folding the vertices that form the endpoints of each side onto each other.

- Rotate your triangle 180° about the midpoint of one of its sides. Trace the new triangle onto your paper and color-code the angles of this image triangle so that corresponding image/pre-image pairs of angles are the same color.

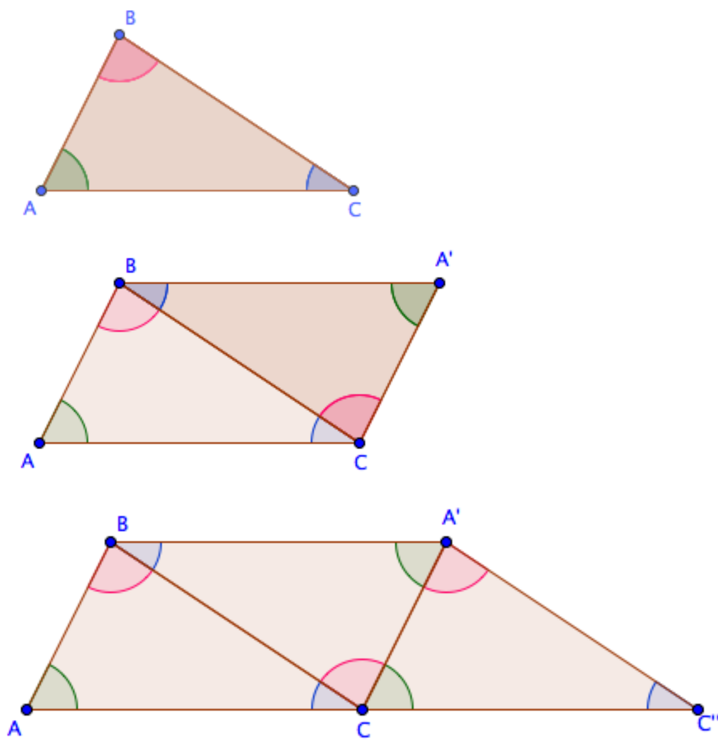
- Now rotate the new "image" triangle 180° about the midpoint of one of the other two sides. Trace the new triangle onto your paper and color-code the angles of this new image triangle so that corresponding image/pre-image pairs of angles are the same color.

3. What does this experiment reveal about the sum of the interior angles of the triangles you cut out, and how does it do so?

4. Do you think you can rotate *all* triangles in the same way about the midpoints of its sides, and get the same results? Why or why not?

Examining the Diagram

Experiment #2 produced a sequence of triangles, as illustrated in the following diagram.



Here are some interesting questions we might ask about this diagram:

5. Will the second figure in the sequence always be a parallelogram? Why or why not?

6. Will the last figure in the sequence always be a trapezoid? Why or why not?

5.1 How Do You Know That? – Teacher Notes

A Develop Understanding Task

Purpose: A major focus of the Mathematics II Geometry Standards is to develop the notion of formal proof—how the mathematics community comes to accept a statement as true. In this and subsequent tasks, students will explore a variety of ways of identifying the underlying reasoning behind a proof, and different formats for writing the proof. At the beginning of this sequence of tasks these proofs may take the form of informal verbal arguments. Over time, students should become more adept at constructing a logical sequence of statements that flow from beginning assumptions to justified conclusions.

In this task students consider the question, "How do you know something is true?" In mathematics, two different types of reasoning are used: inductive reason is the process of examining many examples, noticing a pattern, and stating a conjecture. Deductive reasoning is the process of starting with statements assumed or accepted as true (generally from a previous sequence of deductive reasoning), then creating a logical sequence of statements (if a is true, then b is true; if b is true, then c is true; if c is true, then d is true, etc.), until we arrive at the desired conclusion. Hence, inductive reasoning surfaces conjectures that need to be verified by deductive reasoning. Much of what is proved to be true by deductive reasoning in Mathematics II has already surfaced through experimentation in previous courses. This task should help students distinguish between accepting something as true based on experience or experimentation, and knowing something is true based on logical reasoning.

In this task students explore two different ways of knowing that the sum of the angles in a triangle is 180°, one based on experiments with specific triangles, and one based on a transformational argument that can be applied to all triangles.

Core Standards Focus:

G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°.

Mathematics II Note for G.CO.10: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Related Standards: G.CO.11

The Teaching Cycle:

Launch (Whole Class): [Experiment 1]

Ask students if they know what sum they will get if they add up the three angles in any triangle. It is likely that most will know that the sum is 180°. Ask them how they know that is true. Is it something they can recall from memory, but can't explain? Do they justify their argument by referring to a specific triangle, such as an equilateral triangle? How are they convinced that this property is true for *all* triangles? Perhaps they will refer to some prior experience in a previous math class, such as experiment #1 described in the task.

Point out that we have multiple "ways of knowing." (adapted from Schifter, D., Bastable, V., & Russell, S.J.. *Developing Mathematical Ideas: Reasoning Algebraically About Operations Casebook*, chapter 8). List these ways on the board for reference during this task:

- Accepting it on authority
- Trying it out with multiple examples
- Making an argument based on a diagram or representation
- Making an argument based on previously proved statements and logical ways of reasoning

Point out that most of us know that the sum of the angles of a triangle is 180° based on having been told by a previous teacher, or perhaps they have had an opportunity to try it out with examples.

Describe experiment #1 and let students carry out the experiment, including responding to the two questions. (If students have already had this experience previously, model it with a example or two, and move to the discussion of what we can "know" from this experiment.)

Explore (Small Group): [Experiment 1]

Observe that students are carrying out the experiment correctly: rearranging the vertices torn from the corners of the triangle to form a line. Students should be able to describe that because the three angles come together to form a straight line that the angles sum to 180°.

Discuss (Whole Class): [Experiment 1]

Have a short whole class discussion about what this experiment "proves." Helps students recognize that each individual experiment illustrates that the claim holds for that specific triangle, and even though we have tried several different triangles we haven't tried them all. How do we know that we haven't found one of the triangles for which this statement may not be true? Maybe it isn't true for really large triangles, or perhaps really small triangles, or some really "odd" looking triangles. We really don't know, since we haven't tried them all.

Launch (Whole Class): [Experiment 2]

Introduce experiment #2. An online app is available at http://geogebracentral.blogspot.com/2011/01/triangle-angle-sum-proof.html that can be used to demonstrate the rotation about the midpoint of one of the sides of the triangle. If available, show this app to model the activity, then give students a triangle, a white sheet of paper on which to trace their rotated triangles, and colored pencils to keep track of the three different vertices of their triangles.

Explore (Small Group): [Experiment 2]

Students should color each vertex of their triangle with a different color, and then trace their triangle onto the white paper close to one edge of the paper—so they have room on the paper to draw two additional rotations of the triangle. Watch that they color-code each corresponding vertex of their triangles with the same color. Make sure that they have accurately identified the midpoints of the sides of their triangle, and are rotating their triangle about a midpoint—not reflecting their triangles over a side.

All students should answer questions 3 and 4 that accompany the experiment, and you should allow time for at least some students to work on the "Examining the Diagram" questions.

Discuss (Whole Class): [Experiment 2]
Focus the discussion on question 4, leading to an argument that is based on the rotation of the triangles, and therefore independent of the specific starting triangle. The sequence of images in the "Examining the Diagram" section of the task lay out the general argument: rotating a triangle about the midpoint of one its sides forms a parallelogram; rotating this image a second time forms a trapezoid in which we can see a translated image of the original triangle, with one side of the original triangle extended to form a straight line. This straight line consists of images of the three angles of the triangle, verifying that the sum of the angles of the original triangle is 180°.
There are some subtle issues with this general argument, as pointed out in questions 5 and 6. To know that a straight line exists through points *A*, *C*, and *C''* we need to know that figure *ABA'C* and figure *CBA'C"* are parallelograms that share a common side—therefore, $\overline{AC}$ and $\overline{CC'}$ are either parallel to each other or on the same line—and since the altitudes of both parallelograms are the same length (the altitude of the original triangle), the segments are the same distance from $\overline{BA'}$ and are therefore on the same line. Students might be able to argue that the two quadrilaterals are parallelograms based on the property that opposite sides are congruent. This is a property that they may have observed in Mathematics I, when they experimented with symmetries of quadrilaterals. This is a property that will be proved later in this module. Therefore, at this point, our way of knowing may be more at the level of "making an argument based on a diagram or representation" rather than "making an argument based on previously proved statements."

Aligned Ready, Set, Go: Geometric Figures 5.1

READY, SET, GO! Name Period Date

READY

Topic: Geometric Figures

One of the cool things about geometric figures is that our world is filled with them. For instance, my bathroom mirror is a perfect rectangle and the tiles on my floor are squares. Plus, the edges of these shapes are straight lines or line segments which are pieces of lines, since theoretically a line goes on forever.

1. Look around your world and make a list of the things you see that have a geometric shape. Here are some shapes to begin with. Think of all you can and be prepared to share your lists with the class.

Triangle **Trapezoid** **Parallelogram** **Cube** **Perpendicular lines**

SET

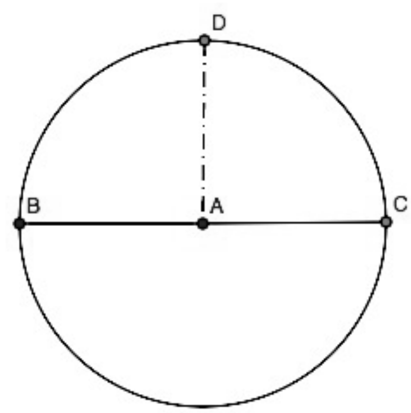
Topic: Linear Pairs

2. Fold a piece of paper, making a smooth crease. Open the paper and examine the shape that you made. Is it a line? Will it always be a line? Justify your thinking.

3. Look at a wall where it meets the ceiling. How would you describe the intersection of the wall and the ceiling?

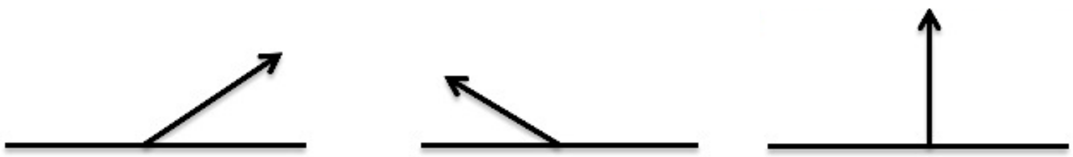
Imagine folding a circle exactly in half so that the fold passes through the center of the circle. This fold is called the diameter of the circle. It is a line segment with a length, but it is also a special kind of angle called a **straight angle***.*

In order to "see" the angle, think of the center of the circle. That point is the vertex of the angle. Either side of the vertex is a radius of the circle. Whenever you draw 2 radii of the circle you make an angle. When the two radii extend in exactly opposite directions and share a common endpoint (the center), they make a line or a **straight angle***.*



14. How many degrees do you think are in a *straight angle*?
Use features of the diagram to justify your answer.

If two angles share a vertex and together they make a straight angle, then the two angles are called a **linear pair**. *(Below are 3 examples of* **linear pairs**.*)*



Examples of linear pairs in real life:

http://www.flicker.com/photos/angle_dore/63650608

http://www.flicker.com/photos/truthlying/3845031/siz

5. Draw at least 2 diagrams of a real life linear pair.

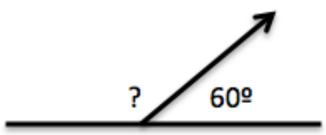
GO

Topic: Algebra of Linear Pairs

For 2 angles to be a **linear pair**, *they must share a vertex and a side, and the sum of their measures must equal 180°.*

Find the measure of the missing angle.

6.



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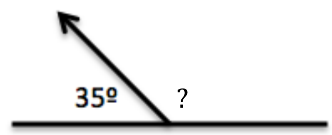

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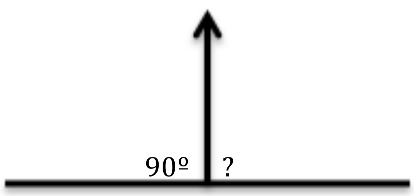
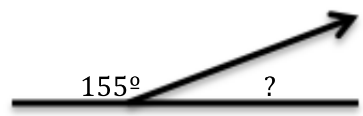


9.



10. Linear pairs could be defined as being **supplementary angles** because they always add up to 180°. Are all supplementary angles linear pairs? Explain your answer.

Find the supplement of the given angle. Then draw the two angles as linear pairs. Label each angle with its measure.

11. m∠ABC = 72° B will be the vertex.

12. m∠GHK = 113° H will be the vertex.

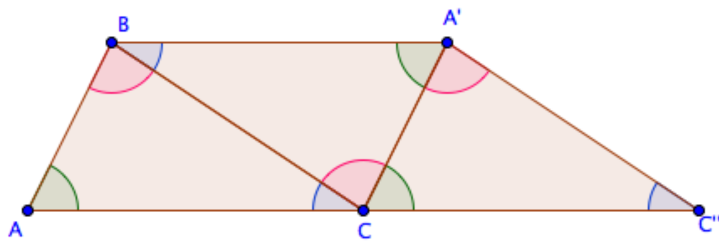
13. m∠XYZ = 24° Y will be the vertex

14. m∠JMS = 168° M will be the vertex

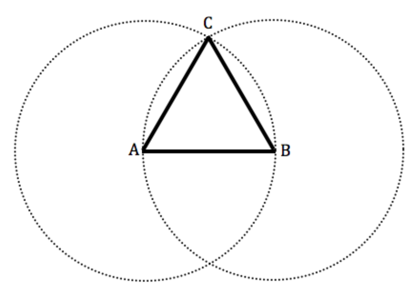
5.2 Do You See What I See?

A Develop Understanding Task

In the previous task, *How Do You Know That*, we saw how the following diagram could be constructed by rotating a triangle about the midpoint of two of its sides. The final diagram suggests that the sum of the three angles of a triangle is 180°. This diagram "tells a story" because you saw how it was constructed through a sequence of steps. You may even have carried out those steps yourself.



Sometimes we are asked to draw a conclusion from a diagram when we are given the last diagram in a sequence steps. We may have to mentally reconstruct the steps that got us to this last diagram, so we can believe in the claim the diagram wants us to see.



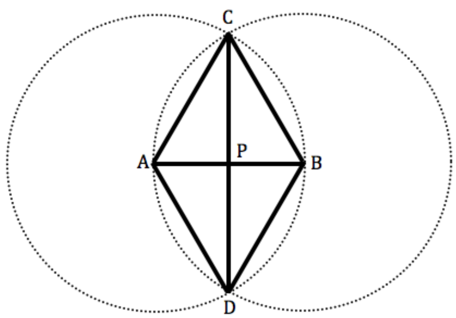
1. For example, what can you say about the triangle in this diagram?

2. What convinces you that you can make this claim? What assumptions, if any, are you making about the other figures in the diagram?

3. What is the sequence of steps that led to this final diagram?

4. What can you say about the triangles, quadrilateral, or diagonals of the quadrilateral that appear in the following diagram? List several conjectures that you believe are true.

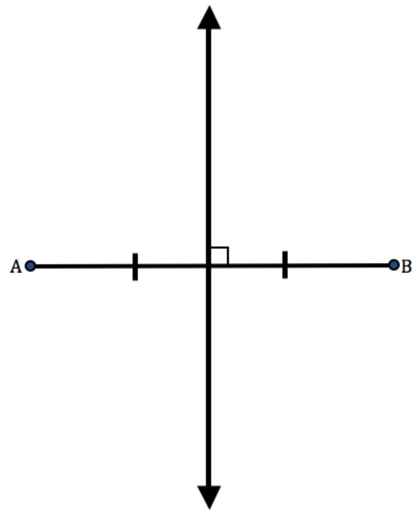
Given: $\odot A \cong \odot B$

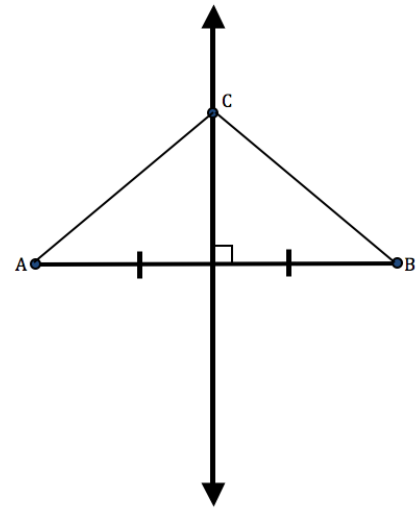


5. Select one of your conjectures and write a paragraph convincing someone else that your conjecture is true. Think about the sequence of statements you need to make to tell your story in a way that someone else can follow the steps and construct the images you want them to see.

6. Now pick a second claim and write a paragraph convincing someone else that this claim is true. You can refer to your previous paragraph, if you think it supports the new story you are trying to tell.

7. Here is one more diagram. Describe the sequence of steps that you think were used to construct this diagram beginning with the figure on the left and ending with the figure on the right.





Travis and Tehani are doing their math homework together. One of the questions asks them to prove the following statement.

The points on the perpendicular bisector of a segment are equidistant from the endpoints of the segment?

Travis and Tehani think the diagram above will be helpful to prove this statement, but they know they will need to say more than just describe how to create this diagram. Travis starts by describing the things they know, and Tehani tries to keep a written record by jotting notes down on a piece of paper.

8. In the table below, record in symbolic notation what Tehani may have written to keep track of Travis' statements. In the examples given, note how Tehani is introducing symbols for the lines and points in the diagram, so she can reference them again without using a lot of words.

Tehani's Notes	**Travis' Statements**
Draw $\overline{AB}$. Locate its midpoint M, and draw a perpendicular line ℓ through the midpoint	We need to start with a segment and its perpendicular bisector already drawn.
Pick any point C on line ℓ	We need to show that *any* point on the perpendicular bisector is equidistant from the two endpoints, so I can pick any arbitrary point on the perpendicular bisector. Let's call it C.
Prove:	We need to show that this point is the same distance from the two endpoints.
First prove:	If we knew the two triangles were congruent, we could say that the point on the perpendicular bisector is the same distance from each endpoint. So, what do we know about the two triangles that would let us say that they are congruent?
	We know that both triangles contain a right angle.
	And we know that the perpendicular bisector cuts segment AB into two congruent segments.
	Obviously, the segment from C to the midpoint of segment AB is a side of both triangles.
	So, the triangles are congruent by the SAS triangle congruence criteria.
	Since the triangles are congruent, segments AC and BC are congruent.
Any point C on line ℓ, the perpendicular bisector of $\overline{AB}$, is equidistant from the endpoints A and B.	And, that proves that point C is equidistant from the two endpoints!

9. Tehani thinks Travis is brilliant, but she would like the ideas to flow more smoothly from start to finish. Arrange Tehani's symbolic notes in a way that someone else could follow the argument and see the connections between ideas.

10. Would your justification be true regardless of where point C is chosen on the perpendicular bisector? Why?

5.2 Do You See What I See? – Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is to continue developing the ideas of formal proof, particularly moving from reasoning with a diagram to reasoning based on a logical sequence of statements that start with given assumptions and lead to a valid conclusion. Reasoning with a diagram is an important geometric thinking skill, and in this task students explore the logic behind the construction of diagrams—what features of a diagram must precede the addition of other features. Students also examine the diagram for possible conclusions that can be made—what else must be true. The last part of the task focuses on writing symbolic statements to match the verbal descriptions we are making about the diagram, and then to sequence those statements into a logical flow of ideas. The format of this work is suggestive of a two-column proof.

Core Standards Focus:

G.CO.9 Prove theorems about lines and angles. Theorems include: points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

G.CO.10 Prove theorems about triangles. Theorems include: base angles of isosceles triangles are congruent.

G.CO.11 Prove theorems about parallelograms. Theorems include: the diagonals of a parallelogram bisect each other

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Mathematics II Note for G.CO.10: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

The Teaching Cycle:

Launch (Whole Class): [Questions 1-3]

Most of the proofs in this task can be justified using the triangle congruence criteria established in Mathematics I: SSS, SAS, and ASA. Review these ideas before beginning this task. Students might also use arguments based on transformations in their proofs. Remind them of the definitions of translation, rotation and reflection developed in Mathematics I. Also remind them that we are accepting as true without proof that these rigid motion transformations preserve distance and angle measure.

Use the first three questions to launch the work of this task. After reading the introductory paragraph together as a class, ask students to answer questions 1-3 individually. After a few minutes, discuss their responses. It is assumed that most students will say that the triangle is an equilateral triangle, and they may support this claim by saying that the sides of the triangle are all radii of congruent circles. Some may say that the sides are radii of the same circle. Point out that there are two distinct circles, and to make sense of the diagram we probably have assumed that the circles are congruent. If this were not the case, we could not conclude that the sides of the triangle are the same length. Point out that the author of such a diagram should avoid forcing the reader of the diagram to make such assumptions by listing additional given information about the diagram, such as stating that the circle centered at A is congruent to the circle centered at B.

It is important that students be able to describe the sequence of steps that would have led to the creation of this diagram. This ability will give them access to the process of thinking through a proof. In this figure, $\overline{AB}$ had to be constructed first, then the congruent circles centered at A and at B were constructed with $\overline{AB}$ as the radius. Next, C, a point of intersection of the two congruent circles was identified, and $\overline{AC}$ and $\overline{BC}$ were constructed. By identifying this sequence of events, students are better able to explain why $\overline{AC} \cong \overline{AB} \cong \overline{BC}$.

Following this discussion, assign students to work on questions 4-6.

Explore (Small Group): [questions 4-6]

As students start to examine the diagram, they will probably note that the four segments that form the sides of the quadrilateral are congruent, based on the discussion of questions 1-3. Encourage students to mark these segments as congruent on the diagram, and to continue to mark congruent segments as angles once they can argue why they are congruent.

Listen to the conjectures students are making about the diagram. Conjectures could include several statements about pairs of congruent triangles, a conjecture that ΔDAC or ΔDBC is isosceles, a conjecture that base angles of an isosceles triangle are congruent, a conjecture that quadrilateral $ACBD$ is a rhombus, and a conjecture that the diagonals of the rhombus are perpendicular or that the diagonals bisect each other. Make sure that students are including conjectures about the quadrilateral, and not just about triangles. As students are selecting which conjectures to write a paragraph proof for, you may want to ask certain students or groups to focus on specific conjectures, such as: base angles of isosceles triangles are congruent, the diagonals of a rhombus bisect each other, and the diagonals of a rhombus are perpendicular. Students who work on these theorems will first need to prove that at least one of the triangles in the diagram is an isosceles triangle, or that the quadrilateral can be decomposed into smaller congruent triangles.

A beginning place for proving many possible conjectures could start by proving that $\Delta DAC \cong \Delta DBC$ using SSS triangle congruence criteria, since both triangles share $\overline{DC}$. Many pairs of corresponding angles can then be marked as congruent, since corresponding parts of congruent triangles are congruent. Alternatively, students could rotate ΔDBC onto ΔDAC in order to identify congruent pairs of angles. As more pairs of angles are marked as congruent, additional pairs of triangles can be identified as congruent using SAS and ASA congruency criteria. Since the quadrilateral has four congruent sides, it is a rhombus. The corresponding segments of the smaller triangles in the rhombus can be used to prove that the diagonals bisect each other. When all four angles at P are marked congruent, it can also be argued that they are right angles, proving that the diagonals of the rhombus are perpendicular.

Discuss (Whole Class): [questions 4-6]

Sequence the sharing of proofs to first establish that $\Delta DAC \cong \Delta DBC$. Then have students prove that several more pairs of triangles in the figure are congruent by drawing upon corresponding pairs of angles from ΔDAC and ΔDBC. Have a student prove that the base angles of ΔDAC are congruent. Have a student prove that the diagonals of the rhombus bisect each other. Conclude with a proof that the diagonals of the rhombus are perpendicular to each other.

Launch (Whole Class): [questions 7-10]

Launch the remainder of the task by pointing out that we have proved that the two diagonals of a rhombus are perpendicular bisectors of each other. Questions 7-10 will examine a proof about perpendicular bisectors in general. Give students a couple of minutes to examine the figure in question 7, and then have students describe what is implied by the diagram. They should note that the markings in the diagram suggest that line l bisects segment AB. Since there are no additional details given in the diagram about point C, we have to assume that it can be *any* point located on line l. Once this arbitrary point was selected, then $\overline{AC}$ and $\overline{BC}$ were constructed.

Once students understand this diagram, read through the Travis and Tehani context and assign students to work on questions 8-10.

Explore (Small Group): [questions 7-10]

Question 8 is essentially a two-column proof, with the reasoning listed. However, some of Travis' statements are out of order in terms of how we will write the final argument. For example, Travis reminds himself of what he is trying to prove in line 3, and in line 4 he mentions something he needs to prove. Students should create and revise the symbolic argument to create a logical flow of ideas (see question 9). If students need help getting started, suggest that the first line of Travis' reasoning could be recorded symbolically as $l \perp \overline{AB}$.

Discuss (Whole Group): [questions 7-10]

Have a selected student present their final two-column proof (question 9) and discuss question 10. Helps students recognize that since point C is an arbitrary point on line l, we have proved that *all* points on the perpendicular bisector of a segment are equidistant from the endpoints of the segment.

Aligned Ready, Set, Go: Geometric Figures 5.2

READY, SET, GO! Name Period Date

READY

Topic: Symbols in Geometry

Throughout the study of mathematics, you have encountered many symbols that help you write mathematical sentences and phrases without using words. Symbols help the mathematician calculate efficiently and communicate concisely.

Below is a set of common mathematical symbols. Your job is to match them to their definitions. Are the symbols logical?

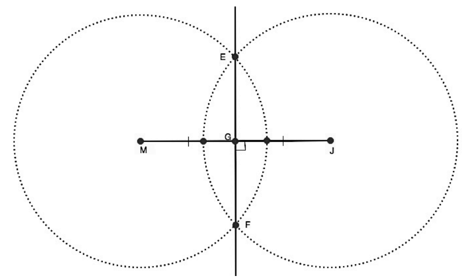
Symbol	Definitions
_____1. $=$	A. **Absolute value** – it is always equal to the positive value of the number inside the lines. It represents distance from zero.
_____2. $m\angle C$	B. **Congruent** – Figures that are the same size and shape are said to be congruent.
_____3. GH	C. **Parallel** – used between segments, lines, rays, or planes
_____4. ΔABC	D. **Line segment with endpoints G and H**. Line segments can be congruent to each other. You would not say they were equal.
_____5. $\perp$	E. **Ray GH** – The letter on the left indicates the endpoint of the ray.
_____6. $\angle ABC$	F. Used when comparing numbers of **equal value**.
_____7. $\overleftrightarrow{GH}$	G. **Plus or minus** – indicates 2 values, the positive value and the negative value
_____8. $\cong$	H. **Triangle ABC**
_____9. $\sim$	J. Indicates the **measure of an angle**. It would be set equal to a number.
_____10. $\overline{GH}$	K. **Perpendicular** - Lines, rays, segments, and planes can all be perpendicular
_____11. $\overrightarrow{GH}$	L. **Angle ABC** – The middle letter is always the vertex of the angle.
_____12. $\vert\vert$	M. **Similar** – Figures that have been dilated are similar.
_____13. $\pm$	N. **The length of GH**. It would equal a number.
_____14. $\vert x\vert$	P. Refers to the **infinite line GH**. Lines are not equal or congruent to other lines.

Need help? Visit **www.rsgsupport.org**

SET

Topic: Construction of midpoint, perpendicular bisector, and angle bisector, using "givens" to solve problems.

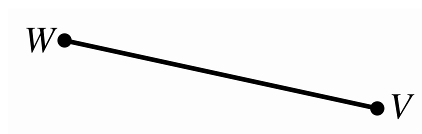
The figure on the right demonstrates the construction of a perpendicular bisector of a segment.



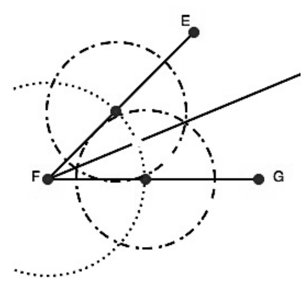
Use the diagram to guide you in constructing the perpendicular of the following line segments. Mark the right angle with the correct symbol for right angles. Indicate the segments are congruent by using slash marks.

15.

16.



The figure on the right demonstrates the construction of an angle bisector.
Use the diagram to guide you in constructing the angle bisector of the following angles. Mark your bisected angles as congruent.



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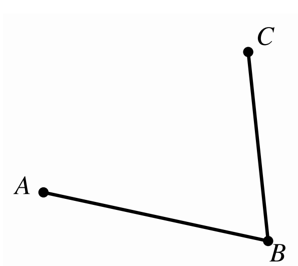

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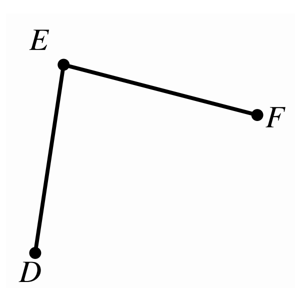

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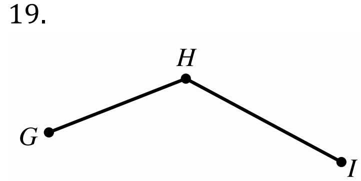
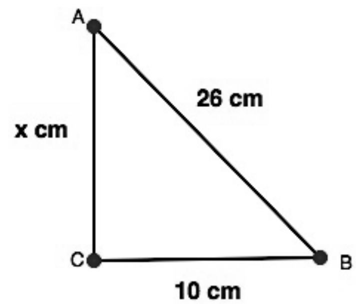


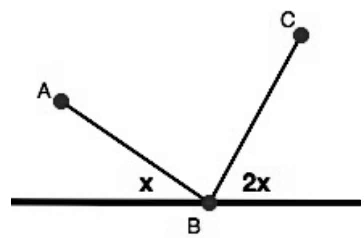
Need help? Visit **www.rsgsupport.org**

Examine the diagram and add any information that you are given. Think how you can use what you have been given and what you know to answer the question. Plan a strategy for finding the value of x. Follow your plan. Justify each step.

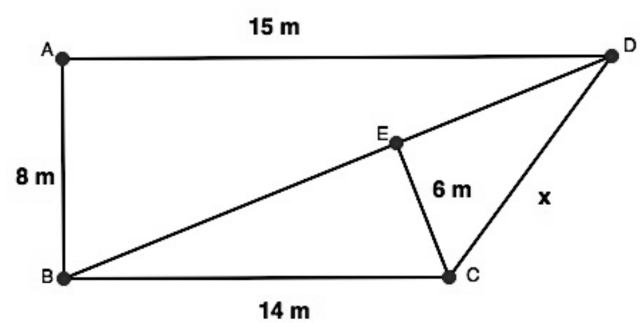
20. Given: $m\angle C = 90^{\circ}$



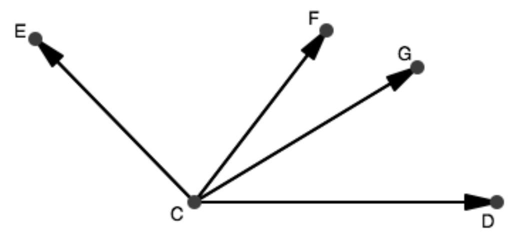
21. Given $m\angle ABC = 90^{\circ}$



22. Given: $\Delta BEC, \Delta CED, and\ \Delta DAB$ are right triangles.



23. Given: $\overrightarrow{CF}$ bisects $\angle ECD, m\angle ECF = 2x + 10$, and $m\angle FCD = 3x - 18$. Find $m\angle FCE$.



Have you answered the question?
This problem asks you to do more than find the value of x.

GO

Topic: Translations, reflections, rotations

Perform the following transformations on the diagram below.

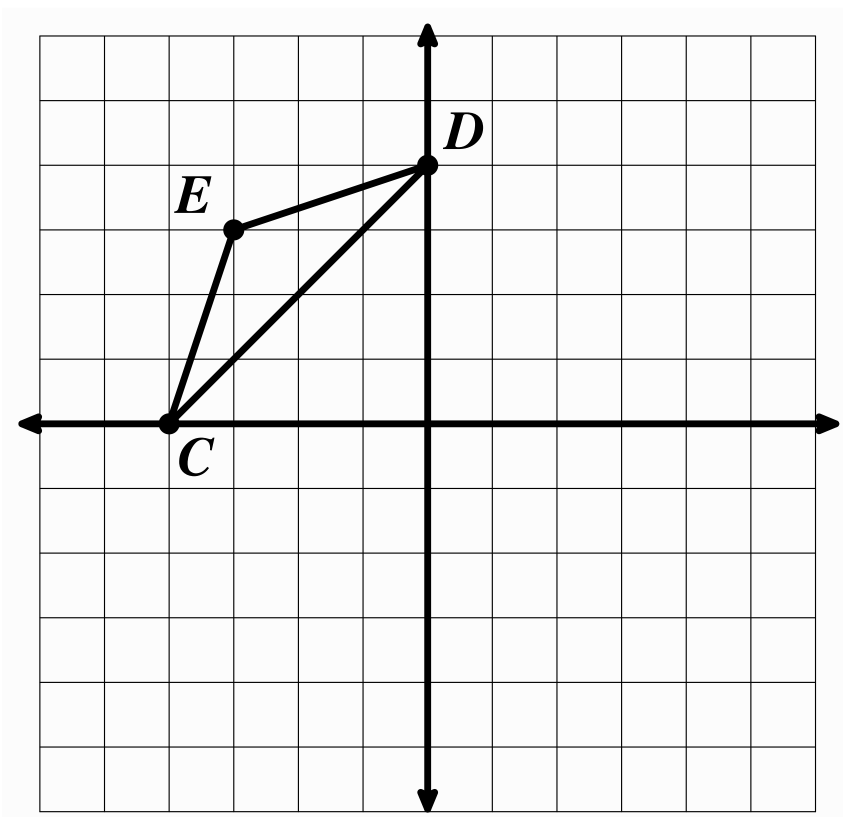
24. Label points C, E, D with the correct ordered pairs.

25. Translate ΔCED down 4 and right 6. Label the image as $\Delta C'E'D'$ and include the new ordered pairs.

26. Draw $\overline{CC'}$, $\overline{EE'}$, and $\overline{DD'}$. What is the slope of each of these line segments?

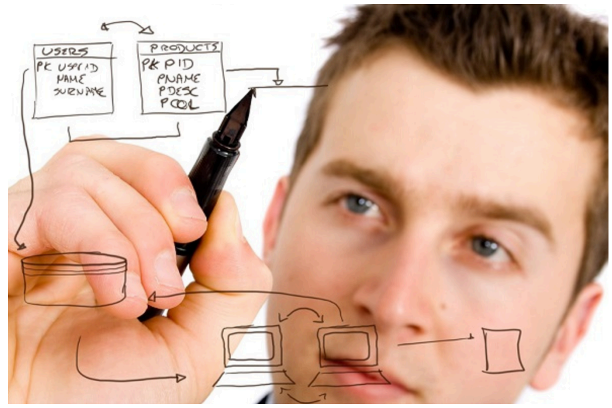
27. Reflect ΔCED across the x = 0 line. Label the image $\Delta C''E''D''$. Include the new ordered pairs.
Draw $\overline{CC''}$ *and* $\overline{EE''}$ Why didn't you need to draw $\overline{DD''}$?
What is the relationship between $\overline{CC''}$ and $\overline{EE''}$ to the $x = 0$ line?

28. Rotate ΔCED 180º about the point (-2, 0). Label the image $\Delta C'''E'''D'''$.
Include the new ordered pairs.



5.3 It's All In Your Head

A Solidify Understanding Task

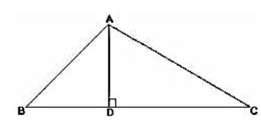


In the previous task you were asked to justify some claims by writing paragraphs explaining how various figures were constructed and how those constructions convinced you that the claims were true. Perhaps you found it difficult to say everything you felt you just knew. Sometimes we all find it difficult to explain our ideas and to get those ideas out of our heads and written down or paper.

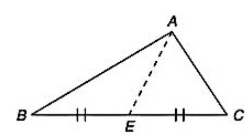
Organizing ideas and breaking complex relationships down into smaller chunks can make the task of proving a claim more manageable. One way to do this is to use a flow diagram.

First, some definitions:

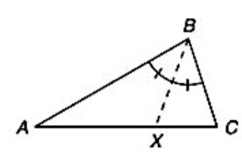
- In a triangle, an **altitude** is a line segment drawn from a vertex perpendicular to the opposite side (or an extension of the opposite side).



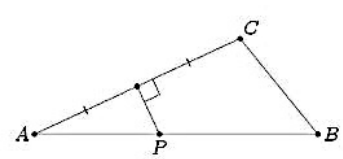
- In a triangle, a **median** is a line segment drawn from a vertex to the midpoint of the opposite side.



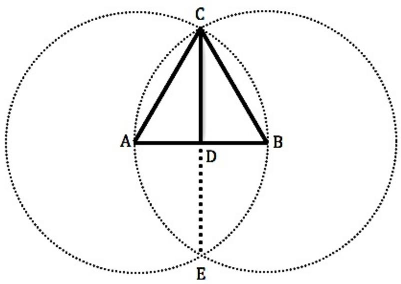
- In a triangle, an **angle bisector** is a line segment or ray drawn from a vertex that cuts the angle in half.



- In a triangle, a **perpendicular bisector of a side** is a line drawn perpendicular to a side of the triangle through its midpoint.



Travis used a compass and straightedge to construct an equilateral triangle. He then folded his diagram across the two points of intersection of the circles to construct a line of reflection. Travis, Tehani, Carlos and Clarita are trying to decide what to name the line segment from C to D.



Travis thinks the line segment they have constructed is also a median of the equilateral triangle. Tehani thinks it is an angle bisector. Clarita thinks it is an altitude and Carlos thinks it is a perpendicular bisector of the opposite side. The four friends are trying to convince each other that they are right.

On the following page you will find a flow diagram of statements that can be written to describe relationships in the diagram, or conclusions that can be made by connecting multiple ideas. You will use the flow diagram to identify the statements each of the students—Travis, Tehani, Carlos and Clarita—might use to make their case. To get ready to use the flow diagram, answer the following questions about what each student needs to know about the line of reflection to support their claim.

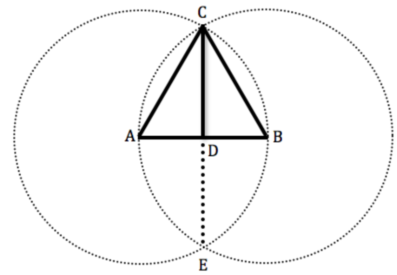
1. To support his claim that the line of reflection is a median of the equilateral triangle, Travis will need to show that:

2. To support her claim that the line of reflection is an angle bisector of the equilateral triangle, Tehani will need to show that:

3. To support her claim that the line of reflection is an altitude of the equilateral triangle, Clarita will need to show that:

4. To support his claim that the line of reflection is a perpendicular bisector of a side of the equilateral triangle, Carlos will need to show that:

Here is a flow diagram of statements that can be written to describe relationships in the diagram, or conclusions that can be made by connecting multiple ideas.



7. Use four different colors to identify the statements each of the students—Travis, Tehani, Clarita and Carlos might use to make their case.

Given: ΔABC is equilateral AND $\overleftrightarrow{CE}$ is a line of reflection

$\overline{AB} \cong \overline{BC} \cong \overline{AC}$

$\angle CDA$ and $\angle CDB$ are right angles

$\overline{CD} \perp \overline{AB}$

D is the midpoint of $\overline{AB}$

$\overline{AD} \cong \overline{DB}$

$\overline{CD} \cong \overline{CD}$

$\angle ACD \cong \angle BCD$

$\Delta ACD \cong \Delta BCD$

therefore, $\overline{CD}$ is an altitude

therefore, $\overline{CD}$ is a median

therefore, $\overline{CD}$ is an angle bisector

therefore, $\overline{CD}$ is a perpendicular bisector

8. Match each of the arrows and braces in the flow diagram with one of the following reasons that justifies why you can make the connection between the statement (or statements) previously accepted as true and the conclusion that follows:

    1. Definition of reflection
    2. Definition of translation
    3. Definition of rotation
    4. Definition of an equilateral triangle
    5. Definition of perpendicular
    6. Definition of midpoint
    7. Definition of altitude
    8. Definition of median
    9. Definition of angle bisector
    10. Definition of perpendicular bisector
    11. Equilateral triangles can be folded onto themselves about a line of reflection
    12. Equilateral triangles can be rotated 60° onto themselves
    13. SSS triangle congruence criteria
    14. SAS triangle congruence criteria
    15. ASA triangle congruence criteria
    16. Corresponding parts of congruent triangles are congruent
    17. Reflexive Property

Travis and his friends have seen their teacher write two-column proofs in which the reasons justifying a statement are written next to the statement being made. Travis decides to turn his argument into a two-column proof, as follows.

Statements	**Reasons**
ΔABC is equilateral	Given
$\overleftrightarrow{CE}$ is a line of reflection	Equilateral triangles can be folded onto themselves about a line of reflection
D is the midpoint of $\overline{AB}$	Definition of reflection
$\overline{CD}$ is a median	Definition of median

9. Write each of Clarita's, Tehani's, and Carlos' arguments in two-column proof format.

5.3 It's All In Your Head – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to solidify ways of thinking about formal proofs, such as reasoning from a diagram and identifying a sequence of statements that start with given assumptions and lead to a valid conclusion. This task introduces the flow diagram as a way of keeping track of the logical connections between given statements and the conclusions that can be drawn from them.

Core Standards Focus:
G.CO.9 Prove theorems about lines and angles. Theorems include: points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

G.CO.10 Prove theorems about triangles.
Mathematics II Note for G.CO.10: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

The Teaching Cycle:
Launch (Whole Class):
Set up the purpose of this task for students: In the previous task, *Do You See What I See*, we learned how to reason from a diagram. In this task, we will learn how to organize information we can gather from a figure into a form called a flow diagram. The flow diagram will help us sort out the details of a proof in a way that is similar to what Travis and Tehani did in the previous task.

Make sure students are familiar with the definitions of median, altitude, angle bisector and perpendicular bisector of a side of a triangle, as given in the task. Then read through the scenario where Travis, Tehani, Carlos and Clarita are trying to decide what to call the line segment created when an equilateral triangle is folded in half. Have them answer the questions and what each person will need to show in order to justify their claim, and then discuss these statements as a class in preparation of students' work with the flow diagram. Give students four different colored pencils to use to identify the statements in the flow diagram that each person would need to use to make their argument for what to call the line segment formed by folding the equilateral triangle. Tell students to start with one person and find his or her logical path through the flow diagram, and then change colors and go back and follow another person's logic through the diagram.

Explore (Small Group):
Observe how students are identifying the statements they choose to include in a particular person's argument. Point out to students that they do not need to start at the beginning and work to the end of the flow diagram; they could also start at the desired conclusion at the end of the diagram and work their way backwards, looking for the statements that allow them to say that the statement they are using in their argument is true. Ask questions like, "Why do you think Carlos needs to include this statement in his argument?" or "What statement would allow Clarita to conclude that the folded line is an altitude?" Encourage students to re-examine the definitions on the first page of the task if they are selecting unreasonable statements for their arguments.

If students are finding this work difficult, have them move to matching reasons with the "flow markers" (arrows and braces) in the diagram. This work is similar to what Travis and Tehani did in the previous task, but recorded in a different format. Once the reasoning has been matched with the diagram, have students read through Travis' two-column proof. Then have them return to the flow diagram and mark Travis' path through the diagram. Now ask them to do a similar type of proof for each of the other students. Working through all three parts of this task simultaneously should offer additional support to students, if they cannot move through the task sequentially.

Discuss (Whole Class):
Have students present each of the other three two-column proofs. Make sure the presenters connect the sequences of statements in their two column-proofs to the related portions of the flow

diagram. Have students discuss the potential values of a flow diagram in helping them organize their thinking before writing a paragraph or two-column proof. Note that a flow diagram can include statements that are known to be true, but are not necessary for a particular proof (for example, Clarita doesn't need all of the same information that Travis does). A flow diagram allows us to write down everything we know to be true, then go back and find the essential elements of a proof. We also don't have to move from the beginning to end of a logical step-by-step argument initially—we can start anywhere we want and think of what would have to come before, or what might come after in the logical flow of the argument. We can even start at the end, with the statement we are trying to prove, and work backwards. To make our *flow diagram* (which is a tool for thinking about the proof) into a *flow proof* (a representation of the actual logic needed to prove a claim), we would need to remove any extraneous information that is not needed for the particular claim we are trying to prove. Have students write Travis' two-column proof as a flow proof, leaving out the unnecessary information relative to his proof.

Aligned Ready, Set, Go: Geometric Figures 5.3

READY, SET, GO! Name Period Date

READY

Topic: Congruence statements and corresponding parts

Remember that when you write a congruence statement such as $\Delta ABC \cong \Delta FGH$, the corresponding parts of the two triangles must be the parts that are congruent.

For instance, $\angle A \cong \angle F$, $\overline{AB} \cong \overline{FG}$, $\angle B \cong \angle G$, $\overline{BC} \cong \overline{GH}$. *Also, recall that the congruence patterns for triangles, ASA. SAS, and SSS, are what we can use to justify triangle congruence.*

The segments and angles in each problem below are corresponding parts of 2 congruent triangles. Make a sketch of the two triangles. Then write a congruence statement for each pair of triangles represented. State the congruence pattern that justifies your statement.

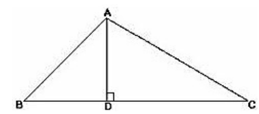
	Congruence statement	Congruence pattern
1. $\overline{ML} \cong \overline{ZJ}, \overline{LR} \cong \overline{JB}, \angle L \cong \angle J$	a.	b.
2. $\overline{WB} \cong \overline{QR}, \overline{BP} \cong \overline{RS}, \overline{WP} \cong \overline{QS}$	a.	b.
3. $\overline{CY} \cong \overline{RP}, \overline{EY} \cong \overline{BP}, \angle Y \cong \angle P$	a.	b.
4. $\overline{BC} \cong \overline{JK}, \overline{BA} \cong \overline{JM}, \angle B \cong \angle J$	a.	b.
5. $\overline{DF} \cong \overline{XZ}, \overline{FY} \cong \overline{ZW}, \angle F \cong \angle Z$	a.	b.
6. $\overline{WX} \cong \overline{AB}, \overline{XZ} \cong \overline{BC}, \overline{WZ} \cong \overline{AC}$	a.	b.

SET

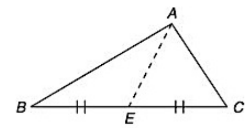
Topic: Special triangle segments and proof.
Recall the following definitions:

In a triangle:

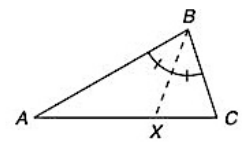
- an **altitude** is a line segment drawn from a vertex perpendicular to the opposite side (or an extension of the opposite side).



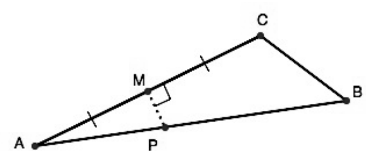
- a **median** is a line segment drawn from a vertex to the midpoint of the opposite side.



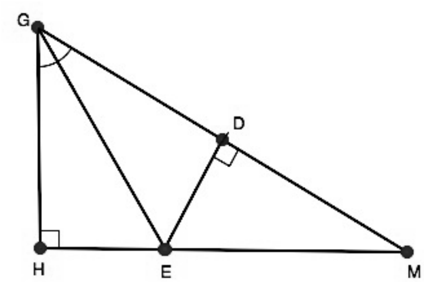
- an **angle bisector** is a line segment or ray drawn from a vertex that cuts the angle in half.



- a **perpendicular bisector of a side** is a line drawn perpendicular to a side of the triangle through its midpoint.



Be sure to use the correct notation for a segment in the following problems.



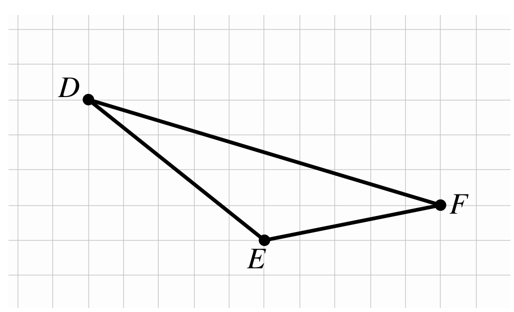
7. Name a segment in ΔGHM that is an altitude.

8. Name a segment in ΔGHM that is an angle bisector.

9. Name a segment in ΔGHM that is NOT an altitude.

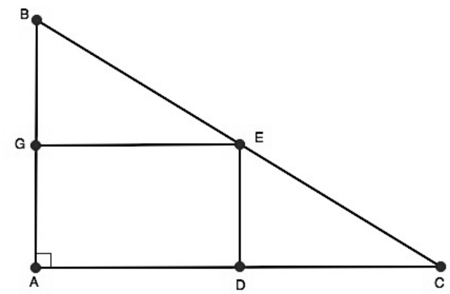
10. Create a perpendicular bisector by marking two segments congruent in ΔGHM . Name the segment that is now the perpendicular bisector.

Use ΔDEF in problems 11 – 13.



11. Construct the altitude from vertex D to $\overleftrightarrow{EF}$.
12. Construct the median from D to $\overline{EF}$.
13. Construct the perpendicular bisector of $\overline{EF}$.

Tehani has been studying the figure below. She knows that quadrilateral ADEG is a rectangle and that $\overline{ED}$ *bisects* $\overline{BC}$. *She is wondering if with that information she can prove* $\Delta BGE \cong \Delta EDC$.
She starts to organize her thinking by writing what she knows and the reasons she knows it.



I know $\overline{ED}$ bisects $\overline{BC}$ because I was given that information

I know that $\overline{BE} \cong \overline{EC}$ by definition of bisect.

I know that $\overline{GE}$ must be parallel to $\overline{AD}$ because the opposite sides in a rectangle are parallel.

I know that $\overline{GA} \parallel \overline{ED}$ because they are opposite sides in a rectangle.

I know that $\overline{AD}$ is contained in $\overline{AC}$ so $\overline{AC}$ is also parallel to $\overline{GE}$.

I know that $\overline{GA}$ is contained in $\overline{BA}$ so $\overline{GA}$ is also parallel to $\overline{BA}$

I know that $\overleftrightarrow{BC}$ has the same slope everywhere because it is a line.

I know the angle that $\overline{BE}$ makes with $\overline{GE}$ must be the same as the angle that $\overline{EC}$ makes with $\overline{AC}$ since those 2 segments are parallel. So $\angle BEG \cong \angle ECD$. I think I can use that same argument for $\angle GBE \cong \angle DEC$.

I know that I now have an angle, a side, and an angle congruent to a corresponding angle, side, and angle. So $\Delta BGE \cong \Delta EDC$ by ASA.

14. Use Tehani's "I know" statements and her reasons to write a two-column proof that proves $\Delta BGE \cong \Delta EDC$. Begin your proof with the "givens" and what you are trying to prove.

Given: quadrilateral ADEG is a rectangle, $\overline{ED}$ bisects $\overline{AC}$
Prove: $\Delta BGE \cong \Delta EDC$

STATEMENTS	REASONS
1. quadrilateral ADEG is a rectangle	given
2. $\overline{ED}$ bisects $\overline{AC}$	given

GO

Topic: Transformations

Perform the following transformations on ΔABC. Use a straight edge to connect the corresponding points with a line segment. Answer the questions.

15. Reflect ΔABC over $\overleftrightarrow{LK}$. Label your new image $\Delta A'B'C'$.
16. What do you notice about the line segments $\overline{AA'}, \overline{BB'}, and\ \overline{CC'}$?

17. Compare line segments $\overline{AB}, \overline{BC}, and\ \overline{CA}\ to\ \overline{A'B'}, \overline{B'C'}, \overline{C'A'}$. What is the same and what is different about these segments?

18. Translate ΔABC down 8 units and right 10 units. Label your new image $\Delta A''B''C''$.

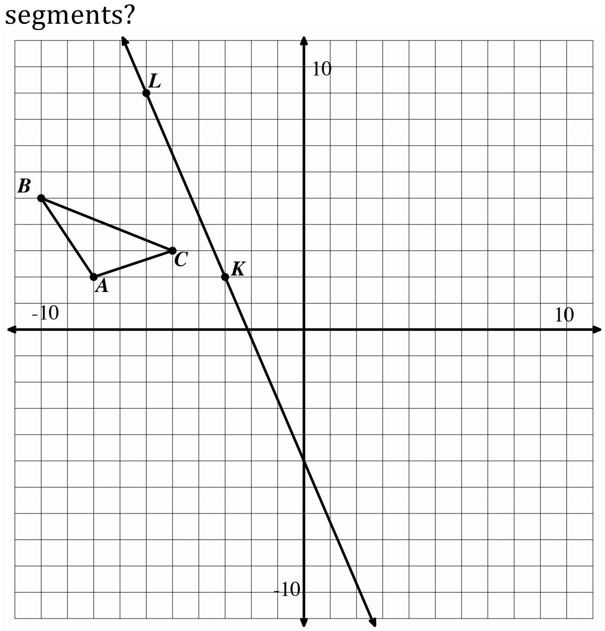
19. What do you notice about the line segments $\overline{AA''}, \overline{BB''}, and\ \overline{CC''}$?

20. Compare line segments $\overline{AB}, \overline{BC}, and\ \overline{CA}\ to\ \overline{A''B''}, \overline{B''C''}, \overline{C''A''}$. What is the same and what is different about these segments?

21. Translate ΔABC down 10 units and reflect it over the Y–axis. Label your new image $\Delta A'''B'''C'''$.

22. What do you notice about the line segments $\overline{AA'''}, \overline{BB'''}, and\ \overline{CC'''}$?

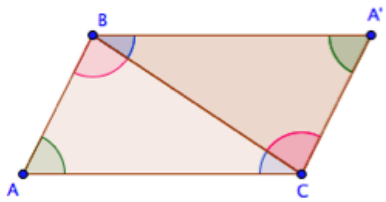
23. Compare line segments $\overline{AB}, \overline{BC}, and\ \overline{CA}\ to\ \overline{A'''B'''}, \overline{B'''C'''}, \overline{C'''A'''}$. What is the same and what is different about these segments?



5.4 Parallelism Preserved and Protected

A Solidify Understanding Task

In a previous task, *How Do You Know That*, you were asked to explain how you knew that this figure, which was formed by rotating a triangle about the midpoint of one of its sides, was a parallelogram.



You may have found it difficult to explain how you knew that sides of the original triangle and its rotated image were parallel to each other except to say, "It just has to be so." There are always some statements we have to accept as true in order to convince ourselves that other things are true. We try to keep this list of statements as small as possible, and as intuitively obvious as possible. For example, in our work with transformations we have agreed that distance and angle measures are preserved by rigid motion transformations since our experience with these transformations suggest that sliding, flipping and turning figures do not distort the images in any way. Likewise, parallelism within a figure is preserved by rigid motion transformations: for example, if we reflect a parallelogram the image is still a parallelogram—the opposite sides of the new quadrilateral are still parallel.

Mathematicians call statements that we accept as true without proof *postulates.* Statements that are supported by justification and proof are called *theorems.*

Knowing that lines or line segments in a diagram are parallel is often a good place from which to start a chain of reasoning. Almost all descriptions of geometry include a *parallel postulate* among the list of statements that are accepted as true. In this task we develop some parallel postulates for rigid motion transformations.

Translations

Under what conditions are the corresponding line segments in an image and its pre-image parallel after a translation? That is, which word best completes this statement?

> *After a translation, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.*

Give reasons for your answer. If you choose "sometimes", be very clear in your explanation about how to tell when the corresponding line segments before and after the translation are parallel and when they are not.

Rotations

Under what conditions are the corresponding line segments in an image and its pre-image parallel after a rotation? That is, which word best completes this statement?

> *After a rotation, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.*

Give reasons for your answer. If you choose "sometimes", be very clear in your explanation about how to tell when the corresponding line segments before and after the rotation are parallel and when they are not.

Reflections

Under what conditions are the corresponding line segments in an image and its pre-image parallel after a reflection? That is, which word best completes this statement?

> *After a reflection, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.*

Give reasons for your answer. If you choose "sometimes" be very clear in your explanation about how to tell when the corresponding line segments before and after the reflection are parallel and when they are not.

5.4 Parallelism Preserved and Protected – Teacher Notes

A Solidify Understanding Task

Purpose: Euclid was right, we can't make much progress in proving statements in geometry without a statement about parallelism. Euclid made an assumption related to parallelism—his frequently discussed and questioned 5th postulate. Non-Euclidean geometries resulted from mathematicians making different assumptions about parallelism. The purpose of this task is to establish some "parallel postulates" for transformational geometry. The authors of CCSS-M suggested some statements about parallelism that they would allow us to assume to be true in their development of the geometry standards: (1) rigid motion transformations "take parallel lines to parallel lines" (that is, parallelism, along with distance and angle measure, is preserved by rigid motion transformations—see 8.G.1), and (2) dilations "take a line not passing through the center of the dilation to a parallel line" (see G.SRT.1a). In this task we develop some additional statements about parallelism for the rigid motion transformations, which we will accept as postulates for our development of geometry: (1) *After a translation, corresponding line segments in an image and its pre-image are always parallel or lie along the same line;* (2) *After a rotation of 180°, corresponding line segments in an image and its pre-image are parallel or lie on the same line;* (3) *After a reflection, line segments in the pre-mage that are parallel to the line of reflection will be parallel to the corresponding line segments in the image.*

These statements about parallelism will lead to the proofs of theorems about relationships of angles relative to parallel lines crossed by a transversal.

Note #1: In transformational geometry, one can take the perspective that an image and its pre-image are distinct figures even when they coincide. Consequently, rotating a line 180° about a point on the line creates an image/pre-image pair of lines that coincide. If we consider the image/pre-image lines as distinct, we might also say that they are parallel to each other. Otherwise, they share all points in common and are the same line. In the wording we have used here for translations and rotations, we are taking the perspective that they share all points in common, and therefore, are the same line.

Note #2: These statements about parallelism could be treated as theorems, rather than postulates, if you wish to pursue more formal proofs about these statements. For example, statement 2 about line segments undergoing a 180° rotation being parallel to each other can be proved by contradiction—assume the lines aren't parallel and show that any assumed point of intersection would contradict the assumption that the line had been rotated 180° since the line segment connecting the point of intersection to the center of rotation and back to the point of intersection does not represent a 180° turn. Such reasoning may be beyond your students, and proof by contradiction is not one of the expected proof formats of the common core standards.

Core Standards Focus:
G.CO.9 Prove theorems about lines and angles. Theorems include: when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent

Mathematics II Note for G.CO.10: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Related Standards: 8.G.1, G.SRT.1a

The Teaching Cycle:
Launch (Whole Class):
Discuss the difference between a theorem and a postulate, as outlined in the first part of the task. You might want to make posters for the room of postulates, definitions and theorems we have established for our geometry work up to this point in time. Alternatively, students should have sections in their notebooks for each of these three types of statements. As we continue our work in this and the

following module, students can add new postulates, definitions and theorems to their notebooks or posters. Remind students that postulates, definitions and previously proved theorems are the tools we use to establish new theorems through deductive reasoning. Here are examples of statements that should be included in our list of tools:

Definitions:
Rigid motion transformations: translation, rotation, reflection
Types of triangles: scalene, isosceles, equilateral
Triangle-related lines and line segments: median, altitude, angle bisector, perpendicular bisector
Quadrilaterals: parallelogram, rhombus, rectangle, square, trapezoid
Polygon-related terms: diagonals, regular polygon, lines of symmetry, rotational symmetry

Postulates:
Rigid motion transformations preserve angle measure and distance

Theorems:
Congruent triangle criteria: SSS, SAS, ASA
The sum of the angles of a triangle is 180°
Points on a perpendicular bisector of a segment are equidistant from the endpoints of the segment
The diagonals of a rhombus are perpendicular and bisect each other

Provide appropriate tools for students to experiment with the ideas generated by each of the questions in this task about parallelism relative to each rigid motion transformation. This would be a good time to use dynamic geometry software, such as Geometer's Sketchpad or Geogebra to check out students' conjectures.

Explore (Small Group):
Allow students time to explore parallelism relative to each rigid motion transformation. Ask questions to push their thinking, such as, "Why do you think corresponding image/pre-image line

segments are always parallel after a translation, why can't they have different slopes?" or "How do you know these corresponding image/pre-image lines will never intersect?"

Discuss (Whole Class):

Based on students' intuitive arguments, add the following postulates to the classroom posters or student notes:

(1) *After a translation, corresponding line segments in an image and its pre-image are always parallel or lie on the same line*

(2) *After a rotation of 180°, corresponding line segments in an image and its pre-image are parallel or lie on the same line.*

(3) *After a reflection, line segments in the pre-image that are parallel to the line of reflection will be parallel to the corresponding line segments in the image.*

Aligned Ready, Set, Go: Geometric Figures 5.4

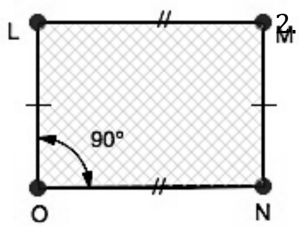
READY, SET, GO! Name Period Date

READY

Topic: Special Quadrilateral

Identify each quadrilateral as a trapezoid, parallelogram, rectangle, rhombus, square, or none of these. List ALL that apply.

1.



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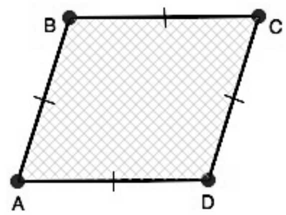

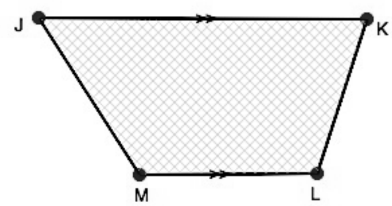

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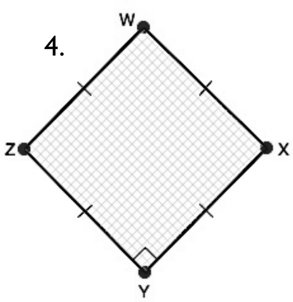
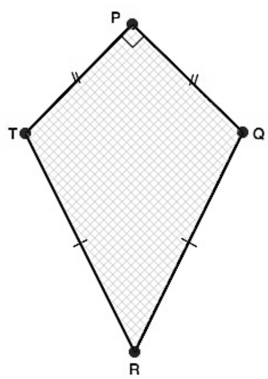
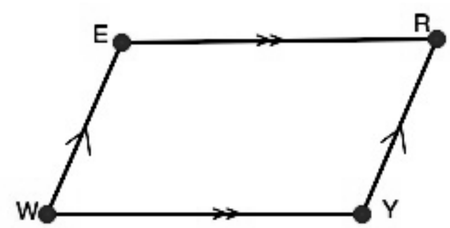




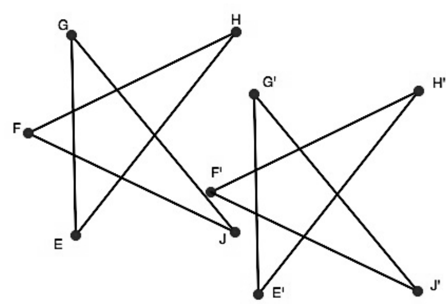


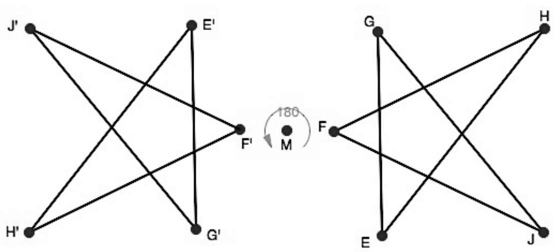
SET

Topic: Identifying parallel segments and lines produced from transformations

7. Verify the parallel postulates below by naming the line segments in the pre-image and its image that are still parallel. Use correct mathematical notation.

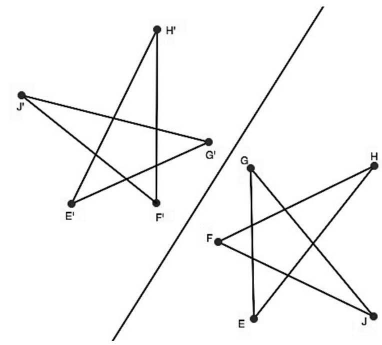
a. After a translation, corresponding line segments in an image and its pre-image are always parallel or lie along the same line.





b. After a rotation of 180°, corresponding line segments in a pre- image and its image are parallel or lie on the same line.

c. After a reflection, line segments in the pre-mage that are parallel to the line of reflection will be parallel to the corresponding line segments in the image.

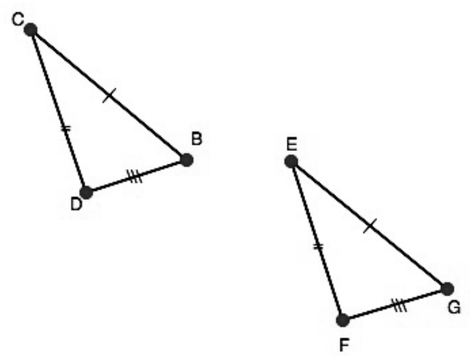


GO

Topic: Identifying congruence patterns in triangles

For each pair of triangles write a congruence statement and justify your statement by identifying the congruence pattern you used. Then justify that the triangles are congruent by connecting corresponding vertices of the pre-image and image with line segments.
How should those line segments look?

8.



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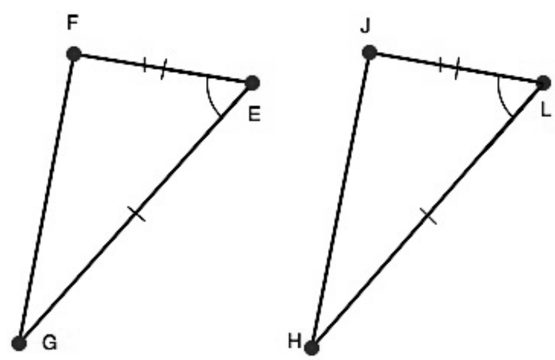

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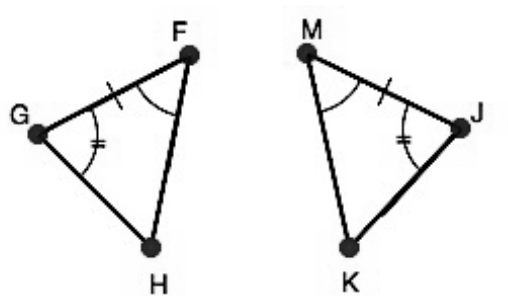

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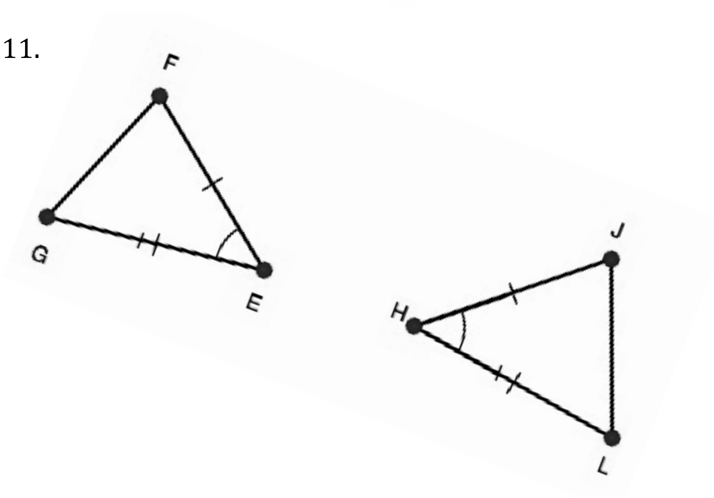
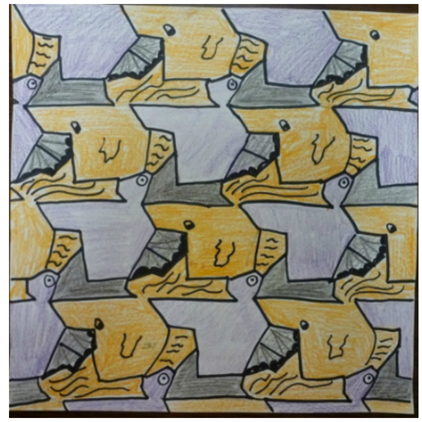
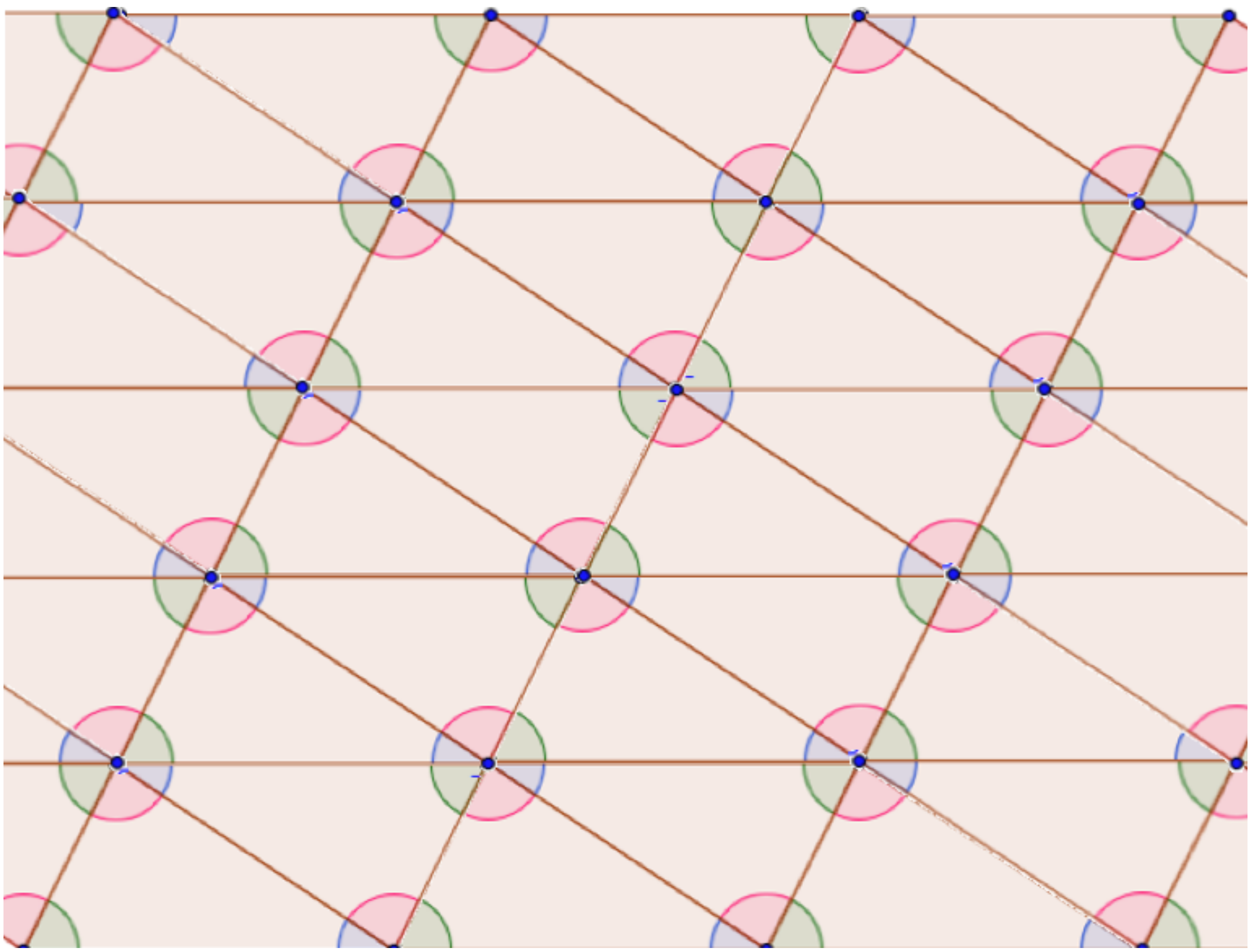


5.5 Claims and Conjectures

A Solidify Understanding Task



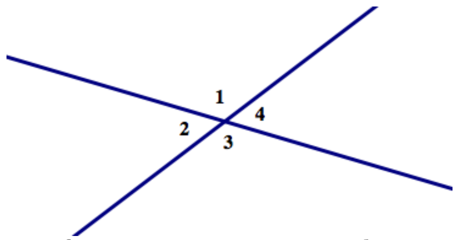
The diagram from *How Do You Know That?* has been extended by repeatedly rotating the image triangles around the midpoints of their sides to form a tessellation of the plane, as shown below.



Using this diagram, we will make some conjectures about lines, angles and triangles and then write proofs to convince ourselves that our conjectures are always true.

Vertical Angles

When two lines intersect, the opposite angles formed at the point of intersection are called *vertical angles*. In the diagram below, $\angle 1$ and $\angle 3$ form a pair of vertical angles, and $\angle 2$ and $\angle 4$ form another pair of vertical angles.



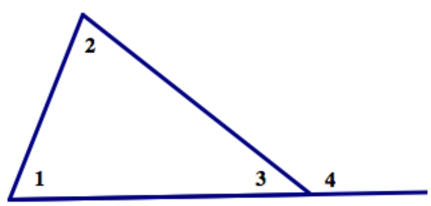
Examine the tessellation diagram above, looking for places where vertical angles occur. (You may have to ignore some line segments and angles in order to focus on pairs of vertical angles. This is a skill we have to develop when trying to see specific images in geometric diagrams.)

Based on several examples of vertical angles in the diagram, write a conjecture about vertical angles.

My conjecture:

Exterior Angles of a Triangle

When a side of a triangle is extended, as in the diagram below, the angle formed on the exterior of the triangle is called an *exterior angle*. The two angles of the triangle that are not adjacent to the exterior angle are referred to as the *remote interior angles*. In the diagram, $\angle 4$ is an exterior angle, and $\angle 1$ and $\angle 2$ are the two remote interior angles for this exterior angle



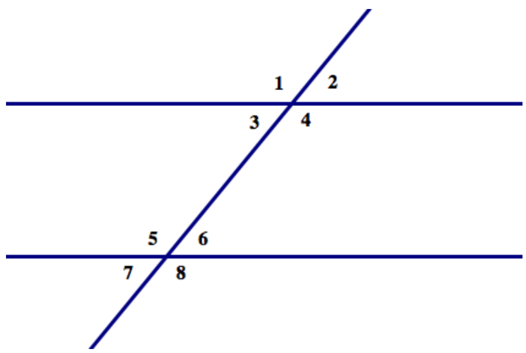
Examine the tessellation diagram above, looking for places where exterior angles of a triangle occur. (Again, you may have to ignore some line segments and angles in order to focus on triangles and their vertical angles.)

Based on several examples of exterior angles of triangles in the diagram, write a conjecture about exterior angles.

My conjecture:

Parallel Lines Cut By a Transversal

When a line intersects two or more other lines, the line is called a *transversal* line. When the other lines are parallel to each other, some special angle relationships are formed. To identify these relationships, we give names to particular pairs of angles formed when lines are crossed (or cut) by a transversal. In the diagram below, $\angle 1$ and $\angle 5$ are called *corresponding angles*, $\angle 3$ and $\angle 6$ are called *alternate interior angles*, and $\angle 3$ and $\angle 5$ are called *same side interior angles*.



Examine the tessellation diagram above, looking for places where parallel lines are crossed by a transversal line.

Based on several examples of parallel lines and transversals in the diagram, write some conjectures about corresponding angles, alternate interior angles and same side interior angles.

My conjectures:

Justifying Our Conjectures

In the next task you will be asked to write a proof that will convince you and others that each of the conjectures you wrote above is always true. You will be able to use ideas about transformations, linear pairs, congruent triangle criteria, etc. to support your arguments. A good way to start is to write down everything you know about the diagram, and then identify which statements you might use to make your case. To get ready for the next task, revisit each of the conjectures you wrote about and record some ideas that seem helpful in proving that the conjecture is true.

5.5 Claims and Conjectures – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this and the following task is to give students practice in analyzing diagrams to identify conjectures, and then writing proofs to show that the conjectures are true. The specific theorems being examined in this and the following task are:

- Vertical angles are congruent
- The measure of an exterior angle of a triangle is the sum of the two remote interior angles
- When a transversal crosses parallel lines, alternate interior angles are congruent, corresponding angles are congruent, and same-side interior angles are supplementary (add to 180°).

Core Standards Focus:

G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent.

G.CO.10 Prove theorems about triangles.

Mathematics II Note for G.CO.10: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

The Teaching Cycle:

Launch (Whole Class):

Introduce the names of the angles and pairs of angles that are used in this task by referring to the numbered diagrams: vertical angles; exterior angle, remote interior angles; alternate interior angles, corresponding angles, same-side interior angles. Have students add these definitions to their notes or classroom posters (see teacher notes from the previous task).

Give students three different colored pencils to use to mark corresponding angles in the tessellation diagram. Remind students that this diagram was formed by rotating each triangle about the midpoint of one of its sides. They might also notice that triangles in the tessellation diagram have been translated in the same direction and the same distance as each of the individual sides (that is, each side of a triangle gives the magnitude and direction of a translated triangle).

Explore (Small Group):

Give students a few minutes to examine the tessellation diagram and to write conjectures about the relationships among the various sets of angles defined above. Students may need to trace over and darken line segments in portions of the diagram in order to see specific examples of vertical angles, exterior angles of a triangle, or parallel lines crossed by a transversal. Encourage them to look for these examples in multiple places in the diagram; for example, not always looking at parallel lines that are horizontal. After a few minutes, verify that students have identified all of the potential conjectures, and then have them read the last paragraph of the task and start to record ideas about why they think each of their conjectures is true.

Discuss (Whole Class):

Have students share the conjectures they have written for each of the three diagrams given in the task. As they do so, they should identify places in the tessellation diagram where they see examples of the claim they are making. Have students who share describe the reasons why they think their conjectures are true, and ask other students in the class to add additional insights, questions or concerns. This will help prepare them to write more formal proofs of these conjectures in the following task.

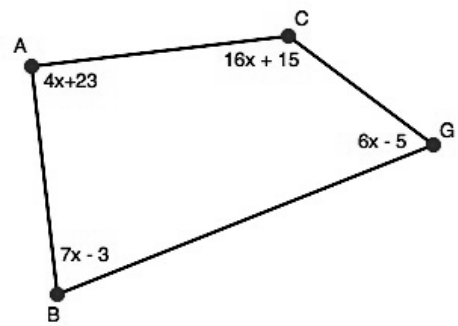
Aligned Ready, Set, Go: Geometric Figures 5.5

READY, SET, GO! Name Period Date

READY

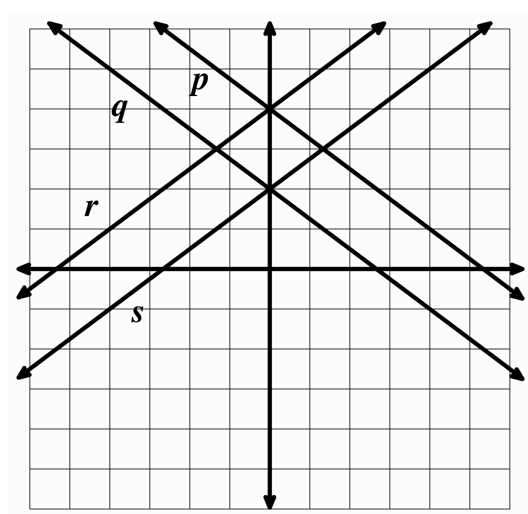
Topic: Properties of Quadrilaterals

1. Use what you know about triangles to write a paragraph proof that proves that the sum of the angles in a quadrilateral is 360º.



2. Find the measure of x in quadrilateral *ABGC*.

Match the equation with the correct line in the graph of lines *p*, *q*, *r*, and *s*.



3. $y = \frac{3}{4}x + 2$

4. $y = -\frac{3}{4}x + 2$

5. $y = \frac{3}{4}x + 4$

6. $y = -\frac{3}{4}x + 4$

7. Describe the shape made by the intersection
4 lines. List as many observations as you
shape and its features.

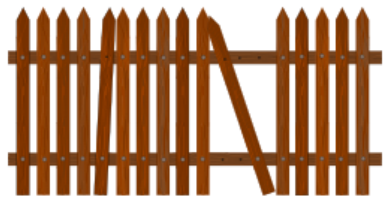
SET

Topic: Parallel lines cut by transversal, vertical angles and exterior angle of a triangle

Label each picture as showing *parallel lines with a transversal, vertical angles*, or an *exterior angle of a triangle*. Highlight the geometric feature you identified. Can you find all 3 features in 1 picture? Where?

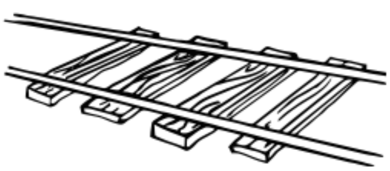
8.

9.



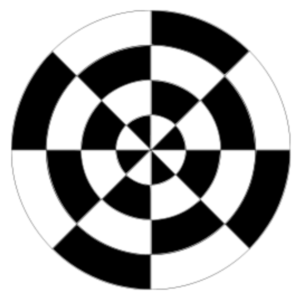
10.

11.



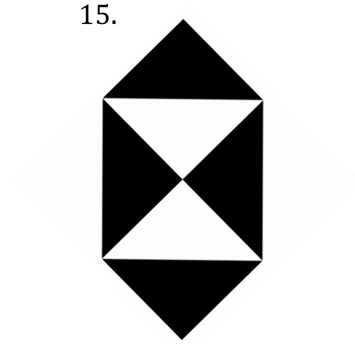
12.

13.



14.

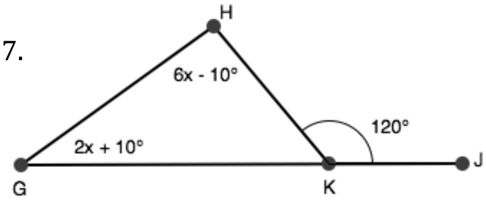
15.



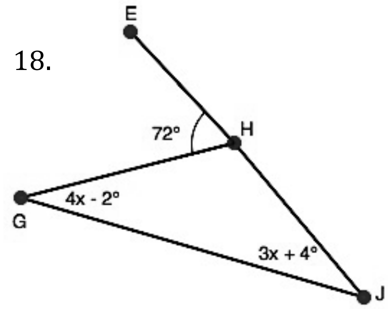
16.

Find the value of the 2 remote interior angles in the figures below.

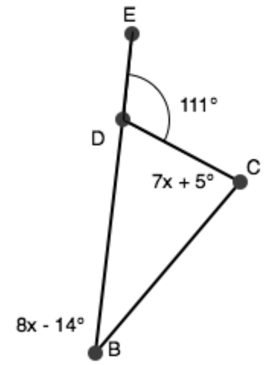
17.



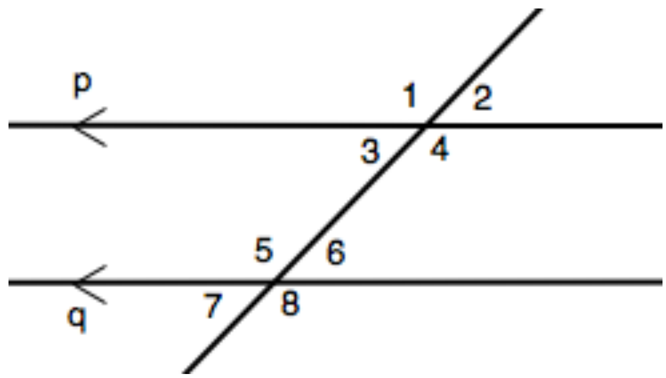
18.



19.



Indicate whether each pair of angles is *congruent* or *supplementary* by trusting how they look. Lines *p* and *q* are parallel.



20. $\angle 5$ *and* $\angle 8$

21. $\angle 2$ *and* $\angle 6$

22. $\angle 2$ *and* $\angle 8$

23. $\angle 4$ *and* $\angle 6$

24. $\angle 3$ *and* $\angle 5$

25. $\angle 1$ *and* $\angle 3$

GO

Topic: Complementary and supplementary angles

Find the complement and the supplement of the given angles. It is possible for the complement or supplement not to exist.

26. 37°

27. 59°

28. 89°

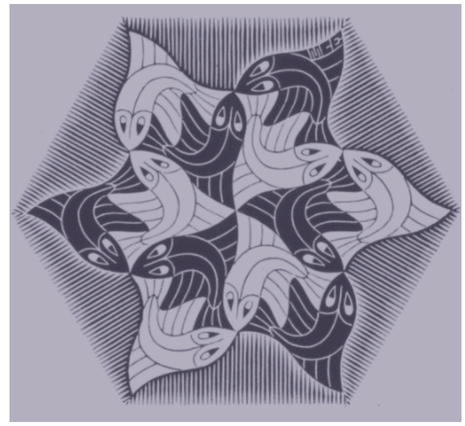
29. 111°

30. 3°

31. 90°

5.6 Justification and Proof

A Practice Understanding Task



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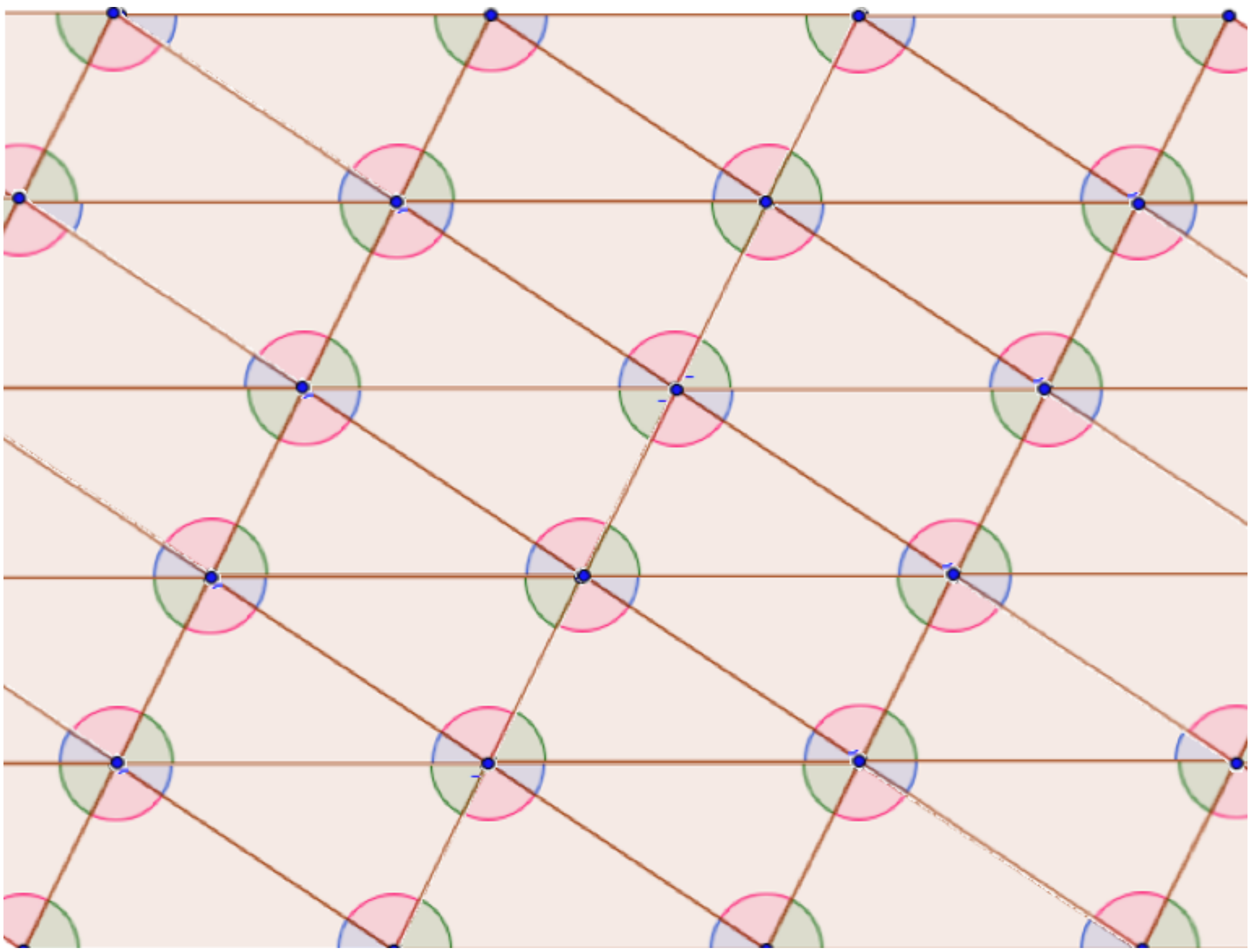
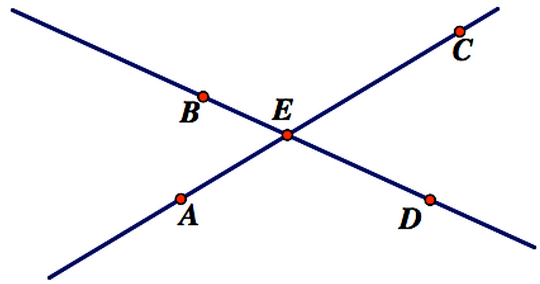


Using this diagram, you have made some conjectures about lines, angles and triangles. In this task you will write proofs to convince yourself and others that these conjectures are always true.

Vertical Angles

When two lines intersect, the opposite angles formed at the point of intersection are called *vertical angles*. In the diagram, $\angle AEB$ and $\angle CED$ form a pair of vertical angles.

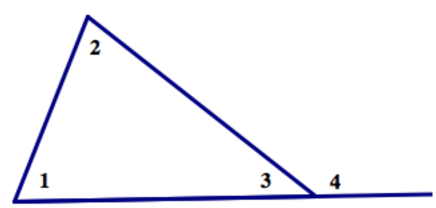


1. Given: $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ intersect at E.
   Prove: $\angle AEB \cong \angle CED$

[Note: For each of the following proofs you may use any format you choose to write your proof: a flow proof diagram, a two-column proof, or a narrative paragraph.]

Exterior Angles of a Triangle

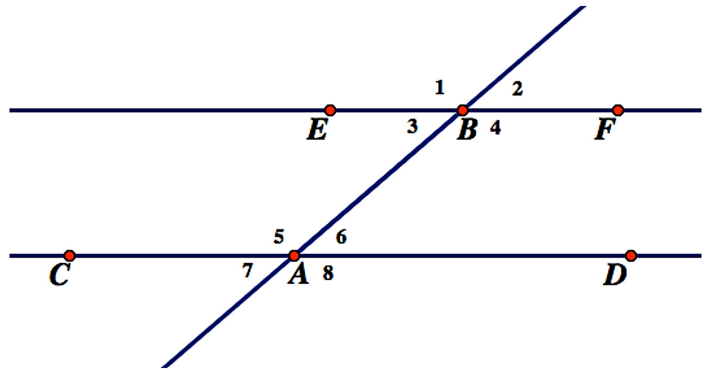
When a side of a triangle is extended, as in the diagram below, the angle formed on the exterior of the triangle is called an *exterior angle*. The two angles of the triangle that are not adjacent to the exterior angle are referred to as the *remote interior angles*. In the diagram, $\angle 4$ is an exterior angle, and $\angle 1$ and $\angle 2$ are the two remote interior angles for this exterior angle.



2. Given: $\angle 4$ is an exterior angle of the triangle
   Prove: $m\angle 4 = m\angle 1 + m\angle 2$

Parallel Lines Cut By a Transversal

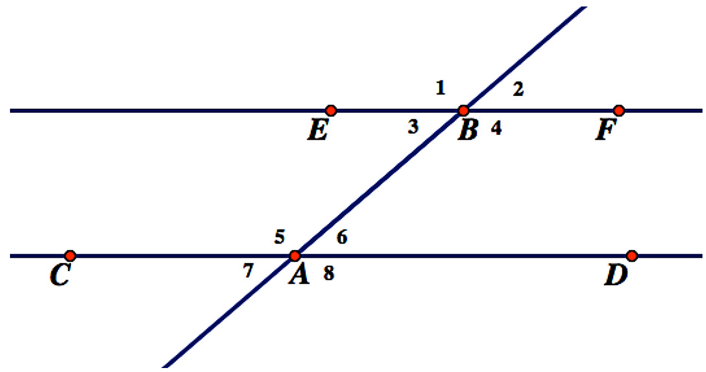
When a line intersects two or more other lines, the line is called a *transversal* line. When the other lines are parallel to each other, some special angle relationships are formed. To identify these relationships, we give names to particular pairs of angles formed when lines are crossed (or cut) by a transversal. In the diagram, $\angle 1$ and $\angle 5$ are called *corresponding angles*, $\angle 3$ and $\angle 6$ are called *alternate interior angles*, and $\angle 3$ and $\angle 5$ are called *same side interior angles*.



3. Given: $\overleftrightarrow{BF} \parallel \overleftrightarrow{AD}$
   Prove: Corresponding angles $\angle 1$ and $\angle 5$ are congruent

4. Given: $\overleftrightarrow{BF} \parallel \overleftrightarrow{AD}$
   Prove: Alternate interior angles $\angle 3$ and $\angle 6$ are congruent

5. Given: $\overleftrightarrow{BF} \parallel \overleftrightarrow{AD}$
   Prove: Same-side interior angles $\angle 3$ and $\angle 5$ are supplementary



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6. Given: Alternate interior angles $\angle 3$ and $\angle 6$ are congruent
   Prove: $\overleftrightarrow{EF} \parallel \overleftrightarrow{CD}$

5.6 Justification and Proof – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to give students practice in writing proofs to show that the conjectures are true. The specific theorems being examined in this task are:

- Vertical angles are congruent
- The measure of an exterior angle of a triangle is the sum of the two remote interior angles
- When a transversal crosses parallel lines, alternate interior angles are congruent, corresponding angles are congruent, and same-side interior angles are supplementary (add to 180°).

In addition to practice writing proofs, the idea of the converse of a statement should be brought up in the discussion of this task. After students prove that alternate interior angles are congruent and corresponding angles are congruent when a transversal crosses parallel lines, they need to examine the converse statements as theorems:

- If corresponding angles are congruent when a transversal crosses two or more lines, then the lines are parallel.
- If alternate interior angles are congruent when a transversal crosses two or more lines, then the lines are parallel.

Core Standards Focus:

G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent.

G.CO.10 Prove theorems about triangles.

Mathematics II Note for G.CO.10: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

The Teaching Cycle:

Launch (Whole Class):

Use the tessellation diagram to review the conjectures students made about vertical angles, the measure of an exterior angle of a triangle, and corresponding, alternate interior, or same side interior angles when parallel lines are intersected by a transversal.

Explore (Small Group):

There are a variety of ways students can approach the proofs of these theorems, both in terms of the conceptual tools they use (transformations, linear pairs, congruent triangle criteria) and in terms of the format they use to write their proofs (two-column, flow diagrams, narrative paragraphs). As you observe student work, select a variety of proof strategies and formats to present during the whole class discussion.

Students will need to create appropriately labeled diagrams to support their work. This will include identifying the given information for the proof and marking it on their diagram, as well as writing a clear statement of what they are trying to prove.

If students are struggling with writing a proof of a particular conjecture, ask questions like, "What did you see in the tessellation diagram that led you to make this conjecture? How is that showing up in your diagram?" or "Can you make a list or flow diagram of things that you know are true about your diagram?"

If some students finish their proofs of these conjectures before others, have them consider developing proofs for the converse of the parallel line theorems:

- If corresponding angles formed by two lines crossed by a transversal are congruent, then the lines are parallel
- If alternate interior angles formed by two lines crossed by a transversal are congruent, then the lines are parallel.

Verify that students recognize the difference between these statements and the ones they have already proved. That is, students should notice that the information we are given and the statement we are to prove have been switched.

Discuss (Whole Class):
If available, have two different students present a proof that vertical angles are congruent: one based on linear pairs, and one based on rotating the figure 180° about the point of intersection of the two lines. Also examine proofs written in different formats, such as narrative form or a flow proof. Note that the same essential ideas are present in each proof.

It is important to discuss the parallel line proofs, since they form the foundation of many future proofs in geometry. A transformational argument can be made for why corresponding angles are congruent: Consider the cluster of 4 angles around the point of intersection of one parallel line and the transversal. Translate this cluster along the transversal until the first point of intersection coincides with the second. Since the image of a line is parallel to its pre-image after a translation (one of our parallel line postulates), the image of the first parallel line will coincide with the second parallel line, and the corresponding angles will also coincide. Once this theorem has been established for corresponding angles, the theorem for alternate interior angles follows by applying vertical angles, and the theorem for interior angles on the same side of the transversal follows by applying linear pairs.

Alternatively, we can prove alternate interior angles are congruent by a transformational argument based on rotation. Locate the midpoint on the transversal between the two points of intersection of the parallel lines and the transversal. Rotate the figure 180° about this point. After the rotation, the image of each parallel line will coincide with the other parallel line, and the alternate interior angles will also coincide. Once this theorem has been established for alternate interior angles, the theorem for corresponding angles follows by applying vertical angles, and the theorem for interior angles on the same side of the transversal follows by applying linear pairs.

The converses of these theorems about parallel lines are also true: If corresponding angles formed by lines crossed by a transversal are congruent, then the lines are parallel; and, if alternate interior angles formed by lines crossed by a transversal are congruent, then the lines are parallel. A transformational argument can also be used to prove the converse statements. For example, assume corresponding angles are congruent when two lines are crossed by a transversal, and we want to prove that the lines are parallel. Translate the cluster of 4 angles around one point of intersection along the transversal until it coincides with the other point of intersection. Since the rays forming the corresponding angles must coincide after the translation, the lines intersecting the transversal must also coincide, and therefore must have been parallel before the translation.

Have the students add the theorems about vertical angles and parallel lines crossed by a transversal (and their converse statements) to their notes or classroom posters.

Aligned Ready, Set, Go: Geometric Figures 5.6

READY, SET, GO! Name Period Date

READY

Topic: Recalling features of the rigid-motion transformations

Complete each statement

1. When I use line segments to connect the corresponding points of a pre-image and the image in a translation, the line segments are ____________________ and ______________________ because ____

2. When I use line segments to connect the corresponding points of a pre-image and the image in a reflection, the line of reflection is the __________________________ of the segments because ____

3. In a rotation, the corresponding points of the pre-image and the image are the same ________________ from the center of rotation because ____________________________

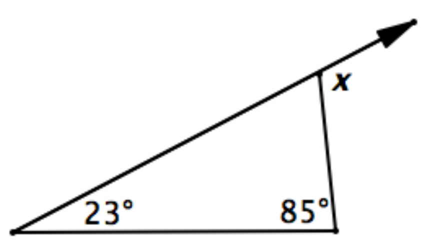
4. Translations, rotations, and reflections are rigid motion transformations because ____

SET

Topic: Solving for missing angles

Use what you know about vertical angles, exterior angles, and the angles formed by parallel lines and transversals to find the value of x in each of the diagrams.

5.



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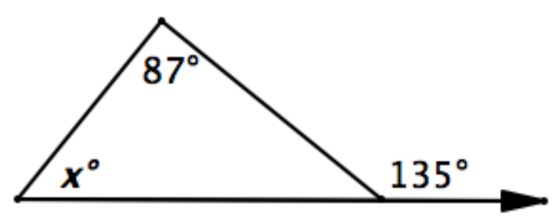

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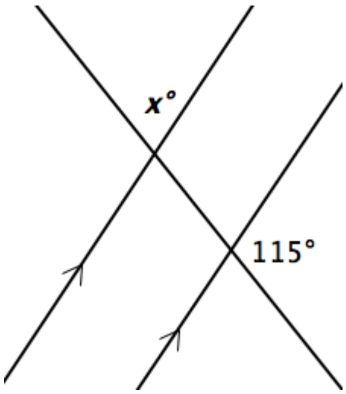

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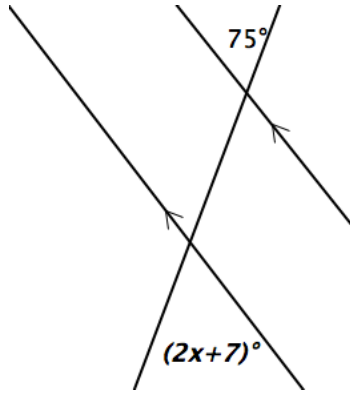
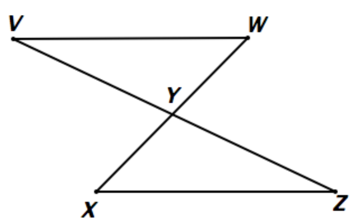
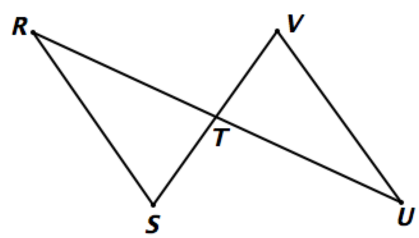


Prove each of the following.

9. Given: Y is the midpoint of $\overline{VZ}$ *and* $\overline{XW}$.
Prove: $\Delta VYW \cong \Delta ZYX$



10. Given $\angle R \cong \angle U$ and $\overline{ST} \cong \overline{VT}$.
Prove: $\Delta SRT \cong \Delta VUT$

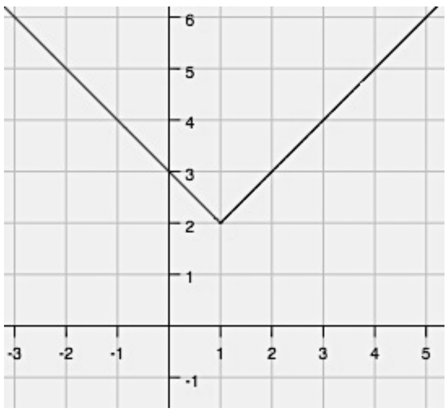


GO

Topic: Connecting a piecewise defined equation with the corresponding absolute value equation

The graph of an absolute value function is given. A) Write the equation using absolute value notation. B) Then write the equation as a piecewise defined function.

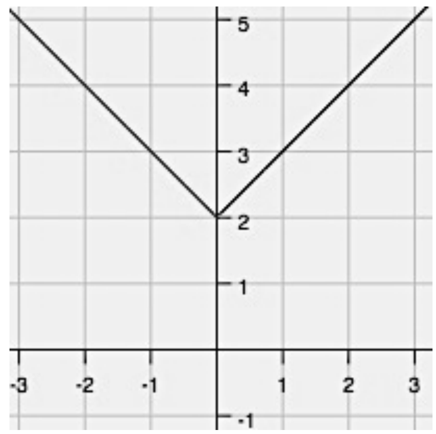
15.



A.

B.

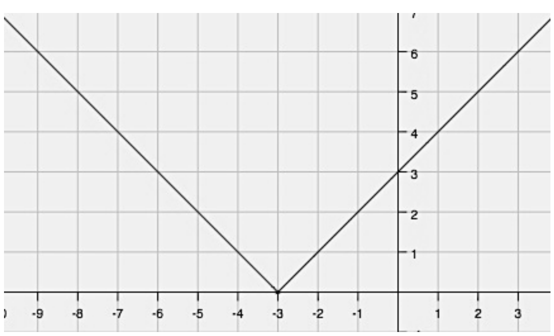
16.



A.

B.

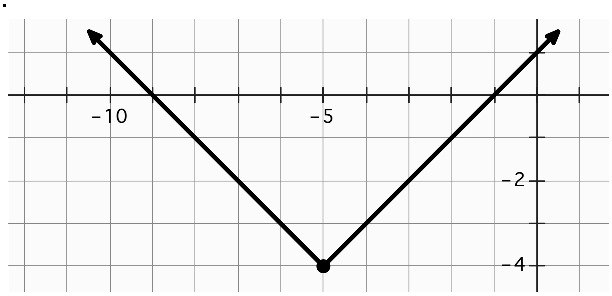
17.



A.

B.

18.

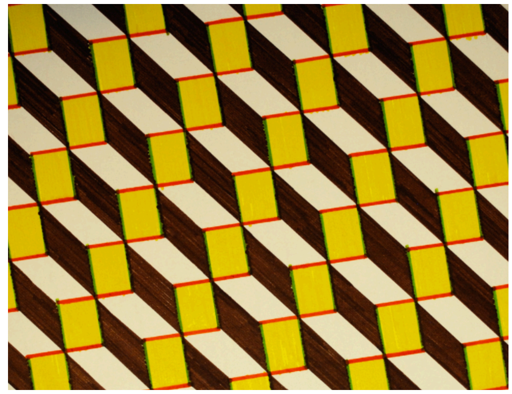


A.

B.

5.7 Parallelogram Conjectures and Proof

A Solidify Understanding Task



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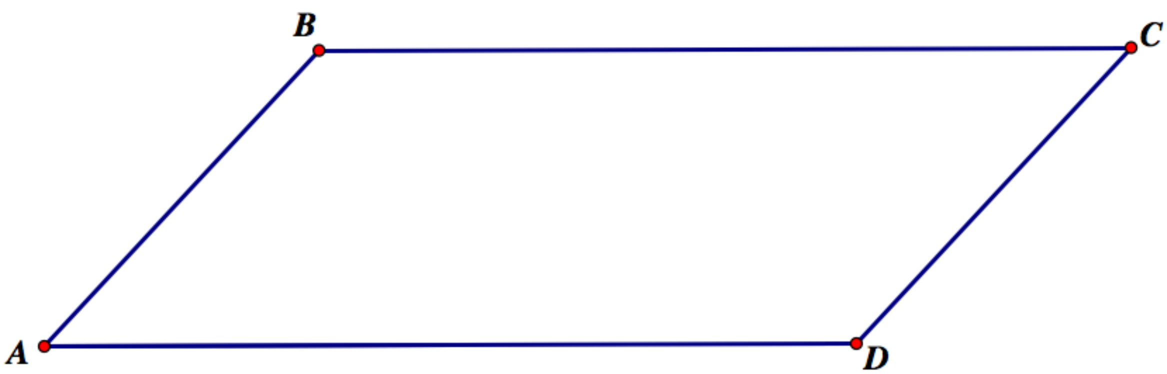
In Mathematics I you made conjectures about properties of parallelograms based on identifying lines of symmetry and rotational symmetry for various types of parallelograms. Now that we have additional knowledge about the angles formed when parallel lines are cut by a transversal, and we have criteria for convincing ourselves that two triangles are congruent, we can more formally prove some of the things we have noticed about parallelograms.

1. Explain how you would locate the center of rotation for the following parallelogram. What convinces you that the point you have located is the center of rotation?



2. If you haven't already, draw one or both of the diagonals in the above parallelogram. Use this diagram to prove this statement: *opposite sides of a parallelogram are congruent*

3. Use this diagram to prove this statement: *opposite angles of a parallelogram are congruent*

4. Use this diagram to prove this statement: *the diagonals of a parallelogram bisect each other*

The statements we have proved above extend our knowledge of properties of all parallelograms: not only are the opposites sides parallel, they are also congruent; opposite angles are congruent; and the diagonals of a parallelogram bisect each other. A parallelogram has 180° rotational symmetry around the point of intersection of the diagonals—the center of rotation for the parallelogram.

If we have a quadrilateral that has some of these properties, can we convince ourselves that the quadrilateral is a parallelogram? How many of these properties do we need to know before we can conclude that a quadrilateral is a parallelogram?

5. Consider the following statements. If you think the statement is true, create a diagram and write a convincing argument to prove the statement.

   a. If opposite sides and opposite angles of a quadrilateral are congruent, the quadrilateral is a parallelogram.

   b. If opposite sides of a quadrilateral are congruent, the quadrilateral is a parallelogram.

   c. If opposite angles of a quadrilateral are congruent, the quadrilateral is a parallelogram.

   d. If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.

5.7 Parallelogram Conjectures and Proof – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to prove theorems about the properties of parallelograms that were surfaced in Mathematics I as students explored the rotational symmetry and line symmetry of various types of quadrilaterals. You may want to review this exploratory work with students. (See *Symmetries of Quadrilaterals* and *Quadrilaterals—Beyond Definition* in the Mathematics Vision Project, Secondary One curriculum.) Students will solidify that these properties of parallelograms are a consequence of the opposite sides of the quadrilateral being parallel to each other. That is, they will draw upon their theorems about parallel lines being cut by a transversal—with the diagonals of the parallelogram forming the transversals—to prove these additional properties of parallelograms.

Core Standards Focus:

G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Mathematics II Note for G.CO.10: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Related Standards: G.CO.9

The Teaching Cycle:

Launch (Whole Class): [questions 1-4]

Review the theorems about angles formed when parallel lines are crossed by a transversal from the previous task and their converses. Remind students of their previous work with symmetries of parallelograms by having them work on question 1. They should recognize that the point of intersection of the two diagonals is the center of a 180° rotation that carries a parallelogram onto itself. Point out that they probably know this based on experimentation. In this task, they will prove that the point of intersection of the diagonals is the center of a 180° rotation by proving that the diagonals bisect each other (see question 4). Assign students to work on the proofs in questions 2-4.

Explore (Small Group): [questions 1-4]

As students work on these proofs you might need to remind them that the diagonals of the parallelogram can be thought of as transversals for the opposite pairs of parallel sides of the parallelogram. All three proofs can be proved by decomposing the parallelogram into congruent triangles using the diagonals, and then identifying appropriate parts of congruent triangles to complete each proof. Suggest that students consider whether it is helpful to draw one diagonal—forming two triangles in the interior of the parallelogram, or both diagonals—forming four triangles in the interior. Remind students they can use previously proven statements to prove additional statements. For example, once we have proved that opposite sides of a parallelogram are congruent, we can use that statement to help prove something else about parallelograms.

If students are struggling with the proofs, have them draw a single diagonal and write down everything they know to be true based on using that diagonal as a transversal between one pair of parallel sides, and then between the other pair of parallel sides. Repeat with the other diagonal. Make sure they mark congruent segments and angles on the diagram and have them record their symbolic statements in a flow diagram. Once they have recorded everything they know to be true in a flow diagram, have them trace out the statements they need for each proof, similar to the work they did in *Its All In Your Head*.

Discuss (Whole Class): [questions 1-4]

It may be helpful to create a class flow diagram as described in the previous paragraph, and then as students present their proofs for statement 2-4 the logic of each proof can be highlighted in different colors on the flow diagram. As ideas are recorded symbolically on the flow diagram, have another student mark congruent angles and segments on the parallelogram diagram.

Launch (Whole Class): [question 5]

Ask students to work on question 5 where they will consider the converses of the statements they have just proved. They should start with a new, unmarked parallelogram diagram for each part a-d, and start their reasoning about the statement by marking on the diagram what is given to be true about each parallelogram.

Explore (Small Group): [question 5]

You may need to help students realize that these proofs require us to show that the beginning quadrilateral with given conditions also fits the definition of a parallelogram. That is, can it be shown that opposite sides are parallel?

Discuss (Whole Group): [question 5]

Given time, have selected students share some of these proofs. These ideas will be considered again in the next task, *Guess My Parallelogram*, so it is not necessary to get through all of these proofs at this time.

Aligned Ready, Set, Go: Geometric Figures 5.7

READY, SET, GO! Name Period Date

READY

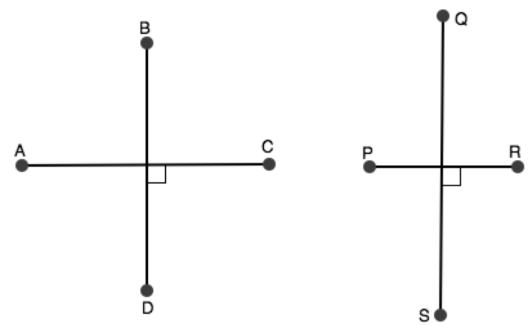
Topic: Sketching quadrilaterals based on specific features

Sketch the quadrilateral by connecting the points in alphabetical order. Close the figure.

1. In both figures, the lines are perpendicular bisectors of each other.

a. Are the quadrilaterals you sketched congruent?

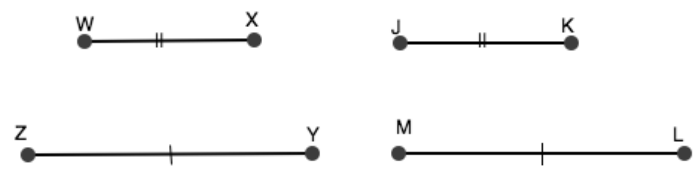
b. What additional requirement(s) is/are needed to make the figures congruent?



2. In both figures one set of opposite sides are parallel and congruent.

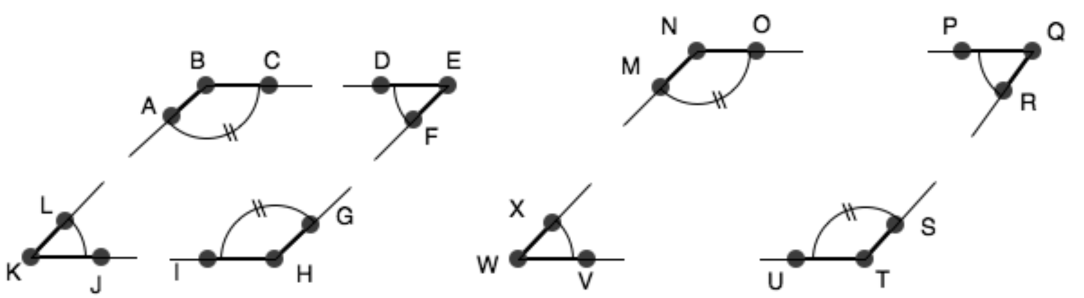
a. Are the quadrilaterals you sketched congruent?

b. What additional requirement(s) is/are needed to make the figures congruent?



3. In both figures corresponding angles are congruent.

a. Are the quadrilaterals you sketched congruent?
b. What additional requirement(s) is/are needed to make the figures congruent?



SET

Topic: Properties of parallelograms

4. Quadrilateral BCDE below was formed by 2 sets of intersecting parallel lines. Figure 2 is the image of figure 1. It has been rotated 180º. Find the center of rotation for figure 1. Make a list of everything that has been preserved in the rotation. Then make a list of anything that has changed.
Is quadrilateral BCDE a parallelogram? How do you know?

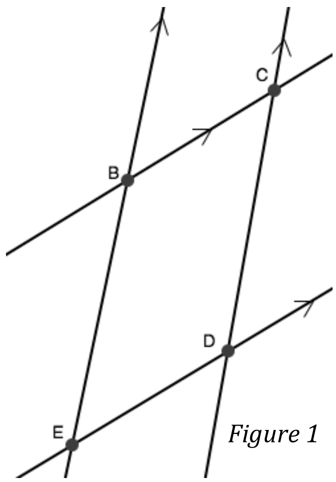


Figure 1

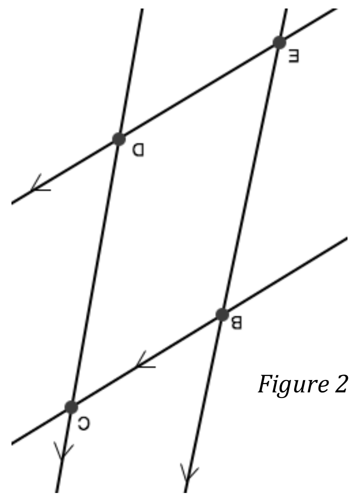


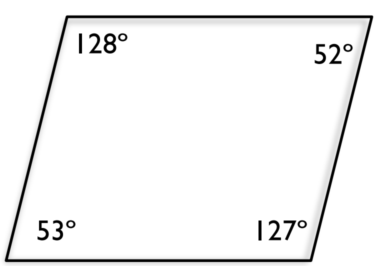
Figure 2

The following theorems all concern parallelograms:

- ❖ Opposite sides of a parallelogram are congruent.
- ❖ Opposite angles of a parallelogram are congruent.
- ❖ Consecutive angles of a parallelogram are supplementary.
- ❖ The diagonals of a parallelogram bisect each other.

Give a reason from the list above that explains why it is NOT possible for each figure below to be a parallelogram. List ALL that apply.

5.



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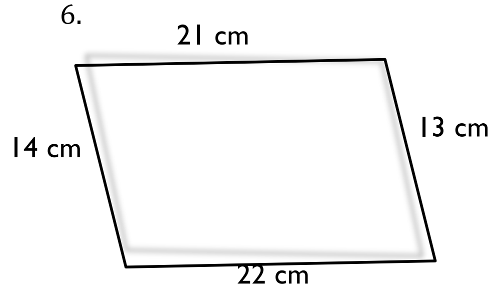
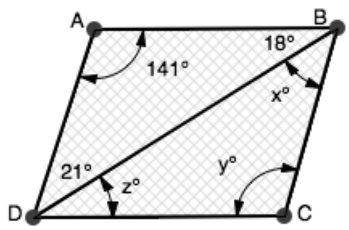


Each quadrilateral below is a parallelogram. Find the values of x, y, and z.

7.



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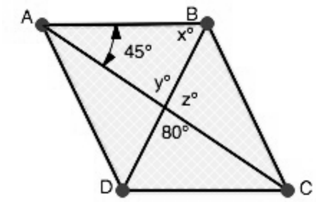

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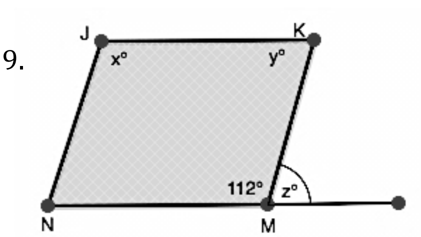

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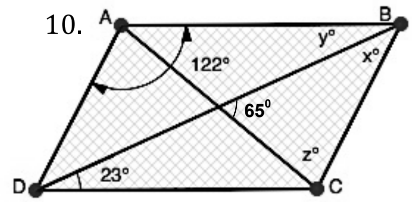

GO

Topic: Using correct mathematical symbols

Rewrite the phrases below using correct mathematical symbols.

Example: Eleven plus eight is nineteen. *11 + 8 = 19*

11. Triangle ABC is congruent to triangle GHJ. ______________________
12. Segment BV is congruent to segment PR. ______________________
13. Three feet are equal to one yard. ______________________
14. Line TR is parallel to line segment WQ. ______________________
15. Ray VP is perpendicular to segment GH. ______________________
16. Angle 3 is congruent to angle 5. ______________________
17. The distance between W and X is 7 feet. ______________________
18. The length of segment AB is equal to the length of TR. ______________________
19. The measure of angle SRT is equal to the measure of angle CDE. ______________________
20. Explain when it is proper to use an equal sign and when it is proper to use the congruent symbol.

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5.8 Guess My Parallelogram

A Practice Understanding Task

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Tehani and Tia are playing a guessing game in which one person describes some of the features of a parallelogram they have drawn and the other person has to name the type of parallelogram: square, rectangle or rhombus.

Here are some of the clues they gave each other. Decide what type of parallelogram they are describing, and explain how you know.

1. The diagonals of this parallelogram are perpendicular to each other.

2. Consecutive angles of this parallelogram are supplementary (that is, they add to 180°).

3. The diagonals of this parallelogram are congruent.

4. When rotated 90°, each diagonal of this parallelogram gets superimposed on top of the other.

5. Consecutive angles of this parallelogram are congruent.

6. The diagonals of this parallelogram are congruent and perpendicular to each other.

5.8 Guess My Parallelogram – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to become fluent in identifying special types of parallelograms based on descriptive features of the parallelogram. Students will also practice explaining the underlying reasoning that allows them to draw these specific conclusions from the given descriptions.

Core Standards Focus:

G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Mathematics II Note for G.CO.10: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

The Teaching Cycle:

Launch (Whole Class):
Explain the context of this game—that students are to identify the type of parallelogram being described based on a single clue. Point out that it is not sufficient to say what special type of parallelogram is being described, but they must also provide a supporting argument.

Explore (Small Group):
Make sure that students aren't over-generalizing the given statements. For example, in question 1 make sure students haven't assumed that the diagonals are congruent in addition to the given

statement that the diagonals are perpendicular. Encourage students to draw diagrams or use other exploratory tools to sort out what is given and what it implies about the parallelogram.

Discuss (Whole Class):

A marked diagram and a verbal argument is sufficient work for a presentation on each of these questions.

Aligned Ready, Set, Go: Geometric Figures 5.8

READY, SET, GO! Name Period Date

READY

Topic: Constructing perpendicular bisectors and angle bisectors

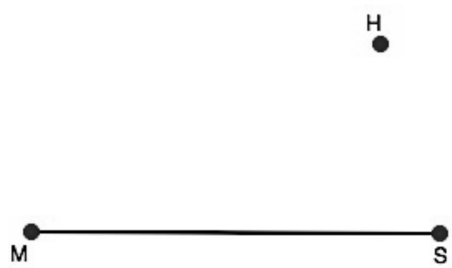
Use a compass and a straightedge to bisect the following line segments.

1. A ________________________ B

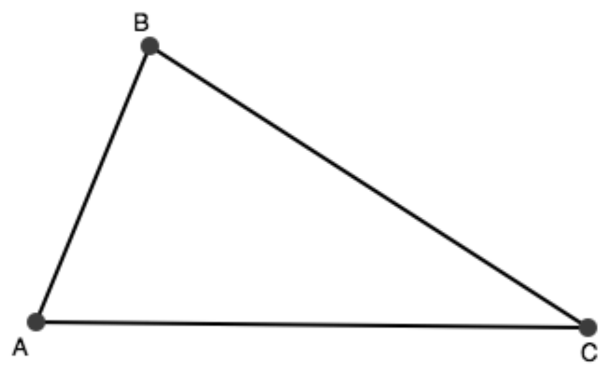
2. T ________________________________ S

3. Often when we construct the bisector of a segment, we are also constructing the perpendicular bisector. Must a bisector of a segment always be a perpendicular line?

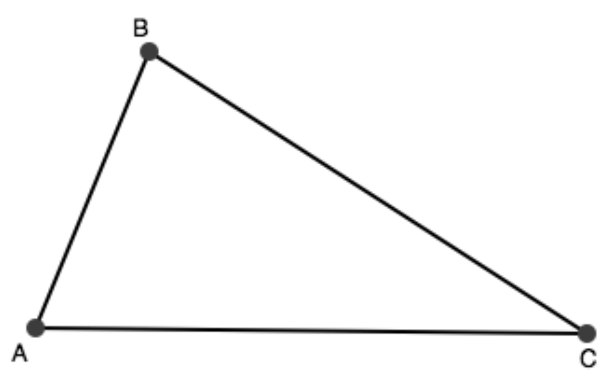
4. Construct the midpoint B of $\overline{MS}$. Then connect point B to point H.



5. Construct the 3 medians of ΔABC.



6. Construct the 3 perpendicular bisectors of ΔABC.



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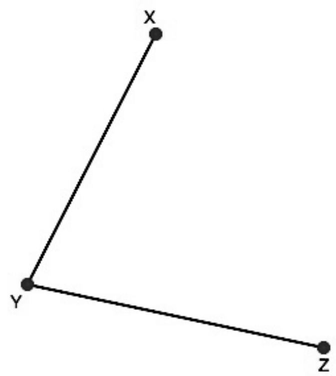
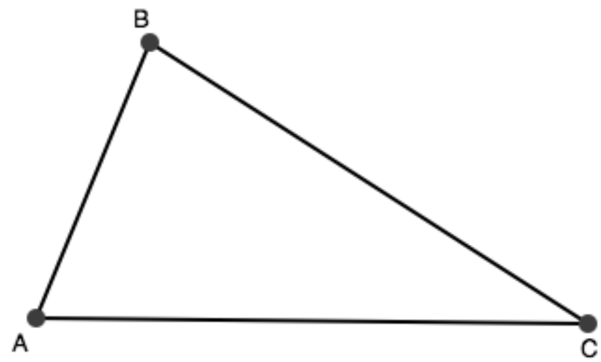
7. Construct the angle bisector of $\angle XYZ$.



8. Construct the 3 angle bisectors of ΔABC.



SET

Topic: Properties of parallelograms

Determine whether each quadrilateral is a parallelogram. Write YES if it is. If you can find an example that is NOT a parallelogram, make a sketch of the non-example.

9. 1 pair of opposite sides is parallel and it has 2 consecutive right angles

10. The quadrilateral has 4 right angles.

11. 1 pair of opposite sides is parallel and congruent

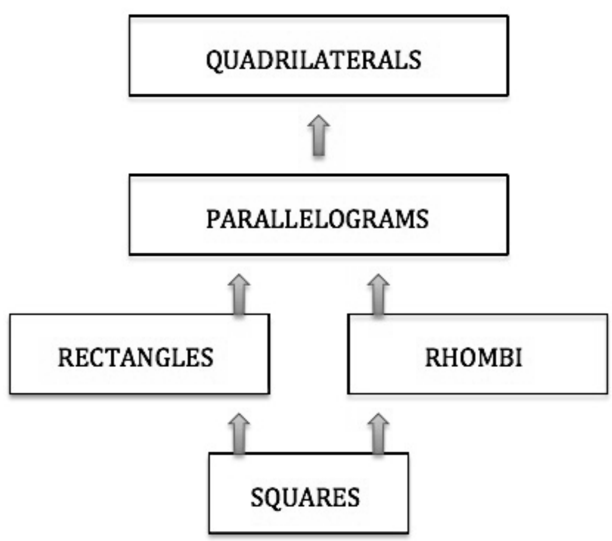
12. 1 pair of opposite sides is parallel. The other pair of opposite sides is congruent.

13. All consecutive angles are supplementary.

14. The diagonals are perpendicular.

15. The flowchart on the right has the most general 4-sided polygon at the top and the most specific one at the bottom. Around each box, write in the details that make the specific quadrilateral unique.

    Explain why the arrows point up instead of down.



GO

Topic: Features of triangles and quadrilaterals

State whether each statement is *true* or *false*. If it is false, explain why or rewrite the statement to make it true.

16. If a triangle is equilateral, then the median and the altitude are the same segments.

17. The perpendicular bisectors of the sides of a triangle also bisect the angles.

18. Some of the angles in a triangle equal 180º.

19. An altitude of a triangle may fall on the exterior of the triangle.

20. The 3rd angle in a triangle is always the supplement to the sum of the other 2 angles.

21. In a right triangle, the 2 acute angles are always complementary.

22. All squares are also rectangles.

23. A rhombus is always a square.

24. If a figure is a trapezoid, then it is also a parallelogram.

25. The diagonals of a rectangle bisect the angles.

26. A parallelogram can have 3 obtuse angles.

27. The figure made by two pair of intersecting parallel lines is always a parallelogram.

28. All of the angles in a parallelogram can be congruent.

29. A diagonal always divides a quadrilateral into 2 congruent triangles.

30. If a quadrilateral goes through a translation, the sides of the pre-image and image will remain parallel.

5.9 Centers of a Triangle

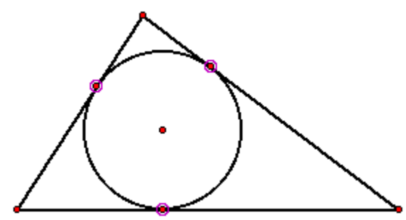
A Practice Understanding Task

Kolton, Kevin and Kara have been asked by their fathers to help them solve some interesting geometry problems.

Problem 1

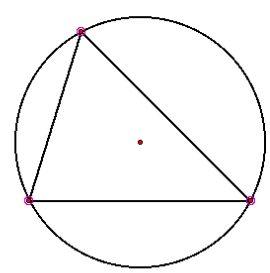
Kolton's father installs sprinkling systems for farmers. The systems he installs are called "Center Pivot Irrigation Systems" since the sprinklers are on a long pipe that rotates on wheels around a center point, watering a circular region of crops. You may have seen such "crop circles" from an airplane.

Sometimes Kolton's father has to install sprinkler systems on triangular shaped pieces of land. He wants to be able to locate the "pivot point" in the triangular field so the circle being watered will touch each of the three fences that form the boundaries of the field. He has asked for Kolton's help with this problem, since Kolton is currently studying geometry in high school.

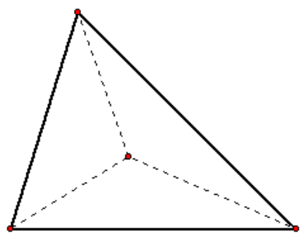


Problem 2

Kara's father installs cell towers. Since phone signals bounce from tower to tower, they have to be carefully located. Sometimes Kara's father needs to locate a new tower so that it is equidistant from three existing towers. He thinks of the three towers that are already in place as the vertices of a triangle, and he needs to be able to find a point in this triangle where he might locate the new tower so that it is equidistant from the other three. He has asked Kara to help him with this problem since she is also studying geometry in school.



Problem 3

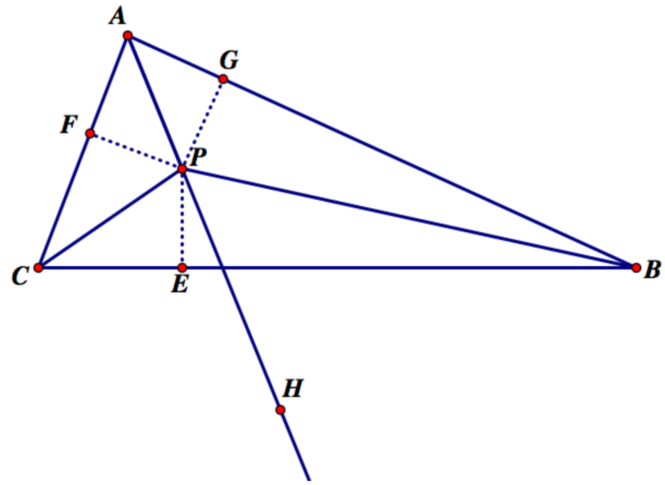


Kevin's father is an artist and has been commissioned by the city to build an art project in the park. His proposal consists of several large pyramids with different shaped triangles balanced on the vertex points of the pyramids. Kevin's father needs to be able to find the point inside of a triangle that he calls "the balancing point." He has asked Kevin to use his knowledge of geometry to help him solve this problem.

Kolton, Kevin and Kara's geometry teacher has suggested they try locating points in the interior of triangles where the three medians, the three altitudes, the three angle bisectors, or the three perpendicular bisectors of the sides intersect.

1. Try out the experiment suggested by the students' geometry teacher. Which set of line segments seem to locate a point in the triangle that best meets the needs of each of their fathers?

Kolton, Kevin and Kara have noticed something interesting about these sets of line segments. To their surprise, they notice that all three medians of a triangle intersect at a common point. Likewise, the three altitudes also intersect at a common point. So do the three angle bisectors, and the three perpendicular bisectors of the sides. They think their fathers will find this interesting, but they want to make sure these observations are true for all triangles, not just for the ones they have been experimenting on. The diagrams and notes below suggest how each is thinking about the proof they want to show his or her father. Use these notes and diagrams to write a convincing proof.



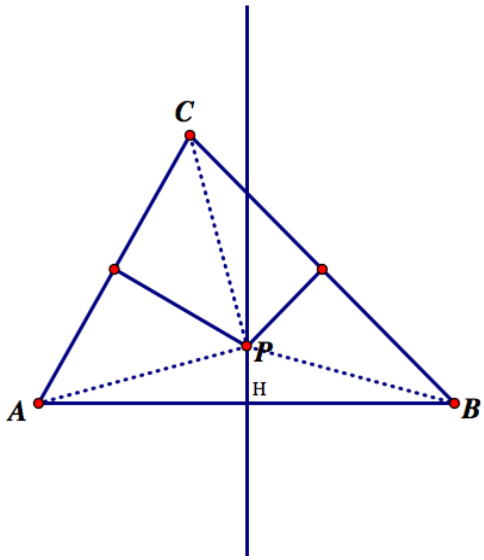
<u>Kolton's Notes</u>

What I did to create this diagram:

I constructed the angle bisectors of angle *ACB* and angle *ABC.* They intersected at point *P*. So I wouldn't get confused by so many lines in my diagram, I erased the rays that formed the angle bisectors past their point of intersection. I then drew ray *AH* through point *P*, the point of intersection of the two angle bisectors. My question is, "Does this ray bisect angle *CAB*?"

While I was thinking about this question, I noticed that I had created three smaller triangles in the interior of the original triangle. I constructed the altitudes of these three triangles (they are drawn as dotted lines). When I added the dotted lines, I started seeing kites in my picture. I'm wondering if thinking about the smaller triangles or the kites might help me prove that ray *AH* bisects angle *CAB.*

2. Complete Kolton's argument:

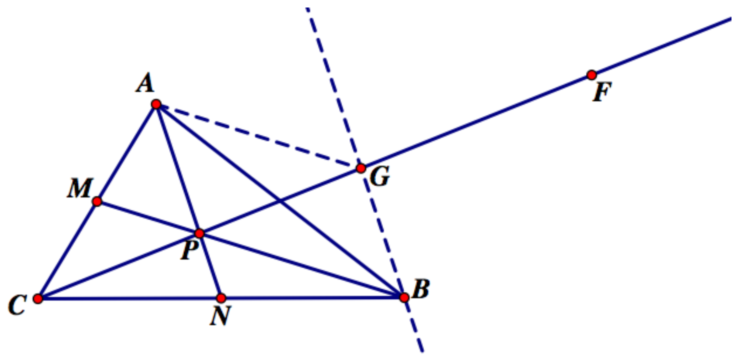


Kara's Notes

What I did to create this diagram:

I constructed the perpendicular bisectors of side *AC* and side *BC*. They intersected at point *P*. So I wouldn't get confused by so many lines in my diagram, I erased the rays that formed the perpendicular bisectors past their point of intersection. I then constructed a line perpendicular to side *AB* through point *P*, the point of intersection of the two perpendicular bisectors. I named the point where this line intersected side *AB* point *H*. My question is, "Does this perpendicular line also bisect side *AB*?" While I was thinking about this question, I noticed that I had creates some quadrilaterals in the interior of the original triangle. Since quadrilaterals in general don't have a lot of interesting properties, I decided to make some triangles by dotting in line segments drawn from *P* to each of the vertices of the original triangle. I'm wondering if thinking about these smaller triangles might help me prove that line *PH* bisects side *AB*.

3. Complete Kara's argument:



Kevin's Notes

What I did to create this diagram:

Point M is the midpoint of side AC, and point N is the midpoint of side CB. Therefore, $\overline{AN}$ and $\overline{BM}$ are medians of the triangle. I then drew ray CF through point P, the intersection of the two medians. My question is, "Does this ray contain the third median?" So, I need to find a way to answer that question. As I was thinking about this, I thought I could visualize a parallelogram with its diagonals, so I drew line GB to be parallel to median AN, and then connected vertex A to point G on the ray. Quadrilateral $AGBP$ looks like a parallelogram, but I'm not so sure. And I am wondering if that will help me with my question about the third median. What do you think?

4. Complete Kevin's argument:

[Hint: Kevin's proof uses similar triangles. Corresponding angles in similar triangles are congruent. Can you find a pair of triangles that you know are similar? Can you find a pair of triangles that you can prove are similar? You will work with similar triangles in module 7, so you may want to revisit this problem as part of your work in that module.]

5.9 Centers of a Triangle – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to practice thinking through and contributing to the reasoning of others when working through a written geometric proof. Students will also practice all of the proof techniques discussed in this module: reasoning from a diagram; organizing a logical chain of reasoning from the given statements to the conclusion of the argument; and drawing upon postulates, definitions and previously proved statements to support the chain of reasoning. Three theorems are proved in this task:

- The medians of a triangle meet at a point.
- The angle bisectors of a triangle meet at a point.
- The perpendicular bisectors of the sides of a triangle meet at a point.

It is not necessary for students to create these theorems from scratch, but they should be able to follow and extend the reasoning of the arguments presented, based on the diagrams and previously proved theorems. The ideas proved here are novel and somewhat surprising. These theorems should demonstrate the power of deductive reasoning to establish the validity of ideas that at first seem counterintuitive.

Core Standards Focus:

G.CO.10 Prove theorems about triangles. Theorems include: the medians of a triangle meet at a point.

Mathematics II Note for G.CO.10: Implementation of G.CO.10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3.

The Teaching Cycle:

Launch (Whole Class):

Read through each of the real-world scenarios described in problems 1-3 with students. Try to bring these problems to life in ways that make them seem interesting and important. Perhaps students have seen crop circles from an airplane or have tried to balance unusual shapes on the tip of their fingers. Here is an opportunity for students to see practical value in the geometry they are studying in areas as diverse as construction, surveying and engineering.

Point out the circles that can be inscribed in and circumscribed about a triangle, as illustrated in problems 1 and 2. Ask students if they think this is possible for all triangles, and how they might anticipate finding the centers of these circles. Then turn their attention to question 1 where they experiment with medians, altitudes, angle bisectors and perpendicular bisectors of the sides of a triangle. Provide appropriate tools for this exploration—either triangles to fold or draw on, or dynamic geometric software, such as Geometer's Sketchpad or Geogebra.

After students have explored these various lines and segments related to a triangle, and conjectured that they are always concurrent, ask if any of the points of concurrency formed by different sets of lines or segments seem to satisfy the conditions of any of the three problems posed by the fathers. After students have made some conjectures turn their attention to the three proofs in question 2. They are to study each diagram and read through each set of notes given by Kevin, Kolton and Kara and then to use the diagrams and notes to complete a proof of each theorem.

Explore (Small Group):
Listen for how students are using the diagram and notes to support their thinking. Suggest that they might translate the notes into a flow diagram and that they should mark angles and segments they know to be congruent on the diagram. Ask questions like, "What would you need to know to determine that this third line that passes through the intersection of the two medians is also a median? What is the definition of a median?"

Discuss (Whole Class):
Select three students to present their proofs of these theorems. Then ask students if they think that each son or daughter has selected the correct "center of a triangle" to answer his or her father's question. Allow students to verify that these diagrams work using dynamic geometry software (or demonstrate these ideas) using Sketchpad or Geogebra. For Kevin's diagram, show that the area of

each of the triangles, ΔAPB, ΔAPC and ΔBPC are equal, so the weight of the triangle is equally distributed around point P. For Kolton's diagram, show that P is the center of a circle that passes through the points E, F and G, each of which lies on a different side of the triangle. For Kara's diagram, show that point P is the center of a circle that passes through points A, B and C, the three vertices of the triangle.

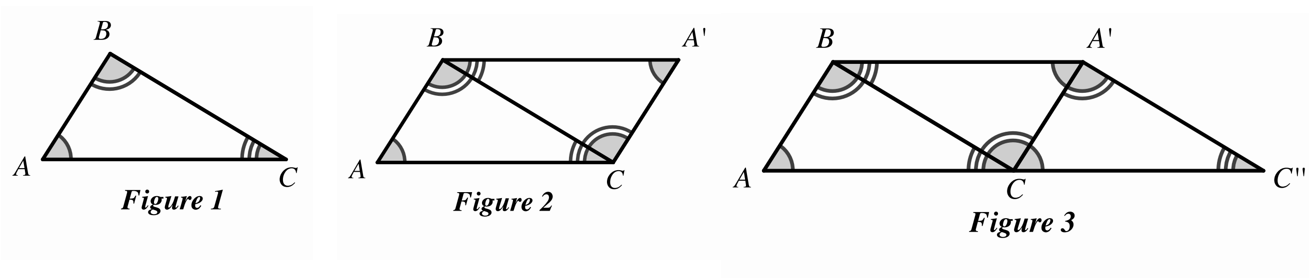
Aligned Ready, Set, Go: Geometric Figures 5.9

READY, SET, GO! Name Period Date

READY

Topic: Are you ready for a test on module 5?

Figure 1 has been rotated 180° about the midpoint in side BC to form figure 2. Figure 1 was then translated to the right to form figure 3.



1. Use figure 3 to explain how you know the exterior angle $\angle BCC''$ is equal to the sum of the 2 remote interior angles $\angle BAC$ and $\angle ABC$.

2. Use figure 3 to explain how you know the sum of the angles in a triangle is always 180º.

3. Use figure 2 to explain how you know the sum of the angles in a quadrilateral is always 360º.

4. Use figure 2 to explain how you know that the opposite angles in a parallelogram are congruent.

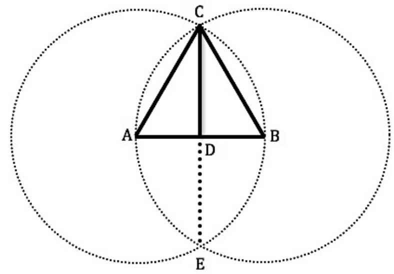
5. Use figure 2 to explain how you know that the opposite sides in a parallelogram are parallel and congruent.

6. Use figure 2 to explain how you know that when two parallel lines are crossed by a transversal, the alternate interior angles are congruent.

7. Use figure 2 and/or 3 to explain how you know that when two parallel lines are crossed by a transversal, the same-side interior angles are supplementary.

SET

Topic: Writing proofs



8. Prove that $\overline{CD}$ is an altitude of ΔABC.
   Use the diagram and write a 2 column proof.

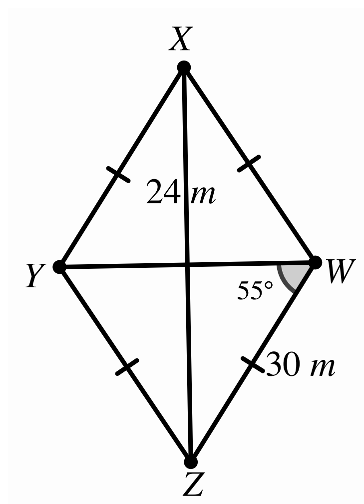
9. Use the diagram to prove that ΔABC is an isosceles triangle. (Choose your style.)

10. Use the diagram to prove that $m\angle A \cong m\angle B$. (Choose your style.)

GO

Topic: Connecting algebra with parallelograms

Use what you know about triangles and parallelograms to find each measure.



11. $\overline{XZ}$

12. $m\angle XYZ$

13. $m\angle XYW$

14. $\overline{YX}$

15. $m\angle YXZ$

16. $\overline{YW}$

17. $\overline{LG}$

18. $\overline{HF}$

19. $m\angle EHG$

20. $m\angle FEH$

21. $m\angle ELF$

22. $\overline{FG}$

23. $\overline{EG}$

24. $m\angle FGE$