Divisibility Rules

Summary

This activity teaches students how to use formulas for dividing numbers. They will learn the formulas for multiples of 3, 2, and 5.

Materials

Invitation to Learn
    Index Cards
 - *Divisibility Test* (pdf)
    Calculators
 - *Divisibility Rules* (pdf)
    Chart paper
    Markers
Instructional Procedures
    Math journals
 - *Divisibility Rocks Cards* (pdf)
 - *Divisibility Pebbles*
 - *Divisibility Key* (pdf)

Background for Teachers

The rules of divisibility are simple formulas for understanding how fair shares can be created from large numbers without practicing long or short division. Students usually come to fifth grade with an implicit understanding about why numbers are divisible by 2, 5, and 10, but it is important in fifth grade to make that understanding explicit. Additionally, the formulas for dividing numbers by 3 and 9 must be taught, since they are rarely discovered by children. It is helpful to separate the formulas for 2, 5, and 10 (which depend on the digit in the ones column) from the formulas for 3 and 9 (which depend on the sum of the digits and the formula for 6, which combines the rules for 2 & 3). Note that there are simple formulas for divisibility by 4, and 8, (as well as more complicated formulas for larger numbers) but they are not part of the Utah fifth grade Core Curriculum requirements. Information about these formulas is included in the curriculum extensions section for interested students.
This lesson should be sequenced after division with whole numbers has been reviewed and practiced, division with remainders has been reviewed and practiced, and students are familiar with vocabulary terms dividend, divisor, and quotient. It may also be used to review prime and composite, since every number greater than 2 that is divisible by 2, 3, 5, 6, 9, or 10 is composite; also, when discussing divisibility, students will probably remember that all numbers are divisible by one and themselves.

Instructional Procedures

Invitation to Learn
Divide the class into teams of three members each. One member is the director, one the recorder, and one the materials coordinator. Each team takes four index cards and writes a different digit from 0-9 on each card. Then, from the four choices of digits, the team makes a list of all the possible four-digit number combinations using each digit once. There will 24 possible number combinations. Next, have the students each take a graphic organizer, *Divisibility Test*, with columns for the numbers they created, plus the columns for 2, 3, 5, 6, 9, and 10 listed across the top. Using calculators if you wish, have the students divide each of their 24 numbers by 2, 3, 5, 6, 9, and 10 to decide if their numbers divide evenly without leaving remainders. If the number divides evenly, have the students write "yes"

in the column on the graphic organizer. If the number does not divide evenly, have the students write "no" in the column on the graphic organizer.

After the graphic organizer is complete, have each team record their "yes" examples on chart paper hanging around the room, one piece for each of the numbers 2, 3, 5, 6, 9, and 10. Once this is done, have each team make a hypothesis about a "rule" for divisibility for each of the numbers 2, 3, 5, 9, and 10. Have them record their hypotheses on the graphic organizer labeled *Divisibility Rules*. It is important that each child have his or her own copy of the two graphic organizers because the next part of the lesson is done as a whole class.

Instructional Procedures

After teams have completed their *Divisibility Test* graphic organizer, recorded their numbers on the chart paper, and made hypotheses about divisibility on their *Divisibility Rules* graphic organizer, have them return to their individual seats for a whole-class lesson.

Using the chart paper lists as summaries of numbers generated by the class teams, discuss each chart and have the students share their hypotheses of divisibility rules. Guide their discussions to the correct rules for each number, and have them write them on the graphic organizer. Then have them trim the edges of their graphic organizers and glue them into their math journals for later referencing.

Ask the students if it is possible to divide their rules into two main categories, using a Venn Diagram to compare and contrast the categories. Lead them to separate the numbers where the ones digit determines the divisibility (2, 5, 10) from the numbers that require adding all the digits (3, 9). Have them complete a Venn Diagram in their math journals while you model one on the board.

Play *Divisibility Rocks* using students' journals as reminders of the divisibility rules. Note: if this game is used as one station in a variety of center activities, fewer sets of the game will need to be produced.

How to play *Divisibility Rocks*:

Divide the class into groups of two to six students per game. (An ideal size game is three students because each player will always have a job.)

Give each group one game set. Each set requires a deck of cards, a bag of rocks, and a *Divisibility Key*.

Divide the cards face down evenly among members of a group. Discard any remaining cards.

Pile the rocks in the center of the game.

Decide which person will be the first Player. The person to his or her right will hold the *Divisibility Key* and the person to his left will be the Challenger.

The first Player turns over his or her top card. The person holding the *Divisibility Key* asks, "is it divisible by 2?" If the Player answers, "yes," then he takes a rock from the pile. The process is repeated with the numbers 3, 5, 6, 9, and 10, with the Player taking a rock for each "yes" answer. (An example is a student would receive three rocks for the number 10 because it is divisible by 2, 5, and 10.)

Then the person with the *Divisibility Key* turns to the Challenger and asks, "do you want to challenge him?" If the Challenger believes any answers were incorrect, he or she may answer "yes," telling what numbers are believed to be incorrect.

If the Challenger is correct, he gets all the rocks from the Player. If the Challenger is incorrect, he forfeits the next turn.

If the Player is wrong and the Challenger refuses to challenge, the person with the *Divisibility Key* corrects the turn and corrects the number of rocks taken.

The play then moves clockwise to the left, with the past Player now responsible for the *Divisibility Key*, and the Challenger becoming the next Player.

At the end of a round, the person with the most rocks collects the cards used in the round and

all the rocks are returned to the center of the game. A new round is played.
At the end of a round, if there is a tie, both Players involved in the tie turn over their next card and collect the rocks for that card. Whoever holds the card that earns the most rocks wins the round.
A player is out when he is out of cards; the Player with all the cards at the end of the game is the winner.
To shorten the game, the teacher may set a time limit; the person with the most cards at the end of the allocated time is the winner.

Strategies for Diverse Learners

Advanced learners may enjoy discovering the rule of divisibility for 4 (last two digits are either 00 or are divisible by 4), and the rule for 8 (last three digits are divisible by 8). Rules for higher numbers are available on the web sites listed in the additional resources.
Why do the rules work? Advanced learners may enjoy hypothesizing about the rules for 3 and 9--why are adding digits meaningful? Explanations are given on the web sites listed in the additional resources.
Heterogeneous grouping for the invitation to learn and the card game help struggling learners through cooperative processes.

Extensions

The scientific method is used to discover mathematical absolutes. Children may recognize science vocabulary as the rules for divisibility are discovered through the formation of hypotheses, the gathering of data, the formation of conclusions, etc. Explicit teaching of these vocabulary terms strengthens both areas of science and mathematics.
Family Connections
Can the rules of divisibility apply to real-life situations? Ask the students to find at least one example after school where the rules of divisibility shorten the task of creating equal shares. An example: mom fries scones and makes 15 scones. She knows they can be divided evenly among the five people in her family. Repeat this assignment for a few days, until everyone has had a chance to discover an example.
Are the rules of divisibility for 3 and 9 unfamiliar enough to mystify people? How many people can you surprise by asking them to tell you a 10-digit number and then you telling them whether it is divisible by 3 and 9? Record their numbers and their comments and report back to class for a discussion.

Assessment Plan

Pre-assessment: Observe the children's hypotheses as they write on their graphic organizers to see if their prior knowledge about divisibility is accurate, especially with numbers 2, 5, and 10.
Formative assessment: Check for accuracy as students write correct rules on their graphic organizers, complete their Venn diagrams, and verbalize their responses during the Divisibility Rocks game.
Final assessment: Using the Divisibility Test graphic organizer as a master, list ten numbers with a variety of divisibilities and have the students complete the chart with "yes" or "no" answers.

Bibliography

Furner, J.M., Yahya, N., Duffy, M.L. (2005). Teach mathematics: Strategies to reach all students. *Intervention in school and clinic, Vol. 41, No. 1, 16-23*.
In 2000 the National Council of Teachers of Mathematics identified "equity" as the first principle for school mathematics, meaning all children have the right to understand mathematical principles. This

article offers 20 teaching strategies to reach the wide variety of learning styles and ability levels in our classrooms as we aim to meet the equity principle. Good lessons may incorporate several of these 20 strategies at one time: we may draw, explain verbally, organize conceptually, demonstrate manipulatively, and practice kinesthetically. Grouping heterogeneously and connecting culturally helps our lessons cross learning barriers and provide opportunities for children to help each other learn.

Ball, D., (1992). Magical Hopes: Manipulatives and the reform of math education: *American educator*, Summer 1992.

Although this article is 15 years old, its concerns are still valid: are we using manipulatives wisely when we teach mathematics to children? What are the relative merits of different concrete objects? Are lessons using manipulatives sensible to adults because we already understand the concepts they are designed to represent? As teachers it is important for us to understand the purpose behind the manipulatives we use when we design instruction, and it is vital for us to link the activities using manipulatives to the mathematical concepts explicitly for children to make important connections.

Authors

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