

Multiplying Matrices for Fun and Profit

Summary

Students will organize data using matrices and use matrix multiplication to generate new data. The lesson is set in the context of running an ice cream production business.

Time Frame

3 class periods of 45 minutes each

Group Size

Small Groups

Life Skills

Thinking & Reasoning, Communication, Employability

Materials

graphing calculators, capable of entering and multiplying matrices

Background for Teachers

This is an introductory lesson on multiplying matrices. Students should not have been introduced to a procedure for multiplying matrices prior to this lesson.

In this lesson, students will organize information about running an ice cream production business into matrices. They will then explore ways to combine the data in the matrices to create new matrices that contain relevant information about their business. Students will share strategies for multiplying and organizing their data and recognize the need to standardize the process of multiplying matrices.

The task is set in the context of organizing data and combining information for making batches of ice cream so that students can use the context to determine which factors need to be multiplied, which terms need to be added, and how data might be arranged to keep track of the operations in an organized way. Since the row-by-column procedure for multiplying matrices is a mathematical convention, students may initially choose to organize the data in a different way. The need for a standard, agreed upon procedure will be motivated by the move to technology where matrix multiplication is performed according to established rules. Inventing their own procedures for multiplying matrices will help students make sense of the conventional row-by-column procedure. To highlight the context of the task, it is recommended that students name each of the matrices using labels such as the "ingredient by recipe" matrix or the "recipe by day" matrix. They will need to be familiar with the convention of describing matrices first by rows, then by columns. Hence, the name "ingredient by recipe" matrix would suggest that the rows contain information about the ingredients used to make the ice cream, and the columns represent the amount of each ingredient used in different ice cream recipes. Using the conventions of matrix multiplication, the result of multiplying an "ingredient by recipe" matrix by a "recipe by day" matrix will be an "ingredient by day" matrix. Teachers will need to introduce the "row-by-column" naming structure to students, but students should sort out how these naming structures help them keep track of the operations to be performed and the meaning of the data in the product matrix.

For additional examples of developing matrix multiplication within a context, see the unit *Meadows or Malls* from the Interactive Mathematics Program published by Key Curriculum Press.

Student Prior Knowledge

Students should be familiar with organizing data into matrices and the "row-by-column" convention for

describing the dimensions of a matrix.

This series of lessons would typically follow lessons on adding matrices within a real world context.

Intended Learning Outcomes

Develop positive attitudes toward mathematics, including the confidence, creativity, enjoyment, and perseverance that come from achievement.

Become proficient problem-solvers by posing appropriate questions, selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches.

Cooperatively and independently explore mathematics, using inquiry and technological skills.

Make connections between mathematical ideas, between mathematics and other disciplines, and to life.

Communicate mathematics through writing, modeling, and visualizing, using precise mathematical language and symbolic notation.

Instructional Procedures

Launch:

Students are introduced to their role as "math consultants" for an ice cream production business that is trying to determine where they should buy the ingredients for their ice cream in order to minimize the costs. Data regarding the amount of ingredients required by various ice cream recipes, the number of batches of different recipes to be produced based on market conditions, the cost of ingredients at different stores, and the sales prices for different recipes will be presented to students in the handout *"Ye Olde-Fashioned Ice Cream Shoppe"* which will need to be reproduced for each student. The handout also includes a series of questions guiding the exploration.

Explore: The first question on the handout asks students to combine two sets of data, one set representing the amount of each ingredient required by different recipes, and a second set of data representing how many batches of each recipe is to be produced on different days of the week. Students are to use this data to determine how much of each ingredient needs to be purchased each day. Because there are lots of pieces of data representing different quantities, and several pieces of data need to be multiplied together and some of the resulting products added, students will need to work on a way to keep the data organized and keep track of what factors need to be multiplied and what terms need to be added. Allow students to develop their own schemes for keeping track of the data and the operations that need to be performed, however encourage them to organize their work in some way. At an appropriate point, the handout will suggest using matrices as a way of organizing the initial two sets of data, as well as the resulting product. Refer to the "Background for Teachers" section for additional suggestions on ways students may be prompted to organize their data.

Discuss: Have students present some of the computations required to find the amount of ingredients needed on different days, such as the number of dozen eggs required on holidays or the number of gallons of cream required on weekends. They should present detailed descriptions of the required computations, not just the result (e.g., (1.5 dozen eggs per batch of "light" ice cream) X (8 batches of "light" ice cream on a holiday) + (2 dozen eggs per batch of "extra rich" ice cream) X (15 batches of "extra rich" ice cream on a holiday)= 42 dozen eggs required on a holiday).

After agreeing on a few of the computations involved, have groups present their ways of organizing the data into matrices, and how they used their organizational scheme to keep track of the operations they performed to get the required results. Watch for groups that may have used an organizational scheme similar to the conventional "across the rows of the first matrix, down the columns of the second" for matrix multiplication. It is not problematic if no group suggests this strategy. At least they will have recognized that some "keeping track" strategy is required and they will have constructed a "multiply and add" strategy for computing the desired results. You can motivate students to use the standard procedure for multiplying matrices by noting that mathematicians have agreed on a "keeping

track" strategy for multiplying matrices and that this procedure is the one programmed into their calculators.

Strategies for Diverse Learners

Students who struggle with representing the data in an organized way may benefit from using the attached *"Using Matrices with Real World Data"* template.

Ask students who are having a difficult time getting started how they would calculate one specific piece of required data, such as the number of gallons of cream needed on a holiday. Help them visualize that this amount is made up of two parts, the amount of cream required for "light" ice cream, and the amount of cream required by "extra rich" ice cream.

Extensions

Have students create their own scenarios that would require tables of data to be manipulated using matrix multiplication. What do they notice about such types of situations and applications? How do they use product matrices to help them make decisions within the context of their scenarios?

Assessment Plan

Assessment Task:

At this point students should have an "ingredients by day" matrix representing the amount of each ingredient required on weekdays, weekend days, and holidays. Ask them to use the additional data on the worksheet to determine with which store *"Ye Olde Fashioned Ice Cream Shoppe"* should sign a contract agreeing to purchase the ice cream ingredients exclusively at that store. (Note: there are a variety of different decisions that students might make here, and based on their decisions, they might argue for different results. For example, do they consider that there are more weekdays than holidays, or do they weigh the three categories of days equally? Do they propose buying some ingredients at one store and the other ingredients at another, or do they choose to purchase all ingredients at the same store?) Pay attention to the ways students use matrices and matrix multiplication to generate the data that will support their claims.

Rubrics

[Math Problem Solving Rubric](#)

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