## Function-al Machines \& Spaghetti Graphs

Summary
Students will learn to identify the relationship between patterns and functions.
Main Core Tie
Mathematics Grade 6
Strand: EXPRESSIONS AND EQUATIONS (6.EE) Standard 6.EE. 9
Materials
Invitation to Learn
Cookie ingredients
Package of store cookies
Function-al Machine

- Functional Machine (pdf)

The Ins and Outs of Functions

- Ins and Outs of Functions cards (pdf)


## Spaghetti Graphs

Play Dough or other clay
Spaghetti
Graph paper
Spaghetti Graphs equations
Additional Resources
Book
The Fly on the Ceiling, by Julie Glass; ISBN 0679886079

## Background for Teachers

There is a powerful pattern identified when any number is put into an equation and consistently follows the rule. This is called a function. When the rule is identified, each number does not have to be solved, but one could simply skip to the input number desired, insert it into the "rule" or equation, and the answer (output) will be given. Functions are easily shown in tables, such as the example below:

| Input <br> $(X)$ | Output <br> $(Y)$ |
| :---: | :---: |
| 1 | 3 |
| 2 | 4 |
| 3 | 5 |
| 4 | 6 |
| 5 | 7 |

It is easy to see that the output increases by one each time. The relationship between the input and output is the key, however. The "rule" is to add 2 . If 2 is added to the input number 1 , the answer is 3 . Therefore, an equation can be formed. An equation is a mathematical sentence that contains an equal sign. The equation for the above example is $x+2=y$. Students should become proficient at spotting the pattern, recognizing the rule, and creating an equation from that rule. The rule or equation should be a one- or two-step problem, or it becomes really difficult to solve.
Another skill that students need to master is the ability to change an equation back into a function table. If the equation is $x+2=y$, the student can choose any number to represent the input ( $x$ ). They will then "plug in" that number to get the output (y). So, if 8 were chosen for the input, then 10 would
be the output. Keep in mind that any variable can and should be used, not just $x$ and $y$ each time. Moving into the final skill, students need to first be able to graph a function table. This is a simple plotting exercise (taught in 5th grade and in 6th grade Standard III Objective 2). If $x$ is 1 and $y$ is 3 , the coordinates will be $(1,3)$. At least 2 (preferably 3 ) coordinates must be plotted, then connected to create a line (for these types of equations, a straight line will be created). The goal is for students to be able to graph an equation. In summary, here are the steps:

Change the equation to a function table
Graph the function table
Connect the plots to form a line
Instructional Procedures
Invitation to Learn
Show students ingredients for cookies. Ask students if they would like to eat each ingredient. They may want the sugar, but not the salt, etc. Explain to students that these ingredients go through a "magical" change from their separate ingredients until they are spit out of a factory machine. The magic, of course, is the mixing of the ingredients and chemical change when they are cooked together. Tell students that today they will be putting numbers through a machine, which will "magically" change the number. The magic, of course, is the function rule. You may give the students a cookie, notifying them that this cookie may stimulate their brain and make them even better mathematicians.
Instructional Procedures

## Function-al Machine

Teach students the basics of functions: give examples of function tables, discuss how to discover the rule, and how to change that rule into an equation.
It may take a few examples for students to catch on, but they will begin to see this as a fun game. Allow students to come up with their own functions with rules. Let a few of them try to stump the class.
Students will pair up. Student A will think of a function table, rule, or equation and secretly write it down on a piece of scratch paper. The rule or equation should be a one or two step problem. Student B will write an input number (x) on the Functional Machine worksheet and Student A will write the output ( $y$ ). The second student will then guess the function. If correct, they switch. If incorrect, Student B guesses again.
After 3 guesses, if Student B has not guessed correctly, Student A will unveil their table, rule, or equation and explain it to Student B.
Students switch roles.
NOTE: You may want to allow students to choose to use a table, rule, or equation, but eventually move students to using equations.
The Ins and Outs of Functions
Using the instructions in Background Information, ensure that students understand the basics of functions.
Students will be in heterogeneous groups of 3 or 4 . Each group will be given 20 Ins and Outs of Functions cards. The first card is the simplest, with each card becoming more difficult. They will work as a group to answer each question on the card:

What's the rule?
What are three more examples?
What's the equation?
When they have finished a card, they may check with you to see if their answers are correct. If they are incorrect, they need to go back to their group to understand where they went wrong. If a group is consistently getting answers wrong, determine what they are missing and reteach the
group.
When they are working, walk around and make sure that everyone is participating. They may split up the cards, but they should also help each other.
The first group to finish all 20 cards is declared the winner and will become "experts" that will go around to help the other groups (not give answers, but assist). You could even give them stickers to put on to show they are the experts.
Spaghetti Graphs
FIRST THINGS FIRST: The prerequisite to this is that students need to have the ability to change an equation to a function table, like this:
$2 x+3=y$
If $x=1$, then $y=5$
If $x=2$, then $y=7$
If $x=3$, then $y=9$

| $X$ | $Y$ |
| :---: | :---: |
| 1 | 5 |
| 2 | 7 |
| 3 | 9 |

Then, they need to plot a function table.
Step 1: Students will put an equation into a function table (at least 3 sets to plot).


Step 2: Students will graph the above table using play dough "dots."
Step 3: They will put the dried spaghetti in all 3 dots. This will ensure that the points are straight.
Step 4: They will repeat the process with the rest of the equations in the set.
Step 5: If done correctly, the 3 lines will intersect with at least one other line on the same graph.
Equation sets to use:
SET A:
$X+1=Y$
$X-2=Y$
$2 X+3=Y$
SET B:
$X+3=Y$
$3 X-2=Y$
$X \div 2=Y$
SET C:
$X-1=Y$
$X \div 3+1=Y$
$5 X-3=Y$
SET D:
$X \cdot X=Y$
$X=Y$
$3 X \div 2=Y$
SET E:
$3 X+7=Y$
$X-7=Y$
$6 X \div 3=Y$

## Strategies for Diverse Learners

The equations used for the spaghetti graphs were positive slopes (lines that go from right to left). Your advanced learners can be exposed to negative slopes, which are lines that go from left to right. If the equation has both a negative number before $X$ and the second number, it will be a negative slope. For example, $-5 X-3=Y$ is a negative slope.

## Extensions

For an introduction to coordinate grids, a great picture book is The Fly on the Ceiling about Rene` Descartes creating the Cartesian coordinate system.
Include ideas for integration for other curricular areas (use appropriate subject area headings).
Family Connections
Using spaghetti, clay, and graph paper, students will show a parent or older sibling how to graph equations.
Students will take home the Ins and Outs of Functions cards, or create their own function tables. They will show family members how to figure out the rule and create an equation.

## Assessment Plan

Give students 6 equations. If they can accurately plot the equations on a graph, they are proficient. This may also be done with function tables and many other math concepts. 0-2 correct: Intervention-These students need direct reteaching instruction
3-4 correct: Practice-These students need extra practice
5-6 correct: Proficient-These students have mastered the content. Give them an enrichment/extension activity to do
In a gym, have students create a coordinate grid, using masking tape as the x and y axes. Students will be the points, and they may use a broomstick, etc. to create a line by connecting the "points."

## Bibliography

Cwikla, J. (2004). Less experienced mathematics teachers report what is wrong with their professional support system. Teachers \& Teaching, 10(2), 181-197.
When less-experienced mathematics teachers interviewed, they expressed disappointment that many of their more experienced colleagues lacked content knowledge. Overall, they were not satisfied with the mentoring or collaboration offered by fellow teachers because they often knew more content than their more experienced peers.
Holly, K. R. (1997). Patterns and functions. Teaching Children Mathematics, 3, 312-313.
This article gives many ideas and activities for teaching patterns and functions in elementary grades K-6. Venn diagrams, function machines, and building cubes are some ideas presented.

## Authors

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