mathematics vision project

Transforming Mathematics Education

SECONDARY MATH TWO

An Integrated Approach

Standard Teacher Notes

MODULE 8

Circles & Other Conics

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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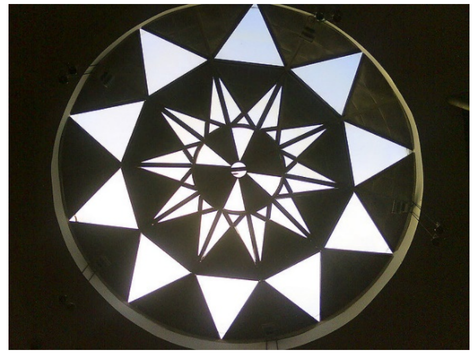
MODULE 8 - TABLE OF CONTENTS

CIRCLES AND OTHER CONICS

8.1 Circling Triangles (Or Triangulating Circles)



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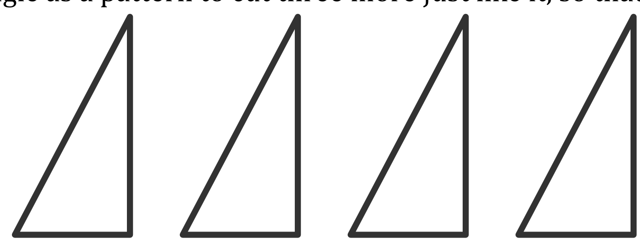
A Develop Understanding Task

Using the corner of a piece of colored paper and a ruler, cut a right triangle with a 6" hypotenuse, like so:

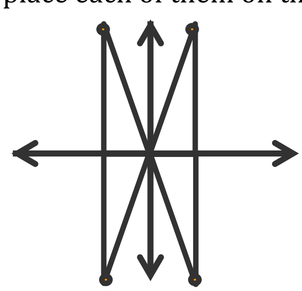
Use this triangle as a pattern to cut three more just like it, so that you have a total of four congruent triangles.



1. Choose one of the legs of the first triangle and label it *x* and label the other leg *y*. What is the relationship between the three sides of the triangle?

2. When you are told to do so, take your triangles up to the board and place each of them on the coordinate axis like this:

   Mark the point at the end of each hypotenuse with a pin.



3. What shape is formed by the pins after the class has posted all of their triangles? Why would this construction create this shape?

4. What are the coordinates of the pin that you placed in:
   a. the first quadrant?
   b. the second quadrant?
   c. the third quadrant?
   d. the fourth quadrant?

5. Now that the triangles have been placed on the coordinate plane, some of your triangles have sides that are of length $-x$ or $-y$. Is the relationship $x^2 + y^2 = 6^2$ still true for these triangles? Why or why not?

6. What would be the equation of the graph that is the set on all points that are 6" away from the origin?

7. Is the point (0, -6) on the graph? How about the point (3, 5.193)? How can you tell?

8. If the graph is translated 3 units to the right and 2 units up, what would be the equation of the new graph? Explain how you found the equation.

8.1 Circling Triangles (Or Triangulating Circles) – Teacher Notes

A Develop Understanding Task

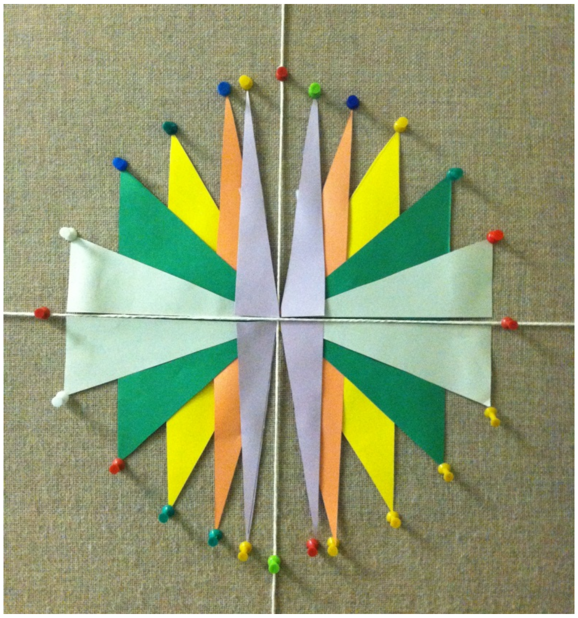
Purpose: This purpose of this task is for students to connect their geometric understanding of circles as the set of all points equidistant from a center to the equation of a circle. In the task, students construct a circle using right triangles with a radius of 6 inches. This construction is intended to focus students on the Pythagorean Theorem and to use it to generate the equation of a circle centered at the origin. After constructing a circle at the origin, students are asked to use their knowledge of translations to consider how the equation would change if the center of the circle is translated.

Core Standards Focus:
G-GPE Expressing Geometric Properties with Equations. Translate between the geometric description and the equation for a conic section.
G-GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

The Teaching Cycle:

Launch (Whole Class):
Be prepared for the class activity by having at least two sheets of colored paper (heavy paper is better), rulers and scissors for students to use. On a board in the classroom, create the coordinate axes with strings or tape. Prepare a way for students to mark the endpoint of their triangles with a tack or some other visible mark so that the circle that will be constructed is visible. Depending on the size of your class, you may choose to have several axes set up and divide students into groups. Ask students to follow the instructions on the first page and post their triangles. Encourage some students to select the longest leg of the triangle to be x and others to select the shortest leg to be x

so that there are as many different points on the circle formed as possible. Watch as students post their triangles to see that they get all four of them into the proper positions. An example of what the board will look like when the triangles are posted is:



Tell students to work on problem #3 as other students finish posting their triangles. When all students are finished, ask students why the shape formed is a circle. They should be able to relate the idea that since each triangle had a hypotenuse of 6", they formed a circle that has a radius of 6". Use a 6" string to demonstrate how the radius sweeps around the circle, touching the endpoint of each hypotenuse.

Explore (Small Group):

Ask students to work on the remaining questions. Monitor student work to support their thinking about the Pythagorean Theorem using x and y as the lengths of the legs of any of the right triangles used to form the circle. Question #5 may bring about confusion about the difference between $(-x)^2$ and $-x^2$. Remind students that in this case, x is a positive number, so $-x$ is a negative number, and the square of a negative number is positive.

Discuss (Whole Class):

Begin the discussion with #6. Ask students for their equation and how their equation represents all the points on the circle. Press for students to explain how the equation works for points that lie in quadrants II, III, and IV.

Turn the discussion to #7. Ask how they decided if the points were on the circle. Some students may have tried measuring or estimating, so be sure that the use of the equation is demonstrated. After discussing the point (3, 5.193), ask students what could be said about (3, -5.193) or (-3, -5.193) to highlight the symmetries and how they come up in the equation.

Finally, discuss the last question. Students should have various explanations for the change in the equation. Some may use the patterns they have observed in shifting functions, although it should be noted that this graph is not a function. Other students may be able to articulate the idea that $x - 3$ represents the length of the horizontal side of the triangle that was originally length x, now that it has been moved three units to the right.

Aligned Ready, Set, Go: ***Circles and Other Conics 8.1***

READY, SET, GO! Name Period Date

READY

Topic: Factoring special products

Factor the following as the difference of 2 squares or as a perfect square trinomial. Do not factor if they are neither.

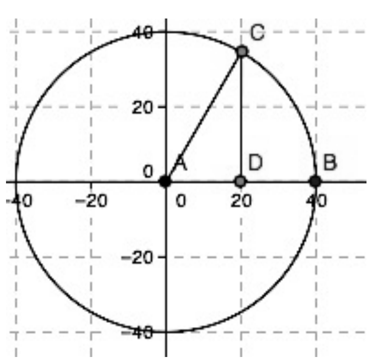
1. $b^2 - 49$
2. $b^2 - 2b + 1$
3. $b^2 + 10b + 25$
4. $x^2 - y^2$
5. $x^2 - 2xy + y^2$
6. $25x^2 - 49y^2$
7. $36x^2 + 60xy + 25y^2$
8. $81a^2 - 16d^2$
9. $144x^2 - 312xy + 169y^2$

SET

Topic: Writing the equations of circles

Write the equation of each circle centered at the origin.

10.



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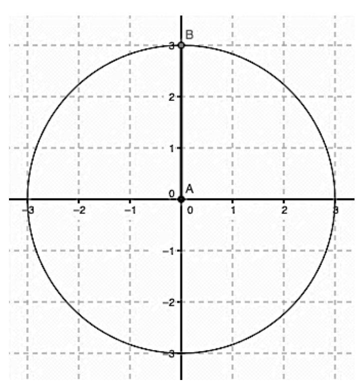

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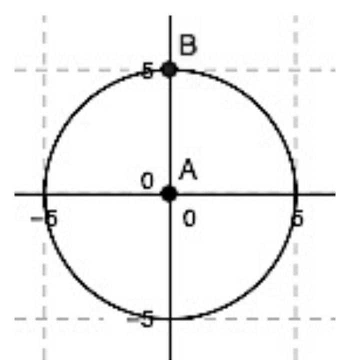

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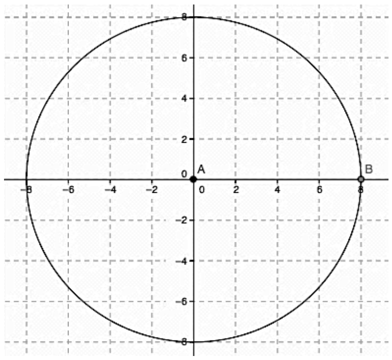

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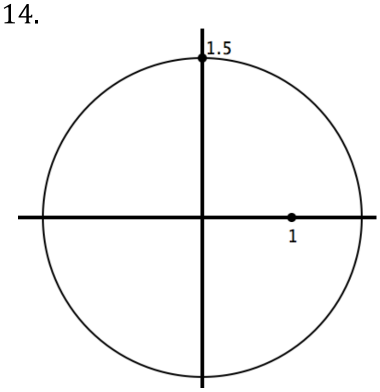

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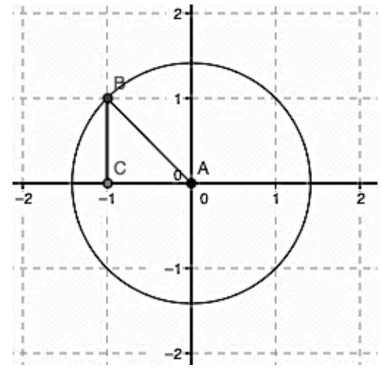


GO

Topic: Verifying Pythagorean triples

Identify which sets of numbers could be the sides of a right triangle. Show your work.

16. $\{9, 12, 15\}$

17. $\{9, 10, \sqrt{19}\}$

18. $\{1, \sqrt{3}, 2\}$

19. $\{2, 4, 6\}$

20. $\{\sqrt{3}, 4, 5\}$

21. $\{10, 24, 26\}$

22. $\{\sqrt{2}, \sqrt{7}, 3\}$

23. $\{2\sqrt{2}, 5\sqrt{3}, 9\}$

24. $\{4ab^3\sqrt{10}, 6ab^3, 14ab^3\}$

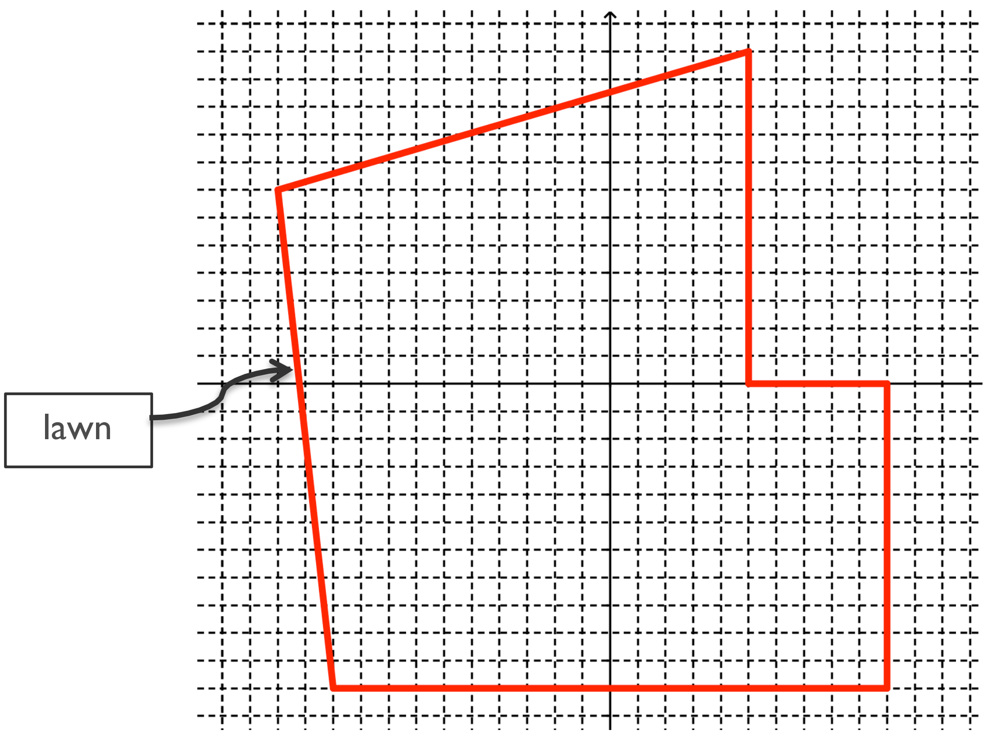
8.2 Getting Centered

A Solidify Understanding Task

Malik's family has decided to put in a new sprinkling system in their yard. Malik has volunteered to lay the system out. Sprinklers are available at the hardware store in the following sizes:

- Full circle, maximum 15' radius
- Half circle, maximum 15' radius
- Quarter circle, maximum 15' radius

All of the sprinklers can be adjusted so that they spray a smaller radius. Malik needs to be sure that the entire yard gets watered, which he knows will require that some of the circular water patterns will overlap. He gets out a piece of graph paper and begins with a scale diagram of the yard. In this diagram, the length of the side of each square represents 5 feet.



1. As he begins to think about locating sprinklers on the lawn, his parents tell him to try to cover the whole lawn with the fewest number of sprinklers possible so that they can save some money. The equation of the first circle that Malik draws to represent the area watered by the sprinkler is:

$$(x + 25)^2 + (y + 20)^2 = 225$$

   Draw this circle on the diagram using a compass.

2. Lay out a possible configuration for the sprinkling system that includes the first sprinkler pattern that you drew in #1.

3. Find the equation of each of the full circles that you have drawn.

Malik wrote the equation of one of the circles and just because he likes messing with the algebra, he did this:

Original equation: $(x - 3)^2 + (y + 2)^2 = 225$

$$x^2 - 6x + 9 + y^2 + 4y + 4 = 225$$

$$x^2 + y^2 - 6x + 4y - 212 = 0$$

Malik thought, "That's pretty cool. It's like a different form of the equation. I guess that there could be different forms of the equation of a circle like there are different forms of the equation of a parabola or the equation of a line." He showed his equation to his sister, Sapana, and she thought he was nuts. Sapana, said, "That's a crazy equation. I can't even tell where the center is or the length of the radius anymore." Malik said, "Now it's like a puzzle for you. I'll give you an equation in the new form. I'll bet you can't figure out where the center is."

Sapana said, "Of course, I can. I'll just do the same thing you did, but work backwards."

4. Malik gave Sapana this equation of a circle:

$$x^2 + y^2 - 4x + 10y + 20 = 0$$

Help Sapana find the center and the length of the radius of the circle.

5. Sapana said, "Ok. I made one for you. What's the center and length of the radius for this circle?"

$$x^2 + y^2 + 6x - 14y - 42 = 0$$

6. Sapana said, "I still don't know why this form of the equation might be useful. When we had different forms for other equations like lines and parabolas, each of the various forms highlighted different features of the relationship." Why might this form of the equation of a circle be useful?

$$x^2 + y^2 + Ax + By + C = 0$$

8.2 Getting Centered – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to solidify understanding of the equation of the circle. The task begins with sketching circles and writing their equations. It proceeds with the idea of squaring the $(x - h)^2$ and $(y - k)^2$ expressions to obtain a new form of an equation. Students are then challenged to reverse the process to find the center of the circle.

Core Standards Focus:

G-GPE Expressing Geometric Properties with Equations. Translate between the geometric description and the equation for a conic section.

G-GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

The Teaching Cycle:

Launch (Whole Class):

Begin the task by helping students to understand the context of developing a diagram for a sprinkling system. The task begins with students drawing circles to cover the yard and writing equations for the circles that they have sketched. Allow students some time to work to make their diagrams and write their equations. However, don't spend too much time trying to completely cover the lawn. The point is to draw four or more circles and to write their equations. As students are working, be sure that they are accounting for the scale as they name the center of their circles. Ask several students to share their equations. After each student shares, ask the class to identify the center and radius of the equation. After several students have shared, ask one student to take the last equation shared and square the $(x - h)^2$ and $(y - k)^2$ expressions and simplify the remaining equation. Tell students that this is what Malik did and now their job is to take the equation back to the form in which they can easily read the center and radius.

Explore (Small Group):

Since students have previously completed the square for parabolas, some students will think to apply the same process here. Monitor their work, watching for groups that have different answers for the same equation (hopefully, one of them is correct).

Discuss (Whole Class):

Begin the discussion by posting two different equations that answer question #4. Ask students how they can decide which equation is correct. They may suggest working backwards to the original equation, or possibly checking a point. Decide which equation is correct and ask that group to describe the process they used to get the answer. Ask another group that has a correct version of #5 to show how they obtained their answer. You may also wish to discuss #6. Wrap up the lesson up by working with the class to create a set of steps that they can follow to get the equation back to center/radius form.

Aligned Ready, Set, Go: *Circles and Other Conics 8.2*

READY, SET, GO! Name Period Date

READY

Topic: Making perfect square trinomials

Fill in the number that completes the square. Then write the trinomial in factored form.

1. $x^2 + 6x +$ ________
2. $x^2 - 14x +$ ________
3. $x^2 - 50x +$ ________
4. $x^2 - 28x +$ ________

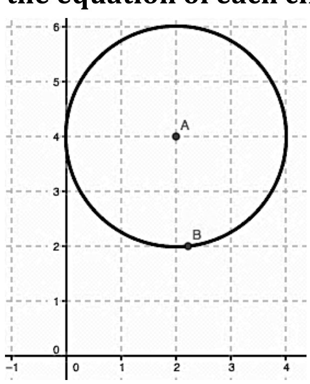
On the next set, leave the number that completes the square as a fraction. Then write the trinomial in factored form.

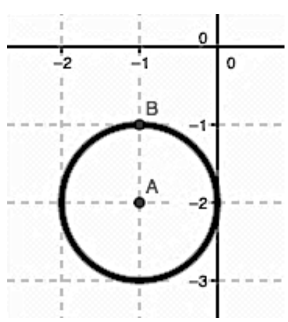
5. $x^2 - 11x +$ ________
6. $x^2 + 7x +$ ________
7. $x^2 + 15x +$ ________
8. $x^2 + \frac{2}{3}x +$ ________
9. $x^2 - \frac{1}{5}x +$ ________
10. $x^2 - \frac{3}{4}x +$ ________

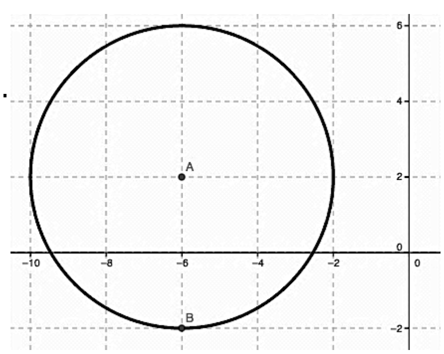
SET

Topic: Writing equations of circles with center (h, k) and radius *r*.

Write the equation of each circle.

11. 

12. 

13. 

Write the equation of the circle with the given center and radius. Then write it in expanded form.

14. Center: (5, 2) Radius: 13

15. Center: (-6, -10) Radius: 9

16. Center: (0, 8) Radius: 15

17. Center: (19, -13) Radius: 1

18. Center: (-1, 2) Radius: 10

19. Center: (-3, -4) Radius: 8

Go

Topic: Verifying if a point is a solution

Identify which point is a solution to the given equation. Show your work.

20. $y=\frac{4}{5}x-2$ a. (-15, -14) b. (10, 10)

21. $y=3|x|$ a. (-4, -12) b. $(-\sqrt{5}, 3\sqrt{5})$

22. $y=x^2+8$ a. $(\sqrt{7}, 15)$ b. $(\sqrt{7}, -1)$

23. $y=-4x^2+120$ a. $(5\sqrt{3}, -180)$ b. $(5\sqrt{3}, 40)$

24. $x^2+y^2=9$ a. $(8,-1)$ b. $(-2, \sqrt{5})$

25. $4x^2-y^2=16$ a. $(-3, \sqrt{10})$ b. $(-2\sqrt{2}, 4)$

8.3 Circle Challenges

A Practice Understanding Task

Once Malik and Sapana started challenging each other with circle equations, they got a little more creative with their ideas. See if you can work out the challenges that they gave each other to solve. Be sure to justify all of your answers.

1. Malik's challenge:

   What is the equation of the circle with center (-13,-16) and containing the point (-10,-16) on the circle?

2. Sapana's challenge:

   The points (0, 5) and (0,-5) are the endpoints of the diameter of a circle. The point (3, y) is on the circle. What is a value for y?

3. Malik's challenge:

   Find the equation of a circle with center in the first quadrant and is tangent to the lines $x = 8$, $y = 3$, and $x = 14$.

4. Sapana's challenge:
   The points (4,-1) and (-6,7) are the endpoints of the diameter of a circle. What is the equation of the circle?

5. Malik's challenge:
   Is the point (5,1) inside, outside, or on the circle $x^2 - 6x + y^2 + 8y = 24$? How do you know?

6. Sapana's challenge:
   The circle defined by $(x-1)^2 + (y+4)^2 = 16$ is translated 5 units to the left and 2 units down. Write the equation of the resulting circle.

7. Malik's challenge:

   There are two circles, the first with center (3,3) and radius r_1, and the second with center (3, 1) and radius r_2.

   a. Find values r_1 and r_2 so that the first circle is completely enclosed by the second circle.

   b. Find one value of r_1 and one value of r_2 so that the two circles intersect at two points.

   c. Find one value of r_1 and one value of r_2 so that the two circles intersect at exactly one point.

8.3 Circle Challenges – Teacher Notes

A Practice Understanding Task

Purpose:
The purpose of this task is for students to practice using the equation of the circle in different ways. In each case, they must draw inferences from the information given and use the information to find the equation of the circle or to justify conclusions about the circle. They will use the distance formula to find the measure of the radius and the midpoint formula to find the center of a circle.

Core Standards Focus:
G-GPE Expressing Geometric Properties with Equations
Translate between the geometric description and the equation for a conic section
G-GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

The Teaching Cycle:
Launch (Whole Class):

Begin the task by telling students that they will be solving the circle challenges by using the information given, ideas that they have learned in the past (like the distance and midpoint formulas), and their logic to write equations and justify conclusions about circles. It will probably be useful to have graph paper available to sketch the circles based on the information given.

Explore (Small Group):

Monitor students as they work, focusing on how they are making sense of the problems and using the information. Encourage students to draw the situation and visualize the circle to help when they are stuck. Insist upon justification, asking, "How do you know?"

Discuss (Whole Class):

Select problems that were challenging for the class or highlighted important ideas or useful strategies. Problem #4 is recommended for this purpose, but it is also important to select the problems that have generated interest in the class.

Aligned Ready, Set, Go: ***Circles and Other Conics 8.3***

READY, SET, GO! Name Period Date

READY

Topic: Finding the distance between two points

Simplify. Use the distance formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ **to find the distance between the given points. Leave your answer in simplest radical form.**

1. $A(18,-12)\ B(10,4)$
2. $G(-11,-9)\ H(-3,7)$
3. $J(14,-20)\ K(5,5)$
4. $M(1,3)\ P(-2,7)$
5. $Q(8,2)\ R(3,7)$
6. $S\left(-11,2\sqrt{2}\right)\ T\left(-5,-4\sqrt{2}\right)$
7. $W\left(-12,-2\sqrt{2}\right)\ Z\left(-7,-3\sqrt{2}\right)$

SET

Topic: Writing equations of circles

Use the information provided to write the equation of the circle in standard form,

$$(\boldsymbol{x}-\boldsymbol{h})^2+(\boldsymbol{y}-\boldsymbol{h})^2=\boldsymbol{r}^2$$

8. Center $(-16,-5)$ and the circumference is 22π
9. Center $(13,-27)$ and the area is 196π
10. Diameter measures 15 units and the center is at the intersection of y = x + 7 and y = 2x – 5
11. Lies in quadrant 2 Tangent to $x=-12\ \mathit{and}\ x=-4$

12. Center (-14 , 9) Point on circle (1 , 11)

13. Center lies on the y axis Tangent to y = -2 and y = -17

14. Three points on the circle are $(-8,5),(3,-6),(14,5)$

15. I know three points on the circle are (-7,6), (9,6), and (-4,13). I think that the equation of the circle is $(x-1)^2+(y-6)^2=64$. Is this the correct equation for the circle? Justify your answer.

GO

Topic: Finding the value of ***B*** in a quadratic in the form of Ax^2+Bx+C in order to create a perfect square trinomial.

Find the value of *B* that will make a perfect square trinomial. Then write the trinomial in factored form.

16. $x^2+\_\_\_\_\_x+36$

17. $x^2+\_\_\_\_\_x+100$

18. $x^2+\_\_\_\_\_x+225$

19. $9x^2+\_\_\_\_\_x+225$

20. $16x^2+\_\_\_\_\_x+169$

21. $x^2+\_\_\_\_\_x+5$

22. $x^2+\_\_\_\_\_x+\frac{25}{4}$

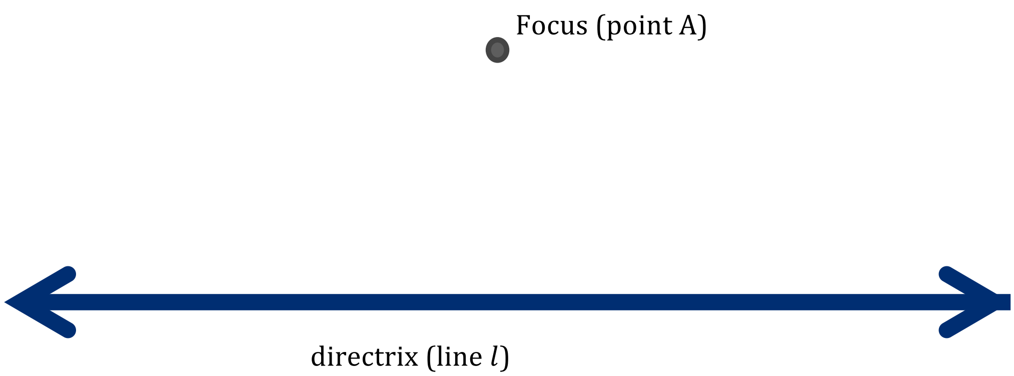
23. $x^2+\_\_\_\_\_x+\frac{9}{4}$

24. $x^2+\_\_\_\_\_x+\frac{49}{4}$

8.4 Directing Our Focus

A Develop Understanding Task

On a board in your classroom, your teacher has set up a point and a line like this:

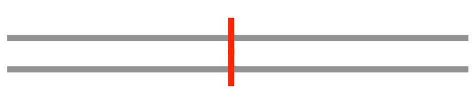


We're going to call the line a directrix and the point a focus. They've been labeled on the drawing.

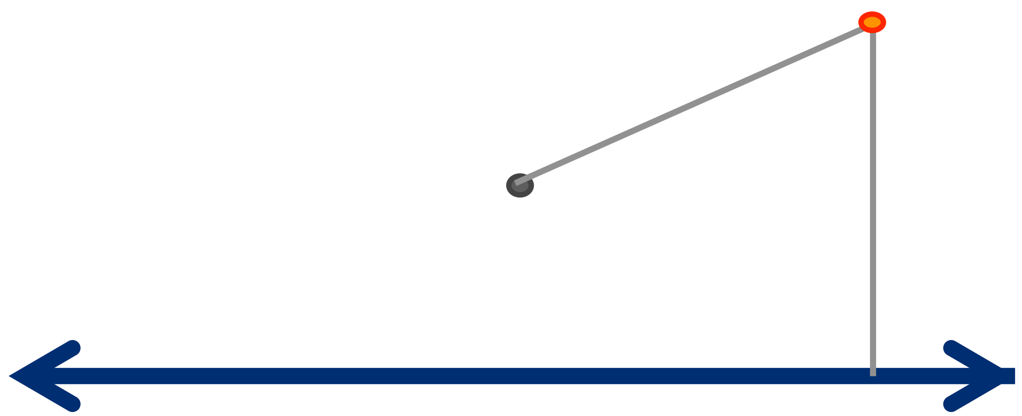
Similar to the circles task, the class is going to construct a geometric figure using the focus (point A) and directrix (line l).

1. Cut two pieces of string with the same length.

2. Mark the midpoint of each piece of string with a marker.



3. Position the string on the board so that the midpoint is equidistant from the focus (point A) and the directrix (line l), which means that it must be perpendicular to the directrix. While holding the string in this position, put a pin through the midpoint. Depending on the size of your string, it will look something like this:



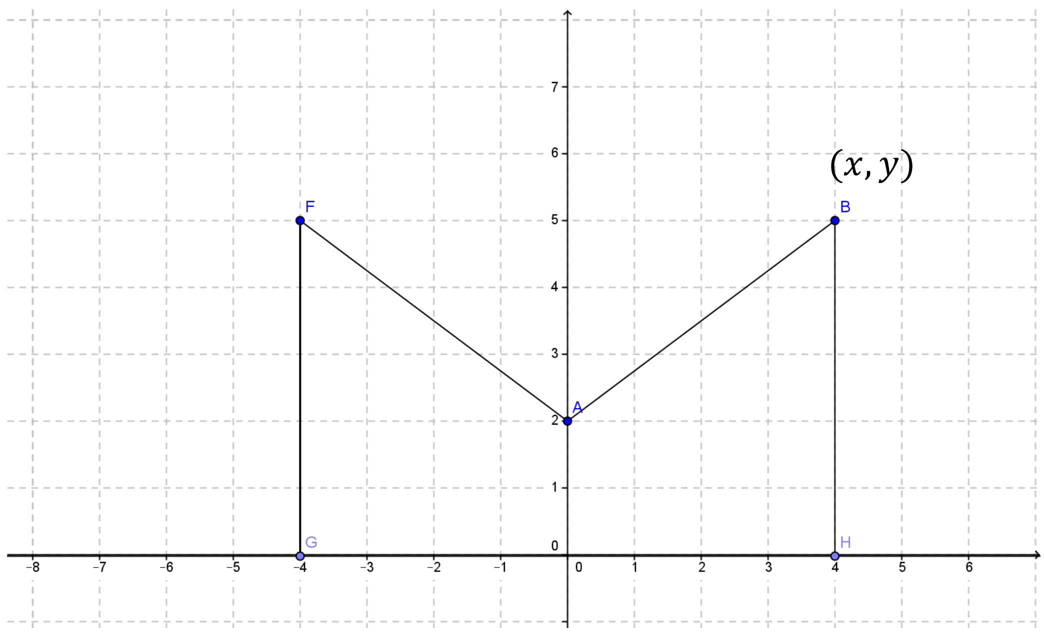
4. Using your second string, use the same procedure to post a pin on the other side of the focus.

5. As your classmates post their strings, what geometric figure do you predict will be made by the tacks (the collection of all points like (x, y) show in the figure above)? Why?

6. Where is the vertex of the figure located? How do you know?

7. Where is the line of symmetry located? How do you know?

8. Consider the following construction with focus point A and the x-axis as the directrix. Use a ruler to complete the construction of the parabola in the same way that the class constructed the parabola with string.



9. You have just constructed a parabola based upon the definition: A parabola is the set of all points (x, y) equidistant from a line l (the directrix) and a point not on the line (the focus). Use this definition to write the equation of the parabola above, using the point (x, y) to represent any point on the parabola.

10. How would the parabola change if the focus was moved up, away from the directrix?

11. How would the parabola change if the focus were to be moved down, toward the directrix?

12. How would the parabola change if the focus were to be moved down, below the directrix?

8.4 Directing Our Focus – Teacher Notes

A Develop Understanding Task

Purpose:

The purpose of this task is to develop the definition of a parabola as the set of all points equidistant from a given point (the focus) and a line (the directrix). Only those parabolas with horizontal directrices are considered in this task. Students develop an equation for a parabola based on the definition, using the distance formula. Students are also asked to consider the relationship between the focus and directrix and how the parabola changes as they are moved in relation to each other.

Core Standards Focus:

G.GPE Expressing Geometric Properties with Equations
Translate between the geometric description and the equation for a conic section

G.GPE.2. Derive the equation of a parabola given a focus and directrix.
Note: *Connect the equations of circles and parabolas to prior work with quadratic equations. The directrix should be parallel to a coordinate axis.*

The Teaching Cycle:

Launch (Whole Class):

Be prepared for the class activity by having scissors, markers, rulers, and string for students to use. Have a large corkboard with focus and directrix set up for students to use, as pictured in the task.

Lead the class in following the directions for cutting and marking the strings and then posting them on the board. Before anyone posts a string, ask students what shape they think will be made and why. Watch as students post their strings to be sure that they are perpendicular to the directrix and pulled tight both directions so that they look like the illustration.

After students have identified that the figure formed is a parabola, have them work individually on completing the diagram in #8. When completed, ask how they find the vertex point on a parabola? Be sure that the discussion includes the fact that the vertex will be the point on the line of symmetry that is the midpoint of the segment between the focus and the directrix. How is the vertex like other points on the parabola? (It is equidistant from the focus and the directrix.) How is it different? (It's the only point of the parabola on the line of symmetry.) Direct the discussion to the line of symmetry. Where is it on the parabola they just made? How could it be found on any parabola, given the focus and directrix?

After their work on #8, explain the geometric definition of a parabola given in #9. Then have students work together to use the definition to write the equation of the parabola.

Explore (Small Group):

Monitor students as they work to be sure that they are using the point marked (x, y) to represent any point on the parabola, rather than naming it (4,5). If they have written the equation using (4,5) then ask them how they would change their initial equation to call the point (x, y) instead. After they have written their equation they may want to test it with the point (4, 5) since they know it is on the parabola. If students need help getting started, help them to focus on the distance between the (x, y) and the focus (0,2) and (x, y) and the directrix, $y = 0$. Ask how they could represent those distances algebraically.

Be sure that students have time to share their ideas about problems 10 -12 so that the class discussion of the relationship of the focus and the directrix is robust.

Discuss (Whole Class):

When students have finished their work on the equation, ask a group to present and explain their work. A possible version is below:

Distance from (x, y) to focus (0, 2) = distance from (x, y) to x-axis

$\sqrt{(x-0)^2+(y-2)^2}$	=	y	
$(x-0)^2+(y-2)^2$	=	y^2	Squaring both sides
x^2+y^2-4y+4	=	y^2	Simplifying

$x^2 - 4y + 4$	=	0	Simplifying
$x^2 + 4$	=	$4y$	Solving for y
$\frac{x^2}{4} + 1$	=	y	Solving for y

Ask students how this equation matches what they already know about the parabola they have drawn. Where is the vertex in the equation? How could they use the equation to predict how wide or narrow the parabola will be?

Turn the discussion to questions 10 -12. Ask various students to explain their answers. Use the parabola applet to test their conjectures about the effect of moving the focus in relation to the directrix.

Aligned Ready, Set, Go: ***Circles and Other Conics 8.4***

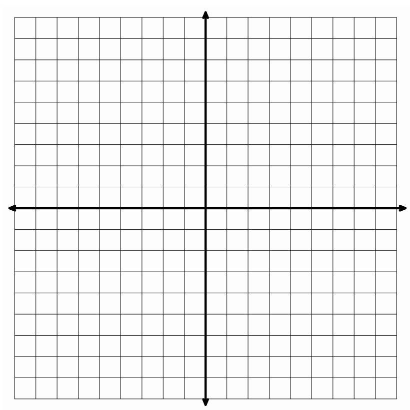
READY, SET, GO! Name Period Date

READY

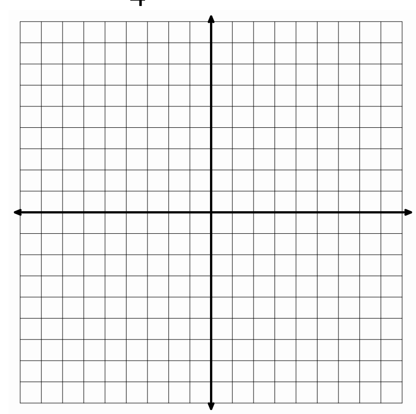
Topic: Graphing Quadratics

Graph each set of functions on the same coordinate axes. Describe in what way the graphs are the same and in what way they are different.

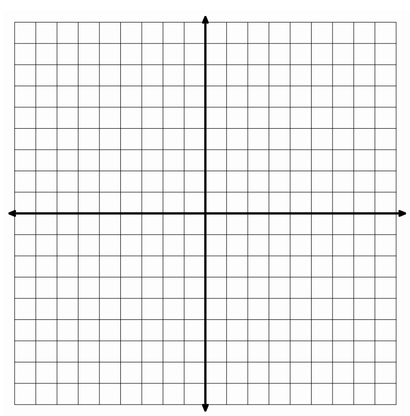
1. $y = x^2, y = 2x^2, y = 4x^2$



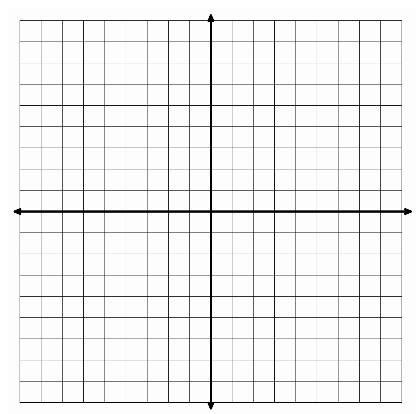
2. $y = \frac{1}{4}x^2, y = -x^2, y = -4x^2$



3. $y = \frac{1}{4}x^2, y = x^2 - 2, y = \frac{1}{4}x^2 - 2, y = 4x^2 - 2$



4. $y = x^2, y = -x^2, y = x^2 + 2, y = -x^2 + 2$



SET

Topic: Sketching a parabola using the conic definition.

Use the conic definition of a parabola to sketch a parabola defined by the given focus *F* and the equation of the directrix.

Begin by graphing the focus, the directrix, and point P_1. Use the distance formula to find FP_1 and find the vertical distance between P_1 and the directrix by identifying point H on the directrix and counting the distance. Locate the point P_2, (the point on the parabola that is a reflection of P_1 across the axis of symmetry.) Locate the vertex V. Since the vertex is a point on the parabola, it must lie equidistant between the focus and the directrix. Sketch the parabola. Hint: the parabola always "hugs" the focus.

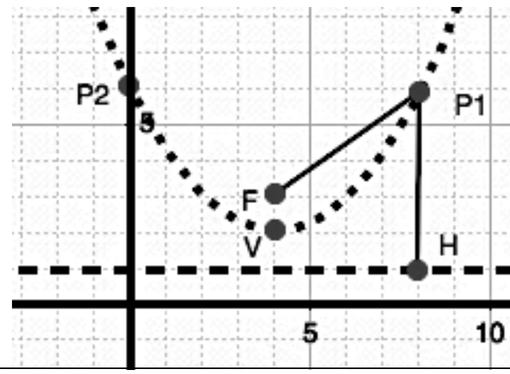
Example: ***F*** (4,3), $\boldsymbol{P_1}$ (8, 6), y = 1

$FP_1 = \sqrt{(4-8)^2+(3-6)^2} = \sqrt{16+9} = \sqrt{25} = 5$

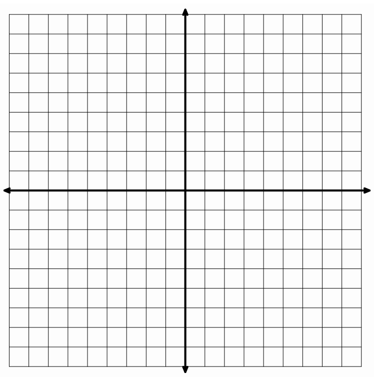
$P_1H = 5$

$\boldsymbol{P_2}$ is located at (0, 6)

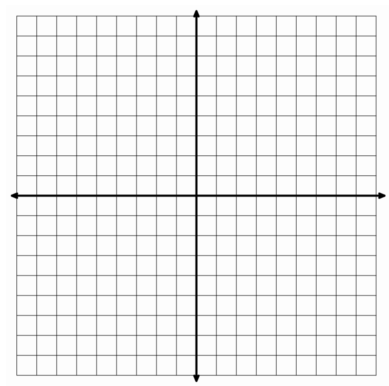
V is located at (4, 2)



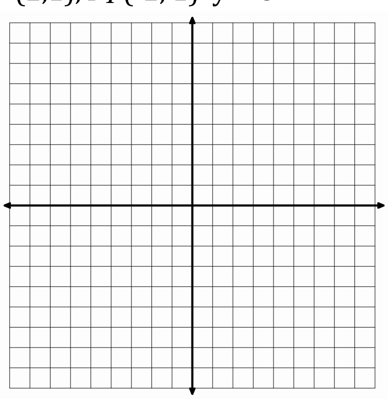
5. ***F*** (1,-1), $\boldsymbol{P_1}$ (3, -1) y = -3



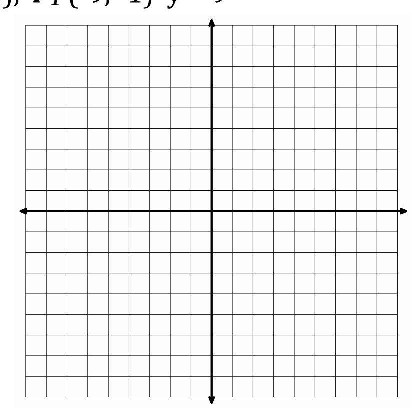
6. ***F*** (-5,3), $\boldsymbol{P_1}$ (-1, 3) y = 7



7. ***F*** (2,1), $\boldsymbol{P_1}$ (-2, 1) y =-3



8. ***F*** (1, -1), $\boldsymbol{P_1}$ (-9, -1) y = 9



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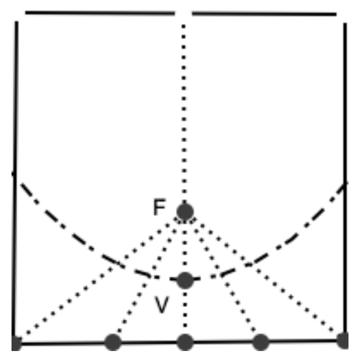

9. Find a square piece of paper (a post-it note will work). Fold the square in half vertically and put a dot anywhere on the fold. Let the edge of the paper be the directrix and the dot be the focus. Fold the edge of the paper (the directrix) up to the dot repeatedly from different points along the edge. The fold lines between the focus and the edge should make a parabola.

Experiment with a new paper and move the focus.
Use your experiments to answer the following questions.

10. How would the parabola change if the focus were moved up, away from the directrix?

11. How would the parabola change if the focus were moved down, toward the directrix?

12. How would the parabola change if the focus were moved down, below the directrix?

GO

Topic: Finding the center and radius of a circle.

Write each equation so that it shows the center (*h, k*) and radius *r* of the circle. This called the standard form of a circle. $(x-h)^2+(y-k)^2=r^2$

13. $x^2+y^2+4y-12=0$

14. $x^2+y^2-6x-3=0$

15. $x^2+y^2+8x+4y-5=0$

16. $x^2+y^2-6x-10y-2=0$

17. $x^2+y^2-6y-7=0$

18. $x^2+y^2-4x+8y+6=0$

19. $x^2+y^2-4x+6y-72=0$

20. $x^2+y^2+12x+6y-59=0$

21. $x^2+y^2-2x+10y+21=0$

22. $4x^2+4y^2+4x-4y-1=0$

8.5 Functioning With Parabolas

A Solidify Understanding Task

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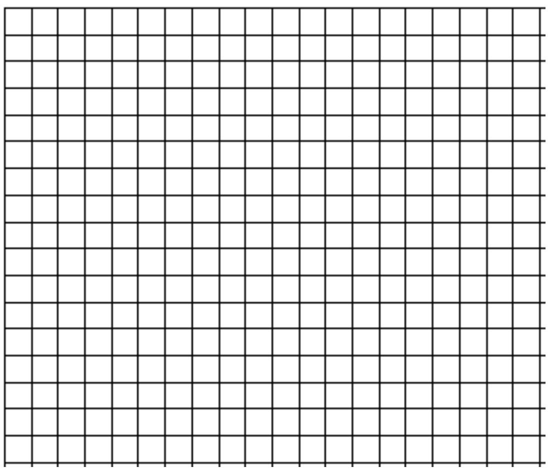
Sketch the graph of each parabola(accurately), find the vertex and use the geometric definition of a parabola to find the equation of these parabolas.

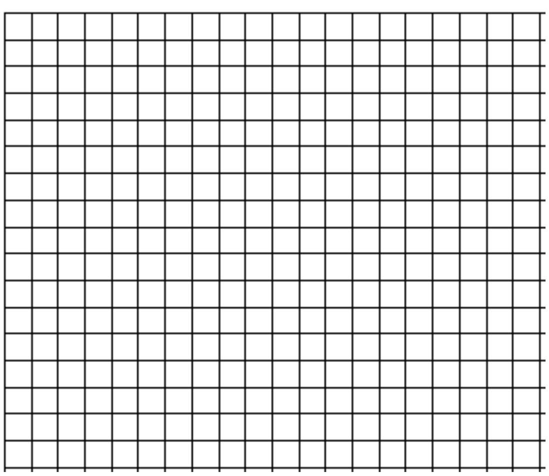
1. Directrix $y = -4$, Focus A(2, -2)



Vertex ________________

Equation:

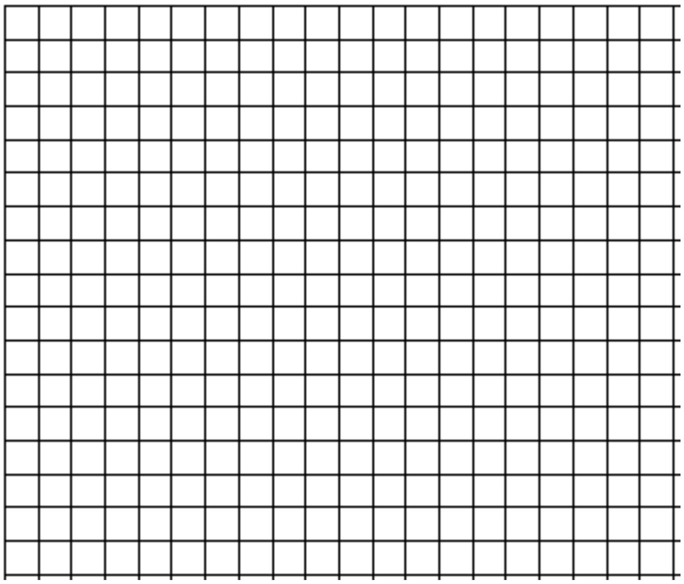
2. Directrix $y = 2$, Focus A(-1, 0)



Vertex ________________

Equation:

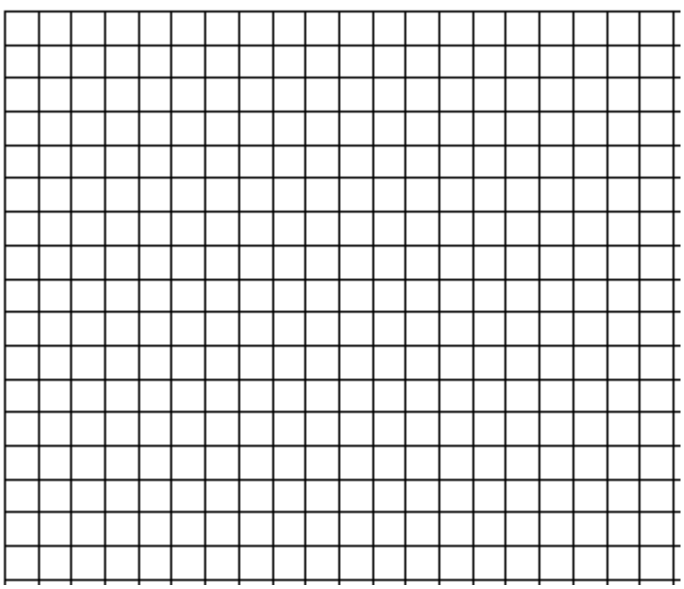
3. Directrix $y = 3$, Focus A$(1, 7)$



Vertex ________________

Equation:

4. Directrix $y = 3$, Focus A$(2, -1)$



Vertex ________________

Equation:

5. Given the focus and directrix, how can you find the vertex of the parabola?

6. Given the focus and directrix, how can you tell if the parabola opens up or down?

7. How do you see the distance between the focus and the vertex (or the vertex and the directrix) showing up in the equations that you have written?

8. Describe a pattern for writing the equation of a parabola given the focus and directrix.

9. Annika wonders why we are suddenly thinking about parabolas in a completely different way than when we did quadratic functions. She wonders how these different ways of thinking match up. For instance, when we talked about quadratic functions earlier we started with $y = x^2$. "Hmmmm. .... I wonder where the focus and directrix would be on this function," she thought. Help Annika find the focus and directrix for $y = x^2$.

10. Annika thinks, "Ok, I can see that you can find the focus and directrix for a quadratic function, but what about these new parabolas. Are they quadratic functions? When we work with families of functions, they are defined by their rates of change. For instance, we can tell a linear function because it has a constant rate of change." How would you answer Annika? Are these new parabolas quadratic functions? Justify your answer using several representations and the parabolas in problems 1-4 as examples.

8.5 Functioning With Parabolas

A Solidify Understanding Task

Purpose: The purpose of this task is to solidify students' understanding of the geometric definition of a parabola and to connect it to their previous experiences with quadratic functions. The task begins with students writing equations for specific parabolas with specific relationships between the focus and directrix. Students use this experience to generalize a strategy for writing the equation of a parabola, solidifying how to find the vertex and to use the distance between the focus and the vertex (or the distance between the vertex and the directrix) in writing an equation. Students are then asked to find the focus and directrix for $y = x^2$ to illustrate that the focus and directrix could be identified for the parabolas that they worked with as the graphs of quadratic functions. Finally, they are asked to verify that parabolas constructed with a horizontal directrix from a geometric perspective will also be quadratic functions, based upon a linear rate of change.

Core Standards Focus:

G.GPE Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section

G-GPE.2. Derive the equation of a parabola given a focus and directrix.

Note: *Connect the equations of circles and parabolas to prior work with quadratic equations. The directrix should be parallel to a coordinate axis.*

The Teaching Cycle:

Launch (Whole Class):

Begin by having students individually work the first problem. Have one student that has done a good job of accurately sketching the parabola demonstrate for the class. The first problems are very similar to the work done in "Directing Our Focus", but each problem has been selected so that students will see different distances between the focus and the directrix and use them to draw conclusions later in the task. After the first problem is done as a class, the rest of the task can be done in small groups.

Explore (Small Group):

As students are working on the task, listen to see what they are noticing about finding the vertex. They should identify that the vertex is on the line of symmetry, which is perpendicular to the directrix, and that the vertex is the midpoint of the segment between the focus and directrix. They should also be noticing how it shows up in the equation, particularly that it is easier to recognize if the $(x-h)^2$ term in the equation is not expanded. They should also notice the distance from the vertex to the focus, *p,* and where that is occurring in the equation. Identify students for the discussion that can describe the patterns that they see with the parabola and the equation and have developed a good "recipe" for writing an equation.

As you monitor student work on #10, identify student use of tables, equations, and graphs to demonstrate that the parabolas they are working with fit into the quadratic family of functions because they have linear rates of change.

Discuss (Whole Class):

Begin the discussion with question #8. Ask a couple of groups that have developed an efficient strategy for writing the equation of a parabola given the focus and directrix to present their work. (Students will be asked to generate a general form of the equation in the RSG). Ask the class to compare and edit the strategies so that they have a method that they are comfortable with using for this purpose. Then ask them to use the process in reverse and tell how they found the focus and directrix for $y = x^2$ (question 9).

Move the discussion to #10. Ask various students to show how the parabolas are quadratic functions using tables, graphs, and equations. Focus on how the linear rate of change shows up in each representation. Connect the equations and graphs to the transformation perspective that they worked with in previous modules.

Aligned Ready, Set, Go: ***Circles and Other Conics 8.5***

Equation:

Graph with features labeled:

Good things to know:

Definition:

Equation:

Graph with features labeled:

Good things to know:

Definition:

READY, SET, GO! Name Period Date

READY

Topic: Standard form of a quadratic.

Verify that the given point lies on the graph of the parabola described by the equation. (Show your work.)

1. $(6,0)$ $y=2x^2-9x-18$

2. $(-2,49)$ $y=25x^2+30x+9$

3. $(5,53)$ $y=3x^2-4x-2$

4. $(8,2)$ $y=\frac{1}{4}x^2-x-6$

SET

Topic: Equation of parabola based on the geometric definition

5. Verify that $(y-1)=\frac{1}{4}x^2$ is the equation of the parabola in *Figure 1* by plugging in the 3 points V (0,1), C (4,5) and E (2,2). Show your work for each point!

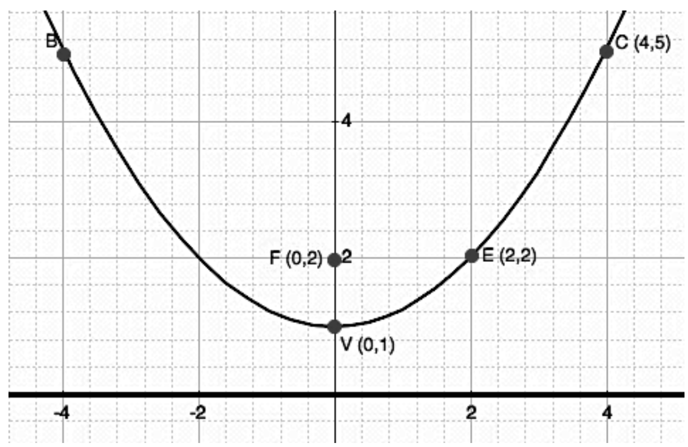
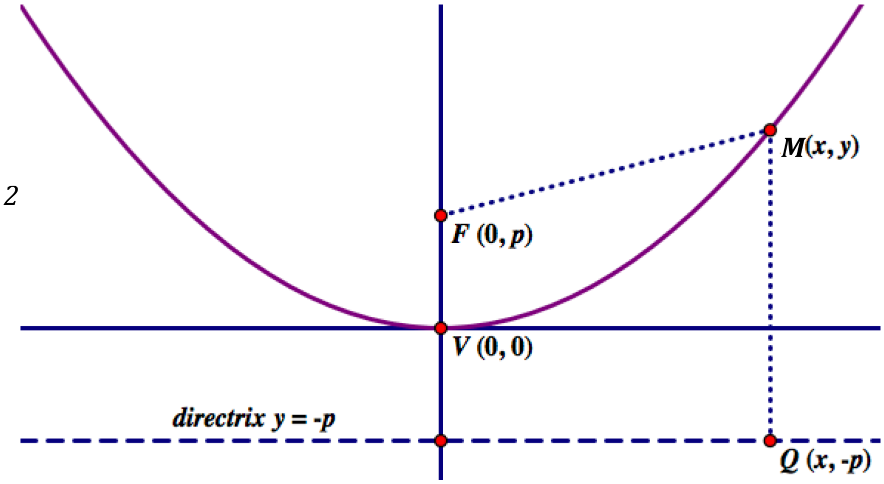


Figure 1

6. If you didn't know that (0,1) was the vertex of the parabola, could you have found it by just looking at the equation? Explain.

7. Use the diagram in *Figure 2* to derive the general equation of a parabola based on the **geometric definition** of a parabola. Remember that the definition states that MF = MQ.

Figure 2

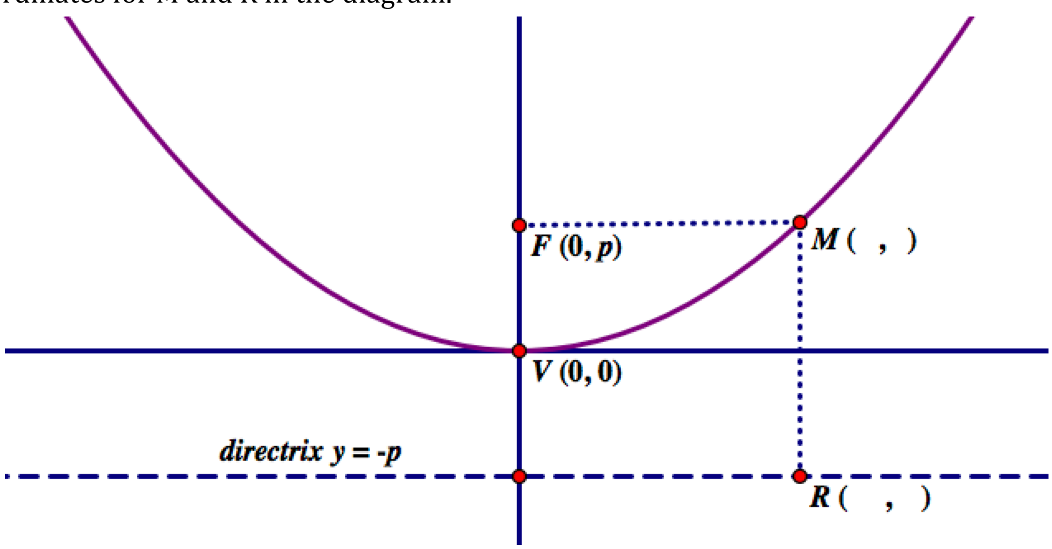


8. Recall the equation in #5, $(y-1)=\frac{1}{4}x^2$, what is the value of $\boldsymbol{p}$?

9. In general, what is the value of $\boldsymbol{p}$ for any parabola?

10. In *Figure 3*, the point M is the same height as the focus and $\overline{FM} \cong \overline{MR}$. How do the coordinates of this point compare with the coordinates of the focus?
Fill in the missing coordinates for M and R in the diagram.

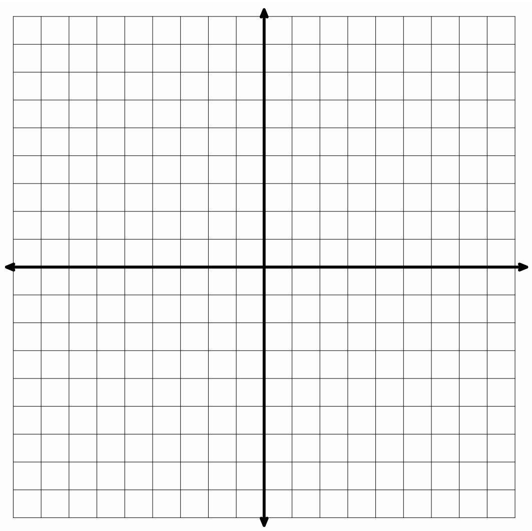
Figure 3



Sketch the graph by finding the vertex and the point M and R (the reflection of M) as defined in the diagram above. Use the geometric definition of a parabola to find the equation of these parabolas.

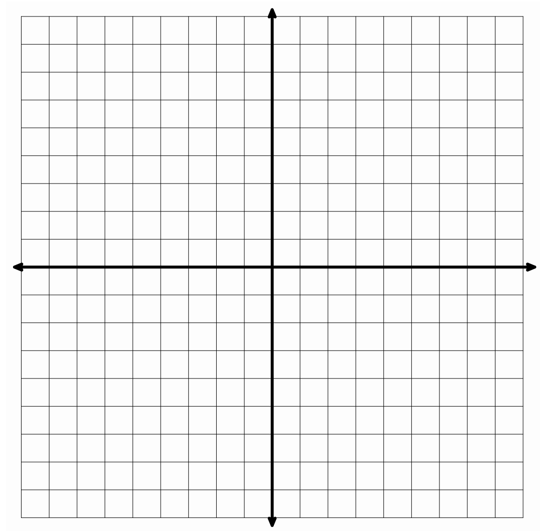
11. Directrix $y = 9$, Focus $F(-3, 7)$

Vertex ___________
Equation ________________________________



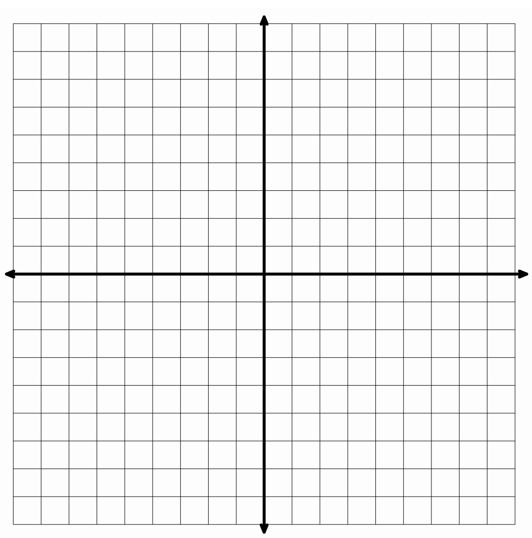
12. Directrix $y = -6$, Focus $F(2, -2)$

Vertex ___________
Equation ________________________________



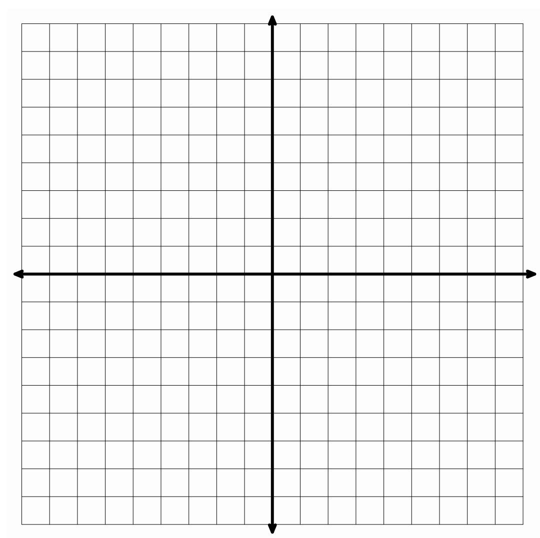
13. Directrix $y = 5$, Focus $F(-4, -1)$

Vertex ___________
Equation ________________________________



14. Directrix $y = -1$, Focus $F(4, -3)$

Vertex ___________
Equation ________________________________



GO

Topic: Finding minimum and minimum values for quadratics

Find the maximum or minimum value of the quadratic. Indicate which it is.

15. $y = x^2 + 6x - 5$

16. $y = 3x^2 - 12x + 8$

17. $y = -\frac{1}{2}x^2 + 10x + 13$

18. $y = -5x^2 + 20x - 11$

19. $y = \frac{7}{2}x^2 - 21x - 3$

20. $y = -\frac{3}{2}x^2 + 9x + 25$

8.6 Turn It Around

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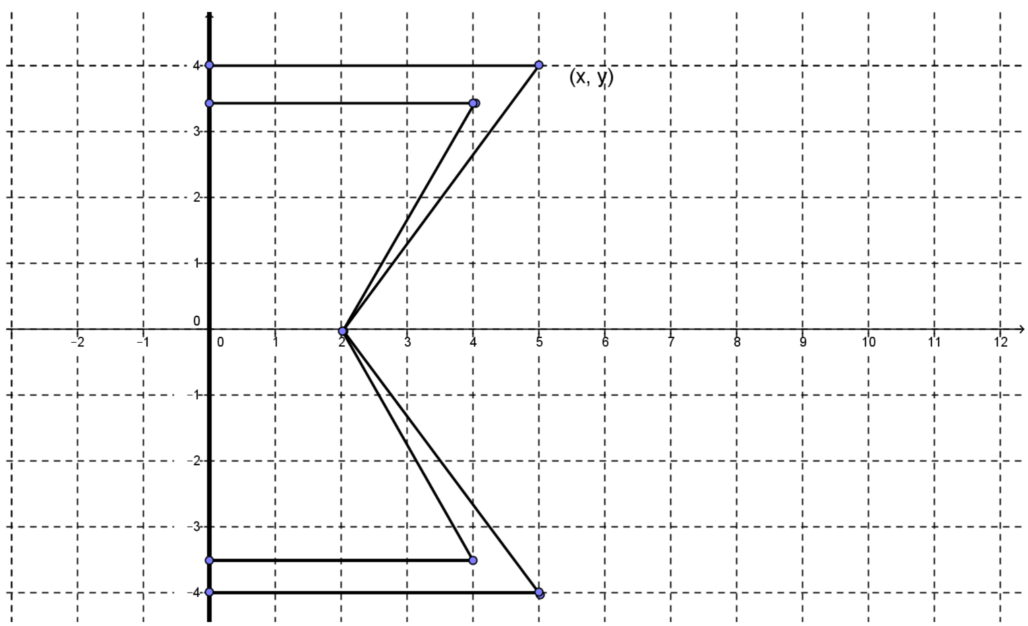
A Solidify Understanding Task

Annika is thinking more about the geometric view of parabolas that she has been working on in math class. She thinks, "Now I see how all the parabolas that come from graphing quadratic functions could also come from a given focus and directrix. I notice that all the parabolas have opened up or down when the directrix is horizontal. I wonder what would happen if I rotated the focus and directrix 90 degrees so that the directrix is vertical. How would that look? What would the equation be? Hmmm…." So Annika starts trying to construct a parabola with a vertical directrix. Here's the beginning of her drawing. Use a ruler to complete Annika's drawing.



1. Use the definition of a parabola to write the equation of Annika's parabola.

2. What similarities do you see to the equations of parabolas that open up or down? What differences do you see?

3. Try another one: Write the equation of the parabola with directrix $x = 4$ and focus (0, 3).

4. One more for good measure: Write the equation of the parabola with directrix $x = -3$ and focus (-2, -5).

5. How can you predict if a parabola will open left, right, up, or down?

6. How can you tell how wide or narrow a parabola is?

7. Annika has two big questions left. Write and explain your answers to these questions.
   a. Are all parabolas functions?

   b. Are all parabolas similar?

8.6 Turn It Around – Teacher Notes

A Solidify Understanding Task

Special Note to Teachers: Rulers should be available for student use in this task.

Purpose: The purpose of this task is to generalize the work that students have done with parabolas that have a horizontal directrix (including those generated as quadratic functions), and extend the idea to parabolas with a vertical directrix. In the task, they graph and write equations for parabolas that have vertical directrices. They are asked to consider the idea that not all parabolas are functions, even though they have quadratic equations. The task ends with constructing an argument that all parabolas, like circles, are similar.

Core Standards Focus:

G.GPE Expressing Geometric Properties with Equations
Translate between the geometric description and the equation for a conic section

G.GPE.2. Derive the equation of a parabola given a focus and directrix.
Note: *Connect the equations of circles and parabolas to prior work with quadratic equations. The directrix should be parallel to a coordinate axis.*

The Teaching Cycle:
Launch (Whole Class): Before handing out the task, ask students to think back to the lesson when they constructed a parabola by placing tacks on a board with a given focus and horizontal directrix. Ask students what shape would be constructed if they did the same thing with the strings and tacks, but the directrix was vertical and the focus was to the right of the directrix. After a brief discussion, distribute the task and have students complete the diagram and write the equation of the parabola. Ask a student to demonstrate how they wrote the equation using the distance formulas, just like they did previously with other parabolas. After the demonstration, students can work together to discuss the remaining questions in the task.

Explore (Small Group): Monitor student work as they write the equations to see that they are considering which expressions to expand and simplify. Since they have previously expanded the y^2 expression, they may not recognize that it will be more convenient in this case to expand the $(x - b)^2$ term.

Listen to student discussion of #7 to find productive comments for the class discussion. Students should be talking about the idea that a function has exactly one output for each input, unlike these parabolas. Some may also talk about the vertical line test. Encourage them to explain the basis for the vertical line test, rather than just to cite it as a rule.

The question about whether all parabolas are similar may be more controversial because they don't seem to look similar in the way that other shapes do. Listen to students that are reasoning using the ideas of translation and dilation, particularly noting how they can justify this using a geometric perspective with the definition or arguing from the equation.

Discuss (Whole Class): Begin the discussion with question #5. Press students to explain how to tell which direction the parabola opens given an equation or focus and directrix. Create a chart that solidifies the conclusions for students.

Move the discussion to question #7a. Ask students to describe why some parabolas are not functions. Be sure that the discussion relies on the idea of a function having exactly one output for each input, rather than simply the vertical line test or the idea that it's not a function if the equation contains a y^2. In either case, press students to relate their idea to the definition of function.

Close the discussion with students' ideas about question #7b. Allow the arguments to be informal, but focused on how they know that any parabola can be obtained from any other by the process of dilation and translation.

Aligned Ready, Set, Go: ***Circles and Other Conics 8.6***

READY, SET, GO! Name Period Date

READY

Topic: Review of Circles

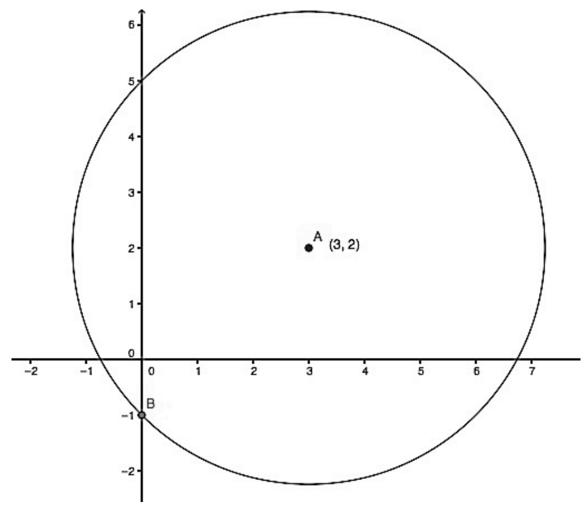
Use the given information to write the equation of the circle in standard form.

1. Center: (-5, -8) , Radius: 11

2. Endpoints of the diameter: (6, 0) and (2, -4)

3. Center (-5, 4) : Point on the circle (-9, 1)

4. Equation of the circle in the diagram to the right.

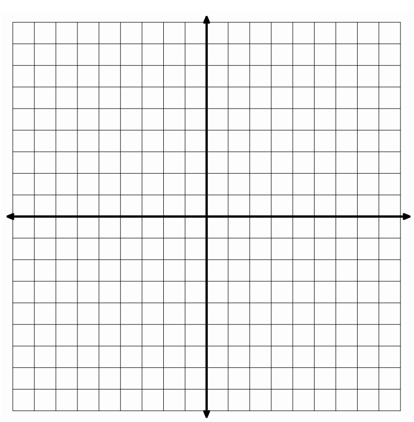


SET

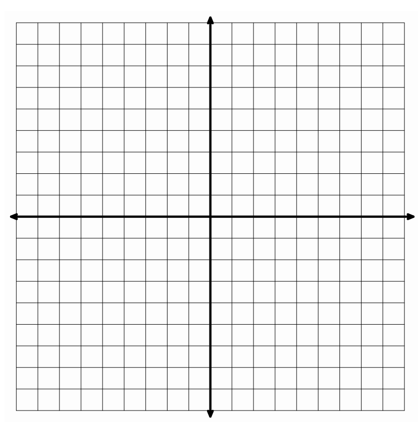
Topic: Writing equations of horizontal parabolas

Use the focus F, point M, a point on the parabola, and the equation of the directrix to sketch the parabola (label your points) and write the equation. Put your equation in the form $x = \frac{1}{4p}(y-k)^2 + h$ where "p" is the distance from the focus to the vertex.

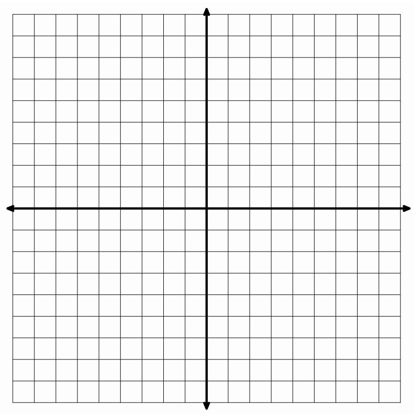
5. ***F*** (1,0), ***M*** (1,4) x = -3



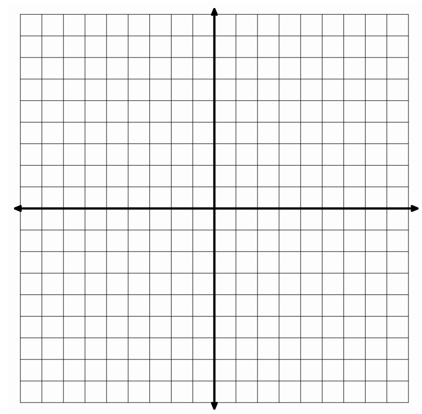
6. ***F*** (3,1), ***M*** (3,-5) x = 9



7. ***F*** (7,-5), ***M*** (4,-1) x = 9



8. ***F*** (-1,2), ***M*** (6,-9) x = -7



GO

Topic: Identifying key features of a quadratic written in vertex form

State (a) the coordinates of the vertex, (b) the equation of the axis of symmetry, (c) the domain, and (d) the range for each of the following functions.

9. $f(x)=(x-3)^2+5$

10. $f(x)=(x+1)^2-2$

11. $f(x)=-(x-3)^2-7$

12. $f(x)=-3\left(x-\frac{3}{4}\right)^2+\frac{4}{5}$

13. $f(x)=\frac{1}{2}(x-4)^2+1$

14. $f(x)=\frac{1}{4}(x+2)^2-4$

15. Compare the vertex form of a quadratic to the geometric definition of a parabola based on the focus and directrix. Describe how they are similar and how they are different.