Multiplication Problem Solving

Summary

The students will be able to take factors apart to make multiplication problems simpler.

Main Core Tie

Mathematics Grade 3

Strand: OPERATIONS AND ALGEBRAIC THINKING (3.OA) Standard 3.OA.3

Materials

Corner piece for each student
Manipulatives such as centimeter cubes, unifix cubes, etc.
2 "L" pieces (pdf) for each student
- *What's My Array worksheet* (pdf)
- *What's My Array recording sheet* (pdf)

1 die per student
- *Number Bonds Activity* (pdf)

-- addition and multiplication
- *Problem Solving Cards* (pdf)

for addition and multiplication
- *Addition, Multiplication, Division*

support
Graph paper
Manipulatives such as corner pieces and unifix cubes to show the number bonds
Journal

Books:
- *Anno's Mysterious Multiplying Jar*

, by Mitsumasa Anno; ISBN-13:9780698117532
- *Best of Times: Math Strategies that Multiply*

, by George Tang; ISBN-13:9780439210447
- *Multiplying Menace: The Revenge of Rumplestiltskin,*

by Pa Calvert; ISBN-13:9781570918902
- *The Hershey's Milk Chocolate Multiplication Book*

, by Jerry Pallotta; ISBN-13:9780439236232
- *Too Many Kangaroo Things To Do!*

by Stuart J. Murphy; ISBN-13:9780064467124

Background for Teachers

Succession in teaching multiplication: Students learn best when starting with the concrete, moving to the pictorial, then continuing to the abstract. When teaching multiplication, give students opportunities to experience the following ways to learn (stated from pictorial to abstract):
Picture representation
Repeated addition -- number sentence
Picture diagram row by column (array)
Rectangular region -- diagram -- array
Corner Piece or "L" piece array
Number sentence

Distributive property
Algorithm
Partitioning strategies = Partial products (number bonds)/Compensation (changing the numbers to get an estimate)
A beneficial way to teach math concepts is to let the Core Curriculum drive the instruction. Below are essential questions written from the core for multiplication, including vocabulary. Let the questions drive the instruction by placing lessons from the district adopted textbook where appropriate to find holes that need to be taught.
Multiplication Enduring Understanding: Students will model problems using multiplication and division.
Vocabulary:

- Sum -- the answer to an addition problem
- Factors -- one of two numbers that when multiplied together, give a product factor
- Squares -- 2 factors that are the same -- they make a square array
- Product -- answer to a multiplication problem
- Array -- a pictorial representation of multiplication
- Multiples -- the product of a number of any other number 5, 10, 15, 20 (multiples of 5)
- Multiplication -- "super" addition -- repeated addition -- skip counting
- Commutative property -- it doesn't matter what order the numbers are, the answer remains the same
- Zero property # x 0 = 0 0 x # = 0
- Distributive property -- the product of a number and a sum is equal to the sum of the individual products of the addends and the number
- Division -- "super" subtraction
- Quotient -- answer to a division problem

Essential Questions:

- What is multiplication? (skip counting, equal groups, arrays, area models, equal jumps on the number line)
- How does repeated addition/subtraction represent multiplication/division?
- How do equal jumps on a number line represent multiplication and division?
- What is an array? How does an array represent multiplication and division?
- What shapes are arrays? What is a square number? Why is it called a square?
- What are the commutative, distributive, and associative properties? (fact families or number bonds)
  - Commutative property: Order doesn't matter when adding or multiplying numbers.
  - Distributive property: Product of a number and a sum is equal to the sum of the individual products of the addends and the number.
  - Associative property: The product of three or more factors will be the same not matter the order
- What is the zero property? What happens to place value when multiplying by 10, 100, or 1,000? (2x10=20, 2x100=200, 2x1,000=2,000, 43x10=430, 43x100=4,300, 43x1,000=43,000
  - Zero property: "0" x any number will always be "0."
- What would happen without multiplication?

Misconceptions:
Multiplying with "0" --,One of the most common mistakes involving zero is the failure by many students to realize that multiplying any number by zero yields zero. This is especially true when using the associative property of multiplication. Rees and Barr (1984) found that 52% of 8,613 people in a public examination wrote that 9 x 0 x 8 = 72.
Students should already have an understanding of things that come in groups before doing this lesson.

Before the lesson, copy *What's My Array* and *L* pieces on cardstock; laminate and cut out the L pieces. Have a dry erase marker and dice available.

Intended Learning Outcomes

2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.
4. Communicate mathematical ideas and arguments and coherently to peers, teachers, and others using the precise language and notation of mathematics.

Instructional Procedures

Invitation to Learn:
What My Array?
Ask "What is the difference between 2x3 and 3x2?"
Tell students they will be creating arrays to prove multiplication problems. They will also learn what each number represents when multiplying and what it means to multiply.
Instructional Procedures:

Give each student a "corner piece," and about 30 unifix cubes or centimeter cubes. With an overhead or document camera, show how to place the corner piece and cubes.
Place two cubes on the left side of the corner piece. These cubes represent the first factor in the multiplication problem (the number of groups). Place 3 cubes on top of the array. These cubes represent the second factor (how many in each group). This will be the array representation for 2x3.
Create the array on the inside of the corner piece to line up the rows and columns.
Ask what the product of 2x3 is. How do they know? What direction (vertical or horizontal) is the array? This is an important step for students to have a firm understanding and "picture" of what the multiplication problem looks like.
Next, have the students show the array for 3x2.l How is it different? How are they the same? What direction is the array?
Do this activity with another multiplication problem: 6x2 and 2x6. Ask the same questions. Make sure students can visualize the direction of the array. Ask: How many rows? How many columns? How do you know?
After this activity, pass out the *What's My Array?* and two *L* pieces to each student. Use the *L* pieces to show different arrays.
Show the array of 5x2 using two *Ls*. What is the difference between 5x2 and 2x5?
In small groups, have students explain and the difference between the two problems using the *L* pieces. Demonstrate how to form arrays using the *Ls*. These 2 "different" activities both show arrays in different ways.
Students should be able to use vocabulary such as "vertical rectangle" or "horizontal rectangle" to explain which multiplication problem it is.

- *Students need to know: The first number represents the number of rows (or number of groups) and the second number represents the columns (or the number in each group). State that 2x3 means two groups of three.*

Pass out 1 die for each student. Students will work with a partner. Each student will roll the die. The two numbers thrown will be the two products to create an array. Students will show the two arrays for the two factors. Students should fill out the recording sheet for each roll and write the multiplication problem in their journals along with an array.
At each table, students should write to explain what they found about multiplication, and draw an array in their journals (the multiplication problem it represents, and write what direction the array is.)

Lesson and Activity Time Schedule:

Each lesson is 55 minutes.

Each activity is 30 minutes.

Total lesson and activity time is 90 minutes.

Activity Connected to Lesson:

Simplify Multiplication: Number Bonds

Some students struggle with memorization of multiplication. This activity will help students pull apart the numbers, multiply them and then add them back together.

For example: Students learn in first and second grades about number bonds (fact families). These bonds can be extremely useful when students have a difficult time memorizing their times tables.

Students do the repeated addition way or take one of the numbers apart to make two simpler problems.

If the two factors are 7 x 6, an easier problem might be to take the 7 apart and make 7 + 2 making the two multiplication problems: 5 x 6 plus 2 x 6 (add the two products together. The 2 times tables are doubles and pretty easy. Counting by 5 is also easy.

If students can pull the numbers apart and then put them back together again, it would be easier than giving up and saying, "I can't do it." Students can also prove it by showing the arrays of the two problems and adding them together.

Here is an example of pulling the numbers apart to do a multiplication problem.

There is more than one way to complete the problem. If students have memorized the "doubles," it is another way to take the problem apart. At first it is necessary for students to prove their answers by showing arrays.

If students have not had the opportunity to take numbers apart and add the problems, this may be difficult at first. However, if students can learn to know the fact families or number bonds, it will help them with their mental math and empower students by helping them learn how to manipulate numbers. An example worksheet is available for extra help.

Now give students the opportunity of doing a couple simpler problems, such as 3 x 2.

3x2 can be pulled apart to be 2 x 2 and 1 x 2 (because 2+1=3). Have students prove their answers by showing the arrays. They can show their arrays with manipulatives or with "L" pieces and a grid.

Differentiation for students who struggle: Begin with adding instead of multiplying the numbers together. For : 5+2=? 5 is the same as adding 3+2 so the problem becomes 3+2+2. The strategy for the answer to that problem becomes 3+(2+2) doubles which is 3 +4 (which is a doubles +1: 3+3+1=6+1=7. The end result is finding strategies students can use so they become fluent in mental math by taking numbers apart.

Students should write in their journals about what they learned about taking numbers apart to multiply or add.

Extensions

Make addition bonds -- take 1 addend apart to make the problem simpler for struggling students.

Use strategies learned in number sense such as "doubles" and numbers to ten to make number bonds

Use manipulatives to make arrays and prove the multiplication problem.

- *Addition, Multiplication, Division*

activity to show understanding of repeated addition/multiplication/division.

Give students opportunities to cut arrays from graph paper, and glue them onto paper, and label the multiplication expression.

As an enrichment activity for students with a firm foundation of multiplication let students create

their own multiplication picture and story problem from a specific multiplication problem.

Family Connections:

Children should explore multiplication at home with parents as they divide cakes, brownies, cookies, plates for dinner, etc., and show arrays to find the product.

Send graph paper home for students to share their learning and understanding of what multiplication means.

Give students opportunities to draw pictures of things that come in groups. Let parents help show arrays for the picture and write story problems that also represent the picture.

For multiplication fluency, use a deck of cards (without face cards) to play multiplication war. Each person lays down a card. Both partners should determine what the product is; however, the partner with the greatest factor wins the set.

Assessment Plan

Formative assessment -- watch if students can create the arrays appropriately, horizontally or vertically.

Ask questions such as:

"How do you know that is the correct array?"

"What direction should the array be going?"

"How do you know?"

Observe students creating numbers bonds for addition or multiplication problems.

Ask questions such as:

"Why did you choose those two numbers for making the number bonds?"

- *What's My Array?*
- *Number Bonds Activity*
- *Problem Solving Cards*
  -- addition and multiplication
- *Addition, Multiplication, Division*
  worksheet for understanding of multiplication process

Bibliography

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Retrieved from the World Wide Web on January 16, 2010. Multiplication 2, 4, 6, 8… if you're looking to learn the basics of multiplication, this BrainPOP movie is really great! In the movie, Tim and Moby are at the zoo and need to use simple multiplication skills in order to feed the animals. You'll discover the difference between a factor and a product, what a multiple is, and why learning your multiplication table is so important. You'll also find out how multiplication is really just a shortcut for adding up lots of numbers, and how, when it comes to multiplication, practice makes perfect at this website: BrainPop Multiplication

Retrieved from the World Wide Web on January 16, 2010, *My Learning Process.* Multiplication is a very controversial topic as many feel strongly about how it should be taught. This blog explores recent research which shows successful methods available: Understanding Multiplication.

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