mathematics vision project

Transforming Mathematics Education

SECONDARY MATH TWO

An Integrated Approach

Standard Teacher Notes

MODULE 6

Similarity & Right Triangle Trigonometry

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The Mathematics Vision Project

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READY, SET, GO Homework: Similarity & Right Triangle Trigonometry 6.6

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READY, SET, GO Homework: Similarity & Right Triangle Trigonometry 6.9

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READY, SET, GO Homework: Similarity & Right Triangle Trigonometry 6.10

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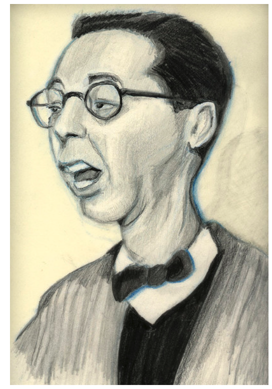
READY, SET, GO Homework: Similarity & Right Triangle Trigonometry 6.11

6. 1 Photocopy Faux Pas

A Develop Understanding Task



Burnell has a new job at a copy center helping people use the photocopy machines. Burnell thinks he knows everything about making photocopies, and so he didn't complete his assignment to read the training manual.

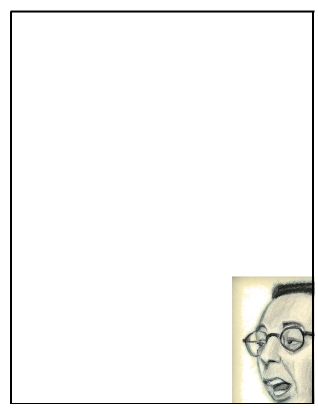
Mr. and Mrs. Donahue are making a scrapbook for Mr. Donahue's grandfather's 75th birthday party, and they want to enlarge a sketch of their grandfather which was drawn when he was in World War II. They have purchased some very expensive scrapbook paper, and they would like this image to be centered on the page. Because they are unfamiliar with the process of enlarging an image, they have come to Burnell for help.

"We would like to make a copy of this image that is twice as big, and centered in the middle of this very expensive scrapbook paper," Mrs. Donahue says. "Can you help us with that?"

"Certainly," says Burnell. "Glad to be of service."

Burnell taped the original image in the middle of a white piece of paper, placed it on the glass of the photocopy machine, inserted the expensive scrapbook paper into the paper tray, and set the enlargement feature at 200%.

In a moment, this image was produced.



"You've ruined our expensive paper," cried Mrs. Donahue. "Much of the image is off the paper instead of being centered."

"And this image is more than twice as big," Mr. Donahue complained. "One fourth of grandpa's picture is taking up as much space as the original."

In the diagram below, both the original image—which Burnell taped in the middle of a sheet of paper—and the copy of the image have been reproduced in the same figure.

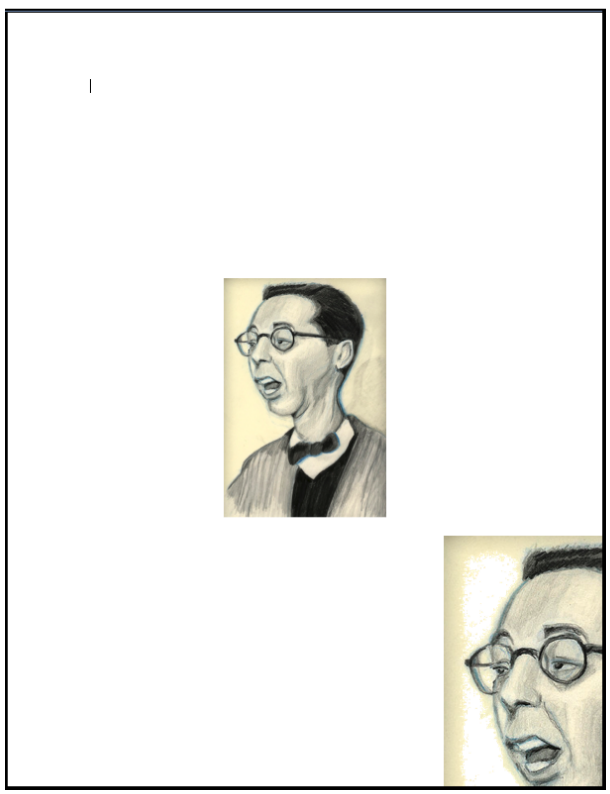
1. Explain how the photocopy machine produced the partial copy of the original image.

2. Using a "rubber band stretcher" finish the rest of the enlarged sketch.

3. Where should Burnell have placed the original image if he wanted the final image to be centered on the paper?

4. Mr. Donahue complained that the copy was four times bigger than the original. What do you think? Did Burnell double the image or quadruple it? What evidence would you use to support your claim?

5. Transforming a figure by shrinking or enlarging it in this way is a called a *dilation*. Based on your thinking about how the photocopy was produced, list all of the things you need to pay attention to when dilating an image.



6. 1 Photocopy Faux Pas – Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is to develop a description of the essential features of a dilation:

a. Lines are taken to lines, and line segments to line segments of proportional length in the ratio given by the scale factor.
b. Angles are taken to angles of the same measure.
c. A line not passing through the center of dilation is taken to a parallel line, and lines passing through the center of dilation are unchanged.
d. To describe a mathematical dilation we need to specify a center of dilation and a scale factor. The center of dilation is a fixed point in the plane about which all points are expanded or contracted. It is the only invariant point under a dilation.
e. Dilations create similar figures—the image and pre-image are the same shape, but different sizes (unless the scale factor is 1, then the image and pre-image are congruent).

Throughout the next few tasks students should formalize the definition of dilation: A dilation is a transformation of the plane, such that if O is the center of the dilation and a non-zero number k is the scale factor, then P' is the image of point P if O, P and P' are collinear and $\frac{OP'}{OP} = k$.

Core Standards Focus:

G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Mathematics II Note for G.GMD.1, G.GMD.3 Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.

Related Standards: G.SRT.2, G.GMD.1

The Teaching Cycle:

Launch (Whole Class):
Engage students in the context of this task by reading the scenario with Burnell and the Donahues, including both Mrs. Donahue and Mr. Donahue's complaints about the ruined photocopy image. Then ask students to respond to questions 1-5. Allow students to work individually for a time on these questions before moving students into small groups to share and discuss their results.

You will need to show students how to make and use the rubber band stretchers called for in question 2. These are made by looping one rubber band through a second one of the same size to create a knot where the two bands meet. Hold one end of the rubber band stretcher at the center of dilation and insert the point of a pencil at the other end. Let the knot in the middle of the rubber band stretcher trace over the pre-image while the pencil traces out the image.

Explore (Small Group):
Allow students to grapple with the questions without a lot of intervention. Make sure that students have identified that the center of dilation (by whatever name they refer to it) is located at the top left corner of the paper. Assist students who need help with the rubber band stretchers. It is not necessary for students to trace a perfect image of the Donahue's grandfather with the rubber band stretcher, but they should at least identify where the four corners of the enlarged picture will be located. This will give them a sense of the area occupied by the enlargement. Have them also locate a few other key points on the image, such as the end of the chin or the center of the ear.

Watch for where students decide they would locate the original picture in order for the copy to be centered on the paper and listen to their reasons for suggesting so. Identify any misconceptions students might have about dilations so these can be discussed as a whole class. Be aware of students who are thinking correctly about these concepts so you can call upon them to clarify any misconceptions.

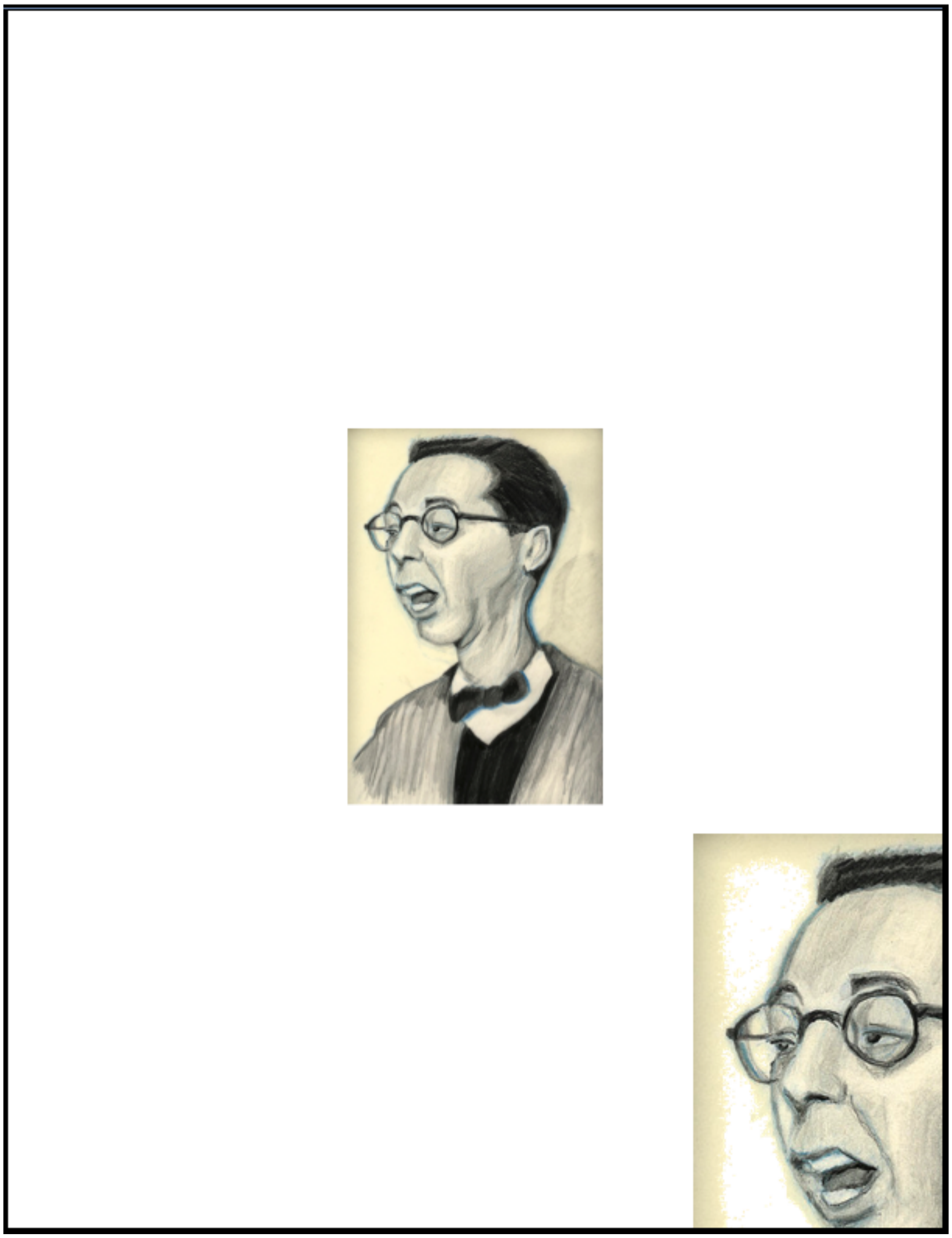
Discuss (Whole Class):

You might consider starting the whole class discussion with question 4—did the image get doubled or quadrupled? Let students debate both perspectives. Evidence for quadrupling would come from the area of the large figure compared to the area of the original. Evidence for doubling would come from measuring distance between corresponding points on the pre-image and image, such as measuring the length of the nose in both figures or the width of the collar on the neck of the shirt. Assist students in recognizing that distances have been doubled, causing the area to be quadrupled since it is the product of length and width.

Next, discuss question 3. This question should surface the issue that scale factors between 0 and 1 shrink the image closer to the center of dilation while scale factors greater than 1 enlarge the image farther from the center.

Finally, discuss question 5. Make sure that the issues listed in a-e in the purpose statement above come out in this discussion.

Aligned Ready, Set, Go: Similarity & Right Triangle Trigonometry 6.1



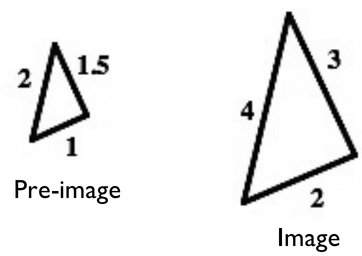
READY, SET, GO! Name Period Date

READY

Topic: Scale factors for similar shapes

Give the factor by which each pre-image was multiplied to create the image. Use the scale factor to fill in any missing lengths.

1.



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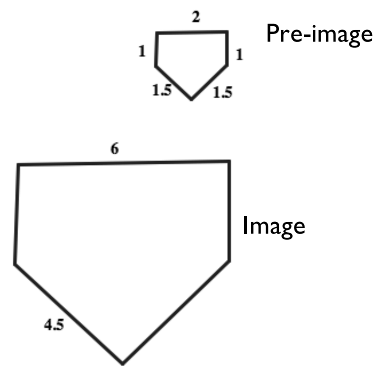

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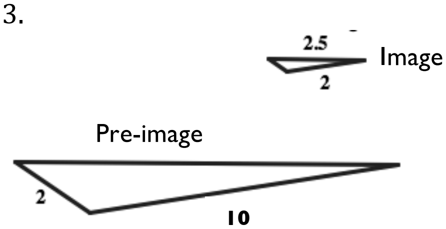

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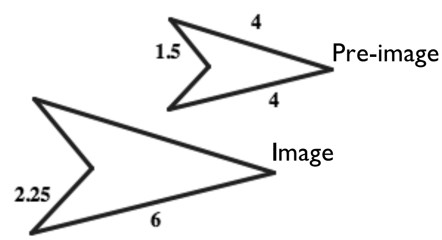
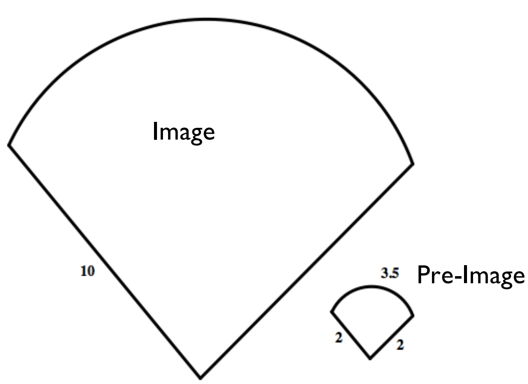
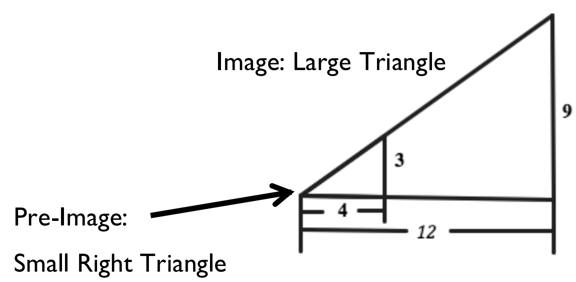


5.



6.



SET

Topic: Dilations in real world contexts

For each real-world context or circumstance determine the center of the dilation and the tool being used to do the dilation.

7.
Fran walks backward to a distance that will allow her family to all show up in the photo she is about to take.

8.
The theatre technician plays with the zoom in and out buttons in effort to fill the entire movie screen with the image.

9.
Melanie estimates the height of the waterfall by holding out her thumb and using it to see how many thumbs tall to the top of the waterfall from where she is standing. She then uses her thumb to see that a person at the base of the waterfall is half a thumb tall.

10.
A digital animator creates artistic works on her computer. She is currently doing an animation that has several telephone poles along a street that goes off into the distance.

11.
Ms. Sunshine is having her class do a project with a rubber-band tracing device that uses three rubber bands.

12.
A copy machine is set at 300% for making a photo copy.

GO

Topic: Rates of change related to linear, exponential and quadratic functions

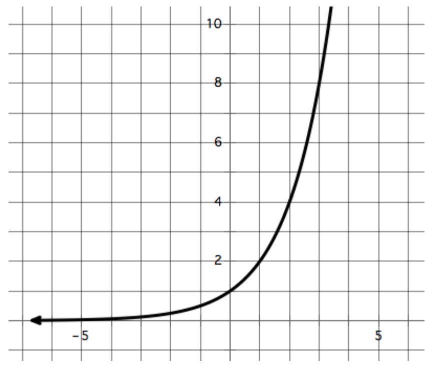
Determine whether the given representation is representative of a linear, exponential or quadratic function, classify as such and justify your reasoning.

13.

X	Y
2	7
3	12
4	19
5	28

Type of function:
Justification:

14.

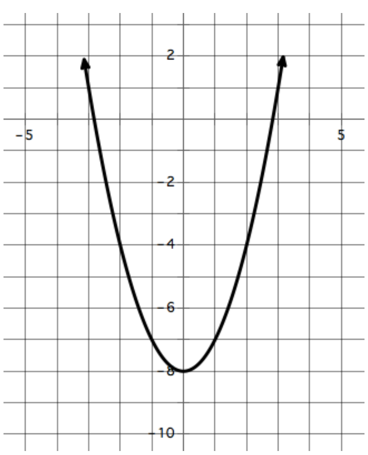


Type of function:
Justification:

15. $y = 3x^2 + 3x$
Type of function:
Justification:

16. $y = 7x - 10$
Type of function:
Justification:

17.



Type of function:

Justification:

18.

X	Y
2	-5
7	5
14	19
25	41

Type of function:

Justification:

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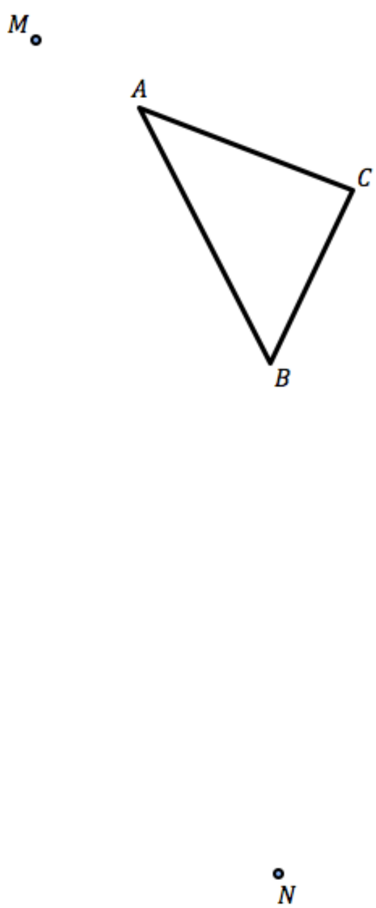
6. 2 Triangle Dilations

A Solidify Understanding Task

1. Given ΔABC, use point M as the center of a dilation to locate the vertices of a triangle that has side lengths that are three times longer than the sides of ΔABC.

2. Now use point N as the center of a dilation to locate the vertices of a triangle that has side lengths that are one-half the length of the sides of ΔABC.



3. Label the vertices in the two triangles you created in the diagram above. Based on this diagram, write several proportionality statements you believe are true. First write your proportionality statements using the names of the sides of the triangles in your ratios. Then verify that the proportions are true by replacing the side names with their measurements, measured to the nearest millimeter.

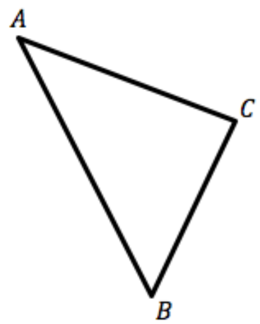
   My list of proportions: (try to find at least 10 proportionality statements you believe are true)

4. Based on your work above, under what conditions are the corresponding line segments in an image and its pre-image parallel after a dilation? That is, which word best completes this statement?

   After a dilation, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.

5. Give reasons for your answer. If you choose "sometimes", be very clear in your explanation about how you can to tell when the corresponding line segments before and after the dilation are parallel and when they are not.

Given ΔABC, use point A as the center of a dilation to locate the vertices of a triangle that has side lengths that are twice as long as the sides of ΔABC.



6. Explain how the diagram you created above can be used to prove the following theorem:

The segment joining midpoints of two sides of a triangle is parallel to the third side and half the length.

6. 2 Triangle Dilations – Teacher Notes

A Solidify Understanding Task

Purpose: One purpose of this task is to continue to solidify the definition of dilation: A dilation is a transformation of the plane, such that if O is the center of the dilation and a non-zero number k is the scale factor, then P' is the image of point P if O, P and P' are collinear and $\frac{OP'}{OP} = k$.

.

A second purpose of this task is to examine proportionality relationships between sides of similar figures by identifying and writing proportionality statements based on corresponding sides of the similar figures.

A third purpose is to examine a similarity theorem that can be proved using dilation: *a line parallel to one side of a triangle divides the other two proportionally.*

Core Standards Focus:

G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally.

Related Standards: G.SRT.1, 8.G.4

The Teaching Cycle:

Launch (Whole Class): [questions 1-3]

Remind students that in previous math classes they have studied proportionality relationships. Discuss what it means to say that quantities are proportional and review how to write proportionality relationships symbolically. In Math 8 students learned that dilations produce similar figures and remind them of the definition they used for similar figures: (CCSS-M 8.G.4) *Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations*. Point out to students that questions 1 and 2 ask them to create some similar figures using dilations, and that question 3 will ask them to write some proportionality statements based on these similar figures.

Explore (Small Group): [questions 1-3]

As students identify the vertices of the two dilated triangles on question 1 and 2 encourage them to label corresponding vertices as *A'*, *B'*, *C'* and *A"*, *B"*, *C"* so they can refer to the sides of the triangles by name when writing proportions for question 3. Verify that they are correctly locating the vertices of the two triangles with scale factor 3 centered at *M* and scale factor ½ centered at *N*. Observe the proportions that students write for question 3, and try to identify (and press for) a variety of ways of writing these proportions. For example, in one type of proportion the ratios would consist of comparing corresponding sides from each of two triangles $\frac{AB}{A'B'} = \frac{BC}{B'C'}$; in another type of proportion the ratios would consist of two sides taken from one triangle compared to the same two sides of another triangle $\frac{AB}{BC} = \frac{A'B'}{B'C'}$. Watch for students who write proportions between the sides of the largest triangle and the smallest triangle, recognizing that these triangles are also similar to each other. Students may also create proportions that include the center of dilation $\frac{MC}{MC'} = \frac{MA}{MA'}$. This leads right to the solidification of the definition of dilation, so if it does not come up it would be good to be prepared to suggest it.

Discuss (Whole Class): [questions 1-3]

Begin the discussion by posing this formal definition of dilation: *A dilation is a transformation of the plane, such that if O is the center of the dilation and a non-zero number k is the scale factor, then P' is the image of point P if O, P and P' are collinear and* $\frac{OP'}{OP} = k$, and ask students how this definition showed up in their work with questions 1 and 2. Have students record this definition in their notes or on a class poster. It is good to contrast this definition with the definition of similarity that was reviewed at the beginning of the task, helping students to see that dilation is a component of the similarity definition. They may realize with a question or two to guide them that all congruent figures are similar however not all similar figures are congruent.

Select several students to share proportions they wrote for question 3, making sure that a variety of ways of identifying proportionality statements are discussed, as outlined in the exploration above. End this discussion by reviewing the definition of similarity (*a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations*) and ask students to verify that the largest and smallest triangles are similar by describing a sequence of transformations that exhibits the similarity between them. (One possibility is to translate the smallest triangle to the largest triangle so that vertex *A"* corresponds with vertex *A'*, then dilate the small triangle with the dilation centered at *A"* to superimpose it on top of the largest triangle. Ask students to identify the scale factor of this dilation.)

Launch (Whole Class): [questions 4-6]

The last part of the previous discussion will prepare students to work on questions 4-6. Remind them that in the task *Parallelism Preserved and Protected* they identified some parallel postulates for the rigid motion transformations: translation, rotation and reflection. In question 4 and 5 they will propose a parallel postulate for dilation. They will also have an opportunity to create a diagram and write a proof about a similarity theorem in question 6.

Explore (Small Group): [questions 4-6]

Encourage students to examine the three similar triangles they created by dilation on the previous part of the task. Ask, "What relationships do you notice between corresponding line segments after these dilations? Do you think this will always be the case? Why do you think so?"

On question 6 verify that students have created the diagram correctly, and that they are using features of the diagram to guide their thinking on the proof. Help them focus on using the sides of the triangle as transversals for the third sides of the triangles. Students should notice that since dilations preserve angle measure, pairs of corresponding angles in their diagram are congruent when the sides of the triangle are treated as transversals. This proves that the third sides of the triangles are parallel. Since *C* and *B* are midpoints of the larger triangle's sides, the scale factor of the dilation is 2 (going from the smaller triangle to the larger) or ½ (going from the larger triangle to the smaller).

Discuss (Whole Class): [questions 4-6]

Start the discussion by having students state their parallel postulate for dilations as requested in questions 4 and 5. Note that in question 6 they have actually proved this postulate for one type of dilation: a triangle dilated about one of its vertices. Based on our experiments in questions 1 and 2 we are going to accept this statement as a postulate for our approach to transformational geometry, as suggested by the common core standards.

Have a student share his proof for question 6. This is a special case of a more general theorem that will be explored in detail in *6.4 Cut By a Transversal.* Have students record it in their notes or on a classroom poster.

Aligned Ready, Set, Go: Similarity & Right Triangle Trigonometry 6.2

Diagram for use with questions 1 and 2.

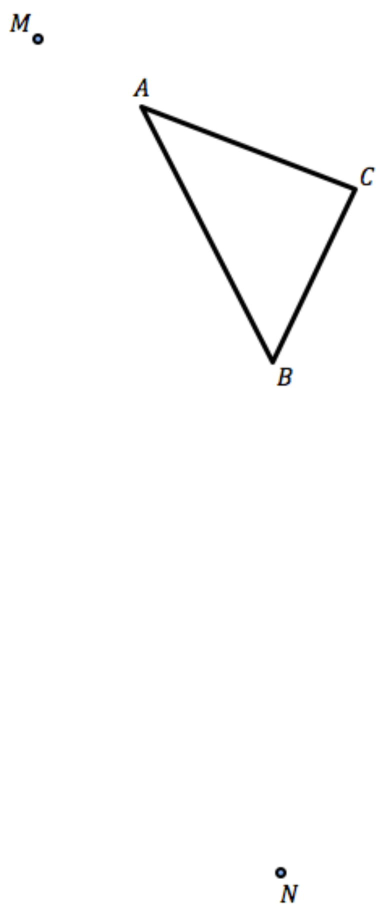
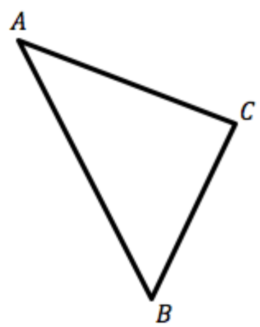


Diagram for use with question 6.

Given ΔABC, use point A as the center of a dilation to locate the vertices of a triangle that has side lengths that are twice as long as the sides of ΔABC.



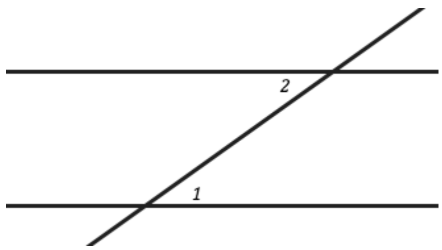
READY, SET, GO! Name Period Date

READY

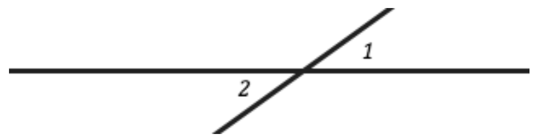
Topic: Basic angle relationships

Match the diagrams below with the best name or phrase that describes the angles.

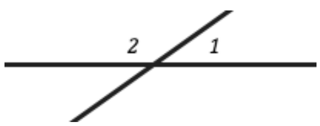
________1.



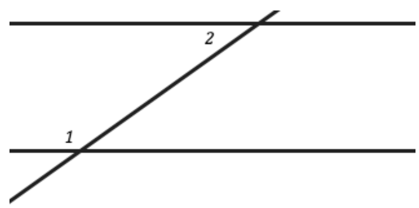
________2.



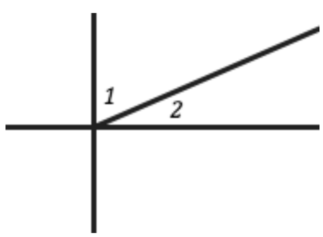
________3.



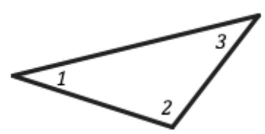
________4.



______5.



________6.



a. Alternate Interior Angles b. Vertical Angles c. Complementary Angles

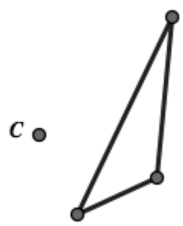
d. Triangle Sum Theorem e. Linear Pair f. Same Side Interior Angles

SET

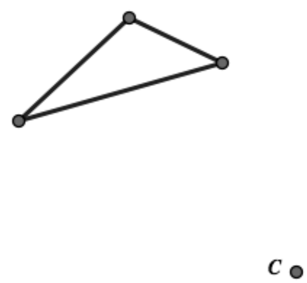
Topic: Performing mathematical dilations and finding the center of dilations.

Use the given pre-image and point *C* as the center of dilation to perform the dilation that is indicated below.

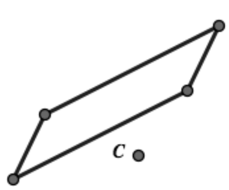
7. Create an image with side lengths twice the size of the given triangle.



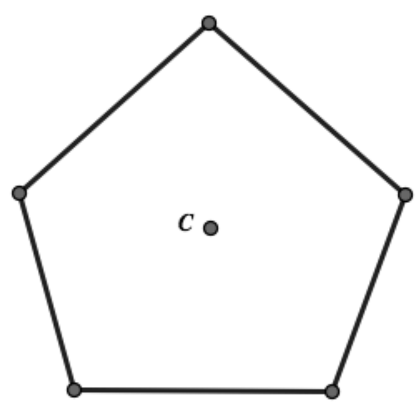
8. Create an image with side lengths half the size of the given triangle.



9. Create an image with side lengths three times the size of the given parallelogram.

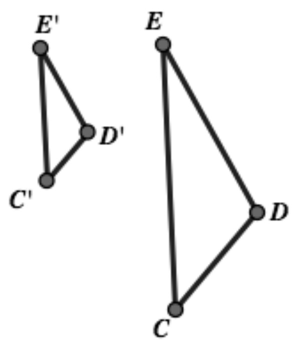


10. Create an image with side length one fourth the size of the given pentagon.

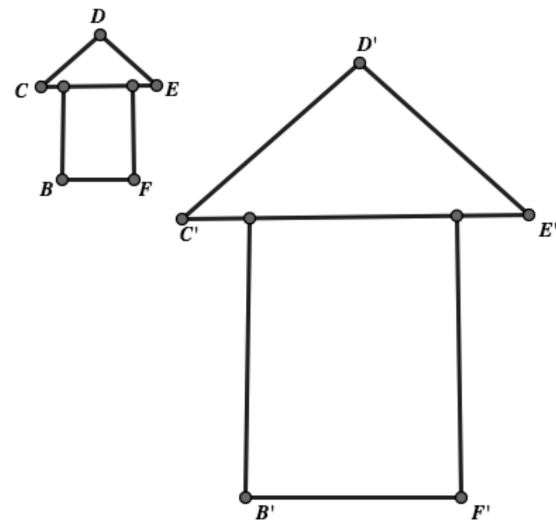


Use the given pre-image and image in each diagram to define the dilation that occurred. Include as many details as possible such as the center of the dilation and the ratio.

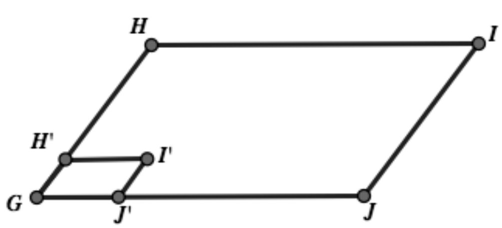
11.



12.



13.



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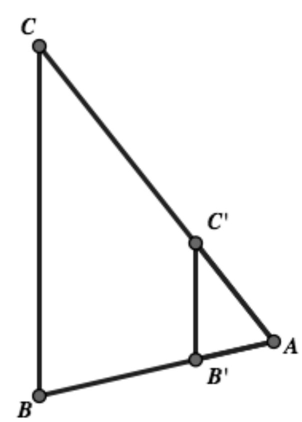


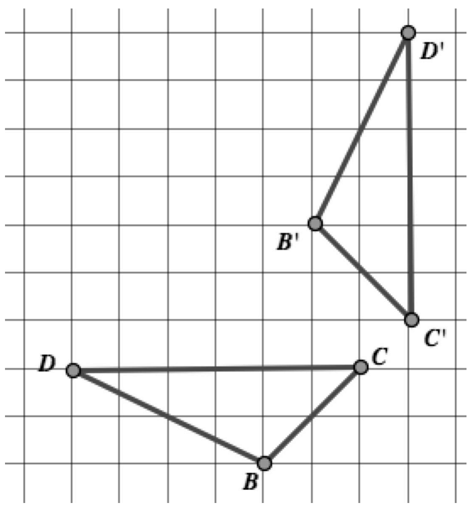
GO

Topic: Classifying mathematical transformations.

Based on the given image and pre-image determine the transformation that occurred. Further, prove that the transformation occurred by showing evidence of some kind.

(*For example, if the transformation was a reflection show the line of reflection exists and prove that it is the perpendicular bisector of all segments that connect corresponding points from the image and pre-image. Do likewise for rotations, translations and dilations.*)

15.



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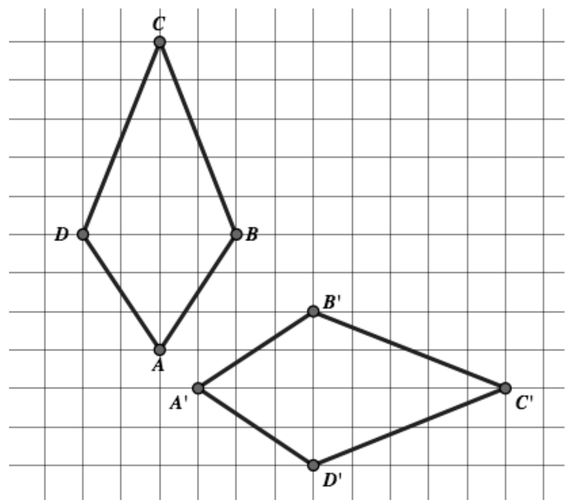

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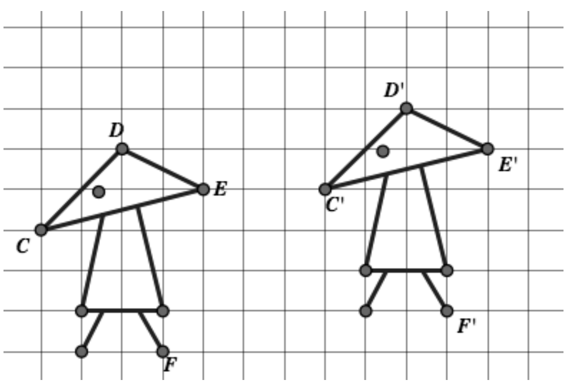

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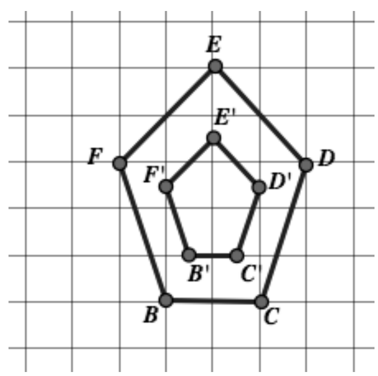


6.3 Similar Triangles and Other Figures

A Solidify Understanding Task

Two figures are said to be *congruent* if the second can be obtained from the first by a sequence of rotations, reflections, and translations. In Mathematics I we found that we only needed three pieces of information to guarantee that two triangles were congruent: SSS, ASA or SAS.

What about AAA? Are two triangles congruent if all three pairs of corresponding angles are congruent? In this task we will consider what is true about such triangles.

Part 1

Definition of Similarity: Two figures are *similar* if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.

Mason and Mia are testing out conjectures about similar polygons. Here is a list of their conjectures.

Conjecture 1: *All rectangles are similar.*

Conjecture 2: *All equilateral triangles are similar.*

Conjecture 3: *All isosceles triangles are similar.*

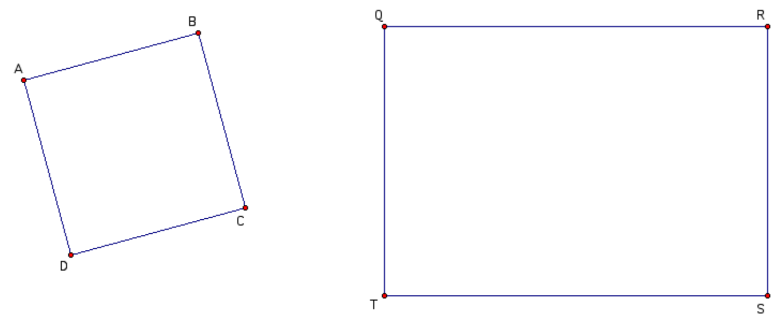
Conjecture 4: *All rhombuses are similar.*

Conjecture 5: *All squares are similar.*

1. Which of these conjectures do you think are true? Why?

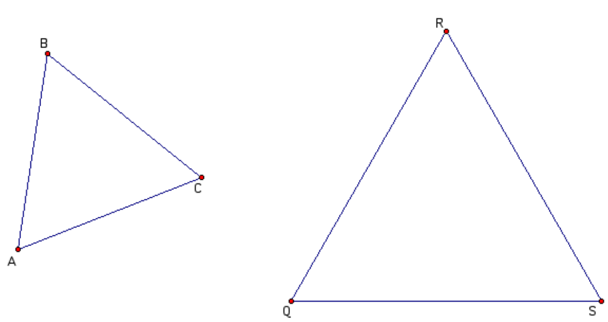
Mason is explaining to Mia why he thinks conjecture 1 is true using the diagram given below.

"All rectangles have four right angles. I can translate and rotate rectangle *ABCD* until vertex *A* coincides with vertex *Q* in rectangle *QRST.* Since $\angle A$ and $\angle Q$ are both right angles, side *AB* will lie on top of side *QR*, and side *AD* will lie on top of side *QT.* I can then dilate rectangle *ABCD* with point *A* as the center of dilation, until points *B*, *C*, and *D* coincide with points *R*, *S*, and *T.*



2. Does Mason's explanation convince you that rectangle *ABCD* is similar to rectangle *QRST* based on the definition of similarity given above? Does his explanation convince you that *all rectangles are similar*? Why or why not?

Mia is explaining to Mason why she thinks conjecture 2 is true using the diagram given below.



"All equilateral triangles have three 60° angles. I can translate and rotate ΔABC until vertex *A* coincides with vertex *Q* in $\Delta QRS.$ Since $\angle A$ and $\angle Q$ are both 60° angles, side *AB* will lie on top of side *QR*, and side *AC* will lie on top of side *QS.* I can then dilate ΔABC with point *A* as the center of dilation, until points *B* and *C* coincide with points *R* and *S.*"

3. Does Mia's explanation convince you that ΔABC is similar to ΔQRS based on the definition of similarity given above? Does her explanation convince you that *all equilateral triangles are similar*? Why or why not?

4. For each of the other three conjectures, write an argument like Mason's and Mia's to convince someone that the conjecture is true, or explain why you think it is not always true.

a. Conjecture 3: *All isosceles triangles are similar.*

b. Conjecture 4: *All rhombuses are similar.*

c. Conjecture 5: *All squares are similar.*

While the definition of similarity given at the beginning of the task works for <u>all</u> similar figures, an alternative definition of similarity can be given for polygons: **Two polygons are similar if all corresponding angles are congruent and all corresponding pairs of sides are proportional.**

5. How does this definition help you find the error in Mason's thinking about conjecture 1?

6. How does this definition help confirm Mia's thinking about conjecture 2?

7. How might this definition help you think about the other three conjectures?

    a. Conjecture 3: *All isosceles triangles are similar.*

    b. Conjecture 4: *All rhombuses are similar.*

    c. Conjecture 5: *All squares are similar.*

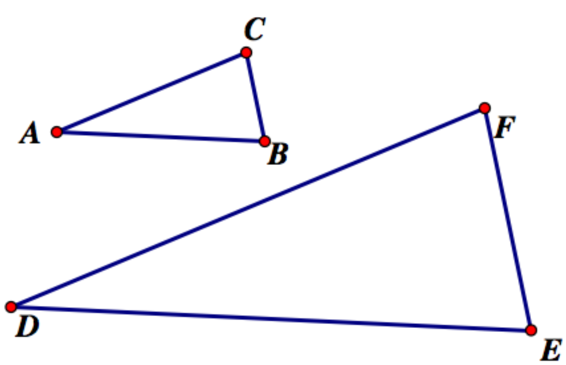
Part 2 (AAA, SAS and SSS Similarity)

From our work above with rectangles it is obvious that knowing that all rectangles have four right angles (an example of AAAA for quadrilaterals) is not enough to claim that all rectangles are similar. What about triangles? In general, are two triangles similar if all three pairs of corresponding angles are congruent?

8. Decide if you think the following conjecture is true.

   Conjecture: *Two triangles are similar if their corresponding angles are congruent.*

9. Explain why you think the conjecture—*two triangles are similar if their corresponding angles are congruent*—is true. Use the following diagram to support your reasoning, Remember to start by marking what you are given to be true (AAA) in the diagram.



Hint: *Begin by translating A to D.*

10. Mia thinks the following conjecture is true. She calls it "*AA Similarity for Triangles*." What do you think? Is it true? Why?

    Conjecture: *Two triangles are similar if they have two pair of corresponding congruent angles.*

11. Using the diagram given in problem 9, how might you modify your proof that $\Delta ABC \sim \Delta DEF$ if you are given the following information about the two triangles:

a. $\angle A \cong \angle D,\ DE = k \cdot AB,\ DF = k \cdot AC$; that is, $\frac{DE}{AB} = \frac{DF}{AC}$

b. $DE = k \cdot AB,\ DF = k \cdot AC$ and $EF = k \cdot BC$; that is, $\frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC}$

6. 3 Similar Triangles and Other Figures – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to solidify the concepts of similarity for polygons. The definition of similarity that students have been introduced to prior to this task is: *Two figures are similar if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.* This definition works for all geometric figures (such as the Photocopy Faux Pas images from task 6.1.) When a figure is made up entirely of line segments, such as in a polygon, an alternative definition of similarity can be used: *Two polygons are similar if all corresponding angles are congruent and all corresponding pairs of sides are proportional.* In this task students explore the equivalence of these two definitions for similar polygons. Students also prove the AA, SAS and SSS Similarity Theorems for triangles.

Core Standards Focus:

G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

The Teaching Cycle:

Launch (Whole Class): [part 1]

Remind students of the definition of similarity based on transformations: *Two figures are similar if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.* Read through Mia and Mason's conjectures about similar polygons, and then have students individually decide which conjectures they believe are true (see question 1) before assigning them to work with a partner on questions 2-4.

Explore (Small Group): [part 1]

Listen to how students react to Mason and Mia's "proofs" (questions 2 and 3) and what issues are being raised for students in question 4. You may need to help students understand how Mason and Mia are thinking about translation and dilation, but allow students to sort out their own misconceptions through their own conversation and exploration.

Discuss (Whole Class): [part 1]

One goal of this discussion is to help students notice that the use of the word "similar" in mathematics is different from the common use of the word where, for example, all rectangles would be considered similar because they fit in the same class of objects. A second goal is to help students notice the two criteria that have to be met for members of the same class of polygons to be similar: all pairs of corresponding sides must be proportional, and all pairs of corresponding angles must be congruent.

Begin by discussing Mason and Mia's "proofs."
In Mason's argument students should recognize that since a dilation stretches or shrinks each side of a rectangle by the same scale factor, two rectangles will only be similar if both the length and width of one is multiplied by the same factor to obtain the length and width of the other. Therefore, not all rectangles are similar. In Mia's argument students should recognize that stretching or shrinking the sides of an equilateral triangle by the same scale factor will still produce an equilateral triangle, and therefore, all equilateral triangles are similar.

Use conjectures 3 and 4 to highlight that corresponding angles need to be congruent for polygons to be similar. For example, corresponding sides of two rhombuses may be proportional, but not all rhombuses are similar.

Use conjecture 5 to highlight that when all corresponding angles are congruent and all corresponding sides are proportional we have similar figures based on the transformation definition of similarity. Introduce the new definition of similarity for polygons: *Two polygons are similar if all corresponding angles are congruent and all corresponding pairs of sides are proportional.* Use this new definition to discuss questions 5-7 as a class.

Launch (Whole Class): [part 2]

Inform students that while for polygons in general we need to know all corresponding angles are congruent and all corresponding pairs of sides are proportional before we can say they are similar, it turns out that we can say triangles are similar with a lot less information. Assign students to work on part 2 of the task.

Explore (Small Group): [part 2]

The transformational proof of the AAA Similarity Theorem for triangles is very similar to what Mia did (and Mason attempted) in part 1 of this task. It is also related to the proof of the theorem in the previous task: *The segment joining midpoints of two sides of a triangle is parallel to the third side and half the length*, although the scale factor of the dilation need not be ½ (or 2). Remind students of this work if they are having problems completing the proof.

Discuss (Whole Group): [part 2]

Have a student start with the hint and complete the proof of the AAA Similarity Theorem for Triangles. (If we translate A to D and align segment AB with segment DE, C must lie on segment DF since the angles are congruent. Scale up segment AB so that B coincides with E. Then segment BC and segment EF must coincide since the angles are all congruent.) Have another student explain why we don't even need to know all three pairs of corresponding angles are congruent to say two triangles are similar, but that two pairs of congruent corresponding angles are enough. Have students record the AA Similarity Theorem for Triangles in their notebooks or on a classroom poster.

Question 11 asks students to prove the SAS and SSS Similarity Theorems for Triangles. Students should be able to prove SAS Similarity using a modified version of the proof for AAA similarity (i.e., Translate A to D and align segment AB with segment DE, C must lie on segment DF since the angles are congruent. The scale factor k will scale up segment AB so that B coincides with E and segment AC so that C coincides with F). Record the SAS Similarity Theorem for Triangles as follows: Two triangles are similar if two pairs of corresponding sides are proportional and the included angles are congruent.

To prove SSS Similarity, students might begin by dilating ΔABC by a factor of k using point A as the center of dilation, and then showing they can superimpose the dilated triangle, $\Delta A'B'C'$, on top of ΔDEF by first transforming segment $A'B'$ to coincide $D'E'$ and then convincing themselves that point C' must coincide with F since the segments (i.e., distances between points) are all congruent. Record the SSS Similarity Theorem for Triangles as follows: Two triangles are similar if all three pairs of corresponding sides are proportional.

Aligned Ready, Set, Go: Similarity & Right Triangle Trigonometry 6.3

READY, SET, GO! Name Period Date

READY

Topic: Solving proportions in multiple ways

Solve each proportion. Show your work and check your solution.

1. $\frac{3}{4} = \frac{x}{20}$

2. $\frac{x}{7} = \frac{18}{21}$

3. $\frac{3}{6} = \frac{8}{x}$

4. $\frac{9}{c} = \frac{6}{10}$

5. $\frac{3}{4} = \frac{b+3}{20}$

6. $\frac{7}{12} = \frac{a}{24}$

7. $\frac{a}{2} = \frac{13}{20}$

8. $\frac{3}{b+2} = \frac{6}{5}$

9. $\frac{\sqrt{3}}{2} = \frac{\sqrt{12}}{c}$

SET

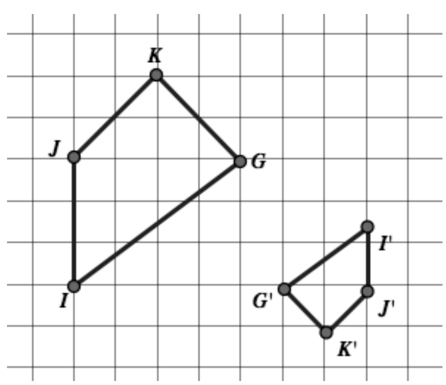
Topic: Proving Shapes are similar

Provide an argument to prove each conjecture, or provide a counterexample to disprove it.

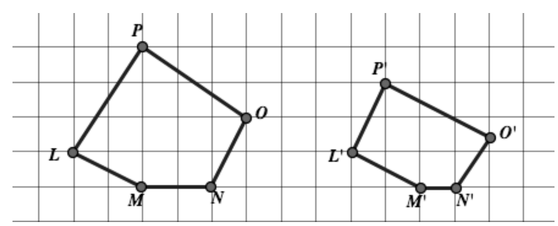
10. All right triangles are similar

11. All regular polygons are similar to other regular polygons with the same number of sides.

12. The polygons on the grid below are similar.



13. The polygons on the grid below are similar.



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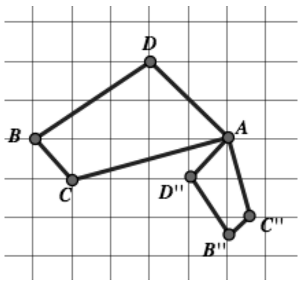
A sequence of transformations occurred to create the two similar polygons. Provide a specific set of steps that can be used to create the image from the pre-image.

14.



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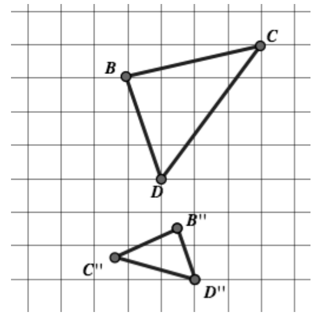

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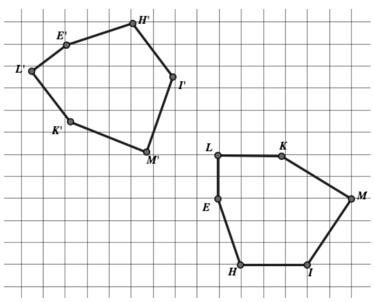

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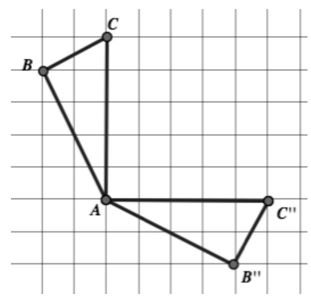
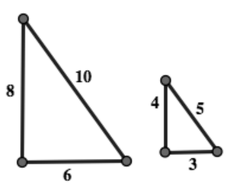


GO

Topic: Ratios in similar polygons

For each pair of similar polygons give three ratios that would be equivalent.

18.



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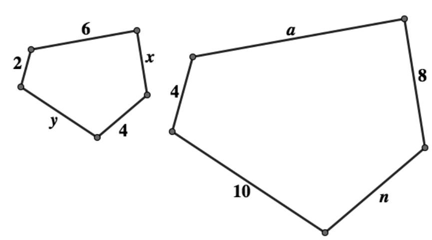

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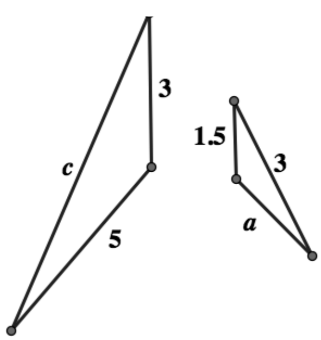

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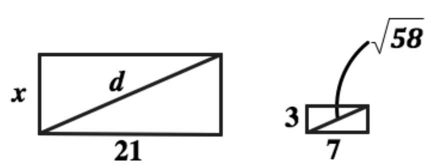
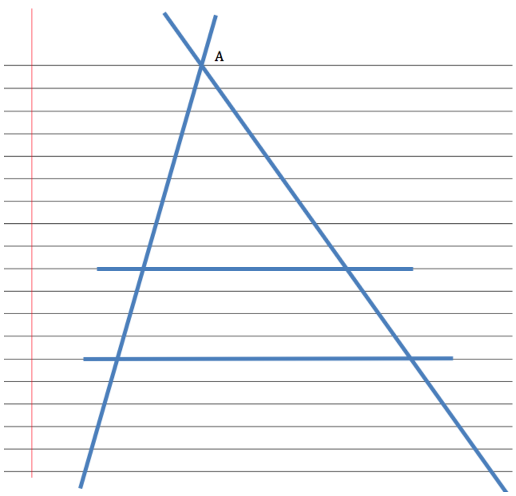


6. 4 Cut by a Transversal

A Solidify Understanding Task

Draw two intersecting transversals on a sheet of lined paper, as in the following diagram. Label the point of intersection of the transversals *A*. Select any two of the horizontal lines to form the third side of two different triangles.



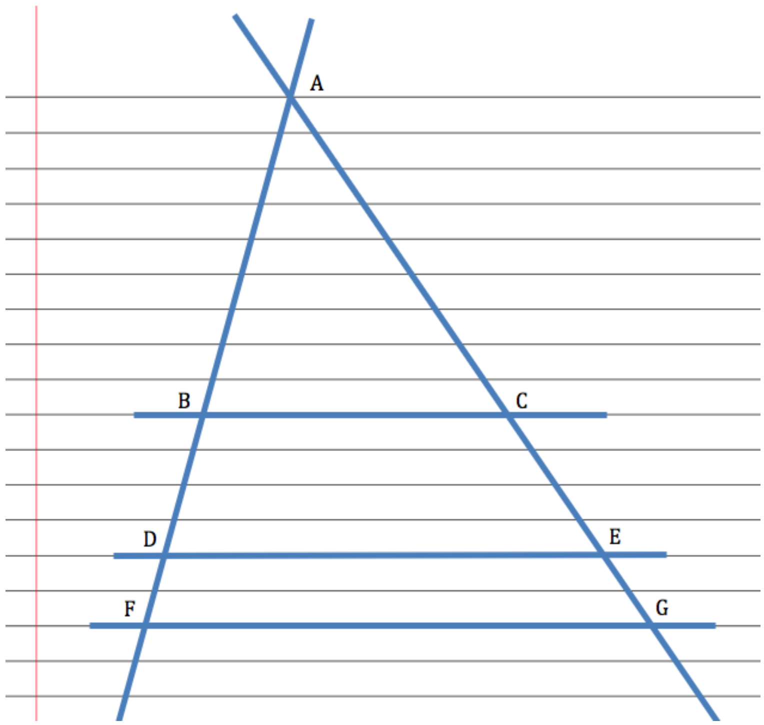
1. What convinces you that the two triangles formed by the transversals and the horizontal lines are similar?

2. Label the vertices of the triangles. Write some proportionality statements about the sides of the triangles and then verify the proportionality statements by measuring the sides of the triangles.

3. Select a third horizontal line segment to form a third triangle that is similar to the other two. Write some additional proportionality statements and verify them with measurements.

Tristan has written this proportion for question 3, based on his diagram below: $\frac{BD}{AB} = \frac{CE}{AC}$

Tia thinks Tristan's proportion is wrong, because some of the segments in his proportion are not sides of a triangle.



4. Check out Tristan's idea using measurements of the segments in his diagram at the left.

5. Now check out this same idea using proportions of segments from your own diagram. Test at least two different proportions, including segments that do not have A as one of their endpoints.

6. Based on your examples, do you think Tristan or Tia is correct?

Tia still isn't convinced, since Tristan is basing his work on a single diagram. She decides to start with a proportion she knows is true: $\frac{AD}{AB} = \frac{AE}{AC}$. (Why is this true?)

Tia realizes that she can rewrite this proportion as $\frac{AB+BD}{AB} = \frac{AC+CE}{AC}$ (Why is this true?)

Can you use Tia's proportion to prove algebraically that Tristan is correct?

6. 4 Cut by a Transversal – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is extend the types of proportionality statements that can be written when two sides of a triangle are crossed by a line that is parallel to the third side. In previous tasks students have written proportionality statements based on the corresponding sides of the smaller and larger triangle. In this task, they observe that corresponding *segments* formed on the sides of the triangle are proportional, even though those segments are not *sides* of the triangles. This is sometimes known as "the side splitter theorem."

Core Standards Focus:
G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally.

The Teaching Cycle:

Launch (Whole Class):
Students will need lined paper and rulers for this task. Point out to students that we are assuming all of the lines on the lined paper we buy at the store are parallel. Ask students to work on questions 1-3 individually. The task is fairly straight forward, so students should be able to follow the instructions on their own to work through much of the task.

Explore (Small Group):
Notice what kinds of proportionality statements students are writing on questions 2 and 3. It is assumed that students will write proportionality statements based on the corresponding sides of the smaller and larger triangles, which are similar. If you notice students using segments of the sides, instead of the complete sides, ask them why they think these proportions are true since they don't involve the actual sides of the triangles, then point them to Tristan's proportion and Tia's

concern and have them begin working on that part of the task. If students are only writing proportions based on corresponding sides of the similar triangles, acknowledge their correct application of the theorems we have proved so far, and then point them to the dilemma created by Tristan and have them consider examples that might support or disprove his claim.

Since most of the work of this task is about testing proportions by measuring and calculating, it is assumed that students will not need a lot of help until they get to the algebraic proof following question 6. If some students can work through the algebra, great! If not, call the class together to start the whole class discussion.

Discuss (Whole Class):
Ask students what is different about the proportion that Tristan wrote, and the proportions they wrote on questions 2 and 3. Ask them if such proportions seem to be true, based on their own measurements and calculations, and have a couple of students share their work.

If no one has proposed a proportion like $\frac{BD}{DF} = \frac{CE}{EG}$ for the diagram that accompanies question 4, propose it yourself and ask students to test it out by measuring the segments and calculating the ratios. Help students see that this is rather surprising, since these are not corresponding sides of similar triangles. Therefore, to decide if this is always true, we need a proof.

Point out that this is the first time students have seen a geometric proof using algebra. If there is a student who can complete the algebraic proof, let him or her do so. Otherwise, provide some assistance for the algebraic steps, starting with $\frac{AB+BD}{AB} = \frac{AC+CE}{AC}$ and ending with $\frac{BD}{AB} = \frac{CE}{AC}$.

Aligned Ready, Set, Go: Similarity & Right Triangle Trigonometry 6.4

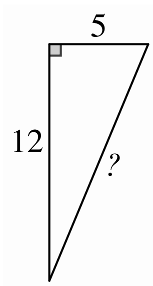
READY, SET, GO! Name Period Date

READY

Topic: Pythagorean theorem and proportions in similar triangles.

Find the missing side in each right triangle

1.



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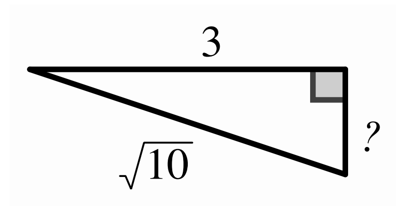

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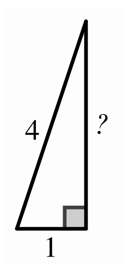

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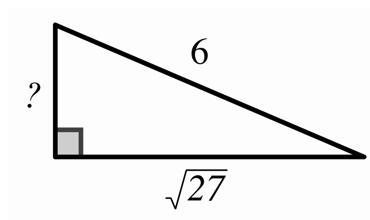
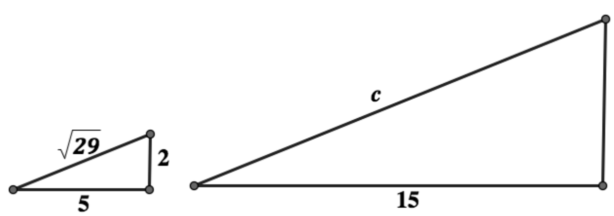


Create a proportion for each set of similar triangles. Then solve the proportion.

5.



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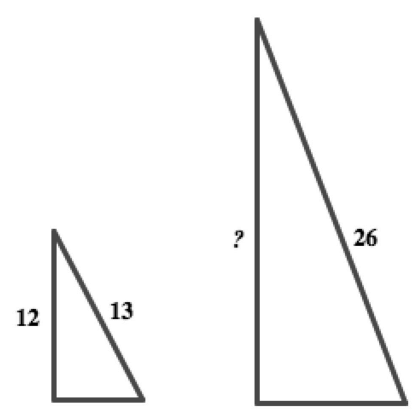
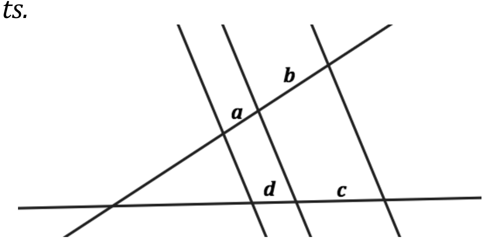


SET

Topic: Proportionality of transversals across parallel lines

For questions 7 and 8, write three equal ratios.

7. *The letters a,b,c and d represent lengths of line segments.*



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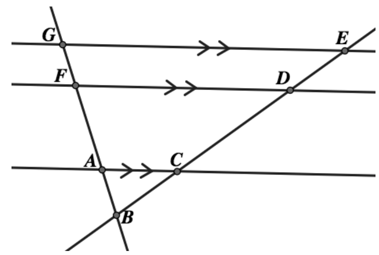
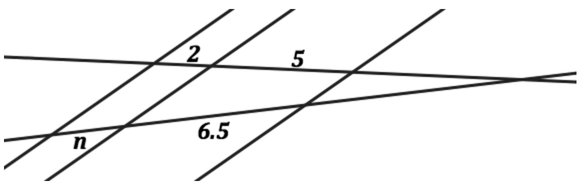
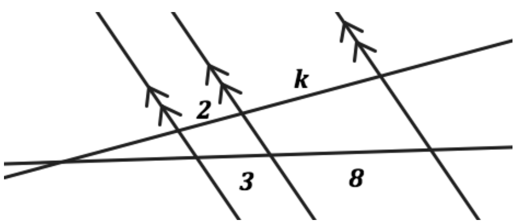


9. Write and solve a proportion that will provide the missing length.

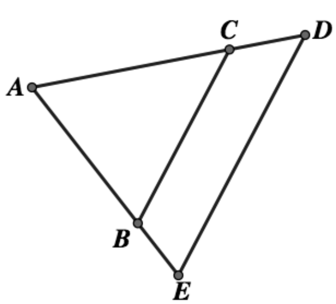


10. Write and solve a proportion that will provide the missing length.

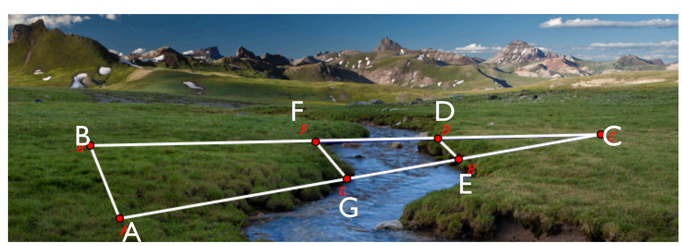


For questions 11 – 14 find and label the parallel lines. (i.e. $\overline{AB} \parallel \overline{CD}$) Then write a similarity statement for the triangles that are similar. (i.e. $\Delta\, ABC \sim \Delta\, XYZ$)

11.



12.



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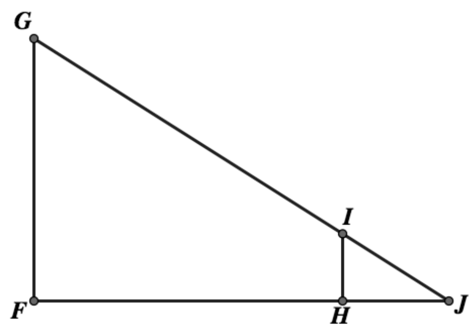

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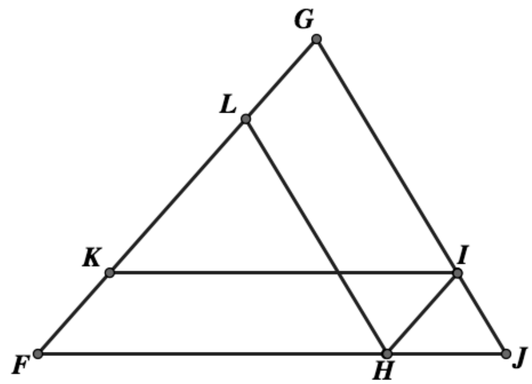
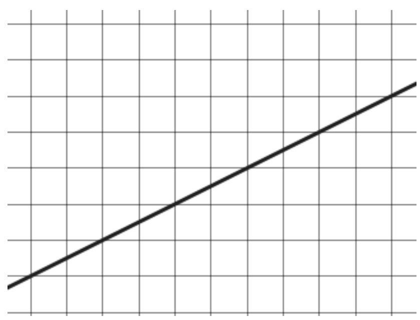


GO

Topic: Similarity in slope triangles

Each line below has several triangles that can be used to determine the slope. Draw in three slope-defining triangles of different sizes for each line and then create the ratio of rise to run for each.

15.



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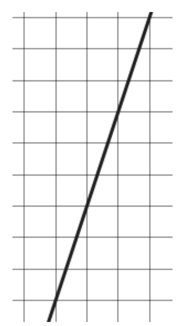

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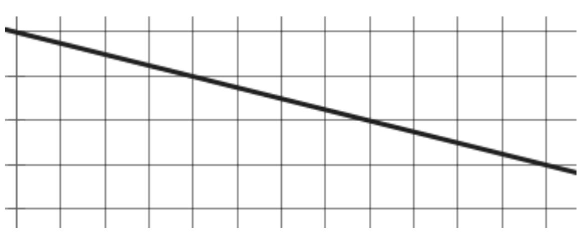

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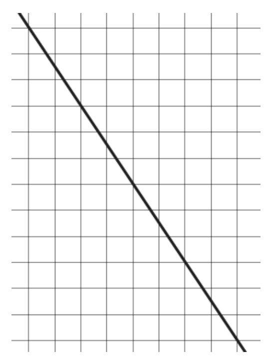
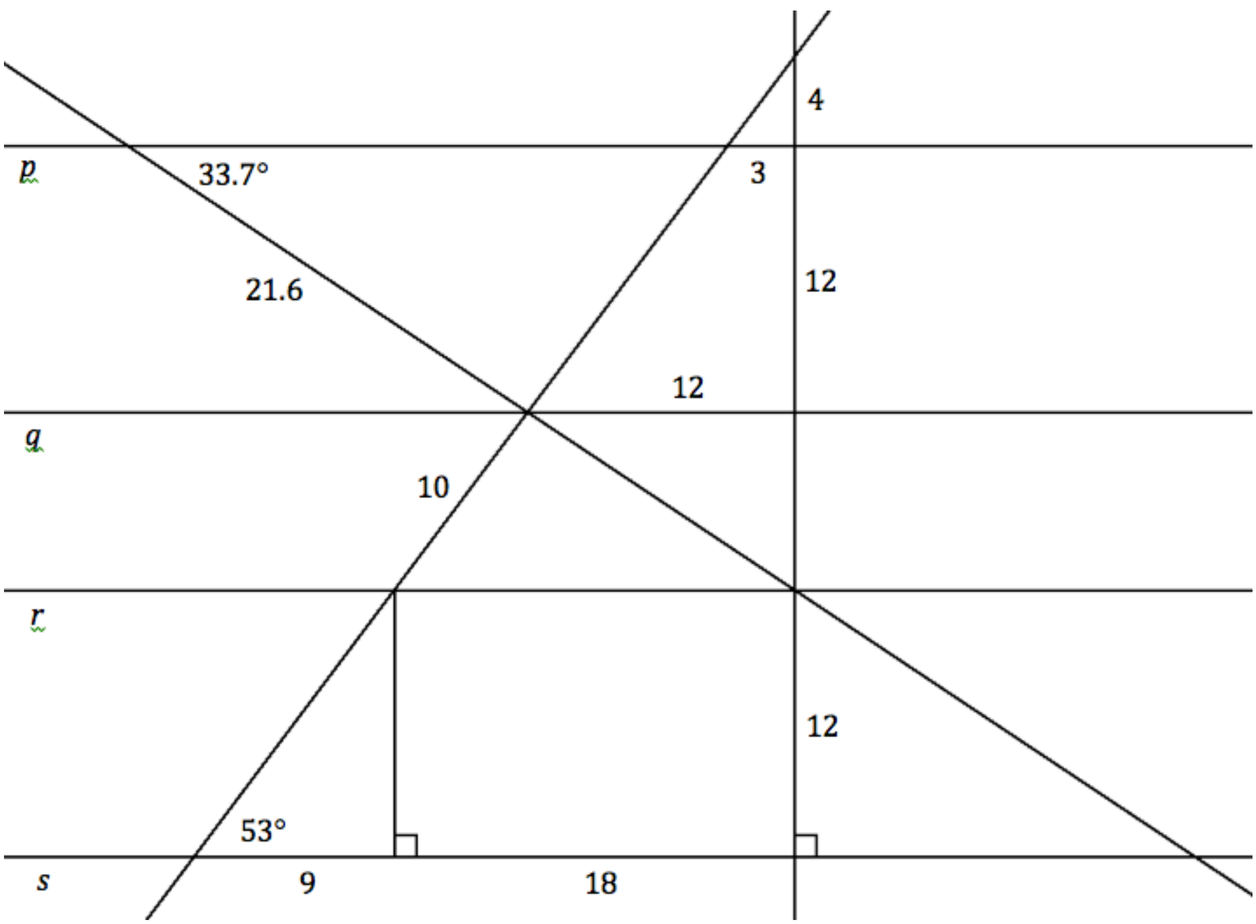


6. 5 Measured Reasoning

A Practice Understanding Task

Find the measures of all missing sides and angles by using geometric reasoning, not rulers and protractors. If you think a measurement is impossible to find, identify what information you are missing.

Lines *p*, *q*, *r*, and *s* are all parallel.



1. Identify at least three different quadrilaterals in the diagram. Find the sum of the interior angles for each quadrilateral. Make a conjecture about the sum of the interior angles of a quadrilateral.

   Conjecture:

2. Identify at least three different pentagons in the diagram. (Hint: The pentagons do not need to be convex.) Find the sum of the interior angles for each pentagon. Make a conjecture about the sum of the interior angles of a pentagon.

   Conjecture:

3. Do you see a pattern in the sum of the angles of a polygon as the number of sides increases? How can you describe this pattern symbolically?

4. How can you convince yourself that this pattern holds for all *n*-gons?

6. 5 Measured Reasoning – Teacher Notes

A Practice Understanding Task

Purpose: This task gives students opportunities to practice applying the theorems of this and the previous module. The theorems students will draw upon include:

- Vertical angles are congruent.
- Measures of interior angles of a triangle sum to 180°.
- When transversals cross parallel lines, alternate interior angles are congruent and corresponding angles are congruent.
- A line parallel to one side of a triangle divides the other two sides proportionally.

Students will also apply the Pythagorean theorem to find the missing sides of right triangles, and conversely, to determine if a triangle is a right triangle.

The last part of the task allows students to review their "ways of knowing" something is true through inductive and deductive reasoning. Students will collect data about the sums of the measures of the interior angles of quadrilaterals and pentagons. When combined with their knowledge of the sum of the measures of the interior angles of a triangle, students make a conjecture about the sum of the measures of the interior angles of polygons with any number of sides. Students are then asked to use deductive reasoning to prove their conjecture for *n*-sided polygons.

Core Standards Focus:

G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent.

G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent.

G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely.
G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

The Teaching Cycle:

Launch (Whole Class):
Distribute the task and inform students that their goal is to use the theorems and ideas developed in this and the previous module to find all of the missing lengths and angle measures in the diagram. Point out to students the instruction to "use geometric reasoning, not rulers and protractors." Also, point out the given statement that lines p, q, r, and s are all parallel. Inform students that the last part of the task will give them an opportunity to use this diagram to make a conjecture and prove a new theorem about the sum of the interior angles of any polygon.

Explore (Small Group):
Monitor the work of students so you can clarify any misconceptions that may arise in the ways students are reasoning through the missing lengths and angles in the diagram. You might prompt some discussion with groups of students by asking if the triangle at the bottom of the figure that has been decomposed into two right triangles is itself a right triangle. Students may assume it is without verifying their assumption by using the Pythagorean theorem. Remind them that "it looks like it is" is not sufficient reasoning.

Students will come to recognize that they need the lengths of at least one pair of corresponding segments on the transversals in order to set up proportions to find the other lengths. This should lead them to finding the lengths of segments on the transversals by first finding the length of each hypotenuse of the right triangles at the bottom of the diagram. From there, the rest of the lengths can be found using proportions.

One new idea that should come up in students' discussion is the fact that if a transversal is perpendicular to one line in a set of parallel lines, it is perpendicular to all of the parallel lines in the set.

As students move to gathering data for their conjectures about the sum of the interior angles in a quadrilateral or pentagon, make sure that they find more than one example of each in the diagram. There is at least one concave pentagon in the diagram that students might not notice. Students should observe that the sum of the interior angles in a polygon increases by 180° each time we add another side to the polygon. This can be represented symbolically as $180(n - 2)$ and verified with a diagram in which a polygon is decomposed into triangles by drawing all of the diagonals of the polygon emanating from one vertex.

Discuss (Whole Class):
You may not need to discuss the first part of the task if you give appropriate feedback to individual students and groups during the exploration. The second part of the task will need some whole class discussion.

Have students outline the quadrilaterals and pentagons they found on a projected image of the diagram. Organize the data into an in-out table with the "in" representing the number of sides of the polygon and the "out" representing the sum of the angles. Ask students if there is a pattern to this table, and if so, what the "out" would be when the "in" is 6. Then ask students to describe the "out" when the "in" is n. Ask students how they might illustrate the rule $180(n - 2)$ in a diagram of a hexagon, an octagon and a decagon. If no one can do so, suggest that since the sum of the interior angles of a triangle is 180°, we could try to decompose the hexagon into 4 triangles, the octagon into 6 triangles, and the decagon into 8 triangles, so that the sum of the angles of the triangles also represents the sum of the angles of the polygon. Give students a few more minutes to try to create such a decomposition.

As you monitor their work, look for students who create triangles by drawing diagonals from one vertex. Alternatively, students might pick a point in the interior of the polygon, and draw line segments from that point to each of the vertices. This will create 6 triangles in the hexagon, 8 in the octagon, and 10 in the decagon. However, the triangles drawn contain a "circle of angles" in the interior of the polygon that are not part of the sum of the interior angles of the polygon. Consequently, we need to subtract 360° from the sum of the angles of all of the triangles. Students who see the polygons decomposed in this way might write $180n - 360$ as their rule for finding the sum of the interior angles in a polygon. Note the equivalence of the two formulas, and how different ways of visualizing a situation can lead to different, but equivalent rules.

Aligned Ready, Set, Go: Similarity & Right Triangle Trigonometry 6.5

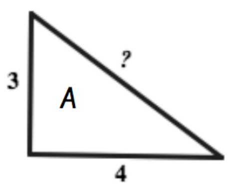
READY, SET, GO! Name Period Date

READY

Topic: Pythagorean Theorem and ratios of similar triangles

Find the missing side in each right triangle. Triangles are not drawn to scale.

1.



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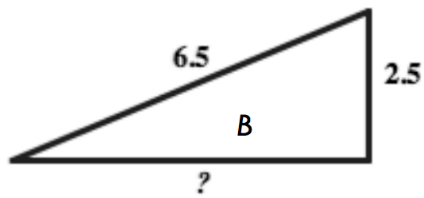

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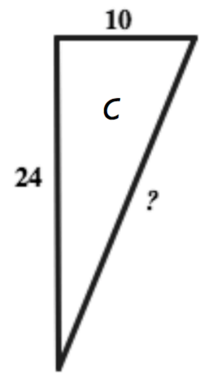

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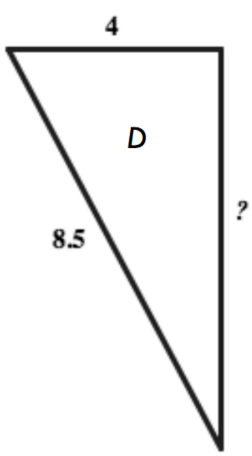

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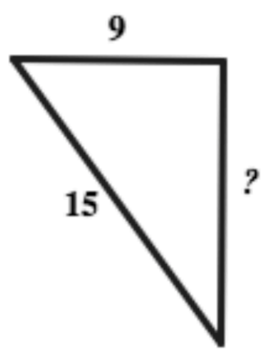

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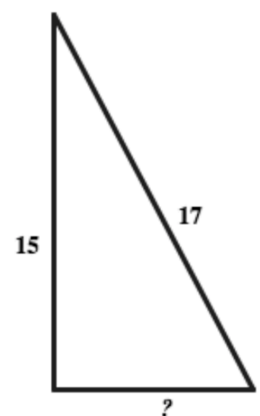


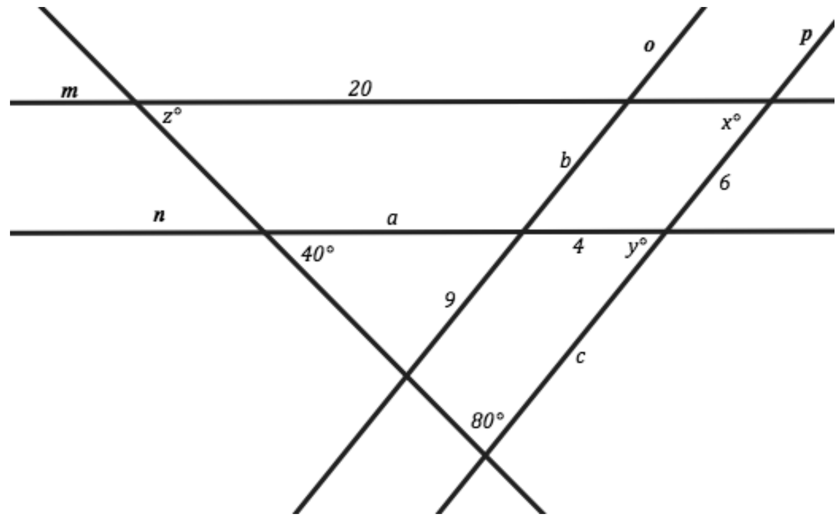
7. Based on ratios between side lengths, which of the right triangles above are mathematically similar to each other? Provide the letters of the triangles and the ratios.

SET

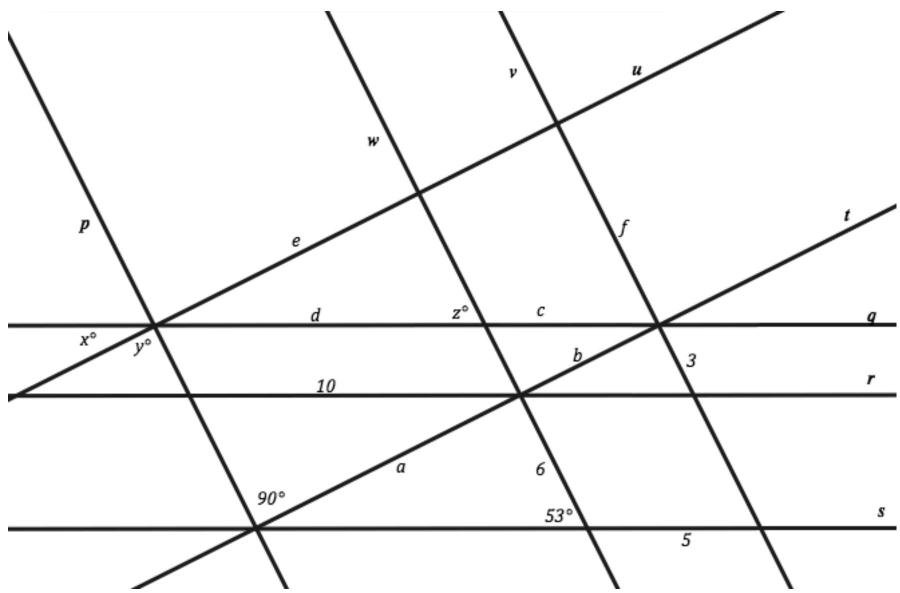
Topic: Using parallel lines, and angle relationships to find missing values.

In each of the diagrams use the given information provided to find the missing lengths and angle measurements.

8. Line $m \parallel n$ and $o \parallel p$, find the values of angles *x, y* and *z*. Also, find the lengths of *a, b* and *c*.



9. Line $q \parallel r \parallel s$ and $t \parallel u$ and $p \parallel w \parallel v$, find the values of angles *x, y* and *z*. Also, find the lengths of *a, b, c, d, e, f*.



GO

Topic: Solve equations including those including proportions

Solve each equation below.

10.

$$3x - 5 = 2x + 7$$

11.

$$\frac{5}{7} = \frac{x}{21}$$

12.

$$\frac{3}{x} = \frac{18}{5x + 2}$$

13.

$$\frac{1}{2}x - 7 = \frac{3}{4}x - 8$$

14.

$$17 + 3(x - 5) = 2(x + 3)$$

15.

$$\frac{x + 5}{6} = \frac{3(x + 2)}{9}$$

16.

$$x + 2 + 3x - 8 = 90$$

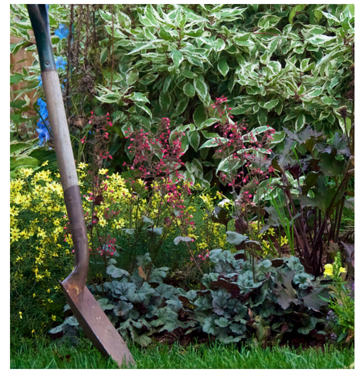
17.

$$\frac{5}{12} = \frac{x}{8}$$

18.

$$\frac{4}{5} = \frac{x + 2}{15}$$

6. 6 Yard Work in Segments



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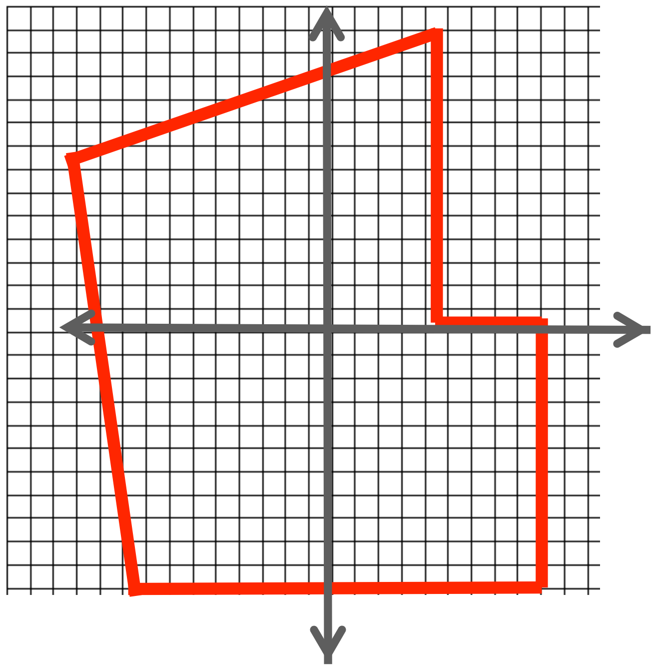
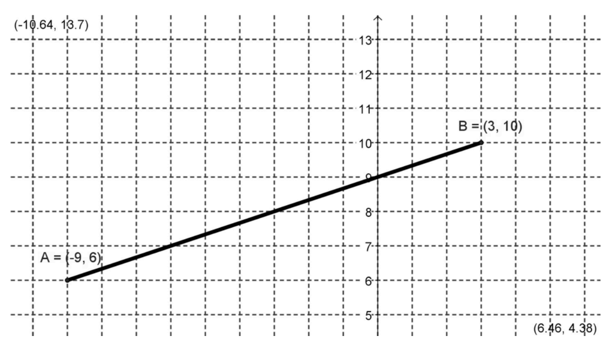
A Solidify Understanding Task

Malik's family has purchased a new house with an unfinished yard. They drew the following map of the back yard:

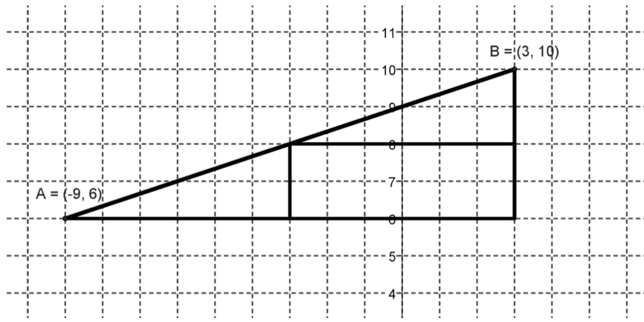


Malik and his family are using the map to set up gardens and patios for the yard. They plan to lay out the yard with stakes and strings so they know where to plant grass, flowers, or vegetables. They want to begin with a vegetable garden that will be parallel to the fence shown at the top of the map above.

1. They set the first stake at (-9, 6) and the stake at the end of the garden at (3, 10). They want to mark the middle of the garden with another stake. Where should the stake that is the midpoint of the segment between the two end stakes be located? Using a diagram, describe your strategy for finding this point.



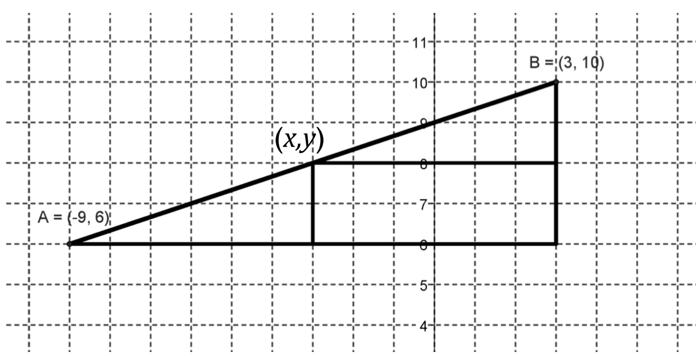
Malik figured out the midpoint by saying, "It makes sense to me that the midpoint is going to be halfway over and halfway up, so I drew a right triangle and cut the horizontal side in half and the vertical side in half like this:"



Malik continued, "That put me right at (-3, 8). The only thing that seems funny about that to me is that I know the base of the big triangle was 12 and the height of the triangle was 4, so I thought the midpoint might be (6, 2)."

2. Explain to Malik why the logic that made him think the midpoint was (6, 2) is almost right, and how to extend his thinking to use the coordinates of the endpoints to get the midpoint of (-3,6).

Malik's sister, Sapana, looked at his drawing and said, "Hey, I drew the same picture, but I noticed the two smaller triangles that were formed were congruent. Since I didn't know for sure what the midpoint was, I called it (*x, y*). Then I used that point to write an expression for the length of the sides of the small triangles. For instance, I figured that the base of the lower triangle was x – (-9).



3. Label all of the other legs of the two smaller right triangles using Sapana's strategy.

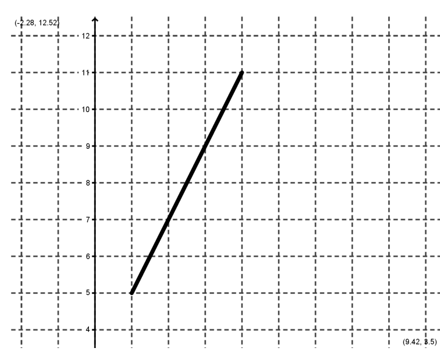
Sapana continued, "Once I labeled the triangles, I wrote equations by making the bases equal and the heights equal."

4. Does Sapana's strategy work? Show why or why not.

5. Choose a strategy and use it to find the midpoint of the segment with endpoints (-3, 4) and (2, 9).

6. Use either strategy to find the midpoint of the segment between (x_1, y_1) and (x_2, y_2).

The next area in the garden to be marked is for a flower garden. Malik's parents have the idea that part of the garden should contain a big rose bush and the rest of the garden will have smaller flowers like petunias. They want the section with the other flowers to be twice as long as the section with the rose bush. The stake on the endpoints of this garden will be at (1, 5) and (4, 11). Malik's dad says, "We'll need a stake that marks the end of the rose garden."



7. Help Malik and Sapana figure out where the stake will be located if the rose bush will be closer to the stake at (1, 5) than the stake at (4, 11).

There's only one more set of stakes to put in. This time the endpoint stakes are at (-8, 5) and (2, -10). Another stake needs to be placed that partitions this segment into two parts so that the ratio of the lengths is 2:3.

8. Where must the stake be located if it is to be closer to the stake at (-8, 5) than to the stake at (2, -10)?

6. 6 Yard Work in Segments – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is for students to apply their understanding of similar triangles and proportionality to find the point on a line segment that partitions the segment in a given ratio. Students are first asked to find the midpoint of a segment using two possible strategies that involve congruent triangles. The task continues by asking students to extend those strategies and use similar triangles to find segments in ratios other than 1:1. The formula for finding the midpoint of a segment is formalized during the discussion. The discussion can also be extended to derive a formula for finding the point that partitions a segment in any given ratio.

Core Standards Focus:
G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

The Teaching Cycle:

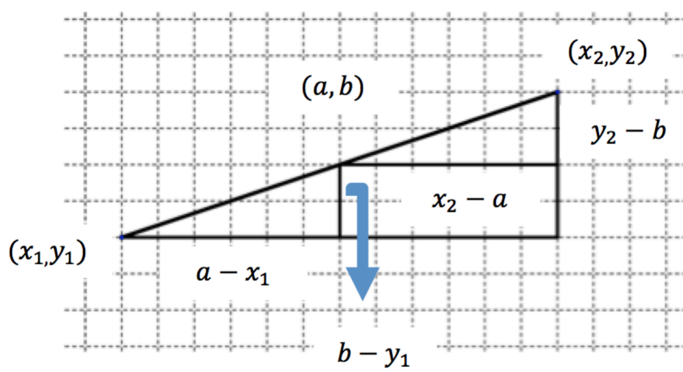
Launch (Whole Class):
Begin by ensuring that students understand the layout of the yard and how the coordinate axes are placed on the diagram of the yard. Ask students to work individually to find the answer to problem 1. Ask students what they found and confirm that the answer is (-3, 8). Have students explain to a partner the strategy that they used to find the point that split the segment into segments of equal length. After students have shared, explain that the point that they found is called the midpoint. Tell students that their strategy for finding this point may have been used in problems 2 or 3. Their job is to understand these strategies so that they can use them to find a general formula for finding a midpoint (problem 5) and then to extend their reasoning to find a point that divides the segments into parts in various ratios to each other (problems 6 and 7).

Explore (Small Group):

While students are working, support their reading of problems 2 and 3. Encourage them to draw upon their knowledge of congruent triangles for problem 3 and their knowledge of similar triangles for problems 6 and 7. Identify students to present their work in problem 4 using Malik's strategy and Sapana's strategy. If students are having trouble with problems 6 and 7, you may wish to stop their exploration and discuss problems 4 and 5, and then have them go back to work on problems 6 and 7.

Discuss (Whole Class):

Begin the discussion by asking a student to use Malik's strategy to work problem 4. Follow-up with a student that used Sapana's strategy for problem 4. Ask students to state the basic idea behind each strategy. Move the discussion to problem 6. Ask a student to label the various parts of the diagram.



Ask students to use the labels on the drawing to write and solve the literal equations in order to find the value of *a* and *b*. This work yields the formula for the midpoint (*a*,*b*):

$$(a, b) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Ask students to give a justification of this formula. You may also choose to show that Malik's strategy gives an equivalent formula.

Move the discussion to problem 7. In this case, students are asked to divide the segment in a ratio of 1:2. Some students may describe thinking about this in three parts and finding the point that is

1/3 of the way along the segment, making the point apparent in the diagram as (2, 7). Ask a student that has used Sapana's strategy to label the diagram and write the equations. Using (m, n) as the unknown point, the equations would be:

$$m - 1 = \frac{1}{2}(4 - m) \text{ and } 11 - n = 2(n - 5)$$

Ask for students to justify the equation that they have written. You may also find that students used similar triangles to write equations using the ratio of sides as 1:2. This strategy will easily lead to a general formula.

Aligned Ready, Set, Go: Similarity & Right Triangle Trigonometry 6.6

READY, SET, GO! Name Period Date

READY

Topic: Averages, measures of center, arithmetic mean

For each set of numbers find the mean (average). Explain how the mean of the set compares to the values in the set.

1. 6, 12, 10, 8
2. 2, 7, 12
3. -13, 21
4. 3, -9, 15
5. 43, 52
6. 38, 64, 100

Find the value that is exactly half way between the two given values. Explain how you find this value.

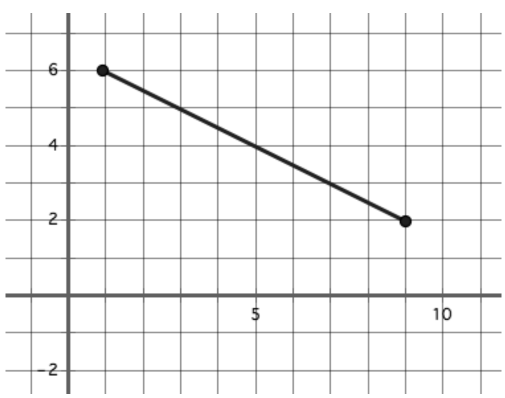
7. 5, 13
8. 26, 42
9. 57, 77
10. -34, -22
11. -45, 3
12. -12, 18

SET

Topic: Midpoints of segments and proportionality of sides in embedded similar triangles

Find the coordinates of the midpoint of each line segment below. If multiple line segments are given then give the midpoints of all segments.

13.



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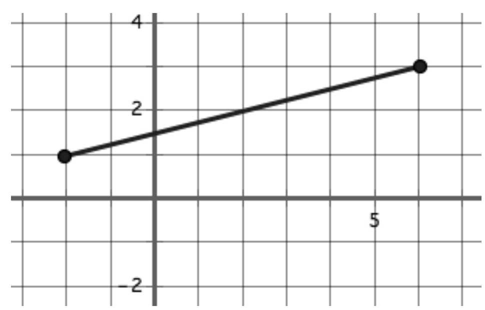

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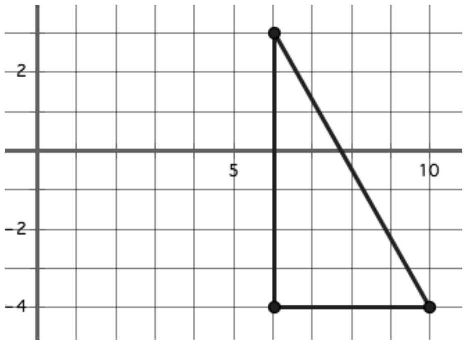

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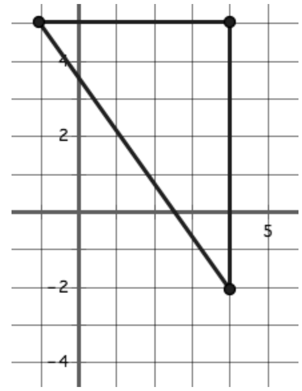
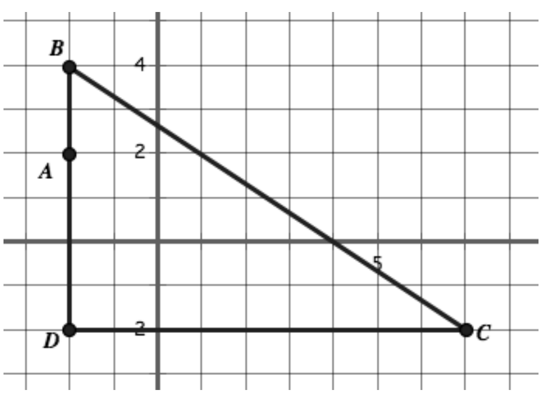


17. A line segment between (2, 3) and (10, 15)

18. A line segment between (-2, 7) and (3, -8)

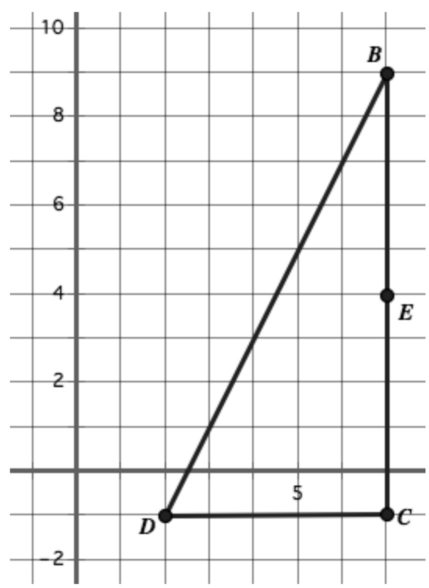
Use proportional relationships to find the desired values.

19.



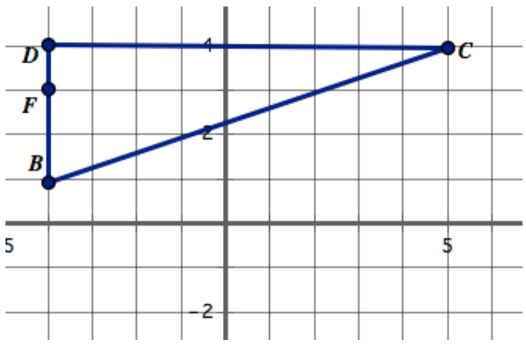
If a line is drawn parallel to $\overline{BC}$ and through point A. At what coordinate will the intersection of this parallel line be with $\overline{DC}$?

20.



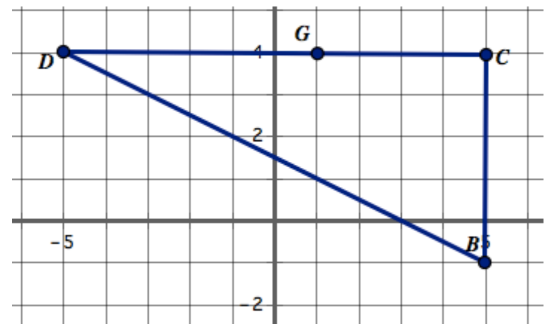
If a line is drawn parallel to $\overline{BD}$ and through point E. At what coordinate will the intersection of this parallel line be with $\overline{DC}$?

21.



If a line is drawn parallel to $\overline{BC}$ and through point F. At what coordinate will the intersection of this parallel line be with $\overline{DC}$?

22.



If a line is drawn parallel to $\overline{BD}$ and through point G. At what coordinate will the intersection of this parallel line be with $\overline{BC}$?

23. When a line is drawn parallel to one side of a triangle so that it intersects the other two sides of the triangle, how do the measures of the parts of the two intersected sides compare? Explain.

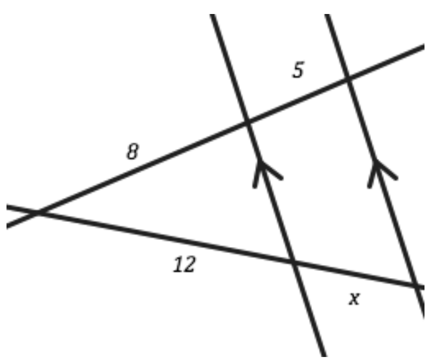
24. Problems 19-22 provided right triangles. Could a determination of the coordinates be made if they were not right triangles? Why or why not?

GO

Topic: Proportionality with parallel lines.

Write a proportion for each of the diagrams below and solve for the missing value.

25.



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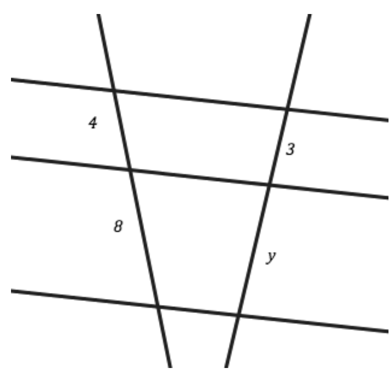
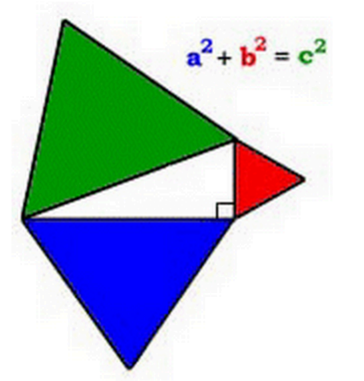


6. 7 Pythagoras by Proportions

A Practice Understanding Task

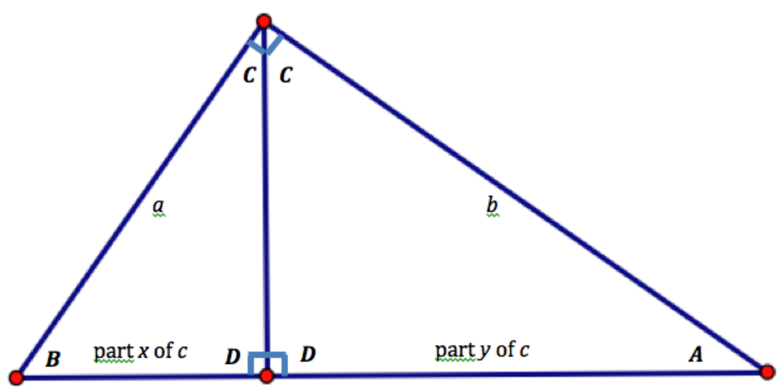


There are many different proofs of the Pythagorean theorem. Here is one based on similar triangles.

Step 1: Cut a 4×6 index card along one of its diagonals to form two congruent right triangles.

Step 2: In each right triangle, draw an altitude from the right angle vertex to the hypotenuse.

Step 3: Label each triangle as shown in the following diagram. Flip each triangle over and label the matching sides and angles with the same names on the back as on the front.



Step 4: Cut one of the right triangles along the altitude to form two smaller right triangles.

Step 5: Arrange the three triangles in a way that convinces you that all three right triangles are similar. You may need to reflect and/or rotate one or more triangles to form this arrangement.

Step 6: Write proportionality statements to represent relationships between the labeled sides of the triangles.

Step 7: Solve one of your proportions for x and the other proportion for y. (If you have not written proportions that involve x and y, study your set of triangles until you can do so.)

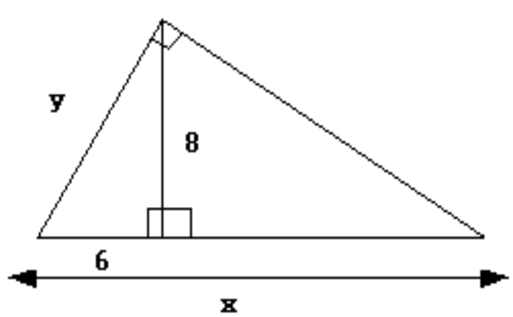
Step 8: Work with the equations you wrote in step 7 until you can show algebraically that $a^2 + b^2 = c^2$. (Remember, $x + y = c$.)

Use your set of triangles to help you prove the following two theorems algebraically. For this work, you will want to label the length of the altitude of the original right triangle *h*. The appropriate legs of the smaller right triangles should also be labeled *h.*

Right Triangle Altitude Theorem 1: If an altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean between the lengths of the two segments formed on the hypotenuse.

Right Triangle Altitude Theorem 2: If an altitude is drawn to the hypotenuse of a right triangle, the length of each leg of the right triangle is the geometric mean between the length of the hypotenuse and the length of the segment on the hypotenuse adjacent to the leg.

Use your set of triangles to help you find the values of x and y in the following diagram.



6. 7 Pythagoras by Proportions – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to give students additional practice with writing proportionality statements about similar triangles. Students will generate a new proof of the Pythagorean theorem that is based on similar triangles, rather than area. They will also explore a geometric way of representing the geometric mean between two numbers. Students may have previously worked with the geometric mean algebraically in the task *Geometric Meanies* found in the Mathematics Vision Project, Secondary Mathematics I course.

Core Standards Focus:

G.SRT.4 Prove theorems about triangles. Theorems include: the Pythagorean theorem proved using triangle similarity.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

The Teaching Cycle:

Launch (Whole Class):

Give each student a 4×6 index card and provide rulers and scissors for the construction of the set of three triangles described in steps 1-4 of the task. Make sure that students label the sides and angles of the triangles correctly, according to the diagram. (Note: Students are to label the two triangles formed from cutting the index card along one of its diagonals *before* they cut one of the triangles along its altitude to form two smaller triangles.) Once students have correctly created and labeled their set of three triangles, have them work on steps 5-8. Students who finish this work quickly should work on the Right Triangle Altitude Theorems, but not all students need to do so.

Explore (Small Group):

If students arrange all three right triangles on top of each other with the right angles superimposed on top of each other, it will be apparent that the triangles are similar. This can be verified using the AA Similarity Theorem for Triangles. With the triangles arranged in this way, students should be able to write the following proportions by comparing the two smaller triangles to the largest triangle: $\frac{c}{b}=\frac{b}{y}$ and $\frac{c}{a}=\frac{a}{x}$. The remainder of the proof may take a lot of prompting and guidance. Don't be discouraged by this, but try to use as much student thinking as possible as you help students work through the theorem. Decide when it might be appropriate to bring the class together for a whole class discussion.

Discuss (Whole Class):

You might begin the discussion by having students list all of the proportionality statements they found. If no one has written these two proportions, $\frac{c}{b}=\frac{b}{y}$ and $\frac{c}{a}=\frac{a}{x}$, ask students to re-examine their triangles to see if they can find how these proportions show up in the triangles they have constructed. Ask students how they might rearrange these proportions to solve them for *x* and *y*. This should lead to the equations $b^2 = cy$ and $a^2 = cx$. Adding these equations together yields $a^2 + b^2 = cx + cy$ or $a^2 + b^2 = c(x + y)$. Since $x + y$ is another name for side *c* of the largest triangle, we can rewrite this equation as $a^2 + b^2 = c^2$ by substitution. We have arrived at a relationship between the lengths of the sides of a right triangle—the Pythagorean theorem—without referring to the area of the sides.

Right Triangle Altitude Theorem 1 may already have been included on the list of proportionality statements students have written: $\frac{x}{h}=\frac{h}{y}$. If not, ask students if they can find how this proportion shows up in the triangles they have constructed. (They will need to compare sides of the two smaller triangles.)

The proportions used to prove the Pythagorean theorem, although it may take students awhile to recognize this, represent Right Triangle Altitude Theorem 2.

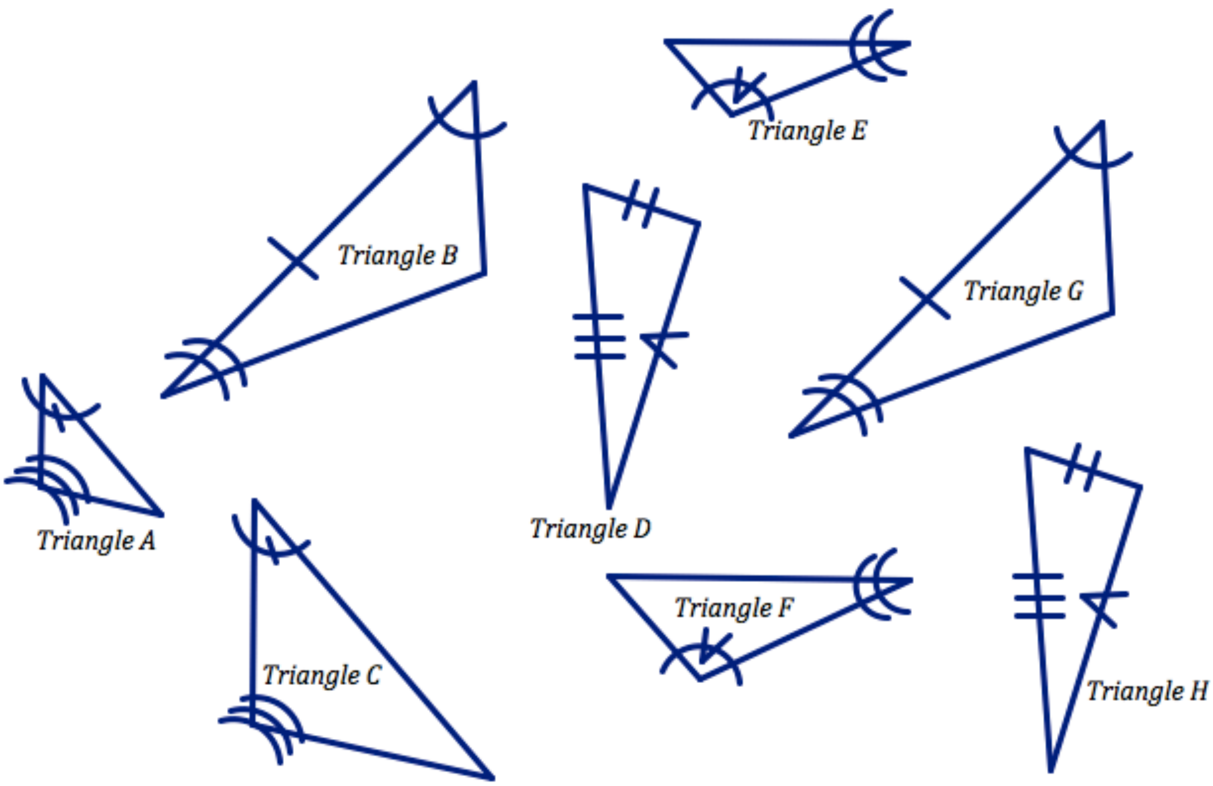
Aligned Ready, Set, Go: Similarity & Right Triangle Trigonometry 6.7

READY, SET, GO! Name Period Date

READY

Topic: Determining similarity and congruence in triangles

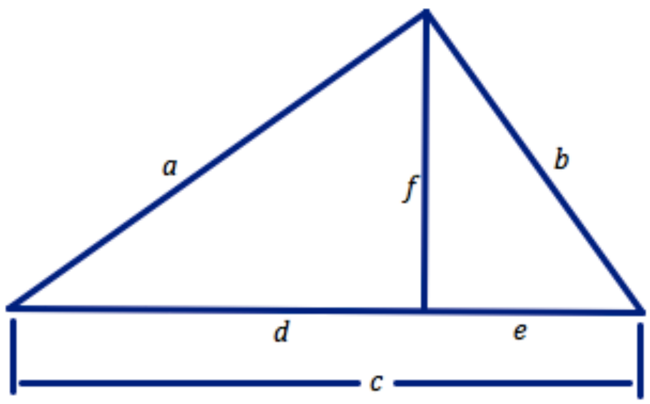
1. Determine which of the triangles below are similar and which are congruent. Justify your conclusions. Give your reasoning for the triangles you pick to be similar and congruent.



SET

Topic: Similarity in right triangles

Use the given right triangles with altitudes drawn to the hypotenuse to correctly complete the proportions.



2. $\frac{a}{c} = \frac{f}{?}$

3. $\frac{a}{f} = \frac{c}{?}$

4. $\frac{a}{b} = \frac{f}{?}$

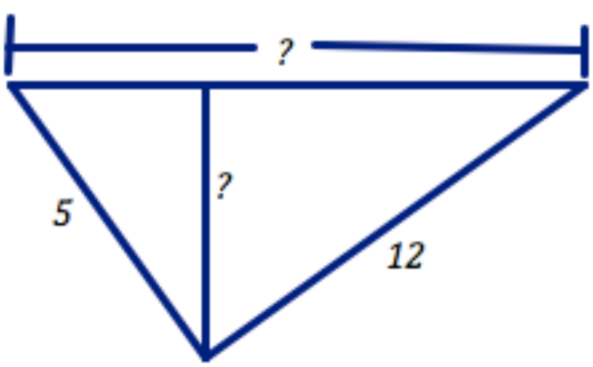
5. $\frac{a}{d} = \frac{c}{?}$

6. $\frac{f}{d} = \frac{e}{?}$

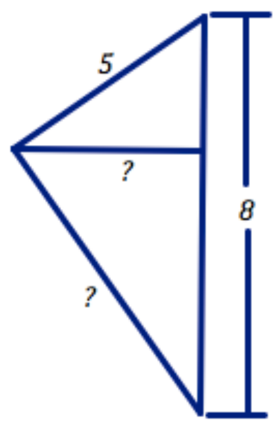
7. $\frac{b}{c} = \frac{e}{?}$

Find the missing value for each right triangle with altitude.

8.



9.

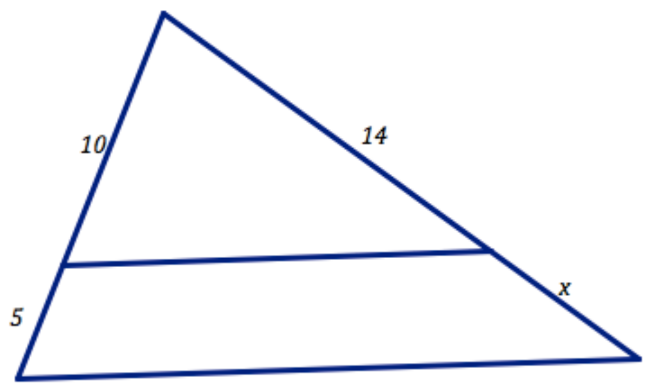


GO

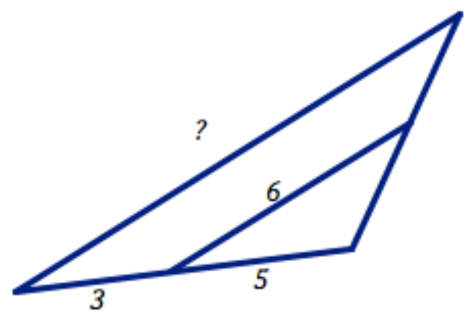
Topic: Use similarity and parallel lines to solve problems.

In each problem determine the desired values using the similar triangles parallel lines and proportional relationships. Write a proportion and solve.

10.



11.



Analyze each table below closely and determine the missing values based on the given information and values in the table.

12. An Arithmetic Sequence

Term	1	2	3	4
Value	7			22

13. A Geometric Sequence

Term	1	2	3	4
Value	7			56

14. An Arithmetic Sequence

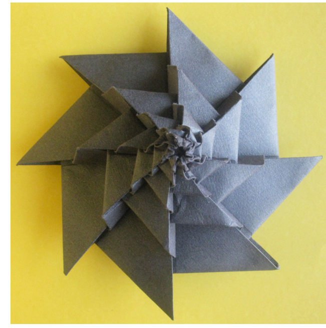
Term	5	6	7	8
Value	10			43

15. A Geometric Sequence

Term	7	8	9	10
Value	3			24

6. 8 Are Relationships Predictable?

A Develop Understanding Task



In your notebook draw a right triangle with one angle of 60°.
Measure each side of your triangle as accurately as you can with a centimeter ruler.
Using the 60^0 angle as the ***angle of reference*** list the measure for each of the following:

Length of the ***adjacent*** side:

Length of the ***opposite*** side:

Length of the ***hypotenuse***:

Create the following ratios using your measurements:

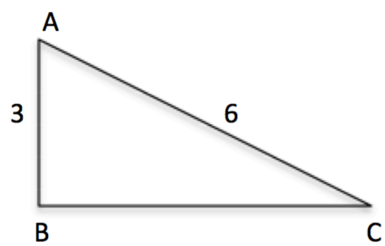
$$\frac{opposite\ side}{hypotenuse} =$$

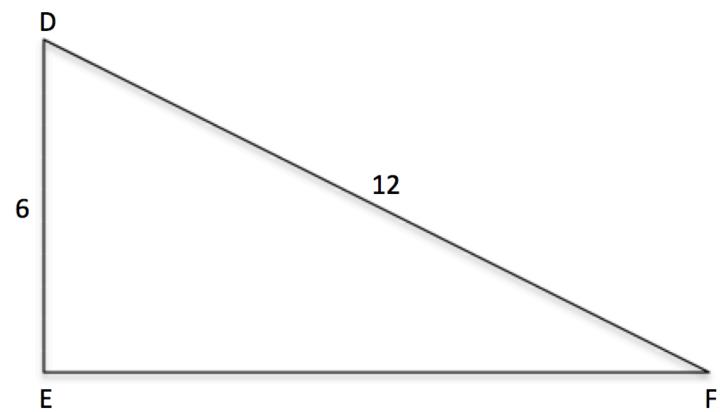
$$\frac{adjacent\ side}{hypotenuse} =$$

$$\frac{opposite\ side}{adjacent\ side} =$$

1. Compare your ratios with others that had a triangle of a different size. What do you notice? Explain any connections you find to others' work?

2. In the right triangles below find the missing side length and then create the desired ratios based on the angle of reference (angle A and angle D).





List the ratios for ΔABC using angle A as the angle of reference.

$$\frac{opposite\ side}{hypotenuse} =$$

$$\frac{adjacent\ side}{hypotenuse} =$$

$$\frac{opposite\ side}{adjacent\ side} =$$

List the ratios for ΔDEF using angle D as the angle of reference.

$$\frac{opposite\ side}{hypotenuse} =$$

$$\frac{adjacent\ side}{hypotenuse} =$$

$$\frac{opposite\ side}{adjacent\ side} =$$

3. What do you notice about the ratios from the two given triangles? How do these ratios compare to the ratios from the triangle you made on the previous page?

4. What can you infer about the angle measures of ΔABC and ΔDEF?

5. Why do the relationships you have noticed occur?

6. What can you conclude about the ratio of sides in a right triangle that has a 60° angle? Would you think that right triangles with other angle measures would have similar relationships among their ratios?

6. 8 Are Relationships Predictable? – Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is to allow students to use their prior understandings of similar triangles to develop an understanding of trigonometric ratios. As the discussion about the trigonometric ratios and their usefulness surfaces, be sure to name the ratios with their names (sine, cosine, tangent) and demonstrate how they are most frequently written as *cos, sin, tan*.

Core Standards Focus:
G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Related Standards: G.SRT.7, **A.SSE.1a**

The Teaching Cycle:
Launch (Whole Class):
The purpose of this task is to develop an understanding of the trigonometric ratios. Students have worked with similarity and similar triangles prior to this and the intent here is to develop an understanding of the three main trigonometric ratios that are connected to each acute reference angle. There is also opportunity to link these ratios to the understandings of similarity and proportionality students have developed in previous tasks and grade levels.

To launch this task you can simply ask students to draw a right triangle with a 60° angle for one of the acute angles. It would be good to encourage them to make their triangle a different size then their partner's or neighbor's. After completing their drawings and measuring the lengths of the sides of their triangle students should continue working through the task.

Explore (Small Group):

It would work well for students to be in pairs for this task. Look for students that may not attend to the angle of reference and help them with the orientation of adjacent and opposite sides with regard to the reference angle.

Students that have a firm grasp on ratio and similarity may finish this task quickly. Look for these students and provide them with the opportunity to extend their thinking. For example, you could have them look at a right triangle with one angle that measures 45° to notice that the cosine and sine ratios are equal, or have them revisit the triangles in question 2 using the other acute angles as the reference angle and see what patterns they might notice. They could also be pressed upon to provide a justification for the last question that could be used later in the whole class discussion.

Discuss (Whole Class):

Begin the whole class discussion by sharing several examples of triangles from problem 1 so that the equivalent ratios can be observed. This will facilitate and promote the question about why these ratios are equivalent.

Questions 5 and 6 should be given enough time in the discussion to surface the idea that two right triangles are similar if they both have the same measure for one of their acute angles. This is because the corresponding right angles are always congruent, and the third angles would be congruent by the theorem about the sums of the interior angles of a triangle adding to 180°. So, by AA similarity, or even AAA similarity, the two right triangles are similar. This means that ratios of corresponding sides of these right triangles will also be equal. It is important for students to bring these attributes of similarity to the surface and discuss how knowing one allows for conclusions to be drawn about the other. Knowing that two triangles have the same angle measures means they must be similar, which in turn means they must have the equivalent ratios of sides and vice versa.

Conclude this discussion by denoting the ratios with their names (sine, cosine, tangent) and demonstrate how they are most frequently written as *cos*, *sin*, or *tan*. That is, if we want the cosine ratio with reference to angle A in a triangle we will see it written *cos* (A).

Given time provide students with another right triangle, possibly a 3-4-5 right triangle and ask them to create each trigonometric ratio given a reference angle you select. Ask them about how these ratios would change if we change the reference angle. Ask students to give you the side lengths of another triangle that would be similar and to justify equivalence of angles or trigonometric ratios for the side lengths they select.

Aligned Ready, Set, Go: Similarity & Right Triangle Trigonometry 6.8

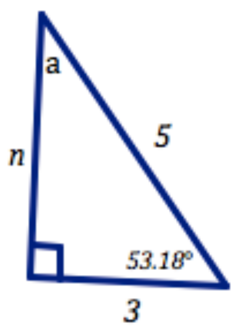
READY, SET, GO! Name Period Date

READY

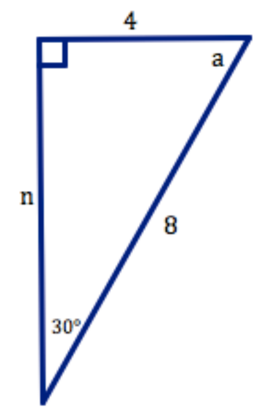
Topic: Properties of angles and sides in right triangles

For each right triangle below find the missing side *n* (Pythagorean Theorem could be helpful) and the missing angle, a (Angle Sum Theorem for Triangles could be useful).

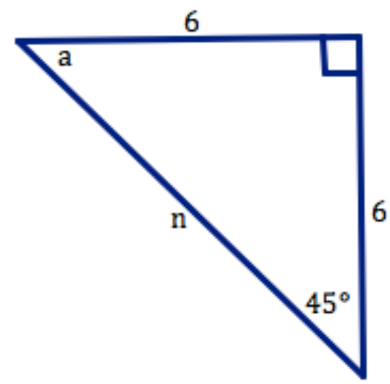
1.



2.



3.



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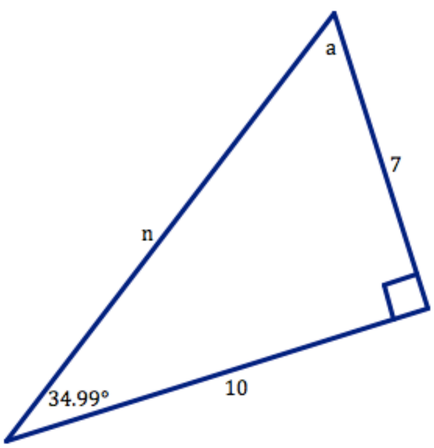
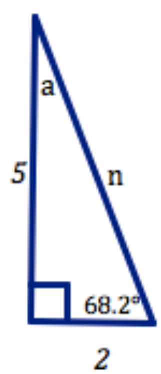
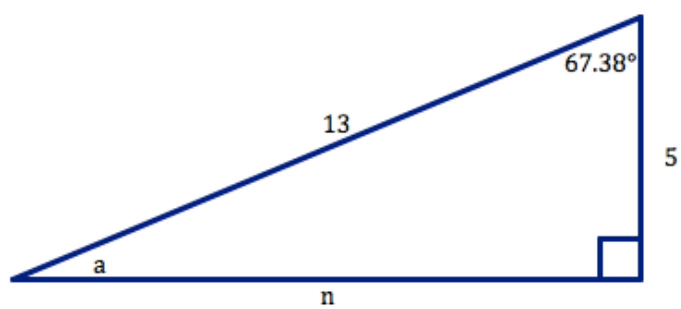


5.



6.



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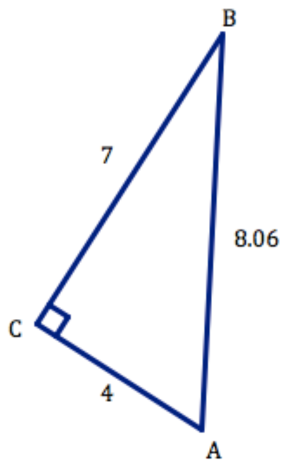
SET

Topic: Creating trigonometric ratios for right triangles

For each right triangle and the identified angle of reference create the desired trigonometric ratios. If any sides of the triangle are missing, find them before determining the ratio.

7.

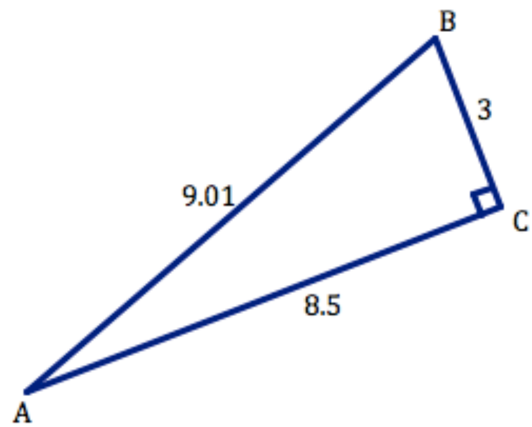


Cos (A)= Cos (B)=

Sin (A)= Sin (B)=

Tan (A)= Tan (B)=

8.

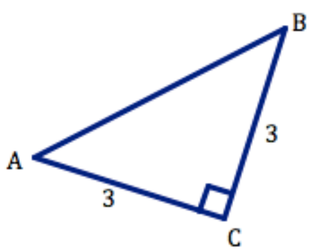


Cos (A)= Cos (B)=

Sin (A)= Sin (B)=

Tan (A)= Tan (B)=

9.

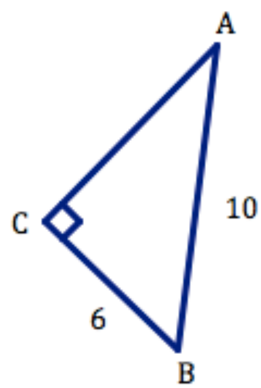


Cos (A)= Cos (B)=

Sin (A)= Sin (B)=

Tan (A)= Tan (B)=

10.



Cos (A)= Cos (B)=

Sin (A)= Sin (B)=

Tan (A)= Tan (B)=

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GO

Topic: Factoring quadratics

Provide the factored form and the x- and y-intercepts

11. $f(x) = x^2 + 9x + 20$

factored form:

x-intercepts:

y-intercept:

12. $g(x) = x^2 + 2x - 15$

factored form:

x-intercepts:

y-intercept:

13. $h(x) = x^2 - 49$

factored form:

x-intercepts:

y-intercept:

14. $r(x) = x^2 - 13x + 30$

factored form:

x-intercepts:

y-intercept:

15. $f(x) = x^2 + 20x + 100$

factored form:

x-intercepts:

y-intercept:

16. $g(x) = x^2 - 8x - 48$

factored form:

x-intercepts:

y-intercept:

17. $h(x) = x^2 + 16x + 64$

factored form:

x-intercepts:

y-intercept:

18. $k(x) = x^2 - 36$

factored form:

x-intercepts:

y-intercept:

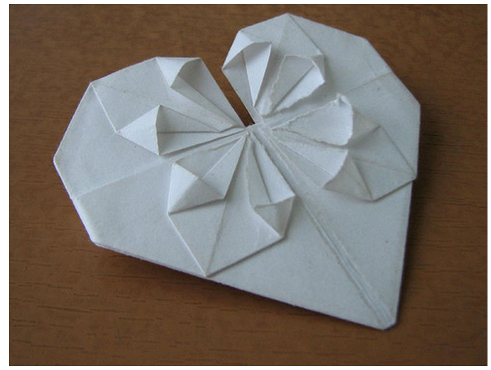
19. $p(x) = x^2 - 2x - 24$

factored form:

x-intercepts:

y-intercept:

6. 9 Relationships with Meaning



A Solidify Understanding Task

Part I

1. Use the information from the given triangle to write the following trigonometric ratios:

$sin\ (A) = \frac{opposite}{hypotenuse} =$

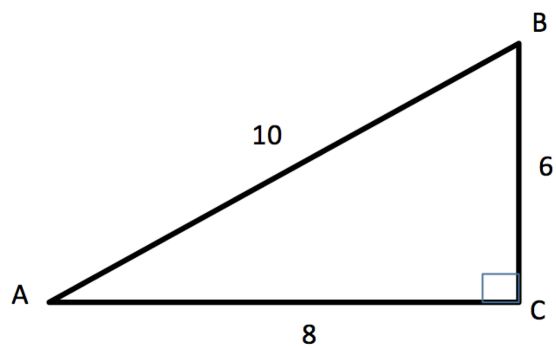
$cos\ (A) = \frac{adjacent}{hypotenuse} =$

$tan\ (A) = \frac{opposite}{adjacent} =$

$sin\ (B) =$

$cos\ (B) =$

$tan\ (B) =$



2. Do the same for this triangle:

$sin\ (A) =$

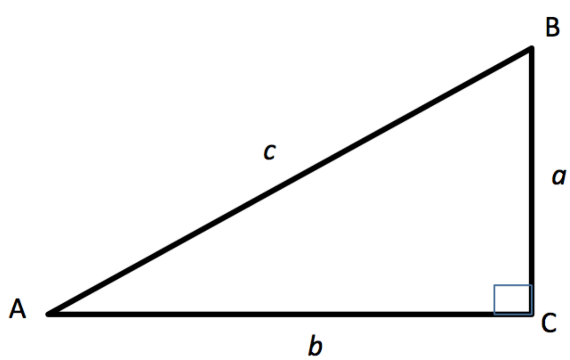
$cos\ (A) =$

$tan\ (A) =$

$sin\ (B) =$

$cos\ (B) =$

$tan\ (B) =$



3. Use the information above to write observations you notice about the relationships between trigonometric ratios of the two different reference angles in these right triangles.

4. Do you think these observations will always hold true? Why or why not?

Part 2

The following is a list of conjectures made by students about right triangles and trigonometric relationships. For each, state whether you think the conjecture is true or false. Justify your answer.

5. $\cos(A) = \sin(A)$

6. $\tan(A) = \frac{\sin(A)}{\cos(A)}$

7. $\sin(A) = \cos(90° - A)$

8. $\cos(A) = \sin(B)$

9. $\cos(B) = \sin(90° - A)$

10. $\tan(A) = \frac{1}{\tan(B)}$

Note the following convention used to write $[\sin(\mathrm{A})]^2 = \sin^2(A)$

11. $\sin^2(A) + cos^2(A) = 1$

12. $1 - \sin(A)^2 = cos^2(A)$

13. $\sin^2(A) = \sin(A^2)$

Part 3

14. Given a right triangle with the following trigonometric ratio: $\sin(30°) = \frac{1}{2}$, find all of the trigonometric ratios for this triangle. How do you know these values are always going to be true when given this angle?

6. 9 Relationships with Meaning – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is for students to find relationships between sine and cosine using their knowledge of right triangles, complementary angles, and the Pythagorean theorem. The focus of this task is on Part II where students reason about conjectures. Emphasis in the discussion should be placed on questions related to the complementary relationship between sine and cosine as well as the Pythagorean identity.

Core Standards Focus:
G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.
F.TF.8 Prove the Pythagorean identity $\sin^2(A) + cos^2(A) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. (note: the High School Geometry course focuses on angles in quadrant I, that is, angles that can be found in a right triangle).

Related Standards: G.SRT.8

The Teaching Cycle:

Launch (Whole Class):
Review trigonometric ratios for sine, cosine, and tangent, then have students complete part 1 on their own. You may even choose to have these questions as the starter activity of the day. After a couple of minutes, have students work together in partners to discuss questions 3 and 4 regarding relationships among trigonometric ratios.

Explore (Small Group):

For part 2 of the task, students are asked to agree or disagree with conjectures made by fellow students. As you monitor, listen for reasoning about agreement regarding the conjectures. If students are not explaining their reasoning, ask them to explain why they agree or disagree using a mathematical argument, perhaps based on reference to the right triangle figures. If they seem stuck, have them test the conjecture on more than one triangle. The focus of this activity is for students to reason about the conjectures in general and to understand the complementary relationship of the acute angles in a right triangle and to connect this to the trigonometric ratios of sine and cosine. Students should also spend time making sense of the Pythagorean identity using their knowledge of the Pythagorean theorem and the trigonometric ratios for sine and cosine. For students who are moving quickly, press them to be more specific in their justification for why they agree or disagree.

Discuss (Whole Class):

By the end of the lesson, all students should know whether each conjecture is true or false, however, depending on time, focus the reasoning and justification portion of the discussion to questions relating to standards **G.SRT.7** and **F.TF.8.** If you have additional time, start the discussion by asking students to explain their reasoning for questions 5 and 6. For example, a student may explain that question 5 is only true when dealing with an isosceles right triangle and that otherwise, if the angles are not congruent, then the sides would be of differing lengths, therefore $\cos(A) \neq \sin(A)$.

Again, the focus of the task and the most important discussions are around questions 7-9 (**G.SRT.7** : complementary angle relationship of sine and cosine) and 11-14 (**F.TF.8:** Pythagorean Identity). Students should be able to express their understanding of the relationship between the sine and cosine of complementary angles. Students should also be able to draw a model of the trigonometric relationships to prove the Pythagorean identity $\sin^2(A) + cos^2(A) = 1$ using their background knowledge of the Pythagorean theorem as well as the meaning of the trigonometric ratios. As a result, students should be able to answer question 14 prior to doing the RSG homework assignment. Conclude the discussion by creating a chart of these relationships, which are also referred to as ***trigonometric identities***.

Aligned Ready, Set, Go: Similarity & Right Triangle Trigonometry 6.9

READY, SET, GO! Name Period Date

READY

Topic: Geometric Formulas for Perimeter, Area and Volume

State the area and volume formulas that are required below.

1.

a. Area of a circle:

b. Volume of a rectangular prism:

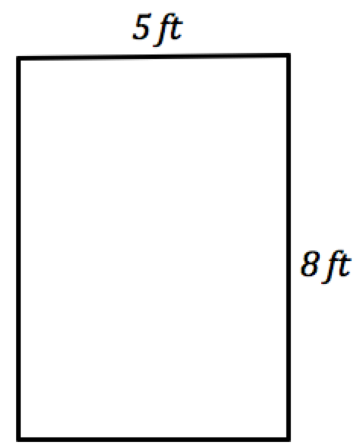
c. Volume of a cylinder:

d. Area of a rectangle:

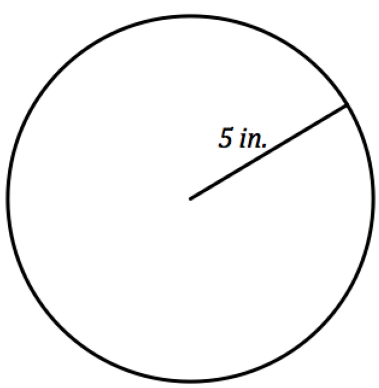
e. Perimeter of a rectangle:

f. Circumference of a circle:

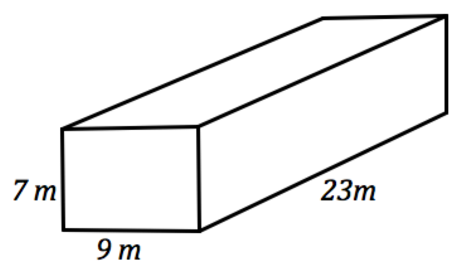
3. Find the perimeter and area for the rectangle.



2. Find the circumference and area for the circle.



4. Find the volume and surface area of the prism.



SET

Topic: Trigonometric Ratios and Connections between them.

Based on the given trigonometric ratio, sketch a triangle and find a possible value for the missing side as well as the other missing trig ratios. Angles A and B are the two non-right angles in a right triangle.

5. tan (A)= $\frac{3}{4}$
   sin (A)=
   cos (A)=

   tan (B)=
   sin (B)=
   cos (B)=

Sketch of Triangle:

6. tan (A)=
   sin (A)=
   cos (A)=

   tan (B)=
   sin (B)= $\frac{8}{17}$
   cos (B)=

Sketch of Triangle:

7. tan (A)=
   sin (A)=
   cos (A)= $\frac{12}{13}$

   tan (B)=
   sin (B)=
   cos (B)=

Sketch of Triangle:

8. tan (A)=
   sin (A)=
   cos (A)=

   tan (B)=
   sin (B)= $\frac{1}{\sqrt{2}}$
   cos (B)=

Sketch of Triangle:

Given a right triangle with angles A and B as the non-right angles. Determine if the statements below are true or false. Justify your reasoning and show your argument.

9. $\cos(A) = \frac{1}{\sin A}$

10. $\tan(B) = \tan(90° - A)$

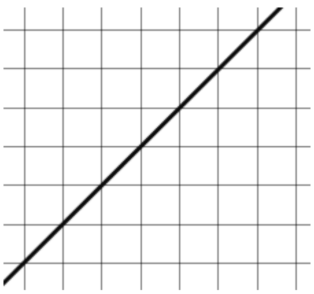
11. $\tan(A) \cdot \cos(A) = \sin(A)$

GO

Topic: Slope as a ratio

Find the slope of each line and write it as a ratio of rise to run.

12.



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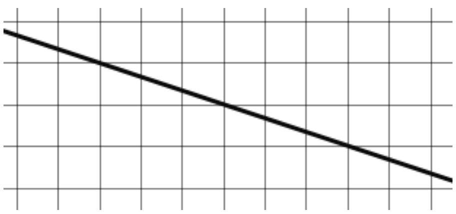

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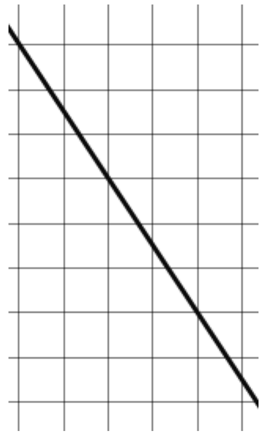
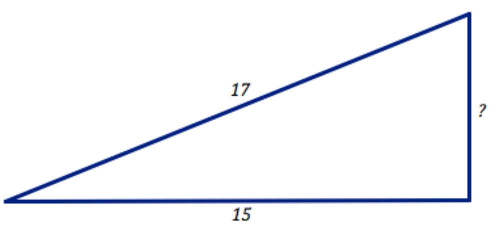


Find the missing length in each right triangle. Then determine the slope of the hypotenuse.

15.



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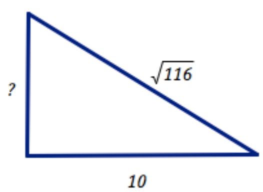
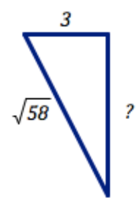


17.



6.10 Finding the Value of a Relationship

CC BY Stuart Heath "tree shadow"
https://flic.kr/p/4Lkc1D

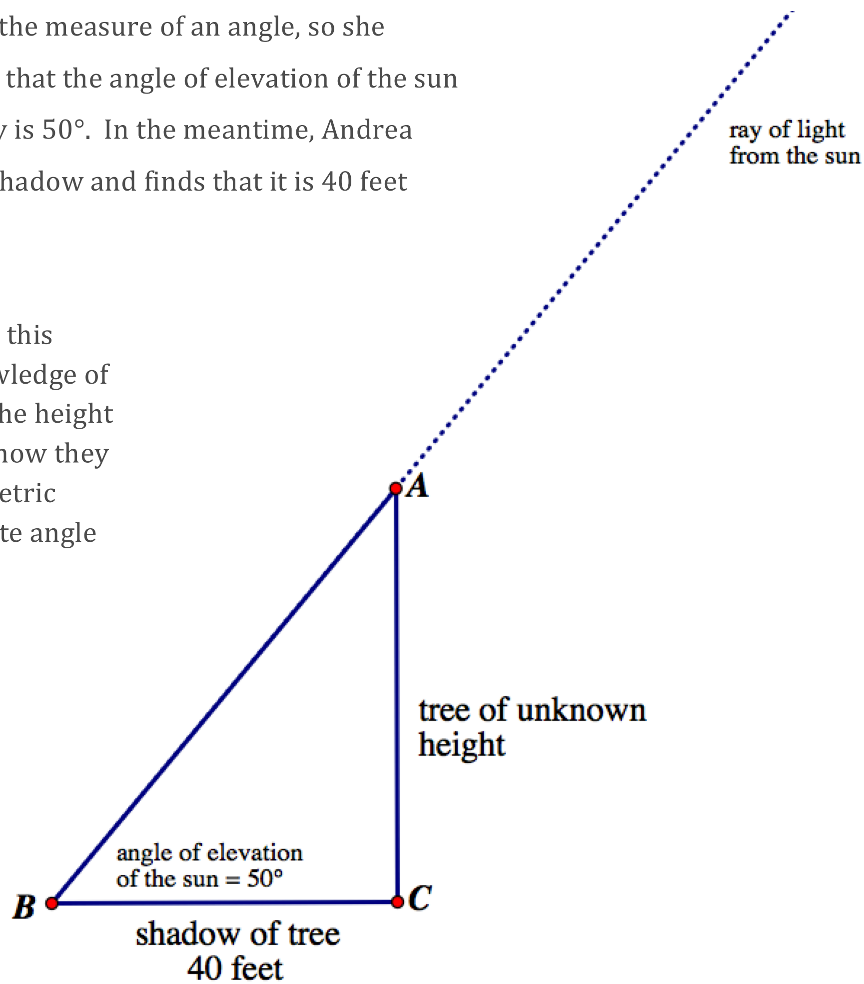
A Solidify Understanding Task

Part 1: Pick a side

Andrea and Bonita are resting under their favorite tree before taking a nature walk up a hill. Both girls have been studying trigonometry in school, and now it seems like they see right triangles everywhere. For example, Andrea notices the length of the shadow of the tree they are sitting under and wonders if they can calculate the height of the tree just by measuring the length of its shadow.

Bonita thinks they also need to know the measure of an angle, so she checks an app on her phone and finds that the angle of elevation of the sun at the current location and time of day is 50°. In the meantime, Andrea has paced off the length of the tree's shadow and finds that it is 40 feet long.

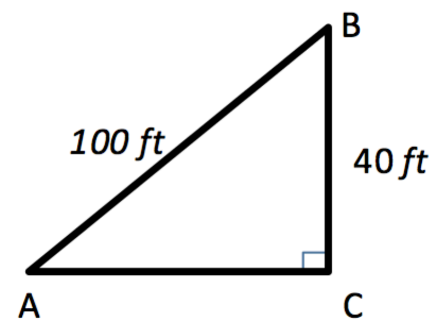
1. How might Andrea and Bonita use this information, along with their knowledge of trigonometric ratios, to calculate the height of the tree? (Andrea and Bonita know they can find the value of any trigonometric ratios they might need for any acute angle using a calculator.)



Part 2: What's your angle?

After their rest, Andrea and Bonita are going for a walk straight up the side of the hill. Andrea decided to stretch before heading up the hill while Bonita thought this would be a good time to get a head start. Once Bonita was 100 feet away from Andrea, she stopped to take a break and looked at her GPS device that told her that she had walked 100 feet and had already increased her elevation by 40 feet. With a bit of time to waste, Bonita wrote down the trigonometric ratios for ∠A and for ∠B.

2. Name the trigonometric ratios for ∠A and for ∠B.



When Andrea caught up, she said "What about the unknown angle measures? When I was at the bottom and looked up to see you, I was thinking about the "upward" angle measure from me to you. Based on your picture, this would be ∠A."

Bonita wrote the trigonometric ratio $\sin A = \frac{40}{100}$ and asked, "So, how do we find angle A?"

Together, the girls talked about how this was like thinking backwards: instead of knowing an angle and using their calculators to find a trigonometric ratio like they did while working on the height of the tree problem, they now know the trigonometric ratio and need to find an unknown angle value. Bonita notices the $sin^{-1}(\theta)$ button on her calculator and wonders if this might work like an "inverse trigonometric ratio" button, undoing the ratio to produce the angle. She decides to try it out, and produces the following output on her calculator:

$$\sin A = \frac{40}{100}$$

$$sin^{-1}\left(\frac{40}{100}\right) = 23.578°$$

$$\sin(23.578°) = 0.4$$

3. How might this output convince Bonita that her assumption about the calculator was correct?

4. Use the trigonometric ratio you found for $\cos B$ to find the value of ∠B.

5. Find all unknown values for the following right triangle:

   a) $\angle\alpha =$ _____

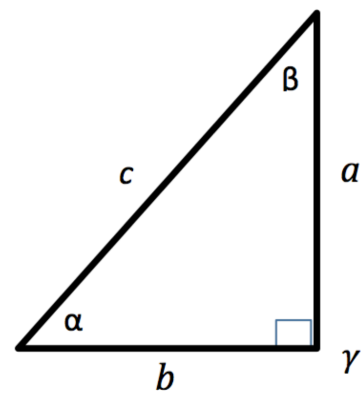
   b) $\angle\beta =$ _____

   c) $\angle\gamma =$ <u>90°</u>

   d) $a =$ <u>*12 m*</u>

   e) $b =$ <u>*8 m*</u>

   f) $c =$ _____



6. Bonita and Andrea started talking about all of the ways to find unknown values in right triangles and decided to make a list. What do you think should be on their list? Be specific and precise in your description. For example, 'trig ratios' is not specific enough. You may use the following sentence frame to assist with writing each item in your list:

   When given ____________________, you can find ____________________ by _______________.

Part 3: Angle of elevation and angle of depression

During their hike, Andrea mentioned that she looked up to see Bonita. In mathematics, when you look straight ahead, we say your line of sight is a horizontal line. From the horizontal, if you look up, the angle from the horizontal to your line of sight is called the **angle of elevation.** Likewise, if you are looking down, the angle from the horizontal to your line of sight is called the **angle of depression.**

7. After looking at this description, Andrea mentioned that her angle of elevation to see Bonita was about 23.5°. They both agreed. Bonita then said her angle of depression to Andrea was about 66.5°. Andrea agreed that it was an angle of depression but said Bonita's angle of depression was also 23.5°. Who do you think is correct? Use drawings and words to justify your conclusion.

8. What conclusion can you make regarding the angle of depression and the angle of elevation? Why?

6. 10 Finding the Value of a Relationship – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is for students to solidify their understanding about the various ways to solve for unknown values of right triangles. Students will build on their prior knowledge of using the Pythagorean theorem to find unknown side lengths as well as their knowledge of setting up trigonometric ratios to find unknown side lengths. In this task, students will also learn how to use inverse trigonometric relationships to find unknown angle measures. Students will also further their knowledge of solving application problems using trigonometry by discussing the angle of elevation and the angle of depression.

Core Standards Focus:
G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.
G.SRT.8 Use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems.★

Related Standards: G.SRT.6, F.TF.8

The Teaching Cycle:

Launch, Part 1 (Whole Class):
Read the scenario together and make sure students understand how the angle of elevation of the sun can be used to impose a right triangle on a situation where one doesn't really exist. Also point out that Andrea and Bonita have access to a calculator that displays the values of trigonometric ratios, and spend a couple of minutes showing students how to access that information on their own calculators. Then set students to work on devising a strategy for finding the height of the tree using the given information.

Explore, Part 1 (Small Group):

Since the length of only one side of the right triangle is known, students may wonder how they can find either of the other sides since their strategy for finding missing sides of a right triangle consists solely of using the Pythagorean theorem. So far, they have used trigonometry only to find ratios of sides where both side lengths are known. Listen for students to make note of the following key ideas:

- Since we know the angle of elevation, we can use a calculator to find the value of each of the three trigonometric ratios
- We can represent unknown lengths in a trigonometric ratio using variables
- The cosine ratio and the tangent ratio both involve the side that measures 40 feet
- The tangent ratio also includes the side whose length we want to know
- We can write an equation of the form $\tan(50°) = \frac{the\ height\ of\ the\ tree}{40\ feet}$
- This is equivalent to the equation $40 \cdot \tan(50°) = the\ height\ of\ the\ tree$, which can be solved by replacing $\tan(50°)$ with a value that can be found on the calculator

Identify students who can share this line of thinking with the whole class.

Discuss, Part 1 (Whole Class):

Use selected students' thinking to bring out the key ideas listed above. Refer to the illustration of the tree as each key idea discussed.

Launch, Part 2 (Whole Class):

Read the scenario given in part 2 and have students find the trigonometric ratios for $\angle A$ and $\angle B$. This should only take a couple of minutes (finding the unknown side length either by recognizing the 3-4-5 triple or by using the Pythagorean Theorem, then writing the ratios). Afterward, go through the process of finding the unknown angle measure using the inverse trigonometric keys on the calculator. Be sure to show where the 23.5° angle measure is on the triangle (this is in preparation for part 3 of the task). Then have students work in pairs to answer questions 4-6 of part 2.

Explore, Part 2 (Small Group):

As you monitor, look for student understanding of questions 2 and 3 and guide students if misconceptions arise. If students are stuck with how to find unknown values, ask prompting questions that remind them to think about how they have recently solved for unknown values (Pythagorean theorem, trigonometric ratios, and more recently, inverse trigonometric ratios). For the most part, focus on the answers to question 6 where students are writing their list of how to solve for unknown values of right triangles.

Discuss, Part 2 (Whole Class):

The goal for the first part of this discussion is to get out the different ways to solve for unknown values of right triangles. Start by having a student share something from his or her list. As each student shares, make sure their communication is specific. Examples may include:

- "When given two sides, you can find the third side by using the Pythagorean theorem."
- "When given one side and the angle opposite, you can find the hypotenuse (or another side) by using the trig ratio $\sin\theta = \frac{opposite}{hypotenuse}$ "

Let several students provide statements related to 'given one side and an angle, you can use a trigonometric ratio to solve for an unknown side. Have them articulate this several times (you may even wish to have them say these to their partners).

- "When given two sides, you can find one of the acute angles by using a trigonometric ratio, then using the related inverse trig ratio to solve for the unknown angle.
- "When given one acute angle, you can find the other acute angle by using your knowledge that the two acute angles are complementary

After several items are on the list, have students record these notes in their journals in an organized fashion so they can be referred to in the next task where they will become more comfortable using trigonometric ratios to solve for unknown values in a right triangle.

Explore, Part 3 (Small Group):

Have students move on to part 3 and answer questions 7 and 8. Press students to model or draw the situation and justify their reasoning. At this time, it is not as important that students are 'correct' and realize that Andrea is right and that the angle of depression is 23.5° as much as it is important that they are thinking and reasoning about their answer. This understanding will be solidified in the whole group discussion as the relationships and definitions of angle of elevation and angle of depression are discussed. Before moving to the whole group discussion, select one or two groups who have a good justification for why Andrea is correct to share out as well as one group who thinks Bonita is correct, if available.

Discuss, Part 3 (Whole Class):

For the whole group discussion, you will most likely have students who think Andrea is correct and perhaps some students who think Bonita is correct. If you noticed the class is split when you monitored, you may wish to bring this to the whole group's attention. This can help those who will present feel more at ease knowing they are not alone in their thinking.

First, select a group who chose Bonita's answer (if available) to show their model and explain their reasoning. (If no one was thinking this way, ask students to suggest what Bonita might have been thinking when she made her claim.) This discussion should highlight the issue that they are confusing the complementary angle with the angle of depression. [Note: If you have a safe environment in your classroom, one where it is alright for students to adjust their reasoning as they learn, starting with a misconception is a good way to help all students have a better understanding of the common misconception, in this case regarding the angle of depression. If your class is typically focused on only sharing ideas when students are certain they are correct, you may wish to not have a group share who thought Bonita was correct.]

Second, have a group show their model and explain their reasoning for agreeing with Andrea's statement. Focus on the model presented and the definition of angle of depression. Once all students agree that Andrea was correct and understand angle of depression, select a group to present who not only showed that Andrea was correct, but that the angle of depression is always equal to the angle of elevation using an informal proof, based on alternate interior angles formed when a transversal cuts two parallel lines. There is a level of abstraction here, since the parallel lines representing horizontal lines of sight, and the transversal representing the actual line of sight only exist in one's mental representation of the situation. If no one can produce an argument, challenge the class to do this along with the RSG homework.

Aligned Ready, Set, Go: Similarity & Right Triangle Trigonometry 6.10

READY, SET, GO! Name Period Date

READY

Topic: Modeling real world problems with triangles.

For each story presented below *sketch a picture* of the situation *and label as much of the picture as possible.* No need to answer the question or find the missing values, simply represent the situation with a sketch.

1. Jill put a ladder up against the house to try and reach a light that is out and needs to be changed. She knows the ladder is 10 feet long and the distance from the base of the house to the bottom of the ladder is 4 feet.

2. Francis is a pilot of an airplane that if flying at an altitude of 3,000 feet when the plane begins its descent toward the ground. If the angle of decent of the plane is 15^0 how much farther will the plane fly before it is on the ground?

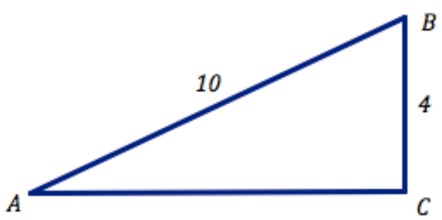
3. Abby is standing at the top of a very tall skyscraper and looking through a telescope at the scenery all around her. The angle of decline on the telescope says 35^0 and Abby knows she is 30 floors up and each floor is 15 feet tall. How far from the base of the building is the object that Abby is looking at?

SET

Topic: Solving triangles using Trigonometric Ratios

In each triangle find the missing angles and sides.

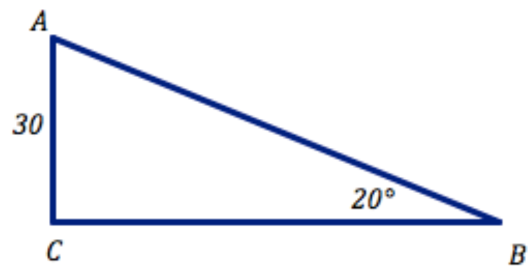
4.



$m\angle A =$ $m\angle B =$

$m\angle C = 90^o$ $AC =$

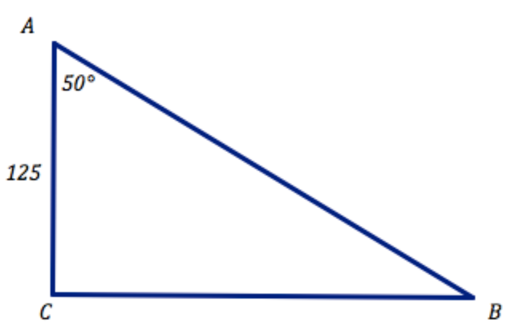
5.



$m\angle A =$ $AB =$

$m\angle C = 90^o$ $BC =$

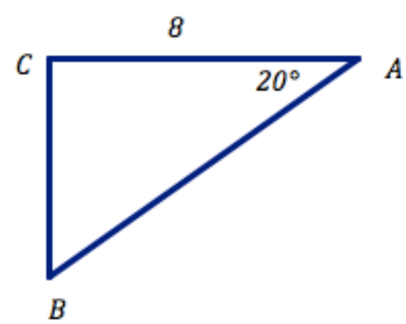
6.



$AB =$ $m\angle B =$

$m\angle C = 90^o$ $BC =$

7.



$AB =$ $m\angle B =$

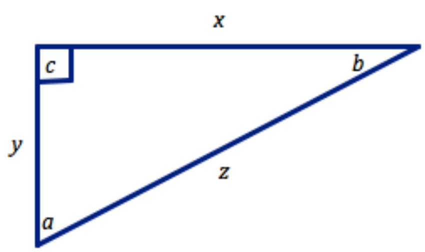
$m\angle C = 90^o$ $BC =$

GO

Topic: Trigonometric ratios

Use the given right triangle to identify the trigonometric ratios. And angles were possible.

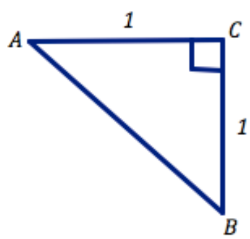
8.



$\sin(a) =$ $\cos(a) =$ $\tan(a) =$

$\sin(b) =$ $\cos(b) =$ $\tan(b) =$

9.

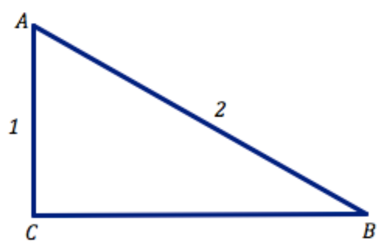


$\sin(A) =$ $\cos(A) =$ $\tan(A) =$

$\sin(B) =$ $\cos(B) =$ $\tan(B) =$

m∠A= m∠B=

10.



$\sin(A) =$ $\cos(A) =$ $\tan(A) =$

$\sin(B) =$ $\cos(B) =$ $\tan(B) =$

m∠A= m∠B= m∠C= 90°

6. 11 Solving Right Triangles Using Trigonometric Relationships

A Practice Understanding Task

I. For each problem:

- ***make a drawing***
- ***write an equation***
- ***solve (do not forget to include units of measure)***

1. Carrie places a 10-foot ladder against a wall. If the ladder makes an angle of 65° with the level ground, how far up the wall is the top of the ladder?

2. A flagpole casts a shadow that is 15 feet long. The angle of elevation of the sun at this time is 40°. How tall is the flagpole?

3. In southern California, there is a six-mile section of Interstate 5 that increases 2,500 feet in elevation. What is the angle of elevation?

4. A hot air balloon is 100 feet straight above where it is planning to land. Sarah is driving to meet the balloon when it lands. If the angle of elevation to the balloon is 35°, how far away is Sarah from place on the ground where the balloon will land?

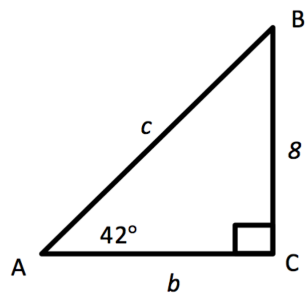
5. An airplane is descending as it approaches the airport. If the angle of depression from the plane to the ground is 7°, and the plane is 2,000 feet above the ground, what is the distance from the plane to the airport?

6. Michelle is 60 feet away from a building. The angle of elevation to the top of the building is 41°. How tall is the building?

7. A ramp is used for loading equipment from a dock to a ship. The ramp is 10 feet long and the ship is 6 feet higher than the dock. What is the angle of elevation of the ramp?

II. For each right triangle below, find all unknown side lengths and angle measures:

8.



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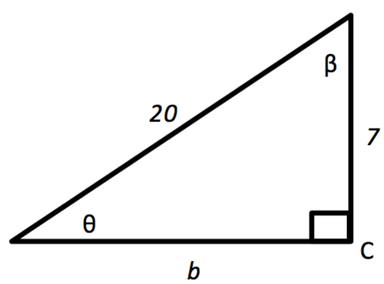
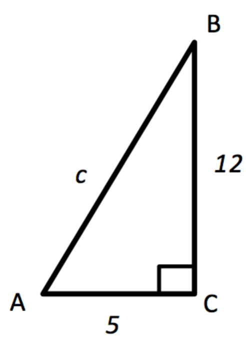
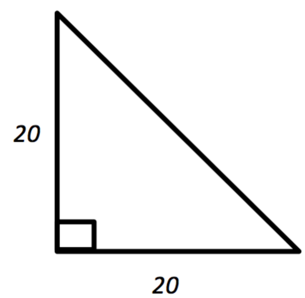


10.



11.



12. Draw and find the missing angle measures of the right triangle whose sides measure 4, 6, and 8.

III. Determine the values of the two remaining trigonometric ratios when given one of the trigonometric ratios.

13. $\cos(\alpha) = \frac{3}{5}$

14. $\tan(\theta) = \frac{8}{3}$

15. $\sin(\beta) = \frac{4}{7}$

6. 11 Solving Right Triangles Using Trigonometric Relationships – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is for students to practice setting up and solving right triangles for unknown sides and unknown angles. Students will use the Pythagorean Identity as well as trigonometric ratios to set up and solve equations.

Core Standards Focus:

G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

F.TF.8 Prove the Pythagorean identity $\sin^2(A) + cos^2(A) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.

The Teaching Cycle:

Launch (Whole Class):

Explain to students that this task provides an opportunity to practice solving problems using their knowledge of trigonometric ratios. Read the directions and have students solve problems in their journal. Suggest that students work independently for a few minutes, and then compare their work with a partner.

Explore (Small Group):

As you monitor, look for student understanding of setting up and solving right triangle trigonometry application problems. For part I, pay attention to the drawings students produce, and listen for how students are attending to precision (units, calculated vs. measured lengths, etc.). For parts II and III, assist students, provide feedback, and redirect them in areas where they are not firm in their understanding of right triangle trigonometry.

Discuss (Whole Class):
Choose questions to go over to make sure students comprehend all aspects of solving application problems involving right triangle trigonometry. Also, choose questions that highlight the trigonometric ratios of similar right triangles. Be sure students understand that the ratios in part III do not imply that we know the actual lengths of the triangles.

Aligned Ready, Set, Go: Similarity & Right Triangle Trigonometry 6.11

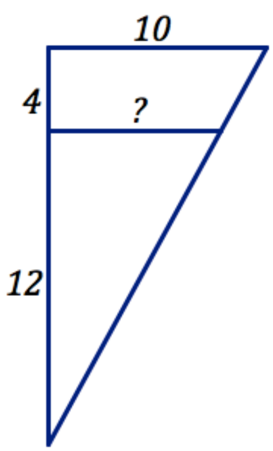
READY, SET, GO! Name Period Date

READY

Topic: Similar triangles and proportional relationships with parallel lines

Based on each set of triangles or parallel lines create a proportion and solve it to find the missing values.

1.



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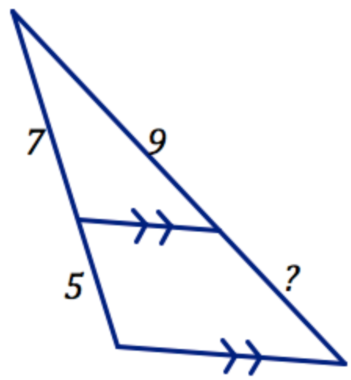

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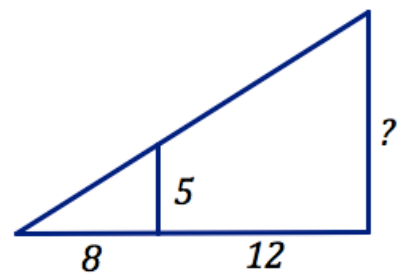

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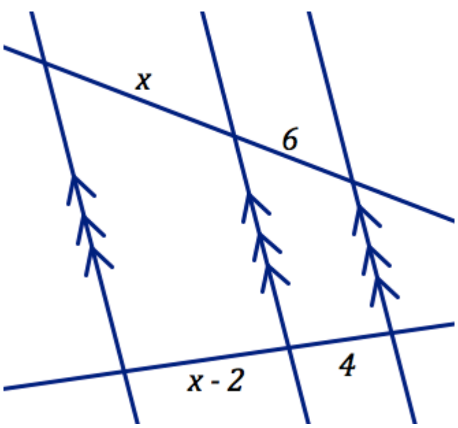

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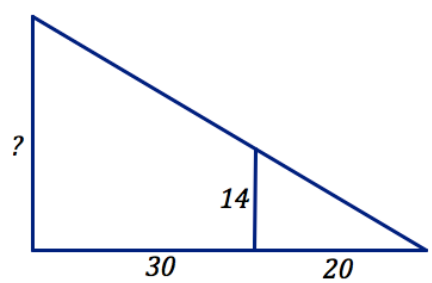

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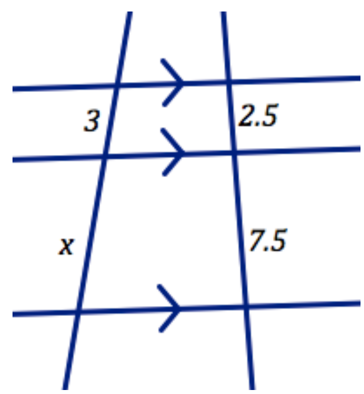
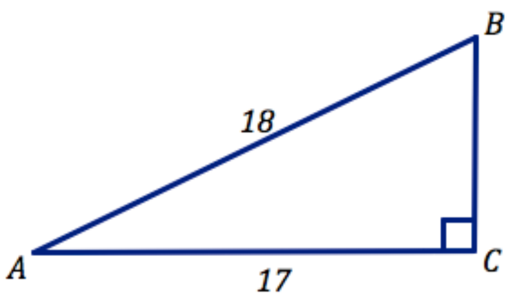


SET

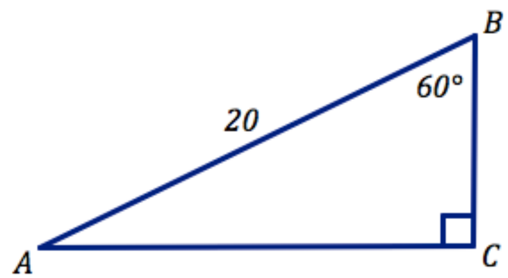
Topic: Solving triangles with trigonometric ratios and Pythagorean theorem

Solve each right triangle. Give any missing sides and missing angles.

7.



8.



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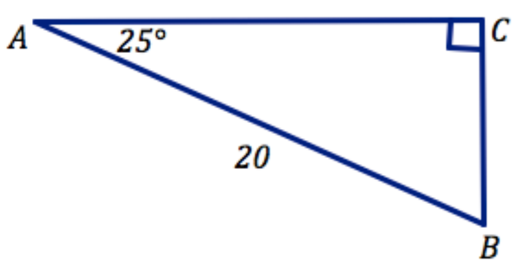

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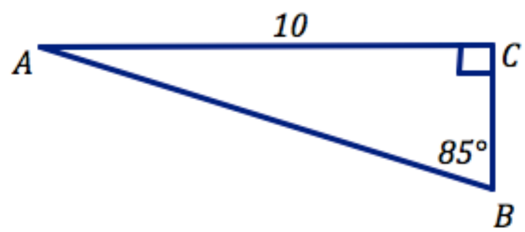

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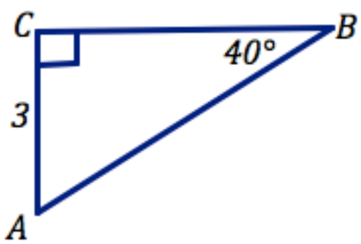
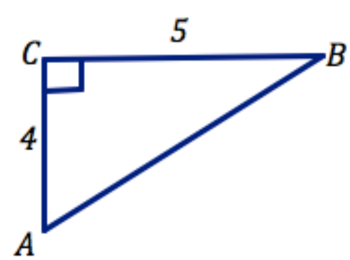


12.



Use the given trigonometric ratio to sketch a right triangle and solve the triangle.

13. $\sin(A) = \frac{1}{2}$

14. $\cos(B) = \frac{3}{5}$

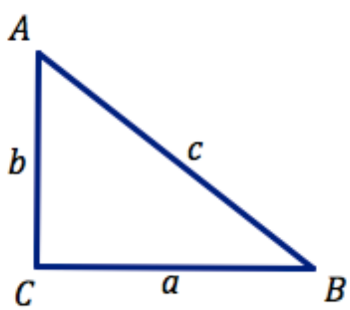
15. $\tan(B) = \frac{6}{7}$

16. $\sin(B) = \frac{7}{10}$

17. $\cos(A) = \frac{5}{8}$

18. $\tan(A) = \frac{4}{15}$

Use the right triangle below to determine which of the following are equivalent.



19. $\sin(A)$
20. $\cos(A)$
21. $\tan(A)$
22. $\sin(B)$
23. $\cos(B)$
24. $\tan(B)$
25. $\frac{\sin(A)}{\cos(A)}$
26. $\frac{1}{\tan(A)}$
27. 1
28. $a^2 + b^2$
29. c^2
30. $sin^2(\text{B}) + cos^2(B)$

GO

Topic: Applying trigonometric ratios and identities to solve problems.

Solve each problem. Sketch a drawing of the situation.

31. Mark is building his son a pitcher's mound so he can practice for his upcoming baseball season in the back yard. Mark knows that the league requires an incline of 12° and an elevation of 8 inches in height. How long will the front of the pitcher's mound need to be?

32. Susan is designing a wheelchair ramp. Wheelchair ramps require a slope that is no more than 1-inch of rise for every 12-inches of ramp length. Susan wants to determine how much horizontal distance a ramp of 6-feet in length will span? She also wants to know the degree of incline from the base of the ramp to the ground.

33. Michael is designing a house with a roof pitch of 5. Roof pitch is the number of inches that a roof will rise for every 12 inches of run. What is the angle that will need to be used in building the trusses and supports for the roof? What is the angle of a roof with 5/12 pitch increase? At the peak of the roof what angle will there be when the front and the back of the roof come together?